Analytical and experimental studies on the rigidities of orthogonally and non-orthogonally rib-stiffened concrete slabs.

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ANALYTICAL AND EXPERIMENTAL STUDIES
ON THE RIGIDITIES OF ORTHOGONALLY AND
NON-ORTHOGONALLY RIB-STIFFENED CONCRETE SLABS

by

Satish Kumar Bali

A Thesis
submitted to the Faculty of Graduate Studies
through the Department of
Civil Engineering in Partial Fulfillment
of the requirements for the degree
of Master of Applied Science at
The University of Windsor

Windsor, Ontario, Canada
1980
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To my parents
In this study, analytical expressions for the anisotropic rigidities of reinforced concrete orthogonally rib stiffened slab structures are presented. These expressions are valid for both the precracking and post-cracking stages of concrete and are, therefore, applicable to reinforced as well as prestressed concrete structures. The existing experimental method for determining the rigidity constants of orthogonally stiffened plates is modified to properly account for the coupling rigidities of the slab structure. The method utilizes the fact that pure bending and twisting moments can be expressed in terms of the curvature and twist of the surface, respectively. Flexural and twisting tests on several reinforced concrete orthogonally rib stiffened structures were conducted. The experimental results verify and substantiate the analytical expressions for both the precracking and post-cracking rigidities of the structure and are also compared with the existing theories.

Theoretical expressions are also given to calculate the elastic rigidity constants of non-orthogonally rib stiffened reinforced concrete slab structures. These formulae are also valid for the elastic flexural and torsional rigidities of orthog-
onally rib stiffened slabs. An experimental procedure has also been devised whereby the elastic rigidities of non-orthogonally stiffened slabs can be determined by applying pure bending and twisting moments to the test specimens. The experimental procedure and the necessary precautions that must be taken to help insure accurate results are discussed. The use of realistic estimates for the anisotropic rigidity constants of rib stiffened reinforced concrete slab structures will lead to better design as well as economy.
ACKNOWLEDGEMENTS

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Many thanks are due to Mr. G. Michalezuk and Mr. P. Feimer for their assistance in the experimental work.

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**LIST OF ABBREVIATIONS**

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_s$ ($A'_s$)</td>
<td>Area of tension steel in longitudinal (transverse) rib</td>
</tr>
<tr>
<td>$A_{sv}$</td>
<td>Area of tension steel in skew rib</td>
</tr>
<tr>
<td>$b_x$ ($b_y$)</td>
<td>Width of longitudinal (transverse) rib</td>
</tr>
<tr>
<td>$b_v$</td>
<td>Width of the skew rib</td>
</tr>
<tr>
<td>$D$</td>
<td>Flexural rigidity of the flange plate with respect to its middle plane</td>
</tr>
<tr>
<td>$D_{x*}$ ($D_{y*}$)</td>
<td>Flexural rigidity of the orthogonally rib stiffened slab in the $x$- ($y$-) direction</td>
</tr>
<tr>
<td>$D_{xy}$ ($D_{yx}$)</td>
<td>Torsional rigidities of the orthogonally rib stiffened slab</td>
</tr>
<tr>
<td>$D_{11}$, $D_{12}$, $D_{16}$</td>
<td>Rigidity constants of the cross-section of the non-orthogonally rib stiffened slabs</td>
</tr>
<tr>
<td>$D_{22}$, $D_{26}$, $D_{66}$</td>
<td></td>
</tr>
<tr>
<td>$d_x$ ($d_y$)</td>
<td>Depth of the longitudinal (transverse) rib</td>
</tr>
<tr>
<td>$d'$ ($d''$)</td>
<td>Concrete cover to the centre of longitudinal (transverse) reinforcement</td>
</tr>
<tr>
<td>$d_v$</td>
<td>Depth of the skew rib</td>
</tr>
<tr>
<td>$d_v'$</td>
<td>Concrete cover to the centre of reinforcement in the skew rib</td>
</tr>
</tbody>
</table>
E  Modulus of elasticity of concrete

e_x, e_y, e_v  Depth of neutral plane of the uncracked section from extreme compression fibre for bending in the x-, y- and v- direction respectively

f'_c  28 day compressive strength of concrete (psi)

G  Shear modulus of concrete

h  Thickness of flange plate

J  Torsional constant

K  Torsional parameter

k_{dx}, k_{dy}, k_{dv}  Depth of neutral plane of the cracked section from extreme compression fibre for bending in the x-, y- and v- direction respectively

M_x, M_y, M_{xy}  Bending and torsional moments associated with the x- and y- axes

n  Modular ratio

q(x,y)  Intensity of lateral load on the stiffened slab.

S_x, S_y  Spacing of the longitudinal (transverse) ribs

S_v  Spacing of the skew ribs

x
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$w$</td>
<td>Deflection in the $z$-direction</td>
</tr>
<tr>
<td>$w_{xx}$, $w_{yy}$, $w_{xy}$</td>
<td>Curvatures and twist of the surface associated with the $x$- and $y$-axes</td>
</tr>
<tr>
<td>$w_{uu}$, $w_{vv}$, $w_{uv}$</td>
<td>Curvatures and twist of the surface associated with the $u$- and $v$-axes</td>
</tr>
<tr>
<td>$w_{rr}$, $w_{tt}$</td>
<td>Curvatures associated with $r$- and $t$- axes</td>
</tr>
<tr>
<td>$x$, $y$, $z$</td>
<td>Rectangular co-ordinates system</td>
</tr>
<tr>
<td>$u$, $v$, $z$</td>
<td>Oblique co-ordinate system</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of inclination of the skew ribs ($v$-axis) to the $y$-axis</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Poisson's ratio of concrete</td>
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</table>
CHAPTER I

INTRODUCTION

1.1 General

Thin plates stiffened by a system of orthogonal or non-orthogonal ribs have found wide application for aircraft, bridge, building and ship bottom structures as well as in many other branches of contemporary structural engineering. These stiffened elements, representing a relatively small part of the total weight of the structure, substantially influence its strength, stiffness and stability leading to economy and other advantages. Studies of orthogonally and non-orthogonally stiffened slabs have been of particular interest and practical importance in bridge structures constructed for reasons of economy and structural efficiency. It becomes essential to use realistic estimates of the rigidity constants of the structure in order to predict accurately the behaviour of such construction to an applied load.

1.2 Object

The object of this investigation is to develop theoretical expressions for determining flexural and torsional rigidities of orthogonally and non-orthogonally rib stiffened reinforced concrete slab structures. Since concrete is a non-homogeneous material, it is
necessary to verify the expressions developed by conducting experimental studies. Thus, the overall objectives of this study are:

a) To develop simple and rational expressions to predict flexural and torsional rigidities of anisotropic concrete slabs.

b) To develop mathematical expressions to calculate the anisotropic rigidities of reinforced concrete rib-stiffened slabs by means of laboratory tests.

c) To substantiate and verify the analytical expressions by experimental results from the tests on orthogonally rib stiffened slabs only, since the analytical expressions for non-orthogonally rib stiffened slabs are also applicable for orthogonally rib-stiffened slabs.

1.3 Scope

Simple expressions are proposed for the precracking and postcracking rigidities of orthogonally rib-stiffened reinforced concrete slabs. The analysis is verified by experimental results from bending and twisting tests on reinforced concrete waffle-slab elements. Analytical expressions and experimental procedure are also developed for the determination of the rigidity constants of non-orthogonally rib-stiffened reinforced concrete slab structures.
Historically, the development of stiffened structural elements is one of slow growth. In the early stages of development, man probably learned of the existence of such forms from nature. Sea shells, trees, leaves, vegetables - all of these are, in fact, stiffened structures. The wide use of stiffened structural form in engineering began in the nineteenth century, mainly with the application of steel plates for hulls of ships and with the development of steel bridges and aircraft structures.

The study of elastic constants of plates by bending and twisting tests began in 1927 when Bergstrasser (4) used a procedure suggested by Nadai for experimentally applying pure bending and twisting moments on a plate of constant thickness. For applying the bending moments, he used a rectangular plate supporting it at three points and loading it at three other points. A square plate was used for applying a twisting moment that was supported on two diagonally opposite corners and loaded on the other two corners. Bergstrasser assumed that the shape of the deflected surface for an isotropic plate due to a pure bending moment \( M_x \) could be expressed as
\[ w = \frac{6M_x}{Eh^3}(x^2 - \mu y^2) \quad (1.2.1) \]

where, \( x \) and \( y \) are the rectangular co-ordinates.

Thielemann (34) and Hearmon and Adams (14) used essentially the same procedure. The principal difference between these various investigators is in the manner in which they measured the displacements caused by the applied bending and twisting moments or in the equations used to determine the elastic constants. Bergstrasser (4) and Hearmon and Adams (14) measured their displacements by placing a measuring device on the deflected plate and thus calculated relative displacements. Thielemann (34) and Hearmon and Adams (14) assumed that the deflected shape for a "specially orthotropic" plate (principal axes parallel to the sides of the plate) had the form

\[ w = \frac{6M_x}{h^3}(S_{11} x^2 + S_{12} y^2) \quad (1.2.2) \]

where \( S_{11} \) and \( S_{12} \) are the elastic constants which can be expressed as \( 1/E_x \) and \( -\mu_{xy}/E_x \), respectively. For the case of a homogeneous isotropic plate, \( E_x = E \) and \( \mu_{xy} = \mu \). Both equations (1.2.1) and (1.2.2) described a hyperbolic-paraboloid surface.

Witt, Hoppmann and Buxbaum (38) have given the theoretical basis and an experimental method for determining the anisotropic elastic constants of a material by measuring deflections of the thin
plate of the material subjected to couples on its boundary. They measured the displacements relative to a fixed plane and described the shape of the deflected surface of the plate by

\[ w = \frac{6M_x}{h^3} (S_{ii} x^2 + S_{ij} y^2) + Ax + By + C \quad (1.2.3) \]

where \( S_{ii} \) and \( S_{ij} \) are the elastic constants. The constants \( A, B \) and \( C \) were determined from the boundary conditions in which \( w = 0 \) at the three support points.

In 1956, Huffington (18) investigated theoretically and experimentally the method for the determination of rigidities for metallic rib-reinforced deck structures. It was applied to the case of equally spaced stiffeners, of rectangular cross-section, and symmetrically placed with respect to its middle plane.

Beckett et al (3) presented a curvature method for the experimental determination of elastic constants of orthogonally stiffened plates. The method utilized the fact that pure bending and twisting moments can be expressed in terms of the curvature and twist of a surface, respectively.

In 1968, Jackson (20) proposed a method to estimate the torsional rigidities of concrete bridge decks using membrane analogy and accounting for the junction effect. The effect of the continuity of the slab on the flange plate was not accounted for.
et al (5) investigated the in-plane and flexural stiffnesses of isotropically and non-isotropically reinforced concrete plates. Results indicated that the stiffness of such plates was related quantitatively to the relative orientation of the reinforcement with respect to the applied forces, the combinations of the applied forces and the amounts of the reinforcement in the two orthogonal directions. Kennedy and Gupta (22) used the elastic constants of the equivalent structure which closely resembled the twisting and bending behaviour of an orthogonally stiffened plate structure to analyse the original structure by means of the orthotropic plate theory.

