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Analysis of plates stiffened by mutually perpendicular ribs of variable depths by using the method of finite differences.

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ANALYSIS OF PLATES STIFFENED BY MUTUALLY PERPENDICULAR RIBS OF VARIABLE DEPTHS BY USING THE METHOD OF FINITE DIFFERENCES

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ABSTRACT

This investigation is mainly concerned with the application of finite difference method to analyse plates stiffened by mutually perpendicular ribs of variable depths.

The stiffeners are integrally connected to one side of a thin isotropic slab of constant thickness. The material of the plate is considered to be perfectly homogeneous and linearly elastic. Analysis is based on the usual assumptions underlying the two dimensional flexure theory of a thin elastic plate.

For orthogonally stiffened plate of constant rigidity in each direction the solution is usually obtained by using Huber's differential equation for ideal orthotropic plates. For the plate in this investigation having variable depths of the stiffeners, the Huber equation cannot be used without modification. Therefore, the solution is approached by the energy method.

In this investigation the value of apparent torsional rigidity in the potential energy expression has been taken from ref. 10. The energy expression of the plate is approximated by using the method of modified central differences. The function product obtained in the terms of the deflection functions of a number of discrete points is minimized with respect to the deflection of any interior point to obtain the difference operator of that particular point. Using appropriate boundary conditions, difference operator for any point near the plate boundary has also been derived. After obtaining

all the necessary difference operators, a general matrix equation has been formed and utilized to solve the plate problem.

Prior to the solution of the main plate problem of this investigation, the validity of the proposed method of solution has been investigated by solving a plate and a beam problems of constant and variable rigidities respectively. Comparison of the solutions with those obtained from other standard methods has revealed that a considerable discrepancy arises if whole station (conventional) method of central differences is used to approximate odd order derivatives. Such a discrepancy has been avoided by using half station (modified) central differences method.

The values of deflections obtained by using the modified central differences method are in fair agreement with those obtained from tests on a steel plate model made from an isotropic slab stiffened with mutually perpendicular ribs of variable depths. Tests were performed under two different kinds of loadings: (a) A single concentrated and (b) Two equal concentrated.

From this study it has been clarified that the theory of equivalent orthotropy in conjunction with the modified central differences method can be used as a valuable tool in solving the problem of an orthogonally stiffened plate of variable rigidity.

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INTRODUCTION

An orthogonally stiffened plate is commonly composed of a thin isotropic plate stiffened on one side with mutually perpendicular ribs.

In general, bending moment is variable in plates and, since bending rigidity is a function of depth, it is possible to reduce the weight of materials used in a plate by providing appropriate depth of ribs. Because weight is a very important factor in the design of aircrafts, space vehicles, and ship-bottom structures, where orthogonally stiffened plates are mainly used, plates with variable depth of ribs are gaining more importance over the ones stiffened with ribs of constant depth.

The differential equation of an orthotropic plate, first formulated by Huber [8], is commonly used to solve the problem of stiffened plates. One of the first suitable solutions of the Huber differential equation applicable to such plates was published by Cornelius [5] in 1952, based on the concept of "equivalent orthotropy". Later, the theory of "equivalent orthtropy" gained more importance when Giencke [7] and Massonnet [10] formulated the "equivalent rigidity of torsion" on a rational basis.

Besides them, other authors have contributed to the further development of both the analytical and experimental methods of solution of orthogonally stiffened plates. But, most of them were for plates of constant rigidities. To the knowledge of the writer, very few works have been done on orthogonally stiffened plate with variable rigidity.

Based on the method suggested by Witteveen [16] for plates of abruptly varying thickness, Aquilino [2] derived finite difference equations for plates having rib sizes changeable both ways. These equations are also applicable to orthogonally arranged beam-gratings of variable sections. But he did not include the effect of stresses induced in the middle plane of the plate. Mazurkiewicz [12] used the method of Double Fourier series to solve simultaneous bending and compression (or tension) problems of rectangular plates reinforced by ribs of variable rigidities. He assumed that the plate and the ribs have common axes of bending and disregarded the effect of shear pressure. Also, his solution is practically applicable to plates reinforced in one direction by limited number of stiffeners.

It may be noted that most of the works to date either did not take middle plane stresses into consideration or is subject to cumbersome numerical works and convergence difficulties. The work presented herein stems from the need for a more accurate and rapidly converging method of solution for problems in the theory of orthogonally stiffened plates of variable rib sizes.

Based on the theorem of minimum potential energy and using the method of half station central differences, modified finite difference equations for a plate with variable rigidity have been developed for calculations of deflections. Solutions thus obtained are also verified with experimental results.

NOMENCLATURE

$AS = (\lambda_y / \lambda_x)^2$	Aspect ratio
B	Flexural rigidity of isotropic slab
B_x, B_y	Orthotropic flexural rigidities per unit width in x and y directions
B_{xy}, B_{yx}	Orthotropic torsional rigidities per unit width in x and y directions
a, b	Length and width of the plate
b_x, b_y	Spacing of ribs lying parallel to x and y axes respectively
C	Torsional rigidity constant
$D = Et/(1-\nu^2)$	Isotropic plate constant
D_x, D_y	Strain rigidities per unit width in x and y directions per
$d(y)$	Depth of ribs in y direction of the plate model
E	Modulus of Elasticity
e_x, e_y	Eccentricities of the cenbroids of the stiffened plate elements from the middle surface of the slab in x and y directions
$F(x), F(y)$	Elementary areas in x and y directions of the plate-rib repeating sections
G	Shear Modulus of Rigidity
H	Apparent tortional rigidity of the orthogonally stiffened plate
I_x	Moment of inertia with respect to x axis
K	Any interior point
ℓ	Length of beam
M_x, M_y	Bending moments per unit width of cross section in x and y directions
M_{xy}, M_{yx}	Twisting moments per unit width of cross section about x and y axes

N_x, N_y	Normal axial force per unit width of cross section along the middle plane in x and y directions
N_{xy}, N_{yx}	Shearing forces per unit length on planes perpendicular to x and y axes
q	Distributed load/area or length
Q_x, Q_y	Shear forces per unit length on planes parallel to z axis but perpendicular to y and x axes respectively
R	Region within the contour of plate boundary
t	Thickness of the isotropic slab
U	Total potential energy
u, v, w	Displacements in x,y and z directions respectively
$\epsilon_x, \epsilon_y, \epsilon_z$	Unit strain components in x,y and z directions respectively
θ	Angle of twist per unit length
λ_x, λ_y	Distances of node or mesh points in x and y directions respectively
ν	Poisson's ratio
ρ	Radius of curvature
σ_x, σ_y	Unit normal stresses on planes perpendicular to x and y axes respectively
τ_{xy}, τ_{yx}	Unit shearing stresses on planes perpendicular to z axis but parallel to y and x axes respectively
$( )_{,x}, ( )_{,y}, ( )_{,yx}$	Differentiation with respect to x,y and xy respectively

I - FORMULATION OF THE PROBLEM

A. Plate With Constant Rigidity In Each Direction

An ideal orthogonally anisotropic (or orthotropic) plate can be defined as a plate having constant thickness, but different elastic properties in two mutually perpendicular directions.

The small deflection theory of an ideal orthotropic plate is based on the usual limitations applied to isotropic plates, for example, deformation of a loaded plate is purely elastic, follows Hook's Law and Bernouli's hypothesis. The small deflection behaviour of such a plate was first formulated by Huber [8] in the form of the following partial differential equation:

$$B_x \omega_{,xxxx} + 2H \omega_{,xxyy} + B_y \omega_{,yyyy} = q \qquad (1)$$

where,

B_x = orthotropic flexural rigidity in x-direction

B_y = orthotropic flexural rigidity in y-direction

H = apparent torsional rigidity of an orthotropic plate.

The same form of governing differential equation is also applied to an orthogonally stiffened plate, since, in such a plate, the discrete ribs are assumed to be continuously distributed in two mutually perpendicular directions to yield an "equivalent orthotropic structure".

"Equivalent orthotropic structures" may be defined in the following ways [9] :

(a) The equivalence might be based on the equality of deflections at similar points within the boundaries of the actual and the hypothetical plates.

(b) It might be based on the equality of strain components or upon minimization of differences of strains at specified points of the actual and hypothetical plates.

(c) It might be based on equality of the strain energies of the actual and hypothetical plates.

Massonnet [10] has shown that the assumption of an equivalent orthotropic plate to replace an actual ribbed plate is excellent for a plate with symmetrical ribs, provided that the ratios of stiffener spacings to plate boundary dimensions are small (bx/a, by/b $\ll$ 1) to ensure approximate homogenity of stiffness [9] . But, for a plate with asymmetrical ribs, such an assumption is theoretically imperfect. More exact analysis shows that an eighth order partial differential equation is necessary to determine the exact behavior of such a plate. Since Huber's differential equation is only of the fourth order, there exists no Huber-type orthotropic plate rigourously equivalent to the real asymmetrically ribbed plate. Huber's equation, unless modified, can only give an approximate solution of the actual state of stress and strain.

Giencke [7] attempted to solve such a plate problem in equivalent orthotropy on the basis of Huber's equation (1) :

where $H = B+B_{xy} + B_{yx} + \nu e_x e_y D + (e_x+e_y)^2 \frac{(1-v)}{4}$ (2)

Massonnet [10] has shown that the apparent torsional rigidity, H, in Giencke's solution, is vastly exaggerated. He (Massonnet) proposed:

$$H = B + \frac{B_{xy}+B_{yx}}{2} + v\, e_x e_y D + (e_x+e_y)^2 \left(\frac{1-v}{4}\right) \qquad (3)$$

In this investigation the value of H as given in expression (3) has been used.

B. Orthogonally Stiffened Plate Reinforced By Ribs Of Variable Rigidity In Each Direction

If an isotropic plate is reinforced by an orthogonal set of ribs of variable rigidities in its mutually perpendicular directions, x and y, (Figs.1 & 2), the force equilibrium equations due to a uniformly distributed load, q, can be written as follows: [10]:

$$N_{x,x} + N_{yx,y} = 0 \quad , \quad N_{xy,x} + N_{y,y} = 0 \quad \text{and,}$$

$$M_{x,xx} + (-M_{xy} + M_{yx})_{,xy} + M_{y,y} + q = 0 \qquad (4,a\text{-}c)$$

where, $M_x = -B_x\,\omega_{,xx} - \nu B\,\omega_{,yy} + e_x D_x\,(u_{,x} - e_x\,\omega_{,xx})$

$M_y = -B_y\,\omega_{,yy} - \nu B\,\omega_{,xx} + e_y D_y\,(v_{,y} - e_y\,\omega_{,yy})$

$M_{xy} = [(1-\nu)B + B_{xy}]\,\omega_{,xy}$

$M_{yx} = -[(1-\nu)B + B_{yx}]\,\omega_{,xy}$

$N_x = D_x(u_{,x} - e_x\,\omega_{,xx}) + \nu D v_{,y}$

$N_y = D_y(v_{,y} - e_y\,\omega_{,yy}) + \nu D u_{,x}$

and $N_{xy} = N_{yx} = \frac{(1-\nu)D}{2}(u_{,y} + v_{,x})$

Introducing the stress-strain relations and the compatibility conditions in the above fundamental equations, one obtains the following equation:

$$B_x\,\omega_{,xxxx} + B_{x,xx}\,\omega_{,xx} + B_{y,yy}\,\omega_{,yy} + B_y\,\omega_{,yyyy} + 2(\nu B\,\omega_{,xxyy} + B_{x,x}\,\omega_{,xxx} + B_{y,y}\,\omega_{,yyy}) + \{B_{xy} + B_{yx} + 2(1-\nu)B\}\,\omega_{,xxyy} + (B_{xy,xy} + B_{yx,xy})\,\omega_{,xy} + (B_{xy,x} + B_{yx,x})\,\omega_{,xyy} + (B_{xy,y} + B_{yx,y})\,\omega_{,xxy} = q \qquad (5)$$

For simply supported plates, the solution of the above equation becomes too much involved. Therefore, instead of proceeding with such an equation, the principle of minimum potential energy has been used for derivation of finite difference operators as involved in chapter III of this investigation.

II METHOD OF SOLUTION

The boundary value problem in partial differential equations, as involved in this investigation will be solved by using the method of finite differences. In this method, partial derivative is replaced by an approximating difference operator in the continuous region in which the solution is desired for a set of discrete points. This process of replacement yields a system of simultaneous equations involving the same number of unknowns. By solving these equations, when the values for a finite set of points are available, data for intermediate points may be obtained by interpolation or any other analytical method.

The method of finite differences can be applied in the three different ways [14, 15, 17]:

(1) Central Differences

(2) Forward Differences

(3) Backward Differences

Among these three, the most commonly applied is the method of Central Differences. There are two different formulations of this method:

A. Conventional or Whole Station Method:

A whole station method is one in which derivatives of function products are expanded first and then the finite difference approximations are made.

B. Modified or Half Station Method:

In this method finite difference approximations are made before expanding derivatives of function products.

Whole station method can be conveniently employed to approximate even order differential operators. But for odd order derivatives which frequently occur in structural problems with non-uniform rigidity, whole station central difference approximation leads to a considerable error, while, excepting a few cases, half station method always yields better results.

A comparison of the root-mean square values of the errors involved in both the methods shows that half station method is usually superior to whole station method in calculating deflections and bending curvations of such structures [3] .

III - APPLICATION OF THE METHOD OF FINITE DIFFERENCES

This chapter deals with the application of finite difference method in solving beam and plate problems. In section 1, both the conventional and the modified finite differences are applied to the problem of a beam with variable rigidity and the results are varified with those obtained from a solution by using Castigliano's second theorem. In section 2, the conventional method of central differences is applied to approximate both the even order and the odd order derivatives in the equation of an orthotropic plate of constant rigidity in each direction and the discrepancy arised thereby has been detected and corrected by using half station or modified method.

In the conclusions of sections 1 and 2, the validity of application of the method of finite differences, in particular the modified method, has been discussed and the application of the method has been further extended to the problem of an orthogonally stiffened plate having ribs of variable rigidity in each direction in section 3.

A - Application To A Beam Problem With Variable Rigidity

A beam with variable rigidity is a special case of a two-dimensional plate problem with variable rigidity. If the derivative with respect to the cartesian coordinate y is put equal to zero, equation (5) reduces to the differential equation of a beam having variable rigidity in the direction x and subjected to a uniformly distributed load, q, as given below:

$$B_x \omega_{,xxxx} + B_{x,x} \omega_{,xxx} + B_{x,xx} \omega_{,xx} = q \qquad (6)$$

or

$$[B_x \omega_{,xx}]_{,xx} = q \qquad (6a)$$

Equations (6) and (6a) can be approximated by using the method of whole station and half station central differences in the following manner:

1. <u>Approximation By Using Whole Station Central Differences Method For Any Interior Point, K, (Fig.3)</u>

$$q_K = \frac{1}{\lambda_x^4} \cdot B_{x(K)} \left[\omega_{K-2} - 4\omega_{K-1} + 6\omega_K - 4\omega_{K+1} + \omega_{K+2}\right]$$
$$- \frac{2}{\lambda_x^3} \cdot B_{x(K),x} \left[-\omega_{K-2} + 2\omega_{K-1} - 2\omega_{K+1} + \omega_{K+2}\right]$$
$$- \frac{1}{\lambda_x^2} \cdot B_{x(K),xx} \left[\omega_{K-1} - 2\omega_K + \omega_{K+1}\right] \qquad (7)$$

2. <u>Approximation By Using Half Station Central Differences Method For Any Interior Point, K, (Fig.3)</u>

$$q_K = \left[B_x \omega_{,xx}\right]_{K,xx}$$
$$\simeq \frac{1}{\lambda_x^2}\left[B_{x(K-1)}(\omega_{K-1})_{,xx} - 2B_{x(K)}(\omega_K)_{,xx} + B_{x(K+1)}(\omega_{K+1})_{,xx}\right]$$
$$= B_{x(K-1)} \frac{\omega_{K-2} - 2\omega_{K-1} + \omega_K}{\lambda_x^4} - 2B_{x(K)} \frac{\omega_{K-1} - 2\omega_K + \omega_{K+1}}{\lambda_x^4}$$
$$+ B_{x(K+1)} \frac{\omega_K - 2\omega_{K+1} + \omega_{K+2}}{\lambda_x^4} \qquad (8)$$

In a similar manner finite difference equations can be derived for any point near the edge of the beam and appropriate boundary conditions can be applied to evaluate the deflections of fictitious external node points in terms of those of the points of interest. In the case of the simply supported beam,

$$w(\text{support}) = 0$$
$$M(\text{support}) = 0 \qquad (9a,b)$$

For convenience of the reader a number of finite difference equations have been given for both interior and exterior points in the Appendix A.

Numerical Example:

Deflections at different node points of a simply supported tapered beam (Fig.4), of length ℓ and depth 2b/5 on one end and 2b on the other, have been calculated by using both equations (7) and (8) for ten and five segments and the results are compared with those obtained by using Castigliano's Second Theorem in Fig. (7)&(8) respectively.

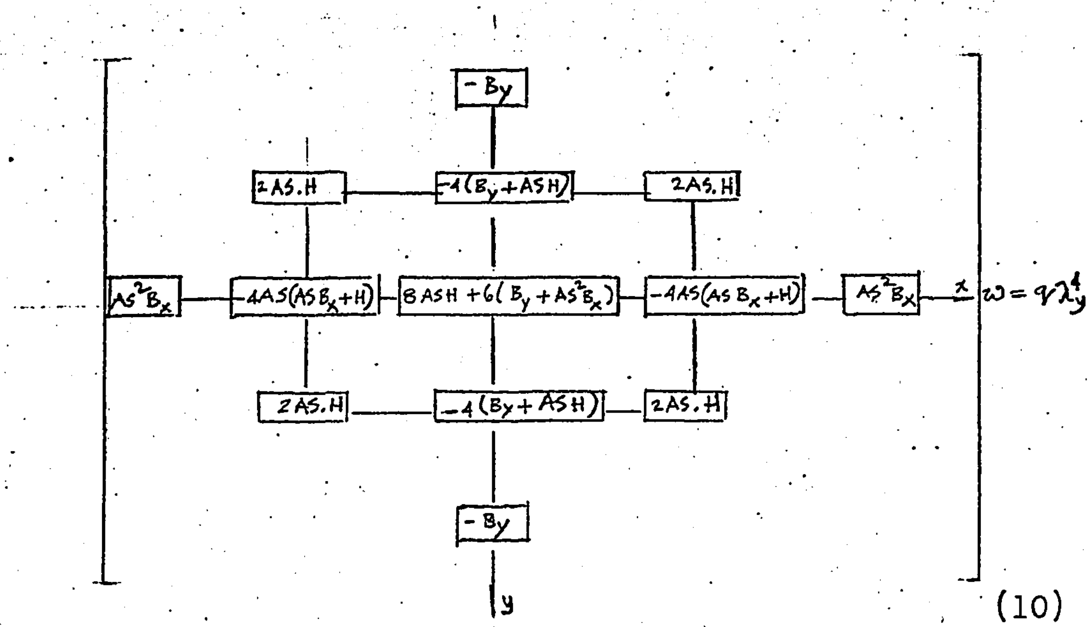
It is observed that deflections obtained for ten segments by using half station method are very close to those obtained by using Castigliano's Second Theorem. Whereas, whole station solutions are quite far away from Castigliano's solution. This also applies for a five segment solution. A comparison between Fig. (7)&(8), shows that in case of half station method, though deflections by using five segments seem to be closer to Castigliano's solution, at some node points than those by using ten segments; the pattern of deflections, in case of ten segment solution, is much closer to Castigliano's solution. A comparison between the same Figures also shows that five segment half station solutions are more accurate than ten-segment whole station solutions.

From the above results, it is clear that though in case of conventional finite difference solutions, the use of large number of segments usually yields more refined results than by using a fewer number of segments, it is also possible to obtain results of the same degree of accuracy by using the method of modified finite differences with a fewer number of segments.

So, it can be concluded that using only a few mesh points and applying the method of modified finite differences as applied in the case of a tapered beam, sufficiently precise solution of deflections of an orthogonally stiffened plate with ribs of variable rigidities may be obtained. Because such a plate is nothing but a generalized version of a one-dimensional problem of a tapered beam.

B - Application Of The Conventional (Or Whole Station) Method To An Orthotropic Plate Problem With Constant Rigidity In Each Direction and Detection Of Discrepancy

If the partial derivatives of the governing differential equation (1) of an orthotropic plate are approximated by the conventional finite differences, the resulting difference equation may be written in the following form:



(10)

where, $AS = (\lambda_y/\lambda_x)^2$

Instead of using equation (1), if the principle of minimum potential energy is utilized, one should end up with the same finite difference operator as in equation (10).

The total potential energy, U, of such a plate can be expressed as [10] :

$$U = \frac{1}{2}\iint \left(B_x w_{,xx}^2 + 2H w_{,xy}^2 + B_y w_{,yy}^2 \right) dx\,dy - \iint q w \,dx\,dy \tag{11}$$

If the quantities $w_{,xx}$, $w_{,yy}$ and $w_{,xy}$ are approximated by using the whole station central differences, equation (11) reduces to:

$$U \cong \frac{\lambda_x \lambda_y}{2} \sum \left\{ \frac{B_x}{\lambda_x^4} \left[\begin{array}{ccc} 1 & -2 & 1 \end{array} \right]^2 w + \frac{B_y}{\lambda_y^4} \left[\begin{array}{c} 1 \\ -2 \\ 1 \end{array} \right]^2 w + \frac{2H}{16 \lambda_x^2 \lambda_y^2} \left[\left[\begin{array}{cc} -1 & 1 \\ 1 & -1 \end{array} \right] \right]^2 \right\} - \lambda_x \lambda_y \sum w q \tag{11a}$$

Equation (11a) can be written in the following form:

$$U \cong T_1 + T_2 + T_3 - T_4 \tag{12}$$

Retaining only the terms containing the deflection, w_K , of an interior point, K, (Fig.5), the finite difference expressions for the terms T_1, T_2, T_3, and T_4 reduce to the following forms:

$$\begin{aligned} T_1 = \cdots + \frac{\lambda_x \lambda_y B_x}{2\lambda_x^4} \Big[& \{ w_{K-2} - 2w_{K-1} + w_K \}^2 \\ & + \{ w_{K-1} - 2w_K + w_{K+1} \}^2 \\ & + \{ w_K - 2w_{K+1} + w_{K+2} \}^2 \Big] \\ & + \cdots \\ & + \cdots \end{aligned}$$

$$\begin{aligned} T_2 = \cdots & + \cdots \\ & + \frac{\lambda_x \lambda_y B_y}{2\lambda_y^4} \Big[\{ w_Q - 2w_N + w_K \}^2 \\ & + \{ w_N - 2w_K + w_M \}^2 \\ & + \{ w_K - 2w_M + w_P \}^2 \Big] + \cdots \\ & + \cdots \end{aligned}$$

$$T_3 = \text{-------} + \frac{\lambda_x\lambda_y 2H}{2\times 16\lambda_x^2\lambda_y^2}\Big[\{-\omega_{\ell-2} - \omega_K + \omega_\ell + \omega_{K-2}\}^2$$

$$+\{-\omega_\ell - \omega_{K+2} + \omega_{\ell+2} + \omega_K\}^2$$

$$+\{-\omega_{K-2} - \omega_P + \omega_K + \omega_{P-2}\}^2$$

$$+\{-\omega_K - \omega_{P+2} + \omega_{K+2} + \omega_P\}^2\Big]$$

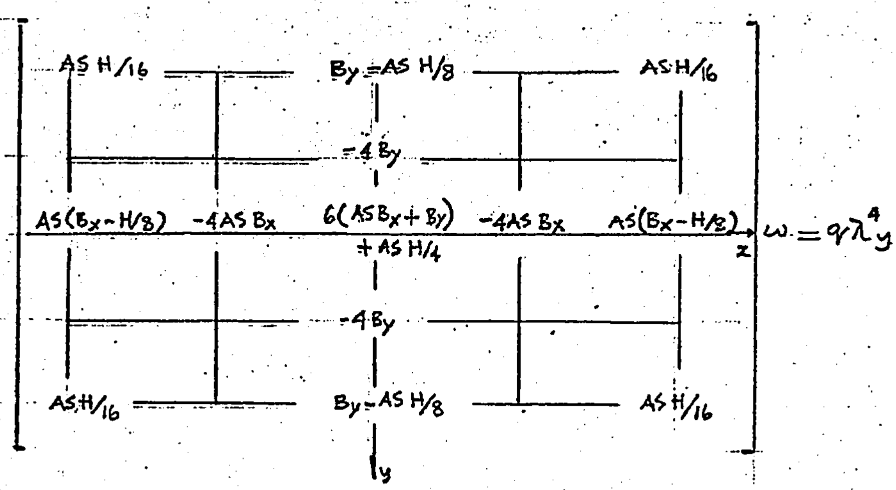
$$+ \text{-----------------}$$

$$T_4 = \text{-----} + \lambda_x\lambda_y q_K\omega_K + \text{-----------}$$

Minimizing U with respect to ω_K , and putting it equal to zero:

$$\frac{\partial U}{\partial\omega_K} = \frac{\partial T_1}{\partial\omega_K} + \frac{\partial T_2}{\partial\omega_K} + \frac{\partial T_3}{\partial\omega_K} - \frac{\partial T_4}{\partial\omega_K} = 0 \qquad (13)$$

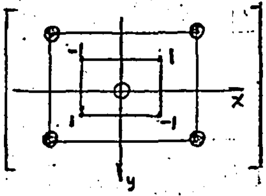
the following difference equation for an interior point, K, is obtained:



(14)

It is observed that equation (14) does not tally with equation (10). But instead of taking whole station central differences, if half station central difference technique is applied, other terms in the potential energy equation (12) remain the same, while only term, T_3, containing an odd order derivatives $\omega_{,xy}$, needs the following modification:

$$T_3 = \frac{2H\lambda_x\lambda_y}{2\lambda_x^2\lambda_y^2}\,\Sigma\left[\quad\right]^2_{\omega}$$



$$= - - - - - - - + \frac{H}{\lambda_x\lambda_y}\Big[\{\omega_{N-1}+\omega_K-\omega_N-\omega_{K-1}\}^2$$

$$+\{-\omega_N+\omega_{N+1}+\omega_K-\omega_{K+1}\}^2$$

$$+\{-\omega_{K-1}+\omega_K+\omega_{M+1}-\omega_M\}^2$$

$$+\{\omega_K+\omega_{M+1}-\omega_{K+1}-\omega_M\}^2\Big] + - - - - - - - - -$$

Minimizing with respect to ω_K ;

$$\frac{\partial T_3}{\partial\omega_K} \cong \frac{H}{\lambda_x\lambda_y}\Big[4\omega_K - 2\{\omega_{K-1}+\omega_N+\omega_M+\omega_{K+1}\} + \{\omega_{N-1}+\omega_{N+1}+\omega_{M-1}+\omega_{M+1}\}\Big] \tag{15}$$

Replacing the term $\frac{\partial T_3}{\partial\omega_K}$ in equation (13) by the right hand side of equation (15), and putting it equal to zero, one obtains a finite difference equation for any interior mesh point which is same as equation (10). A comparison of equations (10) and (14), shows that the discrepancy arises due to the fact that the procedure applied in the conventional central difference method to obtain approximations for odd order derivatives is not consistent with that used to obtain even order differences. Such inconsistencies were also reported by Cyrus and Fulton [3], Melin and Robinson [1] , and Chuang [4] .

