Simplified method for basement heat loss prediction.

Murty Satyanarayana Kompella

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/6818
SIMPLIFIED METHOD FOR BASEMENT HEAT LOSS PREDICTION

by

MURTY SATYANARAYANA KEMPPELLA

A thesis
presented to the University of Windsor
in partial fulfillment of the
thesis requirement for the degree of
MASTER OF APPLIED SCIENCE
in
THE DEPARTMENT OF MECHANICAL ENGINEERING

Windsor, Ontario, 1997
UMI Number: EC54807

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI

UMI Microform EC54807
Copyright 2010 by ProQuest LLC
All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
I hereby declare that I am the sole author of this thesis. I authorize the University of Windsor to lend this thesis to other institutions or individuals for the purpose of scholarly research.

MURTY SATYANARAYANA KOMPELLA

I further authorize the University of Windsor to reproduce this thesis by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

MURTY SATYANARAYANA KOMPELLA

- iii -
DEDICATION

TO NANA, AMMA AND AKKA

- iv -
A simplified method has been developed to calculate basement heat losses. The method is able to accommodate a wide range of physical and thermal properties of soil, basement depths, wall insulation levels, and ground surface temperatures. The method is also compatible with microcomputers and calculators.

A finite difference computer program (HEATING5) was used to simulate a typical basement. It was observed that when a sinusoidally varying ground surface temperature was used as the forcing function, the wall heat flux, averaged over the height of the wall, also varied sinusoidally and had a lag with respect to the ground surface temperature. This was also true for the floor, except that the variation of the floor heat flux extended up to 3 m from the corner, and the heat flux from the centre of the floor was approximately constant over the annual cycle. It was observed that the average mean annual heat flux was very nearly equal to the steady-state value. To predict the average heat flux from the wall (or floor), the values of three parameters are required - the average mean annual heat flux, the amplitude of the heat flux and the lag.
Transient and steady-state two-dimensional simulations of a simplified basement model were carried out using HEATINGS with the following range of variables: basement depth, 1.5 to 3.0 m; soil conductivity, 0.88 to 2.5 W/m·K; soil diffusivity, $3.34 \times 10^4$ to $11.94 \times 10^{-4}$ m$^2$/s. No insulation and full wall insulation on the inside (up to $2.9 \text{ m}^2\cdot\text{K}/\text{W}$ insulation resistance) were treated. Floor insulation was not considered. The basement was considered to be completely below-grade. The ground water level was taken to be 10 m below the ground surface.

A non-linear curve-fitting program, DPENLN, was used to curve-fit the results of the simulations, and simplified correlations were obtained for the values of the three parameters in terms of the basement depth, soil properties, wall insulation resistance (if any) and the ground surface temperature.

The new method was compared to seven other existing methods by calculating the heat loss from a typical basement with climatic conditions of three northern U.S. cities. The results showed that the new method compared very well with the methods proposed by Mitalas and Yard et al.

The new method has the advantage of accommodating various basement depths, soil properties, and using ground temperatures.
ACKNOWLEDGEMENTS

I am very grateful to Dr. H.J. Tucker for his excellent supervision, constant encouragement and timely help. Working under him was a very pleasant experience.

I wish to thank Dr. T.W. McDonald for his valuable suggestions, and fruitful discussions.

My thanks are due to Dr. W.G. Colborne for his help in the early stages of this thesis. I also wish to express my gratefulness to him for offering me financial support for my graduate studies.

I would like to thank Dr. W.T. Kierkus for his many valuable suggestions and help.

I am deeply grateful to Dr. Shih-I Tang and Ms. Jayalakshmi Sreedharan for their help with HEATINGS5. Without their continued help this work would not have been possible.

I am forever indebted to Dr. M.K.S. Madugula and his family for their constant help and moral support. They provided me with a home away from home.

Finally, I wish to thank my parents and my sister for their constant encouragement and faith in me.
CONTENTS

DEDICATION........................................................................................................................................ iv

ABSTRACT......................................................................................................................................... v

ACKNOWLEDGEMENTS....................................................................................................................... vii

Chapter I: INTRODUCTION.............................................................................................................. 1

FACTORS AFFECTING BASEMENT HEAT LOSS.............................................................................. 2
GROUND TEMPERATURES.................................................................................................................... 2
SOIL THERMAL PROPERTIES........................................................................................................ 4
OTHER FACTORS............................................................................................................................ 6
GROUND SURFACE TEMPERATURE VERSUS AIR TEMPERATURE............................................. 9
LITERATURE SURVEY....................................................................................................................... 10
EARLY STUDIES.................................................................................................................................. 10
STUDIES USING COMPUTER SIMULATION..................................................................................... 11
EXISTING SIMPLIFIED METHODS FOR CALCULATING BASEMENT HEAT LOSSES.................. 19
OBJECTIVES...................................................................................................................................... 23

Chapter II: THE HEATINGS PROGRAM............................................................................................ 25

DESCRIPTION OF THE PROGRAM...................................................................................................... 25
DESCRIPTION OF THE COMPUTER MODEL....................................................................................... 27
MODEL ASSUMPTIONS....................................................................................................................... 27
MODEL OPERATION........................................................................................................................... 28
VALIDATION...................................................................................................................................... 30
VALIDATION USING McBRIE'S BASEMENT.................................................................................. 30
VALIDATION USING DBR/NRC BASEMENT IN OTTAWA.............................................................. 33

Chapter III: DEVELOPMENT OF THE NEW METHOD..................................................................... 44

INTRODUCTION.................................................................................................................................. 44
PRELIMINARY STUDIES....................................................................................................................... 46
DEVELOPMENT OF THE METHOD....................................................................................................... 55
STEADY STATE (UNINSULATED WALL).............................................................................................. 59
STEADY STATE (INSULATED WALL).................................................................................................... 61
TRANSIENT (UNINSULATED WALL).................................................................................................... 63
TRANSIENT (INSULATED WALL).......................................................................................................... 64
# LIST OF TABLES

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Building and Soil Properties</td>
<td>32</td>
</tr>
<tr>
<td>2.</td>
<td>Surface Convection Film Coefficients</td>
<td>32</td>
</tr>
<tr>
<td>3.</td>
<td>Ground Temperatures Adjacent to the Basement Wall</td>
<td>34</td>
</tr>
<tr>
<td>4.</td>
<td>Ground Temperature Horizontally 0.91 m from Wall and Vertically 0.61 m from the Earth Surface</td>
<td>35</td>
</tr>
<tr>
<td>5.</td>
<td>Ground Temperatures at the Basement Floor and Earth Interface</td>
<td>36</td>
</tr>
<tr>
<td>6.</td>
<td>Monthly Average Wall Heat Losses</td>
<td>37</td>
</tr>
<tr>
<td>7.</td>
<td>Monthly Average Floor Heat Losses</td>
<td>38</td>
</tr>
<tr>
<td>8.</td>
<td>Thermal and Physical Properties of the Materials used, in DBR/NRC Basement</td>
<td>40</td>
</tr>
<tr>
<td>9.</td>
<td>Monthly Average Heat Losses</td>
<td>43</td>
</tr>
<tr>
<td>10.</td>
<td>Test Basement Input Values</td>
<td>83</td>
</tr>
<tr>
<td>11.</td>
<td>Ground and Air Temperature Input Data for Five U.S. Cities</td>
<td>84</td>
</tr>
<tr>
<td>12.</td>
<td>Data used as Input to DPENLN</td>
<td>155</td>
</tr>
<tr>
<td>15.</td>
<td>to</td>
<td>158</td>
</tr>
<tr>
<td>16.</td>
<td>Comparison of original data and data predicted by the regression equations</td>
<td>164</td>
</tr>
<tr>
<td>26.</td>
<td></td>
<td>174</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Basement Model Showing Boundary Conditions</td>
<td>29</td>
</tr>
<tr>
<td>2.</td>
<td>Basement Model for the DBR/NRC Basement</td>
<td>41</td>
</tr>
<tr>
<td>3.</td>
<td>Inside Surface Temperature of the Wall for Various Depths Below the Ground Surface</td>
<td>48</td>
</tr>
<tr>
<td>4.</td>
<td>Inside Surface Temperature of the Floor for Various Distances from the Corner</td>
<td>49</td>
</tr>
<tr>
<td>5.</td>
<td>Heat Flux for Various Points on the Inside Wall Surface that are at Different Depths Below the Ground Surface</td>
<td>50</td>
</tr>
<tr>
<td>6.</td>
<td>Heat Flux for Points on the Floor at Various Distances from the Corner</td>
<td>51</td>
</tr>
<tr>
<td>7.</td>
<td>Time Lag of the Wall Heat Flux for Various Depths Below the Ground Surface</td>
<td>52</td>
</tr>
<tr>
<td>8.</td>
<td>Time Lag of the Floor Heat Flux for Various Points on the Inside Surface of the Floor</td>
<td>53</td>
</tr>
<tr>
<td>9.</td>
<td>Ground Surface Temperature, Wall Surface Temperature, and Wall Heat Flux Variation</td>
<td>57</td>
</tr>
<tr>
<td>10.</td>
<td>The Variation of the Soil Resistance with the Basement Depth for Various Soil Conductivities</td>
<td>69</td>
</tr>
<tr>
<td>11.</td>
<td>The Variation of the Slope of the &quot;Soil Resistance versus Basement Depth&quot; Curve with Soil Conductivity</td>
<td>70</td>
</tr>
<tr>
<td>12.</td>
<td>The Variation of the Soil Resistance (to the Floor Heat Loss) with the Basement Depth for Various Soil Conductivities</td>
<td>72</td>
</tr>
<tr>
<td>13.</td>
<td>The Variation of Coefficients &quot;a&quot; and &quot;b&quot; in Eqn. 4.4, with Soil Conductivity</td>
<td>73</td>
</tr>
<tr>
<td>14.</td>
<td>The Variation of Coefficient &quot;c&quot; in Eqn. 4.4, with Soil Conductivity</td>
<td>74</td>
</tr>
<tr>
<td>15.</td>
<td>Total Annual Basement Heat Loss</td>
<td>86</td>
</tr>
<tr>
<td>16.</td>
<td>Annual Basement Heat Loss from Walls Only</td>
<td>87</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
NOMENCLATURE

\( C_p \)  
soil specific heat (J/kg·K)

\( K \)  
soil conductivity (W/m·K)

\( L \)  
basement depth (m)

\( q \)  
heat flux (W/m²)

\( q_a \)  
amplitude of the heat flux (W/m²)

\( q_f \)  
average mean annual floor heat flux for the uninsulated case (W/m²)

\( q_{f,a} \)  
amplitude of the average floor heat flux for the uninsulated case (W/m²)

\( q_{f,a,i} \)  
amplitude of the average floor heat flux for the insulated case (W/m²)

\( q_{f,c} \)  
steady-state heat flux for the floor centre for the uninsulated case (W/m²)

\( q_{f,c,i} \)  
steady-state heat flux for the floor centre for the insulated case (W/m²)

\( q_{f,i} \)  
average mean annual heat flux for the floor for the insulated case (W/m²)

\( q_m \)  
mean annual heat flux (W/m²)

\( q_w \)  
average mean annual wall heat flux for the uninsulated case (W/m²)

\( q_{w,a} \)  
amplitude of the wall heat flux for the uninsulated case (W/m²)

\( q_{w,a,i} \)  
amplitude of the wall heat flux for the insulated case (W/m²)
average mean annual wall heat flux for the insulated case (W/m²)

heat flux at any location x (W/m²)

thermal resistance of wall insulation only (m²·K/W)

thermal resistance of soil (m²·K/W)

thermal resistance of wall and insulation (including film coefficient) (m²·K/W)

thermal resistance of wall (including film coefficient) (m²·K/W)

amplitude of first harmonic of ground surface temperature (°C)

amplitude of the wall or floor surface temperature (°C)

basement air temperature (°C)

ground surface temperature (°C)

mean annual ground temperature (°C)

mean wall or floor surface temperature at any location (°C)

surface temperature of wall or floor at location x (°C)

soil diffusivity (m²/s)

difference between the basement air temperature and the mean annual ground temperature (°C)

phase lag of the floor heat flux (uninsulated case) (rad)

phase lag of the floor heat flux (insulated case) (rad)

phase lag of the wall heat flux (uninsulated case) (rad)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{w,i}$</td>
<td>phase lag of the wall heat flux (insulated case) (rad)</td>
</tr>
<tr>
<td>$\delta_x$</td>
<td>phase lag at any location $x$ with respect to the ground surface temperature (rad)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency (rad/day)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>soil density ($\text{kg/m}^3$)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>phase (rad)</td>
</tr>
</tbody>
</table>
Chapter I

INTRODUCTION

The energy crisis in recent years has led to a growing awareness of the value of conserving energy resources. This has given rise to an increased interest in the reduction of heat losses from or to basements as evidenced by numerous papers appearing in the literature. Most basements today are partially finished, heated and used as an integral part of the living space of the home. Heat losses from uninsulated basement areas constitute a major conservation opportunity.

About 90% of low-rise residential buildings in Canada have a full basement [9]. Those, of older construction, are rarely insulated and are responsible for a considerable proportion of the heat loss from the building. Provided that no major structural changes are required, the thermal upgrading of basements can be one of the most cost effective ways of conserving energy, and therefore the technology of basement retrofit, as well as new retrofit design, is important in the field of energy conservation.

Studies of heat loss from conventional basements have been conducted for over 40 years. The following section contains a discussion of important factors influencing heat
loss from basements. This is followed by a comprehensive up-to-date literature survey.

1.1 FACTORS AFFECTING BASEMENT HEAT LOSS

1.1.1 GROUND TEMPERATURES

The heat transfer in soil is a process driven either directly or indirectly by solar heat flux at the surface. The ground temperature is one of the most important parameters affecting heat transfer in basements. The earth temperature varies with latitude, weather conditions, time of year, altitude, landscaping, shading, neighbouring buildings, earth surface conditions, and rainfall. It is also dependent on snow cover, cloud cover, thermal and physical properties of soil, and the level and movement of ground water in the soil. Although the exact ground temperature at a specific site can be obtained only by direct measurements, statistical information on the general distribution of the ground temperature is worth obtaining. The first recorded measurement of soil temperature studies in Canada were made at McGill University in Montreal and reported by Callender [22] and Callender and McLecd [23]. Temperature measurements were taken and observations made on the temperature equalizing effect of rainfall and the insulating effect of snow cover on the soil temperatures near the surface. The studies showed that daily variations in temperatures occur only to depths of a few centimeters, while changes below 25.4 cm follow an annual cycle.
Crabb et al [24] studied the influence of vegetative cover on soil temperatures at East Lansing, Michigan. Their studies showed that solar radiation has a marked effect on the soil temperatures. Crawford [25] measured ground temperatures and his studies indicated that ground temperatures are significantly affected by surface conditions, such as snow cover, etc., and solar radiation. Gold [26] showed the effect of snow cover and the convective coefficient value on the difference between the average air and corresponding ground surface temperatures. Kusuda [27] studied soil temperatures under five different soil covers. Bhardwaj [32] covers the above studies in more detail.

The most influential ground climate work has been attributed to Kusuda and Achenbach [28]. The authors did extensive analyses on earth temperature data from 63 stations located in fifty different areas throughout the United States. Annual cycles of monthly average earth temperatures were used to study and correlate their annual averages, amplitudes, phase angles and thermal diffusivities. It was found that a simplified harmonic presentation of earth temperature provides an acceptable approximation of the monthly average earth temperatures at various depths. Their analysis indicated that the annual average earth temperature in the range studied was practically constant with respect to depth and was very closely approximated by the annual average air temperature.
Raff [29] examined those factors which might raise the average ground temperature and thus make underground buildings in the northern and central regions of the U.S. comfortable with less need for space heating. He reports that the deep ground temperature is quite close to the annual average ground surface temperature. It was shown that that paving the surface increases the average ground temperature by 2°C by causing the precipitation to run off, thus avoiding the cooling effect of the evaporation of 72% of it which normally occurs in the Baltimore-Washington area. If the ground slopes to the south, its average temperature will be further increased.

1.1.2 SOIL THERMAL PROPERTIES

The soil thermal conductivity and diffusivity are the two most important properties that govern earth-coupled heat transfer. The thermal conductivity of a soil, at a given density and moisture content, varies in general with the texture of a soil, being relatively high for coarse textured soils and relatively low for fine textured soils. The mineral composition of the soils also affects the conductivity. Unlike a homogeneous substance, such as copper, for which the experimental measurement of thermal conductivity is simple and well defined, soils are a nonhomogeneous mixture of minerals, organic matter, air and water. Kersten [30] conducted experiments to determine the conductivity of soils. Tests were made on 19 different
soils. It was found that an increase in moisture content caused an increase in thermal conductivity. This was found to be true up to the point of saturation. For moisture content above 6 to 12\%, the conductivity of frozen soils was found to become progressively greater than that of unfrozen soil. Density affected the conductivity of a soil in about the same manner for all soils, at any moisture content, and for either the frozen or unfrozen condition. On the average each one pound per cubic foot increase in density increased the thermal conductivity by about 3\%. Kersten developed four charts to predict soil conductivity and claimed that these charts would give the values within an accuracy of 25\%. The work of Van Duin [37] too shows the strong dependency of soil conductivity on moisture content. MacDonald [31] has shown that the existing data on soil conductivity are frequently contradictory and need further work aimed specifically at ground-coupled heat transfer.

A knowledge of the soil thermal diffusivity is required in any heat transfer model that takes thermal capacitance into account. Diffusivity, \( \alpha \), is defined as

\[
\alpha = \frac{K}{\rho C_p}
\]

where

\( K \) = soil conductivity
\( \rho \) = soil density
\( C_p \) = soil specific heat
All the above variables are functions of moisture content. It has been found that for a wide range of soils, an increase in moisture content from 0 to 10% causes a sharp rise in diffusivity, followed by a gradual reduction thereafter. Since diffusivity is a dual-valued function of moisture, the prediction of soil heat transfer requires a full knowledge of soil density, conductivity and specific heat and moisture content. The most extensive regional data on soil diffusivity comes from Kusuda.

1.1.3 OTHER FACTORS

Bhardwaj [32] conducted a parametric study using a finite difference model to find the influence of various parameters on basement heat loss. The parameters considered were (1) soil conductivity, (2) proximity of adjacent basements, (3) ground water level, and (4) ground surface cover (asphalt or snow). He found that the soil conductivity exerted a significant influence on the heat loss. Bhardwaj also studied the influence of using yearly average values of soil conductivity rather than seasonally varying values in the Columbus, Ohio area. He found that using fixed mid-range values of soil conductivity in his model resulted in a reduction of 8.3% in the wall heat loss during February and a corresponding reduction of 50% for the floor loss. The average wall and floor heat losses during the heating season were reduced by 5.4% and 17.9% respectively. It was concluded from the study that seasonal values of soil
conductivity should be used for heat loss calculations. Different fixed values were then used. It was found that the variation of the soil conductivity had a greater influence on basement wall heat loss during the winters than during the summers. On the other hand, the basement floor loss was affected throughout the year. Structures similar to the basement, not closer than 3 m, were not found to affect the wall heat losses much. The floor losses, however, were reduced by about 42% during the heating season. The level of the ground water was found to have a considerable effect on the floor losses. When the ground water level was put 4 m below the basement floor, the floor loss during the heating season increased by 18.8% while the wall loss increased by about 1% only. When the level was put 1 m below the floor, the floor loss increased by 150% while the wall loss increased by about 6%.

On an average, the wall and floor losses decreased by about 11% and 2.4% respectively during the heating season for an average thickness of snow cover of 5.7 inches. It was seen that the snow cover reduced the wall losses by as much as 33% during the month of February. The corresponding values for 4 inches of asphalt cover were found to be 19% and 9.4% respectively.
1.2 GROUND SURFACE TEMPERATURE VERSUS AIR TEMPERATURE

All earth coupled heat transfer models must estimate earth temperatures either as direct input or as values calculated from, or linked to, other driving climatic inputs. In reality, air and ground surface temperatures may not be equal. In colder regions, there may be a seasonal insulating layer of snow. In moist, temperate regions, moisture transpiration may lead to a soil cooling effect. Several authors (Swinton and Platts 1981; Kusuda and Achenbach 1965; Labs 1979) recommend using annual mean air temperatures and amplitudes to approximate ground temperatures when ground temperature data are unavailable. This option is attractive since this data is readily available for virtually any location. The air temperature could also be used directly in conjunction with surface convective coefficients. But the values of these coefficients are not known precisely. Also they are time varying, being highly dependent on ground surface conditions and wind velocity and direction.

For many Canadian sites, ground surface temperature measurements have been made. These temperatures implicitly take into account the complex heat transfer mechanisms occurring at the ground surface, including the effects of solar radiation, evapo-transpiration, rainfall, convection, snow cover, etc. So, when these temperatures are available, they should be used.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
While the use of average annual air temperature to approximate average ground temperature is an expedient solution, there may be some inherent error. Baggs [33] has extensively mapped Australian soil temperatures in an attempt to relate annual air temperature to annual ground temperature. His isotherms show that annual average ground temperatures range from 1.5°C less to 5.0°C greater than air temperatures, a range of 6.5°C. Kusuda's data show a similar range of 6.4°C. MacDonald has used linear regression analysis to quantify the difference with respect to climatic regions. The analysis found the northern snowbelt and western regions of the U.S.A. show average annual ground temperatures 1.9°C to 2.8°C higher than the average annual air temperature. The elevated ground temperature is attributed to the probable insulating snow cover. Ground and air temperature amplitudes also have regional differences. This difference is thought to cause a difference of 10% to 15% in the basement heat loss predictions in snowbelt regions. MacDonald et al [21] concluded that any method of basement heat loss prediction that uses air rather than ground temperature as input will be subject to error, the magnitude of which may or may not be acceptable.
1.3 LITERATURE SURVEY

1.3.1 EARLY STUDIES

Houghten et al [1] carried out experimental work at the ASHVE Research Laboratory in 1940-41. One of the earliest methods for calculating heat loss is derived from this work and is described in the ASHRAE Handbook of Fundamentals, 1972. The method specifies fixed heat-flux rates through the basement walls based solely on the local groundwater temperatures. The basement floor loss is related to the ground temperatures at depths of 30 to 60 ft, and the wall loss taken as twice the floor loss and assumed to be uniform over the height of the wall. No distinctions are made for different types of construction, depth below the ground surface, or the presence of perimeter insulation.

Recognizing the need to improve upon the accuracy of this method, a revised procedure was published in the ASHRAE Handbook of Fundamentals, 1977, based on the work of Boileau and Latta [2] carried out in 1968. This method prescribes circular arc pathlines centred about the intersection of the basement wall and the ground surface to describe the heat flow from the basement. The total thermal resistance of the pathline for a particular depth is calculated from the sum of the resistances of each building element traversed by the arc and extending through the basement wall. The total peak heat loss can then be estimated by integrating the product of the temperature difference for winter design conditions.
and the equivalent path conductance (the reciprocal of the thermal resistance) over the depth of the basement wall. While this procedure does offer the flexibility of permitting different wall insulation configurations, the assumption of heat flow along a circular arc, and the inability to include thermal storage effects present serious limitations.

1.3.2 STUDIES USING COMPUTER SIMULATION

A digital computer was used by Kusuda and Achenbach [3] to solve the time iteration finite difference equation of the three-dimensional heat conduction problem. The boundary conditions were such that simultaneous transfer of heat and water vapour took place inside a parallelepiped cavity buried underground at a finite depth. The program was applied to calculate the thermal environment of underground fallout shelters where the heat and water vapour generated by the occupants exchange heat with the surrounding earth as well as with the ventilation air. The illustrative calculations performed for four different test conditions in two different shelters showed good agreement with observed conditions. This study was carried out to analyze short-term (two weeks) transient thermal conditions in underground survival shelters and is not directly applicable to basements.

As part of an extensive research project McBride et al [4] investigated subgrade heat losses from basement walls

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
and floors with the objective of developing a calculational procedure. In order to develop and validate the procedure, a series of measurements were made for a full year on an unoccupied house with a full basement. A finite difference modelling method was employed which consisted of a set of transient two-dimensional heat transfer equations. The earth seasonal thermal conductivities used in the model were selected as mid-range values for the Ohio area as reported in the literature. The soil thermal conductivity and the convection film coefficient at the boundary of the ground and air were changed during the year: mid-range values for four seasons were used. Solar radiation intensity at the ground surface was neglected in an effort to reduce the complexity of the model. The deep ground water temperature was considered constant for the entire year and was obtained from average deep well measurements taken near the site. The calculated results agreed very well with the experimental data.

