A test for time-reversal invariance in nuclear forces.

Gary E. Small
University of Windsor

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A TEST FOR TIME-REVERSAL INVARIANCE
IN NUCLEAR FORCES

by
Gary E. Small

A Thesis
Submitted to the Faculty of Graduate Studies through
the Department of Physics in Partial Fulfillment
of the Requirements for the Degree of Master
of Science at the University of Windsor.

Windsor, Ontario
1967
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223655

(Copy to Faculty of Graduate Studies, University of Windsor)
ABSTRACT

Time reversal invariance in nuclear forces is investigated using the electromagnetic interaction present in a $\beta - \gamma_1$ (mixed) - $\gamma_2$ triple angular correlation experiment. If time reversal invariance does not hold, a term in the triple angular correlation function odd under $\gamma$'s interchanging must be present. The asymmetry is measured in the isotope Mn$^{56}$. The result is $\sin \theta = 0.33 \pm 0.27$. 

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ACKNOWLEDGEMENTS

I would like to express my appreciation to Dr. E. E. Habib for his supervision of my work. Thanks are also offered to Dr. H. Ogata for discussions I had with him during the preparation of this thesis. Lastly, I would like to extend my thanks to Dr. M. W. Johns, Dr. L. Sujkowski and Reactor Personnel for aid and facilities offered to me during the course of the experimental work carried out at McMaster University, Hamilton, Canada.
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I INTRODUCTION

With the advent of special and general relativity and particularly the development of quantum mechanics, the role played by symmetry principles in physics has been greatly enhanced. These symmetry principles are the invariance of a physical process to the parity operator, $P$, to the charge conjugation operator, $C$, and to the time-reversal operator, $T$. Other operators, to which a physical process may be invariant, may be constructed from these basic three; viz., $CP$, $CT$, $PT$, and $CPT$.

The law of conservation of parity is a result of a physical process being invariant to the space reversal operator, $P$, defined as reversing the sense of the three co-ordinate axes, but retaining particle → particle, and $t → t$. Mach's principle (C. S. Wu, 1966), which has been long accepted, states that the laws of physics are completely symmetrical to the $P$ operation. The charge conjugation operator, $C$, is defined as particle → antiparticle, $\vec{F} → \vec{F}'$, and $t → t$. Invariance to the charge conjugation operation means that to every state of a system of particles, there corresponds another state identical in all respects except for particle antiparticle interchange. The time-reversal operator, $T$, is defined as $t → -t$, particle → particle, and $F → \vec{F}$.

Recently, it was unequivocally proven (C. S. Wu et al., 1957) that the law of conservation of parity is violated in beta decay. The decay particles from radioactive Co$^{60}$ showed
a preferential direction with respect to the nuclear spin in violation of Mach's principle. Furthermore, in the same experiment, it was also proven conclusively (C. S. Wu, 1966) that invariance to C is violated. It was then suggested that rather than processes being invariant to P and C, they were invariant to the combined operation CP. CP is defined as particle→antiparticle, \( \bar{\mathbf{F}} \rightarrow \mathbf{F}, \mathbf{t} \rightarrow \mathbf{t} \).

Cronin and Fitch (E. P. Wigner, 1965) have observed that CP seems to be violated in \( K^0 \) meson decay. It is possible for a physical process to violate any of C, P, T or combinations of two of these; however, according to the CPT theorem (C. S. Wu, 1966), processes must be invariant to the triple operator, CPT. One may deduce from the CPT theorem that if CP is violated, then T must be violated. Also if P is violated, then one of the other operators C or T should be violated.

The possibility of testing time reversal in nuclear forces was suggested by Jacobsohn and Henley (Jacobsohn and Henley, 1959). The procedure involved the study of a \( \beta^\gamma_1^\gamma_2 \) triple cascade to determine whether T violating terms existed in the triple angular correlation expression.
II THEORY

A. The Theory of Angular Correlation

1. Introduction. When a nucleus undergoes successive emissions (cascade) of quanta, there exists a correlation between their directions of emissions. To understand this physically, one may consider a large number of identical nuclei, about to undergo a single emission, with their spin vectors orientated in one direction. The direction of emission of each quantum with respect to the spin axis is observed. After a large number of observations, it will become evident that certain emission directions are favoured over others: the radiation pattern may not be isotropic. To describe the radiation pattern, we introduce a probability angular distribution function, $F_L^M(\Theta)$ (Frauenfelder and Steffen, 1966). This function describes the angular probability of emission of a particle with total and z-component (along the spin axis) angular momentum $L$ and $M$ respectively. If $L=1$, the radiation pattern is dipole and for $L=2$, the pattern is quadrupole.

Next, one may consider a system consisting of a large number of nuclei randomly orientated undergoing the same kind of transition as above. In this case, the radiation pattern must be isotropic. The random orientation of the nuclei will result in the random superposition of the individual radiation patterns. Now let each of the randomly orientated nuclei undergo two successive emissions and allow that the first
quantum may be differentiated from the second. Furthermore, since the emissions are successive and almost simultaneous, the second quantum may be genetically connected to the first by means of a coincidence-in-time observation. We construct an arbitrary axis through the source and detect the first quantum along this axis. Certain of the individual radiation patterns will be larger (higher probability) than others along this axis. The observation of the first quantum along this axis, then, is equivalent to weighting the selection of the individual radiation patterns. Assuming the first quantum has unique angular momentum, one deduces that the individual radiation patterns in this case differ only according to spatial orientation, i.e. the M-value. The M-value is related to the nuclear spin's intermediate m-value describing the nuclear orientation with respect to the arbitrary axis. Hence weighting the selection of the individual radiation patterns is equivalent to weighting the selection of nuclear orientations. We call the first quantum the polarizer and say that the magnetic sublevels (the m-values) of the nuclei are not equally populated after the first emission.

Because of the unequal populations of the m-levels, certain of the radiation patterns for the second quantum in coincidence with the first quantum will have a higher overall intensity leading to an observable anisotropy. We call the second quantum emitted the analyser.

Angular correlation experiments identify the various mag-
netic sublevels of nuclei spatially by unequal populations. These experiments may be contrasted to Zeeman effect experiments in atomic spectroscopy where sublevels of atomic states are distinguished energetically. In nuclear spectroscopy, the Zeeman effect has only recently (Frauenfelder and Steffen, 1966) been observed.

ii. Quantum Mechanical Theory. The quantum mechanical theory for angular correlation was first developed by Hamilton (Hamilton, 1940) who considered a double cascade of quanta from a nucleus. The emission of a quantum constituted a transition between two states of the system (system = nucleus + quantized radiation field). The quantization of the radiation field enabled it to be represented by an assembly of quantum mechanical oscillators, which corresponded to a travelling plane wave, with a well-defined propagation direction, \( \mathbf{K} \), and polarization, \( \mathbf{\epsilon} \).

