Computation of electric fields in and around high voltage insulators.

Mohammad Javed Khan
University of Windsor

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COMPUTATION OF ELECTRIC FIELDS
IN AND AROUND
HIGH VOLTAGE INSULATORS

A Thesis
Submitted to the Faculty of Graduate Studies
Through the Department of Electrical Engineering
in Partial Fulfilment of the Requirements for the
Degree of Master of Applied Science at the
University of Windsor

by

Mohammad Javed Khan

Windsor, Ontario
Canada

November, 1980
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TO MY MOTHER
ABSTRACT

The application of the charge simulation technique to multiple dielectric systems (two-dielectric) is the focus of this effort. The electric field distribution is evaluated in and around insulator elements used on practical high voltage transmission lines. For this purpose, two insulators namely a Suspension Insulator section and a Long Rod Polymer Insulator were studied. The agreement of some experimental results obtained elsewhere with the computed ones for the long rod insulator is of the order of the accuracy with which measurements of potential can be made. A detailed study of the errors and discrepancies arising in the field conditions in the resulting models is presented. Suggestions for the application of the method in high voltage insulator design are made.
ACKNOWLEDGEMENTS

I am most grateful to Professor Philip Alexander and express my sincere appreciation to him for suggesting the problem, for his supervision of this work and of course for his tireless reading of this dissertation in manuscript.

I am also indebted to Mr. M. D. Baillargeon, academic programmer, Computer Centre, University of Windsor for his help in detecting errors in the computer programs.

Thanks are also extended to Mr. W. A. Chisholm, Engineer Transmission and Special Projects Section, Electrical Research Department, Ontario Hydro, for allowing the presentation of their experimental results for the long rod polymer insulator.

Finally, I would like to thank my parents and my complete family tree, not for having anything to do with the compilation of this thesis, but for helping to further my career either deliberately or by accident.
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I. INTRODUCTION

1.1 Preamble

The term "high voltage insulator" designates, in general, all devices that serve for the insulation of conductors at high potential from ground. They can be grouped as bushings, interior supports and line insulators. In the early days of electrical distribution of power the insulator problem was unimportant. The insulator gave more satisfactory service than the rest of the apparatus essential to the generation and distribution systems. As long as the voltages were low the electric field distribution was of relatively small importance. As the transmission distances and therefore, the economic transmission line voltages, increased the insulator problem became more acute. As a result, with the increased voltages came an increasing amount of insulator trouble resulting in frequent failures of electric power systems at a rate that for a time threatened the success of high voltage transmission of electrical energy [1].

During the past few years there has been a remarkable development throughout the world in high voltage transmission systems. With the growing demand for electrical energy, the present trend is to transmit large blocks of power at an ultra high voltage (UHV*) level which further necessitates the thorough investigation of the insulation problem of transmission lines for successful operation with minimum interruptions.

* (UHV - 1,000 KV and above)
The problems associated with transmission at extra high-voltage are the appearance of extremely high electric fields near the conductor surface as well as on some grounded surfaces. Due to extremely high electric field, radio interference and audible noise generated may reach levels higher than those permitted by regulations or higher than desirable. An important problem associated with EHV/UHV lines is the occurrence of corona around the conductor surface [2]. Corona is considered as the partial breakdown of air surrounding the metallic parts of the insulator and line conductors. The electric power used in ionizing the air surrounding the conductor parts of the insulator is known as corona loss [3]. The corona loss represents a loss of revenue and therefore it must be minimized for economical reasons.

When high voltage transmission lines are in normal operation, small spark discharges occur on the insulators even in clean dry conditions [4]. These discharges, in common with all spark discharges, contain components of radio frequency, so that a radio-frequency disturbing field is radiated from the line conductors. If the discharges became more intense because of humid weather, contaminated insulators, or faulty contact between the hardware and the porcelain, the intensity of the radiation increases, and may interfere with the reception of radio signals by receivers in the vicinity of the line.

In recent years, the problem of radio interference has become more acute, partly owing to the extension of high voltage
systems and partly owing to the high sensitivity of modern receivers. A well insulated high voltage line, properly maintained, is not a serious source of interference, but intense interference can arise if an abnormal discharge is present. The insulator should therefore be free from discharges at a voltage which is usually specified at 20 to 40 percent above the phase to earth voltage [5].

Recognizing the importance of providing adequate insulation strength to high voltage transmission systems, it is essential to have an accurate knowledge of the electric field distribution in and around high voltage insulators.

This knowledge of the electric field distribution can also be applied to the determination of breakdown strength of the insulator material as well as corona noise generation susceptibility and is of great help to the designer of EHV and UHV transmission lines.

1.2 Statement of Problem

In general there are two types of transmission line insulators - Pin type and Suspension type.

The use of Pin type insulators is limited to voltages below 50 kV since at high voltages they become uneconomical; the cost increases rapidly as the voltage increases and is proportional to $V^x$ where $x$ is greater than 2 [6]. A further disadvantage of Pin type insulators is that replacements are expensive. For these reasons high voltage lines are insulated by suspension insulators in which case, as the name indicates, the line conductor is suspended below
the support by means of the insulator or insulators.

Several important advantages result from this system:

1) Each insulator is designed for a comparatively low working voltage and the required total insulation is obtained by using a "string" of a suitable number of such insulators.

2) In the event of failure of an insulator, one unit instead of the whole string, may need to be replaced.

3) The mechanical stresses are reduced since the line is suspended flexibly; with Pin type insulators the rigid nature of the attachment results in fatigue and ultimate brittleness of the conductor due to the intermittent nature of the stress due to wind loads. Also, since the string is free to swing, there is an equalization of the tensions in the conductors of successive spans.

4) In the event of an increase in the operating voltage of a line, the insulation requirements can be met by adding the appropriate number of units to the string instead of replacing all insulators as would be made necessary with the Pin type.

Owing to the present day wide application of Suspension insulators for high voltage transmission systems, a Suspension insulator of Canadian Ohio Brass Company Limited (Catalogue -71, page 11, 1977) as shown in Fig. (1.1) is considered for the analysis of electric field distribution.

As an extension of the work, a newly developed insulator
shown in Fig. (1.2) called a long rod polymer insulator is also analyzed. This unit is under development by Ontario Hydro and has not yet been installed on actual high voltage transmission lines, being in the experimental process.

Transmission line insulators must bear the flashover stress under very difficult circumstances, namely dry, wet and polluted conditions. Line insulator's shapes have become complicated chiefly because of this requirement. The thickness of the insulation between the metallic parts of the insulator and the length of the sparking distance are not the only factors which decide the electrical performance of an insulator, but the distribution of the electrostatic field and equipotential surfaces is also of considerable importance in determining corona formation and flashover voltage.

The purpose of this thesis is therefore to compute the electric field distribution in and around high voltage suspension insulators used on practical high voltage transmission lines. This knowledge of the electric field distribution also enables us to determine the breakdown strength of the insulator material.

1.3 Organization of Thesis

Chapter II gives a detailed and thorough review of the available literature on the Charge Simulation Technique used for the calculation of electric fields in high voltage systems. The application of the technique to single dielectric systems -- high voltage electrodes, transmission lines and multiple dielectric systems are reviewed critically in separate sections.
PRINCIPAL FEATURES
OF A C-O-B SUSPENSION INSULATOR

Figure 1.1: Practical Suspension Insulator
shown in Fig. (1.2) called a long rod polymer insulator is also analyzed. This unit is under development by Ontario Hydro and has not yet been installed on actual high voltage transmission lines, being in the experimental process.

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Figure 1.2: Long Rod Polymer Insulator
The accuracy of the method and the computation time requirements are also reviewed and conclusions have been derived.

Chapter III presents a preliminary analysis. A simple example of a hemispherically capped cylinder is demonstrated using a combination of point and line charges. The effect on the potential distribution of the number of charges, position of the boundary point (on the hemisphere of the cylinder), and the length of the line charge segment is studied and conclusions have been derived.

Chapter IV is on the mathematical formulation of the problem. The equations necessary for simulation (two dielectric system only) are presented. The charge distribution in the suspension and long rod insulator are illustrated in separate sections. The rotationally non-symmetric problem of a rod insulator is approached making use of the partial symmetry present.

Chapter V is devoted to 'Results and Discussions'. Results for the suspension and long rod insulator are presented and discussed in separate sections. The experimental results obtained by Ontario Hydro for the long rod insulator are compared with the computed ones.

Chapter VI provides 'Conclusions and Recommendations'. In this chapter, important conclusions are presented and suggestions are made for the extension of the work.
II. REVIEW OF LITERATURE

2.1 General

The knowledge of the electric field distribution in the design of any high voltage system component is important. An error of a few percent in the estimation of the field and potential distribution may result in an unacceptable loss of accuracy/efficiency in the manufactured device or even the failure of the device. The prior knowledge of the field distribution will therefore accurately predict the design parameters and performance evaluation of devices by means of purely computational techniques.

There are several techniques that have been used to numerically calculate electrostatic field distributions. The calculation of electric fields requires the solution of Laplace's and Poission's equations with boundary conditions satisfied. This can be done either by analytical or numerical methods. However, in many cases physical systems are so complicated that analytical solutions are seldom applicable and hence numerical methods are commonly used for such applications [7, 8].

There are two distinct numerical techniques reported in the literature. The first method is based on difference techniques employing Laplace's and Poission's equations in the space where the field is desired to be evaluated. This is done by dividing the whole space into small regions and
Laplace's equation is then approximated within each small region by equations which relate the unknown potential of the region to the unknown potential in other regions and to known boundary potentials.

Many papers have been published on the solution of Laplace's equations by the Finite Difference Method and the Finite Element Method and they have been very extensively described in the literature [9, 10].

Difference techniques are very powerful when the region of interest contains a number of different materials or a dielectric constant which varies in space [11]. In more simple cases (such as the case where only conductors are present), Difference techniques may be more cumbersome than necessary due to the large number of linear equations that must be solved [12]. Furthermore, finite element and difference techniques are useful only in bounded regions, whereas, many physical problems of interest are unbounded. However, attempts have been made to use Difference techniques for the solution of electric field problems where the field is not bounded in space but extends infinitely far [13, 14]. In this method, an artificial finite boundary condition is initially introduced to allow solution and then iteratively removed, as the solution proceeds. In order to have a minimum effect of errors in the known potentials on the calculated potential values in the region of interest, the artificial boundary must be located far enough away from the region of interest. The price paid for doing this is that
Laplace's equations must be solved for a larger region than the region of interest. Thus a larger number of equations must be solved which adds to the computational time [15].

The second approach to the computation of fields is based on integral equations [16, 17].

By an application of Green's identity, Laplacian's equation \( \nabla^2 V = 0 \) can be expressed in integral form [18]. This can be done in two ways.

2.1.1 Electrode Surface Subsectioning

The electrode surface is divided into subsections with their associated charges. It will be assumed here that the medium consists of several different homogeneous materials and that as a result, the only charges are the surface charges. If this is not the case, then volume charges must be considered [19]. The integral equation can be solved for the surface charge densities by approximating the integral as a sum on subsections considered on the entire electrode surface. The sum is set equal to the known potential at the centre of each subsection. As a result of this discretization process, a set of linear algebraic equations is obtained in terms of unknown segments of charges which when solved with the digital computer gives values of these charges. This technique is a special case of the Moment method [20]. Once the charge densities are obtained, potential and electric field values can be evaluated at any point.
2.1.2 Interior Placement of Charges

Discrete charges are placed inside the electrode surface. The potential of the unknown surface charges is approximated by line, ring and point charges (instead of assuming them to be surface charges) placed at some distance behind the surface on which the potential is to be matched. The potentials due to these charges will be well behaved at, and in front of, the surface and thus can simulate the field of a surface charge (at least approximately).

This method of discretization is known as the Charge Simulation technique and was first presented by H. Singer [21]. In addition to two dimensional field calculations, Singer has reported results for three dimensional geometries with and without axial symmetry. He also studied problems with dielectric-dielectric boundaries.

The work presented in this thesis is based on Singer's technique of Charge Simulation. A survey of the available literature on the Charge Simulation technique was therefore carried out in order to observe the applicability of the method to coping with field problems associated with High Voltage devices.

The review is presented in the following sections.

2.2 Charge Simulation Technique

2.2.1 Conventional Method

The Conventional Charge Simulation technique is well presented by H. Singer [21]. In his method, the potential
of the unknown surface charges is approximated by three forms of charge arrangements i.e. line, ring and point charges depending upon the geometry of the High Voltage device at hand. The only difficulty with this idea of approximating the surface charges is that potentials near a uniformly charged surface are bounded, but potentials near line, ring and point charge are unbounded. In order to overcome this difficulty, Singer placed equivalent charges at some distance behind the surface on which the potential is matched.

The three forms of charge arrangements for a particular High Voltage electrode configuration have been seen to cover almost all possible needs of simulation to cope with the System Configurations [21, 22]. The point charge, due to its spherically symmetrical field behaviour, suits spherically symmetrical surfaces; the line charge (finite or infinite length) suits cylindrical configurations and the ring charge is used to model axially symmetrical profiles of High Voltage apparatus. An adequate combination of the three forms of charge can be made to simulate almost any practical electrode system [23].

In this method, the actual charge distribution on the conductor surface is represented by several fictitious line charges inside the conductor surface and their images represent the effect of the ground (if considered). These charges can be placed at any desired position inside the conductor surface. To satisfy the boundary conditions at the conductor
surface, one of the equipotential surfaces resulting from the fictitious charges and their images must coincide with the conductor surface. The potential at any point can be calculated by the superposition of the potential due to the individual initially unknown fictitious charges. Specifying the potential at various points on the conductor surface to be the values of potential desired there, yields a number of simultaneous equations in terms of the unknown line/ring/point charges depending upon the geometry at hand. These equations, when solved give the values of the desired charges. Having calculated the values of these charges, the potential at many other points on the conductor surfaces are computed to check whether the conductor surface results in an equipotential surface. Thus considering a set of "m" points selected on a surface at potential "$\phi$" and "m" charges considered inside the conductor(s) leads to a system of "m" linear equations for the "m" charges as shown below:

$$\sum_{j=1}^{n} P_{ij} \cdot Q_j = \phi_i \quad \text{... 2.1}$$

where,

- $n$ = number of charges in the system
- $m$ = number of contour points at which the potential is specified
\( P_{ij} \) = the associated potential coefficient

in matrix form,

\[
\begin{bmatrix} P \end{bmatrix} \cdot \begin{bmatrix} \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{\phi}_c \end{bmatrix}
\]

... 2.2

Ordinarily, the number of boundary points "m" is equal to the number of charges "n". Given a particular configuration, the co-efficients \( P_{ij} \) will be determined by the boundary conditions. Once the system of linear equations is solved for the charges \( \mathbf{Q} \), then it must be determined whether the calculated set of charges fits the boundary conditions. For instance, the potential at a number of check points on the boundary can be calculated. The difference between these potentials is a measure of the accuracy of the simulation.

The basic principle described above is well known in Field Theory. Together with suitable ways of discretization, this known principle forms the basis of electric field computation of two and three dimensional systems as presented by H. Singer et al [21].

2.2.2 Application to Single Dielectric Systems

2.2.2.1 High Voltage Electrodes

With the increasing variety in the profiles of high voltage apparatus, more attention is being paid to surfaces in proximity to these bodies. Sharply tipped termination with high voltage electrodes are of special importance as they are the possible sources of undesirable local ionization.
The charge simulation technique is applied for the calculation of two-dimensional and three-dimensional fields with and without axial symmetry. Considerable attention has been given to electrode configurations used in high voltage experimentation by many authors. H. Singer [21] calculated the electric field between a conductor strip and a plane and the influence of an earthed cage on the field of a sphere-gap was analyzed as an example of an axial symmetrical problem. He also explained the principle of the method applied to three-dimensional fields without axial symmetry by a single example, a rod-rod gap with a trigger electrode.

Abu-Seada [22] applied the charge simulation method to the calculation of electric fields of a rod-rod gap. H. Parekh et al [23] computed electric fields for rotationally symmetric electrodes and A. Yializis et al [24] calculated the potential distribution for a rod-plane electrode configuration and presented an optimized version of the charge simulation technique. Masanori et al [25] examined the field distribution of multiple axisymmetric electrodes displacing and rotating the plane of symmetric model charges. They made calculations of a three-dimensional axisymmetric gap for a tilted upper rod-plane electrode arrangement and the rod-plane with lower rod arrangement.

2.2.2.2 Transmission Lines

The application of the charge simulation technique to single dielectric systems finds its greater use in the evaluation of electric fields around high voltage

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overhead transmission lines. Since the phenomena of corona and radio interference are more acute with transmission at EHV and UHV levels, electric field calculations are made for bundle conductors used for such transmission lines. The first attempt was made in the early thirties by Crary and reported by Clarke [26]. Crary's formulation of electric field was limited to a ratio of spacing to diameter of subconductors equal to or greater than five. His formulation was modified by Poristspy, reported by Clarke, using complex variables and the transformation techniques but still was not applicable for a bundle of more than two subconductors. Adams [27] calculated the electric field for a three phase AC transmission line. He represented the charge on each conductor by a single line charge at the centre of the conductor. The effect of ground was taken into account by considering an image of each line charge below the ground plane. He calculated first the values of the line charges by solving simultaneous equations which related the line charge with the potentials on the conductor. Once these values of line charges were obtained, the electric field at any desired point was calculated by superimposing the contributions due to the individual line charges. The results obtained by Adams were not accurate because of the fact that the equipotential surfaces resulting from the line charges (assumed to be at the centre of the subconductor) did not coincide with the subconductors surfaces. This meant that one of the boundary conditions which required that subconductor surfaces be equipotential.
surfaces was not satisfied.

Sarma and Janischewskyj [28] used the method of successive images to calculate the electrostatic field of a system of parallel cylindrical conductors. Their method was based on the concept of representing the distributed charge on the conductor's surface by a system of line charges inside the conductor's surface in such a way that one of the equipotential surfaces resulting from the line charges coincided with the conductor surface. Using the process of successive images, Sarma and Janischewskyj calculated the electric field around the bundle conductors. The ground effect was taken into account by placing images of line charges below the ground plane. Although the values of electric field obtained by this method were accurate for higher ratios of \( r/2s \) (\( r \) = radius of cylinder, \( 2s \) = distance between line charge and the centre of the cylinder), the number of line charges required to simulate the parallel conductor system became very large for the electric field computation for bundle conductor transmission lines.

Abu-Seada and Nasser [29, 30] used the method of charge simulation to calculate the electric field and potential around a twin-subconductor bundle. In this method, the actual charge distribution on each subconductor surface was represented by several fictitious charges placed inside each subconductor. These line charges of unknown magnitude were placed on a fictitious cylinder whose radius was half of the subconductor radius. The correct angular
positions of these line charges on the fictitious cylinder were determined by a trial and error method to achieve better accuracy of the results. The results obtained by Abu-Seada and Nasser gave an error of about one percent in the values of the direction of electric field. However, their method was applicable for the case of twin subconductor bundles. Parekh [31] described a method based on charge simulation which used arbitrary specifications of location of the images and boundary points and was applied to bundles of up to eight cylindrical subconductors.

2.2.3 Application to Multiple Dielectric Systems

In the design of any electrical system, a potential distribution of the system is essential for the calculation of electric stresses involved. The design may vary from a simple situation to a more complex one. The advent of ever faster digital computers has made it possible to synthesize in great detail hundreds of alternative designs to give specifications for many types of electrical machines. Electric machines and high voltage devices often contain multi-dielectric insulation layers between conductors or cores etc. For the design of electric insulation of any such device and for the withstand voltage tests of dielectrics, it is important to have accurate information of the electric field distribution in such systems.

Different approaches have been presented by various authors [32, 33, 34] to calculate electric fields in electric
systems involving multiple dielectrics, the discussion of which is beyond the scope of this review.

H. Singer et al. [21] developed a method of calculating electric fields in two-dielectric arrangements which formed the basis of this work. This method will be explained fully in Chapter IV. As an application of this method, they applied it to a sphere electrode with a dielectric slab. For studies of the electric strength of solid insulating material, flat-slab specimens are often tested by such arrangements (i.e. a dielectric slab between a sphere and a plane) [34]. As an example, they applied the method to a practical electrode arrangement used for shielding of high voltage apparatus. They also applied this method to the calculation of the field strength at the shielding electrodes of UHV testing transformers.

Mukerjee and Roy [35] applied the same method of computation as proposed by Singer to the calculation of fields in a multiple dielectric, three dimensional system. They applied the method to a parallel plate arrangement as a test example and then applied it to a disc insulator. The insulator shape was simplified to a truncated cone as shown in Fig. (2.1).

The method of computation in two dielectric media proposed by Singer received further encouragement from the researchers. More recently T. Sakakibara [36] applied this method to the calculation of three dimensional asymmetric fields for a post type spacer with two dielectric media. A
Figure 2.1: Suspension Insulator Simplified to a Truncated Cone [35]
simplified post type spacer used in SF₆ gas insulated apparatus was selected as a calculation model. A high voltage impulse test was also carried out by the authors to verify the validity of the method and the results were found to be in good agreement. Takeshi and Oyozo [37] described a method to determine the electric field of a point charge by the method of images in three or more dielectric layers on a plane conductor. They applied this method in combination with charge simulation to calculate the field of a spherical conductor situated in three dielectric layers on a plane conductor. In this method ring charges in the sphere together with their images were used for representing the field.

Tadasu [38] described a method to study the field behaviour near singular points in composite dielectric arrangements. This paper analyzes the field behaviour of a contact point of the boundary with an electrode numerically by the charge simulation method. A comparison with the analytical solution gave fairly good agreement between analytical and numerical values.

2.3 Accuracy of the Method

A measure for the accuracy of the model is the "Potential Error" at various check points on the surface of the electrode between two adjacent contour points [21]. This "Potential Error" is defined as the difference between the specified potential of the electrode and the computed potential. Experience shows that the error of the electric field is up to ten times greater than the corresponding potential error.
Therefore, the potential error should be less than 0.1% in an area of the electrode if a field strength accuracy of 1% is desired in this area.