Wang [5] used a finite element program to analyze the heat loss of a basement wall with and without insulation. The objectives were to examine partial and full height insulation systems and interior and exterior applications. The conclusions were:

1. **Exterior foundation wall insulation applications** are 25 to 32% more efficient than interior applications.
2. Insulation located near the top of the wall is more beneficial than insulation placed at the bottom of the wall.

Meixel et al [6] developed a transient finite difference computer program and utilized it to investigate the dynamic thermal performance of basement and earth-sheltered walls to determine the spatial and temporal characteristics of the heat-flux through these walls. Calculations using the program were compared with experimental values and the seasonal heat loss through basement walls was predicted for several insulation configurations. For weather conditions typical of Minneapolis, Minnesota, computer calculated monthly average heat fluxes showed that insulated underground walls have less winter heat loss than comparably insulated aboveground walls. It was found that a basement wall with 10cm additional inside insulation (R-25) performs better than an average R-40 wall. However, the results indicated that insulating the inside of basement walls results in sub-freezing temperatures inside the basement walls and the surrounding soil which have the potential for frost-heave damage to the walls. The calculations also demonstrated that placing the insulation horizontally outside the basement wall results in good winter performance, reduces the possibility of foundation damage, and as an additional benefit, allows the underground wall to lose heat to the soil during the summer cooling period.
Szydlowski and Kuehn [7] developed a transient two-dimensional model sufficiently general to accommodate numerous earth-sheltering configurations under variable boundary and soil conditions. The model was used for calculations on McBride's basement and showed reasonable agreement with the experimental data. Several factors were then investigated and the results found were as follows:

1. Ceiling ground coupling is minimal with 0.75 m soil cover, but is more pronounced with 2.05 m soil cover.

2. Seasonal wall and ceiling heat loss, though dramatically reduced by extended soil cover, was not reduced to values typical of conventional construction without the addition of insulation.

3. Although heating season heat losses are reduced by added insulation, the beneficial cooling season heat losses are also reduced. The basement floor insulation reduces heat losses more in the cooling season than in the heating season, thereby actually increasing the annual energy requirements.

Shipp et al [9] monitored the heat transfer through the walls of an earth-sheltered building by means of wall heat-flux gauges and soil temperature measurements at the University of Minnesota. Measurements were carried out over a two-year period. The thermal effects of paved and grassy ground surface cover, as well as four different soil types, were examined in the study. The ground surface cover was
found to be the most influential element in determining the building's external thermal environment. During the course of the investigations, it was demonstrated that a region of soil surrounding the building warms to form a thermal buffer zone, which moderates the effect of weather-related fluctuations in the wall energy fluxes and enhances the effective thermal mass of the building envelope. Seasonal weather effects were observed to exert a measurable influence on the upper 2.5 m of the building. Below this level, the wall heat flux was found to be relatively small, being influenced primarily by interior building conditions.

In comparing the effects of using four different types of soils as backfill around the building, it was found that the exterior surface treatment produced a larger influence on the thermal performance of the walls than the variations in the thermophysical properties of the soil. A transient two-dimensional finite program was also developed to study the thermal performance of earth-sheltered structures. Results of the program were compared with experimental data and reasonable agreement was obtained. From the program results it was observed that while property variations between different types of soil produce marked changes in the winter heat loss performance of the structure, the variations produced by a single soil type are not pronounced once the mean property values have been specified. It was also found that specification of the soil properties is only critical within a narrow region surrounding the structure.
Black et al [10] studied the technical and economic practicality of reducing energy consumption in underground buildings by placing insulating materials directly in the soil or by applying insulation to the exterior portion of the building. The heat transfer analysis used in this study employed a steady-state, two-dimensional finite element formulation. Monthly and annual heat transfer rates from a residential and commercial-size building at two extreme locations representing northern and southern climates in the United States were analyzed. At both locations the monthly air temperature, soil temperature profiles and solar heat fluxes were used as input and the resulting heat transfer rates were converted into costs using estimated prices for typical heating fuels. The influence of room thermostat settings during the summer and winter months on conserving energy was also investigated.

Richmond et al [11] studied heat loss through the basement floors of two houses. Data was collected through the use of remote data acquisition systems over a period of several months. The measured heat fluxes were compared to models given by Mitalas [12] and ASHRAE [38]. While there was general agreement in the seasonal variation of basement floor heat loss between experimental data and Mitalas's method, a wide discrepancy in the magnitude of the heat loss is reported. It was found that there was little agreement between measured heat fluxes and the design values predicted by ASHRAE.
Heat losses from basement floors were investigated by measuring the temperature distributions and heat rates for a period of one year across the basement floor insulation in two houses on the Canadian prairies by Richmond et al [13]. These experimental results were compared with two existing design models as well as a finite model. The two models used were the Mitalas method and the Yard method [14]. Two-dimensional finite element studies showed the importance of radiation in the heat transfer processes occurring in a basement. Studies also showed that the elimination of thermal contact between the basement floor slab and the basement wall footing reduces basement floor heat loss while increasing the floor temperature. Use of the Yard model for high levels of insulation on basement walls for houses in the Canadian prairies was not possible. The model, when compared to the measured results from a house with lower insulation levels, had a much higher amplitude of heat loss over the year and disagreed with the trends and amount of heat loss exhibited in the measured data. Compared on a yearly basis with the measured data, the error in the total yearly heat loss was greater than any of the other models.

Mitalas's model, as well, was found to be incapable of modelling the transient phenomenon exhibited in the measured data. However, when yearly totals were compared over both the year and the heating season, agreement with the measured data was found to be within plus or minus 5%.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
The transient finite element model was found to be the most promising of the three models tested. Its predictions were seen to closely model what was occurring in the physical data on a monthly basis. Energy totals for the year and heating season were within 9% of the measured data.

1.3.3 EXISTING SIMPLIFIED METHODS FOR CALCULATING BASEMENT HEAT LOSSES

While the use of numerical schemes, such as the finite difference and finite element methods, has led to a greater understanding of earth-contact heat transfer from basements and earth-sheltered buildings, these methods are rather limited as practical design tools. Because of their large computer storage requirements and long run times, these methods are not compatible with the microcomputers now available to design engineers. In view of this a number of simplified methods have been developed to estimate basement heat loss.

Shen et al [17] proposed a simplified earth-sheltered building thermal analysis program, which makes use of an analytically obtained Fourier-series solution to the governing heat conduction equation. The Fourier coefficients were obtained numerically by imposing the surface conditions along the boundary. The least squares method requires very little computer storage and run time. It is capable of simulating slab-on-grade, basement, berm, and earth-covered building configurations. Results of the
method were compared with those from a fully implicit finite difference program. The wall heat loss values for the two methods showed good agreement during the summer months, while for the winter months, the least squares results were about 20% less than the finite difference values. The floor heat loss calculated by the least squares method had a much larger yearly variation than that obtained from the finite difference analysis.

Another method developed by Swinton et al [15] is an empirical curve fit of experimental data from several Canadian sites. This method predicts heating season loss using degree-days, wall insulation R-value, basement temperature, and height of the above-grade wall. Field histories of heat losses were taken from the "Mimic Box" test program [16]. These tests involved placing a calorimeter device to monitor the losses of a 4-ft wide strip of basement wall from a few inches above grade to about 5 ft below. Identical boxes were placed on the basement slab to record its losses to the ground below. Eighteen such boxes were installed in various basements across a wide range of climates, in Saskatoon, Ottawa and Charlottetown. Various insulation levels were included, including bare walls. This method should probably be used only in cold climates where winter snow cover is typical, since the data used to develop the method were restricted to such climates.
A method proposed by Mitalas [12] has received wide use. The development of the method is based on both experimental and analytical studies of basement heat loss. The essential data needed to calculate the basement heat loss are the steady-state and periodic shape factors, the amplitude attenuation factor and the time-lag factors and the ground surface temperatures. Finite element analyses have been used to arrive at a set of shape factors for a variety of basement configurations. These factors were derived using a combination of two-dimensional and three-dimensional heat conduction calculations. Different factors are assigned to different basement segments to account for the varying heat fluxes. The method predicts the heat loss through specific basement segments to get the total basement heat loss. The results of this method were found by Mitalas to be within 10% of measured values for several basements located in Ottawa, Saskatoon and Charlottetown. Over 13 different insulation configurations may be modelled. Serious limitations to the method are that only two different soil conductivities, 0.8 \( \text{W/m-K} \) and 1.2 \( \text{W/m-K} \), can be modelled, and the depth of the basement cannot be taken into account.

The method developed by Shipp [18] utilizes a regression expression to calculate the seasonal heating or cooling impact of a basement as a function of heating degree-days, cooling degree-days and basement thermal resistance. The regression coefficients were arrived at by using a
transient, two-dimensional finite difference program. The method assumes 1 ft of above grade basement wall, does not consider the fin effect of corners, and considers heating and cooling seasons whose length implicitly varies as a function of basement thermal resistance. It contains implicit assumptions of 22.8°C basement temperature and soil properties. Regression coefficients are also given for a basement that is only half below grade. Energy use is calculated as a function of perimeter length. The major limitations are the invariant basement temperature and soil properties.

The method given by Akridge [19] is known as the Decremented Average Ground Temperature method for predicting heat loss from below grade walls. This method accounts for the increase in ground temperature over time due to the presence of the building. This method consists of a set of decrement factors that are used with the wall UA, interior temperature, and the undisturbed ground temperature to calculate the average heat loss for a month or other time period. The decrement factors were developed from parametric studies using the GROCS/TRNSYS computer model [20]. This method does not consider basement floor heat loss. It can treat different external vertical insulation schemes using varying soil conductivities.

Yard [14] developed a new method to calculate the monthly and annual values of heat losses from basement walls and
floors. The overall wall and floor conductances are calculated first. These were determined from a two-dimensional finite element analysis. The numerical values were correlated in terms of the governing nondimensional parameters. These conductance values are then combined with a representative ground temperature based on local ambient temperature profiles. As reported by Yard, the results of this method compared well with those obtained using the method given in the ASHRAE Handbook of Fundamentals 1981, and those obtained from the Mitalas's method. This method can accept a wide range of soil thermal properties, basement temperatures and insulation values. It also accommodates variable width to depth ratios but does not allow partially insulated walls or floors. Since this method uses air temperatures it may not be suitable for Canadian sites which have winter snow cover.

The work done by MacDonald et al [21] involves the comparison of seven basement heat loss calculation methods. The seven methods were applied to a prototypical basement in five U.S. cities to compare predicted annual energy usage. Heat losses were calculated for R-1.5, R-6.5 and R-16.5 walls. The sensitivity of each method to basement depth, insulation R-value, and soil conductivity was also investigated. It was found that predicted annual energy loss varied by more than a factor of two in spite of claims of instrumented field verification for most methods. When
the wall loss was considered separately, the agreement was much better than when floor losses were included. The parametric studies performed pointed to the differences in the computer models used to generate the predictions. It was concluded that at the time of the investigation there was no standard against which to measure the accuracy of any basement heat loss method. The seven methods studied consist of two variations on the Latta and Boileau method, the Akridge method, the Mitalas method, the Shipp method, the Swinton method, and the Yard method. It was also observed that a generalized method applicable to a wide range of locations must allow input of a complete set of soil thermal properties and use ground temperature data.

1.4 OBJECTIVES

It is seen from the literature survey that currently there is no method which uses ground temperature as input and is capable of accommodating a wide range of soil properties, basement temperatures, insulation values and basement depth.

The objective of this study is to develop a simplified method for calculating basement heat losses for Canadian sites for use in retrofit studies. The method should be compatible with microcomputers and calculators, and accept the following input parameters:

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
1. Soil thermal properties: soil density, conductivity and specific heat.
2. Basement indoor air temperature.
3. Average annual ground surface temperature, amplitude of the sinusoidally varying ground surface temperature and the day of occurrence of the minimum temperature.
4. Basement depth below grade.
5. Basement wall insulation resistance value.
Chapter II

THE HEATING5 PROGRAM

2.1 DESCRIPTION OF THE PROGRAM

HEATING5, a modification of the generalized heat conduction code HEATING3, was developed by W.D. Turner, D.C. Elrod and I.I. Siman-Tov. It is designed to solve steady-state and/or transient heat conduction problems in one-, two-, and three-dimensional Cartesian or cylindrical coordinates or one-dimensional spherical coordinates. The thermal conductivity, density, and specific heat may be both spatially and temperature dependent. The thermal conductivity may be anisotropic. Materials may undergo a change of phase. Heat generation rates may be dependent on time, temperature and position, and boundary temperatures may be time-dependent. The boundary conditions, which may be surface-to-boundary or surface-to-surface, may be fixed temperatures or any combination of prescribed heat flux, forced convection, natural convection, and radiation. The boundary condition parameters may be time- and/or temperature-dependent. The mesh spacing can be variable along each axis.

The point successive overrelaxation iterative method and a modification of the "Aitken extrapolation process" are
used to solve the finite difference equations which approximate the partial differential equations for a steady-state problem. The transient problem may be solved using any one of several finite difference schemes. These include an implicit technique which can range from Crank Nicolson to the Classical Implicit Procedure, an explicit method which is stable for any time step size, and the Classical Explicit Procedure which involves the first forward time difference. The solution of the system of equations arising from the implicit technique is accomplished by point successive overrelaxation iteration, and includes procedures to estimate the optimum "acceleration" parameter. The time step size for implicit transient calculations may be varied as a function of the maximum temperature change at a node.

Analytical and tabular functions have been included to aid in the definition of input parameters. An option allows the user to write his own subroutines to evaluate many of the input parameters. Thus, if an input parameter cannot be described with the built-in analytical or tabular functions, then the user may easily supply his own algorithm to evaluate the parameter. Appendix A describes HEATING5 in greater detail. Input preparation is described in Appendix B. A detailed description of the card input is given in Appendix C. Appendix D contains a sample input with the necessary JCL cards.
It can be seen that HEATING5 accommodates a wide variety of boundary conditions and input parameters. The complex situation of earth-coupled heat transfer can easily be handled by HEATING5 without making any simplifications with respect to time-varying boundary conditions, as well as time- and space-varying soil properties. The implicit solution technique allows the simulation of the basement over a full year or more using large time steps without fear of loss of numerical stability. These important features of HEATING5 led to its choice for use in this research work.

2.2 DESCRIPTION OF THE COMPUTER MODEL

2.2.1 MODEL ASSUMPTIONS

Several initial simplifying assumptions are necessary in order to develop a manageable yet reasonably accurate model. Only two dimensional heat flow is considered. Changes in moisture content and heat transfer by convection and liquid migration in the soil are not accounted for. Eckert and Pfender [34] observed that moisture transport plays an insignificant role in the total soil heat flux within the range of soil temperatures surrounding occupied earth-sheltered structures. The latent heat of fusion of soil moisture under freezing and thawing conditions and the latent heat of vaporization during surface drying are not considered. All soil/air interface boundary heat transfer is characterized by associated uniform convective film
coefficients, thus excluding radiation and water vapour condensation/evaporation heat exchange. The basement is assumed to be rectangular, symmetric about a centreline, and set in a location sufficiently removed from other ground-based heat sources/sinks to minimize their effects. The model considers a two-dimensional cross-section, illustrated in Figure 1, bounded by a vertical adiabatic centreline and a second vertical adiabatic boundary sufficiently removed (4 m away from the basement wall) to minimize its impact on the heat loss. The lower thermal boundary consists of a horizontal isothermal sink, typically set at the deep ground water temperature. The outdoor air and basement indoor air temperatures, coupled with their associated convective film coefficients, are the final two boundary conditions.

2.2.2 Model Operation

Fourier curve fits of temperatures were used to describe the annual outdoor air and basement indoor air temperatures. The transient simulation model involves the use of both the steady-state and transient models. The steady-state model (initialized at some constant temperature) is used to obtain a temperature distribution, using the boundary conditions and soil properties defined at the start of the analysis period. This steady-state temperature distribution is used as the initial temperature distribution for the transient model. The transient simulation is conducted over
Fig. 1: Basement model showing the boundary conditions.
successive one-year time periods. The temperature distributions are compared after each year to determine if they have reached a quasi-steady-state. When two successive sets of results are within an acceptable error bound, the simulation is stopped and heat losses are calculated based on the latest temperature distributions. The time step is then shortened and the mesh refined by decreasing the grid spacing and the procedure repeated until no significant change occurs in the results.

2.3 VALIDATION

Any numerical scheme requires thorough validation before confidence can be obtained in the solution. For the validation of the approximate model, experimental data acquired by McBride et al [4] from an instrumented basement in Columbus, Ohio has been used. Validation has also been done using a test basement that was instrumented and maintained by the DBR/NRC in Ottawa.

2.3.1 VALIDATION USING McBRIDE'S BASEMENT

An unoccupied residence in Columbus, Ohio was instrumented and measurements were taken for a full year. Temperatures at two locations in the soil, the outside surface of the basement wall, ambient air, and the interior basement air were recorded daily at 4:00 p.m. A complete description of the residence and instrumentation is given in [4]. Now, an attempt has been made to reproduce McBride's
measured and calculated (from a finite difference program) results by using his data in HEATING5. Table 1 gives the building and soil properties described by McBride. The basement wall (concrete block) and floor (poured concrete) thermal properties have been extracted directly from ASHRAE (1972), and the soil thermal properties extracted from Smith [35] as midrange values for the soils prevalent in Columbus. Table 2 presents the convective film coefficients, which have also been taken from ASHRAE (1977). The soil/ambient outdoor air convection term is varied to account for seasonal changes, as recommended by ASHRAE. The interior wall and floor convection coefficients are those recommended by ASHRAE for design heat loss calculations.

The forcing functions for the simulation model are the deep ground water temperature, the dry-bulb basement and ambient air temperatures. The deep ground water temperature was reported as 11.67°C. A third order Fourier curve fit of both the ambient and basement air temperatures was used. Monthly mean values from McBride's report were used for the Fourier curve fits.

A time step of 5 days was initially used and the transient model was run for 6 years. The maximum temperature difference between the fourth and fifth year temperature distributions was found to be about 1%. The time step was reduced to 3 days and the mesh spacing near the wall and floor was reduced to 15 cm from 20 cm. No appreciable
### TABLE 1: BUILDING AND SOIL PROPERTIES

<table>
<thead>
<tr>
<th>Element</th>
<th>Density $\text{kg/m}^3$</th>
<th>Specific Heat $\text{kJ/kg-} ^\circ \text{C}$</th>
<th>Thermal Conductivity $\text{W/m-} ^\circ \text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basement Wall</td>
<td>977</td>
<td>0.84</td>
<td>1.16</td>
</tr>
<tr>
<td>Basement Floor</td>
<td>2243</td>
<td>0.84</td>
<td>1.73</td>
</tr>
<tr>
<td>Soil (Nov-Mar)</td>
<td>1922</td>
<td>1.67</td>
<td>1.99</td>
</tr>
<tr>
<td>Soil (Apr)</td>
<td>1922</td>
<td>1.67</td>
<td>1.38</td>
</tr>
<tr>
<td>Soil (May-Sep)</td>
<td>1922</td>
<td>1.67</td>
<td>1.21</td>
</tr>
<tr>
<td>Soil (Oct)</td>
<td>1922</td>
<td>1.67</td>
<td>1.38</td>
</tr>
</tbody>
</table>

### TABLE 2: SURFACE CONVECTION FILM COEFFICIENTS

<table>
<thead>
<tr>
<th>Boundary</th>
<th>Convective Coefficient $\text{W/m}^2- ^\circ \text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil/Ambient Air</td>
<td></td>
</tr>
<tr>
<td>Nov-Mar</td>
<td>34.07</td>
</tr>
<tr>
<td>Apr</td>
<td>28.39</td>
</tr>
<tr>
<td>May-Sep</td>
<td>22.71</td>
</tr>
<tr>
<td>Oct</td>
<td>28.39</td>
</tr>
<tr>
<td>Building Interior</td>
<td></td>
</tr>
<tr>
<td>Floor</td>
<td>6.13</td>
</tr>
<tr>
<td>Wall</td>
<td>8.29</td>
</tr>
</tbody>
</table>
changes in the distributions were noticed. The results of the fourth year for the 5-day time step simulation were used for comparison with McBride's results. Tables 3 to 7 contain the comparisons.

The tables show that HEATING5 results compare well with McBride's measured and simulated results. There are some discrepancies for the wall heat loss values. These can be attributed to the error in extracting weekly temperatures from monthly mean values. The ground temperatures at the basement floor and earth interface show very good agreement. Considering the assumptions made for the temperatures because of lack of sufficient documentation, the results indicate that HEATING5 is quite capable of simulating basements adequately. It should be pointed out that the soil thermal conductivity used may not closely represent the actual values at the site. The good agreement may be somewhat illusory.

