An investigation of a micro-scale Ranque-Hilsch vortex tube.

Amar F. Hamoudi
University of Windsor

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AN INVESTIGATION OF A MICRO-SCALE RANQUE-HILSCH VORTEX TUBE

by

Amar F. Hamoudi

A Thesis
Submitted to the Faculty of Graduate Studies and Research through Mechanical, Automotive and Materials Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

Windsor, Ontario, Canada
2006

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ABSTRACT

The results of a parametrical experimental investigation of the energy separation performance of a micro-scale Ranque-Hilsch vortex tube are presented. The tube is 2 mm in diameter and constructed using a layered technique from multiple pieces of Plexiglas and aluminum. Four inlet slots, symmetrically located around the tube, form the vortex flow. The hydraulic diameter of each inlet slot is 229 microns. Three different orifice diameters for the cold exit are used in this experiment. They are 0.5, 0.8 and 1.1 mm. The multilayer technique enables the researcher to simply extend the length of the micro-scale vortex tube to any size ranging from 20 to 120 mm. The working fluid is filtered and dehumidified compressed air approximately at room temperature. The rate of the hot gas flow is varied by means of a control valve to achieve different values of cold mass fraction. The mass flow rates, temperatures and pressures of the supply and outlet flows are measured and the performance of the device presented in a dimensionless manner. The supply channel Reynolds numbers is varied over a considerable range which extends into the laminar regime in order to determine the minimum conditions for cooling.

Experiments conducted on a micro-scale vortex tube, for a fixed geometry and control valve setting, at low Reynolds numbers, based on the inlet tube hydraulic diameter and average velocity, exhibit an increase in dimensionless temperature in both the hot and the cold outlets as the Reynolds number increased from zero reaching maximum values before a Reynolds number of 500 and 800 respectively. In the case of the hot outlet, the dimensionless temperature decreases after reaching its maximum and achieves a minimum value at a Reynolds number below 1500. It then increases steadily with further increases in Reynolds number. The cold outlet dimensionless temperature decreases steadily after the maximum to become negative at a Reynolds number of approximately 1800. This implies that there is cooling effect at Reynolds numbers consistent with laminar flow. Except for very low Reynolds numbers the cold mass fraction is approximately constant as the Reynolds number increases for a fixed geometry and control valve setting. For smaller orifice diameter and hence smaller $d_c/D$ ratio, the cold air mass fraction is decreased due to the increase of the resistance in the cold flow.
The cold air mass fraction also reduced with higher $L/D$ ratio due to high resistance in the cold flow.

The performance of the micro-scale vortex tube is also investigated at a number of higher inlet pressures for different values of the cold air mass ratio. The inlet pressures considered are 200, 300 and 400 kPa at an average inlet temperature of 293.6 K and the cold air mass ratio is systematically varied from 0.05 to 0.95. An increase in the inlet pressure causes the values of the dimensionless cold temperature difference to increase over the whole range of the cold air mass fraction. Unstable operation is observed at small $L/D$ that causes the shape of the dimensionless cold temperature difference versus cold mass flow fraction plot to be different than the conventional plot.

The effect of dimensionless tube length and cold exit orifice diameter on micro-scale vortex tube performance is found to be similar to that in the conventional devices.
DEDICATION

To my beloved parents,
my wife, my son Yaser and my daughters
Zinah and Rana.
ACKNOWLEDGEMENT

The author would like to acknowledge his sincere gratitude to Dr. A. Fartaj and Dr. G. W. Rankin for their valuable advice, outstanding supervision during the term of this research and for the time commitments, without which this work would never be achieved. It is a privilege to study and to work under their supervision.

The author also gratefully acknowledges the efforts of Mr. Timothy Bolger from the Technical Support Center (TSC) at the University of Windsor for his excellent work in constructing the micro-scale vortex tube.

Many thanks to my family for their support, encouragement and patience.
TABLE OF CONTENTS

ABSTRACT iii
DEDICATION v
ACKNOWLEDGMENT vi
LIST OF TABLES x
LIST OF FIGURES xi
NOMENCLATURE xiv

CHAPTER

1. INTRODUCTION 1
   1.1 Background 1
   1.2 Applications of Vortex Tube 5
   1.3 Thesis Outline 6

2. LITERATURE REVIEW 7
   2.1 Experimental Studies 7
       2.1.1 Vortex Tube Geometry 8
       2.1.2 The Internal Flow Field 10
       2.1.3 Effect of Working Fluid 13
   2.2 Theoretical Studies 15
   2.3 Numerical Studies 19
   2.4 Research Objective 20

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LIST OF TABLES

Table 1  Optimum design parameters for Soni and Thomson [25]  10
Table 2  Correction used for temperature measurements  27
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Vortex tube schematic drawing</td>
<td>2</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Hot and cold rotating streamlines in a counter flow vortex tube</td>
<td>2</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Sketch of the supposed flow pattern in a counter flow vortex tube</td>
<td>3</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Typical performance curves of the vortex tube</td>
<td>4</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Uniflow vortex tube with one opening at the hot end</td>
<td>4</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Swirl velocity variation with radius</td>
<td>12</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Effect of specific heat ratio, $\kappa$, on the maximum cold temperature drop [20]</td>
<td>14</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Secondary flow of the vortex tube [18]</td>
<td>17</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Expansion and contraction of turbulent eddies as they move radially</td>
<td>18</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Ideal reversed Brayton cycle</td>
<td>19</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Inlet nozzle section</td>
<td>23</td>
</tr>
<tr>
<td>Figure 12</td>
<td>Cold end orifice section</td>
<td>23</td>
</tr>
<tr>
<td>Figure 13</td>
<td>Longitudinal cross-sectional view of the vortex tube assembly</td>
<td>24</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Expanded view of the micro-scale vortex tube</td>
<td>25</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Vortex tube control volume</td>
<td>26</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Schematic of the experiment setup</td>
<td>28</td>
</tr>
<tr>
<td>Figure 17</td>
<td>Dimensionless cold and hot temperatures vs. Reynolds number for $L/D = 10$</td>
<td>33</td>
</tr>
<tr>
<td>Figure 18</td>
<td>Dimensionless cold and hot temperatures vs. Reynolds number for $L/D = 30$</td>
<td>33</td>
</tr>
</tbody>
</table>
Figure 19  Dimensionless cold and hot temperatures vs. Reynolds number for $L/D = 50$

