Biological signals analysis: A proposal of methods for quantifying patterns in biological signals.

Walter Zubrycky

University of Windsor

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BIOLOGICAL SIGNALS ANALYSIS
A PROPOSAL OF METHODS
FOR QUANTIFYING PATTERNS
IN BIOLOGICAL SIGNALS

BY
WALTER ZUBRYCKY

A Thesis
Submitted to the Faculty of Graduate Studies through the
Department of Electrical Engineering in Partial Fulfillment
of the Requirements for the Degree of
Master of Applied Science at
University of Windsor

Windsor, Ontario
1969
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ABSTRACT

This thesis considers the problems involved in the analysis of biological signals and suggests that the goal of such analyses should be a unique representation of the biological signal.

A proposal is made for a system which attempts to reach this goal by representing characteristic patterns or waveforms in biological signals quantitatively with a set of parameters derived from a Fourier series representation of the patterns.

To support the proposal hardware implementations and digital computer simulations were carried out investigating certain aspects of the system.

It was found that if a biological signal, which is basically periodic in nature, can be segmented accurately into its fundamental periods, a precise quantitative description of the signal can be achieved.
ACKNOWLEDGEMENTS

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I. INTRODUCTION

The results of measurements on fluctuations in physiological parameters constitutes a biological signal. (physiological - related to the study of the functions and activities of living matter as such, and of the physical and chemical phenomena involved\(^1\)).

Many of these fluctuations, and thus the resultant biological signals that represent them, are rhythmic in nature, and occur in a basically periodic mode. A few examples of such signals are given in Fig. (1-1). The list is by no means exhaustive and further examples can be cited throughout the natural world, ranging from signals derived from micro-organisms to man\(^2\).

It is the interpretation of such signals that constitutes a major problem in Bio-medical Engineering\(^6\). To interpret a biological signal implies that one can accurately distinguish it from other signals in the same class; this, in turn, implies that one can give a quantitative description of the signal, that to a certain degree of accuracy uniquely represent the signal. The key word in this thesis is quantification (the transformation of qualitative into quantitative data in scientific methodology\(^1\)).

Much of the output from bio-medical instrumentation is processed by physicians or medical technicians who rely on a great deal of previous experience with such instrumentation to recognize patterns and characteristics in these signals that are significant.
FIG. 1-la. PRESSURE AND AREA DATA OBTAINED FROM A COMPETENT
VENOUS VALVE DURING PULSATILE FLOW.

FIG. 1-lb. RADIOELECTROCARDIOGRAM OF WOMAN IMMEDIATELY AFTER
EXERCISE.
FIG. 1-1c. TACHOGRAMS OF LEVEL WALKING

FIG. 1-1d. FETAL ELECTOCARDIOGRAM
Fig. 1-1e. PLOT OF A NORMAL ECG

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Although man is a very superior extractor of information by pattern recognition, he cannot be completely objective, nor can he "completely express with any accuracy the average amplitude, the median, the maxima and minima, nor can he describe the average frequency, the deviations about this frequency" of some biological signal.

This absence of an accurate representation of a biological signal makes technical communications dealing with such signals very difficult.
A. Literature Survey

Much effort in Bio-medical Engineering today is directed toward the development of automated systems to analyze and interpret a wide class of biological signals. Such systems, in addition to eliminating the need for skilled human analysts or interpreters, attempts to produce more objective, accurate, quantitative data about the characteristics of biological signals. The following section considers several existent systems which attempt to solve the problem of biological signals analysis.

Balm conducted a study to see if a single easily recorded electrocardiogram can be used, employing a crosscorrelation scheme, to screen abnormal adult ECG waveforms from normal waveforms. Balm considered six standard waveforms, two characteristic of a wide range of normal patterns and four others, each representing a certain disease category. He let $f_s(t)$ represent a certain standard waveform, and $f_x(t)$ the unknown waveform. He used $N + 1$ samples of both waveforms, spaced at $N$ equal intervals to form the discrete time correlation given by $\phi_{xs}(\tau)$,

$$\phi_{xs}(\tau) = \frac{1}{N+1} \sum_{K=1}^{N} X_K S_{K+N+\tau}$$

(1-1)

where $X_K$ and $S_K$ are the discrete sampled amplitudes of $f_x(t)$ and $f_s(t)$ respectively, and $\tau$ is the time delay.

Depending on the degree of correlation with $f_s(t)$, $f_x(t)$ was then interpreted as being normal or abnormal, and more significantly, if abnormal, classified into one of the four given disease categories. Since ECG waveforms vary in magnitude and time duration, and since correlation is shape comparison, Balm
performed certain time and amplitude normalization procedures on the ECG signals to be processed in order to make the correlations meaningful.

The implementation of this procedure involved manual extraction of 36 evenly spaced samples from an analogue ECG waveform (derived from a strip chart recording) and the introduction of this data, by means of key punched cards, into an IBM 1440 computer. The computer then performed the amplitude normalization and crosscorrelations.

Autocorrelation and crosscorrelation techniques are more often applied to a class of noisy signals like the electroencephalogram (electrical signals from the scalp) in order to study rhythmic activity in EEG, and to compare two EEG signals from different parts of the scalp.

Another statistical technique, namely the determination of power spectra and its variance, was used by Berkhout and Walter to discriminate individual EEG from a collection of EEG data taken from 47 subjects. The power spectral criterion was also used by Wyatt to describe and compare quantitatively finger tremors.

Caceres suggests the development of a computer program which simulates the subjective processes that a physician would use to extract clinically significant parameters from an electrocardiographic signal or any other electrophysiological waveform.

The term clinically significant parameter, with regard to the ECG, refers to the amplitudes of P, Q, R, S and T waves, and the durations of the ST, PQ, QT and RR segments of the waveform as shown in Fig. 1-2. These parameters were arbitrarily established by Einthoven in 1900.
Such a program of parameter extraction was produced by Steinberg et al. The input data to this program took the form of a 5 second segment of an ECG signal, which was sampled at a rate of 625 samples per second, converted into digital form, and recorded on magnetic tape. This data was then subjected to a preprocessing program using a least-squares parabolic smoothing scheme for the elimination of noise, before it was run in the main body parameter recognition program.

The main body program begins by detecting the point of greatest negative rate of change in 0.64 second subsegments of the data which are shorter than the fundamental period of the ECG waveform and establishes this as a point of reference for each cycle within the signal. This point reference is a valid one because it is repeatable in any subject and always occurs after the peak of the
R wave and before the peak of the S wave. The program then continues, according to a complex series of logical operations, to locate and measure the various peaks and intervals which constitute the set of clinical parameters, and thus automates a tedious process of measurement. The output of this process is used by a physician as a diagnostic aid, to classify waveforms into categories of normality or abnormality.

Lowenberg discusses the problem of making quantitative measurements on EEG signals, and emphasizes the point that a unique representation should be the goal of analysis of such signals. He suggests that statistical characteristics of EEG signals do not necessarily include all significant information within the signal, and that problems arise when the signal from a patient is not stationary due to changes in the patient's physical state or environment. This is supported by the fact that in many cases indications of abnormalities in EEG signals appear for only brief periods of time and thus may be lost in the averaging process.

Lowenberg proposed a method of quantifying EEG signals by representing finite intervals of EEG records with a discrete Fourier series. In his work, a one second segment of some EEG signal, recorded on magnetic tape, is formed into a tape loop and replayed repetitively. This segment of the signal is thereby transformed into a repetitive signal with a fundamental frequency of 1Hz. A tuneable high-Q bandpass filter was used to determine the amplitudes of the components of the discrete Fourier spectra that make up the signal, and thereby, according to
Lowenberg, giving a unique representation of that segment of the EEG waveform.

Lowenberg states that this method should be applied only to short intervals of the EEG signal, that may be of clinical interest, rather than to the entire EEG record obtained from a patient.

Recently an efficient digital technique called the Fast Fourier Transform has been developed to compute the discrete Fourier transform of a waveform from a time series of samples taken from the waveform. Coefficients of the DFT can be directly related to the coefficients of a Fourier series representation of a continuous waveform.

Parzen, in his paper, considers three separate aspect of spectral analysis; how to define the spectrum, how to compute it, and how to interpret the computed data. With regard to the latter, he states that spectrum analysis techniques could be applied to recognize and classify patterns (whether they be biological signals or variations in some economic cycle) given by a time varying signal. Furthermore he maintains that, in any instance, if one considers the time domain approach to a problem one should also consider applying a Fourier transform to the signal represented in the time domain.
B. Critique

The pattern recognition program developed by Steinberg\textsuperscript{15} extracted clinically significant parameters from ECG waveforms. However, the fact that these parameters were originally established in an arbitrary manner, and are presently subject to a broad range of interpretations, limits this analysis to an essentially subjective process. In addition, no allowance is made in this analysis for the discovery and establishment of new significant parameters. These problems arise due to the limited nature of the parameters representing the ECG waveform. The resolution of a process that attempts to define a complex waveform by 8 time-amplitude samples is not very sharp, and can result in the loss of a great deal of significant information within the signal.

To apply this technique to a general class of biological signals would necessitate the development of a separate program for every specific type of signal requiring processing. As a result a large library of such programs would be required to make this system a comprehensive one.

In Balm's work\textsuperscript{8} similar problems arise. Again the first step in the process is a broad subjective one, in that typical waveforms were defined to represent classes of normal and abnormal ECG waveforms. Furthermore, only the QT segment of the entire ECG waveform was considered significant, and only this portion was subjected to cross-correlation with the QT segments of the typical waveforms.

A tedious process of time normalizing all test waveforms to correspond to the time base of the control waveforms was
carried out by hand rather than being automated.

The resultant output was a classification of each test waveform into five categories, one normal, and four abnormal, diseased categories, and it must be said that the process was carried out successfully on a number of specimen waveforms.

This basic technique has the advantage of being flexible since it can be applied to correlate any two biological signals. The process, in its present state, however, is limited to a taxonomic field of five categories. The field could be expanded to include more typical waveforms but at some point the resolution of the technique would break down. This is because no precise quantitative description of the waveforms was derived, and comparisons made upon that basis.

Lowenberg treated EEG signals in a deterministic manner by representing a segment of an EEG signal with a Fourier series representation. However, in his experimental work he makes no attempt to obtain phase information for each of the frequency components, and only extracts an amplitude spectrum for the signal, thereby rendering an incomplete representation of the signal segment.