Gusens et al (19) presented an elastic analysis for plates reinforced with an orthogonal system of rectangular ribs which enables the determination of rigidities of the structure in flexure and torsion. The analysis did not account for the stiffening effect of the orthogonal ribs on the torsional rigidity of the flange plate; also the method is not applicable for reinforced concrete bridge decks where some cracking is expected or for structures with relatively thick flange plates. Little information is available as to how these rigidities might be assessed for the cracked sections of a concrete structure. Desayi and Kulkarni (10) suggested the empirical reduction
factors for the flexural rigidities of reinforced concrete slabs up to the yield load.

In 1972, the Concrete Reinforcing Steel Institute (8) recommended that the average gross moment of inertia be used for two way joist and waffle slabs. Lampert (26) derived theoretical expressions for the post-cracking stiffness of rectangular reinforced concrete beams in torsion and bending using a space truss model. Jofriet and McNeice (19) used empirical bilinear moment curvature relationships to incorporate the influence of cracking in the finite element analysis of slab structures. Recently, Clark and White (6) carried out tests to determine the torsional stiffness of flexurally cracked slab elements.
3.1 General Concept

In this chapter, the analytical method to obtain flexural and torsional rigidities of orthogonally and non-orthogonally stiffened plates is developed. If a homogeneous material has three mutually perpendicular planes of symmetry with respect to its elastic properties, it is called orthotropic, i.e. materials which are orthogonally anisotropic. For instance, two-way reinforced concrete slabs are intrinsically anisotropic. Other materials with natural anisotropy are plywood, fiber-reinforced plastics and wood. In certain cases, structural anisotropy is introduced by means of corrugations or ribs such as decks of steel bridges, corrugated plates, composite beam grid frameworks, concrete slabs reinforced with closely spaced ribs, etc. These structures can be analyzed using orthotropic plate theory which assumes that the orthotropy of the structure may be replaced by the orthotropy of the constituent material. Although the actual structural behaviour of a stiffened plate cannot be entirely replaced by that of an equivalent orthotropic plate, previous theoretical and experimental investigations indicate good agreement (12).
3.2 Assumptions

In dealing herein with orthotropic concrete structures, it is assumed that orthotropy is a result of geometry and not of material. Except for the assumptions made with respect to the rib-stiffened construction, the assumptions for orthotropic plates are based on the same assumptions used in the analysis of isotropic plates, and they are outlined below.

a) The material of the plate is considered to be continuous and homogeneous, by transforming the steel area into an equivalent area of concrete.

b) The plate thickness is uniform and small compared with other dimensions of the plate. Thus, the shearing stresses and stresses normal to the plane of symmetry are small and can be neglected.

c) The deflections of the loaded plate are small in comparison to its thickness, so that membrane stresses in the plate can be neglected.

d) The material of the plate is elastic, i.e. the stress-strain relationship is governed by Hooke's Law.

e) Straight lines normal to the middle surface of the plate remain straight and normal to the middle surface of the plate after loading.

f) The ratio of stiffener spacing to plate boundary dimensions is small enough for the real structure
to be replaced by an idealized one with continuous properties.

g) Flexural and torsional rigidities do not depend on the boundary conditions of the plate or on the distribution of the vertical load.

h) The neutral plane in each of the two orthogonal directions coincides with the centre of gravity of the total section in the corresponding direction.

i) The area of the flange plate is magnified by the factor \(1/(1-\mu^2)\) to allow for the effect of Poisson's ratio, \(\mu\).

### 3.3 Governing Differential Equation

Adopting the assumptions explained in 3.2, the following expressions are obtained for the bending and torsional moments.

\[
\begin{align*}
M_x &= - (D_{xx} W_{,xx} + D_{yy} W_{,yy}) \\
M_y &= - (D_{yy} W_{,yy} + D_{xx} W_{,xx}) \\
M_{xy} &= D_{xy} W_{,xy}
\end{align*}
\]  

(3.3.1)

where,

- \(D_{xx}, D_{yy}\) = flexural rigidities of the plate per unit width in \(x\) and \(y\) directions, respectively;
- \(D_{xy}, D_{yx}\) = coupling rigidities of the plate per unit width due to Poisson's ratio;
- \(D_{xx}, D_{yy}\) = torsional rigidities of the plate.
The equation of equilibrium of the moments, after neglecting the shearing forces acting on the element, is given by

\[ M_{xx} + 2M_{xy}xy + M_{yy} = -q(x,y) \]  \hspace{1cm} (3.3.2)

By substituting expressions (3.3.1) in the moment Eq. (3.3.2), a fourth order differential equation governing the deflection of the orthotropic plate is obtained in rectangular co-ordinates:

\[ DxW_{xxxx} + 2HW_{xxyy} + DyW_{yyyy} = q(x,y) \]  \hspace{1cm} (3.3.3)

where,

\[ 2H = D_1 + D_2 + D_{xy} + D_{yx}, \] which is the effective torsional rigidity of the orthotropic plate.

3.4 **Rigidities of orthogonally rib-stiffened Uncracked Sections**

3.4.1 **Flexural Rigidities**

Figure 3.1 shows a typical section of a rib-stiffened slab. Based on the assumptions made earlier, the orthotropic flexural rigidities, \( D_x \) and \( D_y \), as well as the coupling rigidities \( D_1 \) and \( D_2 \) due to Poisson's effect, of an uncracked concrete section (Figure 3.1) can be expressed as

\[
\begin{align*}
D_x &= D + \left[ \frac{Eh(e_x - h/2)^2}{(1 - \mu^2)} \right] + \frac{EI_x}{S_x} \\
D_y &= D + \left[ \frac{Eh(e_y - h/2)^2}{(1 - \mu^2)} \right] + \frac{EI}{S_y} \\
D_1 &= v D_x' \\
D_2 &= v D_y' 
\end{align*}
\]  \hspace{1cm} (3.4.1.1)
D = flexural rigidity of the flange plate with respect to its middle plane, $Eh/12 \left(1 - \mu^2\right)$,

$E = \text{modulus of elasticity of concrete} = 57000 \left(f'_c\right)^{\frac{3}{2}}$

$f'_c = 28$-day concrete cylinder strength in psi

$h = \text{thickness of the flange plate}$

$\mu = \text{Poisson's ratio of concrete} = \left(f'_c\right)^{\frac{1}{3}}/350 \quad (23)$

$S_x(S_y) = \text{spacing of the longitudinal (transverse) ribs}$

$e_x(e_y) = \text{depth of neutral plane from the top fibre}$

for bending in the $x - (y -)$ direction, i.e.,

\[
e_x = \{b_x d_x (h + d_x/2) + (n - 1)A_s(h + d_x - d') + S_x h^2/2(1 - \mu^2)\}/\{b_x d_x + (n - 1)A_s + S_x h/(1 - \mu^2)\};
\]

\[
e_y = \{b_y d_y (h + d_y/2) + (n - 1)A'_s(h + d'_y - d'') + S_y h^2/2(1 - \nu^2)\}/\{b_y d_y + (n - 1)A'_s + S_y h/(1 - \mu^2)\}; \quad (3.4.1.2)
\]

$I'_x (I'_y) = \text{moment of inertia of longitudinal (transverse) rib with respect to the assumed neutral axis,}$

i.e.,

\[
I'_x = b_x d_x \{(h + d_x/2) - e_x\}^2 + (n - 1)A_s \{(h + d_x - d') - e_x\}^2 + b_x d_x^3/12 \quad (3.4.1.3)
\]

\[
I'_y = b_y d_y \{(h + d_y/2) - e_y\}^2 + (n - 1)A'_s \{(h + d'_y - d'') - e_y\}^2 + b_y d_y^3/12
\]
in which,

\[ n = \text{modular ratio} = \frac{E_s}{E_c} \]

\[ b_x (b_y) = \text{width of longitudinal (transverse) rib}; \]
\[ d_x (d_y) = \text{depth of the longitudinal (transverse) rib}; \]
\[ d' (d'') = \text{concrete cover to the centre of the longitudinal (transverse) reinforcements}; \]
\[ A_s (A_s') = \text{area of tension steel in the longitudinal (transverse) rib}; \]
\[ D_x (D_y) = \text{flexural rigidity of the flange plate with respect to the neutral plane of the gross cross section associated with bending in the x - (y-) direction}. \]

### 3.4.2 Torsional Rigidity

The twisting rigidity of an uncracked section of an orthogonally rib-stiffened concrete slab is estimated by using the membrane analogy method as proposed by Timoshenko and Goodier (35). The stiffening effect afforded by the rib stiffeners (beam stems) in the orthogonal direction to the one under consideration is taken into account. Referring to the tee-section shown in Figs. 3.2 (a) and 3.2 (b) and considering the geometry of the deflected membrane, the torsional constants for the rectangular sections 1, 2, and 3 are calculated and added to yield the torsional constant of the cross-section. Thus for a section normal to y-axis (Figs. 3.1 and 3.2(b)).
\[ J'_y = J_1 + J_2 + J_3 \]  

(3.4.2.1)

where, \( J_1, J_2 \) and \( J_3 \) are the contributions of areas \( 1, 2, \) and \( 3 \) respectively, defined as

\[
\begin{align*}
J_1 &= \frac{1}{2} K_1 S_y h^3 \\
J_2 &= K_1 d_y b_y^3 \quad \text{for } d_y > b_y \\
J_3 &= 4K_1 (n - 1) (A_s')^2
\end{align*}
\]

(3.4.2.2)

in which, \( K_1 \) is the constant for rectangular sections in torsion (35). The contribution \( J_3 \) of the reinforcing steel is calculated by transforming the area of steel into equivalent area of concrete. The contribution \( J_3 \), being relatively small, can be ignored in a practical design. For the calculation of \( J_1 \), a reduction factor of \( 1/2 \) is used which accounts for the continuity of the slab, this being distinct from the top flange of an isolated tee-section. Because the slabs considered herein, are stiffened by ribs in two orthogonal directions, the torsional contribution \( J_1 \) of the slab in one direction (see hatched area in Fig. 3.1) is augmented by the stiffening rib in the orthogonal direction in the following manner:

Considering the section normal to the x-axis (Figs. 3.1 and 3.2 (a)), the presence of the transverse rib \( W \) will increase the torsional contribution \( J_s \) of the
slab $S$ to a value denoted by $J_{(s+w)}$. Thus, the modified value of $J_1$ for the hatched area (normal to the $y$-axis) becomes,

$$(J_1)_{\text{modified}} = (J_1) \left( \frac{J_{(s+w)}}{J_s} \right)$$

(3.4.2.3)

Thus, Eq. (3.4.2.1) is modified to,

$$J_y = (J_1)_{\text{modified}} + J_2 + J_3$$

(3.4.2.4)

where,

$J_y$ is the torsional constant of the cross-section normal to the $y$-axis.

