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An algorithm for linear magnification of patterns.

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AN ALGORITHM FOR LINEAR MAGNIFICATION OF PATTERNS

by

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ABSTRACT

Linear expansion and contraction of the input is a basic requirement for many pattern recognition schemes. An algorithm to achieve this has been developed when the values of the pattern are known at a finite number of sampling locations in a two-dimensional array. The finite area of a sampling device has been taken into account. The values of the magnified pattern are calculated over the same array of locations for a given contraction/expansion. Error analysis has been carried out for arbitrary pattern configurations. Visual representation of the pattern, from its samples, has been obtained with the help of an on-line digital plotter. Experimental results indicating the actual error have been tabulated. A modification for reducing the effective error has been outlined for the expansion of patterns.

ACKNOWLEDGMENTS

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CHAPTER I

INTRODUCTION

A pattern is known to belong to a certain set of classes. The process of identifying it as a member of a particular class is known as pattern recognition. By a pattern we mean here any two-dimensional representation of coded information.

With the advent of high-speed information-processing machines, a great deal of mental burden of a very routine nature has been taken off the human mind; still the cost and time spent in communicating with the machine is quite large. Preparation of data and the input is the slowest operation in the present-day computer technology.

The object is to achieve an intimate and immediate man-to-machine interaction. This requires that the machine be capable of accepting and quickly responding to patterns which a human being can generate easily without any sophisticated, costly and time-consuming equipment.

Conventional links of communication with the machine (card readers, punches and printers) are undergoing marked improvements and are already many times more efficient than their predecessors. Much attention is at the same time being given to the development of novel means of communicating with the machine. A major goal is the transmission of handwritten programmes and data to the computers without the need of key-punching operators. This requires a recognition of spatial characters. A major portion of the work done has been in the field of optical character recognition, probably due to the fact that nature has provided human beings with optical sensors for this purpose.

Modern trend is toward on-line communication with time-shared machines. This makes desirable the recognition of handwritten characters in addition to typewritten and printed information. Thus, on-line recognition of hand-printed information has been the subject of much recent investigation.[1]

Most of the investigations have been made in two-dimensional black and white patterns. Colour, the three-dimensionality, overall reflectivity and even shading have been ignored; still the problem remains loaded with difficulties. Firstly, the pattern may be poorly formed or excessively distorted and, secondly, there is the ever-present noise, in the form of irregularly distributed spots, which adds to the confusion.

Another problem which most of the investigators face is the problem of normalization. Their systems can perform only when the input patterns are presented in a standard size and are placed in a standard position. Thus, even in the complete absence of noise and distortions the system fails to recognize an enlarged version of a previously-recognized pattern. The process of making a non-standard input compatible with the system is here referred to as normalization.

Having positioned the character in a standard form and after normalizing, the simplest of the approaches consists of matching this with a set of masks and finding out the correlation. The recognition is achieved when maximum coefficient of correlation has been found and happens to be more than a preset threshold.

The approach has proved highly successful in cases where the data consists of a regular font as in the case of printed or typewritten material. As early as 1962 machines working on this principle were reported[2,3] operational. Even with a variable font size and serial decisions, the recognition problem becomes very complex. This system is unsuitable for the handwritten characters.

Instead of looking for a high degree of overlap between the standard mask and the input, another approach is to look for certain salient features and make decisions on the basis of these features. Determination of unique, invariant features to be associated with different characters is central in this approach. With distortions and increased number of input characters, the decisions become difficult and ambiguous.

Attempts have been made to define the problem mathematically and thus to find out optimal solutions.

One of the mathematical models regards the patterns as points in the multi-dimensional input space. The process of recognition consists of identifying the boundaries that subdivide this input space into different regions such that every member of each pattern class maps inside the prescribed boundaries. Linear decision functions have been investigated[4] for their relationship to other selection processes, and their relevance to pattern recognition. Assuming a statistical distribution of the input set, tests have been developed[5] for the effectiveness of a set of measurements on a pattern. Investigations[6] carried out on realistic pattern measurements, rather than assuming statistical independence, have treated the effects of neighbour dependence in the element masks as well.

Some of the approaches[7] seek transformations which are invariant to size and location.

An interesting approach[8,9] maintains that a small number of moments is adequate to characterize certain patterns and discriminate among the patterns of a certain set, such as alphabetic and numerical characters. This method is incapable of making distinctions on local variations of features, but the fact that it recognizes as identical two patterns which differ in location, size, etc. makes it especially valuable.

Attempts have been made[10] to derive valuable characteristics from the time derivatives of spatial contours which could aid in the recognition process. The results of these investigations are not clearly stated in [10].

If the boundaries of a character are followed clockwise or anti-clockwise, a unique sequence of change of directions occurs for each character. Investigation[11] has been made to achieve recognition by the use of these "edge-sequences".

Work is being done[14] using an entirely new approach. Each and every step of machine function is not determinate. The machine derives its own statistics from the input samples in the learning phase. In the recognition phase this previously-acquired statistic is used and a new statistic is built up. Thus, the machine is constantly undergoing the process of learning. Such machines are known as adaptive, i.e., they change their behaviour with experience.

The problem of normalization is vital to many[12,16] character recognition schemes. In this investigation we wish to develop algorithms for deriving a normalized pattern of a given input. In a broad proposal[12] for the recognition of handprinted capital letters of the Roman script, the essential first step is that of the normalization of the input. A sequential logic has been designed and simulated[13] which determines whether the pattern needs expansion or contraction. The required process is then to be performed manually.

In this analysis, starting with the quantized values of a two-dimensional black and white pattern, contraction-expansion is carried out inside the computer, i.e., the quantized values that specify the contracted or expanded pattern are actually calculated. No properties of the pattern are assumed and for any possible pattern configuration an error analysis has been carried out. The effect of the number of sampling sensors on error in computed values has also been attempted.

CHAPTER II

OUTLINE OF THE PROBLEM

2.1 Given a plane retina bounded by $x = \pm a$ and $y = \pm b$, the pattern occupies the retina and lies wholly in it. The pattern sets up a two-dimensional light (flux) intensity field on the retina.

The contraction or expansion of the pattern is assumed to take place in such a way that any point P on the retina maps onto a point that lies on the straight line joining P to the origin. The origin maps onto itself. Contraction or expansion is to be linear.

We define the expansion coefficient Ek as the ratio of the distance from the origin of a point on the retina after magnification to its distance from the origin before magnification. Thus, a point P at a distance x from the origin will lie on the straight line joining P to the origin and at a distance $(Ek * x)$ on expansion (contraction) by a factor Ek.

For $Ek > 1$ we get expansion and $Ek < 1$ the pattern undergoes contraction.

For expansion coefficient $Ek > 1$, only that portion of the pattern which was previously bounded by $x = \pm a_{/Ek}$ and $y = \pm b_{/Ek}$ occupies the retina. The process of contraction brings onto the screen the pattern field lying in the plane of the retina and bounded by $x = \pm a_{/Ek}$ and $y = \pm b_{/Ek}$.

Since it is the relative flux intensity distribution that defines a pattern, it is desirable to normalize the light intensities. We represent maximum possible intensity by number 1 and the minimum possible intensity by number 0. Thus, any intensity on the screen is represented by a number lying between 0 and 1 inclusive.

Our object is to determine the light (flux) intensity distribution on the retina after expansion or contraction of the pattern by a known factor Ek.

It is assumed, in the discussion that follows, that the flux intensity distribution on the plane of retina is constant for a homogeneous pattern and remains unchanged for different values of Ek.

If the light flux intensity could be represented by a single function f (x,y) of two variables, the problem would be trivial.

For example, a pattern is projected on a rectangular retina. Point sensors measure light intensity at (Xi,Yj) for i = 1, 2, . . . m; j = 1, 2, . . . n.

The pattern undergoes linear expansion (contraction). It is desired to assign values of intensity at the sample points (Xi,Yj) for a coefficient of enlargement Ek.

The solution can be attempted in the following way. Keep Y constant at j = 1, 2, . . . n. Then the intensity I is a function of x alone. With m values of this function known at the sample points, we can fit a polynomial of degree (m - 1) to have representative intensities at the sample points. Thus we have

$$I_y = \text{constant} = \sum_{k=1}^{m} a_{k-1}\, x^{k-1}$$

The coefficients a_o, a_1, . . . a_{m-1} can be determined from m linear equations by substituting the values of the measured intensities at m points.

Similarly, keeping x constant at i = 1, 2, . . . m, we can write

$$I_x = \text{constant} = \sum_{k=1}^{n} b_{k-1}\, Y^{k-1}$$

The values of b_o, b_1, . . . b_{m-1} being again determined from the measured intensities at the sample points.

Thus, we have n pol nomials that represent $I_{y = \text{constant}}$ and m polynomials that represent $I_{x = \text{constant}}$ for different values of x and y.

For a given coefficient of enlargement Ek, the problem thus reduces to the determination of light flux intensities at points ($Xi_{/Ek}$, $Yj_{/Ek}$) i = 1, 2, . . . m; j = 1, 2, . . . n of the given pattern.

The integral parts of $Xi_{/Ek}$ and $Yi_{/Ek}$ refer to the position of a sampling point where the intensity is assumed to be known. The fractional part x and y are useful in the determination of the intensity at the required point.

We have

$$\Delta I = \frac{\partial I}{\partial x} \Delta x + \frac{\partial I}{\partial y} \Delta y$$

While taking the partial derivatives of I with respect to x we can replace I by $I_{y = \text{constant}}$. Thus, it being a function of x alone, when taking partial derivative with respect to y we can replace I by $I_{x = \text{constant}}$. Thus, the values of I can be interpolated from the known polynomials. It is important to keep Δx and Δy small. This means that the interpolation should start from a sampling point which is nearest from the point where the intensity is being calculated. Thus, we can keep both Δx and Δy to be ≤ 0.5.

This approach, however, fails to represent the practical situation since: (a) it assumes continuity of the function representing the pattern intensity distribution; and (b) it does not take into account the finite size of the sampling elements.

The patterns we wish to recognize will have sharp boundaries, i.e., there will be abrupt changes in intensity distribution. In the latter discussion we assume, unless otherwise specified, only two level patterns, i.e., either black or white.

Thus, the first assumption of the continuity of the function representing the intensity distribution is not valid. The assumption of continuity of this function can lead us to assign unacceptable, in fact meaningless, values of intensities in certain specific situations. For example, let us consider three sampling points A, B and C. Let the value of intensity at A be $Ia \neq 0$, and the values of intensities at B and C be both zero. Let it be required to determine the intensity at any point between B and C. It is obvious that such a polynomial representation for this situation will assign negative values of intensities for all points that lie between B and C. Similarly, it is easy to conceive of situations when the calculated value of intensity will be greater than unity, using the above approach.

The situation can be slightly improved by having more than one function to represent the intensity distribution over different regions of the retina. Moreover, we can agree to make all intensities zero that turn out to be negative on interpolation and make all intensities one that are calculated to be more than one. This simply makes the problem complex without providing any answer to the still complex problem of finite area sensors.

The light intensity is measured, in practical cases, with sensors that have a finite area and the average intensity over this area is determined. Individual points thus lose their identity and we must instead think in terms of small areas over which the average intensity is determined.

The problem of the finite area of the sensors is more complex as well as more realistic. Thus, on magnification it is not the intensity at a particular point that will map onto a sensor element, but the average intensity over an area that maps onto the sensor element.

It has been shown (Appendix I) that in linear transformation of the type we are considering, the shape of the area is conserved on magnification. Thus, a straight line will map into another straight line and a square will map into another square.

Conversely, an area that is a square after magnification was a square before magnification. This property is very useful because instead of locating each and every point of the mapped area, we need only locate a small number of points in case of regular figures. The modified problem can now be stated as follows.

The pattern is projected on a rectangular retina which consists of square sensors of unit dimension. Let there be n elements in each row and let there be m such rows. The sensors measure the light intensity averaged over the sensor area. Since we have assumed that the pattern has only two levels (either black or white) and since the intensity remains constant over the retina for a homogeneous pattern under any value of expansion coefficient, this in effect means that the output of a sensor is directly proportional to the pattern coverage over that sensor. It is required to determine the output of the sensors when the pattern undergoes expansion or contraction by a factor Ek. Thus, it is required to determine the pattern coverage over each sensor when the pattern is magnified by any known factor Ek.

Let us have a cartesian co-ordinate system such that the axes are parallel to straight lines passing through the centres of square sensors in a row and in a column. Let us fix the origin of this cartesian co-ordinate system so that the lowermost sensor to the left has the co-ordinates of its centre as (1,1). Every sensor is identified with the co-ordinates of its centre. Thus (2,2) represents the sensor shown shaded in Fig. 1.

The value of the pattern coverage we wish to assign to any sensor (X,Y) on contraction or expansion by Ek is precisely the pattern coverage on a square area which on contraction or expansion will map into the sensor under consideration. For different values of Ek the location of this square area with each side equal to $1_{/Ek}$ will vary on the plane of the retina, the centre of this square lying at a point $(X_{/Ek}, Y_{/Ek})$. In general, this mapping area will be located over more than one sensor.

Thus, the solution of the problem of normalization requires the determination of pattern coverage over specified areas of the retina. These areas, in general, will lie over more than one sensor.

2.2 The problems of contraction and expansion are quite similar and a common programme is able to deal with both of

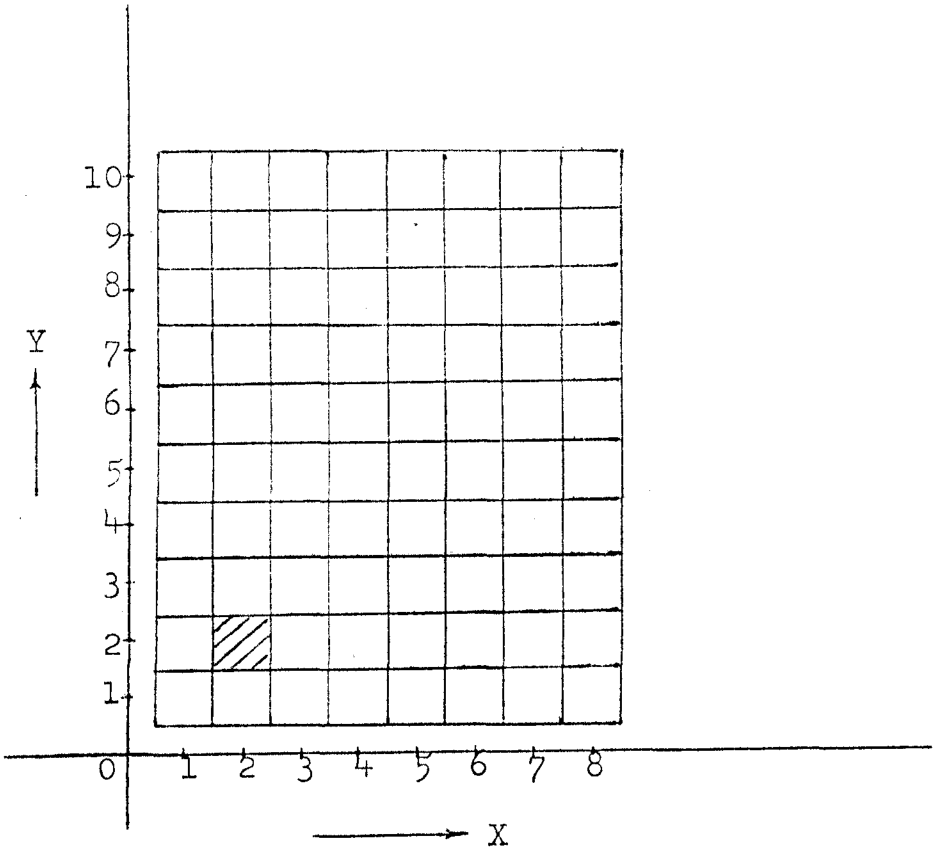


FIG. 1.--A REPRESENTATION OF THE RETINA WITH THE CO-ORDINATE SYSTEM.

them. However, for the sake of clarity the two problems have been dealt with separately. Let us first consider the case of expansion ($Ek > 1$).

The first part of the problem consists of locating for each individual sensor the location of the area that on expansion will map onto it. It has been shown (Appendix I) that this area will itself be a square with sides parallel to the co-ordinate axes. The sides of this mapping area will be equal to $1/_{Ek}$. Since $Ek > 1$, the mapping area can lie on at most four adjacent sensors.

A portion of the retina is shown in Fig. 2. The centres of four adjacent sensors are marked as S1, S2, S3 and S4. The shaded square area on expansion maps onto the sensor (X,Y). The coefficient of expansion is Ek. Since on expansion the centre of this shaded square will map onto the point (X,Y), the co-ordinates of this centre (marked *) are $(X/_{Ek}, Y/_{Ek})$.

Let us denote by X_I and X_F the integral and the fractional parts of $X/_{Ek}$. Similarly, Y_I and Y_F represent the integral and fractional parts of $Y/_{Ek}$. Thus, we are able to determine the co-ordinates of the four possible sensors S1 to S4 over which the mapping area can lie. It is to be noted that the area of this shaded square is $1/_{Ek^2}$.