A more fundamental problem in Lowenberg's work stems from the type of signals he is processing. Electrical signals derived from the nervous system are not well defined and basically periodic but are essentially random signals and thus cannot be treated effectively in a deterministic manner. A characteristic point of reference is difficult to establish within this class of random signals, and thus segments from an EEG record must be extracted for analysis in a rather arbitrary manner. This makes
meaningful comparisons between segments of EEG signals hazardous. In addition, the procedure of forming tape loops and tuning in on each of the harmonics manually does not lend itself readily to processing large amounts of data.

Fast Fourier Transform methods could be used to quantify segments of biological signals by means of a Fourier series representation, however, large computer facilities like the IBM 7094 system are required to carry out the FFT computations quickly.

C. Summary

To conclude, the concept of using a Fourier representation to quantize biological signals appears feasible, but only if less cumbersome Fourier analysis techniques can be developed to process such signals. The objective of this thesis is to present a proposal for a biological signals analysis system which will quantify biological signals, based upon a Fourier series representation of characteristic segments within the signal.

The system proposed will be limited in scope to the analysis of signals that are basically periodic, and well defined or deterministic in nature, and excludes the class of signals which are random, and therefore must be treated by non-deterministic methods of analysis.

The proposed system is implemented in two phases. The first is a digital pre-processing phase that transforms the signal by time normalization into a form suitable for Fourier
analysis. The second phase uses analogue techniques to carry out a Fourier analysis of the time normalized signal.

Digital computer simulations as well as hardware implementations are used to support the proposal.
II. THEORY

A. Sectioning Biological Signals Into Characteristic Patterns.

Consider a hypothetical signal \( f(t) \), shown in Fig. 2-1, which can be taken to represent a class of well-defined basically periodic biological signals, like those illustrated in the introduction of this thesis.

\[ f(t) \]

\[ \text{Fig. 2-1. Hypothetical Biological Signal} \]

It can be seen that in many cases despite the fact that the frequency and intensity of the signal may vary, due to certain physical or chemical changes at the source, the signal remains in a basically periodic mode. If the signal is sectioned at certain characteristic points along the time axis, as shown in Fig. 2-2, it is transformed into a series of characteristic patterns representing a fundamental period in the variations of some physiological process.
To quantify a biological signal it is required to produce an analytic description of each of the characteristic patterns or waveforms within the signal. These patterns may vary in shape and duration; and it is these two criteria upon which the analytic description must be based. One parameter in the analytic description of the pattern, namely duration, given by $\Delta T_1, \Delta T_2, \Delta T_3 \ldots \ldots$, can be readily established by simply measuring the intervals which define the beginning and end of each fundamental period within the signal.
B. Time Normalization

Once the characteristic patterns or waveforms within the signal have been segmented and their durations measured, it is necessary to consider the shape of each of these patterns to render an analytic representation of the signal.

To make the shape criterion independent of the duration of the pattern, and suitable for further processing, each interval $\Delta T_1$, $\Delta T_2$, $\ldots$ must be normalized to some standard interval $\Delta T$ so that each characteristic waveform assumes a fundamental period of $T$ seconds duration. The resultant normalized signal $f_n(t)$, would be a sequence of patterns occurring at a fundamental frequency of $\frac{1}{T}$ Hz. Each waveform retains its original shape, and thus the information implicit within it, being merely expanded or contracted in time as required to accommodate a normalized period of $T$ seconds duration.

1. Variable Rate Sampling

A method of time normalization can be implemented by means of a variable rate sampling system, the rate being dependent upon the values of the duration parameters $\Delta T_1$, $\Delta T_2$, $\Delta T_3$, $\ldots$ of each waveform in the signal. If one considers the hypothetical biological signal $f(t)$ to have a maximum frequency component or bandwidth of $f_m$, the sampling theorem states that the signal must be sampled at a minimum rate of $2f_m$ Hz in order to be able to eventually reproduce the signal from the sampled values without distortion. This means that a minimum of $2f_m T_i$ number of samples are required to completely characterise a $T_i$ second segment of this signal. If we choose an arbitrarily large number of samples $N$ to represent each characteristic waveform such that;
the fidelity of a sampled representation is insured.

The time normalization process is implemented by sampling the amplitude of each characteristic pattern at a rate given by $S_i$,

$$S_i = \frac{N}{\Delta T_i} \tag{2-2}$$

where $\Delta T_i$ is the value of the duration of the particular waveform being sampled, and subsequently storing the sampled data. The problem of aliasing resulting from limitations upon Eqn. (2-2) will be discussed in the next section. The data thus appears as a sequence of $N$ numbers for each characteristic pattern in the signal. Every block of $N$ numbers corresponds to the values of the amplitudes of a waveform at equally spaced intervals of $\frac{\Delta T_i}{N}$ seconds along the time axis.

If the data is then read out in sequence, at a rate $\frac{N}{T}$ Hz where $T$ is the normalized period, a discrete or sampled representation of each characteristic pattern assumes a duration equal to $T$ seconds. The entire signal has thereby been time normalized, and a constant pattern repetition rate or fundamental frequency of $\frac{1}{T}$ for the signal has been achieved.

The time normalized signal at this stage can be represented by a series of impulses $f_S(t)$ shown in Fig. 2-3.
FIG. 2-3. SAMPLED REPRESENTATION OF TIME NORMALIZED SIGNAL

In order to implement the quantification of these patterns they must be reconverted to a continuous time normalized form given by \( f_n(t) \).

2. Reconstruction of Sampled Data

The series of impulses \( f_s(t) \) (Fig. 2-3) corresponds to a pulse amplitude modulated representation of the entire original signal \( f(t) \), in a time normalized form. The amplitude spectrum \( F_n(j\omega) \) of the signal, bandlimited to \( f_m \) can be characterized by Fig. 2-4,
where:

$$\omega'_M = 2\pi f'_M$$  \hspace{1cm} (2-3)

The corresponding spectrum of \( f_s(t) \) given by \( F_s(j\omega) \) is shown in Fig. 2-5, and is
given by Eqn. (2-4)

\[ F_s(j\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} F_n(j\omega - jn\omega_s) \]  

(2-4)

where \( \omega_s \) is the pulse repetition rate and \( T_s \) is the sampling interval.

\[ \omega_s = \frac{2\pi}{T_s} = 2\pi f_s = \frac{2\pi N}{T} \]  

(2-5)

\[ \omega_s \gg 2\omega'_M \]  

(2-6)

If the conditions given by Eqn. (2-1), and reflected in Eqn. (2-6) cannot be met due to certain limitations upon the sampling rate such that;

\[ \frac{N}{\Delta T_i} \gg 2f_M \]  

(2-7)

distortion occurs as a result of the attenuation of the signal spectrum caused by too low a sampling frequency \( \omega_s \). To overcome this problem the signal will have to be constrained to a smaller bandwidth by means of low pass filtering. This is a compromise situation in which a certain portion of the higher frequencies of the signal will have to be rejected in order to insure an undistorted representation of the remaining portion of the band.

To reconstruct a continuous signal \( f_s(t) \) from \( f_n(t) \) the PAM signal low pass filter techniques must be applied to pass signal frequencies in the range \( 0 - \omega'_M \) and attenuate the rest.
of the band. Low pass filtering, for reconstruction of sampled data, can be accomplished by using a zero order hold system, in conjunction with additional filtering to smooth the signal\textsuperscript{20}. A zero order hold system desamples an impulse train by clamping the amplitude of each impulse to a constant level until a subsequent impulse arrives. If the PAM signal $f_s(t)$ is subjected to a zero order hold system the resultant signal $f_H(t)$ has a staircase effect as shown in Fig. 2-6.

$$f_H(t)$$

![FIG. 2-6. OUTPUT OF A ZERO ORDER HOLD SYSTEM](image)

The zero order hold frequency response characteristics $G(j\omega)$ are given by Eqn. (2-8)\textsuperscript{22}.

$$G(j\omega) = T_s \frac{\sin(T_s\omega/2)}{T_s\omega/2} e^{-jT_s\omega/2}$$

(2-8)

The amplitude spectrum of $f_s(t)$ is superimposed upon the amplitude characteristics of $G(j\omega)$ in Fig. 2-7, to show the filter characteristics of the zero order hold system.
FIG. (2-7). AMPLITUDE AND PHASE CHARACTERISTICS OF $G(j\omega)$

The amplitude and phase characteristics $F_H(j\omega)$ of $f_H(t)$ are given by Eqn. (2-9).
\[ F_H(j\omega) = G(j\omega) \cdot F_S(j\omega) \]

\[ = T_s \frac{\sin(T_w/2)}{T_w/2} \cdot \frac{1}{T_s} \sum_{n=-\infty}^{\infty} F_n(j\omega - jn\omega_s) \]

\[ = e^{jT_w/2} \frac{\sin(T_w/2)}{T_w/2} \cdot \sum_{n=-\infty}^{\infty} F_n(j\omega - jn\omega_s) \]

\[ (2-9) \]

Additional filtering is necessary to smooth \( f^*_H(t) \) by further attenuating the portion of the sidebands of the PAM signal which have been passed by the zero order hold desampler. Ideally, a low pass filter \( H(j\omega) \) should be used which has unity gain characteristics in the passband \( 0 - \omega_m \) and infinite attenuation outside the passband as shown in Fig. (2-8) and given by Eqn. (2-10).

![Ideal Low Pass Filter Characteristics](image)

**FIG. (2-8). IDEAL LOW PASS FILTER CHARACTERISTICS**

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\[ H(j\omega) = A(\omega) e^{j\theta(\omega)} \]

\[ A(\omega) = 1.0 \quad |\omega| \leq |\omega_M| \]

\[ A(\omega) = 0 \quad |\omega| > |\omega_M| \]

\[ \theta(\omega) = K_i \omega \]

\[ K_i = \text{Constant} \quad (2-10) \]

Smoothing \( f_H(t) \) by such a filter would result in the complete reconstruction of the signal in its time normalized form as shown by Eqn. (2-11).

\[ F(j\omega) \cdot H(j\omega) = e^{jT_s \omega/2} \frac{\sin(T_s \omega/2)}{T_s \omega/2} \sum_{n=-\infty}^{\infty} F_n(j\omega - jn\omega) \cdot A(\omega) e^{j\theta(\omega)} \]

\[ = e^{j(T_s \omega/2 + \theta(\omega))} \cdot \frac{\sin(T_s \omega/2)}{T_s \omega/2} \cdot F_n(j\omega) \quad (2-11) \]
Now with:

\[
\frac{\sin(T_s \omega/2)}{T_s \omega/2} \approx 1.0
\]

\[\omega \ll \omega_s \quad (2-12)\]

\(F_n(j\omega)\) with a linear phase term results. This corresponds to \(f_n(t)\) with a time delay.

