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Heat and mass transfer in soils, Part I — Thermal capacitance measurements.

Ray Shing Wu
University of Windsor

HEAT AND MASS TRANSFER IN SOILS

PART I -

THERMAL CAPACITANCE MEASUREMENTS

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By

RAY SHING WU
M.A.Sc., The Research Institute of Mech. Eng. of
Cheng-Kung University, Tainan, Taiwan. 1968

Windsor, Ontario, Canada
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Approved By: [illegible signature]

H. J. Tucker

Murray Imber

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ABSTRACT

The phenomenological equations and the governing differential equations were derived along with their numerical counterparts for heat and mass transfer in soils. An apparatus was designed to measure the specific heat of soils over a temperature range from 10 to 50 °F. Tests were carried out with this apparatus using one sample of grey Windsor clay, primarily to study the behaviour of the specific heat when the soil was subjected to alternate freezing and thawing conditions below 32 °F. It was observed that a plot of specific heat vs temperature in the range 32 °F to 15 °F showed a hysteresis effect with the specific heat jumping suddenly between the heating and cooling curves. From this behaviour it was concluded that the hysteresis loop should vanish with repeated cycles below 32 °F.

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NOTATION

Symbol	Description	Dimensions
A	= surface area,	L^2
C	= capillary constant,	ML/Ft
C_p	= specific heat at constant pressure,	FL/MT
h	= enthalpy,	FL/MT
k	= thermal conductivity,	FL/LTt
G	= transferable quantity per unit volume,	M or FL
F	= force,	ML/t^2
p	= pressure,	F/L^2
Q	= energy,	FL
T	= temperature,	T
t	= time,	t
U	= moisture content by dry weight of soil,	M/M
Ø	= the rate of generation of transferable quantity per unit volume,	$(M \text{ or } FL)/tL^3$
k_f	= coefficient of filtration,	L^4/Ft
k_m	= coefficient of migration,	L^4/Ft
j_q	= heat flux,	FL/L^2t
j_m	= mass flux,	M/L^2t
x, y, z	= the space coordinates,	L

Symbol	Description	Dimensions
m_k	= accumulation of mass at nodal point k,	M
Q_k	= accumulation of heat at nodal point k,	FL
m_{ij}	= mass flow from nodal point i to nodal point j,	M
Q_{ij}	= heat flow from nodal point i to nodal point j,	FL
C_{11}	= phenomenological coefficient,	FL/tTL
C_{12}	= phenomenological coefficient,	FL/tL
C_{21}	= phenomenological coefficient,	M/tTL
C_{22}	= phenomenological coefficient,	M/tL
C_{ijk}	= coefficient of C_{ij} at nodal point k,	as C_{ij}
ρ	= average density of porous bodies,	M/L^3
ρ_o	= dry soil density,	M/L^3
$\frac{\partial}{\partial t}$	= time rate of change,	1/t
∇	= gradient of scalar,	
∇^2	= Laplacian operator,	
φ	= capillary potential,	F/L^2
$P_{ext.}$	= external pressure, the transfer force of filtration,	F/L^2
$P_{int.}$	= internal pressure, the vapour pressure,	F/L^2

I. METHOD OF ANALYSIS

A. PHYSICAL PHENOMENA

Transfer of non-condensable gases, vapours and liquids can occur in porous bodies (soils) due to both temperature and moisture content gradients. Transfer of vapour can take place by different methods: by molecular means in the form of diffusion or by molar means by a filtration of the whole vapour-gas mixture. Liquid transfer can take place by means of diffusion, capillary absorption, migration, and filtration.

The process of heat transfer takes place mainly by:

(1) A conductive mechanism, in which the transfer of thermal energy is not accompanied by means of transfer of substance, and

(2) A convective mechanism, in which the transport of heat is effected by the movement of water and air in the pores of the soil.

Each of these mechanisms has its specific distinguishing features, a fact that makes it necessary to consider them separately.

A further complication in any heat transfer study is the need to recognize three zones, the thawed zone, the phase transition zone, and the frozen zone.

(1) The Conductive Mechanism of Heat Transfer

(a) Processes of Heat Transfer in the Thawed and Frozen Zone

From the point view of heat conduction these two zones may be considered together since they are both characterized by no phase changes taking place in the soil as a function of temperature.

The fundamental law governing the conductive mechanism is the Fourier-Biot Law which states that the heat flow is proportional to the gradient of temperature, that is:

$$\bar{Q} = -k A \nabla T \qquad (1\text{-}1)$$

where k is called the coefficient of thermal conductivity of the material. It represents an average material characteristic, thus it is a function not only of temperature but also of its moisture content, composition, and porosity.

(b) Heat Transfer Processes in the Phase Transition Zone

Although the processes of water freezing and ice thawing require a great deal of energy, and exert a very strong influence on the properties of the soil, especially on the texture, the basic equation of heat conduction, Eq.(1-1), is still valid for this zone. However, k is now also a function of the past history of the soil since the soil texture and porosity are affected.

(2) The Convective Mechanism of Heat Transfer

(a) Processes of Heat and Moisture Transfer in the Thawed Zone

The convective heat transfer in this zone is carried out mainly by the liquid. The motion of the water is due to filtration, migration, and capillary absorption.

The fundamental law of filtration is Darcy's Law which states that the rate of filtration is proportional to the gradient of the external pressure, that is:

$$\bar{j}_m = - k_f \rho \nabla P_{ext.} \tag{1-2}$$

where k_f is the coefficient of filtration and is a function of temperature, moisture content, and the properties of soil.

The fundamental law of migration is Buckingham-Lykov's Law which states that the rate of flow is proportional to the gradient of internal pressure, that is:

$$\bar{j}_m = - k_m \rho \nabla P_{int.} \qquad (1\text{-}3)$$

where k_m is the coefficient of migration and $P_{int.}$, the partial vapour pressure, is a function of temperature, moisture content, and the properties of soil.

Under the capillary motion of liquid, the flow is proportional to the gradient of capillary potential, that is:

$$\bar{j}_m = C \nabla \varphi \qquad (1\text{-}4)$$

where C is the capillary conductance and φ, the capillary potential, is a function of temperature, and moisture content.

(b) Processes of Heat and Moisture Transfer in the Phase Transition Zone

The movement of water is similar in the thawed zone and phase transition zone, but the causes of migration are different. The characteristics of the movement of water in this zone has not yet been understood. However, the Buckingham-Lykov Law is still valid.

(c) Processes of Heat and Moisture Transfer in the Frozen Zone

The movement of water is by vapour diffusion in this zone, and it is described by Buckingham-Lykov Law.

To sum up the above discussion, the total moisture flow caused by the influence of all transfer forces will have the form

$$\bar{j}_m = - k_f \rho \nabla P_{ext.} - k_m \rho \nabla P_{int.} + C \nabla \varphi$$

However, $P_{int.} = P_{int.}(T, U)$ and $\varphi = \varphi(T, U)$

so

$$\nabla P_{int.} = \frac{\partial P_{int.}}{\partial T}\Big)_U \nabla T + \frac{\partial P_{int.}}{\partial U}\Big)_T \nabla U$$

$$\nabla \varphi = \frac{\partial \varphi}{\partial T}\Big)_U \nabla T + \frac{\partial \varphi}{\partial U}\Big)_T \nabla U$$

Hence,

$$\bar{j}_m = - k_f \rho \nabla P_{ext.} + \left[C \frac{\partial \varphi}{\partial T}\Big)_U - k_m \rho \frac{\partial P_{int.}}{\partial T}\Big)_U\right] \nabla T$$

$$+ \left[C \frac{\partial \varphi}{\partial U}\Big)_T - k_m \rho \frac{\partial P_{int.}}{\partial U}\Big)_T\right] \nabla U$$

$$= - k_f \rho \nabla P_{ext.} + C_{21} \nabla T + C_{22} \nabla U$$

where

$$C_{21} = C \frac{\partial \varphi}{\partial T}\Big)_U - k_m \rho \frac{\partial P_{int.}}{\partial T}\Big)_U$$

$$C_{22} = C \frac{\partial \varphi}{\partial U}\Big)_T - k_m \rho \frac{\partial P_{int.}}{\partial U}\Big)_T$$

If the soil is unsaturated and in direct contact with the atmosphere, then $P_{ext.}$ = constant, so

$$\bar{j}_m = C_{21} \nabla T + C_{22} \nabla U \qquad (1\text{-}5)$$

The total heat transfer occurs by conduction and convection. Therefore the heat flux is

$$\bar{j}_q = \bar{j}_{q,\ cond.} + \bar{j}_{q,\ conv.}$$
$$= - k \nabla T + \bar{j}_m h$$

Substituting Eq. (1-5) into above equation, get

$$\bar{j}_q = - k \nabla T + h (C_{21} \nabla T + C_{22} \nabla U)$$
$$= (- k + hC_{21}) \nabla T + h C_{22} \nabla U$$

and set

$$C_{11} = (- k + h C_{21})$$
$$C_{12} = h C_{22}$$

Hence,

$$\bar{j}_q = C_{11} \nabla T + C_{12} \nabla U \qquad (1\text{-}6)$$

The Eq. (1-5) and Eq. (1-6) are called the phenomenological governing equations and the C_{ij} are called the phenomenological coefficients. They are functions of the temperature, the moisture content, and the properties of soil, and are determined by experiment.

