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Backscattering of light atomic projectiles from thin gold films in the energy range 50-110 keV.

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BACKSCATTERIM6 OF LIGHT ATOMIC PROJECTILES FROM THIN Au FILMS

IN THE ENERGY RANGE 50-110 keV

by

Everett Brimner

A Thesis

Submitted to the Faculty of Graduate Studies through the Department of Physics in Partial Fulfillment of the Requirements for **the Degree of Master of Science at the University of Windsor**

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1970

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ABSTRACT

Thin gold films, vacuum deposited onto a beryllium substrate, have been irradiated with l_H^+ , l_H^+ , l_H^+ , $7Li^+$, and l_l^+ projectiles in the energy range 50-110 keV. The backscatter intensity at a laboratory **scattering angle of 136.4 degrees was measured. The intensity is not exactly proportional to film thickness but increases more rapidly.** This effect has been attributed to multiple small-angle collisions. Using estimates for the angular divergence of an ion beam as it traverses a thin film, the backscatter intensity has been corrected for compound **scattering. The observed scattering cross sections are in good agreement** with theory for scattering in a shell-shielded Coulomb potential.

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

The energy loss of atomic projectiles in solids can be **attributed to two separate processes: nuclear recoil and electronic** excitation. In a nuclear collision the target nucleus together with its orbital electrons recoils, and the projectile is deflected. In general slow ions lose energy primarily to target atoms and their **trajecto ry consists of a devious path, while fast ions dissipate energy** mainly through electronic excitation and travel, therefore, in nearly straight lines. Although the major energy loss mechanism for light **k atomic p rojectiles above about 10 eV is the electronic one, some nuclear** collisions occur and backscattering of incident ions can result. At **energies above 10⁶-10⁷ eV, such interactions are almost purely** Coulombic for the lightest atomic projectiles, and the scattering, **therefore,exhibits Rutherford behaviour. At lower energies the Incident** particle at its point of maximum potential energy in the collision sees a weakened Coulomb potential, due to electronic screening between the two **nuclei. One of the aims of this investigation is to study the interaction** potential for nearly head-on collisions of light atomic projectiles with gold. In such collisions the incident ions can be scattered back through **the surface of the target sample into the surrounding vacuum.**

Backscattering of light atomic projectiles is used to investigate **the location of impurity atoms in crystals (Mayer et a l. 1968), and to explain sputtering (Behrisch 1969; Sigmund 1968 and 1969; van** Wijngaarden et al. 1970). McCracken and Freeman (1969) have studied

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the energy distribution of hydrogen ions backscattered from thick heavy targets. They assumed a model for the process in which "the incident ion slows down in the target material without undergoing any scattering, that it is then scattered *in a pure Coulomb potential* through a large angle and that it returns to the surface without further scattering." The energy dependence of their observed backscatter intensity agrees fairly well with predictions based on their model, but the intensity is high by a factor of about 4. A normalization by 3.1 is required to fit Behrisch's angular distribution of H^+ backscattered from copper. They therefore concluded that a single-collision model was inadequate. To determine the significance of multiple small-angle collisions in this **work, an analysis of the dependence of backscatter yield on target thickness has been made.**

CHAPTER 11

THEORY

Consider an ion beam incident on a target film of several atomic layers thickness. If the ion mass m₁ is less than the target atom mass m_o , sing le-collision backscattering can occur. In such a violent collision only the target atom and the projectile need be considered. This is so **because the target atom is nearly free (Bergstrom and Domeij I** 966**) since** bond energy of neighbouring atoms is small for keV projectiles. The projectile does not see the target atom's neighbours since the transit time across the interaction region of the target atom (typically about 10⁻¹⁶ sec) is much shorter than normal vibrational periods (10⁻¹³ -10⁻¹⁴ sec) in the solid. Multiple scattering results whenever the projectile sufficiently penetrates the electron clouds of more than one atom, and **occurs therefore In a ll thick target media. The two-body scattering problem, however, must be understood before we can proceed to study** multiple scattering effects.

The scattering of keV projectiles from atoms is not strictly elastic, since electronic excitation of both the incident ion and the **target atom usually results. The energy loss to electrons, however, is a small fractio n of the incident ion energy and the single scattering** event can be considered elastic.

Because of electronic screening, the actual potential energy of two interacting atoms is unknown. At high energies, when the colliding **nuclei interpenetrate each other's electron clouds, electronic screening is small and the potential energy is almost purely Coulombic:**

 $\overline{\mathbf{3}}$

$$
V(R) = Z_1 Z_2 e^2 / R \tag{1}
$$

The scattering is described by the familiar Rutherford cross section

$$
(d\sigma/d\Omega)_{R} = (b/4)^{2} \csc^{4}(0/2) * \qquad (2)
$$

where b is the collision diameter, the distance of closest approach of two unscreened nuclei in a head-on collision:

$$
b = Z_1 Z_2 e^2 / (\frac{1}{2}uv^2)
$$
 (3)

Here u is the reduced mass of the projectile-target system, and v is their initial relative velocity. The Rutherford cross section is accurate only as long as the minimum distance of approach $\frac{1}{2}$ b (1+csc θ /2) is much less than the effective screening radius of the interacting particles. For ¹H backscattered from Au at θ ₁=136.4°, for example, we find deviations from Rutherford behaviour at energies as high as 100 keV. For more massive projectiles and or smaller scattering angles, the lower energy bound for Rutherford behaviour increases.

