An experimental study of steady non-Darcy flow in crushed rock.

Hung Chee Ng  
*University of Windsor*

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AN EXPERIMENTAL STUDY OF
STEADY NON-DARCY FLOW
IN CRUSHED ROCK

A Thesis
Submitted to the Faculty of Graduate Studies through the
Department of Civil Engineering in Partial Fulfilment
of the Requirements for the Degree of
Master of Applied Science at the
University of Windsor

by

Hung Chee Ng

Windsor, Ontario
1969
This thesis describes a study of one and two dimensional, steady, non-Darcy flow in crushed rock. Only the macroscopic properties of the porous media and the fluid are interpreted. The friction equations proposed by Porchheimer and Rose are compared and it is shown that the Porchheimer's equation is statistically as good as the Rose's equation. For simplicity, the Porchheimer's equation is used in the two dimensional case. An investigation of the porosity wall effect was made and for crushed rock, it was found that wall effect is negligible. Two general dimensionless friction equations based on the dry and drainable porosity respectively for crushed rock and an equation for the estimation of the seepage height are proposed. The finite element method is verified experimentally and found to give satisfactory results when applied to the solution of seepage problems with a phreatic surface.
ACKNOWLEDGMENTS

The writer is greatly indebted to Professor J. A. McCorquodale for his valuable advice and bright instruction, especially in the application of the finite element method to solve the two dimensional seepage problems as well as his patience in the correction of the thesis. The writer wishes to express his appreciation to the Department of Civil Engineering for the opportunity and an assistantship to pursue post-graduate work in the University of Windsor. Thanks are extended to the technicians Mr. G. Michalczuk and Mr. P. Feimer for their help in building the model for study. Acknowledgment is made to the National Research Council of Canada for sponsoring the work under grant No. 685. Thanks are due to Dr. S. P. Chee for his warmest regards and encouragement.
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CHAPTER ONE

INTRODUCTION

During the past two decades the topic of the fluid flow through porous media has been receiving much attention. This topic is related to many fields of engineering such as hydraulic engineering, chemical engineering, fuel engineering etc. Many papers but only a few books have been published. Unfortunately, most of the research work has been carried out for the linear case rather than the non-linear cases.

The factors governing this phenomenon are those arising from hydrodynamic considerations, from the characteristics of the bed material and from the statistical nature of the packing of the bed. As one can see, the variables are large in number and the researcher has to put certain limitations on his work. Besides, the experimental difficulties are further increased because many variables are closely related. Therefore, all equations proposed can only be recognized as empirical formulae that have good approximations for the restrictions made.

The material used in this study was crushed rock with geometric mean ranging from 0.962 cm. to 3.522 cm. and the cases considered were the one and two dimensional, steady, non-linear flow. In the one dimensional flow, the object of the work was to compare the proposed friction laws; to find the wall effect upon porosity; and to find a general friction law for all the experimental results. In the two dimensional flow, the applicability of the finite element method to find the position of the free surface and the non-linear flow field...
was verified. Besides, investigation was made of the relationship among the height of the upstream and downstream water level as well as the seepage height. All experimental results were analysed by the 1620 and 360/40 IBM computers.
CHAPTER TWO
LITERATURE SURVEY

2.1. Flow Regimes in Porous Media

Porous media is defined as a solid containing holes or voids, either connected or disconnected, dispersed within it in either a regular or random manner provided that such holes occur relatively frequently within the solid (1). Therefore, it is very easy to visualize that, in a layer of porous media, pores vary widely in magnitude, shape, and orientation. Curvilinear flow paths are then offered by the porous media to the fluid flowing through it. One might consider the solution of fluid flow through porous media as an analogy to the solution of fluid flow through curved pipes. But it was found that the flow pattern in porous media is different from the curved pipes in some respects.

Ward (2) in 1966 showed there to be four flow regimes in porous media that are given in Table 1.

Table 1. - Porous Media Flow Regime

<table>
<thead>
<tr>
<th>Flow Regime</th>
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<tr>
<td>Linear laminar</td>
<td>0.0182</td>
</tr>
<tr>
<td>Non-linear laminar</td>
<td>1.42 ± 0.08</td>
</tr>
<tr>
<td>Transition</td>
<td>4.23 ± 0.51</td>
</tr>
<tr>
<td>Turbulent</td>
<td></td>
</tr>
</tbody>
</table>

a See equation (2.3.4.)
This was confirmed by Wright (3). He used the hot-wire anemometers as a tool to measure the turbulence intensities within the pores of a gravel bed and proposed the following flow regimes:

(a) A "laminar" regime in which at every point the micro-velocity is stationary and the head loss is directly proportional to the velocity. The viscous forces predominate and the max. velocity occurs nearly in the centre of each flow passage.

(b) A "steady inertial" regime, (i.e. equivalent to the non-linear laminar proposed by Ward), in which at every point the micro-velocity is still stationary, but the head loss has ceased to vary linearly with the velocity. Both viscous forces and inertial actions influence the motion. Stationary vortices may be formed at upper end of the regime.

(c) A "turbulent transition" regime, in which the micro-velocity fluctuates at any point with increasing but regular frequency, and the head loss approaches dependence on the square of the velocity. Inertial actions predominate and vortices are shed at regular intervals from individual grains. Turbulence may occur in parts of the flow at the upper end of the regime.

(d) A "fully turbulent" regime in which all parts of the flow are turbulent, the micro-velocity fluctuating randomly about a mean. The head loss is now close to dependence on the square of the velocity.
Table 2. - Porous Media Flow Regime

<table>
<thead>
<tr>
<th>Flow Regime</th>
<th>$R_e$</th>
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<tr>
<td>Laminar</td>
<td>1 - 5</td>
</tr>
<tr>
<td>Steady inertial</td>
<td>90 - 120</td>
</tr>
<tr>
<td>Turbulent transition</td>
<td>800 -</td>
</tr>
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a  See equation (2.3.6.)

Besides the flow regimes Todd (4) showed on a log-log plot of the Fanning friction and Reynold number (Fig. 1.) that the transitions from laminar flow where resistive forces govern to laminar flow where inertial forces govern and to turbulent flow are gradual as evidenced by the mild curvature in Fig. 1. between $N_r = 1$ and $N_r = 1000$. The explanation for the gradual transition into turbulent flow hinges upon the microstructure of porous media flow. As the hydraulic gradient increases, turbulent flow occurs initially in only isolated pores associated with highest velocities. With further increases turbulence occurs in a correspondingly larger portion of the medium; the result is a gradual transition in flow type.

2.2. The Reasons for The Deviation of Darcy's Law

Since Darcy's Law states that the velocity is directly proportional to the potential gradient, it does not apply to all cases of filtration. The reason for deviation from the linear Darcy resistance law can be explained in several ways.

A. As the hydraulic gradient exceeds a certain limit, the flow
Fig. 1 Relation of Fanning friction factor to Reynolds number for flow through granular porous media

\[ \frac{\Delta P}{L} V_f = \frac{g d_4}{d^4} \]

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of water percolating through the material becomes turbulent; should this be true it would follow that beyond this limit the discharge rises at a smaller rate than the slope, because part of the energy is consumed in the formation of eddies. (5).

B. Porchheimer stated that when the discharge was not proportional to the gradient, this was probably due to the pressure exercised by the filtering water upon the grains themselves of which the material was composed. If the velocity of flow were high, the grains were more tightly pressed against each other, and this naturally affected the size of the passage existing between them (5).

C. Rose (6,7,8) stated that in laminar flow the energy losses will be due to the viscous forces acting on the area of surface and the tortuosity of the bed is not significant. At high Reynolds numbers, since the distribution of the particles in the bed influences tortuosity, the tortuosity of the bed will have great effect on the inertia forces.

Photographs of cross-sections of a bed show that the porosity of a bed is greater in the layer of particles in contact with the walls of the vessel. This layer is less tortuous than those towards the interior of the bed, because the particles in the layer adjacent to the wall are placed at random in two degrees of freedom only, the wall providing one constraint. This annular passage offers relatively low resistance and gives the fluid flow in it greater velocity than at any other point above the bed. This is known as the wall effect.

Rose also stated that the correction for wall effect depends upon the Reynolds numbers defining the flow, but for values of $(D/d)$ greater
than about 50, the correction may, for most purposes, be neglected; that for \( \text{Re} = 400 \) the wall effect is practically non-existent and that the minimum of the curves, for \( \text{Re} > 400 \), occurs at \((D/d) = 3.5\) whatever the value of Reynolds numbers. The above mentioned limits are applied for beds of spherical materials only. The \( D \) is the diameter of bed tube and \( d \) is the diameter of spherical particle.

D. Since the flow path is curvilinear in porous media, the fluid flow in it undergoes continual cycles of acceleration and deceleration. These would result in increased energy loss caused by secondary flow and increased shear stresses created by the faster moving fluid filaments fanning out close to the grain surfaces.

E. The flow through a bed of granular material can be regarded as flow past a number of individual solid bodies. It is evident that as fluid flows through the granular materials, wakes are formed and as the velocity increases, both the wake and the turbulent layer thickness on the surface of the rock increases.

F. Electrochemical reactions may cause deviation from Darcy's Law at very low Reynolds numbers.

2.3. Discussion on The Upper Limit of Darcy's Law

There is no doubt that Darcy's Law is not a universal law for all flow regimes and for all kinds of bed materials. What is the upper limit of Darcy's Law then? Many efforts had been made to find this limit but it seems there to be no consistent answers.

E. Prinz (5) in 1930 derived from his tests a very clear graphical representation of this limit as Fig. 2 shows.
Besides E. Prinz, most of the researchers have tried to represent the upper limit of the Darcy's equation by a "Critical Reynold" number. The representation by Reynolds numbers was chosen originally because of the assumption of an analogy between the flow in tubes and the flow in a porous medium; i.e., the latter is thought to be equivalent to an assemblage of capillaries. Therefore researchers have been looking for a phenomenon in porous media similar to the onset of turbulence in tubes, which takes place at a definite Reynolds number.

It was expected that above a certain number, which would be universal for all porous media, deviations from Darcy's law would occur. But one must recognized that Reynolds number is a valid criterion of dynamical similarity only if \( \nabla \) \( \frac{\partial V}{\partial t} \) \( - \mathbf{F} = \left( \frac{1}{\rho} \right) \nabla P - \left( \frac{\mu}{\rho} \right) \nabla \times \nabla \times \mathbf{V} \) vanishes in the systems to be compared,

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = \frac{1}{\rho} \nabla P - \left( \frac{\mu}{\rho} \right) \nabla \times \nabla \times \mathbf{V} \quad (2.3.1)
\]
where \( \mathbf{v} \) is the local-velocity-vector of a point of the fluid; \( T \) is time; \( F \) the volume force per unit mass; \( p \) is pressure; \( \mu \) is viscosity and \( \rho \) is density.

Unfortunately, the flow path in a porous medium is curvilinear so that the term \( \mathbf{v} \cdot \text{grad} \mathbf{v} \) is not zero. Besides, even for the same portion of rocks, it is not easy to achieve dynamical similarity for two setups. Thus the Critical Reynold number is not a good criterion for judging the upper limit of Darcy's law (9). Moreover, different definitions for the characteristic length term for the Reynolds number have been chosen for different works. For example:

A. Fancher (10)

\[
R = \frac{Vd\rho}{\mu}
\]

where

\[
d = \sqrt{\frac{\sum n_s d_s^2}{\sum n_s}} \tag{2.3.2.}
\]

and \( d \) = arithmetic mean of the openings in any two consecutive sieve sizes;

\( n \) = number of grains of diameter;

B. Rose

\[
R = \frac{Vd_i\rho}{\mu}
\]

where

\[
d_i = \frac{6 (1-E)}{S} \tag{2.3.3.}
\]

and \( d_i \) = the surface mean diameter;

\( E \) = the porosity;

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S = surface per unit volume of porous medium.

C. Ward (1966)

\[ R = \frac{\rho K^2 V}{\mu} \quad (2.3.4.) \]

where

\[ K = \frac{\phi_s M_g^2 E^3}{36 k_I (1 - E) \sigma_3^2 n} \quad (2.3.5.) \]

and \( E \) = the porosity;

\( \phi_s \) = the particle shape factor;

\( M_g \) = the geometric mean particle size, in centimeters;

\( k \) = a dimensionless constant that depends on the shape of the cross section of flow;

\( T \) = the tortuosity;

\( \sigma_3 \) = the geometric standard deviation.

D. Wright (1968)

\[ R = \frac{0.6 V d_s}{\mu (1 - E)} \quad (2.3.6.) \]

where \( d_s \) = surface mean diameter of grain;

\( i \) = hydraulic gradient;

\( E \) = porosity.

These criteria give rise to the great discrepancy in the answers found by different researchers. The values range between 0.1 and 75. The range of uncertainty of the factor is 750. Therefore, attempts to find the upper limit for the Darcy's law by introducing a Reynolds number which would become critical about a certain value universal for all porous media, have failed (9).
2.4. One Dimensional Flow Equations

The solution of the non-linear flow in porous media can be put into mathematical terms in two ways, i.e. by theoretical developments or by trial fitting of equations to the experimental data so as to obtain a correlation between the hydraulic gradient and flow velocity.

2.4.1. Theoretical Developments

A. Based on the Kozeny equation and the Blake's group:

Carman (11) started from an equation postulated by Reynolds (11) for the flow in linear tubes where the resistance offered by friction to motion of the fluid is represented as the sum of two terms, i.e.

\[ \frac{\Delta P}{L} = a V + b V^2 \quad (2.4.1.1.) \]

The factors \( a \) and \( b \) are functions of the system.

Carman then assumed that this equation is also valid for flow in porous media which implies that the medium is assumed to be equivalent to an assembly of capillaries. He proceeded to interpret the two terms as representing viscous and kinetic energy losses respectively. Since the Kozeny equation for laminar flow is

\[ \Gamma = \frac{5 \mu S}{\rho V} \quad (2.4.1.2.) \]

where \( S \) = particle surface for unit volume of the bed = \( S_\ast (1 - E) \);

\( S_\ast \) = specific surface;

\( \Gamma \) = the Blake's group = \( \frac{\Delta P E^3}{L^2 \rho V^3 S} \);

therefore using the Reynolds equation (2.4.1.1.), the Kozeny equation
for turbulent flow would be

\[ \frac{\Delta P E^3}{L \rho V^4 S} = \frac{5 \mu S}{\rho V} + b \]  

(2.4.1.3.)

on substitution, it becomes

\[ \frac{\Delta P}{L} = \frac{5(1-E)^{5}}{E^5} \mu S v + \frac{b(1-E)}{E^5} \rho S v \]  

(2.4.1.4.)

This equation is valid for those porous media that have pore texture uniform enough to satisfy the Kozeny equation.

**B. Reynold number and friction factor relationship**

Linguist (10) used a friction factor

\[ \zeta = \frac{\Delta P d}{2L \rho V^2} \]  

(2.4.1.5.)

and Reynolds number

\[ R = \frac{\rho V d}{\mu} \]  

(2.4.1.6.)

where \( d \) = diameter of the grains.

Linguist ran experiments with shot of uniform size and then used \( R \zeta \) as \( y \) axis and \( R \) as \( x \) axis and he found that the data for \( R > 4 \) being well represented by the linear relationship

\[ R \zeta = a + b R \]  

(2.4.1.7.)

or

\[ \zeta = a + b, R \]  

(2.4.1.8.)

He found

\[ \frac{b}{a} = \frac{1}{60} \]
C. Statistical Theories

Gibbs (12) assumed that a particle in a specific system (porous medium) will, in time, encounter all the conditions that are present in the many systems (porous media) representing the ensemble. In other words, he assumed that one can interchange time-averages and ensemble averages. This is called the ergodic hypothesis. It enables one to treat the path of a particle through a system (porous medium) by statistics, although the latter is, in fact, entirely determined by the boundary conditions.

There are three steps in the statistical analysis:
First, one chooses an ensemble representing all the porous media that we wish to call "identical".
Secondly, one chooses the type of statistics to be employed. By virtue of the ergodic hypothesis, this can also be expressed by saying that one has to choose "what happens in each time step". For instance, if one assumes a random distribution of residence times, then the Central Limit Theorem, gives a Gaussian distribution for the probability-density \( \Psi \), for a specific particle being at the position \( X \) at time \( T \) where \( \overline{X} \) is the average position of the particle. One obtains

\[
\Psi_p(X,T) = \left( 4 \pi D_0 T \right)^{-\frac{3}{2}} \exp \left[ -\frac{(X-\overline{X})^2}{4 D_0 T} \right]
\]  

(2.4.1.9.)

This automatically implies that the function \( \Psi \), expressed in "mean" coordinates \( \overline{X} \), (i.e. co-ordinates in which there is no mean flow) is subject to a diffusivity equation

\[
\frac{\partial \Psi}{\partial T} = D_0 \Delta \Psi
\]  

(2.4.1.10.)

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where $D_s$ is a "factor of dispersion".

Finally, consider what happens in each small time-step. This is called the "microdynamics". The microdynamical assumption specifies whether we are considering laminar flow, turbulent flow, Knudsen (molecular) flow, or something else in each microscopic flow channel. The microdynamic assumption does not change the fundamental diffusivity equation, but gives a means of correlating the constants occurring therein with other properties of the porous medium.

Following these three steps, an equation for turbulent flow is given as follows:

$$\frac{\partial \rho}{\partial t} = \text{lap}(D_s \rho) + \text{div}\left[m \rho^{\frac{1}{2}} n \sqrt{\text{grad } P}\right]$$  \hspace{1cm} (2.4.1.11.)

where $m$ is a constant of the porous medium; lap is the Laplace operator; $\rho$ the mass density $n_v$ is the unit vector. For in compressible flow equation (2.4.1.11) becomes

$$\text{lap}(D_s \rho) - \text{div}\left[m \rho^{\frac{1}{2}} n \sqrt{\text{grad } P}\right] = 0$$  \hspace{1cm} (2.4.1.12.)