The amount of area on these sensors S1 to S4 has been shown (Appendix II) to be:

Area on $(X_I, Y_I) = A1 = (1/_{2\,Ek} - X_F + 0.5)\ (1/_{2\,Ek} - Y_F + 0.5)$

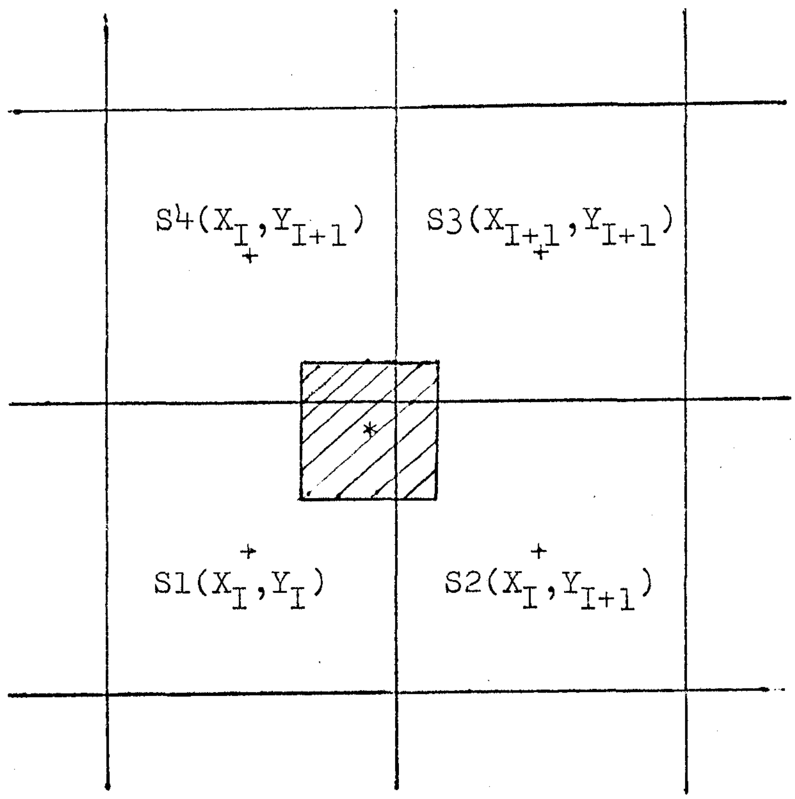


FIG. 2.---A PORTION OF THE SCREEN SHOWING THE AREA THAT ON EXPANSION MAPS INTO A SINGLE SENSOR

Area on $(X_{I+1},Y_I) = A_2 = (1/_{2\ Ek} + X_F - 0.5)(1/_{2\ Ek} - Y_F + 0.5)$

Area on $(X_{I+1},Y_{I+1}) = A_3 = (1/_{2\ Ek} + X_F - 0.5)(1/_{2\ Ek} + Y_F - 0.5)$

Area on $(X_I,Y_{I+1}) = A_4 = (1/_{2\ Ek} - X_F + 0.5)(1/_{2\ Ek} + Y_F - 0.5)$

Each one of these two factors which on multiplication produce areas A1 to A4 can be recognized as the sides of the rectangular areas lying on individual sensors. The length of the sides on individual sensors must be between zero and $1/_{Ek}$. The way these sides are calculated (Appendix II) can make them negative or even greater than $1/_{Ek}$. Thus, having calculated each side, it is essential to check that each side length is within the permissible range. If the side length comes out to be negative, it must be made equal to zero and in case this length turns out to be greater than $1/_{Ek}$ this length should be made equal to $1/_{Ek}$.

So for each (Xi,Yj) i = 1, 2, . . . n; j = 1, 2, . . . m, we are able to determine the location of areas A1 to A4 and their amount.

In general, if the coverage on sensors S1 to S4 (known from their output) is equal to I1, I2, I3 and I4 respectively, the value of the coverage we would assign to the sensor over which this area maps will be proportional to $\sum_{i=1}^{4}$ Ai Ii. The constant of proportionality comes about because of the fact that the area over which this assignment is made is greater than the mapping area. Since the sensor area is Ek^2 times the area that is mapping into it, the assigned coverage I would be

$$I = Ek^2 \sum_{i=1}^{4} Ai\ Ii$$

Thus, the new assignment can be calculated from the known values of sensor coverages (outputs) and the expansion coefficient Ek.

Let us now consider the case of contraction. On contraction ($Ek < 1$) the pattern shrinks towards the centre, i.e., towards the fixed point of the mapping. Each sensor square will map onto a square of side length Ek. Since $Ek < 1$ the portion of the pattern lying on individual sensor will map on at most four adjacent sensors. Thus, the coverage on each sensor has to be divided into four parts proportional to these four areas mapping on four different sensors. The areas on these four squares can be calculated in a manner exactly similar to the manner in which the four areas were calculated in case of expansion (Appendix II). Thus, in the final assignment of coverage on each sensor, it is only required to sum up these contributions from different sensors that have a non-zero area mapping on the sensor under consideration.

For the sake of a computer programme, the two cases can be handled simultaneously. Thus, a square area of side length Ek that results from contraction of a sensor square will map onto the same sensor if the expansion coefficient is $1/Ek$. Thus, as far as the calculation of four different areas on four different sensors is concerned, it can be made by properly modifying the factor Ek. For example, if in the calculation of these areas we are making each unit length to map into a length Ek ($Ek < 1$), the areas for contraction are straightaway calculated. For the case of expansion, it is only necessary to replace Ek ($Ek > 1$) to $1/_{Ek}$ which will be less than one and determine the required areas.

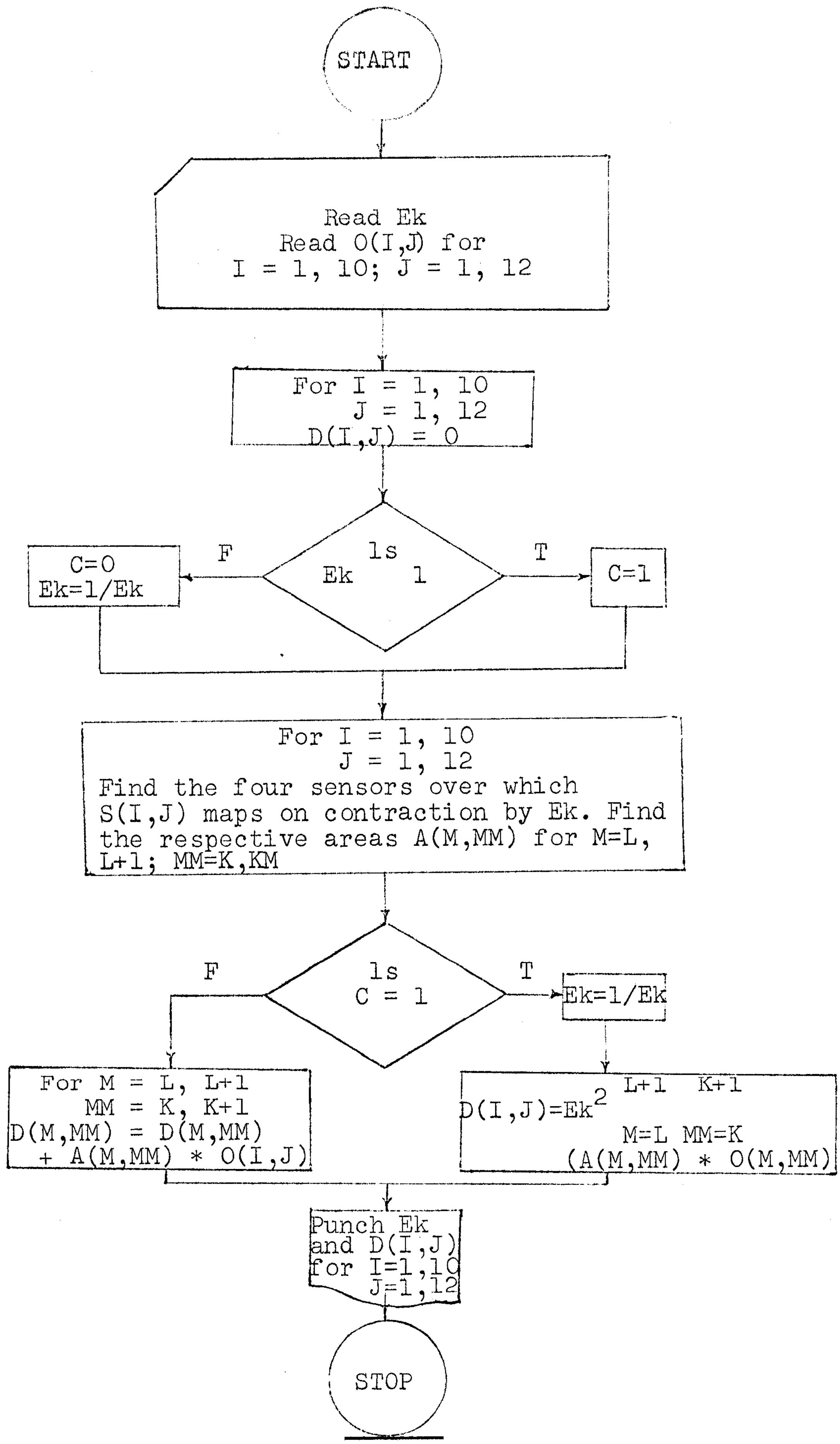
The flow chart for the computer programme is shown in Fig. 3.

2.3 Computer Programme

The above algorithm for contraction and expansion was programmed using FORTRAN language. The following points in the programme deserve mention.

(1) The question of the origin or the fixed point of the mapping is central in any scheme of contraction or expansion. Logically, the contraction or expansion should take place about the centre of the pattern. Thus, it is very important to locate the centre of the pattern. For an arbitrary location of the pattern on the retina, the pattern has to be shifted such that the centre of the retina and the centre of the pattern coincide. This could be done, for example, by describing the largest rectangle around the pattern with the sides of the rectangle parallel to the co-ordinate axes and each side touching a portion of the pattern. Then the centre of the pattern can be identified with the centre of this

FIG. 3.--FLOW DIAGRAM FOR CONTRACTION/EXPANSION



rectangular area. However, in the present programme it is assumed that the pattern centre coincides with the physical centre of our retina and normalization has been attempted keeping the centre of the retina as the fixed point of the mapping.

(2) In cases where the mapping area does not lie on all the possible four adjacent sensors, it might turn out that some side lengths of the partial areas assume negative values or values greater than the length a unit length (our sensors are assumed to be of unit length) should assume on mapping. In such cases it is necessary to make all lengths zero that turn out to be negative. At the same time, we should make all lengths equal to the maximum permissible length, which turn out to be greater than this maximum value.

(3) Space has been reserved in the computer memory for storing two sets of data, the original coverage (input values) and the derived coverage (calculated values). The reason for this is that the operations of contraction and expansion, as outlined above, are not stable, i.e., the pattern undergoes transformations that are not reversible. If we obtain the same amount of contraction or expansion directly and in two or more steps, the results are, in general, different. Similarly, error will result when a pattern is subjected to expansion and contraction by equal amounts (Ek (exp.) * Ek (cont.) = 1).

This shows that if we store only the most recently derived pattern coverage and are contracting or expanding it a number of times, the pattern will soon be deformed to such an extent that it completely loses its identity. These operations of contraction and expansion, when repeated a number of times, do not preserve the original or, in other words, they are unstable.

The reason for the unstable nature of these transformations is that the pattern coverage as designated by the output of a particular sensor is the average coverage over that area. Thus, when a portion of a sensor area is taking part in the mapping process, the actual coverage on this area will be, in general, different from the average coverage over the whole sensor area. This error goes on accumulating when the process of contraction or expansion is repeated a number of times. It seems that as the number of operations of contraction and expansion will increase beyond limit, the pattern coverage on each individual sensor will approach a limiting value equal to the total pattern coverage on the whole screen divided by the number of sensors on the screen.

However, to get over this difficulty we store the original pattern all the time and every time operate on this original pattern to produce a derived pattern and thus do not accumulate the errors.

2.4 Error Analysis

The algorithm for contraction and expansion is susceptible to errors. Again we consider the two cases of expansion and contraction separately.

In case of the expansion of patterns, errors will be made whenever the sensors are partially covered and the average coverage over the areas A1 to A4 on the sensors S1 to S4, taking part in the mapping, is different from the average over the sensors S1 to S4 respectively.

We shall attempt to determine the maximum error made in case of expansion by a given Ek. We limit our discussion to only two level patterns (either black or white) such that the boundaries are sharp and well defined as well as there is abrupt change in pattern from black to white and vice versa.

It has been shown (Appendix III) that maximum error will be made when the area on a sensor that takes part in the mapping process has a coverage just opposite to what the coverage is on the remaining sensor area.

At the most, four adjacent sensors take part in the calculation of the assigned coverage. The location of these areas A1 to A4 is fixed for the individual sensor by the value of expansion coefficient.

Assuming that the mapping areas Ai (i = 1,4) have a value of coverage 1, and the remaining areas on Si (i = 1,4) have a value of zero, the maximum deviation can be calculated as shown below:

Actual coverage we should assign = 1

Average value of coverage on S1 = A1

Average value of coverage on S2 = A2

Average value of coverage on S3 = A3

Average value of coverage on S4 = A4

Since we assign values of coverage proportional to the area mapping and proportional to Ek^2, we have:

Assigned value of the coverage = $Ek^2\ (A1^2+A2^2+A3^2+A4^2)$

Deviation in assignment $D = 1 - Ek^2\ (A1^2+A2^2+A3^2+A4^2)$

Maximum deviation D_{max} for all the sensors on retina = Max of D over all the sensors.

Thus, we must calculate the deviation on individual sensors for a given value of Ek and find out the maximum value of deviation on an individual sensor.

Let us now consider the errors made in contraction of patterns. Basically, all errors due to the finite areas of the sensors arise from the fact that the measured value of coverage (output of a sensor) represents the average coverage over this area. Thus, the details of pattern configuration can in no way be ascertained from its output. For example, in the case of the two sensors shown in Fig. 4, if the shaded areas are equal the output will be the same and, therefore, from the output alone no information can be obtained about the actual pattern configuration over a sensor.

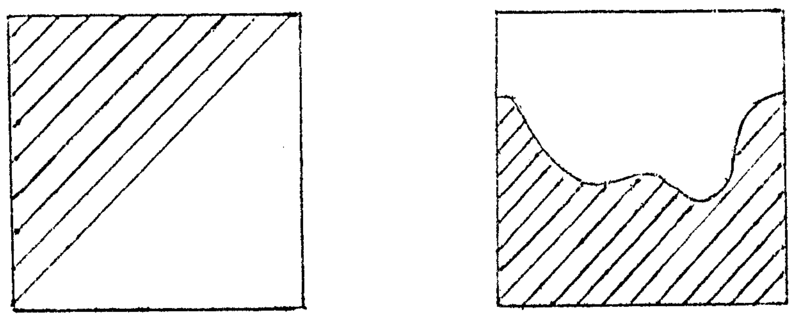


FIG. 4.--TWO SENSORS THAT MAY HAVE THE SAME OUTPUT.

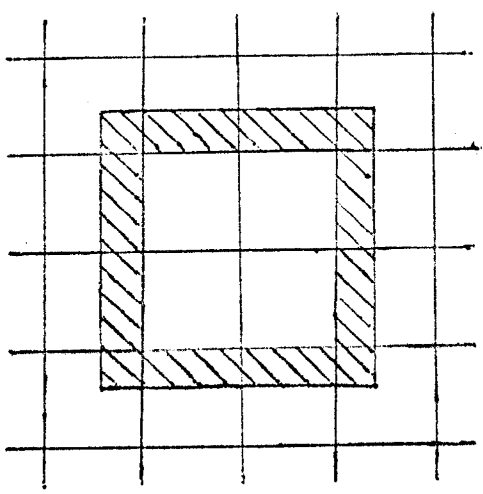


FIG. 5.--A POSSIBLE PATTERN FOR MAXIMUM ERROR ON CONTRACTION HAVING MAXIMUM PARTIALLY COVERED SENSORS.

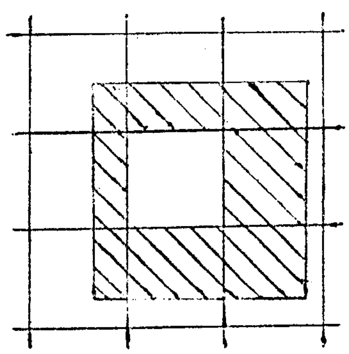


FIG. 6.--ANOTHER POSSIBLE PATTERN WHICH PRODUCES ERROR ON CONTRACTION.

Let the coefficient of contraction be equal to Ek ($Ek < 1$). Any horizontal or vertical line of length $1/_{Ek}$ can map as the side of a sensor. Let k_I represent the integral part of $1/_{Ek}$. Hence $(k_I + 2)^2$ sensors at the most will contribute their areas for mapping into a single sensor. Thus, the averaging process ranges over $(k_I^2 + 2\,k_I + 4)$ sensors at the most. Out of this total number of $(k_I^2 + 2\,k_I + 4)$ sensors, there will be k_I^2 sensors that completely map into one sensor. Now as has been shown (Appendix III), the error results whenever partial areas over a sensor map into one sensor. Thus, these k_I^2 sensors do not produce any error. The remaining $4\,(k_I + 1)$ sensors are responsible for the error in assignment (as in Fig. 5). In case only $(k_I + 1)^2$ sensors are taking part in the mapping, $(k_I - 1)^2$ sensors will be mapping completely and only $4\,k_I$ sensors are responsible for error in assignment (as in Fig. 6).

Thus, let the pattern shown in Fig. 5 map into a single sensor. Here $k_I = 2$ and therefore $4\,(2 + 1) = 12$ partially-covered sensors are contributing toward the error made in the assignment of coverage to a single sensor on which this pattern maps on contraction. In Fig. 6, k_I is again equal to 2 but only $4\,k_I$, i.e., eight sensors, are responsible for the error.