To sum up the preprocessing phase of the analysis, the system segments the signal into characteristic patterns, extracts and stores a time duration parameter for each pattern, and subsequently time normalizes each of the patterns or waveforms in the signal without distorting their characteristic shapes, such that a continuous signal \(f_n(t)\) with a fundamental frequency of \(1/N\) hertz results.
C. Fourier Analysis

A function $y(t)$ defined over an interval $0 - T$ seconds can be expressed in terms of a Fourier series as given in Eqn. (2-13),

$$y(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos \omega_k t + b_k \sin \omega_k t \right)$$  \hspace{1cm} (2-13)

$$\omega_k = \frac{2\pi k}{T}$$  \hspace{1cm} (2-14)

where the coefficients are defined as follows:

$$\frac{a_0}{2} = \frac{1}{T} \int_{0}^{T} y(t) dt$$  \hspace{1cm} (2-15)

$$a_k = \frac{2}{T} \int_{0}^{T} y(t) \cos \omega_k t \, dt$$  \hspace{1cm} (2-16)

$k = 0, 1, 2, 3, \ldots$

$$b_k = \frac{2}{T} \int_{0}^{T} y(t) \sin \omega_k t \, dt$$  \hspace{1cm} (2-17)

$k = 1, 2, 3, 4, \ldots$
This Fourier series representation is not dependent upon the periodicity of \( y(t) \) defining the function only on the interval \( 0 - T \) seconds. The periodicity occurs in the Fourier series representation itself, which converges to the function \( y(t) \) in the interval \( 0 - T \) and to its periodic equivalent outside the interval.

If \( f_n(t) \) is considered as a series of functions \( f_{n_1}(t), f_{n_2}(t), \ldots \), defined over the intervals \( 0 - T, T - 2T, \ldots \), each \( T \) second segment of \( f_n(t) \) can be described by the Fourier series of Eqn. (2-13). Furthermore, if the signal is bandlimited, only a finite number of terms in the series are required to define the characteristic waveforms given by \( f_{n_1}(t), f_{n_2}(t), \ldots \).

1. **Amplitude Normalization**

The Fourier series representation of a characteristic pattern or waveform depends not only upon the shape of the pattern but on its amplitude.

Two patterns \( y_1(t), y_2(t) \) identical in shape but with different amplitudes as given by Eqn. (2-18), will give different sets of Fourier coefficients.

\[
y_1(t) = K_2 y_2(t)
\]

\[
K_2 = \text{CONSTANT}
\]

In order to have a more efficient method of representation another normalization procedure must be applied to make the Fourier series representation of a waveform dependent upon
shape and independent of amplitude.

The average power $P$ of a voltage waveform $y(t)$ dissipated by a 1 ohm resistor in the interval $0 - T$ seconds is given by Eqn. (2-19).

$$ P = \frac{1}{T} \int_{0}^{T} y^2(t) \, dt \quad (2-19) $$

In terms of the Fourier coefficients $P$ is given by Eqn. (2-20).\(^{23}\)

$$ P = \frac{1}{2} \left[ \frac{a_0^2}{2} + \sum_{k=1}^{\infty} \left( a_k^2 + b_k^2 \right) \right] \quad (2-20) $$

Amplitude normalization can be accomplished by first computing the average power $P_1$, $P_2$, ..., $P_l$, ... for each of the characteristic waveforms $f_{n_1}(t)$, $f_{n_2}(t)$, ..., $f_{n_l}(t)$, ..., respectively. If each of the characteristic waveforms is constrained to a normalized average power $P$, such that;

$$ P = P_1 C_i \quad (2-21) $$

$$ C_i = \text{CONSTANT} $$

the resultant Fourier coefficient power equation for each $f_{n_i}(t)$ is given in Eqn. (2-22).
Therefore, to amplitude normalize \( f_{n_1} (t) \) and make the Fourier series representation independent of amplitude requires that each of the Fourier coefficients be multiplied by a factor \( \sqrt{C_i} \).

The normalized Fourier coefficients in the series describing \( f_{n_1} (t) \) can be expressed by \( a_k \) and \( b_k \), \( k = 1, 2, 3... \)

\[
\begin{align*}
a_o^I &= \sqrt{C_i} a_o \\
a_k^I &= \sqrt{C_i} a_k \\
b_k^I &= \sqrt{C_i} b_k
\end{align*}
\]

(2-23)

To summarize, each characteristic waveform \( f_{n_1} (t) \) in a bandlimited biological signal can be expressed uniquely by a set of parameters given by \( \Delta T_i \) the duration parameter, \( P_i \) the power parameter and a finite number of shape parameters.
given by \( a_k \) and \( b_k \), where \( k = 1, 2, 3...n \); \( n \) being determined by the bandwidth of the signal.

2. Computation of Fourier Coefficients.

Consider the response of a linear undamped oscillator to a forcing function \( g(t) \) as described by Eqn. (2-24).\(^{24}\)

\[
\ddot{X}(t) + \omega_k^2 X(t) = g(t) \quad (2-24)
\]

\( X(0) = 0 \), \( \dot{X}(0) = 0 \)

Now let:

\[
\omega_k = \frac{2\pi k}{T} \quad (2-25)
\]

\[
g(t) = \frac{2\pi k}{T} y(t) \quad (2-26)
\]

Eqn. (2-24) becomes:

\[
\ddot{X}(t) + \frac{2\pi k}{T} X(t) = \frac{2\pi k}{T} y(t) \quad (2-27)
\]

The solution to this equation\(^{24}\) is given by Eqn. (2-28) and Eqn. (2-29),

\[
X(t) = \int_{0}^{t} y(\tau) \sin \frac{2\pi k}{T} (t-\tau) d\tau \quad (2-28)
\]

\[
\dot{X}(t) = \int_{0}^{t} y(\tau) \cos \frac{2\pi k}{T} (t-\tau) d\tau \quad (2-29)
\]

These solutions bear a direct relationship to the expressions describing the Fourier coefficients \( a_k \) and \( b_k \) given by Eqn. (2-16) and Eqn. (2-17) respectively.

The analogue system shown in Fig. (2-9) mechanizes Eqn. (2-27).
FIG. (2-9) ANALOGUE SYSTEM MECHANIZING EQUATION (2-27)

The output of integrator 1 at time $t = T$ is given in Eqn. (2-30).

$$\frac{T}{2\pi k} \ddot{x}(T) = \int_{0}^{T} y(\tau) \cos \frac{2\pi k}{T} (T - \tau) d\tau$$

$$= \int_{0}^{T} y(\tau) \cos \frac{2\pi k}{T} \tau d\tau$$

$$= \frac{T}{2} a_k \quad (2-30)$$
Similarly, the output of integrator 2 at time $t = T$ is given by Eqn. (2-31).

\[
-X(T) = \int_{0}^{T} y(\tau) \sin \left( \frac{2\pi k}{T} (T - \tau) \right) d\tau
\]

\[
= \int y(\tau) \sin \left( \frac{2\pi k}{T} \tau \right) d\tau
\]

\[
= \frac{T}{2} b_k
\]  \hspace{1cm} (2-31)

Therefore, it can be seen that any particular set of Fourier coefficients $a_k$ and $b_k$ can be solved by mechanizing Eqn. (2-27) using simple analogue techniques. Thus $n$ sets of coefficients $a_k$ and $b_k$, $k = 1, 2, 3...n$, can be simultaneously determined by using $n$ parallel analogue computer modules of the type shown in Fig. (2-9). Each module is identical except for the potentiometer setting $\frac{2\pi k}{T}$, in which $k = 1, 2, 3...n$, corresponding to the particular Fourier coefficients of the harmonic $k$, being solved.

The DC term $a_0$ of $y(t)$ can be solved by mechanizing Eqn. (2-15) as given by Fig. 2-10.
3. Computation of Average Power Parameter

Another simple mechanization can solve for $P$, the average power for any particular waveform $y(t)$.

\[ \frac{2}{T} \int_{0}^{T} y(t) \, dt = a_0 \quad (2-32) \]
The output of integrator 6 at time $t = T$ is given by

Eqn. (2-33).

$$\frac{1}{T} \int_{\mathcal{A}} y^2(\mathcal{A}) d\mathcal{A} = P$$  \hspace{1cm} (2-33)

Thus it can be seen that once the biological signal is in a time normatized form, with fundamental period $T$, analogue techniques can be applied to solve for all of the parameters necessary for quantifying each of the characteristic patterns within the signal. The beauty of these techniques lies in their ability to carry out these computations within the duration $T$ of each waveform. This allows the entire signal to be processed in a smooth continuous manner.
III. PROPOSED IMPLEMENTATION

A. Time Normalization
   1. Segmentation

Online recording of large amounts of biological signal data can be carried out by the use of magnetic tape recorders. FM records have bandwidths that go down to DC and are especially suitable for recording a class of biological signals that are slowly varying or that have DC components. Once a biological signal has been recorded, the problem of segmenting the signal into characteristic patterns arises. Steinberg, et al., as discussed previously, solved the problem for a particular class of ECG signals by using a computer program to detect a characteristic point in the signal which is recurrent and delimits one cardiac cycle from another. Segmenting a general class of biological signals by computer techniques would require either a separate program for each particular class of signals to be processed, or a generalized pattern recognition capability by a computer. A less analytic, yet simpler approach to the problem would be to use a semi-automatic system incorporating the pattern recognition capabilities of a human operator.