Several interatomic potentials have been proposed to allow for **the electronic screening of the nuclear Coulomb fie ld . Bohr (1948) suggested that the interaction between two atomic structures is given by the screened Coulomb potential energy**

$$
V(R) = (Z_1 Z_2 e^2 / R) \exp(-R/a_e).
$$
 (4)

*** A ll Center-of-Hass (CM) quantities w ill be w ritten without** a subscript. Laboratory (lab) quantities will be denoted **by the subscript** *J l* **.**

The range of the screening, and hence a measure of the overall size of the two interacting particles, is given by the Bohr screening length

$$
a = a_0 / \sqrt{2_1^{2/3} + 2_2^{2/3}}
$$
 (5)

Here $a_0 = 0.529$ Å, the Bohr radius. Firsov (1958) employs a screening factor calculated from the Thomas-Fermi statistical model of the atom

$$
a = 0.8853 a_B.
$$
 (6)

We shall refer to and use this latter value exclusively as the screening **parameter in the remainder of this thesis.**

For $R \ll a$, the Bohr potential energy is approximately given as

$$
V(R) = (Z_1 Z_2 e^2 / R) (1 - R/a) H(1 - R/a)
$$
 (7)

where H(x) is the Heavi side step function

$$
H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}.
$$

Eq. 7 represents the interaction energy between a charge Z_1e and a charge Z₂e when the latter is surrounded by a spherical shell of **negative charge density**

$$
-Z_2e (4\pi a^2)^{-1} \delta (R-a)
$$
 (See Appendix 1).

An important advantage of the shell-shielded potential is that the corresponding cross section has a simple analytic form:

$$
\frac{d\sigma}{d\Omega}=\left[\frac{ab(a+b/2)}{4a(a+b)\sin^2(\theta/2)+b^2}\right]^2=k\left(\frac{d\sigma}{d\Omega}\right)_R
$$
 (8)

where the correction factor k Is

$$
k = \left[\frac{1 + \frac{1}{2}b/a}{1 + b/a + b^2/(4a^2 \sin^2 \frac{1}{2}\theta)}\right]^2.
$$
 (9)
(See Appendix 11).

Eq. 8 is readily integrated to give the classically expected finite **to ta l cross section**

$$
\sigma^{-} = \int_{\mu_{\text{TT}}} \mathrm{d}\alpha \, \frac{\mathrm{d}\sigma}{\mathrm{d}\alpha} = \mathrm{e}^{2} \, \pi \quad . \tag{10}
$$

In the limit of high incident energy (small b) and large screening **radius, b/a is small, and /dG7\(Eq. 8) approaches / (Eq. 2), \ d / i j idXLjp;**

Smith et al. (1967) have used variational methods to obtain the best empirical fit of the functional form

$$
V(R) = Ae^{2}/R \exp(-R/C)
$$
 (11)

to experimental data for scattering of He on Ne and Ar in the energy **range of 10 eV to 100 keV. In Eq. 11 the variational parameters are** A and C. The empirical values for C are about twice as large as the **corresponding Thomas-Fermi screening parameters (Eq. 6). Furthermore** Abrahamson (1963) has calculated Thomas-Fermi-Dirac potentials for the **interaction of noble-gas atoms, and has found good agreement with experiment** at small R. In Figure 1 Abrahamson's results for Ne-Ne, Kr-Kr, and **Rn-Rn interactions are shown, along with the analytic potentials of Eqs.** 1, 4 and 7, for both screening lengths a and 2a. Curve number 3, **representing the shell-shielded Coulomb potential energy**

 $\overline{7}$

Figure I . Various potentials as a function of internuclear distance R. The points are Abrahamson's (1963) calculations for interactions between like noble-gas atoms in the Thomas-Fermi-Dirac theory.

$$
V(R) = (Z_1 Z_2 e^2/R) (1-R/2a) H(1-R/2a)
$$
 (12)

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gives the best fit for R_{Ka}. The corresponding cross section is given **by Eqs. 8 and 9 with a replaced by 2a.**

Cross sections for Bohr's screened potential have been calculated numerically for several values of b/a (Everhart et al. 1955). For $\pi/2\leq \theta \leq \pi$ and $b/a \leq l$, the Rutherford cross section is as much as a factor **of 2.5 larger than Everhart's values, whereas the shell-shielded cross section (Eq. 8) always lies w ithin 10% of the numerical results.**

Our experimental scattering cross sections (to be discussed below) for ¹H, ⁴He, ⁷Li and ¹¹B on Au for a fixed laboratory scattering angle of 136.4[°] and at laboratory energies from 50 to 110 keV are presented as circles in Fig. 2. The Rutherford cross sections (dash-dot **lines) lie well above the shell-shielded (dashed lines) and Everhart's** exponentially-screened Coulomb values $(\diamondsuit$ ¹s). The good agreement between **the shell-shielded and exponentially-screened Coulomb cross sections is evidence that backscattering is governed prim arily by the potential at** distances R \leq a. This is so because the former interaction is zero for R > a while the latter is not. The solid curve, presenting the cross **sections of the shell-shielded Coulomb potential with the screening radius doubled, agrees fa ir ly well with the reduced experimental data (Chapters** V and V1) both in energy dependence and absolute value.

 $\bar{\beta}$

 $\overline{9}$

CHAPTER 111

EXPERIMENTAL

Figure 3 is a schematic diagram of the apparatus. Honoenergetic ion beams at energies below 120 keV are obtained from a magnetic analyzer (van Wijngaarden et al. 1970). The angular divergence of the emerging beam is limited to within 0.2⁰ by a circular slit system. **A fter collim ation the ion beam enters the target chamber which is** maintained at a pressure of the order of 10^{"/} Torr. Regularly spaced thin target films of Au, vacuum deposited onto a thick Be plate, can be **moved across the path of the ion beam as indicated by the arrows. A Faraday cup connected to a bellows can be moved into the path of the** ion beam. The absolute value of the current is measured to within **2% by a Keithley 410 electrom eter. An Ortec Model E-013-025-100 surface -4 b a rrie r detector, subtending a solid angle û,n = (5 .2 5 ± 0 .0 3) x 10** sr. is positioned at 136.4⁰ with respect to the incident ion beam. The pulses from the detector are fed through a preamplifier (Ortec Model 109a), **a main am p lifier (Ortec Model 485), a single channel analyzer (Ortec Model** 406a), and are recorded by a digital ratemeter (Ortec Model 434).