Now maximum error results (Appendix III) when the mapping area of a sensor has the value of coverage just opposite to the value of coverage that is not taking part in the mapping process. Thus, for the calculation of maximum error made in calculating the coverage on a single sensor, we must determine for each individual sensor all those areas on different sensors which map into this sensor on contraction. For those sensors which have only partial areas mapping into a particular sensor, we must assign value of coverage such that the mapping area has coverage one and the remaining area has coverage equal to zero or vice versa. This assignment will give us the maximum error on each individual sensor of our retina. The maximum of all these errors on individual sensors is the maximum error made in contraction for a given value of Ek.

The flow chart for a computer programme capable of finding the maximum error is similar to the flow chart for finding coverages.

Theoretically, it has been shown (Appendix IV) that maximum error results when in addition to the conditions specified by Appendix III the mapping area is symmetrically located with respect to the sensors over which it lies.

It will be noted from the maximum error thus calculated that under special pattern configurations the error can be so large that it renders the calculated value of coverage on individual sensors entirely useless. For the characters of the Roman alphabet, however, such special configurations are not common. This fact is borne out by the experimental results obtained as described in the following chapter.

CHAPTER III

EXPERIMENTAL RESULTS

3.1 The scheme was tested on a number of handwritten capital letters of the Roman alphabet. (An on-line digital-plotter available was also used.)

In some earlier work[12] done at this university, the retina was made up of a rectangular array of eighty sensor elements arranged in ten rows of eight sensors. In order to see the results of the contraction-expansion programme, it was decided to have a retina size of ten columns * twelve rows of square sensors of a unit dimension. The need for a larger retina arose from the fact that a pattern which originally occupies the retina completely would have some of its portions falling out of the retina on expansion.

In practice, the input to the programme will be the normalized coverage on each of the sensors. The algorithm calculates the coverage on each of the sensors after contraction or expansion of the pattern by a known factor Ek.

The first part of the problem consists of obtaining a graphical representation of the input and output. The on-line digital plotter was used for this purpose as explained below.

The basic idea is to represent the coverage, as measured (for inputs) or as calculated (for outputs) after expansion or contraction by a known factor Ek on each individual sensor of our retina. It should be noted that the above values represent the average coverage over a sensor under consideration. A rectangular grid of size 10 x 12 drawn on paper and divided into 120 squares of a unit dimension represents our retina. In order to represent the coverage on individual sensors, two different methods were tried as described.

(1) The coverage can be represented by the number of points proportional to the coverage marked on a particular sensor. These points have to be distributed over the unit area representing a sensor as evenly as possible. Thus, the density of these points on a particular sensor is a measure of the relative coverage on this sensor. This distribution of points makes the visual representation fairly clear. It was decided to represent the maximum coverage by 100 points evenly distributed on the area representing a sensor. For any other coverage the number of points to be marked is reduced proportionally, i.e., a coverage of, say, 0.5 is represented by 50 points. The problem of evenly distributing the points requires some sacrifice on the exact number of points to be marked in a given area. Thus, for example, if we have to mark 50 points in a square area, the best way is to put only 49 points in seven rows having seven points each. This representation will give better distribution

than any other arrangement. Again for the sake of visual clarity, it is required to set a threshold of minimum coverage such that any coverage below this threshold is neglected.

The number of points P, to be marked, is first calculated by multiplying the coverage on a particular sensor by 100. Next a whole number k is found which is as close to P as possible. Finally, another whole number M is determined such that $k * M$ is as nearly close to P as possible. Thus, we can plot $k * M$ points in a square area representing a sensor in k rows of M points each, evenly distributed.

The Fortran programme listing and the representation of letter S obtained by this method are shown in Appendix V.

(2) Marking points such that their density is a representation of the average coverage requires distribution adjustments in two dimensions. In addition to some sacrifice in accuracy for the exact number of points to be marked, it is observed that the visual representation is not very clear. Thus, it was decided to modify the problem by drawing horizontal lines on the area representing a sensor. This requires adjustment only in one dimension. The distance between the lines or, in other words, the density of lines in a given area is made proportional to the average coverage over that area. Thus, for full coverage it was decided to put sixteen lines equally spaced in the square area representing a sensor. For any other coverage the number of lines is proportionally reduced, thus decreasing their

density and providing a much better visual representation.

The results obtained for the visual representation using this technique are shown in Appendix V and are used for the representation of contracted and expanded patterns.

The second portion of the problem consists of the determination of the accuracy with which the assignment of coverage is made for a given value of expansion coefficient. Thus, we are required to find out experimentally the actual coverage on the sensors that results when a given pattern undergoes expansion or contraction by a known amount. This coverage, when compared with the calculated coverage, indicates the deviation in assignment.

For experimentally determining the coverage on individual sensors, the pattern under consideration was projected, with the help of a projector, on a rectangular screen divided into 120 square areas representing sensors arranged in twelve rows of ten elements each. Each of these square areas representing a sensor was further divided into 64 small squares. The pattern coverage is determined by counting the number of small squares that are covered by the pattern. Those small squares which are partially covered are approximated to whole number coverage. The projection requires the alignment of the centre of the pattern and the centre of the screen. For this purpose, six rectangles of different sizes were drawn around the pattern such that the centre of the pattern coincides with the centre of each of the rectangles. Thus, for the alignment of the centre of the screen and the centre of the pattern it was only necessary to make the sides of a rectangle coincide with the boundaries of the screen. Six rectangles give us the actual coverage of the pattern plus the values of coverage for five different values of contraction-expansion coefficient Ek.

3.2 Error Analysis

In addition to the inherent error (Appendices III, IV) to which the algorithm is susceptible, there are five more sources of error which influence the experimental determination of coverages on individual sensors. These are:

(1) Error in ascertaining the exact amount of coverage.

(2) Error in quantizing the observed values.

(3) Error in quantizing the calculated values.

(4) Error due to inexact value of Ek.

(5) Error due to misalignment which can be further subdivided into:

(a) Misalignment of the centres, i.e., linear translation of the pattern.

(b) Rotation of the pattern.

The effect of these is considered in detail as follows.

<u>Exact Amount of Coverage</u>

As explained earlier, the actual coverage of the pattern on individual sensors was obtained by counting the number of small squares drawn on the area representing a sensor. Those small squares which are partially covered are approximated to the nearest whole number. In practice, the boundaries of a projected pattern are hazy and are not sharply defined. Thus, there is an error in estimating the exact amount of coverage. The approximation of partially-covered small squares to the nearest whole number introduces a personal error which varies even with the same observer when the observations are made at two different times. To reduce the personal error the coverage on each sensor was determined twice at different times and the mean of these two values was recorded. The error can be further reduced if some more sophisticated method of determining coverages is employed, say, the use of a planimeter. It is difficult to make any mathematical estimate of the error made in determining the exact value of coverage.

In some earlier work[12] the values of coverage were required to be quantized into sixteen levels. We have sixty-four small squares in each of our sensor areas. Thus, the number of small squares covered by the pattern on a particular sensor when divided by four and rounded to the nearest whole number gives us directly the quantized value of coverage on that sensor on a level of sixteen.

<u>Error Due to Quantization of Observed Values</u>

Any error in the observed values of coverage produces an error in the calculated values. It is required to calculate the maximum deviation in the calculated values of coverage for coefficient of expansion Ek for a given maximum deviation d in the observed value. If we have:

Dj = value of the calculated coverage on j^{th} sensor.

Oi = value of observed coverage on i^{th} sensor.

Ai^{j} = area on the i^{th} sensor which maps on the j^{th} sensor for expansion coefficient Ek.

N denotes the total number of sensors that take part in the mapping process.

The scheme of expansion assigns coverages as:

$$Dj = Ek^2 \sum_{i=1}^{N} (Oi) * (Ai) \tag{1}$$

For fixed value of Ek and hence Ai we obtain from (1):

$$\Delta Dj = Ek^2 \sum_{i=1}^{N} (Ai) * (\Delta Oi)$$

$$\text{Max } \Delta Dj = Ek^2 \sum_{i=1}^{N} (Ai) * (\text{Max } \Delta Oi)$$

Max ΔOi = d, the maximum deviation in the observed value.

$$\text{Max} \quad Dj = Ek^2 \sum_{i=1}^{N} (Ai) * d = d * Ek^2 \sum_{i=1}^{N} Ai = d$$

Because $Ek^2 \sum_{i=1}^{N} Ai = 1$. In case of contraction there is no factor like Ek^2 on the right-hand side of (1). But then Ai represents the area of i^{th} sensor actually mapping onto the j^{th} sensor and clearly $\sum_{i=1}^{N}$ Ai equals the area of a sensor which is again equal to unity.

Thus, maximum error in the calculated value of coverage is equal to the maximum error in observed value.

In quantization we have the maximum error equal to the rounding error which is equal to one step of quantization level.

Error Due to Quantization of Calculated Values

The calculations inside the computer are done on the basis of continuous variation of coverage (the digital nature of the machine produces insignificant error), and the resulting coverage is quantized. Again the quantization error is equivalent to the rounding error, i.e., one step of quantization level.

Error Due to Linear Translation of Pattern

It is quite difficult in practice to align the centre of the pattern with the centre of the screen. In general, the centre of the pattern will lie within a circle of radius r centred at the centre of the screen where r denotes the magnitude of linear displacement of the pattern.

It is required to find out the maximum error due to a linear shift r.

Let x and y represent the components of r in the x and y directions respectively. For the position of sensors fixed on the retina, the extra area which maps into a sensor but does not belong to it is equal to $(x+y - xy)$.

As shown hatched in Fig. 7, the area can be a source

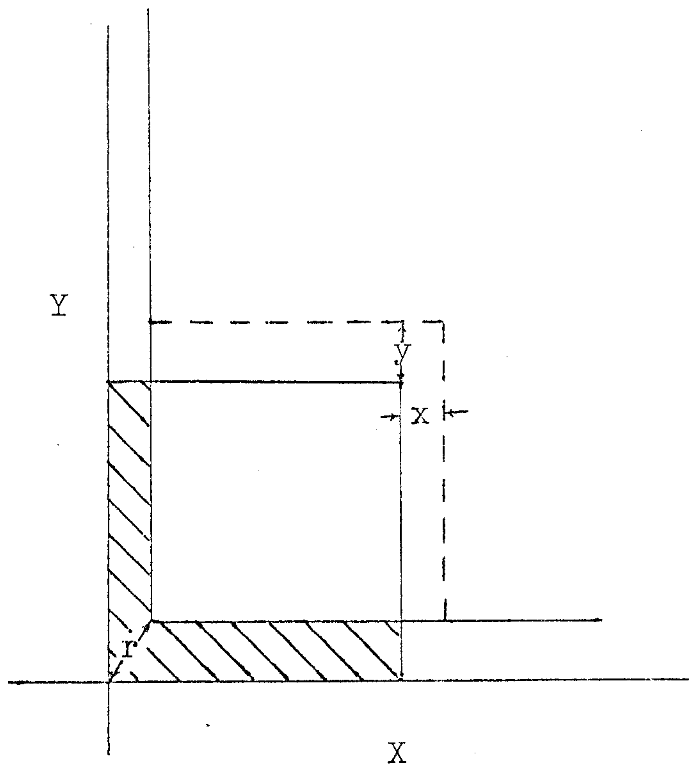


FIG. 7.--LINEAR TRANSLATION OF THE PATTERN AND THE RESULTING ERROR.

of error in case the portion of pattern coverage on this newly-acquired area is of opposite value than the pattern coverage on the remaining area. Suppose the coverage on this area is of value 1 and the coverage on the remaining sensor area has a value of zero, then:

$$\text{Actual value} = 0$$

$$\text{Observed value} = x+y - xy$$

$$\text{Error } E = x+y - xy$$

We want to find the maximum value of this error constrained by:

$$x^2 + y^2 = r^2 = \text{constant}$$

$$\text{Let } x = r \operatorname{Cos} \theta$$

$$y = r \operatorname{Sin} \theta$$

$$E = r (\operatorname{Cos} \theta + \operatorname{Sin} \theta) - \frac{r^2}{2} \operatorname{Sin} 2\theta$$

$$\frac{dE}{d\theta} = r (\operatorname{Cos} \theta - \operatorname{Sin} \theta) - r^2 \operatorname{Cos} 2\theta = 0$$

$$r (\operatorname{Cos} \theta - \operatorname{Sin} \theta) [1 - r (\operatorname{Cos} \theta + \operatorname{Sin} \theta)] = 0$$

$$\text{Either } \operatorname{Sin} \theta = \operatorname{Cos} \theta$$

$$\text{or } \operatorname{Cos} \theta + \operatorname{Sin} \theta = 1/r$$

For values of r much less than 1. The only value for θ = 45° (other values will be imaginary).

$$x = r/\sqrt{2}$$

$$y = r/\sqrt{2}$$

$$\text{Error} = \sqrt{2}\, r - r^2/2$$

Thus, we find that for a shift of 0.05 units the error will be 0.07, i.e., approximately one step of quantization on a level of sixteen.

Error Due to Rotation of the Pattern

Even when the centre of the pattern and the centre of the screen coincide, there may be a rotation of the whole pattern clockwise or anti-clockwise through a small angle $\emptyset$.

As in the case of linear translation, the error is introduced by that additional area mapping on a particular sensor which really does not belong to it and the deviation is equal to this area. For $\emptyset$ (in radians) to be small, this area is approximately $\emptyset$ units for sensors in the immediate neighbourhood of the centre as shown in Fig. 8. However, the maximum change of area occurs on a sensor farthest away from the centre. Thus, sensors lying near the boundary of the screen are very sensitive to errors produced on small rotation. For a given screen size the error can be easily calculated by finding out the maximum change in area occurring for the sensor which is farthest away from the centre of the screen.

Error Due to Changes in Ek

In practice, it is quite difficult to expand or contract a given pattern by an exact amount Ek. We wish to determine the maximum deviation in assignment values of coverage for a small change Ek in Ek. A length $1/_{Ek}$ along the horizontal direction can map on the side of a sensor on

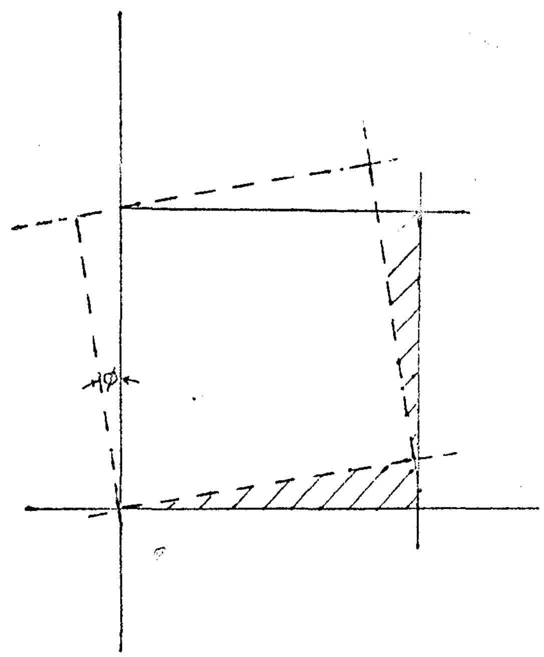


FIG. 8.--EFFECT OF THE ROTATION OF THE PATTERN ON THE ERROR ON THE SENSOR ADJACENT TO THE CENTRE.

magnification by Ek, i.e., an area $1/_{Ek^2}$ maps into the sensor. If ΔEk is the change in Ek, then an area equal to $\frac{1}{(Ek + \Delta Ek)^2}$ will now map into the sensor. The additional area is $1/_{Ek^2} - 1/_{(Ek + \Delta Ek)^2} \simeq \frac{2 \Delta Ek}{Ek^3}$. This additional area $\frac{2 \Delta Ek}{Ek^3}$ is a source of error. Now in the process of normalizing coverages the mapping area gets multiplied by a factor Ek^2. Thus, the error will be equal to $\frac{2 \Delta Ek}{Ek}$. For Ek = 1.5 approximately 0.05 change in Ek produces one step of error on a quantization level of sixteen.

We have thus seen the various sources of error affecting experimental determination of exact coverage and the extent to which they can influence the assignments.

The results of experimental observation of coverage along with the calculated values are shown in Appendix VI for some sample letters. It is to be noted that for most of the calculated values the results are within the experimental errors. However, certain assignments (underlined in Appendix VI) the error is quite large. This is due to the inherent error in the algorithm. We will see in the next chapter how these results can be improved.

CHAPTER IV

ERROR PERFORMANCE IMPROVEMENT

4.1 For certain special patterns the inherent error in the algorithm can make the assigned value on a particular sensor, in expansion or contraction, completely unreliable. For the pattern class of Roman alphabet such special pattern configurations are not common. However, even with this pattern class, some of the individual assignments are not satisfactory.

The actual recognition of a pattern is envisaged by interrogating portions of normalized patterns and establishing a reasonable match as a whole or in parts (as in the case of matching certain salient features) with some predetermined stored values. Thus, it is essential that in the process of normalization the error on these areas being interrogated should not exceed a known maximum. With the error in assignment on individual sensors becoming large, it is obvious that the sensor area should be smaller than the area under interrogation. Additionally, the error over the area being interrogated should be less than the prescribed upper limit.

In the determination of the maximum error over a single sensor, a particular pattern configuration is assumed.

