Beta-gamma directional correlation measurements in the decay of gallium-72.

Mohammed M. Seddik

University of Windsor

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Beta-Gamma Directional Correlation Measurements

In The Decay Of $^{72}$Ga

By

Mohammed M. Seddik

A Thesis

Submitted to the Faculty of Graduate Studies through the Department of Physics in Partial Fulfillment of the Requirements for the Degree of Master of Science at the University of Windsor

Windsor-Ontario

1969
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ABSTRACT

The beta-gamma directional correlation involving the first forbidden, 2.52 Mev, $\beta$-group in coincidence with the 1.465 Mev $\gamma$ - transition, in the 14-hours decay of $^{72}$Ga, has been measured as a function of energy of the beta-particles. The correlation coefficient $A_2$ was determined at five $\beta$ - energies ranging from 1.839 to 2.248 Mev as given below:

<table>
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<th>$E$(Mev)</th>
<th>$W(m_0c^2)$</th>
<th>$A_2$</th>
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<tr>
<td>1.839</td>
<td>4.6</td>
<td>-0.24 ± 0.015</td>
</tr>
<tr>
<td>1.941</td>
<td>4.8</td>
<td>-0.25 ± 0.011</td>
</tr>
<tr>
<td>2.044</td>
<td>5.0</td>
<td>-0.234 ± 0.009</td>
</tr>
<tr>
<td>2.146</td>
<td>5.2</td>
<td>-0.29 ± 0.0217</td>
</tr>
<tr>
<td>2.248</td>
<td>5.4</td>
<td>-0.294 ± 0.017</td>
</tr>
</tbody>
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Determinations of the nuclear matrix elements are planned for the future.
ACKNOWLEDGMENTS

I would like to express my appreciation to Dr. E. E. Habib for his guidance and instruction throughout the duration of this work and under whose supervision this project was carried out.

Also, I would like to thank Dr. H. Ogata for the useful discussion. I am indebted to the Graduate Studies, University of Windsor, and the Physics Department for the financial support in the form of research and teaching assistantship.
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CHAPTER I

INTRODUCTION

The refinement of experimental technique during the last ten years has resulted in a large amount of relatively accurate data on the observables in $\beta$- decay. Many attempts have been made to use this information in order to extract the nuclear matrix elements. Measurements of the beta-gamma directional correlation, the beta-gamma circular polarization correlation, the shape factor, and the log ft value have been employed to obtain quantitative values for the matrix elements. These matrix elements are used to test models that describe the structure of the nuclear states involved.

In most cases, however, the analysis was based upon theoretical expressions simplified by approximations involving the electron radial wave functions (Kotani(1959)). The availability of tabulated exact electron wave functions and high speed electronic computers make the use of more accurate expressions just as easy as the approximated formulas.

Many theoretical papers about first forbidden $\beta$-decay contain expressions in which the exact (Morita(1958)) as well as the approximated (Kotani and Ross(1958), Matumoto(1960)) electron wave functions are symbolically represented.
These formulas have rarely been used, due to the difficulty of actually evaluating the electron wavefunctions. But the tables of electron wavefunctions by Bhalla and Rose (1961) now make it feasible to analyze the experimental results in terms of more accurate theoretical expressions.

All the data that can be obtained from the angular correlation work thus provide very useful information for understanding the nuclear structure. But on the other hand, the information, that can be obtained from these experiments, depends on the type of radiation observed, the type of experiment which is used, and also the extranuclear fields acting on the nucleus. In the case of gallium 72, the intermediate nuclear state has a very short half life and the extranuclear fields are not expected to have a significant effect on the results ($T_{1/2} = 3 \times 10^{-12}$ sec.)

For the $\beta$- transition investigated in this work, an effort was made to obtain precise measurements of the beta-gamma directional correlation as a function of the $\beta$- energy. Such differential energy measurements are useful because many terms in the theoretical expressions are functions of the energy of the emitted beta-particles.

**CONSERVED VECTOR CURRENT THEORY:**

This theory is based on its analogy with electromagnetism where all particles are coupled to the electromagnetic field by the same strength determined by the elementary charge $e$. 

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This universality of the electromagnetic interaction follows from the conservation of the electromagnetic current. If the vector current of the weak interaction is similarly conserved, then the vector coupling constant would be a universal constant and

$$g^\omega_v = g_0 = g''$$

The C.V.C. theory gives the relationship between the nuclear matrix elements $y$ and $x$ (Fujita(1962), Eichler(1963)). If we assume that this relationship is valid, then this will facilitate the analysis of the data. Support for the C.V.C. theory as applied to first forbidden decay is reported by Ogata and Habib.

The $3^-$ ground state of the $^{72}$Ga decays to two excited $2^+$ states of $^{72}$Ge at $0.835(2^+)_1$ and $1.465(2^+)_2$. For the $(3^-\rightarrow 2^+_1)$ transition to the $0.835$ level, the spectrum shape factor (Langer and Smith(1960)), and the beta-gamma angular correlation (Alberghini and Steffen(1961), Lipnik and Sunier(1964), Newsome and Fischbeck(1964)) have been measured. If the values of the beta-gamma circular polarization experiment (Grenacs and Deutsch(1964), Camp et al (1965)) are used, the matrix elements can be determined. The results are shown in table (I).

The decay to the second excited state $(2^+_2)$ has a spectrum (Langer and Smith(1964)), and a beta-gamma angular anisotropy which are similar to those obtained by the $\int B_{ij}^2$ alone (Grenacs and Deutsch (1964)).
The modified $\int B_{ij}$-approximation has been used to analyze the data and the results (II) are also given in table (I).

It was shown (Grenacs and Deutsch (1964)) that both forbidden decays in $^{72}$Ga can be understood on the basis of the j-j coupling shell model with configuration mixing. However, the results of Grenacs and Deutsch are not supported by adequate description of the apparatus used or the corrections applied, and cannot therefore be accepted. Also the shape factor measured by Langer and Smith was obtained by subtracting of the beta I spectrum, which was assumed to be pure $\int B_{ij}$. The beta-gamma angular correlation measurements of Fischbeck and Newsome show that this is not the case.

The present research concerns the beta-gamma directional correlation for the transition $3^- \rightarrow 2^+ \rightarrow 0^+$ in $^{72}$Ga. It is clear that the $\hat{\epsilon}$-approximation cannot be applied to this case because of the non-allowed shape of the beta energy spectrum (Langer and Smith (1960)), and of the relatively large beta-gamma directional correlation coefficient (Newsome and Fischbeck (1964)).

The beta-gamma correlations in addition to the spectrum shape factor and the log ft value can be used to determine the four nuclear matrix elements $\int M, \int r, \int x r$ and $\int B_{ij}$. $\int B_{ij}$ is preferred usually as a reference matrix element ($z=1$) and $y, u, x$ are the unknown parameters.

It is hoped that with the use of more accurate data, the matrix elements associated with this decay can be determined.
TABLE I

72^Ga

Experimental Matrix Elements of $3^- \rightarrow 2^+$ Transition

$(z = 1)$

| References | $Y$ | $u$ | $x$ | $|\bar{B}\rangle / R$ (x10^3) | $|\bar{T}\rangle / R$ (x10^3) | $|\bar{IR}\rangle / R$ (x10^3) | $|\bar{X}\rangle / R$ (x10^3) |
|------------|----|----|----|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 72^Ga I.   | ≈0 | ≈0.2 | ≈30 | 44.4 | 4.26 | 13.2 | 0.09 |
| 2.59       | -0.30 | -0.005 | 44.4 | 0.26 | 13.2 | 0.09 |
| 72^Ga II.  | ≈10^-2 | ≈10^-2 | ≈100 | 44.4 | 0.26 | 13.2 | 0.09 |

Grenacs&Deut. (1964)
Camp et al (1965)
Grenacs&Deut. (1964)
(i) Introduction:

Angular correlations have two main areas of application. One is to test the fundamental laws of nature such as parity non-conservation and the other is to study the nuclear structure. The formulas for the beta-gamma angular correlation are expressed in terms of certain nuclear matrix elements. We gain insight into the nuclear structure by knowing the relative magnitudes of the nuclear matrix elements responsible for the beta transitions.

If a nucleus undergoes emission of two radiations in succession ($\alpha, \beta, \gamma$ or $e^-$), their correlations are an attractive and useful field of study. Such studies can be divided into essentially two kinds:

"Spatial Correlations" and "Time Correlations".

The first one (angular correlations) constitutes the topic of this chapter.