The significance of keeping the "Potential Error" below 0.1% is important since all corona calculations are very sensitive to the values of electric fields, a very small error in the values of electric field might result in a very large error in the values of the corona onset voltage, corona loss and radio interference (as estimated by Parekh [39] even a very small maximum error in the potential (for example 0.2%) does not ensure a small error which can, in the same region be as large as 3.5 percent). The practical goal for accuracy in the simulation of electrodes is limited by the manufacturing tolerences of conductors. Similarly the accuracy of the simulation of dielectrics has as its practical goal, the accuracy of the measurement of dielectric constant values.

One difficulty in using the charge simulation technique is that the location of equivalent charges is difficult to obtain analytically [24]. As a result, the location of the equivalent charges is guided by experience. Thus the accuracy is usually checked after the problem is solved by determining how closely the boundary conditions are matched along all the interfaces. Also the accuracy in the solution is sensitive to the number of charges chosen for a particular problem. The larger the number of charges, the better is the solution, provided computer time and memory
requirements are of relative importance. Roy and Mukerjee [35] reported that for an Air-Dielectric arrangement for a Disc insulator, an increase in the number of charges modifies the results but a point is reached beyond which this increase has no appreciable bearing on the solution on the conductor or dielectric interfaces. This indicates that the limit of accuracy has been attained. More recently Beasley et al. [40] made a comparative study of the three methods, namely the Charge Simulation Method, the Finite Element Method and the Monte Carlo method and applied them to different geometries of a practical engineering nature. They concluded that for some very large and complex configurations it may not be possible to obtain satisfactory solutions using only one method. They suggested that in such cases the Monte Carlo Method or the Charge Simulation Method may be used to derive a first approximation followed by the Finite Element Method within some reduced subregion of interest.

2.4 Computation Time Requirements

The significance of precision in computing potential at the electrode surface and thereby calculating the error can not be denied but improving the precision of results beyond a given limit may not be worth as much as obtaining a given precision with a maximum saving of computer time and memory requirements. In this connection, any algorithm aimed at reducing overall computer time and memory requirements, precision being equal, will appear to be extremely valuable since, in this way, either the cost of a given computation
could be reduced, or at a given cost, a more complex problem could be solved.

Keeping in view this aspect of the problem attempts have been made by different authors proposing optimization techniques to reduce notably the cost of numerical field computation on the basis of a fixed degree of precision required. A fitting-oriented modification to the charge simulation method in estimating electric fields is introduced by H. Anis et al. [41] which appreciably reduces computation time and cost. In this method, the multiple linear regression makes it possible to reduce the size of the simulating charge system without altering the selected potential boundary points. Another optimization approach for the computation of electric fields, based on Charge Simulation technique is described by A. Yializis et al. [24]. The objective function for optimization in this method was the accumulated square error. The position of charges and their values were chosen to be the variables of optimization subject to constraints imposed according to the nature of the problem described. The objective function was minimized to 1%. However, the optimization algorithm (Rosenbrok's Optimization Technique) used by the authors is not the most rapidly converging technique and therefore savings in computation time did not result. Fast convergence techniques such as Davidson's method modified by Fletcher and Powel have been suggested for reducing computation time with comparable accuracy. Other factors that could influence the computation time according
to the authors [24] are the initial values of the optimization parameters and the effectiveness of the objective function. The latter factor is important, since for more complex configurations it is possible that the minimum accumulated square error may not be an efficient criterion.

A similar approach but with unconstrained optimization is presented by Y. L. Chow et al. [42] and has been applied to a number of normally encountered geometries in engineering applications. The objective function assumed in this is the mean square error of the potentials and the optimization technique used is due to Fletcher [43] which is one of the most powerful techniques for unconstrained optimization. The advantage of using Fletcher's algorithm is that as the gradient "g" \((g = \nabla U, \ U \text{ being the objective function})\) is computed, the electric field intensity \((\mathbf{E} = \nabla \phi)\) on the conducting boundary is implicitly obtained. Therefore it can be extracted without further computation.

A. Mohsen and M. Salam [44] presented a development of the charge simulation technique for the calculation of electric field around conductor bundles of EHV transmission lines. In this method a known initial set of charges is introduced which are deduced from the analysis of a similar but simpler problem. These initial charges may be used in addition to a set of unknown charges which are determined from the boundary conditions. The better the initial charges, the lower will be the number of unknown charges. This will lead to a considerable saving in computational time. More recently Sakakibara
et al. [36] described a modification of the charge simulation method applicable to three-dimensional asymmetric fields with two dielectric media (initially proposed by Singer [21]). Since the formulation of a large potential coefficient matrix results in extensive computation time and memory requirements, this paper takes advantage of symmetry in the problem wherever it exists.

A comparison with the finite difference method shows that the charge simulation technique leads to shorter computation times in many geometries used in high voltage technology [21].

2.5 Conclusions

The Charge Simulation Technique is a powerful tool for solving Laplace's equation. It has been successfully applied to practical engineering problems of two and three dimensional configurations and also to geometries involving multiple dielectric layers.

The following conclusions are derived from the review presented in this chapter.

1. The field strength can be calculated analytically using the numerically obtained charge values for a variety of electrode and dielectric arrangements.

2. It is not necessary that the field region be limited by a closed boundary.

3. The computation of three-dimensional fields without symmetry is possible with a reasonable amount of computation. The Charge Simulation technique leads to computation times
which have been observed to be one quarter of the value for the finite difference technique in many geometries used in high voltage technology.

4. For some very large and complex configurations, the Charge Simulation method may be used to derive a first approximation followed by the Finite Element method within some reduced subregion of interest.

5. The accuracy of the Charge Simulation method is sensitive to the location of the charges, the boundary points where the boundary conditions are satisfied, and the number of charges.
III. PRELIMINARY ANALYSIS

3.1 General

Methods based on charge simulation of electrode systems have been frequently employed to obtain a numerical solution for the non-uniformly distributed field around the system of particular interest. Assuming one form of symmetry or another for the electrode configuration, three forms of charge arrangement have been seen to be suitable for modelling, in most cases. These are the point charge which tend to suit spherically shaped surfaces, the axial line charge which accommodates cylindrical configurations, and the ring charge for axially symmetrical profiles. However, an adequate combination of the three forms of charge can be made to simulate many practical electrode systems having symmetrical or non-symmetrical configurations.

The pin of the suspension insulator shown in Fig.(3.1) has a major degree of cylindrical symmetry. Therefore for the calculation of electrostatic fields, the distributed charge on the cylindrical surface of the pin is replaced by discrete line charges arranged inside the pin (It is assumed that the field configurations at power frequencies can be considered to be quasi-static for the system components of interest so that static field solutions are appropriate). In order to analyze the effect of the number of discrete charges and the length of line charges on the field distribution, an
Figure 3.1: Pin of a Practical Suspension Insulator
example of a hemispherically capped cylinder shown in Fig. (3.2) is considered.

3.2 Basic Principles

The equation for an electrostatic field in a homogeneous medium is the well known Poisson equation.

\[ \nabla^2 \phi = -\rho/\varepsilon \] ... 3.1

where,

\[ \phi = \text{Potential} \]
\[ \rho = \text{Volume charge density} \]
\[ \varepsilon = \text{Permittivity of the medium} \]

For zero charge density Eqn. (3.1) reduces to the Laplace equation:

\[ \nabla^2 \phi = 0 \]

Laplace's equation in cylindrical polar co-ordinates with axial symmetry is,

\[ \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = 0 \] ... 3.2

Equation (3.2) governs the field distribution of the devices of interest. Any function that satisfies these equations can be taken as the solution, provided that the boundary conditions are satisfied. The potential function for any charge distribution satisfies Laplace's equation.
Figure 3.2: Hemispherically Capped Cylinder.
Discrete line and point charges are placed on the symmetric axis of the cylinder.
outside the volume it occupies. Therefore, if the fictitious charges are placed outside the space in which the field is to be computed, the combined potential function due to these and the real space charges automatically satisfy the required Laplace or Poisson equation everywhere inside the region of interest. The magnitude and positions of these fictitious charges are chosen such that they satisfy the specified field conditions at the boundaries.

3.2.1 Principle of Calculation

For the calculation of electrostatic field, the distributed charge on the surface of conductors is replaced by "n" discrete charges located within the conductor surface. These charges may be any combination of charge types depending upon the geometry of the high voltage apparatus at hand.

In Fig. (3.3a), the electrode surface charges are replaced by the simulating charges $Q_j \ (j = 1, 2, \ldots, n)$ located inside the electrode and the same number of contour points $P_i \ (i = 1, 2, \ldots, n)$ as simulating charges are assumed on the electrode surface. The potential at the contour point $P_i \ (r_i, z_i)$ is calculated using the superposition of potential due to the simulating charges inside the electrodes. This potential is equal to the potential $\phi_c$ on the electrode boundary surface. If the conductor is located in the vicinity of the ground, then the effect of ground must be taken into account. Since the potential of ground is assumed to be zero, for each fictitious charge inside the conductor, an
Figure 3.3: Principle of the Charge Simulation Method

(a) Fictitious charges on electrode with contour points

(b) A point charge and its image
image charge of opposite polarity is appropriately placed with respect to the ground surface, Fig. (3.3b).

The potential at the i-th contour point of the conductor can be expressed by the linear relationship;

\[ \phi_i = \sum_{j=1}^{n} P_{ij} Q_j \quad \ldots \quad 3.3 \]

where,

\( \phi_i \) = Potential at the i-th contour point
\( P_{ij} \) = Potential co-efficient which depends on the type of charge, its location \((r_j, z_j)\) and the location of the point at which the potential is being specified \((r_i, z_i)\)
\( Q_j \) = Charge at the j-th location

For a point charge in a two dimensional arrangement the potential co-efficient can be expressed by Eqn. (3.4).

\[ P_{ij} = \frac{1}{4\pi\varepsilon_0} \left( \frac{1}{r_{ij}^+} - \frac{1}{r_{ij}^-} \right) \quad \ldots \quad 3.4 \]

where, \( r_{ij}^+ \) and \( r_{ij}^- \) (Fig. (3.3b)) are defined as,

\[ r_{ij}^- = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]
\[ r_{ij}^+ = \sqrt{(x_i - x_j)^2 + (y_i + y_j)^2} \]

\( x_i, y_i \) are the co-ordinates of the i-th contour point and
\( x_j, y_j \) are the co-ordinates of the \( j \)-th charge.

\[ \varepsilon_0 = \text{Dielectric constant of vacuum} \]

\[ = 8.854187 \times 10^{-12} \text{ F/m} \]

The application of equation (3.3) to \( n \) contour points leads to a system of linear equations for \( n \) charges which can be expressed in matrix form:

\[ \phi_C = [P] \cdot [Q] \quad \ldots \quad 3.5 \]

The charge values are obtained as the solution of Eqn. (3.5). The potential at any point in space is due to the integrated effect of these charges and their images. To satisfy the boundary conditions at the conductor surface, one of the equipotential surfaces resulting from the fictitious charges must coincide with the conductor surface. The measure of accuracy for the calculation can be defined to be the difference between the known potential of the electrode and the computed potential. This can be expressed as,

\[ \delta = \sum_{j=1}^{m} (\phi_{ij}) - \phi_C \quad \ldots \quad 3.6 \]

where,

\( \phi_{ij} = \text{Computed potential at the } i \text{-th contour point due to a charge located at the } j \text{-th position.} \)

\( \phi_C = \text{Conductor potential} \)

\( \delta = \text{Accuracy in solution} \)
3.2.2 Application Example

The principle of the charge simulation technique discussed above is applied to the hemispherically capped cylindrical electrode of Fig. (3.2) which shows the arrangement of the fictitious charges and the corresponding boundary points. Discrete line charges of finite length are placed on the axis for the cylindrical electrode portion and two point charges are located at the centre of the hemisphere to model the hemispherical parts of the electrode. The selection of the number of fictitious charges is restricted by the storage capacity of the computer but it is generally expected that increased accuracy of the charge simulation technique is obtained with an increased number of charges. In order to achieve higher accuracy, the above method can be applied in the following modified way for determination of the electric field and potential taking advantage of any symmetry existing in the problem.

For each contour point one fictitious charge is selected if the arrangement is wholly asymmetric. Then the term \( P_{ij} \) represents the effect of only one charge. If the arrangement is symmetrical about the X-X or Y-Y axis, then the two charges of equal value can be positioned symmetrically about the axis of symmetry as shown in Fig. (3.4a) and only one half of the system need to be analyzed for the evaluation of potential and electric field distribution. Then \( P_{ij} \) is computed as the combined effect of both charges on the potential of the \( i \)-th contour point and the storage require-
Figure 3.4: Charge Distribution in Symmetric System

(a) Symmetry about X-axis
(b) Symmetry about Y-axis

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ment is reduced appreciably. Again if the arrangement is symmetrical about both the X-X and Y-Y axis, then for each contour point, four equal charges can be positioned as in Fig. (3.4b) and the term $P_{ij}$ can represent the total effect of all four charges. For "n" selected contour points in one quadrant, effectively $4n$ charges are positioned and the computer storage requirement is proportionately reduced as compared to a totally asymmetrical system with $4n$ charges.

The hemispherically capped cylinder is rotationally symmetrical about the Z-axis and symmetric above the axis of symmetry (A.O.S.). Therefore only one quadrant of the system is considered for the analysis of the field distribution.

The Eqn. (3.4) for evaluating potential values on the conductor surface is therefore accordingly modified:

$$\phi = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{m} Q_j \left( \frac{1}{r_{ij}} + \frac{1}{r_{ij}} \right) \quad \ldots \quad 3.7$$

The expression for potential due to a finite line charge is:

$$P_{ij} = \frac{1}{4\varepsilon_0 \lambda} \cdot \ln \frac{z_{j2} - z_i + \gamma_1}{z_{j1} - z_i + \delta_1} \quad \ldots \quad 3.8$$

where,

$$\lambda = \text{length of line charge}$$

$$= z_{j2} - z_{j1}$$

$$\gamma_1 = \sqrt{r^2 + (z_{j2} - z_i)^2}$$
\[ \delta_i = \sqrt{x_i^2 + (z_j - z_i)^2} \]

The expressions for potential due to a point charge and line charge of finite length and their electric field components are presented in Appendix A.

3.2.3 Programming

A computer program was written for computing the potential distribution for the hemispherically capped cylindrically shaped electrode shown in Fig. (3.2). The program is written in the FORTRAN IV language for the IBM 3031 computer available at the computer centre, University of Windsor. Double precision arithmetic is used throughout the main program. The simulataneous Eqns. (3.5) representing the boundary conditions are solved using a subroutine which operates on the method of Gauss Elimination with complete pivoting. The main computational steps are illustrated in the flowchart of Fig. (3.5). The discrete line charges are placed at the axis of symmetry of the electrode. The point charges are located at the centre of the hemispherical cap. This choice is partly due to considerations of convenience, since the non-spherical symmetry of the charge arrangement means that the potential cannot be exactly correct on the hemisphere; for example, a thorough optimization would probably have the point charge location displaced from the centre of the sphere toward the interior of the rod. Because of the symmetry existing in the problem, the field is
START

READ AND WRITE CYLINDER'S RADIUS LENGTH AND NUMBER OF CHARGES

COMPUTE POTENTIAL COEFFICIENT MATRIX

SOLVE SIMULTANEOUS EQUATIONS

WRITE VALUES OF LINE AND POINT CHARGES

COMPUTE AND WRITE ERROR IN POTENTIAL ON THE CYLINDER'S SURFACE

PLOT EQUIPOTENTIAL AND FLUX LINES

STOP

Figure 3.5: Flow Chart
analyzed only in one quadrant of the system. The total number of line charges is varied from 1 to 12 above the axis of symmetry, the position of the point charge being fixed at the centre of the hemisphere. The total number of simultaneous equations to be solved is 13 instead of 26, as 24 line and 2 point charges are used to model the whole system. The solution of these equations gives the values of the unknown line and point charges. These are then used in computing the potential and the field anywhere on the cylinder's surface and in the outside space.

The best way to illustrate the distribution of the electric field and potential around the electrode is to plot the equipotential lines. For this purpose, a contour plotting subroutine "CONTUR" available at the computer centre, University of Windsor is used to plot the equipotential and flux lines around the electrode. These plots are shown in Figs. (3.6-3.7) for the cylinder shown in Fig. (3.2) and are discussed in section 3.3.

3.3 Results

The following values of geometrical parameters for the cylinder shown in Fig. (3.2) are chosen to illustrate the results.

Radius of the cylinder = 1.0 Inch.
Radius of the hemisphere = 1.0 Inch.
Length of the cylinder = 6.0 Inch.

Because of the symmetry existing in the problem, the
electric field distribution was calculated in only one quadrant. The values of the line and point charges obtained by solving the simultaneous Eqns. (3.4) are used to compute potential at a number of points on the cylinder's surface other than the selected boundary points where the boundary conditions were imposed. The conductor potential is assumed to be one unit. The computed values of the potential at the specified points are found to be one unit. This means that the boundary conditions are satisfied very well. The error in potential is then evaluated according to the following expression.

\[ \delta = \sum_{j=1}^{n} (\phi_{ij} - \phi_c) \]

\((i = 1, \ldots, m)\) Generally \(m = n\)

where,

- \(\phi_{ij}\) = Computed potential
- \(\phi_c\) = Conductor potential

As discussed in Chapter II, (conclusion 5), the accuracy of the charge simulation technique is sensitive to the location of charges, the boundary points where boundary conditions are satisfied, and the number of charges.

To observe the effect of these factors on the simulating potential, the positions of the charges were considered to be suitable at the axis of the cylinder because of symmetry.
existing in the problem. One boundary point each for the line charge and the point charge were chosen arbitrarily on the cylindrical and hemispherical part of the cylinder respectively. Firstly, only one line charge and a point charge placed at the symmetric axis of the cylinder and centre of the hemisphere were considered. The effect of the image charge given by Eqns. (3.7, 3.8) was taken into account. The length of the line charge was then varied from 0.5 inch to 2.0 inch above the axis of symmetry, the point charge being at the center of the hemisphere.

Table (3.1) shows a comparison of error (Mean Square, Root Mean Square) on the cylindrical and hemispherical portions of the cylinder for different lengths of line charges. As seen the R.M.S. and M.S. error both increase as the extremity of the line charge approaches the centre of the hemisphere where the point charge is placed. (This effect is explained in the forthcoming paragraph.)

Figures (3.6(a-e)) show the equipotential and field lines for all the cases shown in Table (3.1). As seen from these figures, in none of the cases does any one of the equipotential surfaces resulting from the fictitious charges coincide with the conductor surface. This can be ascribed to the fact (as explained in the previous sections) that the field distribution due to a point charge is spherically symmetrical and that due to a line charge is rotationally symmetrical, the individual field behaviour influences the combined effect and therefore the cylinder's surface is not an equipotential surface. As
Table 3.1: Effect of Length of Line Charge on Error (M.S., R.M.S.)*

<table>
<thead>
<tr>
<th>Length of Line Charge</th>
<th>Hemispherical Portion</th>
<th>Cylindrical Portion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M.S Error $\times 10^{-2}$</td>
<td>R.M.S Error $\times 10^{-1}$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0781</td>
<td>0.279</td>
</tr>
<tr>
<td>1.0</td>
<td>0.350</td>
<td>0.592</td>
</tr>
<tr>
<td>1.5</td>
<td>0.703</td>
<td>0.838</td>
</tr>
<tr>
<td>1.8</td>
<td>0.823</td>
<td>0.927</td>
</tr>
<tr>
<td>2.0</td>
<td>0.846</td>
<td>0.919</td>
</tr>
</tbody>
</table>

*The system is solved for one line charge and a point charge; the position of the point charge is fixed at the centre of the cap of the cylinder.
Figure 3.6a: Equipotential and Field Lines due to a Line Charge of Length 0.5 inch and a Point Charge
Figure 3.6b: Equipotential and Field Lines due to a Line Charge of Length 1.0 Inch and a Point Charge
Figure 3.6c: Equipotential and Field Lines due to a Line Charge of Length 1.5 inch and a Point Charge
Figure 3.6d: Equipotential and Field Lines due to a Line Charge of Length 1.8 inch and a Point Charge
Figure 3.6e: Equipotential and Field Lines due to a Line Charge of Length 2.0 inch and a Point Charge
seen from Fig. (3.6a)) when the line charge is terminated far from the point charge, one of the equipotential surfaces tends to coincide with the cylinder's surface. But the accuracy in the solution is still very poor. In order to improve the model, a larger number of charges (discrete line charges) were taken along the axis of the cylinder. This improved the error in potential on the cylindrical portion of the cylinder, but the error on the hemispherical part still had to be improved. This also had an impact on the cylindrical part.

To improve the overall error, manual optimization was carried out to select an appropriate boundary point on the hemisphere of the cylinder. Table (3.2) shows a comparison of error (M.S., R.M.S.) in potential distribution on the cylinder's surface. The point charge was placed at the centre of the hemisphere and varying length of line charge used above the axis of symmetry. As seen from Table (3.2) the error in potential distribution is better when the boundary point on the hemispherical portion of the cylinder above the axis of symmetry is (0.924, 2.382). Furthermore, there is no significant difference in error when the length of the line charge was varied with increasing/decreasing lengths or when equal in length, as can be seen from Table (3.3). The point (0.924, 2.382) on the hemisphere portion obtained as a suitable point from the previous computer runs was chosen as a boundary point.

Figures (3.7(a-c)) show equipotential and field lines for 5, 7, 12 line charges and one point charge respectively.