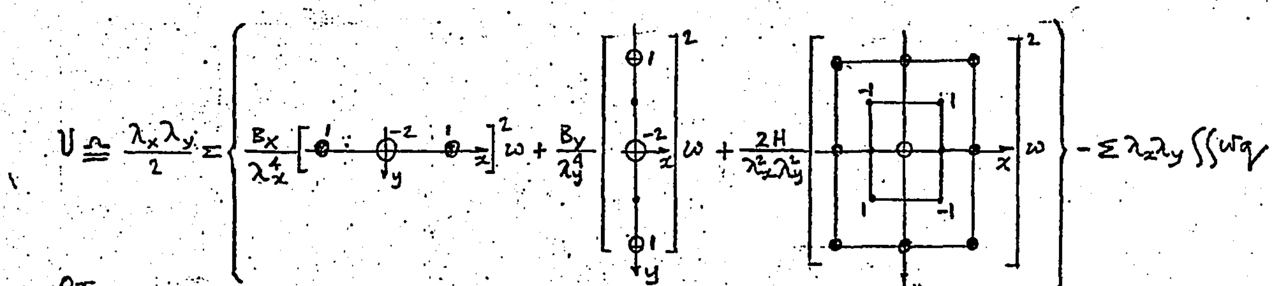
During this present investigation, such an inconsistency was first observed while solving a few test problems of orthogonally stiffened plates of constant rigidity, originally solved by N.Ando [1] , by using equation (14), had no correlation with those obtained by N. Ando. After modified central differences were used to approximate the odd order derivative as involved in equation (11), both of the difference equations coincided and the deflections obtained by them were fairly close to those obtained by N. Ando. (Table 2A).

Due to the above facts, in the rest of this investigation, half station (or modified) central difference method has been utilized to approximate odd order derivatives. For convenience, some modified operators are given in the appendix.

IV - ORTHOGONALLY STIFFENED PLATE WITH RIBS OF VARIABLE RIGIDITIES IN EACH DIRECTION

A - Derivation Of Modified Central Difference Operators Based On The Principle Of Minimum Potential Energy: (Fig.5)

For a given load, q, uniformly distributed over an isotropic plate orthogonally stiffened with ribs of variable rigidities in each direction, and having no initial curvature, the equation (11) for total potential energy at the middle plane due to bending, torsion and external load can be approximated over a region, R, as:



or,

$$U \doteq T_1 + T_2 + T_3 - T_4 \tag{17}$$

Expanding the terms T_1, T_2, T_3, and T_4, and retaining the terms required to minimize each of them with respect to the deflection, ω_K', of an interior point K, one obtains,

$$T_1 = - - - - + \frac{B_{x(K-1)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_{K-2} - 2\omega_{K-1} + \omega_K}{\lambda_x^2}\right\}^2 + \frac{B_{x(K)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_{K-1} - 2\omega_K + \omega_{K+1}}{\lambda_x^2}\right\}^2$$
$$+ \frac{B_{x(K+1)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_K - 2\omega_{K+1} + \omega_{K+2}}{\lambda_x^2}\right\}^2 + - - - -$$

$$T_2 = - - - - + \frac{B_{y(N)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_Q - 2\omega_N + \omega_K}{\lambda_y^2}\right\}^2 + \frac{B_{y(K)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_N - 2\omega_K + \omega_M}{\lambda_y^2}\right\}^2$$
$$+ \frac{B_{y(M)}\lambda_x\lambda_y}{2}\left\{\frac{\omega_K - 2\omega_M + \omega_P}{\lambda_y^2}\right\}^2 + - - - -$$

$$T_3 = - - - + H_a \lambda_x \lambda_y \left\{\frac{\omega_{N-1} - \omega_N - \omega_{K-1} + \omega_K}{\lambda_x \lambda_y}\right\}^2 + H_b \lambda_x \lambda_y \left\{\frac{-\omega_N + \omega_{N+1} + \omega_K - \omega_{K+1}}{\lambda_x \lambda_y}\right\}^2$$
$$+ H_c \lambda_x \lambda_y \left\{\frac{-\omega_{K-1} + \omega_K + \omega_{M-1} - \omega_M}{\lambda_x \lambda_y}\right\}^2 + H_d \lambda_x \lambda_y \left\{\frac{\omega_K - \omega_{K+1} - \omega_M + \omega_{M+1}}{\lambda_x \lambda_y}\right\}^2$$
$$+ - - - - - -$$

$$T_4 = - - - - - q_K \omega_K \lambda_x \lambda_y - - - - - -$$

Minimizing U with respect to ω_K, and putting it equal to zero, ie.

$$\frac{\partial U}{\partial \omega_K} = \frac{\partial T_1}{\partial \omega_K} + \frac{\partial T_2}{\partial \omega_K} + \frac{\partial T_3}{\partial \omega_K} - \frac{\partial T_4}{\partial \omega_K} = 0, \text{ one obtains,}$$

$$\frac{B_{x(K-1)}}{\lambda_x^4}\{\omega_{K-2} - 2\omega_{K-1} + \omega_K\}$$
$$-\frac{2B_{x(K)}}{\lambda_x^4}\{\omega_{K-1} - 2\omega_K + \omega_{K+1}\}$$
$$+\frac{B_{x(K+1)}}{\lambda_x^4}\{\omega_K - 2\omega_{K+1} + \omega_{K+2}\}$$
$$+\frac{B_{y(N)}}{\lambda_y^4}\{\omega_Q - 2\omega_N + \omega_K\}$$
$$-\frac{2B_{y(K)}}{\lambda_y^4}\{\omega_N - 2\omega_K + \omega_M\}$$
$$+\frac{B_{y(M)}}{\lambda_y^4}\{\omega_K - 2\omega_M + \omega_P\}$$
$$+\frac{2H_a}{\lambda_x^2\lambda_y^2}\{\omega_{N-1} - \omega_N - \omega_{K-1} + \omega_K\}$$
$$+\frac{2H_b}{\lambda_x^2\lambda_y^2}\{-\omega_N + \omega_{N+1} + \omega_K - \omega_{K+1}\}$$
$$+\frac{2H_c}{\lambda_x^2\lambda_y^2}\{-\omega_{K-1} + \omega_K + \omega_{M-1} - \omega_M\} =$$
$$+\frac{2H_b}{\lambda_x^2\lambda_y^2}\{\omega_K - \omega_{K+1} - \omega_M + \omega_{M+1}\} = q_K \quad \text{------------------(18)}$$

Equation (18) can be written in the following form:

$$BEX_{K-1}\{\omega_{K-2} - 2\omega_{K-1} + \omega_K\}$$
$$-2BEX_K\{\omega_{K-1} - 2\omega_K + \omega_{K+1}\}$$
$$+BEX_{K+1}\{\omega_K - 2\omega_{K+1} + \omega_{K+2}\}$$
$$+BEY_N\{\omega_Q - 2\omega_N + \omega_K\}$$
$$-2BEY_K\{\omega_N - 2\omega_K + \omega_M\}$$
$$+BEY_M\{\omega_K - 2\omega_M + \omega_P\}$$
$$+BEC\,a\{\omega_{N-1} - \omega_N - \omega_{K-1} + \omega_K\}$$
$$+BEC\,b\{-\omega_N + \omega_{N+1} + \omega_K - \omega_{K+1}\}$$
$$+BEC\,c\{-\omega_{K-1} + \omega_K + \omega_{M-1} - \omega_M\}$$
$$+BEC\,d\{\omega_K - \omega_{K+1} - \omega_M + \omega_{M+1}\} = q_K \quad \text{------------------(19)}$$

where, $(\lambda_y/\lambda_x)^2 = AS$

$BEX = B_x AS^2$

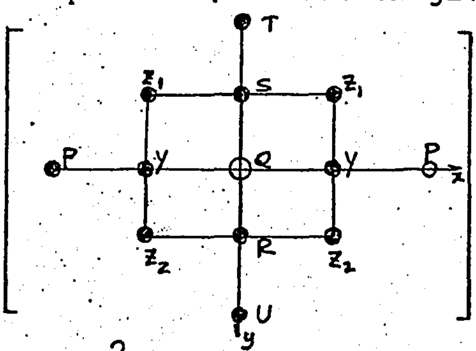
$BEY = -B_y$

$BEC = -2H\,AS$

The equation (19) has been derived using equation (11), which is based on the condition:

$$\iint \frac{\partial^2 \omega}{\partial x^2} \cdot \frac{\partial^2 \omega}{\partial y^2}\, dx\,dy = \iint \left(\frac{\partial^2 \omega}{\partial x \partial y}\right)^2 dx\,dy$$

To avoid such a condition one may use the following finite difference equation (Derivation given in the Appendix A):



$$\omega = q\lambda_y^4 \qquad (20)$$

where, $P = AS^2 B_{x(K)}$

$Q = 6AS^2 B_{x(K)} + 4B_{y(K)} + B_{y(M)} + B_{y(N)} + 2\,BEC_K + BEC_M$
$\quad + BEC_N + 4\,BEZ_K$

$R = -2\left[B_{y(M)} + B_{y(K)}\right] - BEC_K - BEC_M$
$\quad - BEZ_K - BEZ_M$

$S = -2\left[B_{y(N)} + B_{y(K)}\right] - BEC_K - BEC_N$
$\quad - BEZ_K - BEZ_N$

$T = -B_{y(N)}$

$U = B_{y(M)}$

$Y = -4AS^2 B_{x(K)} - BEZ_K - \{BEC_N + BEC_M\}/2 - BEC_K - BEZ_K$

$Z_1 = \{BEC_K + BEC_N\}/2 + \{BEZ_K + BEZ_N\}/2$

$Z_2 = \{BEC_K + BEC_M\}/2 + \{BEZ_K + BEZ_M\}/2$

In the above equation, K, is the interior point where the derivative is approximated and N and M (Fig.5) are two adjacent points whose rigidities were also necessary in this approximation. Rigidities for other adjacent points shown by black circles (⊕), except for N and M, were not necessary.

Equations (18), (19) or (20) can be applied to an orthogonally stiffened plate with variable rigidities of ribs in each direction to get the deflections of any point within the boundary. These equations are based on small deflection theory. All the assumptions of a thin isotropic plate are also applicable to them. To apply these equations at the exterior mesh points of a plate, suitable boundary conditions have to be applied. In case of a clamped plate, Lagrange's Multiplier method may be used [6]. In case of a free edge, Kirchoff-Love condition may be applied. For a simply supported plate along the entire contour, as involved in this investigation, the following conditions are applied:

$\omega = 0$ and $M_x = 0$ at $x = 0$ and $x = a$

$\omega = 0$ and $M_y = 0$ at $y = 0$ and $y = b$

Since $M_x = -B_x \omega_{,xx} - B\omega_{,}\omega_{,xx} + e_x D_x (u_{,x} - e_x \omega_{,xx})$

and since there is no curvature in the y-direction, $\omega_{,yy} = 0$, and also according to Massonnet's approximation [10] , $u = e_x \omega_{,x}$ or $u_{,x} = e_x \omega_{,xx}$ the last term in the equation of moment is equal to zero, and the zero moment condition reduces to $\omega_{,xx} = 0$ at x=0 and x=a, and in the y-direction, $\omega_{,yy} = 0$ at y= 0 and y= b. (21)

In the case of a plate with constant rigidity in each direction, equation (19) coincides with equation (10).

In case of a plate with variable rib-dimensions in the y-direction only, as used in this investigation, the terms,

$$BEC_a = BEC_b = \frac{BEC_N + BEC_K}{2} \quad \text{(approx.)}$$

$$BEC_c = BEC_d = \frac{BEC_M + BEC_K}{2} \quad \text{(approx.)}$$

B. <u>Numerical Solution of an Orthogonally Stiffened Plate With Ribs of Variable Rigidity</u>

<u>Introduction</u>

In this section, finite difference solutions by both whole station (conventional) and half station (modified) methods have been presented for an isotropic plate (Fig.6), having dimensions 3'x2'x¼" and stiffened in the x-direction with five equally spaced ribs, each having different but constant stiffness. In the y-direction, five ribs are used, each of constant width but depth varying as a half sine wave, d(in inches) = $1.5(1 + \sin\frac{\pi y}{b})$. Width of all stiffness was ¼". Material used in the solution was hot rolled structural grade steel having Young's modulus of elasticity, E= 30×10^6 p.s.i., and Poisson's ratio, 0.3. The solutions presented herein for the above plate with simply supported boundary conditions along the entire contour are under three different conditions of loading:

(a) Uniformly Distributed Doad, q

(b) A Single Concentrated Load, P, located at center

(c) Two Concentrated Loads, each equal to P/2, located at (a/3, b/2 and (2a/3, b/2)

Since the loadings considered and the plate geometry are symetrical about the central axes, only one quadrant of the plate was considered. Fig.(6) shows the layout of the meshes for application of both the conventional and the modified method of finite difference techniques.

In both cases, pertinent difference operators in accordance with equation (20) are applied for interior mesh points (5,6,8,9) and for mesh points near the boundary (1,2,3,4,7). Boundary conditions (21) are applied to evaluate the deflection functions outside the plate boundary, in terms of those within the boundary and the resulting equations are written in terms of deflections of different interior mesh points.* The resulting matrix of the above simultaneous equations are solved with the help of a 1620-II compiler of the University of Windsor. To maintain proper accuracy digits up to eight significant figures were taken into consideration.

The Solution

The finite difference operator (20) can be used to solve the orthogonally stiffened plate shown in Fig.(6) by forming the general matrix equations in the following way:

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{Bmatrix} \\ w \\ \\ \end{Bmatrix} = \begin{bmatrix} \\ c \\ \\ \end{bmatrix} \lambda_y^4 \qquad (22)$$

where, $w^{(T)} = w_1, w_2, w_3, w_4, \ldots\ldots. w_9$, the deflections at mesh points 1,2,3,4,......9

$C^{(T)} = C_1, C_2, C_3, C_4, \ldots\ldots.. C_9$, the loads acting on

* To clarify the application, some of the equations for both interior and exterior mesh points are given in the appendix.

region R_1, R_2, R_3, ...R_9.
and the square matrix A is,

$Q-T-P$	y	P	R	Z_2	0	U	0	0
y	$Q-T+P$	y	Z_2	R	Z_2	0	U	0
$2P$	$2y$	$Q-T$	0	$2Z_2$	R	0	0	U
S	Z_1	0	$Q-P+U$	y	P	R	Z_2	0
Z_1	S	Z_1	y	$Q+P+U$	y	Z_2	R	Z_2
0	$2Z_1$	S	$2P$	$2y$	$Q+U$	0	$2Z_2$	R
$T+U$	0	0	$S+R$	Z_1+Z_2	0	$Q-P$	y	P
0	$T+U$	0	Z_1+Z_2	$S+R$	Z_1+Z_2	y	$Q+P$	y
0	0	$T+U$	0	$2(Z_1+Z_2)$	$S+R$	$2P$	$2y$	Q

By inverting the co-efficient matrix A and carrying out multiplication with the vector C, deflections ω_1, ω_2, ...ω_9 can easily be computed. In this section three different cases of loading are considered.

a. <u>Distributed Load</u>

For uniformly load of intensity, q, per unit area, equation (22) assumes the form:

$$\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_9 \end{Bmatrix} = \begin{bmatrix} \\ A^{-1} \\ \\ \end{bmatrix} \begin{bmatrix} q \\ q \\ \vdots \\ q \end{bmatrix} \qquad (23)$$

where, $W^{(T)} = W_1$, W_2, W_3, ...W_9. =deflections at mesh points 1,2,3,.....9.

A^{-1} = inverse of the stiffness co-efficient matrix.

b. <u>Concentrated load</u>

For a single concentrated load, P, at the centre (mesh point 9), equation (22) assumes the form:

$$\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_9 \end{Bmatrix} = \begin{bmatrix} & A^{-1} & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ P/\lambda_x\lambda_y \end{bmatrix} \qquad (24)$$

where, A^{-1}=same as in equation (23) and the vector W refers to deflections at mesh points 1,2,3, ...9.

c. <u>Two-Point Load Symmetrical About The Axes</u>

For two concentrated loads each having a magnitude of p/2 and located at (a/3, b/2) and (2a/3, b/2), (mesh points 8-8') equation (22) assumes the following form:

$$\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_9 \end{Bmatrix} = \begin{bmatrix} & A^{-1} & \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ P/2\lambda_x\lambda_y \end{bmatrix} \qquad (25)$$

Where, the vector W represents deflections at points 1,2,3,...9 and A^{-1} is the same as in previous equations.

By using the three equations (23), (24) & (25), the plate problem as described in the introduction of this section has been solved and the results are given in Table 3.A.

V - EXPERIMENTAL VARIFICATION OF THE PROBLEM OF AN ORTHOGONALLY STIFFENED PLATE OF VARIABLE RIGIDITY

A. Description Of The Model

The model as described in chapter four, section "B", was built up from a rectangular plate (¼" x 36" x 24") made of hot rolled structural steel having Young's modulus of elasticity, $E = 30 \times 10^6$ p.s.i. and Poisson's ratio, 0.3, and ten ¼" thick stiffeners of sizes as shown in Fig.(3.1). The stiffeners were welded to one side of the plate to form rectangular meshes, each having a size of 6" x 4" (centre to centre). To prevent local warping, the plate used was a little oversized (¼" x 40" x 28") and the exterior stiffeners were ½" thick. (Photo plate 1)

B. Supporting Structure

The edges of the plate were simply supported on four polished special machine steel rods of 3/4" diameter. These rods in return were supported on a rectangular steel frame built up with four 15" x 3" x 3/8" channels, the top surface of which was specially ground to form uniform support. This frame was finally supported by four standard steel bases on the steel floor of the testing structure. (Photo Plate II)

C. Loading Device

The plate was tested in the elastic range under two types of loading:

1. Concentrated Load At The Centre

For this loading, a 40-ton load cell was specially built at the Central Research Shop and was callibrated with a Budd Portable type strain indicator and a PCA-300,000 lb.

testing machine. (Calibration Curve, Fig. 12). The load was concentrically applied from the top of the plate by a hydraulic ram supported under the beam of the testing structure. (Photo Plate III).

2. Two Concentrated Equal Loads Applied at (a/3,b/2 and (2a/3,b/2)

For this loading the same ram as in case 1 was used. Load was applied through a Thawing=Albert 20,000 lb. load cell (Calibration Curve, Fig.13), on the center of a 5 x 3 x 7/32 I-beam resting on two rollers (12" centre to centre) symmetrically about the axes. The rollers in return were supported on two 3" x 4" grooved steel blocks. Between these two steel blocks and the surface of the plate two pieces of plywood were placed to remove any inaccuracy of loading. (Photo Plate IV).

D. Testing Method and Recording of Data

Loads in both the cases were applied from a testing machine through a hydraulic ram mounted from the loading beam of the testing structure. The rate of loading was constantly kept at 6.

Deflections at different points along the transverse axis was recorded, for both the cases of loading, with the help of extra sensitive dial gauges (10^{-4}). Corrections have been incorporated in the result for the inaccuracy arose due to the vertical deflections of the supporting structure and the steel floor. To do this, vertical deflections of supporting structure and the steel floor were also measured during the application of the load. Deflections in the latter case were measured relative to the floor adjacent to steel strips. A digital strain indicator was used to record strains (Photo Plates V and VI).

Discussion and Conclusion

Comparison of Fig. 9-11 shows that the experimental values of deflections for the plate subjected to a single concentrated load and to a two point loading are respectively 11.45% (max.) and 18% (max.) higher than those obtained by using modified finite difference method. Whereas, the solution obtained by using the method of conventional finite difference has hardly any correlation with the experimental solution.

The deviation of the theoretical solutions from the experimental results may be attributed to the following:

A. Calculations of Rigidities

The effective width of plate used in calculating rigidities could be taken about 90% of the full width [13]. Since the coefficient matrix depends on the values of rigidity, lower rigidities would result in higher deflections. The above fact is one of the major causes of deviation between theoretical and experimental results.

B. Experimental and Constructional Inaccuracies of the Model

The model was having uneven initial curvatures in its neutral planes induced due to welding shrinkage, which gave rise to unsymmetrical deflections.

To maintain uniform support on all sides, the depths of ribs, provided over the supports were 1.5 inches, which resulted in lower torsional rigidity and higher deflections.

No foolproof method could be devised to take measure against such errors in the theoretical investigation.

From the foregoing treatment of the subject it has been clarified that the theory of equivalent orthotropic plate could be used to analyse orthogonally stiffened plates having not only constant rigidity but also variables rigidities.

It has also been clarified that the modified central differences method could be conveniently used to solve problems of varational calculus with the aid of high speed electronic computers. Since in this method, the number of equations or the time required for solutions is considerably less than other commonly used methods.