(ii) Angular Correlation of Successive Radiations:

(Unperturbed Angular Correlations):

The probability of emission of a particle or quantum by a radioactive nucleus depends in general on the angle between the nuclear spin axis and the direction of emission.
Under ordinary circumstances, the radiation \( R \) emitted from a radioactive sample for the individual transition \( I_a \longrightarrow I_b \) will be isotropic in the laboratory coordinates because the nuclei are randomly oriented in space. If the transition \( I_a \longrightarrow I_b \) is followed by a second transition \( I_b \longrightarrow I_c \), the individual radiations from the second transition are likewise isotropic in the laboratory coordinates (Note that \( I_a, I_b, \) and \( I_c \) are the angular momenta of the states involved).

However, in a two-step cascade transition, such as \( I_a \longrightarrow R_1 \longrightarrow I_b \longrightarrow R_2 \longrightarrow I_c \), there is often an angular correlation between the directions of emission of the successive radiations \( R_1 \) and \( R_2 \) which are emitted from the same nucleus. The existence of an angular correlation arises because the directions of the first radiation is related to the orientation of the angular momentum \( I_b \) of the intermediate level. This orientation can be expressed in terms of the magnetic angular momentum quantum number \( m_b \) with respect to some laboratory direction such as that of the first radiation. If \( I_b \) is not zero, and if the lifetime of the intermediate level is short enough so that the orientation of \( I_b \) persists, then the direction of emission of the second radiation will be related to the direction \( I_b \) and that of the first radiation.

An anisotropic radiation pattern can be observed only from an ensemble of nuclei that are not randomly oriented. One method of arriving at such an ensemble consists in picking out only those nuclei whose spins happen to lie in a preferred direction.
This can be realized if the nuclei decay through successive emissions of two radiations $R_1$ and $R_2$. The observation of $R_1$ in a fixed direction, say, $k_1$, selects an ensemble of nuclei that has a non-isotropic direction of spin orientations. The succeeding radiation $R_2$ then shows a definite angular correlation with respect to $k_1$.

(iii) The Purpose of Angular Correlation Measurement:

A study of the angular correlation of radiations emitted or absorbed in nuclear processes has been one of the principal methods available for the nuclear spectroscopy of excited states, since in many $\alpha$-decay transitions, the daughter nucleus is left in an excited state and emits a gamma photon in a subsequent transition to a lower energy state. The information that can be obtained from angular correlation work depends on the type of radiation observed ($\alpha, \beta, \gamma, e^-$), on the properties that are singled out by the experiment (direction, polarization, energy), and on the extranuclear fields acting on the nucleus.

If we assume the decaying nuclei are free, i.e., no extranuclear fields act on the nucleus and disturb its orientation in the intermediate state, then, to the extent that this can be realized in practice, angular correlation measurements provide information about the properties of the nuclear level involved and about the angular momenta carried away by the radiations. To be more precise, the $\gamma-\gamma$ directional correlation yields the spins of the nuclear levels and the multipole orders of the gamma transitions.
The gamma matrix elements which describe the transitions and hence the directional correlation, provide no information on nuclear structure which would check the validity of the nuclear models. The relative parities of the nuclear levels can be determined if one measures the directional correlation involving an $e^- - \gamma$ cascade. The $e^- - \gamma$ directional correlation depends in general on combination of normal conversion matrix elements and gamma matrix elements. If in addition to the normal conversion matrix elements, there appear penetration matrix elements then some information on nuclear structure can be derived.

The directional correlation of a $\beta - \gamma$ cascade depends not only on the spins and parities of the nuclear levels and the multipole orders of the transitions but also very strongly on the matrix elements for the $\beta$-transition which in turn are dependent on the details of nuclear structure. A unique determination of the beta matrix elements could provide a test for a nuclear model which describes the states involved.

(iv) General Theory of Angular Correlation:

(a) Introduction:

In the following, we will present a simple description of the directional correlation theory which provides some insight into the correlation mechanism. Some restrictions are kept for this theory:

The nuclear states are assumed to have well defined symmetry properties, i.e. to be characterized by single values of
the total angular momentum and the parity. In other words, the initial nuclear state must be of sharp angular momentum $I_1$ and parity which undergoes successive transitions, either emitting or absorbing radiations through the intermediate states of sharp angular momenta $I_a, I_b, \ldots$ and sharp parity and terminating as a nucleus with sharp angular momentum $I_2$ and parity. Furthermore, the radiations are assumed to be emitted in succession and not simultaneously.

For example, we will consider here the directional correlation of two successive radiations for the case of an unperturbed intermediate state. The schematic picture of a directional correlation measurement is shown in figure (1). A nuclear cascade involving states $a, b$ and $c$ with spins $I_a, I_b, I_c$ occurs through the successive emission of particles $R_1$ and $R_2$.

We denote by $W(\theta)\, d\Omega$, the relative probability that the radiation $R_2$ is emitted into the solid angle $d\Omega$ at an angle $\theta$ with respect to $R_1$.

(b) Directional Correlation Function $W(\theta)$

We consider first a $\gamma-\gamma$ cascade $I_a \rightarrow I_b \rightarrow I_c$ in which both $\gamma$-rays, of multipole order $L_1$ and $L_2$, respectively, are pure. The most convenient form in which to express the directional correlation function $W(\theta)$ between $\gamma_1$ and $\gamma_2$ is

$$W(\theta) = \sum_{K=0}^{K_{\text{max}}} A_K P_K(\cos \theta) \quad (2.1)$$
Figure (I) Schematic of Directional Correlation Measurement

(a) Nuclear Cascade

(b) $W(\theta)\, d\omega$ is the relative probability that the radiation $R_2$ is emitted into the solid angle $d\omega$ at an angle $\theta$ with respect to $R_1$.

(c) The two counters 1 and 2 subtend an angle $\theta$ at the source. From the coincidence count rate as a function of angle, $C(\theta)$, one obtains after suitable correction the correlation function $W(\theta)$. 
The functions $P_k$ are Legendre polynomials. The coefficients $A_k$ contain all the physical information. Usually one chooses $A_0 = 1$, so that the correlation function integrated over all angles is unity.

$$\int W(\theta) \, d\Omega = 1 \quad (2.2)$$

There are three general rules, for the coefficient $A_k$, used for most of the radiations:

1. $k$ integer even \hspace{2cm} (a)
2. $0 \leq k_{\text{max}} \leq \min(2I_b, 2L_1, 2L_2) \hspace{2cm} (b)$
3. $A_k = A_k(a,b \xi)A_k(b, c \xi) \hspace{2cm} (c)$

Rule (a) expressing the fact that only even Legendre polynomials appear, holds as long as one measures only directions and linear polarizations of radiations. The observation of circular polarization, however, introduces odd integers. The expansion of the directional correlation function is

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + \ldots + A_{k_{\text{max}}} P_{k_{\text{max}}}(\cos \theta) \quad (2.3)$$

The highest term in the expansion is determined by the rule (b).

Rule (b) is given by:

$$k_{\text{max}} = \min(2I_b, 2L_1, 2L_2) \quad (2.4)$$

This rule implies an isotropic correlation if the angular momentum of either of the radiations or the intermediate state equals zero or half. In the event that cascade involves mixed radiations, i.e., $L_1 \rightarrow L_1 + L'_1$ and $L_2 \rightarrow L_2 + L'_2$, rule (b) becomes:
The explicit calculation of the coefficients $A_k$ is greatly facilitated by the fact that they can be broken up into two factors each depending on only one transition of the cascade (rule(c)). The factors $A_k(a,b \xi)$, $A_k(b,c \xi)$, depend only on the properties of the levels $a,b$, and $b,c$ and their associated radiations $\xi$ and $\xi_2$.

The values of these factors have been calculated for most cases of interest (Biedenharn and Rose (1953)). As an example of the above, consider the directional correlation between successive gamma rays in the cascade shown in figure (2). The second gamma ray in this cascade must be pure electric quadrupole $E_2$, and from the references in the literature, we find the corresponding directional correlation factor $A_2 = -0.5976$. If we assume the first transition to be pure also, for instance, dipole radiation ($L_1 = 1$), we find immediately $A_2 = -0.4183$ and hence

$$A_2 = A_2(a,b \xi) A_2(b,c \xi)$$

(2.6)

This value has to be compared with experiment.