It is to be noted that this particular pattern configuration, although it results in maximum error over the said sensor, at the same time the error resulting on the adjacent sensors is small. Thus, when the area over which error is calculated is larger than the sensor area, the average error on this area will be reduced. We assumed sampling-sensors of unit dimension or, in other words, we had an average of one sensor per unit area of the retina. If the average error over an area reduces when the area under consideration is greater than the sensor area, we may have reduced error in calculating the coverage if we have on an average more than one sensor in a unit area.

In this investigation we will study the effect on the maximum average error when the area over which this average error is calculated becomes greater than the area of a sensor. We will hereafter refer to this area over which the error is averaged as the area of interest. We will consider the case when the area of interest is square and concentric with a sensor with its sides parallel to the sensor sides.

The first part of the problem is to determine the pattern components configuration which makes the error on the area of interest to be as large as possible. The second part consists of calculating this maximum error for the assumed pattern. We consider the case of the expansion of patterns only.

When the area of interest becomes larger than the sensor area, it lies on at least nine sensors. Let us denote these sensors as S1, S2, . . . S9. In the process of expansion a portion of the pattern lying on some sensors S1′, S2′, . . . Sn′ maps into the area of interest where $n \leq 9$. It is required to determine the pattern configuration on sensors S_i' (i = 1, n) such that the average error over the area of interest is maximum. For finding the average error the weighting function is proportional to the area over which this error occurs.

Thus, it is clear that for the average error to be maximum, the individual error components should be as large as possible. Moreover, if there is a choice in maximizing error over two components of the area of interest, we should first maximize error over the larger area.

However, the error components on the area of interest cannot be maximized independently of one another. This is due to the fact that for maximizing the error we choose coverage values on the mapping sensor such that the mapping area has the value of coverage just opposite to the remaining area. Now for a given Ek two or more components of the area of interest might contain parts of the pattern which before expansion lay on a single sensor. Thus, it is not possible to assume pattern coverage on the mapping sensor according to the method given in Appendix III. The determination of the pattern which maximizes the error components is beyond the scope of our discussion. However, to get an idea of the problem, the following example is considered.

The area of interest has components on different sensors which are numbered such that the larger number refers to an area which is either less than or equal to the area referred by a smaller number. This is shown in Fig. 9. Starting with area number 1, we make assignments for pattern configuration according to Appendix III and go on to higher number areas. If the assignment on a particular sensor has already been made, we leave it as it is and make a new assignment of pattern component configuration only on those sensors which do not have the pattern defined on them. Thus, we are able to determine a pattern which maximizes the error on larger areas first, thus producing a relatively larger average error than any other sequential assignment (since all errors $\geqslant 0$). The area of interest is centred successively on all the sensors on the screen. When the area of interest is greater than the sensor area and is centred on the sensors on the boundary of the screen, a portion of the area of interest falls outside the screen. To get over this difficulty we have added two additional rows and two additional columns of sensors to the effective screen. Thus, the effective screen size is 10 rows * 8 columns and the area of interest is only centred with each of these sensors. For the sake of calculations, however, the screen size is 12 rows * 10 columns.

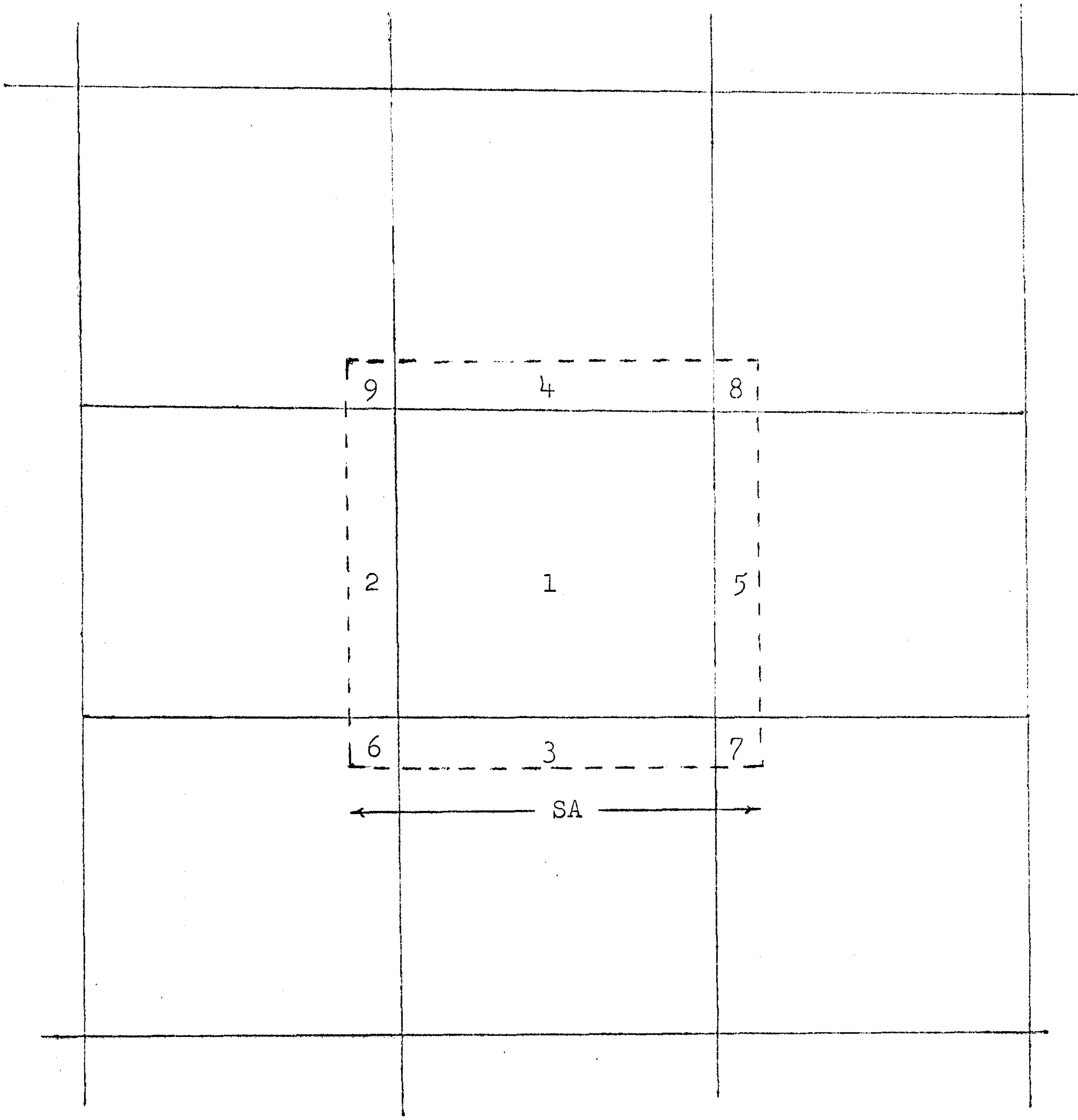


FIG. 9.--AREA OF INTEREST AND ITS COMPONENTS ON DIFFERENT SENSORS.

The calculation of error is made with the help of a computer programme. The error on each of the components of the area of interest is calculated separately. These errors are weighed proportionally to the area over which they occur and finally the effective error for the assumed pattern component configuration is calculated. For a given value of Ek and a given area of interest the maximum of these effective errors is calculated by successively centering the area of interest over all the sensors of the effective screen. This maximum error has been calculated for values of Ek ranging from one to two and the area of interest ranging from one sensor area up to the area of nine sensors.

4.2 Computer Programme

The calculation of the error has been made with the help of a computer programme. A complete flow chart is shown in Fig. 10.

A three-dimensional array A (I, J, K) deserves some explanation. Here the first subscript I refers to the number of the component of the area of interest. The numbering scheme is shown in Fig. 9. The two subscripts J and K refer to the location of a sensor (J, K). The function A (I, J, K) is that area on sensor (J, K) which for a given Ek maps into the I^{th} component of the area of interest. SA represents the side of the area of interest.

The Fortran programme listing and the results are shown in Appendix VI.

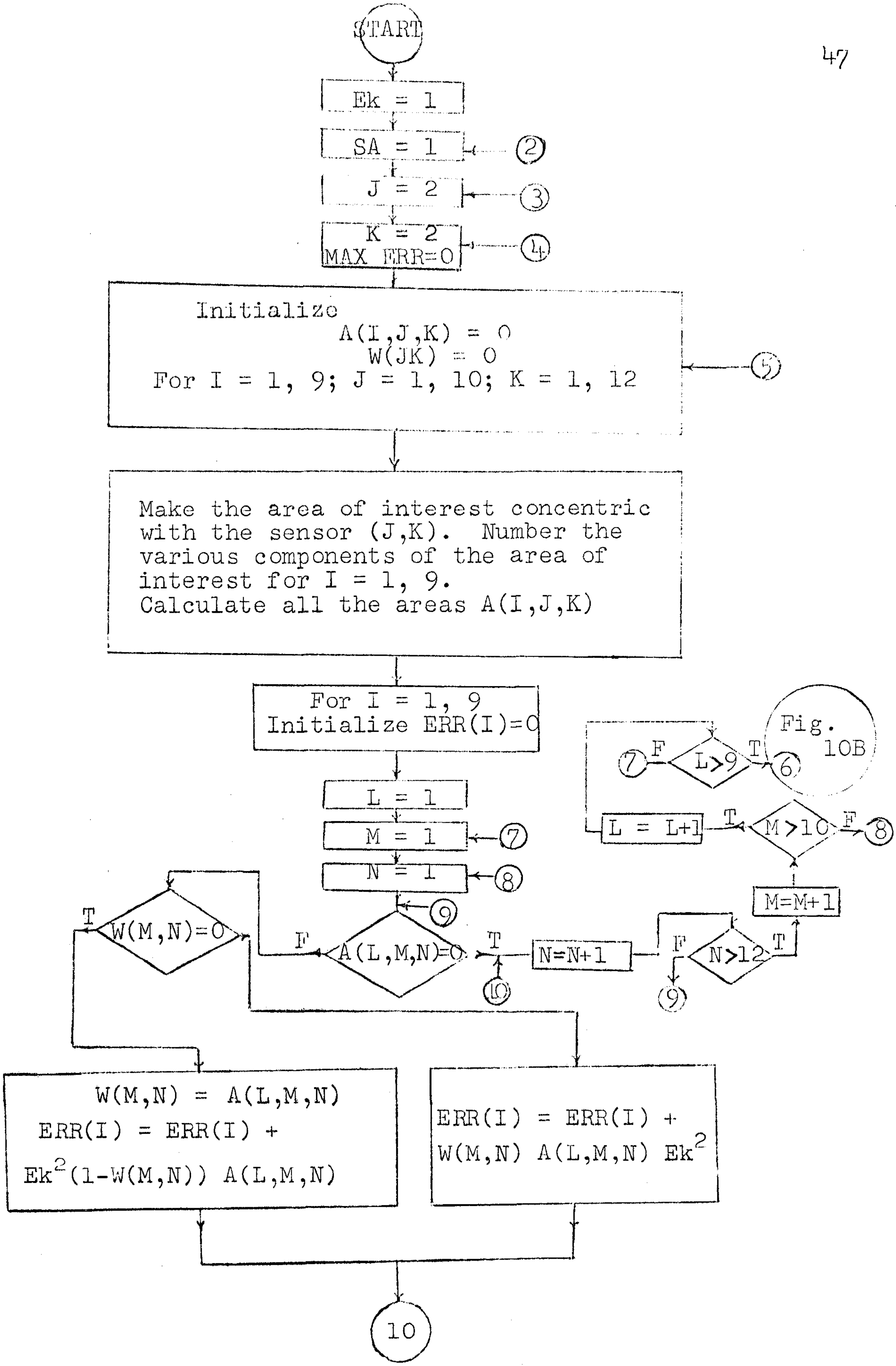


FIG. 10A.--FLOW CHART FOR THE EFFECTIVE ERROR OVER THE AREA OF INTEREST IN THE CASE OF EXPANSION OF PATTERNS.

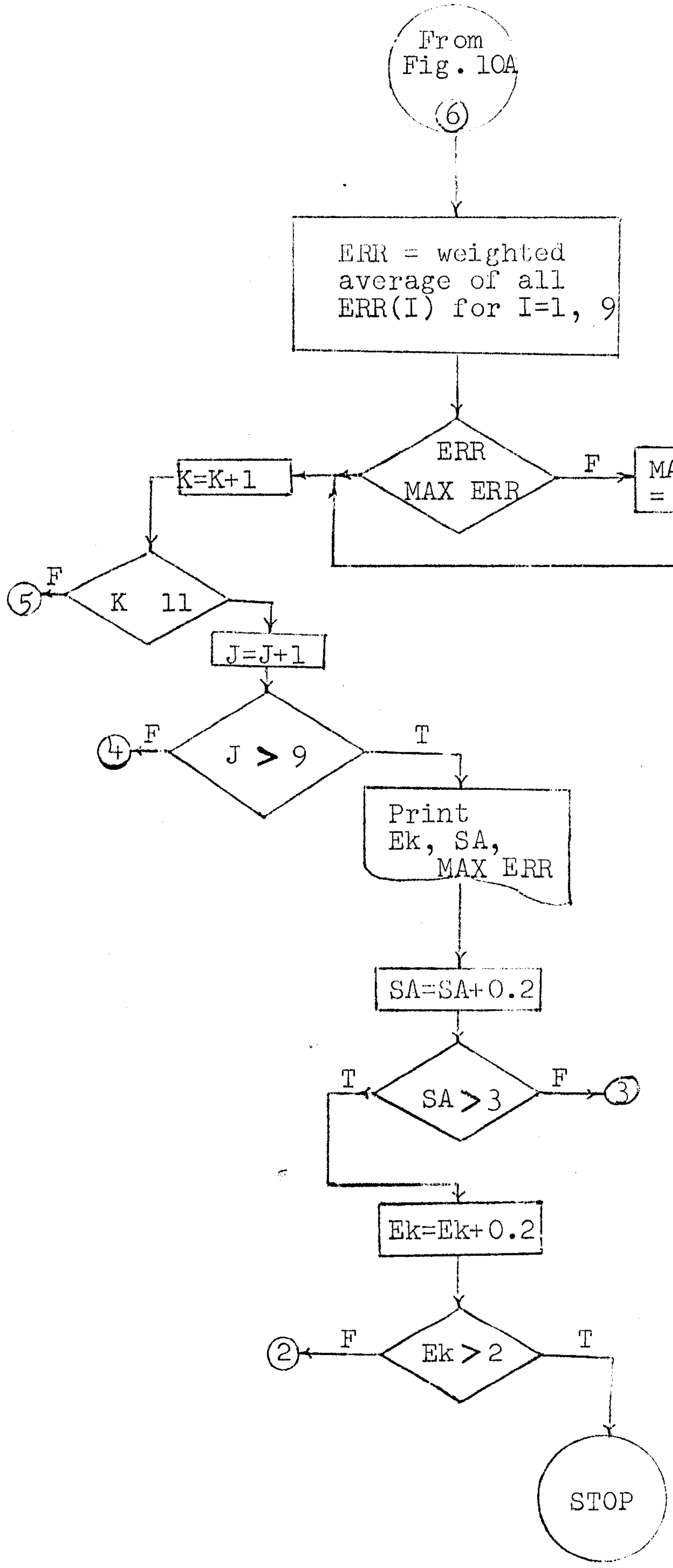


FIG. 10B.--FLOW CHART FOR THE EFFECTIVE ERROR OVER THE AREA OF INTEREST IN THE CASE OF EXPANSION OF PATTERNS

4.3 Discussion of the Results

The effective error has been plotted against SA for various values of Ek.

For SA = 1 the results agree with the calculation of maximum error carried out earlier.

It can be seen that the effective error decreases with an increase in SA and the minimum value of this error occurs near SA = 2.

An interesting result of this investigation is that even with Ek = 1 an error is introduced for values of SA between 1 and 3. This is due to the fact that in this case, even for Ek = 1, we are dealing with partial coverages over sensors which give rise to errors, as in Appendix III.

In the neighbourhood of SA = 2 the error, in addition to being minimum, changes slowly for different values of Ek. This minimum error is considerably less than the value of error for the area of interest equal to one sensor area. As SA increases beyond 2 the error again starts to increase, but the value of this error for SA = 3 is less than the error for SA = 1. It appears that for values of SA greater than 3, the error will again pass through a minimum value which will be less than the value of error in the neighbourhood of SA = 2. If this error pattern is repeated it seems possible to determine the exact number of sampling sensors required in a unit area for a given maximum error even for arbitrary pattern configurations. The calculations for SA > 3 were not made because of the length of time required on the computer. The calculations for SA ≤ 3 took forty-five minutes on the IBM 7040.

These results have established the validity of using more sensors per unit area for improved error performance by the algorithm.

CHAPTER V

DISCUSSION

From the experimental results it is apparent that for the class of patterns defined by the block capital letters of the Roman alphabet, the calculated values of maximum possible error on a single sensor represents a highly pessimistic estimate of error. This is due to the patterns in the Roman alphabet having an important property of connectivity, i.e., the areas covered by a pattern are connected to one another. For calculating the maximum error, the assumption of an arbitrary pattern component configuration results in patterns that are, in general, devoid of this property.

The fact that the error decreases, when the area of interest becomes greater than the sensor area, suggests that the actual error for the modified scheme will result in a lower error for the class of patterns in the Roman alphabet.