Figure 20  Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 10$

Figure 21  Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 30$

Figure 22  Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 50$

Figure 23  Reynolds number as a function of the inlet pressure for $L/D = 10$

Figure 24  Reynolds number as a function of the inlet pressure for $L/D = 30$

Figure 25  Reynolds number as a function of the inlet pressure for $L/D = 50$

Figure 26  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.25$

Figure 27  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.4$

Figure 28  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.55$

Figure 29  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.25$

Figure 30  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.4$

Figure 31  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.55$

Figure 32  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.25$

Figure 33  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.4$

Figure 34  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.55$

Figure 35  Optimum cold air mass fraction vs. inlet pressure for $L/D = 10$

Figure 36  Optimum cold air mass fraction vs. inlet pressure for $L/D = 30$

Figure 37  Optimum cold air mass fraction vs. inlet pressure for $L/D = 50$

Figure 38  Optimum conditions vs. dimensionless orifice diameter for $L/D = 10$

Figure 39  Optimum conditions vs. dimensionless orifice diameter for $L/D = 30$

Figure 40  Optimum conditions vs. dimensionless orifice diameter for $L/D = 50$
Figure 41  Optimum conditions vs. dimensionless tube length for $d_c/D = 0.25$  50
Figure 42  Optimum conditions vs. dimensionless tube length for $d_c/D = 0.40$  51
Figure 43  Optimum conditions vs. dimensionless tube length for $d_c/D = 0.55$  51
## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>P</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
<td>P</td>
<td>pressure</td>
</tr>
<tr>
<td>$c_v$</td>
<td>specific heat at constant volume</td>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$D$</td>
<td>vortex tube inner diameter</td>
<td>$\dot{Q}$</td>
<td>Volumetric flow rate</td>
</tr>
<tr>
<td>$d_c$</td>
<td>cold exit (orifice) diameter</td>
<td>r</td>
<td>radius</td>
</tr>
<tr>
<td>$d_a$</td>
<td>slot holes diameter</td>
<td>R</td>
<td>gas constant</td>
</tr>
<tr>
<td>$h$</td>
<td>enthalpy</td>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>$L$</td>
<td>vortex tube length</td>
<td>$\Delta T$</td>
<td>temperature difference</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate</td>
<td>u</td>
<td>axial velocity</td>
</tr>
<tr>
<td>$Ma$</td>
<td>Mach number</td>
<td>w</td>
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### Greek symbols

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<thead>
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<th>Symbol</th>
<th>Description</th>
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<th>Description</th>
</tr>
</thead>
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<tr>
<td>$\rho$</td>
<td>density</td>
<td>$\eta$</td>
<td>efficiency</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity</td>
<td>v</td>
<td>kinematics viscosity</td>
</tr>
</tbody>
</table>

### Subscripts

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
<th>P</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Cold gas mass flow ratio</td>
<td>c</td>
<td>cold</td>
</tr>
<tr>
<td>$s$</td>
<td>static</td>
<td>h</td>
<td>hot</td>
</tr>
<tr>
<td>$t$</td>
<td>total</td>
<td>o</td>
<td>inlet</td>
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The development of new micro-fabrication techniques has led to a resurgence of research in micro-fluidic devices. The devices that have received most attention in the literature include pumps, valves, flow sensors and heat exchangers [1]. The performance of micro-scale Ranque-Hilsch tubes, non-moving part pneumatic devices that separate cold fluid from hot fluid for the purpose of cooling, has not received much attention. Traditionally, the vortex tube has been used in many low temperature applications where the efficiency is not the most important factor. A micro-scale Ranque-Hilsch tube in combination with a micro-fluidic pump has potential application in the cooling of electronic chips.

1.1 Background

The phenomenon of temperature separation occurring inside a cylindrical tube was reported for the first time by a French physicist, George J. Ranque who applied for a US patent in December 1932 [2] and subsequently presented a paper to the French Society of Physics in 1933 [3]. The discovery was further advanced, in 1947, by R. Hilsch [4] who published some details of the construction of the vortex tube along with the performance of the device for different tube diameters at various operating conditions. The vortex tube is a very simple device without moving parts (i.e. diaphragm, pistons, shafts, etc.) as shown in Figure 1.

In this arrangement a stream of a compressed gas (i.e.: air) is injected tangentially into the vortex tube having diameter $D$ using one or more nozzles symmetrically located around the tube. The injected flow expands and accelerates at the entrance establishing a strong swirl flow which causes a region of increased pressure near the wall and a region of decreased pressure near the axis. The presence of an end wall alongside the inlet nozzles forces some of the injected gas to flow axially in a helical motion toward the far end of the tube (the hot end) where a control valve is located. If the control valve is open too much, a certain amount of out air is
drawn into the tube through the cold end opening and leaves the tube through the hot end. This is obviously not a desirable operating condition.

Figure 1  Vortex tube schematic drawing

In the usual operation of this device, the flow through the hot end is restricted by partially closing the control valve. This causes some of the flow that had been directed to the far end of the tube to reverse direction along the center of the tube and leave the tube through the central orifice (Figure 2). The partial restriction of the flow leaving through the hot end causes even the low pressure at the center of the tube to be higher than atmospheric and hence flow exits the central orifice in the end wall. The gas escaping near the tube wall at the far end of the inlet nozzles has higher stagnation temperature than the incoming gas and the gas exiting through the central orifice has a lower stagnation temperature than the inlet gas.

Figure 2  Hot and cold rotating streamlines in a counter flow vortex tube
A possible flow pattern of the counter flow vortex tube proposed by Fulton [5] is shown in Figure 3. The outward flow of energy is represented by the radial arrows. The stagnation point that exists on the axis of the vortex tube indicates the starting point of reverse flow that represents the cold air stream which eventually leaves the tube through the orifice.

![Figure 3 Sketch of the supposed flow pattern in a counter flow vortex tube](image)

A very low temperature flow can be made to exit from the cold end by operating the device at a supply pressure of a few atmospheres. For example, Hilsch [4] operated a 9.6 mm diameter tube using compressed air at approximately 600 kPa inlet pressure and 293 K inlet temperature to produce a cold stream temperature, $T_c$, of 245 K and a hot stream temperature, $T_h$, of 353 K. The temperatures of the hot and the cold stream are varied by properly changing the ratio of the cold mass flow rate to the total inlet mass flow rate, $y_c$, which can be regulated using the control valve located at the hot exit. The temperature difference of the hot flow, $\Delta T_h = T_h - T_o$, and the cold flow, $\Delta T_c = T_c - T_o$, can be plotted versus the cold mass fraction, $y_c$, to obtain the performance curve of the vortex tube as indicated in Figure 4.

An alternate design of the vortex tube is shown in Figure 5. Both the hot and the cold gas flow are exhausted through one side of the tube. The end wall adjacent to the inlet nozzles is completely sealed and the cold flow leaves the other end of the tube either through the same opening of the hot gas or from an opening within the control valve. The former type was used for flow field investigation only. However, it
has been observed by many investigators that this type of vortex tube, which is also called “uniflow”, has a poor performance compared to the equivalently proportioned counter flow vortex tube type.