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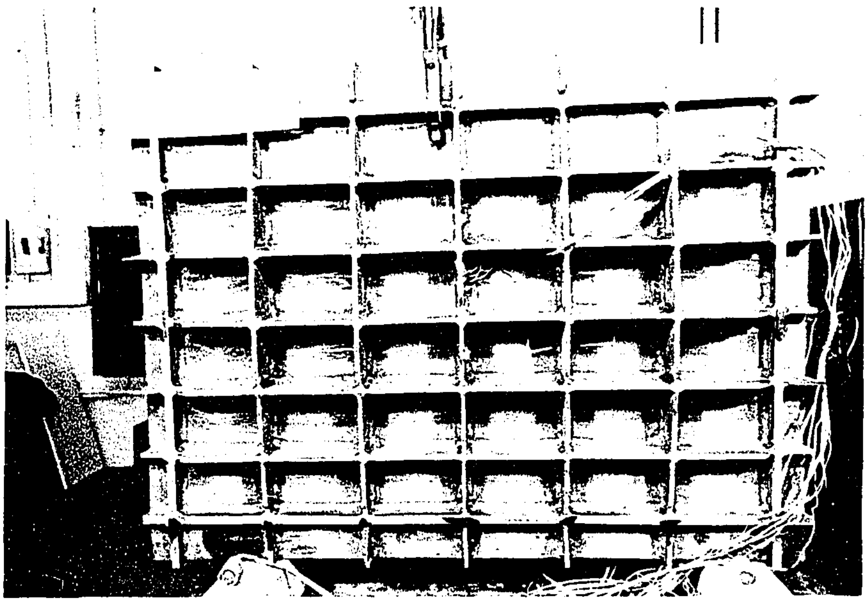


PLATE-I

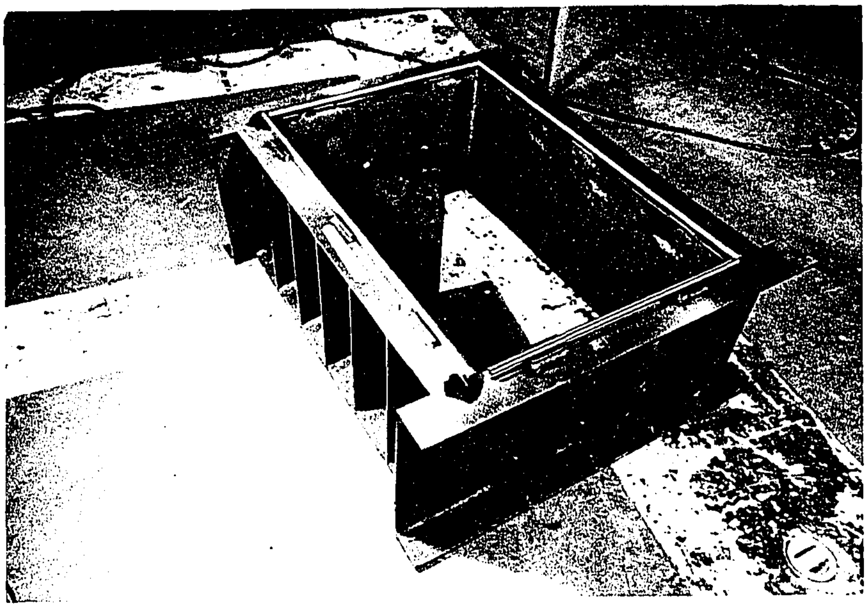


PLATE-II

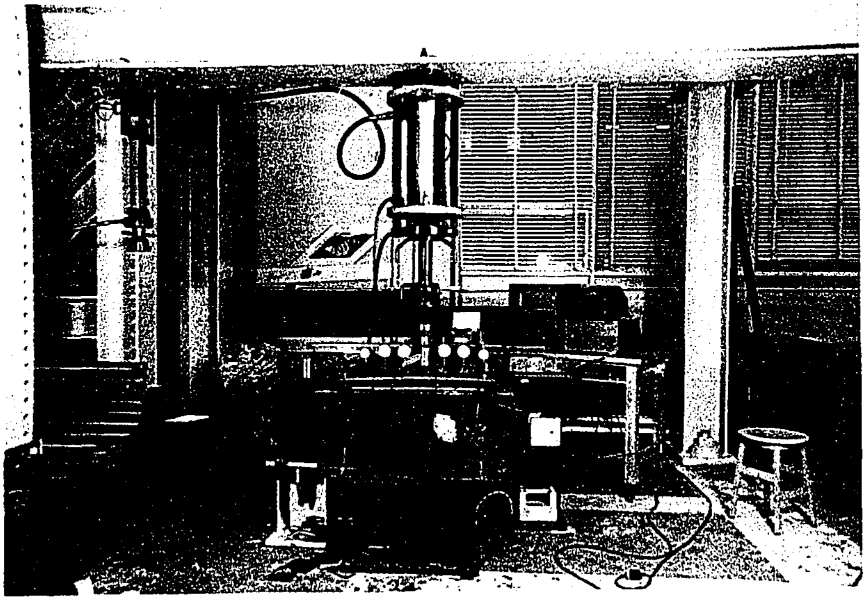


PLATE-III

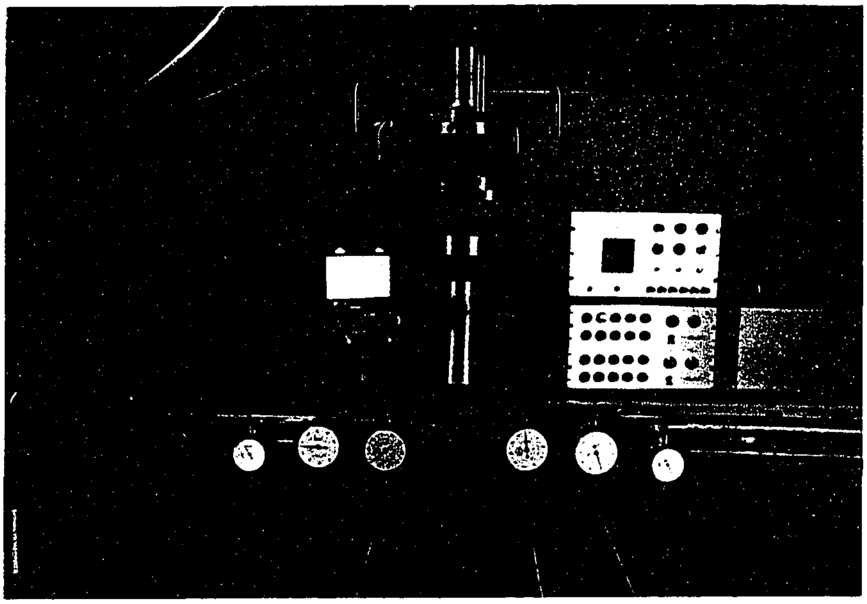


PLATE-IV

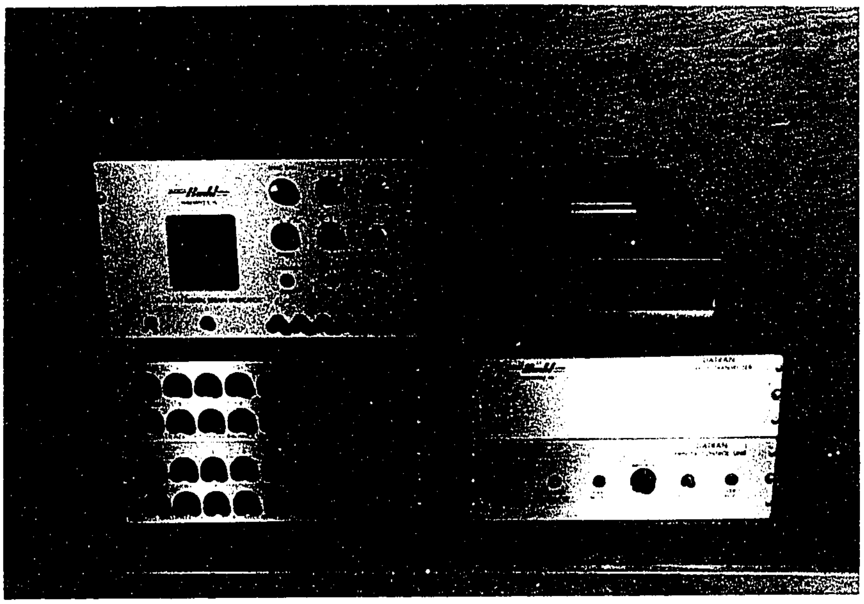


PLATE-V

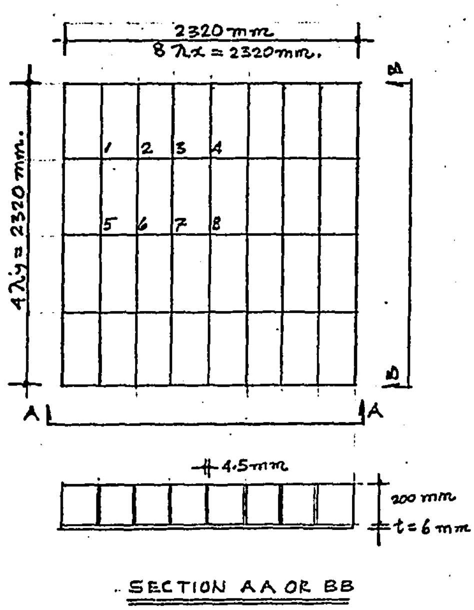
PHOTOGRAPHIC PLATES

&

TABLES

COMPARISON OF DEFLECTIONS

MESH POINTS	(1) WHOLE OR HALF STATION METHOD* (mm)	(2) ANDO'S EXPERIMENTAL VALUES (mm)	(3) ANDO'S THEORETICAL VALUES* (mm)	PERCENTAGE DIFFERENCE FROM (2) (1)	(3)
1	·1995	DATA NOT AVAILABLE	DATA NOT AVAILABLE	-	-
2	·3598	" " "	" " "	-	-
3	·4614	" " "	" " "	-	-
4	·4960	·5500	·5000	-9.8	-9.1
5	·2756	·3500	·2800	-21.3	-20.0
6	·4988	·6100	·5000	-18.4	-18.0
7	·6412	·7200	·6400	-10.0	-11.1
8	·6899	·7500	·6900	-8.0	-8.0



DR. ANDO'S PLATE

*RIGIDITIES CALCULATED PER ONE PITCH BREADTH. TOTAL LOAD 10-TONS UNIFORMLY DISTRIBUTED ON ENTIRE PLATE (SHORT TONS). $E = 30\times10^6$ PSI $\nu = \cdot 3$. $G = E/(1+\nu)2$. ALL QUANTITIES TAKEN IN METRIC SYSTEM OF MEASUREMENTS.

TABLE 2.A

DEFLECTION INFLUENCE COEFFICIENTS

MESH POINTS	DISTRIBUTED LOAD q = 1#	CONCENTRATED LOAD P = 1# AT PT.9	2-POINT CONC. LOADS P/2 = 0.5# AT 8 − 8'.
1	.15191304 E −03	.34023410 E − 06	.34614074 E −06
2	.24791875-03	.65228175 E −06	.57594340 E −06
3	.27994557-03	.81165299 E −06	.65228170 E −06
4	.23187214 −03	.54710902 E − 06	.53320509 E −06
5	.38327060-03	.10664104 E − 05	.96148852 E −06
6	.43441059-03	.13758685 E − 05	.10664104 E −05
7	.25627655-03	.61576562 E − 06	.60848511 E −06
8	.42535185-03	.12169704 E − 05	.11322790 E −05
9	.48272281-03	.16487932 E − 05	.12169706 E −05

FOR LOCATION OF LOADS REFER TO FIGURE 6.

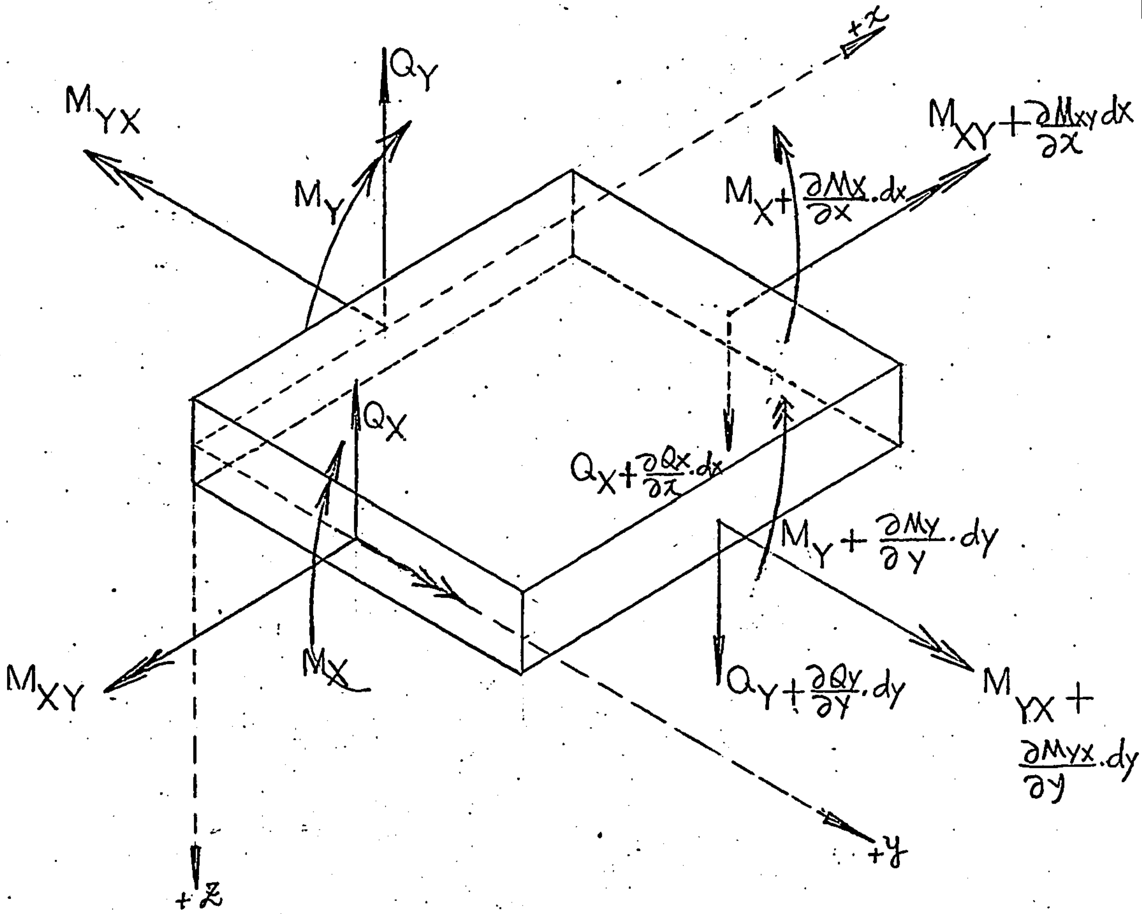
YOUNG'S MODULUS OF ELASTICITY = 30×10^6 PSI

POISSON'S RATIO = 0.3. ALL LINEAR MESUREMENTS ARE IN INCHES.

PLATE BOUNDARY SIMPLY SUPPORTED. DEFLECTIONS WITH RESPECT TO THE MIDDLE PLANE OF PLATE

TABLE 3.A

FIGURES



+x
Q_Y
M_{YX}
M_Y
$M_{XY}+\frac{\partial M_{xy}}{\partial x}dx$
$M_X+\frac{\partial M_X}{\partial x}.dx$
Q_X
$Q_X+\frac{\partial Q_X}{\partial x}.dx$
$M_Y+\frac{\partial M_Y}{\partial y}.dy$
M_{XY}
M_X
$Q_Y+\frac{\partial Q_Y}{\partial y}.dy$
$M_{YX}+\frac{\partial M_{YX}}{\partial y}.dy$
+y
+z

FIGURE I

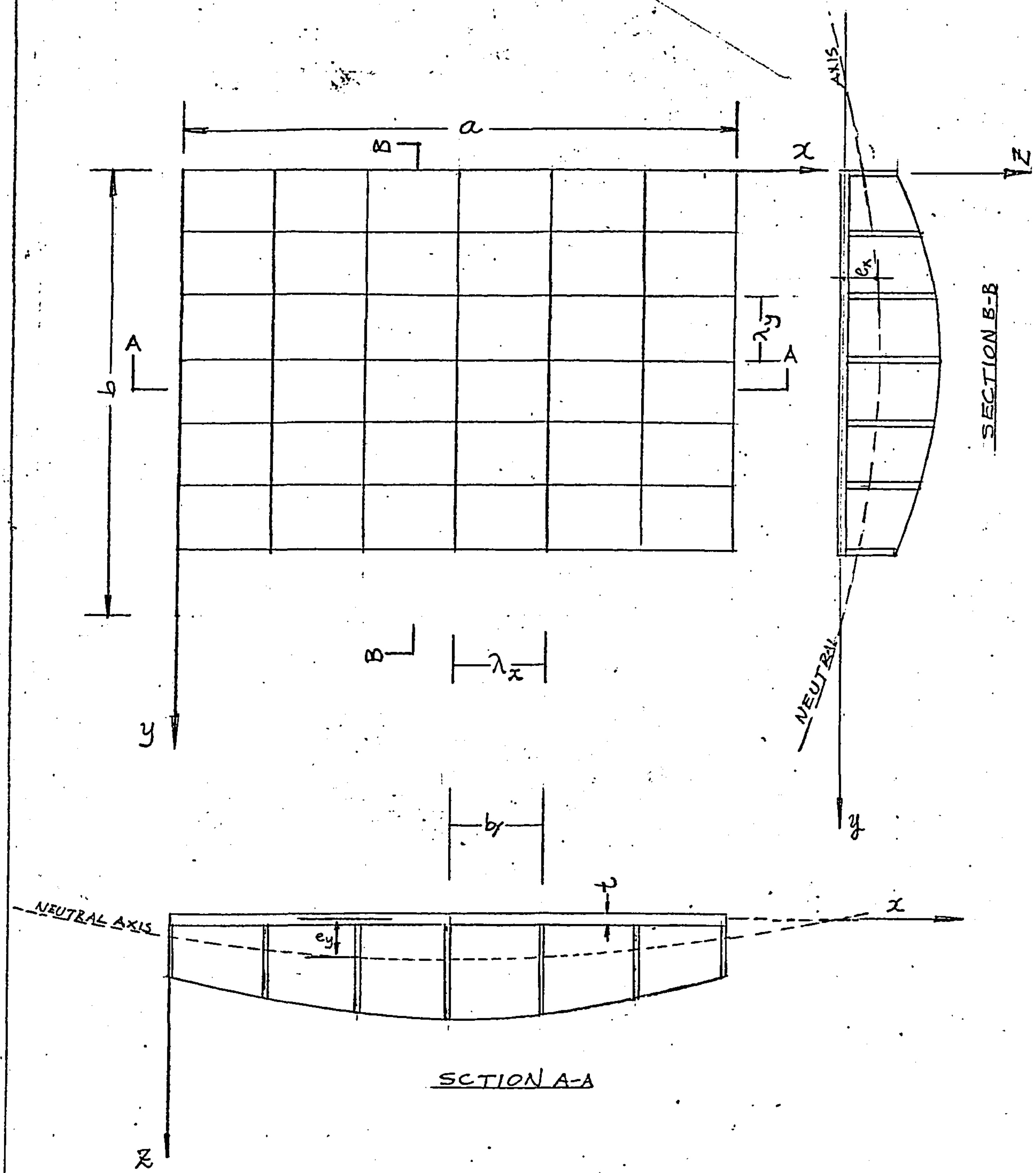

a
B
x
Z
A
b
λ_y
A
e_x
SECTION B-B
AXIS
NEUTRAL
B
λ_x
y
y
b_f
t
NEUTRAL AXIS
e_y
x
SCTION A-A
Z

FIGURE 2

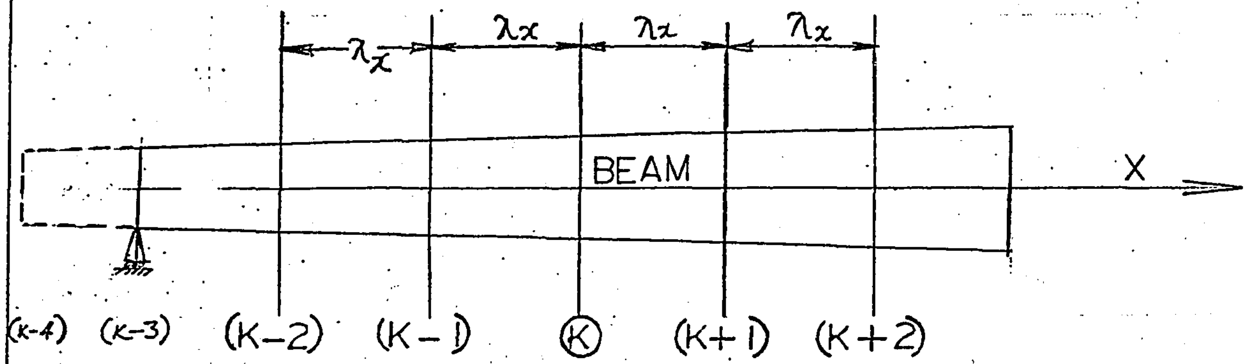


FIGURE 3

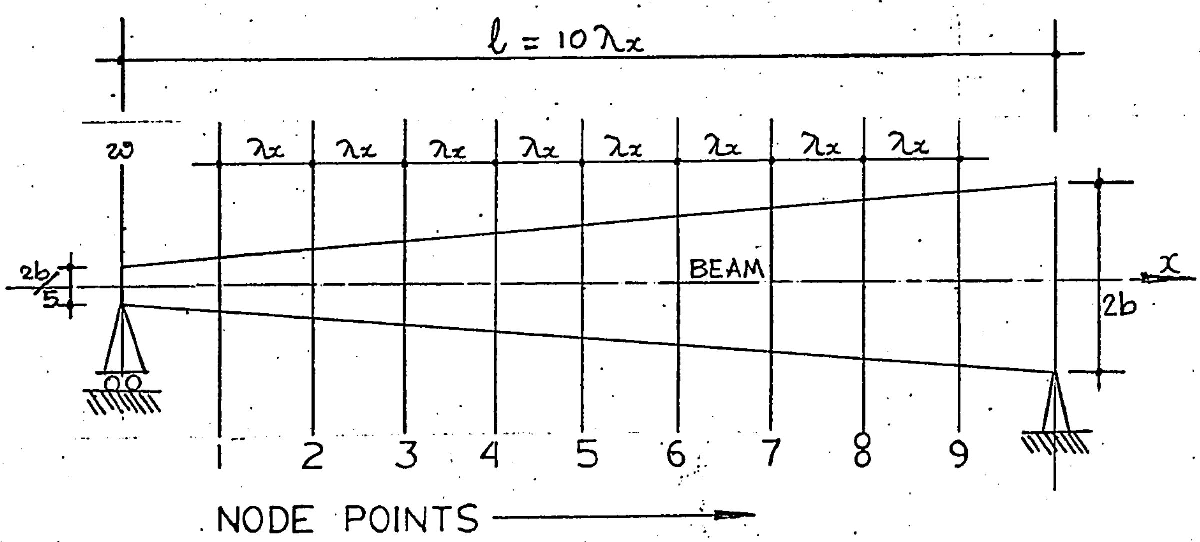
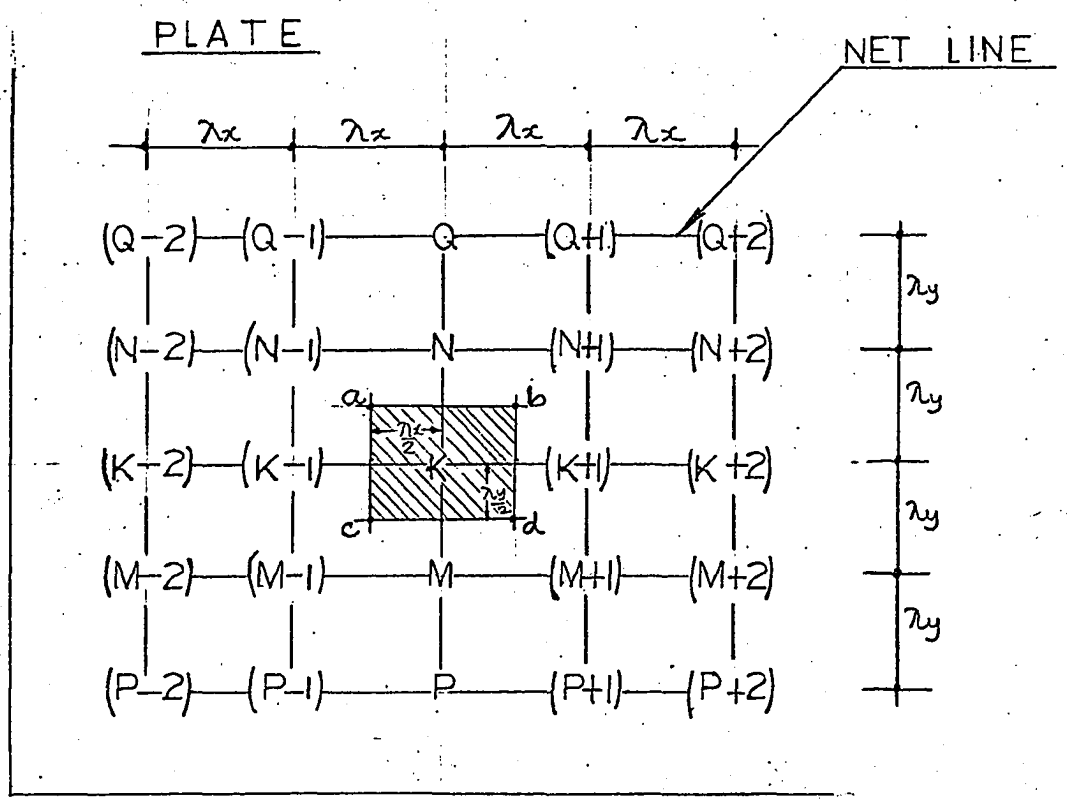
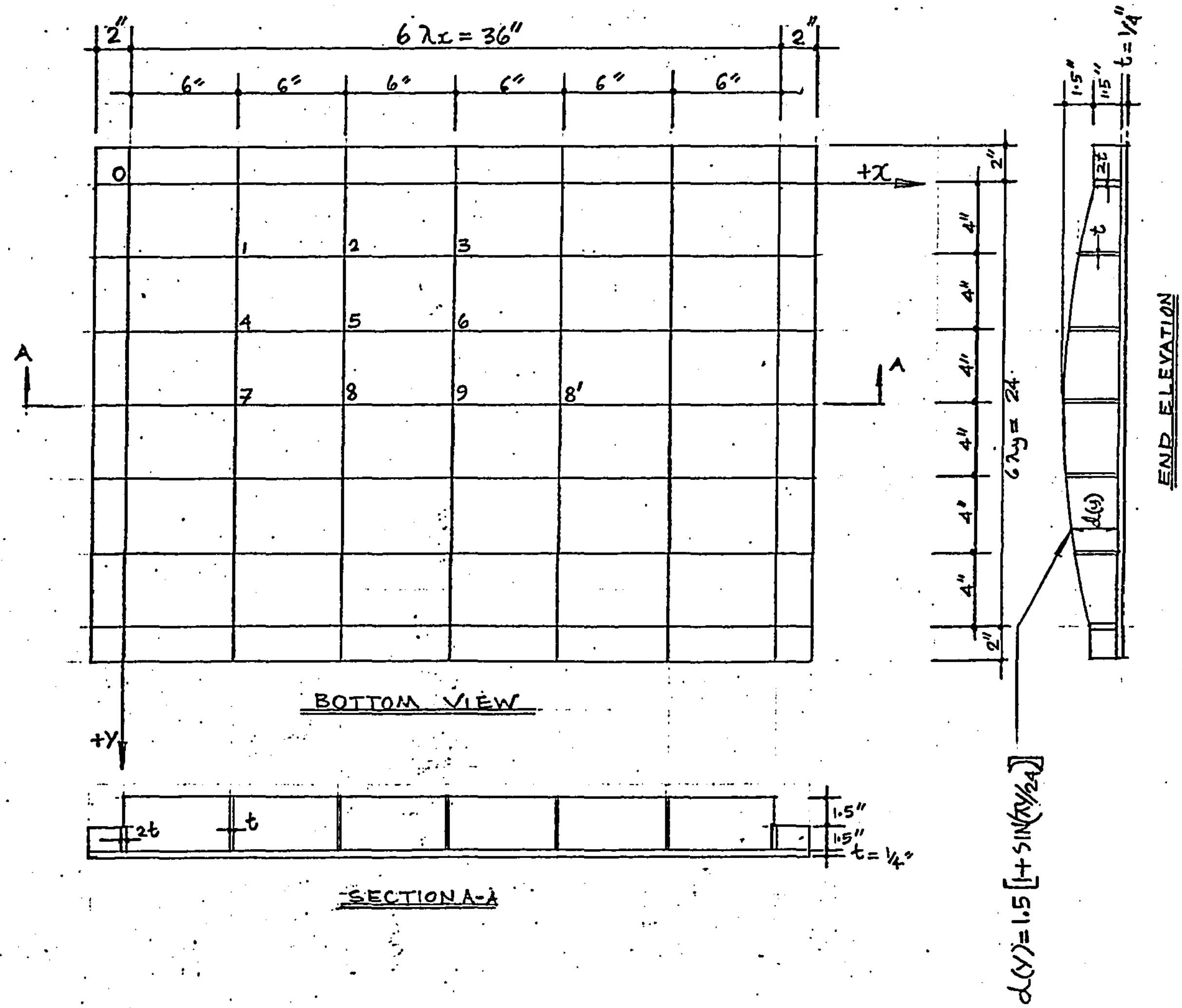