3:1 Detector Efficiency

Any ion which is detected has passed through the Au electrode (228 *%* **thickness) of the surface b a rrie r detector, and then produced** sufficient ionization to create a pulse discernible above the noise of the detector. To find the minimum energy for 100% detection, the detector was placed directly in the path of monoenergetic ^lH⁺ and ⁴He⁺ **beams. With the lower discrim inator level of the single channel analyzer**

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cnID *1* **u </> 2 at** fixed just above the detector noise level, the detector efficiency (number of counts per incident projectile) was observed to be essentially **perfect for protons at primary energies E** \geqslant **40 keV and for** 4 **He at E** \geqslant **50 keV. As the primary energy decreased below these values, so did the efficien cy of the detector. Below 10 keV no particles could be detected.**

It is the counting efficiency for the backscattered projectiles which is of interest in the present experiment. The energy loss for these projectiles in the thin target films is quite small (Chapter V), and 100% backscattered-projectile counting efficiency was expected for primary energies down to about the same limits. To verify this, only incident energies were used for each projectile and each film for which the counting rate of the backscattered ions into the detector $(\theta_L = 136.4^{\circ})$ remained constant for a small range of the lower discriminator settings.

3 :2 FiIm Thickness

Five thin Au films were vacuum deposited simultaneously onto Be **and glass substrates using an Edwards Coating Unit (Model I2EA/722). The thicknesses of the Au film s on the glass substrate were measured using optical techniques (Marton and Schleslnger 1969). To check that** corresponding Au films on the two substrates were of equal thickness, the **following experiment was performed. The glass-backed and Be-backed Au** films were successively irradiated with ⁴He projectiles and the intensities of the backscattered projectiles from the various thin films and the two **substrates were measured. The backscatter yield (number of backscatter** counts per incident projectile) from the Be substrate was 0.2% of **that from the thinnest Be-backed Au film , while that of the glass was** about 15% of the yield from the thinnest glass-backed film. After

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subtraction of the substrate backscatter yields from those of the thin films, the resulting corrected yields were equal, within experimental error of about 2%, for corresponding films on glass and Be. This agreement was found over the primary energy range 50-110 keV and for various film **thicknesses In the range 75-350 X. It was, therefore, concluded that** (1) corresponding films on glass and Be had the same thickness, and (2) **the observed backscatter yields could be corrected by subtraction of the substrate y ie ld .**

The measured thicknesses of the five Au films are 75, 120, 170, 265, and 350 \hat{R} . These are estimated to be correct to within 5 \hat{R} .

CHAPTER IV

RESULTS

4:1 Backscatter of

The five Au films and the Be substrate were bombarded in succession by proton beams of the order of 10⁻¹⁰ A at various fixed primary energies in the range 50-110 keV. The total backscatter counts during 10 sec intervals were recorded for each film and the substrate at each primary energy. Just prior to and immediately after each counting interval the **incident ion current was measured to an accuracy of better than 2%. If** these two ion current measurements differed by more than 5%, the data were discarded. In order to minimize statistical errors a minimum total of 3000 backscatter events were recorded for each film at each energy. The **gross backscatter y ie ld , b' , was obtained by dividing the backscatter** counts per second by the projectile particle current.

For a given film b" includes contributions from both the Au film and the substrate. At the highest primary energy the B' value of the substrate reached a maximum and equalled about 5% of the B^ value of the thinnest Au film. The corrected backscatter yield, B_o, was calculated (Sec 3:2) by subtracting the substrate yield from each recorded film yield. The energy dependence of the observed backscatter yield, B_o, for the various Au films is presented in Fig. 4 by the solid curves through the points marked as circles. The dashed curves, which represent the same data corrected for energy loss in the films and for multiple small-angle scattering effects, will be discussed in Chapter VI.

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Figure 4. The solid curves through the experimental points (circles) show the dependence of the observed backscatter yield on primary energy for ¹H impinging on various gold films. The dashed curves, without **experimental points, are plots of the corresponding backscatter yields** corrected for multiple scattering effects versus the average energy.

4:2 Backscattering of $\frac{L_1}{16}$, $\frac{7}{16}$, and $\frac{11}{16}$

The five Au films and the Be substrate were also irradiated with monoenergetic beams of $\frac{4}{1}$ + $\frac{1}{6}$, $\frac{1}{1}$ and $\frac{11}{8}$ ions. With these projectiles the substrate yield was either zero, or negligible, being **three orders of magnitude smaller than the thinnest film yield in the** case of ⁴He. The observed B_o values for the various thin films are presented as circles in Figs. 5-7 as a function of primary energy. As the projectile mass increased, the primary energy for which 100% detection could be obtained increased rapidly, limiting the energy range of our **investigations.**

Figure 5. The solid curves through the experimental points (circles) **show the dependence of the observed backscatter yield on primary energy** for ⁴He impinging on various gold films. The dashed curves, without **experimental points, are plots of the corresponding backscatter yields** corrected for multiple scattering effects versus the average energy.

Figure 6. The solid curves through the experimental points (circles) **show the dependence of the observed backscatter yield on primary energy for ^Li impinging on various gold film s. The dashed curves, without experimental points, are plots of the corresponding backscatter yields** corrected for multiple scattering effects versus the average energy.

Figure 7. The solid curves through the experimental points (circles) **show the dependence of the observed backscatter yield on primary energy** for ¹¹B impinging on gold films of 75 Å and 120 Å thickness. The dash**ed curves, without experimental points, are plots of the corresponding** backscatter yields corrected for multiple scattering effects versus the **average energy.**

CHAPTER V

ENERGY LOSS IN THE FILM

The observed B_o value consists of projectiles backscattered anywhere inside the film material. Since the ion continually loses energy as it traverses the target medium, there is a range of energies **over which the main scattering event can occur. In the absence of** multiple scattering, the backscatter yield is directly proportional to film thickness, and on the average all violent collisions can be considered to occur at the centre of the thin film. To each B_o value we assign an average energy of interaction $E = E_0 - \Delta E$, where E_0 **Is the primary energy and aE is the energy loss in one-half of the film** thickness. The $\triangle E$ values have been computed using the energy-loss theory of Lindhard and Scharff (1961) (Lindhard et al. 1963). We shall **now summarize some of the important aspects of this theory.**