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Table 3.2: The Effect of the Position of Boundary Point on the Hemisphere on Error with Varying Length of Line Charges

<table>
<thead>
<tr>
<th>POSITION OF BOUNDARY POINT</th>
<th>DECREASING LENGTH OF LINE CHARGE</th>
<th>INCREASING LENGTH OF LINE CHARGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HEMISPHERICAL PART</td>
<td>CYLINDRICAL PART</td>
</tr>
<tr>
<td></td>
<td>M.S. ERROR x 10^{-4}</td>
<td>R.M.S. ERROR x 10^{-2}</td>
</tr>
<tr>
<td>(0.918, 2.195)</td>
<td>0.649</td>
<td>0.606</td>
</tr>
<tr>
<td>(0.924, 2.382)</td>
<td>0.203</td>
<td>0.551</td>
</tr>
<tr>
<td>(0.831, 2.555)</td>
<td>0.857</td>
<td>0.925</td>
</tr>
<tr>
<td>(0.707, 2.707)</td>
<td>1.080</td>
<td>1.045</td>
</tr>
<tr>
<td>(0.555, 2.831)</td>
<td>1.132</td>
<td>1.063</td>
</tr>
<tr>
<td>(0.382, 2.924)</td>
<td>1.132</td>
<td>1.063</td>
</tr>
<tr>
<td>(0.198, 2.981)</td>
<td>1.114</td>
<td>1.051</td>
</tr>
</tbody>
</table>

a) Total Discrete Line Charges = 7  
b) Position of boundary point on the hemisphere is varied, the point charge being located at the centre of the hemisphere  
c) Length of line charge is varied above the A.O.S along the Z-axis  
d) Suitable boundary point
Table 3.3: Effect of the Number of Charges on Error\textsuperscript{a,b}

<table>
<thead>
<tr>
<th>NUMBER OF DISCRETE LINE CHARGES</th>
<th>LENGTH OF EACH LINE CHARGE (INCH)</th>
<th>HEMISPHERICAL PART</th>
<th>CYLINDRICAL PART</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M.S. ERROR $\times 10^{-5}$</td>
<td>R.M.S. ERROR $\times 10^{-2}$</td>
</tr>
<tr>
<td>5</td>
<td>0.500 0.450 0.405 0.364 0.280</td>
<td>0.308   0.175</td>
<td>0.547   0.740</td>
</tr>
<tr>
<td>7</td>
<td>0.200 0.230 0.284 0.304 0.350 0.402 0.250</td>
<td>0.334   0.182</td>
<td>0.486   0.697</td>
</tr>
<tr>
<td>10 (Equal Length)</td>
<td>0.1</td>
<td>0.271   0.165</td>
<td>0.391   0.625</td>
</tr>
<tr>
<td>12</td>
<td>0.200 0.110 0.121 0.133 0.146 0.161 0.177 0.195 0.214 0.236 0.259 0.147</td>
<td>0.194   0.139</td>
<td>0.254   0.504</td>
</tr>
</tbody>
</table>

\textsuperscript{a)} Point charge placed at the centre of the hemisphere
\textsuperscript{b)} Position of boundary point on the hemisphere: (0.924, 2.382)
\textsuperscript{c)} Varying length of line charges are considered above the A.O.S.
Figure 3.7a: Equipotential and Field Lines
Number of Discrete Line Charges = 5
Figure 3.7b: Equipotential and Field Lines
Number of Discrete Line Charges = 7
Figure 3.7c: Equipotential and Field Lines

Number of Discrete Line Charges = 12

(One of the potential surfaces is very nearly coincided with the cylinder's surface)
These figures illustrate that the cylinder's surface do result in an equipotential surface and apparently there is no difference in the plots, although the larger number of line charges (12) used in Fig. (3.7c) contributed to the accuracy (Table (3.3)) as was expected (Chapter II, Conclusion 5).

3.3.1 Conclusions

1) The method described in this chapter can be used to model a variety of high voltage electrode arrangements, that is rod-rod, rod-plane, plane-plane, etc.

2) The number of simultaneous equations to be solved is reduced if advantage of symmetry existing in the problems is taken into account.

3) The accuracy of the method is sensitive to the selection of contour points on the conductor surface where the boundary conditions are to be met. Manual optimization can be carried out to obtain a suitable set of boundary points on the conductor surface.

4) The slight variation in the length of the line charges does not have a significant bearing on the error in the potential distribution on the conductor's surface.
IV. PROBLEM FORMULATION

4.1 General

Of the many approaches available for modelling of electrostatic field problems, the integral equation approach appears to be the one most conveniently adaptable to digital computer solution. By using Green's theorem, Laplace's equation can be expressed in integral form [18]. Some additional manipulation yields an expression for the electric potential which is a surface integral over the unknown charges.* An integral equation is obtained by setting the integral equal to known values of potential on conductors. The integral equation can then be solved for the surface charge densities by approximating the integral as a sum over small surface elements with assumed uniform charge densities.

The integral equation formulations did not receive extensive attention from researchers until recently, mainly because numerical solution of physical problems only became feasible with the advent of high speed digital computers. Along with the sophistication of computer hardware and software, integral equations are lending themselves to numerical treatment to provide solutions of practical problems with modest computational effort and cost.

*It will be assumed here that the medium consists of several different homogeneous materials and that as a result, the only charges are the surface charges. If this is not the case, then volume charges must be considered [19].
The partial differential equations of Maxwell have been extensively used for analytical and numerical solution of field problems. The numerical solution of partial differential equations is usually obtained by finite difference or finite element methods. However, the use of these methods may involve limitations of their application in three-dimensional problems because of the requirement of a huge memory allocation and because of the usual practice of locating the boundaries of the region of computation at a distance far from the region of interest in order to achieve a satisfactory solution. This is required when the boundary conditions can only be approximated at finite locations. In the integral equation approach, on the other hand, the discretization is required for the material body only and boundary conditions remain implicit in the way of obtaining the mathematical formulation, thus alleviating to a great extent the burden of large memory requirements. The integral equation techniques are appealing because it is not necessary to compute the potential at points where it is not desired. Another attractive quality is that the analysis of problems which are unbounded does not require additional programming effort or additional unknown quantities as is the case for techniques like the finite difference and finite element methods. Another attractive characteristic of the integral equation formulation is that since the unknowns are surface quantities, the number of unknowns will be proportional to the surface area of the region (excluding the infinite boundary if there is one).
For two-dimensional problems, the number of equations is proportional to the perimeters of the finite boundary. It follows that the matrix equation obtained by discretization of the integral equation will be of small dimension compared to the matrices obtained for finite difference and element techniques. Thus it appears that the computational time for solutions based on integral equations will be smaller than for solutions based on finite difference and element techniques.

The method described in this chapter is known as the Charge Simulation Technique applicable to a system involving two dielectric media. This method was first proposed by H. Singer [21]. The application of this method to practical geometries of high voltage transmission line insulators, namely suspension insulators, and a recently developed long rod polymer insulator is discussed in separate sections.

4.2 Calculation for Two-Dielectric Arrangements

The behaviour of the electric field in a medium depends on the structure of the material. The electric charges in a conducting medium are free to move and therefore in the presence of an electric field the free charges appear as surface charge density on the exterior surface of the conductor. In insulators or dielectric materials, charges are not free to move, but can only be displaced by small distances locally. A dielectric in an electric field can be viewed as a free space arrangement of microscopic electric dipoles or positive and negative bound charges, the centres of which do not coincide.
The actual mechanism of the charge displacement differs in the various dielectric materials [45]. Some molecules called polar molecules, have a permanent displacement existing between the centroid of the positive and negative charges, and each pair of charges acts as a dipole. A nonpolar molecule does not have this dipole arrangement until after a field is applied. The negative and positive charge shift in opposite directions against their mutual attraction and produce a dipole which is aligned with the electric field. Usually the dipoles are oriented in a random way in the interior of the material, and the action of the external field is to align these molecules in the direction of the applied field. In the interior they tend to cancel the effect of each other, but on the surface of the dielectric boundary they appear as surface charges.

The effect of any surface charge distribution is equivalent to (or can be replaced by) the identical effect of other charges located beyond the surface as far as effects on "this side" of the surface are concerned. Therefore a dielectric surface can be simulated by discrete charges located appropriately at either side of the dielectric boundary. The magnitude of these charges should be such that they satisfy the following conditions on the dielectric boundary.

1. The potential at any point on the dielectric surface must be the same when computed from either side of the dielectric surface.
2. The normal component of the electric flux density at any point in either medium at the surface must be the same.

4.2.1 Computation Procedure

The procedure for computation of fields and potentials of a two-dielectric arrangement with relative permitivity \( \varepsilon_r \) is as follows:

(As shown in Fig. 4.1, a simple example with a small number of discrete charges is chosen to illustrate the method of computation).

1. Conductors are replaced by a number of discrete charges \( N_e \) with the same number of contour points on the conductor of which \( N_a \) contour points are on the side of air and \( N_e - N_a = N_d \) contour points are on the side of dielectric. These \( N_e \) charges contribute to the potential and field strength in both media.

2. The dielectric boundary is represented by \( N_b \) contour points with \( N_b \) charges in the dielectric and \( N_b \) in the air. The charges in the dielectric contribute to the potential when it is evaluated at contour points on the air side of the conductors. While evaluating potential at contour points on the conductor — dielectric interfaces, the contribution of the charges in the air is taken into account.

There are thus \( N_e + 2N_b \) unknown charges. Their values can be determined if \( N_e + 2N_b \) equations are solved simultaneously. These charges can be subscripted as follows;
Figure 4.1: Procedure for Computation of Fields and Potentials of a Two Dielectric System

(a) Conductor interfaced with dielectric material and air
(b) Charges contributing to potential at contour points on air side of the conductor.
(c) Charges contributing to potential at contour points on dielectric side of the conductor
In conductor .................. 1, .......... Ne
In air ......................... (Ne + 1), ...... Ne + Nb
In dielectric .................. (Ne + Nb + 1) ... Ne + 2Nb
The points on the dielectric side of the conductor are labelled: 1, 2, ....... Nd;
The points on the air side of the conductor are labelled: (Nd + 1), (Nd + 2), ....... Ne;
The points on the air-dielectric interface are labelled: (Ne + 1), (Ne + 2), ...... Ne + Nb.

The symbols used are:

i: The counter for contour points
j: The counter for the charges
Pij: The potential coefficients. These depend on the type of the charge, their location and the location of the point at which the potential is being specified.
Qj: The discrete charge at the j-th location
fij: The normal component of the field vector. It is defined as the contribution of the j-th charge to that component of the field vector, which is normal to the dielectric boundary.
φc: The conductor potential

4.2.2 Equation Formulation

The system of equations required for determination of the simulation charges is formed by defining the boundary conditions which must be satisfied.

These equations can be formulated in the following
manner:

1. The potential \( \phi_c \) at the contour points on the dielectric side of the conductor is the integrated effect of all the Ne charges on the conductor and the Nb charges in air, that is,

\[
\sum_{j \in \text{conductor}} P_{ij} \cdot Q_j + \sum_{j \in \text{air}} P_{ij} \cdot Q_j = \phi_c \quad \text{For } i = 1, 2, \ldots, Nd
\]

2. The potential \( \phi_c \) at the contour points on the air side of the conductor is the integrated effect of all the Ne charges modelling the conductor and Nb charges in the dielectric:

\[
\sum_{j \in \text{conductor}} P_{ij} \cdot Q_j + \sum_{j \in \text{dielectric}} P_{ij} \cdot Q_j = \phi_c \quad \text{For } i = Nd + 1, \ldots, Ne
\]

3. The potential at contour points on the dielectric boundary is unknown but the potential at each point on the boundary must be the same whether it is calculated from the region of air or from the dielectric region. Thus the equations can be written as:

\[
\sum_{j \in \text{conductor}} P_{ij} \cdot Q_j + \sum_{j \in \text{dielectric}} P_{ij} \cdot Q_j = \phi_c \quad \text{For } i = Ne + 1, \ldots, Ne + Nb
\]
\[ \sum_{j \in \text{conductor}} P_{ij} \cdot Q_j + \sum_{j \in \text{dielectric}} P_{ij} \cdot Q_j = \sum_{j \in \text{conductor}} P_{ij} \cdot Q_j + \sum_{j \in \text{air}} P_{ij} \cdot Q_j \]

or simplifying,

\[ \sum_{j \in \text{dielectric}} P_{ij} \cdot Q_j - \sum_{j \in \text{air}} P_{ij} \cdot Q_j = 0 \]

4. \( P_{ij} \) has been defined as the potential coefficient considering the effect of the charges \( Q_j \) on the potential at a given contour point. Similarly \( f_{ij} \) is defined as the contribution of the charge \( Q_j \) to that component of the electric field vector, which is normal to the dielectric boundary at a given contour point. Then at the contour points of the dielectric boundary, the normal field strength in the air must be \( \varepsilon r \) times greater than in the dielectric, that is,

For \( i = Ne + Nb + 1, \ldots, 2Nb \)
\[ e_r \sum_{j \in \text{conductor}} f_{ij} \cdot Q_j + \sum_{j \in \text{air}} f_{ij} \cdot Q_j \]

\[ = \sum_{j \in \text{conductor}} f_{ij} \cdot Q_j + \sum_{j \in \text{dielectric}} f_{ij} \cdot Q_j \]

or,

\[(e_r - 1) \cdot \sum_{j \in \text{conductor}} f_{ij} \cdot Q_j + e_r \cdot \sum_{j \in \text{air}} f_{ij} \cdot Q_j \]

\[ - \sum_{j \in \text{dielectric}} f_{ij} \cdot Q_j = 0 \]

... 4.4

All the above equations can be written in a matrix form as shown in Fig. (4.2).

One equation is written for every contour point on the conductor and two are written for each contour point on the dielectric boundary. Thus in the matrix, the row number corresponds to the number of the contour point and the column number represents the number of the charge. The pin of the insulator is at line conductor potential and the cap at ground potential.
Figure 4.2: Matrix Representation of Equations Described in Section 4.2.2 for Suspension Insulator

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4.2.3 Choice of the Method for Solving Linear System of Equations

It is impossible to devise a single computer program that will solve any type of matrix equation representing a linear system of equations. Equation solving methods are generally categorized as either direct techniques, which yield answers in a finite, predictable number of operations, or as iterative techniques, which give answers that become increasingly more accurate as the number of iterations becomes large.

The bad point about the direct techniques used for solving large dense and unsymmetric matrices is the 'round-off error'. The number of arithmetic operations using these techniques is proportional to $n^{3/3}$ (n is the order of the matrix). Thus additional round-off error can be incurred with each mathematical operation, and it may simply accumulate until the final step is reached. On the other hand, in the iterative techniques, the round-off error in the final converged values (answer) is only that accumulated in the final iteration. However, the iterative techniques carry the risk of being divergent instead of convergent.

To check the applicability of iterative techniques on our particular problem, the Gauss-Siedel method was used to solve 71 linear equations of the partial configuration of the long rod insulator, Fig. (4.4) for the unknown values of ring charges. The technique diverged after 850 iterations consuming a CPU time of 120 secs.

Gauss-Siedel technique, though one of the most powerful iterative techniques for the solution of sets of
linear equations, does not guarantee convergence unless

\[ \sum_{j=1}^{n} |P_{ij}| > |P_{ii}| \]

for each value of i (each row). However, convergence can be obtained with much weaker diagonal dominance than this. An example of a known solution was tested to illustrate the convergence and divergence behaviour of Gauss-Siedel technique:

Consider the following diagonally dominant matrix:

\[
\begin{bmatrix}
56&22&11&-18 \\
17&66&-12&7 \\
3&-5&47&20 \\
11&16&17&10 \\
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
\end{bmatrix}
=
\begin{bmatrix}
34 \\
82 \\
18 \\
26 \\
\end{bmatrix}
\]

Starting with the initial guess of \(x^{(0)} = 1.0\), and the convergence criterion of \(\delta = 1.0 \times 10^{-10}\) (in double precision), the process converges after 106 iteration with CPU time of 0.80 secs. However, when one of the elements on the main diagonal '\(P_{44}\)' was made 9 or smaller, then the procedure is divergent.

Clearly, the presence of any small main diagonal elements can pose a significant threat to the convergence of Gauss-Siedel technique. A different process, called Jacobi iteration has no significant advantages compared with Gauss-Siedel and has a considerably slower convergence rate [46].

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For these reasons, the application of this technique to our problem was not considered.

Whether the iterative process is convergent or divergent does not depend on the initial guess supplied for the unknowns, but depends on the character of the equations themselves [46]. Thus the choice of a particular technique depends on the properties of the problem at hand. The matrix of Fig. (4.2) will contain 107 linear equations for the suspension insulator and 150 equations for the long rod insulator. Both matrices are non-symmetric, large and dense. They might have small real numbers on their diagonal, as the potential coefficient at a contour point due to charges close to it is large (10^-25) and small for other charges. For such dense matrices whose elements are stored in high speed memory, no class of solution algorithms has been found which is better, either in time or in accuracy, than the systematic elimination methods of C.F. Gauss [47]. Gaussian elimination occurs in many variants which are algebraically the same. The methods differ according to how the matrices are stored, the order of the elimination, the precautions taken against large rounding errors, and the way calculated solutions are improved.

The method used for solving the linear system of equations for the suspension and long rod insulators is a modified form of the Gauss elimination technique. This technique employs the Crout algorithm for decomposition of the coefficient matrix and performs partial pivoting, scaling and
equilibration [47]. The subroutine LEQT1F or LEQT2F incorporating these features, is available at the Computer Centre, University of Windsor and yields a solution accurate to the number of digits specified [48]. The subroutine also performs extended precision computation of the residual. The first iteration is as follows:

\[ r_0 = b - AX_0 \]

then the routine solves,

\[ Ay_0 = r_0 \text{ for } y_0 \]

the improved solution is,

\[ X = X_0 + y_0 \]

This method is a modification of the Forsythe-Moller iterative improvement subroutine [47, Chapter 13, 14]. For a comparison purpose this routine was used to solve 71 linear equations of the long rod insulator, which resulted in the solution in 23.76 seconds of CPU time. The memory requirements are approximately the same in both cases.
4.3 Mathematical Formulation

The term $P_{ij}$ used in the previous section has been defined as the potential coefficient considering the effect of a ring or line charge at position $(r_j, z_j)$ at a given contour point $(r_i, z_i)$. In the same way $f_{ij}$ is defined as the contribution of the charge at position $r_j$ to that component of the field vector, which is normal to the dielectric boundary at the $i$th contour point.

In dealing with the high voltage insulator configurations shown in Figs. (1.1) and (1.2), it is apparent that complete rotational symmetry exists in the former and partial rotational symmetry exists in the latter configuration. In order to take advantage of this symmetry, ring charges were used for the simulation. The use of ring charges effectively reduces a rotationally symmetrical insulator configuration to a two-dimensional problem. The advantage of this nature is twofold: in the case of Suspension Insulator firstly the complexity of simulation is greatly reduced and secondly, the electric field can be analyzed in a half plane of the configuration. However, the right circular cylindrical portions of the suspension insulator are modelled by discrete line charges placed at the axis of symmetry of the pin. The expressions for the potential co-efficient $P_{ij}$ and field strength co-efficient $f_{ij}$ for a ring and line charge are presented in Appendix A.

4.3.1 Cubic Spline Interpolatory Scheme

The metallic cap and dielectric skirt of the suspension insulator have fairly curved boundaries. The computer program
written for this geometry stores the co-ordinates of the charge locations and contour points obtained from a digitizer.

A system of linear equations are set up satisfying the boundary conditions and solved simultaneously as explained earlier. In order to carry out detailed investigations and compute the potential at contour points other than the points where the boundary conditions were satisfied, a discrete set of points defining these boundaries is necessary. This can be done in two ways:

1. Digitize the contour points on the irregular boundaries. The only difficulty in doing so is that if the data must be manually entered into computer files from a digitizer-generated listing (as was the case before the digitizer was interfaced directly with the main computer system), the number of errors which might have to be worked out could be excessive.

2. The coordinates could be generated within the program by employing an appropriate interpolating scheme for the curved boundaries.

The second approach is advantageous over the first one if changes in the shape of the metallic cap or dielectric skirt of the suspension insulator are to be studied.

There are several interpolation algorithms available. One of the most direct ways to approximate a function defined at a finite set of points is to fit a polynomial of suitable degree to the discrete data. In addition to its simplicity,
polynomial approximation has some advantageous properties. But the chief drawback is that it may become excessively oscillatory between the nodes as the number of points to be interpolated increases. This drawback can be eliminated by considering a set of piecewise polynomials. These piecewise polynomial functions together with some continuity conditions on them and their derivatives at the nodes are known as splines. A cubic spline \( S_i(x) \) is a piecewise continuous function on a mesh of \( n \) points with the following properties;

1. the cubic spline is cubic in each sub-interval \( x_{i-1} \leq x \leq x_i, \quad i = 1, 2, \ldots, n \)

2. The cubic splines \( S(x) \) are continuous together with their first and second derivative. On an interval \([a, b]\) the second derivative is also piecewise linear.

3. \( S(x) \) satisfies the equation \( S(x_i) = f(x_i) \), \( i = 0, 1, \ldots, n \). Where \( f(x) \) is the function being approximated.

The subject of spline theory is much too long and complex to cover in detail here. Entire books devoted to the subject include Rice [49] and Ahlberg et al. [50].

4.4 Application of the Method to Practical High Voltage Insulators

4.4.1 Suspension Insulator

The practical geometry of a Suspension Insulator unit is shown in Fig. (1.1). It is usually connected in a string
of such units on EHV transmission lines. The upper metallic cap of the insulator is connected in the string to the crossarm end of the tower and therefore, it is assumed to be at ground potential. The lower part that is the pin of the insulator, is connected to the conductor end of the string and is at a higher potential. In between is shown a porcelain dielectric cemented to the metallic parts.

Figure (4.3) shows the distribution of charges in the insulator. This distribution is specified arbitrarily. Using optimization, the best location for these charges could be found. Rather than get into the involved and costly (in terms of computer resources) business of extensive optimization, we have chosen to specify the locations arbitrarily, and solve for the necessary values of charge magnitude to satisfy the boundary conditions. Furthermore, this arbitrary distribution of charges can be varied for better results as the technique is sensitive to the location of charges.

The insulator is rotationally symmetrical about the Z-axis if the presence of the line conductor is ignored. Since the metallic cap and the dielectric skirt both have rotational symmetry, ring type charges are used to model them. The pin of the insulator is cylindrical, therefore, line charges are considered suitable to model this part of the device. Again, the tip of the pin has axial symmetry, a few ring charges are used to model this part of the pin. Also at the upper conductor-dielectric interface and lower dielectric-conductor interface and conductor-air interface, some ring charges are
Figure 4.3: Distribution of Charges in the Suspension Insulator Model

Only half the plane symmetric about Z-axis is considered for the analysis of field distribution.
used to simulate the potential near these parts of the insulator.