The torsional rigidity $D_{yx}$ is now given by,

$$D_{yx} = GJ_y/S_y$$

(3.4.2.5)

where, $G = \text{shear modulus} = E/(1+\mu)$

Similarly, the torsional rigidity, $D_{xy}$, of a section normal to the $x$-axis (Figs. 3.1 and 3.2(a)) can also be calculated from the equation,

$$D_{xy} = GJ_x/S_x$$

(3.4.2.6)

where,

$J_x$ is the torsional constant of the cross-section normal to the $x$-axis.
3.5 Rigidities of Orthogonally Rib-Stiffened Cracked Sections

3.5.1 Flexural Rigidities

After cracking of the concrete section, the structure continues to behave elastically, provided the stress in the steel is below the yield point and the compressive stress in the concrete does not exceed 0.5 $f_c'$. In addition to the assumptions made in section 3.2, it is assumed that the tension cracks have progressed to the neutral axis (assumed to be in the flange plate, which is generally the case). For the computation of the rigidities, the transformed section consisting of concrete [and $(n - 1)$ times area of the compression steel, if provided] in compression and $n$ times the area of the tension steel is used. Based on the above simplifying assumptions, the rigidities of the cracked section can be expressed as,

$$
\begin{align*}
D_x &= E \left[ I_{sx} + I_{cx} \left(1 - \mu^2\right) \right]/S_x \\
D_y &= E \left[ I_{sy} + I_{cy} \left(1 - \mu^2\right) \right]/S_y \\
D_1 &= \nu D_x \\
D_2 &= \nu D_y
\end{align*}
$$

(3.5.1.1)

where,

$I_{cx}$ ($I_{cy}$) = moment of inertia of the concrete [and $(n - 1)$ times area of compression steel, if any] in compression about the neutral axis for

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bending in the \( x^- (y^-) \) direction, respectively, i.e.,

if the neutral axis lies in the flange plate (which

is generally the case),

\[
\begin{align*}
I_{cx} &= S_x (kd_x)^3/3 \\
I_{cy} &= S_y (kd_y)^3/3
\end{align*}
\]

\( I_{sx} \) (or \( I_{sy} \)) = moment of inertia of the transformed steel

section about the neutral axis for bending

in the \( x^- (y^-) \) direction respectively, i.e.,

\[
\begin{align*}
I_{sx} &= nA_s \{ (h + d_x - d^*) - kd_x \}^2 \\
I_{sy} &= nA_s' \{ (h + d_y - d'') - kd_y \}^2
\end{align*}
\]

\( D_x' (D_y') \) = flexural rigidity of the flange plate with

respect to the neutral plane of the gross cracked sec-

tion associated with bending in the \( x^- (y^-) \) direction.

The location of the neutral axis \( kd_x \) or \( kd_y \)

is determined by equating the tension force to the

compression force on the section. Thus, by assuming

that the neutral axis lies in the flange plate, \( kd_x \)

is given by,

\[
\begin{align*}
nA_s \{ (h + d_x - d^*) - kd_x \} - S_x (kd_x)^2/2(1 - \mu^2) &= 0 \\
\text{and } kd_y \text{ by,} \\
nA_s' \{ (h + d_y - d'') - kd_y \} - S_y (kd_y)^2/2(1 - \mu^2) &= 0
\end{align*}
\]

\( (3.5.1.4) \)
3.5.2 Torsional Rigidity

For a rib-stiffened slab construction, it can be shown (31) that,

\[(D_{xy} + D_{yx} + D_1 + D_2) = 2K(D_xD_y)^{\frac{1}{2}} \quad (3.5.2.1)\]

or,

\[K = (D_{xy} + D_{yx} + D_1 + D_2) / 2(D_xD_y)^{\frac{1}{2}} \quad (3.5.2.2)\]

Experimental studies conducted have shown that for this type of slabs, \(D_{xy} \approx D_{yx}\). Thus, Eq. (3.5.2.2) reduces to,

\[K = (2D_{xy} + D_1 + D_2) / 2(D_xD_y)^{\frac{1}{2}} \quad (3.5.2.3)\]

Hence,

\[D_{xy} = K(D_xD_y)^{\frac{1}{2}} - (D_1 + D_2) / 2 \quad (3.5.2.4)\]

The coefficient \(K\) is a torsional parameter with a lower limit of zero (for a grillage with members having no torsional rigidity) and an upper limit of one (for a true orthotropically reinforced slab).

It is reasonable to assume that the coefficient \(K\) remains the same before and after cracking of the concrete. Thus, the coefficient \(K\) can be calculated from Eq. (3.5.2.3) using pre-cracking...
flexural and torsional rigidities from Eqs. (3.4.1.1) and (3.4.2.5), respectively. Now, the post-cracking torsional rigidity, $D_{xy}$, can be determined from Eq. (3.5.2.4) in which $D_x$, $D_y$, $D_\perp$, and $D_2$ are estimated from Eq. (3.5.1.1) for the post-cracking condition.

3.6 Rigidities of Non-Orthogonally Rib-Stiffened Slabs

In the structures considered herein, anisotropy is provided by a parent isotropic plate to which are securely attached one or more distinct systems of parallel straight reinforcing rib members, as shown in Figs. 3.3 and 3.4. The parent plate is sufficiently thin that it is considered in a state of plane stress and the stress in the rib stiffeners is assumed constant through their cross-section. The rib stiffeners are made from the same material as the parent plate and they provide more stabilization to the structure which makes the structure more efficient in resisting compression. For estimating the rigidities $D_{11}$, $D_{12}$, $D_{16}$, $D_{22}$, $D_{26}$ and $D_{66}$ of such structures, appropriate contributions of slab, swept ribs and ribs in the orthogonal directions are required to be taken into account.
Referring to Figs. 3.3 and 3.4, it is seen that the typical system of skew ribs is inclined at an angle $\alpha$, to the OY axis. If $EI_v'$ defines the "effective rigidity" of the uniformly distributed skew ribs under the assumptions already made, then this resultant $(EI_v')$ can be resolved into direct components by using the transformation:

$$
\begin{bmatrix}
D_{11}' \\
D_{12}' \\
D_{16}' \\
D_{22}' \\
D_{26}' \\
D_{66}'
\end{bmatrix} = \begin{bmatrix}
m^4 & 0 & 0 & 0 & 0 & 0 \\
0 & 1^2m^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 1^3m^3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1^4 & 0 & 0 \\
0 & 0 & 0 & 0 & 1^3m & 0 \\
0 & 0 & 0 & 0 & 0 & 1^2m^2
\end{bmatrix}
$$

(3.6.1)

where:

$$
\begin{align*}
1 &= \cos \alpha \\
m &= \sin \alpha
\end{align*}
$$

(3.6.2)

$D_{11}', D_{12}', D_{16}', D_{22}', D_{26}', D_{66}'$ are the contributions from the skew ribs to the rigidities $D_{11}, D_{12}, D_{16}, D_{22}, D_{26}$ and $D_{66}$ respectively,
\[ I'_v = \text{moment of inertia of the skew ribs with respect to the assumed neutral axis, i.e.}, \]
\[
I'_v = I_v \left\{ (h + \frac{d_v}{2}) - e_v \right\}^2 + (m - 1)A_{sv} \]
\[ (3.6.3) \]
\[
\left\{ (h + \frac{d_v}{2}) - e_v \right\}^2 + b_v \frac{d^3_v}{12}
\]
in which,
\[ e_v = \text{depth of neutral plane from the top fibre for bending in the V- direction, i.e.}, \]
\[
e_v = \left\{ b_v \frac{d_v}{2} (h + \frac{d_v}{2}) + (n - 1)A_{sv} (h + \frac{d_v}{2} - \frac{d_v^*}{2}) \right\} / \left\{ (b_v \frac{d_v}{2} + (n - 1)A_{sv} \right\}
\[
+ S_v h^2 (1 - \mu^2) \right\} \]
\[ (3.6.4) \]
\[ h = \text{thickness of flange plate}; \]
\[ S_v = \text{spacing of the skew ribs}; \]
\[ b_v = \text{width of the skew ribs}; \text{ and} \]
\[ d_v = \text{depth of the skew rib}. \]
\[ d_v^* = \text{concrete cover to the centre of reinforcement in the skew rib}. \]
\[ A_{sv} = \text{area of tension steel in the skew rib} \]
Thus, for a non-orthogonally rib-stiffened plate with skew stiffeners and rib stiffeners along the x-direction (Figs. 3.3, 3.5), the anisotropic rigidities can be calculated by the following equations:

\[ D_{11} = D + E h \left( e_v - h/2 \right)^2 / (1 - \mu^2) + EI_v' m^4 / S_v + EI_x' / S_x \]

\[ D_{12} = \mu D + E h \left( e_v - h/2 \right)^2 / (1 - \mu^2) + EI_v' l^2 m^2 / S_v \]

\[ D_{16} = \mu D + E h \left( e_v - h/2 \right)^2 / (1 - \mu^2) + EI_v' l^3 m / S_v \]

\[ D_{22} = D + E h \left( e_v - h/2 \right)^2 / (1 - \mu^2) + EI_v l^4 / S_v \]

\[ D_{26} = \mu D + E h \left( e_v - h/2 \right)^2 / (1 - \mu^2) + EI_v' l^3 m / S_v \]

\[ D_{66} = GJ_x / S_x + EI_v' l^2 m^2 / S_v \]  \hspace{1cm} (3.6.5)

where

- \( D_{11}, D_{22} \) = bending rigidities in the x- and y-directions, respectively;
- \( D_{66} \) = twisting-rigidity of the plate;
- \( D_{12}, D_{16}, D_{26} \) = additional coupling rigidities of the plate.

The rest of the parameters used in Eqs. (3.6.5) are the same as defined in sections 3.4.1 and 3.4.2.
If a structural system is reinforced with skew ribs and ribs along the y direction (Figs. 3.4 and 3.5), then the above equations are modified to account for ribs in the y direction and rigidities are found from the expressions,

\[
D_{11} = D + Eh (e_v - h/2)^2 / (1 - \mu^2) + EI_v m^4 / S_v
\]

\[
D_{12} = \mu D + \mu Eh (e_v - h/2)^2 / (1 - \mu^2) + EI_v l^2 m^2 / S_v
\]

\[
D_{16} = \mu D + \mu Eh (e_v - h/2)^2 / (1 - \mu^2) + EI_v l^3 m / S_v
\]

\[
D_{22} = D + Eh (e_v - h/2)^2 / (1 - \mu^2) + EI_v l^4 / S_v
\]

\[
+ EI_y / S_y
\]

\[
D_{26} = \mu D + \mu Eh (e_v - h/2)^2 / (1 - \mu^2) + EI_v l^3 m / S_v
\]

\[
D_{66} = GJ_y / S_y + EI_v l^2 m^2 / S_v
\]

(3.6.6)

As a check about the accuracy of the Eqs. (3.6.5) and (3.6.6), by substituting \( \alpha = 0 \) (i.e. skew ribs are oriented along y-axis), or \( \alpha = 90 \) (i.e. skew ribs are oriented along the x-axis), these equations become identical to the ones for orthogonally stiffened slabs as illustrated in section 3.4.
CHAPTER IV

MATHEMATICAL FORMULATION FOR THE EXPERIMENTAL STUDIES

4.1 Rigidities of Orthogonally Rib-Stiffened Slabs

The bending and twisting moments associated with an orthotropic slab structure can be expressed as (36),

\[ M_x = -(D_{xw,xx} + D_{1w,yy}) \]
\[ M_y = -(D_{yw,yy} + D_{2w,xx}) \]
\[ M_{xy} = D_{xyw,xy} \]

in which,

- \( M_x \) = moment vector in the \( y \)-direction;
- \( M_y \) = moment vector in the \( x \)-direction;
- \( M_{xy} \) = twisting moment vector in the \( x \)- and \( y \)-directions;

\( w,xx, w,yy \) = curvatures associated with the moments \( M_x \) and \( M_y \), respectively;

\( w,xy \) = twisting curvature associated with the twisting moment \( M_{xy} \); and

\( w \) = lateral deflection in the \( z \)-direction of a
point on the structure (Fig. 3.1).