Generally, however, the first transition in a $2^+ \rightarrow 2^+ \rightarrow 0^+$ cascade will be mixed, i.e., the radiation carried away involves more than one single value of angular momentum. As a rule the two lowest multipoles, $L_1 = 1$ and $L_1^* = 2$, since the angular momentum selection rules allow $L_1 = 1, 2, 3,$ or $4$, occur with measurable intensity (figure(2b)).
The coefficient $A_2(a, b \gamma)$ and, hence, $A_2$ become continuous functions of the mixing ratio $\xi$, which in this case indicates the ratio of amplitude of the quadrupole to that of the dipole contribution.

$$\xi^2 = \frac{\text{Total intensity of the } L'\text{ pole}}{\text{Total intensity of the } L\text{ pole}} \quad (2.7)$$

Experimentally, one determines the coefficients $A_k$ by assuming an appropriate value for $k_{\text{max}}$ (usually 4) and making a least square fit of equation (2.3) to the values of $W^{\exp}(\theta)$ measured at several angles. If $A_4$ and higher terms can be neglected, one restricts the observations to the angles $\theta = 90^\circ$ and $\theta = 180^\circ$. This experiment then determines the anisotropy, $a(W)$, as a function of the relativistic beta-particle energy, where

$$a(W) = \frac{W(180^\circ) - W(90^\circ)}{W(90^\circ)} \quad (2.8)$$

and

$$A_2 = \frac{2a(W)}{3 + a(W)} \quad (2.9)$$

Equation (2.1) is very general indeed and with the proper evaluation of $A_k$, it applies to all two step cascade $\alpha - \gamma$, $\beta - \gamma$, $\gamma - e^-$, $e^- - e^-$, ............. etc, as well as to nuclear scattering experiments and nuclear disintegrations.
A $\gamma - \gamma$ Cascade

(a) $2^+ \longrightarrow 2^+ \longrightarrow 0^+$ Cascade

(b) The first gamma ray $\gamma_1$ is usually mixed in this case; the two dominant angular momenta being $L_a = 1$, $L_b = 2$. 
(c) Theory of Beta-Gamma Angular Correlation:

The correlation between a beta-particle and a subsequent gamma-ray involves problems much more complex than which we have encountered in the previous sections. The reasons for this complexity are, in the first transition of a $\beta-\gamma$ cascade, an electron and a neutrino are emitted simultaneously. Formally this event is described by treating the process as if a neutrino (anti-neutrino) enters the nucleus and a negatron (positron) is emitted.

In a $\beta-\gamma$ correlation experiment one measures the direction of the electron while the neutrino escapes unobserved. Theoretical calculation of the angular correlation thus necessitates an averaging over all neutrino directions and over the spins of the electron.

In order to understand the general theory of $\beta-\gamma$ angular correlation, a fairly detailed knowledge of the relativistic electron theory (Dirac Theory) is required. A complete account can be found in a book by Rose (1960). The theory of the $\beta-\gamma$ directional correlation was first developed by Falkoff and Uhlenbeck. Their work was limited by three important restrictions:

* A zero nuclear charge $Z$
* Pure interaction only
* Only one nuclear matrix element responsible for the $\beta-$ transition.

Later calculations by Fuchs (1951), by Morita (1958), by Biedenharn and Rose (1953) overcame these restrictions.
After the discovery of the non-conservation of parity in $\beta$-decay, the theory of the $\beta$-\$ angular correlations was extended by Alder(1957), Morita(1957-58), Kotani and Ross(1958), and others.

The expressions of Kotani are particularly useful for the analysis of the experimental data on first forbidden transition. More accurate expressions are given by Buhring(1962). A review paper by Weidenmuller(1961) surveys the theory of first forbidden $\beta$-decay and discuss some experimental data.
CHAPTER III

NUCLEAR BETA-DECAY

(i) Recent Developments

Nuclear $\beta$-decay results from a weak interaction involving the decay of a nucleon with the subsequent production of a nucleon, a beta particle and a neutrino. One or more of three basic reactions can occur. These are:

- $(a) \ n \rightarrow e^- + p + \bar{\nu} \quad (\beta^- \ \text{decay}) \quad (3.1)$
- $(b) \ p \rightarrow e^+ + n + \nu \quad (\beta^+ \ \text{decay}) \quad (3.2)$
- $(c) \ p + e^- \rightarrow n + \nu \quad (\text{e-capture}) \quad (3.3)$

where $n$, $p$, $e^-$, $e^+$, $\nu$ and $\bar{\nu}$ are the neutron, proton, positron, neutrino, and anti-neutrino, respectively. All these particles are fermions, having intrinsic spins of $\frac{1}{2} \hbar$. The evidence that the neutrino (anti-neutrino) exists became really satisfactory when it was detected (after leaving its source in $\beta$-radioactivity) through the inverse $\beta^-$ decay interaction

$$p + \bar{\nu} \rightarrow n + e^+$$

The experiment was difficult because the particle has never been found to undergo any interaction with matter except through the weak $\beta$-process.

After the discovery of the non-conservation of parity in $\beta$-decay, a period of tremendous activity in the study of the laws of $\beta$-decay occurred.
As a result of this period, it is well established that the interaction is a mixture of vector and axial vector, (V-A), (Weidenmuller (1961)). The fact that the parity is not conserved has enlarged the number of possible experiments on nuclear $\beta$-decay. The measurements of the correlation between the direction of the $\beta$-particle and of the circular polarization of the emitted photons after the $\beta$-decay, for example, has become a new source of information.

This increase in the number of experimental possibilities, together with a knowledge of the interaction law, has given the $\beta$-decay a new aspect. It can be applied to the study of nuclear structure. In the same way in which we can use knowledge of the electromagnetic interaction to measure $E1$, $M1$ matrix elements, etc., we can use the knowledge of the $\beta$-interaction in many cases to obtain the nuclear matrix elements involved in the transition.

Also, after the detection of parity violation the two-component theory proved to be the most successful formulation of the nuclear $\beta$-interaction. In this theory, the neutrino is described by two-component spinors. All particles possess a definite helicity defined by the sign of the expectation value of the operator $\langle \sigma \cdot p \rangle$. 

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(ii) Beta - Decay Theory

Guided by the analogy to the electromagnetic interaction, the $\beta$-interaction can be given as:

$$H_i = g_i \left( \bar{\psi}_f O_i \psi_i \right) \left( \bar{\psi}_e O_i \psi_e \right) + h.c. \quad (3.4)$$

where $g_i$ is the coupling constant. The operators $O_i$ can be grouped into five classes according to their transformation properties. The essential point is that the five invariants, $H_i$, are obtained by multiplying two quantities with the same transformation property which is determined by the operator $O_i$. According to this transformation property of $(\bar{\psi}_f O_i \psi_i)$ one speaks of scalar, vector, tensor, axial vector and pseudoscalar interaction. It has been mentioned as the result of the parity non-conservation and the experimental results that the vector (V) and axial vector (A) are responsible for the beta-decay interaction.

Quantum mechanically, the probability of finding $\beta$-particle with energy $E$ per unit time is given by the well known expression:

$$N(E) \, dE = \frac{2 \gamma}{h} \left| H_{fi} \right|^2 \frac{dN_f}{dE_0} \quad (3.5)$$

where $\frac{dN_f}{dE_0}$ is the density of the final states available to the system per unit range of total energy $E_0$. $H_{fi}$ is the matrix element of the beta interaction between the initial and final states.

The matrix element $H_{fi}$ can be expressed in terms of the lepton wavefunctions (and other factor) which can be expanded in a power series in terms of its argument. According to this expansion of $H_{fi}$, the beta-decay classifies into two main groups:
(1) The Allowed Transition

The first (largest) term in the expansion of $H_{fi}$ gives the transition probability of the so-called allowed transition which corresponds to the emission of the leptons with zero orbital angular momentum ($I$). The leptons may be emitted with spins ($\bar{s}$) parallel or anti-parallel. The total angular momentum, $J$, of the leptons must obey the conservation laws of angular momentum

$$J = L + \bar{s} \quad \text{----- (3.6)}$$

and

$$|I_f - I_i| \leq J \leq |I_f + I_i| \quad \text{----- (3.7)}$$

where $I_i$ and $I_f$ are the initial and final nuclear spins of the transition states.

If $S = 0;\quad L = 0$ then $J = 0$ 

$$\triangle I = I_f - I_i = 0 \quad \text{----- (3.8)}$$

Also if $S = 1;\quad L = 0$ then $J = 1$

$$\triangle I = 0, 1 \quad \text{----- (3.9)}$$

where $\pi_i$ and $\pi_f$ are the parity of the initial and final states, respectively. Equation (3.8) represents the Fermi interaction while equation (3.9) represents the Gamow-Teller interaction.

The first term in the expansion of $H_{fi}$ yields two nuclear matrix elements: The Fermi matrix element $\int 1$ and The Gamow-Teller matrix element $\int 0$.
In terms of these nuclear matrix elements, the transition probability becomes

\[ N(E) \, dE = \frac{2\pi \hbar^2}{h^2} \left[ |c_V| \sqrt{1 + |c_A|^2} \right]^2 \frac{dN_F}{dE} \]  \hspace{1cm} (3.10)

where \( c_A \) and \( c_V \) are the axial-vector and vector coupling constants, respectively.