The boundary of the cap is replaced with a number of contour points and the same number of ring charges. Similarly the boundary of the lower electrode, that is the pin, is replaced by a number of contour points with the same number of discrete line charges and ring charges. At the dielectric boundary, there are as many ring charges in air as in the dielectric and the same number of contour points lie on the dielectric boundary.

Now applying the method of computation as explained in previous sections, the potential at the contour points on the air side of the cap and pin of the insulator is the integrated effect of all the ring charges modelling the cap, the line and ring charges modelling the pin and the ring charges in the dielectric. In the same way, the potential at contour points on the dielectric-conductor interface of the cap and pin is the integrated effect of all the charges in the air. The potential at contour points on the dielectric boundary due to charges in the conductor and dielectric is the same as if calculated due to charges in the conductors and air.

The results obtained for the suspension insulator and the computer programming are discussed in Chapter V.

4.4.2 Long Rod Polymer Insulator

Voltages and mechanical loads on insulators are increasing as the trends toward higher voltages, use of large conductors, multiple conductors and longer spans continue.
These factors bring increasing need for higher reliability in materials and components for use in transmission lines. The need for an even better suspension insulator to meet these requirements becomes readily apparent. These needs can be defined in terms of the following practical design considerations [51]:

1. A lighter but stronger insulator would facilitate construction and hot line maintenance.

2. An insulator with a much higher resistance to impact would reduce the susceptibility to breakage during handling and erection.

3. The ability to sustain power-arc follow currents without damage would reduce outages due to lightning and switching surges.

In this connection, the long rod polymer insulators have shown promising results in the worst conditions (contaminated) but the life expectancy of such a unit is undetermined [52]. The long rod design eliminates the many metal-to-insulating material junctions and therefore modifies the voltage distribution along the string. This reduces the possibility for dryband arcing.

The potential advantages of these designs are [51, 52].

1. Approximately 70 percent weight reduction, lessening construction costs and handling expense.

2. Flexibility of design to permit a wide range of
strength requirements.

3. Greatly reduced risk of insulator breakage due to hardware corrosion.

4. High thermal shock resistance, reducing damage from power flashover.

5. The skirts have less surface adhesion than ceramic materials.

6. Insulator surfaces that wash easily and give promise of improved performance in contaminated atmospheres.


8. High leakage to strike ratio.

The charge distribution for the insulator is shown in Fig. (4.4). Only ring charges were used to model the entire insulator. For preliminary analysis, the insulator was initially modelled including one dielectric unit labelled "A" as shown in Fig. (4.4). However, the computer program developed is extended to include the remaining dielectric units labelled B, C, etc. and the grounded metal part of the insulator.

This geometry differs from the suspension insulator in that the line conductor is included in the modelling. The arrangement has no rotational symmetry but can be deemed as approximately symmetrical near both the X-X and Y-Y axes (except
Figure 4.4: Distribution of Charges in the Partial Configuration of Long Rod Polymer Insulator
near their intersection).

In order to model the conductor clamp and infinitely extended high voltage conductor having symmetry about the $X-X$ axis, positive images of the ring charges modelling these parts were placed on the left side of the $Y-Y$ axis as shown in Fig. (4.5). This principle of image charge modelling is discussed in detail in Chapter III and was used in modelling the hemispherically capped cylinder.

The results obtained for the long rod polymer insulator and the computer programming are discussed in Chapter V.
Figure 4.5: Charge distribution in the Conductor Clamp of the Polymer Insulator and their Positive Images

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V. RESULTS AND DISCUSSIONS

5.1 General

In the electrical design of any insulation system, a knowledge of the potential distribution of the system is essential for the calculation of electric stresses involved. Accurate information about the electric field in some specified regions (normally those near the electrodes and on the dielectric interfaces) will enable a manufacturer engineer to establish a criterion of design parameters for the electrical component to be manufactured. The knowledge of the electric field distribution can be used in the determination of breakdown strength of the insulating material provided the experimental value of the puncture voltage $U_b$ of the insulator is obtained with the insulator submerged in a better insulating medium such as oil, whose dielectric constant is much less than that of the insulator material [35]. The breakdown value of the insulator $E_b$ is obtained by the relation,

$$E_b = E_{\text{max}} \cdot \frac{U_b}{U}$$

where,

$U_b$ is the value of the applied voltage at which breakdown occurs experimentally,

$E_{\text{max}}$ is the maximum computed field in the insulator with the applied electrode potential $U$.

Furthermore, if the flashover voltage of the insulator is determined experimentally, the tangential field for surface flashover of the particular insulator can be computed by the
method of computation explained in the previous chapters.

In view of the importance of the knowledge of field distributions in and around the insulators under investigation, emphasis is placed on the determination of the following criteria for evaluation of the quality of the solution.

1. **Potential Error***

The error in potential distribution on various metallic parts of the insulator interfaced with air and solid materials is evaluated.

2. **Potential Discrepancy**

The discrepancy in potential as well as in tangential component of the electric field is evaluated along all dielectric skirts of the suspension and long rod polymer insulator.

---

* The potential error is defined as the difference between the known conductor potential and the computed potential at the location of the conductor surface.

** The potential discrepancy is defined as the difference in solutions for potential at the dielectric boundary obtained by considering all the simulating charges modelling the insulator excluding the charges outside the dielectric skirts and the solution obtained due to all the charges modelling the insulator excluding the charges inside the dielectric.
5.2 Suspension Insulator

5.2.1 Metallic Parts of the Insulator Exposed to Air

The cap and lower part of the pin are the metallic parts of the insulator exposed to the air dielectric. The cap of the insulator is connected in the string of insulators on the cross arm end of the tower. Therefore the potential at the cap is assumed to be at ground potential. Only seventeen ring charges were used to simulate the potential at this part of the insulator. Figure (5.1) shows the potential error along the cap of the insulator. As seen from this figure, at most parts of this cap, the potential error is less than 1%. The significance of keeping the 'Potential Error' to a small value is that since all corona calculations are sensitive to the values of electric fields, a very small error in the values of electric field might result in a very large error in the values of the Corona ONSET voltage, Corona loss and Radio Interference. The pin of the insulator is connected in the string on the side of the conductor. Therefore, potential at the pin is assumed to be at the conductor potential. Sixteen line charges were located at the axis of symmetry of the pin pertaining to the air part. Four ring charges were placed along the radial axis of the tip of the pin and three charges (ring) were used to model the conductor-air interface, Fig. (4.3).

The potential of the pin was assumed to be at one unit (normalized). The boundary conditions at all boundaries were found to be satisfied and the potential at several points
Figure 5.1: Potential Error along the Air Side of the Cap
(also known as check points on the conductor's surface other than those where boundary conditions were satisfied) was evaluated. The difference between the known conductor potential and the computed potential is the measure for the accuracy of the calculations and is previously defined as 'Potential Error'. Figure (5.2) illustrates the potential error along the pin of the insulator exposed to air. As can be seen, the maximum potential error is about 1.2% near point 'A' on the tip of the pin. The error is less than 1.0% along the cylindrical part. Figure (5.3) illustrates the potential error along the conductor-air interface where it rises to about 1.4%.

5.2.2 Metallic Parts Interfaced with the Dielectric

The porcelain dielectric of the suspension insulator is assembled with its metallic hardware with steam cured, neat portland cement. In this work, the cement layers adjacent to the metallic parts are assumed to be conductive. Therefore, the surface charge density on the cemented surface (labelled ABC in Fig. (5.4)) instead of the actual surface of the steel pin (embedded in the porcelain dielectric) is considered for discretization. Straight line charges of finite length located along the axis of symmetry were used to simulate the cylindrical part (labelled AB) and toroidal line charges (ring charges) were arbitrarily chosen to simulate the potential at the upper dielectric-conductor interface.

In Fig. (5.4) is shown the potential error along the embedded part of the pin. Eighteen line charges for the
Figure 5.2: Potential Error along the Air Part of the Pin
Figure 5.3: Potential Error along the Conductor-Air Interface of the Pin
Figure 5.4: Potential Error Along the Vertical Surface of the Pin Embedded in the Dielectric
cylindrical part and five ring charges for the horizontal interface were used. As can be seen that the error in potential along this part is reasonably small which indicates that this part of the insulator is accurately modelled. This result also gave us a clue that the potential distribution on the dielectric embedded part of the conductor can be evaluated to a reasonable amount of accuracy. The maximum potential error is about 0.3% near point 'B'.

Figure (5.5) illustrates the potential error along the top horizontal conductor-dielectric interface of the pin. The maximum error is about 0.6% near point 'B'.

Figure (5.6) illustrates the potential error along the cement-porcelain interface housed by the cap (labelled ABCD). Ten ring charges were used to model the entire interface. As seen from Fig. (5.6), the maximum potential error is about 1.0% along the part labelled BC.

Figure (5.7) illustrates the potential error along the vertical interface of the cap with the porcelain. As seen, the error in potential is within 1.0%.

5.2.3 Potential Discrepancy along the Dielectric Skirt

The potential at the dielectric boundary is not known. However, in the charge simulation method described in Chapter IV, the potential at any point on the dielectric boundary must be the same when computed from either side of the dielectric surface. The substitute charges in the cap and pin of the insulator contribute to potential and field strength in both the media. In order to check accuracy in
Figure 5.5: Potential Error along the Top Horizontal Conductor-Dielectric Interface of the Pin

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Figure 5.6: Potential Error along the Horizontal Conductor-Dielectric Interface of the Cap
Figure 5.7: Potential Error along the Vertical Interface of the Cap with Dielectric
the solution, the check points on the upper and lower (corrugated) surfaces of the dielectric skirt were interpolated among the boundary points (where the boundary conditions were satisfied). The potential at these interpolated points were then evaluated. While satisfying the potential matching conditions on the boundary points of the dielectric skirt, the contribution of charges in the cap and pin were not taken into account according to Eqn. (4.2.2, (3)), whereas, the effect of all the charges modelling the insulator were considered when satisfying the flux matching conditions on the dielectric boundaries.

Since the potential on the dielectric surface is not known, the term 'Potential Error' cannot be used as was the case for conductors (cap and pin) where the potential was known. The difference of the two approximate solutions (i.e. one due to charges in the conductors and inside the dielectric skirt and the other due to all the charges in the conductors and air) has been previously introduced as the 'Potential Discrepancy'. The discrepancy in the tangential field can also be defined in a similar way. For a reasonably good model, the potential discrepancy must be minimum. Figure (5.8) illustrates the potential discrepancy along the upper dielectric skirt of the insulator. This figure indicates that the potential discrepancy along this part of the skirt is fairly small. The maximum discrepancy in potential is about 0.24% near part 'B'.

Figure (5.9) illustrates the potential discrepancy along the lower dielectric skirt. As seen from this figure,
Figure 5.8: Potential Discrepancy along the Upper Skirt
Figure S.9: Discrepancy in Potential along the Lower Skirt
the maximum potential discrepancy is about 9.0% near part BC whereas it is less than 3.0% on most of the skirt. Only eight ring charges placed at appropriate distances from the tip of each corrugation were used, Fig.(5.10a). To improve the potential discrepancy, six more charges were inserted among these eight charges, raising the total to fourteen charges modelling the lower skirt as shown in Fig. (5.10b). The maximum discrepancy in the potential was found to be 10.0% near the corrugation BC in Fig.(5.10b). In order to see the effect of the position of charges on the potential discrepancy, the ring charges modelling the lower skirt in Fig. (5.10b) were repositioned as shown in Fig. (5.10c). This figure is a representative example of several such attempts that have been made to improve the discrepancy in potential along this part. The maximum potential discrepancy was about 11.0% near the part BC, Fig. (5.10c). In both charge arrangements, for the lower skirt shown in Fig.(5.10b,c), the charge distribution in the rest of the insulator was not disturbed. The results seem very insensitive to changes in the number of charges and their positions used to model the lower dielectric skirt.

5.2.4 Discrepancy in the Tangential Field along the Dielectric Skirts

Figure (5.11) illustrates the discrepancy in the tangential field along the upper dielectric skirt. As seen from this figure, corresponding to 0.24% discrepancy in potential near part 'B' on the upper skirt, the maximum discrepancy in the tangential field is about 2.2%.
Figure 5.10 Ring Charge Representation in the Lower Dielectric Skirt
Figure 5.11: Discrepancy in Tangential Field along the Upper Skirt

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Figure (5.12) illustrates the discrepancy in the tangential field along the lower skirt. As was expected, the discrepancy in tangential field is as large as about 17.0%.

One of the boundary conditions imposed on the system was that the normal component of the flux density (D-field) is continuous across the dielectric-dielectric boundary. The D-field was found to be continuous across both the upper and lower dielectric skirt of the suspension insulator with a discrepancy of less than 0.1%.

5.2.5 Potential Distribution along the Dielectric Skirts

The potential along the dielectric boundary is not known. However, the potential at any point on the dielectric boundary can be determined if the system of linear equations of section 4.2.2 is solved for the unknown ring and line charges modelling the insulator. Then the values of the ring charges inside and outside of the dielectric can be used to compute potential at the dielectric boundary. The charges in the conductors contribute to potential values in the porcelain as well as air dielectric. Since the potential across a dielectric-dielectric boundary is continuous, therefore, at any point on the dielectric boundary, the two sets of charges (one—conductor charges and the charges in the dielectric and the other—the conductor charges and the charges outside of the dielectric) should produce the same value of potential. The potential distribution on the surface of the upper and lower skirt of the suspension insulator presented in this

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Figure 5.12 Discrepancy in Tangential Field along the Lower Skirt

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section are obtained by taking the average of the potentials obtained due to the two sets of charges.

Figure (5.13) illustrates the potential distribution along the upper dielectric skirt. As seen, the potential close to point 'B' is about 40% of the total applied voltage.

Figure (5.14) illustrates the potential distribution along the lower dielectric skirt of the insulator. The potential values computed at the boundary points are labelled on the extremities of each corrugation of the skirt. As seen from this figure, the maximum potential difference (33%) appears across the corrugation labelled 2, 3. Also the potential at the exterior part of the corrugation is found to be more than at the interior part.

5.2.6 Programming

A computer program was written for computing the electric field and potential distribution for the suspension insulator used on practical high voltage transmission lines. The program is written in the FORTRAN IV language for the IBM 3031 computer available at the Computer Centre, University of Windsor.

The simultaneous equations given in section 4.2.2 representing the boundary conditions are solved using a subroutine which performs Gaussian elimination (Crout algorithm) with equilibration and partial pivoting. The main computational steps are shown in Fig. (5.15). The solution of the simultaneous equations (section 4.2.2) gives the values of the unknown ring and line charges. These charges are used in computing the
Figure 5.13: Potential Distribution along the Upper Skirt
Figure 5.14: Potential Distribution along the Lower Skirt
START

READ INSULATOR'S DIMENSIONS, COORDINATES OF CHARGE LOCATIONS AND CONTOUR POINTS

COMPUTE CHARGE COEFFICIENT MATRIX

SOLVE SIMULTANEOUS EQUATIONS

WRITE VALUES OF THE RING AND LIKE CHARGES

INTERPOLATE AND STORE THE CO-ORDINATES OF THE CONTOUR POINTS ON THE CURVED BOUNDARIES OF THE INSULATOR

EVALUATE POTENTIAL ERROR ON CONDUCTING PARTS OF THE INSULATOR

EVALUATE POTENTIAL DISCREPANCY AND DISCREPANCY IN TANGENTIAL FIELD ON THE DIELECTRIC BOUNDARIES

WRITE AND PLOT THE POTENTIAL AND FIELD DISTRIBUTIONS

STOP

Figure 5.15: Flow Chart
potential and the field anywhere on the conductor's surface and in the outside space.

The main features of the program developed are:

1. Allows digitizer input of the actual geometry.
2. Uses cubic spline interpolatory scheme for interpolating points along the curved boundaries of the insulator.
3. Allows convenient testing for several charge locations.

The listing of the computer program is given in Appendix B.

5.3 Long Rod Polymer Insulator

5.3.1 General

The method of computation for the long rod polymer insulator shown in Fig. (1.2) has been explained in Section 4.4.2.

Only ring charges were used to simulate the potential in the vicinity of (and within) the insulator. The computer program developed was initially run for one unit of the polymer rod without considering the grounded conductor. The following number of charges were used to model the various parts of the insulator as shown in Fig. (5.16(a)).

Conductor-Dielectric interface (A-B): 3
Metal Shank (C-D): 2
Conductor clamp and part of conductor (E-F): 10
Vertical column of the dielectric (I-J): 5
Horizontal interface of the first dielectric unit (K-L): 5
Figure 5.16: Distribution of Charges in the Long Rod Insulator

(a) Partial Configuration
(b) Insulator with three dielectric units between high voltage and grounded conductors

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Vertical and sloped skirt of the dielectric unit

\( (M-N) : 9 \)

Thus 71 linear equations were solved for the calculation of the same number of unknown charges. The contribution of positive images of the charges in the clamp and part of the high voltage conductor (explained in chapter IV) have been taken into account in the calculation of the electric field distribution. Table (5.1) presents the R.M.S. and M.S. error along the various parts of the insulator considered in Fig. (5.16(a)). As seen from this table, these values of R.M.S and M.S error are within a reasonable limit of accuracy. The computer program was then extended to include the entire polymer rod and the grounded conductor for the same number of charges in the high voltage metal shank, clamp, conductor, vertical polymer column and the first dielectric unit. The positions of the charges were not disturbed. Since the rest of the units in the rod are identical to the first one, the same number of charges as for the first unit were also used for all the other units in the rod.

According to the preliminary calculations, fourteen ring charges (total charges = 28, considering those inside and outside the dielectric) were found to be an adequate number to model one unit of the rod. If all the charges in the rod (374* charges) and 23 charges in the grounded conductor (same number of charges were used to model the high voltage metal

* Total number of units in the rod is 13, number of charges modelling the vertical column of the rod adjacent to high voltage shank is 10

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Table 5.1: M.S. and R.M.S. Error on Parts of the Partial Configuration of the Rod Insulator (Fig.5.16a)

<table>
<thead>
<tr>
<th>CONDUCTOR PART*</th>
<th>DIELECTRIC PART **</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conductor-Dielectric Interface (AB)</td>
</tr>
<tr>
<td>M.S. Error</td>
<td>$0.297 \times 10^{-5}$</td>
</tr>
<tr>
<td>R.M.S. Error</td>
<td>$0.172 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

* Mean square error on the conductor = $0.174 \times 10^{-4}$  
Root mean square error on the conductor = $0.419 \times 10^{-2}$

** Mean square error on the dielectric = $0.131 \times 10^{-5}$  
Root mean square error on the dielectric = $0.115 \times 10^{-2}$

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shank) are considered to model the entire insulator, would result in 430 linear equations that will have to be solved for the same number of unknown charges. Although the computer program developed is flexible enough to accommodate any number of dielectric units between the high voltage conductor and the grounded conductor, the large number of simultaneous equations places rather severe demands on the memory requirements and the computer processing time. Both the computer time and the memory storage considerations present limitations on the number of charges used to model the system.

In the following sections, results are presented for the polymer rod insulator considering only three units of the rod between the high voltage conductor and the grounded conductor as shown in Fig. (5.16(b)).

5.3.2 Metallic Parts of the Insulator Exposed to Air

The high voltage conductor is rigidly connected to the polymer insulator by means of a clamp. The potential of the high voltage conductor and hence the clamp and the metal shank is assumed to be of one unit. The difference between the known conductor potential and the computed potential has been deemed a criterion of accuracy of the calculation.

Figure (5.17) illustrates the potential error along the metal shank (A-B) clamped with the high voltage conductor. The contribution of all the charges in the conductors as well as in the polymer rod were taken into account while simulating potential at the conductor parts exposed to air. As seen from
Figure 5.17: Potential Error along the Air Part of the High Voltage Metal Shank
Fig. (5.17) the maximum error of the potential is about 1.0% near point 'A'. The error is less than 1.0% at all other points on the shank. Figure (5.18) illustrates the potential error along the clamp and part of the high voltage conductor (5.0 cm). Ten ring charges in the clamp and four ring charges in the high voltage conductor were used to model these parts. As seen from Fig. (5.18), the maximum error in potential is about 1.2% near point 'B' on the clamp and 1.4% on the high voltage conductor close to the clamp near point 'C'.

Figure (5.19) illustrates the potential error along the metal shank (A-B) of the grounded conductor. The potential of the cap of the unit was assumed to be at ground potential. The potential was evaluated at several check points along the shank. As indicated by Fig. (5.19), the maximum potential error is about 0.26% near point 'A'. The error is less than 0.2% at all other points on the shank which indicates that the number of charges used to model the grounded conductor have produced a solution of very good accuracy there.

5.3.3 Metallic Parts Interfaced with the Dielectric

Figure (5.20) illustrates the potential error along the dielectric surface interfaced with the high voltage conductor. Only three ring charges were used to model this dielectric-conductor interface. The contribution of all the charges in the conductors and the charges in air modelling the polymer rod were taken into account. As seen from Fig. (5.20), the potential error is less than 0.5%.
Figure 5.18: Potential Error along the Conductor Clamp and a Section of the High Voltage Conductor (5.0 cm)
Figure 5.19: Potential Error along the Metal Shank of the Grounded Conductor
Figure 5.20: Potential Error along the Conductor-Dielectric Interface of the High Voltage Conductor
Figure (5.21) illustrates the potential error along the dielectric surface interfaced with the grounded conductor. Three ring charges were used to simulate the potential of this interface. As indicated by this figure, the potential error is less than 0.15%.

5.3.4 Potential Discrepancy along the Dielectric Skirts

As explained in section 5.2.5 the potential at the dielectric boundary is not known but it must be the same when computed from either side of the dielectric surface. The charges in the high voltage conductor and the grounded conductor contribute to the potential and field strengths in both media i.e. in the air and in the solid dielectric. The term 'Potential Discrepancy' on the dielectric boundary instead of 'Potential Error' on the conductor surfaces has been deemed to be a criterion for the evaluation of accuracy in the calculations.

Figure (5.22) illustrates the 'Potential Discrepancy' along the vertical column of the polymer rod. As seen from the figure, the maximum potential discrepancy is 0.3% close to point 'B'. Figure (5.23) illustrates the potential discrepancy along the horizontal interface of the first dielectric unit. As seen the maximum discrepancy in potential is about 0.34% close to point 'B'.