However, the utility of a particular expansion/contraction scheme is dependent on the decision scheme of the actual pattern recognition system. Thus, the limits of the tolerable error will be decided by the mode of making measurements on the normalized pattern and the decisions which are to be made on the basis of these measurements.

The results obtained with the algorithm seem to be satisfactory when correlation techniques are to be employed for the recognition process.

The algorithm can be modified to take into account the connectivity of the patterns in the Roman alphabet. When considering the mapping of a particular area on a given sensor, we have treated this mapping in isolation irrespective of the pattern configuration on the adjacent sensors. We have considered the average of the partial coverage on a sensor only whereas we know that for a two-level pattern, the output of a sensor specifies the exact coverage. The information regarding the exact location of this coverage on the sensor is lacking. Thus, by looking at the coverages on adjacent sensors, a reasonably fair estimate can be made of the location of this partial coverage on the sensor. We can, in general, expect more accurate assignments if this connectivity of the patterns is taken into account.

The optimal density of sensors for a given maximum error can be determined for a given value of pattern component size and distribution. Further work in this direction will be very useful.

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APPENDIX I

SHAPE INVARIANCE UNDER LINEAR MAGNIFICATION

In Fig. 11, A (x_1, y_1) and B (x_2, y_2) are two points in a plane. Any point C (x,y) on the line AB is given by:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} \qquad \ldots (1)$$

On magnification by k, A maps onto A^1 (kx_1, ky_1) and B maps onto B^1 (kx_2, ky_2). The equation of a straight line through A^1 and B^1 is given by:

$$\frac{X-kx_1}{kx_2-kx_1} = \frac{Y-ky_1}{ky_2-ky_1} \qquad \ldots (2)$$

Point C maps onto C^1 (kx, ky). If C^1 is to lie on A^1 B^1 then we must have (replacing X by kx and Y by ky):

$$\frac{kx-kx_1}{kx_2-kx_1} = \frac{ky-ky_1}{ky_2-ky_1}$$

which is true by virtue of (1).

Hence, when two points (A and B) map onto two other points (A^1 and B^1) under any expansion (contraction) coefficient (k), then any point (C) on the original line (AB) maps onto the new line (A^1 B^1).

A straight line on mapping remains a straight line.

It is to be noted from the above that the two lines AB and A^1 B^1 have the same slope $\left(\frac{y_2-y_1}{x_2-x_1}\right)$ and length

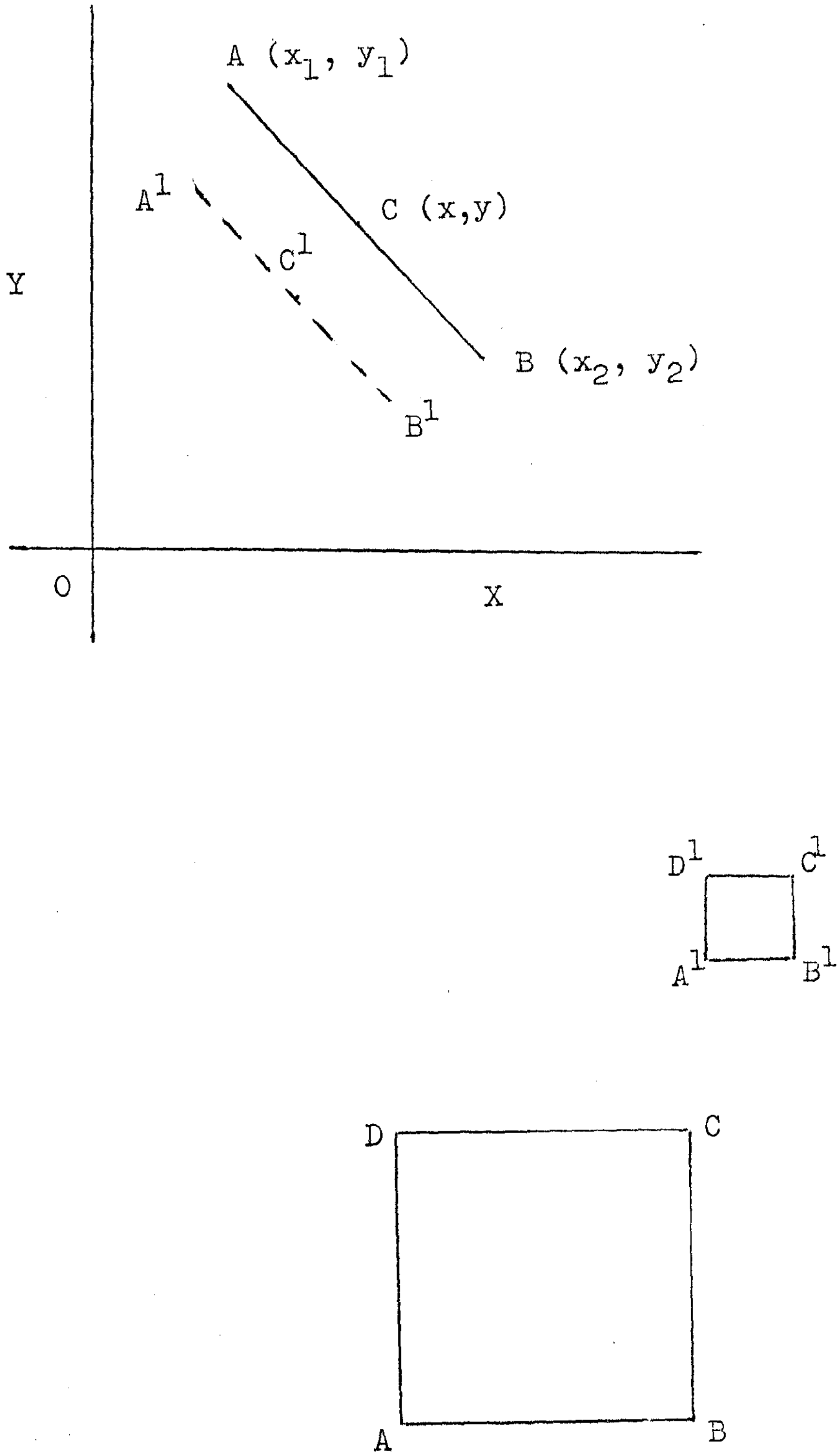


FIG. 11.--MAPPING UNDER LINEAR MAGNIFICATION.

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\text{length } A^1 B^1 = \sqrt{(kx_2-kx_1)^2 + (ky_2-ky_1)^2}$$

$$A^1 B^1/_{AB} = k.$$

Mapping of a Square Area

Let us find out the shape of a square ABCD shown in Fig. 11, on mapping under expansion (contraction), by a factor

Line AB maps into line $A^1 B^1$. Similarly, BC maps into $B^1 C^1$, CD maps into $C^1 D^1$ and DA maps into $D^1 A^1$.

$$\text{Now } \frac{A^1B^1}{AB} = \frac{B^1C^1}{BC} = \frac{C^1D^1}{CD} = \frac{A^1D^1}{AD} = k$$

But AB = BC = CD = DA

Hence, $A^1B^1 = B^1C^1 = C^1D^1 = D^1A^1$

Also, line A^1B^1 is parallel to AB (same slope)

Also, B^1C^1//BC, C^1D^1//CD and A^1D^1//AD

All the angles $D^1A^1B^1 = A^1B^1C^1 = B^1C^1D^1 = C^1D^1A^1$

each equal to one right angle

Hence, $A^1B^1C^1D^1$ is a square.

Finally, any arbitrary area can be considered as a limiting case of straight line segments around its boundary and thus linear magnification will preserve the shape of this area under the above mapping.

APPENDIX

Square area ABCD, shown in Fig. 12, on expansion by Ek maps onto sensor (X,Y).

$O(X_{/Ek}, Y_{/Ek})$ is the centre of ABCD. X_I and Y_I denote integral parts, X_F and Y_F are the fractional parts of $X_{/Ek}$ and $Y_{/Ek}$ respectively. S_1 is the sensor (X_I, Y_I), other adjacent sensors S_2, S_3 and S_4 are shown.

From S_1 draw lines parallel to the co-ordinate axis meeting the boundary between sensor S_1 and sensor S_2 in I and the boundary between S_1 and S_4 in J. From O draw lines $\|$ to the co-ordinate axis to meet S,I in M and S,J in N. Extend AD and AB to meet S,I and S,J in K and L respectively.

$$S(1) = \text{Part of AB lying on } S_1 = KI = KM + MI$$

$$\text{Now } S_1M = X_F \text{ and } S_1N = Y_F$$

$$MI = 0.5 - X_F$$

$$LN = KM = 1/2Ek$$

$$KI = 1/2Ek - X_F + 0.5$$

$$S(3) = \text{Parts of AD on } S_1 = LJ = LN + NJ$$

$$NJ = 0.5 - Y_F$$

$$LJ = 1/2Ek - Y_F + 0.5$$

$$S(2) = \text{Part of AB on sensor } S_2 = \frac{1}{Ek} - S(1) = \left(\frac{1}{2Ek} + X_F - 0.5\right)$$

$$S(4) = \text{Part of AD on sensor } S_4 = \frac{1}{Ek} - S(3) = \left(\frac{1}{2Ek} + Y_F - 0.5\right)$$

$$\text{Area on } S_2 = S(2)*S(3) = \left(\tfrac{1}{2Ek}+X_F-0.5\right)\left(\tfrac{1}{2Ek}-Y_F+0.5\right)$$

$$\text{Area on } S_1 = S(1)*S(3) = \left(\tfrac{1}{2Ek}-X_F+0.5\right)\left(\tfrac{1}{2Ek}-Y_F+0.5\right)$$

$$\text{Area on } S_3 = S(2)*S(4) = \left(\tfrac{1}{2Ek}+X_F-0.5\right)\left(\tfrac{1}{2Ek}+Y_F-0.5\right)$$

$$\text{Area on } S_4 = S(1)*S(4) = \left(\tfrac{1}{2Ek}-X_F+0.5\right)\left(\tfrac{1}{2Ek}+Y_F-0.5\right)$$

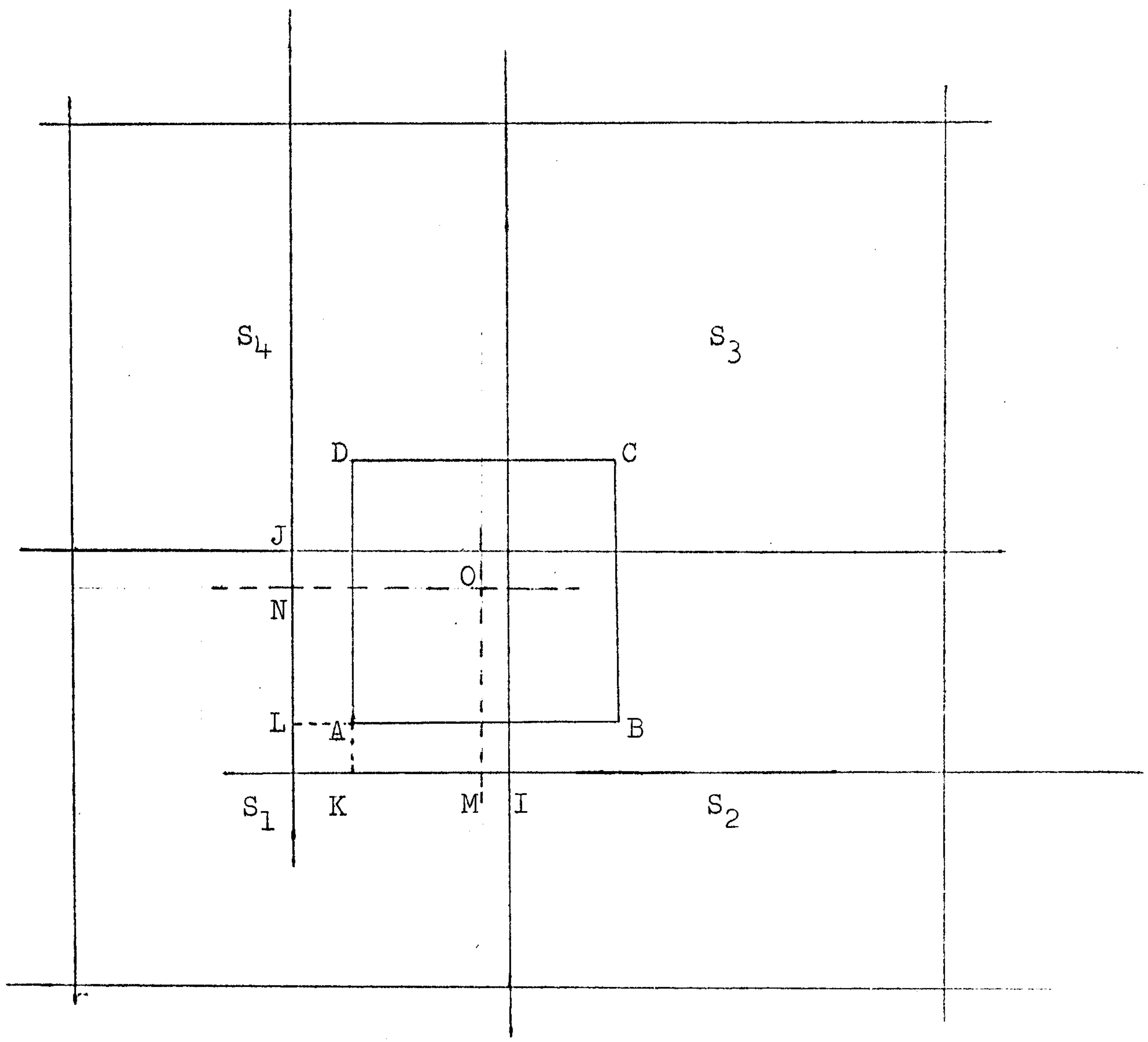
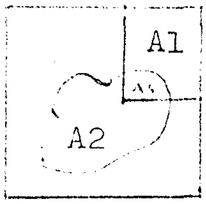


FIG. 12.--CALCULATION OF THE AREAS ON DIFFERENT SENSORS.

APPENDIX III

PATTERN CONFIGURATION FOR MAXIMUM DEVIATION OF ASSIGNED INTENSITY

On a given sensor S only area A1 takes part in the mapping process. Considering only two level patterns, let any arbitrary area A2 have the value of intensity equal to 1 so that the intensity over the remaining area (1 - A2) is zero. $A3 = A1 \cap A2$.



Sensor S

$$\text{Average intensity on the sensor S} = A2$$

$$\text{Actual intensity over the mapping area A1} = A3$$

$$\text{Intensity contribution of A1 on mapping} = (A1 * A2)\, Ek^2$$

Actual value of contribution that should be assigned =

$$(A1 * A3)\, Ek^2$$

$$\text{Deviation in assignment} = Ek^2[A1 * (A2 - A3)]$$

Ek and A1 are fixed for a given mapping. Thus, maximum deviation occurs when (A2 - A3) is maximum.

$A2, A3 \geq 0$. Thus (A2 - A3) is maximum when A2 is maximum and A3 is minimum.

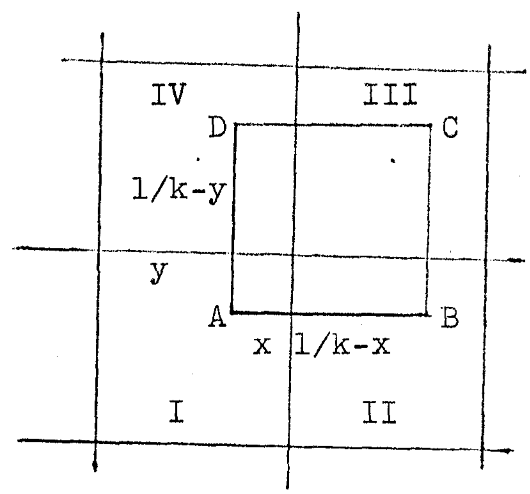
Minimum value of A3 is zero. For this value of A3 maximum A2 is (1 - A1).

$$\text{Hence, maximum deviation} = Ek^2 * [A1 * (1 - A1)]$$

Thus, we prove that for maximum deviation in intensity assignment the area A1 should have value of intensity that is opposite to the value of intensity on the remaining area.

APPENDIX IV

For maximum error the area is symmetrically located with respect to the four sensors.



The square ABCD on enlargement by a factor k maps onto a single sensor. According to Appendix III, let the coverage inside the square be denoted by 1, then the coverage on the remaining areas on the four sensors is zero.