We will briefly review Hamilton's theory since it is the starting point for more complex angular correlation calculations. Consider three states, initial, intermediate (formed after the first quantum is emitted), and final (formed after the second quantum is emitted) with associated probability amplitudes \( a_\alpha, b_\beta, c_\rho \) respectively. Symbols for the different states are listed in Table II-I. We are interested in \( \gamma \)-transitions between these states. Hence the initial radiation field state corresponds to the various possibilities.
for the first \( \gamma \)-emission, \( \eta \), called the occupation number, describes the number of oscillators vibrating with the same frequency and having similar directions of propogations and polarizations.

Table II-I

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\( l, n, \) and \( p \) represent the magnetic substates of the nuclear spin states \( \lambda \rangle, \eta \rangle \) and \( \epsilon \rangle \).

The probability amplitudes satisfy an equation equivalent to the Schrodinger equation,

\[-i \hbar \frac{d}{dt} \psi_j = \sum_k \langle j | H' | k \rangle \psi_k + E_j \psi_j \quad (II-I)\]

where \( H' \) is the small, time dependent interaction Hamiltonian. The \( \psi_j \) are the probability amplitudes of the eigenfunctions \( \psi_j \rangle \). Applying equation (1) to the double cascade, we obtain

\[-i \hbar \frac{d}{dt} a_\lambda = \sum_{\eta e} \langle A_\lambda | H_\epsilon | B_\eta \rangle b_{\eta e} + E_\lambda a_\lambda \quad (II-2a, b, c)\]

\[-i \hbar \frac{d}{dt} b_{\eta e} = \sum_\epsilon \langle A_\lambda | H_\epsilon | B_\eta \rangle a_\lambda + \sum_\rho \langle B_\eta | H_\epsilon | C_\rho \rangle c_{\rho e} + (\lambda \nu_\epsilon - E_{A_\lambda B_\eta}) b_{\eta e}\]

\[-i \hbar \frac{d}{dt} c_{\rho e} = \sum_\eta \langle B_\eta | H_\epsilon | C_\rho \rangle a_\lambda + (\lambda \nu_\rho + \lambda \nu_\epsilon - E_{A_\lambda C_\rho}) c_{\rho e}\]

In equation (2a), the summation over \( \eta \) allows for all the possible nuclear states after the first transition. Summation over \( \epsilon \) allows for all the possible oscillators into which the
first quantum may be emitted. $E_{A_k}$ is the energy of the nuclear state $A_k$ and may be taken as zero. $E_{A_k B_n}$ is the energy difference between nuclear states $A_k$ and $B_n$.

Solutions to these probability amplitudes are then assumed and resemble damped exponential functions of the nuclear state transition probabilities and of the energy differences between the nuclear states. The solution for $\alpha_k$ is of the form

$$\alpha_k \sim \alpha_k e^{-(\gamma(A_k) t / 2)}, \quad (II-3)$$

where $\alpha_k$ is a constant and $4\pi \gamma(A_k)$ is the total radiative transition probability from the nuclear state $A_k$. The actual solutions after some manipulation are

$$A_k = \alpha_k, \quad (II-4a, b, c)$$

$$b_n = \sum_k \alpha_k \langle A_k | H_i (\hat{R}_p \hat{e}) | B_n \rangle^*,$$

$$C_p = \sum_k \alpha_k \left\{ \sum_n \langle A_k | H_i (\hat{R}_p \hat{e}) | B_n \rangle^* \langle B_n | H_i (\hat{R}_p \hat{e}) | C_p \rangle^* \right\},$$

where $a_k$, $b_n$, and $C_p$ differ from $a_k$, $b_n$, and $C_p$, respectively by a factor independent of the direction of propagation and polarization of either radiation. $\alpha_k$ specifies the initial distribution of nuclei among the states $A_k$. The form of the Hamiltonian will be explained below.

Let $W_{\epsilon \sigma}$ be the probability of emission of two quanta into the oscillators $\epsilon$ and $\sigma$ (the quanta have well-defined frequency, direction of propagation and polarization) -

$$W_{\epsilon \sigma} = \lim_{t \to \infty} \sum_p |C_p|^2. \quad (II-5)$$
\( W_{\rho \sigma} \) is a limit since we are interested not in the actual time dependent process of the transition but in the final result of the transition. Substitution for \( C_p \) yields,
\[
W_{\rho \sigma} = \sum_{p} \left| \sum_{k} \alpha_{k} \left[ \sum_{n} \langle A_{k} | H_{\rho}^{(P, \hat{E}_{P})} | B_{n} \rangle \chi_{B_{n}}^{*} | H_{\sigma}^{(\hat{E}_{\sigma})} | C_{P} \rangle \right] \right|^{2}. (II-6)
\]

It can be shown (Hamilton, 1940) that the cross terms in the squared bracket must vanish if the initial distribution of nuclei among states \( A_{k} \) is random. Furthermore, the \( \alpha_{k} \) may then be set equal to unity (\( W_{\rho \sigma} \) is a relative probability). The \( l \) summation may then be removed from the bracket and we obtain,
\[
W = \sum_{\rho \sigma} \left| \sum_{n} \langle A_{k} | H_{\rho}^{(P, \hat{E}_{P})} | B_{n} \rangle \chi_{B_{n}}^{*} | H_{\sigma}^{(\hat{E}_{\sigma})} | C_{P} \rangle \right|^{2}. (II-7)
\]

If the polarization of the \( \gamma \)-rays is not measured, the summation should extend over \( \hat{E}_{P} \) and \( \hat{E}_{\sigma} \). The relative probability of two quanta being emitted in directions \( \hat{K}_{P} \) and \( \hat{K}_{\sigma} \) with polarizations \( \hat{E}_{P} \) and \( \hat{E}_{\sigma} \) into the solid angles \( d\omega_{P} \) and \( d\omega_{\sigma} \) is \( W d\omega_{P} d\omega_{\sigma} \).

The emission of a quantum is a result of small matter radiation coupling. We characterize the radiation field of the emitted quantum by its vector potential, \( \hat{A} \), and the nucleons associated with the nuclear transition by their charge density, \( \hat{r}_{N} \). The coupling or interaction Hamiltonian is
\[
H' = -\frac{1}{c} \int \rho_{N} \hat{r}_{N} \cdot \hat{A} \ dV \quad (II-8)
\]

The vector potential of a harmonic oscillator is given by
\[ \hat{A} = q(t) \cdot \hat{E} \cdot \exp(i \hat{\mathbf{r}} \cdot \hat{\mathbf{R}}), \quad (\text{II-9}) \]

where \( q(t) \) is the time dependent amplitude of the oscillator. Since \( q(t) \) is not a function of \( \hat{\mathbf{r}} \), the Hamiltonian may be split into two parts,

\[ H' = H(q) \cdot H_0(\hat{\mathbf{r}}, \hat{\mathbf{\hat{E}}}). \quad (\text{II-10}) \]

Because we are interested in the probability amplitude's dependence upon \( \hat{\mathbf{r}}, \hat{\mathbf{r}}_p, \hat{\mathbf{r}}_s, \hat{\mathbf{\hat{E}}}_p \) and \( \hat{\mathbf{\hat{E}}}_s \), we drop the first factor retaining only the second factor of \( H' \).