FIGURE 4



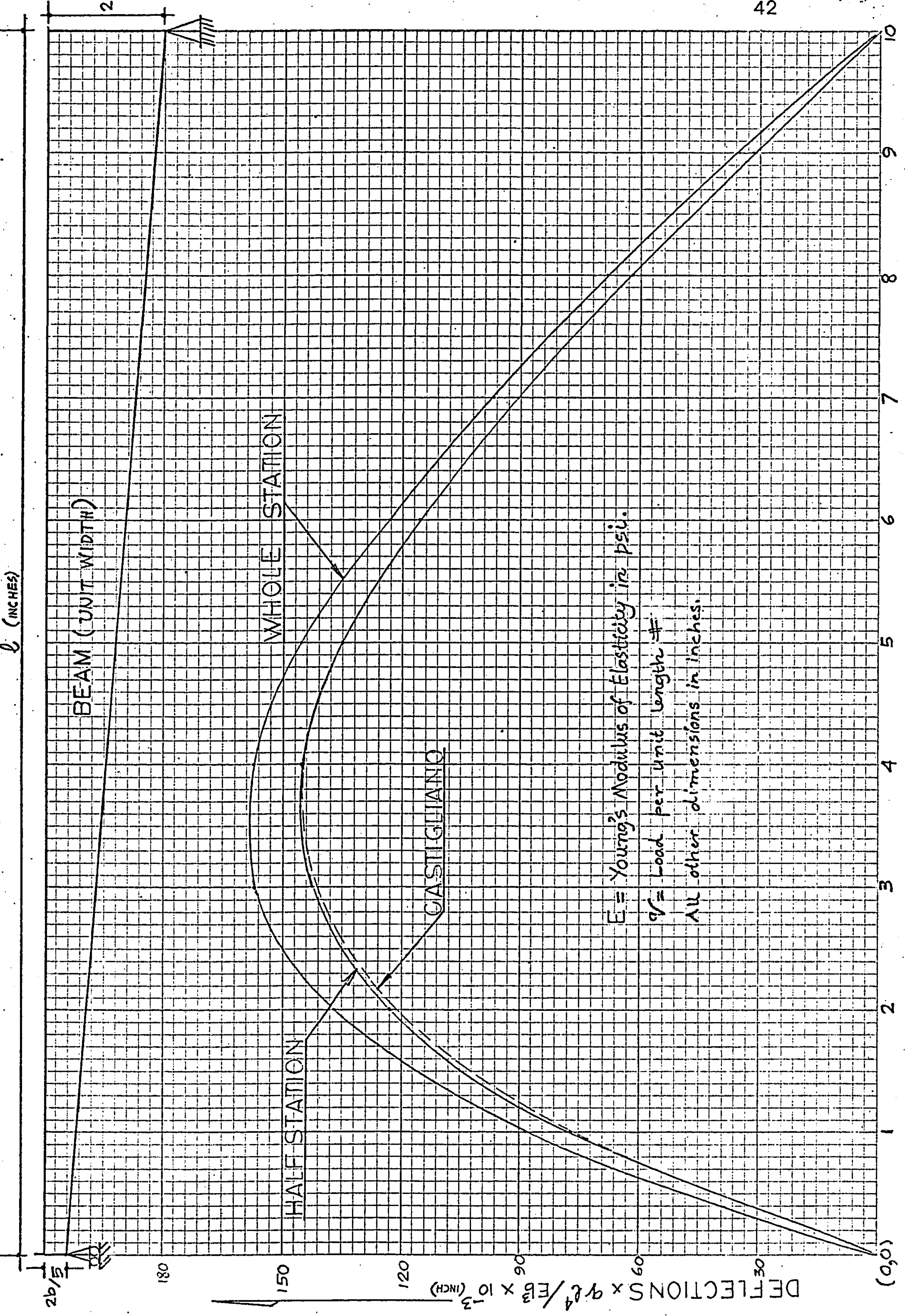
K IS AN INTERIOR POINT

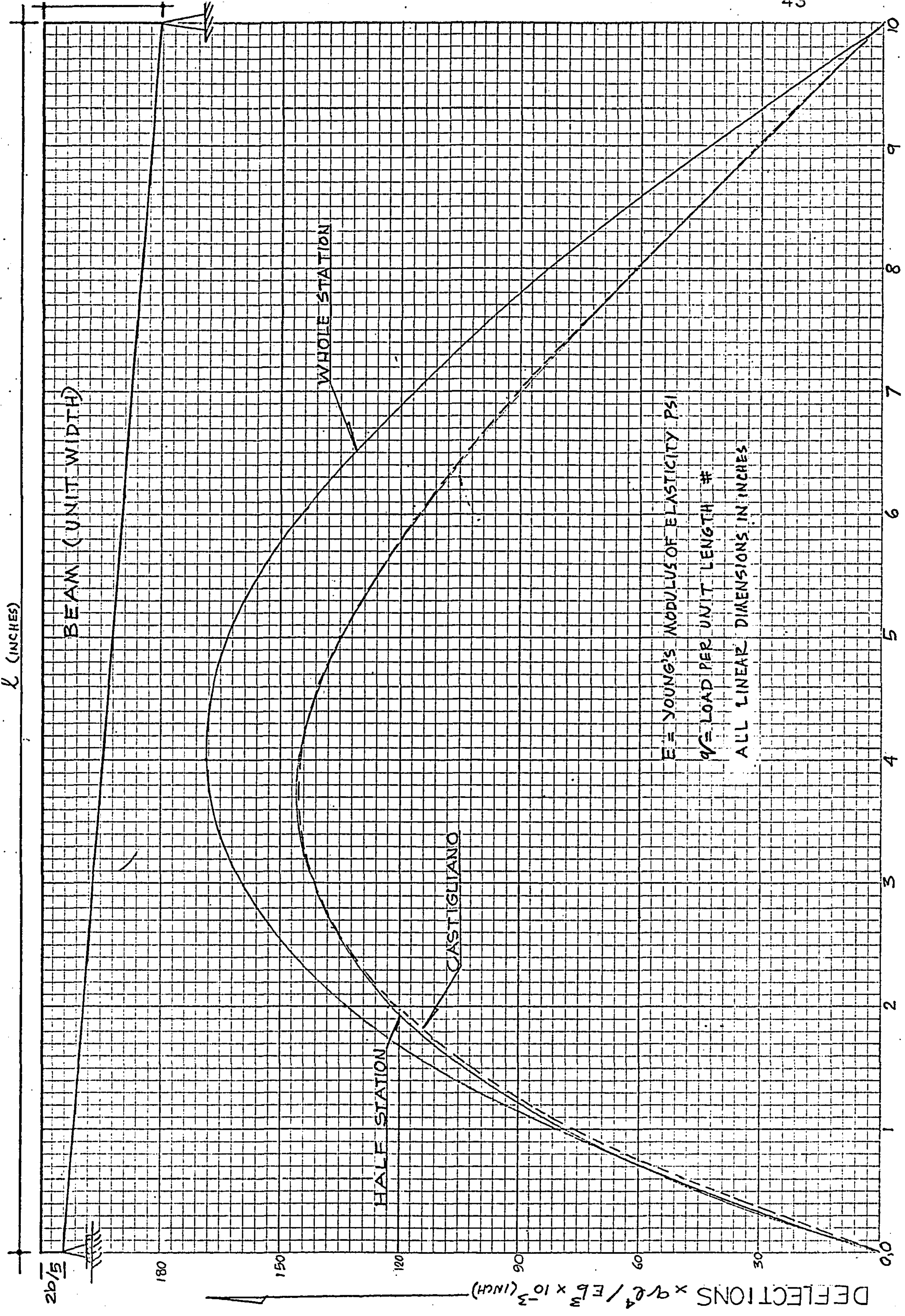
FIGURE 5



6 λx = 36"
2"
6"
0
+x
1
2
3
4
5
6
7
8
9
8'
A
BOTTOM VIEW
+y
SECTION A-A
2t
t
1.5"
t = 1/4"
END ELEVATION
6λy = 24
4"
d(y)
d(y) = 1.5 [1 + SIN(πy/24)]

FIGURE 6


BEAM (UNIT WIDTH)
l (INCHES)
2
2b/5
WHOLE STATION
HALF STATION
CASTIGLIANO
E = Young's Modulus of Elasticity in psi.
q = Load per unit length #
All other dimensions in inches.
(0,0)
1
2
3
4
5
6
7
8
9
10
30
60
90
120
150
180
DEFLECTIONS × qℓ⁴/EB³ × 10⁻³ (INCH)


ℓ (INCHES)
BEAM (UNIT WIDTH)
WHOLE STATION
CASTIGLIANO
HALF STATION
E = YOUNG'S MODULUS OF ELASTICITY PSI
q = LOAD PER UNIT LENGTH #
ALL LINEAR DIMENSIONS IN INCHES
2b/5
180
150
120
90
60
30
0,0
1
2
3
4
5
6
7
8
9
10
DEFLECTIONS × qℓ⁴/Eb³ × 10⁻³ (INCH)

DEFLECTION AT POINT 3 FOR A LOAD AT CENTER

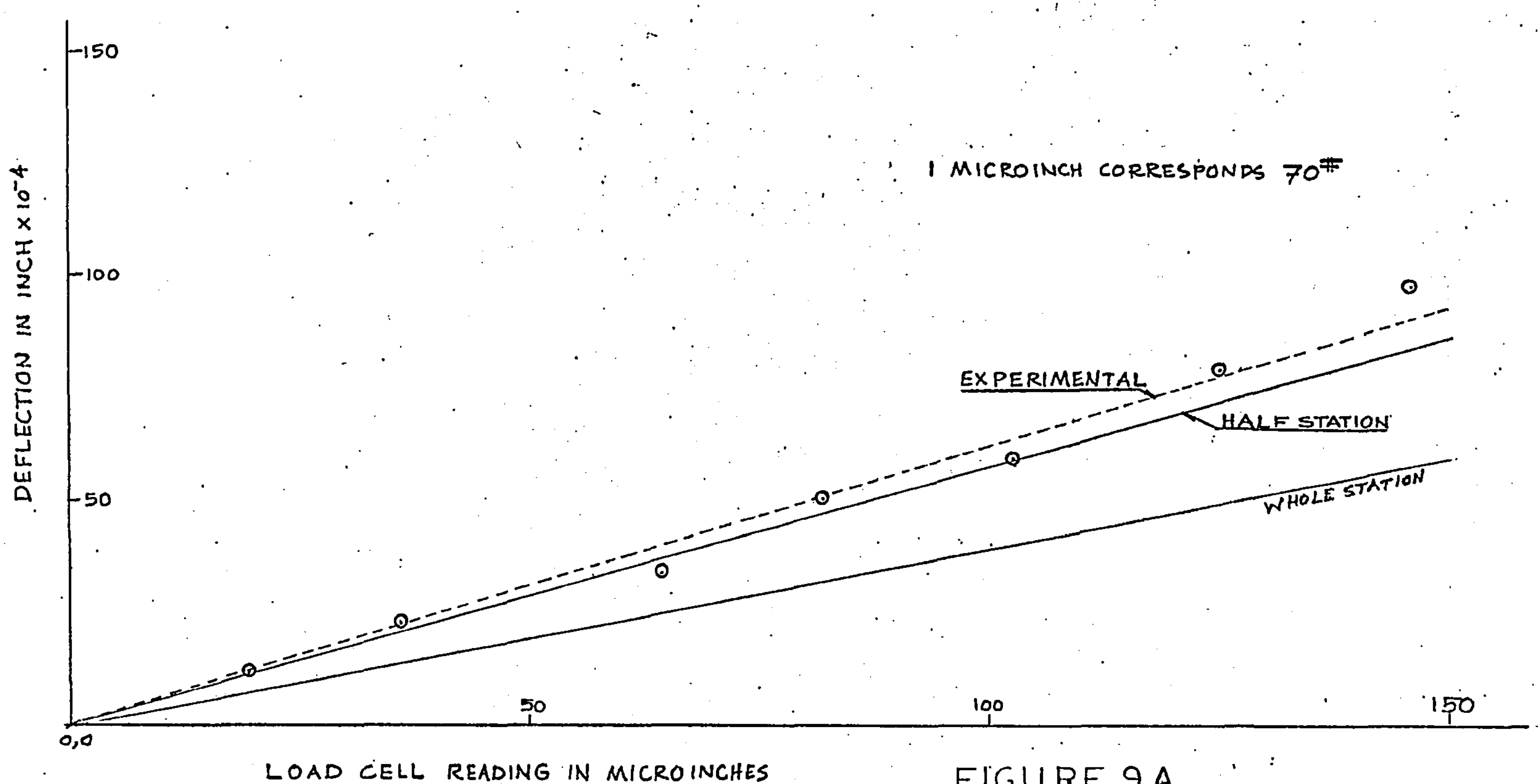
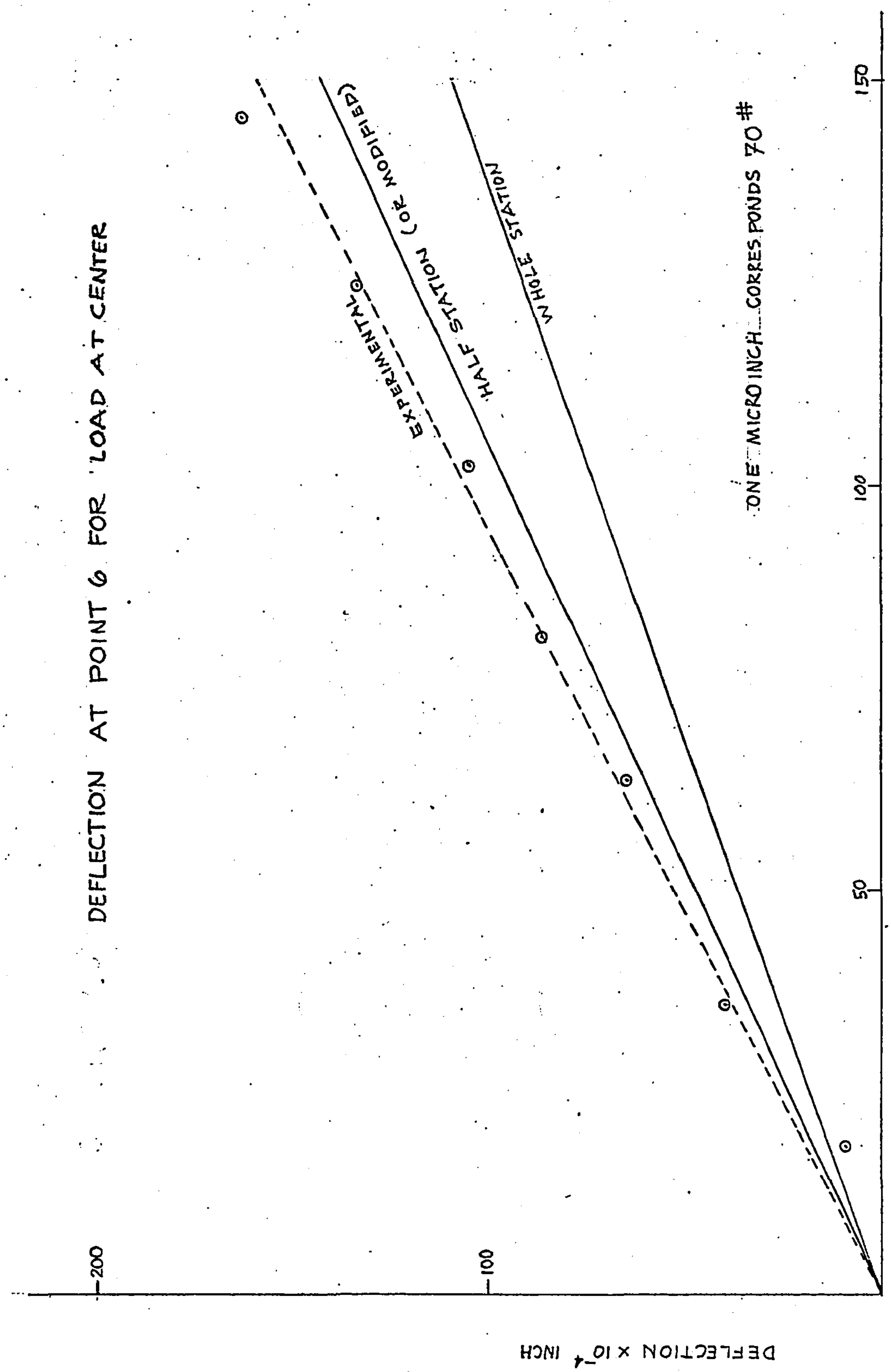


FIGURE 9A



LOAD CELL READING μ

FIGURE 9 B

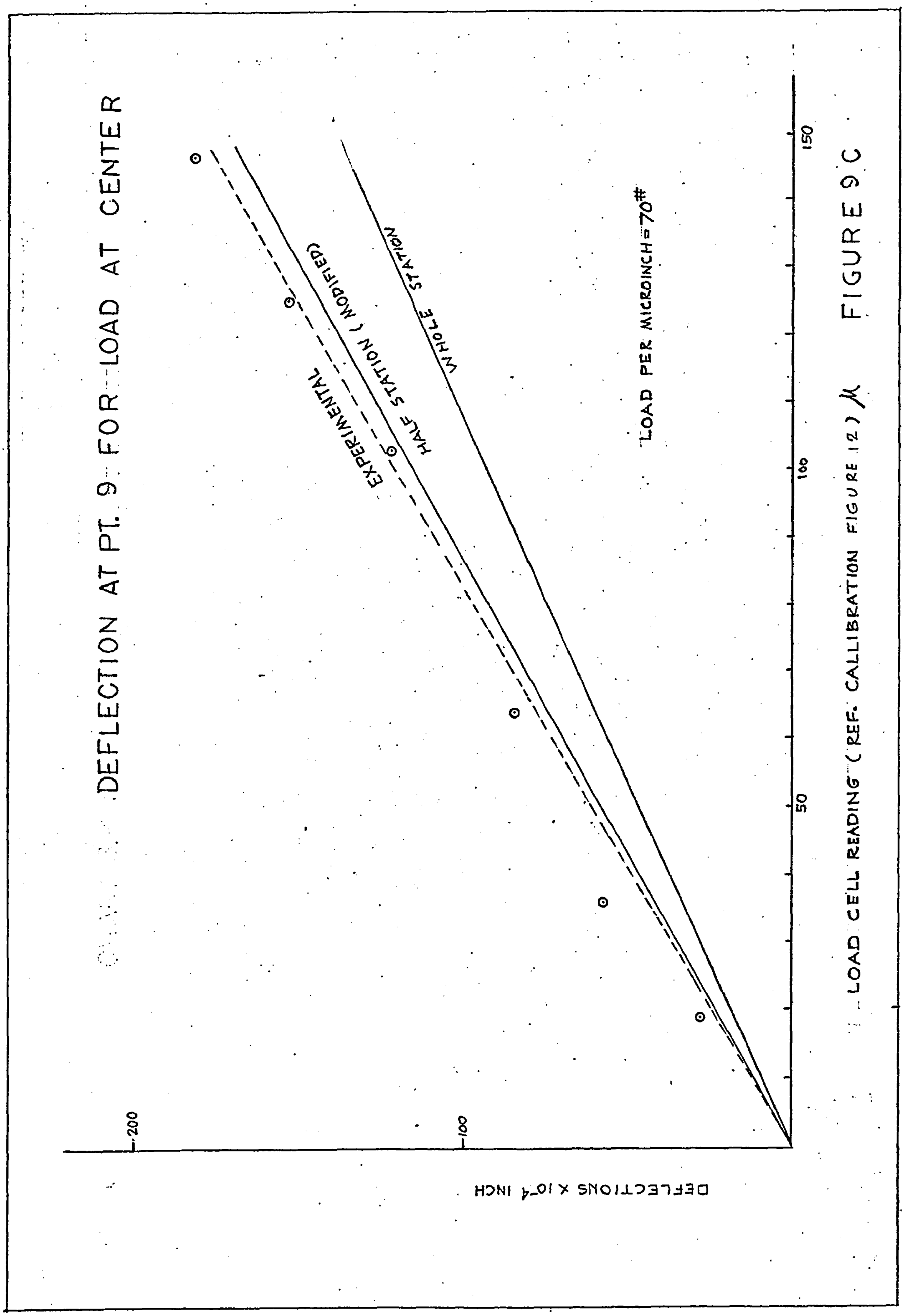

DEFLECTION AT PT. 9 FOR LOAD AT CENTER
WHOLE STATION
HALF STATION (MODIFIED)
EXPERIMENTAL
LOAD PER MICROINCH = 70#
150
100
50
200
100
DEFLECTIONS x 10^-4 INCH
LOAD CELL READING (REF. CALLIBRATION FIGURE 12)

FIGURE 9C

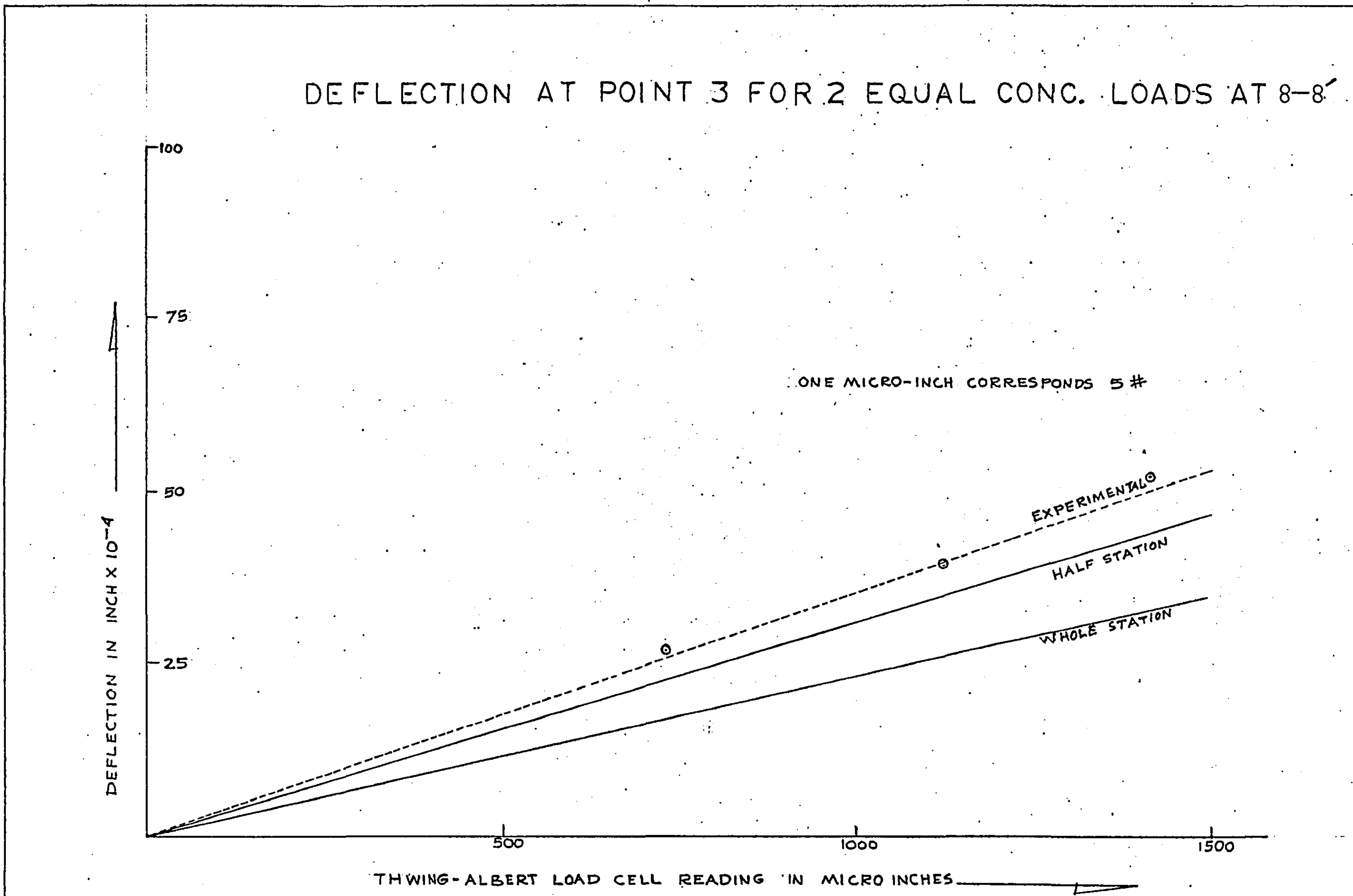

DEFLECTION AT POINT 3 FOR 2 EQUAL CONC. LOADS AT 8–8'
ONE MICRO-INCH CORRESPONDS 5 #
EXPERIMENTAL
HALF STATION
WHOLE STATION
100
75
50
25
500
1000
1500
DEFLECTION IN INCH X 10^-4
THWING-ALBERT LOAD CELL READING IN MICRO INCHES