![Typical performance curves of the vortex tube](image1)

**Figure 4** Typical performance curves of the vortex tube

![Uniflow vortex tube with one opening at the hot end](image2)

**Figure 5** Uniflow vortex tube with one opening at the hot end

Most of the previous investigators referred to the Ranque-Hilsch vortex tube effect as a temperature or energy separation process. This term will also be used in this work.
although the terminology might not be quite accurate to describe the process. It is important to note that there are no hot and cold molecules to be separated from each other as imagined by James Clerk Maxwell [6]. In his thought experiment, Maxwell imagined a container filled with gas and separated by a wall into two compartments. A little “demon” guards a trap door between the two compartments, looking at oncoming molecules on both sides. Depending on their speeds, the demon opens or closes the door so that when a faster than average molecule flies from one compartment toward the door, the demon opens it, and the molecule will fly to the other compartment. Eventually, the molecules faster than average will be collected in one compartment and the slower one will be collected in the other compartment. Since average molecular speed corresponds to temperature, this will result in a temperature difference between the two compartments. The result is a hot, high pressure gas on one side, and a cold, low pressure gas on the other. Although the net effects of the Ranque-Hilsch vortex tube are similar to Maxwell demon, the effect of the former is to establish a total radial temperature gradient produced by energy transfer from the axis of the tube to the periphery and not to separate hot, fast moving molecules from slower, cold ones.

1.2 Applications of Vortex Tube

Despite its low efficiency, the simplicity, robustness and the feature of no moving parts makes the vortex tube attractive for many low temperature applications (below zero degree Celsius) where the use of conventional cooling processes is not viable due to technical requirements. Such low temperature applications would include cooling machine parts, dehumidification of gas samples as well as cooling of electronic control enclosures and environmental chambers. There has also been an attempt [7,8] to replace the conventional expansion valve in a refrigeration cycle with the vortex tube.
1.3 Thesis Outline

In Chapter 2, a literature review is presented covering the experimental, theoretical and numerical work previously conducted. Chapter 3 describes the design of the micro-scale vortex tube, the component details, the instrumentation used and the experimental setup as well as experiment methodology. The experimental results are presented in Chapter 4 followed by an extensive thermodynamic analysis and discussion. Detailed designs drawings of the micro-scale vortex tube, an uncertainty analysis as well as flow rate and Reynolds number calculations are included in the appendices.
CHAPTER 2 - LITERATURE REVIEW

The most recent review of the literature related to the Ranque-Hilsch vortex tube is given in the dissertation by Gao [9]. Previous reviews [10,11 and 12] reveal hundreds of papers dealing with this topic.

In spite of the large volume of research that has been devoted to this subject over the years, there is still disagreement regarding the mechanism that accounts for the operation of this device. Although it has been shown that energy separation can occur in laminar flow [13,14 and 15] most explanations involve turbulent fluctuations [16]. Kurosaka [17] gives an acoustic streaming explanation while others claim that the operation is based on secondary flows and a thermodynamic refrigeration cycle [18].

The only study that was specifically directed towards micro-scale vortex tube devices was that of Dyskin and Kramarenko [19]. They reported experimentally determined performance characteristics (adiabatic efficiency) for vortex tubes operating with a pressure ratio, $P_v/P_c$, of 6 with diameters of 1, 2 and 3 mm. The corresponding mass flow rates were $14.3 \times 10^{-5}$, $40.0 \times 10^{-5}$ and $94 \times 10^{-5}$ kg/sec respectively. Although details of the geometry are not given, an estimate of the inlet Reynolds numbers used for these cases yield values greater than 6000. It was, however, noted that the cooling effect decreased with decreasing flow rate. This is consistent with the speculation of Negm et al. [20] that the cooling effect should decrease with decreasing Reynolds number.

The previous investigations of the Ranque-Hilsch vortex tube can be divided basically into three main groups. These groups are experimental, theoretical, and numerical studies and they will be discussed briefly in the following sections.

2.1 Experimental Studies

The experimental studies are concerned with the effects of varying the geometry of the vortex tube components; focus on the details of the flow field inside the vortex tube and investigating the effect of using different working fluids other than air. Most of the experimental studies aim to determine the physical geometry of the tube required for optimum performance.
2.1.1 Vortex Tube Geometry

Following Hilsch's publication in 1947 [4], many investigators studied the effects of varying the vortex tube geometry such as tube diameter ($D$), tube length ($L$), orifice diameter ($d_o$) and nozzle diameter ($d_n$).

Dyskin and Kramarenko [19] found that by using a conical tube having $3^\circ$ of tapered angle, the adiabatic efficiency was 10-12% lower than that when a cylindrical tube is used. Piralishvili and Polyaev [21] use what they called a “double circuit vortex tube” with a larger conical angle tube to improve the performance.

Keyes [22] found that the tube diameter is the most important geometrical variable influencing the vortex strength as the periphery Mach number increases with the reduction of the tube diameter. He reported that this improvement is due to a decrease in the turbulent wall shear as a result of the decrease in the tangential peripheral Reynolds number. The vortex tube diameters used in the study were 16, 25 and 50 mm.

Negam et al. [20] stated that the vortex tube performance depends only on Reynolds number for geometrically similar tubes and for the same operating conditions. They found that a vortex tube diameter gives the maximum cold temperature drop at different inlet pressures of 16 mm. Their finding agrees well with Hilsch [4] experiment conducted on 4.6, 9.6 and 17.6 mm tube diameters. They concluded that the tube performance improves with an increase in its diameter.

Many investigators reported that the tube length should be many times longer than the tube diameter for better temperature separation. Hilsch [4] for example found that the $L/D$ ratio should be 50 for optimum performance. Saidi and Valipour [23] found that the optimum value for the tube length to the tube diameter ratio is $20 \leq L/D \leq 55.5$. Furthermore, they found that for $L/D \leq 20$ the energy separation decreases leading to a decrease in both cold air temperature difference and efficiency. In the case of the micro-scale vortex tube, the optimum $L/D$ ratio found by Dyskin and Kramarenko [19] was around 60. Contrary to what other investigators reported, Dombrand [14] found that tube length need not be as long as what was believed necessary. Although for certain tube geometry, the $L/D$ ratio used by him was 1.4, 2.6
and 3.8, he found, however, that the maximum efficiency would occur at $L/D$ greater than 20-25.

There is more agreement on the orifice diameter, $d_c$, which should be around $0.5D$ for maximum cold air temperature difference. Most of the previous studies found that the optimal orifice diameter and hence the $d_c/D$ ratio would be in the range of 0.4 to 0.6. Dombrand [14] found that if a shorter tube length is used with greater cold air mass ratio, a larger orifice diameter should be used to obtain maximum efficiency and a smaller orifice diameter to be used in case of low cold air mass ratio. He further investigated the effect of using a tapered orifice in diffusing the rotating flow to lower the back pressure. The results he obtained, however, do not show any appreciable gain in vortex tube performance. In explanation, he stated that either the rotating flow through the orifice diffuses efficiently without tapering it or the tapered orifice does not diffuse the rotating air.

Sublikin [24] concluded that the cold orifice diameter, $d_c$, is not an important parameter and has little effect on vortex tube performance.

The other parameter that influences the performance of the vortex tube is the inlet nozzle. Dombrand [14] investigated five different types of nozzles. Among them are round tangent, round offset, and rectangular tangent. The round tangent nozzle gave the best results and a similar round nozzle which is offset by a very slight amount was the poorest. The optimum vortex tube performance was obtained at an equivalent nozzle diameter ratio, $d_{n}/D$, between 0.25 and 0.27. Sibulkin [24] reported that the inlet nozzle height is one of the important geometrical parameters having a significant effect on the vortex tube performance. He noticed that by increasing the inlet nozzle height both hot and cold temperature differences increase.