Formula (7) is the starting point for angular correlation calculations for double cascades. It is easy to write the angular correlation function for triple cascades knowing the form of (7) -

\[ W(\hat{\mathbf{r}}, \hat{\mathbf{r}}_p, \hat{\mathbf{r}}_s) = \sum_{(l_p, 0, l_s, 0)} \left| A_{l_p} H_f(\hat{\mathbf{r}}_p, \hat{\mathbf{\hat{E}}}_p) B_{0} \times B_{n} H_f(\hat{\mathbf{r}}_s, \hat{\mathbf{\hat{E}}}_s) D_m \times D_n H_f(\hat{\mathbf{r}}_p, \hat{\mathbf{\hat{E}}}_p) | C_{l_s, l_p} \right|^2. \quad (\text{II-11}) \]

To reduce formulae (7) and (11) to a stage whereby they admit numerical computation, group theory, Racah algebra and the density matrix formulation (Frauenfelder and Steffen, 1966) is used.

Both formulae (7) and (11) assume the excited intermediate nuclear states, \( B_n \) and \( D_m \), have well defined energies, spins, etc. This is equivalent to saying the excited states are short-lived \( (< 10^{-12} \text{ sec.}) \). Otherwise, the intermediate state will be perturbed and formulae (7) and (11) must be altered.

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B. Beta Minus Decay

Beta minus ($\beta^-$) decay is the decay of a neutron into a proton, electron and antineutrino -

$$n \rightarrow p^+ + e^- + \bar{\nu}.$$  

The antineutrino is a particle of vanishing rest mass possessing an intrinsic half unit spin in a direction antiparallel to its momentum. $\beta^-$ decay may be divided into several groups: allowed, first forbidden, second forbidden, etc. The amount of orbital angular momentum removed by the leptons provides the criterion for the division.

For the allowed cases, the leptons are emitted as s-waves possessing no orbital angular momentum. Allowed cases are characterized by a short comparative half life when compared to forbidden cases in which orbital angular momentum is removed. The selection rules (Konopinski, 1966) for allowed decay are:

$$\Delta I = |I_f - I_i| = 0, 1$$

$$\lambda = 0$$

$$\Delta \pi = 1.$$  

$\Delta I$ is the change in the magnitude of the nuclear spin vector before and after emission, $\lambda$ is the orbital angular momentum removed by the leptons, and $\Delta \pi$ is the change in parity of the wave functions describing the initial and final states of the nucleus. $\Delta \pi = 1$ indicates no parity change.

Each of the emitted leptons possesses a half unit spin.
The resultant of the spins, $\sum$, may be either $0$ (components antiparallel) or $1$ (components parallel). If the spins be antiparallel, the radiation is called Fermi singlet. For the parallel case, the radiation is known as Gamow-Teller (G.T.) triplet.

Allowed G.T. $\beta^-$ decay may be employed as a means of polarizing nuclei. For this type of decay, the spin of the emitted electron must be parallel or antiparallel to the nuclear spin. Furthermore, the spin of the electron is antiparallel to the electron's velocity. The observation of an electron emitted in a fixed direction (from a randomly orientated source) is equivalent to selecting only those nuclei, for successive emissions in coincidence, with spins parallel or antiparallel to the fixed observation direction. The nuclei before the successive emissions occur are therefore polarized.

C. Mixing Ratio

It is convenient to classify electromagnetic radiation by multipole orders according to the angular momentum, $L$, carried off by each quantum. For each multipole order, there are two classes of radiation. Electric $2^L$ pole radiation (EL) is due to the coupling between the charges of the nucleons and the electromagnetic field of the emitted quantum. Magnetic $2^L$ pole radiation (ML) arises from coupling between the electromagnetic field of the emitted quantum and nuclear currents. For the same $L$, EL and ML radiation differ in respect to the change in parity of the nuclear levels. The conservation of
angular momentum and parity for the system of nucleus plus emitted photon imposes selection rules (Moszkowski, 1966) on the possible multipolarities of a $\delta$-transition between two nuclear states of specified angular momentum $(I_\omega, I_f)$ and parities $(\pi_\omega, \pi_f)$. These are:

\[
|I_\omega - I_f| \leq L \leq |I_\omega + I_f|, \\
\Delta \pi = (-1)^{L} \text{ for } EL, \quad \Delta \pi = (-1)^{L-1} \text{ for } ML \\
\Delta \pi = 1 \quad \text{ for no parity change and}, \\
\Delta \pi = -1 \quad \text{ for a parity change.}
\]

Let us consider the matrix element describing the vector addition, $\bar{I}_\omega = \bar{I}_f + \bar{L}$, and $m_\omega = m_f + M$,

\[
\langle I_f \ m_f \ L \ M \ \pi | H | I_\omega \ m_\omega \rangle,
\]

where $H$ is a scalar operator. It is possible to factorize this matrix element into a Clebsch-Gordon coefficient (geometrical factor) and a reduced matrix element (nuclear factor) (Frauenfelder and Steffen, 1966):

\[
\langle I_f \ m_f \ L \ M \ \pi | H | I_\omega \ m_\omega \rangle = C^{I_f \ I_\omega \ L}_{m_f \ m_\omega \ M} \langle I_f || L \ | I_\omega \rangle. \quad (II-12)
\]

If a transition between $(I, \pi_\omega)$ and $(I, \pi_f)$ results in two $L$ values for the radiation, we term the transition 'mixed'. The ratio of the two reduced matrix elements is defined as the mixing ratio, $\eta$, between the two nuclear levels or states, $|I, \pi_\omega \rangle$ and $|I_f, \pi_f \rangle$,

\[
\eta = \frac{\langle I_f || L \ \pi_\omega \ || I_\omega \rangle}{\langle I_f || L \ \pi_f \ || I_\omega \rangle}. \quad (II-13)
\]

$L \ \pi_\omega$ is the appropriate multipole operator yielding the
\[ L_d \pi_d \text{ radiation, and } L_m \pi_m \text{ is the operator yielding } L_m \pi_m \text{ radiation. For electric quadrupole radiation, the multipole operator is written as } E2, \text{ for magnetic dipole as } M1, \text{ etc.} \]

**D. Time Reversal**

The operation of time reversal is defined as:

- **particle** → **particle**
  - \[ \vec{r} \quad \rightarrow \quad \vec{r}' \]
  - \[ t \quad \rightarrow \quad -t. \]

A physical law describing a system is invariant to this operation if for every possible state of the system, there exists a time reversed state satisfying the same physical law. For example, the motion of a planet around the sun is invariant under time reversal. If a given orbit, \( \vec{r} = \vec{r}(t) \), satisfies Newton's equations, then the time reversed orbit, \( \vec{r}' = \vec{r}(-t) \), is also a possible orbit for the planet.