FIGURE 10 A

DEFLECTION AT POINT 6 FOR 2 EQUAL CONC. LOADS AT 8-8′

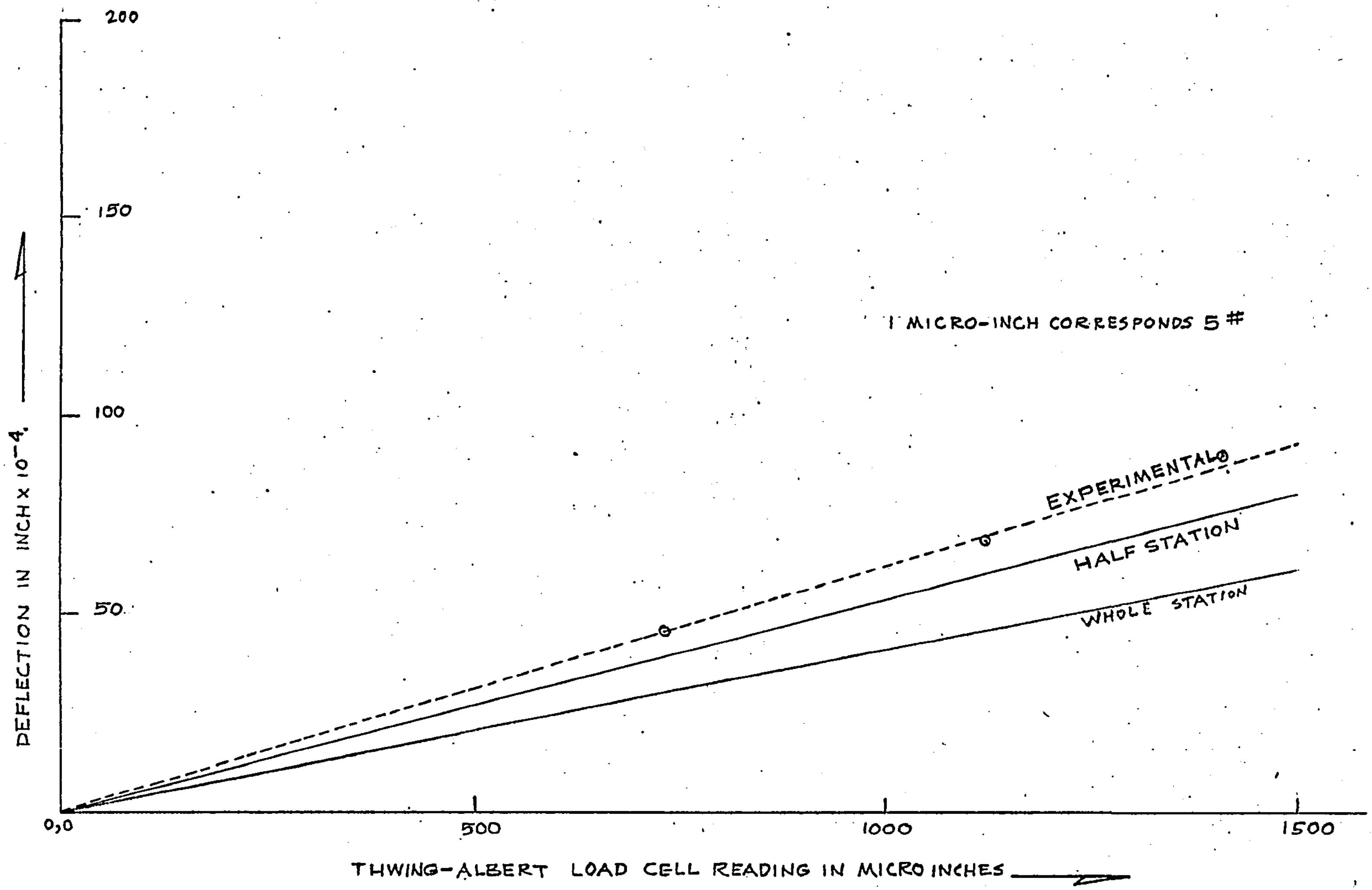


FIGURE 10B

DEFLECTION AT POINT 9 FOR TWO EQUAL CONC. LOADS AT 8-8′

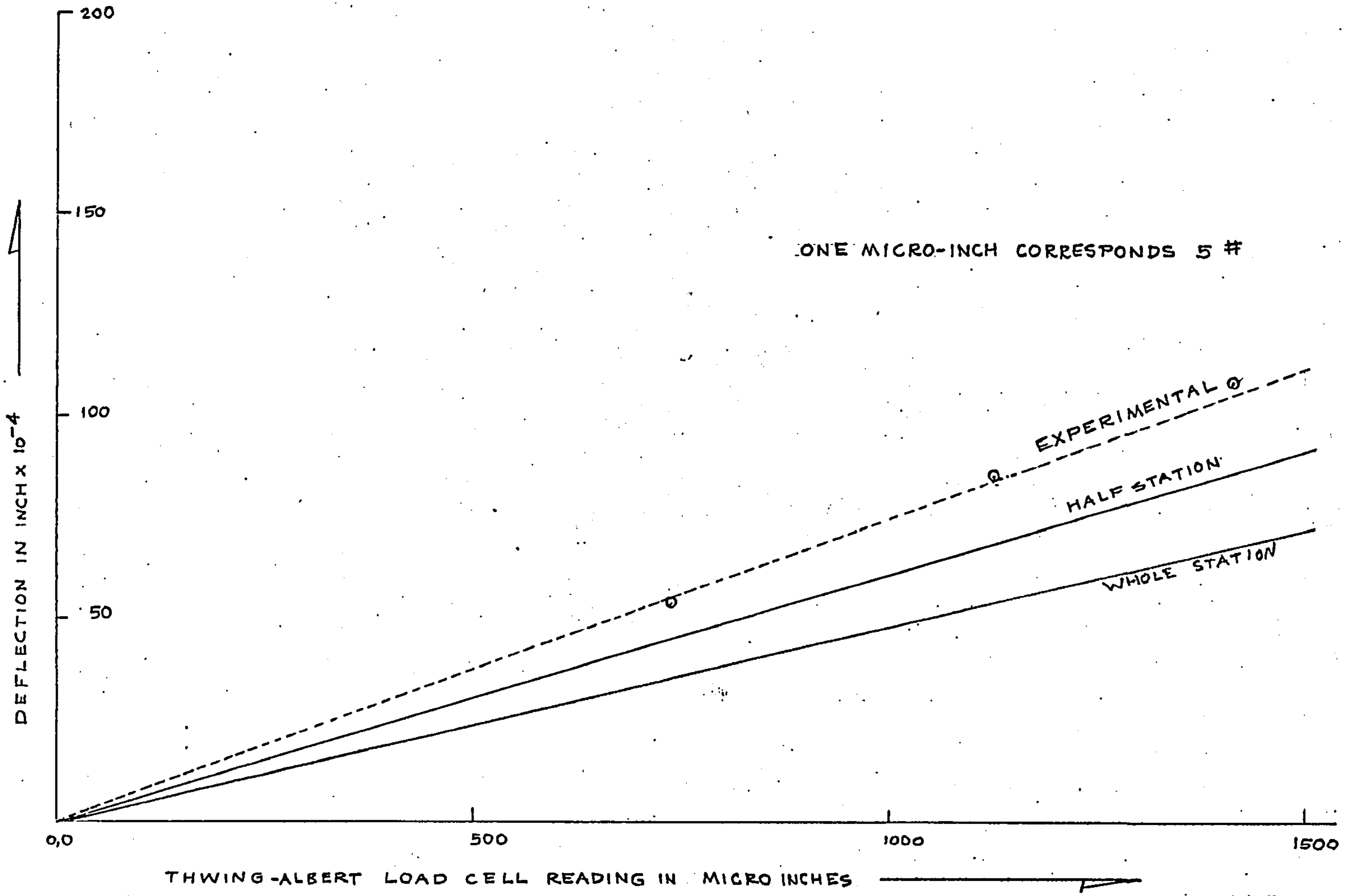


FIGURE 10C

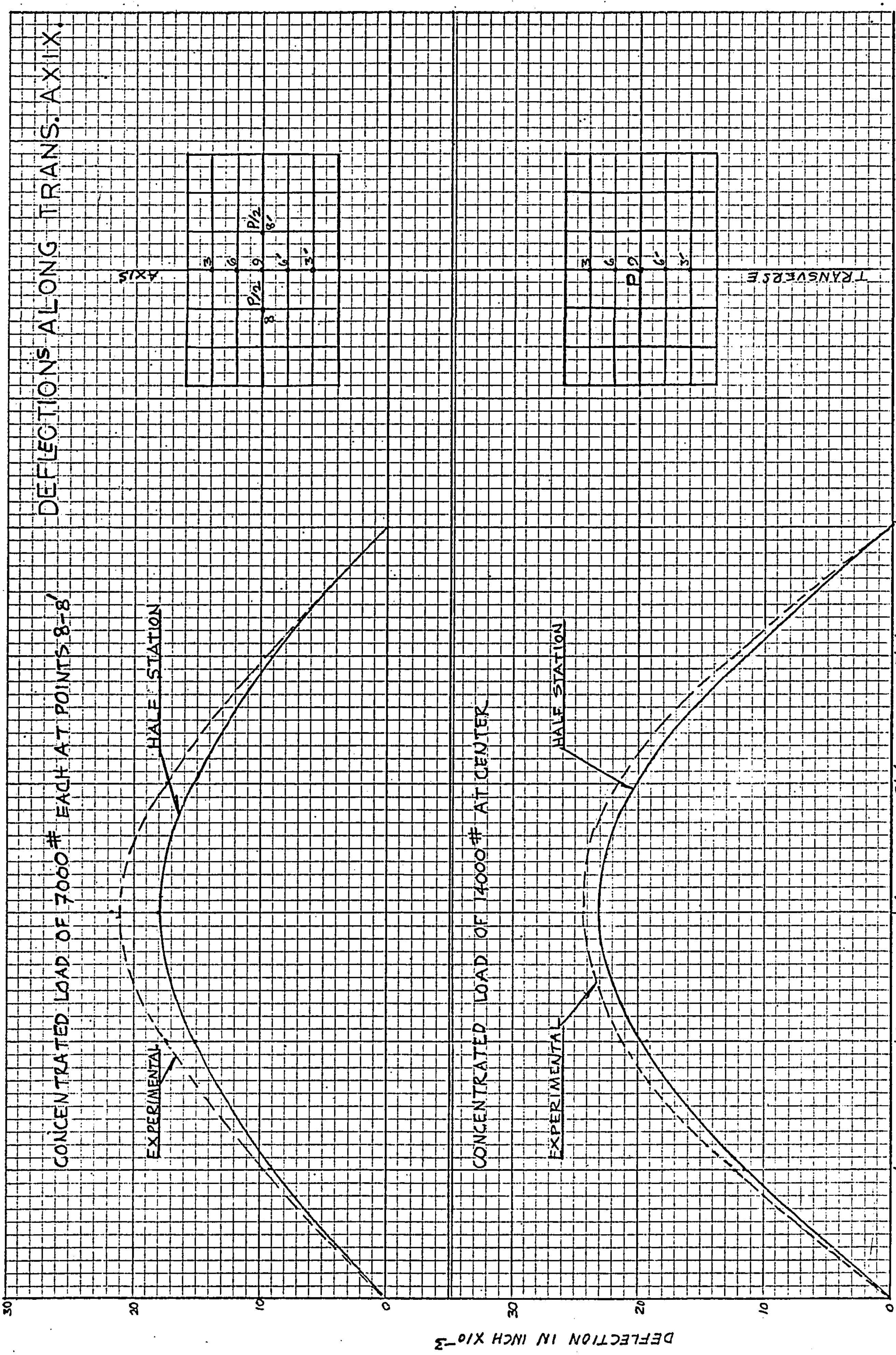

DEFLECTIONS ALONG TRANS. AXIX.
CONCENTRATED LOAD OF 7000# EACH AT POINTS 8-8'
HALF STATION
EXPERIMENTAL
CONCENTRATED LOAD OF 14000# AT CENTER
HALF STATION
EXPERIMENTAL
AXIS
TRANSVERSE
P/2
P
b/6
b/3
b/2
2b/3
5b/6
b
0
10
20
30
DEFLECTION IN INCH X10-3

FIGURE 11

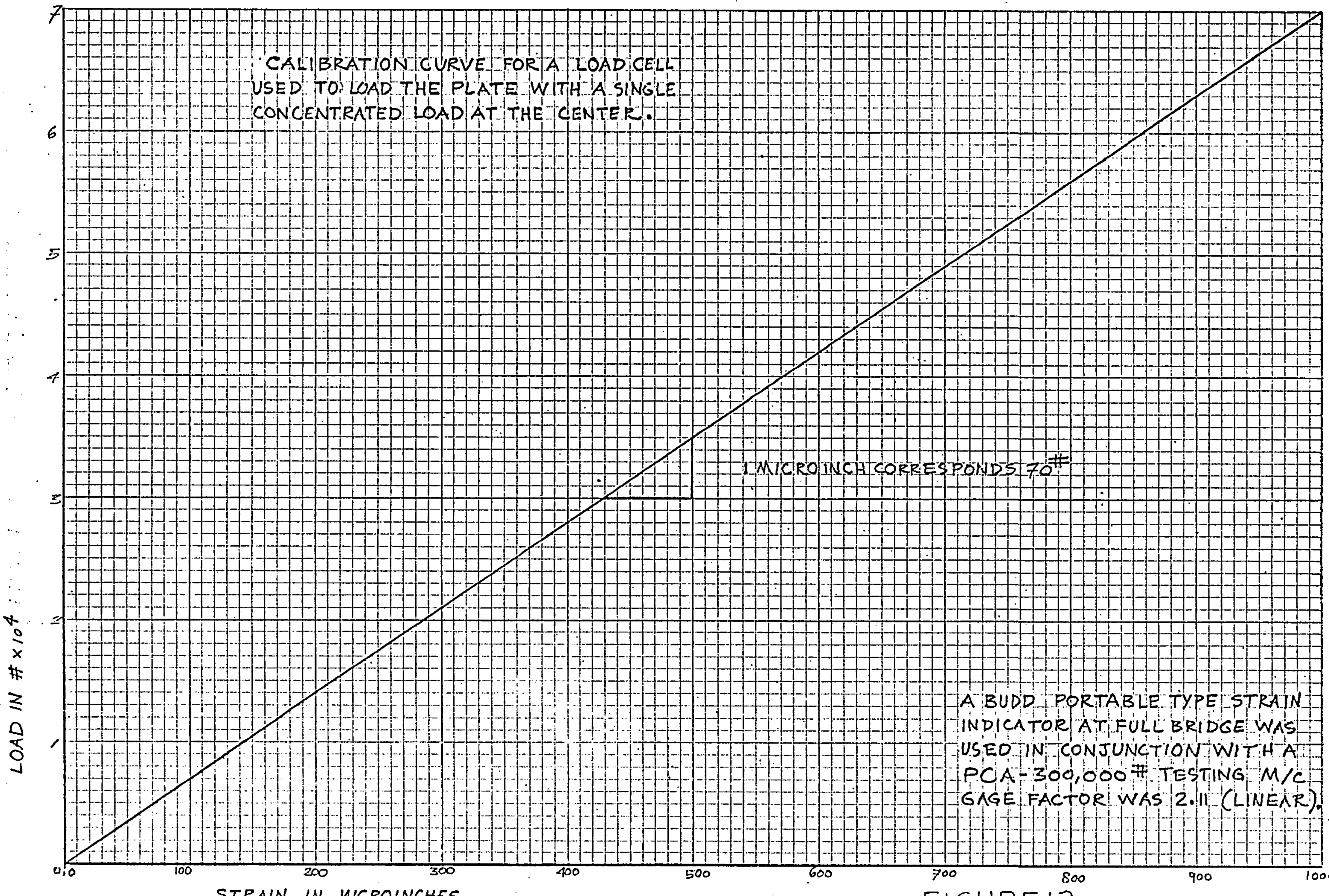

CALIBRATION CURVE FOR A LOAD CELL
USED TO LOAD THE PLATE WITH A SINGLE
CONCENTRATED LOAD AT THE CENTER.
1 MICROINCH CORRESPONDS 70#
A BUDD PORTABLE TYPE STRAIN
INDICATOR AT FULL BRIDGE WAS
USED IN CONJUNCTION WITH A
PCA-300,000# TESTING M/C
GAGE FACTOR WAS 2.11 (LINEAR).
LOAD IN # x 10^4
STRAIN IN MICROINCHES
0,0
100
200
300
400
500
600
700
800
900
1000
1
2
3
4
5
6
7

FIGURE 12

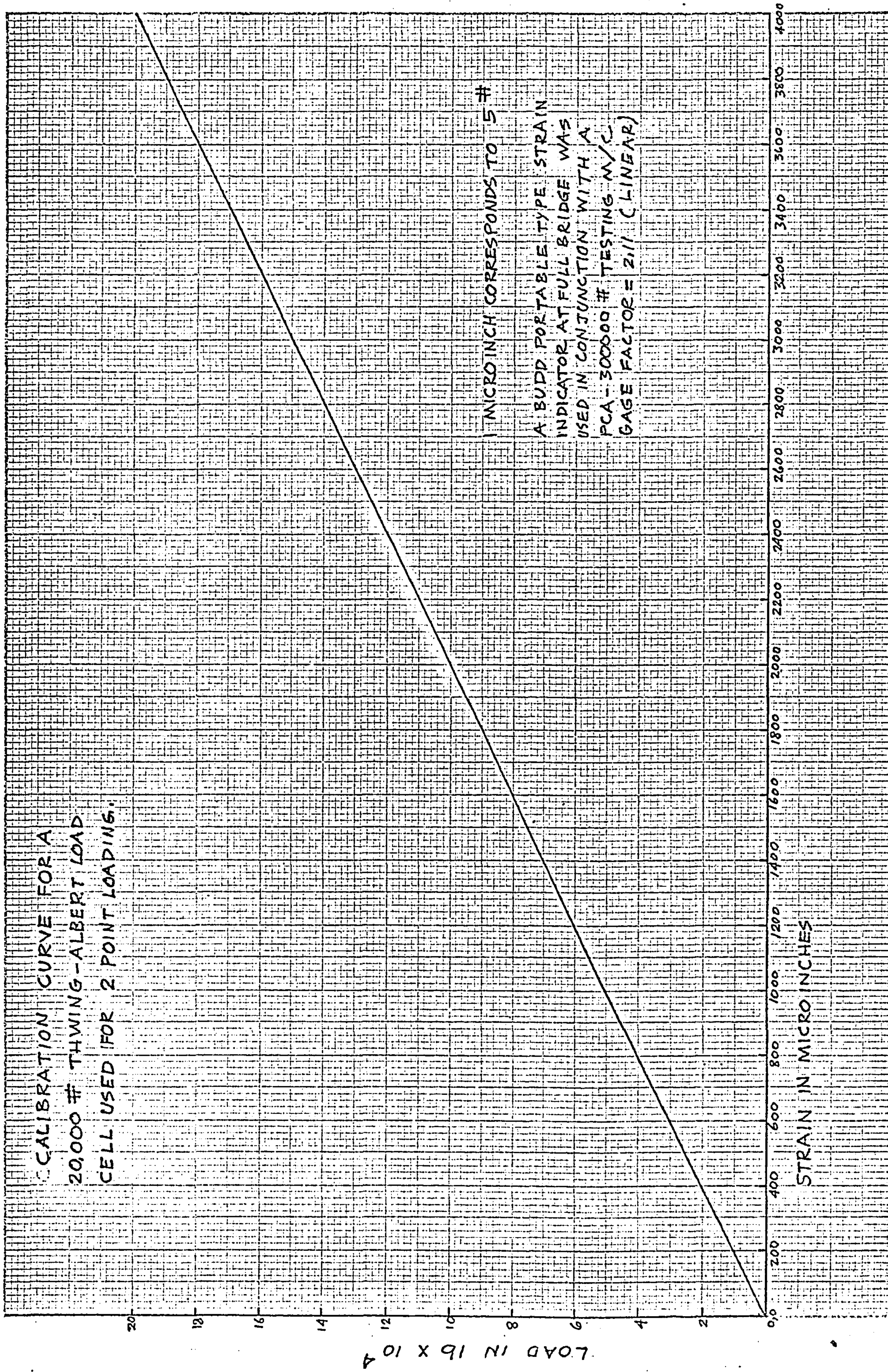

CALIBRATION CURVE FOR A
20,000 # THWING-ALBERT LOAD
CELL USED FOR 2 POINT LOADING.
1 MICROINCH CORRESPONDS TO 5 #
A BUDD PORTABLE TYPE STRAIN
INDICATOR AT FULL BRIDGE WAS
USED IN CONJUNCTION WITH A
PCA - 300000 # TESTING M/C
GAGE FACTOR = 2.11 (LINEAR)
STRAIN IN MICRO INCHES
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200 3400 3600 3800 4000
LOAD IN lb X 10 4
0 2 4 6 8 10 12 14 16 18 20

FIGURE 13

APPENDIX-A

Modified Central Difference Operators: [4]

The following notations have been used:

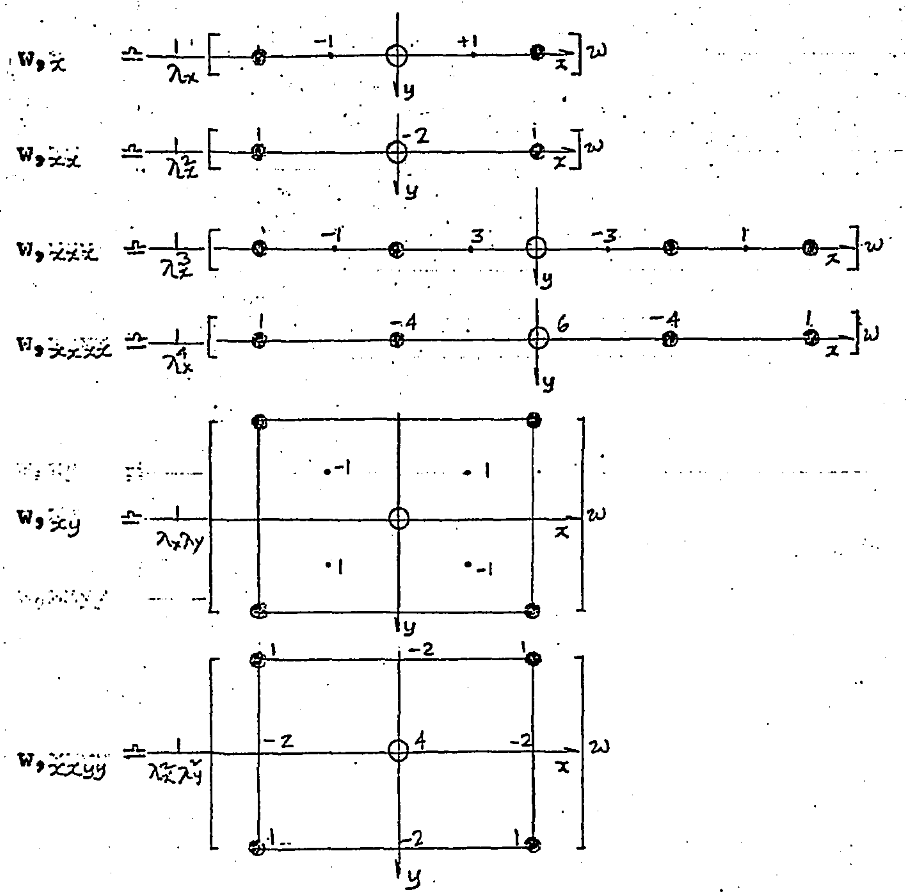
A Full Mesh Point -- ⊗

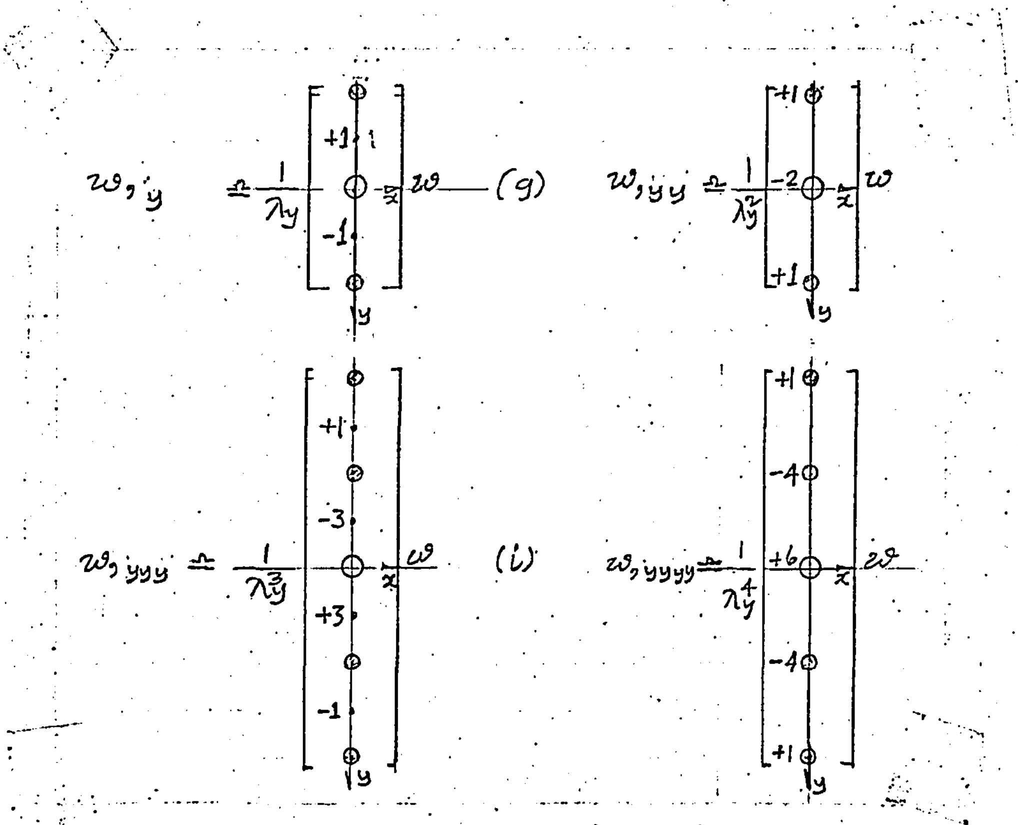
An Intermediate Mesh Point-------------------------------- •

Mesh Point at which the derivative is approximated -------- ○

To evaluate the derivative of any point within the plate boundary each of the arithmetical figures,with its proper sign, quoted on each mesh point has to be multiplied by the appropriate deflection function. The function products thus obtained have to be added algebrically to get the difference equation of that interior point.