(2) The Forbidden Transitions

The second and following terms in the expansion of \( H_{fi} \) represent the forbidden transition spectra for which the orbital angular momentum \( L = 1, 2, \ldots \), corresponding to first forbidden, second forbidden, \ldots.

(a) First Forbidden Transition

The second term in the expansion of \( H_{fi} \) is much smaller than the first term, and its effects can only be observed if the allowed transition cannot take place. This term gives the transition probability for the so-called "First-Forbidden" transition.

The selection rules for the first forbidden transitions are:

1\textsuperscript{st} forbidden: \( \Delta I = 0, \pm 1 \) \hspace{1cm} (no \hspace{0.5cm} 0 \rightarrow 0, \pi_{nf} = -1 \hspace{1cm} (3.11)

Unique 1\textsuperscript{st} forbidden: \( \Delta I = \pm 2 \) \hspace{1cm} (no \hspace{0.5cm} 0 \rightarrow 0) \hspace{1cm} (no \hspace{0.5cm} 0 \rightarrow 1, \pi_{nf} = -1 \hspace{1cm} (3.12)

The first forbidden decay is governed by six nuclear matrix elements. They are summarized in Table II together with the allowed one.
TABLE II

Allowed and First-Forbidden Nuclear Matrix Elements and Their Selection Rules

<table>
<thead>
<tr>
<th>Type of Transition</th>
<th>Matrix Element</th>
<th>$\lambda$</th>
<th>$\Delta I$</th>
<th>$\Delta \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowed</td>
<td>$c_v \int_1$</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td>$c_A \int_{\sigma^*}$</td>
<td>1</td>
<td>$0, \pm 1$</td>
<td>+1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 0(\leftrightarrow)0)</td>
<td></td>
</tr>
<tr>
<td>First Forbidden</td>
<td>$c_A \int_{\sigma^*}$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>$c_A \int_{\sigma \cdot \tau}$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>$-c_v \int_{\sigma^*}$</td>
<td>1</td>
<td>$0, \pm 1$</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 0(\leftrightarrow)0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-c_v \int_{\tau}$</td>
<td>1</td>
<td>$0, \pm 1$</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 0(\leftrightarrow)0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_A \int_{\sigma \cdot \tau} \cdot \tau$</td>
<td>1</td>
<td>$0, \pm 1$</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 0(\leftrightarrow)0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_A \int B_{ij}$</td>
<td>2</td>
<td>$0, \pm 1, \pm 2$</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 0(\leftrightarrow)0)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(no 1(\leftrightarrow)1)</td>
<td></td>
</tr>
</tbody>
</table>

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The form of the $\beta$-decay interaction is now well established to be

$$H_\beta = \sum_i \sum_\mu \int \left[ \gamma_f^{(i)} \gamma_i^{(i)} (C_V - C_A \gamma_5) \gamma_i^- \gamma_\nu \right]$$

$$\left[ \gamma_\nu^c \gamma_4 \gamma_\mu \left( 1 + \gamma_5 \right) \gamma_\nu^c \right] d\gamma + h.c. \quad (3.13)$$

where $H_\beta$ is the interaction density, $\gamma_i^c$ and $\gamma_\nu^c$ are the initial and final wavefunctions, $C_V$ and $C_A$ the vector and axial-vector coupling constants. The index $(i)$ refers to the nucleons building up the initial and final wavefunctions. $\gamma_i^c$ and $\gamma_\nu^c$ are the wavefunctions for the electron and neutrino, respectively. The above form of interaction has the following meaning: The lepton current

$$L_\mu(r) = \left[ \gamma_c^{(i)} \gamma_\mu \left( 1 + \gamma_5 \right) \gamma_\nu^c \right] \quad (3.14)$$

is a four vector, dependent on a space coordinate $r$. The bracket only means that the scalar product with respect to the spinor indices has to be taken. $L_\mu(r)$ also depends on the magnetic quantum numbers of electron and neutrino.

This lepton current interacts with the baryon current

$$B_\mu(r) = \left[ \gamma_c^{(i)} \gamma_\mu (C_V - C_A \gamma_5) \gamma_\nu^c \right] \quad (3.15)$$

and the form of the interaction is

$$\sum_\mu \int B_\mu(r) L_\mu(r) d\gamma \quad (3.16)$$

which is in analogous to the electromagnetic interaction.
The theoretical determination of $\Lambda_2$ requires detailed information concerning the interaction describing the decay process. All allowed transitions have isotropic $\beta$-angular distributions and if the shape of the energy spectrum of the forbidden transition is strictly identical to that for an allowed case, then the distribution will be also isotropic.

Many non-unique first forbidden transition, $\Delta J=0,1; \Delta L=1$ and $\Delta m=(-)$, have allowed shape beta ray energy spectra. The $\xi$-approximation may be applicable to this case.

(a.1) The $\xi$-approximation

In this approximation the Coulomb energy of the electron at the nuclear radius is larger than the total energy of the electron. Mathematically we have

$$\frac{\alpha Z}{R} \gg W_0$$

(3.17)

where $R =$ nuclear radius
$\alpha =$ fine structure constant
$Z =$ number of protons
$W_0 =$ maximum total energy of the $\beta$-particle

$\xi$ takes the form

$$\frac{\alpha Z}{2R} = \xi$$

(3.18)

This assumption is the same as saying that the distortion due to Coulomb forces in the wavefunction of the electron, $\psi_e$, are much
more important than the next term in the expansion of the plane-wave, which is of order \( kr \). Therefore, all terms of order \( Z \) are kept whereas, terms of order \( QR \) and \( kR \) are dropped (\( k \) and \( q \) are the wave numbers of the electron and neutrino, respectively).

As a result of this, all first forbidden quantities in the \( \xi \) -approximation have the same energy and angular dependence as the allowed ones.

The spectrum shape factor, \( \beta - \gamma \) angular correlation coefficient, \ldots etc, can be obtained from the allowed case by making the following substitutions for the allowed matrix elements (Kotani (1959)):

\[
- C_V \int l \rightarrow C_A \int s + \frac{1}{2} C_A \int (\phi, r)/i = V
\]

\[
C_A \int \phi \rightarrow - C_V \int x + \frac{1}{2} C_A \int \phi x + \frac{1}{2} C_V \int r = -Y
\]

The notation used for the matrix elements is

\[
\eta w = C_A \int \phi x, \quad \eta \xi^\prime V = C_A \int i \delta_x \text{ for } \lambda = 0
\]

\[
\eta u = C_A \int i \phi x, \quad \eta \xi^\prime y = -C_V \int i x, \quad \eta x = -C_V \int r \text{ for } \lambda = 1 \quad (b)
\]

\[
\eta z = C_A \int B_{ij} \text{ for } \lambda = 2
\]

The \( \xi \) stands for \( \beta^2 \). The nuclear parameters \( u, v, w, x, y, \) and \( z \) are the ratios of the various matrix elements compared to a standard matrix element, \( \eta \), so that \( |\eta|^2 \) can be taken out as a common factor in the transition probability. The magnitude of \( |\eta|^2 \) is determined only from the \( \log ft \) value (Kotani (1959)).

\[
f_c t = \pi^3 \ln \frac{2}{|\eta|^2}
\]

where

\[
f_c = \int_0^\infty F(Z, W) p W q^2 \left( \frac{p^2 - q^2}{12} \right) \, dW
\]

is called the corrected integrated Fermi function.
f = \int_{W_0}^{W} \mathcal{F}(Z,W) p \, W \, q^2 \, dW

is called the integrated Fermi-function. \( \mathcal{F}(Z,W) \) is the Fermi function and \( W_0 \) is the end point energy of the transition. The factor appearing in the definition of \( V \) and \( Y \), is introduced so that \( Y \) and \( V \) are of order unity.

By substituting equation (3.19 b) into equation (3.19 a) we get

\[
V = \xi V + \xi W \quad \text{for } \lambda = 0 \quad \text{(3.21)}
\]

\[
Y = \xi Y - \xi (u+x) \quad \text{for } \lambda = 1 \quad \text{(3.22)}
\]

The \( \xi \)-approximation corresponds to the assumption that

\[
|V| \sim |Y| \sim \xi \ll |W| \sim |u| \sim |x| \sim |z|
\]

If the \( \xi \)-approximation holds exactly, then all the measurable quantities will have the same behaviour as in the allowed case, and will depend only on the ratio of \( V \) to \( Y \). Therefore, transitions which show deviations from the \( \xi \)-approximation are to be examined very thoroughly.