Equation (2.4.1.11) shows two effects in turbulent flow through porous media:

1. a dispersivity effect;
2. the average turbulent flow through a set of small channels.

D. Capillaric Models

From the theory of capillaric models (9), the laminar flow equation can be expressed as
and the turbulent flow term can be expressed as

\[ dP = c' \frac{\rho V^2}{\delta} \, dX \]  \hspace{1cm} (2.4.14.)

where \( \frac{dP}{dX} \) is the pressure gradient; \( c \) and \( c' \) are certain constants; \( V \) is the microscopic velocity; \( \delta \) is the pore diameter, \( \rho \) is the density.

Therefore

\[ P_2 - P_1 = -\int \frac{c \mu V}{\delta^2} \, ds + \int \frac{c' \rho V^2}{\delta} \, ds \]  \hspace{1cm} (2.4.15.)

By further substitutions, equation (2.4.15) becomes

\[ \text{grad} \, P = -V \frac{3c \mu T^2}{E} \left\{ \int_0^{\infty} \frac{\alpha(\delta) \, d\delta}{\delta^3} \right\} \left[ \int \delta^2 \alpha(\delta) \, d\delta \right]^2 + \]

\[ V^2 \frac{9c' \rho T^2}{P} \left\{ \int_0^{\infty} \frac{\alpha(\delta) \, d\delta}{\delta^3} \right\} \left[ \int \delta^2 \alpha(\delta) \, d\delta \right]^3 \]  \hspace{1cm} (2.4.16.)

where \( T_0 \) is the tortuosity; \( \alpha \) is a constant.

Equation (2.4.16) appears to be of the form

\[ \text{grad} \, P = aV + bV^2 \]  \hspace{1cm} (2.4.17.)
2.4.2. Empirical Developments

A. The method of dimensional analysis

Ward (2) assumed the fluid flow through porous media is governed by \( \frac{dP}{dL} = \text{pressure gradient}; \ K = \text{permeability}, \ \rho = \text{mass density of the fluid}; \ \mu = \text{absolute viscosity of the fluid and} \ V = \text{macroscopic velocity.} \) He expressed these in an equation

\[
\frac{dP}{dL} = f(V, K, \rho, \mu) \tag{2.4.2.1.}
\]

Introducing the fundamental units of mass \( M \), length \( L \), and time \( T \) of the various parameters and solving for \( x, y \) and \( z \) in terms of \( w \), yields

\[
\frac{dP}{dL} = f\left(V^w K^{w-1} \rho^{w-1} \mu^{2-w}\right) \tag{2.4.2.2.}
\]

Then he used the equation suggested by M. Muskat (10) for turbulent flow in porous media, i.e.

\[
\frac{dP}{dL} = aV + bV^2 \tag{2.4.2.3.}
\]

for his rational development. He combined equations (2.4.2.2.) and (2.4.2.3.) yields

\[
\frac{dP}{dL} = \sum_{w-1}^{2} C \ V^w K^{w-1} \rho^{w-1} \mu^{2-w} = C_1 \frac{MV}{K} + C_2 \frac{PV^2}{K^2} \tag{2.4.2.4.}
\]

in equation (2.4.2.4.) \( C_1 = 1; \ C_2 = C \). Then

\[
\frac{dP}{dL} = \frac{MV}{K} + C \frac{PV^2}{K^2} \tag{2.4.2.5.}
\]
in which $C = 0.550 \pm 0.024$ represents a dimensionless constant that was the same value for all porous media.

Equation (2.4.2.5) is supposed to be universal for all porous media and flow regimes.

Rose (6) and co-workers made a thorough study of the possible variables that might influence flow and the dimensionless combinations in which they might occur in a flow equation.

They assumed that the macroscopic velocity $V$ through a bed of granular material depends upon the density $\rho$; viscosity $\mu$; the thickness of the bed $h_o$; grain diameter $d$ of the bed; the diameter $D$ of the container which the bed is packed; the porosity $E$; the gravity $g$; the difference of hydraulic head $H_o$ across the bed; the height $e$ of the surface roughness of the pores; the shape of the particle $Z$; the distribution of the grain $U$. Thus the relationship governing "turbulent" flow in porous media may be written symbolically as follows

$$H_o = f(V, h_o, d, D, E, e, Z, U, \mu, \rho, g) \quad (2.4.2.6)$$

As a result by the method of dimensional analysis, equation becomes

$$\frac{H_o}{d} = f \left\{ \left( \frac{V d \rho}{\mu} \right)^3 \left( \frac{h_o}{d^2} \right)^8 \left( \frac{d g}{V^2} \right)^\frac{e}{d} \left( \frac{D}{d} \right)^{\frac{e}{d}} E^z Z^w U^w \right\} \quad (2.4.2.7)$$

Unfortunately, dimensional analysis is not able to yield more information than is expressed by equation (2.4.2.7).

The form of the function has to be determined experimentally.
B. Intuitive Formulae

Forcheimer (13) suggested that

\[ i = aV + bV^2 \] (2.4.2.8.)

in which \( a \) and \( b \) were coefficients depending on the physical properties of the system. The term \( bV^2 \) is the term for eddy loss correction and the term \( aV \) is the term for laminar flow.

Later Forcheimer (13) added a third term giving

\[ i = aV + bV^2 + cV^3 \] (2.4.2.9.)

Rose (6) suggested that the friction for non-linear flow can be expressed as follow

\[ f = \frac{1000}{R} + \frac{160}{R^\frac{1}{4}} + 14 \] (2.4.2.10.)

in which \( \frac{1000}{R} \) is a term for laminar flow; \( \frac{160}{R^\frac{1}{4}} \) is a term for transition flow; 14 is an absolute constant for the turbulent flow.

Since

\[ f = \frac{\Delta h}{L} \cdot \frac{4R^\frac{1}{2}}{V^2} \] (2.4.2.11.)

and

\[ R = \frac{PV}{\mu} \left[ \frac{S(1-E)}{S} \right] \] (2.4.2.12.)

It can be shown that equation (2.4.2.10.) is equivalent to

\[ i = aV + bV^2 + cV^{\frac{3}{2}} \] (2.4.2.13.)

Krober (3) proposed the equation

\[ i = aV^n \quad (1 < n < 2) \] (2.4.2.14.)
in which $a$ and $n$ are functions of velocity, porosity, diameter of the rock, viscosity, shape factor and the distribution of the rock.

2.5. Two Dimensional Flow Equations

2.5.1. Finite Difference Solution

A general equation for flow over and through rockfill banks that is valid for all flow regimes was given by Parkin (14) as follows:

$$\left(\phi_{xx} + \phi_{yy}\right)\left(\phi_x^2 + \phi_y^2\right) + (N - 1)\left(\phi_x^2 \phi_{xx} + 2\phi_x \phi_y \phi_{xy} + \phi_y^2 \phi_{yy}\right) = 0 \quad (2.5.1.1)$$

in which $\phi$ = piezometric head and $N = \frac{1}{n}$. The values for $a$ and $n$ are obtained from the empirical equation $i = v^a v^n$

Curtis and Lawson (15) suggested that the equation can be solved by successive approximations applied to a finite number of node points. The main steps in the development of the successive approximations approach for a square grid system are given below.

\[
\begin{array}{c|c|c}
(I-1,J-1) & (I,J-1) & (I+1,J-1) \\
4 & 3 & 2 \\
(I-1,J) & (I,J) & (I+1,J) \\
5 & 0 & 1 \\
(I-1,J+1) & (I,J+1) & (I+1,J+1) \\
6 & 7 & 8 \\
\end{array}
\]

\[
\text{Fig. 3 Square Grid System}
\]

Using the method of finite differences, it is found that

\[
\phi = \frac{1}{2(1+N)}(\phi_1 + \phi_2 + \phi_3 + \phi_4) + \frac{N-1}{2(N+1)}.
\]

\[
(\phi + \phi_2)(\phi - \phi_2) + \frac{1}{2}[(\phi - \phi_2)(\phi + \phi_2)(\phi + \phi_4 - \phi_3) + (\phi_4 + \phi_2)(\phi_3 - \phi_4)]
\]

\[
(\phi - \phi_4)^2 + (\phi_3 - \phi_4)^2
\]

(2.5.1.2)
2.5.2. Method of Transformation

Engelund (9) stated that the streamlines and the equipotential lines form an orthogonal system in non-linear seepage flow. Engelund started with the following assumed equations

\[- \frac{\partial P}{\partial x} = F(V)V_x \quad (2.5.2.1.)*
\]

and

\[- \frac{\partial P}{\partial y} = F(V)V_y \quad (2.5.2.2.)*
\]

in which \(P\) is the pressure, \(V\) is the macroscopic velocity, \(V_x\) and \(V_y\) are the velocity components in the \(x\) and \(y\) directions respectively.

Both of the equations (2.5.2.1.) and (2.5.2.2.) are non-linear and are therefore inexpedient for direct calculation. He then considered an infinitesimal element of flow confined by two neighbouring streamlines and two lines of constant pressure. The equation of continuity can then be written

\[- \frac{1}{\Delta n} \frac{\partial (\Delta n)}{\partial s} = \frac{1}{V} \frac{\partial V}{\partial s} \quad (2.5.2.3.)*
\]

where \(n\) and \(s\) are the length of arc along curves \(\psi = \text{constant}\) and \(\rho = \text{constant}\) respectively. Since curl grad = 0, the flow equations (2.5.2.1.) and (2.5.2.2.) can be written as follows:

\[\text{curl} [F(V)\nabla V] = 0 \quad (2.5.2.4.)*\]

By means of Stokes's theorem this condition may be expressed by

\[- \frac{1}{\Delta s} \frac{\partial (\Delta s)}{\partial n} = \frac{1}{FV} \frac{\partial F}{\partial n} (FV) \quad (2.5.2.5.)*\]

He then proceeds to introduce as more convenient independent variables the macroscopic velocity \(V\) and the angle \(\theta\) between the velocity vector \(V\) and the \(x\) axis. The angle difference between the two
Streamlines of the element is given by
\[ \frac{\partial \Theta}{\partial n} \Delta n = -\frac{1}{\Delta s} \frac{\partial (\Delta n)}{\partial s} \Delta s \] (2.5.2.6.)

Furthermore,
\[ \frac{\partial}{\partial s} \left( \frac{\Pi}{2} - \Theta \right) \Delta s = -\frac{1}{\Delta n} \frac{\partial (\Delta s)}{\partial n} \Delta n \] (2.5.2.7.)

these expressions reduce to
\[ \frac{1}{\Delta n} \frac{\partial (\Delta n)}{\partial s} = \frac{\partial \Theta}{\partial n} \] (2.5.2.8.)

and
\[ \frac{1}{\Delta s} \frac{\partial (\Delta s)}{\partial n} = \frac{\partial \Theta}{\partial s} \] (2.5.2.9.)

Substitution into equations (2.5.2.3.) and (2.5.2.5.) yields
\[ \frac{\partial \Theta}{\partial n} = \frac{1}{\sqrt{F}} \frac{\partial}{\partial n} (\sqrt{F}) = \left( \frac{1}{\sqrt{V}} - \frac{F'}{F} \right) \frac{\partial V}{\partial n} \] (2.5.2.10.)

\[ \frac{\partial \Theta}{\partial s} = \frac{1}{V} \frac{\partial V}{\partial s} \] (2.5.2.11.)

where \( F' \) denotes the derivative \( \frac{\partial F}{\partial V} \).

The quantities \( P \) and \( \Psi \) can be introduced into these equations by substitution of \( \frac{\partial \Theta}{\partial n} = \frac{\partial \Theta}{\partial \Psi} \cdot \frac{\partial \Psi}{\partial n} = V \frac{\partial \Theta}{\partial \Psi} \) etc. and thus become
\[ \frac{\partial \Theta}{\partial \Psi} = \frac{V}{V} \frac{\partial V}{\partial P} \] (2.5.2.12.)

\[ \frac{\partial \Theta}{\partial P} = \frac{1}{F} \left( \frac{1}{V} \frac{F'}{F} \right) \frac{\partial V}{\partial P} \] (2.5.2.13.)

We may equally well express the functions \( P \) and \( \Psi \) in terms of \( V \) and \( \Theta \), and substitute this into the flow equations. The equations (2.5.2.12.) and (2.5.2.13.) then become
\[ \frac{\partial \Psi}{\partial \Theta} = -\frac{V}{F} \frac{\partial P}{\partial V} \] (2.5.2.14.)
\[ \frac{\partial \psi}{\partial V} = \frac{1}{F} \left( \frac{1}{V} + \frac{F'}{F} \right) \frac{\partial P}{\partial \theta} \]  

(2.5.2.15.)

Furthermore, \( \psi \) can be eliminated from these equations by appropriate differentiation, which leads to

\[ \frac{\partial}{\partial V} \left( \frac{V}{F} \frac{\partial P}{\partial V} \right) + \frac{1}{F} \left( \frac{1}{V} + \frac{F'}{F} \right) \frac{\partial^2 P}{\partial \theta^2} = 0 \]  

(2.5.2.16.)

This equation describes the linear as well as non-linear steady state flow in porous media.

2.5.3. The Method of Transformation and Complex Variables

Khristianovich (16) considers equations for turbulent flow in the following form

\[ i = - \text{grad } \phi = - \frac{V f(V)}{V} = \frac{V}{K} \]  

(2.5.3.1.)

where \( V \) is the vector of filtration velocity; \( i \) is the hydraulic gradient; \( K = \frac{V}{f(V)} \); \( \phi = \frac{P}{\gamma} + y \).

For plane motion, eliminating \( \phi \) from these equations and including the continuity equation of incompressible fluid, we get the system

\[ \frac{\partial}{\partial Y} \left( \frac{V_x}{K} \right) - \frac{\partial}{\partial X} \left( \frac{V_y}{K} \right) = 0 \]  

(2.5.3.2.)

\[ \frac{\partial V_x}{\partial X} + \frac{\partial V_y}{\partial Y} = 0 \]  

(2.5.3.3.)

Introducing the velocity vector \( V \) and its angle \( \theta \) with the \( X \) axis, instead of \( V_x \), \( V_y \) and taking \( \phi \) and \( \psi \) as the independent variables, the flow function \( \psi \) will satisfy the equations.
\[
\frac{\partial \psi}{\partial x} = -V_y = -V \sin \theta \quad (2.5.3.4.)
\]

\[
\frac{\partial \psi}{\partial y} = V_x = V \cos \theta \quad (2.5.3.5.)
\]

and we get

\[
\frac{\partial \theta}{\partial \psi} - \frac{f(V)}{V^2} \frac{\partial V}{\partial \theta} = 0 \quad (2.5.3.6.)
\]

\[
\frac{\partial \theta}{\partial \theta} + \frac{V f'(V)}{f(V)} \frac{\partial V}{\partial \psi} = 0 \quad (2.5.3.7.)
\]

Further, the author reduces this system to a system of four equations, in which the functions \( V, \psi, \theta \) and \( \theta \) are expressed in terms of the auxiliary independent variables \( F \) and \( G \).

\[
\frac{\partial \psi}{\partial G} + \frac{V}{f'(V)} \sqrt{\frac{V f'(V)}{f(V)}} \frac{\partial \theta}{\partial F} = 0 \quad (2.5.3.8.)
\]

\[
\frac{\partial \psi}{\partial F} - \frac{V}{f'(V)} \sqrt{\frac{V f'(V)}{f(V)}} \frac{\partial \theta}{\partial G} = 0 \quad (2.5.3.9.)
\]

\[
\sqrt{\frac{f'(V)}{V f(V)}} \frac{\partial V}{\partial G} - \frac{\partial \theta}{\partial F} = 0 \quad (2.5.3.10.)
\]

\[
\sqrt{\frac{f'(V)}{V f(V)}} \frac{\partial V}{\partial F} + \frac{\partial \theta}{\partial G} = 0 \quad (2.5.3.11.)
\]

Finally, a fictitious filtration velocity is introduced.
\[ \log V = \int \frac{f'(V)}{V f(V)} \, dV \quad (2.5.3.12) \]

and then the system of equations from (2.5.3.8) to (2.5.3.11) is replaced by

\[ \frac{\partial \log V}{\partial G} - \frac{\partial \theta}{\partial F} = 0 \quad (2.5.3.13) \]

\[ \frac{\partial \log V}{\partial F} + \frac{\partial \theta}{\partial G} = 0 \quad (2.5.3.14) \]

\[ \frac{\partial \phi}{\partial F} = -L \frac{\partial \psi}{\partial G} \quad (2.5.3.15) \]

\[ \frac{\partial \phi}{\partial G} = L \frac{\partial \psi}{\partial F} \quad (2.5.3.16) \]

where

\[ L = \frac{f(V)}{V} \int \frac{f(V)}{f'(V)} \, dV \quad (2.5.3.17) \]

Here \( F + iG = f(X + iY) \), where \( f \) is an analytic function.