B. CONSERVATION LAWS

The conservation equation for any transport phenomenon may be written as follows:

$$-\nabla \cdot \bar{j}_G + \phi = \frac{\partial}{\partial t}(G) \qquad (1\text{-}7)$$

where G = the transferable quantity per unit volume,

j_G = the flux of "G",

ϕ = the rate of generation of "G" per unit volume,

$\frac{\partial}{\partial t}$ = time rate of change,

For the thawed and frozen zone, $\phi = 0$, so the governing energy and mass differential equations can be derived by substituting the appropriate terminology into Eq.(1-7)

$$-\nabla \cdot \bar{j}_q = \frac{\partial}{\partial t}\left(\int_0^T \rho \, c_p \, dT\right) \qquad (1\text{-}8)$$

$$-\nabla \cdot \bar{j}_m = \frac{\partial}{\partial t}(\rho) \qquad (1\text{-}9)$$

In the phase transition zone, the mass balance equation is the same as Eq. (1-9), but the energy equation is different due to the effect of the latent heat of water. Thus for this zone the energy equation has the form

$$-\nabla \cdot \bar{j}_q + \phi = \frac{\partial}{\partial t}\left(\int_0^T \rho \, c_p \, dT\right) \qquad (1\text{-}10)$$

where the term ϕ may be calculated as follows,

$$m_{water} = m_{ice} + m_{liq.} + m_{vap.} \qquad (1\text{-}11)$$

$$\phi = h_{ice}\frac{\partial m_{ice}}{\partial t} + h_{liq.}\frac{\partial m_{liq.}}{\partial t} + h_{vap.}\frac{\partial m_{vap.}}{\partial t} \qquad (1\text{-}12)$$

It is very complex to make the above calculation. However, it is possible to combine the term ϕ into the term $\frac{\partial}{\partial t}(\int_0^T \rho\, C_p\, dT)$ and define the effective specific heat $\bar{C}_p$, so the Eq.(1-10) has the same form of Eq.(1-8) as,

$$-\nabla \cdot \bar{j}_q = \frac{\partial}{\partial t}(\int_0^T \rho\, \bar{C}_p\, dT) \qquad (1\text{-}13)$$

For small intervals of temperature and moisture content, if the C_{ij} can be considered as constants, then the governing to conservation equations can be written in the following form

$$-C_{11}\nabla^2 T - C_{12}\nabla^2 U = \frac{\partial}{\partial t}(\int_0^T \rho\, \bar{C}_p\, dT) \qquad (1\text{-}14)$$

$$-C_{21}\nabla^2 T - C_{22}\nabla^2 U = \frac{\partial}{\partial t}(\rho) \qquad (1\text{-}15)$$

C. NUMERICAL SOLUTION

In engineering an approximate numerical method is used to solve many two-dimensional problems. Usually, in this method the continuous physical system is divided into a number of discrete elements, see Fig. (1-1), with thermal properties considered to be concentrated at the central nodal point of each element. Heat and moisture are assumed to be transferred between these nodes. Finite-difference equations may be set up to replace the governing differential equations with the temperature and moisture contents of a given nodal point expressed as function of its previous temperature and moisture content and the previous temperature and moisture content of its adjacent nodal point.

Consider a portion of the two dimensional network as shown in Fig. (1-2). For a typical nodal point 5, the moisture and heat balance equation over an infinitesmal increment of time Δt would be:

$$m_5 = m_{15} + m_{25} + m_{35} + m_{45} \qquad (1\text{-}16)$$

$$Q_5 = Q_{15} + Q_{25} + Q_{35} + Q_{45} \qquad (1\text{-}17)$$

where the subscripts m_{ij} and Q_{ij} indicate the flow from nodal point i to nodal point j per unit depth and m_5 and Q_5 represent the increment of moisture content and heat energy respectively at nodal point 5. Hence, by Eq.(1-5) and Eq. (1-6) get

$$\frac{m_{15}}{\Delta t} = \frac{T_1^n - T_5^n}{\dfrac{\Delta x_3}{2C_{211}\Delta z_2} + \dfrac{\Delta x_2}{2C_{215}\Delta z_2}} + \frac{U_1^n - U_5^n}{\dfrac{\Delta x_3}{2C_{221}\Delta z_2} + \dfrac{\Delta x_2}{2C_{225}\Delta z_2}} \qquad (1\text{-}18)$$

$$\frac{Q_{15}}{\Delta t} = \frac{T_1^n - T_5^n}{\dfrac{\Delta x_3}{2C_{111}\Delta z_2} + \dfrac{\Delta x_2}{2C_{115}\Delta z_2}} + \frac{U_1^n - U_5^n}{\dfrac{\Delta x_3}{2C_{121}\Delta z_2} + \dfrac{\Delta x_2}{2C_{125}\Delta z_2}} \qquad (1\text{-}19)$$

where C_{ijk} means the C_{ij} at the nodal point k and superscript "n" means that the value which exists at time $n\Delta t$. Thermal properties were assumed constant and isotropic within the boundary of that element.

In the thawed and frozen zones, the moisture and heat gain are used to increase the moisture content and the temperature respectively, but in the phase transition zone, some of the energy is used in phase transformations. Make use of previous definition of $\overline{C}_p$ of the soil, the numerical solution of the moisture content and temperature at the time (n+1) t after starting are respectively

$$\frac{m_5}{\Delta t} = \Delta x_2 \Delta z_2 \frac{\rho_{o5}\,(U_5^{n+1} - U_5^n)}{\Delta t} = \sum_{i=1}^{4} m_{i5} \qquad (1\text{-}20)$$

Thus $$U_5^{n+1} = U_5^n + \frac{\Delta t}{\rho_{05} \Delta x_2 \Delta z_2} \sum_{i=1}^{4} m_{i5} \qquad (1\text{-}21)$$

and

$$\frac{Q_5}{\Delta t} = \Delta x_2 \Delta z_2 \frac{\rho_{05} \bar{C}_p (T_5^{n+1} - T_5^n)}{\Delta t} = \sum_{i=1}^{4} Q_{i5} \qquad (1\text{-}22)$$

Thus $$T_5^{n+1} = T_5^n + \frac{\Delta t}{\rho_{05} \bar{C}_p \Delta x_2 \Delta z_2} \qquad (1\text{-}23)$$

So in a two-dimensional problem, if the initial and boundary conditions are given, then it will be able to predict the moisture content and the temperature as a function of time.

II. EMPIRICAL DATA REQUIRED FOR THE ANALYSIS

In order to carry out any numerical solution of Eq. (1-21) and Eq. (1-23), the effective specific heat of the soil and the phenomenological coefficient have to be determined empirically for different soils as functions of temperature and moisture content.

The C_{ij} can be obtained experimentally as follows:

Test (1), isothermal condition; set $\nabla T = 0$ and measure j_m and ∇U.

Now

$$\bar{j}_q = C_{12} \nabla U \tag{2-1}$$

Thus

$$C_{12} = \bar{j}_q / \nabla U \tag{2-2}$$

where j_q is called the transfer energy under isothermal condition and equal to

$$\bar{j}_q = \bar{j}_m \cdot h \tag{2-3}$$

where the energy datum is set arbitrary but is consistent through the system.

And

$$\bar{j}_m = C_{22} \nabla U \tag{2-4}$$

Thus

$$C_{22} = j_m / \nabla U \tag{2-5}$$

This test can be performed by a transient method using the arrangement shown in Fig. (2-1).

Initially, the insulated soil sample is fully saturated, heated dry air is passed over one end in order to cause it to dry. The temperature of the heated dry air depends upon the energy required to evaporate the moisture without creating the temperature gradient. The one dimensional flow rate can be calculated by analyzing moisture content profiles as a function of time. The moisture content profiles can be obtained by using a gamma-ray densitometer using radioactive cesium-137 to measure the mean radial density at different positions along the axis of the sample.

Test (2), non-flow condition; set $j_m = 0$ and measure j_q, ∇T, and ∇U.

Now

$$\bar{j}_q = C_{11} \nabla T + C_{12} \nabla U \qquad (2\text{-}6)$$

Thus

$$C_{11} = \bar{j}_q / \nabla T - C_{12} \nabla U / \nabla T \qquad (2\text{-}7)$$

and

$$\bar{j}_m = C_{21} \nabla T + C_{22} \nabla U = 0 \qquad (2\text{-}8)$$

Thus

$$C_{21} = - C_{22} \nabla U / \nabla T \qquad (2\text{-}9)$$

This test can be carried out by a steady state method using the arrangement shown in Fig. (2-2).

The soil sample initially is isothermal with a homogeneous moisture content. A temperature gradient is then created by using two end heaters. After the system achieves a steady state condition, the temperature and moisture content are measured as functions of positions. The temperature is measured by thermocouples or thermistors inserted into the sample. The heat flux can be measured by the electrical input power.

III. MEASUREMENT OF EFFECTIVE SPECIFIC HEAT OF POROUS BODIES WITH VARIOUS DEGREES OF MOISTURE CONTENT

A. EQUIPMENT

A calorimeter has been constructed as shown in Fig. (3-1), that permits measurement of the amount of heat added to or removed from a specimen in order to raise or lower its temperature by a small amount.

A high vacuum thermos is used as the outer shell of the calorimeter.

The soil sample is contained in the copper vessel shown in Fig. (3-2).

The surrounding temperature is controlled close to that of the test temperature by placing the entire calorimeter in the cold chamber.

The magnetic stirrer is used to mix the freon in order to make its temperature homogenous and to speed the heat transfer.

Within the sample, two temperatures are measured by two thermistors and one is by a thermocouple which is connected to a Moseley Model 7100B Strip Chart Recorder. The temperature of the freon bath is measured by one thermistor probe and one thermocouple which is connected to the recorder.

A bottle with two glass tubes, in which one is short and the other is long, is used to charge the calorimeter with freon. The bottle is insulated by one inch styrofoam and is sealed. This charging method was used in order to avoid the evaporation of freon. Many other methods were tried before, but due to the evaporation of the freon the results were not sufficiently accurate.

Because the enthapy change of freon from liquid to vapour is equal to 70.0 Btu per pound, and the heat capacity of the sample above 32 F can be as low as 0.08 Btu per degree Fahrenheit, an evaporation of as little as 0.001 pounds (approximately 0.03 % of the total mass of freon) freon will cause an error of as much as 100 %. In the design, the freon system is sealed in the bottle and comes into contact only with saturated air from the thermos, hence this method prevents any freon evaporation.

A Wheatstone Bridge with 0.01% accuracy of reading is used to measure the resistance of thermistors. And a constant resistor is in series the thermistor in order to avoid self-heating of the thermistor.