Soni and Thomson [25] conducted an extensive parametric study to describe the optimal performance. They utilized an empirical regression analysis to derive a functional relationship between maximum cold temperature drop and the independent design variables. Optimal design parameters obtained by them are shown in Table 1. They reported that to obtain maximum cold temperature drop a smaller nozzle diameter with a larger orifice diameter should be used.
Table 1  Optimum design parameters for Soni and Thomson [25]

<table>
<thead>
<tr>
<th>Tube performance</th>
<th>Design Parameter</th>
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<tr>
<td></td>
<td>nozzle area</td>
</tr>
<tr>
<td></td>
<td>tube area</td>
</tr>
<tr>
<td>Maximum (\Delta T_c)</td>
<td>0.11 ± 0.01</td>
</tr>
<tr>
<td>Maximum (\eta)</td>
<td>0.084 ± 0.001</td>
</tr>
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Different types of material were used to construct the vortex tube. The most important factor in selecting the material is the surface roughness of the inner tube wall. As reported by Dombrand [14], the vortex tube performance would decrease with an increase of the surface roughness. The material of construction of the vortex tube can be either steel, copper and aluminum or even a plastic or glass to allow flow visualization.

2.1.2 The Internal Flow Field

The desire to optimize the tube parameters for maximum cold temperature drop led to investigations regarding the mechanism behind the temperature separation inside the vortex tube. Investigations of the internal flow pattern and measurements of the velocity vector components, pressure and temperature distribution inside the vortex tube were conducted to shed some light on the mechanism of the energy separation. These can be used, at the same time, as a basis for the theoretical analysis of the vortex tube phenomenon. The earliest investigation of the flow field inside the vortex tube was the one conducted by Dombrand [14] in 1948. He uses a 1 mm close end tube with a very small hole drilled on the wall near the closed end. A number of these tubes were inserted at specific locations along the tube length with the ability to be moved radially inside the vortex tube. This technique enabled a qualitative indication of the velocity field. He reported that the insertion of too many probes broke the vortex pattern. Other major investigations of this type were published by Hartnett and Eckert [26], Keyes [22] and Bruun [27]. The vast majority of this type of experiment utilize probes such as thermocouples to measure the temperature.
distribution inside the vortex tube. To obtain the pressure distribution inside the vortex tube, a number of Pitot tubes are inserted at different sections along the length of the vortex tube with the capability to move the Pitot tube radially toward the axis of the vortex tube. Some investigators, such as Ahlborn and Groves [28], developed a technique to rotate the Pitot tube 360° around its axis to obtain the flow angles (or velocity vectors). In order to reduce the error caused by inserting these probes inside the vortex tube, most of the internal studies of the flow field were conducted on a relatively larger tube diameter. It should be noted, however, that the swirling flows are very sensitive to the disturbance created by inserting relatively large pressure probes. Furthermore, when these probes are inserted in a turbulent flow, they are subject to errors caused by turbulent fluctuations [29]. Therefore, more recent internal investigations by [9 and 29] utilize hot-wire anemometers to measure the velocity components. The advantages of the hot-wire probes over the pressure probes are its small size (causing minimal disturbance to the flow), excellent spatial resolution, good signal sensitivity and high frequency response which enables the study of turbulence [30]. Sibulkin [24] was the earliest investigator to utilize the hot-wire anemometer to measure the velocity in a uniflow and a counter flow vortex tube but could not obtain the circumferential velocity components. Fitouri et al. [29] utilized a single yaw hot-wire anemometer to measure the three velocity components and the turbulence intensity.

The results obtained from the internal study enabled the investigators to draw a map of the flow field inside the vortex tube. As reported by many investigators such as [26, 27 and 29], the tangential velocity distribution determined at different locations inside the vortex tube show that the flow in the inner region rotates like a rigid body (forced vortex). The magnitude of the velocity is constant at any point and is a function of the radius:

\[
\frac{W}{r} = \text{constant} \tag{1}
\]

where \(W\) is the tangential velocity and \(r\) is the distance from the center of the flow. In the outer region, the tangential velocity decreases due to wall shear stress and reaches zero at the wall. This region is referred to a “wall bounded” region.
Forced vortex

\[ W \sim r \]

0

Wall bounded
region

Radius

Forced vortex

\[ W \sim r \]

0

Wall bounded
region

Radius

Tangential velocity

Figure 6    Swirl velocity variations with radius

It should be recalled, however, that the flow inside the vortex tube is three
dimensional, confined and rotating at a high velocity so that inserting probes at any
location could alter the flow field significantly causing inaccurate readings. As
reported by Bruun [27], for instance, inserting any probe 30 to 40 mm radially in any
of the ten cross sections along the vortex tube increased the total temperature of the
cold stream measured in the cold outlet by 4 °C. Keys [22] mentioned that the data
obtained by inserting a probe directly in the flow field is of uncertain validity due to
the influence of the probe itself. An alternate solution to this problem is the use of
non-contact measurement technique such as laser Doppler anemometer (LDA). This
method requires the flow to be seeded in order to produce the necessary Doppler burst
signals that are used to deduce the flow velocity. The ideal particles are small enough
to accurately track the flow, yet large enough to produce an adequate signal to noise
ratio (SNR) [31]. Real seed particles, however, are seldom ideal. By using the largest
particles, the data is much more easily obtained and the measurements of the particle
velocities are more accurate because of the improved SNR but the particles velocities
are not representative of the fluid velocity. Although this technique constitutes some
technical difficulties such as distortion of the optic beam due to it’s interaction with
the curved surface of the cylindrical chamber and the time required in setting up the
equipment, it is still considered the more accurate method of measuring the velocity components inside the tube.

2.1.3 Effect of Working Fluid

The vast majority of the work concerning the vortex tube used compressed air as the working fluid. A number of investigators, however, studied the effect of gas other than air. Keys [22] used nitrogen and helium instead of air. Linderstrom [32] used the vortex tube as a gas separator to separate a different composition of gas mixtures. He injected a mixture of oxygen and nitrogen, a mixture of oxygen and carbon dioxide and a mixture of oxygen and helium. A mixture of compressed gases flown into the vortex tube separate into individual gas streams by the effect of differential centrifugal forces acting on them. In their attempt to separate methane and nitrogen gases using vortex tubes, Kulkarni and Sardesai [33] found that there was partial gas separation leading to a higher concentration of methane at one exit in comparison to the inlet and a lower concentration at the other exit.

Ambrose [7] injected a saturated liquid of carbon dioxide in the vortex tube in an attempt to replace it with the throttling device used in the refrigeration cycle in order to increase the coefficient of performance of the refrigerator. He proposed a refrigeration cycle utilizing CO$_2$ as a refrigerant instead of Freon due to its high potential refrigeration gain at a fairly low pressure ratio. A large temperature difference was generated by expanding a saturated liquid carbon dioxide in the vortex tube but the temperature of the hot flow was not sufficiently high to permit heat rejection through the gas cooler.