Figure (5.24) illustrates the potential discrepancy along the vertical and sloped skirt of the first dielectric unit. The maximum discrepancy along the vertical skirt labelled A-B is about 0.01% whereas the maximum discrepancy
Figure 5.21: Potential Error along the Conductor-Dielectric Interface of the Grounded Conductor

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Figure 5.22: Potential Discrepancy along the Vertical Column of the Polymer Rod
Figure 5.23: Potential Discrepancy along the Horizontal Interface (Unit #1)
Figure 5.24: Potential Discrepancy along the Vertical and Sloped Skirt (Unit #1)
along the sloped part is about 0.26% near point 'D'.

Figure (5.25) illustrates the potential discrepancy along the horizontal interface of the second dielectric unit. The maximum discrepancy is about 0.12% near point 'A'.

Figure (5.26) illustrates the potential discrepancy along the vertical and sloped skirt of the second unit of the rod. The maximum discrepancy along the vertical skirt labelled A-B is less than 0.01% whereas the maximum discrepancy in potential along the sloped skirt is about 0.19% near point 'D'.

Figure (5.27) illustrates the potential discrepancy along the horizontal interface of the third unit of the polymer rod. As seen from this figure, the maximum potential discrepancy is about 0.17% near point 'A'.

Figure (5.28) illustrates the potential discrepancy along the vertical and sloped skirt of the third dielectric unit. The maximum discrepancy on the vertical skirt is about 0.05% near point B whereas the maximum discrepancy along the sloped skirt is about 0.31% near point 'D'.

5.3.5 Discrepancy in the Tangential Field along the Dielectric Skirts

Figure (5.29) illustrates the discrepancy in the tangential field along the vertical column of the polymer rod. As seen from this figure, corresponding to a maximum potential discrepancy of 0.3%, the maximum discrepancy in the tangential field is about 0.22%.

Figure (5.30) illustrates the discrepancy in the tangential field along the horizontal interface of the first
Figure 5.25: Potential Discrepancy along the Horizontal Interface (Unit #2)
Figure 5.26: Potential Discrepancy along the Vertical and Sloped Skirt (Unit #2)
Figure 5.27: Potential Discrepancy along the Horizontal Interface (Unit # 3)
Figure 5.28: Potential Discrepancy along the Vertical and Sloped Skirt (Unit # 3)
Figure 5.29: Discrepancy in Tangential Field along the Vertical Column of the Polymer Rod

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Figure 5.30: Discrepancy in Tangential Field along the Horizontal Interface (Unit #1)
dielectric unit. The maximum discrepancy is 0.8% near point 'B' compared to 0.34% maximum discrepancy in potential.

Figure (5.31) illustrates the discrepancy in the tangential field along the vertical and sloped skirt of the first dielectric unit. The maximum discrepancy along the vertical skirt is about 0.01% whereas the maximum discrepancy in the tangential field along the sloped skirt is 0.5% near point 'D' compared to 0.26% maximum discrepancy in potential in the same region.

Figure (5.32) illustrates the discrepancy in the tangential field along the horizontal interface of the second dielectric unit. As seen from this figure, the maximum discrepancy in the tangential field is about 0.14% near point 'A' corresponding to 0.12% discrepancy in potential at this part.

Figure (5.33) illustrates the discrepancy in the tangential field along the vertical and sloped skirt of the second unit of the polymer rod. The maximum discrepancy along the vertical skirt is 0.1% whereas the maximum discrepancy along the sloped skirt is found to be about 0.5% near point 'D' corresponding to 0.19% maximum discrepancy in potential.

Figure (5.34) illustrates the discrepancy in the tangential field along the horizontal interface of the third dielectric unit. As seen from this figure, the maximum discrepancy is about 0.19% near part 'A' corresponding to 0.17% discrepancy in the potential.

Figure (5.35) illustrates the discrepancy in the
Figure 5.31: Discrepancy in Tangential Field along the Vertical and Sloped Skirt (Unit #1)
Figure 5.32: Discrepancy in Tangential Field along the Horizontal Interface (Unit #2)
Figure 5.33: Discrepancy in Tangential Field along the Vertical and Sloped Skirt (Unit #2)
Figure 5.34: Discrepancy in Tangential Field along the Horizontal Interface (Unit #3)
Figure 5.35: Discrepancy in Tangential Field along the Vertical and Sloped Skirt (Unit #3)
tangential field along the vertical and sloped skirt of the third dielectric unit. As indicated by this figure, the maximum discrepancy along the vertical skirt is less than 0.3% whereas the maximum discrepancy along the sloped skirt is 1.0% near point 'D' corresponding to 0.31% maximum discrepancy in potential in the same vicinity.

5.3.6 Potential Distribution along the Polymer Rod

As explained earlier, the potential at the dielectric boundary is not known. In order to model a dielectric-dielectric boundary, equal numbers of charges are placed at appropriate distances on either side of the dielectric interface. Then the potential at any point on the dielectric boundary due to charges in one dielectric must be equal to the potential due to charges in the other dielectric. The charges in the conductors contribute to potential values in both the dielectric media.

The results for potential distribution on the surface of the polymer rod presented in this section are obtained by taking the average of the potential due to the charges inside and outside of the polymer rod.

Figures (5.36-5.42) illustrate the potential distribution on various parts of the polymer rod. The largest potential difference appears across the vertical column of the polymer rod as shown in Fig. (5.43). The impressed voltage across this section is about 45% of the applied voltage. The potential distribution across the other units of the rod are approximately equally distributed.

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Figure 5.36: Potential Distribution along the Vertical Column of the Polymer Rod
Figure 5.37: Potential Distribution along the Horizontal Interface of the Polymer Rod (Unit #1)
Figure 5.38: Potential Distribution along the Vertical and Sloped Skirt (Unit #1)
Figure 5.39: Potential Distribution along the Horizontal Interface (Unit #2)
Figure 5.40: Potential Distribution along the Vertical and Sloped Skirt (Unit #2)
Figure 5.41: Potential Distribution along the Horizontal Interface (Unit #3)
Figure 5.42: Potential Distribution along the Vertical and Sloped Skirt (Unit #3)
Figure 5.43: Computed Potential Distribution of a Long Rod Insulator
5.3.7 Experimental Results

For the several advantages of the polymer rod insulator mentioned in section 4.4.2, Ontario Hydro is considering their use for transmission lines at voltages of 230 KV and 500 KV. To this end, experiments were conducted in the 'transmission and special projects section of Electrical Research Development of Ontario Hydro'. The potential was measured by means of AC potentiometer of mechanical resonance type [53] along the surface of the polymer rod and in the vicinity of the insulator.

Figure (5.44) presents the experimental results along with our computed results for comparison purpose. As seen, the experimental results appear to match the calculated ones to within the experimental error [54]. The agreement of the two results indicate the degree of validity of the charge simulation technique and encourage confidence in its application to the design of insulation systems in power apparatus.

5.3.8 Programming

A computer program is written for computing the electric field and potential distribution for the long rod polymer insulator. The computer program is written in FORTRAN IV language for the IBM 3031 computer available at the Computer Centre, University of Windsor. Double precision arithmetic is used throughout the main program and the subroutines. The simultaneous equations given in section 4.2.2 representing the boundary conditions are solved using the modified Gaussian technique which employs decomposition of the coefficient matrix.
Figure 5.44: Comparative Results of Potentials on First Three Surfaces of Polymer Rod

(a) Experimental
(b) Computed
using Crout algorithm and performs partial pivoting, scaling and equilibration, computing solution which is without round off error to the number of digits specified [48].

The solution of the simultaneous equations (section 4.2.2) gives the values of the unknown ring charges. These are used in computing the potential and the field anywhere on the conductor's surface and in the outside space.

The main computational steps are similar to those shown in the flow chart of Fig. (5.15). The strategy of interpolating the boundary points on the curved boundaries has not been used in this program. Instead, an appropriate number of discrete points on different parts of the insulator have been digitized and stored in the arrays.

The main features of the program are:

1. **Flexibility** - accommodates any number of dielectric units of the polymer rod between the high voltage conductor and the grounded conductor.

2. The program needs the coordinates of the charge locations and contour points for the partial configuration of the insulator shown in Fig. (5.16(a)). The data for the rest of the rod and the grounded conductor is automatically generated within the program.

3. The program is controlled by logical commands, thus the following sections of the program can be independently operated.

   i) Checking of boundary conditions on all boundary points
on the conductor parts.

ii) Checking of boundary conditions on all boundary points on the polymer rod.

iii) Potential Error on several check points on the conductor parts of the insulator

iv) Potential discrepancy on check points on the dielectric skirts.

v) Discrepancy in tangential field on all check points of the dielectric skirt.

vi) Plotting of electric field distribution.

The listing of the Computer Program is given in Appendix C.
VI. CONCLUSIONS AND RECOMMENDATIONS

6.1 General

The predetermination of discharge voltages needs an accurate knowledge of the electric field in the vicinity of the configuration of interest. Experimental procedures, e.g. the use of an electrolytic tank or semiconductive paper, are cumbersome and not suitable for three dimensional problems. To this end, both computer hardware and software sophistication has made it possible to synthesize and evaluate the performance of devices of interests by means of purely computational techniques. However, the range of problems and the range of computational features that they present are wide, and it is unlikely that one single mathematical method will deal successfully with them all. Among various analysis methods, the charge simulation technique is advantageous over the finite difference technique and finite element method because of the relative ease with which boundary conditions can be set and the reduction of calculation time. The implementation of the Charge Simulation Technique requires:

a) A sketch or description of the configuration to be modelled.

b) Generation of co-ordinates of the arbitrarily distributed charge locations and contour points in two or three dimensions.

c) Construction of the linear system of simultaneous equations, (Eqn. (3.5)) and its numerical solution giving the values of charges.
d) Calculation of the desired potential and fields.

Furthermore, it may be desired to calculate the location of the equipotential surfaces representing the electrodes and to make a comparison with the electrode surface location to check the accuracy of the solution.

From the practical program user's point of view, the electrode and dielectric surfaces are specified by the co-ordinates of a series of contour points. In most cases these co-ordinates are calculated within the computer program (a procedure which may be quite cumbersome or rather impractical in some complicated geometries of practical high voltage devices). This might tempt a programmer to either alter the geometry of the device or introduce assumptions/approximations in the calculations. The use of the digitizer has alleviated this problem somewhat, and it is now possible to use large numbers of co-ordinates of the charge locations and contour points for practical geometries. This has also reduced the computer time (a little).

6.2 Conclusions

The present study of the Charge Simulation technique applied to practical geometries of power apparatus involving two dielectric systems indicate that the method is a useful tool in the design of the insulation system of a high voltage device. It can be recognized, however, that the results obtained by this method are approximate; but this method is easy to use and it is possible to apply it for the electric field computation for a variety of practical configurations of
devices in electric power apparatus.

The following conclusions are derived from the work presented in this thesis.

1. The method described can be used to model the complicated practical geometry of certain electric power apparatus. The values of potential and tangential field can be computed in areas of particular interest.

2. Previously the 'POTENTIAL ERROR' has been used by many authors as a measure of accuracy of the method on conductor surfaces and has been defined by Singer [21] as the difference between the known conductor potential and the computed potential.

   The term 'POTENTIAL DISCREPANCY' for the dielectric boundary is introduced in this work to convey a similar concept at dielectric interfaces.

3. Symmetry can be utilized to reduce the modelling effort and cut-up time for some problems.

4. The accuracy of the method is sensitive to the charge location, the contour points where the boundary conditions are imposed and the number of charges. However, these factors seem to have the least bearing on the accuracy when the shape of the device is complicated like the lower skirt of the suspension insulator. Thus the accuracy achieved for a complicated situation will be different from that when the method is tested for a simple system.

5. The time for computation with an IBM 3031 computer at the University of Windsor for the suspension insulator with
107 charges and 444 check points at which potential and field strength values are calculated is 148 seconds; for the Long Rod Insulator with 150 charges (and 610 check points at which potential and field strength values are calculated) it is 248 seconds.

6. The analysis of section 4.2.3 shows that although the iterative techniques are popularly used for large sets of equations mainly because of better round-off error characteristics and less storage requirements, the iterative techniques (Gauss-Siedel) did not converge when applied to equations of section 4.2.3 after the subroutine [48] in the computer library for solving linear systems of equations (using the decomposition technique of the Crout algorithm [47] instead of the elimination technique of the Gauss algorithm) with partial pivoting, scaling and equilibration was used.

7. The use of the digitizer provides convenient testing of the method for different sets of data.

6.3 Recommendations

It must not be overlooked that the present study examines only one aspect of the properties for a good design of an insulation system in power apparatus. The whole electrical aspect itself is only one of the many which must be taken into account in electrical insulation equipment design. A great deal of engineering lies behind the production of even the simplest electrical component and modifications, desirable and simple as they may appear electrically, may be impractical on account of other difficulties entailed in their realization.
The following recommendations are made for possible continuation of this work:

1. The impact of the present study of the charge simulation technique and its applicability to practical geometries of high voltage devices is important for a design engineer. On the other hand, information regarding performance of the insulator in field service is essential for an operation and maintenance engineer.

   It is therefore now necessary to carry out laboratory tests to verify the computational results (especially for the suspension insulator in a dry, clean atmosphere). These results if matched will increase the confidence in the applicability of the method. A thought should therefore be given to modelling an insulator in a polluted environment by the charge simulation technique. It should be noted here, that other computational techniques (e.g. the finite difference technique) have been used to calculate the potential distribution of insulator systems in polluted conditions.

2. One of the problems with the polymer insulation is the high electric field in the polymer material [54]. High gradients may cause damage to the polymer material and eventual degradation of the units. To alleviate this, 'stress control rings' are usually used at 500 KV. These rings are presently expensive and eliminate any cost advantage. Thus an utmost need is evident to extend the charge simulation technique to optimize the size and shape of these gradient control rings for 230 KV and 500 KV polymer insulators.

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3. The accuracy of the charge simulation technique is in fact dependent on the location of charges and the boundary points and on the density of charges. A number of developments to the method have been suggested but all are expensive in terms of computer time. Thorough research and testing of the method may be required to simplify the method to the degree of least dependency on the factors mentioned above.

4. A useful way to visualize the electric field distribution of a device is to map the equipotential and field lines. The subroutine for this purpose, available at the Computer Centre, University of Windsor, was used exhaustively to draw equipotential surfaces, but it seems to be limited in its applicability to a case involving multiple dielectric. Furthermore, the package is inefficient and does not exist even if the routine steps off the equipotential surface. In fact the subroutine was originally developed for simple charge configurations and need modification to incorporate special features for handling more complicated ones.

5. The results obtained by the charge simulation technique are approximate and it may not be possible to obtain satisfactory solutions for some large and complex configurations using this method. In these cases as suggested by Beasley et al [40], the charge simulation method could be used to derive a first approximation followed by the finite element method within some reduced subregion of interest.
APPENDIX A

EXPRESSIONS FOR POTENTIAL AND FIELD STRENGTH DUE TO POINT, LINE, RING CHARGE [18, 21]

1. **Point Charge**

\[
P_i = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{j=1}^{n} \frac{Q_j}{|\vec{r}_i - \vec{r}_j|}
\]

\[
E_x = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{j=1}^{n} \frac{Q_j \cdot (x_i - x_j)}{|\vec{r}_i - \vec{r}_j|^3}
\]

\[
E_y = \frac{1}{4\pi\varepsilon_0} \cdot \sum_{j=1}^{n} \frac{Q_j \cdot (y_i - y_j)}{|\vec{r}_i - \vec{r}_j|^3}
\]

2. **Line Charge**

\[
P_i (r_i, z_i)
\]

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\[ P_i = \frac{1}{4\pi \varepsilon(z_{j2} - z_{j1})} \cdot \ln \left( \frac{z_{j2} - z_i + \delta_1}{z_{j1} - z_i + \delta_1} \right) \]

\[ E_r = \sum_{j=1}^{n} \frac{Q_j}{4\pi \varepsilon(z_{j2} - z_{j1})} \cdot \left[ \frac{z_{j2} - z_i}{r \cdot 1} - \frac{z_{j1} - z_i}{r \cdot 1} \right] \]

\[ E_z = \sum_{j=1}^{n} \frac{Q_j}{4\pi \varepsilon(z_{j2} - z_{j1})} \cdot \left[ \frac{1}{\delta_1} - \frac{1}{\delta_1} \right] \]

\[ \delta_1 = \sqrt{r^2 + (z_{j2} - z_i)^2} \]

\[ \delta_1 = \sqrt{r^2 + (z_{j1} - z)^2} \]

3. **Ring Charge**
\[ p_i = \frac{1}{4\pi \varepsilon} \cdot \frac{2}{\pi} \cdot \frac{K(Kl)}{n} \]

\[ E_r = \sum_{j=1}^{n} \frac{-Q_j}{4\pi \varepsilon} \cdot \frac{1}{\pi r} \cdot \left[ \frac{r_j^2 - r_i^2 + (Z_i - Z_j) \cdot E(Kl)}{\alpha l \cdot \beta^2 l} \right] \]

\[ E_z = \sum_{j=1}^{n} \frac{Q_j}{4\pi \varepsilon} \cdot \frac{2}{\pi} \cdot \left[ \frac{(Z_i - Z_j) \cdot E(Kl)}{1 \cdot \beta^2 l} \right] \]

\[ \alpha l = \sqrt{(r_i + r_j)^2 + (Z_i - Z_j)^2} \]

\[ \beta l = \sqrt{(r_i - r_j)^2 + (Z_i - Z_j)^2} \]

\[ Kl = \frac{2 \sqrt{r_j \cdot r_i}}{\alpha l} \]

\[ K(Kl) \text{ and } E(Kl) \text{ are the complete elliptic integrals of the first and second kind respectively.} \]
APPENDIX B

Computer Program for Suspension Insulator

IMPLICIT INTEGER*4 (I-H), REAL*8(A-H,O-Z)
REAL*4 SRC(450), ZSC(450), SERT(450), SERB(450), XC(90), YC(90), RW, ZW
X, DIFF(300)
DIMENSION RQ(110), ZQ(110), P(110, 110), WAREA(110),
1 CX(110), X9(20), ZB(20), XQ(90),
XY(90), ZUM(450)
COMMON RW(200), ZW(200), I31, I13, I41, I14, KC
COMMON /HLK2/KC(450), ZC(450), LL1, LLX, LLL1, LLL
DATA N1, N2, N3, N4, N5, NL, NDL, NRH, NAL, DL, RA, RD, HD/17, 5, 3, 12, 8,
X16, X5, X3, 1.276D0, 2.541D0, 0.312D0, 0.767D0, 0.362D0/
X, PI/3.141592653D0/
X, EPSIR/.01/.D-9/
LOGICAL DPRINT/.FALSE./, DPLLOT/.FALSE./, SEC1/.FALSE./, SEC2/
X, FALSE./, CONFG/.FALSE./
I36 = N1 + N2 + N3 + 2*(N4 + N5)
NX = 10
READ 200, (X0(I), Y0(I), XC(I), YC(I), I = 1, I36)
200 FORMAT (4(G7.4))
READ 201, (XB(I), ZB(I), I = 1, NX)
201 FORMAT (2(G7.4))
NAMELIST/UN/DPRINT
READ 3, (UN(I), END = 9999)
I6 = N1 + N2
I61 = I6 + N3
XA = (AL + DL + HD)
U = 1.D0

131 = N1
133 = 2*(N4 + N5)
122 = I65 - I6
12 = I11 + N2 + I22 + N4 + N5
13 = I2 + 1
131 = 13
133 = I2 + N4
141 = I3 + N4
144 = I3 + N4 + N5

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I4=I2+N4+N5
I5=I1+1
I7=I65
I8=I7+1
I9=I6+1
NBD=I65-I63
I10=I7-NBD
I11=I10+1
N41=N4
N51=N5

WRITE(*,101) I6,I22,I33,I4
101 FORMAT(*,'NUMBER OF CHARGES MODELLING TOP ELECTRODE:',I3//
       'NUMBER OF CHARGES MODELLING THE BOTTOM ELECTRODE:',I3//
       'TOTAL CHARGES MODELLING THE SYSTEM:',I3)
DO 199 I=1,I61
RQ(I)=XQ(I)
ZQ(I)=YQ(I)
RC(I)=Xc(I)
ZC(I)=YC(I)
199

C SPECIFY THE CONTOUR POINTS ON THE DIELECTRIC BOUNDARY.
DO 202 I=1,N4
RQ(I65+I)=XQ(I61+I)
RQ(I65+N4+N5+I)=XQ(I61+N4+N5+I)
ZQ(I65+I)=YQ(I61+I)
ZQ(I65+N4+N5+I)=YQ(I61+N4+N5+I)
RC(I65+I)=XC(I61+I)
RC(I65+N4+N5+I)=RC(I65+I)
ZC(I65+I)=YC(I61+I)
ZC(I65+N4+N5+I)=ZC(I65+I)
M22=I65+N4
M23=I65+2*N4+N5
DO 140 I=1,N5
RQ(M22+I)=XQ(I61+N4+I)
RQ(M23+I)=XQ(I61+N4+N5+I)
ZQ(M22+I)=YQ(I61+N4+I)
ZQ(M23+I)=YQ(I61+N4+N5+I)
RC(M22+I)=XC(I61+N4+I)
ZC(M22+I)=YC(I61+N4+I)
RC(M23+I)=RC(M22+I)
ZC(M23+I)=ZC(M22+I)
140 IF(CONFG).G0T07777
PRINT 139
139 FORMAT(*,'RQ',10X,'ZQ',12X,'RC',12X,'ZC',/)
DO 141 I=1,I4
141 WRITE(*,142) RQ(I),ZQ(I),RC(I),ZC(I)
142 FORMAT(*,'RQ',10X,'ZQ',12X,'RC',12X,'ZC',/)