Output of Sensor I = xy

Output of Sensor II = y(1/k-x)

Output of Sensor III = (1/k-y)(1/k-x)

Output of Sensor IV = x(1/k-y)

The assigned value of coverage to the sensor over which the area ABCD maps will be proportional to the output of the sensors Si, i=I, N and the coverage on these sensors.

$$\text{Contribution due to Sensor I} = x^2y^2$$

$$\text{Contribution due to Sensor II} = y^2(1/k-x)^2$$

$$\text{Contribution due to Sensor III} = (1/k-y)^2\ (1/k-x)^2$$

$$\text{Contribution due to Sensor IV} = x^2\ (1/k-y)^2$$

$$\text{Assigned value} = k^2[x^2y^2+y^2(1/k-x)^2 + x^2(1/k-6)^2 + (1/k-x)^2 (1/k-y)^2]$$

The actual value = 1.

$$\text{Deviation in assignment} = D = 1-k^2[x^2y^2+y^2(1/k-x)^2 + (1/k-x)^2\ (1/k-y)^2]$$

For D to be maximum, $\frac{\partial D}{\partial x} = \frac{\partial D}{\partial y} = 0$

$$k^2[2xy^2+2y^2(1/k-x)(-1)+2x(1/k-y)^2 + 2(1/k-x)(-1)(1/k-y)^2] = 0$$

$$k^2[2x^2y+2y(1/k-x)^2 + 2x^2(1/k-y)(-1)+2(1/k-y)(1/k-x)^2] = 0$$

Or, $2xy^2-2y^2(1/k-x) + 2x(1/k-y)^2 - 2(1/k-x)(1/k-y)^2 = 0$ . . . (1)

$2x^2y-2x^2(1/k-y) + 2y(1/k-x)^2 - 2(1/k-y)(1/k-x)^2 = 0$ . . . (2)

Solving (1) and (2) simultaneously for x and y gives

$$x = y = 1/2k$$

APPENDIX V

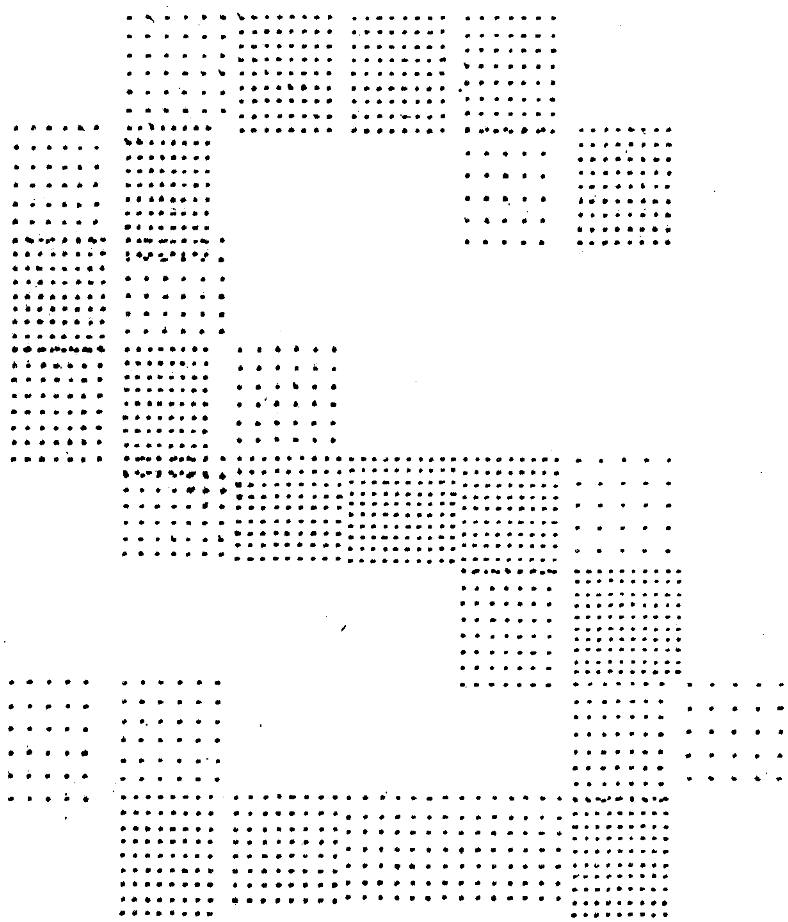
FORTRAN PROGRAMME LISTING

FOR MARKING POINTS

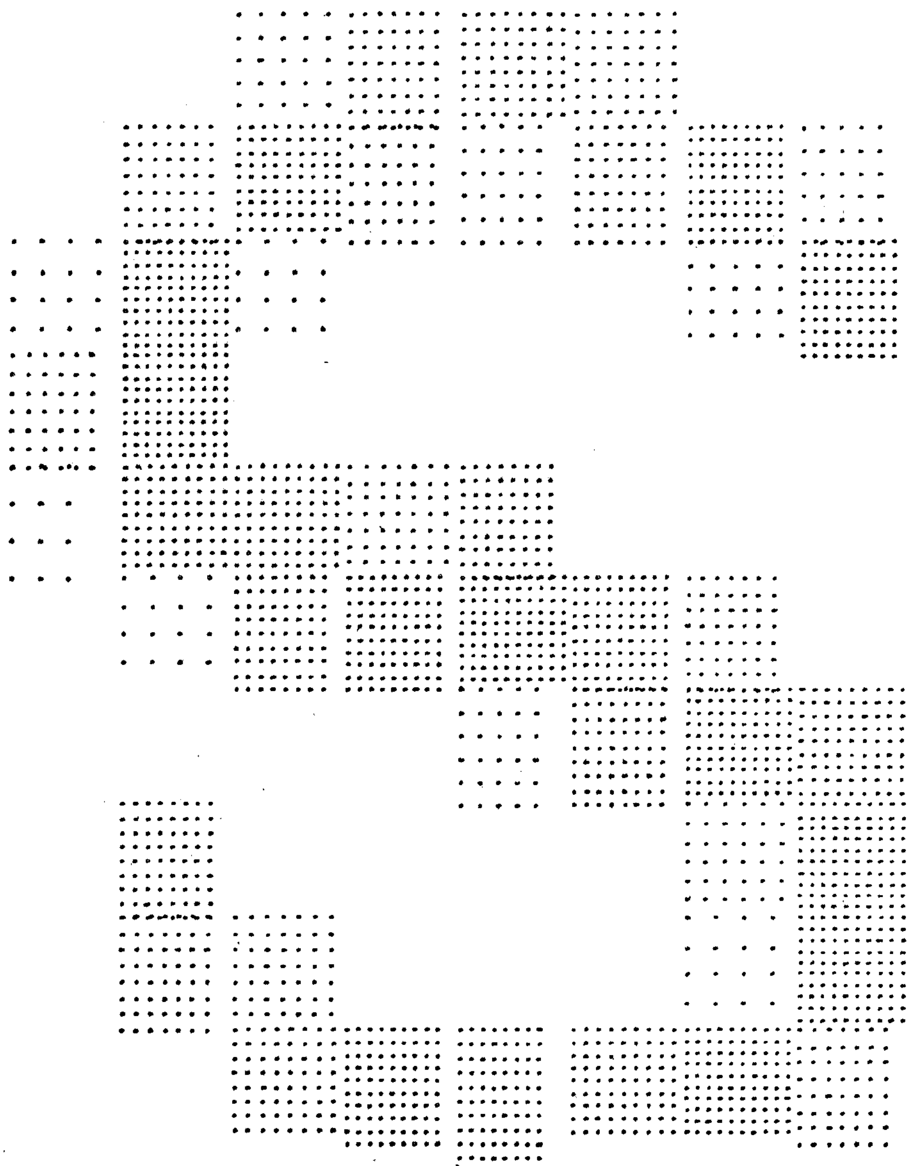
AND THE RESULTS ON THE DIGITAL PLOTTER

FORTRAN PROGRAMME LISTING FOR MARKING POINTS

```
      DIMENSION D(10,10)
      CALL PLOT(101,0.,12.,12.,12.,0.,10.,10.,10.)
      DO 1 I=1,10
    1 READ 2,(D(I,J),J=1,8)
    2 FORMAT(8F10.4)
      DO 3 I=1,10
      DO 3 J=1,8
      P=D(I,J)*100.
      IF(P)3,3,6
    6 K=SQRTF(P)
      IF(K)3,3,5
    5 BK=K
      L=P/BK
      BL=L
      DELX=1./BK
      DELY=1./BL
      X=I-1
      DO 4 II=1,K
      X=X+DELX
      Y=J-1
      DO 4 JJ=1,L
      Y=Y+DELY
    4 CALL PLOT(9,X,Y)
    3 CONTINUE
      END
```