The total stopping power of a target material for a projectile is related to the total stopping cross section S by

$$
\frac{dE}{dR} = NS
$$
 (13)

where N is the atomic density of the target medium. Although an energetic atomic projectile is deflected only by nuclear collisions (Bohr 1948), the energy of the projectile is dissipated to both electrons and recoiling atoms in the stopping medium. Thus the total stopping cross section consists of the stopping cross section S_n for less in energy in nuclear collisions and the stopping cross section S_e for loss in **energy to electrons:**

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$$
S = S_{\mathbf{e}} + S_{\mathbf{n}} \tag{14}
$$

From Eqs. 13 and 14 we see that

$$
\frac{dE}{dR} = \left(\frac{dE}{dR}\right)_R + \left(\frac{dE}{dR}\right)_e \tag{15}
$$

where
$$
\left(\frac{dE}{dR}\right)_n = NS_n
$$
 and $\left(\frac{dE}{dR}\right)_e = NS_e$. (16)

To obtain a universal description for energy loss, which is valid for all atomic projectile-target combinations, Lindhard et al. introduced **the dimensionless variables**

$$
\epsilon = E \frac{am_2}{Z_1 Z_2 e^2 (m_1 + m_2)}
$$
 (17)

and
$$
\varphi = R \frac{N m_2 4 \pi a^2 m_1}{(m_1 + m_2)^2}
$$
 (18)

for energy and range respectively. In terms of these variables Eqs. 16 can be written as

$$
\left(\frac{dE}{dR}\right)_n = \frac{4\pi aZ_1 Z_2 e^2 Nm_1}{(m_1 + m_2)} \left(\frac{d\epsilon}{d\Omega}\right)_n
$$
 (19)

and
$$
\left(\frac{dE}{dR}\right) = \frac{4m aZ_1 Z_2 e^2 N m}{(m_1 + m_2)} \left(\frac{d}{d\varphi}\right) e^{\varphi}
$$
 (20)

Lindhard e t a l. present a universal curve (F ig . 8) fo r */ § £ * **versus (d?/n** $\epsilon^{\frac{1}{2}}$ which is based on the Thomas-Fermi statistical model of the colliding **atoms. In terms of the new variables the electronic stopping power is given by**

$$
\left(\frac{d\epsilon}{d\hat{\gamma}}\right) = K\epsilon^{\frac{1}{2}}\tag{21}
$$

22

and

where
$$
K = Z_1^{1/6} \frac{0.0793 \ Z_1^{\frac{1}{2}} \ Z_2^{2}}{(Z_1^{2/3} + Z_2^{2/3})^{3/4} A_1^{3/2} A_2^{1/2}}
$$
 (22)

in which A₁ and A₂ are the atomic numbers of the projectile and target atoms respectively. The values of K for ¹H, ⁴He, ⁷Li and ¹¹B on Au are 15.13, 3.00, 1.71, and 1.216 respectively. Thus using Fig. 8 for /<u>de</u>\ and Eqs. 21 and 22 for /<u>de</u>\, one can compute the <u>dE</u> values from **Id find the latter of the drive drive** drive d **Eqs.** 15**,** 19 **and 20.**

The energy loss ΔE in a thin stopping region of thickness ΔR (much smaller than the range of the projectile) is then found from the relation**ship**

$$
\Delta E \simeq \left(\frac{dE}{dR}\right)^{\Delta R} \tag{23}
$$

where $\triangle E = \triangle E_n + \triangle E_n$, (24)

$$
\Delta E_{\mathbf{h}} \simeq \left(\frac{\mathrm{d}E}{\mathrm{d}R}\right)_{\mathbf{h}} \simeq R, \tag{25}
$$

 $\Delta E_{\rm e} \approx \left(\frac{dE}{dR}\right)_{\rm e} \Delta R_{\rm e}$ (26)

The simple analytic form (Eq. 21) applies only for projectile **2/3 velocities** $\mathsf{v}_{\beta} \mathsf{v}_{\mathsf{t}} = \mathsf{v}_{\mathsf{o}} \mathsf{z}_{\mathsf{1}}^T$ **where** v_{o} **is the velocity of an electron in the** first Bohr orbit of ¹H. Only at these low velocities $(v < v₁)$ is the electronic stopping nearly proportional to v. The upper limit for the validity of the Lindhard and Scharff theory therefore increases with **projectile mass, being** \sim **25 keV for protons and** \sim **250 keV for** 4° **He.** For protons travelling in Au, however, the theory is in agreement with **experiment (see Whaling 1958, and references contained therein) up to**

energies of 120 keV, allowing us to compute $\triangle E$ for protons as well as for heavier projectiles over the entire energy range investigated.

For velocities $v > v_1 = v_0 Z_1^{2/3}$ electronic stopping completely dominates, but it is no longer proportional to v. For $v < v_1$ electronic stopping still dominates for $Z_1 < Z_2$. For heavier projectiles and or **decreasing energy, nuclear stopping gradually overtakes electronic stopping** to become the major mechanism of energy loss. However even for our heaviest projectile $\binom{11}{B}$, $\Delta E^{}_{\alpha}$ remains a factor of 4 larger than $\Delta E^{}_{n}$. For this **reason the model described by McCracken and Freeman (1969), where the** incident ion travels in straight lines except for violent collisions, should be correct to a first approximation for light ions.