(a,2) Cancellation and Selection Rule Effects:

The cancellation effect means, for example, that \( \xi Y \) in \( Y \), equation (3.22), is nearly equal to \( \xi (u+x) \). Thus this effect makes either \( V \) or \( Y \) (or both) be of order of the other nuclear parameter, i.e.,

\[
|V| \sim |Y| \sim \xi \ll |W| \sim |u| \sim |x| \sim |z| \quad \text{(3.23)}
\]
Let us consider the characteristics of two possibilities to account for the selection rule effect, by which the parameter \( z \) becomes the same order of, or larger than, \( V \) and \( Y \). One of them is K-forbiddenness, introduced by Alaga, Alder, Bohr, and Mottelson (1955) and another is due to the configuration character of the Mayer-Jensen shell model which is called j-forbiddenness.

(A) K-forbiddennes:
K being the projection of the total angular momentum (I) of the nucleus on the nuclear axis of symmetry. According to the Bohr-Mottelson model, we have one more selection rule for \( \lambda \) beside the total angular momentum selection rule (3.7); namely

\[
|K_0 - K_1| \leq \Delta K \leq |K_0 + K_1| \tag{3.24}
\]

for the transition from a state with quantum number \((K_0, I_0, \pi_0)\) to another state with \((K_1, I_1, \pi_1)\), where \( \pi \) stands for the parity and \( \lambda \) designate the rank of the transition operator, when regarded as a tensor. The regions established especially well for this model are \( 150 \leq A \leq 190 \) and \( A > 225 \). There is no clear experimental evidence for the applicability of the Bohr-Mottelson model to nuclei with \( A < 150 \) but some lighter nuclei may deform so that the K-forbiddenness is applicable.

Due to the K-selection rule, we have the relations

\[
|Z| > |X| \sim |U| \sim |W| \tag{3.25}
\]

and \( |Y| > |W| \) if there is no cancellation in \( Y \).
Since $Y$ includes the large numerical factor, we cannot say which of $z$ and $Y$ is larger, unless the reduction factor due to the $K$-forbiddenness and its perturbation are known. Anyhow, this type of forbiddenness could well explain the large $\log ft$ value for the decay of some nuclei.

(B) $j -$ forbiddenness:

The selection rule associated with the total angular momentum of the nucleons is

$$|j_p - j_n| \leq \lambda \leq |j_p + j_n|$$

(3.26)

Where $j_p$ and $j_n$ stand for the proton and neutron spins, i.e., the initial and final spins of the nucleon undergoing $\beta$-decay. The essential point of this selection rule is the following:

Let us consider nuclei in the region $50 \leq Z, N \leq 82$. According to the Jensen-Mayer shell model the nucleons outside the closed shell $Z=N=50$ belong to the $h_{1/2}$, $g_{7/2}$, $d_{5/2}$, $d_{3/2}$ and $s$ states. Among these only the $h_{1/2}$ has odd parity. Since we are considering a $\beta$-decay with parity change, the number of nucleons in the $h$-state has to be changed by one. Consequently, the change of $j$ is at least 2 and the matrix element with $\lambda = 2$ makes the main contribution (Morita(1958)). One expects

$$|z| > |x|, |u|, |w|$$

(3.27)

Nothing can be said about the relative magnitude of $V$, $Y$ and $z$. The same argument can be extended to the regions $28 \leq Z, N \leq 50$ and $82 \leq Z, N \leq 126$ but cannot be applied to nuclei where $N$ and $Z$ belong to different major shells.
The cancellation or selection rule effect gives a relatively large coefficient (A) for the \( \beta - \gamma \) directional correlation. When the \( \delta^* \) - approximation is applicable then we have:

(*) A large \( Y \) and \( V \).

(*) A constant shape factor.

(*) An angular correlation with a \( p^2/W \) energy dependence, and a \( \log ft \) value around 6.0 (a large value will indicate a deviation).

In the case where the \( B_{ij} \) term is predominant we expect:

(**) A large \( \log ft \) value, with unique first forbidden transition.

\[
\log ft = 7 \rightarrow 9
\]

(**) A large \( \beta - \gamma \) anisotropy.

(**) A non-statistical spectrum shape.

In order to see the typical effects due to both selection rules, it may be convenient to examine the following special case suggested by Morita and Yamada (1958).

(a.3) The Modified \( B_{ij} \) - Approximation

In this approximation we assume that

\[
z \neq 0, \quad Y \neq 0, \quad V \neq 0; \quad x \sim u \sim w \sim 0
\]

In other words, there are contribution from matrix elements of rank 0 and 1, which are negligible in comparison with the \( B_{ij} \) matrix of rank 2. In the modified \( B_{ij} \)-approximation we need only two parameters, \( V \) and \( Y \) (since \( iB_{ij} \) is used as standard matrix element, one puts \( z = 1 \)).
Exact Calculations

We will use for the analysis of the data of our \( \beta-\gamma \)
angular correlation experiment, the exact expressions of Kotani
(1959). These expressions are particularly useful for the
evaluation of experimental data on the first forbidden transitions
and are free of the restrictive conditions stated above.

(a.5) The \( 3^- (\beta) 2^+ (\gamma) 0^+ \) Transition

Most of these \( \beta- \) transitions lead to the first excited
state of the daughter nucleus with a subsequent \( \gamma- \) ray to the
ground state and one has the sequence \( 3^- \rightarrow 2^+ \rightarrow 0^+ \). Again
\( \beta-\gamma \) correlations in addition to the spectrum and the log \( \text{ft} \) value
have been employed to yield information from which the four matrix
elements

\[
\begin{align*}
\int x, \int r, \int \sigma x r \quad \text{and} \quad \int B_{ij}
\end{align*}
\]

can be determined. \( \int B_{ij} \) is preferred usually as a reference
matrix element (\( z=1 \)) and \( Y, u \) and \( x \) are the unknown parameters.

The angular correlation coefficient \( \mathcal{E}(W) \) and the shape
factor \( C(W) \) for the transition \( 3^- (\beta) 2^+ (\gamma) 0^+ \) cascade are given by:

\[
\mathcal{E}(W) = \frac{p^2}{W} \left[ \left( \frac{1}{2} \right)^2 \frac{1}{2} Y \gamma \gamma - (\frac{1}{2} \gamma) \gamma \gamma - (1/112) W \right] C^{-1} \quad (3.30)
\]

\[
C(W) = Y^2 + \gamma^2 - (\frac{1}{2} \gamma) \left[ p^2 + q^2 \right] \quad (3.31)
\]

Note that \( V = 0 \) for this cascade.
TABLE (III)

Special Cases of First-Forbidden $\beta$-Decay

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions for Matrix Ele's</th>
<th>Applicable for</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$-approximation</td>
<td>$</td>
<td>V</td>
</tr>
<tr>
<td>Cancell Effect</td>
<td>$</td>
<td>V</td>
</tr>
<tr>
<td>K-selection rule</td>
<td>$</td>
<td>z</td>
</tr>
<tr>
<td>J-selection rule</td>
<td>$</td>
<td>z</td>
</tr>
<tr>
<td>Mod. $B_{ij}$-approx$^n$</td>
<td>$z \neq 0$, $X \neq 0$, $V \neq 0$; $x=u=v=0$</td>
<td>$B_{ij}$ enhanced by selection rule</td>
</tr>
</tbody>
</table>
(a.6) The Method of Using the Experimental Data

to Distinguish between Two Models:

To see the effects due to both selection rules"j" and "K" for
forbiddenness, the modified $B_{ij}$-approximation may be used. The
nuclear matrices for this approximation are given in table III.
The expression for the $\phi-\gamma$ directional correlation is

$$N(W,\Theta) = 1 + A_2 P_2(\cos \Theta) = 1 + \varepsilon(W)(3/2 \cos^2 \Theta - \frac{1}{2}) \quad (3.32)$$

where $A_2 = \varepsilon(W)$.

The asymmetry

$$\varepsilon(w) = \frac{W(\pi) - W(\pi/2)}{W(\pi/2)} \quad (3.33)$$

is measured and is related to $A_2$ by the equation

$$A_2 = \frac{2\varepsilon(w)}{3 + \varepsilon(w)} \quad (3.34)$$

where $W(\pi)$ and $W(\pi/2)$ are the number of coincidence counts at 180°
and 90°, respectively.