Assuming that in the laboratory tests, it is possible to apply pure bending moments $M_x$ and $M_y$ and a pure twisting moment $M_{xy}$, then for the case when only a pure bending moment, $M_x$, is applied to a slab and $M_y = M_{xy} = 0$, Eqs. (4.1.1) yield,

\[
\begin{align*}
\frac{w_{,xx}}{c} &= S_{xx} M_x \\
\frac{w_{,yy}}{c} &= -S_{xy} M_x
\end{align*}
\]

in which,

\[
\begin{align*}
S_{xx} &= \frac{D_y}{(D_1D_2 - D_xD_y)} \\
S_{xy} &= \frac{D_2}{(D_1D_2 - D_xD_y)}
\end{align*}
\]

Similarly, when only $M_y$ is applied and $M_x = M_{xy} = 0$,

\[
\begin{align*}
\frac{w_{,yy}}{c} &= S_{yy} M_y \\
\frac{w_{,xx}}{c} &= -S_{yx} M_y
\end{align*}
\]

in which,

\[
\begin{align*}
S_{yy} &= \frac{D_x}{(D_1D_2 - D_xD_y)} \\
S_{yx} &= \frac{D_1}{(D_1D_2 - D_xD_y)}
\end{align*}
\]
Solving Eqs. (4.1.3) and (4.1.5) yields,

\[ D_x = \frac{S_{yy}}{(S_{xy}S_{yx} - S_{xx}S_{yy})} \]
\[ D_y = \frac{S_{xx}}{(S_{xy}S_{yx} - S_{xx}S_{yy})} \]  

Thus, by applying a known moment \( M_x \) to one slab specimen and a known moment \( M_y \) to another identical specimen and measuring the resulting curvatures, the values for \( S_{xx} \), \( S_{xy} \), \( S_{yy} \) and \( S_{yx} \) can be calculated from Eqs. (4.1.2) and (4.1.4). Then, the flexural rigidities \( D_x \) and \( D_y \) can be determined from the Eqs. (4.1.6). The bending and twisting curvatures can be calculated by measuring deflections of the bent surface of the slab (3).

To deduce the torsional rigidity \( D_{xy} \), the specimen is subjected to a known twisting moment, \( M_{xy} \). The twist of the surface \( w_{xy} \) cannot be determined directly; thus, Mohr's circle for curvature is used in this determination. Consider a set of rectangular axes, \( r \) and \( t \), inclined to the \( x \)- and \( y \)-axes by an angle \( \theta \) (Figs. 3.1 and 4.1 (b)). The curvature and twist in the \( r \) and \( t \) direction, at a point where \( r \) and \( t \) are normal to each other can be expressed in terms of the known curvature and twist at the same point along \( x \)- and \( y \)-axes which are normal to each other.
and in the same plane as the r- and t- axes. This
relationship is given by (3)

\[
\begin{align*}
\omega_{rr} &= \omega_{xx} \cos^2 \theta + \omega_{xy} \sin 2 \theta + \omega_{yy} \sin^2 \theta \\
\omega_{tt} &= \omega_{xx} \sin^2 \theta - \omega_{xy} \sin 2 \theta + \omega_{yy} \cos^2 \theta
\end{align*}
\]

(4.1.7)

where,

\(\omega_{rr}, \omega_{tt}\) are the curvatures in the r- and t-directions.

Thus, assuming that a pure twisting moment, \(M_{xy}\), is applied to the slab specimen (Fig. 4.1 (b)) and that the curvatures are measured in x, y, r and t directions, the value of the twist of the surface, \(\omega_{xy}\), can be calculated from Eq. (4.1.7). If \(\theta = 45\), Eq. (4.1.7) yields,

\[
\omega_{xy} = (\omega_{rr} - \omega_{tt})/2
\]

(4.1.8)

Hence, the torsional rigidity, \(D_{xy}\), can be determined from the last of Eqs. (4.1.1).

4.2. Rigidities of Non-Orthogonally Rib-Stiffened Slabs

In order to estimate the rigidity constants of non-orthogonally rib-stiffened slabs by measurements of bending and twisting curvatures of test specimens, it is assumed that plane sections which in the un-
deformed state of the plate are normal to its surface remain plane and normal to the bent middle surface during the bending. Also the normal stress σ_z in the cross-section parallel to the middle plane is assumed to be small as compared with the stress in the transverse cross-section.

If u and v are the displacements of any point in the direction of x- and y- axes and w (x,y) is the deflection of the middle plane, it follows from the first assumption,

\[ u = -zw, x \quad v = -zw, y \quad (4.2.1) \]

then,

\[ \varepsilon_x = -zw, \varepsilon_y = -zw, \gamma_{xy} = -2zw \quad (4.2.2) \]

From the generalized Hooke's Law,

\[ \varepsilon_x = a_{11} \sigma_x + a_{12} \sigma_y + a_{16} \gamma_{xy} \quad (4.2.3) \]

\[ \varepsilon_y = a_{12} \sigma_x + a_{22} \sigma_y + a_{26} \gamma_{xy} \]

\[ \gamma_{xy} = a_{16} \sigma_x + a_{26} \sigma_y + a_{66} \gamma_{xy} \]

From Eqs. (4.2.2) and (4.2.3), the stress components are given by,
\[
\begin{align*}
\sigma_x &= -z(B_{11}w,xx + B_{12}w,yy + 2B_{16}w,xy) \\
\sigma_y &= -z(B_{12}w,xx + B_{22}w,yy + 2B_{26}w,xy) \\
\tau_{xy} &= -z(B_{16}w,xx + B_{26}w,yy + 2B_{66}w,xy)
\end{align*}
\] (4.2.4)

where,

\[
\begin{align*}
B_{11} &= 1/N (a_{22} a_{66} - a_{26}^2) \\
B_{12} &= 1/N (a_{16} a_{26} - a_{12} a_{66}) \\
B_{16} &= 1/N (a_{12} a_{26} - a_{22} a_{16}) \\
B_{22} &= 1/N (a_{11} a_{66} - a_{16}^2) \\
B_{26} &= 1/N (a_{12} a_{16} - a_{11} a_{26}) \\
B_{66} &= 1/N (a_{11} a_{22} - a_{12}^2)
\end{align*}
\] (4.2.5)

\[
N = \begin{vmatrix}
a_{11}' & a_{12}' & a_{16}' \\
a_{12}' & a_{22}' & a_{26}' \\
a_{16}' & a_{26}' & a_{66}'
\end{vmatrix}
\] (4.2.6)

Now the bending and twisting moments can be expressed as,
\[
M_x = \int_{-h/2}^{h/2} \sigma_x zdz
\]
\[
M_y = \int_{-h/2}^{h/2} \sigma_y zdz
\]
\[
M_{xy} = M_{yx} = \int_{-h/2}^{h/2} r_{xy} zdz
\]

From Eqs. (4.2.4) and (4.2.7)
\[
\begin{align*}
M_x &= - (D_{11} w_{xx} + D_{12} w_{yx} + 2D_{16} w_{xy}) \\
M_y &= - (D_{12} w_{xx} + D_{22} w_{yy} + 2D_{26} w_{xy}) \\
M_{xy} &= - (D_{16} w_{xx} + D_{26} w_{yy} + 2D_{66} w_{xy})
\end{align*}
\]

Rigidities of the anisotropic plate, \(D_{ij}\), are related to \(B_{ij}\) as,
\[
D_{ij} = B_{ij} h^3/12 \quad (4.2.9)
\]

For the anisotropic plate under consideration, skew ribs are inclined to the \(OY\) axis at an angle of \(\alpha\). The co-ordinates of the rectangular \((x,y,z)\) and oblique \((u,v,z)\) systems of axes (Fig. 4.2) are related by,
\[
\begin{align*}
    u &= x + y \tan \alpha \\
    v &= y/\cos \alpha
\end{align*}
\]

(4.2.10)
\[ u_x = 1, \quad u_y = \tan \alpha \quad (4.2.11) \]
\[ v_x = 0, \quad v_y = \frac{1}{\cos \alpha} \]

The displacement function \( w(x,y) \) can be expressed as,

\[ w_x = w, u_x + w, v_x \]
\[ \Rightarrow w_x = w, u \]

\[ w_y = w, u_y + w, v_y \]
\[ \Rightarrow w_y = w, u \tan \alpha + \frac{w}{\cos \alpha} \quad (4.2.12) \]

After a second differentiation with respect to \( x \) and \( y \), Eqs. (4.2.12) yield,

\[ w_{xx} = w, uu \]
\[ w_{xy} = w, uu \tan \alpha + w, uv \frac{1}{\cos \alpha} \quad (4.2.13) \]
\[ w_{yy} = w, uu \tan^2 \alpha + 2w, uv \frac{\tan \alpha}{\cos \alpha} + w, vv \frac{1}{\cos^2 \alpha} \]

where,

\[ w, uu, w, vv = \text{curvatures due to bending in the u and v- directions}. \]
\\( w, uv = \text{twist of the surface with respect to the} \)
\\( u \text{ and } v \text{-axes}. \)

By substituting in Eqs. (4.2.8), the values of curvatures from Eqs. (4.2.13), the following expressions for the bending and twisting moments are obtained:

\[
M_x = -\left\{ D_{11} w, uu + D_{12} \left( w, uu \tan \alpha + 2 w, uv \tan \alpha / \cos \alpha \right)^2 + w, vv / \cos \alpha \right\} + 2 D_{16} \left( w, uu \tan \alpha + w, uv / \cos \alpha \right)
\]

\[
M_y = -\left\{ D_{12} w, uu + D_{22} \left( w, uu \tan \alpha + 2 w, uv \tan \alpha / \cos \alpha \right)^2 + w, vv / \cos \alpha \right\} + 2 D_{26} \left( w, uu \tan \alpha + w, uv / \cos \alpha \right)
\]

\[
M_{xy} = -\left\{ D_{16} w, uu + D_{26} \left( w, uu \tan \alpha + 2 w, uv \tan \alpha / \cos \alpha \right)^2 + w, vv / \cos \alpha \right\} + 2 D_{66} \left( w, uu \tan \alpha + w, uv / \cos \alpha \right)
\]

\[(4.2.14)\]

To determine the rigidities of the structure from the above equations, pure bending moments, \( M_x \) and \( M_y \), and a pure twisting moment, \( M_{xy} \), are applied separately to the test specimen (Fig. 4.3).

**1st Case**

When only \( M_y \) is applied and \( M_x = M_{xy} = 0 \), and let \( w = w_1 \). This represents a specimen bent about
the x-axis (Fig. 4.3 (a)).

Thus, Eqs. (4.2.14) will be as follows:

\[-D_{11}w_{1,uu}^2 - D_{12} \left(w_{1,uu} \tan \alpha + 2w_{1,uv} \tan \alpha / \cos \alpha + \right.
\left. w_{1,vv} \tan \alpha / \cos \alpha \right) - 2D_{16} \left(w_{1,uu} \tan \alpha + w_{1,uv} / \cos \alpha \right) = 0 \]

\[-D_{12}w_{1,uu}^2 - D_{22} \left(w_{1,uu} \tan \alpha + 2w_{1,uv} \tan \alpha / \cos \alpha + \right.
\left. w_{1,vv} \tan \alpha / \cos \alpha \right) - 2D_{26} \left(w_{1,uu} \tan \alpha + w_{1,uv} / \cos \alpha \right) = M_y \]

\[-D_{16}w_{1,uu}^2 - D_{26} \left(w_{1,uu} \tan \alpha + 2w_{1,uv} \tan \alpha / \cos \alpha + \right.
\left. w_{1,vv} \tan \alpha / \cos \alpha \right) - 2D_{66} \left(w_{1,uu} \tan \alpha + w_{1,uv} / \cos \alpha \right) = 0 \]

(4.2.15)

In the above equations, the curvatures \(w_{1,uu}\), \(w_{1,uv}\) and \(w_{1,uv}\) can be determined as described below.