The average $\triangle E$ values in the thinnest film are 0.7 keV for protons, \sim 1 keV for ⁴He and ⁷Li and \sim 1.8 keV for ¹¹B. Even large uncertainties in these small energy corrections still allows us to obtain the average **energy of interaction,** $E = E_0 - \Delta E$ **, to a sufficiently high degree of precision.**

CHAPTER VI

MULTIPLE SCATTERING EFFECTS

If the incident ions travelled straight-line paths in the target **m aterial u n til they suffered a single vio len t interaction and were scattered, and then proceeded recti I inearl y out of the stopping medium,** only particles scattered into $\Delta \Omega = 5.25 \times 10^{-4}$ sr about $\theta_1 = 136.4^{\circ}$ would reach the detector. As a consequence of multiple small-angle scatterings before the violent collision, the beam gradually spreads out about the direction of initial incidence. In any infinitesimal time in**terval dT, there is a probability** $\sigma(\theta, E)$ **dT for each particle to be scattered back, where 6 is the scattering angle with respect to the direction of motion immediately preceding scattering and E Is the** projectile energy at that instant. The projectile density in the **diverging beam is azimuthally isotropic. Because the Rutherford cross** section is highly preferred in the forward direction, particles travelling at angles $0 < \theta_L = 136.4^{\circ}$ with respect to the detector are more likely to enter the detector than those with $\theta > \theta_i$. Consequently more particles are multiply-scattered into the solid angle of the detector than out of it, resulting in an enhancement of the **backscatter y ie ld . Backscattered particles also suffer small deflections,** further enhancing the backscatter yield. To compare the observed backscatter yield with the theoretically predicted one for a single scattering event, a correction must be applied to B_o.

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6:1 Angular Divergence

To find the correction factor by which the observed backscatter yield must be divided in order to correct for multiple small-angle **deflections, an estimate of the angular divergence of the beam in** travelling through a thin stopping region is required. Suppose the projectile encounters several scattering centers on its passage through **the stopping medium. Since the individual deflections are completely random, the average deflection is zero. The root-mean-square (rms) deflection w ill not vanish, however, since there exists a random walk** in angle away from the direction of initial incidence. The total rms **deflection associated with the p ro je c tile penetrating a depth aR into** the medium can be estimated as follows. Consider an elastic collision **between two atoms in which the particles are deflected through the common** angle of scattering θ in the CM system. From Appendix 111 the projectile **(m^) transfers an amount of energy to the recoiling target (m^) equal to**

$$
T = \frac{4m_1m_2}{(m_1 + m_2)^2} E \sin^2(\theta/2), \qquad (27)
$$

where E is the impact energy. For light projectiles $(m_1 \ll m_2)$, the lab and CM systems are almost identical, and $\theta \approx \theta_0$. Since the vast majority of collisions are small-angle ones Eq. 27 reduces to

$$
T \simeq 4(m_1/m_2) E (0/2)^2.
$$
 (28)

When a swift ion traverses a thin stopping region, it undergoes **several small deflections from its original direction of motion, losing an amount of energy**

$$
\Delta E_n = \sum_i T_i
$$
 (29)

in atomic recoils, where the summation includes all scattering events. Substituting for T_i from Eq. 28, the nuclear energy loss becomes

$$
\sim E_n \simeq (m_1/m_2) \sum_i E_i \theta_i^2 \simeq (m_1/m_2) E \sum_i \theta_i^2
$$
 (30)

where E_i is the projectile energy prior to the ith collision. In a thin film the projectile energy remains nearly constant and has, therefore, been taken outside the summation sign. If all scattering angles are small, the resulting angular distribution is approximately gaussian **(Bohr, 1948) with a root-mean-square width**

$$
\phi_{\text{rms}} \simeq \sqrt{\sum_{i} \theta_{i}^{2}} \simeq \sqrt{\frac{1}{E} \frac{m_{2}}{m_{1}} \Delta E_{\text{n}}}
$$
 (31)

The calculation of ΔE_n has been described in the previous Chapter. **Because a ll scattering events are not small angle ones, the actual** angular distribution will differ from a gaussian. Eq. 31 is, there**fo re, considered to be only a rough estimate of the angular divergence.**

6:2 Correction Factor

The correction factor $(CF = B_0/B)$ is the factor by which the observed backscatter yield, enhanced by the effects of multiple small**angle scattering, must be divided to obtain the yield that would be** observed at a laboratory scattering angle $\theta_l = 136.4^{\circ}$ in the absence of multiple deflections. The correction factor was estimated, by means of a computer program, using the following assumptions:

(1) The Rutherford cross section gives the form of the backscattered angular distribution. Since for large scattering angles the shape of the angular distribution is but little affected by electronic screening, this assumption introduces only small errors.

(2) All backscatter events occur at a film depth $AR = t/2$ where **t is the film thickness.**

(3) Just prior to backscattering the incident ions are distributed over an angular distribution of gaussian form, centered about the incident direction, whose rms width is given by Eq. 31 with $\triangle R = t/2$.

(4) As the backscattered ions travel out of the film they again spread out in a gaussian angular distribution about their original scatter directions, with an rms width at the film surface again given by Eq. 31 but with $\Delta R = (t/2) \sqrt{2}$, the average distance the ions travel through the film after being scattered from the film center towards the **detector.**

The results of the computation are presented in Fig. 9 as a plot of the correction factor versus ϕ , the predicted rms width of the angular distribution for $\Delta R = t/2$. The correction factor was found to be slightly small. A better one was obtained by replacing ϕ , as calculated from Eq. 31, with 1.10 ϕ and reading the correction factor from the same curve (Fig. 9). The new values were found to be adequate (See Section 6:3) for all film thicknesses and all projectiles at all **energies investigated.**

The dependence of B (the B_o value corrected for multiple scattering effects) on the average energy of interaction (the primary energy corrected for energy loss in the film) is represented by the

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dashed curves in Figs. 4 -7 . Both axes in each of these diagrams have a dual meaning. The abscissa represents the primary energy E ⁰ for the solid curves, and the average energy $E = E_o - \Delta E$ for the dashed curves. The ordinate represents the observed backscatter yield B_o for the solid curves, and the corrected yield $B = B_0 / CF$ for the dashed curves.

6:3 Dependence of Backscatter Y ield on FIIm Thickness

The dependence of the observed backscatter yield B_o on the **measured film thicknesses a t an average Interaction energy of 100 keV is shown by the dashed curves through the experimental points marked as** squares in Fig. 10. Since smooth curves can be fitted quite nicely through **these points, the film thickness measurements appear to be accurate well** within the uncertainty of \pm 5 $\hat{\lambda}$.