If the solution of the system of equations from (2.5.3.13) to (2.5.3.16) is found, one can find \( X, Y \) from the equations given as follows:

\[ X = - \int \left( \frac{\cos \theta}{f'(V)} \, d\phi + \frac{\sin \theta}{V} \, d\psi \right) \quad (2.5.3.18) \]

\[ Y = - \int \left( \frac{\sin \theta}{f'(V)} \, d\phi - \frac{\cos \theta}{V} \, d\psi \right) \quad (2.5.3.19) \]

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Kristianovich proposed to solve the four equations in the following manner. Assume in the plane FG a region analogous to the region given in the plane XY or simply coinciding with it, and solve the first two equations (2.5.3.13.) and (2.5.3.14.); then substitute the value of V thus obtained into L, getting thus L(F, G), and solve equations (2.5.3.15.) and (2.5.3.16.).
CHAPTER THREE
ONE DIMENSIONAL FLOW

In this chapter, the one dimensional flow is going to be considered. A general equation for crushed rocks is derived. The comparison of the merits of the two term and three term equations is described by the experimental results. The study and the results of dry porosity, drainable porosity together with the wall effect are mentioned. The facilities, experimental procedure and experimental data are fully described.

3.1. Background and Theory

One can see that the friction equations mentioned in the literature survey can be simplified into three forms as follows:

A. \( i = a V^n \) \hspace{1cm} (3.1.1.)

B. \( i = a V + b V^2 \) \hspace{1cm} (3.1.2.)

C. \( i = a V + b V^{\frac{3}{2}} + c V^2 \) \hspace{1cm} (3.1.3.)

In these equations, \( i \) is the head drop per unit length and \( V \) is the macroscopic velocity.

Equation (3.1.1.) is derived from the fact that in turbulent flow, additional energy is expended in the formulation of eddies, therefore the hydraulic gradient is no longer linearly proportional to velocity but is proportional to a greater power than unity. Therefore, an exponent \( n \) (always bigger than 1) is substituted into the Darcy's
equation instead of 1 to account for turbulence. This condition of laminar and turbulent flow in one term is certainly simple in form but its significance is obviously not as good as equations (3.1.2) and (3.1.3). For this reason equation (3.1.1) is not considered in the comparisons of this study.

Equation (3.1.2) considers laminar and turbulent flow separately and equation (3.1.3) considers a transition flow term \( b V^{\frac{3}{2}} \) also. Since the more terms that are included in the expression, the more general the final expression, equation (3.1.3) is probably better than equation (3.1.2). However, equation (3.1.3) involves one more non-linear term than equation (3.1.2) and this would introduce much complexity in the mathematical solutions for the two dimensional cases. Therefore, it is wise to find to what extent equation (3.1.3) would be better than equation (3.1.2). These equations are compared by their respective correlation coefficients. If the difference in their correlation coefficients is small, equation (3.1.2) would be chosen for simplicity in further applications.

An attempt to define an equation that is suitable to all sizes of crushed rock and all flow regimes was made basically from the Ward's equations (2.3.5) and (2.4.1.4). Since each size of crushed rock has approximately the same shape factor, tortuosity and dimensionless constant \( k \) which depends on the shapes of the cross-section of flow, \( \frac{\phi^2}{36kT} \) can be considered as constant \( C \), in the permeability \( K \) expression. i.e.

\[
K = C \left[ \frac{E^3}{(1 - E)^2} \right] \frac{M^2}{\sigma_2 \ln \sigma_2}
\]  

(3.1.4)
From equation (2.4.2.4.)

\[
\frac{i}{V} = \frac{\mu}{K} + \frac{C \rho \nu}{K_t^2}
\]  

(3.1.5.)

Substituting equation (3.1.4.) into equation (3.1.5.) yields

\[
\frac{i}{V} = \frac{\mu}{C_i} \frac{E^3}{(1 - E)^2} \frac{M_3}{\sigma g \ln \sigma g} + \frac{C \rho \nu}{C_i^2} \left( \frac{E^3}{(1 - E)^2} \frac{M_3}{\sigma g \ln \sigma g} \right)^{\frac{1}{3}}
\]  

(3.1.6.)

On simplification, equation (3.1.6.) becomes

\[
\frac{i M_3^2}{V \mu ^{\frac{1}{3}} \sigma g ^{\frac{1}{3}} \ln \sigma g} = A + B \left\{ \frac{M_3^2}{\left( \frac{E^3}{(1 - E)^2} \right)^{\frac{1}{3}}} \left( \frac{V M_3^2}{\nu} \right) \right\} 
\]  

(3.1.7.)

where

\[
A = \frac{1}{C_i}, \quad B = \frac{C}{C_i^2}
\]  

(3.1.8.)

An equation corresponding to equation (3.1.7.) was found for each size of crushed rock respectively but the slope of these equations were not exactly the same. The probable reason for this is that porosity is the major factor causing deviations; therefore a factor of a certain power of porosity is multiplied by the second term on the right hand side of the equation in order to make the slopes of all equations the same. The equation so found can be used to estimate the hydraulic gradient for any size of crushed rock.

The porosity commonly used is the dry porosity. In the studies of this report, the drainable porosity which is the ratio of the water drained out to the total volume was easier to obtain and is also considered. One reason for considering the drainable porosity
Fig. 4A View of One Dimensional Test Section

Fig. 4B Enlarged View of Rock Section
Fig. 4D  SECTION A–A

Fig. 5  WOODEN BOXES FOR POROSITY TESTS
is that, in turbulent flow, the boundary layer is thicker than in laminar flow, in other words, its effect becomes more significant; therefore this water layer decreases the volume of effective voids. The effective porosity in turbulent flow would be less than the dry porosity. Both the dry and drainable porosities were considered in this study.

Rose stated that porosity could be affected by the wall effect. Therefore an investigation of the wall effect was made for the purposes of precision and correction.

3.2. Test Equipments

3.2.1. Facilities

The facilities used for the studies of the one dimensional friction laws and porosities are described as follows:

A. The centrifugal pump used in the experimental studies has a maximum speed of 1700 R.P.M., minimum speed of 120 R.P.M., a maximum head of 92 feet and a maximum discharge of 216 U.S.G.P.M.

B. A venturimeter is connected to a manometer for flow measurement. The manometer can give readings accurate up to 1 U.S. gallon per minute.

C. The flume used for the experiment is sixteen feet long, one foot wide and four feet high as shown in Fig. 4C & D.

D. A water tight wooden tube or test box having an internal cross sectional area of 11" x 11" and length of 4 feet was filled with the various rock samples for the one dimensional friction studies. The tube was set horizontally on the bottom of the flume and its sides
sealed before the experiments were run. See Fig. 4C.

E. The sliding gate shown in Fig. 4 was set at two inches from the upstream edge of the tube to maintain the required heads for the experiments. The gate was held tightly at the sides by two gloves and at the bottoms by two blocks. It was sealed to prevent leakage.

F. The tailwater gate consisted of a movable weir hinged at the bottom with its position controlled by an adjustable rod as shown in Fig. 4.

G. A baffle was placed two feet upstream from the upstream edge of the tube to smooth the turbulence coming out from the pipe. See Fig. 4.

H. The porosity test boxes of cross sectional areas equal to 18" x 18", 11" x 11" and 6" x 6" are shown in Fig. 5. All the boxes were water tight.

I. Other facilities used included graduated cylinders, a precision balance with 0.01 gram divisions, standard sieves, a mechanical sieve analyser, a thermometer and a manomater.

3.2.2. General Layout of Facilities

The facilities are shown in Figs. 4A & B & C & D and Fig. 5.

3.3. Experimental Procedure

3.3.1. Flow Rates and Head Losses for the Friction Tests

For each rock size, the flow rate and corresponding head loss were determined as follows:

A. The mass of crushed rock was washed and dried before a sieve analysis was made.

B. The dried crushed rock was sieved by a mechanical vibrating
machine through a series of consecutive standard sieves of $1\frac{1}{2}''$, $1''$, 
$\frac{3}{4}''$, $\frac{1}{2}''$, $\frac{1}{8}''$. The rocks were then separated into five groups 
i.e. $1\frac{1}{2}'' \rightarrow 1''$, $1'' \rightarrow \frac{3}{4}''$, $\frac{3}{4}'' \rightarrow \frac{1}{2}''$, $\frac{1}{2}'' \rightarrow \frac{1}{4}''$ and $\frac{1}{4}'' \rightarrow \frac{1}{8}''$, each of which 
was stored individually for future use.

C. Each of the five samples were tested. The rocks were placed into the test box and compacted. Then a screen was placed into the box and held vertically and tightly at the downstream face of the crushed rock by an iron rod through the top and bottom of the box as shown in Fig. 4A.

D. The box was then lifted and put into the flume in the horizontal position as shown in Fig. 4A.

E. The sliding gate shown in Fig. 4C was then sealed so that a certain head difference between the upstream and downstream could be obtained. Leakage along the sides of the gate and the box was prevented by plastcine sealing.

F. Before the flow was run, the venturi meter was zeroed.

G. The pump was then started and water was delivered to head tank. Different flow rates ($Q$) were run and their magnitudes were read and recorded from the venturimeter.

H. The hydraulic gradients between the end points (1) and (2) shown in Fig. 4C were read from the manometer on the side of the flume and recorded for each flow. At low flow rates, the movable weir was raised so that the sample was always be submerged.

I. After an adequate number of flow rates and hydraulic gradients were observed, the sliding gate was removed and the test box was removed from the flume. The test box was then stood upright
and was sealed at the bottom for the determination of drainable porosity as described in the following section (3.3.3.).

J. The rocks tested were emptied and the new sample was placed in the test box and the experiment was repeated.

3.3.2. Rock Weights and Volumes

The weights and volumes of the particles of each rock sample were determined as follows:

A. Thirty particles were taken from each size of rock.

B. Each particle was weighed by a calibrated balance and the weight was recorded.

C. A graduated cylinder was filled with water and the volume of water \( W \) was recorded. Then each particle was dropped into the cylinder and the total volume \( W_t \) was recorded. The volume \( W_s \) of the sample is \( W_s = W_t - W \).

D. After the first sample was finished (B) and (C) were repeated for the other samples.

3.3.3. Drainable porosities

The drainable porosity for each size of rock in the three boxes were determined as follows:

A. Tests were first run on the 6" by 6" box.

B. The first size of rock was placed in the test box and then the box was shaken for compaction.

C. Water was poured onto the test box until drainage occurred from the pipe (B) shown in Fig. 5.

D. A stopper was applied to closed pipe (B).
E. A known volume of water \((W)\) was measured and poured into a container for use.

F. A stopper was applied to pipe \((A)\) shown in Fig. 5.

G. The water in the container was poured to fill the box to just above the level of pipe \((A)\). Then the stopper on pipe \((A)\) was removed and the excess water \((W_x)\) that drained out was measured.

H. The volume of water \((W_v)\) retained by the voids is

\[ W_v = W - W_x \]

I. The above procedure was used for all samples and also for the 18" by 18" and 11" by 11" test boxes.

J. The total volumes of the boxes were determined in the following way. Both of the pipes \((A)\) and \((B)\) were closed and the box filled to the level of pipe \((A)\). Pipe \((A)\) was opened and the water was drained to the last drop. Pipe \((B)\) was then opened and the water that drained out was measured. Its value is the total volume desired.

3.4. Results of Experiments

3.4.1. Flow Rates and Head Losses

The flow rates and head losses for each sample of crushed rock are shown in the following Tables 3 to 7.

3.4.2. Weight and Volume

The weight and volume for each sample of crushed rock are shown in the following Tables 8 to 12.
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<tr>
<td>11</td>
<td>0.30</td>
<td>0.95</td>
</tr>
<tr>
<td>12</td>
<td>0.30</td>
<td>0.55</td>
</tr>
<tr>
<td>13</td>
<td>0.50</td>
<td>1.45</td>
</tr>
<tr>
<td>14</td>
<td>0.15</td>
<td>0.54</td>
</tr>
<tr>
<td>15</td>
<td>0.35</td>
<td>0.85</td>
</tr>
<tr>
<td>16</td>
<td>0.48</td>
<td>1.20</td>
</tr>
<tr>
<td>17</td>
<td>0.32</td>
<td>0.80</td>
</tr>
<tr>
<td>18</td>
<td>0.30</td>
<td>1.00</td>
</tr>
<tr>
<td>19</td>
<td>0.60</td>
<td>1.60</td>
</tr>
<tr>
<td>20</td>
<td>0.80</td>
<td>2.05</td>
</tr>
<tr>
<td>21</td>
<td>0.50</td>
<td>2.10</td>
</tr>
<tr>
<td>22</td>
<td>0.20</td>
<td>0.43</td>
</tr>
<tr>
<td>23</td>
<td>0.20</td>
<td>0.75</td>
</tr>
<tr>
<td>24</td>
<td>0.50</td>
<td>1.72</td>
</tr>
<tr>
<td>25</td>
<td>0.40</td>
<td>0.85</td>
</tr>
<tr>
<td>26</td>
<td>0.50</td>
<td>1.15</td>
</tr>
<tr>
<td>27</td>
<td>0.40</td>
<td>1.35</td>
</tr>
<tr>
<td>28</td>
<td>0.30</td>
<td>0.80</td>
</tr>
<tr>
<td>29</td>
<td>0.50</td>
<td>1.25</td>
</tr>
<tr>
<td>30</td>
<td>0.50</td>
<td>1.45</td>
</tr>
</tbody>
</table>
3.4.3. Porosity

The volume of water required to fill the voids in the drainable porosity tests are recorded in the following Tables 13 to 15:

**TABLE 13. - 6" x 6" TEST BOX**

<table>
<thead>
<tr>
<th>SIZE OF ROCKS</th>
<th>NUMBER</th>
<th>VOLUME OF WATER REQUIRED TO FILL THE VOIDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&quot;</td>
<td>1</td>
<td>3390 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3380</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3340</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3170</td>
</tr>
<tr>
<td>2&quot;</td>
<td>2</td>
<td>3150</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3150</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3230</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>2</td>
<td>3200</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3180</td>
</tr>
</tbody>
</table>

* a Bulk Volume = 7020 c.c.

**TABLE 14. - 18" x 18" TEST BOX**

<table>
<thead>
<tr>
<th>SIZE OF ROCKS</th>
<th>NUMBER</th>
<th>VOLUME OF WATER REQUIRED TO FILL THE VOIDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&quot;</td>
<td>1</td>
<td>11265 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>11200</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11125</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>11700</td>
</tr>
<tr>
<td>2&quot;</td>
<td>2</td>
<td>11600</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11500</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>11430</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>2</td>
<td>11425</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11425</td>
</tr>
</tbody>
</table>

* a Bulk Volume = 26085 c.c.
TABLE 15. - 11" x 11" TEST BOXa

<table>
<thead>
<tr>
<th>SIZE OF ROCKS</th>
<th>NUMBER</th>
<th>VOLUME OF WATER REQUIRED TO FILL THE VOIDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&quot;</td>
<td>1</td>
<td>10750 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10690</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10600</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10810</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>2</td>
<td>10480</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10270</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10700</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>2</td>
<td>10460</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10428</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10580</td>
</tr>
<tr>
<td>5/8&quot;</td>
<td>2</td>
<td>10450</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10360</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>10900</td>
</tr>
<tr>
<td>5/16&quot;</td>
<td>2</td>
<td>10350</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10900</td>
</tr>
</tbody>
</table>

a Bulk Volume = 23500 c.c.

3.5. Calculations

3.5.1. Constants and Correlation Coefficients

The constants and the correlation coefficients for both equation (3.1.2) and (3.1.3) were calculated on c.g.s. units.

The macroscopic velocity (\(v\)) was calculated from the flow rate found divided by the cross section of the tube which is 11" x 11". Since the flow rate was in U.S. gallon per minute, then

\[
V = \frac{Q \cdot 231 \cdot 2.54}{60 \cdot 11 \cdot 11} = 0.0809 \ Q
\]  \hspace{1cm} (3.5.1.1.)

The potential drop (\(i\)) per unit length was found as

\[
i = \frac{\Delta \Psi}{L}
\]  \hspace{1cm} (3.5.1.2.)
where $\phi$ was found from experiment and $L$ is the length between the two tubes (1) and (2) shown in Figure 4.

The line of best fit for both equations (3.1.2.) and (3.1.3.) was found by the method of least squares regression (17).

The principle of least squares states that if $Y$ is a linear function of an independent variable $X$, the most probable position of a best fit line is such that the sum of squares of deviations of all points from the line is a minimum; the deviations are measured in the direction of the $Y$ axis. The solutions of both the equations are shown as follows:

A. Equation (3.1.2.) is written below:

$$I = aV + bV^2$$

$$y = \frac{i}{V}; \quad x = V$$

Then the equation becomes

$$y = a + bx \quad (3.5.1.3.)$$

By transforming the axis of coordinates to a new origin $(X,Y)$, new co-ordinates $(X,Y)$ are introduced, i.e.

$$X = x - \bar{x} \quad (3.5.1.4.)$$

$$Y = y - \bar{y} \quad (3.5.1.5.)$$

Where $\bar{x}$ and $\bar{y}$ are the mean of $x$ and $y$ respectively.

These give

$$b = \frac{\sum XY}{\sum X^2} \quad (3.5.1.6.)$$

and

$$a = \bar{y} - b\bar{x} \quad (3.5.1.7.)$$

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The correlation coefficient $R$ is found as

$$R = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}} \tag{3.5.1.8.}$$

The solution of $b$ and $R$, were obtained on the IBM 1620 computer and the line of best fit was plotted on the line plotter. The program is shown in the appendix I.

B. Equation (3.1.3.) is written as follows:

$$\frac{1}{V} = a + bx + cV$$

Let $y = \frac{1}{V}$; $x_1 = \sqrt{x}$ and $x_2 = V$

then equation (3.1.3.) is then transformed into

$$y = a + bx_1 + cx_2 \tag{3.5.1.9.}$$

In choosing the centroid as the origin

$$Y = y - \bar{y} \tag{3.5.1.10.}$$

$$X_1 = x_1 - \bar{x} \tag{3.5.1.11.}$$

$$X_2 = x_2 - \bar{x} \tag{3.5.1.12.}$$

where $\bar{y}, \bar{x}_1, \bar{x}_2$ are the arithmetic mean of $y, x_1, and x_2$ respectively.