2.3.2 Validation Using DBR/NRC Basement in Ottawa

During a three-year period (Sept. 1979 to Sept. 1981), heat loss experiments were conducted on several basements by the Division of Building Research. One of these basements is a test basement located on the NRC grounds in Ottawa. Both calorimeter and total basement heat loss measurements were recorded for the three-year period. A drainage ditch was dug around the test site in the summer of 1979 to lower the water table uniformly and to reduce the water flow at the
### TABLE 3:
GROUND TEMPERATURES ADJACENT TO THE BASEMENT WALL

<table>
<thead>
<tr>
<th>Month</th>
<th>McBride Measured C</th>
<th>McBride Simulated C</th>
<th>HEATINGS Simulated C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>13.6</td>
<td>13.9</td>
<td>13.5</td>
</tr>
<tr>
<td>Feb</td>
<td>13.5</td>
<td>13.6</td>
<td>13.3</td>
</tr>
<tr>
<td>Mar</td>
<td>14.3</td>
<td>14.4</td>
<td>14.3</td>
</tr>
<tr>
<td>Apr</td>
<td>17.3</td>
<td>17.9</td>
<td>17.6</td>
</tr>
<tr>
<td>May</td>
<td>21.2</td>
<td>21.8</td>
<td>20.4</td>
</tr>
<tr>
<td>Jun</td>
<td>22.4</td>
<td>22.7</td>
<td>22.5</td>
</tr>
<tr>
<td>Jul</td>
<td>24.4</td>
<td>25.3</td>
<td>24.3</td>
</tr>
<tr>
<td>Aug</td>
<td>24.3</td>
<td>24.4</td>
<td>24.5</td>
</tr>
<tr>
<td>Sep</td>
<td>22.9</td>
<td>22.9</td>
<td>22.3</td>
</tr>
<tr>
<td>Oct</td>
<td>19.4</td>
<td>19.3</td>
<td>19.5</td>
</tr>
<tr>
<td>Nov</td>
<td>18.1</td>
<td>18.1</td>
<td>17.2</td>
</tr>
<tr>
<td>Dec</td>
<td>15.9</td>
<td>15.6</td>
<td>15.8</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
**TABLE 4:**
GROUND TEMPERATURE HORIZONTALLY 0.91 m FROM WALL AND VERTICALLY 0.61 m FROM THE EARTH SURFACE

<table>
<thead>
<tr>
<th>Month</th>
<th>McBride Measured °C</th>
<th>McBride Simulated °C</th>
<th>HEATING5 Simulated °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>6.6</td>
<td>7.8</td>
<td>6.0</td>
</tr>
<tr>
<td>Feb</td>
<td>7.1</td>
<td>7.4</td>
<td>6.2</td>
</tr>
<tr>
<td>Mar</td>
<td>8.2</td>
<td>8.7</td>
<td>7.9</td>
</tr>
<tr>
<td>Apr</td>
<td>12.2</td>
<td>11.6</td>
<td>11.6</td>
</tr>
<tr>
<td>May</td>
<td>19.3</td>
<td>18.3</td>
<td>17.5</td>
</tr>
<tr>
<td>Jun</td>
<td>22.5</td>
<td>21.3</td>
<td>22.7</td>
</tr>
<tr>
<td>Jul</td>
<td>24.4</td>
<td>23.6</td>
<td>24.9</td>
</tr>
<tr>
<td>Aug</td>
<td>24.4</td>
<td>23.1</td>
<td>24.3</td>
</tr>
<tr>
<td>Sep</td>
<td>21.3</td>
<td>20.2</td>
<td>22.4</td>
</tr>
<tr>
<td>Oct</td>
<td>16.3</td>
<td>15.6</td>
<td>18.6</td>
</tr>
<tr>
<td>Nov</td>
<td>13.3</td>
<td>13.6</td>
<td>12.9</td>
</tr>
<tr>
<td>Dec</td>
<td>8.1</td>
<td>8.4</td>
<td>8.0</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
### TABLE 5:

**GROUND TEMPERATURES AT THE BASEMENT FLOOR AND EARTH INTERFACE**

<table>
<thead>
<tr>
<th>Month</th>
<th>McBride Simulated $^\circ\text{C}$</th>
<th>HEATING5 Simulated $^\circ\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>18.1</td>
<td>18.8</td>
</tr>
<tr>
<td>Feb</td>
<td>17.8</td>
<td>17.6</td>
</tr>
<tr>
<td>Mar</td>
<td>18.3</td>
<td>18.4</td>
</tr>
<tr>
<td>Apr</td>
<td>21.1</td>
<td>20.8</td>
</tr>
<tr>
<td>May</td>
<td>23.3</td>
<td>22.3</td>
</tr>
<tr>
<td>Jun</td>
<td>23.2</td>
<td>23.2</td>
</tr>
<tr>
<td>Jul</td>
<td>25.7</td>
<td>24.6</td>
</tr>
<tr>
<td>Aug</td>
<td>24.9</td>
<td>25.1</td>
</tr>
<tr>
<td>Sep</td>
<td>24.0</td>
<td>23.5</td>
</tr>
<tr>
<td>Oct</td>
<td>21.2</td>
<td>21.5</td>
</tr>
<tr>
<td>Nov</td>
<td>21.0</td>
<td>20.8</td>
</tr>
<tr>
<td>Dec</td>
<td>20.4</td>
<td>20.4</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>Month</th>
<th>McBride Simulated</th>
<th>McBride Measured</th>
<th>HEATING5 Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>16.03</td>
<td>14.89</td>
<td>18.15</td>
</tr>
<tr>
<td>Feb</td>
<td>17.13</td>
<td>16.75</td>
<td>16.47</td>
</tr>
<tr>
<td>Mar</td>
<td>17.51</td>
<td>16.94</td>
<td>17.40</td>
</tr>
<tr>
<td>Apr</td>
<td>17.32</td>
<td>15.08</td>
<td>12.99</td>
</tr>
<tr>
<td>May</td>
<td>11.29</td>
<td>9.24</td>
<td>6.96</td>
</tr>
<tr>
<td>Jun</td>
<td>5.46</td>
<td>4.32</td>
<td>2.03</td>
</tr>
<tr>
<td>Jul</td>
<td>8.42</td>
<td>5.46</td>
<td>1.80</td>
</tr>
<tr>
<td>Aug</td>
<td>5.08</td>
<td>4.51</td>
<td>2.20</td>
</tr>
<tr>
<td>Sep</td>
<td>4.13</td>
<td>4.13</td>
<td>0.98</td>
</tr>
<tr>
<td>Oct</td>
<td>5.08</td>
<td>5.65</td>
<td>3.19</td>
</tr>
<tr>
<td>Nov</td>
<td>11.86</td>
<td>11.86</td>
<td>12.70</td>
</tr>
<tr>
<td>Dec</td>
<td>17.70</td>
<td>18.64</td>
<td>18.85</td>
</tr>
<tr>
<td>Average</td>
<td>11.42</td>
<td>10.63</td>
<td>9.47</td>
</tr>
</tbody>
</table>
TABLE 7:
MONTHLY AVERAGE FLOOR HEAT LOSSES (W/m²)

<table>
<thead>
<tr>
<th>Month</th>
<th>McBride Simulated</th>
<th>HEATING5 Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.73</td>
<td>0.00</td>
</tr>
<tr>
<td>Feb</td>
<td>3.19</td>
<td>0.58</td>
</tr>
<tr>
<td>Mar</td>
<td>5.17</td>
<td>4.24</td>
</tr>
<tr>
<td>Apr</td>
<td>5.90</td>
<td>5.53</td>
</tr>
<tr>
<td>May</td>
<td>5.43</td>
<td>5.45</td>
</tr>
<tr>
<td>Jun</td>
<td>3.44</td>
<td>5.27</td>
</tr>
<tr>
<td>Jul</td>
<td>5.43</td>
<td>5.80</td>
</tr>
<tr>
<td>Aug</td>
<td>3.94</td>
<td>4.15</td>
</tr>
<tr>
<td>Sep</td>
<td>0.50</td>
<td>0.86</td>
</tr>
<tr>
<td>Oct</td>
<td>-1.23</td>
<td>-0.31</td>
</tr>
<tr>
<td>Nov</td>
<td>2.46</td>
<td>1.38</td>
</tr>
<tr>
<td>Dec</td>
<td>3.19</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
footings to a more "normal" rate. The groundwater table after construction of the drainage ditch was about 0.5 m below the basement floor surface. The basement was constructed in an area of Leda clay. The wall had full insulation on the inside and the floor was uninsulated. A complete description of the basement is given in [12]. The basement space temperature was kept at 21 C. A sinusoidal variation of the ground surface temperature taken from Mitalas's report [12] has been used. The thermal and physical properties of the soil, insulation and wall and floor construction are given in Table 8. The deep ground isothermal level was set 9 m below the basement floor at the mean annual ground surface temperature. The simulation model used, along with the assumptions and the boundary conditions are similar to the one used for the McBride's case with the following exceptions. Ground surface temperatures are used instead of air temperatures. Since the water table was reported to be 0.5 m below the basement floor, two soil conductivities were used in the model: a value of 0.88 W/m-K adjacent to the wall and up to 0.5 below the basement floor, and a value of 1.33 W/m-K from 0.5 m below the floor to the deep ground water level, representing the water table. The latter value of soil conductivity is for saturated Leda clay and has been taken from [21]. Figure 2 shows the model used for the basement. The results from HEATINGS5 have been compared to the values calculated using
<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity W/(m.K)</th>
<th>Density kg/m³</th>
<th>Specific Heat kJ/(kg.K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.73</td>
<td>2243</td>
<td>0.84</td>
</tr>
<tr>
<td>Glass Fibre Insulation</td>
<td>0.0433</td>
<td>32</td>
<td>0.84</td>
</tr>
<tr>
<td>Soil: Leda Clay</td>
<td>0.88 (Average) 1490 @ 42% moisture</td>
<td>Volumetric specific heat = 2.63 MJ/(m³.K)</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2: Basement model for the DBR/NRC basement.
the method outlined by Mitalas. The heat loss values measured by Mitalas have been given in the form of a graph in [12]. The wall losses calculated by Mitalas's method compared very well with the measured values but the floor losses were grossly underpredicted by Mitalas. He attributed this to the groundwater effects. The heat loss values calculated using HEATING5 are presented in Table 9. The wall heat losses agree very well with Mitalas's values. The floor losses are much lower. This could be due to the groundwater effects reported by Mitalas. If groundwater flow exists below the basement floor, as speculated by Mitalas for the basement studied, the floor losses would be substantially more than without any flow.

The above validations involve the reproduction of results for instrumented basements using the information provided in the literature by the persons who conducted the investigations. HEATING5 has been used to reproduce the results. The results indicate that HEATING5 is capable of simulating basement heat loss studies with sufficient engineering accuracy, provided that suitable boundary conditions and property values are used in the simulation.
**TABLE 9:**
MONTHLY AVERAGE HEAT LOSSES (W/m²)

<table>
<thead>
<tr>
<th>Month</th>
<th>A₂ (Top 0.6m)</th>
<th>A₃ (Rest of wall)</th>
<th>A₄ (Floor 1m corner)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>234 (207)</td>
<td>262 (232)</td>
<td>171 (274)</td>
</tr>
<tr>
<td>Feb</td>
<td>226 (194)</td>
<td>270 (244)</td>
<td>231 (299)</td>
</tr>
<tr>
<td>Mar</td>
<td>189 (160)</td>
<td>255 (232)</td>
<td>186 (309)</td>
</tr>
<tr>
<td>Apr</td>
<td>137 (113)</td>
<td>216 (192)</td>
<td>179 (299)</td>
</tr>
<tr>
<td>May</td>
<td>78 (66)</td>
<td>158 (151)</td>
<td>162 (274)</td>
</tr>
<tr>
<td>Jun</td>
<td>35 (32)</td>
<td>109 (104)</td>
<td>143 (238)</td>
</tr>
<tr>
<td>Jul</td>
<td>13 (19)</td>
<td>70 (70)</td>
<td>123 (203)</td>
</tr>
<tr>
<td>Aug</td>
<td>20 (32)</td>
<td>60 (58)</td>
<td>110 (177)</td>
</tr>
<tr>
<td>Sep</td>
<td>53 (66)</td>
<td>77 (70)</td>
<td>106 (167)</td>
</tr>
<tr>
<td>Oct</td>
<td>108 (113)</td>
<td>120 (104)</td>
<td>114 (177)</td>
</tr>
<tr>
<td>Nov</td>
<td>153 (160)</td>
<td>172 (151)</td>
<td>130 (203)</td>
</tr>
<tr>
<td>Dec</td>
<td>212 (194)</td>
<td>227 (198)</td>
<td>152 (238)</td>
</tr>
</tbody>
</table>

NOTE: Values in parentheses are those calculated by Mitalas's method.
Chapter III

DEVELOPMENT OF THE NEW METHOD

3.1 INTRODUCTION

The total basement heat loss in residential basements is comprised of above-grade and below-grade portions. The heat loss through both the components is governed mainly by the laws of conduction and convection, temperature difference, resistance and exposed area, but the application of these laws to analyze below-grade heat loss is made extremely difficult by the lack of uniformity and stability of conditions surrounding the walls and floor. The layers of insulation, concrete, adjacent backfill, undisturbed soil, all have different heat conduction properties which give rise to potentially complex two- and three-dimensional heat flow from the basement to the ground surface or to the deep groundwater surface. Also the significant mass of soil in the path of the heat flow introduces a thermal storage effect. The moisture in the soil provides an additional heat storage mechanism via the formation of ground frost, and a heat transfer mechanism because of the moisture transport. All these factors are important in determining the heat losses from the basement walls and floor. Some of the factors are difficult to quantify and others are difficult
to measure accurately. This has made mathematical and computer modelling very difficult.

However, extensive research has been conducted on basement heat loss studies. Finite difference computer simulations indicate that a two-dimensional analysis with a few simplifying assumptions produces an acceptable solution to the problem. Since these simulations require extensive computational facilities, often involving high first costs, recent interest in this area has centred on the development of simplified calculation methods to aid the design engineer.

Previous studies have shown that the basement can be thought of as a linear thermal system excited by a sinusoidally varying ground surface temperature [12]. The soil thermal capacitance causes the soil temperature to lag behind the ground surface temperature, with the time lag increasing with the depth below the ground surface. The soil temperature still follows a sinusoidal pattern, but the amplitude of variation decreases with increasing depth, so that, at a sufficiently deep level, the soil temperature reaches a constant value which is very nearly equal to the mean annual ground temperature. This level of constant earth temperature has been found to be about 7 to 10 m below the ground surface [29].
3.2 PRELIMINARY STUDIES

Some preliminary studies were made using the finite difference model of a hypothetical basement to get a clear picture of the heat loss characteristics of a typical basement. The following assumptions were incorporated into the model:

1. Soil thermal properties were taken to be constant with respect to both time and space.
2. Uniform and constant wall and floor convective heat transfer coefficients were taken from ASHRAE 1985 for the basement interior.
3. Wall construction was taken to be 20 cm poured concrete.
4. Floor construction was taken to be 10 cm poured concrete. A 10 cm gravel footing under the floor was included. Thermal properties for gravel were taken from ASHRAE 1985.
5. The basement interior air temperature was taken to be constant throughout the year.
6. Ground surface temperatures were used and they were taken to be sinusoidally varying. The variation was described by only the first harmonic, because the second harmonic is small compared to the first harmonic for most Canadian sites [12].
7. The deep ground isothermal plane was set 10 m below the ground surface at the mean annual ground temperature.
3. The effects of freezing and thawing, moisture migration and ground surface conditions like paving, etc. were neglected.

As described in the previous chapter, the steady-state and transient simulations are carried out until a quasi-steady-state has been reached. Figures 3 to 6 illustrate the inside basement wall and floor temperatures and heat fluxes as a function of time for various depths along the wall and different distances from the floor corner for the floor. It can be seen that the temperatures, and consequently the heat fluxes, vary sinusoidally. The decrement in the amplitudes for both the wall and floor temperatures and heat fluxes is quite apparent. Figures 7 and 8 show the time lags for the wall and floor. The time lags are with respect to the ground surface temperature. The time lag has been calculated by noting the time at which the temperature curve passed through its mean value. For example, if the ground surface temperature curve passed through its mean value at time "a", and the wall surface temperature curve passed its mean value at time "b", the time lag is given by "b-a". It can be observed that the lag varies from a few days for the upper portions of the wall to a few months to the centre portions of the floor.

The basement half-width was increased to 5 m from 3 m and a very shallow basement, 1.5 m deep, was simulated with this half-width. It was observed that beyond 3 m from the corner
Fig. 3: Inside surface temperature of the wall for various depths below the ground surface.

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Fig. 4: Inside surface temperature of the floor for various distances from the corner.
Fig. 5: Heat flux for various points on the inside wall surface that are at different depths below the ground surface.
Fig. 6: Heat flux variation for points on the inside floor surface at various distances from the corner.
Fig. 7: Time lag of the wall heat flux for various depths below the ground surface.
Fig. 8: Time lag of the floor heat flux for various points on the inside surface of the floor.
of the floor, the annual variation of the floor heat flux was negligible. In fact the major portion of the variation of the floor heat loss is provided by the corner portion of the floor.

The ground surface temperature consists of the mean annual value and the amplitude of the first harmonic. When the mean annual value of the ground surface temperature was used as the boundary condition and a steady-state simulation was carried out, the steady-state heat fluxes for the wall and the floor were found to be very close to the mean annual values obtained from the transient simulation.

From the above observations, the following important conclusions can be drawn:

1. If a sinusoidal variation of the ground surface temperature is assumed, then the wall and floor heat losses are also sinusoidal variations over the annual cycle, with both a lag and an amplitude decrement. The mean annual heat loss values and the amplitudes of the heat losses depend on the soil properties, depth of the basement and the temperature of the ground and the basement interior. The lags depend on the depth of the basement and the soil properties.

2. The first three metres from the corner of the floor affect the annual variation of the floor losses. The loss from the rest of the floor remains essentially constant throughout the year.
3. The total annual basement heat loss can be broken into two components — a steady-state part and a transient part. The transient part is caused by the first harmonic of the ground surface temperature. The steady-state part is derived by imposing the mean annual ground surface temperature on the ground surface and performing a steady-state simulation.

3.3 DEVELOPMENT OF THE METHOD

The ground surface temperature can be written as:

\[ T_G = T_M + T_A \sin \omega(t + \psi) \]  \hspace{1cm} (3.1)

where

- \( T_M \) = mean annual ground temperature
- \( T_A \) = amplitude of the first harmonic of the ground surface temperature
- \( \omega \) = angular frequency
- \( t \) = time, \( t = 0 \) for 0:00 AM, Jan. 1
- \( \psi \) = phase

The phase \( \psi \) depends on the date of the occurrence of the minimum ground surface temperature, and has a value such that for the value of \( t \) corresponding to the date of minimum temperature, the sine term has a value of -1.0. The surface temperature \( T_x \), of the wall or floor of the basement at any specified location \( x \) along the wall or floor is then given by

\[ T_x = T_m + T_a \sin (\omega(t + \psi) - \delta_x) \]  \hspace{1cm} (3.2)
where

\[ T_m = \text{mean wall or floor surface temperature at that location} \]
\[ T_a = \text{amplitude of the wall or floor surface temperature at that location } \]
\[ \delta_x = \text{lag at that location, with respect to the ground surface temperature} \]

If the heat flux at that location is calculated as the convective heat transfer coefficient multiplied by the difference of the basement air temperature and the wall or floor surface temperature at that location, then the heat flux \( q_x \), is given by

\[ q_x = q_m + q_a \sin (\omega(t + \psi) - \delta_x + \pi) \quad \text{[3.3]} \]

where

\[ q_m = \text{mean annual heat flux} \]
\[ q_a = \text{amplitude of the heat flux} \]

Figure 9 shows the ground surface temperature variation. It also shows the wall surface temperature and the wall heat flux at any depth \( x \). The wall surface temperature lags the ground surface temperature by \( \delta_x \). The heat flux is 190 degrees or \( \pi \) radians out of phase with the wall surface temperature. This means that the maximum heat flux occurs when the wall surface temperature is minimum.

To predict the average heat flux from the wall, the values of three parameters are required - the wall average mean annual heat flux \( q_m \), the amplitude of the heat flux \( q_a \)
Fig. 9: Ground surface temperature, wall surface temperature, and wall heat flux variation.
, and the lag \( \delta \). Similarly, the values of three parameters are required to predict the annual heat flux variation of the floor. The terms "average" and "mean" in the subsequent sections have the following meanings. The former term for the wall means the value is averaged over the depth of the wall. For the floor it stands for the value averaged over the corner 3 m of the floor. The latter term is used to describe the value averaged over one complete year. The knowledge of the phase \( \psi \) is very important since it determines the day of maximum heat loss. This means that specific dates can be assigned to the heat loss values. For a particular location the phase difference can be found from meteorological records of ground surface temperatures if they are available.

An attempt has been made to obtain empirical curve fits for the three parameters in terms of the important factors affecting basement heat losses, for the basement wall and floor separately. The factors affecting basement heat loss are: the basement depth, the soil thermal and physical properties, the insulation resistance (if any), the basement indoor air temperature and the ground surface temperature.

HEATING5 was used to simulate a typical basement (described above in section 3.2, Preliminary Studies), with different depths, soil properties, ground surface temperatures and wall insulation levels. The section of the below grade basement only was considered. Uninsulated walls
and full wall insulation on the inside were treated. Floor insulation was not included in the study. Curve fits were obtained for uninsulated and insulated cases separately. The functional correlations for each case are described in the following sections.

An attempt was made to determine shape factors in the steady state cases for the wall and the floor. The attempt was successful in the case of the wall but it failed for the floor.

### 3.3.1 STEADY STATE (UNINSULATED WALL)

The resistance to the wall heat loss, occurring between the basement air temperature and the ground surface temperature, consists of the wall resistance, the inside convection resistance and the soil resistance. Since the wall construction is unchanged in the typical basement model, the soil resistance depends on the soil conductivity and the heat flow path of the average heat loss, which in turn is governed by the depth of the basement. This leads to the following relation:

\[
\frac{q_w}{\Delta T} = \frac{1}{(R_w + R_s)} \tag{3.4}
\]

where

- \( q_w \) = average mean annual wall heat flux
- \( \Delta T \) = \( T_B - T_M \)
- \( T_B \) = basement air temperature
- \( T_M \) = mean annual ground surface temperature
\[ R_W = \text{resistance of the wall (including the film coefficient)} \]

\[ R_S = \text{soil resistance} \]

and

\[ R_S = F_1 (L, 1/K) \quad \ldots \quad (3.5) \]

where

\[ L = \text{depth of the basement} \]

\[ K = \text{soil conductivity} \]

Eqn. (3.4) is based on the assumption that the inside surface of the basement wall is at a uniform temperature. For the floor, the same form of functional relation can be expected, because the floor construction is unchanged too. A portion of the heat loss from the floor corner is directed towards the ground surface and the rest flows to the deep ground isothermal plane set at \( T_M \). Thus,

\[ \frac{q_f}{\Delta T} = \frac{1}{(R_f + R_S)} \quad \ldots \quad (3.6) \]

where

\[ q_f = \text{average mean annual floor heat flux} \]

and

\[ R_S = F_2 (L, 1/K) \quad \ldots \quad (3.7) \]

Eqn. (3.6) is based on the assumption that the inside surface of the basement floor is at a uniform temperature.
The floor centre portion heat loss is directed towards the deep groundwater surface. The functional relation is given by

\[ \frac{q_{f,c}}{\Delta T} = \frac{1}{(R_f + R_s)} \]  \(3.9\)

where \(R_s = F_3(L, 1/K)\)

\(q_{f,c}\) = average steady state heat flux of the floor centre

3.3.2 STEADY STATE (INSULATED WALL)

The resistance to the heat flow from the wall now consists of the additional insulation resistance. The major portion of the wall heat loss is towards the ground surface and the corner loss flows to the deep groundwater level. So the following relation can be written:

\[ \frac{q_{w,i}}{\Delta T} = \frac{1}{(R_t + R_s)} \]  \(3.9\)

where \(q_{w,i}\) = average mean annual wall heat flux for an insulated wall

\(R_t\) = total wall resistance, including the inside film coefficient and the insulation.

\(R_s\) = soil resistance

Eqn. (3.9) is again based on the assumption that the inside surface of the basement wall is at a uniform temperature. Now the soil resistance is itself a function
of the wall resistance because the inclusion of wall resistance and its value directly influence the wall heat flow path. Thus,

\[ R_s = F_4 (L, 1/K, R_I) \]  \hspace{1cm} ... (3.10)

For the floor, the heat flow is still principally towards the deep ground surface, except for the corner. The wall resistance does not directly form a part of the resistance to the floor heat flow, but nonetheless influences it. As the wall insulation level is increased, the wall heat loss decreases and this lowers the temperatures surrounding the wall and the floor corner portion. This leads to an increase in the heat loss from the corner portion of the floor and consequently in the average floor heat loss. So, the following relation can be written:

\[ \frac{q_{f,i}}{\Delta t} = F_5 (1/L, K, R_I) \]  \hspace{1cm} ... (3.11)

where

\[ q_{f,i} = \text{average mean annual floor heat flux with the wall insulated} \]

\[ R_I = \text{resistance of wall insulation only} \]

The same reasoning applies to the heat loss from the centre of the floor and so

\[ \frac{q_{f,c,i}}{\Delta t} = F_6 (1/L, K, R_I) \]  \hspace{1cm} ... (3.12)
where
\[ q_{f,c,t} = \text{average steady state heat flux of the floor centre} \]

### 3.3.3 TRANSIENT (UNINSULATED WALL)

As previously noted, the transient description of the heat loss consists of the amplitude of the heat loss and the lag with respect to the ground surface temperature. For the wall, the amplitude of the average heat loss depends on the basement depth, the soil properties and the ground surface temperature amplitude. The following functional relation is expected:

\[ \frac{q_{w,a}}{T_A} = F(1/L, \alpha) \quad ... (3.13) \]

where
\[ q_{w,a} = \text{amplitude of the average wall heat flux} \]
\[ T_A = \text{amplitude of the ground surface temperature} \]
\[ \alpha = \text{soil diffusivity} \]

For the floor, the influencing variables remain the same and thus:

\[ \frac{q_{f,a}}{T_A} = F_8(1/L, \alpha) \quad ... (3.14) \]

where
\[ q_{f,a} = \text{amplitude of the average floor heat flux} \]
The lag of the average heat flux with respect to the ground surface temperature depends on the basement depth and the soil properties. The functional relation for the wall heat flux lag is of the form

\[ \delta_w = F_9(L, 1/\alpha) \]  \hspace{1cm} (3.15)

where

\[ \delta_w = \text{lag of the average wall heat flux} \]

A similar relation exists for the floor heat flux lag

\[ \delta_f = F_{10}(L, 1/\alpha) \]  \hspace{1cm} (3.16)

where

\[ \delta_f = \text{lag of the average floor heat flux} \]

3.3.4 TRANSIENT (INSULATED WALL)

The addition of insulation to the basement wall reduces the amplitude of the wall heat flux and increases that of the floor heat flux. This leads to the following two relations

\[ \frac{q_{w,a,i}}{T_A} = F_{11}(1/L, \alpha, 1/R_I) \]  \hspace{1cm} (3.17)

where

\[ q_{w,a,i} = \text{amplitude of the average wall heat flux} \]

\[ R_I = \text{resistance of the wall insulation only} \]

and

\[ \frac{q_{f,a,i}}{T_A} = F_{12}(1/L, \alpha, R_I) \]  \hspace{1cm} (3.18)
where

\( q_{f,a,i} \) = amplitude of the average floor heat flux.