B. PROCEDURE

The following steps were used for calibrating the calorimeter system and for finding the effective specific heat of the soil sample.

(1) Start when the temperature of the soil, the freon in the thermos, and the cold chamber are all the same. Record this temperature as the initial temperature of the soil sample.

(2) Prepare a bottle of new freon for charging. Determine its temperature and mass.

(3) Drain the existing freon from the thermos through a siphon tube inserted through the longer glass tube, see Fig. (3-1), removing the rubber tube and utilizing a rubber bulb. It takes about two minutes to drain 3.5 pounds freon.

(4) Immediately recharge with the new freon. It takes about 10 seconds to empty 3.5 pounds freon from the bottle.

(5) The charging procedures are (a) connect the tube shown as in Fig. (3-1) by dotted lines, and (b) invert the bottle above the calorimeter. The freon flows into the thermos and the air in the thermos, saturated with freon vapour, flows into the bottle.

(6) Measure the temperature of the freon in the thermos and the temperature of the soil, and record the final equilibrium temperature.

(7) Repeat step (1) for a new test.

C. DATA

Data recorded for the calculation of the effective specific heat of Windsor grey clay over a temperature range from 15 F to 50 F for various degrees of moisture content are given in appendix A.

D. RESULTS

The results of the analysis to calculate the heat capacity of the calorimeter and the effective specific heat of the soil are presented in appendix B. The computer program used for this calculation is available in appendix C.

In the program the calculation is based on the energy balance equation, that is, the input energy by freon is equal to the energy absorbed by the calorimeter system and soil sample.

$\bar{C}_p$ is essentially constant in the unfrozen zone and fully frozen region but varies considerably in the phase transition region. The following is a synopsis of the results:

Moisture Content U	$\bar{C}_p$		
		Temperature	
	above 32 F	31.99 to 22	below 22
0.0	0.174	0.170	0.164
0.126	0.285	see Fig. 3-3	0.245
0.252	0.343	see Fig. 3-4	0.300
0.366	0.400	see Fig. 3-5	

E. ANALYSIS

(1) The experimental data for the unfrozen region satisfied the equation

$$(m_s C_p)_{soil} + (m_w C_p)_{water} = (m_s + m_w)\,\bar{C}_p$$

well within the accuracy of the system data.

(2) The temperature at which this soil behaves as though it were fully frozen is a function of moisture content. As the moisture content increases, the fully frozen temperature decreases.

(3) In the phase transition region the effective specific heat is larger for cooling than for heating. A plot of specific heat vs temperature in the range 32 F to 15 F showed a hysteresis effect with the soil property jumping suddenly from one curve to the other depending upon whether the soil temperature is rising or falling. This implies that at a given temperature more liquid is freezing while cooling than would be melting while heating, at least initially. It follows therefore that if the soil is cycled within the phase change region then the two $\bar{C}_p$ vs T curves must approach each other.

(4) The relative error in $\bar{C}_p$ due to experimental inaccuracies varies between 1% and 50% as shown in Fig. 3-6 where the relative error is plotted vs T.

F. DISCUSSION

(1) For the unfrozen zone the specific heat measured for the grey Windsor clay is in close agreement with the value found by M.S. Kersten[4] for three different clay soils.

(2) For the phase transition zone the hysteresis type behavior of the soil is in agreement with the results found by P.J. Williams[1] for clay soils and are of approximately the same magnitude, see Fig.3-7. An exact comparison is not possible since the soils are not identical.

(3) The temperature at which the soil acts as though fully frozen was not established in these tests, except for the dry soil. It is known from the literature[5] that the frozen temperature decreases with increasing moisture content. In these tests $\bar{C}_p$ is within 13.4 % of $(\bar{C}_p)_{frozen}$ at 23.9 F for U = 0.126, and within 22.5 % at 23 F for U = 0.252.

(4) In any numerical solution of the energy equation involving freezing and thawing of a soil the appropriate specific heat (for heating or for cooling) must be used for temperatures below 32 F.

(5) Further studies must be carried out to establish the exact specific heat behaviour under repeated cycling both below 32 F and for complete freezing-thawing cycles (e.g. 35 to 10 F).

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APPENDIX A

The tabulated symbols and units are defined as follows,

RIR = initial resistance of soil thermistor, ohms

FIR = initial resistance of freon, ohms

FFR = final resistance of sample and freon, ohms

POUNDS and OZ = weight of freon,

A 23510.0 ohms resistor was inserted into the bridge unit.

DATA I

Data for Calculation of Calorimeter (mC_p)

No.	RIR	FIR	FFR	POUNDS	OZ
1	29702	29302	29537	1.0	12.30
2	29527	28652	29077	2.0	11.20
3	29060	28210	28620	2.0	8.00
4	31074	30065	30550	3.0	7.23
5	31895	32944	32285	2.0	4.62
6	31755	32854	32145	2.0	2.43
7	30676	31580	31064	2.0	14.19
8	30463	31494	30915	3.0	0.97
9	31300	32940	31950	2.0	10.94
10	32240	30360	31180	3.0	14.89
11	31180	29520	30281	3.0	10.96

No.	RIR	FIR	FFR	POUNDS	OZ
12	30906	31940	31365	3.0	0.25

Note: (a) From No. 1 to No. 3, the container was empty.

(b) From No. 4 to No. 12, the container was filled with 3 ounces distilled water.

DATA II

Data for Calculation of Dry Soil $\bar{C}_p$

No.	RIR	FIR	FFR	POUNDS	OZ
1	32570	30150	31230	3.0	1.44
2	31412	29534	30314	3.0	9.90
3	30320	27584	28885	2.0	6.58
4	28765	29690	29163	2.0	7.05
5	29222	29860	29534	3.0	0.38
6	29943	27995	28865	3.0	0.24
7	36825	35950	36320	3.0	15.87
8	36340	34520	35417	3.0	0.82
9	35462	33650	34562	2.0	11.20
10	34600	32650	33657	2.0	8.07

DATA III

Data of Soil (6 ounces) with Water (0.76 ounces)

No.	RIR	FIR	FFR	POUNDS	OZ
1	30480	31700	31000	2.0	10.51
2	30980	32310	31480	2.0	2.0
3	31082	34050	32430	3.0	3.95
4	32280	33742	32955	3.0	0.83
5	32852	33540	33120	2.0	15.63
6	32844	29000	30771	2.0	12.42
7	30940	29473	30195	2.0	15.64
8	40400	36500	38397	2.0	14.71
9	38400	34970	36833	2.0	6.41
10	36750	33405	35040	2.0	13.15
11	35075	33195	34130	3.0	2.95
12	34120	32530	33530	3.0	2.97
13	33610	31930	33070	3.0	3.81
14	33607	35192	33900	3.0	5.89
15	33850	36130	34540	3.0	4.38
16	34550	36560	35316	3.0	3.910

Data III (Continued)

No.	RIR	FIR	FFR	POUNFS	OZ
17	35330	37110	36110	3.0	11.10
18	35560	36950	36082	2.0	13.66
19	36071	40190	37700	2.0	12.30
20	37895	33835	35700	3.0	5.76
21	35745	34025	35205	2.0	0.00
22	35200	33375	34415	2.0	15.75
23	34420	33145	33910	2.0	11.80
24	34180	33310	33890	2.0	4.60
25	33306	35830	33640	4.0	0.93
26	33620	36127	33960	2.0	0.00
27	33950	36720	34785	2.0	12.85
28	34750	36870	35310	2.0	1.20
29	36100	34630	35620	2.0	0.00
30	35640	33880	34750	3.0	5.39
31	34770	33320	34020	4.0	9.92
32	34030	32820	33490	4.0	12.00
33	33500	33000	33410	3.0	8.80

DATA IV

Data of Soil (5 ounces) with Water (1.26 ounces)

No.	RIR	FIR	FFR	POUNDS	OZ
1	30994	33220	31970	3.0	0.00
2	31860	32500	32110	2.0	2.77
3	32062	33384	32650	2.0	13.34
4	32680	33780	33210	3.0	5.34
5	33510	34883	33725	3.0	2.52
6	33900	36314	34630	3.0	3.25
7	34620	36525	35305	3.0	3.28
8	35154	36335	35600	3.0	7.80
9	35588	37570	36390	3.0	3.90
10	36600	33900	35400	2.0	5.01
11	35130	33525	34570	1.0	10.00
12	34370	33570	33960	3.0	6.50
13	33780	31870	33400	3.0	6.72
14	33546	35896	33900	3.0	6.75

Data IV (Continued)

No.	RIR	FIR	FFR	POUNDS	OZ
15	35170	36860	35740	3.0	2.26
16	35560	37080	36110	3.0	7.71
17	36840	34480	35890	2.0	0.00
18	35715	34185	35000	2.0	10.41
19	34955	33600	34490	1.0	12.00
20	34400	32655	33705	3.0	6.03
21	33715	32850	33500	3.0	4.43
22	33470	31050	33305	3.0	5.44
23	33270	31000	32310	2.0	1.00
24	32116	30410	31410	2.0	1.20
25	31230	30300	30860	2.0	0.18

DATA V

Data of Soil (5 ounces) with Water (1.83 ounces)

No.	RIR	FIR	FFR	POUNDS	OZ
1	31850	29900	30880	2.0	15.22
2	30880	28890	29853	3.0	1.68
3	31486	32530	31980	3.0	7.09
4	31918	33100	32445	2.0	15.86
5	32432	33080	32622	1.0	8.90
6	35270	38400	36610	3.0	4.36
7	35545	33340	34585	2.0	14.73
8	34795	32915	34305	1.0	10.00
9	34100	32000	33650	2.0	14.36
10	33670	32890	33540	3.0	1.40
11	33530	34370	33630	3.0	7.36
12	33650	36000	34000	3.0	4.90
13	34000	36215	34230	2.0	9.79
14	34230	36010	34755	3.0	4.31

Data V (Continued)

No.	RIR	FIR	FFR	POUNDS	OZ
15	34745	36120	35200	2.0	10.73
16	33880	35400	34210	3.0	12.20
17	34205	36020	34820	3.0	13.70
18	33509	35890	33820	3.0	7.58
19	33815	36920	34430	3.0	2.59
20	34420	37430	35355	2.0	12.55
21	35270	37210	35960	3.0	2.03
22	35920	37720	36560	2.0	13.80
23	36285	33300	34880	3.0	8.20
24	34900	33305	34385	2.0	1.90
25	34300	32000	33950	1.0	13.57
26	33950	31850	33550	3.0	0.62

APPENDIX B

The units of the following tables are:

T = temperature, degree of Fahrenheit.