For convenient analysis the equations (3.28),(3.29),(3.30),
and (3.31) can be written in the form,(Matumoto(1963)),

$$(V - V_0)^2 + (Y - Y_0)^2 = R^2 \quad (3.35)$$

where

$$V_0 = -\left(\frac{1}{3}\right)(\frac{3}{2})^{\frac{1}{2}} \left(\frac{p^2}{W}\right) \frac{3 + \varepsilon(W)}{\varepsilon(w)} \quad (3.36)$$

$$Y_0 = -\left(\frac{1}{3}\right)(\frac{3}{2})^{\frac{1}{2}} Y_0 \quad (3.37)$$

and

$$R^2 = V_0^2 + Y_0^2 + (w/8)(3/7)^{\frac{3}{2}} Y_0 - (\frac{1}{2})(q^2 - p^2) \quad (3.38)$$

Equation (3.35) describes a circle in the V-Y plane with
radius $R$ and center $(V_0, Y_0)$.

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For each experimental value of $\varepsilon(W)$, two such circles can be drawn corresponding to the two extreme values of the error limits. Values of $V$ and $Y$ that satisfy all the data points simultaneously lie within a meniscus. Another relationship between $V$ and $Y$ may be obtained from the branching ratios to the ground state and first excited state in the daughter nucleus. This relationship depends on the particular model assumed and is given by Matumoto (1963) as:

$$R^2 = V^2 + Y^2 = \left(\frac{f_{c2}}{f_2}\right)^2 \left[ \frac{a_2 f_{c1}}{5a_1 f_{c2}} \right] \times \left| \frac{\int B_{1j1}}{\int B_{1j2}} \right|^2 - 1$$

(3.39)

The term $\left| \frac{\int B_{1j1}}{\int B_{1j2}} \right|^2$ is the nuclear model dependent term.

$a_1$ and $a_2$ are the branching ratios of $\beta_1$ and $\beta_2$.

$f_{c1}$ is the corrected integrated Fermi function of $\beta_1$.

$f_{c2}$ is the corrected integrated Fermi function of $\beta_2$.

$f_2$ is the integrated Fermi function of $\beta_2$.

The values of $\left| \frac{\int B_{1j1}}{\int B_{1j2}} \right|^2$ have been calculated for different nuclear models by Matumoto (1963). If the theoretical circle for a given model intersect with the meniscus of the experimental circles, then that model is possible. The values of $V$ and $Y$ at the intersections may be used to calculate the value of $\varepsilon(W)$ as a function of $W$. The extent of agreement with the experimental data may confirm or reject the model. The expression used by Matumoto (1963) is given by:

$$a(W) = \frac{W^2}{6(\pi/7)^{\frac{1}{2}} V + (3/4) (\frac{3}{4})^{\frac{1}{2}} Y - (3/224) W}$$

$$V^2 + Y^2 + \frac{1}{\sqrt{5}} \left( \frac{p^2}{w} \right) V - \frac{1}{\sqrt{5}} \left( \frac{p^2}{w} \right) Y + \frac{q^2}{2} - \frac{59}{672} p^2$$

(3.40)

However, this expression assumes $x = u = w = 0$. 

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CHAPTER IV

EXPERIMENTAL ARRANGEMENT AND APPARATUS

(i) The Spectrometers

(a) The Gamma Spectrometers

Gamma photons produce secondary electrons within a scintillation crystal by undergoing Compton scattering, photoelectric absorption or pair production. The total absorption cross-section for gamma photons is highest in inorganic scintillators, such as NaI(Tl), which have a large average atomic number.

Thallium is added to NaI to change the emission wavelength since the pure crystal is partly opaque to its characteristic emission wavelength. The number of photons emitted by a NaI(Tl) scintillator is approximately linear, with respect to the energy of the electron exciting it, from energies of about 2 Kev to 6 Kev. Almost all gamma and x-ray transitions of interest in beta-decay have energies within this range.

The slow pulses from the gamma probe can be passed through a linear pulse amplifier to a pulse height analyzer. A plot of slow pulse rate versus pulse height gives the spectrum for the γ-energies observed. Our gamma detectors were provided with magnetic shielding through the use of Netic and Co-Netic foil. Each gamma detector consisted of a NaI(Tl) scintillator which is optically coupled to the
photocathode of a 56-AVP P.M. (photomultipliers P.M.) tube. A 1.2 x 2 crystal and a 2 x 2 crystal were used. The gamma detectors were mounted at a fixed angle of 90° to one another on a movable arm which rested on the semicircular angular correlation table. The arm could be rotated through an angle of 90° about the source centre.

The physical arrangement was such that one detector (denoted as γ-1 probe) could be set at either the 270° or the 180° position, while the other detector (denoted as γ-2 probe) was set simultaneously at either the 180° or the 90° position as shown in Figure (4). The arm was moved by a stepping motor which was controlled by means of digital electronic circuits, (Young (1964)).

(b) General Properties of Magnetic Spectrometers

Magnetic beta ray spectrometers are used extensively to analyze complex nuclear spectra. They may be classified into two basic types, the flat and helical spectrometer. In the flat type, the magnetic lines of force are mainly in the direction perpendicular to the electron paths, whereas in the helical instrument, the lines of force are mainly in the same direction as the electron paths. The equation of motion of an electron moving in a plane perpendicular to a uniform magnetic field is given by

\[ B e v = \frac{m v^2}{\rho} \]

where

\[ m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \]

\[ m_0 \text{ is the rest mass of the electron.} \]

\[ v = \text{velocity of the electron} \]

\[ \rho = \text{radius of the electron orbit} \]
\begin{itemize}
  \item S - SOURCE
  \item C - CRYSTAL
  \item LC - LUCITE-LIGHT-COLLECTOR
  \item B_1 - ENTRANCE BAFFLE
  \item B_2 - EXIT BAFFLE
  \item PM - PHOTOMULTIPLIER
\end{itemize}
It is convenient to express the momentum in terms of its $B/\rho$ (gauss-cm) value. The corresponding energies can be calculated directly or obtained from tables. The relation

$$P \text{ (momentum)} = mv = eB/\rho \quad (4.2)$$

indicates that in the flat spectrometers, where $\rho$ is constant for all energies and the uniform field, there is a linear relationship between the momentum focused and the analysing field.

The transmission of a spectrometer is defined as the fraction of the electron momentum $P$ emitted from the source which arrive at the detector when the instrument is focussed on the momentum $P$. The resolution of a spectrometer is defined with respect to its response to a source of monoergic electron. The response curve is shown in Figure (5)

![Figure (5)](image)

The resolution is defined as $\frac{\Delta P}{P_0}$. $\Delta P$ is the full width at half the maximum count rate. In a helical (lens) type spectrometer, the electron image of the source at the detector has usually a large spherical aberration. A position of minimum spherical aberration generally occurs before the electrons cross the symmetry axis. This is called the ring focus and a great improvement in
performance is obtained by placing defining baffles at this point rather than at the detector.

Hubert (1952) has shown the combination of baffles as shown in Figure (7). The combination of diaphragms $D_1, D_2$, and $D_3$ define both the solid angle and the momentum band. The diaphragms $D_1$ and $D_2$ can be replaced by a single conical baffle and $D_2$ by a moveable one.

![Figure (7)](image)

The gap between the moveable baffle and the conical baffle defines both the transmission and resolution of the instrument. This leads to a simplified adjustment over other systems and is the type used in the first Gerholm instruments.

(c) Beta Spectrometer

The beta spectrometer used in this experiment is due to T.R. Gerholm. As Figure (4) shows, this instrument can be used for the $(e-\gamma)$, $(\beta-\gamma)$ angular correlations. Where $e$, $\beta$, and $\gamma$ refer to conversion electron, beta-decay electron or positron, and gamma radiation, respectively.

In this type of spectrometers, the magnetic lenses consist of a water cooled copper coils surrounded by iron shielding. The iron end flanges contain shaped pole pieces to give an interior...
field intensity that is roughly triangular in shape, as shown in Figure (6). Magnetic field measurements show that the source is in a field free region. Theoretical as well as experimental studies of this field distribution show that it gives a favourably high luminosity (source area x transmission) which compensates for the small geometrical dimensions of the instrument. Fairly large sources (± 5mm.) can be used without sacrifice in resolution which was 3% at a transmission of about 1.8% for the arrangement utilized in our measurements.

For the electron optics a somewhat modified version of the Hubert ring focus baffle system is used. The envelope baffle consists of a cone and the internal baffle of an axially variable disc. As is shown in Figure (4), this disc baffle defines the extreme rays. It therefore determines the mean angle of emission as well as the acceptance solid angle (transmission).

The beta detector consists of a plastic phosphor, in the shape of a disc, cemented onto a lucite light collector, of ellipsoidal shape, optically coupled to the central part of the photo-cathode of the 56-AVP P.M. tube. The scintillator is covered by a thin layer of MgO serving as a light reflector. The light collector acts as a light guide and renders it possible to obtain the required magnetic shielding.