Consider a biological signal recorder at high tape speeds \( V_R \) and subsequently replayed at greatly reduced speeds \( V_P \). The relationship between the recorded signal frequencies \( F_R \) and the playback frequencies \( F_P \) is given by Eqn. (3-1).

\[
\frac{V_R}{V_P} = \frac{F_R}{F_P} \quad (3-1)
\]

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This tape technique, in most cases, can bring the biological signal down to a frequency which is well within the reaction time of a human operator. The operator, a biomedical technician, could monitor the signal on an oscilloscope and identify points within the signal that define the beginning and end of each characteristic pattern. These characteristic points may take the form of a zero crossing, a maxima or minima, or a point of maximum positive or negative slope within certain segments of the signal. Using his pattern recognition capabilities the operator can, by watching the oscillographic trace of the signal, predict when such a characteristic point is likely to occur.

If a detector is engaged at this time which is sensitive to the particular form of the signal which indicates a characteristic point (i.e. a maxima or minima) an output from the detector will coincide with the occurrence of this particular point in the signal. With such semi-automatic techniques the resolution of the operator's pattern recognition faculties is improved.

Using the output of the detector to trigger a monostable multivibrator, a sequence of synchronization pulses occur. If these pulses are recorded simultaneously on another track of the magnetic tape which contains the signal information, the signal has in effect been segmented into a sequence of characteristic patterns as represented in Fig. 3-1.
2. **Variable Rate Sampling**

The time normalization phase of this biological signal analysis system can be implemented with the use of a small digital computer like the PDP-8 series which has peripheral facilities for A/D and D/A conversion; in addition to magnetic tape data storage facilities.

This step in the procedure requires the computer to process the information on the synchronization track of the tape bearing the signal information. This involves storing in memory the time at which the leading edge of each pulse occurs, and subsequently calculating and storing the time interval between pulses. This step establishes the value of the duration parameter $\Delta T_1$ for each characteristic waveform.
With the timing information in memory, and an additional bit of information specifying N, the number of samples required per waveform, as given by Eqn. (2-1), the computer has all the essential data stored to carry out a variable rate sampling procedure. A provision, at this point, must be made for having a capability of filtering the signal before sampling, if there is any likelihood that aliasing will occur. Some filter techniques are discussed in the next section.

The sampling procedure requires the computer to monitor both the signal and synchronization tracks of the tape. The synchronization pulse controls the sampling rate such that upon the incidence of the i'th pulse the sampling rate of the D/A converter, operating on the signal is \( \frac{N}{\Delta t_1} \), as given by Eqn. (2-2). Alternatives for implementing the capability of varying the sampling rate from period to period could use counters to monitor the number of samples taken, or the clock time elapsed, and use this information as a basis for changing the sampling rate.

The digitized information is sequentially stored on binary magnetic tape such that N samples are recorded per characteristic waveform. A sampled value can be converted into a 12 bit binary number and stored on tape approximately every 250 microseconds, or at a maximum rate of 4Khz, using a type 138E analog to digital converter and the Digital Dectape system²⁷.

3. Data Reconstruction

The time normalization is completed by using the computer to read out in sequence the data stored on the digital tape at
a constant rate of $\frac{N}{T} \text{Hz}$. The data is simultaneously reconstructed into its analogue form by a D/A converter. In addition to its conversion properties the D/A converter acts as a zero order hold system by operating continuously upon the register to which the digital data from the tape is being transferred. Along with the signal information, it is necessary to produce a normalized synchronization pulse which occurs every time $N$ data points are read out of the computer. This timing information will subsequently be used to control the analogue phase of the analysis.

The reconstruction is completed by smoothing the time normalized, analogue signal output from the computer and recording the data on one track of tape while simultaneously recording the normalized synchronization pulses on another track.

The pre-processing is thus accomplished by 3 computer runs; the first run obtains the $\Delta T_i$ parameters, the second samples the $i$'th segment at the correct rate, and the third reads out the data at a constant rate.

The ideal low pass filter characteristics given by $H(j\omega)$ in Eqn. (2-10), can be approximated by using a third order low pass Butterworth filter. The approximation is valid if the low pass filter designed has a 3 db. bandwidth well beyond the bandwidth $\omega_{m}^{i}$ of the signal, yet significantly attenuates the sidebands generated by the zero order hold process. (see Fig. 2-7).

An active 3rd order filter with a normalized transfer function given by Eqn. (3-2) is shown in Fig. 3-2a.
This circuit for a low pass filter was constructed and tested, and a response for a particular set of component values is given in Fig. 3-2b.

The operational amplifier used in this filter is a Ratheon RM 709 integrated circuit operational amplifier. Fig. 3-3 gives a connection diagram for the RM 709 along with the associated compensation circuitry that was used.

\[ H'(s) = \frac{1}{s^3 + (2\delta + 1)s^2 + (2\delta + 1)s + 1} \quad (3-2) \]

![Diagram of 3rd Order Active Butterworth Filter](image-url)
FIG. 3-2b. FREQUENCY RESPONSE AND COMPONENT VALUES OF 3rd ORDER ACTIVE BUTTERWORTH FILTER

\[ |H(j\omega)| \]

\[
\begin{align*}
R_1 &= 60K \\
R_2 &= 625K \\
R_3 &= 40K \\
R_4 &= 20K \\
C_1 &= 0.13 \mu f \\
C_2 &= 0.0016 \mu f \\
C_3 &= 0.625 \mu f \\
\delta &= 0.5
\end{align*}
\]

FIG. 3-3. RM 709 OPERATIONAL AMPLIFIER AND COMPENSATION NETWORK

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B. Analogue System

A test module for one harmonic, was built to illustrate the simplicity of implementing the Fourier analyzer of Fig. 2-9. The analogue module was modified slightly to operate on the signal \( y(t) \) itself rather than on its inverted form \(-y(t)\).

A schematic of the module is given in Fig. 3-4. The operational amplifiers used in this unit are identical to the one used in the active filter network discussed previously.

The timing pulses are used to operate a DPST NO reed relay which resets the computer to zero initial conditions at the onset of each characteristic waveform. This is done by shorting the feedback capacitors on the integrators momentarily and then allowing the computation to proceed for \( T \) seconds at which time another timing pulse occurs, coincident with the next waveform.

To test the module, two signals with known spectral properties were produced by a function generator; namely, a square wave and a cosine wave. In both cases the amplitudes of the signals were set to 1.0 volt peak and the periods to 2.0 seconds. The value of \( T = 2.0 \) seconds was chosen in order to cancel the factor \( T/2 \) of Eqn. (2-30) and Eqn. (2-31), and produce a solution to the \( a_k \) and \( b_k \) coefficients directly. The timing pulses, also produced by a function generator, were synchronized to the test signals.

The cosine and sine coefficients \( a_k \) and \( b_k \) for the first harmonic in the Fourier series are respectively, 1.0 and 0.0 for the cosine, and 0.0 and 1.27 for the square wave. The following figures illustrate the operation of the computer module and the solutions it generated.
FIG. 3-4. SCHEMATIC OF ANALOGUE COMPUTER MODULE FOR FOURIER ANALYSIS
FIG. 3-5a. SIGNAL WAVEFORM AND TIMING PULSES

FIG. 3-5b. COSINE WAVEFORM AND SOLUTION TO $a_k$ COEFFICIENT
FIG. 3-5c.  COSINE WAVEFORM AND SOLUTION TO $b_k$ COEFFICIENT

FIG. 3-5d.  SQUARE WAVE AND SOLUTION TO $a_k$ COEFFICIENT
FIG. 3-5e. SQUARE WAVE AND SOLUTION TO $b_k$ COEFFICIENT

The value of the coefficients $a_k$ and $b_k$ can be read at the point where the signal waveform has completed one cycle. These values coincide with the theoretical values given. The reset time of 1 msec, which is the operating time of the reed relay used in this module does not significantly affect the results of the computation. Faster reset times can be accomplished by using electronic switches, at the expense of greater complexity and leakage current problems.

Using a parallel bank of such modules, one for each harmonic to be analyzed, a complete set of Fourier coefficients can be generated for each of the characteristic waveforms in the signal.
C. Data Collection

Since the Fourier coefficient data, in addition to the average power and $a_0$ parameters, are generated simultaneously by the analogue system every $T$ seconds, a parallel method of monitoring many channels of information must be implemented. If every channel is connected to a holding circuit\textsuperscript{20}, driven by a low impedance source as shown in Fig. 3-6,

![Diagram of a holding circuit](image)

FIG. 3-6. DATA HOLD SYSTEM

and operated by a parallel set of switches, the value of the coefficients can be stored as potentials induced upon the capacitors.

The switches are operated by the same timing pulses which reset the Fourier analyzer, and thereby register the values of the Fourier coefficients, in addition to $a_0$ and $p_1$, at the end of the computation cycle of $T$ seconds. This data is subsequently held until another data set is generated $T$ seconds later. To read this data a PDP-8 digital computer with 138E/139E\textsuperscript{27} General
purpose A/D converter and multiplexer control facilities can be used. With this equipment as many as 64 channels of analogue data can be converted into 12 bit digital numbers at a rate of 415 times per second. At this stage it is necessary to carry out the simple computations given by Eqn. (2-23) to amplitude normalize the Fourier coefficients.

The resultant printed output of the $a_0^1$, $P^1$, and amplitude normalized Fourier coefficients can be accomplished several ways depending upon the rate at which these parameters are generated. (1/T sets per second) A comprehensive method would include the use of binary magnetic tape to store the data, and the subsequent outputting of this information at a rate compatible with a Teletype unit.

The following block diagrams summarize this proposal for the implementation of a system for the quantification of patterns in biological signals.

**FIG. 3-7. SEGMENTATION INTO CHARACTERISTIC PATTERNS**
Fig. 3-8. Measurement, storage and output of duration parameters

FIG. 3-9. VARIABLE RATE SAMPLING SYSTEM
FIG. 3-10. TIME NORMALIZATION AND SIGNAL RECONSTRUCTION
FIG. 3-11. FOURIER ANALYSIS AND WAVEFORM PARAMETER DATA OUTPUT
D. CSMP Simulation Results

Three CSMP (Continuous System Modelling Program) simulations were performed to support the proposal given in this thesis. The first was compared with the experimental results obtained from the Fourier analyzer module, the second simulated the time normalization and Fourier analysis phase of the proposal, and the third considered the effect of errors, in the signal segmentation process upon the final results of the signals analysis. These simulation programs are listed in the appendix.

The results of the first simulation, which carries out a Fourier analysis on a cosine and square waveform, are shown graphically in Fig. 3-12a and Fig. 3-12b. These solutions can be compared with the solutions obtained experimentally on the analogue test module given in Fig. 3-5b to Fig. 3-5e.