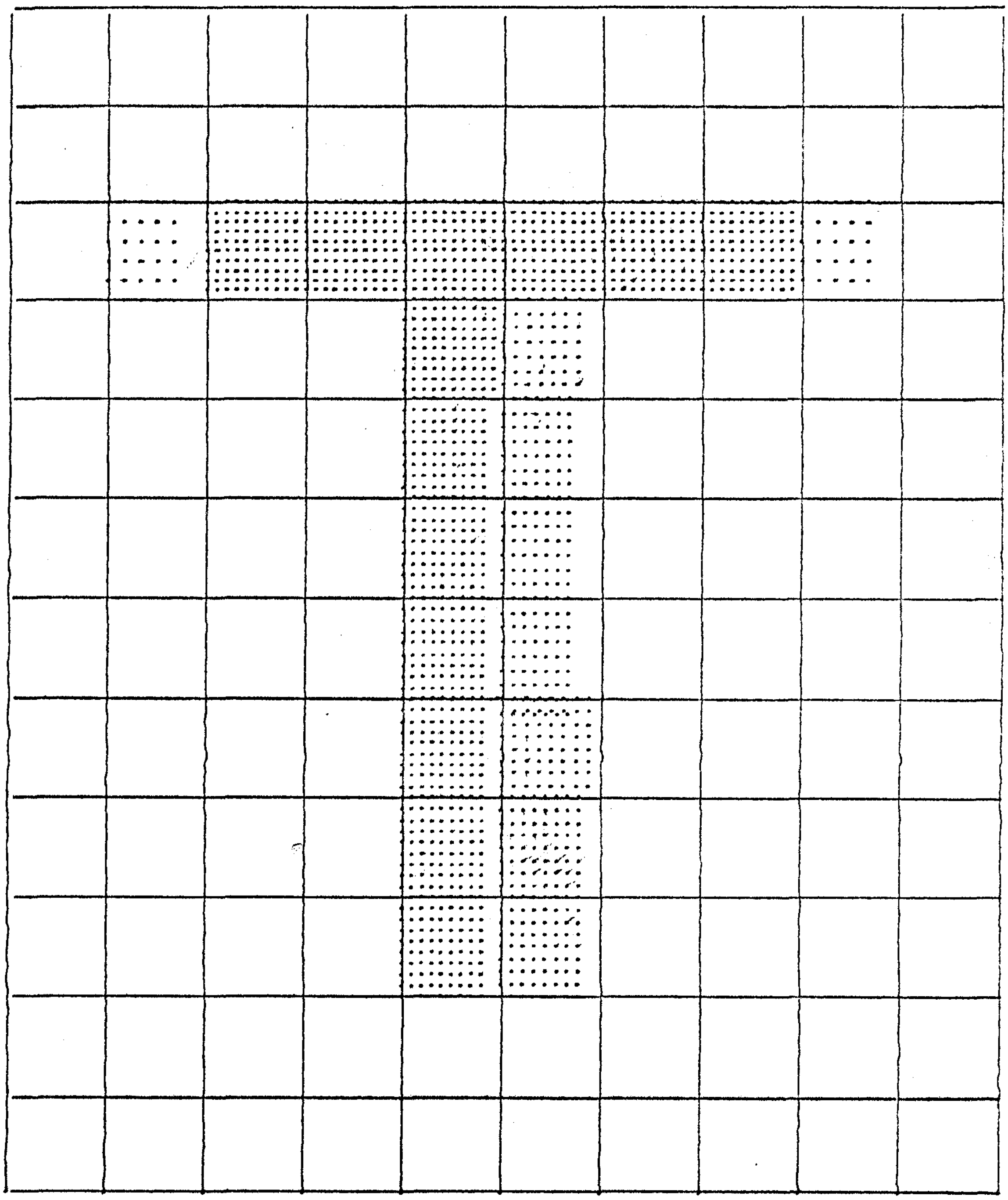


LETTER S FROM OBSERVED VALUES

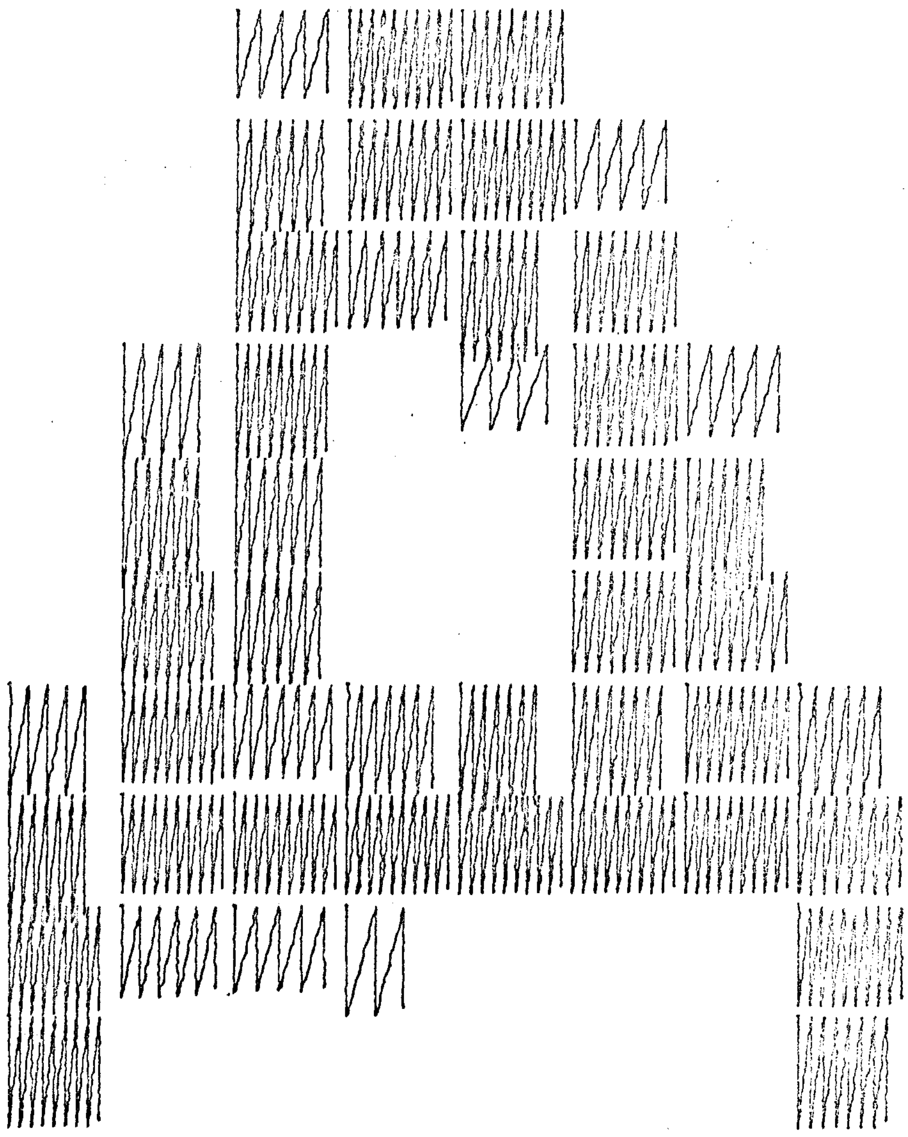


LETTER S AFTER EXPANSION

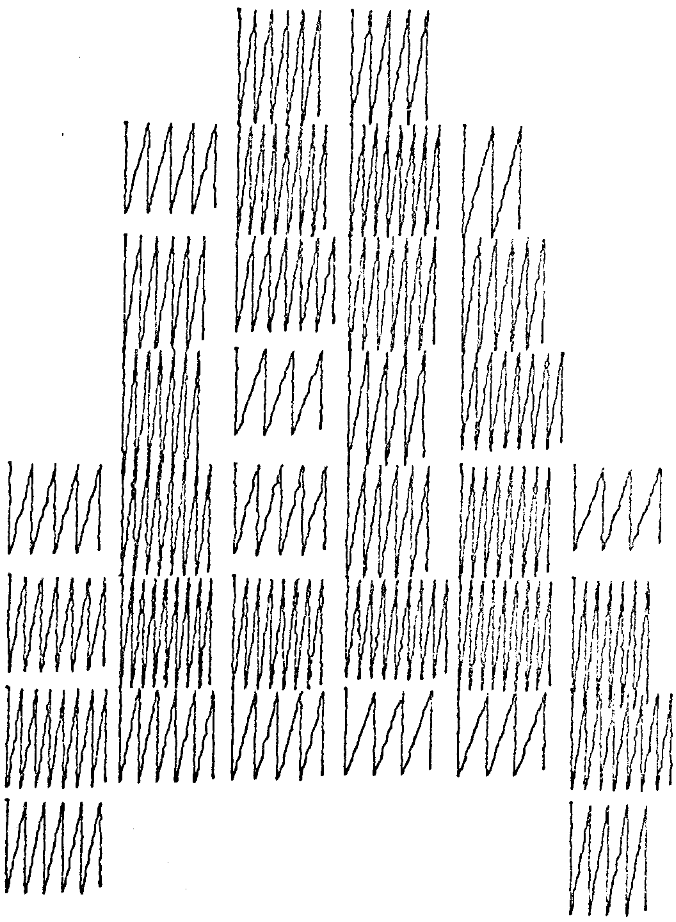
Ek = 1.2



LETTER T

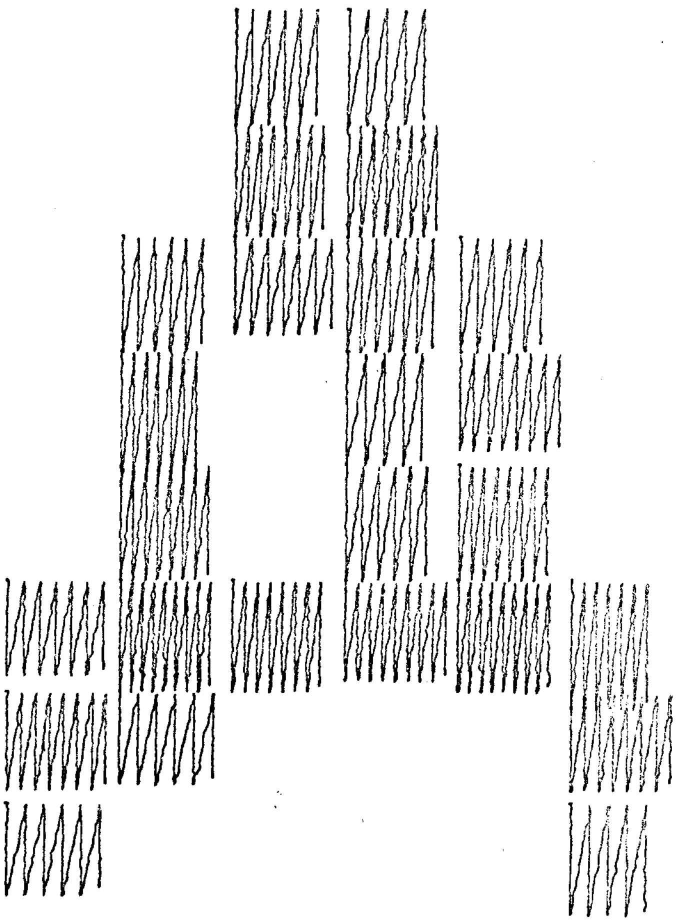


LETTER A FROM OBSERVED VALUES



LETTER A AFTER CONTRACTION

Ek = 0.83

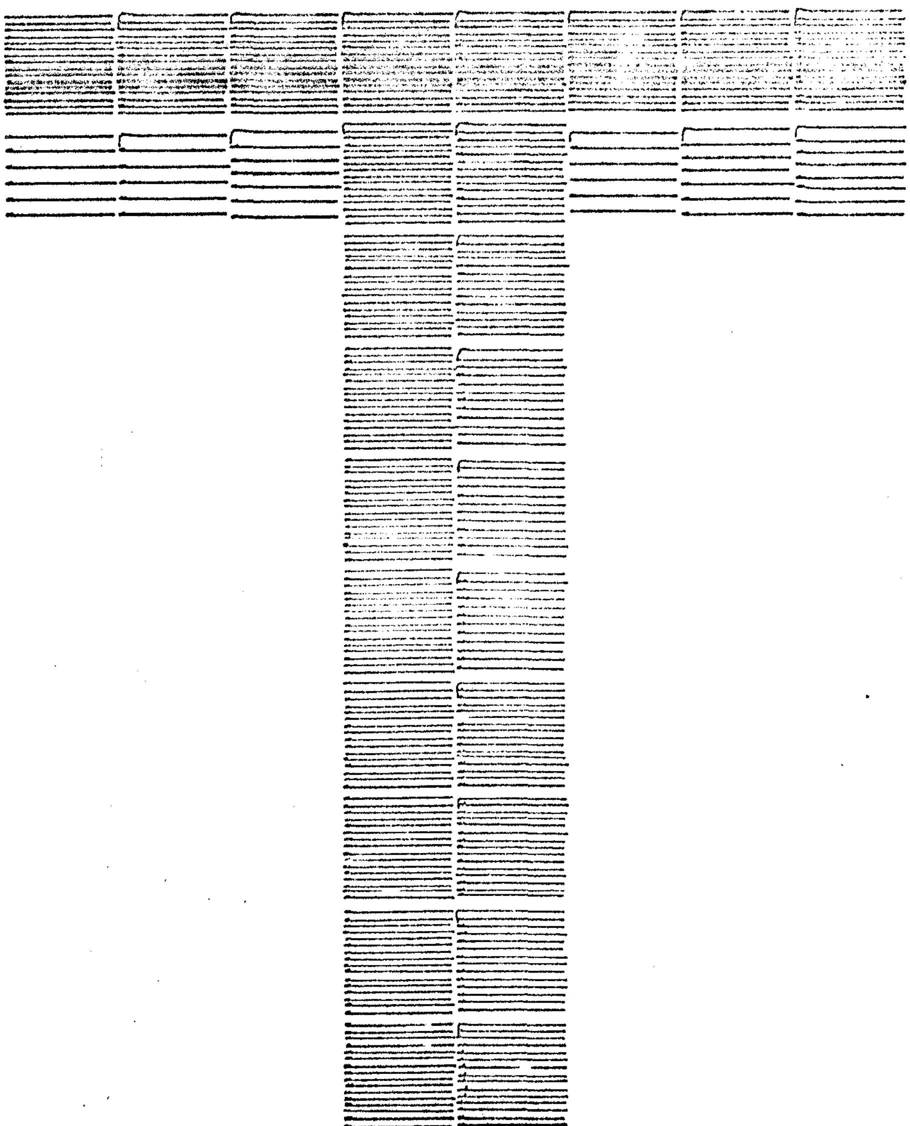


LETTER A AFTER CONTRACTION

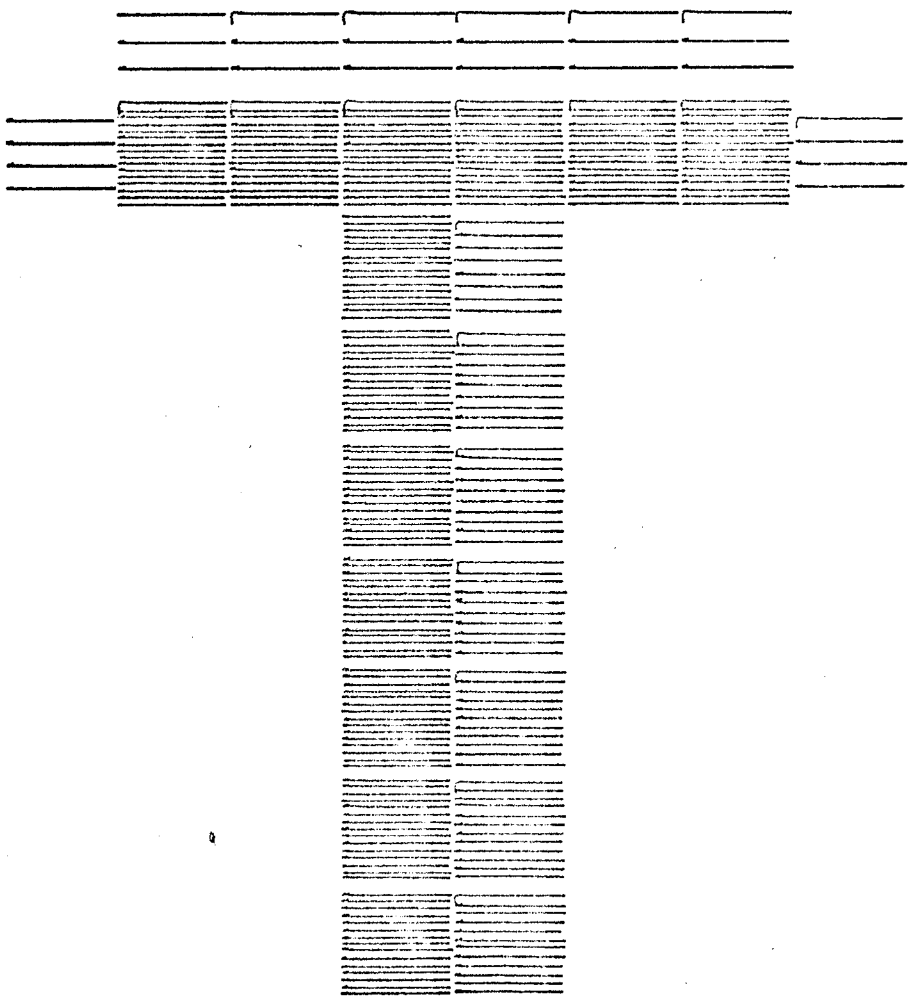
Ek = 0.83

(Threshold at 25 points)

PARALLEL LINES REPRESENTATION OF LETTER S

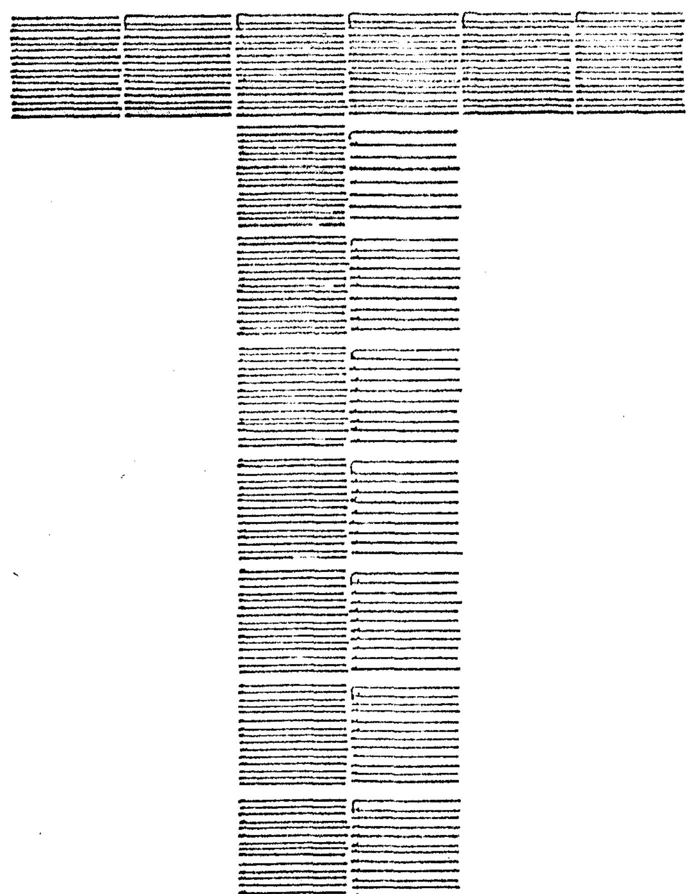


LETTER T FROM OBSERVED VALUES



LETTER T AFTER CONTRACTION

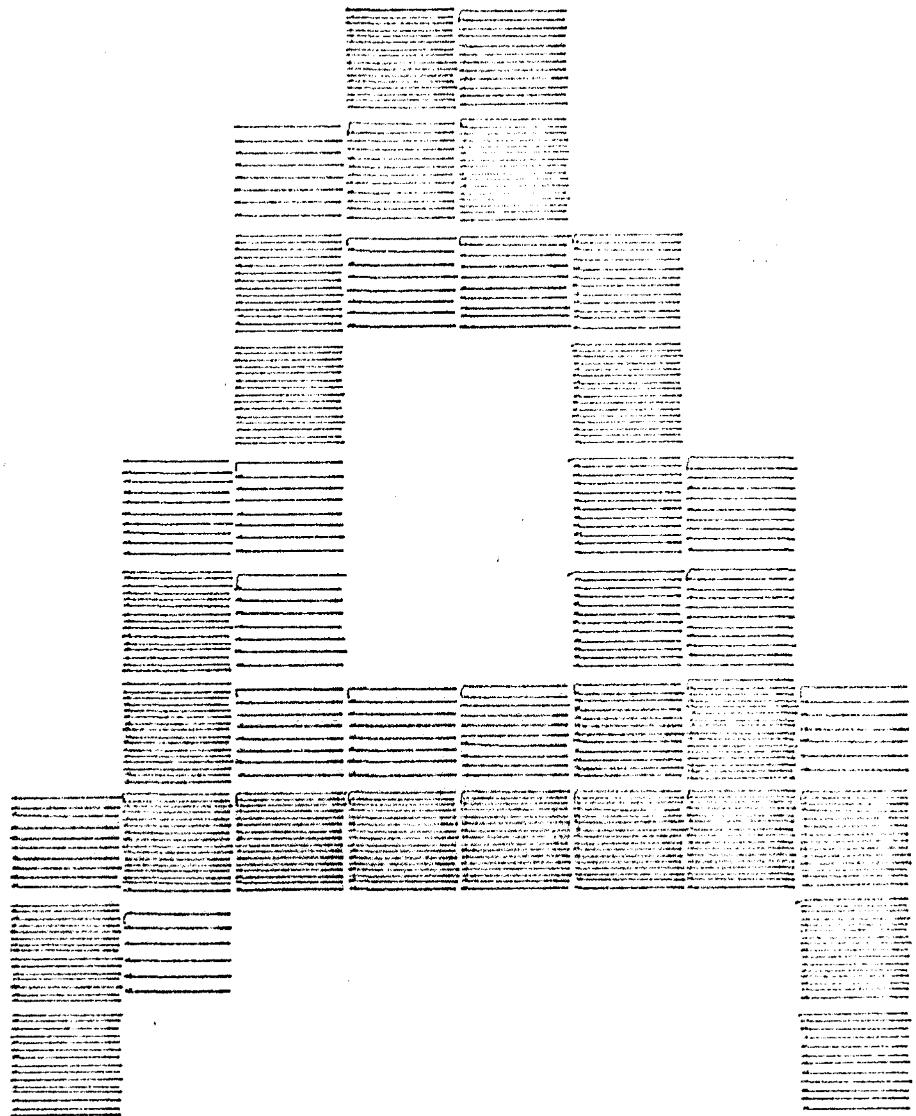
Ek = 0.83



LETTER T AFTER CONTRACTION

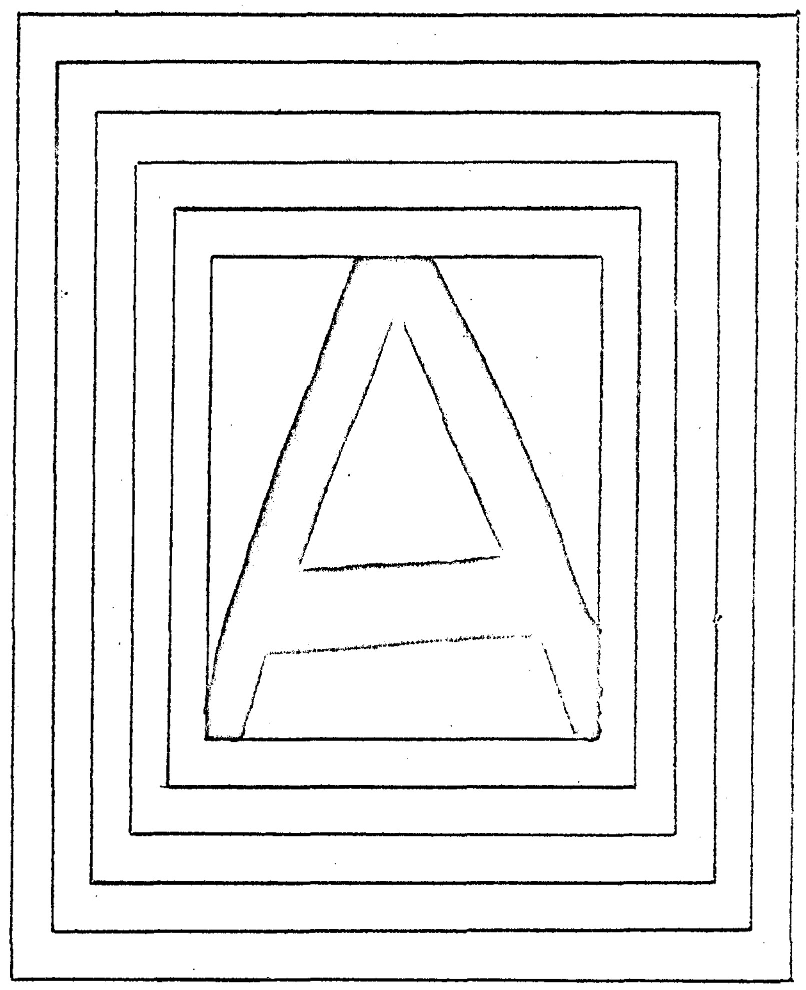
Ek = 0.83

(Threshold at 4 lines)

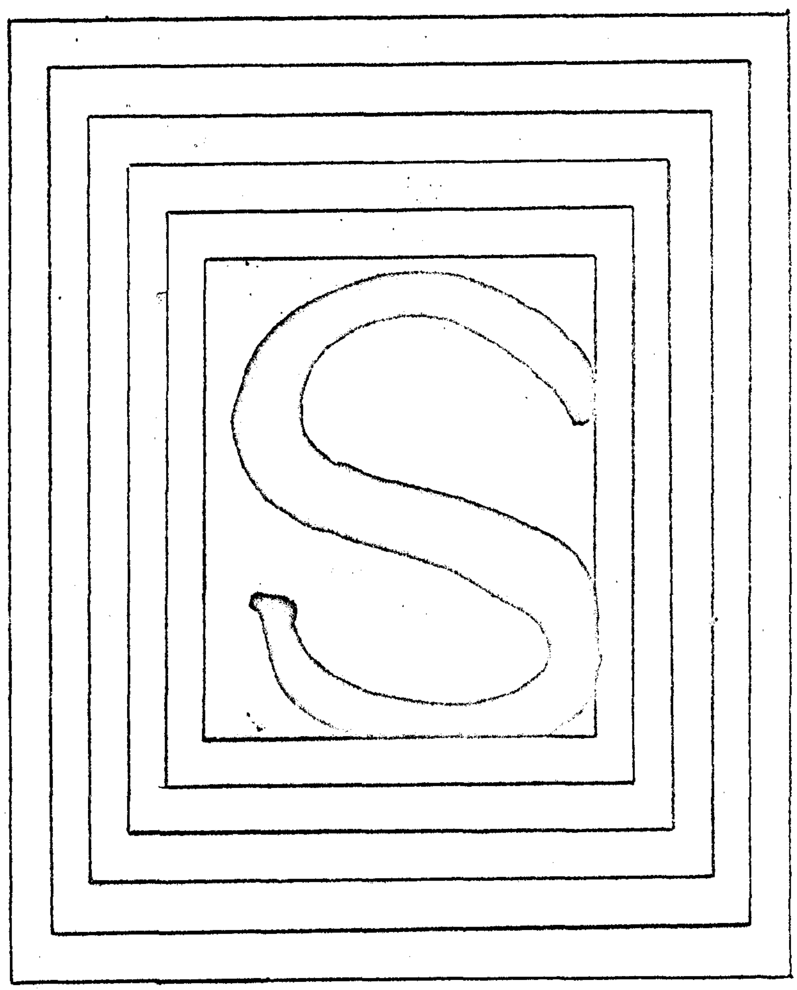


PARALLEL LINES REPRESENTATION OF LETTER A

APPENDIX VI



A SAMPLE USED FOR EXPERIMENTAL VERIFICATION



A SAMPLE USED FOR EXPERIMENTAL VERIFICATION

EXPERIMENTAL RESULTS

All values are on the basis of sixteen levels of quantization.