In the experiment conducted by Stephan et al. [34], no temperature difference was found in the cold flow when air or oxygen is used as a working fluid. The results of their experiment, however, indicated that the cold temperature drop when helium is used as a working fluid is larger than that when air or oxygen is used. This is due to the fact that the molecular weight of the helium is much smaller than the molecular weight of air or oxygen.
The correlation mentioned in Figure 7 by Negam et al.[20] to predict the maximum cold temperature drop indicates that the specific heat ratio, $c_p/c_v$, has a significant influence on the performance of the vortex tube. It was found that gases with a higher specific heat ratio, $k$, have a better performance than the gases having a lower specific heat ratio.

\[
\frac{(T_o - T_c)_{\text{max}}}{T_o} = T_o (1 - n^{(\alpha - 1) (k - 1) / k})
\]

![Figure 7](image)

**Figure 7**  **Effect of specific heat ratio, $k$, on the maximum cold temperature drop [20]**

In the equation indicated in Figure 7, $n$ is the pressure ratio, $P_o/P_c$, $\alpha$ is the minimum change of specific entropy, $(\Delta S)_m$, reported by them to equal 0.67 and $k$ is the specific heat ratio of the working fluid, $c_p/c_v$.

Balmer [35] injected liquid water at high inlet pressure (50 MPa) into a commercially available vortex tube. Although the results showed a significant temperature difference of about 15 °C between the cold and the hot flows, both flows temperature, however, was above the inlet water temperature. From his work, Balmer
concluded that the energy separation in these devices is not limited to compressible
gases. Using the second law of thermodynamics, theoretical analysis establishes that
net entropy producing temperature separation effect is also possible when
incompressible liquids are used in the vortex tube.

2.2 Theoretical Studies

It is important to distinguish a change in the temperature of a fluid that is
associated with a change in the fluid velocity from that due to work done or heat
transfer from the fluid.

Consider the portion of fluid entering the vortex tube that is emitted from the
cold exit. If there is no work done or heat transfer involved in the flow, application of
the energy equation yields:

\[
\text{internal energy} + \text{flow work} + \text{kinetic energy} = \text{enthalpy} + \text{energy} = \text{constant} \quad (2)
\]

\[
h + \frac{u^2}{2} = \text{constant} \quad (3)
\]

For a perfect gas:

\[
h = c_p T \quad (4)
\]

where \( c_p \) is the specific heat of the inlet air at constant pressure, taken as 1005 J/kg K.
Or:

\[
T + \frac{u^2}{2c_p} = \text{constant} \quad (5)
\]

The thermodynamic (or static temperature) \( T \) can therefore be seen to change
if the fluid velocity changes even though no heat is transferred or work is done on the
fluid.
The temperature change that is important in the vortex tube is one that is only due to heat transfer. Defining the total temperature as:

\[ T_o = T + \frac{\mu^2}{2c_p} \]  

(6)

the energy equation becomes \( T_o = \text{constant} \). It will only change if heat is transferred or work done on the working fluid.

Following the qualitative description of Hilsch [4], many theoretical approaches were published in an attempt to analytically predict the distribution of velocity and temperature, and explain the energy transfer process. The theoretical analyses reported are either based on analytical simulations or on results obtained from the experimental work.

The explanation given by Ranque [2] attributes the energy separation process to the expansion of gas from a region of high pressure near the periphery to a low pressure in the region near the axis. Hilsch [4] presented the same thought in explaining the mechanism of the energy separation. He included the effect of the internal friction which causes the energy to flow from the axis region in the outward direction causing the temperature to increase in the periphery. Kassner and Knoernschild [13] explained that the cold temperature drop in the center region is due to the gas expansion and the hot temperature in the outer region due to the shear stresses that cause the flow in the inner region to be a forced vortex. A number of earlier theories suggested that the energy separation can occur in laminar flow as in the work presented by [14 and 15]. They considered the radial velocity instead of the tangential velocity in determining the Reynolds number.

In the two dimensional analytical model presented by Deissler and Perlmutter [16] and Reynolds [36] it was concluded that the turbulent mixing and turbulent shear work done on elements causes the heat transfer between flow layers by temperature and pressure gradients and hence causes the energy separation inside the vortex tube.

Kurosaka [17] suggested that the acoustic streaming of the vortex whistle is responsible for the Ranque-Hilsch effect. He explains that when the whistle is
inaudible, the steady state tangential velocity distribution in the radial direction is not
in the form of a forced vortex and the steady state temperature is uniform. As the
whistle becomes audible, the velocity profile converted to a forced vortex causing the
temperature distribution to be separated into cold stream near the axis and hot stream
near the periphery.

Ahlborn et al. [18] describe the Ranque-Hilsch energy separation phenomenon
as a heat pump mechanism which is enabled by a secondary circulation flow
imbedded into the primary vortex. There are three processes in the heat pump. All
these processes exist in the vortex tube. These are the working fluid (1) as indicated
in Figure 8 which moves heat between a high pressure region and a low pressure
region, the fluid is compressed at a temperature higher than the surrounding
temperature to give off heat (2) and the expanded working fluid is colder than it’s
surrounding to absorb heat (3). The processes of compression (4) and expansion (5)
occurred due to random fluctuations.

![Figure 8: Secondary flow of the vortex tube [18]](image)

Braun [27] and Hinze [37] explained that the mechanism of the energy
separation is mainly caused by adiabatic contraction and expansion of turbulent
eddies in a centrifugal field. To explain this mechanism, particles are considered
fluctuating between the high pressure and the low pressure region as shown in Figure
9. As the particles move from a low pressure region A to a higher pressure region B,
it will undergo an adiabatic compression and the temperature of the particles would increase to a level above the surrounding temperature. As a result, heat transfer will take place from the particles to the surrounding region at B increasing its temperature. At the same time, equivalent particles from the high pressure region B undergo an adiabatic expansion while moving to a lower pressure region A lowering its temperature to a level below the surrounding’s temperature, causing the flow at that region to have a lower temperature. This process of adiabatic compression and expansion is similar to that which occurs in an ideal reverse Brayton cycle as shown in the T-s diagram in Figure 10. With the higher pressure difference between the axis of the tube and its periphery and with the higher level of fluctuation, the level of cooling would increase, lowering further the temperature of the flow leaving through the center of the tube.

Figure 9  Expansion and contraction of turbulent eddies as they move radially
2.3 Numerical Studies

In the numerical studies conducted on vortex tubes, one of the computational fluid dynamics (CFD) packages such as FLUENT, CFX or STAR CD is used to solve the continuity, Navier–Stokes and energy equations. This allows prediction of the internal flow pattern, as well as temperature and pressure distribution inside the tube.

Cockerill [10] uses a two dimensional numerical model to investigate the flow field inside a uniflow and a counter flow vortex tube. The flow is assumed to be an axis-symmetry. He applied a number of modifications to the \( k-e \) model to account for the anisotropic flow inside the vortex tube. The computations give results for the swirl profiles that qualitatively comparable with the experiments conducted by him. The energy separations, however, are not predicted well. His numerical solutions show that the flow near the axis becomes warmer than the periphery.