Then

$$a = \bar{y} - b\bar{x}_1 - c\bar{x}_2 \tag{3.5.1.13.}$$

$$b = e_{11} \sum X_1 Y + e_{12} \sum X_2 Y \tag{3.5.1.14.}$$

$$c = e_{21} \sum X_1 Y + e_{22} \sum X_2 Y \tag{3.5.1.15.}$$

Where

$$e_{11} = \frac{\sum X_2^2}{\sum X_1^2 \sum X_2^2 - (\sum X_1 \sum X_2)^2} \tag{3.5.1.16.}$$
\[ e_{22} = \frac{\sum X_i^3}{\sum X_i^2 \sum X_i^2 - (\sum X_i X_2)^2} \]  
(3.5.1.17.)

\[ e_{22} = e_{21} = \frac{-\sum X_i X_2}{\sum X_i^2 \sum X_i^2 - (\sum X_i X_2)^2} \]  
(3.5.1.18.)

And the multiple correlation coefficient \( R \) is

\[ R = \sqrt{\frac{\sum c^2}{\sum Y^2}} \]  
(3.5.1.19.)

Where

\[ c = b \sum YX_i + \sum YX_1 \]  
(3.5.1.20.)

The solution for \( a, b, c \) and \( R \) were obtained on the IBM 1620 computer and the curve of best fit was plotted by the plotter also. The program is shown in the appendix 1.

3.5.2. Geometric Mean and Geometric Deviation:

The relevant formulae for geometric mean and geometric deviation are shown as follows:

A. The diameter of each rock can be found in two ways:

1. By Volume:

\[ d = \left( \frac{6 \cdot \text{Volume}}{\pi} \right)^{\frac{1}{3}} \]  
(3.5.2.1.)

2. By Weight:

\[ d = \left( \frac{6 \cdot \text{Weight}}{\pi \cdot \text{Density}} \right)^{\frac{1}{3}} \]  
(3.5.2.2.)

Density was found from the total mass divided by the total volume of a sample of rocks.

B. Geometric mean (18) is defined as the antilogarithm of the
sum of the logarithms of the diameters of all the rocks in the sample divided by the number of rocks N.

\[
\text{LOG GM} = \frac{\text{LOG } d_1 + \text{LOG } d_2 + \cdots + \text{LOG } d_N}{N} (3.5.2.3)
\]

The geometric deviation is expressed as follows:

\[
\text{LOG GD} = \sqrt{\frac{\sum_{k=1}^{N} [\text{LOG } d_k - \text{LOG } d_{\text{MEAN}}]^2}{N-1}} (3.5.2.4)
\]

The GM and GD were found on the IBM 1620 computer and the programme is attached in the appendix 1.

3.5.3. Drainable Porosity

The drainable porosity is obtained from the following formula,

\[
E = \frac{\text{Volume of water added to a freshly drained sample}}{\text{total (bulk) volume}} (3.5.2.5)
\]

A plot of \( \frac{d}{D} \) against \( E \) was made to show the wall effect upon porosity.

3.6. Analysis of Results

The results of the computations for the constants and correlation coefficients for both equations (3.1.2) and (3.1.3) are shown in the following Table 16.

<table>
<thead>
<tr>
<th>TABLE 16. - THE VALUES FOR THE CONSTANTS a, b, c and R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( i = aV + bV^2 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( i = aV + bV^{\frac{3}{2}} + cV^2 )</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

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A comparison of $R_i$ shows that equation (3.1.3) is only slightly better than equation (3.1.2). Therefore it can be assumed that the equation (3.1.2) is as good as equation (3.1.3) when applied to crushed rock for Reynolds numbers ranging from 100 to 4500.

The plots of equation (3.1.2) for each size of crushed rock are shown in Fig. 6 to Fig. 10 and the plot of $a$ and $b$ against the size of crushed rock is shown in Fig. 11. From these plots, it can be seen that the slope of the straight line is decreasing with increasing rock size. This implies that the bigger the material, the bigger the flow path. Therefore, under the same head, the macroscopic velocity is greater in a coarse material than in a finer one and so is the turbulent term.

The geometric mean $GM$, geometric standard deviation $GD$ for the various rock samples together with the dynamic viscosity $\mu$, kinematic viscosity $\nu$, drainable porosity $E$ and dry porosity are shown in the following Table 17. These are factors applicable to the friction tests described in section 3.3.1.

Dry porosity is found as the ratio of the water in the void when the crushed rocks are wet on the surface only to the bulk volume. Therefore, dry porosity is found from the volume of water required to fill the voids of the first run in any porosity test divided by the bulk volume.
3.522 cm. CRUSHED ROCKS

\[
\frac{i}{v} = 0.00443 + 0.00433v
\]

Fig. 6 Experimental Friction Equation for 3.522 cm. Crushed Rock
Fig. 9 Experimental Friction Equation for 1.180 CM Crushed Rock

\[
\frac{i}{v} = 0.00671 + 0.01317v
\]
Fig. 11 Experimental Friction Constants Plotted against the Size of Crushed Rock

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Figure 12 General Equation for Crushed Rock Based on Drainable Porosity

\[ Y = 1.022 + 0.0421X \]

\[ Y = \frac{i \cdot Mg^2 Ev^3}{V \cdot \mu \cdot \sigma g \ln \sigma g \cdot (1 - Ev)^2} \]

\[ X = \frac{Ev^{1.5}}{(\sigma g \ln \sigma g) \cdot (1 - Ev)} \cdot \frac{V \cdot Mg \cdot Ev^2}{V} \]
Fig. 13 General Equation for Crushed Rock Based on Dry Porosity

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Fig. 14  Wall Effect on Drainable Porosity

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Fig. 15 Wall Effect on Dry Porosity
TABLE 17. - SOME USEFUL FACTORS IN THE ANALYSIS FLOW THROUGH ROCKS

<table>
<thead>
<tr>
<th>Normal Size</th>
<th>(1'')</th>
<th>(\frac{3}{4}'')</th>
<th>(\frac{1}{2}'')</th>
<th>(\frac{3}{8}'')</th>
<th>(\frac{1}{4}'')</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM by weight</td>
<td>3.522</td>
<td>2.009</td>
<td>1.657</td>
<td>1.180</td>
<td>0.962</td>
</tr>
<tr>
<td>GM by volume</td>
<td>3.521</td>
<td>2.007</td>
<td>1.652</td>
<td>1.181</td>
<td>0.939</td>
</tr>
<tr>
<td>GD by weight</td>
<td>1.089</td>
<td>1.070</td>
<td>1.103</td>
<td>1.117</td>
<td>1.161</td>
</tr>
<tr>
<td>GD by volume</td>
<td>1.090</td>
<td>1.073</td>
<td>1.151</td>
<td>1.175</td>
<td>1.154</td>
</tr>
<tr>
<td>(\mu \cdot 10^{-5}) gram-sec/cm(^2)</td>
<td>0.00862</td>
<td>0.00879</td>
<td>0.00862</td>
<td>0.00873</td>
<td>0.00867</td>
</tr>
<tr>
<td>(\nu \cdot 10^{-5}) cm(^3)/sec.</td>
<td>0.00864</td>
<td>0.00890</td>
<td>0.00864</td>
<td>0.00876</td>
<td>0.00870</td>
</tr>
<tr>
<td>Drainable porosity</td>
<td>0.454</td>
<td>0.439</td>
<td>0.444</td>
<td>0.441</td>
<td>0.436</td>
</tr>
<tr>
<td>Dry porosity</td>
<td>0.458</td>
<td>0.460</td>
<td>0.455</td>
<td>0.450</td>
<td>0.465</td>
</tr>
</tbody>
</table>

The factors described in Table 17 were applied to equation (3.1.7.) to find the A and B for each size of rock. The calculation procedure is the same as for equation (3.1.2.). Firstly, a trial was made based on the drainable porosity without including any additional correction factor for porosity to the second term but the plots of various sizes of rocks showed poor agreement in their slopes. Therefore a series of trials that involved multiplying the term at the left hand side or the second term of the right hand side by various powers of the respective drainable porosity were tried with the aid of the IBM 1620 computer and its on line plotter. Finally, it was found that a factor of the second power of the drainable porosity applied to the second term of the right hand side gave the best result with a correlation coefficient of 0.99999 or almost 1. The equation found is shown as follows and also on Fig. 12.

\[
\frac{i \cdot Mg^2 \cdot E^3}{V \cdot \mu \cdot \sigma_3 \cdot (1 - E)^3} = 1.022 + 0.0421 \frac{E^3 \cdot V \cdot Mg}{(\sigma_3 \cdot (1 - E)^3) (1 - E) \nu} \quad (3.6.1.)
\]
or

\[ Y = 1.022 + 0.0421X \]

In the equation \( X \) and \( Y \) are dimensionless.

Secondly, following the procedure mentioned above, trials were made based on the dry porosity. It turns out that even without a correction factor involving porosity, the slopes of the curves are almost parallel to one another but there is a small deviation in the \( X \) or \( Y \) direction as shown in figure 13. The equation is shown as follows and also on figure 13.

\[
\frac{i \cdot Mg^2 E^3}{V \cdot \mu \cdot \sigma_3 \ln \sigma_3 \cdot (1 - E)^2} = 1.8185 + 0.00875 \frac{E^{1.5} \cdot V \cdot Mg}{(\sigma_3 \ln \sigma_3)(1 - E)V} \quad (3.6.2.)
\]

The correlation coefficient is 0.99999.

Fig. 14. shows that below \( \frac{d}{D} = 0.1 \), the curve of \( \frac{d}{D} \) against drainable porosity becomes perpendicular to the \( X \) axis which means the porosity has a constant value of 0.44 and there is no wall effect. In Fig. 15, the \( \frac{d}{D} \) against dry porosity plot, the value of \( \frac{d}{D} \) under which there will be no wall effect is 0.13. and the constant value for dry porosity is 0.452. The deviation of the drainable porosity from the dry porosity is around 2.8%. Fortunately all the experiments except for the 1" run in the 6"x 6" test box are within the range of no wall effect. Therefore no further corrections for the wall effect were made.
CHAPTER FOUR
TWO DIMENSIONAL FLOW

In this chapter, the dimensional analysis method is applied to find an equation for the relationship of upstream and downstream water levels and the seepage height. The assumed exponential form of the dimensionless equation is then solved by the combination of the least squares method and remainder theorem. Then, the finite element method is described for the solution of seepage problem through a rectangular crushed rock section and a comparison of the numerical results with the experimental results is shown.

4.1. Background and Theory

4.1.1. Dimensional Analysis

For turbulent flow through a rectangular section, the correlation of the heights of upstream water level (H), the height of downstream water level (h) and the height of the seepage surface was derived in the following manner.

\[ \text{Fig. 16 Factors Governing the Two Dimensional Flow} \]

From fig. (16), it is obvious that the governing factors are velocity (\( V \)), the size of the rocks (d), h, h_s, L, H, and kinematic viscosity. The gravitational force is assumed negligible. By
the theory and method of dimensional analysis, five dimensionless products can be written, i.e. (1) \( R = \frac{Vd}{\nu} \); (2) \( \frac{h}{H} \); (3) \( \frac{h_s}{H} \); (4) \( \frac{L}{H} \); and (5) \( \frac{d}{L} \). Therefore it can be established that

\[
\frac{h_s}{H} = f \left( R, \frac{d}{L}, \frac{h}{H}, \frac{L}{H} \right) \quad (4.1.1.1)
\]

or it could be assumed \( \frac{d}{L} = \text{constant} \) and that

\[
\frac{h_s}{H} = CR^\alpha \left( \frac{h}{H} \right)^\gamma \left( \frac{L}{H} \right)^\xi \quad (4.1.1.2)
\]

It can be imagined that as \( \frac{h}{H} \to 1, \frac{h_s}{H} \to 0 \), so that equation (4.1.1.2) can be written as follows,

\[
\frac{h_s}{H} = C(1 + R)^\xi \left( 1 - \frac{h}{H} \right) \left( 1 + \frac{L}{H} \right)^\gamma \quad (4.1.1.3)
\]

Let \( \Theta = \beta \left( 1 + \frac{L}{H} \right)^\xi \); \( C \) is put equal to unity since \( \frac{h_s}{H} \to 1 \) as \( \frac{h}{H} \to 0, \frac{L}{H} \to 0 \) and \( R \to 0 \). Therefore

\[
\frac{h_s}{H} = (1 + R)^\xi \left( 1 - \frac{h}{H} \right) \left( 1 + \frac{L}{H} \right)^\gamma \quad (4.1.1.4)
\]

Equation (4.1.1.4) is assumed to be the form of the equation and \( \alpha, \beta, \gamma, \xi \) must be obtained from experimental data.

4.1.2. The Verification of the Finite Element Method for the Solution of Non-linear Seepage Problem

The finite element method is applied to the solution of the steady, non-linear flow through a rectangular crushed rock section with a free surface. This method has been applied by Zienkiewicz (21, 22), Finn (23) and Taylor (24). From the theory of
continuity, an equation for this kind of flow and isotropic media can be shown as follows:

\[
\frac{\partial}{\partial X} \left( K \frac{\partial \phi}{\partial X} \right) + \frac{\partial}{\partial Y} \left( K \frac{\partial \phi}{\partial Y} \right) - q(X,Y) = 0
\]  \hspace{1cm} (4.1.2.1.)

The derivation of equation (4.1.2.1.) is shown in the appendix (II). In equation (4.1.2.1.), the term \( q(X,Y) \) indicates a certain boundary condition which is the rate of seepage from the seepage face.

The finite element method assumes that the solution of equation (4.1.2.1.) subject to specified boundary conditions is equivalent mathematically to find a function, \( \phi \), which minimizes the following integral taken over the whole region of solution,

\[
\chi = \frac{1}{2} \iint \left[ K \left( \frac{\partial \phi}{\partial X} \right)^2 + K \left( \frac{\partial \phi}{\partial Y} \right)^2 \right] dX \, dY + \int_{c} q \, \phi \, ds
\]  \hspace{1cm} (4.1.2.2.)

The derivation of equation (4.1.2.2.) is shown in the appendix (II). In equation (4.1.2.2.) \( \phi \) is the piezometric head and \( K \) is the hydraulic conductivity which in the case of non-linear flow is a function of the macroscopic velocity and is derived from the Lindquist law, i.e.

\[
K = \frac{1}{a + bV}
\]  \hspace{1cm} (4.1.2.3.)

To solve equation (4.1.2.2.) by the finite element concept, the continuous distribution of pressure head in the crushed rock section of flow is represented by the values of the piezometric head \( \phi \) at a finite number of points or nodes. The region is divided into a network of triangles called finite elements and the piezometric head in each element is specified in terms of the values of the piezometric head.
at the nodes. Thus, the seepage problem is considered solved when the piezometric head at the nodes are known.

The boundary conditions that play an important role in the solution are shown in the following Figure 17.

![Figure 17. Boundary Condition for the Problem](image)

Since the head $H$ and $h$ are kept constant in a specific run, the pressure at the boundaries of the upstream and downstream water-basins obeys the hydrostatic law. In a mathematical form, if the air pressure on the upper face of the water-basin is neglected, this can be written as

$$\phi_1 = H; \quad \phi_2 = h$$

That is, the boundaries along $AB$ and $EF$ are equipotential lines.
CD is assumed to be impervious. Therefore no flux can pass through it and the integral

$$\int \left( V_x \, dY - V_y \, dX \right)$$

taken along any part of such a boundary is equal to zero or

$$\psi = \text{constant}$$

and

$$\frac{\partial \phi}{\partial n} = \frac{\partial \psi}{\partial s} = 0$$

in which \( \phi \) is a potential function, and \( \psi \) is a stream function, \( \partial n \) is the normal derivative and \( \partial s \) is the tangent derivative.

On the seepage areas i.e. the surface \( BG \) of the embankment through which water is steadily leaking and falling into the tail-water, the pressure is assumed to be constant and equal to atmospheric pressure. i.e.

$$\phi = Y$$

BC and DE are the upstream and downstream face of the crushed rock section respectively. They are lines of constant potential

$$\phi = \text{constant}$$
or

$$\frac{\partial \phi}{\partial s} = \frac{\partial \psi}{\partial n} = 0$$

The pressure on the free surface \( BG \) is constant. i.e.

$$\phi = Y$$

Since a steady motion is considered, the flux across a free surface will be equal to zero also, and on the free surface we shall have

$$\psi = \text{constant}$$
or
\[ \frac{\partial \phi}{\partial n} = \frac{\partial \psi}{\partial s} = 0 \]

The relevant theoretical developments are referred in the unpublished paper "Non-Darcy Flow Solved by Finite Element Analysis" attached in the appendix (2). The application of the finite element method to the solution of non-linear flow problems forms part of professor McCorquodale's Ph.D. dissertation (20). The functional equation is expressed as follows:

\[ \frac{\partial x}{\partial \phi_i} = \sum \{ K \left[ 1 - \frac{Kb}{2} \left( \frac{1}{a^2} \frac{\partial \phi}{\partial s} \right)^{\frac{1}{2}} \right] \left[ S_{ij} \right] \left[ \phi^f \right] + \sum P_i = 0 \]

4.2. Experimental Equipments

4.2.1. Facilities

The facilities used in the experiment are listed as follows:

A. The venturi meter for flow measurement was the same as described in section 3.2.

B. The pump was also the same as described in section 3.2.

C. A piezometer rack with 1/10 inch divisions was used for measuring pressures in the two dimensional section. The board consisted of 22 vertical glass tubes of 3 mm. diameter that were used for measuring the pressures at the nodes at the outer face of the rock section as shown in figure (18).