Heat Capacity = Btu per degree of Fahrenheit

Effective Specific Heat = Btu per pound per degree Fahrenheit

Table I

Heat Capacity of Calorimeter

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Heat Capacity
1	48.6408	51.1484	49.6510	49.1454	0.555
2	49.7124	55.6513	52.6453	51.1817	0.562
3	52.7474	59.0925	55.8783	54.3126	0.541
4	41.7033	47.0404	44.3569	43.0301	0.545
5	37.9294	33.6704	36.2827	37.1061	0.561
6	38.5471	34.0157	36.8621	37.7046	0.563
7	43.7052	39.3246	41.7525	42.7288	0.555
8	44.8202	39.7215	42.4852	43.6527	0.560
9	40.6169	33.6852	37.6603	39.1386	0.5612
10	36.4690	45.3872	41.1820	38.8255	0.542
11	41.1820	50.3226	45.8220	43.5020	0.555
12	42.5295	37.7309	40.3169	41.4232	0.553

Table II

Effective Specific Heat of Dry Soil

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
1	34.7765	46.0477	40.5246	37.6505	0.1683
2	39.6846	49.6694	45.1449	42.4148	0.1915
3	45.1119	64.6951	53.9714	49.5417	0.1617
4	54.8286	48.7143	52.0591	53.4438	0.1707
5	51.6645	47.7038	49.6694	50.6670	0.1762
6	47.2250	60.9323	54.1143	50.6696	0.2026
7	21.3860	23.7161	22.7092	22.0476	0.1636
8	22.6489	27.7096	25.2127	23.9308	0.1881
9	25.0810	30.7965	27.7906	26.4358	0.1652
10	27.6719	34.4627	30.7721	29.2220	0.1680

Table III

Effective Specific Heat of Soil (6 ounces) with Water (0.76 ounces)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
1	44.2534	38.3926	41.6246	42.9390	0.2952
2	41.7230	35.8069	39.3708	40.5468	0.2827
3	41.2311	29.4451	35.3255	38.2783	0.2690
4	35.9310	30.4825	33.3148	34.6229	0.3011
5	33.6852	31.1802	32.6852	33.1852	0.8560
6	33.7222	53.1704	42.7557	38.2390	0.2702
7	41.9197	50.0645	45.8275	43.8736	0.2731
8	13.4368	22.2376	17.6185	15.5277	0.2500
9	17.6116	26.5210	21.3729	19.4923	0.28000
10	21.5825	31.6512	26.3161	23.9493	0.2300
11	26.2136	32.4074	29.1813	27.6975	0.3610
12	29.2143	34.9333	31.2151	30.2147	1.6002
13	30.9360	37.3793	32.0000	31.4680	2.4000
14	30.9465	25.8771	29.9395	30.4430	5.2804
15	30.1044	23.2184	27.8594	28.9819	1.9602
16	27.8281	22.0803	25.5083	26.6682	1.0199
17	25.4673	20.6738	23.2737	24.3705	0.8127

Table II. (Continued)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
18	24.7945	21.0672	23.3567	24.0756	0.8850
19	23.3843	13.8432	19.2332	21.3088	0.4200
20	18.7815	30.1538	24.4074	21.5944	0.3500
21	24.2829	29.5275	25.8332	25.0580	1.0042
22	25.8478	31.7558	28.2500	27.0489	0.8000
23	28.2344	32.5926	29.9096	29.0705	0.8300
24	29.0165	31.9825	29.9725	29.4945	1.0202
25	32.0000	24.0479	30.8314	31.5509	7.9995
26	30.9012	23.2267	29.7417	30.3214	4.1313
27	29.7747	21.6611	27.0938	28.4342	1.4503
28	27.2031	21.2681	25.5258	26.3645	1.2391
29	23.3014	27.5469	24.6286	23.9650	0.8175
30	24.5732	30.0055	27.2031	25.8882	0.2176
31	27.1406	31.9476	29.5440	28.3433	0.9500
32	29.5110	33.7963	31.3546	30.4328	1.7748
33	31.3198	33.1296	31.6337	31.4767	7.0000

Table IV

Effective Specific Heat of Soil (5 ounces) with Water (1.26 ounces)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
1	41.6541	32.3148	37.2138	39.4339	0.3408
2	37.6559	35.0510	36.6349	37.1452	0.3382
3	36.8331	31.7244	34.4627	35.6479	0.3302
4	34.3451	30.3430	32.3518	33.3485	0.3420
5	31.2849	26.7875	30.5349	30.9099	6.8342
6	29.9396	22.7249	27.5781	28.7588	2.0255
7	27.6094	22.1760	25.5405	26.5749	1.3213
8	25.9824	22.6699	24.6839	25.3312	1.4187
9	24.7170	19.5426	22.5258	23.6214	0.8965
10	21.9755	29.9396	25.2624	23.6190	0.3000
11	26.0527	31.2325	27.7656	26.9092	0.3105
12	28.3906	31.0756	29.7417	29.0662	0.3650
13	30.3430	37.6426	31.6686	31.0058	6.7665
14	31.1593	23.8654	29.9396	30.5494	7.5073

m Table IV (Continued)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
15	25.9356	21.2943	24.2968	25.1162	1.5868
16	24.7945	20.7475	23.2737	24.0431	1.6114
17	21.3467	28.0469	23.8820	22.6144	0.3022
18	24.3659	29.0000	26.4332	25.3995	0.3152
19	26.5649	30.9709	28.0156	27.2902	0.4641
20	28.2969	34.4431	30.6046	29.4508	1.5588
21	30.5698	33.6852	31.3198	30.9448	4.0527
22	31.4244	41.3787	32.0000	31.7122	27.5873
23	32.1296	41.6246	35.8069	33.9683	0.3212
24	36.6096	44.6165	39.6938	38.1517	0.3419
25	40.5246	45.2220	42.3131	41.4186	0.3372

Table V

Effective Specific Heat of Soil (5 ounces) with Water (1.83 ounces)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
1	37.7309	47.4731	42.2147	39.9728	0.3952
2	42.2147	53.9357	47.7442	44.9795	0.4069
3	39.3431	34.9332	37.1724	38.2577	0.4099
4	37.4309	32.7592	35.2667	36.3488	0.3961
5	35.3176	32.8333	34.5725	34.9451	0.4869
6	25.6429	17.6116	21.9493	23.7961	0.5420
7	24.8380	31.8779	27.8786	26.2784	0.7230
8	27.0625	33.4444	28.6044	27.8334	1.1599
9	29.2802	37.0897	30.7965	30.0384	4.5238
10	30.7267	33.5370	31.1802	30.9535	6.4628
11	31.2151	28.3906	30.8662	31.0407	10.5475
12	30.7965	23.5779	29.6099	30.2032	6.7699
13	29.6099	23.2325	28.8514	29.2307	7.9991
14	28.8516	23.5502	27.1875	28.0196	2.1399

m Table V (Continued)

No.	$T_{initial}$ of soil	$T_{initial}$ of freon	T_{final}	T_{mean} of soil	Effective Specific Heat
15	27.2186	23.2461	25.8478	26.5333	1.1200
16	30.0055	25.2624	28.9176	29.4615	4.7778
17	28.9341	23.5226	26.9844	27.9592	1.9998
18	31.2884	23.8802	30.2035	30.7459	8.4235
19	30.2209	21.1410	28.2031	29.2120	4.0023
20	28.2343	19.8869	25.3941	26.8143	1.2983
21	25.6429	20.4279	23.6885	24.6657	1.2029
22	23.7991	19.1869	22.0803	22.9397	0.9998
23	22.8009	32.0185	26.7963	24.7989	0.8960
24	26.7344	32.0000	28.3437	27.5391	1.0061
25	28.6209	37.0897	29.7747	29.1978	4.3802
26	29.7747	37.7309	31.1453	30.4600	5.7862

APPENDIX C

The symbols in the following program are as follows,

TF	=	the temperature of conversion table, in F
TC	=	the temperature of conversion table, in C
FH	=	the enthalpy of conversion table, in Btu per pound per F
TR	=	the resistance of conversion table, in ohms
SM	=	the mass of soil sample, in pounds
FM	=	the mass of freon for charging, in pounds
RIR	=	the resistance of thermistor in the soil sample, in ohms
FIR	=	the resistance of thermistor in the charging freon, in ohms
FFR	=	the resistance of thermistor in the final, in ohms
CIS	=	the initial temperature of soil sample, in C
FIS	=	the initial temperature of soil sample, in F
CFI	=	the initial temperature of freon, in C
FFI	=	the initial temperature of freon, in F
CFF	=	the final temperature of system, in C
FFF	=	the final temperature of system, in F
HI	=	the initial enthalpy of freon, in Btu/ F lb