The second simulation carried out a time normalization and Fourier analysis on one cycle of a normal ECG shown in Fig. 1-1e. A more complete simulation of the entire system including synchronization pulses and a facility for variable rate sampling was attempted; however, the arguments on the IMPULS function which establish the sampling intervals could not be changed during the course of the simulation. As a result the author resorted to a simpler simulation. The principal results of this program are given in Table 3-1 and illustrated graphically in Fig. 3-13. The cumulative AC power curve given in Fig. 3-14 shows that 99.99% of the signal power of this particular ECG is contained in the first 20 harmonics.
FIG. 3-12a. CSMP SIMULATION RESULTS FOR FOURIER ANALYSIS OF COSINE WAVEFORM (K = 1)

FIG. 3-12b. CSMP SIMULATION RESULTS FOR FOURIER ANALYSIS OF SQUARE WAVEFORM (K = 1)
### TABLE 3-1

$a_0$, $P$, and Fourier Coefficient Parameters for One Cycle of Normal ECG ($T = 2.0$ sec)

<table>
<thead>
<tr>
<th>$a_0$ Component</th>
<th>Average Power</th>
<th>$a_k$</th>
<th>$b_k$</th>
<th>$a_k$</th>
<th>$b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.845</td>
<td>15.834</td>
<td>k</td>
<td></td>
<td>k</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.335</td>
<td>0.151</td>
<td>26</td>
<td>-0.004</td>
<td>0.012</td>
</tr>
<tr>
<td>2</td>
<td>0.431</td>
<td>0.530</td>
<td>27</td>
<td>-0.011</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>-1.625</td>
<td>0.679</td>
<td>28</td>
<td>0.010</td>
<td>0.032</td>
</tr>
<tr>
<td>4</td>
<td>-0.947</td>
<td>0.146</td>
<td>29</td>
<td>-0.038</td>
<td>0.031</td>
</tr>
<tr>
<td>5</td>
<td>-0.752</td>
<td>-0.525</td>
<td>30</td>
<td>0.061</td>
<td>0.012</td>
</tr>
<tr>
<td>6</td>
<td>-0.017</td>
<td>-0.783</td>
<td>31</td>
<td>-0.061</td>
<td>0.016</td>
</tr>
<tr>
<td>7</td>
<td>0.609</td>
<td>-0.664</td>
<td>32</td>
<td>-0.036</td>
<td>-0.040</td>
</tr>
<tr>
<td>8</td>
<td>0.785</td>
<td>-0.274</td>
<td>33</td>
<td>-0.005</td>
<td>-0.041</td>
</tr>
<tr>
<td>9</td>
<td>0.562</td>
<td>0.183</td>
<td>34</td>
<td>0.013</td>
<td>-0.017</td>
</tr>
<tr>
<td>10</td>
<td>0.138</td>
<td>0.413</td>
<td>35</td>
<td>0.011</td>
<td>0.017</td>
</tr>
<tr>
<td>11</td>
<td>-0.144</td>
<td>0.347</td>
<td>36</td>
<td>-0.015</td>
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</tr>
<tr>
<td>12</td>
<td>-0.223</td>
<td>0.150</td>
<td>37</td>
<td>-0.047</td>
<td>0.038</td>
</tr>
<tr>
<td>13</td>
<td>-0.184</td>
<td>-0.024</td>
<td>38</td>
<td>-0.063</td>
<td>0.010</td>
</tr>
<tr>
<td>14</td>
<td>-0.108</td>
<td>-0.072</td>
<td>39</td>
<td>-0.053</td>
<td>-0.018</td>
</tr>
<tr>
<td>15</td>
<td>-0.042</td>
<td>-0.048</td>
<td>40</td>
<td>-0.025</td>
<td>-0.022</td>
</tr>
<tr>
<td>16</td>
<td>-0.001</td>
<td>-0.071</td>
<td>41</td>
<td>-0.014</td>
<td>-0.001</td>
</tr>
<tr>
<td>17</td>
<td>0.009</td>
<td>0.013</td>
<td>42</td>
<td>-0.030</td>
<td>0.021</td>
</tr>
<tr>
<td>18</td>
<td>0.004</td>
<td>-0.010</td>
<td>43</td>
<td>-0.062</td>
<td>0.016</td>
</tr>
<tr>
<td>19</td>
<td>0.010</td>
<td>0.001</td>
<td>44</td>
<td>-0.076</td>
<td>-0.018</td>
</tr>
<tr>
<td>20</td>
<td>-0.012</td>
<td>-0.003</td>
<td>45</td>
<td>-0.056</td>
<td>-0.053</td>
</tr>
<tr>
<td>21</td>
<td>-0.004</td>
<td>0.004</td>
<td>46</td>
<td>-0.015</td>
<td>-0.059</td>
</tr>
<tr>
<td>22</td>
<td>-0.019</td>
<td>0.015</td>
<td>47</td>
<td>0.018</td>
<td>-0.028</td>
</tr>
<tr>
<td>23</td>
<td>-0.015</td>
<td>0.006</td>
<td>48</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>24</td>
<td>0.018</td>
<td>0.005</td>
<td>49</td>
<td>-0.017</td>
<td>0.043</td>
</tr>
<tr>
<td>25</td>
<td>-0.022</td>
<td>-0.001</td>
<td>50</td>
<td>0.056</td>
<td>0.040</td>
</tr>
</tbody>
</table>

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FIG. 3-14. CUMULATIVE AC POWER CURVE FOR ONE CYCLE OF NORMAL ECG
### TABLE 3-2

$a_0$, $P$, and Fourier Coefficient Parameters for One Cycle of Normal ECG ($T = 0.43$ sec)

<table>
<thead>
<tr>
<th>$a_0$ Component</th>
<th>Average Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.862</td>
<td>15.918</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k$</th>
<th>$a_k$</th>
<th>$b_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.335</td>
<td>-0.141</td>
</tr>
<tr>
<td>2</td>
<td>0.437</td>
<td>0.535</td>
</tr>
<tr>
<td>3</td>
<td>-1.623</td>
<td>0.699</td>
</tr>
<tr>
<td>4</td>
<td>-0.942</td>
<td>0.158</td>
</tr>
<tr>
<td>5</td>
<td>-0.755</td>
<td>-0.509</td>
</tr>
<tr>
<td>6</td>
<td>-0.030</td>
<td>-0.772</td>
</tr>
<tr>
<td>7</td>
<td>0.585</td>
<td>-0.666</td>
</tr>
<tr>
<td>8</td>
<td>0.760</td>
<td>-0.291</td>
</tr>
<tr>
<td>9</td>
<td>0.549</td>
<td>0.157</td>
</tr>
<tr>
<td>10</td>
<td>0.143</td>
<td>0.387</td>
</tr>
<tr>
<td>11</td>
<td>-0.127</td>
<td>0.331</td>
</tr>
<tr>
<td>12</td>
<td>-0.205</td>
<td>0.145</td>
</tr>
<tr>
<td>13</td>
<td>-0.173</td>
<td>-0.021</td>
</tr>
<tr>
<td>14</td>
<td>-0.103</td>
<td>-0.067</td>
</tr>
<tr>
<td>15</td>
<td>-0.040</td>
<td>-0.046</td>
</tr>
<tr>
<td>16</td>
<td>-0.004</td>
<td>-0.070</td>
</tr>
<tr>
<td>17</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>18</td>
<td>0.004</td>
<td>-0.015</td>
</tr>
<tr>
<td>19</td>
<td>-0.009</td>
<td>-0.005</td>
</tr>
<tr>
<td>20</td>
<td>-0.010</td>
<td>-0.008</td>
</tr>
</tbody>
</table>

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To confirm that no distortion results from the time normalization process the same ECG was analyzed in its unnormalized form. (T = 0.43 sec). The results are given in Table 3.2. A comparison of the data in Table 3-1 gives almost identical results. Small discrepancies can be attributed to the different integration methods used in the simulations.

In the third simulation the same ECG waveform was used, but it was segmented such that the resultant characteristic waveforms assumed durations of 0.425, 0.42 and 0.41 seconds respectively. A Fourier analysis was then performed on each of these segments in order to establish tolerance criteria for the segmentation process. RMS error calculations were carried out upon the results of the analysis of these segments, using the Fourier coefficients of the 0.43 second segment as the norm. The results and calculated errors are given in Table 3-3.
<table>
<thead>
<tr>
<th>k</th>
<th>a'_k</th>
<th>b'_k</th>
<th>a'_k</th>
<th>b'_k</th>
<th>a'_k</th>
<th>b'_k</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.307</td>
<td>-0.167</td>
<td>-0.268</td>
<td>-0.187</td>
<td>-0.182</td>
<td>-0.220</td>
</tr>
<tr>
<td>2</td>
<td>0.485</td>
<td>0.584</td>
<td>0.521</td>
<td>0.625</td>
<td>0.591</td>
<td>0.732</td>
</tr>
<tr>
<td>3</td>
<td>-1.699</td>
<td>0.582</td>
<td>-1.707</td>
<td>0.440</td>
<td>-1.659</td>
<td>0.166</td>
</tr>
<tr>
<td>4</td>
<td>-1.042</td>
<td>0.111</td>
<td>-1.105</td>
<td>0.044</td>
<td>-1.187</td>
<td>0.128</td>
</tr>
<tr>
<td>5</td>
<td>-0.809</td>
<td>0.580</td>
<td>-0.833</td>
<td>-0.653</td>
<td>-0.830</td>
<td>-0.842</td>
</tr>
<tr>
<td>6</td>
<td>-0.046</td>
<td>0.786</td>
<td>-0.067</td>
<td>-0.799</td>
<td>-0.080</td>
<td>-0.895</td>
</tr>
<tr>
<td>7</td>
<td>0.618</td>
<td>-0.603</td>
<td>-0.608</td>
<td>-0.534</td>
<td>0.552</td>
<td>0.471</td>
</tr>
<tr>
<td>8</td>
<td>0.821</td>
<td>-0.203</td>
<td>0.827</td>
<td>-0.099</td>
<td>0.750</td>
<td>0.070</td>
</tr>
<tr>
<td>9</td>
<td>0.586</td>
<td>0.218</td>
<td>0.585</td>
<td>0.294</td>
<td>0.494</td>
<td>0.449</td>
</tr>
<tr>
<td>10</td>
<td>0.139</td>
<td>0.389</td>
<td>0.131</td>
<td>0.396</td>
<td>0.078</td>
<td>0.442</td>
</tr>
<tr>
<td>11</td>
<td>-0.151</td>
<td>0.305</td>
<td>-0.161</td>
<td>0.274</td>
<td>-0.175</td>
<td>0.239</td>
</tr>
<tr>
<td>12</td>
<td>-0.219</td>
<td>0.122</td>
<td>-0.222</td>
<td>0.096</td>
<td>-0.220</td>
<td>0.047</td>
</tr>
<tr>
<td>13</td>
<td>-0.160</td>
<td>0.037</td>
<td>-0.143</td>
<td>-0.044</td>
<td>-0.126</td>
<td>-0.047</td>
</tr>
<tr>
<td>14</td>
<td>-0.090</td>
<td>0.082</td>
<td>-0.068</td>
<td>-0.086</td>
<td>-0.037</td>
<td>0.065</td>
</tr>
<tr>
<td>15</td>
<td>-0.045</td>
<td>0.052</td>
<td>0.043</td>
<td>0.061</td>
<td>0.025</td>
<td>0.075</td>
</tr>
<tr>
<td>16</td>
<td>0.014</td>
<td>-0.069</td>
<td>0.032</td>
<td>0.056</td>
<td>0.035</td>
<td>0.007</td>
</tr>
<tr>
<td>17</td>
<td>-0.009</td>
<td>0.003</td>
<td>-0.016</td>
<td>-0.011</td>
<td>0.001</td>
<td>0.023</td>
</tr>
<tr>
<td>18</td>
<td>0.005</td>
<td>0.008</td>
<td>0.000</td>
<td>0.004</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>19</td>
<td>-0.009</td>
<td>0.007</td>
<td>-0.008</td>
<td>-0.008</td>
<td>-0.009</td>
<td>-0.008</td>
</tr>
<tr>
<td>20</td>
<td>-0.009</td>
<td>-0.012</td>
<td>-0.004</td>
<td>-0.015</td>
<td>0.007</td>
<td>0.006</td>
</tr>
</tbody>
</table>