The wall insulation isolates the basement wall and floor from the ground surface temperature variation and causes an increase in the lag of the heat fluxes of both the wall and the floor. Thus the functional relations for the wall and floor lags are

\[
\delta_{w,i} = F_{13} \left( L, 1/\alpha, R_t \right) \tag{3.19}
\]

where

\( \delta_{w,i} \) = lag of the average wall heat flux

\( R_t \) = resistance of wall insulation only

and

\[
\delta_{f,i} = F_{14} \left( L, 1/\alpha, R_t \right) \tag{3.20}
\]

where

\( \delta_{f,i} \) = lag of the average floor heat flux.

In the simulations using HEATINGS, a five-day time step was used and the minimum temperature was taken to occur on January 15. The transient simulation was carried out for four years and the fourth year temperatures were used to calculate the heat fluxes. After each time step the heat fluxes were determined so that for a one year period there were 73 such values. A Fourier curve fit, with only the first harmonic of the sine and cosine terms, was carried out.
to find the wall and floor heat flux amplitudes and lags. The steady state simulations were used to calculate the mean annual heat fluxes, since as earlier indicated, these two values were found to be very close to one another.

The heat flux is finally of the form

$$q = q_m + q_a \sin \{ \omega(t + \psi) + \pi - \delta \}$$ \hspace{1cm} \text{(3.21)}

where

- $q_m =$ the mean annual heat flux
- $q_a =$ amplitude of the heat flux
- $\omega = 2 \pi / 73$, rad/5-day
- $\psi =$ phase lag

The phase lag has a value of 51.75 5-day. For each functional relation polynomials consisting of the influencing variables were used. A non-linear curve-fitting computer program was used to determine the unknown coefficients in the polynomials. The program, DPENIN, was developed by D. W. Marquardt and was programmed at the University of Waterloo. It computes the least-squares estimates of the unknown coefficients. Appendix E contains a detailed description of DPENIN, along with a sample input.

From the results of the simulations, it was observed that as the wall insulation resistance increased the soil resistance decreased in the steady-state case. This is because the insulation decreases the heat flow path to the ground surface. For the heat flux amplitudes of the wall and floor in the uninsulated case, the soil diffusivity was
first used as one of the influencing variables. The fits were not very good. Instead, when the soil conductivity and volumetric capacity were used as two separate variables the fits improved considerably. The soil diffusivity worked well with the insulated case. For the lags, the soil conductivity and volumetric capacity provided better fits in both the insulated and uninsulated cases. The final correlations obtained are presented in the next chapter. Appendix F contains the data which was used as input to the regression program DFENLN.
Chapter IV

THE FINAL CORRELATIONS

In this chapter the actual correlations and the regression coefficients are presented. The symbols have already been described in the previous chapter. The nonlinear confidence limits (95% limits) are given for each regression coefficient after the units in the following format: (lower limit, upper limit).

4.1 STEADY STATE (UNINSULATED WALL)

Figure 10 shows the dependence of the soil resistance on the basement depth for fixed soil conductivities. The dependence is linear. The y-intercepts are negligible. Therefore

\[ R_s = b L \ (m^2 \cdot K/W) \]  \hspace{1cm} (4.1)

Figure 11 shows the variation of the slope "b" with the soil conductivity and the relation is given by

\[ b = C_1 + C_2 / K \]  \hspace{1cm} (4.2)

where

\[ C_1 = 0.0489 \ m \cdot K/W \]
\[ C_2 = 0.2465 \]

Thus the average mean annual wall heat flux is given by

\[ \frac{q_w}{\Delta T} = \left( \frac{1}{R_w + C_1 L + C_2 L/K} \right) \ (W/m^2 \cdot K) \]  \hspace{1cm} (4.3)

- 68 -
Fig. 10: The variation of the soil resistance with the basement depth for various soil conductivities.
Fig. 11: The variation of the slope of the "Soil resistance versus Basement depth" curve with soil conductivity.
Figure 12 shows the dependence of the soil resistance (to the floor heat loss) on the basement depth for different soil conductivities, for the uninsulated wall case. The relation is given by

\[ R_s = a + bL + CL^2 \text{ (m}^2\text{K/W)} \]  \( \cdots \) (4.4)

Figure 13 shows the relation between \( a \) and \( K \), and \( b \) and \( K \). Figure 14 shows the relation between \( c \) and \( K \). The relations are given by the following equations

\[ a = a_1/K \]  \( \cdots \) (4.5)

\[ b = b_1/K \]  \( \cdots \) (4.6)

and

\[ C = C_1/K \]  \( \cdots \) (4.7)

where

\[ a_1 = 2.31 \text{ m} \]

\[ b_1 = 1.03 \]

\[ C_1 = -0.15 \text{ m}^{-1} \]

Therefore the average mean annual floor heat flux is given by

\[ \frac{q_f}{\Delta T} = \frac{1}{[R_f + (a_1 + b_1L + C_1L^2)/K]} \text{ (W/m}^2\text{K)} \]  \( \cdots \) (4.8)

where \( a \), \( b \), and \( c \) have the values given above.
Fig. 12: The variation of the soil resistance (to the floor heat loss) with the basement depth for various soil conductivities.
Fig. 13: The variation of coefficients "a" and "b" in Eqn. 4.4, with soil conductivity.
Fig. 14: The variation of coefficient "c" in Eqn. 4.4, with soil conductivity.
The average steady state floor heat flux for the centre is given by Eqn. \(4.8\) and the coefficients are

\[
a_1 = 3.6246 \text{ (m)} \\
b_1 = 1.1906 \\
c_1 = -0.2335 \text{ (m}^{-1}\text{)}
\]

### 4.2 STEADY STATE (INSULATED WALL)

The average mean annual wall heat flux is given by

\[
\frac{q_{w1}}{\Delta T} = (R_T + b_1 + b_2L + b_3/R_T + b_4/K + b_5L/R_T + b_6L/K + b_7/KR_T)^{-1} \text{ (W/m}^2\cdot\text{K)}
\]

where

\[
\begin{align*}
b_1 &= -2.9285 \text{ (m}^2\cdot\text{K/W)}; (-3.0248, -2.7936) \\
b_2 &= 0.7868 \text{ (m} \cdot \text{K/W)}; (0.7407, 0.8505) \\
b_3 &= 4.3486 \text{ (m}^4\cdot\text{K}^2/\text{W}^2); (4.1985, 4.5764) \\
b_4 &= 0.7659 \text{ (m)}; (0.6348, 0.9517) \\
b_5 &= -1.1296 \text{ (m}^3\cdot\text{K}^2/\text{W}^2); (-1.1988, -1.0249) \\
b_6 &= 0.4676; (0.4027, 0.5574) \\
b_7 &= -1.5837 \text{ (m}^2\cdot\text{K/W}); (-1.7706, -1.2981)
\end{align*}
\]

The average mean annual floor heat flux is given by

\[
\frac{q_{f1}}{\Delta T} = b_1/L + b_2K + b_3R_i + b_4K/L + b_5R_i/L + b_6KR_i \text{ (W/m}^2\cdot\text{K)}
\]

where

\[
-75-
\]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
\[ \frac{q_{f,c,i}}{\Delta T} = b_1 + \frac{b_2}{L} + \frac{b_3}{K} + b_4 R_i + \frac{b_5}{L} + \frac{b_6 R_i}{K} + \frac{b_7 R_i}{L} + \frac{b_8}{L^2} + b_9 K^2 + b_{10} R_i^2 \text{ (W/m}^2\text{K)} \]
4.3 Transient (Uninsulated Wall)

The amplitude of the average wall heat flux is given by

\[
\frac{q_{WA}}{T_A} = b_1 + b_2/L + b_3K + b_4/\rho C_p + b_5K/L + b_6/L\rho C_p + b_7K/\rho C_p + b_8/L^2 + b_9K^2 \text{ (W/m}^2\cdot\text{K) }
\]  

where

\[
\begin{align*}
b_1 &= -0.4021 \text{ (W/m}^2\cdot\text{K); (-0.4050, -0.3993)} \\
b_2 &= 2.1176 \text{ (W/m} \cdot \text{K); (2.1123, 2.1229)} \\
b_3 &= 0.3453 \text{ (m}^{-1}; (0.3433, 0.3472) \\
b_4 &= 0.1630 \times 10^6 \text{ (W}^2\cdot\text{s/m}^5\cdot\text{K}^2); (0.1555 \times 10^6, 0.1705 \times 10^6) \\
b_5 &= 0.2945 \\
b_6 &= 0.0031 \times 10^6 \text{ (W}^2\cdot\text{s/m}^4\cdot\text{K}^2); (-0.1070 \times 10^5, 0.1689 \times 10^5) \\
b_7 &= 0.0503 \times 10^6 \text{ (W} \cdot \text{s/m}^4\cdot\text{K); (0.4524 \times 10^5, 0.5541 \times 10^5) \\
b_8 &= -0.7423 \text{ (W/K); (-0.7514, -0.7331)} \\
b_9 &= -0.0485 \text{ (K/W); (-0.4956E-01, -0.4756E-01)}
\end{align*}
\]

The amplitude of the average floor heat flux is given by Eqn. (4.12) and the correlation coefficients are

\[
\begin{align*}
b_1 &= 0.0297 \text{ (W/m}^2\cdot\text{K); (0.2952E-01, 0.2984E-01)} \\
b_2 &= -0.0298 \text{ (W/m} \cdot \text{K); (-0.3007E-01, -0.2947E-01)} \\
b_3 &= -0.0594 \text{ (m}^{-1}; (-0.5949E-01, -0.5927E-01) \\
b_4 &= -0.0936 \times 10^6 \text{ (W}^2\cdot\text{s/m}^5\cdot\text{K}^2); (-0.9399 \times 10^5, -0.9314 \times 10^5) \\
b_5 &= 0.0939; (0.9375E-01, 0.9419E-01)
\end{align*}
\]
\[ b_6 = 0.0783 \times 10^6 \, (W^2 \cdot s/m^4 \cdot K^2); \ (0.7752 \times 10^5, \ 0.7907 \times 10^5) \]
\[ b_7 = 0.1012 \times 10^6 \, (W \cdot s/m^4 \cdot K); \ (0.1009 \times 10^6, \ 0.1015 \times 10^6) \]
\[ b_8 = -0.0064 \, (W/K); \ (-0.6977E-02, \ -0.5929E-02) \]
\[ b_9 = 0.0088 \, (K/W); \ (0.8733E-02, \ 0.8847E-02) \]

The phase lag of the average wall heat flux is given by

\[ \delta_w = b_1 + b_2/K + b_3 \rho C_p + b_4 L \, (rad) \quad \cdots (4.13) \]

where

\[ b_1 = 0.7296 \times 10^{-4} \, (rad); \ (-0.2971E-02, \ 0.3117E-02) \]
\[ b_2 = 0.0082 \, (W/m \cdot K); \ (0.4403E-02, \ 0.1201E-01) \]
\[ b_3 = 0.0269 \times 10^{-6} \, (m^3 \cdot K/J); \ (0.2581 \times 10^{-7}, \ 0.2803 \times 10^{-7}) \]
\[ b_4 = 0.0540 \, (m^{-1}); \ (0.5247E-01, \ 0.5547E-01) \]

The phase lag of the average floor heat flux is given by Egn. [4.13] and the correlation coefficients are

\[ b_1 = -0.3706 \, (rad); \ (-0.3944, \ -0.3469) \]
\[ b_2 = 0.3956 \, (W/m \cdot K); \ (0.3659, \ 0.4253) \]
\[ b_3 = 0.1582 \times 10^{-6} \, (m^3 \cdot K/J); \ (0.1495 \times 10^{-6}, \ 0.1668 \times 10^{-3}) \]
\[ b_4 = 0.4477 \, (m^{-1}); \ (0.4360, \ 0.4594) \]

- 78 -

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
The amplitude of the average wall heat flux is given by

\[
\frac{q_{w,a,i}}{A} = b_1 + b_2/L + b_3\alpha + b_4/R_I + b_5\alpha/L^2 + b_6\alpha/R_I + b_7/LR_I + b_8/L^2 + b_9/R_I^2 + b_{10}\alpha^2 \quad \text{(W/m}^2\cdot\text{K)} \quad \text{--- (4.14)}
\]

where

\[
\begin{align*}
b_1 &= -0.2594 \quad \text{(W/m}^2\cdot\text{K); (-0.2716, -0.2473)} \\
b_2 &= 0.6155 \quad \text{(W/m} \cdot \text{K); (0.5931, 0.6380)} \\
b_3 &= 0.0421 \times 10^7 \quad \text{(J/m}^4\cdot\text{K); (0.4044 \times 10^6, 0.4367 \times 10^6)} \\
b_4 &= 0.2078 \quad (0.1920, 0.2236) \\
b_5 &= 0.0020 \times 10^7 \quad \text{(J/m}^2\cdot\text{K); (-0.2668 \times 10^5, 0.6681 \times 10^5)} \\
b_6 &= 0.0128 \times 10^7 \quad \text{(s/m}^2); (0.1054 \times 10^6, 0.1498 \times 10^6) \\
b_7 &= 0.3068 \quad \text{(m); (0.2783, 0.3352)} \\
b_8 &= -0.4224 \quad \text{(W/K); (-0.4608, -0.3840)} \\
b_9 &= -0.1014 \quad \text{(m} \cdot \text{K/W); (-0.1176, -0.8526E-01)} \\
b_{10} &= -0.0025 \times 10^4 \quad \text{(J} \cdot \text{s/m}^6\cdot\text{K); (-0.2653 \times 10^{12}, -0.2358 \times 10^{12})}
\end{align*}
\]

The amplitude of the average floor heat flux is given by

\[
\frac{q_{f,a,i}}{A} = b_1 + b_2/L + b_3\alpha + b_4R_I + b_5\alpha/L^2 + b_6\alpha R_I + b_7R_I/L + b_8/L^2 + b_9R_I^2 + b_{10}\alpha^2 \quad \text{(W/m}^2\cdot\text{K)} \quad \text{--- (4.15)}
\]

where

\[
\begin{align*}
b_1 &= -0.1870 \quad \text{(W/m}^2\cdot\text{K); (-0.1899, -0.1840)} \\
b_2 &= 0.4093 \quad \text{(W/m} \cdot \text{K); (0.4038, 0.4147)}
\end{align*}
\]

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
\[ b_3 = 0.0256 \times 10^7 \text{ (J/m}^4\text{K)}; \ (0.2520 \times 10^6, 0.2598 \times 10^6) \]

\[ b_4 = 0.0198; \ (0.1826E-01, 0.2132E-01) \]

\[ b_5 = 0.0120 \times 10^7 \text{ (J/m}^2\text{K)}; \ (0.1086 \times 10^6, 0.1312 \times 10^6) \]

\[ b_6 = 0.7347 \times 10^3 \text{ (s/m}^2) ; \ (-0.1053, 0.2523) \]

\[ b_7 = -0.0044 \text{ (W/m}^2\text{K)}; \ (-0.7309E-02) \]

\[ b_8 = -0.3002 \text{ (W/K)}; \ (-0.3095, -0.2909) \]

\[ b_9 = -0.3002 \text{ (W/K)}; \ (-0.3095, -0.2909) \]

\[ b_{10} = -0.0008 \times 10^{14} \text{ (J}\cdot\text{s/m}^-{6}\text{K)}; \ (-0.8696 \times 10^{11}, -0.7984 \times 10^{11}) \]

The phase lag of the average wall heat flux is given by

\[ \delta_{w,i} = b_1 + \frac{b_2}{K} + b_3 \rho C_p + b_4 L + b_5 R_i \text{ (rad)} \quad \text{--- (4.16)} \]

where

\[ b_1 = -0.1242 \text{ (rad)}; \ (-0.1368, -0.1115) \]

\[ b_2 = 0.0616 \text{ (W/m}\cdot\text{K)}; \ (0.4687E-01, 0.7632E-01) \]

\[ b_3 = 0.0563 \times 10^{-6} \text{ (m}^3\text{K}/\text{J)}; \ (0.5144 \times 10^{-7}) \]

\[ b_4 = 0.0954 \text{ (m}^{-1}) ; \ (0.8903E-01, 0.1017) \]

\[ b_5 = 0.0388 \text{ (W/m}^2\text{K)} ; \ (0.3233E-01, 0.4526E-01) \]

The phase lag of the average floor heat flux is given by

Egn. (4.16) and the correlation coefficients are

\[ b_1 = -0.1235 \text{ (rad)}; \ (-0.1788, -0.6808E-01) \]

\[ b_2 = 0.1327 \text{ (W/m}\cdot\text{K)}; \ (0.6841E-01, 0.1969) \]

\[ b_3 = 0.1351 \times 10^{-6} \text{ (m}^3\text{K}/\text{J)}; \ (0.1140 \times 10^{-6}, 0.1562 \times 10^{-6}) \]
\[ b_4 = 0.3978 \text{ (m}^{-1}) \; ; \; (0.3702, 0.4255) \]

\[ b_5 = -0.0108 \text{ (W/m}^2 \cdot \text{K}) \; ; \; (-0.3901E-01, 0.1743E-01) \]
Chapter V

COMPARISON WITH OTHER EXISTING METHODS

MacDonald et al [21] applied seven basement heat loss calculation methods to a prototypical basement in 5 U.S. cities. Dimensions, soil conditions, and climates were chosen with three factors in mind.

1. The parameter values were chosen to be typical of those frequently encountered in common design applications.

2. The configuration chosen was a simple rectangular basement.

3. Parameter values were chosen to minimize the effects of invariant parameters.

Certain parameters, such as soil diffusivity, were frequently invariant in one calculative method but variable in others. In such cases, the invariant value was used. When more than one invariant was encountered for a parameter, the most typical value was chosen and used for all other methods. The input values of all basement parameters are presented in Table 10. Table 11 gives the ground and air temperature data for the U.S. cities.

The prototypical basement configuration, 7.62 m wide by 12.2 m long, was chosen to represent an average residential
<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Length</td>
<td>12.2 m</td>
</tr>
<tr>
<td>2.</td>
<td>Width</td>
<td>7.6 m</td>
</tr>
<tr>
<td>3.</td>
<td>Wall height (entirely below grade)</td>
<td>2.14 m</td>
</tr>
<tr>
<td>4.</td>
<td>Floor - uninsulated concrete and air film, $R = 0.26 \text{ W/m.K}$</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Wall construction - concrete/masonry</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Wall total $R$-value (including air film), three cases:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_T = 0.26 \text{ m.K/W}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_T = 1.12 \text{ m.K/W}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_T = 2.86 \text{ m.K/W}$</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>Soil thermal conductivity</td>
<td>1.3 W/m.K</td>
</tr>
<tr>
<td>8.</td>
<td>Soil density</td>
<td>2100 kg/m³</td>
</tr>
<tr>
<td>9.</td>
<td>Soil specific heat</td>
<td>0.96 kJ/(kg.K)</td>
</tr>
<tr>
<td>10.</td>
<td>Basement temperature</td>
<td>21.1 °C</td>
</tr>
<tr>
<td>City</td>
<td>TG</td>
<td>BG</td>
</tr>
<tr>
<td>--------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>St. Paul, MN</td>
<td>48.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Ft. Collins, CO</td>
<td>50.0</td>
<td>24.7</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td>53.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Decatur, AL</td>
<td>59.0</td>
<td>21.0</td>
</tr>
<tr>
<td>Tucson, AZ</td>
<td>75.0</td>
<td>18.0</td>
</tr>
</tbody>
</table>

TG = mean annual ground temperature, averaged over a ten foot depth

BG = annual ground surface temperature

TA,A = annual mean air temperature

BA = annual air temperature amplitude

(Source: Ref. 21)
basement. The 2.14 m depth was chosen to be entirely below grade. The wall insulation values chosen represent the two most common values for the U.S.A. : 0.96 and 2.60 m².K/W. Full height insulation was assumed, since it is one of the only available strategies common to all methods. The floor was taken to be uninsulated. The soil properties are shown in Table 10. They are characteristic of a moist clay-silt-loam. The five U.S. cities chosen were St. Paul, MN, Ft. Collins, CO, Columbus, OH, Decatur, AL, and Tucson, AZ.

The present method developed in this investigation has been applied to only the three colder cities, St. Paul, Ft. Collins and Columbus, because they best represent the climatic conditions of Canada. The seven existing methods and the new method were used to calculate the annual heat loss in each of the cities for the uninsulated basement and for two different insulation levels (mentioned above). Basement heat loss has been defined by MacDonald et al as the total energy required for basement heating : negative heat loss has been excluded.

The results are given in Figures 15 and 16. The three cities are presented horizontally with the mean annual ground temperature increasing from left to right. The heat loss values predicted by the new method are denoted by a dot placed in the same vertical line as the Mitalas's method. For each city the top line joining the points is for the
Fig. 15: Total annual basement heat loss (Source: Ref. 21)
Fig. 16: Annual basement heat loss from walls only (Source: Ref. 21)
uninsulated case, the middle line is for 0.96 m$^2$.K/W insulation and the bottom line is for 2.60 m$^2$.K/W insulation. Figure 15 shows the annual total basement heat loss. Heat loss values for only six existing methods are shown because the Akridge method predicts wall loss only, so it is omitted.

It can be seen from the figure that the new method is quite close to the Yard method for the insulated cases. The heat loss predicted by the Mitalas method is consistently higher than that given by the new method. The reason for this could be that the shape factors given by Mitalas were adjusted to account for high floor losses observed at some test sites. The new method differs from the Mitalas method at most by 20%.

Figure 16 shows the annual basement heat loss from walls only. The new method compares very well with the Mitalas' method for the uninsulated case and the lower insulation level case. The new method predicts the highest heat loss among the other methods, for the higher insulation level. Agreement is quite good with the Yard method also.
Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The following conclusions can be drawn from the present study:

1. The basement wall and floor heat losses vary sinusoidally if the ground surface temperature is considered as a sinusoidal variation.

2. The heat losses are strong functions of the basement depth, the soil conductivity and the difference between the basement interior temperature and the ground surface temperature.

3. The annual variation of the floor heat loss extends up to 3 m from the corner, and the loss from the centre portion is approximately constant over the annual cycle.

4. The values of three parameters are sufficient to describe the heat loss variation for the wall or the floor: an average mean annual value, the amplitude of the heat loss and the lag of the heat loss with respect to the ground surface temperature.

The method developed in this study agrees quite well with the methods proposed by Mitalas and Yard et al., and the
annual heat loss predictions of this method are within 20% of those predicted by the other two methods. The accuracy of the new method to predict the heat loss from an actual basement cannot be judged based on the comparisons done in this study.

The method developed in this study should be used only in the cases where the deep groundwater level is at least 10 m below the ground surface level. Any adjacent basement should be at least 8 m away. The half-width of the basement should be at least 3 m. The method should not be used for values of wall insulation resistance which are higher than 2.8 m² K/W. These are the essential restrictions on the use of the method.

6.2 RECOMMENDATIONS

1. Floor insulation was not considered in this study. Future work could treat floor insulation.

2. Exterior wall insulation could also be studied.

3. The correlations for the insulated wall case in the present method are valid for insulation resistance values up to 2.8 m² K/W. New correlations could be obtained for higher insulation levels.

4. The correlations in this study were obtained from a simplified two-dimensional finite difference model of a typical basement. Future work should involve more complicated models and consider high groundwater
levels and groundwater flow, space- and time-varying soil conductivity and radiation effects inside the basement.
REFERENCES


Methods Suitable for Variable-Base Degree-Day Calculations. ASHRAE Transactions.


Appendix A
DESCRIPTION OF HEATING5

Some of the important features of HEATING5 have already been outlined in chapter II. This appendix explains the characteristics in greater detail. HEATING5 is a modified version of the generalized heat conduction code HEATING3.

The major improvement in the code is the incorporation of an implicit scheme to solve transient problems. Of the three basic algorithms which are available in HEATING5 to solve transient problems, the implicit scheme is the recommended approach for most problems. This scheme has been written generally to include the Crank-Nicolson finite difference equations, the classical implicit or backwards Euler finite difference equations, or a linear combination of the two. The resulting system of equations is solved by the point successive overrelaxation iterative method, and the technique includes procedures to estimate the optimum acceleration parameter as a function of time. The time step size for the implicit transient calculations can be controlled explicitly through the input data or implicitly by specifying the maximum temperature change or maximum percent of relative change in temperature allowed at a node.