HF = the final enthalpy of freon, in Btu per F per pound

QC = the heat capacity of the calorimeter, in Btu per F

CPS = the effective specific heat of soil sample, in Btu per pound per F

ERROR = the relative error of CPS

```
C CALCULATE THE SPECIFIC HEAT
      DIMENSION TF(80),FH(80),TC(46),TR(46)
      COMMON TF,FH,TC,TR
C READ IN THE TEMPERATURE AND FREON TABLE
      CALL HREAD(TF,FH)
C READ IN THE RESISTANCE AND TEMPERATURE TABLE
      CALL TREAD (TC,TR)
      SM=
      DO 11 I=1,80
      J=I-1
      TF(I)=J
   11 CONTINUE
      DO 12 I=1,46
      AB=I
      TC(I)=AB-21.0
   12 CONTINUE
C READ IN THE RESULTS OF THE EXPERMENT
   31 READ 14, POUND,OZ
   14 FORMAT(2F10.3)
      READ 13, RIS,FIR,FFR
   13 FORMAT (3F10.1)
      FM=POUND+OZ/16.0
C CONVERSE RESISTANCE TO TEMPERATURE
      CALL RTCONV(RIS,CIS,TR,TC)
      CALL RTCONV(FIR,CFI,TR,TC)
      CALL RTCONV(FFR,CFF,TR,TC)
C  CONVERSE CENTIGRADE TO FAHRENHEIT
      FIS=1.8*CIS+32.0
      FFI=1.8*CFI+32.0
      FFF=1.8*CFF+32.0
C INTERPOLATE THE ENTHALPY OF FREON
      CALL HFREON(FFI,HI,TF,FH)
      CALL HFREON(FFF,HF,TF,FH)
C CALCULATE THE SPECIFIC HEAT
      TM=(FFF+FIS)/2.0
      IF (FFF-FIS) 71,71,72
   71 D=FIS-FFF
      QC=0.555*D
      A =FM*(HF-HI)-QC
      B=SM*D
      CPS=A/B
      E=FFF-FFI
      DH=HF-HI
      REA=0.05/(16.0*SM)+0.08/(16.0*FM)+0.008/D+0.008/E
      AT=(FM*DH)/(SM*D)
      ABA=REA*AT
      REB= 0.016/0.555+0.05/(16.0*SM)
      BT=0.555/SM
      ABB=REB*BT
      GO TO 73
   72 D=FFF-FIS
      B=SM*D
      QC=0.555*D
      A=FM*(HI-HF)-QC
      CPS=A/B
      E=FFI-FFF
      DH=HI-HF
      REA=0.05/(16.0*SM)+0.08/(16.0*FM)+0.008/D+0.008/E
      AT=(FM*DH)/(SM*D)
      ABA=REA*AT
      REB= 0.016/0.555+0.05/(16.0*SM)
```

```
      BT=0.555/SM
      ABB=REB*BT
C CALCULATE THE RELATIVE ERROR
   73 CONTINUE
      ACPS=(ABA+ABB)/(AT-BT)
      ERROR=ACPS
C CALCULATE THE LIMIT
      CPSU=CPS*(1.0+ERROR)
      CPSL=CPS*(1.0-ERROR)
C OUTPUT
      PRINT 21, RIS,FIR,FFR
   21 FORMAT(1H , 3F15.5)
      PRINT 22, CIS,CFI,CFF
   22 FORMAT(1H ,4HCIS=,F10.5,10X,4HCFI=,F10.5,10X,4HCFF=,F10.5)
      PRINT 23, FIS,FFI,FFF
   23 FORMAT(1H ,4HFIS=,F10.5,10X,4HFFI=,F10.5,10X,4HFFF=,F10.5)
      PRINT 24, POUND,OZ,FM
   24 FORMAT (1H ,6HPOUND=,F8.5,10X,3HOZ=,F11.5,10X,3HFM=,F11.5)
      PRINT 25, HF,HI
   25 FORMAT(1H ,3HHF=,F11.5,10X,3HHI=,F11.5)
      PRINT 26, TM,CPS
   26 FORMAT(1H ,3HTM=,F11.5,10X,4HCPS=,F10.5)
      PRINT 27, ERROR,CPSL,CPSU
   27 FORMAT(1H ,6HERROR=,F8.5,10X, 5HCPSL=,F9.5,10X,5HCPSU=,F9.5)
      PRINT 28, REA,AT,ABA,REB,BT,ABB,ACPS
   28 FORMAT(1H ,7F12.5)
      IF (POUND) 31,1000,31
 1000 STOP
      END
      SUBROUTINE  HREAD (TF,FH)
      DIMENSION TF(80),FH(80)
      READ 1, (FH(I),I=1,80)
      PRINT 1, (FH(I),I=1,80)
    1 FORMAT (10F8.2)
      RETURN
      END
C SUBROUTINE TO READ THE RESISTANCE TABLE
      SUBROUTINE TREAD(TC,TR)
      DIMENSION TC(46),TR(46)
      READ 2, (TR(I),I=1,46)
      PRINT  2, (TR(I),I=1,46)
    2 FORMAT (10F8.1)
      RETURN
      END
C SUBROUTINE TO INTERPOLATE THE TEMPERATURE FROM THE TABLE
      SUBROUTINE RTCONV(RR,T,TR,TC)
      DIMENSION TC(46),TR(46)
      DO 41 I=1,46
      AB=I
      TC(I)=AB-21.0
   41 CONTINUE
      C=23510.0
      R=RR-C
      IF (R-TR(1)) 100,90,110
   90 T=TC(1)
      GO TO 110
  100 L=1
  180 J=L+1
      IF (R-TR(J)) 170,160,150
  160 T=TC(J)
      GO TO 110
```

```
 150  T=TC(L)+(TC(J)-TC(L))/(TR(L)-TR(J))*(TR(L)-R)
      GO TO 110
 170  L=L+1
      GO TO 180
 110  CONTINUE
      PRINT 10, RR,R,T
  10  FORMAT(3F20.5)
      RETURN
      END
      SUBROUTINE HFREON(FT,H,TF,FH)
      DIMENSION TF(80),FH(80)
      IF (FT-TF(1)) 28,29,20
  29  H=FH(1)
      GO TO 28
  20  L=1
  21  J=L+1
      IF (FT-TF(J)) 24,25,26
  24  H=FH(L)+(FH(J)-FH(L))/(TF(J)-TF(L))*(FT-TF(L))
      GO TO 28
  25  H=FH(J)
      GO TO 28
  26  L=L+1
      GO TO 21
  28  CONTINUE
      PRINT 5, FT,H
   5  FORMAT (2F20.5)
      RETURN
      END
```