Fig. 4.4 shows the points at which the deflections of the slab are measured due to the application of a pure bending moment, \(M_y\). Then the curvatures are given by,

\[w_{1,uu} = \frac{2}{a} \left(w_5 - \frac{w_2 + w_8}{2}\right) \]
\[w_{1,uv} = \frac{2}{b} \left(w_5 - \frac{w_4 + w_6}{2}\right) \]

(4.2.16)

where,
$w_2$, $w_4$, $w_5$, $w_6$, and $w_8$ are deflections measured at points 2, 4, 5, 6 and 8 respectively.

$a = \text{spacing of points 2 and 8 from point 5.}$

$b = \text{spacing of points 4 and 6 from point 5.}$

For determining the curvature $w_{,uv}$, it is necessary to find the curvature $w_{,xy}$, with respect to the $x$- and $y$- axes. As described in section 4.1, the curvature $w_{,xy}$ is calculated by considering a set of orthogonal axes $r$ and $t$, inclined to the $x$- and $y$- axes by an angle $\theta$ (Fig. 4.4). If $\theta = 45^\circ$, from Eqs. (4.1.7),

$$w_{1,xy} = (w_{1,rr} - w_{1,tt})/2 \quad (4.2.17)$$

where,

$w_{1,rr} \text{ and } w_{1,tt}$ = the curvatures in the $r$ and $t$ directions i.e.,

$$w_{1,rr} = 2/c \left( w_5 - (w_3 + w_7)/2 \right) \quad (4.2.18)$$

$$w_{1,tt} = 2/s \left( w_5 - (w_1' + w_9)/2 \right)$$

in which,

$w_3$, $w_7$, $w_1'$, $w_9'$ are the deflections measured at points 3, 7, 1' and 9' respectively.

$c = \text{spacing of points 3 and 7 from point 5.}$

$s = \text{spacing of points 1' and 9' from point 5.}$

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If spacing \( a = b \), the rectangular axis \( t \) will pass through points 1, 5 and 9 and will be perpendicular to axis \( r \) passing through points 3, 5 and 7. In this case, it will not be required to measure deflections at points 1' and 9', in order to obtain curvature \( w_{1,tt} \).

Now, the curvature \( w_{1,uv} \) can be calculated from the second of the Eqs. (4.2.13), which can be expressed as,

\[
w_{1,uv} = (w_{1,xy} - w_{1,uu} \tan \alpha) \cos \alpha \quad (4.2.19)
\]

Having known the curvatures \( w_{1,uu}, w_{1,uv} \) and \( w_{1,vv} \), the unknowns in the Eqs. (4.2.15) are the rigidities of the slab, \( D_{11}, D_{12}, D_{16}, D_{22}, D_{26} \) and \( D_{66} \).

**2nd Case**

Moment \( M_x \) is applied about \( y \)-axis and \( M_y = M_{xy} = 0 \), and let \( w = w_2 \) (Fig. 4.3 (b)). Substituting in Eqs. (4.2.14) yields,

\[
-D_{11} w_{2,uu} - D_{12} \left( w_{2,uu} \tan \alpha + 2w_{2,uv} \tan \alpha / \cos \alpha + w_{2,vv} / \cos \alpha \right) - 2D_{16} \left( w_{2,uu} \tan \alpha + w_{2,uv} / \cos \alpha \right) = M_x
\]

\[
-D_{12} w_{2,uu} - D_{22} \left( w_{2,uu} \tan \alpha + 2w_{2,uv} \tan \alpha / \cos \alpha + w_{2,vv} / \cos \alpha \right) - 2D_{26} \left( w_{2,uu} \tan \alpha + w_{2,uv} / \cos \alpha \right) = 0
\]
- \( D_{16} w_{2,uu} - D_{26} (w_{2,uu} \tan \alpha + 2w_{2,uv} \tan \alpha / \cos \alpha + \)

\[ w_{2,uv} / \cos \alpha ) - 2D_{66} (w_{2,uu} \tan \alpha + w_{2,uv} / \cos \alpha ) = 0 \]

(4.2.20)

From the above three equations, the unknown curvatures \( w_{2,uu}, w_{2,uv} \) and \( w_{2,uv} \) can be determined in the same manner as described earlier in the 1st case.

After substitution of the values of the curvatures in Eqs. (4.2.20), the unknowns are the six rigidities \( D_{11}, D_{12}, D_{16}, D_{22}, D_{26}, \) and \( D_{66} \).

3rd Case

Considering that \( M_x = M_y = 0 \), and a pure torsional moment, \( M_{xy} \) is applied to the structure. Let \( w = w_3 \) (Fig. 4.3 (a)), the Eqs. (4.2.14) take the form,

\[- D_{11} w_{3,uu} - D_{12} (w_{3,uu} \tan \alpha + 2w_{3,uv} \tan \alpha / \cos \alpha + \]

\[ w_{3,uv} / \cos \alpha ) - 2D_{16} (w_{3,uu} \tan \alpha + w_{3,uv} / \cos \alpha ) = 0 \]

\[- D_{12} w_{3,uu} - D_{22} (w_{3,uu} \tan \alpha + 2w_{3,uv} \tan \alpha / \cos \alpha + \]

\[ w_{3,uv} / \cos \alpha ) - 2D_{26} (w_{3,uu} \tan \alpha + w_{3,uv} / \cos \alpha ) = 0 \]
Similarly, the unknown curvatures can also be determined for the above equations.

**Determination of the Rigidities**

There are nine equations in (4.2.15), (4.2.20) and (4.2.21) which contain six unknown rigidities $D_{11}$, $D_{12}$, $D_{16}$, $D_{22}$, $D_{26}$ and $D_{66}$. Thus, three experiments can be conducted by applying pure bending moments, $M_x$ and $M_y$, in the first two experiments and a pure twisting moment, $M_{xy}$ in the third experiment which will result in Eqs. (4.2.15), (4.2.20) and (4.2.21), respectively. Six equations from (4.2.15) and (4.2.20) can be used to determine the six unknown rigidities. The rigidities $D_{11}$, $D_{12}$, $D_{16}$, $D_{22}$, $D_{26}$, and $D_{66}$ can also be determined from Eqs. (4.2.15) and (4.2.21) (or from Eqs. (4.2.20) and (4.2.21)) and the results can be compared with those obtained earlier by using Eqs. (4.2.15) and (4.2.20).
CHAPTER V

EXPERIMENTAL INVESTIGATION

5.1 Scope of the Experimental Programme

The experimental programme consisted of nine tests on orthogonally rib stiffened concrete slabs. These tests were classified into three series of test specimens. Each series consisted of three types of specimens, two were rectangular in plan for bending tests (one in the x- and the other in the y- direction; and the third was square in plan for the pure twisting test. Mathematical formulations developed in section 4.1 were used to calculate the flexural and torsional rigidities.

5.2 Materials

5.2.1 Concrete

High early strength Portland Cement manufactured by Canada Cement Company was used in all the slab specimens. The maximum size of the aggregate was restricted to 0.25 inch (6.4 mm) since the narrowest dimension between the sides of the formwork was 1.25 inch (32 mm) and the concrete cover to the centre of steel reinforcement was 0.5 inch (12.7 mm). The combined aggregate was prepared by mixing 97.7% coarse aggregates and 90.3% fine sand. This gave a well graded combined aggregate with a
fineness modulus of 2.42. The concrete mix used had a water-cement ratio of 0.75 and the aggregate-cement ratio was 5.0, both by weight. Mixing of concrete was done in an Eerich Counter Current Mixer, Model EA2 (2W) with a capacity of five cu. ft. Only one batch of concrete weighing 700 lbs. was required for each bending test specimen. The concrete strengths of the specimens tested are given in Table 5.1.

5.2.2 Reinforcement

Two plain mild steel wires of 1/8 in (3.2 mm) diameter were used as reinforcement. The wires were straightened and twisted together in the laboratory, keeping the cross-sectional area the same along the length. The twisted wires were cut, cleaned from rust and hooks were provided at both ends. The stress-strain relationship for the reinforcing wire is shown in Fig. 5.1. The modulus of elasticity was found to be 30,400 ksi (210 GN/m²).

5.3 Description of the Specimens

A total of nine specimens were tested for the bending and twisting tests. All specimens had a continuous slab with a thickness of 1 in (25.4 mm)
and the depth of the ribs was 3 in (76 mm). In each case, the first layer of steel was placed at 3/8 in (9.5 mm) clear cover, and the second layer orthogonal to the first, was placed just over the first layer. Geometric description and properties of these specimens are given in Table 5.1. These specimens were classified into three series.

There were three specimens in each series. Two specimens were tested in bending and one in torsion. The bending specimens in series A were 80 x 40 inches (2.03 x 1.02 m) in plan and the ribs were placed at 8 in (203 mm) spacing in one direction while in the other orthogonal direction, spacing of the ribs was 4 in (102 mm) c/c. The size of the specimen for the twisting test was 40 x 40 inches (1.02 x 1.02 m). In series B, the size of the bending specimens was 70 x 35 inches (1.78 x 0.89 m) and for the twisting test, the specimen was 35 x 35 inches (0.89 x 0.89 m) in plan. Spacing of the ribs was 7 in (178 mm) c/c in one direction and in the other direction, a spacing of 5 in (127 mm) c/c was provided. In series C, the bending specimens were 60 x 36 inches (1.52 x 0.91 m) in plan and twisting specimen was square in
plan with dimensions of 36 x 36 inches (0.91 x 0.91 m). Ribs were placed at 6 in (152 mm) c/c spacing in one direction and in the other orthogonal direction, a spacing of 4 in (102 mm) c/c was provided. Reinforcement was placed in the centre of the ribs for all the nine specimens tested.

5.4 Casting of the Specimens

All the specimens were cast in forms made of 3/4 in (19 mm) thick plywood. The voids between the ribs were made by gluing styrofoam blocks to the plywood form following a marked pattern on the wood. The reinforcement was then placed between the styrofoam blocks in both directions. For the bending specimens the reinforcement in the longer direction was placed first on small wooden blocks to keep it 3/8 in (9.5 mm) from the bottom of the forms. This provided for the minimum cover required for the steel. The steel in the shorter direction was then placed on the top of the reinforcement in the longer direction. Both layers of reinforcement were tied together with thin wire for stability during casting of the concrete.

Before casting of the concrete, the top surface and the sides of the form were soaked with heavy industrial oil. The concrete was then poured,
tamped, vibrated and troweled to a smooth finish. All the specimens were moist cured for a period of 14 days. To determine the compressive strength of the concrete, three 3 x 6 in (76 x 152 mm) concrete cylinders were cast with each specimen. These cylinders were subjected to a compression test at the time of testing of the test specimen.