**RMS Error**

| 0.039 | 0.083 | 0.146 |

**RMS % Error**

| 2.3%  | 4.8%  | 8.6%  |
IV. EVALUATION OF THE PROPOSAL

The use of the Fourier series to quantify characteristic patterns or waveforms in biological signals yields significant advantages in its flexibility and resolution.

For the particular case of the ECG waveform considered in the CSMP simulations, the resolution of the quantification process can be kept within 1% accuracy if the segmentation is carried out with an accuracy of 0.5%. These figures are implied in the approximately linear relationship between resolution and segmentation error as given in Table 3-3.

Balm, in order to carry out his cross correlation analysis of ECG waveforms had to establish beforehand examples of typical normal and abnormal waveforms and as such injected an element of bias into his work. The capability of quantifying such patterns by means of a Fourier series and using this numerical data to establish norms, as will be discussed later, can eliminate this element of bias.

Furthermore, in the case of biological signals which have not received as much intensive study as ECG, typical waveforms in such signals may be difficult to establish. This fact emphasizes the necessity of having a capability of quantifying patterns in biological signals and thereby establishing a basis upon which precise comparisons of signal waveforms can be made.

Steinberg's pattern recognition program is restricted to a specific class of signals, namely ECG. The Fourier series affords the flexibility of being able to represent a wide range of biological signals with the same set of parameters. This, in turn, offers the advantage of allowing one instrumentation system
to be applied to all such signals in order to compute these parameters.

In addition, one system like a Fourier series synthesizer could also be used to reconstruct a signal waveform from these parameters if such a process was found necessary.

The main problem involved in using a Fourier series representation of patterns within biological signals is the necessity of segmenting the signal into characteristic patterns. Unless automated segmentation techniques like those used by Steinberg are used for every particular class of signal to be analyzed, subjective judgements will have to be used in order to determine the approximate location of characteristic points in the signal which segment the signal into fundamental waveforms.

The entire analysis is dependent upon this first subjective step and thus the resultant data could be criticized as being merely a reflection of it; however, the question of degree must be considered. This judgement of where to segment a signal is not as arbitrary as those made by medical personnel on not only the endpoints of a characteristic waveform, but the significance of what lies in between. Furthermore, if these characteristic points are stable and well defined throughout the signal, it is quite feasible that the segmentation can be implemented with accuracy and consistency insuring that meaningful comparisons can be made between waveforms within a particular class of signals. It must nevertheless be said that to make the entire process completely objective it is necessary to develop an automated method of segmenting a general class of biological signals.

This subjective element appears in varying degrees in the
other signals analysis systems considered in the literature survey and points out the fact that the problem of analyzing biological signals is not yet fully resolved.

The next problem which must be taken into consideration is that of fidelity. If measures are taken to prevent aliasing, due to limitations upon the maximum sampling rate of the A/D converter, the high frequency portion of the signal is eliminated under the assumption that the information contained in this band is not significant. If this assumption is not valid, equipment with higher sampling rate capabilities will have to be used.

For the particular ECG waveform considered in the CSMP simulations (Table 3-1), for all practical purposes was found to be 50 Hz. The sampling rate required for a signal with such a bandwidth is approximately 500 Hz. This rate is well within the capabilities of the computer facilities considered in the proposal.

Implementing the digital portion of the analysis, assuming the hardware is available, is mainly a programming problem. In the analogue phase synchronization is the critical factor. The reset time on the analogue modules must be much smaller than the fundamental period of the waveform to insure that only a minimum amount of signal information is passed when the computer is in the reset mode. Extending the period by means of variable tape speed techniques and using electronic switching to reduce reset times, can in most cases allow this criterion to be met.

With the exception of the initial segmentation phase, the system proposed is a fully automatic one. Balm's and Lowenberg's techniques involved extensive manual operations upon each
characteristic waveform in order to carry out their analyses.

To illustrate the operation of the proposed system the processing of a one-minute record of an ECG signal can be used as an example. Such a record contains approximately 140 cardiac cycles, and can be segmented semi-automatically in approximately 20 minutes. Three digital computer runs to establish duration parameters, time normalize the signal, and output it at a constant rate each require 1 minute of processing. This assumed that the recorded signal information is made available to the computer at the same speed that it was recorded. An additional minute of processing is required to carry out the Fourier analysis on the special purpose analogue computer, and approximately 15 minutes of Teletype operation to print out the final parameter values. The entire procedure would thus require about 45 minutes to perform.

To quantify patterns using FFT techniques would require large digital computer facilities. In this case the signal data would still have to be available in a segmented form. Allowing approximately 3 minutes of computer time to perform the FFT computations results in a total operation time of 23 minutes. This is about a 50% savings, in terms of operation time, over the method proposed, but at the expense of using a large digital computer. Such facilities may not be readily available to a clinic involved in biomedical research.

The system for the quantification of patterns in biological signals given in the proposal can be implemented with relatively simple inexpensive equipment; the main components being a small digital computer and a special purpose analogue computer. The complete system, including all peripheral equipment, could be installed in a clinic for the approximate cost of $40,000 and be available for use at all times.
V. CONCLUSIONS AND RECOMMENDATIONS

A system has been proposed which attempts to quantify the patterns or waveforms that occur in biological signals, based on a Fourier series representation of the waveform. It is outside the scope of this thesis to establish whether or not the parameters used to quantify these waveforms can be related directly to some particular physiological phenomena. These considerations must be left to those trained in the medical and biological sciences.

The feasibility of establishing such relationships nevertheless remains, since the information contained in the waveform is transformed intact into numerical data, and provides a resolution and basis for further analysis which is far superior to any broad subjective judgements that can be made upon the waveform.

Further analysis of this data could include statistical techniques to correlate normal and abnormal physiological states in a patient, and the data that was taken from the analysis of a biological signal generated by the patient. If such correlations do exist a multidimensional space based on the parameters defining the characteristic waveform could be established and partitioned to create a taxonomic space.

The possibilities of implementing this system without the use of time normalization was brought out late in the author's work. The entire point of time normalization was to maintain the period \( T \) of the waveforms constant such that a simple analogue system without variations in the parameter \( T \) could be implemented. However, using the other tactic, variations in the parameter \( T \)
can be accomplished by the use of multipliers as shown in Fig. 5-la.

Fourier analysis of the unnormalized signal can thus be accomplished if three tracks of information are simultaneously available; one specifying the signal, one the synchronization pulses and the third producing a potential equal to \( \frac{1}{\Delta T_i} \) for the duration \( T_i \) of the \( i \)'th waveform as shown in Fig. 5-1b. This third track of information could be generated by a digital computer with D/A conversion facilities, having stored in memory the values of the duration parameters \( \Delta T_i \).

The alternate method, outlined above, offers a possibility of decreasing the complexity involved in implementing this system. The variable sampling rate system and the time normalization and signal reconstruction systems can be replaced by a simpler digital computer system which generates the \( \frac{1}{\Delta T_i} \) signal at the expense of adding two multipliers for each of the analogue computer modules.
FIG. 5-1a. MODIFIED FOURIER ANALYSIS ANALOGUE COMPUTER MODULE

FIG. 5-1b. 3 TRACKS OF INFORMATION REQUIRED TO IMPLEMENT MODIFIED SYSTEM

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### CONTINUOUS SYSTEM MODELING PROGRAM