D. Two vertical screens with one quarter inch openings used to hold the rocks in place, see fig. (18). The screens were slid down to the bottom of the flume along two pre-cut gloves.

E. The tailwater gate used to control back pressure on the sample was the same described in section 3.2 and shown in fig. (18).
Fig. 18A The Front View of the Equipments for Two Dimensional Flow

- 3 mm Diameter Glass Tubes
- Supporting Screen with 1/4" Openings
- 655 CM Crushed Rock
- Tailwater Gate
- 1/4" Diameter Holes for Pliezometers
- Flow
Fig. 18B  View of Two Dimensional Test Section

Fig. 18C  View of Test Equipments
F. The flume for the test was the same as described in section (3.2.) with a flexiglass window of 4 ft. high x 6 ft. long. On the flexiglass, seven columns and six rows of piezometer tubes placed on a 6 inches grid were provided to connect with the manometers for the measurement of piezometric heads across the section as shown in Figure 18.

G. The porous medium was formed by 1.655 cm. crushed rock and it has a dimension of four feet long, 1 foot wide and three and half feet high.

4.2.2. Layout of Equipment

The facilities described in section (4.2.1.) are shown in Figure 18A, B & C.

4.3. Procedure

4.3.1. Measurements of the Required Upstream and Downstream Water-levels and the Seepage Height

The steps of measurements for the relevant heights are described as follows:

A. The rocks were placed into the flume with slight compaction as shown in Figure 18.

B. The venturimeter was zeroed.

C. The pump was started to deliver water to the model.

D. When the water levels were steady, the flow rate was recorded from the venturimeter and the heights of the upstream and downstream water level together with the seepage height were measured by a centimeter scale.

E. Experiments were repeated for different flow rates
ranging from 10 to 250 U. S. gallons per minute.

4.3.2. Measurements of Piezometric Heads and the Position of the Free Water Surface

The following procedure was used to obtain the piezometric heads and free surface profile for the two-dimensional model.

A. The same rock section described in section (4.2.1.) was used.

B. The same procedures described in (A) to (D) of section (4.3.1.) were followed.

C. The co-ordinates of several points on the free water surface were measured to fix the free surface profile.

D. Before the head at each piezometric location (see Figure 18) was read from the piezometers, the rubber tubes were drained to ensure there was no air bubbles inside and then connected tightly to the piezometers grouped on a board attached at the left hand side of the flume as shown in Figure 18. The piezometric heads were recorded.

E. The experiment was repeated for different flow rates.

4.4. Results of Experiments

The heights and piezometric heads measured are tabulated in the following Table 18 and Table 19 and Figure 19.
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<th>FLOW RATE IN U.S. GALLON PER MIN.</th>
<th>UPSTREAM WATER LEVEL IN cm.</th>
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4.5. Calculations

4.5.1. Solution of the Empirical Seepage Height Equation

Taking the logarithm of equation (4.1.1.4.), it gives

\[ \log \frac{h_S}{H} = \alpha \log (1+R) + \beta \left( \frac{1-L}{H} \right) \log \left( 1 - \frac{h}{H} \right) + \gamma \log \left( 1 + \frac{L}{H} \right) \]  

(4.5.1.1)

Let \( X_1 = \log (1+R) \); \( X_2 = \log \left( \frac{1-h}{H} \right) \); \( X_3 = \log \left( 1 + \frac{L}{H} \right) \); \( Y = \log \left( \frac{h_S}{H} \right) \)

then

\[ Y = \alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3 \]  

(4.5.1.2)

Equation (4.5.1.2) is solved by the theory of least squares and the remainder theorem. The steps are as follows:

\[ \sum \epsilon^2 = \sum \left[ Y - (\alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3) \right]^2 \]  

(4.5.1.3)

then

\[ \frac{\partial}{\partial \alpha} \sum \epsilon^2 = -2 \sum \left[ Y - (\alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3) \right] X_1 = 0 \]  

(4.5.1.4)

\[ \frac{\partial}{\partial \beta} \sum \epsilon^2 = -2 \sum \left[ Y - (\alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3) \right] e^{X_2} X_2 = 0 \]  

(4.5.1.5)

\[ \frac{\partial}{\partial \gamma} \sum \epsilon^2 = -2 \sum \left[ Y - (\alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3) \right] e^{X_2} X_2 = 0 \]  

(4.5.1.6)

\[ \frac{\partial}{\partial X_1} \sum \epsilon^2 = -2 \sum \left[ Y - (\alpha X_1 + \beta e^{X_2} X_2 + \gamma X_3) \right] \beta e^{X_2} X_2 X_3 = 0 \]  

(4.5.1.7)
From equations (4.5.1.4.) to (4.5.1.7.) we obtain

\[ \sum \gamma x_i = \alpha \sum x_i^2 + \beta \sum e^{x_3} x_i x_2 + \gamma \sum x_i \tag{4.5.1.8.} \]

\[ \sum \gamma x_3 = \alpha \sum x_i x_2 + \beta e^{x_3} x_2 x_3 + \gamma \sum x_2 \tag{4.5.1.9.} \]

\[ \sum \gamma e^{x_2} x_2 = \alpha \sum e^{x_2} x_2 x_2 + \beta \sum (e^{x_2} x_2)^2 + \gamma \sum e^{x_2} x_2 x_3 \tag{4.5.1.10.} \]

\[ \beta \sum \gamma e^{x_2} x_2 x_3 = \alpha \beta \sum e^{x_2} x_2 x_3 + \beta \sum e^{2x_2} x_2 x_3 + \gamma \beta \sum e^{x_2} x_2 \tag{4.5.1.11.} \]

The four unknowns cannot be found directly by determinant method from the four equations because of the non-linearity in \( \xi \).

It is possible to assume arbitrary values of \( \xi \) in equations (3.5.1.8.) to (3.5.1.10.) and solve directly for \( \alpha \), \( \beta \), and \( \gamma \). These values are then substituted into equation (3.5.1.11.) to find a remainder, which is

\[ \text{remainder} = \beta \sum \gamma e^{x_2} x_2 x_3 - \alpha \beta \sum e^{x_2} x_2 x_3 + \beta \sum e^{2x_2} x_2 x_3 + \gamma \beta \sum e^{x_2} x_2 \]

When the remainder equals zero, the respective values of \( \alpha \), \( \beta \), \( \gamma \) and \( \xi \) are the values desired. This approach was carried out on the IBM 360 computer. The program is shown in the appendix (I).

4.5.2. Determination of the Piezometric Heads and Seepage Height by the Finite Element Method

The numerical solution of the piezometric heads for the nodes is referred to the unpublished paper "Non-Darcy Flow Solved by Finite Element Analysis" in the appendix (III).
Fig. 20  The Arrangement of the Triangular Elements
Fig. 21 The Flow Net for the Estimation of Initial Piezometric Heads

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The assumed position for the free surface at the first iteration is shown in Figure 20.

The initial approximate \( \phi \) values for all the nodes were determined by drawing a flow net shown as Figure 21 with the upstream and downstream water levels found from the experiment. The piezometric head values were then interpolated from the flow net.

The exact results were obtained by substituting the value of the height of the exit point found by the method described later.

The extrapolated Liebmann method, the general flow chart and the computer programs are shown in the appendix (4) and appendix (5).

The flow rate can be calculated accurately using the middle of each element at section A-A and B-B in Figure 20. The velocities for each of the elements at either section A-A and B-B are added together and a mean velocity is calculated. Then the flow rate equals the mean velocity multiplied by the width of the flume and the height of the free surface at section A-A i.e. aa or B-B i.e. bb if the mean velocity is calculated at section B-B.

The height of the exit point cannot be obtained by a single assumption. Keeping all the \( \phi \) values of the nodes the same, the program was run with a new assumed height of the exit point substituted and a new set of values for the velocities for each element was obtained. If the substituted height of the exit point is the right one, the flow calculated at section A-A and section B-B in Figure 20 would be the same or their difference would be zero. Several values for the height of exit point are assumed and for each one the difference between the flow rate at section A-A and B-B is

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calculated and then plotted versus the height of exit point. The points are connected and the intercept on the height of exit point axis gives the correct value for the seepage height.

4.6. Analysis of Results

4.6.1. The Proposed Empirical Equation for the Seepage Height

The values of \( \alpha, \beta, \gamma \) and \( \xi \) in equation (4.1.1.4.) were found on the 360/40 IBM computer. Equation (4.1.1.4) can then be written as

\[
\frac{h_s}{H} = (1 + R)^{0.1897} \left(1 - \frac{h}{H}\right)^{5.803 \left(1 + \frac{L}{H}\right)^{-0.6112} \left(1 + \frac{L}{H}\right)^{-2.0636}}
\]  

(4.6.1.1)

This equation is valid for \( \frac{d}{L} \approx 0.014 \) and \( \frac{V^2}{2gH} \) negligible.

It is valuable to plot graphs based on this proposed equation by assigning a value to \( R \) and values for \( \frac{L}{H} \) ranging from 1 to 4 to see the variation of the seepage height with respect to the tailwater level. Figure 22 was plotted with \( R = 800 \). If \( \frac{L}{H} \) is known, the seepage height can be obtained from the graph. Figure 23 shows the variation of seepage heights with respect to Reynolds number.

The height of the exit point found numerically from section (4.5.2.) can be compared with a plot of equation (4.6.1.1.) by choosing \( R \) and \( \frac{L}{H} \) equal to the exact values used in the computer. The seepage height so found by equation (4.6.1.1.) is a little lower than the one found by the numerical method.

All plots show a mix up of interceptions at the lower part of the curve. This might due to the lack of data for the derivation
Fig. 22 The Range of Applicability and the Variation with respect to $\frac{L}{H}$
Fig. 23 Shows the Variation with respect to Reynold Number for $\frac{HS}{H}$ and $\frac{HT}{H}$

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Fig. 24 Comparison of the Laminar and Turbulent Values for $\frac{HS}{H}$ and $\frac{HT}{H}$

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of the equation. Hence Figure 22 with the experimental data plotted shows the range of applicability of the equation.

It is also worthwhile to compare the turbulent case with the theoretical laminar solution case as given by (29). Figure 24 compares the two cases. One can easily see that the value for the turbulent case is almost double the one for laminar case.

4.6.2. The Seepage Height from the Finite Element Solution

For each assumed initial seepage height, a run of the programme will give a new set of velocities and piezometric heads for the elements. The flow across the crushed rock section for the respective seepage heights can be calculated in this way. In Figure 20, the velocities of the nine elements in section A-A are averaged and then multiplied by the height of the section that is \( a a \). In the same way the flow can be calculated from section B-B. If the seepage height is choosen correctly than the flow calculated from the section A-A will equal the one from the section B-B. Therefore, the difference flow rates between the two sections plotted against the assumed seepage height is a method for the determination of the seepage height. The exact seepage height is the one corresponding to zero difference in flow rate as shown in Figure 25.

The seepage height found numerically is 9.9 cm. as compared to the experimental value of 10.4 cm. the error is 4.7%.

The free surface profile and the equal potential lines both from the experiment and theoretical calculations are plotted on Figure 26. From the graphs one can see that they are quite consistent and the deviations of all the surface nodes are less than 2%.
FIGURE 26 The Comparison of Theoretical and Experimental Values
This indicates that the finite element method is applicable to the solution of steady non-Darcy seepage problems in porous media based on experimental evidence.
CHAPTER FIVE
DISCUSSIONS AND CONCLUSIONS

From the work presented in Chapter 3 and Chapter 4, the following discussions and conclusions are drawn:

5.1. Comparison of Intuitive Friction Equations

Generally, the equations for non-linear flow, regardless of the method of development, can be expressed in one of three forms

\[ i = aV^n \]
\[ i = aV + bV^2 \]
\[ i = aV + bV^2 + cV^3 \]

These intuitive formulae do not contain any term involving the fluid viscosity so that they are necessarily limited to conditions where viscosity has little effect or to a restricted range of Reynolds numbers. Among these three equations, the exponential equation, owing to its convenience in application, has been used as a basic equation by many researchers such as Lawson (15). However, it would appear to be the most unfavorable equation. As one can see, the head loss is expressed in terms of the characteristics of the bed and material as embodied in \( a \) and the macroscopic velocity raised to some power of \( n \). Upon differentiation, it is found that

\[ \frac{di}{i} = n \frac{dv}{v} \]

This shows that the relative change in head loss is \( n \) times the relative change in velocity or the discharge so that the proper value for \( n \) varies continuously with the velocity. In other words, it is not correct to express a wide range of discharge rates related to the hydraulic gradient by a unique value of \( n \). This objection is not serious in the two or three term equation. For these two equations, the important influencing factors are the
characteristics of the bed and the characteristics of the bed materials. Therefore, it is believed that the two and three term equations are better than the exponential equation in their range of applicability. The exponential form should not be used, except, with careful laboratory work to determine n for the specified range of flow rates.

It was shown that the three term equation is slightly better than the two term equation. However, for crushed rock in the range of Reynolds number 600-4000, the two term equation is almost as good as the three term equation. Therefore, the two term equation is used in further applications.

In the three term equation, the exponent \( \frac{3}{2} \) for the transition flow term is quite arbitrary. One might find another value for the exponent to get a better correlation based on analysis of experimental results.

Inasmuch as there are four flow regimes, according to the results and deductions of the three term equation, one might propose a four term equation such as

\[
j = aV + bV^m + cV^{\frac{p}{q}} + dV^2
\]

This equation could be expected to give a better correlation just as occurred with the three term equation.

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5.2. The Characteristics of the Constants $a$ and $b$ in the Two Term Friction Equation and the Permeability for Turbulent Flow

The constants $a$ and $b$ in the two term friction equation have been investigated by a number of researchers. For example:

Lindquist reported that $a$ and $b$ have the following expression respectively:

$$a = \alpha_* (1 - E)^3 \frac{\mu}{E^2 d^2}$$  \hspace{1cm} (5.2.1.)

and

$$b = \beta_0 (1 - E) \frac{\rho}{E^3 d}$$  \hspace{1cm} (5.2.2.)

in which $E$ is porosity, $d$ is the equivalent diameter of the grain, $\rho$ the density of the fluid and for irregular angular grains $\alpha_* \geq 1500$ and $\beta_* \geq 3.6$

Carmen found that

$$a = 5 \mu S_0^2 (1 - E)^2$$  \hspace{1cm} (5.2.3.)

and

$$b = b_0 S_0 (1 - E) \frac{\rho}{E^3}$$  \hspace{1cm} (5.2.4.)

in which $S_0$ is the specific surface of solid, $E$ is the porosity, $\mu$ the viscosity, $\rho$ the density and $b_0$ is constants varied for different porous media.

Ward recently proposed that $a$ and $b$ can be expressed by the permeability $K$ as follow:
\[ a = \frac{\mu}{K} \]

and

\[ b = \frac{0.550 \rho}{K^{\frac{1}{3}}} \]

in which \( \mu \) is the viscosity, \( \rho \) is the density and \( K \) is as expressed as (3.1.7.)

From the above equations, one can say the constant \( a \) and \( b \) are dependent on the properties of fluid and the characteristics of the bed and are not affected by the flow velocity. The most important factor is the porosity. Therefore, a careful study of the porosity effect is necessary.

From the plots, \( a \) is the intercept on the \( \frac{1}{V} \) axis and \( b \) is the slope of the straight line. The value of \( a \) and \( b \) found from the experiment for each size of crushed rock shows a decreasing trend as the size of the rock increases.

The permeability characteristics of the rocks section are of vital importance. The permeability proposed for turbulent flow is derived from the Lindquist's law as follow:

\[ K = \frac{1}{a + bV} \]

The value of \( a \) and \( b \) are determined from experiments. It is obvious that the permeability is a function of the velocity in the turbulent case but in the laminar case it does not.
5.3. Proposed General Resistance Formulae for Crushed Rock

For crushed rock in which the turbulent flow conditions are common, the resistance equation based on drainable porosity can be represented by the relation

\[
\frac{i \cdot M_2 \cdot E_3}{V \cdot \mu \cdot \sigma_g \cdot (1 - E) \cdot (1 - E)^2} = 1.022 + 0.0421 \frac{E_{v}^{15} \cdot V \cdot M_4}{(\sigma_g \cdot \mu \cdot \sigma_g) (1 - E) \nu} \tag{5.3.1}
\]

and if based on dry porosity, it is

\[
\frac{i \cdot M_2 \cdot E_3}{V \cdot \mu \cdot \sigma_g \cdot (1 - E)^2} = 1.8185 + 0.00875 \frac{E_{v}^{15} \cdot V \cdot M_4}{(\sigma_g \cdot \mu \cdot \sigma_g) (1 - E) \nu} \tag{5.3.2}
\]

In equation (5.3.1), a correction factor of \( E_v \) was multiplied to the second term on the right hand side so that the best correlation for all the plots for different sizes of crushed rock was obtained. Equation (5.3.2), which agrees with Ward's results, has not been multiplied by any correction factor because the dry porosities are quite close to each other. Though both of them are derived from crushed rock ranging from 3.522 cm. to 0.965 cm., it is believed that equations (5.3.1) and (5.3.2) can be applied to any size of crushed rock. For confirmations, further experiments on larger crushed rock should be performed.

For accuracy, these equations should be used to predict the quantity of flow under turbulent conditions for crushed rock. The quantities of flow under turbulent conditions could be greatly in error if based on Darcy's law.
5.4. Wall Effect

It was shown by Rose and Dudgeon that the presence of side walls in the packing of non-cohesive granular material causes a zone of higher porosity of the order of half a median particle diameter thick to occur against the wall and the velocity distribution across the section has a form as shown in Figure 27.