Quantum mechanically, the operation of time reversal is closely related to taking the complex conjugate of the wave function (A. Messiah, 1965). We take the complex conjugate of the time dependent Schrödinger equation:

\[
\frac{-i}{\hbar} \frac{\partial \psi^*}{\partial t} = H^* \psi^*. \quad (II-14a)
\]

Upon the substitution, \( H^* = U^* H U \), and rearrangement, we get

\[
\frac{i}{\hbar} \left( \frac{\partial (U \psi^*)}{\partial (-t)} \right) = H (U \psi^*). \quad (II-14b)
\]

The time reversed solutions have the form,

\[
\psi^{fs'}(\vec{r}, t) = U \psi^*(\vec{r}, -t) \quad (II-14c)
\]
where $\mathbf{U}$ is a unitary operator. We may define an operator, $T$, called the time reversal operator such that

$$T \psi(\mathbf{r}, t) = \mathbf{U} \psi^*(\mathbf{r}, -t). \quad (\text{II-14d})$$

Under the $T$ operation, the following quantities are transformed according to,

- $\mathbf{p} \rightarrow -\mathbf{p}$ (linear momentum),
- $\mathbf{L} \rightarrow -\mathbf{L}$ (orbital angular momentum),
- $\mathbf{S} \rightarrow -\mathbf{S}$ (spin),
- $\mathbf{H} \rightarrow -\mathbf{H}$ (magnetic field).

Quantities such as $\mathbf{L} \times \mathbf{H}, \mathbf{L} \cdot \mathbf{H}$, would be invariant to time reversal, whereas quantities like $[\mathbf{L}, \mathbf{H}, \mathbf{L}]$ would change sign and hence be variant.

Invariance to time reversal means that a physical process is independent of the direction of time. Variance to $T$ admits the possibility of assigning a relative 'direction' to time-relation in the sense that one could only ascertain if the 'direction' of time 'changed.

It has been strongly believed that invariance to time reversal is a valid physical symmetry and has been invoked in carrying out theoretical calculations. For example, in the calculation of the decay intensity of the neutron (Konopinski, 1965), the following expression arises:

$$\mathcal{J} = 1 + \alpha_n \hat{q}_n \cdot \hat{\mathbf{v}}_e + \hat{\mathbf{L}}_n \cdot \left[ A_n \hat{\mathbf{v}}_e + B_n \hat{\mathbf{S}}_n + D_n \hat{\mathbf{S}}_n \times \hat{\mathbf{v}}_e \right],$$

where $\hat{\mathbf{v}}_e$ is the velocity of the emitted electron, $\hat{\mathbf{L}}_n$ is the neutron's spin direction and $\hat{\mathbf{q}}_n$ is the antineutrino's...
mentum. All terms except that with coefficient $D_n$ are time reversal invariant. If $D_n$ were non-vanishing, the whole expression, $\mathcal{O}$, would become asymmetric to $T$. By the principle of time reversal invariance, one would assume a vanishing $D_n$. Experiments are at present underway attempting to measure $D_n$.

The reversibility postulate is usually formulated under the name of the Principle of Microreversibility (Messiah, 1965): If the law of motion of a conservative system is reversible to $T$, the Hamiltonian is real and conversely. Furthermore, the matrix element between nuclear states of an interaction Hamiltonian which is invariant under time reversal has a phase factor which is not equal to 0 or $\pi$ unless the nuclear forces are themselves invariant (Henley and Jacobsohn, 1959). Then for $T$ violation in nuclear forces, the mixing ratio will be a complex quantity,

$$\eta = |n| e^{i(\delta_-^i - \delta_r)} = |n| e^{i\delta} \quad \delta \neq 0, \pi \quad (II-16)$$

$\delta$ is the phase difference between the reduced matrix elements. It is possible therefore to test the time reversibility of nuclear forces using electromagnetic forces associated with the interaction Hamiltonian.

E. Triple Angular Correlation

Angular correlation experiments testing $T$ symmetry can be of two kinds (E. Fuschini et al., 1964):

1) Measurement of $\cos \delta$
2) Measurement of $\sin \delta$
A $\chi_1-\chi_2$ angular correlation is an experiment of the first kind. The angular correlation function for a $\chi_1-\chi_2$ cascade, the first transition of which is mixed is, as known

$$W(\varphi) = 1 + A_2P_2(\cos \varphi) + A_4P_4(\cos \varphi) + \cdots ,$$

(II-17)

where the coefficients $A_2$ and $A_4$ may be calculated from the general theory of angular correlation as functions of $|W|$ and $\cos \delta$. It is then possible to obtain $\cos \delta$ from the experimental values of the coefficients. From results in the literature (E. Fuschini et al., 1964), one obtains in the best case

$$\sin \delta < 0.3 .$$

(II-18)

It is difficult to lower this limit measuring $\cos \delta$ because experiments of the first type necessarily look for a second order effect in $\delta$.

Experiments of the second kind will be more sensitive to $\delta$ since they are measuring $\delta$ in the first order ($\sin \delta$). In order to measure $\sin \delta$, a $\chi_1-\chi_2$ angular correlation from initially polarized nuclei is required. As indicated in II-3 allowed Gamow Teller $\gamma^-$ decay is a method of producing the initial polarization.

The angular correlation function for the triple cascade may be written as the sum of two terms (Ling and Falkoff, 1949).

$$W = W_T + W_T'$$

(II-19)

The presence of $W_T$ is independent of $T$, whereas $W_T'$ exists only if $T$ is violated. $W_T'$ arises from interference terms as a result of the mixed radiation in the first $\chi^-$. 

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transition, \( \gamma _1 \). Moreover, we may write,

\[
W = |\alpha|^2 W_{\text{int}} + |\beta|^2 W_{\text{int}} + (\alpha \beta^* + \alpha^* \beta) W_1. \tag{II-20}
\]

\( W_1 \) is the interference combination due to the mixed multipoles in the first \( \gamma \)-transition. \( \alpha \) and \( \beta \) are proportional to the reduced matrix element of the \( L_\alpha \) and \( L_\beta \) operators respectively. On dividing by \( |\alpha|^2 \), only the ratios \( \frac{\beta}{|\alpha|^2}, \frac{\beta^*}{|\alpha|^2} \) (\( \equiv \eta, \eta^* / |\eta| \) respectively) appear in \( W \). This indicates the reason for knowing both the absolute value of the mixing ratio, \( |\eta| \), and its phase, \( \delta \), for the triple angular correlation function, \( W \).

Calculations have been carried out for the specific case of a \( 2^+ (E2 M1) 2^+ (E2) 0^+ \) cascade preceded by an allowed \( G-T \) \( \beta^- \) decay of the type \( I_o - I_f = 1 \) (Jacobsohn and Henley, 1959). The angular correlation function for this case is

\[
W(\hat{p}, \hat{k}_1, \hat{k}_2) = \beta_{3, T} \left\{ (1 + |\eta|^2) + (0.250 + 0.732 |\eta| \cos \delta - 0.1765 |\eta|^2) P_2(\hat{k}_1, \hat{k}_2) + 0.327 |\eta|^2 P_4(\hat{k}_1, \hat{k}_2) + 0.245 (ve/c) |\eta| \sin \delta \cdot \hat{E} \right\}, \tag{II-21}
\]

where

\[
\hat{E} = (\hat{k}_1 \cdot \hat{k}_2) [\hat{k}_1, \hat{k}_2, \hat{p}]. \tag{II-22}
\]

\( \hat{k}_1, \hat{k}_2, \hat{p} \) are unit momentum vectors of the first \( \gamma \)-ray, second \( \gamma \)-ray and electron respectively. \( \beta_{3, T} \) is a factor proportional to the \( \beta^- \) intensity. \( P_2 \) and \( P_4 \) are Legendre polynomials of respective orders 2 and 4.