In the absence of multiple scattering the backscatter yield should be proportional to film thickness for t values much smaller than the range of the projectile in the film. The circles in Fig. 10 represent **the same data as the squares, but the ordinate now represents the yield** corrected for multiple deflections $(B = B_o/CF)$. Straight lines (the solid curves) passing through the origin provide good fits to the B values, indicating that the correction factor for multiple deflections may be fairly accurate. It will be noticed that multiple scattering effects increase rapidly with increasing projectile mass. The reason for this is that the ratio $\Delta E_{n}/m_{1}$ (See Eq. 31) increases with projectile **mass.**

The error in the correction factors is difficult to estimate, **but the deviations from unity of these factors appear to be accurate** to, at least, within 20%. For the thinnest film the average correction

3 0

Figure 10. Each of these diagrams is a composite showing (1) the thickness dependence of the experimental backscatter yield (squares) for the indicated projectile on Au, at an energy $E = 100$ keV, and **(2) the thickness dependence of the corresponding backscatter yield** (circles) corrected for multiple scattering effects.

factors in the energy range investigated are 1.02, 1.03, 1.05 and 1.12 for ¹H, ⁴He, ⁷Li and ¹¹B respectively. A 20% error in the deviations from unity of these numbers results in a change in B of less than 1% for the three lightest projectiles and less than 3% for the fourth. We conclude, therefore, that the uncertainties in B introduced by multiple scattering are practically negligible for the thinnest film.

3 2

CHAPTER V il

DISCUSSION

Because the data for the thinnest film are the most certain, **only the 75 % film results w ill be compared with theory. The laboratory backscatter cross section is given by (See Appendix IV)**

$$
\frac{d\sigma}{d\Lambda} \simeq \frac{B}{N(\Delta \Omega) t}
$$
 (32)

where $\Delta \Omega$ is the solid angle subtended by the detector and N is the atomic density of the stopping medium. The cross sections for the various projectiles were calculated at each energy investigated by substitution of numerical values into Eq. 32. The cross sections for ¹H near 100 keV **were found to be 1.03 times the values predicted by the shell-shielded Coulomb potential with a replaced by 2a (Eq. 12). Since the conversion from B value to cross section involves the measured value of the film** thickness and its uncertainty, and the statistical error in B (less than 2%), no physical significance can be attributed to such a factor. The observed cross sections for all projectiles have been divided by 1.03. This normalizes the theoretical and experimental values for ¹H at 100 keV. **The normalized cross sections are presented as circles in Fig. 2. The** solid curves provide fairly accurate fits to the data, both in magnitude and energy dependence. The shell-shielded Coulomb potential, therefore, **quite accurately describes the interaction involved in backscattering of** light atomic projectiles in the energy range investigated.

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Figure 2, Energy dependence of lab cross sections at 0l= 136.1+"fo r various p ro jec tile s on gold, Rutherford; ____ shell-shielded Coulomb a->2a; ---- shell-shielded Coulomb; \diamond **screened Coulomb; o experimental.**

APPENDIX 1

THE SHELL SHIELDED COULOMB POTENTIAL

Consider a potential

$$
\phi(R) = (Q/R) (1 - R/a) H(1 - R/a)
$$
 (7)

where H(x) Is the Heavi side step function.

Define U(R) = R $\phi(R) = Q$ (1-R/a) H(1-R/a).

The charge distribution Q (R) giving rise to this potential $\phi(R)$ is given **by Polsson's equation**

$$
\nabla^2\phi = -4\pi \mathcal{C}(\mathsf{R}).
$$

For $R > a$, $\nabla^2 \phi$ vanishes since ϕ does. For $R < a$, $\nabla^2 \phi = -4\pi Q \delta(\vec{R})$ **For R = a,** $\nabla^{\mathsf{c}} \phi = \underline{\mathsf{I}}$ **<u>d</u> l R** ϕ **(R)** \vert = <u>1 d</u> U(R) **R dR^ lR=a R dR^ R = a**

Here $\delta(R)$ is the Dirac delta function. **Therefore the total** $\nabla^{\varphi} \varphi(R) = I$ **U"(a)-4πQ** $\delta(R)$ **R**

Note: $U''(R) = 0$ everywhere except at $R = a$.

To find $U''(a)$, let $x = (1 - R/a)$.

Then $\mathbf{d}^2 = \mathbf{l}$ \mathbf{d}^2 **<jr2 dx^**

and
$$
\frac{1}{R} \frac{d^2}{dR^2}
$$
 $U = 0$

$$
= 0
$$

$$
\frac{d^2}{Ra^2} \left[2 \delta(x) + x \delta'(x) \right] \text{ since } H'(x) = \delta(x),
$$

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$$
= \frac{0}{Ra^{2}} \delta(x)
$$
 since f(x) $\delta'(x) = -\delta(x)$ f'(x)

$$
= \frac{0}{Ra^{2}} \delta(1-R/a)
$$

$$
= \frac{0}{Ra} \delta(R-a)
$$
 since $\delta(x) = a \delta(ax)$
and $\delta(x) = \delta(-x)$

$$
= \frac{0}{a^{2}} \delta(R-a)
$$
 since $\delta(x) = 0$ unless R=a.

Therefore V^t(R) =& 8(R-a) - I+ttO, <5(R) a&

 $= -4 \pi C$,

and the charge distribution is

$$
C = Q \, \delta(\vec{R}) - Q(4\pi a^2)^{-1} \, \delta(R-a) ,
$$

corresponding to a point charge Q. at the centre of a spherical shell (radius a) of surface charge density $- Q(4\pi a^2)^{-1}$.

APPENDIX 11

THE CLASSICAL CROSS SECTION FOR THE SHELL-SHIELDED COULOMB POTENTIAL

The validity of the classical calculation has been discussed by Bohr (1948) and by Mott and Massey (1965). For a classical treatment to be valid, two conditions must be satisfied:

(1) $\lambda \ll a$, $\lambda \ll b$ where λ is the de Broglie wavelength of the projectile, **b** is the collision diameter,

and a is the screening length.