5.5 Instrumentation

The deflections of the concrete specimens for bending and twisting tests were measured by means of the Hewlett Packard series 740CDT Displacement Transducers. The concrete surface at the location of the transducers was smoothed using fine sand paper and after removing the dust by means of compressed air, the surface at the transducer locations was cleaned with acetone. Surface cavities were filled with an epoxy resin at the transducer locations. All transducers were calibrated with an electronic voltmeter before testing to establish their calibration factors (Figs. 5.2 through 5.10). The range of the voltmeter was from mV to 200 V. Millivolt range was used in all the experiments to obtain very accurate deflections. A Harrison Laboratories Model 6204 B power supply was connected to the transducer excitation terminals to provide an input voltage of 7 volts.
to each transducer.

A T-shaped loading frame was made for the bending tests and a straight loading frame was fabricated for the twisting tests. They were both made of 3 x 3 x 1/8 in (76 x 76 x 3.2 mm) steel hollow section. A bearing plate with a groove for placing a ball bearing was welded to the legs of the frame to transmit the load to the slab. Loading frames used for the bending and twisting tests are shown in Figs. 5.11 and 5.12. A mechanical turn-screw anchored to a very solid portal frame was used to apply a single central concentrated load to the loading frame on the slab. A load cell, straininsert flat universal type, was attached at the lower end of the loading-screw. This load cell with a maximum load capacity of 5.0 kips (22.25 kN) was calibrated with a portable strain indicator (Budd Model P-350) which indicated the strain in the load cell (Fig. 5.13).

5.6 Experimental Set-Up and Test Procedure

A three-point loading and a three point support arrangement (Fig. 4.1 (a)) was adopted for applying a pure bending moment to the bending specimens. To apply a pure twisting moment to the concrete specimens, the latter were supported at two diagonally opposite corners with downward concentrated loads applied at
the other two corners. The test set-ups for bending and twisting tests are shown in Figs. 5.14 and 5.15. Point supports were provided for the bending specimens by means of ball-bearings and the twisting test specimens were supported on 1 in (25.4 mm) diameter steel rollers. Deflections of the concrete specimens were measured by means of displacement transducers located at nine grid locations some distance from the loading and support disturbance zones (Figs. 4.1 (a) and 4.1 (b)).

Deflections were measured during both loading and unloading of the concrete specimens and were averaged. The loading and unloading were carried out in stages, the level of unloading coinciding with the previous level of loading. Abrupt departure from the initial linear load-deflection relationship indicated cracking of the specimen and hence change in the stiffness properties. At this stage, loading was continued at reduced load increments, until failure of the specimen was reached as evidenced by large deflections and formation of visible cracks on the test specimen.

5.7 Experimental Results

Experimental results of all test specimens for orthogonally rib stiffened concrete slabs are
presented in Tables 5.2 and 5.3, where these results are also compared with the theoretical results.
6.1 **Flexural and Torsional Rigidity**

The theoretical and experimental results for the flexural and torsional rigidities of orthogonally rib stiffened reinforced concrete slab structures are presented in tables 5.2 and 5.3 respectively. The entire formulation of the calculation of rigidities from the tests was automated on a digital computer.

The linear deflections (in/lb) of nine transducer locations for all the specimens for the precracking and post-cracking stages are listed in tables 6.1 and 6.2, respectively. These were obtained statistically by linear regression. The load-centre deflection bilinear relations for all specimens were plotted and are presented in Figs. 6.1, 6.2, and 6.3. From these plots, the abrupt change in stiffness after cracking of the concrete is clearly evident. The load-deflection behaviour of the uncracked specimen appears to be elastic and linear. Furthermore, it is observed that the rate of change of the load-deflection behaviour of the cracked specimen continues to be almost constant over a considerable range of loading. Based on these observations and for simplicity, two linear regression lines were fitted to the measured
load-deflection results for the nine locations on each specimen in order to obtain representative values. Measured deflections close to the collapse load were omitted from the fitted data because these measurements are of no consequence in a quasi-elastic analysis of orthogonally stiffened slab structures as presented herein.

The bending curvatures were calculated from the linear deflections, \( w \), at the nine grid points (Fig. 4.1 (a)). Then the longitudinal curvature along line A-A is given by (Fig. 4.1 (a)),

\[
\frac{2}{b^2} \left[ w_5 - \frac{(w_4 + w_6)}{2} \right]
\]

(6.1.1)

Similarly, longitudinal curvatures along lines B-B and C-C were calculated and, thus, the average longitudinal curvature =

\[
2 \left[ \frac{(w_2 + w_5 + w_8)}{3} - \frac{(w_1 + w_3 + w_4 + w_6 + w_7 + w_9)}{6} \right] b^2
\]

(6.1.2)

and the average transverse curvature =

\[
2 \left[ \frac{(w_4 + w_5 + w_6)}{3} - \frac{(w_1 + w_2 + w_3 + w_7 + w_8 + w_9)}{6} \right] a^2
\]

(6.1.3)

From Eq. (4.1.8), the average twisting curvature of the test specimen (Fig. 4.1 (b)) becomes
Thus, by means of Eqs. (4.1.2) and (4.1.4), relating curvatures to the applied known bending moments, the experimental values of the flexural rigidities were calculated from Eq. (4.1.6). Computing the twist, \( w_{xy} \) from Eq (6.1.4) and for an applied known torsional moment, \( M_{xy} \), the torsional rigidity, \( D_{xy} \), was calculated from the last of Eqs. (4.1.1).

In Table 5.2, the experimental values of the flexural rigidities \( D_x \) and \( D_y \), are compared with the theoretical values as calculated from Eqs. (3.4.1.1) and (3.5.1.1). Close agreement is noted for both the precracking and post-cracking flexural rigidities with an average difference not exceeding 3%. The values of the precracking torsional rigidity estimated by Eqs. (3.4.2.5) agree closely with the experimental results (Table 5.3). The discrepancy between the theoretical values and the experimental ones does not exceed 2%. Table 5.3 also presents values of the post-cracking torsional rigidity calculated from Eq. (3.5.2.4) and these values are approximately 10% above the experimental ones.

Results for the torsional rigidity \( D_{xy} \), estimated by other various methods are also given in Table 5.3. Comparison of these results with the experimental ones

\[
\frac{w}{x} = \frac{(w_1 + w_9 - w_3 - w_7)}{4(a^2 + b^2)^{\frac{3}{2}}}
\]  

(6.1.4)
shows that Jackson's method (20) appears to grossly over-estimate the torsional rigidity. Jackson proposed an approximate method using the membrane analogy to account for the junction effect at the re-entrant corners in the estimation of the torsional rigidities of tee-sections. A detailed study of this method of using the membrane analogy reveals that the membrane deflections are not zero at the re-entrant corners. For a typical cross-section of the specimen used in the experiments described herein, the calculated membrane deflections at these corners are several times higher than the largest membrane deflections in area 1 and in the lower portion of area 2 (see Figs. 3.2 (a) and 3.2 (b)). Also, the membrane deflections in the vicinity of the boundary between areas 1 and 2 are several times larger than those in the flange plate, area 1. These large values of membrane deflections are not only unrealistic, but tend to grossly over-estimate the torsional rigidity of the cross-section as is evidenced by the values calculated by Jackson's method shown in Table 5.3.

The values of the torsional rigidity based on Huber's orthotropic plate theory (17) were also calculated from the equation,

$$D_{xy} = (1 - \mu) \left( D_x D_y \right)^{\frac{1}{2}}$$  \hspace{1cm} (6.1.5)
As expected, these values are highly exaggerated when compared with the experimental results. Results for the torsional rigidity estimated by using Rowe's method (32) are lower than the experimental ones. The discrepancy here can be attributed to ignoring the stiffening effect of the ribs in the orthogonal direction. The higher and realistic estimates predicted for the torsional rigidity, in comparison to those obtained by Rowe's method will result in better load distribution, reduction in the area of reinforcing steel, and the use of shallower ribs, which can be of considerable advantage where depth of construction is of importance. It is not possible to quantify these savings of materials because the analysis of such structures (13) depends on several factors, such as \( \frac{D_y}{D_x} \) ratio, aspect ratio, type of load, boundary conditions and the plan-form of the slab structure i.e., whether rectangular or skew.

Although, no experimental programme was undertaken to calculate the anisotropic rigidities of the non-orthogonally rib stiffened slab structures, the expressions for the rigidities of the orthogonally rib stiffened slabs can be deduced from Eqs. (3.6.5) which verifies the validity of these equations. Bending and twisting tests (Figs. 4.3 (a), 4.3 (b), 4.3 (c)) can be conducted on non-orthogonally rib stiffened slab
structures as described in section 4.2 to further verify and substantiate the proposed theoretical equations (3.6.5).

It should be noted that the rigidities of a concrete structure are affected by creep. The influence of creep can be taken into account by adjusting the modulus of elasticity of the concrete, $E_c$, to a time-dependent modulus of deformation $E_e$ (32). Furthermore, creep of concrete also influences the modular ratio, $n$. To allow for this long term effect, it has been suggested (7) that a value of $n = 10$ be used. Recent analytical studies (12) using the proposed theoretical expressions for bending and torsional rigidities reveal that there is good agreement between the theoretical results for deflection and moments and the experimental ones from tests on prestressed concrete as well as on reinforced concrete orthogonally rib stiffened slabs both for the precracking and post-cracking stages of the concrete. For comparison, results for deflections and stresses were obtained for two types of structures, the orthogonally rib stiffened slab type and the slab type with uniform thickness, both having the same volume of concrete and reinforcing steel. It was found that the rib stiffened slab structure exhibited much smaller deflections and lower
stresses due to its higher values of flexural rigidities. It should be noted that one advantage of the rib stiffened slabs over slabs of uniform thickness is in prestressed construction; where the presence of the ribs contributes to the increased eccentricities for prestressing in rib stiffened slabs. This results in higher efficiency of the prestressed rib-stiffened slab in carrying load in comparison to a slab of uniform thickness, having the same volume of concrete and the same amount of steel.

6.2 Sources of Error

The discrepancies between the experimental and the theoretical values can be attributed to the following sources of error:

1. The assumptions made in the theory to formulate the theoretical expressions.
2. Positioning of the reinforcement in the ribs.
3. Estimates of the strength of concrete from the tests on 3 x 6 inches cylinders.
5. Any deformation of the loading frame will introduce some error.

6. Some inaccuracies in the measurement of deflections may have been introduced due to improper alignment of the transducer.

7. Difficulty in setting-up the twisting test due to movement of the specimen on the roller supports.

8. The calibration of the load cell and transducers.
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The overall objective of this study was to conduct theoretical and experimental studies on the rigidities of orthogonally and non-orthogonally rib stiffened slab systems. To this end, on the basis of an idealization of integrally rib stiffened plates to orthotropic plates of uniform thickness, formulae were derived for the rigidity constants of plates with integral ribs in orthogonal and non-orthogonal directions. Experiments were carried out to support the reliability of theoretical expressions for calculating the orthotropic rigidities of orthogonally rib stiffened slabs. Experimental procedure and mathematical formulations to calculate the experimental values of the rigidities of non-orthogonally rib stiffened slabs were suggested.

Based on the results obtained from the theoretical and experimental studies, the following conclusions are drawn:

1. The good agreement between the theoretical and experimental results supports the reliability of the proposed formulae for estimating the rigidities of orthogonally
rib stiffened slabs for both the pre-cracking and post-cracking stages of concrete.