The source used in this experiments were mounted on thin aluminum rings. These rings were placed in a pedestal holder which
slides against the source facing surface of the spectrometer iron flange. The vacuum system for the spectrometer is a very simple one which has an operating pressures may be reached after five minutes of pumping.

Details concerning the power supplies used with the lenses and linearity and range characteristics of the spectrometer can be found in the reference by Colclough (1963).

(ii) Coincidence Technique

(a) Introduction

The coincidence circuits are used to investigate time relationships between nuclear events. They are the instruments of time spectroscopy. A simple twofold coincidence circuit element gives an output pulse whenever two input pulses are simultaneous within a certain resolving time, but gives no output when only one of the input pulses is present.

If the circuit element detects the overlap of two pulses each of length \( \gamma_0 \), then the resolving time can be quoted as \( 2 \gamma_0 \), because a coincidence can be detected if one pulse lies within \( \pm \gamma_0 \) of the other. If there are two sources of uncorrelated pulses of rates \( n_1 \) and \( n_2 \) per second, the expected rate of chance coincidence is easily seen to be \( 2\gamma_0 n_1 n_2 \) per second. If the recorded coincidence rate differs from the expected chance rate, a correlation is established between the two sources of pulses.
(b) The Description of the Fast-Slow Coincidence Circuit

Bell, Graham and Petch (1949, 1952) have described a fast-slow coincidence circuit which uses a helical-coiled coaxial line to provide a variable delay between two counters. A block diagram of our circuit is given in Figure (8). The outputs of the detectors were divided into a slow and a fast pulses. The slow pulses used in the coincidence circuit were obtained by integrating the signal appearing at the 10th or 11th dynode of a high gain 56-AVP P.M.'s. This gave a positive pulse with a rise time depending upon the scintillator decay time and a peak height proportional to the de-excitation photon intensity.

The slow gamma pulses were routed through a spectrum stabilizer which locked-on the peak and thus provided for drift free measurements. These pulses were then amplified, passed through a single channel analyzer, and then fed to the triple coincidence circuit and a scaler. The slow beta pulse was treated in an identical manner with the exception that it was not routed through the stabilizer.

The fast pulses from the anode of the P.M. tubes, which were negative, cut off a 404A pentode (limiter). Since the P.M. tubes were run at 2,000 volts, the leading edge of this pulse was due to the first photoelectron released from the photocathode. The pulses at the output of the pentode were immediately shaped into very short pulses (≈ 10 nsec) of height = 0.5 volts.
The duration $\gamma$ was obtained by means of a short-circuited clipping cable with a transit $\sqrt{2}$.

The width of these pulse essentially determines the resolving time $2\gamma$ of the fast coincidence circuit, Figure(9). This circuit gave an output pulse for $\gamma-\beta$ and $\gamma_2-\beta$ coincidence only. The output of the fast coincidence was fed to the slow triple coincidence circuit was fed to a scaler. The outputs from all the scalers were printed out with a victor printer (Model 12-10-321) and punched out with a Tally Tape perforator (Type 420).

(iii) Setting Up Procedure

The process of setting up the circuit for the experiment, to obtain maximum counting efficiency for the fast coincidence, involved the following steps. Each spectrometer was set on the position of the spectrum to be investigated. With the cables in the fast channels adjusted to approximate values, a plot of the triples count rate versus discriminator voltage was obtained as shown in Figure (10 a). The discriminator voltage operating point was chosen in the center of the plateau thus assuring that the fast coincidence circuit produces an output for coincidence pulse only.

With the discriminator voltage properly set, a plot was then made of the triples rate as a function of the delay line to obtain a time resolution curve, Figure (10 b). The delay time was varied by altering the lengths of the cables between the P.M. tubes and the fast coincidence unit.
In order to obtain 100% coincidence efficiency, it was essential that the resolution curve has a flat top and that the operating point be chosen near the middle of this region. The width of the resolution curve at half-maximum gave the approximate value of the resolving time $2\gamma$ of the coincidence circuit. An accurate determination of the resolving time was obtained from the relation:

$$2\gamma = \frac{N_c}{N_\beta N_\gamma}$$

where

$N_\beta =$ beta singles rate

$N_\gamma =$ gamma singles rate

and $N_c$ was the chance coincidence rate, which can be obtained by interposing a long delay (~100 nsec) in the beta fast channel and observing the resulting coincidence rate.
Figure (10)

(a) Plateau

(b) Plateau, $2\gamma$
CHAPTER V

EXPERIMENTAL RESULTS AND PROCEDURE OF

THE $^{72}$Ga EXPERIMENT

(a) Introduction

Figure (11) shows the partial decay scheme of $^{72}$Ga isotope. In this chapter, a beta-gamma directional correlation experiment is described to obtain the correlation coefficient $A_2$ of the 2.52 Mev beta group and the subsequent 1.465 Mev gamma ray at the $\beta$-energies:

$W = 4.6, 4.8, 5.0, 5.2, 5.4$ (relativistic units)

of the cascade $^3\rightarrow\beta\rightarrow 2^+\rightarrow 0^+$. The lifetime of the second $2^+$ state of $^{72}$Ge is very short ($T_2 = 3\times10^{-12}$ sec.), therefore, there is no any attenuation problems.

(b) Source Preparation

The gallium source material was obtained in the form of an irradiated powder. It was irradiated in the reactor at McMaster University at a flux of about $(1.8 - 2.0)\times10^{13}$ neutrons per cm$^2$ per second. The irradiation of a given sample was terminated upon reaching an activity of 4 mc/mg. The source material was evaporated in a tantalium evaporation crucible. The crucible was placed in the Balzer (type MBA3) evaporation unit and centered with respect to a 5 mm. in diameter opening in a brass mask set directly above it. The source backing a thin Al foil 0.0003 inch thick, was positioned on the mask.
The temperature of the crucible was then raised slowly and at the same time the temperature was observed by a pyrometer to get the correct temperature at which the material evaporates and to make sure that most of the material is evaporated. The evaporated gallium deposited uniformly on the foil within a circular area 5 mm. in diameter at the center of the foil. The source was then taken right away from the evaporation unit and its activity was then measured (50 - 100 /c/mg) and was mounted in the spectrometer.

(c) Preliminary Adjustments

The source was placed in the holder which aligned the source on the magnetic axis of the spectrometer. The Hubert baffle opening in the lens was set at a 8.0 turns open which corresponded to a transmission of about 3% and a resolution of about 3%. The gamma spectrometers were adjusted so that the axes of rotation passed through the source position. They were also fixed so that the face of the scintillator crystals were about 10 cm from the source.

The fast-slow coincidence circuit was adjusted as described in chapter IV. The gamma single channel pulse height analyzers were set to take in the 1.465 Mev photopeak. Figure (12) is a plot of the gamma spectrum at 90° and 180°.

The single channel pulse height analyzer in the beta channel was set to discriminate against noise. The lens current settings, from the current supply corresponding to the relativistic
Figure (12)

$\gamma_1 (90^\circ)$

$\gamma_2 (180^\circ)$
\( \beta \)-particle energies are available.

Our settings were:

<table>
<thead>
<tr>
<th>Energy(W)</th>
<th>Corresponding</th>
<th>Dial setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td></td>
<td>8149</td>
</tr>
<tr>
<td>4.8</td>
<td></td>
<td>8246</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td>8350</td>
</tr>
<tr>
<td>5.2</td>
<td></td>
<td>8432</td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td>8506</td>
</tr>
</tbody>
</table>

A detailed description of the electronics described in this thesis has been given by Calclough (1963) and Young (1964) and may be referred for further information.

(d) Procedure

Before running the experiment on \(^{72}\text{Ga}\), a calibration of the apparatus was made by measuring the \(\beta-\gamma\) directional correlation coefficient \(A_2\) of the \(^{60}\text{Co}\), in order to check the over-all performance of the directional correlation system. Since the beta decay of the \(^{60}\text{Co}\) nucleus is allowed, beta-gamma angular correlation should be isotropic, i.e., a determination of the \(A_2\) coefficient should yield zero. Any deviation from isotropy would indicate a deficiency in the experimental technique or the presence of faulty equipment.

The directional correlation in \(^{60}\text{Co}\) gave

\[
A_2 = -0.0003 \pm 0.006 \text{ (}\gamma-2 \text{ probe)}
\]

\[
A_2 = +0.005 \pm 0.007 \text{ (}\gamma-1 \text{ probe)}
\]
The experimental run was made with the lens current setting of \( x = 5000 \), which corresponds to a relativistic beta-particle energy of \( W = 1.4 \). After checking the equipment, the experiment was set up as previously mentioned for \( ^{72}\text{Ga} \). A large number of coincidences were obtained. Automatic equipment, for changing the angular positioning and recording the triple coincidences (\( \beta \beta \) and \( \beta \gamma \)), the beta singles, and the gamma singles, was used. The angles were changed at the beginning of the run every 20 minutes and later every 200 minutes. The data were recorded at each position.