### PROBLEM INPUT STATEMENTS

<table>
<thead>
<tr>
<th>TITLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOURIER ANALYSIS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PARAMETER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = (1.0, 4*1.0)$</td>
</tr>
<tr>
<td>TNorm = 2.0, PI = 3.1415927</td>
</tr>
<tr>
<td>WM = 2.0<em>PI</em>20.0/TNorm</td>
</tr>
<tr>
<td>W = 2.0<em>PI</em>K/TNorm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONSTANT</th>
</tr>
</thead>
<tbody>
<tr>
<td>FH = SINE (0.0, PI, PI/2.0)</td>
</tr>
<tr>
<td>D3FN = (WM<strong>3) * FH - 2<em>WM</em>D2FN - 2*(WM</strong>2) * D1FN - (WM**3) * FN</td>
</tr>
<tr>
<td>D2FN = INTGRL (0.0, D3FN)</td>
</tr>
<tr>
<td>D1FN = INTGRL (0.0, D2FN)</td>
</tr>
<tr>
<td>FN = INTGRL (0.0, D1FN)</td>
</tr>
<tr>
<td>CD2X = W X - FN</td>
</tr>
<tr>
<td>CD1X = -INTGRL (0.0, CD2X)</td>
</tr>
<tr>
<td>D1X = W*CD1X</td>
</tr>
<tr>
<td>NEGX = -INTGRL (0.0, D1X)</td>
</tr>
<tr>
<td>X = -NEGX</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TERMINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = CD1X*2.0/TNorm</td>
</tr>
<tr>
<td>B = NEGX*2.0/TNorm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WRITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>WRITE (6, 120)</td>
</tr>
<tr>
<td>WRITE (6, 121) K, A, B</td>
</tr>
<tr>
<td>120 FORMAT (23H HARMONIC A B)</td>
</tr>
<tr>
<td>121 FORMAT (F7.3, 1X, F7.3, 1X, F7.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIMER</th>
</tr>
</thead>
<tbody>
<tr>
<td>DELT = 0.001</td>
</tr>
<tr>
<td>PMDEL = 0.01</td>
</tr>
<tr>
<td>FINTIM = 2.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>PRINT</th>
</tr>
</thead>
<tbody>
<tr>
<td>FH, FN, CD1X, NEGX</td>
</tr>
</tbody>
</table>

| END |

### OUTPUT VARIABLE SEQUENCE

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<tr>
<th>M</th>
<th>W</th>
<th>FH</th>
<th>D3FN</th>
<th>D2FN</th>
<th>D1FN</th>
<th>FN</th>
<th>NEGX</th>
<th>X</th>
<th>CD2X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z0007</td>
<td>CD1X</td>
<td>D1X</td>
<td>ZZ0009</td>
<td>A</td>
<td>B</td>
<td>ZZ0011</td>
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</table>

### OUTPUTS

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>PARAMS</th>
<th>INTEGS</th>
<th>MEM BLKS</th>
<th>PORTTRAN</th>
<th>DATA CDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 (500)</td>
<td>34 (1400)</td>
<td>6 (400)</td>
<td>5 + 0 = 5 (300)</td>
<td>21 (600)</td>
<td>6</td>
</tr>
</tbody>
</table>

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### CONTINUOUS SYSTEM MODELING PROGRAM

#### PROBLEM INPUT STATEMENTS

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<thead>
<tr>
<th>TITLE</th>
<th>FOURIER ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETER</td>
<td>K=((1.0, 0.4*1.0))</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>TNORM(=2.0), PI=3.1415927</td>
</tr>
<tr>
<td>FUNCTION</td>
<td>(y=0.0, 0.0, 0.001, 1.0, 0.5, 1.0, 0.999, 1.0, 1.0, 0.0, \ldots)</td>
</tr>
<tr>
<td>INITIAL</td>
<td>WM=2.0*TNORM/20, 0/TNORM</td>
</tr>
<tr>
<td>DYNAMIC</td>
<td>WM=2.0<em>PI</em>K/TNORM</td>
</tr>
<tr>
<td>TERMINAL</td>
<td>PH=AFGN(Y, TIME)</td>
</tr>
<tr>
<td></td>
<td>D3FN=(WM<strong>3)<em>PH-2</em>WM<em>D2FN-2</em>(WM</strong>2)*D1FN-(WM**3)*FN</td>
</tr>
<tr>
<td></td>
<td>D2FN=INTGRAL(0.0, D3FN)</td>
</tr>
<tr>
<td></td>
<td>D1FN=INTGRAL(0.0, D2FN)</td>
</tr>
<tr>
<td></td>
<td>FN=INTGRAL(0.0, D1FN)</td>
</tr>
<tr>
<td></td>
<td>CD2X=W*X-FN</td>
</tr>
<tr>
<td></td>
<td>CD1X=-INTGRAL(0.0, CD2X)</td>
</tr>
<tr>
<td></td>
<td>D1X=W*CD1X</td>
</tr>
<tr>
<td></td>
<td>NEGX=-INTGRAL(0.0, D1X)</td>
</tr>
<tr>
<td></td>
<td>X=-NEGX</td>
</tr>
<tr>
<td>TIMER</td>
<td>A=CD1X*2.0/TNORM</td>
</tr>
<tr>
<td>PRINT</td>
<td>B=NEGX*2.0/TNORM</td>
</tr>
<tr>
<td></td>
<td>WRITE(6, 120)</td>
</tr>
<tr>
<td></td>
<td>WRITE(6, 121)K, A, B</td>
</tr>
<tr>
<td>PRINT</td>
<td>DELT=0.001, PREDL=0.01, PINTM=2.0</td>
</tr>
<tr>
<td>PRINT</td>
<td>FH, FN, CD1X, NEGX</td>
</tr>
<tr>
<td>END</td>
<td></td>
</tr>
<tr>
<td>STOP</td>
<td></td>
</tr>
</tbody>
</table>

**INPUT VARIABLE SEQUENCE**

<table>
<thead>
<tr>
<th>W</th>
<th>FH</th>
<th>D3FN</th>
<th>D2FN</th>
<th>D1FN</th>
<th>FN</th>
<th>NEGX</th>
<th>X</th>
<th>CD2X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0007</td>
<td>CD1X</td>
<td>D1X</td>
<td>ZZ0009</td>
<td>A</td>
<td>B</td>
<td>ZZ0011</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**OUTPUTS** **INPUTS** **PARAMS** **INTEGS** **+ MEM BLKS** **FORTRAN** **DATA CDS**

| 11(500) | 35(1400) | 7(400) | 5+ | 0= | 5(300) | 21(600) | 8 |
### CONTINUOUS SYSTEM MODELING PROGRAM

**PROBLEM INPUT STATEMENTS**

<table>
<thead>
<tr>
<th>TITLE</th>
<th>BIOLOGICAL SIGNAL ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIMENSION DTAPE(1000)</td>
<td></td>
</tr>
<tr>
<td>FIXED</td>
<td>J, ICOUNT</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>THORM=2.0, PNORM=15.0, T=0.43, PI=3.1415927</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>N=1000.0</td>
</tr>
<tr>
<td>INCON</td>
<td>J=1, ICOUNT=1, K=1.0</td>
</tr>
<tr>
<td>INCON</td>
<td>ACCP=0.0</td>
</tr>
</tbody>
</table>

**FUNCTION**

\[
Y = 0.0, 2.0, 0, 0.01, 1.0, 0.02, 0.1, 0.03, 0.1, 0.04, 0.1, 0.05, 0.1, 0.06, 0.1, 0.07, 0.1, 0.08, 0.1, 0.09, 0.1, 0.10, 0.1, 0.11, 0.1, 0.12, 0.1, 0.13, 0.1, 0.14, 0.1, 0.15, 0.16, 0.17, 0.18, 0.19, 0.20, 0.21, 0.22, 0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.30, 0.31, 0.32, 0.33, 0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.40, 0.41, 0.42, 0.43, 0.44, 0.45, 0.46, 0.47, 0.48, 0.49, 0.50, 0.51, 0.52, 0.53, 0.54, 0.55, 0.56, 0.57, 0.58, 0.59, 0.60, 0.61, 0.62, 0.63, 0.64, 0.65, 0.66, 0.67, 0.68, 0.69, 0.70, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.80, 0.81, 0.82, 0.83, 0.84, 0.85, 0.86, 0.87, 0.88, 0.89, 0.90, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.00 |

**INITIAL**

\[
WM = 2.0 * PI * 70.0 / TNORM \\
NORM = 2.0 * PI * K / TNORM \\
NORM = T / N \\
STN = TNORM / N \\
F = NLFGEN(Y, TIME) \\
IF (ICOUNT - 1) 1, 1, 2 \\
1 CONTINUE |

**DIGITAL PHASE**

**TIME NORMALIZATION**

\[
SAMPL1 = IMPULS(ST, ST) \\
IF (SAMPL1 - 0.1) 20, 20, 10 \\
10 DTAPE(J) = F \\
X1 = J \\
X2 = DTAPE(J) \\
J = J + 1 \\
20 CONTINUE \\
GO TO 3 \\
2 CONTINUE |

**ZERO ORDER HOLD SYSTEM**

\[
RDOUT = IMPULS(STN, STN) \\
IF (RDOUT - 0.1) 40, 40, 30 \\
30 FH = DTAPE(J) \\
X3 = J \\
J = J + 1 \\
40 CONTINUE |

**ANALOGUE PHASE**

**3RD ORDER FILTER**

\[
D3PN = (WM**3) * FH - 2 * WM * D2PN - 2 * (WM**2) * D1PN - (WM**3) * FN \\
--- \\
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* DC AND POWER ANALYSIS

DC=INTGRL(0.0, FN)
FN=FN**2
P=INTGRL(0.0, FN)

* FOURIER ANALYSIS

CD2X=W*X-FN
CD1X=-INTGRL(0.0, CD2X)
D1X=W*CD1X
NEGX=-INTGRL(0.0, D1X)
x=-NPGX

NOSORT

TERMINAL

3 CONTINUE

IP(ICOUNT-1) 4,4,5
4 CONTINUE
WRITF(6,33)
33 FORMAT(/15X,25H CONTENTS OF DIGITAL TAPE)
DO 50 J=1,1000,10
WRITF(6,44) J,DTAPE(J)
50 CONTINUE
44 FORMAT(18X,15,3X,F8.3)
GO TO 6
5 CONTINUE
IP(K-1.1) 7,7,9
7 CONTINUE
A0=DC*TNORM/2.0
P=P/TNORM
C=P/PNORM
WRITF(6,11)
WRITE(6,22) A0,P,C
11 FORMAT(/15X,41H A0 COMPONENT AVERAGE POWER FACTOR C)
22 FORMAT(16X,F8.3,9X,F8.3,7X,F8.3)
8 CONTINUE
A=CD1X*2.0/TNORM
R=NEGX*2.0/TNORM
AN=A*SORT(C)
BN=B*SORT(C)
WRITE(6,55)
WRITE(6,66)K,A,B,AN,RN
55 FORMAT(/15X,9H HARMONIC,8X,1HA,8X,1HR,8X,2HAN,8X,2HBN)
ACCP=(A**2+R**2)/2.0*ACCP
WRITE(6,77)
WRITE(6,88) ACCP
77 FORMAT(/15X,18H ACCUMULATED POWER)
88 FORMAT(21X,F8.3)
J=1
6 CONTINUE