2. The analytical expressions derived for the calculation of rigidity constants of non-orthogonally rib stiffened plates are general in form and, thus, are applicable for orthogonally rib stiffened slabs after properly substituting for the value of the angle of inclination of the skew ribs.

3. The expressions developed for the orthotropic rigidities of orthogonally rib stiffened slabs are valid for reinforced as well as prestressed concrete structures.

4. The discrepancies between the theoretical and experimental results are due to various experimental errors.

5. The analysis of the reinforced and prestressed concrete rib stiffened slab structures by means of classical orthotropic plate theory, using realistic estimates of the rigidity constants, should lead to economy and better design of such structures.

7.2 Recommendations For Future Research

The following suggestions are recommended for future research as an extension of this investigation:
1. The flexural and torsional tests as proposed (Figs. 4.3 (a), 4.3 (b) and 4.3 (c)) should be conducted on non-orthogonally rib stiffened reinforced concrete slab structures.

2. Analytical expressions for calculating the rigidities of non-orthogonally rib stiffened slabs after cracking of the concrete section should be developed and the theoretical results be compared with the experimental values.

3. It is recommended that, for the flexural tests, the length of the specimen be more than twice its width in order to insure sufficient test area between the supports for measurement of deflections and better distribution of loads on the test specimen.
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34. Thielemann, W., "Contribution to the Problem of Buckling of Orthotropic Plates With Special Reference to Plywood," NACA TM 1263, August, 1950.


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<th>Sectional detailsSpacing (in)</th>
<th># Spacing of ribs (in)</th>
<th>Slab details (in)</th>
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<td>80 x 40</td>
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**TABLE 5.1** GEOMETRIES AND MATERIAL PROPERTY OF TEST SPECIMENS

* Refer Fig. 4.1
# Refer Fig. 3.1
<table>
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<tr>
<th>Test Series</th>
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<td>C</td>
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Average percent difference (absolute value)  

<p>| 3% | 1% | 1% | 1% |</p>
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<tr>
<td>B</td>
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<tr>
<td>C</td>
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<td>46.2</td>
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TABLE 5.3 COMPARISON OF THEORETICAL AND EXPERIMENTAL TORSIONAL RIGIDITIES, D_{xy}.

Note: Values in parenthesis are for D_{yx}. 
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<td>4.82</td>
<td>6.23</td>
<td>10.8</td>
<td>4.65</td>
<td>4.79</td>
<td>4.12</td>
</tr>
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</table>

**TABLE 6.1 PRECRACKING LINEAR DEFLECTIONS OF TEST SPECIMENS**

*Note: Upward deflections are considered positive.*
<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Location</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td></td>
<td>2.30</td>
<td>2.53</td>
<td>1.95</td>
<td>2.22</td>
<td>2.53</td>
<td>2.07</td>
<td>2.08</td>
<td>2.77</td>
<td>2.27</td>
</tr>
<tr>
<td>A-2</td>
<td></td>
<td>4.95</td>
<td>5.49</td>
<td>4.14</td>
<td>4.64</td>
<td>5.42</td>
<td>4.11</td>
<td>4.52</td>
<td>5.59</td>
<td>4.29</td>
</tr>
<tr>
<td>A-3</td>
<td></td>
<td>21.7</td>
<td>26.8</td>
<td>33.2</td>
<td>32.8</td>
<td>29.4</td>
<td>28.0</td>
<td>55.5</td>
<td>35.1</td>
<td>22.8</td>
</tr>
<tr>
<td>B-1</td>
<td></td>
<td>2.47</td>
<td>2.70</td>
<td>1.98</td>
<td>2.38</td>
<td>2.82</td>
<td>2.17</td>
<td>2.38</td>
<td>3.0</td>
<td>2.30</td>
</tr>
<tr>
<td>B-2</td>
<td></td>
<td>3.43</td>
<td>3.90</td>
<td>3.22</td>
<td>3.27</td>
<td>3.95</td>
<td>3.28</td>
<td>3.25</td>
<td>4.20</td>
<td>3.55</td>
</tr>
<tr>
<td>B-3</td>
<td></td>
<td>4.23</td>
<td>7.62</td>
<td>23.2</td>
<td>10.3</td>
<td>9.83</td>
<td>9.27</td>
<td>22.5</td>
<td>10.1</td>
<td>3.57</td>
</tr>
<tr>
<td>C-1</td>
<td></td>
<td>2.58</td>
<td>2.36</td>
<td>2.16</td>
<td>2.21</td>
<td>3.02</td>
<td>2.18</td>
<td>2.18</td>
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<td>2.04</td>
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<tr>
<td>C-2</td>
<td></td>
<td>3.04</td>
<td>2.72</td>
<td>2.26</td>
<td>2.45</td>
<td>3.32</td>
<td>2.34</td>
<td>1.58</td>
<td>2.90</td>
<td>2.56</td>
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<tr>
<td>C-3</td>
<td></td>
<td>6.08</td>
<td>11.6</td>
<td>22.7</td>
<td>14.2</td>
<td>11.6</td>
<td>10.4</td>
<td>22.7</td>
<td>12.9</td>
<td>5.04</td>
</tr>
</tbody>
</table>

**Table 6.2** Postcracking linear deflections (in/lb x 10^-5).

Note: Upward deflections are considered positive.
FIGURES
Fig. 3.1 GEOMETRIC SHAPE OF TEST SPECIMEN WITH REFERENCE AXIS.
Fig. 3.2 CROSS SECTION AND METHOD OF PARTITIONING

a) Section normal to X-axis

b) Section normal to Y-axis
Fig. 3.3 TYPICAL NON-ORTHOGONALLY RIB
STIFFENED SLAB
Fig. 3.4  TYPICAL NON-ORTHOGONALLY RIB  
STIFFENED SLAB
Fig. 3.5 CROSS SECTION OF RIBS FOR NON-ORTHOGONALLY RIB-STIPPENED SLABS.

a) Section of ribs along V-axis

b) Section of ribs along X-(Y-) axes.

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Fig. 4.2 OBLIQUE CO-ORDINATE SYSTEM
a) Bending About x-axis

b) Bending about y-axis

c) Twisting Specimen

Fig. 4.3 TOP VIEW OF SPECIMENS FOR TESTS ON NON-ORTHOGONALLY RIB STIFFENED SLABS.
Fig. 4.4 LOCATION OF DEFLECTION MEASUREMENT POINTS
FOR SKEW RIB STIFFENED SLABS
Fig. 5.1. LOAD-ELONGATION RELATIONSHIP FOR REINFORCEMENT
SLOPE = 0.1893 in/volt

Fig. 5.2 DEFLECTION VOLTAGE RELATIONSHIP

(TRANSDUCER NO. 1)
SLOPE = 0.1289 in/volt

Fig. 5.3 DEFLECTION VOLTAGE RELATIONSHIP
(TRANSUCER NO. 2)
SLOPE = 0.1266 in/volt

Fig. 5.4 DEFLECTION VOLTAGE RELATIONSHIP

(TRANSDUCER NO. 3)
SLOPE = 0.1222 in/volt

Fig. 5.5  DEFLECTION VOLTAGE RELATIONSHIP

(TRANSDUCER NO. 4)
SLOPE = 0.1239 in/volt

Fig. 5.6 DEFLECTION VOLTAGE RELATIONSHIP

(TRANSDUCER NO. 5)
DEFLECTION ( in )

SLOPE = 0.1285 in/volt

VOLTAGE (volts)

DEFLECTION VOLTAGE RELATIONSHIP

(TRANSUDER NO. 6)
SLOPE = 0.1224 in/volt

Fig. 5.8  DEFLECTION VOLTAGE RELATIONSHIP

(TRANSUDER NO. 7)
DEFLECTION (in) 85
SLOPE = 0.1389 in/volt

Fig. 5.9 DEFLECTION VOLTAGE RELATIONSHIP
(TRANSDUCER NO. 8)
SLOPE = 0.1387 in/volt

Fig. 5.10 DEFLECTION VOLTAGE RELATIONSHIP

(TRANSDUCER NO. 9)
Fig. 5.11  LOADING FRAME FOR BENDING TESTS.
Fig. 5.12  LOADING FRAME FOR TWISTING TESTS
SLOPE = 1.2415

Fig. 5.13 LOAD STRAIN RELATIONSHIP FOR THE LOAD CELL
Fig. 5.14 TEST SETUP FOR BENDING SPECIMENS
Fig. 5.15  TEST SETUP FOR TWISTING SPECIMENS
Specimen A-1  Bending about y-axis
Specimen A-2  Bending about x-axis
Specimen A-3  Twisting test

Fig. 6.1 LOAD-DEFLECTION RELATIONSHIP FOR A-SERIES SPECIMENS
Specimen B-1  Bending about Y-axis
Specimen B-2  Bending about X-axis
Specimen B-3  Twisting test

Fig. 6.2  LOAD-DEFLECTION RELATIONSHIP FOR
B-SERIES SPECIMENS
Specimen C-1  Bending about y-axis
Specimen C-2  Bending about x-axis
Specimen C-3  Twisting test

Fig. 6.3  LOAD-DEFLECTION RELATIONSHIP FOR
C-SERIES SPECIMENS
COMPUTER PROGRAM
C PROGRAM TO COMPUTE THE FLEXURAL RIGIDITIES (BENDING TEST)

DIMENSION V(4,4),Y(4,4),X(4,4),E(4,4),SD(4,4),SF(4,4),A(40),B(40),
CEEUM(4C),W(40),SXX(2),SXY(2)

REAL K(40),SXX(4C)

READ *N

M = N

DO 30 I=1,N

DO 20 I=1,N

REAC.(X(I),I=1,N),J=1,9)

PRINT 12,(A(I),I=1,9)

PRINT 13,(B(I),I=1,9)

DO 37 I=1,N

PRINT 14,(x(I),y(I),j=1,9)

PRINT 15,(y(I),j=1,9)

CONTINUE

PRINT *,LOAD VS DEFORMATION I TC DEFLECTION 9 (INCHES)

PRINT *,INTERCEPT A = *,SLOPE B = *

REDEFINE B, SLOPE (UNIFORM DEFLECTION) FOR EACH LOCATION, AS MATRIX

C DISPLACEMENTS ARE CONSIDERED POSITIVE DOWNWARD

MOMENT ARE CONSIDERED POSITIVE WHEN PRIMARY CURVATURE IS CONVEX UPWARD

LENGTH OF HALF CHORD OF A CURVATURE, AA = 6 IN.

LENGTH OF HALF CHORD OF A CURVATURE, AA = 7 IN.