- 96 -
over a time step. The temperature-dependent parameters may be reevaluated as a function of the number of iterations for steady-state problems. Another modification allows selected materials to undergo a change of phase. However, the implicit technique for transient problems cannot be used for problems involving materials with change-of-phase capabilities with this version of HEATING5. Another feature which has been added to the code is the capability of solving one-dimensional spherical models. Analytical and tabular functions have been added to aid in the definition of input parameters. An option to allow the user to write his/her own subroutines to evaluate many of the input parameters has been added to the code. Thus, if an input parameter cannot be described with the analytical or tabular functions, then the user may easily supply his/her own algorithm to evaluate the parameter. This concept is referred to as user-supplied subroutines. The boundary condition parameters may be time- and/or temperature-dependent or if they are defined in user-supplied subroutines, they can also be position-dependent. The thermal conductivity, density and heat capacity can also be time-dependent if they are defined in user-supplied subroutines. For two- and three-dimensional problems, the temperatures in each plane are printed in the form of a map which depicts the material boundaries. This feature enables one to monitor the temperature distribution.
in a plane with minimal effort. A nodal map accompanies the first temperature map which allows one to readily locate a node and its temperature.

HEATING5 possesses a variety of boundary conditions to enable the user to model a physical problem as accurately as possible. In general, a boundary condition is applied along a surface of a region and heat is transferred from a surface node to a boundary node or to the corresponding node on the opposing parallel surface. Surface nodes are actually internal nodes which are located on the edge of a region. Boundary nodes are dummy nodes and their temperatures are not calculated by the code but are specified as input to the code. These temperatures are only used to calculate the heat flow across a boundary surface. The boundary conditions which can be applied over the surface of a region in the current version of HEATING5 are listed below.

1. The temperature on the surface of a region can be specified as a constant or a function of time.

2. The heat flux across the surface of a region can be specified directly as a constant or a function of time and/or surface temperature.

3. The heat flux across the surface of a region can be specified indirectly by defining the heat transfer mechanism to be forced convection, radiation and/or natural convection.
simulation is not required for insulated boundaries. Heat is simply not allowed to cross the surface. The boundary conditions are classed as either surface-to-boundary (type 1), isothermal (type 2), or surface-to-surface (type 3). Boundary conditions of the surface-to-boundary type are used to define heat transfer between a surface node and a boundary node. The temperature of the boundary node is specified and can be a function of time. Surface-to-surface boundary conditions are used to define heat transfer between parallel surfaces. In this case, heat is transferred between a node on one surface to the corresponding node on the opposing surface. The temperatures are entered in F or C, and the code converts them to absolute degrees when a radiative boundary condition exists.

HEATING5 is designed so that, simultaneously, one may consider surface-to-surface heat transfer across a region as well as conduction through the region. This is accomplished by defining the region to contain a material as well as by defining surface-to-surface boundary conditions across parallel surfaces of the region. Also, one may consider surface-to-surface heat transfer across a gap as well as surface-to-boundary heat transfer along the edge of the gap. This is done by defining the gap as a gap region (i.e., it does not have a material associated with it) with surface-to-surface boundary conditions applied across parallel surfaces of the region. Then surface-to-boundary
boundary conditions are defined on the adjacent material regions at the surfaces defining the edges of the gap. It is to be noted that a surface- tc-boundary boundary condition can be applied along the surface of a region only if there is no region adjacent to it or the adjacent region is defined as a gap region.
Appendix B
DESCRIPTION OF INPUT TO HEATING5

This section is designed to guide the user through the steps necessary to solve a heat transfer problem using HEATING5. In preparing the input data, any consistent set of units may be used except for problems involving radiation. Then, all temperature units must be in either degrees Celsius or Fahrenheit. The units associated with the algorithms which appear in user-supplied subroutines must be consistent with those of the input data.

B.1 REGIONS

First, the configuration of the problem is approximated by dividing it into regions, depending on the shape, material structure, indentations, cutouts, and other deviations from the general geometry. In some cases, zoning into regions must be done in order to describe a specific boundary condition or a material whose thermal conductivity, density, or specific heat is a function of position. There are three basic rules governing region division:

1. Boundary lines or planes must be parallel to the coordinate axes (two points, four lines, or six planes
are required to enclose a region in one-, two-, or three-dimensional geometry, respectively).

2. A region may contain at most one material (however, many regions may contain the same material). A gap does not contain a material.

3. When a boundary condition is defined along the boundary of a region, it must apply along the full length of the boundary line for two-dimensional problems and over all of the boundary plane for three-dimensional problems.

B.2 LATTICE ARRANGEMENT

The second requirement for describing the overall configuration is to construct a set of lattice lines perpendicular to each axis and extending the entire length of the remaining coordinates. The lattice lines are really points, lines, or planes for a one-, two-, or three-dimensional problem, respectively. The lattice is defined in the following manner. The lattice lines are divided into two classes: gross and fine lattice lines. A gross lattice line must be specified at both region boundaries along each axis. Fine lattice lines, equally spaced, may appear between two consecutive gross lattice lines to create a finer mesh. If unequal mesh spacing is desired within a particular region, then gross lattice lines may appear within that region. A nodal point is defined by
each lattice point in one-dimensional problems, by each intersection of two lattice lines appearing in a material region or on its boundary in two-dimensional problems, and by every intersection of three lattice planes appearing in a material region or on its boundary in three-dimensional problems. The points are numbered consecutively by the program at the intersection of each X- (or R-) , Y- (or θ-) , Z-plane starting with the planes nearest the origin and changing the X- (or R-) plane most rapidly and the Z-plane least. Temperatures are calculated at each such nodal point.

B.3 ANALYTICAL AND TABULAR FUNCTIONS

The analytical and tabular functions are built-in functions which may be used to aid in the description of some of the input parameters. An analytical function is defined by

\[ F(V) = A_1 + A_2 V + A_3 V^2 + A_4 \cos (A_5 V) + A_6 \exp (A_7 V) + A_8 \sin (A_9 V) + A_{10} \ln (A_{11} V) \]

A tabular function is defined by a set of ordered pairs, where the first element of the pair is the independent variable and the second is the corresponding value of the function. In order to evaluate the tabular function at some point, the program uses linear interpolation in the interval containing the point. The set of ordered pairs must be chosen so that the independent variable is arranged in ascending order.
Appendix C

CARD INPUT FOR HEATINGS

A detailed description of the card input is presented below. This portion has been taken from the reference manual provided by the Union Carbide Corporation. Except for Card 1, the M cards, and the deck composed of the IT cards, the input data are arranged on each card in 9 column fields. All integers must be right-adjusted, i.e., the last digit of each integer must appear in a column which is a multiple of 9. Except for the IT cards, columns 73 through 80 of each card are reserved for identification to aid the user in the preparation and handling of the data.

C.1 Card 1: Title of Problem
This card, which can contain alphanumeric characters in the first 72 columns, contains a descriptive title for the problem. The card itself cannot be omitted although it may be left blank.
C.2 Card 2: Input Parameters

All eight entries in this card are integers.

C.2.1 Maximum CPU Time

When one submits a job to the computer, the maximum CPU time that the job is expected to run is indicated on the CLASS card. If the CPU time exceeds this time, the job will be pulled out by the system without printing the current temperature distribution. In order to prevent this, the maximum CPU time (seconds) is specified as the first entry in Card 2. This time should be less than the CPU time specified on the CLASS card.

C.2.2 Type Geometry

The HEATNG5 program offers nine possible geometries (seven and eight are really the same) which are members of either the cylindrical, the Cartesian or the spherical coordinate system. These are listed below.

<table>
<thead>
<tr>
<th>Cylindrical</th>
<th>Rectangular</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 R-θ-Z</td>
<td>6 X-Y-Z</td>
<td>10 R</td>
</tr>
<tr>
<td>2 R-θ</td>
<td>7 X-Y</td>
<td></td>
</tr>
<tr>
<td>3 R-Z</td>
<td>8 X-Z</td>
<td></td>
</tr>
<tr>
<td>4 R</td>
<td>9 X</td>
<td></td>
</tr>
<tr>
<td>5 Z</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

One-, two-, or three-dimensional systems are allowed in either cylindrical or Cartesian coordinates by entering the appropriate number (1-9, as indicated above) as the second
entry of Card 2. A one-dimensional model described in the radial, spherical coordinate system is defined as a type 10 geometry.

C.2.3 Total Number of Regions
The total number of regions of the entire configuration is entered as the third entry. A maximum of 100 regions is allowed.

C.2.4 Total Number of Materials
The fourth entry of this card contains the total number of different materials. There can be a maximum of 50 materials.

C.2.5 Total Number of Materials with Change of Phase Capabilities
The total number of materials with change of phase capabilities is the fifth entry of Card 2. If phase changes are not considered in this problem then this entry must be left blank. There can be a maximum of five such materials.

C.2.6 Total Number of Heat Generation Functions
The total number of different heat generation functions (a maximum of 20) is the sixth entry of Card 2. This entry must be blank if there are no heat generation functions.
C.2.7 **Total Number of Initial Temperature Functions**
This entry (the seventh of Card 2) is the total number of different initial temperature functions, up to a maximum of 25. If there are no initial temperature functions, this entry must be left blank, and the program will assume that the initial temperature distribution is zero degrees.

C.2.8 **Total Number of Boundaries**
The eighth entry of Card 2 is the total number (maximum of 50) of boundary conditions. If there are no boundary conditions explicitly specified in the input data, this entry must be left blank.

C.3 **Card 3: Input Parameters**
Each entry on this card is an integer.

C.3.1 **Gross Lattice Size**
The first three entries of this card contain the total number of gross lattice lines in (1) \( X \) or \( B \) direction, (2) \( Y \) or \( O \) direction, and (3) \( Z \) direction. If any dimension is omitted, a zero is inserted in the appropriate entry for the total number of gross lattice lines or the appropriate entry must be left blank. There can be a maximum of 50 gross lattice lines along each axis.
C.3.2 Total Number of Analytical Functions
The fourth entry is the total number of different analytical functions. There can be maximum of 25 of these functions. If there are no analytical functions, then this entry must be left blank.

C.3.3 Total Number of Tabular Functions
The total number of tabular functions is the fifth entry of this card. There can be a maximum of 25 functions. If there are no tabular functions, then this entry must be left blank.

C.3.4 Temperature Units
For problems involving radiation, the temperature units must be either F or C. Entry 6 is used only for problems involving radiation. It indicates that the units of temperature are either in degrees Fahrenheit (entry is zero or blank) or degrees Celsius (entry is 1).

C.3.5 Three-Dimensional Output Map Flag
The seventh entry on this card is a flag which is only used for three-dimensional problems. Normally, the temperature output map for three-dimensional problems is printed for each XY or RG plane. If this entry is nonzero, then the temperature output map will be printed for each XZ or RZ plane.
C.3.6 Updating Temperature-Dependent Properties

The eighth entry on this card specifies the number of iterations which are allowed before the temperature-dependent thermal properties are reevaluated for steady-state problems. Once the convergence criterion has been satisfied, the code continues to iterate. However, the temperature-dependent thermal properties are now reevaluated after every iteration until the convergence criterion is satisfied a second time. Some nonlinear problems will converge in fewer iterations if the thermal properties are not evaluated at each iteration, and certainly the computing time per iteration will be less. It is recommended that this parameter be of the order of 10 or 20. If it is blank or zero, then the default value is unity for nonlinear problems.

C.4 Card 4: Input Parameters

each entry on this card is an integer.

C.4.1 Transient Output

For transient problems, the output may be specified in either of two ways:

1. The temperature distribution may be printed out at equally spaced times. To specify this option, the first entry on Card 4 must contain the number of initial time steps between outputs, and the second entry must be blank. For example, if a value of 5 is
entered, the temperature distribution will be printed at times whose spacing is equal to five times the initial time step.

2. The temperature distribution may be printed out at unequal time increments. To choose this option, the first entry on Card 4 must be left blank and the second entry must contain the number of times the temperature distribution will be printed out. The actual output times will be entered on the O cards.

The temperature distribution is automatically printed out prior to the first time step for transient calculations and prior to the first iteration for steady-state problems. In addition, the temperature distribution may be printed out up to 100 times for each transient portion of the problem. For steady-state-only calculations, the first and second entries may be left blank.

C.4.2 Graphical Output

The third entry on Card 4 indicates whether or not graphical output is desired. At times it is desirable to have plots of the temperature distribution to aid in the interpretation of the results from BEATINGS. By entering a nonzero integer in this field, a data set will be created which contains the temperature distributions along with certain parameters which identify the problem. The absolute value of the integer identifies the unit number on which the data set is to be created. If it is a positive integer, a new data set
is created. If it is negative, then the temperatures from this run will be added to those from a previous run. The data set containing the temperatures from the previous run will be read from the unit whose number is one less than the absolute value of this entry (or one less than the unit number of the data set to be created). This data set can then be used by a plotting package to create various plots to include temperatures versus time, temperature profiles and isothermal plots. If this option is invoked, the user must supply a DD card to define the data set. The records are variable in length. The longest and most frequently written record is \(8(N+NBC)\) bytes long where \(N\) is the number of nodes and \(NBC\) is the number of boundary conditions.

C.4.3 Frequency of Output for Plots

This entry, the fourth on Card 4, specifies the number of time steps between each output of the temperature distribution on the data set defined above. If this entry is blank or zero and if the preceding entry is nonzeroc, then the temperature distribution will be written on the data set each time that a normal printout, as defined above, occurs.

C.4.4 Special Monitoring of Temperatures

The user may wish to tabulate the temperatures of a few nodes as a function of the number of iterations or time steps. In this way one can keep track of what is happening at a few nodes of interest without getting excessive output
by having to print out the entire temperature distribution. This output option is in addition to the standard output of the temperature distribution. To invoke this option, the number of iterations between printouts for steady-state calculations or the number of time steps between printouts for transient calculations is entered in the fifth field of Card 4. If this entry is nonzero, the node numbers whose temperatures are to be printed out are specified on the S cards. If this option is not desired, this entry is left blank.

C.4.5 Initial Temperature Input Unit

The sixth entry on this card specifies the unit number from which the explicitly specified lattice-point initial temperatures are read. If the entry is a positive integer, then it specifies the unit number from which the initial temperatures are read in formatted form. The records vary in length with the maximum size being 80 characters. If it is a negative number, then its absolute value specifies the unit number from which the initial temperatures are read in unformatted form. If there are no initial temperatures explicitly specified, then this entry is left blank. If the unit specified is other than the standard card input, the user must insure that the appropriate DD card has been supplied to describe the data set.
C.4.6 Final Temperature Output Unit
In addition to the normal output, the user may wish to have
the final temperature distribution saved in some manner to
facilitate the restarting of the problem. If the seventh
entry on Card 4 is a positive integer, then it specifies the
unit number on which the final temperature is to be written
in formatted form. If the entry is a negative number, then
its absolute value specifies the unit number on which the
final temperature distribution is to be written in
unformatted form. This entry must be left blank in case no
final temperature distribution is needed.

C.4.7 Problem Status Unit for Remote Users
If a positive number is entered in the eighth field of Card
4, then this designates the unit number on which error
messages and selected information concerning the status of
the problem is written. The user must supply the appropriate
DD card to define the specified unit.

C.5 Card 5: Input Parameters
Each entry on this card is a floating-point number except
for entries 1, 2, and 6, which are integers.

C.5.1 Type of Problem
The first entry on Card 5 specifies the type of problem. It
may be steady-state only, transient only, or combinations of
steady-state and transient calculations. The number to be
entered is in accordance with the following list:
1 Steady-State (S-S) only  -1 Transient only
2 S-S, Trans.  -2 Trans., S-S
    ...
    ...

If, for example, a 3 is entered, the program will first perform a steady-state calculation at time zero; next the transient calculation; then a steady-state calculation at the final transient time using the final transient temperatures as the initial guess for the steady-state temperatures.

C.5.2 Maximum Number of Steady-State Iterations Allowed

If the maximum number of steady-state iterations is reached and the convergence criterion is not satisfied, then the program will write a message, terminate the calculation and call for the next problem. Normally 200 to 500 iterations are sufficient to converge to the solution. This entry, the second of Card 5, is left blank for pure transient problems. The default value is 500.
C.5.3 Steady-State Convergence Criterion
This entry, the third of Card 5, affects the steady-state type of calculation and may be left blank for a transient-only problem. The steady-state calculation will continue until the convergence criterion is met. The default value is 1.0E-5.

C.5.4 Steady-State Over-Relaxation Factor
The fourth entry of Card 5 must be between 1 and 2. It is left blank for transient-only problems. If it is left blank for steady-state problems, the code assumes that the initial value is 1.9.

C.5.5 Time Increment
This entry (the fifth of Card 5) contains the initial time increment for transient problems that will be solved using one of the explicit techniques. For transient problems which will be solved using the implicit procedure, this entry must be left blank. For steady-state-only problems, this entry may be left blank.

C.5.6 Levy's Explicit Method Option
The sixth entry (an integer) is the factor by which the stable time increment is multiplied to form the time increment for Levy's explicit method. If the entry is blank or less than 2, then Levy's method will not be used. This entry may be left blank for steady-state-only problems.
C.5.7 Initial Time
Entry seven affects both transient and steady-state problems. It indicates the initial time for problems with a negative type and the time at which the time-dependent functions are evaluated for problems whose types are greater than zero. If this entry is left blank, then initial time will be zero.

C.5.8 Final Time
The final time for the first transient calculation is specified as the eighth entry on Card 5. For steady-state problems, this entry may be left blank.

C.6 Region Data (Cards R1 and R2)
Each region is described by two cards which must appear in pairs. The cards are repeated for each region. The number of pairs of cards is the third entry on Card 2. There must be at least one region for each problem.

C.6.1 Card R1
1. Region Number:

This entry contains the number of the region to be described. Regions are to be numbered consecutively beginning with number 1 up to a maximum of 100 regions. The region numbering system does not require that region occupy any particular zone in the overall configuration. This entry is the first on Card R1 and must be an integer.
2. Material in Region:

The second entry of this card indicates by an integer the number of the material which occupies the region named in the first entry of this card. This entry is left blank if the region does not contain a material (gap region).

3. Region Dimensions:

Dimensions of the region boundaries are entered as floating-point numbers and are arranged in the following order:

a) smaller dimension of X or R region boundary
b) larger dimension of X or R region boundary
c) smaller dimension of Y or Q region boundary
d) larger dimension of Y or Q region boundary
e) smaller dimension of Z region boundary
f) larger dimension of Z region boundary

C-7  R2 Card

This card must be included in conjunction with the appropriate R1 card, even if it is blank.

1. Initial Temperature of Region:

The initial temperature function number of the region specified by the first entry of Card R1 is entered as an integer. If this entry is left blank, then the program assumes that the initial temperature for the region is zero. This entry is left blank for a gap region.
2. Heat Generation of Region:

This entry contains the number (an integer) of the heat generation function associated with the region given on the first card of this pair. If this entry is left blank, then the code assumes that the region does not generate heat. This entry is left blank for a gap region.

3. Boundary Numbers:

The remaining entries of this card are the boundary numbers defining the boundary conditions corresponding to the six boundaries of the region described by the first card of this pair. These pairs are integers. Each entry contains the boundary number of the region boundary appearing in the corresponding entry of Card B1. A boundary condition cannot be specified on a boundary dividing two regions unless it is a type 3 boundary condition or unless one of the regions is a gap region. For one- or two-dimensional cases, region dimensions are not specified for the unnecessary coordinate or coordinates, and the corresponding boundary numbers are left blank. The entry is also left blank for boundaries which are insulated.
C.8 Material Data (Cards M and PC)

A group of cards consisting of an M card and possibly a PC card is required to describe each material. The total number of groups is the fourth entry on Card 2. There can be five materials with change of phase capabilities, and they must be the first ones described on the M cards. If the thermal conductivity of a material is anisotropic, then it is specified as being temperature-dependent, and the associated temperature-dependent function is specified as being user-supplied. The user then programs the anisotropic algorithm for that material in subroutine CONDTN. The following labeled common must be added to the subroutine:

```
COMMON /THBSBC/ NBDTP, NDIR
```

The variable NDIR will contain a value of 1, 2, or 3 indicating the thermal conductivity is to be evaluated along the X (or R), Y (or θ) or Z axis, respectively. Both variables NBDTP and NDIR are INTEGER*4.

C.8.1 Card M

1. Material Number:

   The first entry, an integer, contains the number of the material which is to be described. Materials are numbered consecutively (each different material has a number) beginning with number 1 up to a maximum of 50 materials.

2. Material Name:
The second entry, which must begin column ir 11 and may extend through column 18, contains the name of the material. This name, which may consist of up to eight alphanumeric characters, is used to aid in identification of output data.

3. **Constant Thermal Properties:**

Entries 3, 4, and 5 are floating-point numbers and contain the constant thermal conductivity, density and specific heat of the material respectively. Since the density and specific heat are not used in steady-state calculations, entries 4 and 5 may be left blank for type 1 problems.

4. **Temperature-Dependent Thermal Properties:**

Entries 6, 7, and 8 (all integers) identify the analytical or tabular functions describing the thermal conductivity, density and specific heat, respectively, as a function of temperature. Entries 7 and 8 may be left blank for a type 1 problem. A positive entry defines the number of the analytical function. The absolute value of a negative entry defines the number of the tabular function.

**C.8.2 Card PC**

For materials which can undergo a phase change, the phase-change or transition temperature and the corresponding latent heat are entered as floating-point numbers in the first and second fields, respectively, of Card PC. These
materials must be the first ones described on the M cards. The PC card is omitted for those materials which do not undergo a change of phase.

**C.9 Heat Generation Function Data (Card G)**

Each different heat generation function is numbered, beginning with number 1, consecutively up to a maximum of 20 such functions. The heat generation function may be dependent on position, time and temperature. The function data cards indicate the function number, the volumetric heat generation rate, the time-dependent function parameter, the \( X \)- or \( R \)-dependent function parameter, the \( Y \)- or \( \Theta \)-dependent function parameter, and the \( Z \)-dependent function parameter, arranged in that order, respectively. The parameters in entries 3 to 7 refer to analytical and tabular defining the above parameters. The first entry on this card is an integer, the second is a floating-point number and the remaining five are integers. The heat generation rate may be positive (heat source) or negative (heat sink). The G cards are omitted if entry 6 on Card 2 is blank or zero.

**C.10 Initial Temperature Function Data (Card I)**

Each different initial temperature function is given a number. Beginning with number 1, the functions are numbered consecutively up to a maximum of 25. Since the initial temperature function associated with a region can be a
function of position, then this data card consists of five entries. All of them are integers except the second which is a floating-point number. The first entry contains the initial function number. The second entry contains the constant factor describing the initial temperature function. The remaining entries identify analytical or tabular functions and contain the $X$- or $R$-, $Y$- or $\Theta$-, and $Z$-dependent function parameters. If the problem is one- or two-dimensional or if the initial temperature does not vary along a particular axis of the region, then the position-dependent function parameter associated with the coordinate will be left blank, and the corresponding function value will be set equal to 1.0. The total number of cards is the seventh entry of Card 2, and if this entry is blank or zero, then the I cards are omitted, the initial temperature distribution is assumed to be zero.

C.11 Boundary Data (Cards B1, B2, B3 and B4)

Excluding insulated or contact type boundaries, each unique boundary is numbered consecutively up to a maximum of 50. The B1 and B2 cards are omitted if the eighth entry on Card 2 is left blank or zero.
C.11-1 Card B1

1. The first entry on Card B1 is an integer and contains the boundary number.

2. The second entry (an integer) indicates the type of boundary. HEATING5 offers three boundary types which are numbered 1, 2, or 3, corresponding to the following:

   1 implies surface-to-boundary,
   2 implies prescribed surface temperature, and
   3 implies surface-to-surface.

If this entry is blank or zero, then no heat transfer connections will be made and the boundary will be treated as an insulated boundary.

3. The third entry, a floating-point number, contains the boundary temperature. This entry is left blank for a type 3 boundary condition.