Each energy was examined a number of times. At the end of each sequence, the chance coincidence rate was recorded. The chance coincidence rate was obtained by delaying the beta fast channel by means of a delay line cable (\( \approx 150 \) nsec) and the resulting coincidence rate recorded.

(e) Corrections

The coincidence rate corresponds to the correlation function \( W(\theta) \) only under the assumption of centered point sources and point detectors. In order to compare the experimental results with the theoretical calculations, the coefficient \( A_{2}^{\text{exp}} \) must be corrected for the deviations from an ideal arrangement.

The first step consists of correcting the coefficient \( A_{2}^{\text{exp}} \) for the finite solid angle subtended by the gamma and beta detectors. The corrected coefficient is

\[
A_{kk}^{\text{exp}} = \frac{A_{kk}^{\text{exp}}}{Q_{kk}}
\]
The corrected experimental formula (which is different from the theoretical one) used for the β-γ directional correlation, was

\[ W^{\text{exp}}(\theta) = 1 + Q_{22} A_2 P_2(\cos \theta) \]

The factor \( Q_{22} \) is always less than unity. For a circular detector the correction factor \( Q_{22} \) can be expressed as the product of two factors

\[ Q_{22} = Q_2(e^-) \cdot Q_2(\gamma) \]

where \( Q_2(\gamma) \) is the correction factor for the gamma channel. These factors (gamma channel), used for correction, may be easily determined from published graphs by West (1959) and tables (Siegbahn (1966)). The values for \( Q_2(\gamma) \) obtained were

\[ Q_2(\gamma) = 0.965 \quad (\gamma-2 \text{ probe: } 2x2'' \text{ crystal}) \]
\[ Q_2(\gamma) = 0.977 \quad (\gamma-1 \text{ probe: } 1\frac{1}{2}x2'' \text{ crystal}) \]

for the energy 1.465 Mev gamma ray. \( Q_2(e^-) \) is the correction factor for the beta channel. It was determined experimentally with the same equipment by M. Trudel (1968). The value given in his thesis was

\[ Q_2(e^-) = 0.725 \pm 0.003 \]

Secondly, in our correlation experiment, the source was prepared so that no correction for source size is necessary. Also corrections for small variation in gamma solid angle, resulting from the imperfect location of the source on the rotation axis, and radioactive decay of the sample were made by normalizing the coincidence counts to the gamma singles rates.
Finally corrections for the scattering (γ-γ) in the spectrometer and the chance coincidence have to be taken into account.

(f) Method of Analysis of Experimental Data

The observable in this experiment is not the correlation coefficient \( A_2 \), but rather the differential \( \beta-\gamma \) coincidence counting rates at \( 90^\circ - 180^\circ \) (\( \gamma-2 \) probe) and \( 180^\circ - 270^\circ \) (\( \gamma-1 \) probe) where the \( 90^\circ \) and \( 270^\circ \) positions are geometrically equivalent.

The average measured anisotropy at each beta-ray energy was derived from many cycles. Coincidence rates, corrected for chance, were normalized to the gamma singles rates in the usual way as follows:

\[
W(\theta) = \frac{N_T - N_c}{N_\gamma}
\]

where
\( N_T \) is the recorded coincidence rate
\( N_c \) is the chance coincidence rate
\( N_\gamma \) is the gamma singles
\( \theta \) is the angle between the beta and gamma detectors.

The anisotropy
\[
a(W) = \frac{W(180^\circ) - W(90^\circ)}{W(90^\circ)}
\]

was extracted from the data. The directional correlation coefficient (\( Q_2 A_2 \)) was then obtained from the relationship

\[
Q_2 A_2 = \frac{2a(W)}{3 + a(W)}
\]

The error assignment is based on a statistical analysis of the fluctuations in the data.
The probable errors are quoted. The experimental formula used for calculating the directional correlation is then given by:

\[ A_2 = \frac{1}{Q_2} \left( \frac{2\bar{a}(W)}{2 + \bar{a}(W)} \right) \pm \sigma \]

where \( \bar{a}(W) \) is the average measured anisotropy and \( \sigma \) is the standard deviation in \( A_2 \).
The Results

In our experiment on $^{72}$Ga, the maximum analyzed beta and gamma singles rates were 18,000 counts min. and 77,000 counts min., respectively. A total of about 3,000 true coincidence was obtained at each examined energy. Each energy was examined a number of times.

The $\beta-\gamma$ directional correlation of the 2.52 Mev $\beta$- group in coincidence with the 1.465 Mev $\gamma$- transition, in the 14-hr decay of $^{72}$Ga, was measured over the $\beta$- energy range 1.839 to 2.248 Mev. The angular correlation coefficient $A_2$ varied from $-0.24 \pm 0.015$ to $-0.294 \pm 0.017$.

A plot of $A_2$ versus beta-particle energy is shown in Figure (13). The results of the $\beta-\gamma$ directional correlation experiment on $^{72}$Ga, corrected for the finite solid angle effects, are summarized in table IV.

Also, the calculated values of $a(W)$, as pure $\int B_{13}$, are shown in table V and the corresponding graph is shown in Figure (14).
**TABLE IV**

Summary of the Corrected $\beta$-$\gamma$ Directional Correlation

Data for the 2.52 Mev $\beta$-group in $^{72}$Ga

<table>
<thead>
<tr>
<th>$E$(MeV)</th>
<th>$W$(m$_0$c$^2$)</th>
<th>($\gamma$ - 1 probe)</th>
<th>($\gamma$ - 2 probe)</th>
<th>$A_2$ (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.839</td>
<td>4.6</td>
<td>-0.19±0.05</td>
<td>-0.196±0.06</td>
<td>-0.24±0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.10±0.08</td>
<td>-0.22±0.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.24±0.034</td>
<td>-0.37±0.0208</td>
<td></td>
</tr>
<tr>
<td>1.941</td>
<td>4.8</td>
<td>-0.22±0.055</td>
<td>-0.095±0.050</td>
<td>-0.254±0.0106</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.266±0.028</td>
<td>-0.267±0.0205</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.236±0.023</td>
<td>-0.214±0.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.50±0.042</td>
<td>-0.394±0.044</td>
<td></td>
</tr>
<tr>
<td>2.044</td>
<td>5.0</td>
<td>-0.21±0.0133</td>
<td>-0.247±0.014</td>
<td>-0.234±0.0092</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.23±0.06</td>
<td>-0.251±0.054</td>
<td></td>
</tr>
<tr>
<td>2.146</td>
<td>5.2</td>
<td>-0.31±0.04</td>
<td>-0.30±0.035</td>
<td>-0.29±0.0217</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.3±0.045</td>
<td>-0.26±0.039</td>
<td></td>
</tr>
<tr>
<td>2.248</td>
<td>5.4</td>
<td>-0.288±0.038</td>
<td>-0.293±0.029</td>
<td>-0.294±0.017</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.328±0.041</td>
<td>-0.275±0.032</td>
<td></td>
</tr>
</tbody>
</table>
Energy Dependence ($m_0c^2$) of the Correlation Coefficient $A_2$ of the 2.52 Mev $\beta$-group in $^{72}$Ga

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Theoretical Calculation of $a(W)$ Based on Pure $\int B_{1j}$-approximation
CHAPTER VI

CONCLUSION

The beta-gamma correlation coefficient $A_2$ in this experiment was found to be negative and exceptionally large. The average value of the $\beta-\gamma$ directional correlation coefficient was found to be $-0.251 \pm 0.006$ which is in good agreement with that of Fischbeck and Newsome ($A_2 = -0.263 \pm 0.015$).

From the results and by comparison with the measurements that had been taken by Grenacs and Deutsch (1964), we found that their results are in fair agreement with ours, but careful analysis shows that our experimental values are different from the calculated values based on the pure $\sum B_{ij}$ approximation. However, the accuracy of the measurements should be improved and our present results should be regarded only as preliminary results.

If the accuracy of these results are improved, and the shape factor determined, the matrix elements involved in this decay could be calculated unambiguously, (Ogata and Habib).

From the above discussion, we see that the decay concerned is not pure $\sum B_{ij}$, i.e., it is not unique.
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VITA AUCTORIS

The author was born on April 10, 1942 in Cairo-Egypt, where he attended both primary and secondary schools.

In 1964 he received his B.Sc. degree (Honours in applied physics) from the University of Ain Shams, Cairo. From 1965-1967 he participated in a research group in the field of Opto-electronic properties of thin films.

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