(2) θ > λ /2 π a (See Bohr).

For the cases treated in this thesis (H, He, Li, and B at 50-110 keV on Au), the classical calculation is valid at all scattering angles $\theta > 0.1^{\circ}$.

The classical differential scattering cross section,

$$
\frac{d\sigma}{d\Omega} = \frac{2 \pi \rho d\rho}{2 \pi \sin\theta} \frac{1}{d\theta} = \frac{\frac{1}{2} d(\rho^2)}{d(\cos\theta)} = \frac{1}{4} \left[\frac{d(\sin^2\theta/2)}{d(\rho^2)} \right]^{-1}
$$
 (33)

for a central potential, can be calculated from the relationship between **the Center-of-Mass (CM) scattering angle 6 and the impact parameter p. From the conservation of energy Goldstein (1959) finds**

$$
\theta = \pi - 2 \int_{0}^{X_0} d\gamma / \sqrt{1 - V(p/\gamma)/E - \gamma^2}
$$
 (34)

where $y = p/R$ and $y_0 = p/R_o$. Here R_o is the turning point of the orbit, **determined by**

$$
E = L^2/2uR^2 + V(R_o) = Ep^2/R + V(R_o)
$$
 (35)

where E is the CM energy, u is the reduced mass and L is the angular momentum.

> **o** In terms of the new variable **y** and the parameter $\mathtt{Q} = \mathtt{Z}_1\mathtt{Z}_2\mathtt{e}^{\mathtt{-}}$ the

potential (Eq, 7)

$$
V(R) = Q/R (1-R/a), \quad 0 \le R \le a
$$

= 0 , $R > a$ (7)

becomes V

$$
V(p/y) = Qy/p (1-p/ya), y \ge p/a
$$

= 0 , y < p/a

Substitution of this potential into Eq. 34 yields

$$
\theta = \pi - 2 \int_{0}^{P/a} dy / \sqrt{1 - y^2} - 2 \int_{P/a} dy / \sqrt{1 - Qy (1 - p/ya) / (pE) - y^2}.
$$

With $w = Ea/Q$, and $c = p/a$, we obtain

$$
\theta = \pi - 2 \int_{0}^{c} dy / \sqrt{1 - y^{2}} - 2 \int_{c}^{y_{0}} dy / \sqrt{1 + (1 - y/c)1/w - y^{2}}
$$

= $\pi - 2$ Arcsin y $\Big|_{0}^{c} - 2$ Arcsin (2y + 1/wc) / $\sqrt{(wc)^{-2} + 4 + 4/w}$ $\Big|_{y=c}^{y=z}$

Using Eq. 35, the upper limit y_0 becomes

$$
2y_{o} = -1/wc + \sqrt{(wc)^{-2} + 4 + 4/w}
$$

On substitution of this value into the previous equation, we find

$$
\theta = 2 \text{ Arcsin} \left[(1 + 2\text{wc}^2) / \sqrt{1 + (2\text{wc})^2 (1 + 1/\text{w})} \right] - 2 \text{ Arcsin } \text{c}
$$

= 2 (A-B)

where
\nsin A =
$$
(1 + 2wc^2) / \sqrt{1 + (2wc)^2 (1 + 1/w)}
$$

\ncos A = $2wc \sqrt{1 - c^2} / \sqrt{1 + (2wc)^2 (1 + 1/w)}$
\nsin B = c
\ncos B = $\sqrt{1 - c^2}$

From the previous equations it follows that

$$
\sin \theta/2 = \sin(A-B) = \sin A \cos B - \cos A \sin B
$$

$$
= \sqrt{\frac{1-c}{1 + (2wc)^2 (1 + 1/w)}}
$$
 (36)

and

$$
\frac{d(\sin^2{\theta/2})}{d(p^2)} = \frac{1}{a^2} \frac{d(\sin^2{\theta/2})}{d(c^2)}
$$

$$
= \frac{1}{a^2} \left[\frac{1 + 2w}{1 + (2wc)^2 (1 + 1/w)} \right]^2
$$

$$
= \frac{1}{a^2} \left[\frac{1 + 2w}{1 - c^2} \right] = \sin^4{(\theta/2)}.
$$

The last step is easily verified by substituting for sin⁴ (8/2) (Eq. 36). Thus the differential scattering cross section (Eq. 33) becomes

$$
\frac{d\sigma}{d\alpha} = \frac{a^2}{4} \left[\frac{1-c^2}{1-2w} \right]^2 \csc^4(\theta/2). \qquad (37)
$$

In terms of the quantities $Q \left(= Z_1 Z_2 e^2 \right)$ and w $\left(= \text{Ea}/Q \right)$, the collision diameter is given by $b = a/w$, and the Rutherford cross section **becomes**

$$
\left(\frac{d\sigma}{d\Omega}\right)_{R} = (b/4)^{2} \csc^{4}(0/2) = \left(\frac{a}{4w}\right)^{2} \csc^{4}(0/2)
$$

which allows us to rewrite Eq. 37 In the form

$$
\frac{d\sigma}{d\Omega} = \frac{4}{1 + 2w} \left[\frac{1 - c^2}{1 + 2w} \right] w^2 \left(\frac{d\sigma}{d\Omega} \right)_R
$$

$$
= k \left(\frac{d\sigma}{d\Omega} \right)_R
$$

where $k'' = 2w(1-c^2)/(1+2w)$

With the aid of Eq. 36, the k value becomes

$$
k = \left[\frac{1 + b/(2a)}{1 + b/a + b^2/(4a^2 \sin \theta/2)}\right]^2
$$
 (9)

and the differential cross section is

$$
\frac{d\sigma}{d\Omega} = k \left(\frac{d\sigma}{d\Omega} \right)_{R} = \left[\frac{ab (a + b/2)}{4a \sin^{2}(0/2) (a + b) + b^{2}} \right]^{2}.
$$
 (8)

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(38)