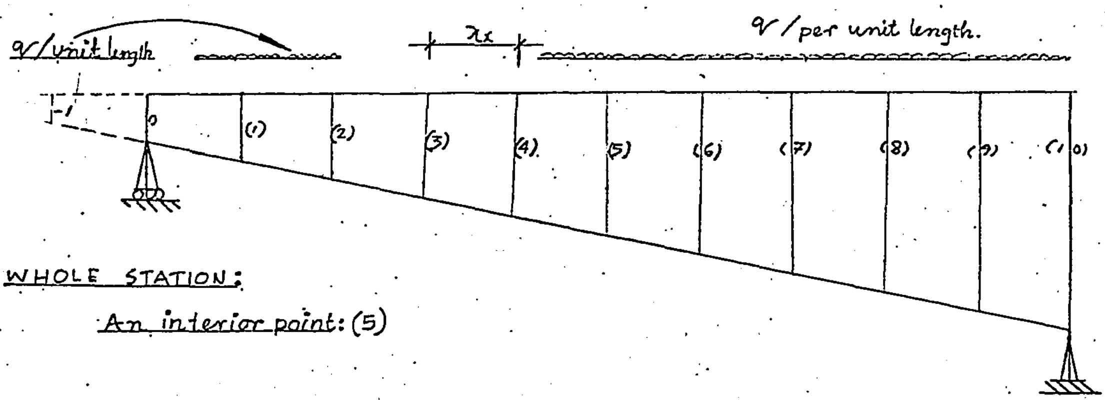
Following are the few modified central difference operators used to evaluate derivatives commonly encountered in plate problems with variable rigidities:





From the above it may be noted that the modified central difference method differs from the conventional central difference formulation only in approximating the odd order derivatives, otherwise both the formulations are similar in approximating even order derivatives.

Difference equations of a beam of variable rigidity under an uniformly distb. load:



WHOLE STATION:

An interior point: (5)

$$\frac{Bx_5}{\lambda_x^4}\left[w_3 - 4w_4 + 6w_5 - 4w_6 + w_7\right] + \frac{2}{\lambda_x^3}\left[Bx_5\right]_{,x}\left[-w_3 + 2w_4 - 2w_6 + w_7\right]$$

$$+ \frac{1}{\lambda_x^2}\left[Bx_5\right]_{,xx}\left[w_4 - 2w_5 + w_6\right] = q$$

Any point near the boundary: (1)

$$\frac{Bx_1}{\lambda_x^4}\left[5w_1 - 4w_2 + w_3\right] + \frac{2}{\lambda_x^3}\left[Bx_1\right]_{,x}\left[-w_1 - 2w_2 + w_3\right] + \frac{1}{\lambda_x^2}\left[Bx_1\right]_{,xx}\left[-2w_1 + w_2\right] = q$$

HALF STATION:

An interior point (5):

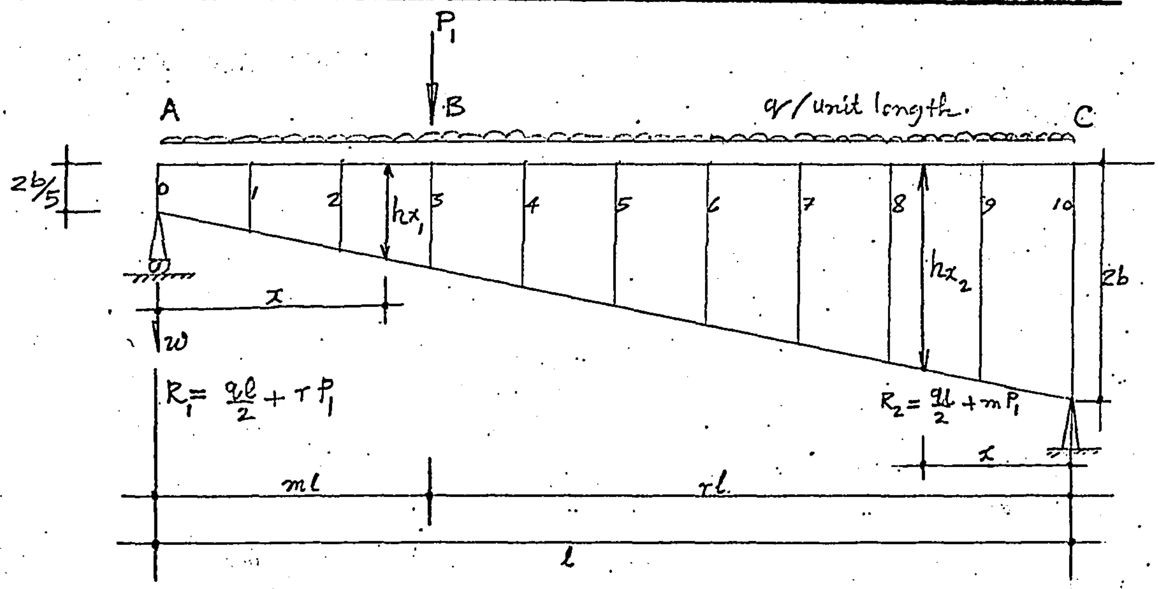
$$\frac{Bx_4}{\lambda_x^4}\left[w_3 - 2w_4 + w_5\right] - 2\frac{Bx_5}{\lambda_x^4}\left[w_4 - 2w_5 + w_6\right] + \frac{Bx_6}{\lambda_x^4}\left[w_5 - 2w_6 + w_7\right] = q$$

Any point near the support (1):

$$-2Mx_1 + Mx_2 = q\lambda_x^2$$

or, $$-2\frac{Bx_1}{\lambda_x^4}\left[0 - 2w_1 + w_2\right] + \frac{Bx_2}{\lambda_x^4}\left[w_1 - 2w_2 + w_3\right] = q$$

Calculation of Deflections of a Beam Using Castigliano's Second Theorem:



$$h_{x_1} = \frac{2b}{5l}(l+4x) \qquad h_{x_2} = \frac{2b}{5l}(5l-4x)$$

$$I_{x_1} = h_x^3/12 = \frac{8b^3(l+4x)^3}{1500\,l^3} \qquad I_{x_2} = \frac{8b^3(5l-4x)^3}{1500\,l^3}$$

$$M_1 = \left(\frac{ql}{2} + rP_1\right)x - \frac{qx^2}{2} \qquad M_2 = \left(\frac{ql}{2} + mP_1\right)x - \frac{qx^2}{2}$$

$$\frac{\partial M_1}{\partial P_1} = rx \qquad \frac{\partial M_2}{\partial P_1} = mx$$

From $A \longrightarrow B$ $(0 < x < ml)$:

$$W_1 = \int \frac{M_1^2\,dx}{2EI_1} \quad \therefore \quad \frac{\partial W_1}{\partial P_1} = \int M_1 \frac{\partial M_1}{\partial P_1} \cdot \frac{dx}{EI_{x_1}}$$

$$\therefore \frac{\partial W_1}{\partial P_1} = \int_0^{ml} \left[\frac{qlx}{2} - \frac{qx^2}{2}\right] rx \times \frac{dx}{E} \times \frac{1500\,l^3}{8b^3(l+4x)^3} = \frac{1500\,ql^3 r}{16Eb^3} \int_0^{ml} \frac{(lx-x^2)x\,dx}{(l+4x)^3}$$

$$= \frac{1500\,ql^3}{16Eb^3} \times \frac{lr}{256}\left[-4m + 7\log(4m+1) + 11\left\{\frac{1}{4m+1} - 1\right\} - \frac{5}{2}\left\{\frac{1}{(4m+1)^2} - 1\right\}\right]$$

From $C \longrightarrow B$ $(0 < x < rl)$:

$$\frac{\partial W_2}{\partial P_1} = -\frac{15\,m\,ql^3}{16Eb^3} \times \frac{lm}{256}\left[-4r - 11\log\left(\frac{5-4r}{5}\right) - 35\left\{\frac{1}{(5-4r)} - \frac{1}{5}\right\} + \frac{25}{2}\left\{\frac{1}{(5-4r)^2} - \frac{1}{25}\right\}\right]$$

Deflection at any point B :

$$= \frac{\partial W_1}{\partial P_1} + \frac{\partial W_2}{\partial P_1}$$

$$= \frac{1500\,r}{4096} \cdot \frac{q l^4}{E b^3}\left[-4m + 7\log(4m+1) + 11\left\{\frac{1}{4m+1} - 1\right\} - \frac{5}{2}\left\{\frac{1}{(4m+1)^2} - 1\right\}\right]$$

$$- \frac{1500\,m}{4096} \cdot \frac{q l^4}{E b^3}\left[-4r - 11\log\left(\frac{5-4r}{5}\right) - 35\left\{\frac{1}{5-4r} - \frac{1}{5}\right\} + \frac{25}{2}\left\{\frac{1}{(5-4r)^2} - \frac{1}{25}\right\}\right]$$

$$= \frac{q l^4}{E b^3} \cdot T_1 + \frac{q l^4}{E b^3} \cdot T_2$$

where $T_1 = \frac{1500\,r}{4096} \cdot \left[-4m + 7\log(4m+1) + 11\left\{\frac{1}{4m+1} - 1\right\} - \frac{5}{2}\left\{\frac{1}{(4m+1)^2} - 1\right\}\right]$

& $T_2 = -\frac{1500\,m}{4096}\left[-4r - 11\log\left(\frac{5-4r}{5}\right) - 35\left\{\frac{1}{5-4r} - \frac{1}{5}\right\} + \frac{25}{2}\left\{\frac{1}{(5-4r)^2} - \frac{1}{25}\right\}\right]$

Let, $m = AM$

$r = AR = 1 - AM$

$4m + 1 = 4AM + 1 = AZ$

$5 - 4r = 5 - 4AR = BZ$

$(5-4r)/5 = (5-4AR)/5 = CZ$

$1500r/4096 = 1500\,AR/4096 = Z_1$

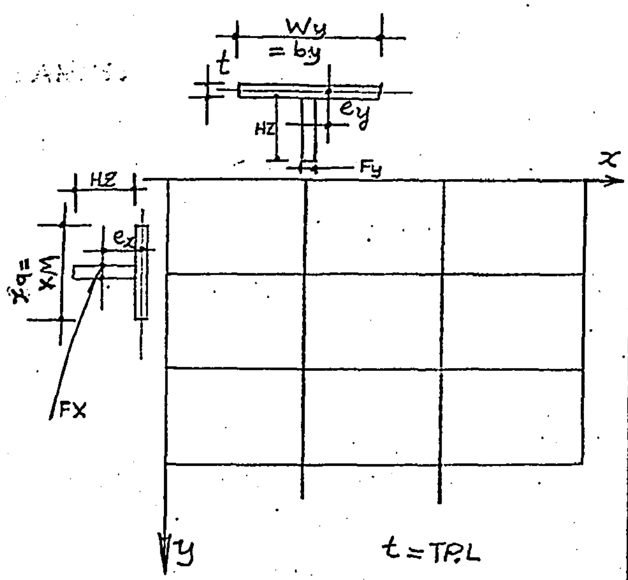
$-1500m/4096 = -1500\,AM/4096 = Z_2$

$\therefore$ $T_1 = Z_1\,[-4\,AM + 7\,LOGF(AZ) + 11(1/AZ - 1) - 2.5(1/AZ^2 - 1)]$

$T_2 = Z_2\,[-4AR - 11\,LOGF(CZ) - 35(1/BZ - 1/5) + 12.5(1/BZ^2 - 1/25)]$

A computer programme has been done with the above notations and solutions of deflections for eighty different nodal points are presented.

Derivation of the Values of Plate Rigidity Constants



$$D_x = \frac{1}{b_x} \int E(z)\, dF(x)$$

$$= \frac{E}{b_x} \left[b_x t + HZ \cdot F_x \right]$$

$$= (WX * TPL + FX * HZ) * E / ABX$$

$$D_y = \frac{1}{b_y} \int E(z)\, dF(y)$$

$$= (WY * TPL + FY * HZ) * E / ABY$$

$$e_x = \frac{1}{b_x D_x} \int E(z)\, z\, dF(x)$$

$$= \frac{E(z)}{b_x D_x} \left\{ \int_{-t/2}^{+t/2} \left[WX \cdot z\, dz \right] + \int_{t/2}^{HZ+t/2} \left[FX \cdot z\, dz \right] \right\}$$

$$= \frac{E}{b_x D_x} \left\{ \left[WX \cdot z^2/2 \right]_{-t/2}^{+t/2} + \left[FX\, z^2/2 \right]_{t/2}^{HZ+t/2} \right\}$$

$$= \frac{E}{b_x D_x} \left\{ 0 + \frac{FX}{2} \left[(HZ + t/2)^2 - (t/2)^2 \right] \right\}$$

$$= \left[AZ^2 - (TPL/2)^2 \right] * E * FX / (ABX * DX * 2)$$

Let AZ=HZ+t/2

$$e_y = \left[AZ^2 - (TPL/2)^2 \right] * E * FY / (ABY * DY * 2)$$

$$B_x = E I_x / W_x = \frac{E}{W_x} \left[FX \cdot HZ^3/12 + W_x t^3/12 + W_x \cdot t\, e_x^2 + FX \cdot HZ \left(\frac{HZ + t}{2} - e_x \right)^2 \right]$$

$$= \left[HZ^3 * FX + TPL^3 * WX + 12 ex^2 * WX * TPL + (HZ + TPL - 2ex)^2 * 3HZ \cdot FX \right] E / (12 * WX)$$

$$B_y = \left[HZ^3\, FY + TPL^3 * WY + 12\, ey^2 * WY * TPL + (HZ + TPL - 2ey)^2 * 3HZ\, FY \right] E / (12 * WY)$$

$$B_{xy} = \frac{1}{3b_x} \int G t^2\, dF(x) = \frac{G t^2}{3 b_x} \left[WX * TPL + HZ * FX \right] = \left[WX * TPL + HZ * FX \right] * TPL^2 * G / (3 * ABX)$$

$$B_{yx} = \left[WY * TPL + HZ * FY \right] * TPL^2 * G / (3 * ABY).$$

Derivation Of Equation (20) Based On Energy Expression (1.21) of Reference [10]:

Instead of using equation (11), if one proceeds with the following energy expression for any interior point K (figure 5):

$$U = \frac{1}{2}\iint \{ D_x u_{,xx}^2 + D_y v_{,yy}^2 + \frac{1-\nu}{2} D(u_{,y} + v_{,x})^2 + 2\nu D\, u_{,x}^2 v_{,y} - 2e_x D_x u_{,x} w_{,xx}$$

$$- 2e_y D_y v_{,y} w_{,yy} + (B_x + e_x^2 D_x) w_{,xx}^2 + (B_y + e_y^2 D_y) w_{,yy}^2 + 2\nu B w_{,xx} w_{,yy}$$

$$+ [2B(1-\nu) + B_{xy} + B_{yx}] w_{,xy}^2 - 2qw \} \, dx\, dy$$

Now using the approximation [13] $u = e_x w_{,x}$ & $v = e_y w_{,y}$ the above eqn can be written as:

$$U = \frac{1}{2}\iint \{ D_x e_x^2 w_{,xx}^2 + D e_y^2 w_{,yy}^2 + [\frac{1-\nu}{2} D(e_x + e_y)^2 + 2B(1-\nu) + B_{xy}$$

$$+ B_{yx}] w_{,xy}^2 + (B_x + e_x^2 D_x) w_{,xx}^2 + (B_y + e_y^2 D_y) w_{,xy}^2 - 2D_x e_x^2 w_{,xx}^2$$

$$- 2D_y e_y^2 w_{,yy}^2 + 2\nu [D e_x e_y + B] w_{,xx} w_{,yy} - 2qw \} \, dx\, dy$$

$$= \iint \{ B_x/_2\, w_{,xx}^2 + B_y/_2\, w_{,yy}^2 + \nu [B + D e_x e_y] w_{,xx} w_{,yy} + [B(1-\nu) + \frac{B_{xy} + B_{yx}}{2}$$

$$+ (1-\nu)/_4 (e_x + e_y)^2] w_{,xy}^2 - qw \} \, dx\, dy$$

$$= \iint \{ B_x/_2\, w_{,xx}^2 + B_y/_2\, w_{,yy}^2 + BZ\, w_{,xx} w_{,yy} + SH\, w_{,xy}^2 - qw \} \, dx\, dy$$

$$= T_1 + T_2 + T_3 + T_5 - T_4 \quad \text{where, } SH = B(1-\nu) + (B_{xy} + B_{yx})/2 + (1-\nu) D (e_x + e_y)^2/4$$

$$\text{and } BZ = \nu (B + D e_x e_y)$$

If the above equation is approximated by using modified finite differences, one observes that the terms T_1, T_2 & T_4 are the same as in equation (17), and both the terms T_3 and T_5 are different which may be approximated in the following ways; retaining only the terms required to minimize with respect to $w(K)$:

$$T_3 = \text{------} + BZ_{K-1} \frac{\{w_{K-2} - 2w_{K-1} + w_K\}\{w_{N-1} - 2w_{K-1} + w_{M-1}\}}{\lambda_x \lambda_y}$$

$$+ BZ_K \frac{\{w_{K-1} - 2w_K + w_{K+1}\}\{w_N - 2w_K + w_M\}}{\lambda_x \lambda_y}$$

$$+ BZ_{K+1} \frac{\{w_K - 2w_{K+1} + w_{K+2}\}\{w_{N+1} - 2w_{K+1} + w_{M+1}\}}{\lambda_x \lambda_y}$$

$$+ BZ_N \frac{\{w_{N-1} - 2w_N + w_{N+1}\}\{w_Q - 2w_N + w_K\}}{\lambda_x \lambda_y}$$

$$+ BZ_M \frac{\{w_{M-1} - 2w_M + w_{M+1}\}\{w_K - 2w_M + w_P\}}{\lambda_x \lambda_y} + \text{-----}$$

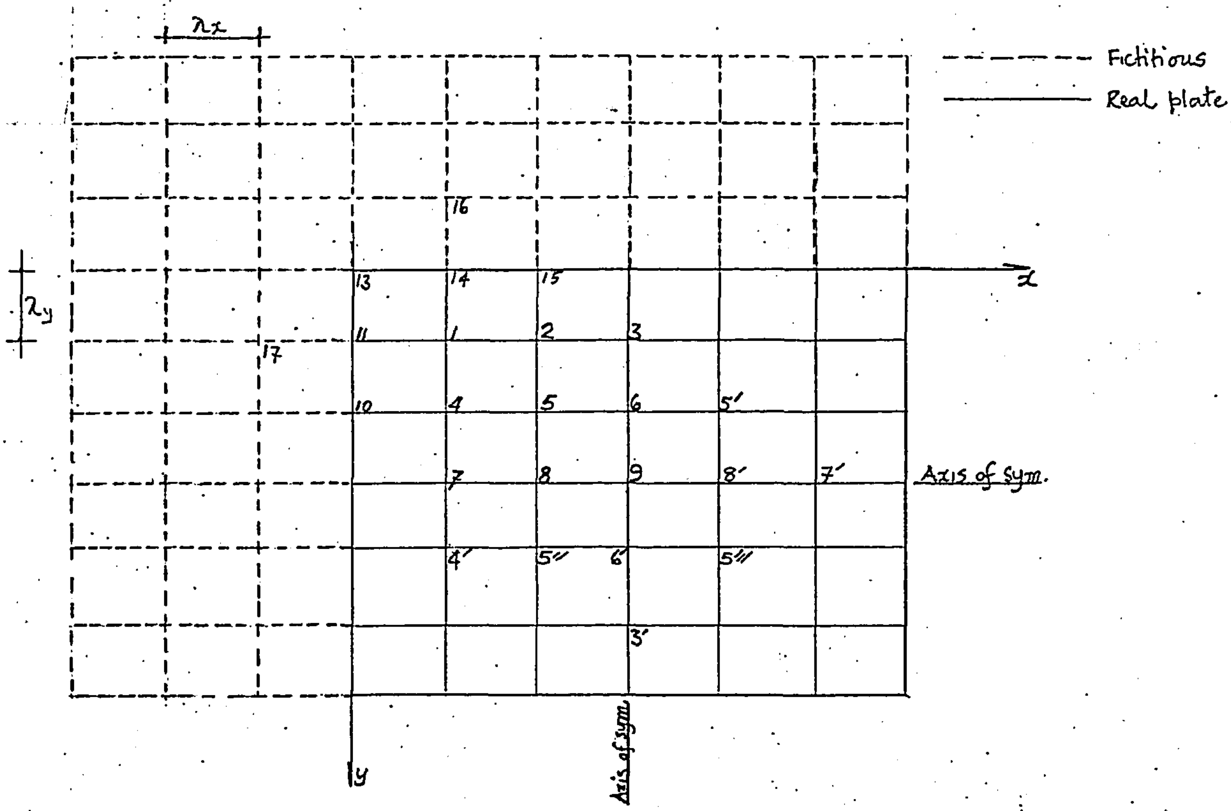
$$T_5 = \text{--------} + SH_a\,\lambda_x\lambda_y\left[\frac{\omega_{N-1} - \omega_N - \omega_{K-1} + \omega_K}{\lambda_x\lambda_y}\right]^2$$
$$+ SH_b\,\lambda_x\lambda_y\left[\frac{\omega_K + \omega_{N+1} - \omega_N - \omega_{K+1}}{\lambda_x\lambda_y}\right]^2$$
$$+ SH_c\,\lambda_x\lambda_y\left[\frac{-\omega_{K-1} + \omega_K + \omega_{M-1} - \omega_M}{\lambda_x\lambda_y}\right]^2$$
$$+ SH_d\,\lambda_x\lambda_y\left[\frac{\omega_K - \omega_{K+1} - \omega_M + \omega_{M+1}}{\lambda_x\lambda_y}\right]^2 + \text{----}$$

Differentiating U with respect to $\omega(K)$ and putting it equal to zero, ie.,

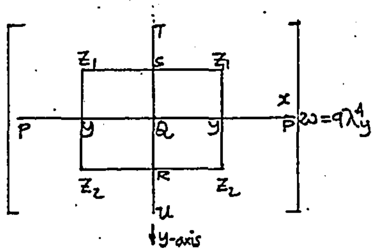
$$\frac{\partial U}{\partial \omega(K)} = \frac{\partial T_1}{\partial \omega(K)} + \frac{\partial T_2}{\partial \omega(K)} + \frac{\partial T_3}{\partial \omega(K)} + \frac{\partial T_5}{\partial \omega(K)} - \frac{\partial T_4}{\partial \omega(K)} = 0,$$ one obtains,

equation (20) where, $AS.BZ = \frac{1}{2}BEZ$ and $2AS.SH = BEC$ and the term SH is averaged as, $SH_a = SH_b \triangleq [SH_N + SH_K]/2$ and $SH_c = SH_d \triangleq [SH_K + SH_M]/2$ since the plate has constant rigidity in x direction,

Application of Difference Equation (20) at Mesh Points 9 and 1 of the Plate:



At Point (9):



$$Q\omega_9 + P\{\omega_7 + \omega_{7'}\} + y\{\omega_8 + \omega_{8'}\} + S\omega_6 + R\omega_{6'} + T\omega_3$$

$$+ u\omega_{5'} + Z_1\{\omega_5 + \omega_{5'}\} + Z_2\{\omega_{5''} + \omega_{5'''}\} = q\lambda_y^4$$

Or $\{T+U\}\omega_3 + 2\{Z_1+Z_2\}\omega_5 + \{S+R\}\omega_6 + 2P\omega_7 + 2y\omega_8 + Q\omega_9 = q\lambda_y^4$