Frohlingsdorf and Unger [38] investigate the compressible and turbulent vortex tube flow numerically by modeling the Bruun experiment [27]. The numerical code used is CFX. The use of the \( k-e \) turbulence model leads to significant differences between measured and calculated tangential velocity profiles. By replacing the \( k-e \) turbulence model with the correlation reported by Keyes [22] in calculating the ratio
of the turbulent to laminar viscosity, \( \frac{\mu_T}{\mu_c} \), a better approximation of the measured results were achieved. They found that by increasing the turbulent Prandtl number from 0.9 to 9.0 produces the same cold gas total temperature difference as in Bruun’s experiment [27].

Bezprozvannykh and Mottl [39] report that various levels of complexity in turbulence modeling are suitable for vortex tube analysis. They use a three dimensional numerical model to investigate the energy separation for incompressible and compressible flows for water and air cases correspondingly. The commercial code used is FLUENT. For air as a working fluid, the maximum total temperature difference obtained, \( \Delta T = T_c - T_o \), is 1 K only. For water, however, no temperature differences were obtained.

Behera et al. [40] implement a three dimensional numerical model in an approach of optimizing the design of the vortex tube using the \( k-\varepsilon \) turbulence model and the Star-CD commercial code. The numerical investigations enable them to obtain the three velocity components of the flow which is difficult to obtain experimentally due to disturbance of flow by measuring probes. The analyses show that the flow has forced and free vortex components. Optimum \( L/D \) ratio that delivered maximum cold temperature difference is found to be in the range of 20 to 30 and for optimum \( d_c/D \) is found to be 0.5.

2.4 Research Objective

The objective of this study is three fold:

1. To investigate the characteristics of a micro-scale vortex tube at low inlet pressure ranging from approximately 2.5 to 75 kPa and to obtain the critical inlet Reynolds number at which the cooling effect will be established.

2. To obtain the performance curves of the micro-scale vortex tube as a function of cold air mass ratio at higher inlet pressure values of 200, 300 and 400 kPa.

3. To determine the effect of \( L/D \) and \( d_c/D \) ratio on the performance of the micro-scale vortex tube.
CHAPTER 3 – EXPERIMENTAL METHODOLOGY

This research is an investigation of a micro-scale vortex tube as an alternative cooling method for electronic micro-chip devices. To achieve this goal, a micro-scale vortex tube is designed, fabricated and tested. The setup is designed for flexible geometrical adjustment. This work provides detailed discussion on the various design parameters. Therefore, the main objective of this research is to experimentally investigate the characteristics of a micro-scale vortex tube at supply channel Reynolds numbers that extend from the laminar into the turbulent flow regime in order to determine the minimum operating conditions of these devices for cooling applications. In addition, the effects of tube length and cold outlet orifice size on the performance characteristics of micro-scale vortex tubes are determined. The experiments are conducted to determine the differences, if any, of the micro-scale vortex tube characteristics to conventional vortex tubes.

3.1 The Micro-Scale Vortex Tube

Most of the previous experimental studies are conducted on fairly large tube diameters (i.e.: 10 – 25 mm) [10,20]. To reduce the effect of inserting measurement probes on the vortex flow pattern in the case of the internal studies, the diameter of the vortex tube used is even as large as 50 - 96 mm in some studies [22,26 and 27]. Investigating a micro-scale vortex tube has not been reported in detail yet. The only study that is specifically directed towards micro-scale vortex tube devices is that of Dyskin and Kramarenko [19]. They report experimentally determined performance characteristics (adiabatic efficiency) for vortex tubes operating with a pressure ratio of 6 with diameters of 1, 2 and 3 mm.

3.2 Apparatus Description

For the purpose of this study, a 2 mm diameter vortex tube is designed and manufactured at the Technical Service Center (TSC) of the University of Windsor. The tube diameter is chosen to be the smallest possible that existing manufacturing
techniques permit. The design and the fabrication of such small a size vortex tube is necessarily different than the conventional one. A layered technique using multiple pieces of Plexiglas and aluminum is used for accurate machining of the inlet, orifice and control valve assembly. The design allows length increment changes hence different \( L/D \) ratios. The cross-sectional area of each piece has a dimension of 30 mm by 30 mm. The length of each piece varies according to its location within the assembly. In the following paragraphs, a detail description of the main parts of the micro-scale vortex tube is presented.

### 3.2.1 Nozzle Section

As the scope of this research is to investigate the vortex tube characteristics at supply channel Reynolds numbers that extend from the laminar into the turbulent flow regime, the supply air slots are designed to be very small to ensure a laminar flow in that region at low supply pressure. The hydraulic diameter of the supply channel is 229 microns. This is the smallest possible dimension that can be machined in the TSC of the University of Windsor. This gives a ratio of nozzle area, \( A_n \), to the tube area, \( A_t \), of 0.11 which is similar to that suggested by Soni and Thompson [25] to obtain a maximum cold temperature difference \( \Delta T_c \).

The main inlet nozzle section is machined from one piece made of Plexiglas material and having square cross section area of 30 by 30 mm and 20 mm length. As depicted in Figure 11, this piece consists of the inlet nozzle for the compressed air line (1) attached to the body of this section of the vortex tube. A 4 by 3 mm channel (3) is connected to the inlet nozzle (1) through a longitudinal hole (2). The channel (3) is provided to act as a pressurized manifold so that the flow enters the inlet slots (4) at a pressure and temperature very close to that measured at the inlet nozzle (1). The vortex is formed by four inlet slots (4) that are symmetrically located around the 2 mm tube diameter (5). The dimensions of each inlet slot (4) are 0.382 mm wide and 0.164 mm in height. The hydraulic diameter of the inlet slot, \( d_n \), is calculated using the following formula:

\[
d_n = \frac{4A}{\text{wetted perimeter}} \tag{7}
\]
where $A$ is the cross-sectional area of the inlet slot. The hydraulic diameter is found to be equal to 229 microns.

![Inlet nozzle section](image1)

(a) Front view of the inlet nozzle section  
(b) Perspective view showing the details of the inlet nozzles

**Figure 11**  
Inlet nozzle section

### 3.2.2 Cold End Orifice Section

Three different sizes of the cold orifice diameter are used in this research to investigate its effect over the vortex tube performance. These sizes are 0.5, 0.8 and 1.1 mm and give $d_i/D$ ratios of 0.25, 0.4 and 0.55 respectively. The cold orifice piece forms one end of the vortex tube while the control valve piece forms the other end of the vortex tube. The material of construction is aluminum.

![Cold end orifice section](image2)

(a) Front view of the orifice section  
(b) Side view of the orifice section

**Figure 12**  
Cold end orifice section
Figure 12 shows the front and the side views of the cold end section. The three pieces are identical except for the size of the orifice diameter.

3.2.3 Extension Pieces

A number of extension pieces are fabricated to allow adjustment of the tube length. The material of construction for all pieces is Plexiglas. The lengths of the extension pieces are 5, 10 and 20 mm to give flexibility in selecting the tube length and hence the $L/D$ ratio as shown in Figure 13.