C FILL THE POTENTIAL MATRIX
DO 203 I=1,I7
DO 203 J=1,I2
IF(J.GT.I11.AND.J.LE.I62)G0T0204
IF(J.GT.I63.AND.J.LE.I64)G0T02041
P(I,J).G0T07777
IF(I.GT.I11.AND.I.LE.I62)G0T0205
IF(I.GT.I62.AND.I.LE.I64)G0T02051
P(I,J).G0T07777
203 ULRITI(A,142) RQ(I),ZQ(I),RC(I),ZC(I)
204 P(I,J).G0T07777
2041
GOTO 203
205 P(I,J)=0.D0
203 CONTINUE
   DO 207 I=1,I1
   DO 207 J=1,J1
207 P(I,J)=0.D0
   DO 209 I=15,I4
   DO 209 J=13,I4
209 P(I,J)=VRING(RCd(I),ZCd(I),RR(J),ZQ(J),Q)
   DO 208 I=19,I10
   IF 208 J=13,I4
208 P(I,J)=0.D0
   DO 902 I=11,I7
   DO 902 J=13,I4
902 P(I,J)=VRING(RCd(I),ZCd(I),RR(J),ZQ(J),Q)
   DO 213 I=18,I2
   DO 213 J=17,I4
IF(J.LE.17)GOT0214
   P(I,J)=VRING(RCd(I),ZCd(I),RR(J),ZQ(J),Q)
   IF(J.GT.17)P(I,J)=P(I,J)
GOT0213
214 P(I,J)=0.D0
213 CONTINUE
   IF(DPRINT)
   1WRITE(6,191)
      K=1
   PRINT 995
995 FORMAT(/15X,'DISCRETE CONTOUR POINTS ALONG THE UPPER DIELECTRIC XPART'//)
   NPTS=8
444 CONTINUE
   KC=1
   IF(K.EQ.2)WRITE(6,996)
996 FORMAT(/15X,'DISCRETE CONTOUR POINTS ALONG THE LOWER DIELECTRIC XSKIRT'//)
   CALL SPLINE(XGdII,YCII,N4,NPTS,RW,ZW,NP)
   DO 66 L=1,NP
       SRC(L+IK)=RW(L)
       SZZC(L+IK)=ZW(L)
   66 IF(K.EQ.1)IK=NP
   DO 220 I=13,I4
       IC=1
       DO 2201 J=1,I4
IF(J.GT.I61.AND.J.LE.I62)GOT0221
IF(J.GT.I63.AND.J.LE.I64)GOT02211
   CALL ERING(RCd(I),ZCd(I),RR(J),ZQ(J),ER,EZ,IC)
   P(I,J)=(ENOR(RCd(I),ZCd(I),ER,EZ,IC))
   IF(J.LE.17) P(I,J)=P(I,J)*EPSIR-1.D0
   IF(J.GT.17) P(I,J)=P(I,J)*EPSIR-1.D0
IF(J.GT.I2) P(I,J)=(EPSIR)*P(I,J)
   IF(I.FQ.(12+N41).AND.J.EQ.I4)GOT0322
GOT02201
221 IF(J.GT.I61.AND.J.LE.I62)SSL=DEL
2211 CALL ELINE(RCd(I),ZCd(I),ZQ(J),SSL,ER,EZ,IC)
   P(I,J)=(ENOR(RCd(I),ZCd(I),ER,EZ,IC))
   P(I,J)=P(I,J)*EPSIR-1.D0
   SSL=DDL
2201 CONTINUE
   KC=KC+NPTS
CONTINUE

II=II+N4
I3=I3+N4
N4=N5
NPTS=10
K=K+1
IF(K.EQ.2)GOTO444
JK=NP
DO 23 I=1,I4
CX(I)=0.D0
IF(I.GT.16.AND.I.LE.17)CX(I)=1.D0
23 CONTINUE

M=1
IDGT=8
IA=110
CALL LEDTIF(P,M+I4,IA,CX,IDGT,WKAREA,IER)
C ERROR IN SOLUTION AT THE TOP ELECTRODE SURFACE.
PRINT 871
871 FORMAT(/15X,'DISCRETE CONTOUR POINTS ALONG THE TOP CONDUCTOR',/)
DO 32 I=1,NLA
32 YQ(I)=ZC(I+I6+I)
DO 33 I=1,NRH
33 XQ(I)=RC(I+I64+I)
DO 44 I=1,NRLH
44 SUM(I)=RC(I+I62+I)
DO 41 I=1,NL
41 SERT(I)=ZC(I+I63+I)
C TOP CONDUCTOR CHECK POINTS.
N11=N1-2
NPTS=5
CALL SPLINE(XC, YC, N11, NPTS, RW, ZW, NP)
DO 303 I=1, NP
RC(I)=RW(I)
303 ZC(I)=ZW(I)
CALL SPLINE(XC(N11), YC(N11), ZC(N11), 2, NPTS, RW, ZW, NP)
NP2=NP1+1
DO 302 I=1, NP2
RC(NP+I)=RW(NP1-I)
302 ZC(NP+I)=ZC(NP1-I)
NTOP=NP+NP2
KOUNT=KOUNT1
NTOP1=NTOP
NTOP2=NTOP+1
C DIELECTRIC INTERFACE OF TOP ELECTRODE.
KOUNT1=0
DO 400 I=1,N2
RC1=XC(N1+I)
RC2=XC(N1+I+1)
IF(I.EQ.N2) RC2=0.D0
RC3=(RC1-RC2)/4.D0
RC(NTOP+1)=RC1
ZC(NTOP+1)=HI
DO 402 J=2,N2
402 ZC(NTOP+J)=HD
NTOP=NTOP+4
KOUNT1=KOUNT1+4
400 CONTINUE
NTOP2=NTOP+1
NTOP3=NTOP
C TIP OF BOTTOM ELECTRODE.
DO 550 I=1,NX
   RC(NTOP+I)=XR(I)
550   ZC(NTOP+I)=ZR(I)
C AIR PART OF THE BOTTOM ELECTRODE.
   NAIR=NTOP+NX
   NAIR1=NAIR+1
   ZC1=(HD+DL+AL)
   KOUNT2=0
DO 500 I=1,NLA
   ZC2=YQ(I)
   ZC3=DBAS((ZC1-ZC2)/4.DO)
   ZC(NAIR+I)=ZC1
   RC(NAIR+I)=RA
500   DO 501 J=2,4
   JJ=J-1
   ZC(NAIR+J)=ZC(NAIR+JJ)+ZC3
501   RC(NAIR+J)=RA
   NAIR=NAIR+4
   ZC1=ZC2
   KOUNT2=KOUNT2+4
500 CONTINUE
C HORIZONTAL INTERFACE OF LOWER DIELECTRIC EMBEDDED CONDUCTOR.(LOWER)
   NHI=0
DO 804 I=1,NLH
   RC1=SUM(I)
   RC2=SUM(I+1)
   IF(I.EQ.NRLH)RC2=RD
   RC3=(RC2-RC1)/3.DO
   RC(NAIR+I)=RC1
   ZC(NAIR+I)=-(HD+DL)
804   DO 885 J=2,3
   JJ=J-1
   RC(NAIR+J)=RC(NAIR+JJ)+RC3
885   ZC(NAIR+J)=ZC(NAIR+1)
   NAIR=NAIR+3
   NHI=NHI+3
804 CONTINUE
   NK1=NX+KOUNT2+NHI
C DIELECTRIC PART OF THE BOTTOM ELECTRODE.(VERTICAL)
   NDIL=NAIR
   NDIL1=NDIL+1
   KOUNT3=0
   ZC1=(HD+DL)
   DO 600 I=1,NLD
   ZC2=SERT(I)
   ZC3=DBAS((ZC1-ZC2)/6.DO)
   ZC(NDIL+I)=ZC1
   RC(NDIL+I)=RD
600   DO 601 J=2,6
   JJ=J-1
   ZC(NDIL+J)=ZC(NDIL+JJ)+ZC3
601   RC(NDIL+J)=RD
   NDIL=NDIL+6
   ZC1=ZC2
   KOUNT3=KOUNT3+4
600 CONTINUE
C HORIZONTAL INTERFACE OF BOTTOM CONDUCTOR (UPPER)
   NDILU=NDIL+1
   KOUNT4=0
DO 800 I=1,NRH
RC1=XG(I)
RC2=XG(I+1)
IF(I.EQ.NRH) RC2=0.D0
RC3=(RC1-RC2)/3.D0
RC(NDIL+I)=RC1
ZC(NDIL+I)=HD
DO 801 J=2,3
JJ=J-1
RC(NDIL+J)=RC(NDIL+JJ)-RC3
801 ZC(NDIL+J)=HD
NDIL=NDIL+3
KOUNT4=KOUNT4+3
800 CONTINUE

LL=NDIL

C DIELECTRIC BOUNDARY.
DISCRETE CONTOUR POINTS ALONG THE PLAIN DIELECTRIC SURFACE.

DO 900 I=1,IK
RC(LL+I)=SRC(I)
900 ZC(LL+I)=SZC(I)
LL=LL+IK
C DISCRETE POINTS ALONG THE LOWER SKIRT OF THE DIELECTRIC.
DO 803 I=1,JK
RC(LAS+I)=SRC(1K+I)
803 ZC(LAS+I)=SZC(IK+I)
LLL=LAS+JK
PRINT 998
998 FORMAT(/5X,'TOTAL CHECK POINTS ON TOP AND BOTTOM CONDUCTORS!','=',
X,'I3','/5X,'TOTAL CHECK POINTS ON CONDUCTORS AND DIELECTRIC BOUNDARY:
X','I3'/)
IF(.NOT.SEC1)G0T0222
C COMPUTATION OF POTENTIAL AT THE DISCRETE POINTS AND EVALUATION OF ERROR AT
THE SYSTEM BOUNDARY.
PRINT 9001
9001 FORMAT(/5X,'POTENTIAL DISTRIBUTION ALONG THE CONDUCTORS'/12X,
X,'RC','12X','ZC','12X','POTENTIAL'/)
DO 320 I=1,LL
SM=0.D0
DO 318 J=1,I2
K=J
Q2=CX(J)*1.D-09
IF(J.GT.I61.AND.J.LE.I62)G0T0324
IF(J.GT.I63.AND.J.LE.I64)G0T0324
IF((I.GE.NTOP1.AND.I.LE.NTOP3).OR.(I.GE.NDIL1.AND.I.LE.LL)).AND.
XJ.GT.17) K=J+41+NS1
Q2=CX(K)*1.D-09
324 PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
G0T0325
3241 PT=ULIN(RC(I),ZC(I),ZR(J),DEL,Q2)
G0T0325
325 SM=SM+PT
318 CONTINUE
SUM(I)=SM
COMPUTATION OF POTENTIAL AT THE CHECK POINTS ON THE DIELECTRIC BOUNDARY.

C COMPUTATION OF POTENTIAL AT THE CHECK POINTS ON THE DIELECTRIC BOUNDARY.

C "SUMIN" USES THE INSIDE SET OF CHARGES.

DO 346 J=1,12
Q2=MX(J)*1.D-09
IF(J.GT.161.AND.J.LE.162)GOTO347
IF(J.GT.163.AND.J.LE.164)GOTO3471
PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
GOTO346
347 PT=VLIN(RC(I),ZC(I),ZQ(J),DEL,Q2)
GOTO346
3471 PT=VLIN(RC(I),ZC(I),ZQ(J),DDL,Q2)
346 SUMIN=SUMIN+PT
DO 350 J=1,12
Q2=MX(J)*1.D-09
K=J
IF(J.GT.161.AND.J.LE.162)GOTO351
IF(J.GT.163.AND.J.LE.164)GOTO3511
IF(J.GT.17) K=J+1
Q2=MX(K)*1.D-09
PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
GOTO350
351 PT=VLIN(RC(I),ZC(I),ZQ(J),DEL,Q2)
GOTO350
3511 PT=VLIN(RC(I),ZC(I),ZQ(J),DDL,Q2)
350 SUMOUT=SUMOUT+PT
ERDIL=(SUMOUT-SUMIN)**2
SRC(I)=RC(I)
SDIF=SUMOUT-SUMIN
DIFF(I-LL1+1)=SDIF
SZC(I)=SDIF*100.D0
ERSUM2=ERSUM2+ERDIL
WRITE(6,430)RC(I),ZC(I),SUMOUT, SUMIN,DIFF(I-LL1+1)
345 CONTINUE
430 FORMAT('18X,I3,5X,5X,G14.5')</ref>
IF(DPRINT)WRITE(6, 868)
868 FORMAT(40X,'X-AGE. POTENTIAL DISTRIBUTION AT THE UPPER DIELECTRIC
XSKIRT')
        KI=IK+1
        LL2=LL+KI
        IF(DPLOT) CALL PLOT3(SRC(LL2),SZC(LL2),JK)
        IF(DPRINT) WRITE(6, 838)
838 FORMAT(40X,'X-AGE. POTENTIAL DISTRIBUTION AT THE LOWER DIELECTRIC
XSKIRT')
C ERROR IN SOLUTION ALONG TOP CONDUCTOR.
910 ERSUM=0.00
        IF(DPRINT) WRITE(6, 823)
823 FORMAT(9A4,4X,'SQD, ERROR ALONG THE TOP ELECTRODE',/25X,'RC(I)',
X29X,'ZC(I)',32X,'ER(I)'//)
        DO 820 I=1,NTOP
                ER=SUM(I)**2
                ERSUM=ERSUM+ER
                IF(.NOT.DPLOT) GOTO 822
                SRC(I)=RC(I)
                SZC(I)=ZC(I)
                SERB(I)=SUM(I)*100.00
        IF(.NOT.DPLOT) GOTO 830
822 IF(DPRINT) WRITE(6, 825) RC(I),ZC(I),ER
820 CONTINUE
        IF(.NOT.DPLOT) GOTO 830
        CALL PLOT3(SZC,SERB,KOUNT)
        WRITE(6, 831)
831 FORMAT('AIR SIDE OF TOP CONDUCTOR')
        CALL PLOT3(SRC(NTOP1),SERB(NTOP1),KOUNT1)
        WRITE(6, 832)
832 FORMAT('HORIZONTAL SIDE OF TOP ELECTRODE ADJACENT TO DIELECTRIC
X')
925 FORMAT(3(20X,G14.5),'/')
830 LL3=NTOP2
C ERROR IN SOLUTION AT BOTTOM CONDUCTOR.
        IF(DPRINT) WRITE(6, 835)
835 FORMAT(9A4,4X,'SQD, ERROR ALONG THE BOTTOM ELECTRODE',/23X,'RC(I)
X',26X,'ZC(I)',27X,'ER(I)'//)
        AX=1.00
        ERSUM1=0.00
        DO 840 I=LL3,LL
                ER=((SUM(I)-AX)**2
                ERSUM1=ERSUM1+ER
                IF(.NOT.DPLOT) GOTO 841
                SRC(I)=ZC(I)
                SZC(I)=RC(I)
                SERB(I)=(SUM(I)-AX)*100.00
        IF(.NOT.DPLOT) GOTO 844
841 IF(DPRINT) WRITE(6, 842) RC(I),ZC(I),ER
840 CONTINUE
842 FORMAT(3(20X,G14.5),'/')
        IF(DPLOT) CALL PLOT3(SZC(NTOP2),SERB(NTOP2),NK1)
        IF(DPLOT) WRITE(6, 853)
853 FORMAT(30X,'AIR PART OF THE BOTTOM ELECTRODE')
        IF(DPLOT) CALL PLOT3(SZC(NDIL1),SERB(NDIL1),KOUNT3)
        IF(DPLOT) WRITE(6, 854)
854 FORMAT(30X,'X-AGE. ERROR IN POTENTIAL DISTRIBUTION ALONG THE
X',45X,'VERTICAL SURFACE OF BOTTOM ELECTRODE ADJACENT TO
XDIELECTRIC.')
IF (DPRINT) WRITE (6, 868)
B68 FORMAT (40X, 'Z-AGE. POTENTIAL DISTRIBUTION AT THE UPPER DIELECTRIC XSKIRT')
KI = IK + 1
LL2 = LL + KI
IF (DPLOT) CALL PLOT3 (SRC(L2), SZC(LL2), JK)
IF (DPRINT) WRITE (6, 838)
B38 FORMAT (40X, 'Z-AGE. POTENTIAL DISTRIBUTION AT THE LOWER DIELECTRIC XSKIRT')

C ERROR IN SOLUTION ALONG TOP CONDUCTOR.

B10 ERSUM = 0.0
IF (DPRINT) WRITE (6, 823)
B23 FORMAT ('-', 40X, 'SQR. ERROR ALONG THE TOP ELECTRODE', '/5X, 'RC(I)', X29X, 'ZC(I)', X32X, 'ER(I)', '/)
DO 820 I = 1, NTOP
ER = SUM(I)**2
ERSUM = ERSUM + ER
IF (.NOT. DPLOT) GOTO 822
SRC(I) = RC(I)
SZC(I) = ZC(I)
SERT(I) = SUM(I) * 100.0
IF (DPRINT) WRITE (6, 825) RC(I), ZC(I), ER
820 CONTINUE
IF (.NOT. DPLOT) GOTO 830
CALL PLOT3 (SRC, SERT, KOUNT)
WRITE (6, 831)
B31 FORMAT (40X, 'AIR SIDE OF TOP CONDUCTOR/')
CALL PLOT3 (SRC(NTOP1), SERT(NTOP1), KOUNT)
WRITE (6, 832)
B32 FORMAT (40X, 'HORIZONTAL SIDE OF TOP ELECTRODE ADJACENT TO DIELECTRIC X')

B25 FORMAT ('-', 3(20X, 014.5))
B30 LL3 = NTOP2

C ERROR IN SOLUTION AT BOTTOM CONDUCTOR.
IF (DPRINT) WRITE (6, 835)
B35 FORMAT ('-', 40X, 'SQR. ERROR ALONG THE BOTTOM ELECTRODE', '/5X, 'RC(I)', X26X, 'ZC(I)', X27X, 'ER(I)', '/)
AX = 1.0
ERSUM1 = 0.0
DO 840 I = LL3, LL
ER = ((SUM(I) - AX)**2
ERSUM1 = ERSUM1 + ER
IF (.NOT. DPLOT) GOTO 841
SRC(I) = RC(I)
SZC(I) = ZC(I)
SERT(I) = SUM(I) - 100.0
IF (DPRINT) WRITE (6, 842) RC(I), ZC(I), ER
840 CONTINUE
B42 FORMAT ('-', 3(20X, 014.5))
IF (DPLOT) CALL PLOT3 (SZC(NTOP2), SERT(NTOP2), NK1)
IF (DPRINT) WRITE (6, 853)
B53 FORMAT (40X, 'AIR PART OF THE BOTTOM ELECTRODE')
IF (DPLOT) CALL PLOT3 (SRC(ND1), SERT(ND1), KOUNT3)
IF (DPRINT) WRITE (6, 854)
B54 FORMAT (40X, 'Z-AGE. ERROR IN POTENTIAL DISTRIBUTION ALONG THE X', 'VERTICAL SURFACE OF BOTTOM ELECTRODE ADJACENT TO DIELECTRIC')
IF (DPLOT) CALL PLOT3 (SRC(ND1), SERT(ND1), KOUNT4)
IF (DPRINT) WRITE (6, 855)
B56 FORMAT (40X, 'Z-AGE. ERROR IN POTENTIAL DISTRIBUTION ALONG THE',/
THIS SECTION CALCULATES NORMAL & TANGENTIAL COMPONENT OF THE E-FIELD AT THE DIELECTRIC BOUNDARIES.