The last term in equation (21) may be written (E. Fuschini et al., 1964) as

\[
\left[ A_1(\beta) \bar{f}(\hat{p}, \hat{k}_1, \hat{k}_2) \right] \sin \delta. \tag{II-23}
\]

This term may be very small relative to the first even if \( \beta \)
is violated. The smallness depends upon the values of \( A_i(\theta) \), \( |\eta| \), and \( \mathcal{F} \) (\( \sim \) to \( \mathcal{J} \) in equation (22)). The factor \( \frac{A_i(\theta)}{A_i(\theta)} \) is a function of the weak interaction coupling constants and of the reduced Fermi and Gamow-Teller matrix elements. For a pure G. T. transition, the ratio becomes (De Sabbata, 1961)

\[
\frac{A_i(\theta)}{A_i(\theta)} = \frac{1}{\sqrt{3}} \frac{\nu e}{C} \frac{I_i(I_i+1) - I_o(I_o+1) + 2}{2 \sqrt{I_i(I_i+1)}}
\]

If \( \Delta I = I_i - I_o \) then

for \( \Delta I = +1 \), \( \sum^\dagger = \sqrt{\frac{I_o + 2}{I_o + 1}} > 1 \), \( \text{(II-25a)} \)

and for \( \Delta I = -1 \), \( \sum^\dagger = \sqrt{\frac{I_o - 1}{I_o}} < 1 \). \( \text{(II-25b)} \)

Hence transitions of the type \( \Delta I = 1 \) magnify the value of the \( T \) variant term in equation.

Furthermore, the geometric factor, \( \mathcal{F} \), has a maximum absolute value for

\[
\cos^{-1}(\mathbf{r}_1 \cdot \mathbf{r}_2) = \mathcal{F} = 45°, 135° \quad \text{(II-26a)}
\]

\[
\cos^{-1}(\mathbf{r}_2 \cdot \hat{p}) = \cos(\mathbf{r}_2 \cdot \hat{p}) = \Theta = 90°. \quad \text{(II-26b)}
\]

No true triple coincidences may be measured without the detection of a \( \beta^- \) particle. The greater the \( \beta^- \) intensity, the greater will be the number of triples collected and the less will be the relative statistical error, \( \mathcal{E} \), in \( \mathcal{W} \).

\[
\mathcal{E} \geq \frac{\sqrt{\mathcal{W}}}{\mathcal{W}} \quad \text{(II-27)}
\]

The last term in equation (21) is a function of both \( \cos \theta \) and \( \sin \theta \), while the remaining terms are functions of \( \cos \theta \) only. To measure an anisotropy, one must measure \( \mathcal{W} \) at two different values of \( \theta \), the angle between \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \).
These may be chosen as
\[
\begin{align*}
\phi_{\phi, \chi_1} &= \phi, \\
\phi_{\phi, \chi_2} &= -\phi.
\end{align*}
\] (II-28)

With this selection, the last term in equation (21) changes sign whereas the remaining terms are not affected. We define the anisotropy as
\[
A_{\text{th}} = \frac{W(\theta, \phi)}{W(\theta, \phi)} = \frac{W_T - W_T'}{W_T + W_T'}.
\] (II-29)

\((*)\) Angle in the \(\beta, \chi_1\) system.

\((***)\) Angle in the \(\beta, \chi' \chi_2\) system.
### III EXPERIMENTAL

#### A. Selection of the Triple Cascade

The Mn$^{56}$ decay scheme is shown in Figure III-1. Table III-1 indicates the selection of components for the triple cascade, $\beta^-\gamma_1\gamma_2$.

<table>
<thead>
<tr>
<th>Component</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^-$ (allowed G.T.)</td>
<td>0.75 Mev. (end point)</td>
</tr>
<tr>
<td>$\gamma_1$ (E2 + M1)</td>
<td>2.12 Mev.</td>
</tr>
<tr>
<td>$\gamma_2$ (E2)</td>
<td>0.845 Mev.</td>
</tr>
</tbody>
</table>

This cascade is the same one used by Jacobsohn and Henley in their calculation of equation II-21.

The energy spectrum for the $\gamma$-transitions in Mn$^{56}$ is shown in Figure III-2. The energy spectrum out of each NaI(T1) detector was fed through a pulse height selector. This permitted the selection of the appropriate transition's photoelectric peak. The selections are indicated in Figure III-2. The energy spectrum for each $\beta^-$ transition is itself a continuum up to a maximum energy. Furthermore, the individual $\beta^-$ spectra are superimposed on one another such that separation by pulse height selectors is not possible. The selection of the appropriate $\beta^-$ spectrum was achieved by coincidence-in-time circuits (coincidence with $\gamma_1$ and $\gamma_2$).
B. Geometry of the Detectors

Figure III-3 shows the geometric arrangement of the detectors. Detectors A and B were differential NaI(Tl) scintillators manufactured by Kyoto industries, Japan. Their function was the detection of γ-rays at distances varying from 1.5 to 3 inches from the radioactive source. Dimensions of the NaI(Tl) crystals were 2 x 2 inches. Detector C was an organic (plastic) phosphor for the detection of the $\beta^-$ particles. This type of scintillator has a weak response to γ-rays. Its distance from the source was fixed (≈ 2 inches). The thickness of the plastic phosphor was approximately 1/16 inches. The source was located in an evacuated chamber which was an extension of the brass casing of detector C. The selection of the photoelectric peaks by detector is listed in Table III-2.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Photoelectric Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.845 Mev. ($\gamma_2$)</td>
</tr>
<tr>
<td>B</td>
<td>2.12 Mev. ($\gamma_1$)</td>
</tr>
<tr>
<td>D</td>
<td>2.12 Mev. ($\gamma'_1$)</td>
</tr>
</tbody>
</table>

The detectors were aligned along radii extending from the radioactive source. The radial direction associated with detector A was $\hat{R}_2$ (the direction of propagation of the 0.845 Mev γ-ray); with detector B, $\hat{R}_1$ (the direction of propagation of the 2.12 Mev γ-ray). The direction associated
Figure III-3. Geometry of the Detectors for $Q_1, Q_2$ System.
For all runs $\Theta_1 = \Theta_2 = \Theta_3 = 0$. 
with detector $C$ was $\hat{p}$, the unit momentum vector of the $\beta$ particles.