![Diagram showing a longitudinal cross-sectional view of the vortex tube assembly.](image)

**Figure 13** Longitudinal cross-sectional view of the vortex tube assembly

3.2.4 Hot End and Control Valve Section

The hot section, as shown in Figure 13, consists of the control valve to adjust the amount of flow leaving the tube through that end. The block section is made of aluminum and the control valve is made of steel. The end of the control valve is tapered at 60°. A lock nut is provided to prevent the movement of the valve at a certain opening where desired.

3.2.5 Assembly

The most important point to be considered at this juncture is the alignment of the different parts of the vortex tube. The minimum number of pieces that can be used
to form a vortex tube with smallest length of 20 mm is three pieces - the main inlet nozzle piece, the cold and the hot end piece. The maximum numbers of pieces that can be used is seven to form a net tube length of 100 mm. Any eccentricity in the position of the cold end opening or the extension pieces may cause a significant disruption to the rotating flow. To avoid this problem, two stainless steel guide pins are used to align the multi-layers forming the vortex tube. The Plexiglas layers are sandwiched between two aluminum pieces which represents the hot and the cold end of the vortex tube as shown in Figure 14. Four bolts and nuts are used to hold the different pieces of the vortex tube and are passed through longitudinal holes at the corners of each piece. Design drawings showing the details of the micro-scale vortex tube using the multi-layer technique are shown in Appendix A.

Figure 14  Expanded view of the micro-scale vortex tube
3.3 Instrumentation

The vortex tube in this case is considered as a control volume having three boundaries as shown in Figure 15. The variables to be evaluated are the pressure, temperature, and mass flow rate of the inlet flow, cold and hot boundaries. The subscripts \( o \), \( c \) and \( h \) denotes for the inlet flow, cold and hot exit respectively.

![Figure 15 Vortex tube control volume](image)

The three important inlet and exit flow quantities that need to be recorded during the experiment are the pressures, temperatures and the flow rates. The results obtained are eventually used to determine the performance of the vortex tube and to investigate the effect of geometry on the cold temperature drop. The following paragraph discusses the instrumentation used during the tests and the applicable calibration procedure.

3.3.1 Pressure Measurement

Low supply inlet pressures ranging from 2 to 17.5 kPa are measured using a water manometer with an uncertainty of \( \pm 0.01 \) kPa. Pressures in the range of 17.5 to 82 kPa are measured using a Bourdon tube gage with an uncertainty of \( \pm 1.7 \) kPa and for higher supply pressures of 200, 300 and 400 kPa, a different Bourdon tube pressure gage is used that had an uncertainty of \( \pm 3.4 \) kPa. The hot and cold flow
pressure measurements are obtained using a digital manometer with an uncertainty of ± 0.01 kPa.

3.3.2 Temperature Measurement

For the process of the energy separation in the vortex tube, the temperature of the cold and hot exit flow is the most important variable required as these values will be used to find the performance of the micro-scale vortex tube. The temperatures of the inlet, cold and hot air are measured using type - T (Copper Constantan) thermocouple probes. These thermocouples are calibrated using an ice bath and boiling water as temperature standards. The correction factors obtained from the correction curves are used to adjust the readings of the temperatures measured as shown in Table 2.

<table>
<thead>
<tr>
<th>Temperatures to be corrected</th>
<th>Correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature of the inlet flow, $T_o$</td>
<td>$T_o \times 1.0081$</td>
</tr>
<tr>
<td>Temperature of the cold flow, $T_c$</td>
<td>$T_c \times 1.0112 - 0.1011$</td>
</tr>
<tr>
<td>Temperature of the hot flow, $T_h$</td>
<td>$T_h \times 1.0071 + 0.1007$</td>
</tr>
</tbody>
</table>

3.3.3 Flow Rate Measurement

The volumetric flow rate, $\dot{Q}$, of air exiting the cold and the hot openings are measured using separate rotameters. The rotameters are calibrated while connected to the apparatus as shown in the experiment setup in Figure 16 so that both are subjected to the same working pressures as encountered in the experiment to avoid the need for corrections. The results of the cold and the hot exit flow are obtained. Assuming an ideal gas, the density of the hot and the cold gas are calculated using the equation of state:

\[ \rho = \frac{M}{R T} \]
\[ \rho = \frac{p}{RT} \]  

(8)

The mass flow rate, \( \dot{m} \), leaving the cold and the hot ends are calculated from:

\[ \dot{m} = \dot{Q} \rho \]  

(9)

The total flow rate entering the apparatus is determined by the summation of the cold and the hot flow.

3.4 Experimental Facility

The experimental test facility used for this study is shown schematically in Figure 16. Dry and filtered compressed air is used throughout this experiment to avoid any condensation of the moisture in the compressed air due to the low cold air temperature exiting through the orifice. The air is dehumidified with a refrigerant dehumidification system. As indicated in Figure 16, the compressed air passes through a control valve (1), 5-micron air filter (2) and a pressure-regulating valve (3) before entering the vortex tube. A water manometer (4) is used to accurately measure the low supply pressure which ranges from 2 to 17.5 kPa. A pressure gage is used for the pressures above this range.

![Figure 16 Schematic of the experiment setup](image-url)
The temperatures of the inlet, cold and hot air are measured using type T thermocouple probes located at (5), (8) and (11) respectively. The compressed air is injected into the vortex tube through the manifold (6) and then the inlet slot (7). The cold exit pressure, \( P_c \), and hot exit pressure, \( P_h \), are measured using digital manometers (9) and (12). The volumetric flow rate of air exiting the cold and the hot openings are measured using rotameters (10) and (13).

### 3.5 Experimental Methodology

Two types of experimental tests are conducted to investigate the characteristic of the micro-scale vortex tube in this research. These are the low pressure test and the high pressure test. Each test is conducted at different tube length, \( L \), orifice diameter, \( d_c \) and inlet pressure, \( P_0 \). A set of nine combinations of vortex tube geometrical parameters consisting of three different \( L/D \) ratios (10, 30 and 50) with three different \( d_c/D \) ratios (0.25, 0.4 and 0.55) form the geometry of the devices under investigation. The following paragraphs discuss the methodology and the aim of each test.

#### 3.5.1 Low Pressure Tests

In the first series of tests, the performance characteristics of the micro-scale vortex tubes are determined at low supply pressure and fixed geometry. This means that the control valve is arbitrarily opened to a certain setting and kept constant throughout the entire set of tests as the supply pressure is varied. The low supply pressures range from approximately 2.5 to 82 kPa.

The measured cold and hot air dimensionless temperatures leaving the vortex tube for various Reynolds numbers, based on the inlet slot of the supply air, are determined using the expression, \( \frac{T_c - T_o}{T_o} \) and \( \frac{T_h - T_o}{T_o} \) respectively. These parameters are chosen to eliminate the effect of the inlet air temperature.