99 FORMAT('1',15X,'D-FIELD (NORMAL & TANGENTIAL COMPONENT) COMPUTATION AT THE DIELECTRIC-AIR INTERFACE.'//)

PRINT 992

992 FORMAT(' ',17X,'RC(I)'t37X,'ZC(I)'t53X,'DIFF.(E.OUT-E.IN)'t
XT76t'ETIN' tT97t 'F.TOUT'tTU I t  'ETD't/)
LL1=LL+1
LLX=LL+IK
DO 555 I=LL1,LLL
II=I
ETI=O.DO
ETO=O.DO
EIN=O.DO
EOUT=O.DO
DO 556 J=1,12
Q2=CX(J)*1.D09
IF(J.GT.161.AND.J.LE.162)G0T0557
IF(J.GT.163.AND.J.LE.164)G0T0558
CALL ERING(RC(I),ZC(I),RQ(J),ZQ(J),ER,EZ,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
G0T0S35

557 IF(J.GT.161.AND.J.LE.162)SSL=DEL
558 CALL EL1NE(RC(I),ZC(I),ZR(J),SSL,ER,EZ,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
SSL=DDL
559 EIN=EIN+EFN
DO 559 J=1,12
K=J
Q2=CX(K)*1.D09
IF(J.GT.161.AND.J.LE.162)G0T0560
IF(J.GT.163.AND.J.LE.164)G0T0561
IF(J.GT.17)K=J+N41+N51
Q2=CX(K)*1.D09
CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZ,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
G0T0S95
560 IF(J.GT.161.AND.J.LE.162)SSL=DEL
561 CALL ELINE(RC(I),ZC(I),ZR(J),SSL,ER,EZ,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
SSL=DDL
562 ETO=ETH+ET
567 EOUT=EOUT+EFN
568 SERC(I)=ETI
569 SERB(I)=ETO
SRC(I)=RC(I)
EOUT=EFSER*EOUT
DIFF(I-LL1+1)=EOUT-EIN
ETD=(ETI-ETO)*100.DO
WRITE(6*564)RC(I),ZC(I),DIFF(I-LL1+1),ETI,ETO,ETD
555 CONTINUE
564 FORMAT(6X,6(8X,G12.5))
IF(DPLOT)CALL PLOT3(SRC(L1),DIFF(IK),IK)
IF(DPRINT)WRITE(6*565)
565 FORMAT(40X,'DIELECTRIC BOUNDARY UPPER SKIRT. (EOUT-EIN)'\ 
       IF(DPLOT)CALL PLOT3(SRC(L1),SERT(L1),IK)
6551 FORMAT(40X,'DIELECTRIC BOUNDARY UPPER SKIRT.\ 
       X'TANGENTIAL FIELD DUE TO OUTSIDE SET OF CHARGES'\ 
       LL2=LL1+IK+1
KI=IK+1
IF(DPLOT)CALL PLOT3(SRC(LL2),DIFF(KI),JK)
IF(DPRINT)WRITE(6*657)
567 FORMAT(40X,'DIELECTRIC BOUNDARY LOWER SKIRT. (EOUT-EIN)'\ 
       IF(DPLOT)CALL PLOT3(SRC(LL2),SERT(LL2),JK)
6552 FORMAT(40X,'DIELECTRIC BOUNDARY LOWER SKIRT.\ 
       X'TANGENTIAL FIELD DUE TO INSIDE SET OF CHARGES'\ 
       LL2=LL1+MKK+1
MK=MKK+1
IF(DPLOT)CALL PLOT3(SRC(LL2),SERT(LL2),JK)
IF(DPRINT)WRITE(6*6553)
5653 FORMAT(40X,'DIELECTRIC BOUNDARY LOWER SKIRT.\ 
       X'TANGENTIAL FIELD DUE TO OUTSIDE SET OF CHARGES.'\ 
1111 CONTINUE
9999 CONTINUE
SH=0.DO
PRINT 777
777 FORMAT(21X,'MAGNITUDE OF THE RING CHARGES.'\ 
DO 776 J=1,14
CH=CH(J)*1.0
776 WRITE(6*774) CH
774 FORMAT(21X,D15.8)
7777 STOP
END
SUBROUTINE IOPC(RQ,ZQ,RC,ZC,NN)
IMPLICIT INTEGER*4(A-H,O-Z)
REAL*4 X(120),Y(120),XX(400),YY(400)
DIMENSION RQ(110),ZQ(110),RC(450),ZC(450)
DO 23 I=1,NN
X(I)=RQ(I)
Y(I)=ZQ(I)
23 CONTINUE
DO 24 I=1,NN
XX(I)=RC(I)
24 YY(I)=ZC(I)
M=NN+1
MN=M+1
MK=MN+1
MX=MK+1
CALL PLOTID('JAVED')
CALL PLOT(0.0,0.5,-3)
CALL SCALE(XX,YY,MN+1)
CALL SCALE(YY,XX,MN+1)

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X(M)=XX(MK)
X(MM)=XX(HKK)
Y(M)=YY(MK)
Y(MM)=YY(HKK)
A=Y(M)/Y(MM)
CALL AXIS(0.0,AXIS,0.0,X(M),X(MM))
A=X(M)/X(MM)
CALL AXIS(0.0,AXIS,0.0,Y(M),Y(MM))
CALL LINE(X,Y,NN,1,1,3)
CALL LINE(XX,YY,NN,1,1,4)
CALL PLTEND(9.)
RETURN

FUNCTION SECOND(P)
REAL*8 SECOND, Z
D1.08
DF.XP = DABS(P)
Z=1.0-P*P
IF(Z .LT. 1.0D-20) GO TO 1
SECOND=1.0D0+Z*(:8.44325141424D0+Z*(:7.3450645122D-2)
1+Z*(1.73450645122D-2))-.DLOG(Z)*Z*(2.4998368310D-1+Z*
2.200180037D-2+Z*(4.069697526D-2+Z*(5.26449639D-3)))
RETURN
1 SECOND=1.0D0
RETURN
END

FUNCTION URING(RHO,Z>RHO0,Z0,R)
IMPLICIT REAL*8(Z)
RHO=DABS(RH0)
ZZ=Z-Z0
K=2.0*SQRT(DABS(RH0*RHO0))/DSQRT(ZZ**2+(RHO+RHO0)**2)
URING=Q/(1.7477D-10)*/ELINK(K)/DSQRT(ZZ**2+(RHO+RHO0)**2)
RETURN
END

FUNCTION ELINK(Z)
REAL*8 ELINK,Z,P,DLOG,DEXP
P=1.0D0-Z*Z
IF(P.LT.1.0D-20) GO TO 1
13.74256373D-2+1.45196212D-2+P))-.DLOG(P)*(0.5D0+P*(1.2498593597
2D-1+P*6.80249576D-2+P*(3.32835536D-2+1.41787012D-3*P))
RETURN
1 ELINK=DEXP(88.D0)
RETURN
END

SUBROUTINE BOTE(Ii,I2,I3,II,RI,ZQ,RN,RA,RC,ZN,DEL,DL)
IMPLICIT REAL*8(A-H,Z)
DIMENSION RQ(11O),ZQ(11O),R(450),ZC(450)
DATA HD,DL,AL,NLA,NLD,NRH,NRLH,RA,RD/0.362D0,2.541D0
X1.276D0,1.185,3.0.312D0,0.767D0
C SPECIFY THE POSITIONS OF LINE CHARGES IN THE AIR PART OF THE B/ELECTRODE.
XA=(AL+DL+RA)
DEL=AL/NLA
RI(I1+1)=0.DO
RC(I1+1)=RA
ZQ(I1+1)=XA+DEL
ZC(I1+1)=XA+DEL/2.DO
DO 201 I=2,NLA
J=1
RQ(I1+1)=0.DO
ZQ(I1+1)=ZQ(I1+1)+DEL
ZC(I1+1)=ZC(I1+1)+DEL/2.DO
201 CONTINUE
RC(I61+I)=RA
201 CONTINUE
PRINT 202*NLA
202 FORMAT(//10X,'NO. OF LINE CHARGES MODELLING THE AIR PART';/10X,'OF
THE BOTTOM ELECTRODE',/10X,'*)
202 CONTINUE
PRINT 202*NLA
C SPECIFY THE POSITIONS OF RING CHARGES IN THE LOWER HORIZONTAL SIDE OF THE
C DIELECTRIC EMBEDDED ELECTRODE.
DO 407 I=1*NRLH
J=1-I
RQ(I62+I)=RA+0.1318(0+1)
ZQ(I62+I)=HD+NL+0.180
RC(I62+I)=RD(I62+I)
407 ZC(I62+I)=(HD+NL)
I63=I62+NRLH
PRINT 450*NRLH
450 FORMAT(//10X,'NO. OF RING CHARGES MODELLING THE LOWER Horizontal PART
OF THE DIELECTRIC EMBEDDED ELECTRODE',/*)
C SPECIFY THE POSITIONS OF LINE CHARGES IN THE DIELECTRIC PART OF THE B/ELECTRODE,
NL=2.041DL/NLD
RC(I63+I)=0.D0
ZQ(I63+I)=(HD+DL+DNL)
ZC(I63+I)=(HD+DNL)/2.D0
DO 203 I=2*NLD
J=1-I
RQ(I63+I)=0.D0
ZQ(I63+I)=ZQ(I62+J)+DNL
ZC(I63+I)=ZQ(I63+J)+HD/2.D0
RQ(I63+I)=RD
203 CONTINUE
PRINT 204*NLD
204 FORMAT(//10X,'NO. OF CHARGES MODELLING THE DIELECTRIC PART OF THE B/ELECTRODE',/*)
C SPECIFY THE POSITIONS OF THE RING CHARGES IN THE DIELECTRIC PART OF THE ELECTRODE.
DO 205 I=1*NRH
RQ(I64+I)=(RD-0.122DL+1)
ZQ(I64+I)=(HD+0.14DL)
RQ(I64+I)=RD(I64+I)
205 ZC(I64+I)=HD
I65=I64+NRH
PRINT 300*NRH
300 FORMAT(//10X,'NO. OF RING CHARGES MODELLING THE UPPER Horizontal PART
OF THE DIELECTRIC EMBEDDED ELECTRODE',/*)
X='/'12)
RETURN
END
SUBROUTINE SPLINE(X,NX,NPTS,T,S,NP)
DIMENSION X(NX),NPTS,C(100),T(200),S(200),IR(30)
X=WK(30)
NPT=NPTS
DO 4 I=1,NX
4 BPAR(I)=0.D0
DO 5 I=1,NX
5 IR(I)=1
MP=0
CALL USRTR(X,NX,IR)
CALL USRTR(Y,1,IR,0,0,WK)
CALL ICORC(X,Y,NX,BPAR,C100,IER)
DO 100 I=2,NX
J=I-1,
INT=(X(I)-X(J))
DELX=INT/NPT
T(NP+1)=X(J)
IF(I.EQ.NX) NPT=(NPT+1)
DO 101 K=1,NPT
N=N+K
D=T(N)-X(J)
S(N)=((C(J,3)+C(J,2))*(C(J,1)+Y(J))
T(N+1)=T(N)+DELX
101 CONTINUE
NP=NP+NPT
100 CONTINUE
DO 88 I=1,NP
WRITE(*,199) T(I),S(I)
199 FORMAT(15X,2(2X,G12.5))
RETURN
END
FUNCTION VLIN(RC,ZC,ZQ,DL,N)
IMPLICIT REAL*8(A-H,O-Z)
ZJ2=ZQ
ZJ1=ZJ2-DL
PI=3.14159265358979D0
EPS=8.854187818D-12
GAMA=DSQRT(RC**2+(ZJ2-ZC)**2)
DELTA=DSQRT(RC**2+(ZJ1-ZC)**2)
SS=0.71/(4.*PI*EPS*DL)
VLIN=SS*DLG(DABS((ZJ2-ZC+GAMA)/(ZJ1-ZC+DELTA)))
RETURN
END
SUBROUTINE ERING(RC,ZC,RQ,ZQ,ER,EZ,G)
IMPLICIT REAL*8(A-H,O-Z)
PI=3.14159265358979D0
ALFA1=DSQRT(((RC+RQ)**2)+((ZC-ZQ)**2))
EPS=8.854187818D-12
ARG1=(2.0*DSQRT(RC*RQ))/ALFA1
BETA1=DSQRT(((RC-RQ)**2)+((ZC-ZQ)**2))
G=Q/(4.0*PI*EPS*RC)
B=((RG**2-RC**2)+((ZC-ZQ)**2))SECOND(ARG1)
C=(ELINK(ARG1))*(BETA1**2)
D=ALFA1*BETA1**2
ER=G*Q/(B-C)/D
F=2.0*X((ZC-ZQ))SECOND(ARG1)
G=Q/(4.0*PI*EPS)
EZ=(G*F)/D
RETURN
END
SUBROUTINE ELINE(RC,ZC,ZJ2,SSL,ER,EZ,G)
IMPLICIT REAL*8(A-H,O-Z)
PI=3.14159265358979D0
EPS=8.854187818D-12
H=1.0
ZJ1=ZJ2-SSL
A=ZJ2-ZC
B=ZJ1-ZC
C=Q/(4.0*PI*EPS*SSL)
GAMA1=1.0/DSQRT(RC**2+A**2)
DELTA1=1.0/DSQRT(RC**2+B**2)
ER=2.0*C/RC*(A*GAMA1)-(B*DELTA1)
RETURN
END
EZ = C* (GAMA1 - DELTA1)
RETURN
END

FUNCTION ENOR(RC, ZC, ER, EZ, II)
IMPLICIT REAL*8(A-H,O-Z)
REAL*4 RU, ZW
COMMON RW(200), ZW(200), I31, I13, I41, I14, KC
J = KC - 1
K = KC + 1
IF(II.EQ.I31 .OR. II.EQ.I41) GOTO 5
IF(II.EQ.I13 .OR. II.EQ.I14) GOTO 6
R1 = RW(J)
R2 = RW(K)
Z1 = ZW(J)
Z2 = ZW(K)
GOTO 8
5 R1 = RC
R2 = RW(3)
Z1 = ZC
Z2 = ZW(3)
GOTO 8
6 R1 = RC
Z1 = ZC
R2 = RW(J - 1)
Z2 = ZW(J - 1)
A = DABS(Z2 - Z1)
B = DABS(R2 - R1)
ENOR = (ER*((-A) + EZ*B)) / (DSQRT(A**2 + B**2))
RETURN
END

SUBROUTINE EFLX(XC, YC, ER, EZ, II, EFN, ET)
IMPLICIT REAL*8(A-H,O-Z)
COMMON /BLK2/ RC, ZC(450), LLX, LLX1, LLL, J = II - 1
K = II + 1
IF(II.EQ.1.LL1 .OR. II.EQ.LLEX) GOTO 5
IF(II.EQ.LLL .OR. II.EQ.LLX) GOTO 6
R1 = RC(J)
R2 = RC(K)
Z1 = ZC(J)
Z2 = ZC(K)
GOTO 8
5 R1 = XC
R2 = RC(K + 1)
Z1 = YC
Z2 = ZC(K + 1)
GOTO 8
6 R1 = XC
Z1 = YC
R2 = RC(J - 1)
Z2 = ZC(J - 1)
A = DABS(Z2 - Z1)
B = DABS(R2 - R1)
EFN = (ER*((-A) + EZ*B)) / (DSQRT(A**2 + B**2))
EFN = 6.00854187818D-12*EFN
ET = (ER*B + EZ*A) / (DSQRT(A**2 + B**2))
RETURN
END
APPENDIX C
COMPUTER PROGRAM FOR LONG ROD POLYMER INSULATOR

IMPLICIT INTEGER*4(I-N), REAL*8(A-H,O-Z)
REAL*4 SDIFF(650), SUM(650), XC(650), YC(650), EMG(40)
DIMENSION RQ(151), ZQ(151), P(151,151), WKAREA(151), CX(151), B(151,1)
X, XQ(20), YQ(20), XX(20), YY(20), BUF(1000)
COMMON RC(650), ZC(650), XCL, YCL, EPSIR, HUNT, IQG, NU
COMMON /BLK1/ EPS, PI
DIMENSION RG(151), ZG(151), P(151), WKAREA(151), CX(151), B(151,1)
X, XQ(20), XX(20), YY(20), BUF(1000)
COMMON RC(650), ZC(650), XCL, YCL, EPSIR, HUNT, IQG, NU
COMMON /BLK1/ EPS, PI
DATA N1,N2,N3,N4,N5,N6,N7,M1,M2,M3,M4,M5,M6,M7,NSEC/3,20,10,
X5.5,3.6,1.6,9.7,5.5,2.5,1.2,4.2,2/,
XG/1.0D-09/,NCLMP/6/,NCOND/4/
HUNT=4.172D0
LOGICAL ALL/.FALSE./, BCND/.FALSE./, BCNDC/.FALSE./, BCNDD/.FALSE./,
XBCNDF/.FALSE./, DPLOT/.FALSE./, POTDE/.FALSE./,
NAMES=OPT/ALL, BCND, BCNDC, BCNDD, XBCNDF, DPLOT, POTDE
READ(5,OPT)
WRITE(6,OPT)
CALL PLOTS(IBUF,1000)
CALL XLIMIT(180.)
NIN=N5+N6+N7
NINS=N4+NIN
NUNIT=2*(N5+N6+N7)
NVC=2*N4
NBT=NVC+NUNIT
NCHS=NIN
NDT=2*NSEC+NDT
NT=N1+N2+N3+2*N4+NUNIT
READ(5,1) (RQ(I), ZQ(I), RC(I), ZC(I), I=1, NT)
1 FORMAT(4(G7.4))
XCL=10.0D0/6.546D0
YCL=21.30D0/13.80D0
V=1.0D0
A=(20.880D0+NSEC*HUNT)*YCL-21.323D0
EPSIR=2.54D0
EPS=EPSIR*8.854187818D-12
PI=3.14159265358979D0
SET THE COUNTERS.
M11=M1+1
M12=M1+M2+1
M13=M1+M2+M3+M4+1
M14=M4+1
M15=M1+M2+M3+M4+M5+1
M16=M4+M5+1
M17=M6+M7
M18=M15+M17
K1=M15+M17
K2=K1+M5
K3=K2+M17
K4=K3+M5
IDF=N1
IDF1=IDF+1
ISH=ISH+1
IG1=IG1+1
IG2=IG1+N3
ICN1=ICN1+1
ICN2=ICN1+N4
IO=IO+NINS
IIS=IIS+1
IT=IT+1
NPO=I0
DO 291 I=IIS,IT
XXQ(I-IIS+1)=RQ(I)
YYQ(I-IIS+1)=ZQ(I)
XX(I-IIS+1)=RC(I)
291
173

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YYC(I-IIS+1)=ZC(I)

291 CONTINUE
DO 300 I=1,NSEC
DO 301 J=1,NCHS
RC(NPO+J)=RC(IDCO+J)
ZC(NPO+J)=I*HUNT+ZC(IDCO+J)
RQ(NPO+J)=RQ(IDCO+J)
ZQ(NPO+J)=I*HUNT+ZQ(IDCO+J)
301 CONTINUE
NPO=NPO+NCHS
300 CONTINUE

NPO=NPO+NCHS
DO 261 I=1,NINS
RQ(NPI+I)=XXQ(I)
ZQ(NPI+I)=YYQ(I)
RC(NPI+I)=XXC(I)
ZC(NPI+I)=YYC(I)
261 CONTINUE
NPO=NPO+NINS
DO 302 I=1,NSEC
DO 303 J=1,NCHS
RC(NPI+J)=XXC(J+N4)
ZC(NPI+J)=I*HUNT+YYC(J+N4)
RQ(NPI+J)=XXQ(J+N4)
ZQ(NPI+J)=YYQ(J+N4)+HUNT*I
303 CONTINUE
NPI=NPI+NCHS

IGC=NPQ
IIC1=IGC+1
IIS=NPO+1
IDC=NPO+N4
IDH=IDC+N5
IDV=IDH+N6
IDS=IDV+N7
ITT=IDS
ITI=IT+I
NINS=N4+N1*NINS+NINS*NSEC
N12=N1+N2
IDF2=IGC+3
IDF3=IDF2+1
DO 305 I=1,N12
DZ=ZC(I)-ZC(I)
DO=ZC(I)-ZG(I)
RQ(NPI+I)=RC(I)
ZC(NPI+I)=20.88*D0+NSEC*HUNT+DZ
RQ(NPI+I)=RQ(I)
305 ZQ(NPI+I)=20.88*D0+NSEC*HUNT+DQ
NT=NPI+N12
IT=NT
ILI=IT-NINS
DO 505 I=1,NT
IF(I.GT.ISH.AND.I.LE.ICND)GOTO506
RQ(I)=XCL*RQ(I)
RC(I)=XCL*RC(I)
ZQ(I)=YCL*ZQ(I)
ZC(I)=YCL*ZC(I)
GOTO505
506 RC(I)=YCL*RC(I)
ZC(I)=XCL*ZC(I)  
RQ(I)=XCL*RQ(I)  
ZQ(I)=XCL*ZQ(I)  

505 CONTINUE  
PRINT 219, IT, NT, ITT, IIS, NPO, IGC, IDF2, IDF3  
219 FORMAT(//20X,6(2X, I3))  
WRITE(6, 140) ISH, NCMP, NCND, NVC, NUNIT, ICND, WDT, IT  
140 FORMAT(//10X, 'NUMBER OF CHARGES MODELLING THE SHANK:', I3/)  
X1OX, 'NUMBER OF CHARGES MODELLING THE CLAMP:', I3/  
X1OX, 'NUMBER OF CHARGES MODELLING THE CONDUCTOR:', I3/  
X1OX, 'DIELECTRIC COLUMN:', I3/  
X1OX, 'NUMBER OF CHARGES MODELLING ONE DIELECTRIC UNIT:', I3/  
X1OX, 'TOTAL NUMBER OF CHARGES MODELLING THE CONDUCTORS:', I3/  
X1OX, 'TOTAL NUMBER OF CHARGES MODELLING THE DIELCETRIC:', I3/  
X1OX, 'TOTAL NUMBER OF CHARGES MODELLING THE SYSTEM:', I3/)  
PRINT 139  
139 FORMAT(//12X, 'RQ', 12X, 'ZQ', 12X, 'RC', 12X, 'ZC', /)  
DO 141 I=1, NT  
141 URITE(6, 142) RQ(I), ZQ(I), RC(I), ZC(I)  
142 FORMAT(5X, 4(2X, 5F12.5))  
DO 100 I=1, IDF1  
IF(J.GT.ISH.AND.J.LE.ICND)GOTO99  
IF(J.GT.IO.AND.J.LE.IGC)GOTO88  
P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO100  
99 P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)+  
VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO100  
88 P(I,J)=0.DO  
100 CONTINUE  
DO 101 I=IDF1, ICND  
DO 101 J=1, IT  
IF(J.GT.ISH.AND.J.LE.ICND)GOTO102  
IF(J.GT.IO.AND.J.LE.IGC)GOTO103  
P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO101  
102 P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)+  
VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO101  
103 P(I,J)=0.DO  
101 CONTINUE  
DO 104 I=ICND1, IO  
DO 104 J=1, IT  
IF(J.LE.ICND)OR.(J.GT.IGC)GOTO105  
P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
IF(J.GT.IO.AND.J.LE.IGC)P(I,J)=-P(I,J)  
GOTO104  
105 P(I,J)=0.DO  
104 CONTINUE  
DO 315 I=IGC1, IDF2  
DO 315 J=1, IT  
IF(J.GT.ISH.AND.J.LE.ICND)GOTO990  
IF(J.GT.IO.AND.J.LE.IGC)GOTO880  
P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO315  
990 P(I,J)=VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)+  
VRING(RC(I), ZC(I), RQ(J), ZQ(J), 0)  
GOTO315  