[Since, $\omega_3 = \omega_{3'}$, $\omega_5 = \omega_{5'} = \omega_{5''} = \omega_{5'''}$, $\omega_6 = \omega_{6'}$, $\omega_7 = \omega_{7'}$ & $\omega_8 = \omega_{8'}$]

At Point (1):

$$Q\omega_1 + P\{\omega_{17} + \omega_3\} + y\{\omega_{11} + \omega_2\} + S\omega_{14} + R\omega_4 + T\omega_{16} + U\omega_7 + Z_1\{\omega_{13} + \omega_{15}\}$$

$$+ Z_2\{\omega_{10} + \omega_5\} = q\lambda_y^4$$

or $\{Q-T-P\}\omega_1 + y\omega_2 + P\omega_3 + R\omega_4 + Z_2\omega_5 + U\omega_7 = q\lambda_y^4$

[Since, for boundary conditions, $\omega_{10} = \omega_{11} = \omega_{13} = \omega_{14} = \omega_{15} = 0$ and $\omega_{16} = -\omega_1 = \omega_{17}$]

APPENDIX-B

```
ZZJOB 5
ZZFORX5
C       DEFLECTIONS OF A BEAM OF VARIABLE MOMENT OF INERTIA ---80-SEGMENTS
C       BY USING CASTIGLIANO.S SECOND THEOREM
C                  DEFLECTIONS.U=V*(L**4*Q)/(B**3*E)
        DO 4M=1.80
        AM=M
        AM=AM/80.
        AR=1.-AM
        AZ=4.*AM+1.
        BZ=5.-4.*AR
        CZ=BZ/5.
        Z1=AR*1500./4096.
        Z2=-AM*1500./4096.
        T1=Z1*(-4.*/ 1+7.         *LOGF(AZ)+11.*(1./AZ-1.)-2.5*(1./(AZ**2)-1.)
       1)
        T2=Z2*(-4.*AR-11.          *LOGF(CZ)-35.*(1./BZ-1./5.)+12.5*(1./(BZ**2
       1)-1./25.))
        V=T1+T2
        PRINT 11.V
     11 FORMAT(E16.8)
      4 CONTINUE
        CALL EXIT
        END
ZZZZ8
```

```
34000320070136000320070249024025119636113001020O
ZZJOB 5                         SIMPLY SUPPORTED BEAM-VARIABLE M.I.-10 SEG
ZZFOR 5
C       SOLUTION OF BEAM PROBLEMS BY FINITE DIFFERENCE METHOD
C       SIMPLY SUPPORTED BEAM-VARIABLE DEPTH-TEN SEGMENTS-MODIFIED
        DIMENSION X(10)
        DIMENSION A(25,25),F(25,25),W(25),W1(25)
        COMMON A,MA,NA,F,MF,NF,W
        XN=1.
        XX=9.
        XL=8.
        XD=1.
        YN=.0
        YX=.2
        YL=10.
        YD=.02
        CALL PLOT(101,XN,XX,XL,XD,YN,YX,YL,YD)
        NA=9
        MA=9
        AR=1500./80000.
        DO 4M=1,10
        AM=M
        X(M)=(1.+.4*AM)**3
      4 CONTINUE
        PUNCH 102,(X(M),M=1,10)
    102 FORMAT(27H          X(M)                X(M)/2E16.8)
        K=1
        L=2
        P=1.
        R=-2.*(1.+X(L))
        Q=1.+4.*X(K)+X(L)
        S=-2.*(X(K)+X(L))
        T=X(L)
        A( 1, 1)=Q-P
        A( 1, 2)=S
        A( 1, 3)=T
        A( 1, 4)=0.
        A( 1, 5)=0.
        A( 1, 6)=0.
        A( 1, 7)=0.
        A( 1, 8)=0.
        A( 1, 9)=0.
        K=2
        L=3
        N=1
        P=X(N)
        R=-2.*(X(N)+X(K))
        Q=X(N)+4.*X(K)+X(L)
        S=-2.*(X(K)+X(L))
        T=X(L)
        A( 2, 1)=R
        A( 2, 2)=Q
        A( 2, 3)=S
        A( 2, 4)=T
        A( 2, 5)=0.
        A( 2, 6)=0.
        A( 2, 7)=0.
        A( 2, 8)=0.
        A( 2, 9)=0.
        K=3
```

```
      N=2
      P=X(N)
      Q=X(N)+4.*X(K)+X(L)
      R=-2.*(X(N)+X(K))
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 3, 1)=P
      A( 3, 2)=R
      A( 3, 3)=Q
      A( 3, 4)=S
      A( 3, 5)=T
      A( 3, 6)=0.
      A( 3, 7)=0.
      A( 3, 8)=0.
      A( 3, 9)=0.
      K=4
      L=5
      N=3
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 4, 1)=0.
      A( 4, 2)=P
      A( 4, 3)=R
      A( 4, 4)=Q
      A( 4, 5)=S
      A( 4, 6)=T
      A( 4, 7)=0.
      A( 4, 8)=0.
      A( 4, 9)=0.
      K=5
      N=4
      L=6
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 5, 1)=0.
      A( 5, 2)=0.
      A( 5, 3)=P
      A( 5, 4)=R
      A( 5, 5)=Q
      A( 5, 6)=S
      A( 5, 7)=T
      A( 5, 8)=0.
      A( 5, 9)=0.
      K=6
      L=7
      N=5
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 6, 1)=0.
      A( 6, 2)=0.
      A( 6, 3)=0.
      A( 6, 4)=P
```

```
      A( 6, 5)=R
      A( 6, 6)=Q
      A( 6, 7)=S
      A( 6, 8)=T
      A( 6, 9)=0.
      K=7
      N=6
      L=8
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 7, 1)=0.
      A( 7, 2)=0.
      A( 7, 3)=0.
      A( 7, 4)=0.
      A( 7, 5)=P
      A( 7, 6)=R
      A( 7, 7)=Q
      A( 7, 8)=S
      A( 7, 9)=T
      K=8
      N=7
      L=9
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 8, 1)=0.
      A( 8, 2)=0.
      A( 8, 3)=0.
      A( 8, 4)=0.
      A( 8, 5)=0.
      A( 8, 6)=P
      A( 8, 7)=R
      A( 8, 8)=Q
      A( 8, 9)=S
      K=9
      L=10
      N=8
      P=X(N)
      R=-2.*(X(N)+X(K))
      Q=X(N)+4.*X(K)+X(L)
      S=-2.*(X(K)+X(L))
      T=X(L)
      A( 9, 1)=0.
      A( 9, 2)=0.
      A( 9, 3)=0.
      A( 9, 4)=0.
      A( 9, 5)=0.
      A( 9, 6)=0.
      A( 9, 7)=P
      A( 9, 8)=R
      A( 9, 9)=Q-T
      PUNCH   111
  111 FORMAT(11HMATRIX A IS)
      PUNCH 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(3E16.8)
```

```
      DO 30 I=1,NA
      W(I)=0.
      DO 7J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PUNCH 444
  444 FORMAT(10HINVERSE IS)
      PUNCH40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(3E16.8)
      DO 90 KA=1,9
      AKA=KA
      W1(KA)=W(KA)*AR
      CALL PLOT(0,AKA,W1(KA))
   90 CONTINUE
      PRINT 333
  333 FORMAT(43HDEFLECTIONS AT POINTS 1,2,3,4,5,6,7,8,9 ARE)
      PRINT 50,(W1(I),I=1,MA)
   50 FORMAT(E16.8)
      PUNCH 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL PLOT (99)
      CALL EXIT
      END
ZZZZ8
```

```
34000320070136000320070249024025119636113001020O
ZZJOB 5                                  SIMPLY SUPPORTED-10 SEGMENTS -CONVENTIONAL
ZZFORX5
C       SOLUTION OF BEAM PROBLEMS BY FINITE DIFFERENCE METHOD
C       SIMPLY SUPPORTED BEAM-VARIABLE DEPTH-TEN SEGMENTS-CONVENTIONAL
        DIMENSION A(25,25),F(25,25),W(25),W1(25)
        COMMON A,MA,NA,F,MF,NF,W
        NA=9
        MA=9
        AR=1500./80000.
        XN=1.
        XX=9.
        XL=8.
        XD=1.
        YN=.0
        YX=.2
        YL=10.
        YD=.02
        CALL PLOT(101,XN,XX,XL,XD,YN,YX,YL,YD)
        DO 4M=1,10
        AM=M
        BX=1.+.4*AM
        ABX=BX**3
        BBX=BX**2*2.4
        CBX=.96*BX
        PUNCH 102,BX,ABX,BBX,CBX
    102 FORMAT(10H STEP A OK/4E16.8)
        P=ABX-BBX
        R=CBX+2.*BBX-4.*ABX
        Q=6.*ABX-2.*CBX
        S=CBX-2.*BBX-4.*ABX
        T=ABX+BBX
        IF (M-9)500,500,4
    500 GO TO(141,142,143,144,145,146,147,148,149),M
    141 A( 1, 1)=Q-P
        A( 1, 2)=S
        A( 1, 3)=T
        A( 1, 4)=0.
        A( 1, 5)=0.
        A( 1, 6)=0.
        A( 1, 7)=0.
        A( 1, 8)=0.
        A( 1, 9)=0.
        GO TO  4
    142 A( 2, 1)=R
        A( 2, 2)=Q
        A( 2, 3)=S
        A( 2, 4)=T
        A( 2, 5)=0.
        A( 2, 6)=0.
        A( 2, 7)=0.
        A( 2, 8)=0.
        A( 2, 9)=0.
        GO TO  4
    143 A( 3, 1)=P
        A( 3, 2)=R
        A( 3, 3)=Q
        A( 3, 4)=S
        A( 3, 5)=T
        A( 3, 6)=0.
        A( 3, 7)=0.
```

```
      A( 3, 8)=0.
      A( 3, 9)=0.
      GO TO 4
  144 A( 4, 1)=0.
      A( 4, 2)=P
      A( 4, 3)=R
      A( 4, 4)=Q
      A( 4, 5)=S
      A( 4, 6)=T
      A( 4, 7)=0.
      A( 4, 8)=0.
      A( 4, 9)=0.
      GO TO 4
  145 A( 5, 1)=0.
      A( 5, 2)=0.
      A( 5, 3)=P
      A( 5, 4)=R
      A( 5, 5)=Q
      A( 5, 6)=S
      A( 5, 7)=T
      A( 5, 8)=0.
      A( 5, 9)=0.
      GO TO 4
  146 A( 6, 1)=0.
      A( 6, 2)=0.
      A( 6, 3)=0.
      A( 6, 4)=P
      A( 6, 5)=R
      A( 6, 6)=Q
      A( 6, 7)=S
      A( 6, 8)=T
      A( 6, 9)=0.
      GO TO 4
  147 A( 7, 1)=0.
      A( 7, 2)=0.
      A( 7, 3)=0.
      A( 7, 4)=0.
      A( 7, 5)=P
      A( 7, 6)=R
      A( 7, 7)=Q
      A( 7, 8)=S
      A( 7, 9)=T
      GO TO 4
  148 A( 8, 1)=0.
      A( 8, 2)=0.
      A( 8, 3)=0.
      A( 8, 4)=0.
      A( 8, 5)=0.
      A( 8, 6)=P
      A( 8, 7)=R
      A( 8, 8)=Q
      A( 8, 9)=S
      GO TO 4
  149 A( 9, 1)=0.
      A( 9, 2)=0.
      A( 9, 3)=0.
      A( 9, 4)=0.
      A( 9, 5)=0.
      A( 9, 6)=0.
      A( 9, 7)=P
      A( 9, 8)=R
```

```
      A( 9, 9)=Q-T
    4 CONTINUE
      PUNCH   111
  111 FORMAT(11HMATRIX A IS)
      PUNCH 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(3E16.8)
      CALL P13(DETRM)
      DO 30  I=1,NA
      W(I)=0.
      DO 7J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PUNCH 444
  444 FORMAT(10HINVERSE IS)
      PUNCH40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(3E16.8)
      DO 90 KA=1,9
      AKA=KA
      W1(KA)=W(KA)*AR
      CALL PLOT(0,AKA,W1(KA))
   90 CONTINUE
      PRINT 333
  333 FORMAT(43HDEFLECTIONS AT POINTS 1,2,3,4,5,6,7,8,9 ARE)
      PRINT 50,(W1(I),I=1,MA)
   50 FORMAT(E16.8)
      PUNCH 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL PLOT (99)
      CALL EXIT
      END
ZZZZ8
```

```
34000320070136000320070249024025119636113001020O
ZZJOB 5                                   BEAM-VARIABLE M.I.- 5 SEGMENTS-MODIFIED
ZZFORX5
      DIMENSION X(10)
      DIMENSION A(25,25),F(25,25),W(25),W1(25)
      COMMON A,MA,NA,F,MF,NF,W
      XN=.5
      XX=4.5
      XL=8.
      XD=.5
      YN=0.
      YX=.2
      YL=10.
      YD=.02
      CALL PLOT(1(1,XN,XX,XL,XD,YN,YX,YL,YD)
      NA=4
      MA=4
      AR=1500./5000.
      DO 4M=1,10
      AM=M
      X(M)=(1.+.4*AM)**3
    4 CONTINUE
      PUNCH 102,(X(M),M=1,10)
  102 FORMAT(27H          X(M)                X(M)/2E16.8)
      P=1.
      Q=1.+4.*X(2)+X(4)
      S=-2.*(X(2)+X(4))
      T=X(4)
      A( 1, 1)=Q-P
      A( 1, 2)=S
      A( 1, 3)=T
      A( 1, 4)=0.
      N=2
      K=N+2
      L=N+4
      T=X(L)
      S=-2.*(X(K)+X(L))
      Q=X(N)+4.*X(K)+X(L)
      R=-2.*(X(N)+X(K))
      P=X(N)
      A( 2, 1)=R
      A( 2, 2)=Q
      A( 2, 3)=S
      A( 2, 4)=T
      N=4
      K=N+2
      L=N+4
      T=X(L)
      S=-2.*(X(K)+X(L))
      Q=X(N)+4.*X(K)+X(L)
      R=-2.*(X(N)+X(K))
      P=X(N)
      A( 3, 1)=P
      A( 3, 2)=R
      A( 3, 3)=Q
      A( 3, 4)=S
      N=6
      K=N+2
      L=N+4
      T=X(L)
      S=-2.*(X(K)+X(L))
```

```
      Q=X(N)+4.*X(K)+X(L)
      R=-2.*(X(N)+X(K))
      P=X(N)
      A( 4, 1)=0.
      A( 4, 2)=P
      A( 4, 3)=R
      A( 4, 4)=Q-T
      PUNCH  111
  111 FORMAT(11HMATRIX A IS)
      PUNCH 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(4E16.8)
      CALL P13(DETRM)
      DO 30 I=1,NA
      W(I)=0.
      DO 7J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PUNCH 444
  444 FORMAT(10HINVERSE IS)
      PUNCH40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(4E16.8)
      DO 90 KA=1,4
      AKA=KA
      W1(KA)=W(KA)*AR
      CALL PLOT ((,AKA,W1(KA))
   90 CONTINUE
      PRINT 333
  333 FORMAT(65H      MODIFIED METHOD-5 SEGMENTS/DEFLECTIONS AT POINTS 2,
     14,6,8 ARE)
      PRINT 50,(W1(I),I=1,MA)
   50 FORMAT(E16.8)
      PUNCH 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL PLOT(99)
      CALL EXIT
      END
ZZZZ8
```

```
34000320070136000320070249024025119636113001O200
ZZJOB 5                                   BEAM-VARIABLE M.I.- 5 SEGMENTS-CONVENTIONAL
ZZFORX5
      DIMENSION BX(10),ABX(10),BBX(10),CBX(10)
      DIMENSION A(25,25),F(25,25),W(25),W1(25)
      COMMON A,MA,NA,F,MF,NF,W
      XN=.5
      XX=4.5
      XL=8.
      XD=.5
      YN=0.
      YX=.2
      YL=10.
      YD=.02
      CALL PLOT(101,XN,XX,XL,XD,YN,YX,YL,YD)
      NA=4
      MA=4
      AR=1500./5000.
      DO 4M=1,10
      AM=M
      BX(M)=1.+.4*AM
      ABX(M)=BX(M)**3
      BBX(M)=BX(M)**2*1.2
      CBX(M)=.96*BX(M)
      PUNCH 102,BX(M),ABX(M),BBX(M),CBX(M)
  102 FORMAT(4E16.8)
    4 CONTINUE
      M=2
      P=ABX(M)-BBX(M)
      R=CBX(M)+2.*BBX(M)-4.*ABX(M)
      Q=6.*ABX(M)-2.*CBX(M)
      S=CBX(M)-2.*BBX(M)-4.*ABX(M)
      T=ABX(M)+BBX(M)
      A( 1, 1)=Q-P
      A( 1, 2)=S
      A( 1, 3)=T
      A( 1, 4)=0.
      M=4
      P=ABX(M)-BBX(M)
      R=CBX(M)+2.*BBX(M)-4.*ABX(M)
      Q=6.*ABX(M)-2.*CBX(M)
      S=CBX(M)-2.*BBX(M)-4.*ABX(M)
      T=ABX(M)+BBX(M)
      A( 2, 1)=R
      A( 2, 2)=Q
      A( 2, 3)=S
      A( 2, 4)=T
      M=6
      P=ABX(M)-BBX(M)
      R=CBX(M)+2.*BBX(M)-4.*ABX(M)
      Q=6.*ABX(M)-2.*CBX(M)
      S=CBX(M)-2.*BBX(M)-4.*ABX(M)
      T=ABX(M)+BBX(M)
      A( 3, 1)=P
      A( 3, 2)=R
      A( 3, 3)=Q
      A( 3, 4)=S
      M=8
      P=ABX(M)-BBX(M)
      R=CBX(M)+2.*BBX(M)-4.*ABX(M)
      Q=6.*ABX(M)-2.*CBX(M)
```

```
      S=CBX(M)-2.*BBX(M)-4.*ABX(M)
      T=ABX(M)+BBX(M)
      A( 4, 1)=0.
      A( 4, 2)=P
      A( 4, 3)=R
      A( 4, 4)=Q-T
      PUNCH 111
  111 FORMAT(11HMATRIX A IS)
      PUNCH 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(4E16.8)
      CALL P13(DETRM)
      DO 30 I=1,NA
      W(I)=0.
      DO 7J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PUNCH 444
  444 FORMAT(10HINVERSE IS)
      PUNCH40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(4E16.8)
      DO 90 KA=1,4
      AKA=KA
      W1(KA)=W(KA)*AR
      CALL PLOT (0,AKA,W1(KA))
   90 CONTINUE
      PRINT 333
  333 FORMAT(65H    MODIFIED METHOD-5 SEGMENTS/DEFLECTIONS AT POINTS 2,
     14,6,8 ARE)
      PRINT 50,(W1(I),I=1,MA)
   50 FORMAT(E16.8)
      PUNCH 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL PLOT(99)
      CALL EXIT
      END
ZZZZ8
```

```
340003200701360003200702490240251196361130010200
ZZJOB 5
ZZFORX5
C         ORTHOTRPIC  PLATE  SOLUTION  BY  FINITE  DIFFERENCE  METHOD
C         DR. ANDO.S PLATE BY USING MODIFIED FINITE DIFF. METHOD
          DIMENSION A(25,25),F(25,25),W(25),W1(25)
          COMMON A,MA,NA,F,MF,NF,W
          XN=4.
          XX=8.
          XL=8.
          XD=1.
          YN=0.0
          YX=1.0
          YL=10.
          YD=.1
          CALL PLOT (101,XN,XX,XL,XD,YN,YX,YL,YD)
          MA=8
          NA=8
          TPL=6.
          WX=580.
          ABX=580.
          WY=290.
          ABY=290.
          HZ=200.
          FX=4.5
          FY=4.5
          E=13608./(25.4*25.4)
          G=E/2.6
          AZIZ=10./(2320.**2)
          DIANNE=580.
          AR=DIANNE**4*AZIZ
          HY=580.
          AZ=HZ+TPL/2.
          BXY=(WX*TPL+HZ*FX)*G*(TPL**2)/(3.*ABX)
          BYX=(WY*TPL+HZ*FY)*G*(TPL**2)/(3.*ABY)
          DY=(WY*TPL+HZ*FY)*E/ABY
          EY=(AZ**2-(TPL/2.)**2)*E*FY/(ABY*DY*2.)
          DX=(WX*TPL+HZ*FX)*E/ABX
          EX=(AZ**2-(TPL/2.)**2)*E*FX/(ABX*DX*2.)
          DD=E*TPL/.91
          B=TPL**3*E/(12.*.91)
          C=B+(BXY+BYX)/2.
          PRINT 555
      555 FORMAT(70H          DX                    DY                    EX                    EY
         1          C)
          PRINT 6,DX,DY,EX,EY,C
        6 FORMAT(5E16.8)
          H=C+.3*EX*EY*DD+(EX+EY)**2*.7*DD/4.
          BX=(HZ**3*FX+TPL**3*WX+EX**2*WX*TPL*12.+(HZ+TPL-2.*EX)**2*HZ*FX*
         13.)*E/(12.*WX)
          BY=(HZ**3*FY+TPL**3*WY+EY**2*WY*TPL*12.+(HZ+TPL-2.*EY)**2*HZ*FY*
         13.)*E/(12.*WY)
          PRINT 10,BX,BY,BXY,BYX,H
       10 FORMAT(71H          BX                    BY                    BXY                   BYX
         1          H/5E16.8)
          HX=290.
          AK=HY/HX
          AS=AK**2
          R=AS**2*BX
          P=6.*(AS**2*BX+BY)+8.*AS*H
          Q=-4.*(AS*H+BY)
```

```
U=-4.*(AS**2*BX+AS*H)
S=2.*AS*H
T=BY
A( 1, 1)=P-R
A( 1, 2)=U
A( 1, 3)=R
A( 1, 4)=0.
A( 1, 5)=Q
A( 1, 6)=S
A( 1, 7)=0.
A( 1, 8)=0.
A( 2, 1)=U
A( 2, 2)=P
A( 2, 3)=U
A( 2, 4)=R
A( 2, 5)=S
A( 2, 6)=Q
A( 2, 7)=S
A( 2, 8)=0.
A( 3, 1)=R
A( 3, 2)=U
A( 3, 3)=P+R
A( 3, 4)=U
A( 3, 5)=0.
A( 3, 6)=S
A( 3, 7)=Q
A( 3, 8)=S
A( 4, 1)=0.
A( 4, 2)=2.*R
A( 4, 3)=2.*U
A( 4, 4)=P
A( 4, 5)=0.
A( 4, 6)=0.
A( 4, 7)=2.*S
A( 4, 8)=Q
A( 5, 1)=2.*Q
A( 5, 2)=2.*S
A( 5, 3)=0.
A( 5, 4)=0.
A( 5, 5)=P-R
A( 5, 6)=U
A( 5, 7)=R
A( 5, 8)=0.
A( 6, 1)=2.*S
A( 6, 2)=2.*Q
A( 6, 3)=2.*S
A( 6, 4)=0.
A( 6, 5)=U
A( 6, 6)=P
A( 6, 7)=U
A( 6, 8)=R
A( 7, 1)=0.
A( 7, 2)=2.*S
A( 7, 3)=2.*Q
A( 7, 4)=2.*S
A( 7, 5)=R
A( 7, 6)=U
A( 7, 7)=P+R
A( 7, 8)=U
A( 8, 1)=0.
A( 8, 2)=0.
```

```
      A( 8, 3)=4.*S
      A( 8, 4)=2.*Q
      A( 8, 5)=0.
      A( 8, 6)=2.*R
      A( 8, 7)=2.*U
      A( 8, 8)=P
      PRINT 111
  111 FORMAT(11HMATRIX A IS)
      PRINT 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(4E16.8)
      CALL P13(DETRM)
      DO 30 I=1,NA
      W(I)=0.
      DO 7 J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PRINT 444
  444 FORMAT(10HINVERSE IS)
      PRINT  40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(4E16.8)
      DO 90KA=5,8
      AKA=KA
      W1(KA)=W(KA)*AR
      CALL PLOT(0,AKA,W1(KA))
   90 CONTINUE
      PRINT 666
  666 FORMAT(21HDEFLECTIONS  ARE      )
      PRINT 51,(W1(I),I=1,MA)
   51 FORMAT(E16.8)
      PRINT 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALLPLOT(99)
      CALL EXIT
      END
ZZZZ8
```

```
34000320070136000320070249024025119636113001 0200
ZZJOB 5
ZZFORX5
C       ORTHOTRPIC PLATE SOLUTION BY FINITE DIFFERENCE METHOD
C       DR. ANDO.S PLATE BY USING NON-MODIFIED FINITE DIFF. METHOD
        DIMENSION A(25,25),F(25,25),W(25),W1(25)
        COMMON A,MA,NA,F,MF,NF,W
        MA=8
        NA=8