4. Since the boundary temperature can be a function of time, the fourth entry on this card contains the time-dependent parameter (an integer) which identifies an analytical or tabular function. If the boundary temperature is independent of time or if the boundary type is 3, then this entry will be left blank.
C.11.2 Card B2

Each entry on Card B2 is a floating-point number except entry 6 which is an integer. This card is left blank for a type 2 boundary condition.

1. Entry 1 contains the constant heat transfer coefficient for forced convection.

2. Entry 2 contains the constant coefficient for radiation.

3. Entry 3 contains the constant coefficient for natural convection.

4. The fourth entry contains the exponent for natural convection (or other nonlinear heat transfer process).

5. The fifth entry contains the constant prescribed heat flux across the boundary.

6. The time- and temperature-dependent parameter flag, an integer, is the sixth and final entry on the B2 card. If any of the five preceding parameters are functions of time or temperature, then additional information must be entered on B3 and/or B4 cards. The time- and temperature-dependent flag indicates whether or not the B3 and B4 cards are present for this particular boundary condition. Its value is determined according to the following table:

<table>
<thead>
<tr>
<th>Entry Six</th>
<th>Additional Cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>1</td>
<td>B3 Only</td>
</tr>
<tr>
<td>2</td>
<td>B4 Only</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
C.11.3 Card B3
All five entries on Card B3 are integers. Each integer identifies the analytical or tabular function that defines the time-dependent function associated with the respective parameter on Card B2. If an entry is zero, then the associated parameter is not time-dependent.

C.11.4 Card B4
This card is just like Card B3 except each integer identifies the analytical or tabular function that defines the temperature-dependent function associated with the respective parameter on Card B2.

The B1 and B2 cards must appear in pairs, and a pair is entered for each boundary. B3 and B4 cards, if any, must follow their respective B1 and B2 cards.

C.12 Lattice Description (Cards L1, N1, L2, N2, L3 and N3)
For each axis, gross lattice data are entered on two sets of cards, the first set specifying the lattice dimensions and the second indicating the mesh division between gross lattice lines. All of the numbers on the cards on the first set (L cards) are of the floating-point type and are entered, by specifying all of the gross lattice dimensions in each direction, sequentially on one or more cards. The
cards of the second set (N cards) specify the number (an integer > 1) of equal increments which are between the gross lattice lines—whose dimensions are given on the cards of the first set. For degenerate geometries, the corresponding unnecessary sets of cards must be omitted.

1. Card L1:

   The L1 cards correspond to the X or R coordinate, and the number of entries corresponds to Entry 1 of Card 3. If there are more than 8 entries, subsequent cards are used.

2. Card N1:

   The N1 cards correspond to the X or R coordinate. There will be one less entry here than on the L1 cards. Additional cards are used for more than eight entries.

3. Card L2:

   The L2 cards correspond to the Y or Θ coordinate, and the number of entries corresponds to Entry 2 of Card 3. If there are more than eight entries, subsequent cards are used.

4. Card N2:

   The N2 cards correspond to the Y or Θ coordinate. There will be one less entry here than on the L2 cards.

5. Card L3:
The L3 cards correspond to the Z coordinate, and the number of entries corresponds to Entry 3 of Card 3. If there are more than eight entries, subsequent cards are used.

6. Card N3:

The N3 cards correspond to the Z coordinate. There will be one less entry here than on the L3 cards.

C.13 Analytical Function Data (Cards A1 and A2)

Each analytical function is described by an A1 card and one or more A2 cards.

C.13.1 Card A1

Each different analytical function is numbered, and there can be a maximum of 25 such functions. The first entry on Card A1 is the unique analytical number, an integer. The second entry, also an integer, is the number of coefficients, \( A_i \), which are on the A2 cards. If this entry is blank or zero, then the code assumes that a user-supplied function will be supplied for the parameter which uses this particular analytical function.

C.13.2 Card A2

The A2 cards contain from one to four ordered pairs, where each ordered pair is defined as follows: the first element of an ordered pair consists of an integer \( i \); the second element consists of the value of the coefficient \( A_i \). The A2
cards will be continued until each coefficient in the analytical function is defined. If the second entry on Card A1 is blank or zero, then the related A2 card is omitted. The A cards are omitted if the fourth entry on Card 3 is blank or zero.

C.14 Tabular Function Data (Cards T1 and T2)
Each tabular function is numbered consecutively beginning with one up to a maximum of 25 functions. The tabular function is assumed to be a set of linearly connected points. The function is described by specifying a set of ordered pairs. Each ordered pair contains an independent variable and its functional value. A maximum of 25 points (pairs) is allowed, and linear interpolation is performed between the points by the program. The values of the independent variable must be entered in ascending order.

C.14.1 Card T1
The first entry on the T1 card (an integer) is the tabular function number. The number of points (an integer) is the second entry.

C.14.2 Card T2
The T2 card contains the first four ordered pairs, all floating-point numbers. If there are more than four ordered pairs in the function, they are entered on subsequent T2 cards.
The T cards are omitted if the fifth entry of Card 3 is blank or zero.

C.15 Output Times (Card 0)
Each entry on this card is a floating-point number. Since the second entry on Card 4 indicates the total number of output times which are to be read, the 0 cards are omitted if this entry is blank or zero. The transient output times are entered in chronological order on the 0 cards. There can be eight entries per card, and the 0 card is repeated as often as necessary to describe the output array. The maximum number of output times is 100, not counting the automatic printout which occurs prior to the initial time step.

C.16 Node Numbers for Special Monitoring of Temperatures (Card S)
As optional output, one may specify up to 20 nodes (now increased to 50) whose temperatures will be printed out as a function of the number of iterations for steady-state calculations or the number of time steps for transient calculations. The first entry on Card S contains the total number of nodes whose temperatures are to be tabulated. The remaining fields contain the actual node numbers. If more than seven nodes are specified, their numbers will appear on additional cards. All entries on the S cards are integers. The frequency for printing out the temperatures of such
nodes appears as the fifth entry on Card 4, and the S card(s) is (are) omitted if this entry is blank or zero.

C.17 Initial Temperatures and Melting Ratios
These cards (or card images) are generated as output by HEATING5 if a positive number appears in entry 7 on Card 4; and generally, they are used only when restarting a job by merely inserting the generated deck at this location in the original deck and resubmitting the job. One must enter the unit number of the card reader in the sixth field on Card 4. Entry 6 on Card 4 specifies the unit on which these data are to be read. If the unit specified is other than the standard card input, the user must insure that the appropriate DD card has been supplied to describe the unit. If entry 7 on Card 4 is nonzero, then the code generates these data at the end of a problem and writes it on the unit specified as entry 7. However, the user must insure that the appropriate DD card has been supplied to correctly identify the unit specified as entry 7. If entry 6 on Card 4 is blank or zero, then these cards are omitted.

Since the user may wish to explicitly specify the initial temperature or melting ratio at some point or points, a description of these data is given below.

1. Job Description (Card IT1)
   This card image gives a descriptive title and can contain alphanumeric characters in the first 72
columns. The card may be left blank but it cannot be omitted.

2. Initial Time and Lattice Point Numbers (Card IT2)

The first entry, a floating-point number which occupies the first ten columns on the IT2 card, specifies the initial problem time. This value overrides the initial time which appears on entry 7 of Card 5.

Normally, this deck will have been generated by the code on a previous run. Thus, the code supplies this value as the time for which the following temperature distribution and melting ratios occur. If the user generates the IT deck, then he must insure that the initial problem time is entered here. The second entry on the IT2 card is an integer and contains the total number of lattice points whose initial temperatures are explicitly specified. It occupies columns 11 through 15 of Card IT2. The third entry, also an integer, contains the total number of nodes whose initial melting ratios are explicitly specified. It also occupies a five-column field, namely columns 16 through 20.

C.18 Lattice Point Temperatures (Card IT3)

The IT3 card can contain up to five pairs of numbers with each pair defined as follows:

1. The first member of the pair is a lattice point number. It is an integer and occupies a five-column field.
2. The second member is the initial temperature of the lattice point whose number appears in the first member. It is a floating-point number and occupies a ten-column field.

The number of pairs to be entered on the IT2 cards is specified on entry 2 of Card IT2. Card IT3 is repeated until all pairs have been described. The temperatures specified by this input data override the corresponding temperatures generated by the I cards.

C.18.1 Lattice Point Initial Melting Ratios (Card IT4)
The IT4 card contains initial melting ratios for each node which is currently undergoing a phase change. The format of the IT4 card is as follows:

1. The first entry is the number of a node which is currently undergoing a phase change. It is an integer and the field occupies the first five columns.

2. The second entry is the material number currently undergoing a phase change for the node which was defined in the previous field. This entry is an integer which occupies the sixth through the tenth columns.

3. The third entry, a floating-point number occupying the eleventh through the twentieth columns, contains the initial melting ratio for the portion of the material associated with the node defined on the first two entries of this card as currently changing phase.
The total number of lattice points with initially specified melting ratios or the total number of IT4 cards is the third entry on Card IT2.

C.19 Implicit Transient Technique Parameters

If the problem involves transient calculations (entry 1 on Card 5 is not equal to 1), and if the implicit technique is to be used to calculate estimates to the transient temperature distribution, then the fifth entry (time step) of Card 5 must be left blank and additional data must be supplied on the IP and TP cards. If it is anticipated that similar problems will be run a large number of times, such as ones arising in a parametric study, then it is recommended that the variables on the IP and TP cards be optimized since it could significantly reduce the overall computer time.

C.19.1 Card IP

This entire card or any of its entries may be left blank, and the default values will be used. They are based on experiences with a few two-dimensional BZ models. They are certainly not the best values that can be used in a given problem, but they are probably good starting points. The first five are floating-point numbers, and the last three are integers.

1. The first entry contains one of two convergence criteria which must be met in order for the iterative
technique to terminate successfully at each time step.
The default is 1.0E-5.

2. The second entry contains the second convergence criterion. The default for this parameter is 1.0E-3 meaning the maximum normalized residual must decrease by three orders of magnitude.

3. The third entry contains the convergence criterion for problems involving temperature-dependent parameters. The default is 1.0E-5.

4. The fourth entry defines the implicit technique which will be used to solve the transient problem and its value lies between 0.5 and 1.0. The default is 0.5.

5. The fifth entry defines the initial value of the point successive overrelaxation iteration acceleration parameter. It also defines the method that will be used to update the acceleration parameter. If this entry is positive, then the acceleration parameter will remain constant throughout the calculations and will be equal to the value of this entry. If it is blank or zero, then the parameter will be optimized empirically as a function of time. This appears to be the best option for nonlinear problems. If it is negative, then the acceleration parameter will be calculated using Carre's technique. The absolute value of this entry must be less than 2.0.
6. This entry, an integer, defines the number of time steps between attempts to optimize the acceleration parameter empirically. It is used only when entry 5 is zero or blank. The default value is 1.

7. For the case when the acceleration parameter will be updated empirically (entry 5 is blank or zero), then this entry defines the change-in-number-of-iterations criterion which must be met before the parameter will be updated. The default is 5. For the case when the SOR acceleration parameter will be updated using Carre's technique, this entry defines the number of iterations between updates. The default is 12.

8. The last entry is the change-in-number-of-iterations criterion which is used to determine when a good estimate to the optimum acceleration parameter has been found. This entry is used only when the acceleration parameter will be updated empirically. The default is 2.

C.20 Card TP
When an implicit scheme is used to solve a transient problem, the time step may be variable. This allows the time step to increase as the solution smooths out and to decrease when some parameter varies rapidly with time. The information controlling the value of the time step is automatically adjusted in order to get printouts of the
temperature distribution at the specified time. If the size of the coefficients in the system of equations varies by orders of magnitude (1.0E05 or greater), it has been observed that point-successive overrelaxation iteration may converge very slowly (it may appear to not converge at all). This occurs when the grid spacing or thermal properties vary by orders of magnitude over the problem. It can be observed by examining the stability criterion table in the output. If this appears to be happening, either further subdivide some of the larger nodes or combine some of the smaller ones. In some cases, it may help to use a larger time step size. All seven entries are floating-point numbers.

1. The first entry is the initial time step.

2. After the temperature distribution has been calculated, the current time step is multiplied by a factor. The value of this factor is entered in the second field of the TP card. The default value is 1.0. For many problems whose parameters vary mildly with time and/or temperature, values between 1.0 and 1.3 have been acceptable.

3. The maximum value of the time step is the third entry. Once the time step reaches this value, it is no longer increased. The default is 1.0E50.

4. The fourth entry contains the maximum time that the time step information on this card applies. If the time reaches this value, then a new TP card is read. The default is 1.0E50.
5. The fifth entry contains the maximum temperature change allowed at a node over a time step. The time step size is adjusted according to the procedure outlined before. If this entry is blank or zero, then this feature is not invoked in calculating the time step size.

6. The sixth entry contains the maximum percent of relative temperature change allowed at a node from one time level to the next one. The time step size is adjusted according to the procedure outlined before. If this entry is blank or zero, then this feature is not invoked in calculating the time step size.

7. The seventh and final entry on this card contains the minimum value of the time step. Once the time step size reaches this value, it is no longer decreased. The default is one-tenth of the initial time step size.

C.21 Blank Card

If the user wishes to solve several problems with one run, he merely inserts a blank card between each problem deck.
Appendix D

JCL AND SAMPLE INPUT FOR HEATINGS5

This appendix gives the format of the JCL required to run HEATINGS5 on the mainframe computer at the University of Windsor under the Wylbur system. A sample input is also included after the JCL.

The description of the basement, for which the sample input has been given, is described below, along with other important features.

The wall is 2.443 m deep and 0.41 m of it is above grade. The wall thickness is 0.203 m and its properties are listed in the program. The convection coefficient between the wall and basement air is 8.29 W/m°C. The floor is 0.203 m thick and its properties are listed in the program. The convection coefficient between the floor and basement air is 6.13 W/m°C.

The soil thermal properties are listed in the input. The soil thermal conductivity is a function of time and is described by a tabular function. The convective coefficient between the ground and air is also a function of time and is described by another tabular function. The outdoor and basement indoor air temperatures are inputted as third order
Fourier fits and are described by the built-in analytical function.

The final results are stored on Unit 4. The JCL and the sample input are presented below.

//BASEM JOB (XXX1,YYY),'NAME',CLASS=A,MSGLEVEL=(1,1),
// REGION=3000K
//HEATING5 PROC
//*HEATING5 CREATED JUNE/86 DEPT: ACADEMIC
//*PROGRAMMING
//*LIB: UOW.APROCLIB TANG-DR-TUCKER (MECH)
//FORT EXEC PGM=1EFORT
//SYSPRINT DD SYSOUT=*,DCB=(RECFM=FBF,LRECL=120,BLKSIZE=840)
//SYSLIN DD DSN=6&ICADSET,UNIT=SYSDA,SPACE=(3200,(5,2)),
// DCB=(RECFM=FEB,LRECL=80,BLKSIZE=3200),DISP=(MOD,PASS)
//LKD EXEC PGM=IEWL,PARM='LET,LIST,OVLY',COND=(4,IT,FORT)
//SYSLIB DD DSN=SYS1.FORTLIB,DISP=SHR
// DD DSNNAME=COMP.ACAD#A.FORT.SUBR,DISP=SHR
// DD DSNNAME=COMP.ACAD#A.USEF.FORT.SUBR,DISP=SHR
//OLCLIB DD DSNNAME=MENG.TUCKER.LOADLIB,UNIT=3380,
// VCL=SER=USER02,DISP=SHR
//SYSPRINT DD SYSOUT=*,DCB=(LRECL=121,RECFM=FBK,BLKSIZE=947)
//SYSLIN DD DSN=6&LOADSET,DISP=(OLL,DELETE),
// DCB=(RECFM=FBK,LRECL=80,ELKSIZE=3200)
// DC DSNNAME=WYL.XXXXYY.LKEDLIB,UNIT=3380,VOL=SER=USER03,
// DISP=SHR
//SYSLMOD DD DSNNAME=EGOSET (MAIN),DISP=(NEW,PASS),UNIT=SYSDA,
// SPACE=(1024,(20,10,1)),DCB=BLKSIZE=7294
//SYSUT1 DD UNIT=SYSDA,SPACE=(1024,(20,10),BLSE).

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
DCB=BLKSIZE=7294

GO EXEC PGM=*.LKED.SYSLECD

//FT01F001 DD UNIT=SYSDA,SPACE=(TRK,(1,3))
//FT02F001 DD UNIT=SYSDA,SPACE=(TRK,(1,3))
//FT03F001 DD UNIT=SYSDA,SPACE=(TRK,(1,3))
//FT04F001 DD UNIT=3380,Vol=SER=USEB04,DISP=(NEW,KEEP),
// DSN=MENG..XXXYYY.NAME1,SPACE=(TRK,(1,3)),
// DCB=(RECFM=VBS,IRECL=3620,BLKSIZE=3624)
//FT11F001 DD UNIT=3380,Vol=SER=WORKEK,DISP=(NEW,KEEP),
// DSN=VYL..XXXYYY.NAME2,SPACE=(TRK,(1,3)),
// DCB=(RECFM=VBS,IRECL=100,BLKSIZE=104)
//FT10F001 DD UNIT=3380,Vol=SER=USER04,DISP=(NEW,KEEP),
// DSN=MENG..TUCKER.NAME3,SPACE=(TRK,(1,3)),
// DCB=(RECFM=VBS,IRECL=3620,BLKSIZE=3624)
//FT05F001 DD UNIT=SYSDA,SPACE=(TRK,(1,3))
//FT06F001 DD SYSOUT=A
//FT07F001 DD SYSOUT=B
//FT08F001 DD DDNAME=SYSIN
//PEND
// EXEC HEATING5
//PT0...SYIN DD *

USER-SUPPLIED SUBROUTINES ARE INSERTED HERE.

//GO..SYIN DD *

SAMPLE PROBLEM FOR HEATING5

<table>
<thead>
<tr>
<th>1800</th>
<th>7</th>
<th>9</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer</td>
<td>ID</td>
<td>Soil</td>
<td>Rock</td>
</tr>
<tr>
<td>-------</td>
<td>----</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.0</td>
<td>2.44</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.0</td>
<td>2.44</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2.44</td>
<td>2.643</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2.44</td>
<td>2.643</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>2.44</td>
<td>2.643</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2.44</td>
<td>2.643</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2.643</td>
<td>8.32</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>2.643</td>
<td>8.32</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>2.643</td>
<td>8.32</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>FLOOR</td>
<td>6228.0</td>
</tr>
<tr>
<td>2</td>
<td>SOIL</td>
<td>1.0</td>
<td>1922.0</td>
</tr>
<tr>
<td>3</td>
<td>WALL</td>
<td>4176.0</td>
<td>977.0</td>
</tr>
<tr>
<td>1</td>
<td>18.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>14.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>15.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>1.0</th>
<th>1.0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>22068.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>29844.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>29844.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>11.67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>11.67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>11.67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.32</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>0.0</td>
<td>0.41</td>
<td>1.01</td>
<td>2.24</td>
<td>2.443</td>
<td>2.64</td>
</tr>
<tr>
<td>4.44</td>
<td>8.44</td>
<td>14.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>22.355</td>
<td>2</td>
<td>3.328</td>
<td>3</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>0.735</td>
<td></td>
<td></td>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td>---</td>
<td>------</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>----</td>
</tr>
<tr>
<td>6</td>
<td>1.263</td>
<td>7</td>
<td>0.334</td>
<td>10</td>
<td>1.0719</td>
</tr>
<tr>
<td>11</td>
<td>193.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13.6472</td>
<td>2</td>
<td>13.2061</td>
<td>3</td>
<td>1.1805</td>
</tr>
<tr>
<td>5</td>
<td>0.8706</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.2709</td>
<td>8</td>
<td>0.425</td>
<td>9</td>
<td>1.1805</td>
</tr>
<tr>
<td>11</td>
<td>183.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>7164.0</td>
<td>2160.0</td>
<td>7164.0</td>
<td>2161.0</td>
</tr>
<tr>
<td>2880.0</td>
<td>4968.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2881.0</td>
<td>4356.0</td>
<td>6552.0</td>
<td>4356.0</td>
<td>6553.0</td>
<td>4968.0</td>
</tr>
<tr>
<td>7296.0</td>
<td>4968.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7297.0</td>
<td>7164.0</td>
<td>10920.0</td>
<td>7164.0</td>
<td>10921.0</td>
<td>4968.0</td>
</tr>
<tr>
<td>11640.0</td>
<td>4968.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11641.0</td>
<td>4356.0</td>
<td>15312.0</td>
<td>4356.0</td>
<td>15313.0</td>
<td>4968.0</td>
</tr>
<tr>
<td>16056.0</td>
<td>4968.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16057.0</td>
<td>7164.0</td>
<td>17520.0</td>
<td>7164.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>122652.0</td>
<td>2160.0</td>
<td>122652.0</td>
<td>2161.0</td>
</tr>
<tr>
<td>2980.0</td>
<td>102204.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2881.0</td>
<td>81756.0</td>
<td>6552.0</td>
<td>81756.0</td>
<td>6553.0</td>
<td>102204.0</td>
</tr>
<tr>
<td>7296.0</td>
<td>102204.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7297.0</td>
<td>122652.0</td>
<td>10920.0</td>
<td>122652.0</td>
<td>10921.0</td>
<td>102204.0</td>
</tr>
<tr>
<td>11640.0</td>
<td>102204.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11641.0</td>
<td>81756.0</td>
<td>15312.0</td>
<td>81756.0</td>
<td>15313.0</td>
<td>102204.0</td>
</tr>
<tr>
<td>16056.0</td>
<td>102204.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16057.0</td>
<td>122652.0</td>
<td>19680.0</td>
<td>122652.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

//

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Appendix E

DESCRIPTION OF DPENLN

E.1 PURPOSE

Given a model

$$\hat{Y}_i = f(x_{i1}, x_{i2}, \ldots x_{im}; b_1, b_2, \ldots b_k)$$

which predicts the value, \( \hat{Y} \), of a dependent variable \( Y \), where the model \( f \) contains \( k \) independent variables \( x_i \) and \( k \) parameters \( b_j \), and given \( n \) observations

$$(Y_i, x_{i1}, x_{i2}, \ldots x_{im}), \ i = 1, 2, \ldots n,$$

this program will compute the least-squares estimates \( b_j \). That is, the program will adjust the to minimize

$$\phi = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

The program provides options to use analytic or estimated derivatives \( \frac{\partial f}{\partial b_j} \), to control printing, to omit parameters (i.e., fix their values and obtain a constrained minimization of \( \phi \)), and to plot the observed and predicted values. Confidence limits for the estimated parameters are also computed. The program is written in FORTRAN IV, employing the maximum neighbourhood algorithm and makes no use of machine-code inserts.

- 144 -
**E.2 PROCEDURE**

The user must supply the following items:

1. A FORTRAN subroutine named "PCODE" to evaluate \( \hat{Y} \) for a specified combination of \( x_{jl} \), and \( b_j \). F coding evaluates the function \( \hat{Y}_i \) for one data point each time it is executed.

2. A FORTRAN subroutine named "PCODE" to evaluate for \( b_1, \ldots, b_k \). P coding evaluates the analytic derivatives of the function \( \hat{Y}_i \) for the same data point just processed by F coding.

3. A FORTRAN subroutine named "SUBZ" which is to be performed once as soon as the data are read in, e.g., to compute constants needed by F coding and P coding.

4. The data \( (y_i, x_{i1}, \ldots, x_{im}), i = 1, \ldots, n \).

5. A call to the subroutine DPENLN: CALL DPENLN.

During execution of the program, F coding will sometimes be used alone and will sometimes be followed immediately by P coding. If analytic derivatives (in P coding) are not supplied, or if it is desired not to use them, the program will calculate estimates of \( \frac{\partial \hat{Y}_i}{\partial b_j} \) using finite-difference approximations. It should be recognized that use of estimated derivatives will usually increase the computing time relative to the use of analytic derivatives. The initial guesses for the \( b_j \) must be different from zero if estimated derivatives are used, since the estimated derivatives are calculated by the form:
where the multiplier $\Delta = 10^{-5}$ is used unless overridden by input option.