The detectors were so arranged that the geometric factor, $\mathcal{G}$, exhibited a maximum. This occurred when detectors $A$ and $B$ were in a plane perpendicular to detector $C$, with an angle between $A$ and $B$ of $135^\circ$. To measure an anisotropy, $A_{\text{an}}$, the triple coincidences must be counted for two different angles between $\hat{R}_1$ and $\hat{R}_2$. The counting was accomplished simultaneously by employing a third NaI(Tl) detector, $D$ (selecting $y_1' = 2.12$ Mev.), located in the plane of detectors $A$ and $B$ and source but at $-135^\circ$ with respect to detector $A$. This arrangement also maximizes $\mathcal{G}$. We then speak of two detection arrangements or systems: $\beta-\hat{R}_1-\hat{R}_2$ and $\beta-\hat{R}_1'\hat{R}_2$.

C. Electronics

A schematic circuit diagram is shown in Figure III-4. The NaI(Tl) scintillation detectors and the organic phosphor were mounted on 53 A.V.F. phototubes. The positive pulses from each phototube were passed through a preliminary linear amplifier and then through a double delay amplifier. Here, the pulses were shaped such that the output pulse consisted of a negative pulse immediately followed by a positive pulse. The cross-over point contained the pulse's time information. This output pulse was then fed into a pulse height selector with a dead-time of $2.2\mu$ sec. The discriminator fired only for incoming pulses within a specified amplitude band. The timing information of the incoming pulse was retained in the
output pulse of the pulse height selector (P.H.S.) by means of cross-over-pick-off.

The output pulses of the four P.H.S. units were fed into a time to amplitude converter (T.A.C.). The time difference between input pulses was converted into the amplitude of a negative square output pulse. The range of each T.A.C. was adjustable by means of a discriminator. The outputs of the T.A.C. units yielded coincidences between detectors A and B, detectors A and C, and detectors A and D. Associated with each of these outputs, was a logic output. If a signal appeared at the T.A.C. output, a positive square pulse of constant amplitude appeared at the logic output.

The T.A.C. pulses \( \gamma_z \) and \( \gamma''_z \) were fed through an inverter and 2 \( \mu \)sec. delay to a 400 channel T.M.C. analyser. The logic pulses \( \gamma_x, \gamma'_x, \) and \( \gamma_\beta \) were fed into two double coincidence circuits each with a resolving time of about one microsecond. Into the first slow coincidence circuit was fed the doubles \( \gamma_x \) and \( \gamma_\beta \); into the second slow coincidence circuit was fed the doubles \( \gamma'_x \) and \( \gamma_\beta \). Outputs of these slow coincidence circuits were fed into four gating inputs of the T.M.C. analyser.

The 400 channels were split into groups of one hundred. In the first one hundred channels was recorded the \( \gamma_z \) time spectrum with no gating \( \gamma_z \) pulse (anticoincidence); in the second one hundred channels was recorded the \( \gamma'_z \) time spectrum with no gating \( \gamma'_z \) pulse (anticoincidence); in the third one
hundred channels was recorded the $\gamma_1 \gamma_2$ time spectrum with a gating $\gamma_1 \gamma_2 \beta$ pulse (coincidence); in the fourth one hundred channels was recorded the $\gamma_1 \gamma_2$ time spectrum with a gating $\gamma_1 \gamma_2 \beta$ pulse (coincidence).

D. Chance Events

Figure III-5 shows a typical time spectrum. Both doubles and triples time spectra were approximately 200 nanoseconds wide. Because of the large number of nuclei decaying in the source, it is possible that a $\gamma_1$ from one nuclei will be in coincidence with a $\gamma_2$ from another nuclei. Since we are interested in genetic coincidences only, it is necessary to differentiate between these chance coincidences and the true genetic coincidences. The true coincidences are related in time and hence will not appear with equal probability throughout the time spectrum. The chance coincidences are not related time-wise and therefore will appear with equal probability throughout the time spectrum. The doubles time spectra consist of a flat region upon which is superimposed a peak. The chance double coincidences are responsible for the flat part of the spectra whereas the true double coincidences yield the peak. Since the chance counts/channel are constant across the time spectra, it is relatively simple to separate the true from the chance doubles.

The triples spectra are more complicated as there exist several possibilities to yield the chance triples. These are enumerated in Table III-3.
Table III-3

<table>
<thead>
<tr>
<th>Case</th>
<th>Coincidence Between ( \gamma_1 ) and ( \gamma_2 )</th>
<th>Contribution to Triples Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Double ((\gamma_1 \gamma_2)_{\text{chance}})</td>
<td>((\gamma_1 \gamma_2)_{\text{chance}})</td>
</tr>
<tr>
<td></td>
<td>Double ((\gamma_1 \gamma_2)_{\text{true}})</td>
<td>((\gamma_1 \gamma_2)_{\text{true}})</td>
</tr>
<tr>
<td>i</td>
<td>((\gamma_1 \gamma_2)_{\text{chance}})</td>
<td>((\gamma_1 \gamma_2)_{\text{chance}})</td>
</tr>
<tr>
<td>ii</td>
<td>((\gamma_1 \gamma_2)_{\text{true}})</td>
<td>((\gamma_1 \gamma_2)_{\text{true}})</td>
</tr>
<tr>
<td>iii</td>
<td>((\gamma_1 \gamma_2)_{\text{chance}})</td>
<td>((\gamma_1 \gamma_2)_{\text{true}})</td>
</tr>
<tr>
<td>iv</td>
<td>((\gamma_1 \gamma_2)_{\text{chance}})</td>
<td>((\gamma_1 \gamma_2)_{\text{true}})</td>
</tr>
</tbody>
</table>

Each case has been subdivided into the cases where the same \( \gamma_2 \) pulse is present in the \((\gamma_1 \gamma_2)\) and \((\gamma_2 \gamma_2)\) coincidences. If the \( \gamma_2 \) pulses are not the same, the \((\gamma_1 \gamma_2)\) and \((\gamma_2 \gamma_2)\) pulses must be separated in time by at least 2.2 sec. (because this is the dead time of the pulse height selector), and the slow coincidence circuit will give no output. For related \( \gamma_2 \)'s, the case ii is the only one which does not give a flat chance spectrum. Its contribution to the peak is given by \((\gamma_2 \gamma_2)_{\text{true}} \cdot \varepsilon \cdot \omega_i\), where \( \varepsilon \cdot \omega_i \) is the photoelectric efficiency and transmission of the \( \gamma_2 \) detector. The true triples peak rate, iv, is given by \((\gamma_2 \gamma_2)_{\text{true}} \cdot \varepsilon \cdot \omega_i\). The observed peak \((\gamma_2 \gamma_2)\)' will be the sum of these contributions. Therefore the true triples rate will be given by

\[
(\gamma_2 \gamma_2) = (\gamma_2 \gamma_2)_{\text{true}} \cdot \varepsilon \cdot \omega_i \cdot (\gamma_2 \gamma_2)' = R \cdot (\gamma_2 \gamma_2)'
\]

The same result is obtained for the \((\gamma_1 \gamma_2)(\gamma_2 \gamma_2)\) time spectrum,

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i.e. both observed triples peaks are multiplied by the same factor, $\bar{R}$, to obtain the true triple coincidences.