Determination of Curvatures

K1 = CURVATURE 1-2-3 AT POINT 2
K2 = CURVATURE 4-5-6 AT POINT 5
K3 = CURVATURE 7-8-9 AT POINT 8
K4 = CURVATURE 1-4-7 AT POINT 4
K5 = CURVATURE 2-5-8 AT POINT 5
K6 = CURVATURE 3-6-9 AT POINT 6
K7 = CURVATURE 1-5-9 AT POINT 5
K8 = CURVATURE 3-5-7 AT POINT 5
K9 = CURVATURE 4-5-9 AT POINT 6

K1 = (W(2) - (W(1) + W(3))/2)*2./AA**2
K2 = (W(5) - (W(4) + W(6))/2)*2./AA**2
K3 = (W(8) - (W(7) + W(9))/2)*2./AA**2
K4 = (W(4) - (W(7) + W(1))/2)*2./AA**2
K5 = (W(6) - (W(8) + W(2))/2)*2./AA**2
K6 = (W(9) - (W(7) + W(2))/2)*2./AA**2
K7 = (W(9) - (W(1) + W(8))/2)*2./AA**2
K8 = (W(5) - (W(4) + W(6))/2)*2./AA**2

IMPROVED BY Dr. L. A. W. REID AND C. M. E. HILTON.
DETERMINATION OF ELASTIC CONSTANTS SXX, SXY, SYY

\[ SXX(I) = \frac{K(I) + (I)}{M(I)} \]

\[ SXX(I) = \frac{SXX(1)}{3} + \frac{SXX(2)}{3} + \frac{SXX(3)}{3} \]

\[ SXY(I) = \frac{SXY(1) + SXY(2) + SXY(3)}{3} \]

PRINT50, SXX(I), SXY(I)

FORMAT('AVERAGE VALUES', SXX=E15.6, SXY=E15.6)

PRINT51, DX, DY

FORMAT('BENDING RIGIDITIES', DX=E15.6, DY=E15.6)

STOP

END

SUBROUTINE LINE (N, X, Y, A, B, SD)

METHOD OF LINEAR REGRESSION

DIMENSION X(40), Y(40), A(40), B(40), SD(40), EESUM(40), C(40), E(40),
CXXSLM(40), YSUM(40), YSUM(40), XSUM(40)

K = 0

DO 1 I = 1, K

XSUM(I) = 0.0

YSLM(I) = 0.0

YSUM(I) = 0.0

YSLM(I) = 0.0

1 FORMAT('EESUM(1) = 0.0')

DO 2 J = 1, N

XSUM(I) = XSUM(I) + X(J)

YSUM(I) = YSUM(I) + Y(J)

YSUM(I) = YSUM(I) + Y(J) Y(J)

2 XSUM(I) = XSUM(I) + X(J)**2

C(I) = N*XSUM(I) - XSUM(I)**2

A(I) = (YSUM(I)*XSUM(I) - XYSUM(I)*XSLM(I))/C(I)

CONTINUE

DO 3 I = 1, K

E(I) = Y(J) - (A(I) + B(I)*X(J))

3 EESUM(I) = EESUM(I) + E(I)**2

CONTINUE

RETURN

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<th>3</th>
<th>4</th>
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<th>6</th>
<th>7</th>
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<td>70.60</td>
<td>199.10</td>
<td>58.29</td>
<td>94.50</td>
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<td>101.80</td>
<td>154.60</td>
<td>74.30</td>
<td>145.00</td>
<td>156.00</td>
<td>120.80</td>
<td>102.00</td>
<td>165.20</td>
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<td>-0.005520 -0.006505 0.004550 0.005171</td>
</tr>
<tr>
<td>2420.9</td>
<td>-0.013402 0.013176 0.011316 0.012174</td>
</tr>
<tr>
<td>3731.3</td>
<td>0.012413 0.002661 0.010211 0.012645</td>
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</table>

| INTERCEPT A = -0.486E-01 -0.441E-01 -0.404E-01 -0.358E-01 |
| SLOPE B = 0.258E-04 0.216E-04 0.221E-04 0.302E-04 |

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<td>-0.258E-04 -0.216E-04 -0.221E-04 -0.302E-04</td>
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<th>VI TO V9 (IN/M)</th>
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<td>56.20 107.40 102.30 77.00 68.20 104.50 57.20 44.60 66.30</td>
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<tr>
<td>1409.8</td>
<td>92.00 148.00 116.50 115.00 115.40 140.00 88.10 82.00 107.20</td>
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<td>1730.1</td>
<td>120.00 209.00 178.00 162.00 188.00 182.00 114.00 141.00 154.00</td>
</tr>
<tr>
<td>2066.4</td>
<td>177.00 262.00 236.00 225.00 266.00 241.00 156.00 198.00 206.00</td>
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<th>DEFORMATION 1 TO DEFORMATION 9 (INCHES)</th>
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<tr>
<td>1409.8</td>
<td>0.000777 0.000536 0.004330 0.000464</td>
</tr>
<tr>
<td>1730.1</td>
<td>0.013529 0.011506 0.009584 0.011120</td>
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<tr>
<td>2066.4</td>
<td>0.022167 0.019228 0.016926 0.012666</td>
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</tbody>
</table>

| INTERCEPT A = -0.302E-01 -0.341E-01 -0.297E-01 -0.301E-01 |
| SLOPE B = 0.304E-04 0.272E-04 0.226E-04 0.245E-04 |

<table>
<thead>
<tr>
<th>LINEAR DEFORMATION w IN/LU</th>
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</thead>
<tbody>
<tr>
<td>-0.304E-04 -0.272E-04 -0.226E-04 -0.245E-04</td>
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<table>
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<th>AVERAGE VALUES</th>
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<tr>
<td>y = -0.42485E-06</td>
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<th>BENDING RIGIDITIES</th>
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<tbody>
<tr>
<td>L = 0.14023E-07</td>
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<tr>
<td>Y = 0.8627E-06</td>
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<th>CURE USAGE</th>
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<tr>
<td>DIAGNOSTICS</td>
<td>NUMBER OF ERRORS = 6</td>
<td>NUMBER OF WARNINGS = 0</td>
<td>NUMBER OF EXTENSIONS = 0</td>
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<tr>
<td>COMPIL TIME</td>
<td>0.61 SEC</td>
<td>EXECUTION TIME</td>
<td>0.34 SEC</td>
<td>WATFIV - JUL 1975 VLA 23.47.1A TUESDAY 30 AUG 77</td>
<td></td>
</tr>
</tbody>
</table>
SJOB LATFLV X<xxxxxxx

PROGRAM TC COMPUTE TORSINAL RIGIDITY (TWISTING TEST)

SCALE FACTOR OF TRANSDUCER, SF IN/V

LAD X LBS

INPT TRANSDUCERS READINGS, V MV

DIMENSION V(I),V2(I),V3(I),V4(I),V5(I),V6(I),V7(I),V8(I),

CV4(I),CV5(I),CV6(I),CV7(I),CV8(I)

REAL XXY,XX,XXI,XXH,

READ A

REAL V(I),V2(I),V3(I),V4(I),V5(I),V6(I),V7(I),V8(I)

READ (V(I),V2(I),V3(I),V4(I),V5(I),V6(I),V7(I),V8(I))

PRINT 3C

10 FC*AT (-F6.1,6X,9F11.2)

11 CONTINUE

12 PRINT 30

13 FORMAT (2X,9F10.2)

14 PRINT 'LAD VS V1 TO V9 (MV/INCH)',

PRINT, 11.X(I),V1(I),V2(I),V3(I),V4(I),V5(I),V6(I),V7(I),V8(I),V9(I)

15 CONTINUE


17 PRINT 13.B1,B2,B3,B4,B5,B6,B7,B8)

18 CONTINUE

19 PRINT 14.P1,P2,P3,P4,P5,P6,P7,P8)

20 CONTINUE

21 PRINT 15.P9,P10,P11,P12,P13,P14,P15,P16)

22 CONTINUE

23 PRINT 16.P17,P18,P19,P20,P21,P22,P23,P24)

25 CONTINUE

26 CONTINUE

27 CONTINUE

28 CONTINUE

29 CONTINUE

30 CONTINUE

31 CONTINUE

32 CONTINUE

33 CONTINUE

34 CONTINUE

35 CONTINUE

36 CONTINUE

37 CONTINUE

38 CONTINUE

39 CONTINUE

40 CONTINUE

41 CONTINUE

42 CONTINUE

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90 CONTINUE

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96 CONTINUE

97 CONTINUE

98 CONTINUE

99 CONTINUE

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14 FORMAT ('-1.', 4X, 'LINEAR DEFLECTION, W IN/LB/17X, 9E11.3)
Determination of curvatures
Length of half chord of a curvature, AA = AB = 8 in.
KA - curvature 1-5-7 at point 5
KB - curvature 1-6-9 at point 5
According to Timoshenko, twisting moment, WXY = (P/2)/2
P is the load

AA = 8
BB = 8
WXY = -.22
WAC = W5 = (W7 + W9)/2.
W6 = W9 - (W7 + W1)/2.
KA = 2.*WAC/(AA**2 + BB**2)
KB = 2.*W6/(AA**2 + BB**2)

KAVCB = (KA-KB)/2.

Determination of torsional rigidity

CPJ = 0.*WXY/AVGAB

RXY = W7/6.

K1 = (W5 - (W6 + W9)/2.)*2./AA**2
K2 = (W5 - (W6 + W9)/2.)*2./BB**2

CXYAPS = ABS(DXY)

Print 'results'
Print 'curvatures K'
Print 'torsional rigidity DXY'
Print '
Print 15, KA, KB, K1, K2, DXYAPS

15 FORMAT ('-1.', 'KAI = ','E15.6/', 'KBI = ','E15.6/', 'K1 = ','E15.6/', 'K2 = ','E15.6/', 'DXY = ','E15.6/)

END

SUBROUTINE STLINE (N, X, Y, A, B, SD)

Method of linear regression

CIVEN'SCH X(2C), Y(2C), E(2C)
XXSLM = 0
XYSLM = 0
YYSLM = 0
XXSLM = 0
XYSLM = 0
YYSLM = 0
XSUM = 1, N
XXSLM = XSLM + X(I)
YYSLM = YSLM + Y(I)
XYSLM = XYSUM + X(I)*Y(I)

CALCULATE THE INTERCEPT A AND THE SLOPE M OF THE STRAIGHT LINE

C = WXYYSUM - XYSUM**2
A = (XYSUM**2 - XXSLM*XYSUM)/C
Ec = (N*XYSUM - XSLM*YSUM)/C
Ec = I = 1, N
E(i) = Y(I) - (A + BX(I))

2 ECPSM = EESUM + E(I)**2
Ec = SORT(EESUM)
END

ENTRY
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<th>Load Vf</th>
<th>Load Vf</th>
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| INTERCEPT A | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 | -0.4997E-01 |
| SLOPE B | 9.65E-04 | 9.11E-01 | 0.227E-03 | 0.142E-03 | 0.104E-03 | 0.227E-03 | 0.112E-03 | 0.504E-04 |

LINEAR DEFLECTION, W IN/IN
-0.66E-04 -0.114E-04 -0.227E-04 -0.142E-03 -0.104E-03 -0.227E-03 -0.112E-03 -0.504E-04
VITA AUCTORIS

SATISH KUMAR BALI

1951 Born on 16th of April in Hoshiarpur, India.

1967 Completed secondary school education attending different schools including St. Paul's School, Palampur (India) and was awarded merit scholarship.

1972 Graduated with Bachelor of Science in Civil Engineering from University of Delhi, Delhi, India.

1974 In June, graduated from Punjab University, Chandigarh, India, with a M.Sc. degree in Structural Engineering.

1974 Joined the National Industrial Development Corporation, New Delhi, India, as a Junior Design Engineer.

1976 Enrolled as a graduate student to pursue a Master's degree in Civil Engineering at the University of Windsor, Windsor, Ontario, Canada.

1978 In March, appointed as an Assistant Design Engineer with Detroit Water Board, Detroit, Michigan, U.S.A.

1978 In September, joined as a Research Engineer in the Structural Analysis Department of Ford Motor Company, Dearborn, Michigan, U.S.A.