### E.3 INITIALIZATION OF CODING

The FORTRAN coding to read in case constants, perform preliminary calculations required by a case, or any other one-time operations desired, is called for by CALL SUBZ(Y, X, B, PRNT, NPBNT, N). The dimensioned variables are $Y(500)$, $X(500,10)$, $B(50)$, $PBNT(5)$. Variables used in more than one subroutine must be linked through COMMON statements. As described in the Output section, as many as five words of auxiliary information (e.g., values of independent variables, etc.) may be printed alongside the values. NPBNT is the number of such auxiliary words ($NPBNT < 5$). The program assumes $NPBNT = 0$ unless SUBZ sets $NPBNT = 0$.

### E.4 F Coding

The FORTRAN coding to evaluate the function $F$ is called for by CALL FCODE (Y, X, B, PRNT, F, I) with the dimensioned variables being $Y(500)$, $X(500,10)$, $B(50)$, $PBNT(5)$.

<table>
<thead>
<tr>
<th>Mathematical Symbol</th>
<th>FORTRAN Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(I,L)$</td>
<td>$X(I,L)$</td>
</tr>
<tr>
<td>$Y(I)$</td>
<td>$Y(I)$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$B(J)$</td>
<td>$B(J)$</td>
</tr>
</tbody>
</table>
If NPENT = 0, then P coding must set up PBNT(JJ), JJ = 1,...,NPENT, to contain the desired output information.

E.5 P Coding

The FORTRAN coding to evaluate \( \frac{dy_i}{db_j} \), \( j = 1,...,k \) is called for by CALL PCODE(F, X, B, PBNT, F, I). The dimensioned variables are P(50), X(500,10), B(50), PBNT(5). The functions \( \frac{dy_i}{db_j} \) are labeled P(J). This subroutine must be included even if it is empty.

E.6 Input preparation

The input cards for a particular case are assembled in the following order, all items being required always except where indicated otherwise:

<table>
<thead>
<tr>
<th>Input Item No.</th>
<th>Mathematical Symbol</th>
<th>FORTRAN Label</th>
<th>Format Card Columns</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>n</td>
<td>N</td>
<td>I3 1-3</td>
<td>No. of data points</td>
</tr>
<tr>
<td></td>
<td>k</td>
<td>K</td>
<td>I3 4-6</td>
<td>Total No. of parameters (includes fixed parameters)</td>
</tr>
<tr>
<td></td>
<td>p</td>
<td>IP</td>
<td>I3 7-9</td>
<td>No. of omitted parameters</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>M</td>
<td>I3 10-12</td>
<td>No. of independent variables</td>
</tr>
<tr>
<td></td>
<td>IPP</td>
<td>I3 13-15</td>
<td>IFP= 000 to tabulate ( y_i, \hat{y}_i, (Y_i - \hat{Y}_i) )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>PRNT(1),...,PBNT(5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>IFP=001 to plot ( y_i, \hat{y}_i )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>IWS1</td>
<td>I3 1-3</td>
<td>Does not apply</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IWS2</td>
<td>I3 4-6</td>
<td>0 Analytic Der.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 Estimated Der.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IWS3</td>
<td>I3 7-9</td>
<td>0 Abbreviated Printout</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>= 1 Detailed Prt.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IWS4</td>
<td>I3 10-12</td>
<td>0 No Branch off</td>
<td></td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
= # Forced Branch to
Confidence Region Calculation
after # Iterations
IWS5 I3 13-15 = 0
IWS6 I3 16-18 = 0 Nonlinear Confidence Limits Desired
= 1 Omit Nonlinear Confidence Limits

3

YMN F10.0 1-10 Left Side of Plot
SPRD F10.0 11-20 Spread of Plot

(Card 3 is required only if IFP = 001; Format statement for
Card 3 is 930. Card 3 is omitted if IFP = 000."

4

IB(1) I3 1-3 Subscripts of
omitted b's
IB(2) I3 4-6
 . .
IB(IP) . .

(Item 4 is required only if IP>0. Omit item if IP=0. Format
statement is 900; note that two cards are required if IP>25.
A zero or blank subscript will give an error message and
cause program to go to next case.)

5

FF F10.0 1-10 Variance ratio Stat.

T F10.0 11-20 Student's t
E F10.0 21-30 Convergence Crit.
TAU F10.0 31-40 Convergence Crit.
XL F10.0 41-50 Program Parameter
GAMCR F10.0 51-60 Critical Angle
DEL F10.0 61-70 Used in finite-difference Ders.
ZETA F10.0 71-80 Singularity Crit.
for Matrix Inversion

(The F and T statistical test values can be found in the
Tables and the others set by the programmer. However, any or
all of the quantities in Item 5 may be left blank on the
Card. If this is done, the program will supply the following
values as being reasonable for most situations:

\[
\begin{align*}
FF &= 4.0, \ T = 2.0, \ E = 5 \times 10^{-5}, \ TAU = 10^{-3}, \ XL = 10^{-3}, \\
GAMCR &= \pi/4, \ DEL = 10^{-5}, \ ZETA = 10^{-31}
\end{align*}
\]

6

B(1) F10.0 1-10 Initial guess for
parameters, 8/card
B(2) F10.0 11-20

etc. F10.0 thru 61-70

(If some of the parameters are fixed and hence are omitted
parameters, then their values can be input here.)

7 This item is a single card containing the format state-
ment according to which the data (Item 8) are to be
read. Column 1 contains the opening parenthesis of the
format statement; the statement must end at or before
column 60.

8 This item consists of n sub-items. Each sub-item is the
input data for one observation, punched according to the
format statement in Item 7. The sequence of the varia-
tions in a sub-item must be: Y(I), X(I,1), X(I,2),
through X(I,M). Each Y(I) must begin on a new card.

As supplied the program has the dimensions N=50C, K=50,
M=10. These limits can readily be altered by changing
the dimension statements.

9 Any case data read in from subroutine SUBZ should go
in here.

10 Sequential cases may be stacked by repeating Items 1
through 9. A blank after the last case will cause a
normal stop without error messages. Without the blank
card, a systems stop indicating end of file on data
will occur.

E.7 Operating Procedure

Assemble the main program deck (a call to the subroutine:
CALL DPENLN) as well as subroutines SUBZ, FCODE, PCODE
according to usual procedure. Data for as many cases as
desired can follow.
**E.8 Output**

The program initially prints

```
N=  K=  P=  M=  GAMMA CEIT=  DEL=
FF=  T=  E=  TAU=  XL=  ZETA=
```

With IWS3=1, and IPP=000, detailed printout is obtained at each iteration. The observed value, the predicted value of the dependent variable, the difference between the two, PRNT(1), ..., PRNT(5), are printed out in that order. If IPP=001, then the lines containing Y(I), F, etc. are replaced by a plot showing observed and predicted values for visual monitoring of the fit. With IWS3=0, the abbreviated printout at each iteration is in the format:

```
PARAMETERS  B(1) .....  B(K)
PHI  SE LENGTH  GAMMA LAMDA  ANALYTIC PARTIALS USED
```

The program may branch to the following confidence-region calculations by any of four routes. One of the following messages will appear on the printout to identify the route taken:

<table>
<thead>
<tr>
<th>Message</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPSILON TEST</td>
<td>Standard Convergence route</td>
</tr>
<tr>
<td>GAMMA LAMBDA TEST</td>
<td>Alternate Convergence route</td>
</tr>
<tr>
<td>GAMMA EPSILON TEST</td>
<td>Alternate Convergence route</td>
</tr>
<tr>
<td>FORCE OFF</td>
<td>Via IWS4</td>
</tr>
</tbody>
</table>

One complete iteration of detailed printout is printed, either with tabulation of Y(I), F, etc. (IPP=0) only, or with plotting (IPP>0) as well.
Symbol definition: \( \Phi \) : Sum of squares of residuals;

\( SE \) : Standard error of the estimate

\( \mathrm{GAMMA \ CRIT} \) : The critical direction in which the parameters are incremented.

**Example**

It is desired to fit the model

\[
Y_i = b_1 e^{b_2 x_{i1}} + b_3 x_{i2}^2
\]

to the following data.

<table>
<thead>
<tr>
<th>( Y )</th>
<th>( X )</th>
<th>( X )</th>
<th>( Y )</th>
<th>( X )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.6487</td>
<td>0.5</td>
<td>0.5</td>
<td>4.6065</td>
<td>-0.5</td>
<td>3.0</td>
</tr>
<tr>
<td>6.7183</td>
<td>1.0</td>
<td>1.0</td>
<td>16.6065</td>
<td>-0.5</td>
<td>-2.0</td>
</tr>
<tr>
<td>13.4817</td>
<td>1.5</td>
<td>1.5</td>
<td>64.3679</td>
<td>-1.0</td>
<td>-4.0</td>
</tr>
<tr>
<td>23.3891</td>
<td>2.0</td>
<td>2.0</td>
<td>36.1353</td>
<td>-2.0</td>
<td>-3.0</td>
</tr>
<tr>
<td>37.1820</td>
<td>2.5</td>
<td>2.5</td>
<td>100.6498</td>
<td>-3.0</td>
<td>5.0</td>
</tr>
<tr>
<td>18.7183</td>
<td>1.0</td>
<td>2.0</td>
<td>11.3891</td>
<td>2.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>24.0860</td>
<td>3.0</td>
<td>1.0</td>
<td>24.0860</td>
<td>3.0</td>
<td>-1.0</td>
</tr>
<tr>
<td>37.1150</td>
<td>3.5</td>
<td>1.0</td>
<td>144.2231</td>
<td>-1.5</td>
<td>6.0</td>
</tr>
<tr>
<td>70.5980</td>
<td>4.0</td>
<td>2.0</td>
<td>16.1353</td>
<td>-2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>126.0170</td>
<td>4.5</td>
<td>3.0</td>
<td>16.1353</td>
<td>-2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

First, the subroutine FCCDE is prepared to evaluate the \( Y \) value for a specific combination of \( x_{i1} \) and \( b_j \). Hence, \( \text{FCODE becomes:} \)

```plaintext
SUBROUTINE FCCDE (Y, X, B, PRNT, F, I)
DIMENSION Y(500), X(500,10), B(50)
F=B(1)*EXP(B(2)*X(I,1))+B(3)*X(I,2)*X(I,2)
PRNT(1)=I(I,1)
PRNT(2)=X(I,2)
RETURN
END
```

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Next subroutine **PCODE** is prepared to evaluate the derivatives of the model with respect to each of the $K$ parameters, in this case, the three parameters, $B(1)$, $B(2)$, and $B(3)$. Analytic derivatives are used, since the model is readily differentiable. The subroutine **PCODE** is:

```
SUBROUTINE PCODE (P, X, B, PRNT, F, I)
DIMENSION P(50), X(500,10), B(50), PRNT(5)
P(1)=EXP (B(2)*X(I,1))
P(2)=B(1)*X(I,1)*EXP (B(2)*X(I,1))
P(3)=X(I,2)**2
RETURN
END
```

In the present problem the data and constants are in the desired form, hence **SUBZ** will only be used to indicate that two extra variables are to be printed.

```
SUBROUTINE SUBZ(P, X, B, PRNT, F, I)
DIMENSION P(500), X(500,10), B(50), PRNT(5)
NPRNT=2
RETURN
END
```

A sample set of input data for the above problem is given below (b represents a blank and the first digit or letter starts in column 1).

**Item 1:** 020bb3bb0002000; This specifies 20 data points, 3 parameters, that there are no omitted parameters, that there are two independent variables, and that a tabulation is desired.

**Item 2:** bbbbb0bb0bb0bb0000; This indicates that analytic derivatives are being used, an abbreviated print out is required, that there is to be no force off, and that non-linear confidence limits are to be given.

**Item 3:** omitted (no card)

**Item 4:** omitted (no card)

**Item 5:** omitted, blank card is inserted
Item 6: 2.bbbbbbb4.bbbbbbbbb6. This specifies our initial guesses for the parameters.

Item 7: In this case, 1 point can be put on each card, hence the following is adequate:

\[ \{F12.6, 2X, 2F10.3\} \]

Item 8: The first point will be punched as:

2.6487bbbbbb51bbbbbb5 i.e. Y value and the two X values etc.

Item 9: card omitted, since there is no input for subroutine SUBZ.

Item 10: A blank card is inserted since only one set of data is being fit.
Appendix F

DATA USED AS INPUT FOR DPENLN

In this appendix the data, which was used as input to DPENLN, is presented in a tabular manner. Table 12 contains the data for the steady state case, with the wall uninsulated. Table 13 contains the data for the steady state case, with the wall insulated. Table 14 contains the data for the transient case, with the wall uninsulated. Table 15 contains the data for the transient case, with the wall insulated.
TABLE 12: STEADY STATE CASE (WALL UNINSULATED)

<table>
<thead>
<tr>
<th>L (m)</th>
<th>K (W/m·K)</th>
<th>ΔT (°C)</th>
<th>q_w (W/m²)</th>
<th>q_f (W/m²)</th>
<th>q_f,c (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>12.0</td>
<td>17.464</td>
<td>3.134</td>
<td>2.084</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>6.0</td>
<td>10.259</td>
<td>2.281</td>
<td>1.532</td>
</tr>
<tr>
<td>1.5</td>
<td>2.0</td>
<td>15.0</td>
<td>28.725</td>
<td>7.358</td>
<td>5.456</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
<td>19.0</td>
<td>39.654</td>
<td>11.31</td>
<td>8.582</td>
</tr>
<tr>
<td>1.5</td>
<td>3.0</td>
<td>10.0</td>
<td>22.355</td>
<td>6.932</td>
<td>5.394</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>8.0</td>
<td>9.554</td>
<td>1.936</td>
<td>1.348</td>
</tr>
<tr>
<td>2.0</td>
<td>1.5</td>
<td>12.0</td>
<td>17.181</td>
<td>4.260</td>
<td>3.126</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>13.0</td>
<td>21.196</td>
<td>5.997</td>
<td>4.597</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5</td>
<td>16.0</td>
<td>28.818</td>
<td>8.965</td>
<td>6.988</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>5.0</td>
<td>9.751</td>
<td>3.269</td>
<td>2.513</td>
</tr>
<tr>
<td>2.5</td>
<td>1.0</td>
<td>19.0</td>
<td>19.328</td>
<td>4.426</td>
<td>3.310</td>
</tr>
<tr>
<td>2.5</td>
<td>1.5</td>
<td>6.0</td>
<td>7.424</td>
<td>2.048</td>
<td>1.471</td>
</tr>
<tr>
<td>2.5</td>
<td>2.0</td>
<td>10.0</td>
<td>14.259</td>
<td>4.439</td>
<td>3.433</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>12.0</td>
<td>19.096</td>
<td>6.487</td>
<td>5.210</td>
</tr>
<tr>
<td>2.5</td>
<td>3.0</td>
<td>15.0</td>
<td>26.026</td>
<td>9.468</td>
<td>7.724</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>7.0</td>
<td>6.2175</td>
<td>1.604</td>
<td>1.165</td>
</tr>
<tr>
<td>3.0</td>
<td>1.5</td>
<td>5.0</td>
<td>5.478</td>
<td>1.678</td>
<td>1.287</td>
</tr>
<tr>
<td>3.0</td>
<td>2.0</td>
<td>12.0</td>
<td>15.278</td>
<td>5.231</td>
<td>4.168</td>
</tr>
<tr>
<td>3.0</td>
<td>2.5</td>
<td>10.0</td>
<td>14.307</td>
<td>5.302</td>
<td>4.291</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>15.0</td>
<td>23.550</td>
<td>9.302</td>
<td>7.662</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>$K$ (W/m·K)</td>
<td>$\Delta T$ (°C)</td>
<td>$R_T$ ($m^2·K/W$)</td>
<td>$q_{W,i}$ (W/m$^2$)</td>
<td>$q_{f,i}$ (W/m$^2$)</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8</td>
<td>8.0</td>
<td>1.1594</td>
<td>4.449</td>
<td>2.055</td>
</tr>
<tr>
<td>1.5</td>
<td>2.0</td>
<td>10.0</td>
<td>1.6209</td>
<td>5.734</td>
<td>5.666</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>12.0</td>
<td>2.0825</td>
<td>5.942</td>
<td>5.428</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>14.0</td>
<td>2.5441</td>
<td>5.140</td>
<td>4.559</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
<td>16.0</td>
<td>3.0057</td>
<td>6.667</td>
<td>10.945</td>
</tr>
<tr>
<td>2.0</td>
<td>1.0</td>
<td>10.0</td>
<td>1.1594</td>
<td>5.166</td>
<td>2.912</td>
</tr>
<tr>
<td>2.0</td>
<td>1.5</td>
<td>12.0</td>
<td>1.6209</td>
<td>5.684</td>
<td>5.124</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>14.0</td>
<td>2.0825</td>
<td>6.088</td>
<td>7.710</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5</td>
<td>8.0</td>
<td>2.5441</td>
<td>3.228</td>
<td>5.304</td>
</tr>
<tr>
<td>2.0</td>
<td>0.8</td>
<td>16.0</td>
<td>3.0057</td>
<td>4.357</td>
<td>4.100</td>
</tr>
<tr>
<td>2.5</td>
<td>0.8</td>
<td>8.0</td>
<td>1.1594</td>
<td>3.428</td>
<td>1.815</td>
</tr>
<tr>
<td>2.5</td>
<td>1.0</td>
<td>10.0</td>
<td>1.6209</td>
<td>3.805</td>
<td>2.884</td>
</tr>
<tr>
<td>2.5</td>
<td>1.5</td>
<td>12.0</td>
<td>2.0825</td>
<td>4.419</td>
<td>5.049</td>
</tr>
<tr>
<td>2.5</td>
<td>2.0</td>
<td>14.0</td>
<td>2.5441</td>
<td>4.908</td>
<td>7.609</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>16.0</td>
<td>3.0057</td>
<td>5.318</td>
<td>10.480</td>
</tr>
<tr>
<td>3.0</td>
<td>2.5</td>
<td>8.0</td>
<td>1.1594</td>
<td>4.677</td>
<td>4.950</td>
</tr>
<tr>
<td>3.0</td>
<td>2.0</td>
<td>10.0</td>
<td>1.6209</td>
<td>4.383</td>
<td>5.232</td>
</tr>
<tr>
<td>3.0</td>
<td>1.5</td>
<td>12.0</td>
<td>2.0825</td>
<td>4.107</td>
<td>4.970</td>
</tr>
<tr>
<td>3.0</td>
<td>1.0</td>
<td>14.0</td>
<td>2.5441</td>
<td>3.741</td>
<td>4.095</td>
</tr>
<tr>
<td>3.0</td>
<td>0.8</td>
<td>16.0</td>
<td>3.0057</td>
<td>3.603</td>
<td>3.872</td>
</tr>
</tbody>
</table>
TABLE 14 : TRANSIENT ( WALL UNINSULATED )

<table>
<thead>
<tr>
<th>L</th>
<th>$K$ (W/m·K)</th>
<th>$\rho$ (kg/m³)</th>
<th>$C_p$ (J/kg·K)</th>
<th>$T_M$ (°C)</th>
<th>$T_A$ (°C)</th>
<th>$q_{w,a}$ (W/m²)</th>
<th>$q_{f,a}$ (W/m²)</th>
<th>$\delta_w$ (rad)</th>
<th>$\delta_f$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 m</td>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>8.9</td>
<td>11.4</td>
<td>13.657</td>
<td>0.158</td>
<td>0.397</td>
<td>1.115</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1490</td>
<td>1765</td>
<td>8.9</td>
<td>11.4</td>
<td>14.328</td>
<td>0.160</td>
<td>0.476</td>
<td>1.093</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1800</td>
<td>1200</td>
<td>8.9</td>
<td>11.4</td>
<td>14.504</td>
<td>0.150</td>
<td>0.532</td>
<td>1.036</td>
</tr>
<tr>
<td></td>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>5.7</td>
<td>8.9</td>
<td>10.662</td>
<td>0.159</td>
<td>0.310</td>
<td>1.114</td>
</tr>
<tr>
<td></td>
<td>1.38</td>
<td>1922</td>
<td>1670</td>
<td>4.6</td>
<td>12.1</td>
<td>17.018</td>
<td>0.172</td>
<td>0.711</td>
<td>1.098</td>
</tr>
<tr>
<td>2.0 m</td>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>8.9</td>
<td>11.4</td>
<td>10.771</td>
<td>0.185</td>
<td>0.253</td>
<td>1.396</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1800</td>
<td>1200</td>
<td>8.9</td>
<td>11.4</td>
<td>11.556</td>
<td>0.175</td>
<td>0.356</td>
<td>1.277</td>
</tr>
<tr>
<td></td>
<td>1.38</td>
<td>1922</td>
<td>1670</td>
<td>4.6</td>
<td>12.1</td>
<td>13.709</td>
<td>0.203</td>
<td>0.477</td>
<td>1.341</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>1820</td>
<td>1150</td>
<td>11.3</td>
<td>8.5</td>
<td>11.814</td>
<td>0.170</td>
<td>0.803</td>
<td>1.074</td>
</tr>
<tr>
<td></td>
<td>2.50</td>
<td>1922</td>
<td>1670</td>
<td>4.6</td>
<td>12.1</td>
<td>18.026</td>
<td>0.195</td>
<td>1.198</td>
<td>1.179</td>
</tr>
<tr>
<td></td>
<td>2.50</td>
<td>1820</td>
<td>1150</td>
<td>10.6</td>
<td>11.0</td>
<td>16.912</td>
<td>0.165</td>
<td>1.453</td>
<td>1.002</td>
</tr>
<tr>
<td>2.5 m</td>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>11.3</td>
<td>8.5</td>
<td>6.594</td>
<td>0.208</td>
<td>0.126</td>
<td>1.478</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1490</td>
<td>1765</td>
<td>5.9</td>
<td>14.6</td>
<td>12.021</td>
<td>0.211</td>
<td>0.270</td>
<td>1.534</td>
</tr>
<tr>
<td></td>
<td>1.38</td>
<td>1922</td>
<td>1670</td>
<td>4.6</td>
<td>12.1</td>
<td>11.421</td>
<td>0.229</td>
<td>0.333</td>
<td>1.566</td>
</tr>
<tr>
<td></td>
<td>2.00</td>
<td>1922</td>
<td>1670</td>
<td>5.7</td>
<td>8.9</td>
<td>10.151</td>
<td>0.227</td>
<td>0.462</td>
<td>1.436</td>
</tr>
</tbody>
</table>
### TABLE 15: TRANSIENT (WALL INSULATED)

<table>
<thead>
<tr>
<th>$K$ (W/m·K)</th>
<th>$\rho$ (kg/m³)</th>
<th>$C_p$ (J/kg·K)</th>
<th>$R_T$ (m²·K/W)</th>
<th>$T_A$ (°C)</th>
<th>$q_{w,a,i}$ (W/m²)</th>
<th>$q_{f,a,i}$ (W/m²)</th>
<th>$\delta_{w,i}$ (rad)</th>
<th>$\delta_{f,i}$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>1.1594</td>
<td>11.4</td>
<td>5.130</td>
<td>0.259</td>
<td>0.680</td>
<td>0.921</td>
</tr>
<tr>
<td>0.88</td>
<td>1490</td>
<td>1765</td>
<td>2.5441</td>
<td>11.4</td>
<td>2.958</td>
<td>0.321</td>
<td>0.741</td>
<td>0.933</td>
</tr>
<tr>
<td>1.00</td>
<td>1490</td>
<td>1765</td>
<td>1.6209</td>
<td>8.9</td>
<td>3.284</td>
<td>0.282</td>
<td>0.636</td>
<td>0.922</td>
</tr>
<tr>
<td>2.50</td>
<td>1820</td>
<td>1150</td>
<td>3.0057</td>
<td>11.0</td>
<td>3.419</td>
<td>0.272</td>
<td>2.423</td>
<td>0.849</td>
</tr>
<tr>
<td>2.50</td>
<td>1820</td>
<td>1150</td>
<td>1.1594</td>
<td>14.6</td>
<td>8.926</td>
<td>0.212</td>
<td>3.148</td>
<td>0.821</td>
</tr>
</tbody>
</table>

$L = 1.5$ m

| 1.38        | 1922           | 1670           | 1.1594         | 12.1       | 5.088               | 0.322               | 0.769               | 1.186               |
| 2.00        | 1922           | 1670           | 1.6209         | 8.9        | 3.288               | 0.329               | 0.939               | 1.144               |
| 0.88        | 1490           | 1765           | 2.5441         | 11.4       | 2.417               | 0.381               | 0.531               | 1.160               |
| 2.50        | 1820           | 1150           | 3.0057         | 11.0       | 2.888               | 0.311               | 2.043               | 0.971               |
| 0.88        | 1490           | 1765           | 3.0057         | 11.4       | 2.135               | 0.398               | 0.538               | 1.168               |

$L = 2.0$ m

| 1.00        | 1490           | 1765           | 1.1594         | 14.0       | 4.634               | 0.352               | 0.473               | 1.380               |
| 1.38        | 1922           | 1670           | 1.6209         | 11.4       | 3.227               | 0.402               | 0.567               | 1.407               |
| 0.88        | 1490           | 1765           | 2.0825         | 10.1       | 2.103               | 0.414               | 0.330               | 1.390               |
| 2.50        | 1820           | 1150           | 3.0057         | 12.1       | 2.797               | 0.351               | 1.920               | 1.091               |

$L = 2.5$ m

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Appendix G

SAMPLE CALCULATION

A sample calculation, illustrating the use of the method developed, is presented in this appendix. A description of the basement, soil properties and temperature data used in the calculation follows. The wall and floor are both uninsulated.