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880 P(I,J)=0.DO  
315 CONTINUE  
DO 316 I=IDF3,NT  
DO 316 J=1,IT  
IF(J.GT.ISH.AND.J.LE.ICND)G0T01021  
IF(J.GT.ICND.AND.J.LE.ID)G0T01031  
P(1,J)=VRING(RC(I),ZC(I),RQ(J),ZQ(J),G)  
G0T0316  
1021 P(I,J)=VRING(RC(I),ZC(I),RQ(J),ZQ(J),G)  
X  
+VRING(RC(I),ZC(I),RQ(J),-ZQ(J),G)  
G0T0316  
1031 P(I,J)=0.DO  
316 CONTINUE  
C  
FLUX MATCHING CONDITIONS.  
DO 106 I=IIS,ITT  
II=I  
NU=0  
IGO=ITT  
DO 106 J=1,IT  
IF(I.LE.IDC).OR.(I.GT.IDH.AND.I.LE.IDU)G0T071  
IF(I.GT.IDC.AND.I.LE.IDH)G0T081  
CALL ERINC(RC(I),ZC(I),RQ(J),ZQ(J),ER,EZ,G)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
G0T0106  
71 P(I,J)=ERNR(RC(I),ZC(I),RQ(J),ZQ(J),G)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
G0T0106  
81 P(I,J)=ERNR(RC(I),ZC(I),RQ(J),ZQ(J),G)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
IF(J.GT.ISH.AND.J.LE.ICND)G0T050  
P(I,J)=DNOR(RC(I),ZC(I),ER,EZ,II)  
G0T0106  
50 CALL ERINC(RC(I),ZC(I),RQ(J),-ZQ(J),ER,EZ,G)  
P(I,J)=(ERNR-1.DO)*DNOR(RC(I),ZC(I),ER,EZ,II)+  
XDNOR(RC(I),ZC(I),ER,EZ,II))  
106 CONTINUE  
DO 1060 NUI=1,NSEC  
NU=NUI  
ITT1=IDS+1  
IDH=IDS+N5  
IDV=IDH+N6  
IDS=IDV+N7  
DO 1061 I=ITT1,IDS  
II=I  
IF(I.EQ.IDS)IGO=II  
DO 1061 J=1,IT  
IF(I.LE.IDH)G0T0281  
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IF(I.GT.IDH.AND.I.LE.IDV)GOTO271
CALL ERINGRC(I),ZC(I),RQ(J),ZQ(J),ER,EZ,Q)
IF(I.GT.ICND.AND.I.LE.IOD)P(I,J)=EPSIR*P(I,J)
IF(I.GT.IOD.AND.I.LE.ICNGP(I,J)=-P(I,J)
GOTO1061
271 P(I,J)ERNRRC(I),ZC(I),RQ(J),ZQ(J),Q)
IF(J.LE.ISH)P(I,J)=(EPSIR-1.D0)*P(I,J)
IF(J.GT.ISH.AND.J.LE.ICND)P(I,J)=(EPSIR-1.D0)*(EZNRC(I),
XZC(I),RQ(J),ZQ(J),Q)+ERNRRC(I),ZC(I),RQ(J),ZQ(J),Q)
IF(J.GT.ICND.AND.J.LE.IOD)P(I,J)=EPSIR*P(I,J)
IF(J.GT.IOD.AND.J.LE.ICNGP(I,J)=-P(I,J)
IF(J.GT.ICNGP(I,J)EPSIR*P(I,J)
GOTO1061
281 P(I,J)ERNRRC(I),ZC(I),RQ(J),ZQ(J),Q)
IF(J.LE.ISH)P(I,J)=(EPSIR-1.D0)*P(I,J)
IF(J.GT.ISH.AND.J.LE.ICND)P(I,J)=(EPSIR-1.D0)*(EZNRC(I),
XZC(I),RQ(J),ZQ(J),Q)+ERNRRC(I),ZC(I),RQ(J),ZQ(J),Q)
IF(J.GT.ICND.AND.J.LE.IOD)P(I,J)=EPSIR*P(I,J)
IF(J.GT.IOD.AND.J.LE.ICNGP(I,J)=-P(I,J)
IF(J.GT.ICNGP(I,J)EPSIR*P(I,J)
GOTO1061
250 CALL ERINGRC(I),ZC(I),RQ(J),ZQ(J),ER,EZI,0)
PC(I,J)=EPSIR-1.D0)*CDN0RC(I),ZC(I),ER,EZI,II)
GOTO1061
1061 CONTINUE
1060 CONTINUE
IDH=IDC+N5
IDV=IDH+N6
IDS=IDV+N7
DO 107 I=1,IT
B(I,1)=0.D0
IF(I.LE.ICND)B(I,1)=1.D0
107 CONTINUE
KOUNT=1
M=1
IDGT=8
IA=151
CALL LEQT1FCP(M,IT,IA,B,IDGT,WKAREA,IER)
PRINT 777
777 FORMAT(/21X,'MAGNITUDE OF THE RING CHARGES.'/
CAPSUM=0.D0
DO 108 I=1,IT
CX(I)=B(I,1)*1.D-09
CAPSUM=CAPSUM+CX(I)
108 WRITE(6,109) I,CX(I)
109 FORMAT(21X,13,2X,D15.8)
PRINT 201,CAPSUM
201 FORMAT(/20X,'CAPACITANCE OF THE INSULATOR.'G15.8,/
IF(.NOT.BCND)GOTO797
IF(BCND)GOTO797
740 CONTINUE
IDF=M1
ICL=M1+M2+M3
I1S=ICL+1
MSH=M1+M2
IDS=ICL
NINSID=M4+M5+M6+M7

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IT=ICL+NINSID
READ(5,818)(RC(I),ZC(I),I=1,IT)
818 FORMAT(2(G7.4))
UNIT=H5+H6+H7
ITT=ICL+NINSID
ITT=ITT+1
NIN=UNIT
ITE=IT
DO 704 NU=1,NSEC
DO 705 J=1,UNIT
RC(ITT+J)=RC(ICL+M4+J)
705 ZC(ITT+J)=NU*HUNT+ZC(ICL+M4+J)
IT=IT+UNIT
704 CONTINUE
IGC=IT
IGCI=IGC+1
IDF2=IGC+M1
MM2=H1+H2
IDC=ICL+M4
IDH=IDC+M5
IDV=IDH+M6
IDS=IDV+H7
DO 707 I=1,MM2
DZ=ZC(I)-ZC(I)
RC(IT+J)=RC(ITT+J)
ZC(IT+J)=20.880D0+NSEC*HUNT+DZ
707 CONTINUE
MC=M2-4
IT=IT+MM2-4
DO 414 I=1,IT
IF( I.GT. MSH. AND. I.LE. ICL)GOT0415
RC(I)=XCL*RC(I)
ZC(I)=YCL*ZC(I)
XC(I)=RC(I)
YC(I)=ZC(I)
GOT0414
415 RC(I)=YCL*RC(I)
ZC(I)=XCL*ZC(I)
XC(I)=RC(I)
YC(I)=ZC(I)
414 CONTINUE
797 CONTINUE
IF(KCOUNT.EQ.2)GOT0151
IF(.NOT.BCNDG)GOT0741
IF(BCNDG)WRITE(6,9001)
151 IF(KCOUNT.EQ.2)WRITE(6,634)
634 FORMAT(/'CHECK POTENTIAL AT DISCRETE POINTS ON CONDUCTORS'/,
X/12X,'RC',12X,'ZC',12X,'POTENTIAL'/)
9001 FORMAT(/'CHECK POTENTIAL AT BOUNDARY POINTS ON CONDUCTORS'/,
X12X,'RC',12X,'ZC',12X,'POTENTIAL'/)
K01=1
KG=0
IF(.NOT.POTDE)GOT0770
502 CONTINUE
DO 110 I=KG1,ICL
SM=0.0D0
DO 111 J=1,ILI
K=J
Q2=CX(K)
IF(I.LE.IDF)GOT0112

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IF(J.GT.ICND.AND.J.LE.I0)K=J+NINS
Q2=CX(K)
IF(J.GT.ISH.AND.J.LE.ICND)GOTO114
PT=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
GOTO115
112 IF(J.GT.ID)K=(J-I0+NINS+I0)
Q2=CX(K)
IF(J.LE.ISH)PT=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
IF(J.GT.ISH.AND.J.LE.ICND)PT=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
X,02=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
IF(J.GT.ICND.AND.J.LE.I0)PT=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
GOTO115
114 PT=VRING(RC(I),ZC(I),RO(K),ZO(K),Q2)+XVRING(RC(I),ZC(I),RO(K),ZO(K),Q2)
115 SM=SM+PT
110 CONTINUE
XCD=RC(I)
YCD=ZC(I)
SUM(I)=(SM-V)/V*100.DO
IF(I.GT.IGC)SUM(I)=SM*100.DO
WRITE(6,116)RC(I),ZC(I),SM
110 CONTINUE
KG=IGC+1
ICL=IT
IDF=IDF+1
IF(KG.EQ.1)GOTO502
ICL=ICND
501 FORMAT(//10X,'CHECK POTENTIAL AT THE GROUNDED CONDUCTOR.',//
X12X,'RC',12X,'ZC',12X,'POTENTIAL',//)
116 FORMAT(5X.3C3X.G12.5)
IF(.NOT.DPLOT)GOTO741
IF(BCNDD)GOTO741
CALL CALC02(XC,SUMM1,H1,H3,H4,H5,H6,H7,H8,H9,H0,H1,H2,H3)
CALL CALC02(YC,SUMM1,H1,H3,H4,H5,H6,H7,H8,H9,H0,H1,H2,H3)
CALL CALC02(XC,KG1,sumKG1,H1,H3,H4,H5,H6,H7,H8,H9,H0,H1,H2,H3)
CALL CALC02(YC,KG1,sumKG1,H1,H3,H4,H5,H6,H7,H8,H9,H0,H1,H2,H3)
CALL CALC02(YC,KG1,sumKG1,H1,H3,H4,H5,H6,H7,H8,H9,H0,H1,H2,H3)
X1=1.0
741 CONTINUE
IF(KOUNT.EQ.2)GOTO152
IF(.NOT.BCNDD)GOTO770
IF(BCNDD)WRITE(6,900)
152 IF(KOUNT.EQ.2)WRITE(6,637)
637 FORMAT(//10X,'CHECK POTENTIAL AT DISCRETE POINTS ON THE DIELECTRIC X',//)
900 FORMAT(//10X,'CHECK POTENTIAL AT BOUNDARY POINTS ON THE DIELECTRIC X',//)
PRINT 4
4 FORMAT(12X,'RC',12X,'ZC',12X,'SUMIN',12X,'SUMOUT',12X,'DIFF',//
X12X,'AVERAGE',//)
DO 118 I=IGC,IGC
SUMIN=0.DO
SUMOUT=0.DO
DO 119 J=1,IGC
K=J
IF(J.GT.ISH.AND.J.LE.ICND)GOTO120
IF(J.GT.ICND.AND.J.LE.I0)K=J+NINS
IF(J.GT.I0)K=(J-I0+NINS+I0)
Q2=CX(K)
PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
GOTO122
120 Q2=CX(J).
PT=VRING(RC(I),ZC(I),RQ(J),ZQ(J),Q2)
X+VRING(RC(I),ZC(I),RQ(J),ZQ(J),Q2)
GOTO125
122 SUMIN=SUMIN+PT
119 CONTINUE
DO123 J=1,ILI
K=J
Q2=CX(K)
IF(J.GT.I0)K=(J-I0+NINS+I0)
IF(J.GT.ISH.AND.J.LE.ICND)G0T0124
Q2=CX(K)
PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
GOTO122
124 PT=VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
X+VRING(RC(I),ZC(I),RQ(K),ZQ(K),Q2)
125 SUMOUT=SUMOUT+PT
123 CONTINUE
XC(I)=RC(I)
YC(I)=ZC(I)
SUM(I-IIS+1)=(SUMOUT+SUMIN)/2.D0
SDIF=SUMOUT-SUMIN
SDIFF(I-IIS+1)=SDIF*100.D0
WRITE(6,16)RC(I),ZC(I),RQ(K),ZQ(K),02)
GOTO127
118 CONTINUE
IF(.NOT.DPLQT)G0T0770
616 FORMAT(5X*G12.5)
IF(BCNDF)G0T0770
CALL CALC02(YC(IIS),SDIFF,M4,8.,6.6.,20.,1.,-1.5,0.5,1,-1.0)
CALL CALC02(XC(M13),SDIFF(M14),M5,8.,6.6.,0.1.,-1.5,0.5,1,-1.0)
CALL CALC02(YC(M15),SDIFF(M16),M17,8.,7.7.,25.,1.,-1.5,0.5,1,-1.0)
CALL CALC02(XC(K1),SUM(M16+M17),M5,8.,7.7.,25.,1.,-1.5,0.5,1,-1.0)
CALL CALC02(YC(K2),SUM(M),M17,8.,7.7.,32.,1.,-1.5,0.5,1,-1.0)
CALL CALC02(XC(K3),SUM(MS),M5,8.,6.6.,0.1.,-1.5,0.5,1,-1.0)
M=M16+M17+M5
CALL CALC02(YC(K4),SDIFF(MH),M17,8.,7.7.,32.,1.,-1.5,0.5,1,-1.0)
M=M4+2*(M5+M6+M7)+1
CALL CALC02(XC(K5),SDIFF(MS),M5,8.,6.6.,0.1.,-1.5,0.5,1,-1.0)
M=M5+M5
CALL CALC02(YC(K6),SDIFF(MH),M17,8.,7.7.,32.,1.,-1.5,0.5,1,-1.0)
M=M5+M6+M7+1
CALL CALC02(XC(K7),SUM(MH),M17,8.,7.7.,32.,1.,0.5,0.5,1,-1.0)
CALL CALC02(YC(K8),SUM(MS),M5,8.,6.6.,0.1.,0.5,0.5,1,-1.0)
CALL CALC02(XC(K9),SUM(MH),M17,8.,7.7.,32.,1.,0.5,0.5,1,-1.0)
M=M5+M5
CALL PLTEND(9.)
IF(POTDE)G0T0101
770 CONTINUE
IF(KOUNT.EQ.2)G0T0153
IF(.NOT.BCNDF)G0T0797
IF(BCNDF)WRITE(6,666)
153 IF(KOUNT.EQ.2)WRITE(6,638)
638 FORMAT(/12X,'CHECK NORMAL D FIELD AT DISCRETE POINTS ON THE DIELECTRIC SURFACE,'/)}

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666 FORMAT('//12X,'CHECK NORMAL D FIELD AT BOUNDARY POINTS ON THE DIELECTRIC SURFACE.,'
')
PRINT 992
992 FORMAT(' ',T17,'RC(I)',T37,'ZC(I)',T53,'DIFF.(D_OUT-D_IN)',
XT76,'ETIN',T97,'ETOUT',T116,'ETD','/)
DO 555 I=I0,ITT
II=I
N=0
IQ=ITT
ETI=0.0
ET0=0.0
DIN=0.0
DTO=0.0
DO 556 J=1,ILI
K=J
Q2=OX(J)
IFI.LE.IVD)GOTO801
IFI.J.GT.IEND.AND.J.LE.IO)K=J+NINS
IFI.J.GT.IO)K=(J-IO+NINS+IO)
Q2=OX(K)
CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZ,Q2)
IFI.J.GT.IISH.AND.J.LE.IEND)GOTO401
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
GOT0835
401 CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZI,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFNI,ETII)
EFN=EFN+EFNI
ET=ET+ETII
GOT0835
801 IF(I.LE.IVD)GOTO805
IFI.J.GT.IISH.AND.J.LE.IO)K=J+NINS
IFI.J.GT.IO)K=(J-IO+NINS+IO)
Q2=OX(K)
CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZ,Q2)
IFI.I.LE.IDH.AND.I.LE.IDT)OR.(I.GT.IDH.AND.I.LE.IDV)GOTO405
IFI(I.LE.IDC).OR.(I.GT.IDH.AND.I.LE.IDV)GOTO406
EFN=EF
ET=ET+ETI
GOT0835
405 CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZI,Q2)
IFI(I.LE.IDC).OR.(I.GT.IDH.AND.I.LE.IDV)GOTO407
EFN=EF+ERI
ET=ET+ETI
GOT0835
407 EFN=EF+ERI
ET=ET+ERI
535 DIN=DIN+EFN
ETI=ETI+ET
556 CONTINUE
DO 559 J=1,ILI
K=J
IFI.J.GT.IO)K=(J-IO+NINS+IO)
Q2=OX(K)
CALL ERING(RC(I),ZC(I),RQ(K),ZQ(K),ER,EZ,Q2)
IFI.I.LE.IDV)GOTO901
IFI.J.GT.IISH.AND.J.LE.IEND)GOTO902
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
CALL ERING (RC(I), ZC(I), RQ(J), -ZQ(J), ERI, EZI, 02)
CALL EFLX (RC(I), ZC(I), ER, EZ, II, EFN, ET)
CALL EFLX (RC(I), ZC(I), ERI, EZI, II, EFN, ETII)
EFN = EFN + EFNI
ET = ET + ETII
GOTO 939

IF (J .GT. ISH .AND. J .LE. ICHD) GOTO 905
IF (I .LE. IDC) OR (I .GT. IDH .AND. I .LE. IDV) GOTO 906
EFN = EZ
ET = ER
GOTO 939

EFN = ER
ET = EZ
GOTO 939

CALL ERING (RC(I), ZC(I), RQ(J), -ZQ(J), ERI, EZI, 02)
IF (I .LE. IDC) OR (I .GT. IDH .AND. I .LE. IDV) GOTO 910
EFN = ERI
ET = EZ + EZI
GOTO 939

EFN = EZ + EZI
ET = ER + ERI
GOTO 939

CONTINUE
XCI = RC(I)
YCI = ZC(I)
SDIF = EPSR + DOUT - DIN
ETD = ETI - ETO
SDIFF (I-IIS + 1) = SDIF * 100.D0
SUM (I-IIS + 1) = ETD * 100.D0
WRITE (*, 1564) RC(I), ZC(I), SDIF, ETI, ETO, ETD

CONTINUE
IF (.NOT. DPL0T) GOTO 5432
CALL CALC02 (YC(IIS), SDIFF, M4, 8, 7, 7, 0, 1, 0, 0, 0, 0, 0, 0, -1, -1, -1)
CALL CALC02 (XC(M13), SDIFF, M14, M5, 8, 7, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
CALL CALC02 (YC(M15), SDIFF, M16, M17, 8, 7, 7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
CALL CALC02 (YC(IIS), SUM, M4, 7, 7, 0, 20, 1, -15, 0, 5, 1, 1, 0)
CALL CALC02 (XC(M13), SUM, M14, M5, 8, 1, -15, 0, 5, 1, 1, 0)
CALL CALC02 (YC(M15), SUM, M16, M17, 8, 7, 25, -15, 0, 5, 1, 1, 0)

CONTINUE
I = 0
DO 420 NUI = 1, NSEC
NU = NUI
IF (.NOT. ALL) GOTO 421
ITI = IDS + 1
IDH = IDS + M5
IDV = IDH + M6
IDS = IDV + M7
GOTO 422

421 IDH = IDS + M5
IDV = IDH + M6
IDS = IDV + M7
DO 422 CONTINUE
I = ITI + IDS
II = I
IF (I .EQ. IDS) IGG = II
ETI = 0.D0
ETO = 0.D0
DIN = 0.D0

5432 CONTINUE
DOUT=0.DO
DO 424 J=1,ILI
K=J
Q2=CY(K)
IF(I.LE.IDV)GOTO8015
IF(J.GT.ICND.AND.J.LE.IDO)K=J+NINS
IF(J.GT.IDO)K=(J-IDO+NINS+IO)
Q2=CY(K)
CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,Q2)
IF(J.GT.ISH.AND.J.LE.ICND)GOTO4015
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
GOTO5355
4015 CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,II,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ETII)
EFN=EFN+EFNI
ET=ET+ETII
GOTO5355
8015 IF(J.GT.ICND.AND.J.LE.IDO)K=J+NINS
IF(J.GT.IDO)K=(J-IDO+NINS+IO)
Q2=CY(K)
CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,Q2)
IF(J.GT.ISH.AND.J.LE.ICND)GOTO4055
IF(J.GT.ISH.AND.J.LE.IDO)GOTO4065
EFN=EFN+EFNI
ET=ET+ETII
GOTO5355
4065 EFN=ER
ET=ETZ
GOTO5355
4055 CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,II,Q2)
IF(J.GT.ISH.AND.J.LE.IDO)GOTO4075
EFN=EFN+EFNI
ET=ETII
GOTO5355
4075 EFN=EFN+EFNI
ET=ETII
GOTO5355
5355 DIN=DIN+EFN
ETI=ETI+ET
424 CONTINUE
DO 595 J=1,ILI
K=J
IF(J.GT.IDO)K=(J-IDO+NINS+IO)
Q2=CY(K)
CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
GOTO9015
9025 CALL ERING(RC(I),ZC(I),RC(K),ZQ(K),ER,EZ,II,Q2)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ET)
CALL EFLX(RC(I),ZC(I),ER,EZ,II,EFN,ETII)
EFN=EFN+EFNI
ET=ETII
GOTO9015
9015 IF(J.GT.ISH.AND.J.LE.ICND)GOTO9055
EFN=EFN
ET=ET
GOTO9035
9035
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9045 EFN=ER
ET=EZ
GOT09395
9055 CALL ERING(RC(I),ZC(I),RD(K),-20(K),ERI,EZI,Q2)
IF(I,GT,IBH,AND,I,LE,IDV)GOT09105
EFN=ER+ERI
ET=EZ+EZI
GOT09395
9105 EFN=ER+ERI
ET=ER+ERI
9395 DOUT=DOUT+EFN
ETO=ETO+ET
5595 CONTINUE
XC(I)=RC(I)
YC(I)=ZC(I)
SDIF=(EPSIR)*DOUT-DIN
ETD=ETI-ETO
SDIFF(I-ITII+IJ)=SDIF
SUM(I-ITII+IJ)=ETD+100.DO
WRITE(*,564) RC(I),ZC(I),SDIF,ETI,ETO,ETD
423 CONTINUE
II=II+M5+M6+M7
420 CONTINUE
787 KOUNT=KOUNT+1
IF(ALL).BCND=.FALSE.
IF(KOUNT,EQ,3)GOT01011
IF(ALL)GOT0740
1011 CONTINUE
IF(.NOT.DPLOT)GOT01010
IJ=2*M5+M17+1
CALL CALC02(XC(K1),SDIFF,M5,R7,R7,0,0,0,0,0,1,-1,0)
CALL CALC02(YC(K2),SDIFF(M5+1),M17,R7,R7,0,0,0,0,1,-1,0)
CALL CALC02(XC(K3),SDIFF(M5+M17+1),M5,R7,R7,0,0,0,0,1,-1,0)
CALL CALC02(YC(K4),SDIFF(IJJ),M17,R7,R7,0,0,0,0,1,-1,0)
CALL CALC02(XC(K1),SUM,M5,R6,R6,0,1,-1.5,0.5,1,-1,0)
CALL CALC02(YC(K2),SUM(M5+1),M17,R6,R6,0,1,-1.5,0.5,1,-1,0)
CALL CALC02(XC(K3),SUM(M5+M17+1),M5,R6,R6,0,1,-1.5,0.5,1,-1,0)
CALL CALC02(YC(K4),SUM(IJJ),M17,R6,R6,38,1,-1.5,0.5,1,-1,0)
CALL PLTEND(9.)
1010 STOP

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