E. Experimental Procedure and Analysis

Manganese (Mn$^{55}$) dioxide samples of approximately 1.5 mgms. were irradiated with thermal neutrons to an activity of about 12 millicuries (half-saturation). By means of evaporation at high temperature and low vacuum, a thin film of MnO was deposited upon thin Al foil glued to a circular Al source ring. The thickness of the sources was calculated to be in the vicinity of 0.2 mgm./cm.$^2$. Because of the short mean life of the isotope Mn$^{56}$ (2.58 hrs.), a new source was prepared for each experimental run.

Data was collected initially for 1 minute for the $\gamma_1\gamma_2$ time spectrum with the discriminator on the T.A.C wide open. The chance on both sides of the $\gamma_1\gamma_2$ true doubles peak was then cut off and the run was initiated. The following time spectra were collected:

- $\gamma_1\gamma_2$, true + chance,
- $\gamma'_1\gamma_2$, true + chance,
- $\gamma_1\gamma_2 (\gamma'_1\gamma_2)$, true + chance,
- $\gamma'_1\gamma_2 (\gamma'_1\gamma_2)$, true + chance.

From the flat portion of each doubles spectrum, $\gamma_1\gamma_2$ and $\gamma'_1\gamma_2$, collected over a $\tau$ minute interval ($\tau =$ duration of a run), an average value for the chance counts/channel was calculated. The spectrum was summed by channel over the peak
and slightly into the chance region on both sides. From the number of channels in the summation, the number of chance events in the summation was determined. The number of true double coincidences was equal to the counts in the summation less the chance counts in the summation channel domain.

The preliminary analysis of the triples proceeded in the same way as for the doubles. To separate the true from chance events in the observed triples peak \( (\beta_2 \gamma_2)' \), the ratio \( R \), was computed from the \( \beta \gamma \) doubles spectrum collected initially.

Since the intensity of the source is rapidly decreasing throughout the run, the ratio, \( R \), is a function of time. Its dependence on time is

\[
R(t) = \frac{1}{1 + \left( \frac{(\beta_2 \gamma_2)_{\text{chance}}}{(\beta_2 \gamma_2)_{\text{true}}} \right) e^{-\frac{t}{\tau_{2g}}} \to 1, (\text{III-2})
\]

where \( \left( \frac{(\beta_2 \gamma_2)_{\text{chance}}}{(\beta_2 \gamma_2)_{\text{true}}} \right) \) is the initial value calculated from the time spectrum. If the initial value of \( R \) is denoted by \( R(0) \), the ratio, \( \bar{R} \), used in equation (1) was

\[
\bar{R} = \frac{R(T) - R(0)}{2}. (\text{III-3})
\]

The statistical errors in the doubles and triples counts were assumed Poisson.

The following ratio together with its associated error was calculated:

\[
A_{\text{exp}} = \frac{(\gamma_1 \gamma_2)_{\text{true}}' (\gamma_1' \gamma_2 \beta)_{\text{true}}}{(\gamma_1 \gamma_2 \beta)_{\text{true}}' (\gamma_1' \gamma_2)_{\text{true}}}. (\text{III-4})
\]
$(\gamma_1 \gamma_2)_{\text{true}}$ and $(\gamma'_1 \gamma'_2)_{\text{true}}$ are the number of true doubles coincidences collected over the running time, $T$. $(\gamma_1 \gamma_2 \gamma_3)_{\text{true}}$ and $(\gamma'_1 \gamma'_2 \gamma'_3)_{\text{true}}$ are the number of true triple coincidences collected over the same running time.

F. Geometrical and Counter Efficiency Considerations

The anisotropy defined by equation II-29 depends only on the ratio of the triples collected. In this equation, it is assumed that each detection system $(\beta \gamma_1 \gamma_2)$ and $(\beta' \gamma'_1 \gamma'_2)$ is equivalent except for the angle between $\hat{\mathbf{k}}$, and $\hat{\mathbf{k}}'$. This means that the geometry, detector efficiency and electronics for the two systems must be the same. It is difficult to achieve this experimentally. The normalization factor,

$$N_M = \frac{(\gamma_1 \gamma_2)_{\text{true}}}{(\gamma'_1 \gamma'_2)_{\text{true}}} \quad (\text{III-5})$$

in equation (I) brings about the equivalence of the two detection systems in the calculations.

Let

$N =$ number of disintegrations/sec. in the source,

$\delta_3 =$ branching ratio for the 0.75 Mev. $\beta^-$ Spectrum,

$\delta_1 =$ branching ratio from 2.660 Mev. level to 0.845 Mev. level,

$K_1 =$ fraction of decay between 2.660 Mev. and 0.845 Mev. levels occurring by $\delta = 2.12$ Mev. transition,

$K_2 =$ fraction of decay between 0.845 Mev. and ground levels occurring by $\delta = 0.845$ Mev. transition,

$\epsilon_{\gamma_1} =$ photoelectric detection efficiency of detector A,
\( \varepsilon_{\gamma_1} \) = photoelectric detection efficiency of detector B,
\( \varepsilon_{\gamma_1'} \) = photoelectric detection efficiency of detector D,
\( \eta_\beta \) = momentum spread in \( \beta \) spectrum detected by C,
\( \omega_{\gamma_2} \) = solid angle subtended by detector A with source,
\( \omega_{\gamma'_2} \) = solid angle subtended by detector B with source,
\( \omega_{\gamma_3} \) = solid angle subtended by detector C with source,
\( \omega_{\gamma'_3} \) = solid angle subtended by detector D with source,
\( E_{\gamma_2 \gamma_3} \) = efficiency of the \( \gamma_2 \gamma_3 \) T.A.C.,
\( E_{\gamma'_2 \gamma'_3} \) = efficiency of the \( \gamma'_2 \gamma'_3 \) T.A.C.,
\( E_{\beta \gamma_2} \) = efficiency of the \( \beta \gamma_2 \) T.A.C.

Since the doubles rate is small compared to the single rates, we assume the efficiency of the slow coincidence circuit is equal to unity.

The number of true coincidences between \( \gamma_1 \) and \( \gamma_2 \) per sec. are

\[
(\gamma_1 \gamma_2)_{\text{true}} = N \delta_\theta \langle \delta_{\gamma_1} \omega_{\gamma_1} \varepsilon_{\gamma_1} K_{\gamma_1} \rangle \langle \omega_{\gamma_2} \varepsilon_{\gamma_2} K_{\gamma_2} \rangle E_{\gamma_1 \gamma_2} f_{\gamma} (\theta, + \theta) \tag{III-6}
\]

where \( f_{\gamma} \) is the normalized angular correlation function yielding the angular dependence between \( \gamma_1 \) and \( \gamma_2 \). Similarly the number of true coincidences detected between \( \gamma_1' \) and \( \gamma_2 \) per sec. are

\[
(\gamma_1' \gamma_2)_{\text{true}} = N \delta_\theta \langle \delta_{\gamma'_1} \omega_{\gamma'_1} \varepsilon_{\gamma'_1} K_{\gamma'_1} \rangle \langle \omega_{\gamma_2} \varepsilon_{\gamma_2} K_{\gamma_2} \rangle E_{\gamma'_1 \gamma_2} f_{\gamma'} (\theta, - \theta) \tag{III-7}
\]