Length $H = 12.2$ m
Width $W = 7.62$ m
Basement Wall Depth $L = 2.14$ m
Wall resistance (including film coefficient)
$$R_w = 0.26 \text{ m}^2 \cdot \text{K/W}$$
Floor Resistance (including film coefficient)
$$R_f = 0.26 \text{ m}^2 \cdot \text{K/W}$$
Soil Conductivity, $K = 1.3$ W/m·K
Soil Density, $\rho = 960$ kg/m
Soil Specific Heat, $C_p = 2100$ J/kg·K
Mean Annual Ground Surface Temperature, $T_M = 10.0$ °C
Amplitude of the Ground Surface Temperature, $T_A = 12.0$ °C
Basement Indoor Air
Temperature, $T_B = 21.1 \degree C$

Day of Occurrence of Minimum

Ground Surface Temperature = Jan. 20

The ground surface temperature is described by eqn. (3.1). The time $t$ is now measured in 5-day steps. For Jan. 20, $t$ has a value of 4. When $t$ equals 4, the sine term must have a value of $-1.0$. This gives $\psi$ a value of 50.75.

From eqn. (4.3), the value of the average mean annual wall heat flux is

$$q_w = 14.4077 \text{ (W/m}^2\text{)}$$

From eqn. (4.8), the value of the average mean annual floor heat flux (for the $3 \text{ m}$ of the floor from the corner) is

$$q_f = 3.4644 \text{ (W/m}^2\text{)}$$

The value of the steady state heat flux from the floor centre is calculated from eqn. (4.9) (using the appropriate regression coefficients) to be

$$q_{f,c} = 2.9601 \text{ (W/m}^2\text{)}$$

The value of the amplitude of the average wall heat flux is calculated from eqn. (4.12) to be

$$q_{w,a} = 13.0221 \text{ (W/m}^2\text{)}$$
From eqn. (4.12) (using the appropriate regression coefficients), the amplitude of the average floor heat flux (for the 3 m of the floor from the corner) is

\[ q_{f,a} = 0.1957 \text{ (W/m}^2\text{)} \]

The phase lag of the average wall heat flux is calculated from eqn. (4.13) to be

\[ \delta_w = 0.1762 \text{ (rad)} \]

The phase lag of the average floor heat flux is calculated from eqn. (4.13) (using the appropriate coefficients), to be

\[ \delta_f = 1.2107 \text{ (rad)} \]

The wall area \( A_w \) is given by

\[ A_w = 2.0 \times (12.2 + 7.62) \times 2.14 \text{ (m}^2\text{)} \]
\[ = 84.83 \text{ (m}^2\text{)} \]

The floor area \( A_f \) (consisting of the 3 m of the floor from the corner) is given by

\[ A_f = 12.2 \times 7.62 - (12.2-6.0) \times (7.62-6.0) \text{ (m}^2\text{)} \]
\[ = 82.92 \text{ (m}^2\text{)} \]

The floor centre area \( A_{f,c} \) is given by

\[ A_{f,c} = (12.2-6.0) \times (7.62-6.0) \text{ (m}^2\text{)} \]
\[ = 10.04 \text{ (m}^2\text{)} \]
Therefore the annual variation of the total wall heat flux is described by

$$A_w [q_w + q_w, a \sin (\omega(t + \psi) + \pi - \delta_w)]$$

where

$$\omega = \frac{2\pi}{73} \text{ rad/5-day}$$

Time t goes from 1 to 73 for one full year. The value \( t=1 \) covers a 5-day period from Jan. 1 to Jan. 5. The value of the heat flux for \( t=1 \) is the average value for this 5-day period. A positive value of the heat flux means a heat loss from the basement; a negative value corresponds to a heat gain by the basement. Thus only positive values of heat fluxes should be used in calculating annual basement heat losses.

The annual variation of the total floor heat flux (for the 3 m of the floor from the corner) is described by

$$A_f [q_f + q_f, a \sin (\omega(t + \psi) + \pi - \delta_f)]$$

The total steady state heat loss from the floor centre is given by

$$A_{f,c} * q_{f,c} = 29.72 \text{ (W)}$$

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Appendix H

COMPARISON OF ORIGINAL AND PREDICTED DATA

This appendix contains the comparison of the original data and the data predicted by the various regression equations. The comparisons are presented in a tabular manner. The tables are given on the following pages.
<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>100* (orig - pred)/orig</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0842</td>
<td>2.3238</td>
<td>-11.49</td>
</tr>
<tr>
<td>1.5325</td>
<td>1.6970</td>
<td>-10.74</td>
</tr>
<tr>
<td>5.4557</td>
<td>5.5120</td>
<td>-1.03</td>
</tr>
<tr>
<td>8.5820</td>
<td>8.5094</td>
<td>0.85</td>
</tr>
<tr>
<td>5.3944</td>
<td>5.2434</td>
<td>2.80</td>
</tr>
<tr>
<td>1.3486</td>
<td>1.4951</td>
<td>-10.86</td>
</tr>
<tr>
<td>3.1263</td>
<td>3.2787</td>
<td>-4.87</td>
</tr>
<tr>
<td>4.5975</td>
<td>4.6186</td>
<td>-0.46</td>
</tr>
<tr>
<td>6.9882</td>
<td>6.9339</td>
<td>0.78</td>
</tr>
<tr>
<td>2.5133</td>
<td>2.5389</td>
<td>-1.02</td>
</tr>
<tr>
<td>3.3102</td>
<td>3.5051</td>
<td>-5.89</td>
</tr>
<tr>
<td>3.4328</td>
<td>3.5092</td>
<td>-2.22</td>
</tr>
<tr>
<td>5.2105</td>
<td>5.1381</td>
<td>1.39</td>
</tr>
<tr>
<td>7.7238</td>
<td>7.5274</td>
<td>2.54</td>
</tr>
<tr>
<td>1.2873</td>
<td>1.3604</td>
<td>-5.67</td>
</tr>
<tr>
<td>4.1684</td>
<td>4.2458</td>
<td>-1.86</td>
</tr>
<tr>
<td>4.2910</td>
<td>4.3163</td>
<td>-0.59</td>
</tr>
<tr>
<td>7.6625</td>
<td>7.5868</td>
<td>0.99</td>
</tr>
</tbody>
</table>
### TABLE 17: COMPARISON OF ORIGINAL DATA \( q_{w,1} \) AND DATA PREDICTED BY EQU. (4.9)

<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>( 100\times (\text{orig} - \text{pred})/\text{orig} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1660</td>
<td>5.1502</td>
<td>0.31</td>
</tr>
<tr>
<td>5.6840</td>
<td>5.8890</td>
<td>-3.60</td>
</tr>
<tr>
<td>6.0880</td>
<td>6.3602</td>
<td>-4.47</td>
</tr>
<tr>
<td>3.2280</td>
<td>3.2761</td>
<td>-1.48</td>
</tr>
<tr>
<td>4.3570</td>
<td>4.1955</td>
<td>3.70</td>
</tr>
<tr>
<td>4.4490</td>
<td>4.3789</td>
<td>1.57</td>
</tr>
<tr>
<td>5.7340</td>
<td>5.6974</td>
<td>0.64</td>
</tr>
<tr>
<td>5.4920</td>
<td>5.7695</td>
<td>-5.04</td>
</tr>
<tr>
<td>5.1400</td>
<td>5.2164</td>
<td>-1.48</td>
</tr>
<tr>
<td>6.6670</td>
<td>6.3578</td>
<td>4.63</td>
</tr>
<tr>
<td>3.4280</td>
<td>3.5972</td>
<td>-4.93</td>
</tr>
<tr>
<td>3.8050</td>
<td>3.9097</td>
<td>-2.75</td>
</tr>
<tr>
<td>4.4190</td>
<td>4.5523</td>
<td>-3.01</td>
</tr>
<tr>
<td>4.9080</td>
<td>4.9330</td>
<td>-0.51</td>
</tr>
<tr>
<td>5.3180</td>
<td>5.1371</td>
<td>3.40</td>
</tr>
<tr>
<td>4.6770</td>
<td>4.5972</td>
<td>1.71</td>
</tr>
<tr>
<td>4.3830</td>
<td>4.4629</td>
<td>-1.82</td>
</tr>
<tr>
<td>4.1070</td>
<td>4.1180</td>
<td>-0.27</td>
</tr>
<tr>
<td>original</td>
<td>predicted</td>
<td>100(\frac{\text{orig} - \text{pred}}{\text{orig}})</td>
</tr>
<tr>
<td>----------</td>
<td>-----------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>2.9120</td>
<td>2.9896</td>
<td>-2.66</td>
</tr>
<tr>
<td>5.1240</td>
<td>5.0709</td>
<td>1.03</td>
</tr>
<tr>
<td>7.7100</td>
<td>7.6376</td>
<td>0.94</td>
</tr>
<tr>
<td>5.3040</td>
<td>5.3429</td>
<td>-0.73</td>
</tr>
<tr>
<td>4.1000</td>
<td>4.1646</td>
<td>-1.57</td>
</tr>
<tr>
<td>2.0550</td>
<td>2.1436</td>
<td>-4.31</td>
</tr>
<tr>
<td>5.6660</td>
<td>5.6495</td>
<td>0.29</td>
</tr>
<tr>
<td>5.4280</td>
<td>5.3149</td>
<td>2.08</td>
</tr>
<tr>
<td>4.5590</td>
<td>4.5012</td>
<td>1.27</td>
</tr>
<tr>
<td>10.945</td>
<td>11.029</td>
<td>-0.77</td>
</tr>
<tr>
<td>1.8150</td>
<td>1.9180</td>
<td>-5.67</td>
</tr>
<tr>
<td>2.8840</td>
<td>2.9088</td>
<td>-0.86</td>
</tr>
<tr>
<td>5.0490</td>
<td>4.9636</td>
<td>1.68</td>
</tr>
<tr>
<td>7.6090</td>
<td>7.5003</td>
<td>1.42</td>
</tr>
<tr>
<td>10.480</td>
<td>10.515</td>
<td>-0.33</td>
</tr>
<tr>
<td>4.9500</td>
<td>5.1181</td>
<td>-3.39</td>
</tr>
<tr>
<td>5.2320</td>
<td>5.2271</td>
<td>0.09</td>
</tr>
<tr>
<td>4.9700</td>
<td>4.8758</td>
<td>1.89</td>
</tr>
</tbody>
</table>
TABLE 19: COMPARISON OF ORIGINAL DATA ($q_{r,c,i}$) AND DATA PREDICTED BY EQUATION (4.11)

<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>$100 \times \frac{(orig - pred)}{orig}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7777</td>
<td>1.7892</td>
<td>-0.64</td>
</tr>
<tr>
<td>3.3715</td>
<td>3.3871</td>
<td>-0.46</td>
</tr>
<tr>
<td>5.3944</td>
<td>5.3637</td>
<td>0.57</td>
</tr>
<tr>
<td>3.8006</td>
<td>3.8521</td>
<td>-1.35</td>
</tr>
<tr>
<td>2.3907</td>
<td>2.3953</td>
<td>-0.19</td>
</tr>
<tr>
<td>1.1647</td>
<td>1.1744</td>
<td>-0.84</td>
</tr>
<tr>
<td>3.8619</td>
<td>3.8777</td>
<td>-0.41</td>
</tr>
<tr>
<td>3.5554</td>
<td>3.5114</td>
<td>1.24</td>
</tr>
<tr>
<td>2.6972</td>
<td>2.7105</td>
<td>-0.49</td>
</tr>
<tr>
<td>7.8464</td>
<td>7.8488</td>
<td>-0.03</td>
</tr>
<tr>
<td>1.1034</td>
<td>1.0952</td>
<td>0.73</td>
</tr>
<tr>
<td>1.8390</td>
<td>1.8157</td>
<td>1.26</td>
</tr>
<tr>
<td>3.4328</td>
<td>3.4236</td>
<td>0.27</td>
</tr>
<tr>
<td>5.4000</td>
<td>5.4118</td>
<td>-0.22</td>
</tr>
<tr>
<td>7.7851</td>
<td>7.7655</td>
<td>0.25</td>
</tr>
<tr>
<td>3.7393</td>
<td>3.7315</td>
<td>0.21</td>
</tr>
<tr>
<td>3.7993</td>
<td>3.7841</td>
<td>0.39</td>
</tr>
<tr>
<td>3.3715</td>
<td>3.4383</td>
<td>-1.98</td>
</tr>
</tbody>
</table>
### Table 20: Comparison of Original Data ($q_{w,a}$) and Data Predicted by Eqn. (4.12)

<table>
<thead>
<tr>
<th>Original</th>
<th>Predicted</th>
<th>$100 \times (\text{orig} - \text{pred})/\text{orig}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.658</td>
<td>13.661</td>
<td>-0.02</td>
</tr>
<tr>
<td>14.328</td>
<td>14.303</td>
<td>0.17</td>
</tr>
<tr>
<td>14.504</td>
<td>14.507</td>
<td>-0.02</td>
</tr>
<tr>
<td>10.663</td>
<td>10.665</td>
<td>-0.03</td>
</tr>
<tr>
<td>17.018</td>
<td>17.034</td>
<td>-0.09</td>
</tr>
<tr>
<td>10.771</td>
<td>10.788</td>
<td>-0.16</td>
</tr>
<tr>
<td>11.556</td>
<td>11.566</td>
<td>-0.08</td>
</tr>
<tr>
<td>13.709</td>
<td>13.688</td>
<td>0.15</td>
</tr>
<tr>
<td>11.815</td>
<td>11.806</td>
<td>0.08</td>
</tr>
<tr>
<td>18.026</td>
<td>18.023</td>
<td>0.02</td>
</tr>
<tr>
<td>16.912</td>
<td>16.917</td>
<td>-0.03</td>
</tr>
<tr>
<td>6.5944</td>
<td>6.5906</td>
<td>0.61</td>
</tr>
<tr>
<td>12.021</td>
<td>12.005</td>
<td>0.13</td>
</tr>
<tr>
<td>11.421</td>
<td>11.441</td>
<td>-0.18</td>
</tr>
<tr>
<td>10.151</td>
<td>10.152</td>
<td>-0.10</td>
</tr>
</tbody>
</table>
### TABLE 21: COMPARISON OF ORIGINAL DATA (q_{f,a}) AND DATA PREDICTED BY EQN. (4.12)

<table>
<thead>
<tr>
<th>Original</th>
<th>Predicted</th>
<th>100* (orig - pred)/orig</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3970</td>
<td>0.3966</td>
<td>0.09</td>
</tr>
<tr>
<td>0.4766</td>
<td>0.4763</td>
<td>0.05</td>
</tr>
<tr>
<td>0.5326</td>
<td>0.5328</td>
<td>-0.04</td>
</tr>
<tr>
<td>0.3100</td>
<td>0.3096</td>
<td>0.11</td>
</tr>
<tr>
<td>0.7114</td>
<td>0.7120</td>
<td>-0.09</td>
</tr>
<tr>
<td>0.2527</td>
<td>0.2538</td>
<td>-0.44</td>
</tr>
<tr>
<td>0.3559</td>
<td>0.3562</td>
<td>-0.10</td>
</tr>
<tr>
<td>0.4774</td>
<td>0.4765</td>
<td>0.17</td>
</tr>
<tr>
<td>0.8034</td>
<td>0.8023</td>
<td>0.14</td>
</tr>
<tr>
<td>1.1984</td>
<td>1.1982</td>
<td>0.02</td>
</tr>
<tr>
<td>1.4535</td>
<td>1.4540</td>
<td>-0.04</td>
</tr>
<tr>
<td>0.1257</td>
<td>0.1239</td>
<td>1.50</td>
</tr>
<tr>
<td>0.2705</td>
<td>0.2710</td>
<td>-0.18</td>
</tr>
<tr>
<td>0.3327</td>
<td>0.3332</td>
<td>-0.14</td>
</tr>
<tr>
<td>0.4625</td>
<td>0.4628</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>100* (orig - pred)/orig</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0926</td>
<td>1.1126</td>
<td>-1.83</td>
</tr>
<tr>
<td>1.0357</td>
<td>1.0382</td>
<td>-0.24</td>
</tr>
<tr>
<td>1.0977</td>
<td>1.0954</td>
<td>0.21</td>
</tr>
<tr>
<td>1.3958</td>
<td>1.3904</td>
<td>0.38</td>
</tr>
<tr>
<td>1.2775</td>
<td>1.2621</td>
<td>1.17</td>
</tr>
<tr>
<td>1.3415</td>
<td>1.3193</td>
<td>1.64</td>
</tr>
<tr>
<td>1.0737</td>
<td>1.0532</td>
<td>1.87</td>
</tr>
<tr>
<td>1.1794</td>
<td>1.1908</td>
<td>-0.93</td>
</tr>
<tr>
<td>1.0021</td>
<td>1.0137</td>
<td>-1.20</td>
</tr>
<tr>
<td>1.5345</td>
<td>1.5603</td>
<td>-1.70</td>
</tr>
<tr>
<td>1.5665</td>
<td>1.5431</td>
<td>-1.47</td>
</tr>
<tr>
<td>1.4365</td>
<td>1.4543</td>
<td>-1.24</td>
</tr>
</tbody>
</table>
**TABLE 23 : COMPARISON OF ORIGINAL DATA (\( q_{w,a,i} \)) AND DATA PREDICTED BY EQN. (4.14)**

<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>(100^\ast (\text{orig} - \text{pred})/\text{orig})</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1296</td>
<td>5.1616</td>
<td>-0.62</td>
</tr>
<tr>
<td>2.9577</td>
<td>2.9295</td>
<td>0.95</td>
</tr>
<tr>
<td>3.2843</td>
<td>3.2949</td>
<td>-0.32</td>
</tr>
<tr>
<td>3.4194</td>
<td>3.4040</td>
<td>0.44</td>
</tr>
<tr>
<td>8.9263</td>
<td>8.9284</td>
<td>-0.02</td>
</tr>
<tr>
<td>5.0879</td>
<td>4.9745</td>
<td>2.22</td>
</tr>
<tr>
<td>3.2878</td>
<td>3.3468</td>
<td>-1.79</td>
</tr>
<tr>
<td>2.4172</td>
<td>2.4289</td>
<td>-0.48</td>
</tr>
<tr>
<td>2.8885</td>
<td>2.9247</td>
<td>-1.24</td>
</tr>
<tr>
<td>2.1351</td>
<td>2.1628</td>
<td>-1.27</td>
</tr>
<tr>
<td>4.6341</td>
<td>4.7002</td>
<td>-1.42</td>
</tr>
<tr>
<td>3.2275</td>
<td>3.1980</td>
<td>0.90</td>
</tr>
<tr>
<td>original</td>
<td>predicted</td>
<td>(100 \times (\text{orig} - \text{pred})/\text{orig})</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>0.6799</td>
<td>0.6706</td>
<td>1.37</td>
</tr>
<tr>
<td>0.7416</td>
<td>0.7320</td>
<td>1.30</td>
</tr>
<tr>
<td>0.6358</td>
<td>0.6568</td>
<td>-3.30</td>
</tr>
<tr>
<td>2.4230</td>
<td>2.4255</td>
<td>-0.10</td>
</tr>
<tr>
<td>3.1481</td>
<td>3.1482</td>
<td>-0.00</td>
</tr>
<tr>
<td>0.7696</td>
<td>0.7636</td>
<td>0.78</td>
</tr>
<tr>
<td>0.9393</td>
<td>0.9387</td>
<td>0.06</td>
</tr>
<tr>
<td>0.5312</td>
<td>0.5504</td>
<td>-3.58</td>
</tr>
<tr>
<td>2.0436</td>
<td>2.0335</td>
<td>0.49</td>
</tr>
<tr>
<td>0.5378</td>
<td>0.5351</td>
<td>0.49</td>
</tr>
<tr>
<td>0.4736</td>
<td>0.4848</td>
<td>-2.34</td>
</tr>
<tr>
<td>0.5675</td>
<td>0.5596</td>
<td>1.41</td>
</tr>
</tbody>
</table>
### Table 25: Comparison of Original Data ($\delta_{w,l}$) and Data Predicted by Eqn. (4.16)

<table>
<thead>
<tr>
<th>Original</th>
<th>Predicted</th>
<th>$100 \times (\text{orig} - \text{pred})/\text{orig}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2589</td>
<td>0.2726</td>
<td>-5.27</td>
</tr>
<tr>
<td>0.3215</td>
<td>0.3264</td>
<td>-1.56</td>
</tr>
<tr>
<td>0.2820</td>
<td>0.2821</td>
<td>-0.06</td>
</tr>
<tr>
<td>0.2718</td>
<td>0.2685</td>
<td>1.18</td>
</tr>
<tr>
<td>0.2126</td>
<td>0.1969</td>
<td>7.43</td>
</tr>
<tr>
<td>0.3218</td>
<td>0.3276</td>
<td>-1.81</td>
</tr>
<tr>
<td>0.3294</td>
<td>0.3317</td>
<td>-0.69</td>
</tr>
<tr>
<td>0.3809</td>
<td>0.3740</td>
<td>1.79</td>
</tr>
<tr>
<td>0.3111</td>
<td>0.3162</td>
<td>-1.64</td>
</tr>
<tr>
<td>0.3980</td>
<td>0.3919</td>
<td>1.50</td>
</tr>
<tr>
<td>0.3527</td>
<td>0.3596</td>
<td>-2.00</td>
</tr>
<tr>
<td>0.4025</td>
<td>0.3932</td>
<td>2.32</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>original</th>
<th>predicted</th>
<th>100* (orig - pred)/orig</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9211</td>
<td>0.9694</td>
<td>-5.22</td>
</tr>
<tr>
<td>0.9329</td>
<td>0.9545</td>
<td>-2.32</td>
</tr>
<tr>
<td>0.9226</td>
<td>0.9464</td>
<td>-2.61</td>
</tr>
<tr>
<td>0.8496</td>
<td>0.7789</td>
<td>8.32</td>
</tr>
<tr>
<td>0.8215</td>
<td>0.7988</td>
<td>2.80</td>
</tr>
<tr>
<td>1.1863</td>
<td>1.1921</td>
<td>-0.49</td>
</tr>
<tr>
<td>1.1440</td>
<td>1.1573</td>
<td>-1.14</td>
</tr>
<tr>
<td>1.1599</td>
<td>1.1534</td>
<td>0.55</td>
</tr>
<tr>
<td>0.9714</td>
<td>0.9778</td>
<td>-0.66</td>
</tr>
<tr>
<td>1.1682</td>
<td>1.1485</td>
<td>1.68</td>
</tr>
<tr>
<td>1.3801</td>
<td>1.3492</td>
<td>2.24</td>
</tr>
<tr>
<td>1.4069</td>
<td>1.3861</td>
<td>1.48</td>
</tr>
</tbody>
</table>
VITA AUCTORIS

Name: Murty Satyanarayana Kompella

Date of Birth: April 10, 1963

Place of Birth: Palakollu, Andhra Pradesh, India

Education:


(b) Undergraduate: B. Tech. (Hons) in Mechanical Engineering from Indian Institute of Technology, Kharagpur, India (1981-1985)

(c) Graduate: M. A. Sc. in Mechanical Engineering, University of Windsor, Windsor, Ontario, Canada (1986 January to date)