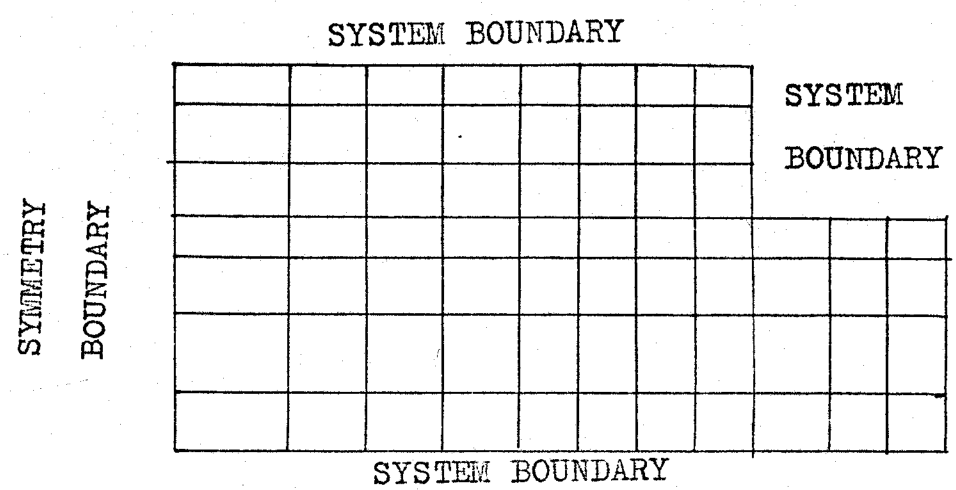


Fig. 1-1 Finite Element Subdivision of System

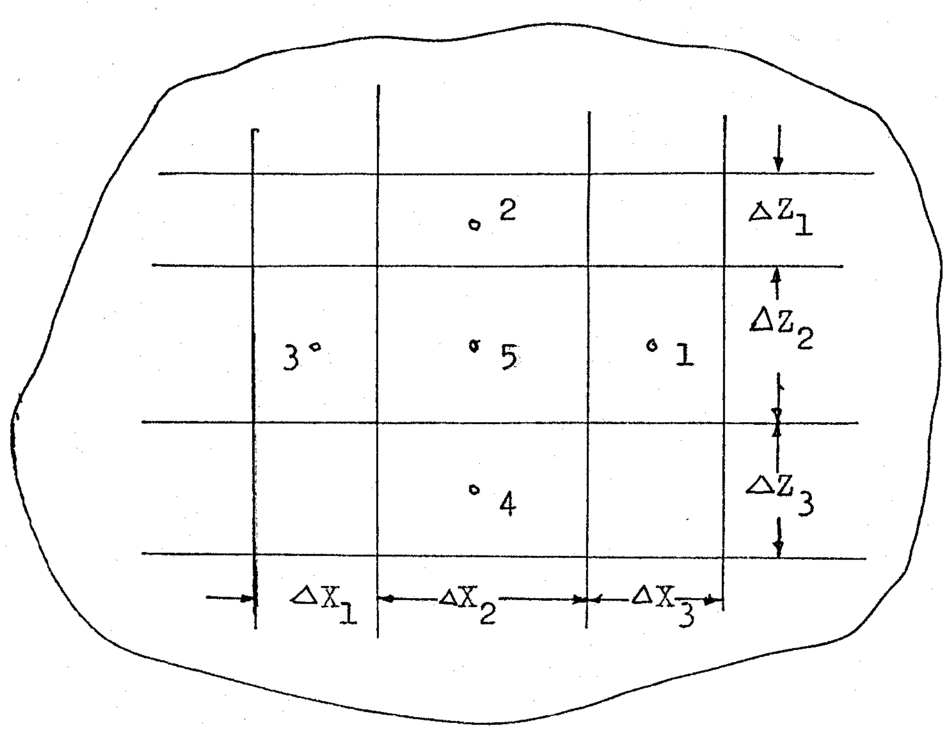


Fig. 1-2 Detail Elements about one Node

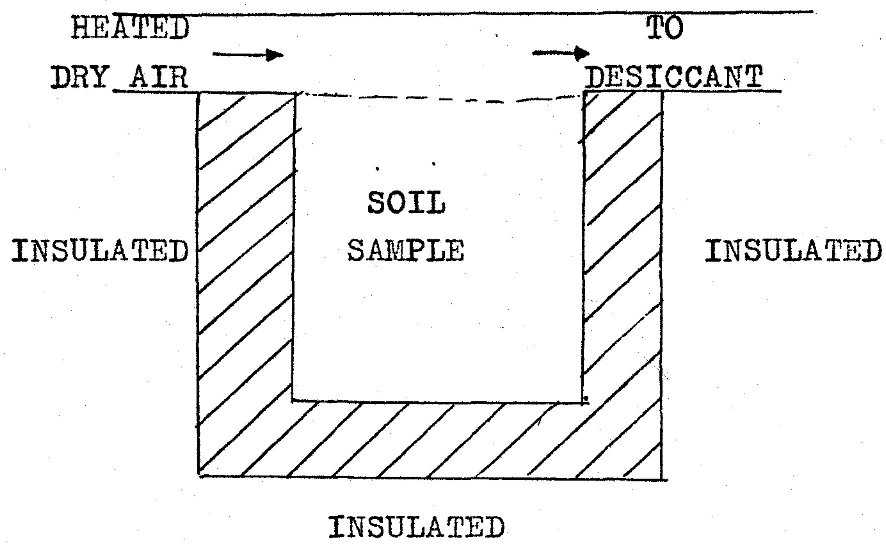


Fig. 2-1 Isothermal Test Configuration

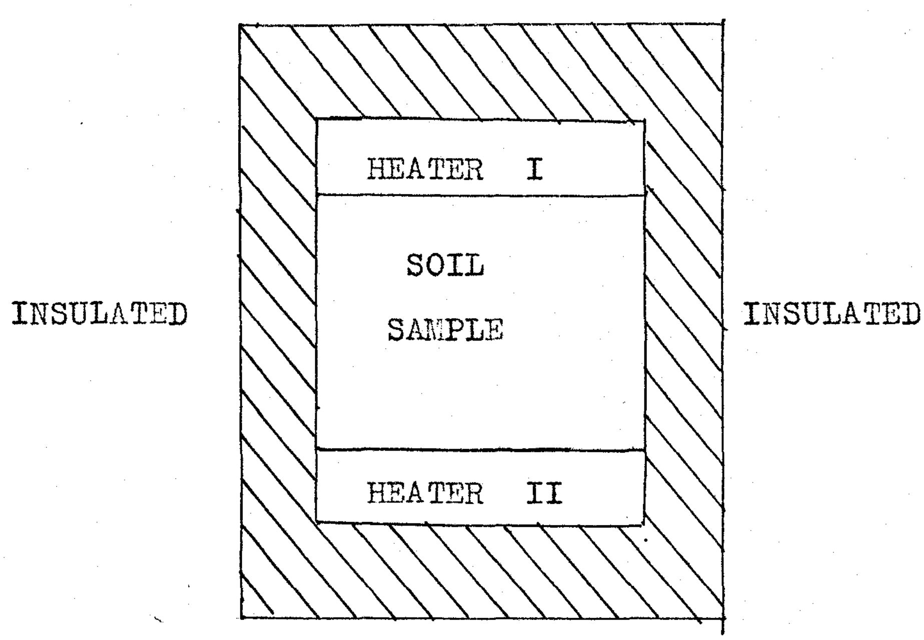


Fig. 2-2 Non-flow Test Configuration

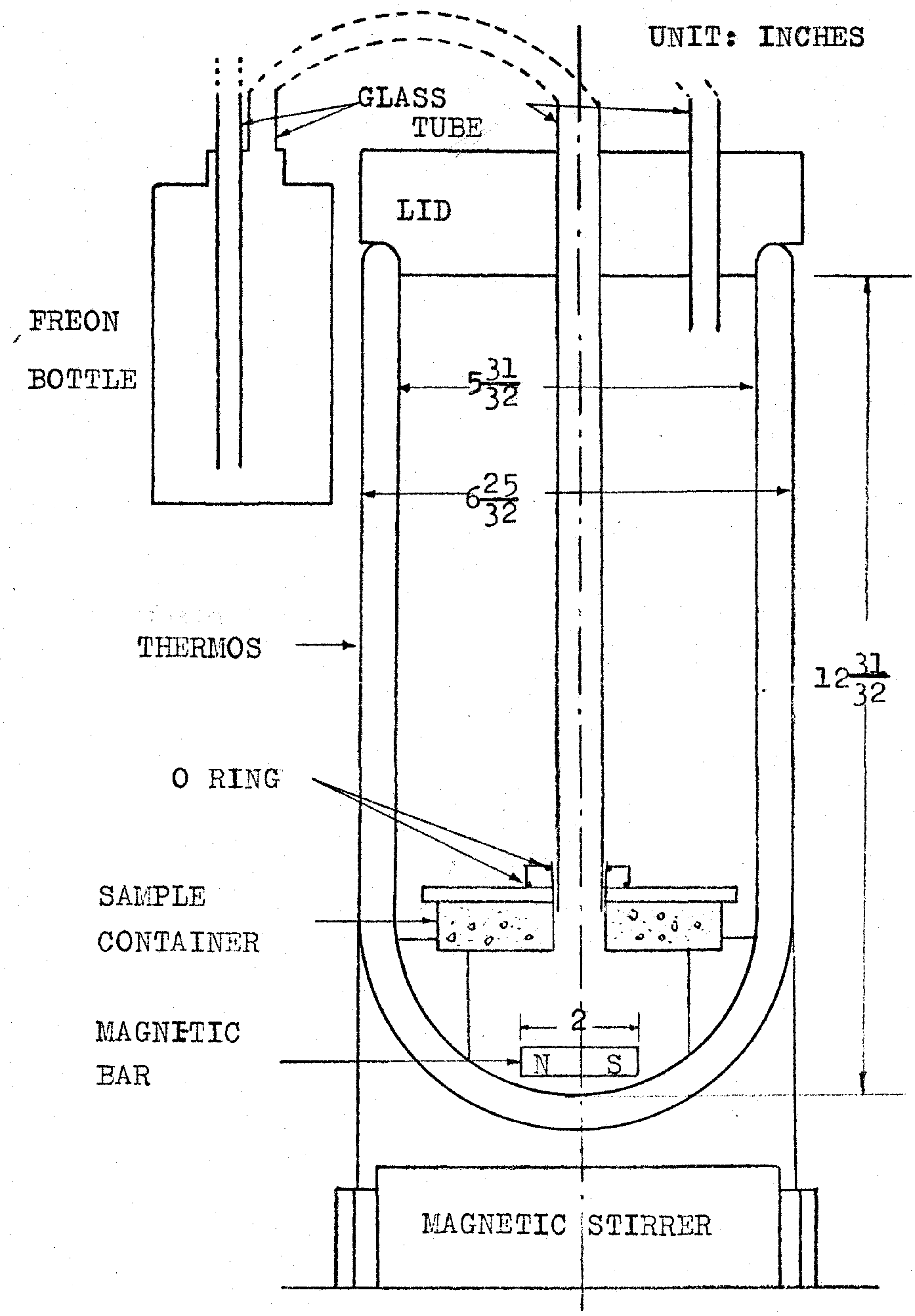