The number of triple coincidences detected per sec. are

\[
(\gamma_1 \gamma_2 \beta)_{\text{true}} = N \delta_\theta \eta_\beta \langle \delta_{\gamma_1} \omega_{\gamma_1} \varepsilon_{\gamma_1} K_{\gamma_1} \rangle \langle \omega_{\gamma_2} \varepsilon_{\gamma_2} K_{\gamma_2} \rangle E_{\gamma_1 \gamma_2} E_{\beta \gamma_3} f_{\gamma} (\theta, + \theta) \tag{III-8}
\]

\[
(\gamma'_1 \gamma'_2 \beta)_{\text{true}} = N \delta_\theta \eta_\beta \langle \delta_{\gamma'_1} \omega_{\gamma'_1} \varepsilon_{\gamma'_1} K_{\gamma'_1} \rangle \langle \omega_{\gamma'_2} \varepsilon_{\gamma'_2} K_{\gamma'_2} \rangle E_{\gamma'_1 \gamma'_2} E_{\beta \gamma_3} f_{\gamma'} (\theta, - \theta) \tag{III-9}
\]
where $f_T$ is the normalized triple angular correlation function.

Substitution of these expressions into equation (11) yields

$$A_{\text{exp}} = \frac{f_d(\theta) f_+(\theta, \phi)}{f_+(\theta, \phi) f_d(-\phi)}$$  \hspace{1cm} (III-10)

The angular correlation function for the doubles is an even function in $\phi$ (see equation II-17). Then

$$f_d(\phi) = f_d(-\phi)$$  \hspace{1cm} (III-11)

and equation (10) becomes

$$A_{\text{exp}} = \frac{f_+(\theta, -\phi)}{f_+(\theta, \phi)} = A_{\text{th}}$$  \hspace{1cm} (III-12)

This is identical to equation ____________.

G. Co$^{60}$ Control Experiments

Before and after the Mn$^{56}$ runs, a series of Co$^{60}$ runs were carried out. The decay scheme of this isotope is shown in Figure III-6. Its decay scheme is similar to that of Mn$^{56}$ in that the $\gamma_1 \gamma_2$ cascade ($\gamma_1 = 1.17$, E2; $\gamma_2 = 1.33$, E2) is preceded by an allowed G.T. $\beta^-$ decay. In contrast to Mn$^{56}$, however, there is no mixing in the first $\gamma$-transition. Consequently, it would be impossible to observe a violation of time reversal in a triple angular correlation experiment measuring $\sin \delta$ with the isotope Co$^{60}$. For Co$^{60}$, $f_+(\theta, -\phi) = f_+(\theta, \phi)$, and the experimental value of the anisotropy, $A_{\text{exp}}$, must be unity. The experimentally measured anisotropy for Co$^{60}$ therefore, provides a systematic check of the experimental apparatus.
Figure II-6. Co-60 Decay Scheme (Nuclear Data Sheets-II)
IV RESULTS

Forty eight Mn$^{56}$ runs were carried out in which 40,000 triples were collected. At the conclusion of the runs sequence, the individual anisotropies were averaged. In the averaging, the individual results were weighted according to the inverse square of their errors. The Co$^{60}$ control experiments consisted of sixty eight runs in which 70,000 triples were collected. These runs were analysed in the same way as those for Mn$^{56}$. The anisotropies with associated errors for both Co$^{60}$ and Mn$^{56}$ are listed in Table IV-1.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>$A_{exp.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn$^{56}$</td>
<td>0.9860 ± 0.0111</td>
</tr>
<tr>
<td>Co$^{60}$</td>
<td>0.9997 ± 0.0082</td>
</tr>
</tbody>
</table>
DISCUSSION OF RESULTS

The \( \beta^- \) transition selected by the true coincidences is that with an end point energy of 0.75 Mev. This energy corresponds to a \( \mu_e/c \) value of 0.91 for the \( \beta^- \) particles. The \( \beta^- \) energy spectrum was cut off at the lower end at about 100 Kev. and therefore electrons with \( \mu_e/c \) values less than 0.55 were excluded. As a compromise between these two limits, we take

\[
\mu_e/c = 0.7. \quad (V-1)
\]

In Figure V-1 is plotted a graph of \( A_{\alpha\theta} \) (equation II-29) versus \( \delta \). The experimental value, 0.9860, for the anisotropy indicates that \( \delta \) lies near \( \pi \) or \( 2\pi \). We choose the neighbourhood of \( \pi \), and write

\[
\delta' = \delta - \pi \quad (V-2)
\]

From equation II-29 using the experimental value of the anisotropy for \( \ln^{56} \), one obtains

\[
\sin \delta' = 0.33 \pm 0.27 \quad (V-3)
\]

It was straightforward to determine the number of counts in the observed triples peak; however, the determination of the true triples counts was more difficult because of the factor \( \bar{R} \). With the choice of \( \bar{R} \) as in equation III-3, the number of triples in an individual run is about equal to the number of chance triples collected. While \( \bar{R} \) does not affect the individual anisotropies ( \( \bar{R} \) cancels), it does affect the individual errors and hence the overall final result.
Figure V-1. Anisotropy Function, $A_{\gamma h}$, versus Phase Angle $\delta$

(Equation II-29)

$|\eta| = 28\%$
$u/c = 0.7$
$\phi = 135^\circ$
$\theta = 90^\circ$
The error in each of the anisotropies was about 1%. This error corresponds to the accumulation of about 50,000 triples. In our case, the accumulation of 50,000 triples required 13 days of continuous running and 48 source changes. To reduce the error to 0.1%, one must accumulate $10^6$ triples. Furthermore, because $\sin \delta$ is small, it would be desirable to have an error in the vicinity of 0.01%. This small error would require the accumulation of at least $10^8$ triples. An increase in the $\beta^-$ singles rate would increase the triples rate. This increase may be accomplished by enlarging the solid angle subtended by the plastic phosphor with the source. At present, this is being done.

The choice of the 2.12 Mev. transition rather than the 1.81 Mev. transition for $\gamma_1$ was made principally to decrease the chance coincidences due to Compton scattering in the scintillators. Furthermore, the mixing ratio of the 2.12 Mev. transition ($|\eta| = 28\%$) is larger than that for the 1.81 Mev. transition ($|\eta| = 19\%$). The $\beta^-$ intensity, however, leading to the 2.12 Mev. transition is 63\% of that leading to the 1.81 Mev. transition. If $\sin \delta$ and the experimental running time are assumed the same for both selections, the time violation term should be larger for $\gamma_1 = 2.12$ Mev. The relative statistical error should also be larger for this selection.

Various other chance events other than those mentioned in III-D could also be present in the doubles and triples spectra. These are triple $\gamma-\gamma-\gamma$ coincidences, $\beta-\gamma$ cascades with the $\gamma$-ray Compton scattered from one into another $\gamma$. 

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detector, $\beta$-$\gamma$ cascades with a third $\gamma$-ray due to bremsstrahlung of the $\beta^-$ particle. All these effects are made negligibly small by the suitable choice of discriminator biases and by the thinness of the plastic scintillator.
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