APPENDIX 111

TRANSFORMATION FORMULAE BETWEEN LAB AND CM FRAMES

Because the experim entalist takes measurements in the laboratory (lab) system, and the theoretician finds it simpler to make his calculations **in the Center-of-Mass (CM) system, conversion formulae from one frame of reference to the other are necessary.**

Consider a nonrelativistic binary elastic collision which **asym ptotically appears in the lab system as in Fig. 11.**

(a) before collision (b) after collision Fig. 11. Asymptotic views of an elastic collision in the lab system. Initially a particle of mass m_1 is moving with velocity $v^{\alpha}_{0\ell}$ and impact parameter p towards a particle of mass m₂ at rest in the lab frame. After scattering through angles θ_{α} and ϕ_{α} , m₁ and m₂ have velocities **v**_{il} and v_{2l} respectively. In the lab frame the center of mass of the

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two particles moves with constant velocity $\overline{V}_{\hspace{-0.05cm}\text{C}^m}$. The magnitude and direction of \bar{V}_{CM} are determined by the conservation theorem for total linear **momentum**

$$
(\mathbf{m}_1 + \mathbf{m}_2)\overline{\mathbf{V}}_{cm} = \mathbf{m}_1 \overline{\mathbf{V}}_{OL}
$$

The CM system moves with respect to the lab system so that the origin of the CM system is always coincident with the center of mass of the colliding particles. In the CM frame the total energy is

$$
E = E_{\lambda} - \frac{1}{2} (m_{\lambda} + m_{\lambda}) V_{cm}^{2} = \frac{m_{\lambda}}{m_{\lambda} + m_{\lambda}} E_{\lambda}
$$

Because the total linear momentum is zero at all times in this frame, the particles are travelling in opposite directions to one another before and after the collision. Hence they share a common angle of scattering, **e (F ig . 12).**

From a vector addition diagram relating the final velocities **in the CM and lab frames, the relationship between the CM and lab scattering angles can be obtained.**

$$
\tan \theta_q = \frac{(V_{oq} - V_{cm}) \sin \theta}{V_{cm} + (V_{oq} - V_{cm}) \cos \theta} = \frac{\sin \theta}{m_1/m_2 + \cos \theta}
$$

The second step follows by the definition of the center of mass velocity: $m_2 V_{cm} = m_1 (v_{og} - v_{cm})$. Note that if $m_1 \ll m_{2'}$ $\theta \approx \theta_{\mathcal{L}}$ and the lab and CM systems are almost identical. For the case $\theta_4 = 0_L = 136.4^{\circ}$, the CM **scattering angles are 136,6°, 137.2°, 137.8°, and 138.6° for *H, ^He,** 7_{Li} and ¹¹B respectively, if $m_2=197$.

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Also from Fig, 13 we see that

$$
\mathsf{v}_{2\,g} = 2 \; \mathsf{V}_{cm} \; \sin(\pi/2 - \phi_g)
$$
\nand

\n
$$
2 \; \phi_g = \pi - \theta.
$$

Hence
$$
v_{22} = 2 V_{cm} \sin(\theta/2)
$$

and the kinetic energy transferred to the target if the incident projectile is elastically scattered through θ is

$$
T_{g} = \frac{1}{2} m_{2} v_{2\ell}^{2} = \frac{4 m_{1} m_{2}}{(m_{1} + m_{2})^{2}} E \sin^{2}(\theta/2). \qquad (27)
$$

111:1 Relationship of the Cross Sections

In an actual experiment the counting rate of a fixed detector is independent of the reference frame of the observer,

i.e.
$$
\left(\frac{d\sigma}{d\Omega}\right) d\Omega = \left(\frac{d\sigma}{d\Omega}\right)_R d\Omega
$$

For scattering by spherically symmetric potentials, the scattering distribution is independent of the azimuthal angle, so that

$$
\frac{d\Omega_c}{d\Omega}=\frac{\sin\theta_c}{\sin\theta}\frac{d\theta_s}{d\theta}.
$$

Substituting in $\theta_{\rm g} = 0$ _L = 136.4⁰ and the CM scattering angles determined above, the ratios $(d \Omega_{\ell}/d\Omega)$ for 1 H, ⁴He, ⁷Li and 11 B on Au are found **to be** 1**.** 0074**,** 1**.** 0302**,** 1.0538 **and 1.0956 respectively. The cross sections calculated in the CM system are converted to the lab system by dividing by these factors.**

 $\mathcal{O}_{\mathcal{A}^{\prime}}$

APPENDIX IV

EXPERIMENTAL DIFFERENTIAL SCATTERING CROSS SECTION

Consider a thin target film of thickness t and density N atoms per unit volume. In a thin film no target atom is blocked by any other. Suppose a collimated beam of monoenergetic projectiles and cross-sectional area A strikes the film at normal incidence. The number of scattering **centers in the path of the beam Is given by the product AtN. Denoting** the differential cross section per target atom for scattering into a **unit solid angle in the direction** θ_{ℓ} **in the laboratory system by d** σ (θ , E_i), the effective target area presented to the beam for such \overline{a}

scattering is

AtN
$$
\frac{d\sigma(\theta_L, E_R)}{d\Omega}
$$

where E₂ is the lab energy. The fraction of the incident particles **backscattered into the unit solid angle is**

$$
\begin{array}{c}\n\text{Nt} & \text{d} \subset (\theta_1, E_2) \\
\text{d} \subset \Omega\n\end{array}
$$

If a detector subtends a lab solid angle $\triangle \Lambda$, and the fraction of the **incident ions detected is denoted by B (the backscatter y ie ld), then**

$$
\mathsf{B}\simeq\mathsf{N}\mathsf{t}\ \underline{\mathsf{d}\subset(\theta_{\ell},E_{\ell})}\,\,\mathop{\triangle\mathcal{A}}\limits_{\mathsf{d}\ \mathsf{A}\subset\mathsf{L}}\,.
$$

The lab differential scattering cross section is then

$$
\frac{d \sigma(\theta_3, E_3)}{d \Omega} \simeq \frac{B}{N(\Delta \Omega)t} \qquad (32)
$$

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