        TPL=6.
        WX=580.
        WY=290.
        HZ=200.
        FX=4.5
        FY=4.5
        ABX=580.
        ABY=290.
        E=13608./(25.4*25.4)
        G=E/2.6
        AZI=10./(2320.*2320.)
        SAM=580.
        AR=SAM**4*AZI
        AZ=HZ+TPL/2.
        BXY=(WX*TPL+HZ*FX)*G*(TPL**2)/(3.*ABX)
        BYX=(WY*TPL+HZ*FY)*G*(TPL**2)/(3.*ABY)
        DY=(WY*TPL+HZ*FY)*E/ABY
        EY=(AZ**2-(TPL/2.)**2)*E*FY/(ABY*DY*2.)
        DX=(WX*TPL+HZ*FX)*E/ABX
        EX=(AZ**2-(TPL/2.)**2)*E*FX/(ABX*DX*2.)
        DD=E*TPL/.91
        B=TPL**3*E/(12.*.91)
        C=B+(BXY+BYX)/2.
        PRINT 6,DX,DY,EX,EY,C
      6 FORMAT(27HVALUES OF DX,DY,EX,EY,C ARE/5E16.8)
        H=C+.3*EX*EY*DD+(EX+EY)**2*.7*DD/4.
        BX=(HZ**3*FX+TPL**3*WX+EX**2*WX*TPL*12.+(HZ+TPL-2.*EX)**2*HZ*FX*
       13.)*E/(12.*WX)
        BY=(HZ**3*FY+TPL**3*WY+EY**2*WY*TPL*12.+(HZ+TPL-2.*EY)**2*HZ*FY*
       13.)*E/(12.*WY)
        PRINT 10,BX,BY,BXY,BYX,H
     10 FORMAT(71H        BX                BY                BXY              BYX
       1              H/5E16.8)
        HY=580.
        HX=290.
        AK=HY/HX
        AS=AK**2
        BZ=.3*EX*EY*DD*AS
        SH=H-BZ/AS
        BEX=AS**2*BX
        BEC=2.*AS*SH
        BEZ=2.*BZ
        R=BEX
        P=6.*(BEX+BY)+4.*(BEZ+BEC)
        Q=-2.*(2.*BY+BEC+BEZ)
        S=BEC+BEZ
        T=BY
        U=-2.*(2.*BEX+BEC+BEZ)
        A( 1, 1)=P-R
        A( 1, 2)=U
        A( 1, 3)=R
        A( 1, 4)=0.
```

```
      A( 1, 5)=Q
      A( 1, 6)=S
      A( 1, 7)=0.
      A( 1, 8)=0.
      A( 2, 1)=U
      A( 2, 2)=P
      A( 2, 3)=U
      A( 2, 4)=R
      A( 2, 5)=S
      A( 2, 6)=Q
      A( 2, 7)=S
      A( 2, 8)=0.
      A( 3, 1)=R
      A( 3, 2)=U
      A( 3, 3)=P+R
      A( 3, 4)=U
      A( 3, 5)=0.
      A( 3, 6)=S
      A( 3, 7)=Q
      A( 3, 8)=S
      A( 4, 1)=0.
      A( 4, 2)=2.*R
      A( 4, 3)=2.*'J
      A( 4, 4)=P
      A( 4, 5)=0.
      A( 4, 6)=0.
      A( 4, 7)=2.*S
      A( 4, 8)=Q
      A( 5, 1)=2.*Q
      A( 5, 2)=2.*S
      A( 5, 3)=0.
      A( 5, 4)=0.
      A( 5, 5)=P-R
      A( 5, 6)=U
      A( 5, 7)=R
      A( 5, 8)=0.
      A( 6, 1)=2.*S
      A( 6, 2)=2.*Q
      A( 6, 3)=2.*S
      A( 6, 4)=0.
      A( 6, 5)=U
      A( 6, 6)=P
      A( 6, 7)=U
      A( 6, 8)=R
      A( 7, 1)=0.
      A( 7, 2)=2.*S
      A( 7, 3)=2.*Q
      A( 7, 4)=2.*S
      A( 7, 5)=R
      A( 7, 6)=U
      A( 7, 7)=P+R
      A( 7, 8)=U
      A( 8, 1)=0.
      A( 8, 2)=0.
      A( 8, 3)=4.*S
      A( 8, 4)=2.*Q
      A( 8, 5)=0.
      A( 8, 6)=2.*R
      A( 8, 7)=2.*U
      A( 8, 8)=P
      CALL P13(DETRM)
```

```
      DO 30 I=1,NA
      W(I)=0.
      DO 7 J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   30 CONTINUE
      PRINT 111
  111 FORMAT(11HMATRIX A IS)
      PRINT 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(4E16.8)
      PRINT 444
  444 FORMAT(10HINVERSE IS)
      PRINT  40,((A(I,J),J=1,NA),I=1,MA)
   40 FORMAT(4E16.8)
      PRINT 333
  333 FORMAT(21HDEFLECTIONS   ARE      )
      PRINT 50,(W1(I),I=1,MA)
   50 FORMAT(E16.8)
      PRINT 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL EXIT
      END
ZZZZ
```

```
34000320070136000320070249024025119636113001O200
ZZJOB 5
ZZFORX5
C       SOLUTION OF AN ORTHOGONALLY STIFFENED PLATE WITH VARIABLE MOMENT
C       OF NERTIA OF THE STIFFENERS IN THE DIRECTION OF SHORT SPAN
C        MODIFIED
        DIMENSION BEZ(6)
        DIMENSION BEC(6)
        DIMENSION BEX(6)
        DIMENSION BEY(6)
        DIMENSION A(9,9),F(9,9),W(9)
        COMMON A,MA,NA,F,MF,NF,W
        DIMENSION W1(9)
        MA=9
        NA=9
        E=30.*(10.**6)
        G=E/2.6
        TPL=1./4.
        B=TPL**3*E/(12.*.91)
        DD=E*TPL/.91
        FX=1./4.
        FY=1./4.
        HX=6.
        HY=4.
        WY=6.
        ABY=6.
        ABX=4.
        WX=4.
        ALOAD=1.
        AR=HY**4*ALOAD
        DO 4M=1,6
        AM=M
        AM=AM/6.
        HZ=1.5+1.5*SIN (3.14159*AM)
        AZ=HZ+TPL/2.
        BXY=(WX*TPL+HZ*FX)*G*(TPL**2)/(3.*ABX)
        BYX=(WY*TPL+HZ*FY)*G*(TPL**2)/(3.*ABY)
        DX=(WX*TPL+HZ*FX)*E/ABX
        DY=(WY*TPL+HZ*FY)*E/ABY
            EX=(AZ**2-(TPL/2.)**2)*E*FX/(ABX*DX*2.)
              EY=(AZ**2-(TPL/2.)**2)*E*FY/(ABY*DY*2.)
        C=B+(BXY+BYX)/2.
        H=C+.3*EX*EY*DD+(EX+EY)**2*.7*DD/4.
        BX=(HZ**3*FX+TPL**3*WX+EX**2*WX*TPL*12.+(HZ+TPL-2.*EX)**2*HZ*FX*
       13.)*E/(12.*WX)
        BY=(HZ**3*FY+TPL**3*WY+EY**2*WY*TPL*12.+(HZ+TPL-2.*EY)**2*HZ*FY*
       13.)*E/(12.*WY)
        AK=HY/HX
        AS=AK**2
        BEX(M)=AS**2*BX
        BEY(M)=BY
        BEC(M)=2.*AS*(C+(EX+EY)**2*.7*DD/4.)
        BEZ(M)=.6*AS*EX*EY*DD
      4 CONTINUE
        N=6
        K=1
        M=2
        P=BEX(K)
        T=BEY(N)
        U=BEY(M)
        Q=6.*BEX(K)+4.*BEY(K)+BEY(M)+BEY(N)+2.*BEC(K)+BEC(M)+BEC(N)
```

```
1+4.*BEZ(K)
  R=-2.*(BEY(M)+BEY(K))-BEC(K)-BEC(M)-BEZ(K)-BEZ(M)
  S=-2.*(BEY(N)+BEY(K))-BEC(K)-BEC(N)-BEZ(K)-BEZ(N)
  Y=-4.*BEX(K)-BEZ(K)-(BEC(N)+BEC(M))/2.-BEC(K)-BEZ(K)
  Z1=(BEC(K)+BEC(N))/2.+BEZ(K)/2.+BEZ(N)/2.
  Z2=(BEC(K)+BEC(M))/2.+BEZ(K)/2.+BEZ(M)/2.
  A( 1, 1)=Q-T-P
  A( 1, 2)=Y
  A( 1, 3)=P
  A( 1, 4)=R
  A( 1, 5)= Z2
  A( 1, 6)= 0.
  A( 1, 7)=U
  A( 1, 8)= 0.
  A( 1, 9)= 0.
  A( 2, 1)=Y
  A( 2, 2)=Q-T+P
  A( 2, 3)=Y
  A( 2, 4)= Z2
  A( 2, 5)= R
  A( 2, 6)= Z2
  A( 2, 7)= 0.
  A( 2, 8)= U
  A( 2, 9)= 0.
  A(3,1)=2.*P
  A( 3, 2)= 2.*Y
  A( 3, 3)= Q-T
  A( 3, 5)= 2.*Z2
  A( 3, 4)= 0.
  A( 3, 7)= 0.
  A( 3, 8)= 0.
  A( 3, 6)= R
  A( 3, 9)= U
  N=1
  K=2
  M=3
  P=BEX(K)
  T=BEY(N)
  U=BEY(M)
  Q=6.*BEX(K)+4.*BEY(K)+BEY(M)+BEY(N)+2.*BEC(K)+BEC(M)+BEC(N)
1+4.*BEZ(K)
  R=-2.*(BEY(M)+BEY(K))-BEC(K)-BEC(M)-BEZ(K)-BEZ(M)
  S=-2.*(BEY(N)+BEY(K))-BEC(K)-BEC(N)-BEZ(K)-BEZ(N)
  Y=-4.*BEX(K)-BEZ(K)-(BEC(N)+BEC(M))/2.-BEC(K)-BEZ(K)
  Z1=(BEC(K)+BEC(N))/2.+BEZ(K)/2.+BEZ(N)/2.
  Z2=(BEC(K)+BEC(M))/2.+BEZ(K)/2.+BEZ(M)/2.
  A( 4, 1)=S
  A( 4, 2)= Z1
  A( 4, 3)= 0.
  A( 4, 4)=Q-P+U
  A( 4, 5)= Y
  A( 4, 6)= P
  A( 4, 7)=R
  A( 4, 8)= Z2
  A( 4, 9)= 0.
  A( 5, 1)= Z1
  A( 5, 2)= S
  A( 5, 3)= Z1
  A( 5, 4)= Y
  A( 5, 5)= Q+U+P
  A( 5, 6)= Y
```

```
      A( 5, 7)= Z2
      A( 5, 8)= R
      A( 5, 9)= Z2
      A( 6, 1)= 0.
      A( 6, 2)= 2.*Z1
      A( 6, 3)= S
      A( 6, 4)= 2.*P
      A( 6, 5)= 2.*Y
      A( 6, 6)= Q+U
      A( 6, 7)= 0.
      A( 6, 8)= 2.*Z2
      A( 6, 9)= R
      N=2
      K=3
      M=4
      P=BEX(K)
      T=BEY(N)
      U=BEY(M)
      Q=6.*BEX(K)+4.*BEY(K)+BEY(M)+BEY(N)+2.*BEC(K)+BEC(M)+BEC(N)
     1+4.*BEZ(K)
      R=-2.*(BEY(M)+BEY(K))-BEC(K)-BEC(M)-BEZ(K)-BEZ(M)
      S=-2.*(BEY(N)+BEY(K))-BEC(K)-BEC(N)-BEZ(K)-BEZ(N)
      Y=-4.*BEX(K)-BEZ(K)-(BEC(N)+BEC(M))/2.-BEC(K)-BEZ(K)
      Z1=(BEC(K)+BEC(N))/2.+BEZ(K)/2.+BEZ(N)/2.
      Z2=(BEC(K)+BEC(M))/2.+BEZ(K)/2.+BEZ(M)/2.
      A( 7, 1)= T+U
      A( 7, 2)= 0.
      A( 7, 3)= 0.
      A( 7, 6)= 0.
      A( 7, 4)= S+R
      A( 7, 5)= Z1+Z2
      A( 7, 7)= Q-P
      A( 7, 8)= Y
      A( 7, 9)= P
      A( 8, 1)= 0.
      A( 8, 2)= T+U
      A( 8, 3)= 0.
      A( 8, 4)= Z1+Z2
      A( 8, 5)= S+R
      A( 8, 6)= Z1+Z2
      A( 8, 7)= Y
      A( 8, 8)= Q+P
      A( 8, 9)= Y
      A( 9, 1)= 0.
      A(9,2)=0.0
      A( 9, 3)= T+U
      A( 9, 4)= 0.
      A( 9, 5)= 2.*(Z1+Z2)
      A( 9, 6)= S+R
      A( 9, 7)= 2.*P
      A( 9, 8)= 2.*Y
      A( 9, 9)= Q
      PRINT 111
  111 FORMAT(11HMATRIX A IS)
      PUNCH  2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(3E16.8)
      CALL P13(DETRM)
  101 DO 311 I=1,NA
  102 W(I)=0.
  103 DO 7 J=1,MA
  104 W(I)=W(I)+A(I,J)
```

```
  105 W1(I)=W(I)*AR
    7 CONTINUE
   31 CONTINUE
      PRINT 444
  444 FORMAT (10HINVERSE IS)
      PRINT 41,((A(I,J),J=1,NA),I=1,MA)
   41 FORMAT(3E16.8)
      PRINT 333
  333 FORMAT(21HDEFLECTIONS   ARE       )
      PRINT 51,(W1(I),I=1,MA)
   51 FORMAT(E16.8)
  106 PRINT 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL EXIT
      END
ZZZZ
340003200701360003200702490240251196361130010200
ZZJOB 5
ZZFORX5
      DIMENSION W1(25)
      DIMENSION A(25,25),B(25,25),W(25)
      COMMON A,MA,NA,B,MB,NB,W
      READ 1,MA,NA,((A(I,J),J=1,NA),I=1,MA)
    1 FORMAT (2I2/(3E16.8))
      AR=256.
      CALL P13(DETRM)
      DO 4 I=1,NA
      W(I)=0.0
      DO 5 J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    5 CONTINUE
    4 CONTINUE
      PRINT 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(3E16.8)
      PRINT 6,(W(I),I=1,MA)
    6 FORMAT(E16.8)
      PRINT 333
  333 FORMAT(36HDEFLECTIONS FOR DISTRIBUTED LOAD ARE)
      PRINT 7,(W1(I),I=1,MA)
    7 FORMAT(E16.8)
      AA=32./3.
      BA=A(1,9)*AA
      BB=A(2,9)*AA
      BC=A(3,9)*AA
      BD=A(4,9)*AA
      BE=A(5,9)*AA
      BF=A(6,9)*AA
      BG=A(7,9)*AA
      BH=A(8,9)*AA
      BI=A(9,9)*AA
      PRINT 909
  909 FORMAT(29HDEFLECTIONS FOR CONC LOAD ARE)
      PRINT 99,BA,BB,BC,BD,BE,BF,BG,BH,BI
   99 FORMAT(E16.8)
      CC=16./3.
      QW=A(1,8)*CC
      QE=A(2,8)*CC
      QR=A(3,8)*CC
      QT=A(4,8)*CC
      QY=A(5,8)*CC
```

```
      QU=A(6,8)*CC
      QI=A(7,8)*CC
      QO=A(8,8)*CC
      QP=A(9,8)*CC
      PRINT 808
  808 FORMAT(33HDEFLECTIONS FOR TWO-POINT LOADING)
      PRINT 88,QW,QE,QR,QT,QY,QU,QI,QO,QP
   88 FORMAT(E16.8)
      PRINT 3,DETF 1
    3 FORMAT(E16.8)
      CALL EXIT
      END
 9 9
   .36740947E+08   -.73820048E+07    .11926107E+07
  -.28214287E+08    .18171858E+07    .00000000E-99
   .78683545E+07    .00000000E-99    .00000000E-99
  -.73820048E+07    .39126167E+08   -.73820048E+07
   .18171858E+07   -.28214287E+08    .18171858E+07
   .00000000E-99    .78683545E+07    .00000000E-99
   .23852214E+07   -.14764009E+08    .37933557E+08
   .00000000E-99    .36343716E+07   -.28214287E+08
   .00000000E-99    .00000000E-99    .78683545E+07
  -.28214287E+08    .18171858E+07    .00000000E-99
   .74415416E+08   -.12960144E+08    .21020476E+07
  -.39940851E+08    .26489488E+07    .00000000E-99
   .18171858E+07   -.28214287E+08    .18171858E+07
  -.12960144E+08    .78619510E+08   -.12960144E+08
   .26489488E+07   -.39940851E+08    .26489488E+07
   .00000000E-99    .36343716E+07   -.28214287E+08
   .42040952E+07   -.25920288E+08    .76517463E+08
   .00000000E-99    .52978976E+07   -.39940851E+08
   .15736724E+08    .00000000E-99    .00000000E-99
  -.79881740E+08    .52979003E+07    .00000000E-99
   .77053475E+08   -.15528581E+08    .25176363E+07
   .00000000E-99    .15736724E+08    .00000000E-99
   .52979003E+07   -.79881740E+08    .52979003E+07
  -.15528581E+08    .82088747E+08   -.15528581E+08
   .00000000E-99    .00000000E-99    .15736724E+08
   .00000000E-99    .10595800E+08   -.79881740E+08
   .50352726E+07   -.31057162E+08    .79571111E+08

ZZZZ8
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```
34000320070136000320070249024025119636113001O200
ZZJOB 5
ZZFORX5
      DIMENSION W1(25)
      DIMENSION A(25,25),F(25,25),W(25)
      COMMON A,MA,NA,F,MF,NF,W
      E=30.*(10.**6)
      G=30.*(10.**6)/2.6
      MA=9
      NA=9
      TPL=1./4.
      B=TPL**3*E/(12.*.91)
      DD=E*TPL/.91
      FY=1./4.
      FX=1./4.
      HX=6.
      ABY=6.
      WY=6.
      ABX=4.
      WX=4.
      HY=4.
      ALOAD=1.
      AR=HY**4*ALOAD
      DO 4 M=1,6
      AM=M
      AM=AM/6.
      HZ=1.5+1.5*SIN (3.14159*AM)
      PRINT 53,HZ
   53 FORMAT(8HVALUE HZ/E16.8)
      AZ=HZ+TPL/2.
      BXY=(WX*TPL+HZ*FX)*G*(TPL**2)/(3.*ABX)
      BYX=(WY*TPL+HZ*FY)*G*(TPL**2)/(3.*ABY)
      DX=(WX*TPL+HZ*FX)*E/ABX
      DY=(WY*TPL+HZ*FY)*E/ABY
          EX=(AZ**2-(TPL/2.)**2)*E*FX/(ABX*DX*2.)
            EY=(AZ**2-(TPL/2.)**2)*E*FY/(ABY*DY*2.)
      C=B+(BXY+BYX)/2.
      PRINT 555
  555 FORMAT(70H            DX              DY              EX              EY
     1          C)
      PRINT 6,DX,DY,EX,EY,C
    6 FORMAT(5E16.8)
      H=C+.3*EX*EY*DD+(EX+EY)**2*.7*DD/4.
      BX=(HZ**3*FX+TPL**3*WX+EX**2*WX*TPL*12.+(HZ+TPL-2.*EX)**2*HZ*FX*
     13.)*E/(12.*WX)
      BY=(HZ**3*FY+TPL**3*WY+EY**2*WY*TPL*12.+(HZ+TPL-2.*EY)**2*HZ*FY*
     13.)*E/(12.*WY)
      PRINT 54,BX,BY,BXY,BYX
   54 FORMAT(13HBX BY BXY BYX/4E16.8)
      AK=HY/HX
      AS=AK**2
      Q=6.*(BY+AS**2*BX)+8.*AS*H
      R=-4.*(BY+AS*H)
      S=-4.*AS*(AS*BX+H)
      T=2.*AS*H
      U=AS**2*BX
      V=BY
      PRINT55,Q,R,S,T,U
   55 FORMAT(13HVAL Q R S T U/5E16.8)
      IF (M-3)500,500,4
  500 GO TO (30,40,50),M
```

```
   30 A(1,1)=Q-U-V
      A(1,2)=S
      A(1,3)=U
      A(1,4)=R
      A(1,5)=T
      A(1,6)=0.0
      A(1,7)=V
      A(1,8)=0.0
      A(1,9)=0.0
      A(2,1)=S
      A(2,2)=Q+U-V
      A(2,3)=S
      A(2,4)=T
      A(2,5)=R
      A(2,6)=T
      A(2,7)=0.0
      A(2,8)=V
      A(2,9)=0.0
      A(3,1)=2.*U
      A(3,2)=2.*S
      A(3,3)=Q-V
      A(3,4)=0.0
      A(3,5)=2.*T
      A(3,6)=R
      A(3,7)=0.0
      A(3,8)=0.0
      A(3,9)=V
      GO TO 4
   40 A(4,1)=R
      A(4,2)=T
      A(4,3)=0.0
      A(4,4)=Q+V-U
      A(4,5)=S
      A(4,6)=U
      A(4,7)=R
      A(4,8)=T
      A(4,9)=0.0
      A(5,1)=T
      A(5,2)=R
      A(5,3)=T
      A(5,4)=S
      A(5,5)=Q+U+V
      A(5,6)=S
      A(5,7)=T
      A(5,8)=R
      A(5,9)=T
      A(6,1)=0.0
      A(6,2)=2.*T
      A(6,3)=R
      A(6,4)=2.*U
      A(6,5)=2.*S
      A(6,6)=Q+V
      A(6,7)=0.0
      A(6,8)=2.*T
      A(6,9)=R
      GO TO 4
   50 A(7,1)=2.*V
      A(7,2)=0.0
      A(7,3)=0.0
      A(7,4)=2.*R
      A(7,5)=2.*T
```

```
      A(7,6)=0.0
      A(7,7)=Q-U
      A(7,8)=S
      A(7,9)=U
      A(8,1)=0.0
      A(8,2)=2.*V
      A(8,3)=0.0
      A(8,4)=2.*T
      A(8,5)=2.*R
      A(8,6)=2.*T
      A(8,7)=S
      A(8,8)=Q+U
      A(8,9)=S
      A(9,1)=0.0
      A(9,2)=0.0
      A(9,3)=2.*V
      A(9,4)=0.0
      A(9,5)=4.*T
      A(9,6)=2.*R
      A(9,7)=2.*U
      A(9,8)=2.*S
      A(9,9)=Q
    4 CONTINUE
      PRINT 111
  111 FORMAT(11HMATRIX A IS)
      PRINT 2,((A(I,J),J=1,NA),I=1,MA)
    2 FORMAT(3E16.8)
      CALL P13(DETRM)
      DO 31I=1,NA
      W(I)=0.
      DO 7 J=1,MA
      W(I)=W(I)+A(I,J)
      W1(I)=W(I)*AR
    7 CONTINUE
   31 CONTINUE
      PRINT 444
  444 FORMAT(10HINVERSE IS)
      PRINT 41,((A(I,J),J=1,NA),I=1,MA)
   41 FORMAT(3E16.8)
      PRINT 333
  333 FORMAT(36HDEFLECTIONS FOR DISTRIBUTED LOAD ARE)
      PRINT 51,(W1(I),I=1,MA)
   51 FORMAT(E16.8)
      AA=32./3.
      BA=A(1,9)*AA
      BB=A(2,9)*AA
      BC=A(3,9)*AA
      BD=A(4,9)*AA
      BE=A(5,9)*AA
      BF=A(6,9)*AA
      BG=A(7,9)*AA
      BH=A(8,9)*AA
      BI=A(9,9)*AA
      PRINT 909
  909 FORMAT(29HDEFLECTIONS FOR CONC LOAD ARE)
      PRINT 99,BA,BB,BC,BD,BE,BF,BG,BH,BI
   99 FORMAT(E16.8)
      CC=16./3.
      QW=A(1,8)*CC
      QE=A(2,8)*CC
      QR=A(3,8)*CC
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```
      QT=A(4,8)*CC
      QY=A(5,8)*CC
      QU=A(6,8)*CC
      QI=A(7,8)*CC
      QO=A(8,8)*CC
      QP=A(9,8)*CC
      PRINT 808
  808 FORMAT(33HDEFLECTIONS FOR TWO-POINT LOADING)
      PRINT 88,QW,QE,QR,QT,QY,QU,QI,QO,QP
   88 FORMAT(E16.8)
      PRINT 16,DETRM
   16 FORMAT(14HDETERMINANT IS/E16.8)
      CALL EXIT
      END
ZZZZ8
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AUTOBIOGRAPHY

I, Azizul Haque Khondker, was born in a middle class family of the District of Faridpur, East Pakistan. My father, late Mvi. Abul Quasem Khondker was the Vice Chairman of Faridpur District Board and the President of Gatti Union Board. I recieved my primary-school education at Gatti Primary School, secondary school education at Krishnapur High English School and Faridpur Zila School and obtained my Matriculation Certificate in 1957. I received my Intermediate Science Certificate from Rajendra College (under Dacca University) in 1959. I received my B.Sc. Engg. (Civil) in 1963 from East Pakistan University of Engineering and Technology, Dacca, East Pakistan. I underwent the 1964-1965 Preventive Maintenance Training at Tokyo, sponsored by The Asian Productivity Organisation and Ford Foundation.

From 1963-67 I worked for The Engineers Ltd. (Pak.), Department of Communications and Buildings (Govt. of East Pakistan), Associated Consulting Engineers Ltd. (Pak.)--Engineering Consultants Inc. (Colorado, U.S.A.) Joint Venture World Bank Project at Pakistan as a Civil Engineer in different capacities.

I was working as a Graduate (Teaching and Research) Assistant from 1967-69 at the University of Windsor while completing the requirements for the Degree of Master of Applied Science in Civil Engineering at the same University. The expense of my research work was supported by a grant from the National Research Council of Canada.

I also worked as a structural Engineer at the Canadian Bridge Division at Windsor , Canada for a short time.

I am a member of the Asian Productivity Organisation Society, Tokyo, Japan.