![Figure 27. Postulated Velocity Distribution across a Tube](image)

The wall effect for crushed rock as shown from Figure 27, is not quite as significant for crushed rock as for the spherical materials which Rose and Dudgeon investigated. This is because the shapes of the crushed rocks are angular and there are different sizes of crushed rock mixed together. Therefore, the presence of the side wall will not cause as much voids between the materials and the wall. The higher porosity near the wall resulting from the effect on local packing is then greatly eliminated.

For crushed rock, if \( \frac{d}{D} < 0.14 \), wall effect is not significant but for spheres, Rose found that \( \frac{d}{D} < 0.02 \). The wall effect is almost independent of flow rate.
5.5. Dry and Drainable Porosity

The dry and drainable porosity have been studied and their effects on the resistance equations have been compared as shown in equations (5.3.1) and (5.3.2). The dry porosity found for all kinds of crushed rock remains almost the same. But the drainable porosity varies and there seems to be no specific trend in the variations.

The procedure for the determination of dry porosity required more effort than the determination of drainable porosity for this study. The experimental results show the deviation of the drainable porosity from the dry porosity to be about 3%. Since the resistance equations are so sensitive to the variation of porosity, the idea of using the drainable porosity instead of the dry porosity is questionable. Nevertheless, the drainable porosity is important in unsteady flow.

The drainable porosity is lower than the dry porosity. This comes from the fact that water sticks on the surface of the rocks and hence reduces the volume of the voids in the drainable case.

5.6. Seepage Height and Face

An equation for the calculation of seepage height based on the upstream and downstream water level, the length of the rock sample and the Reynolds number has been proposed as shown below:

\[
\frac{h_s}{H} = (1 + R)^{0.1897} \left(1 - \frac{h}{H}\right)^{5.803} \left(1 + \frac{L}{H}\right)^{-0.6112} \left(1 + \frac{L}{H}\right)^{-2.0636}
\]

In this equation, the effect of gravitational force is neglected.
because the velocity head in each element is very small as compared to the piezometric head. Also $\frac{d}{L}$ is assumed approximately equal to 0.014. In laminar flow, regardless of the downstream slope, the line of seepage must be tangent to the downstream boundary as shown in Figure 28.

![Figure 28](image)

**Figure 28** Transfer Conditions for Laminar Flow

In turbulent flow, experiments indicate that the tangent condition does not hold. The transfer condition for turbulent flow is illustrated in Figure 29.

![Figure 29](image)

**Figure 29** Transfer Conditions for Turbulent Flow
It was observed that the relative height of the seepage face increased with increasing Reynolds number but decreased with increasing tailwater level. The downstream face formed by the surface of rock and the screen is very rough so that as water drops along the face, it is dispersed out of the face. If the face is smooth, the seepage line might be closer to being tangent to the downstream face.

Graphs have been plotted from the measurements of the seepage height and are shown in Figure 22 to Figure 24. The range of applicability of the proposed equation is indicated by the scatter in Figure 22.

5.7. The Finite Element Method

The finite element approach gave a very good solution for the two dimensional non-Darcy seepage problem as evidenced by the comparison of the numerical results with the experimental measurements shown in Figure 26.

It seems that the original approach using this method cannot give a satisfactory solution of the exit point. A possible reason for this is that the effect of convergence of the macrostreamlines in lowering the hydraulic conductivity in the exit region has not been taken into account. An alternate method of reasoning based on the constant flow through each section was applied to find the exit point and was successful.

The assumptions made for the application of the finite element method are:

A. The flow across each element is uniform.
B. The flow and pressure are continuous from element to element.

C. The velocity head is neglected.

The principal advantage of the finite element approach over the usual finite difference method lies in the arbitrariness with which node point locations may be selected to satisfy various boundary and interface conditions for problems of interest.
APPENDIX I

COMPUTER PROGRAMS
C  CORRELATION OF Y=FX

DIMENSION Y(50), X1(50), CY(50), CX1(50)

1 READ 5, N
PRINT 5, N
READ 6, Q, HT
AA=231.2*54/(63.*121.)
X1(I)=AA*Q
Y(I)=HT/(X1(I)**21.)
SSY=SSY+Y(I)
SSX1=SSX1+X1(I)
SSC=SSC+X1(I)**2
SSCY=SSCY+CY(I)**2

3 FORMAT (2F7.3)

53X 1=SSX 1+X 1(I)
33Y = SSY/F
53X 1=SSX 1+X 1(I)
Y(I)=MT/(X1(I)**31.)
SSY=SSY+Y(I)
SSX1=SSX1+X1(I)

2 FORMAT (12HKEYNOLD NO.=F12.3)

4 CONTINUE

PRINT 9, RN
Y(I)=SSY/PN
X1M=SSX1/PN
DO 10 I=1,N
CY(I)=Y(I)-YM
CX1(I)=X1(I)-X1M
10 CONTINUE

3 CONTINUE

SSY=0
SSX1=0
SSC=0
SSCY=0
DO 20 I=1,N
SSY=SSY+X1(I)**2
SSX1=SSX1+X1(I)**2
SSC=SSC+X1(I)**2
SSCY=SSCY+Y(I)**2
20 CONTINUE

6=SSY/SSX1
6=SQRT(SSX1*SSCY)
K=SSY/6
PRINT 20, K

9 FORMAT (2F7.3)
9 FORMAT (2F7.3)
9 FORMAT (2F7.3)
9 FORMAT (2F7.3)

GO TO 1
END
DIMENSION Y(2,1), X1(2,1), A2(2,1), CY(2,1), SXY(2,1), CX(2,1)
DIMENSION A(2,1), UV(2,1), W(2,1), AT(2,1), U(2,1)
DIMENSION VT(2,1)

SSY =
SSX1 = 0
SSX2 = 0

1 READ B*N*K
2 FORMAT (SX*10*13X*12)
READ 3, GM, VS
3 FORMAT (2F7*3)
DO 4 I = 1, N
READ 5, G, HT
PRINT 6, G, HT
4 FORMAT (6X*F7*3)
AA = 231.0*2.0+0.0*121.0
X1(I) = AA
Y(I) = HT/(X1(I)*31.0)
X2(I) = SQRT(X1(I))
SSY = SSY + Y(I)
SSX1 = SSX1 + X1(I)
SSX2 = SSX2 + X2(I)
CONTINUE
5 CONTINUE
PNEN
YM = SSY/PN
X1F = SSX1/PN
X2F = SSX2/PN
DO 10 I = 1, N
CY(I) = Y(I) - YM
CX1(I) = X1(I) - X1F
CX2(I) = X2(I) - X2F
CONTINUE
10 CONTINUE
SSY = 0
SSX1 = 0
SSX2 = 0
CONTINUE
15 CONTINUE
SSX1 = 0
SSX2 = 0
DO 20 I = 1, N
SSY = SSY + CY(I) + CY(I)
SSX1 = SSX1 + CX1(I)
SSX1 = SSX1 + CX1(I) + CX1(I)
SSX2 = SSX2 + CX2(I)
SSX2 = SSX2 + CX2(I) + CX2(I)
CALK = SSX1 + CX1(I) + CX1(I)
CALK = SSX2 + CX2(I) + CX2(I)
CY(I) = SSX1 + CX1(I) + CX1(I)
CY(I) = SSX2 + CX2(I) + CX2(I)
CONTINUE
20 CONTINUE
RETURN
END

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$z_2 = 3z_3 x_1 + z_3 t_3$
$z_1 = 2x_1 x_2 + z_1 t_1$
$t_1 = 11 x_1 y + t_1 x_2 x_5 y
x_2 = x_2 x_1 y + z_2 x_5 x_5 y
z_5 y = y_1 x_1 z_1 + z_5 x_5$
PRINT 42, 01, 02, 03
42 FORMAT (4X,ilded=15.5, eild=15.5, eild=15.5)
SCC = 01 x_1 y + 02 x x_5 y
SR = SCC / SCy
RESORT(SR)
PRINT 43, H
43 FORMAT (14X, eild=15.5)
GO TO 1
1234 CALL EXIT
9991 STOP
END
CALCULATION OF GEOMETRIC MEAN AND GEOMETRIC DEVIATION BY ALGOL

DIMENSION X(30), Y(30), Z(30), X1(30), Y1(30), Z1(30), W(30), Y(13)
PUNCH 53

33 FORMAT (15N4,1X,12F13.1)
  1 READ 2,N
  2 FORMAT (5X*13)
     A=LOGF(10.*A)
     G1=0
     WTT=0
     DO 3 I=1,N
       READ 4, WT(I)
       PUNCH 4, WT(I)
  4 FORMAT (5X*F6.2)
     X1(I)=(6.8*WT(I)/(3.1416*2.677))**0.233333333
     Y1(I)=A*LOGF(X1(I))
     PUNCH 12, Y1(I)
  12 FORMAT (65X*F6.2)
     G1=G1+Y1(I)
     WTT=WTT+WT(I)
  3 CONTINUE
   PT=N-1
   U1=W1/PT
   SIGMA=SQRTF(U1)
   SIGMA2=10.*SIGMA
   PUNCH 77, SIGMA
  35 FORMAT (4X*12HEXAGEOMETRIC MEAN =F10.2)
  77 FORMAT (4X* 2HEXAGEOMETRIC DEVIATION=F10.3)
  830 CALL EXIT
END.
C  CALCULATION OF GEOMETRIC MEAN AND GEOMETRIC DEVIATION BY VOLUME

1 READ 2, N
2 FORMAT (5X*13)
   $=\log((10^A)/3)
   A=1.372
   G=0
   VT=G
   READ 6, V(I)
   PUNCH 6, V(I)
3 FORMAT (5X*F6.2)
   X(I)=(G*V(I)/3.1416)**0.5508333333
   Y(I)=A*LOGF(X(I))
   PUNCH 11, Y(I)
11 FORMAT (5Y(I)=F10.5)
   G=G+Y(I)
   VT=VT+V(I)
   PUNCH 444
444 FORMAT (10H AFTER 666)
   GL=G/PN
   G=10.0**GL
   PUNCH 5, GAG
   $=0
   Z(I)=(Y(I)-GL)**2
   $=$+$Z(I)
   U=$/PT
   SIGMAT=SQRTF(U)
   SIGMA=1.3**SIGMAT
   PUNCH 7, SIGMA
5 FORMAT (4X*18HGEOMETRIC MEAN MG=F10.5)
7 FORMAT (4X*20HGEOMETRIC DEVIATION=F10.5)
GO TO 1
END
GENERAL EQUATION FOR CRUSHED ROCK

DIMENSION X(20), Y(20), A(20), VV(20), GV(20), GV(20), CM(20)
DIMENSION X(20), D(20), VT(20)
XMIN=0.0
XMAY=1000.0
XL=1.0
XJ=(XMAY-XMIN)/10.
YMIN=0.0
YMAX=50.
YL=1.0
YO=(YMAX-YMIN)/10.
2 CALL PLOT (1, XMIN, XMAY, XL, XJ, YMIN, YMAX, YL, YO)
M=20
SSY=0
SSX1=0
L=1
J=1
READ 3, GM, EV, GD, VD1, VK1
3 FORMAT (5F7.3)
VD=VD1#453.0#331./1.##d/30.4##2
VK=VK1#33.48#2#10.##d
PRINT 3006, VD, VK
2008 FORMAT (2F12.8)
CX=0.4##2
DV=EV##3
EEV=(1.-EV)**2
PV=DV/EEV
GV=LOGF(GD)
AV=GJ##3GV
PV=DV#HV
GV=CK#PV
RV=DV#PV
READ 3, N, RR
5 FORMAT (5X,13,13X,12)
DO 4 I=1,N
4 READ 6, O, HT
6 FORMAT (6X,F7.2,9X,F7.2)
AA=231.##3#4/(L##21))
X1(I)=AA*3
HL=HT/31.
Y(I)=HT/(X1(I)#31))
Z(I)=Y(I)*31
K=3+1
K(J)=K(I)
SV=SVRTF(PV)
TV=3RTF(HV)
UV(I)=X1(I)#GV#VK
VV(I)=SV#UV(I)#TV
VT(I)=VV(I)
D(J)=VT(I)
SSY=SSY+A(J)
SSX1=SSX1+D(J)
CALL PLOT (3+VT(I),L(I))
4 CONTINUE

K=K+1
1 IF(K<-5) 1,1,7
7 PN=M
YM=SSY/PN
X1M=SSX1/PN
PRINT 10, YM, X1M
10 FORMAT (4X, 10HMEAN OF Y=F10.3, 11HMEAN OF X1=F10.3)
DO 16 I=1,M
CY(I)=A(J)-YM
CX1(I)=D(J)-X1M
16 CONTINUE
SXY=0
SSCX1=0
SSCY=0
DO 20 I=1,M
SXY=SXY+CX1(I)*CY(I)
SSCX1=SSCX1+CX1(I)*CX1(I)
SSCY=SSCY+CY(I)*CY(I)
20 CONTINUE
PRINT 1751, SXY
1751 FORMAT (1H+, 4HSXY=F10.6)
SW=SSCX1*SSCY
PRINT 1851, SW
1851 FORMAT (1H+, 3HSW=F10.4)
B=SXY/SSCX1
PRINT 24, B
24 FORMAT (1H+, 2HS=B=F12.2)
DO 50 J=1,M
PY=B*(D(J)-X14)+YM
CALL PLOT (Y,J,D(J),PY)
50 CONTINUE
CALL PLOT (99)
CALL PLOT (7)
1234 CALL EXIT
33 END
SOLUTION OF THE EQUATIONS OF THE位於SEGMENT LOCATION

DIMENSION X23(74), X23(74), Y23(74), T23(74), Z23(74), RE(74)
DIMENSION X12(74), X23(74), Y12(74), T12(74), Z12(74), ALP(74), G34(74)
DIMENSION G(74), H(74), R(74), H3(74)
DIMENSION C11(74), C11(74), C12(74), C13(74)
DIMENSION Y(74), X1(74), X2(74), Z(74)
DIMENSION V(74), R(74)
DIMENSION C12(74), Y2(74)
DIMENSION A(74), U(74), U(74), H(74)

S1 = 74
S2 = 74
S3 = 74
S4 = 74
DO 20 I = 1, 74
READ 1, (H(I), H(I), H(I), H(I))
PRINT 1, (H(I), H(I), H(I), H(I))
1 FORMAT (4F7.2)
V(I) = (V(I) * 231.92 * 342.32 / (H(I) * 111.43))
R(I) = (V(I) * 1.852 * 11.852 / (U(1) * 1.852 * 11.852))
PRINT 3, R(I)
3 FORMAT (1H1, 7HC12(I) = F12.6)
C12(I) = HS(I) / H(I)
PRINT 1112, C12(I)
1113 FORMAT (1H1, 7HC12(I) = F12.6)
Y(I) = ALOG(C12(I))
C1(I) = 1, 7HC(I)
X1(I) = ALOG(C11(I))
ALP = 435.36
T3(I) = 1, 7HC12(I)
X3(I) = ALOG(T3(I))
C13(I) = HT(I) / H(I)
PRINT 1113, C13(I)
1112 FORMAT (1H1, 7HC12(I) = F12.6)
C13(I) = 1, 7HC13(I)
PRINT 1113, C13(I)
1111 FORMAT (1H1, 7HC12(I) = F12.6)
X6(I) = ALOG(C13(I))
S1 = S1 + Y(I) * X1(I)
S2 = S2 + X3(I) * S2
S3 = S3 + Y(I) * X3(I)
S4 = S4 + X3(I) * X1(I)
S5 = S5 + X3(I) * X1(I)
S6 = S6 + X3(I) * X1(I)
X12(I) = X1(I) * X2(I)
X23(I) = X2(I) * X3(I)
Y23(I) = Y(I) * X3(I)
X12(I) = X1(I) * X2(I)
X23(I) = X2(I) * X3(I)
X31(I) = X3(I) * X1(I)
Y23(I) = Y(I) * X3(I)

CONTINUE
DO 20 I = 1, 74
371 = 7
374 = 7

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E P 3

21 CONTINUE

D = 0.5 * (ST0 * ST2 - ST2 * ST2) - ST1 * (ST1 * ST2 - ST1 * ST2) + ST4 * (ST4 * ST4 - ST4 * ST4)
A(J) = 0.5 * (ST0 * ST2 - ST2 * ST2) - ST1 * (ST1 * ST2 - ST1 * ST2) + ST4 * (ST4 * ST4 - ST4 * ST4)
ALP(J) = A(J) / D
B(J) = 0.5 * (ST7 * ST2 - ST2 * ST2) - ST1 * (ST1 * ST2 - ST1 * ST2) + ST4 * (ST4 * ST4 - ST4 * ST4)
GAM(J) = G(J) / D
REM(J) = ST3 * ALP(J) * ST4 + B(J) * ST5 + GAM(J) * ST6)
PRINT 2, EPS(J), ALP(J), B(J), GAM(J), REM(J)
2 FORMAT (1H1, 4HEPS=F12.6, 1H5, 4HALP=F12.6, 1H5, 4HREM=F12.6)
22 CONTINUE
23 CALL EXIT
STOP
33 END
C   PLOTTING ON THE SIMPLE HEIGHT EQUATION

XMIN=0
XMAX=1.0
XL=7
XU=(XMAX-XMIN)/10.
YMIN=0
YMAX=1.0
YL=7
YU=(YMAX-YMIN)/10.
CALL PLOT (1,XMIN,XMAX,XL,XU,YMIN,YMAX,YL,YU)
R=2000.
HLH=4.0
AL=4.133343
DO 2 J=1,30
READ 1, HT
1 FORMAT (F7.2)
M=AL/HLH
AA=(1.+R)**5.1097
AZ=(1.+HLH)
AJ=1.0/AB
AY=AZ**0.6112
AB=AZ**2.9636
AW=1.0/AY
AJ=AW**0.803
AF=HT/H
AQ=(1.+AF)**AJ
HS=(AA*AQ*AD)*H
AG=HS/H
CALL PLOT (9, AF, AG)
2 CONTINUE
CALL PLOT (99)
1111 CALL EXIT
END
APPENDIX II

DERIVATION OF THE RELEVANT EQUATIONS
OR THE FINITE ELEMENT ANALYSIS
APPENDIX II
DERIVATION OF THE RELEVANT EQUATIONS
FOR THE FINITE ELEMENT ANALYSIS

The development of the continuity equation follows the work of Jumikis (25)

Figure A  Hydrostatic Pressure and Velocity Conditions at Four Faces of an Elementary Soil Prism

Consider an elementary, permeable rock prism, the volume of which is \(dx \, dy \, dz\).