				2	2				
			4	13	12	1			
			8	11	12	6			
		2	11	5	7	11	1		
		5	$\underline{10}$	2	3	12	5		
	1	8	11	3	4	$\underline{12}$	7		
	2	11	12	11	12	14	13	3	
	4	12	10	9	8	8	12	5	
	5	9	2	1	0		7	4	
	1	1					1	1	

Calculated values letter A
Ek = 0.83

				3	2				
			2	16	14				
			8	9	14	4			
			13	3	7	13			
		3	$\underline{14}$		1	15	3		
		8	8			$\underline{8}$	10		
		14	13	13	14	16	15	1	
	2	15	12	10	9	8	13	6	
	5	7					5	6	
	1	1							

Observed values for letter A
Ek = 0.83

Values differing by four or more steps are underlined

EXPERIMENTAL RESULTS

All values are on the basis of sixteen levels of quantization.

		2	5	5	5	5	2		
		3	7	9	8	7	3		
		1	5	10	8	4	1		
			2	9	6	1			
			2	9	7	1			
			1	6	5	1			
			2	2					

Calculated values for T
Ek = 1.2

		2	8	8	8	8	2		
		1	4	14	10	4	1		
			0	10	7	0			
				10	7				
				10	8				
				8	6				

Observed values for T
Ek = 1.2

Values differing by four or more steps are underlined.

EXPERIMENTAL RESULTS

All values are on the basis of sixteen levels of quantization.

	1	3	3	3	3	3	3	1	
	4	13	13	16	15	13	13	4	
	1	3	4	14	11	3	4	1	
				13	11				
				13	11				
			1	13	11				
			1	13	12				
			1	13	13				
			1	13	13				
				2	2				

Calculated values for T
Ek = 0.83

	1	2	2	3	3	3	3	1	
	5	16	16	16	16	16	16	5	
			0	16	9				
				15	10				
				15	10				
				15	10				
				15	11				
				15	12				
				15	12				
				2	1				

Observed values for T
Ek = 0.83

Values differing by four or more steps are underlined.

```
C THIS PROGRAMME FINDS THE MAX DEVIATION IN ASSIGNMENT ON
C AREA A GREATER THAN ONE SENSOR AREA AND UP TO NINE SENSORS
  FOR EXPANSION OF PATTERNS
      DIMENSION A(9,11,13),W(10,12),S(4),ERO(9),DEL(9)
      IX=10
      IY=12
      XXI=IX
      YYI=IY
      AFIX=(XXI+1.)/2.
      YFIX=(YYI+1.)/2.
      DO 500 LL=1,6
      DO 500 MM=1,11
      DK=LL-1
      DS=MM-1
      EK=1.+DK/5.
      SA=1.+DS/5.
      ERRM=0.
      D=(SA-1.)/2.
      SS=(SA+1.)/(4.*EK)
      DO 600 I=2,9
      DO 600 J=2,11
      DO 100 IA=1,9
      DO 100 IB=1,10
      DO 100 IC=1,12
  100 A(IA,IB,IC)=0.
      DO 200 IA=1,10
```

```
      DO 200 IB=1,12
  200 W(IA,IB)=0.
      X=I
      Y=J
      XC=(X-XFIX)/EK+XFIX
      YC=(Y-YFIX)/EK+YFIX
      CALL AREA(XC,YC,1.,1.,EK,1,A)
      XC2=XC-SS
      YC3=YC-SS
      YC4=YC+SS
      XC5=XC+SS
      CALL AREA(XC2,YC,D,1.,EK,2,A)
      CALL AREA(XC,YC3,1.,D,EK,3,A)
      CALL AREA(XC,YC4,1.,D,EK,4,A)
      CALL AREA(XC5,YC,D,1.,EK,5,A)
      CALL AREA(XC2,YC3,D,D,EK,6,A)
      CALL AREA(XC5,YC3,D,D,EK,7,A)
      CALL AREA(XC5,YC4,D,D,EK,8,A)
      CALL AREA(XC2,YC4,D,D,EK,9,A)
      DO 701 KK=1,9
  701 ERO(KK)=0.
      DO 111 IJ=1,9
      DO 111 JK=1,10
      DO 111 KL=1,12
      IF(A(IJ,JK,KL))111,111,112
  112 IF(W(JK,KL))113,113,114
```

```
113 W(JK,KL)=A(IJ,JK,KL)
    DEL(IJ)=(1.-W(JK,KL))*A(IJ,JK,KL)*EK*EK
    GO TO 116
114 DEL(IJ)=W(JK,KL)*A(IJ,JK,KL)*EK*EK
116 ERO(IJ)=ERO(IJ)+DEL(IJ)
111 CONTINUE
    ERR=ERO(1)+ D*(ERO(2)+ERO(3)+ERO(4)+ERO(5))+D * D*
      (ERO(6)+ERO(7)
  1 +ERO(8)+ERO(9))
    ERR=ERR/(SA*SA)
    IF(ERRM-ERR)121,600,600
121 ERRM=ERR
600  CONTINUE
    PUNCH 99,EK,SA,ERRM
 99 FORMAT(25X,2F10.2,F10.3)
500 CONTINUE
    CALL EXIT
    END
    SUBROUTINE AREA(XC,YC,SIDEX,SIDEY,EK,N,A)
    DIMENSION A(9,11,13),S(4)
    IXC=XC
    IYC=YC
    XCI=IXC
    YCI=IYC
    XCF=XC-XCI
    YCF=YC-YCI
```

```
      S(1)=SIDEX/(2.*EK)-XCF+0.5
      S(2)=SIDEX/EK-S(1)
      S(3)=SIDEY/(2.*EK)-YCF+0.5
      S(4)=SIDEY/EK-S(3)
      DO 3 JJ=1,4
      IF(S(JJ))4,4,3
    4 S(JJ)=0.
    3 CONTINUE
      DO 5 JJ=1,2
      IF(S(JJ)-SIDEX/EK)5,6,6
    6 S(JJ)=SIDEX/EK
    5 CONTINUE
      DO 7 JJ=3,4
      IF(S(JJ)-SIDEY/EK)7,8,8
    8 S(JJ)=SIDEY/EK
    7 CONTINUE
      IXC1=IXC+1
      IYC1=IYC+1
      A(N,IXC,IYC)=S(1)*S(3)
      A(N,IXC1,IYC)=S(2)*S(3)
      A(N,IXC1,IYC1)=S(2)*S(4)
      A(N,IXC,IYC1)=S(1)*S(4)
      RETURN
      END
```

EK	SA	MERR
1.00	1.00	0.000
1.00	1.20	0.025
1.00	1.40	0.068
1.00	1.60	0.110
1.00	1.80	0.145
1.00	2.00	0.172
1.00	2.20	0.188
1.00	2.40	0.187
1.00	2.60	0.163
1.00	2.80	0.105
1.00	3.00	0.000
1.20	1.00	0.812
1.20	1.20	0.569
1.20	1.40	0.430
1.20	1.60	0.345
1.20	1.80	0.311
1.20	2.00	0.309
1.20	2.20	0.329
1.20	2.40	0.364
1.20	2.60	0.394
1.20	2.80	0.443
1.20	3.00	0.511
1.40	1.00	0.862

EK	SA	MERR
1.40	1.20	0.602
1.40	1.40	0.451
1.40	1.60	0.357
1.40	1.80	0.317
1.40	2.00	0.332
1.40	2.20	0.369
1.40	2.40	0.414
1.40	2.60	0.465
1.40	2.80	0.518
1.60	1.00	0.894
1.60	1.20	0.624
1.60	1.40	0.465
1.60	1.60	0.367
1.60	1.80	0.325
1.60	2.00	0.328
1.60	2.20	0.356
1.60	2.40	0.401
1.60	2.60	0.455
1.60	2.80	0.518
1.60	3.00	0.586
1.80	1.00	0.917
1.80	1.20	0.639
1.80	1.40	0.474

EK	SA	MERR
1.80	1.60	0.379
1.80	1.80	0.342
1.80	2.00	0.341
1.80	2.40	0.372
1.80	2.60	0.416
1.80	2.80	0.478
1.80	3.00	0.550
2.00	1.00	0.750
2.00	1.20	0.538
2.00	1.40	0.433
2.00	1.60	0.380
2.00	1.80	0.356
2.00	2.00	0.351
2.00	2.20	0.358
2.00	2.40	0.375
2.00	2.60	0.400
2.00	2.80	0.433
2.00	3.00	0.472

Ek	Max Err.	Ek	Max Err.
1.000	0	1.00	0
.950	8	1.10	12
.900	11	1.20	13
.850	12	1.30	13
.800	12	1.40	14
.750	12	1.50	14
.700	12	1.60	15
.650	12	1.70	15
.600	13	1.80	15
.550	13	1.90	14
.500	12	2.00	12
For Contraction		For Expansion	

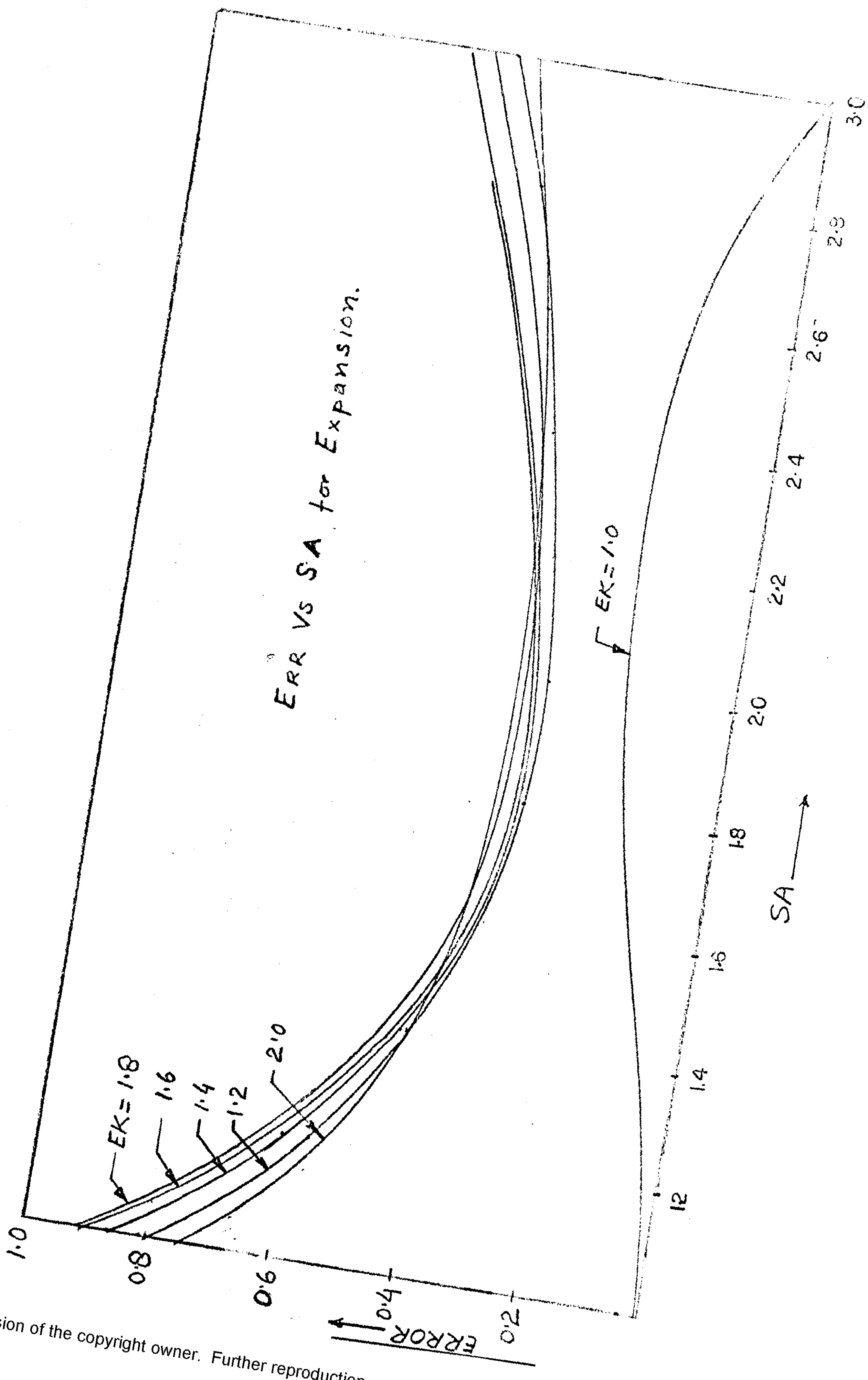
MAXIMUM POSSIBLE ERROR FOR ARBITRARY PATTERNS

SA = 1

```
C     THIS PROGRAMME FINDS THE DISTRIBUTION OF LIGHT FLUX
C     INTENSITIES ON LINEAR CONTRACTION AND EXPANSION OF
      PATTERNS
      DIMENSION DIN(10,12), OIN(10,12), S(4), K(10,12)
      EK=1.2
      IX=10
      IY=12
      XXI=IX
      YYI=IY
      XFIX=(XXI+1.)/2.
      YFIX=(YYI+1.)/2.
   38 FORMAT (10F8.2)
      PUNCH 79
   79 FORMAT (29H THE ORIGINAL INTENSITIES ARE)
      DO 9 J=1,IY
    9 READ 38,(OIN(I,J),I=1,IX)
      DO 98 J=1,IY
   98 PUNCH 38,(OIN(I,J),I=1,IX)
   18 PUNCH 53,EK
   53 FORMAT (30H THE COEFFICIENT OF EXPANSION =,F6.2)
      DO 454 I=1,10
      DO 454 J=1,12
  454 DIN(I,J)=0.
      C=0.
      IF(EK-1.)22,22,21
   21 EK=1./EK
      C=1.
```

```
22 DO 60 I=1,IX
   DO 60 J=1,IY
   X=I
   Y=J
   YC=(Y-YFIX)*EK+YFIX
   XC=(X-XFIX)*EK+XFIX
   IXC=XC
   IYC=YC
   XCI=IXC
   YCI=IYC
   XCF=XC-XCI
   YCF=YC-YCI
   S(1)=1./ 2.*EK -XCF+0.5
   S(2)=1./ 2.*EK -YCF+0.5
   S(3)=1./ 2.*EK +XCF-0.5
   S(4)=1./ 2.*EK +YCF-0.5
   DO 3 JJ=1,4
   IF(S(JJ))4,4,7
 4 S(JJ)=0.
 7 IF(S(JJ)-EK)3,5,5
 5 S(JJ)=EK
 3 CONTINUE
   A1=S(1)*S(2)
   A2=S(2)*S(3)
   A3=S(3)*S(4)
   A4=S(4)*S(1)
```

```
      IXC1=IXC+1
      IYC1=IYC+1
      IF(IX-IXC1)61,62,62
   61  IXC1=IX
          IF(IY-IYC1)63,64,64
   63 IYC1=IY
   64 IF(C)65,66,65
   65 DIN(I,J)=OIN(IXC,IYC)*A1+OIN(IXC1,IYC)*A2+OIN(IXC1,IYC1)
      *A3+1OIN(IXC,IYC1)*A4
      DIN(I,J)=DIN(I,J)*1./(EK*EK)
      EK = 1/EK
      GO TO 60
   66 DIN(IXC,IYC)=DIN(IXC,IYC)+OIN(I,J)*A1
      DIN(IXC,IYC1)=DIN(IXC,IYC1)+OIN(I,J)*A4
      DIN(IXC1,IYC1)=DIN(IXC1,IYC1)+OIN(I,J)*A3
      DIN(IXC1,IYC)=DIN(IXC1,IYC)+OIN(I,J)*A2
   60 CONTINUE
      PUNCH 81
   81 FORMAT (36H AREA ON A SCALE OF 16 AS CALCULATED,/)
      DO 118 J=1,IY
      DO 1181=1,IX
  118 K(I,J)=DIN(I,J)*16.+0.5
      DO 10 J=1,IY
   10 PUNCH 37,(K(I,J),I=1,IX)
   37 FORMAT (1018)
      EK=EK+0.2
      IF(EK-2.)18,18,182
  182 CONTINUE
      END
```


ERR VS SA for Expansion.
EK = 1.8
1.6
1.4
1.2
2.0
EK = 1.0
1.0
0.8
0.6
0.4
0.2
ERROR
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
SA

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