TIMER
METHOD
PRINT
PARAMETER
INCON
TIMER
PRINT
END
STOP
### CONTINUOUS SYSTEM MODELING PROGRAM

#### PROBLEM INPUT STATEMENTS

<table>
<thead>
<tr>
<th>TITLE</th>
<th>BILOGICAL SIGNAL ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>ONE CYCLE OF A NORMAL FCN</td>
</tr>
<tr>
<td>PARAMETER</td>
<td>K = (1.0, 1.9*1.0)</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>TNorm = 0.43, PNorm = 15.0, PI = 3.1415927</td>
</tr>
<tr>
<td>INCON</td>
<td>ACCP = 0.0</td>
</tr>
<tr>
<td>FUNCTION</td>
<td>Y = 0.0, 2.0, 0.01, 1.0, 0.02, 0.1, 0.022, 0.1, 0.026, 0.4, ...</td>
</tr>
<tr>
<td></td>
<td>0.03, 1.5, 0.034, 4.5, 0.038, 6.3, 0.04, 8.0, 0.044, 10.3, ...</td>
</tr>
<tr>
<td></td>
<td>0.048, 11.5, 0.05, 11.6, 0.054, 10.1, 0.058, 8.5, 0.06, 8.0, ...</td>
</tr>
<tr>
<td></td>
<td>0.064, 5.0, 0.07, 3.0, 0.074, 2.0, 0.08, 1.6, 0.09, 1.8, ...</td>
</tr>
<tr>
<td></td>
<td>0.10, 1.9, 0.11, 1.6, 0.12, 1.7, 0.13, 1.8, 0.14, 2.0, ...</td>
</tr>
<tr>
<td></td>
<td>0.15, 2.4, 0.16, 2.8, 0.17, 3.3, 0.18, 4.2, 0.19, 4.6, ...</td>
</tr>
<tr>
<td></td>
<td>0.20, 5.2, 0.21, 5.4, 0.22, 5.3, 0.23, 5.2, 0.24, 4.6, ...</td>
</tr>
<tr>
<td></td>
<td>0.25, 4.0, 0.26, 3.5, 0.27, 2.8, 0.28, 2.7, 0.29, 2.6, ...</td>
</tr>
<tr>
<td></td>
<td>0.30, 2.7, 0.31, 3.1, 0.32, 3.4, 0.33, 3.5, 0.34, 3.6, ...</td>
</tr>
<tr>
<td></td>
<td>0.35, 4.2, 0.36, 4.8, 0.37, 4.7, 0.38, 3.8, 0.39, 2.6, ...</td>
</tr>
<tr>
<td></td>
<td>0.40, 2.0, 0.41, 2.0, 0.42, 2.0, 0.43, 1.9</td>
</tr>
</tbody>
</table>

**INITIAL**

| W = 2.0*PI*70.0/TNorm |
| = 2.0*PI*K/TNorm |

**DYNAMIC**

* ANALOGUE PHASE

**3RD CEDER FILTER**

FH = NLFGEN(Y, TIME)

D1PN = (WM**3) * FN - 2*WM*DFN - 2*(WM**2)*D1FN - (WM**3)*FN

D2FN = INTGRL(0.0, D3FN)

D1FN = INTGRL(0.0, D2FN)

FN = INTGRL(0.0, D1FN)

* DC AND POWER ANALYSIS

DC = INTGRL(0.0, FN)

FN2 = FN**2

P = INTGRL(0.0, FN2)

* FOURIER ANALYSIS

C2DN = WM*FN

C1DX = -INTGRL(0.0, C2DN)

D1X = WM*C1DX

NEGX = -INTGRL(0.0, D1X)

X = -NEGX

**TERMINAL**

IF (K-1.1) 7, 7, 8

7 CONTINUE

A0 = DC*2.0/TNorm

P = P/TNorm

C = P/PNorm

WRITE (5, 11)

WRITE (6, 22) A0, P, C

11 FORMAT (/15X, 41HA0 COMPONENT, AVERAGE POWER FACTOR C)

22 FORMAT (16X, P3.3, 6X, F8.3, 7X, F8.3)

8 CONTINUE

A = CD1X*2.0/TNorm

B = NEGX*2.0/TNorm

AM = A*SORT(C)

BN = B*SORT(C)

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WRITE(6,66) K,A,B,AN,BN
55 FORMAT(/'15X,2H HARMONIC,8X,1HA,8X,1HB,8X,2HAN,8X,2HPN) 74
   ACCP=(A**2+B**2)/2.0+ACCP
WRITE(6,77)
WRITE(6,88) ACCP
77 FORMAT(/'15X,18H ACCUMULATED POWER)
88 FORMAT(21X,P2.3)

J=1
TIMER DELT=0.001,PRDEL=0.005,PINTIM=0.43
METHOD RSFX
PRINT FH,FN,CD1X,NEGX
END
STOP

TPUT VARIABLE SEQUENCE

<table>
<thead>
<tr>
<th>W</th>
<th>FH</th>
<th>D3FN</th>
<th>D2FN</th>
<th>D1FN</th>
<th>FN</th>
<th>DC</th>
<th>FN2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>GX</td>
<td>X</td>
<td>CD2X</td>
<td>ZW0011</td>
<td>CD1X</td>
<td>D1X</td>
<td>ZW0013</td>
<td>ZW0015</td>
<td>A0</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>AN</td>
<td>FN</td>
<td></td>
<td>ACCP</td>
<td>J</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

INPUTS

1(500) 48(1400) 9(400) 7* 0= 7(300) 42(600) 20

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***CONTINUOUS SYSTEM MODELING PROGRAM***

***PROBLEM INPUT STATEMENTS***

<table>
<thead>
<tr>
<th>TITLE</th>
<th>BIOLOGICAL SIGNAL ANALYSIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>ONE CYCLE OF A NORMAL ECG</td>
</tr>
<tr>
<td>PARAMETER K</td>
<td>(1.0, 19*1.0)</td>
</tr>
<tr>
<td>CONSTANT TNORM</td>
<td>0.43, FNORM=15.0, PI=3.1415927</td>
</tr>
<tr>
<td>INCON ACCP</td>
<td>0.0</td>
</tr>
<tr>
<td>FUNCTION Y</td>
<td>0.0, 0.01, 1.0, 0.02, 0.1, 0.022, 0.1, 0.026, 0.1, 0.03, 1.5, 0.034, 4.5, 0.038, 6.3, 0.04, 8.0, 0.044, 10.3, 0.048, 11.6, 0.05, 11.6, 0.054, 10.1, 0.058, 8.5, 0.06, 8.0, ...</td>
</tr>
<tr>
<td>INITIAL WM</td>
<td>2.0<em>PI</em>70.0/TNORM</td>
</tr>
<tr>
<td>DYNAMIC W</td>
<td>2.0<em>PI</em>K/TNORM</td>
</tr>
</tbody>
</table>

* ANALOGUE PHASE

* 3RD ORDER FILTER

FH=NL.FGEN(Y,TIME)
D3FN=(WM**3)*FH-2*WM*D2FN-2*(WM**2)*D1FN-(WM**3)*FN
D2FN=INTGRL(0.0,D3FN)
D1FN=INTGRL(0.0,D2FN)
FN=INTGRL(0.0,D1FN)

* DC AND POWER ANALYSIS

DC=INTGRL(0.0,FN)
FN2=FN**2
P=INTGRL(0.0,FN2)

* FOURIER ANALYSIS

CD2X=W*X-FN
CD1X=-INTGRL(0.0,CD2X)
D1X=W*CD1X
NEGX=-INTGRL(0.0,D1X)
X=-NEGX

TERMINAL

IF K-1,117,7,9
7 CONTINUE
A0=DC*2.0/TNORM
P=P/TNORM
C=P/PNORM
WRITE(6,11)
WRITE(6,22)A0,P,C
11 FORMAT(/15X,4I4A0 COMPONENT AVERAGE POWER FACTOR C)
22 FORMAT(16X,F8.3,9X,F8.3,7X,F8.3)
8 CONTINUE
A=CD1X*2.0/TNORM
B=NEGX*2.0/TNORM
AN=A*SORT(C)
BN=B*SORT(C)

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WRITE (6,66) K, A, B, AN, BN
55 FORMAT (/15X,9H HARMONIC,9X,1HA,8X,1HB,8X,2HAN,8X,2HBN)
ACCP = (A**2+B**2)/2.0+ACCP
WRITE (6,77)
WRITE (6,88) ACCP
77 FORMAT (/15X,18H ACCUMULATED POWER)
88 FORMAT (21X,F8.3)

J = 1
DELT = 0.001, PRDEL = 0.005, FINTIM = 0.43
METHOD = RKSPX
PRINT = FH, FN, CD1X, NEGX

K = (1.0, 1.0)

TNORM = 0.425
DELT = 0.001, PRDEL = 0.005, FINTIM = 0.425

K = (1.0, 1.0)
TNORM = 0.42
DELT = 0.001, PRDEL = 0.005, FINTIM = 0.42

STOP

TPUT VARIABLE SEQUENCE
\[
\begin{array}{cccccccccc}
W & FH & D3FN & D2FN & D1FN & FN & DC & FN2 & P \\
Gx & X & C2D & Z0011 & C1X & D1X & Z0013 & Z0015 & A0 & P \\
A & B & AN & BN & ACCP & J \\
\end{array}
\]

INPUTS | OUTPUTS | PARMS | INTEGS | MEM | BLKS | FORTRAN | DATA | CDS
---|---|---|---|---|---|---|---|---
48 (1400) | 1 (500) | 9 (400) | 7+0=7 (300) | 42 (600) | 28 | 28

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VITA AUCTORIS

1945  Born on March 5th, in Stryj, Western Ukraine.

1950  Came to Canada -- went to General Brock Public School and Forster Collegiate Institute.

1968  Graduated from the University of Windsor, Windsor, Ontario, Canada, with the degree of B.A.Sc. in Electrical Engineering.

1969  Candidate for the degree of M.A.Sc. in Electrical Engineering at the University of Windsor.

1969  Working for Energy Mines & Resources - Oceanographic, Ottawa, Ontario. Plan to do further graduate work in future either on Ph.D. or M.B.A.