Fig. 3-1 Calorimeter Assembly

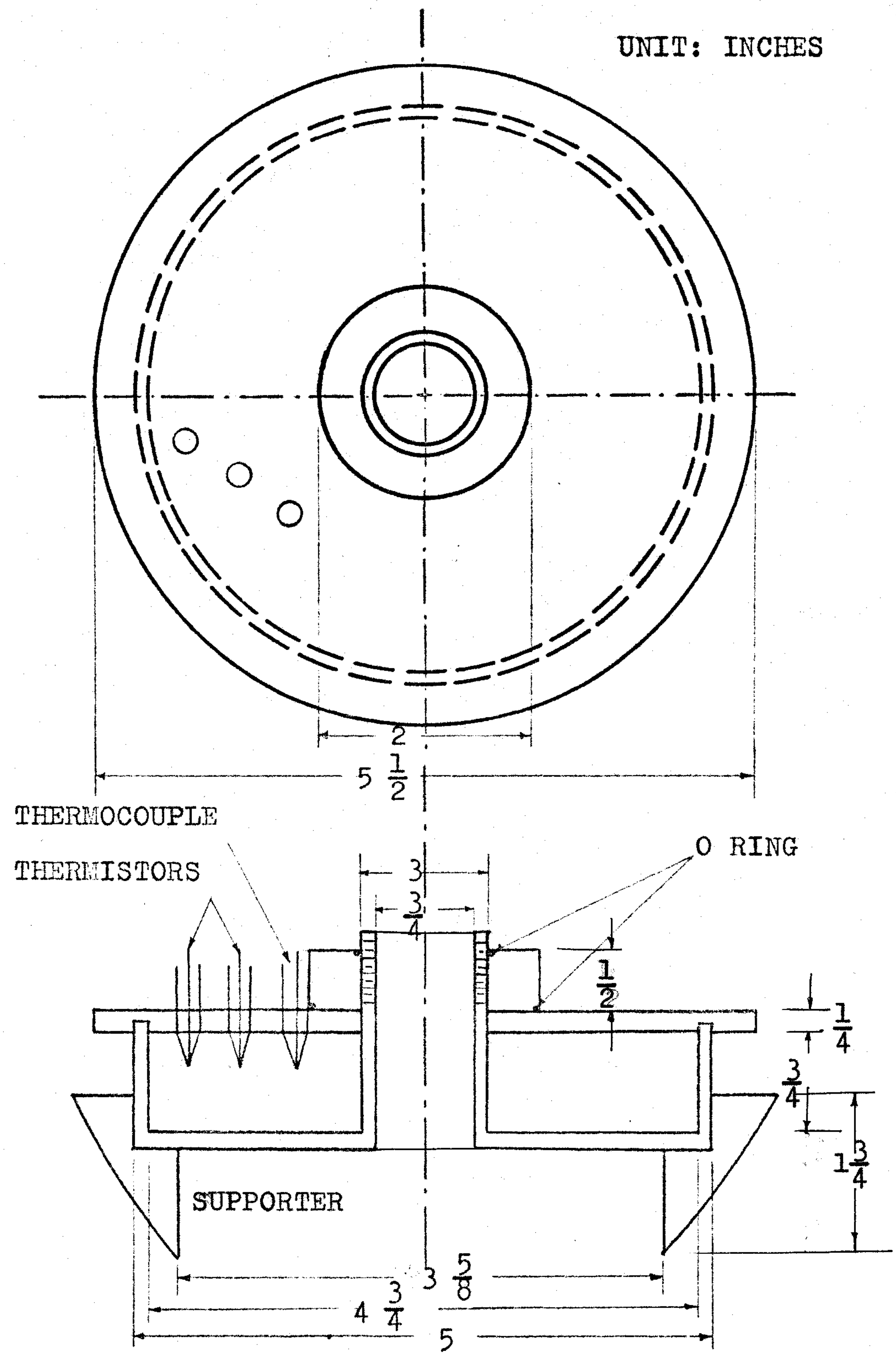


Fig. 3-2 Sample Container

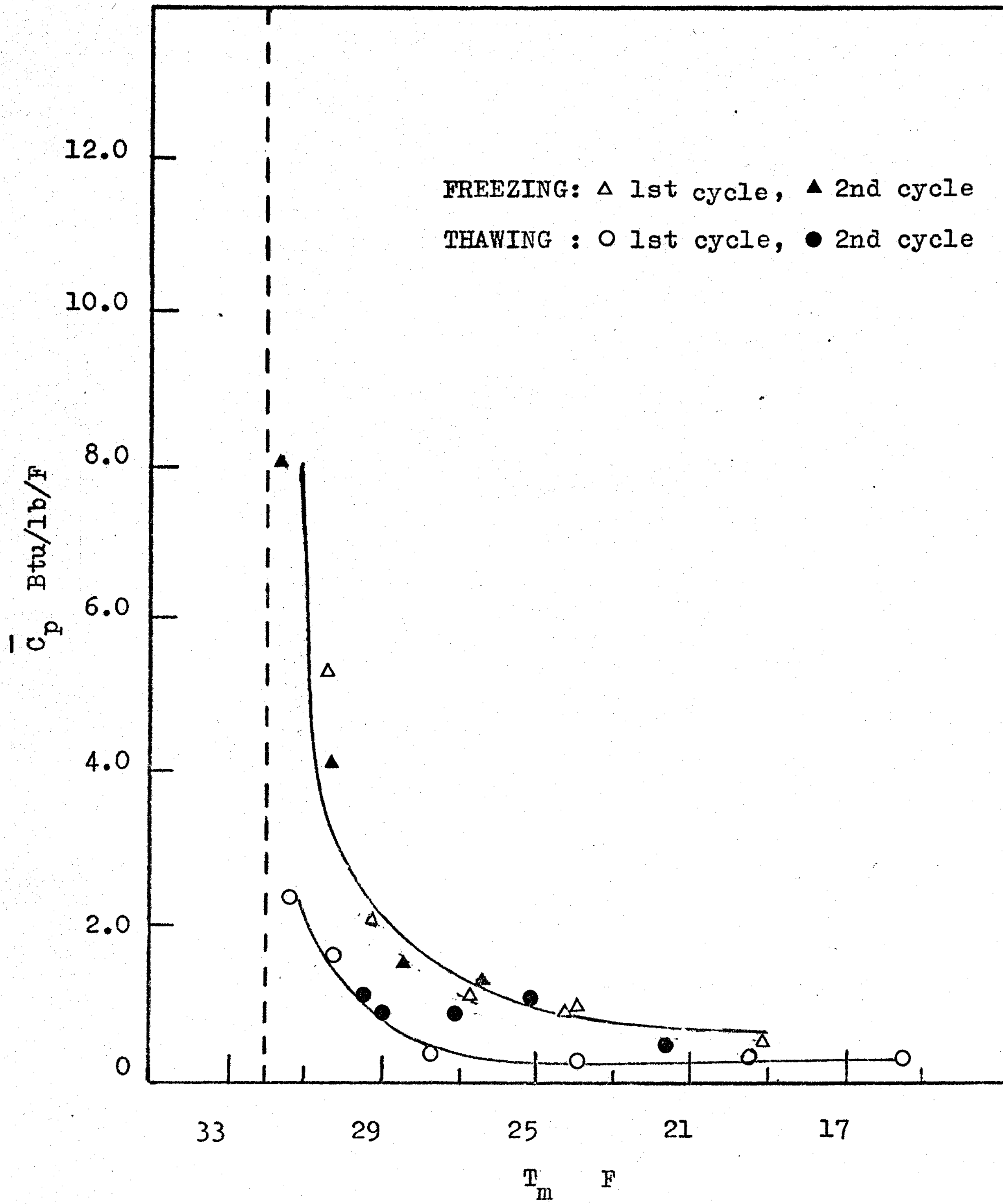


Fig. 3-3 Effective Specific Heat of Soil (6 ounces) with Water (0.76 ounces)

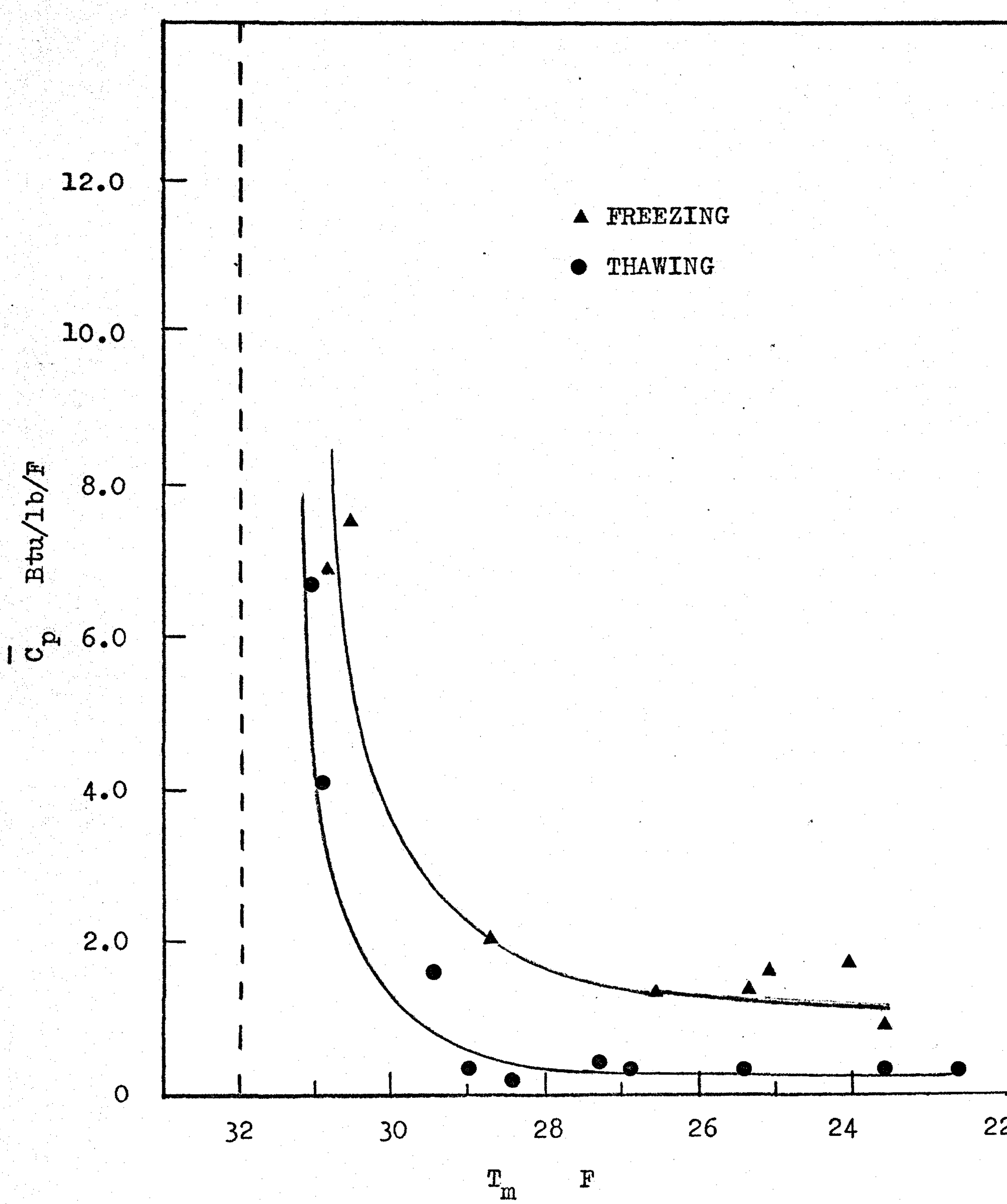


Fig. 3-4 Effective Specific Heat of Soil (5 ounces) with Water (1.26 ounces)