Imagine the prism equipped with piezometric tubes which show the hydrostatic pressure conditions at the four faces of the elementary soil prism.

The quantities

\[
\begin{align*}
  i_x &= -\frac{\partial \phi}{\partial x} \quad ; \quad i_y &= -\frac{\partial \phi}{\partial y}
\end{align*}
\]  

(II.1.)

are the hydraulic gradients in the X and Y direction, respectively.
The total amount of water entering the elementary prism through its two entrance faces per unit of time is calculated by the conventional equation of \( Q = VA \) as

\[
Q = u dy dz + v dx dz
\]  

(II.2.)

and the amount of water leaving the prism through its exit faces is

\[
Q = u dy dz + \frac{\partial u}{\partial x} dx dy dz + v dx dz + \frac{\partial v}{\partial y} dx dy dz
\]  

(II.3.)

However, by the principle of continuity of flow, the amount of water leaving the elementary prism of rock must be equal to the amount of water entering it and the loss of fluid in the plane. Therefore, the continuity equation is

\[
\frac{\partial u}{\partial x} dx dy dz + \frac{\partial v}{\partial y} dx dy dz = -q dx dy dz
\]  

(II.4.)

or

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -q
\]  

(II.5.)

Based on the Lindquist's Law for turbulent flow the friction equation is

\[
v = \frac{1}{a + bv}
\]  

(II.6.)

Let

\[
k = \frac{1}{a + bv}
\]

which is the permeability then

\[
v = k i
\]  

(II.7.)

The two flow velocity components, \( u \) and \( v \), can be written as follows:
Then the equation of motion

\[ \frac{\partial}{\partial x} \left( k \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial \phi}{\partial y} \right) - q(x, y) = 0 \]  

(II.10.)

According to Berg (26), the Euler theorem states that if the integral or functional

\[ X = \int \int f(x, y, \phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}) \, dx \, dy \]  

(II.11.)

is to be minimized, then the necessary and sufficient condition for this minimum to be reached is that the unknown function should satisfy the following differential equation.

\[ \frac{\partial}{\partial x} \left\{ \frac{\partial f}{\partial x} \right\} + \frac{\partial}{\partial y} \left\{ \frac{\partial f}{\partial y} \right\} - \frac{\partial f}{\partial \phi} = 0 \]  

(II.12.)

within the same region, provided \( \phi \) satisfied the same boundary conditions in both cases.

Then the equivalent formulation to that of equation (1) in appendix (III) is the requirement that the area integral given by equation (2) in appendix (III) and taken over the whole region, should be minimized subject to \( \phi \) obeying the same boundary conditions.
Derivation of the relevant characteristics of finite element method are described by Zienkiewicz (22) and are summarized here. Consider the characteristics of a triangular element

![Figure B](image)

**Figure B** The Characteristics of a Triangular Element

Figure (B) shows the typical triangular element considered with the nodes i, j, k numbered in an anti-clockwise order.

The \( \phi \) within an element has to be uniquely defined by the \( \phi \) values of the three nodes, i.e.

\[
\{ \phi \}^e = \begin{pmatrix} \phi_i \\ \phi_j \\ \phi_k \end{pmatrix} \quad (II.13)
\]

Assume the \( \phi \) values can be represented by the \( X, Y \) co-ordinates as

\[
\phi = \alpha + \beta x + \gamma y \quad (II.14.)
\]

Then for each node thus it will give

\[
\phi_i = \alpha + \beta x_i + \gamma y_i \quad (II.15.)
\]
\[ \phi_j = \alpha + \beta x_j + \gamma y_j \quad (\text{II.16}) \]
\[ \phi_k = \alpha + \beta x_k + \gamma y_k \quad (\text{II.17}) \]

Solving \( \alpha \), \( \beta \) and \( \gamma \) in terms of the nodal values \( \phi_i \), \( \phi_j \), \( \phi_k \) by the method of determinants and are obtain finally
\[ \phi = \frac{1}{2\Delta} \left[ (a_i + b_ix + c_iy)\phi_i + (a_j + b_jx + c_jy)\phi_j + (a_k + b_kx + c_ky)\phi_k \right] \quad (\text{II.18}) \]

in which
\[ a_i = x_i x_k - x_k y_j \quad (\text{II.19}) \]
\[ b_i = y_i - y_k = y_j k \quad (\text{II.20}) \]
\[ c_i = x_k - x_j = x_k j \quad (\text{II.21}) \]

and where
\[ 2\Delta = \det \begin{vmatrix} x_i & y_i \\ x_j & y_j \\ x_k & y_k \end{vmatrix} = 2\text{(area of triangle } i.j.k) \quad (\text{II.22}) \]

Let
\[ N_i = \frac{a_i + b_ix + c_iy}{2\Delta} \quad ; \quad N_j = \frac{a_j + b_jx + c_jy}{2\Delta} \]
\[ N_k = \frac{a_k + b_kx + c_ky}{2\Delta} \]

Then \( \phi \) can be expressed in matrix form
\[ \phi = \left[ N_i, N_j, N_k \right] \begin{bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{bmatrix} \quad (\text{II.23}) \]
APPENDIX III

THE PUBLISHED PAPER "NON-DARCY FLOW SOLVED BY FINITE ELEMENT ANALYSIS"
INTERNATIONAL ASSOCIATION FOR HYDRAULIC RESEARCH

NON-DARCY FLOW SOLVED BY FINITE ELEMENT ANALYSIS

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SUMMARY:

The finite element method is applied to the solution of non-Darcy flow with a free surface. An effective hydraulic conductivity for each element is defined in terms of the best available solution of the system. The introduction of the effective conductivity linearizes the system of simultaneous equations thus making it possible to obtain a better solution. A theoretical solution is compared with experimental results for flow through a rectangular section.

RÉSUMÉ:

La méthode des éléments finis est appliquée à resoudre l'écoulement qui ne répond pas à la loi de Darcy et qui a une surface libre. Une conductivité hydraulique effective est définie, pour chaque élément, par une fonction de la meilleure solution disponible du système. L'introduction de la conductivité effective fait les équations simultanées, linéaires; donc une meilleure solution est possible. Une solution théorique est comparée avec les résultats expérimentaux pour l'écoulement dans une section rectiligne.
1. INTRODUCTION

The finite element method was originally developed for structural analysis but has proven useful in many continuum mechanics problems. Zienkiewicz and Cheung [1] have discussed the application of this method to the solution of differential equations of the type:

\[ \frac{\partial}{\partial x} \left( k \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial \phi}{\partial y} \right) - q(x,y) = 0 \]  

\[ \cdots \cdots (1) \]


The finite element method involves expressing equation (1) as a functional

\[ \chi = \frac{1}{2} \iint [k \left( \frac{\partial \phi}{\partial x} \right)^2 + k \left( \frac{\partial \phi}{\partial y} \right)^2 ] \, dx \, dy + \int_c q \phi \, ds \]  

\[ \cdots \cdots (2) \]

and then determining the values of \( \phi \) which minimize \( \chi \) subject to certain boundary conditions. To accomplish this, the flow field is divided into a number of finite elements, usually triangles, characterized by their coordinates and nodal values of \( \phi \). The minimization is obtained by differentiating \( \chi \) by each of the unknown nodal \( \phi \)'s and summing over all the elements. This gives rise to a set of simultaneous equations which can be solved to obtain the unknown values of \( \phi \).

In seepage problems \( K \) represents the hydraulic conductivity, \( \phi \) is the piezometric head and \( q \) is the seepage per unit length of the boundary. If \( K \) is constant Darcy flow is obtained and the minimization of \( \chi \) yields a system of linear simultaneous equations in \( \phi \).

For non-Darcy flow, \( K \) is a function of the velocity \( v \), for example Lindquist's law gives,

\[ K = \frac{1}{a + bv} \quad \text{where } a \text{ and } b \text{ are constants.} \]  

\[ \cdots \cdots (3) \]


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When \( K \) is a function of \( v \), equation (1) is non-linear and the minimization of equation (2) yields a system of non-linear simultaneous equations. This paper is concerned with the formation and solution of these non-linear simultaneous equations.

2. THE EQUATIONS FOR NON-DARCY FLOW

Using Lindquist's Law, equation (2) can be written

\[
\chi = \frac{1}{2} \int \int k(v) \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right] \, dxdy + \int_c q \phi \, ds
\]

\[ ............(4) \]

Now let us choose triangular elements and assume that the gradient of \( \phi \) is constant over each element. Thus the velocity and \( K(v) \) are constant within each element. The contribution to the functional of one element is

\[
\chi^e = k(v) \frac{1}{2} \int \int \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right] \, dxdy + \int_c q \phi \, ds
\]

\[ ............(5) \]

The minimization of \( \chi \) over the whole flow field is given by

\[
\frac{\partial \chi}{\partial \phi_i} = \sum \frac{\partial \chi^e}{\partial \phi_i} = 0
\]

\[ ............(6) \]

where \( i = 1 \) to \( m \)

and \( m \) is the number of unknown nodal \( \phi \)'s.

Differentiating equation (5) gives

\[
\frac{\partial \chi^e}{\partial \phi_i} = \frac{1}{2} \frac{\partial k(v)}{\partial \phi_i} \int \int \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 \right] \, dxdy
\]

\[
+ k(v) \int \int \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial \phi_i} \left( \frac{\partial \phi}{\partial x} \right) + \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial \phi_i} \left( \frac{\partial \phi}{\partial y} \right) \right] \, dxdy
\]

\[
+ \frac{\partial}{\partial \phi_i} \left( \int_c q \phi \, ds \right)
\]

\[ ............(7) \]

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From equation (3)

\[ \frac{\partial k}{\partial \phi_i} = -bk^2 \frac{\partial}{\partial \phi_i} \]

and from

\[ v_x = -k \frac{\partial \phi}{\partial x} \]

\[ v_y = -k \frac{\partial \phi}{\partial y} \]

we obtain

\[ v = \left( \frac{1}{a + b v} \right) \sqrt{\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2} \]

and hence

\[ v = -\frac{a}{2b} + \frac{a}{2b} \sqrt{1 + \frac{4b}{a^2} \sqrt{\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2}} \]

Now

\[ \frac{\partial v}{\partial \phi_i} = \frac{1}{a} \left[ \left( 1 + \frac{4b}{a^2} |v \phi| \right) \left( |v \phi| \right)^{-2} \right]^{-\frac{1}{2}} \]

\[ \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial \phi_i} + \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial \phi_i} \left( \frac{\partial \phi}{\partial y} \right) \right] \]

where

\[ |v \phi| = \sqrt{\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2} \]
and equation (7) becomes
\[ \frac{\partial \phi^e}{\partial \phi_i} = \frac{1}{2} \left\{ - k^2 \frac{b}{a^2} \left[ (1 + \frac{4b}{a^2} |v \phi|) \left( |v \phi| \right)^2 \right]^{-\frac{1}{2}} \right\}. \]

\[ \{ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} \} + \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial y} \} \int \int \left[ (\frac{\partial \phi}{\partial x})^2 + (\frac{\partial \phi}{\partial y})^2 \right] dx dy \]

\[ + k \int \int \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} \} + \frac{\partial \phi}{\partial y} \frac{\partial \phi}{\partial y} \} \right] dx dy \]

\[ + \frac{\partial}{\partial \phi_i} \int q \phi ds \]

\[ \cdots \cdots \text{(11)} \]

Since

\[ \phi = [N_i, N_j, N_k] \left\{ \begin{array}{c} \phi_i \\ \phi_j \\ \phi_k \end{array} \right\} \]

where

\[ N_i = \left[ \begin{array}{c} x_j y_k - x_k y_j \\ y_j - y_k \\ x_k - x_j \end{array} \right] / 2 \Delta 
= (a_i + b_i x + c_i y) / 2 \Delta \]

where \( \Delta \) is the area of the triangular element, and \( i, j \) and \( k \) are the vertices of the element, equation (11) becomes

\[ \frac{\partial \phi^e}{\partial \phi_i} = \frac{k}{4 \Delta} \left\{ 1 - k b \left[ \frac{1}{2} \frac{|v \phi|^2}{a^2 + 4b |v \phi|} \right]^{\frac{1}{2}} \right\} \]

\[ \left\{ (b_i b_i + c_i c_i), (b_i b_j + c_i c_j), (b_i b_k + c_i c_k) \right\} \left\{ \begin{array}{c} \phi_i \\ \phi_j \\ \phi_k \end{array} \right\} + p_i \]

\[ \cdots \cdots \text{(12)} \]

where \( p_i \) is the contribution of boundary seepage from one element to node \( i \).
Now we must sum this over all the elements and all the nodes as indicated by equation (6);

$$\frac{3}{\delta} \sum_{i} \sum_{j} \left\{ k \left[ 1 - \frac{k b}{2} \left( \frac{|v \phi|^2}{a^2 + 4 b |v \phi|} \right)^{1/2} \right] [S_{ij}][\phi^e] + \sum p_i = 0 \right\}$$

where $[S_{ij}]$ is the 'stiffness' matrix.

3. NUMERICAL SOLUTION

Since

$$\frac{k b}{2} \left( \frac{|v \phi|^2}{a^2 + 4 b |v \phi|} \right)^{1/2}$$

is small compared to unity its influence on the solution is also small. Therefore in solving (13) the term

$$k(\phi)_e = k \left[ 1 - \frac{k b}{2} \left( \frac{|v \phi|^2}{a^2 + 4 b |v \phi|} \right)^{1/2} \right]$$

was approximated from the preceding solution for $\phi$ thus linearizing the system of equations which can be solved for a better estimate $\phi$ and thus a better estimate of $k(\phi)$. This procedure is repeated until $\phi$ is obtained to the required accuracy.

The method described above was applied to flow through the rectangular rockfill section as shown in figures 1 and 2. An approximate solution for $\phi$ was required to start the solution; this was obtained by assuming Darcy flow, from which $k(\phi)_e$ could be calculated for each element. $k(\phi)$ was then treated as an effective conductivity for each element and the system then solved for a better approximation of $\phi$.

The position of the free surface was found by successive approximation until the boundary conditions

$$\phi_s = y_s$$

and

$$\frac{\partial \phi}{\partial n} = 0$$

were satisfied. The initial assumed position for the free surface is shown in figure 1.

An extrapolated Liebmann method was used to solve the simultaneous equations. The resulting solution obtained on an IBM 360/40 computer is compared with an
experimental study in figure 2. Despite the non-linearity of system, the solution converges with about three times the effort required for Darcy flow.

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CAPTIONS

Figure 1. Definition of nodes, elements and assumed free surface.

Définition des nœuds, des éléments et du profil assumé de la surface libre.

Figure 2. Comparison of experimental with theoretical results.

Comparaison des résultats expérimentaux avec les résultats théoriques.
APPENDIX IV

THE EXTRAPOLATED LIEBMANN METHOD
APPENDIX IV

THE EXTRAPOLATED LIEBMANN METHOD

The extrapolated Liebmann method (26,27) is also known as the 'extrapolated Gauss-Seidel' and 'successive overrelaxation' method which is one of the most powerful and simplest ways to improve the convergence of an iteration solution of linear simultaneous equations.

The method can be written in the form

$$U_{new} = \omega U_L + (1 - \omega) U_{old}$$

where $U_L$ is the value calculated by the Liebmann or the Gauss-Seidel method. The parameter $\omega$ is known as the relaxation parameter and for overrelaxation the value of $\omega$ lies between 1 and 2. Any optimum value of $\omega$ can be used but if the right one is chosen the rapidest convergence will be obtained.