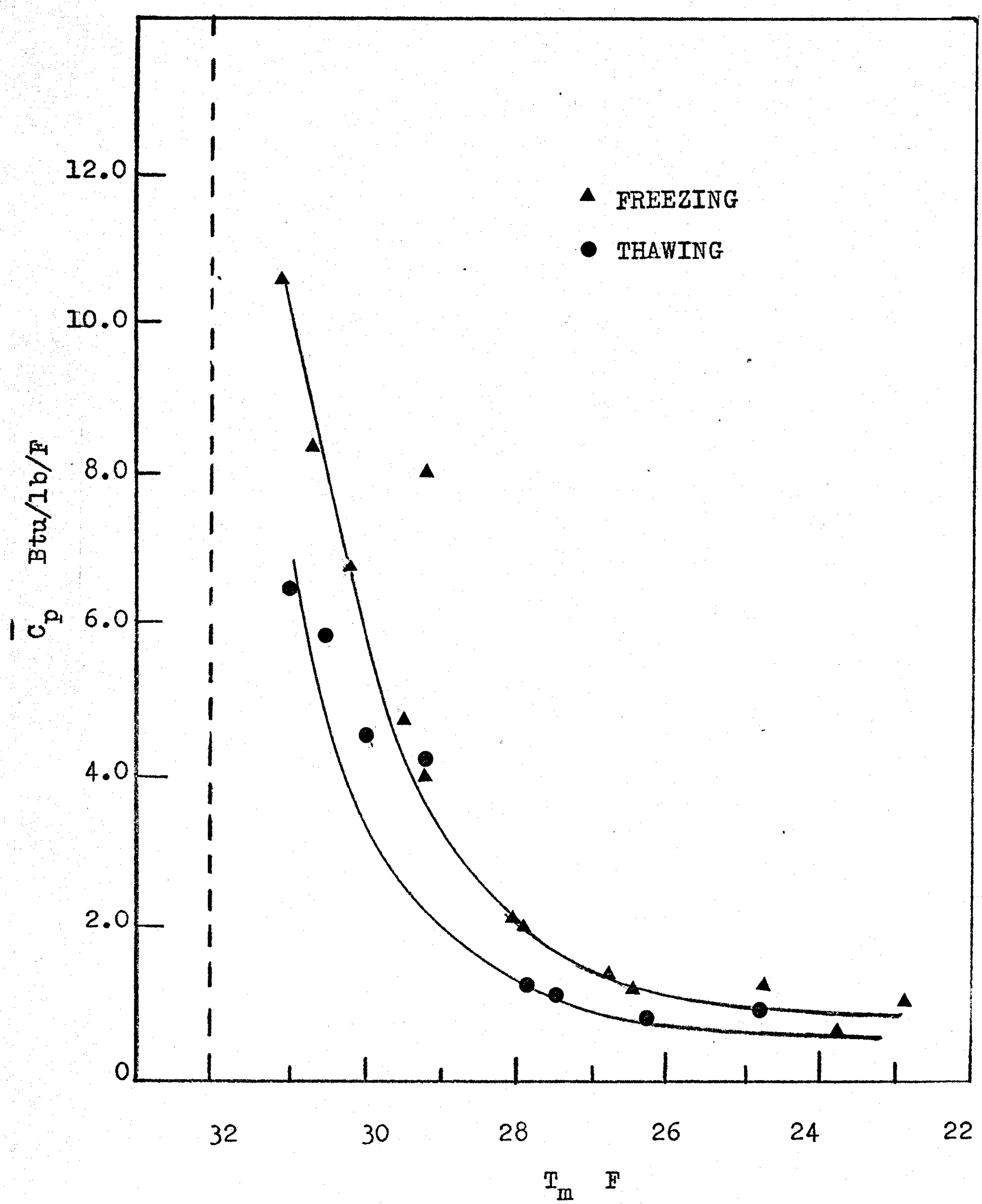


Fig. 3-5 Effective Specific Heat of Soil (5 ounces) with Water (1.83 ounces)

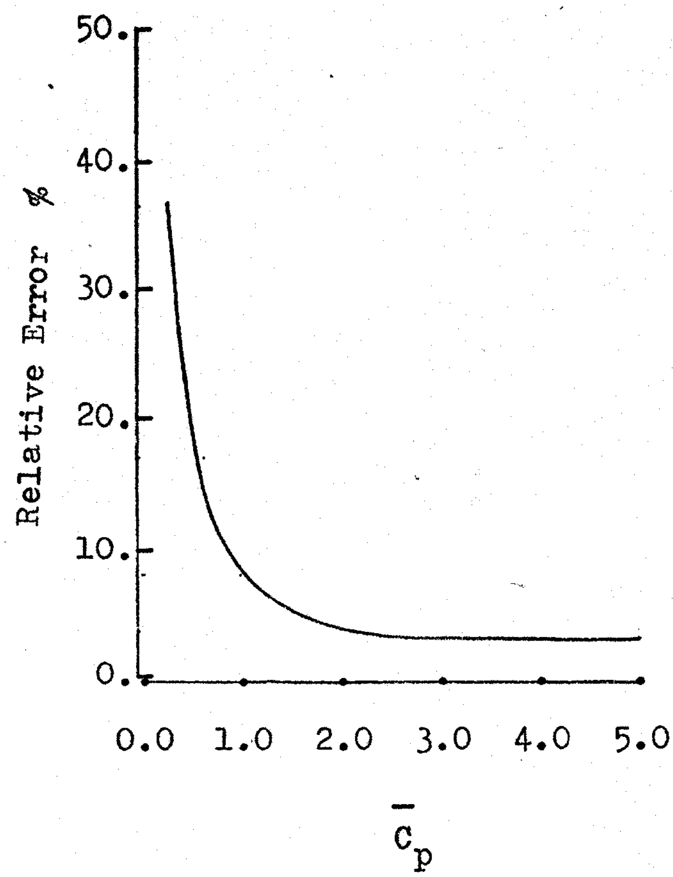
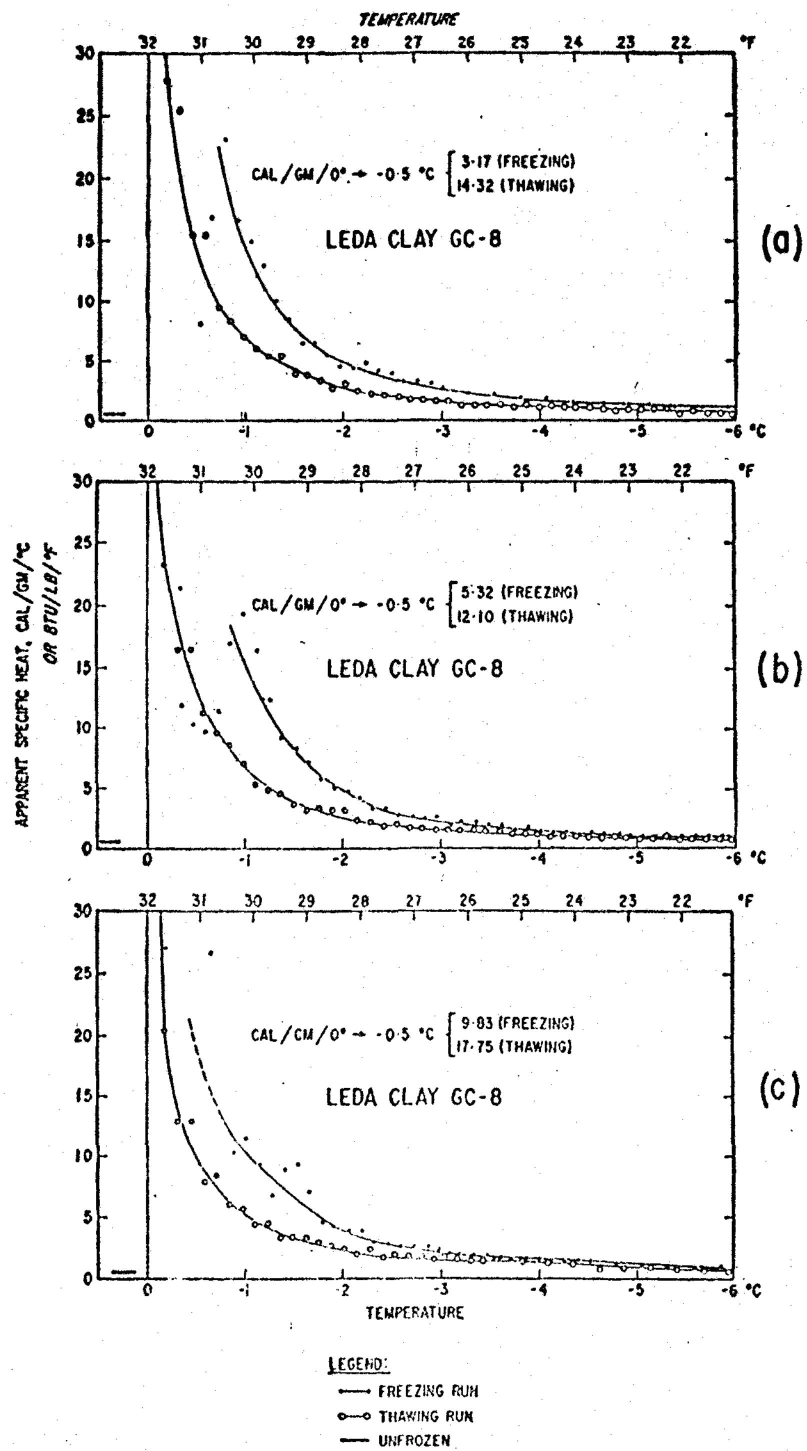


Fig. 3-6 The Relative Error of $\bar{C}_p$

APPARENT SPECIFIC HEATS OF FROZEN SOILS



. Specific heats and apparent specific heats of Leda clay GC-8. (a) and (b) show the results of identical tests on identical samples, and illustrate the reproducibility of the calorimetric observations. (b) and (c) illustrate the different results obtained on first freezing (b) and second freezing (c) of a sample

Fig. 3-7 P.J. WILLIAMS' RESULTS

VITA AUCTORIS

1942 Born in Taiwan, on October 24.

1961 Completed high school education at Miaoli High School, Miaoli, Taiwan.

1965 Received the Degree of Bachelor of Science in Mechanical Engineering from the Cheng-Kung University, Tainan, Taiwan. After graduation, enlisted in the service in the Jet Engine Maintenance Department of Hsin-Chu Airport of China Air Force for one year.

1968 Received the Degree of Master of Applied Science from the Research Institute of Mechanical Engineering of Cheng-Kung University. Currently a candidate for the Degree of Master of Applied Science in Mechanical Engineering at the University of Windsor.