The sequence of steps in the solution is described below:

A. Assume a value for $\omega$ which is between 1 and 2.
B. Assign as initial values for each unknown.
C. Perform the steps for Gauss-Seidel method.
D. If the rate of convergence is very slow, a new value for $\omega$ should be tried to speed up the convergence.
E. Continue iterating until the value of each unknown deter-
mined in a particular iteration differs from its respective value obtained in the preceding iteration by an amount less than some arbitrarily selected epsilon. The procedure is then complete.
APPENDIX V

THE FLOW CHART AND PROGRAM OF
THE FINITE ELEMENT METHOD
Fig. C The Flow Chart for the Finite Element Analysis
THE FINITE ELEMENT METHOD

DIMENSION PHI(X(25), Y(40), PHI(25))
DIMENSION AREA(50), I(B), J(B), K(B), V(B), D1(B), D2(B), D3(B), D4(B)
DIMENSION AREF(50), G(X(50), C(S), L(E), M, H(E), J(E), K(E), E)
DIMENSION ARE(B, G(X(50))
PRINT 83
88 FORMAT (13HSHANG-CHIL NO)
C FORM STIFFNESS MATRIX
887 FORMAT (2I3)
   READ 1111, T, W
1111 FORMAT (FS*I)
   READ 33, A1, a
33 FORMAT (2F6.2)
   READ 33, L1, A1, AM, AN, M, L1, LIT, LMT
353 FORMAT (9I7)
   READ 1122, A1, TOL, AL
1122 FORMAT (3F6.2)
   READ 30, (1(E), J(E), K(E), 1(E), I, J, K, M, N)
30 FORMAT (I3, 3X, I3, 3X, I3)
   READ 31, (1(E), 1(E), I, J, L, E)
   READ 31, (1(E), 1(E), I, J, L, E)
31 FORMAT (I3)
   READ 32, (X(IN), Y(IN), PHI(IN), IN)
   READ 32, (X(IN), Y(IN), PHI(IN), IN)
32 FORMAT (3F7.2)
   READ 33, (X(IN), Y(IN), PHI(IN), IN)
   READ 33, (X(IN), Y(IN), PHI(IN), IN)
33 FORMAT (3F7.2)
   READ 34, OMEGA, EPS
34 Format (2F8.3)
   READ 31, X(19), Y(19), PHI(19)
   READ 31, X(19), Y(19), PHI(19)
31 Format (3F7.2)
   L1=1
2030 IE=1
50 IR=1(I)
   JR=J(IE)
   KRP=K(IE)
   BI(IR)=Y(JR)-Y(KR)
   C(IR)=X(JR)-X(KR)
   DJ(IR)=Y(KR)-Y(IE)
   C(IR)=X(KR)-X(IR)
   DI(IR)=Y(KR)-Y(IE)
   DI(RP)=X(KR)-X(IR)
   ARE(R)=X(JR)*(Y(KR)-Y(IE))-X(IR)*(Y(JR)-Y(IE))-
   ARE(R)=X(JR)*(Y(KR)-Y(IE))-X(IR)*(Y(JR)-Y(IE))-
   ARE(RE)=ARE(R)
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   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
   ARE(RE)= ARE(RE)
IST=2
GO TO 52
52: X=50+1
GO TO 51
102: BUPCC(IST,IRT,IE)=BUPCC(IRT,IST,IE)
   IST=K(IE)
   IRT=1
   IE=K(IE)
   GO TO 52
103: BUPCC(IST,IRT,IE)=BUPCC(IRT,IST,IE)
   IST=J(IE)
   IRT=1
   IE=J(IE)
   GO TO 52
104: IE=K(IE)
   IST=3
   GO TO 52
105: BUPCC(IST,IRT,IE)=BUPCC(IRT,IST,IE)
   IRT=K(IE)
   IE=J(IE)
   GO TO 52
106: IF(LL1-1)>S•S3•SA
   53: IE=IE+1
   IF(IE=ME)>S•S3•SA
   54: IF(LN-LNT)>S•S3•SA
   55: LN=LN+1
   56: IE=IE(S•LN)
   GO TO 50
55: CONTINUE

C FORM NON-LINEAR COEFFICIENT
200 D40=A1/(2*π*B1)
   D41=A••B1
   D42=31/2•B
291: IE=1
291: IE=IE+1
   J=K(IE)
   J(JR)=Y(IR)-Y(KR)
   K(IR)=X(JR)-X(KR)
   J(3R)=Y(KR)-Y(IR)
   K(IR)=X(IR)-X(KR)
   L=K(IR)*X(KR)+K(IR)*X(JR)+K(KR)*X(IR)+K(3R)*X(3R)
   M=K(IR)*Y(KR)-K(IR)*Y(JR)-K(KR)*Y(IR)+K(3R)*Y(3R)
   AGA=4(IE)+=5RT(GRT-EX)*2+UPRMYCE)/AREA(IE)
   0•D40=0•D40(IE)
   W(T)=0•D40(DRT(IE)+5RT50/AR)+1••
   JT=1••/AI(MAY(IE))
   IE=IE(BSRT(IR)+2•D40/AREA(IE))
JX(L.1) = X \times X + X \times \ldots \times X (L.1)

OY(L.1) = \text{INT}^2 (L.1)

IF (L.1 - (L.1) = 0) THEN

2.1 DE(1) = (L.1)

3.1 CONTINUE

4.1 DO WHILE (X < (L.1))

4.2 X = X + 1

4.3 IF (X = (L.1)) THEN

4.4 X = (L.1)

4.5 IF (X = (L.1)) THEN

4.6 X = (L.1)

4.7 X = (L.1)

5.1 DO TO 4.1

6.1 DE(1) = (L.1)

6.2 DE(1) = (L.1)

6.3 DE(1) = (L.1)

6.4 DE(1) = (L.1)

6.5 DE(1) = (L.1)

6.6 DE(1) = (L.1)

6.7 DE(1) = (L.1)

6.8 DE(1) = (L.1)

6.9 DE(1) = (L.1)

14.1 \text{X = X + (INT \times 10)}$

14.2 \text{TO (INT \times 10)}

14.3 \text{X = X + 10}

14.4 \text{TO (INT \times 10)}

14.5 \text{X = X + 10}

14.6 \text{TO (INT \times 10)}

14.7 \text{X = X + 10}

14.8 \text{TO (INT \times 10)}

14.9 \text{X = X + 10}

14.10 \text{TO (INT \times 10)}

14.11 \text{X = X + 10}

14.12 \text{TO (INT \times 10)}
GO TO 41
147 CONTINUE
C
SOLVE FOR PHI(IN)
ITER=1
DO 7 IN=1,MA
7 GOTO 5
DO (IN)=0,MAXA/A(IN,IN)
BIT=(1.0-Omega)
71 ITER=ITER+1
D1=0.0
IN=1
72 EX=0.0
IT=1
IF((IN-0.0).LT.75,74,75
73 EX=IN
74 IF(IT-IN).LT.75,74,75
75 IF(IT-XA).LT.75,74,75
76 IT=IT+1
IF((IN+IT)-0.0).LT.75,74,75
77 EX=SU(IN)EX+BIT*PHI(IN)
IF((ABS(PHI(IN)-EX)-U1)=SU,80,81
81 D1=AUS(PHI(IN),EX)
90 PHI(IN)=EX
IF(IN-MA).LT.32,33,33
92 IN=IN+1
GO TO 72
83 IF(D1-EP3).LT.34,84,71
84 PRINT 85*(PHI(IN),IN=1,MA)
85 FORMAT (1H1,4PH1=F10.3)
C
SOLVE FOR Y(IN)
ALT=ALT+AL
IF(ALT-3.0).LT.291,291,400
400 D2=0.0
DO 405 LS=1,11
403 IN=IN+LS
IF((ABS(PHI(IN)+TW2)-Y(IN))=21402,402,41
401 D2=AUS(PHI(IN)+TW2)-Y(IN))
402 Y(IN)=PHI(IN)+TW2
PHI(IN)=Y(IN)-TW2
403 CONTINUE
404 IF(D1-TOL).GT.408,408,408
408 PRINT 409*X(I),I=1,NL
409 FORMAT (1H1,4PH1=F10.10)
406 PRINT 407*(PHI(IN),Y(IN),IN=1,MA)
407 FORMAT (1H1,5PH1=F10.3*,110,20,20,20,20,20,20,
405 CONTINUE
406 STOP
147 CALL EXIT
STOP
END
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<table>
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<th>Time (s)</th>
<th>Current (mA)</th>
<th>Voltage (V)</th>
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<td>10.70</td>
<td>29.70</td>
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<td>0.00</td>
<td>0.00</td>
<td>16.07</td>
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<tr>
<td>0.00</td>
<td>10.90</td>
<td>7.00</td>
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<tr>
<td>0.00</td>
<td>12.30</td>
<td>7.30</td>
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<tr>
<td>0.00</td>
<td>13.20</td>
<td>17.19</td>
</tr>
<tr>
<td>0.00</td>
<td>18.95</td>
<td>31.26</td>
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<tr>
<td>0.00</td>
<td>36.10</td>
<td>32.82</td>
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<tr>
<td>0.00</td>
<td>39.40</td>
<td>15.24</td>
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<tr>
<td>0.00</td>
<td>26.66</td>
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<td>72.45</td>
<td>52.10</td>
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<td>48.30</td>
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<td>52.10</td>
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<tr>
<td>0.00</td>
<td>1.00</td>
<td>52.10</td>
</tr>
<tr>
<td>1.75</td>
<td>8.05</td>
<td></td>
</tr>
<tr>
<td>121.92</td>
<td>45.80</td>
<td>1.00</td>
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<tr>
<td>121.92</td>
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<td>121.92</td>
<td>31.50</td>
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<tr>
<td>121.92</td>
<td>55.50</td>
<td>11.00</td>
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</table>

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REFERENCES


NOMENCLATURE

Dimensions are given in terms of mass \((M)\), length \((L)\) and time \((T)\).

- **D**
  - Factor of dispersion
- **d**
  - Diameter of grain \(L\)
- **D**
  - Diameter of bed tube \(L\)
- **d**
  - The surface mean diameter \(L\)
- **E**
  - Dry porosity
- **E_v**
  - Drainable porosity
- **e**
  - The height of the surface roughness of the pores
- **F**
  - Force per unit mass \(L/T^2\)
- **g**
  - Gravitational acceleration \(L/T^2\)
- **H**
  - Upstream water level \(L\)
- **H_o**
  - The difference of hydraulic head across the bed \(L\)
- **h**
  - Downstream water level \(L\)
- **h_s**
  - Seepage height \(L\)
- **i**
  - Hydraulic gradient
- **K**
  - Permeability \(L/T\)
- **k**
  - Dimensionless constant that depends on the shape of the cross section of flow

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Units</th>
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<tr>
<td>$L$</td>
<td>Length of the section</td>
<td>$L$</td>
</tr>
<tr>
<td>$M_g$</td>
<td>Geometric mean</td>
<td>$L$</td>
</tr>
<tr>
<td>$m$</td>
<td>A constant of the porous medium</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Number of grains</td>
<td></td>
</tr>
<tr>
<td>$n_v$</td>
<td>Unit vector</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>$M/LT^2$</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Pressure difference</td>
<td>$M/LT^2$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Flow rate</td>
<td>$L^3/T$</td>
</tr>
<tr>
<td>$q$</td>
<td>Rate of seepage</td>
<td>$L^3/T$</td>
</tr>
<tr>
<td>$R$</td>
<td>Reynolds number</td>
<td></td>
</tr>
<tr>
<td>$R_e$</td>
<td>Correlation coefficient</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>Surface per unit vol. of porous medium</td>
<td>$1/L$</td>
</tr>
<tr>
<td>$S_o$</td>
<td>Specific surface</td>
<td>$1/L$</td>
</tr>
<tr>
<td>$T$</td>
<td>Time</td>
<td>$T$</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Tortuosity</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>The distribution of the grain</td>
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<tr>
<td>$V$</td>
<td>Local velocity vector</td>
<td>$L^2/T$</td>
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<tr>
<td>$V$</td>
<td>Macroscopic velocity</td>
<td>$L^2/T$</td>
</tr>
<tr>
<td>$V_x$</td>
<td>Velocity component in the $x$ direction</td>
<td>$L^2/T$</td>
</tr>
<tr>
<td>$V_y$</td>
<td>Velocity component in the $y$ direction</td>
<td>$L^2/T$</td>
</tr>
<tr>
<td>$w$</td>
<td>Volume of water</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$w_T$</td>
<td>Total volume</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$w_s$</td>
<td>Volume of the sample</td>
<td>$L^3$</td>
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<td>Symbol</td>
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<td>Units</td>
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<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$W_x$</td>
<td>Volume of excess water drained out</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$W_y$</td>
<td>Volume of water retained</td>
<td>$L^3$</td>
</tr>
<tr>
<td>$x$</td>
<td>Co-ordinate in the $x$ direction</td>
<td>$L$</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Average co-ordinate in the $x$ direction</td>
<td>$L$</td>
</tr>
<tr>
<td>$y$</td>
<td>Co-ordinate in the $y$ direction</td>
<td>$L$</td>
</tr>
<tr>
<td>$z$</td>
<td>The shape of the particle</td>
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</tr>
<tr>
<td>$T$</td>
<td>Black's group</td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>The pore diameter</td>
<td>$L$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The angle between the velocity vector and the $x$ axis</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>$M/LT$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
<td>$L^2/T$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>$M/L^3$</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>The geometric standard deviation</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Piezometric head</td>
<td>$L$</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Particle shape factor</td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>A functional</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>Stream function</td>
<td>$L^2/T$</td>
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<tr>
<td>$\psi_p$</td>
<td>Probability density</td>
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<tr>
<td>$\omega$</td>
<td>Relaxation parameter</td>
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</tbody>
</table>
APPENDIX VIII

SOURCES OF ERRORS IN THIS STUDY
APPENDIX VIII

SOURCEs OF ERRORs IN THIS STUDY

1. One Dimensional Case

A. The errors that might occur in the determination of flow rate and head losses for the friction tests are as follows:

a. The minor leakage between the side of the tube and the wall of the flume.

b. The venturi meter was read to \( \pm 0.25 \) U.S. gallon/minute.

c. Manometer was read up to \( \pm 0.5 \) mm.

d. The orientation of the crushed rock around the entrance of the piezometers at the wall could affect the local pressure.

B. The errors that might occur in measuring rock weights and volumes are as follows:

a. The balance was read to \( \pm 0.005 \) gram.

b. The cylinder was read to \( \pm 0.05 \) c.c.

c. The water adheres on the cylinder would cause a small error in the volume measurements.

C. The errors that might occur in measuring porosities are as follows:

a. The volume readings were accurate up to \( \pm 0.5 \) c.c.

b. The degree of compaction was difficult to control.

c. The water adhering to the measuring containers would
cause a small error.

d. The variability in the moisture retained by the particles after drainage.

2. Two Dimensional Case

A. The following errors could occur in the determination of the seepage height equation:

a. The precision of the scale readings for different heights was ± 0.5 mm.

b. The calibration error for the venturimeter is approximately of 2%.

c. The venturimeter could only be read to ± 0.25 U.S. gallon/minute which could cause large relative errors in the low flow data.

d. The limited range of data (R 810 to 2900, h/H 0.22 to 0.65, L/H 1.4 to 3.2, hs/H 0.023 to 0.21).

B. The errors that might happen in the solution and experimental verification of the finite element method are as follows:

a. The round off errors in the computer.

b. The EPS = 0.05 cm. in the interaction for solving the set of simultaneous equations.

c. The TOL = 0.05 cm. as the limit for the comparison of the piezometric head to the potential heads in the determination of the location of the free surface.

d. Velocity head and the effect of the convergence of macroscopic streamline are neglected.
e. Prof. McCorquodale found that the functional equation cannot be reduced by the Euler-Lagrange equation to the original partial differential equation but with an additional very small term \( \xi(K, |\mathbf{V}|) \) is introduced. This term vanished in laminar and fully turbulent case but not in the transitional case.

g. Manometer readings were read to \( \pm 0.5 \) mm.

h. Scale readings for free surface were read to \( \pm 0.5 \) mm.

3. Observed Scatter

A. Fig. 6 \( S = 0.002 \times 0.22 = 0.00044 \text{ sec./cm.} \)

B. Fig. 7 \( S = 0.002 \times 0.22 = 0.00044 \text{ sec./cm.} \)

C. Fig. 8 \( S = 0.004 \times 0.22 = 0.00088 \text{ sec./cm.} \)

D. Fig. 9 \( S = 0.003 \times 0.25 = 0.00075 \text{ sec./cm.} \)

E. Fig. 10 \( S = 0.005 \times 0.22 = 0.0011 \text{ sec./cm.} \)

F. Fig. 11 \( S = 3 \times 0.19 = 0.57 \)

G. Fig. 12 \( S = 3 \times 0.19 = 0.57 \)

H. Fig. 13 Range of dry porosity is 0.432 to 0.465.

I. Fig. 14 Range of drainable porosity is 0.426 to 0.45.

4. Measurement Errors

A. Possible Errors in the Readings:

a. \( i = \pm 0.5 \) mm, out of a minimum of 10.0 mm.

b. \( Q = \pm 0.25 \text{ U.S.G.P.M.} \), out of a minimum of 20 U.S.G.P.M.

Therefore the maximum possible relative error due to readings in \( \left( \frac{i}{V} \right) \) is
\[
\left( \frac{0.5}{10} + \frac{0.25}{20} \right) \times 100 = 6.25\%
\]

which is consistent with the observed scatter. There is a possible calibration error about \( \pm 2\% \).
APPENDIX IX

NOTE ON SIMILITUDE
For parallel flow and the velocity in the $y$-$z$ direction is zero. Then the Navier-Stokes equation becomes (for a pipe)

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial x^2}$$

let $u = u'u_o$; $x = x'x_o$; $t = \frac{t'x_o}{u_o}$; $p = \rho u_o^2 p'$

and substitute into equation (1) to obtain

$$\frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial x'} + \frac{1}{R} \frac{\partial^2 u'}{\partial x'^2}$$

but for curvilinear flow this equation is

$$q' \cdot \nabla q' + \frac{\partial q'}{\partial t'} = -\rho' + \frac{1}{R} \nabla \cdot q'$$

where

$q = q'q_o$; $p = \rho q'q_o p'$

Now in a porous media local geometric similarity does not exist therefore the critical $R$ is not a good criterion.
VITA AUCTORIS

1942        Born, March 4.
1948-54      Primary School of St. Joseph's College in Macau.
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1960-62      Matriculation school of Winston College in Hong
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1962-66      Cheng Kung University in Tainan, Taiwan. Received
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1967-69      Candidate for M.A.Sc. in civil engineering at
              University of Windsor.