The aim of this test is firstly to determine the critical Reynolds number at which a cold temperature drop will be established and secondly to observe changes in the cold mass fraction, \( y_c \), with changes in Reynolds number for different tube
geometry with a fixed hot end valve setting. The inlet Reynolds number is determined from:

\[ Re = \frac{\dot{m} d_n}{4 A \mu} \]  

(10)

where \( \dot{m} \) is the air mass flow rate entering the vortex tube through one slot calculated using Equation (9), \( d_n \) is the equivalent diameter of the inlet slot and it is equal to 229 microns, \( A \) is the cross-sectional area of one inlet slot and found to be \( 6.25 \times 10^{-8} \) m\(^2\) and \( \mu \) is the viscosity of the inlet air taken as \( 1.82 \times 10^{-5} \) kg/m.s.

The resistance of the rotameters located at each of the hot and cold outlets combined with the low operating pressures in the experiments significantly alter the exit pressures of the cold and hot exits. Each is altered by a different amount which effectively creates a different exit pressure in each case. To avoid this problem, the flow of the cold or the hot stream is restricted using a flow restriction device in such a manner that the pressure of the cold and hot air exit is approximately equal.

### 3.5.2 High Pressure Tests

The second series of tests is conducted using the same tube geometrical combinations as in the low pressure test. In this case, however, the characteristic performance of the micro-scale vortex tube at higher inlet pressures and at different cold air mass ratio, \( y_c \), is determined. The inlet pressure of the supply is 200, 300 and 400 kPa and the value of \( y_c \) varied from approximately 0.05 to 0.95 by adjusting the control valve at the hot end.
CHAPTER 4 - RESULTS AND DISCUSSION

The results obtained from the low and high pressure tests are presented and discussed in this chapter. The uncertainties in the figures are indicated by the error bars unless they are less than the size of the symbol. Detailed calculations of the uncertainties are presented in Appendix B for reference. Details of the experimental results for low and high pressure tests are presented in Appendix C and Appendix D respectively.

4.1 Results of Low Pressure Tests

The temperatures measured using the thermocouples are assumed to be equal to the total temperature which can be justified as follows. The total temperature defined in Equation 6, the term $u^2/2c_p$, represents the dynamic temperature of the flow. Within the range of the working pressure used in this experiment which is approximately 2.5 to 75 kPa, the cold and the hot exit velocities are found to be in the range of 0.15-1.84 m/s and 0.14 - 1.08 m/s respectively. The maximum dynamic temperatures calculated for both cold and hot temperature are found to be $1.7 \times 10^{-3}$ and $5.8 \times 10^{-4}$ K respectively and hence negligible compared to the static temperature.

The plots of dimensionless cold and hot air temperature as a function of the inlet Reynolds number for $L/D = 10, 30$ and $50$ are presented in Figures 17, 18 and 19 respectively. The minimum total flow rate is $9.72 \times 10^{-6}$ kg/s which results in a Reynolds number of approximately 500. It can be seen that, at this low Reynolds number, both hot and cold exit temperatures are higher than the inlet temperature, $T_o$. The trend of the curves at Reynolds number below 500 is estimated as shown in the dashed line as it is known that all temperatures must be equal for the case of no flow. At this low Reynolds number, the vortex motion is not likely well established and the effect of the viscous term is the dominating factor. The viscous dissipation, therefore, causes the rise in the temperatures of both outlet streams. In the case of the hot outlet, the dimensionless temperature decreases after reaching its maximum at a Reynolds number of about 500 after which it achieves a minimum value at a Reynolds number of approximately 1200, 1300 and 1500 for $L/D = 10, 30$ and $50$ respectively. The
curve then increases steadily with further increase in Reynolds number. The cold outlet dimensionless temperature decreases steadily after reaching the maximum at a Reynolds number of about 800 to become negative at a Reynolds number of approximately 1800. In both the cases of hot and cold flow, the trend of increasing temperatures with Reynolds number is reversed at Reynolds numbers (approximately 500 for the hot flow and 800 for the cold flow) consistent with the critical Reynolds number estimated for tube length shorter than that required for fully developed flow which is common to micro-fluidic devices [1]. It is speculated that the reversal of temperature increase is due to the initiation of turbulence either in the supply nozzle or in the jet that forms in the vortex chamber just downstream of the supply nozzle.

The uncertainties in the Reynolds number are in the range of ± 73 to 196. While the uncertainty in both the dimensionless cold and hot temperature is found to be ± 0.0019. It can be seen from Figures 17 through 19 that the cross-over Reynolds number and the curve shapes are approximately the same and they are within the data uncertainty. It can be concluded that the inlet nozzle geometry is most important factor as it affects the Reynolds number.
Figure 17  
Dimensionless cold and hot temperatures vs. Reynolds number for \( L/D = 10 \)

Figure 18  
Dimensionless cold and hot temperatures vs. Reynolds number for \( L/D = 30 \)

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The effects of the orifice diameter and the tube length on the cold air mass ratio are shown in Figures 20 to 22. It should be recalled that this part of the test is conducted at an arbitrary fixed hot control valve opening. One would expect that the cold air mass fraction, $y_c$, for such a case would remain the same if the resistance to flow out the cold and hot end is constant. It is clearly shown from the data obtained, however, that:

- For Reynolds number below approximately 2000, the value of cold air mass fraction, $y_c$, decreases with an increase of the Reynolds number due to flow resistance changing. It is constant for Reynolds number values above 2000. This is similar to the behavior of other fluid mechanic quantities such as drag coefficient and friction factor.

- For smaller orifice diameter and hence smaller $d_c/D$ ratio, the cold air mass fraction is decreased. This is likely due to the increase of the resistance in the cold flow.
The cold air mass fraction also reduced with higher values of $L/D$ ratio. This variation is not as obvious as in the case of $d_c/D$. An increase in the $L/D$ value changes the internal flow pattern allowing the cold air to travel toward the hot end before reversing direction and passing through cold exit. This increased cold flow path would an increase in resistance.

![Figure 20](image.png)  

**Figure 20**  
**Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 10$**
Figure 21  Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 30$

Figure 22  Cold mass ratio, $y_c$, vs. Reynolds number for $L/D = 50$
The change of the inlet Reynolds number with increasing supply pressure is shown in Figures 23 to 25. They are as expected with supply pressure increasing with Reynolds number. This reflects the total resistance of the flow through both hot and cold air outlets. In general, for the same inlet pressure, there was a slight shift to the right of the curve in the Reynolds number as the \( d_c/D \) increases. This is as expected as decreasing \( d_c \) has the effect of increasing the total resistance. It would be expected that increasing the \( L/D \) ratio would also increase the pressure required for any Reynolds number. For the shortest \( L/D \) ratio, a higher pressure is however required to achieve the same Reynolds number value obtained for longer \( L/D \) ratio. This anomaly may be due to the pressure fluctuation observed for this case and mentioned later.

Figure 23  Reynolds number as a function of the inlet pressure for \( L/D=10 \)
Figure 24  Reynolds number as a function of the inlet pressure for \( L/D = 30 \)

Figure 25  Reynolds number as a function of the inlet pressure for \( L/D = 50 \)
4.2 Results of High Pressure Tests

When investigating the vortex tube as a cooling device, researchers are more interested in obtaining the value of the cold flow temperature. However, both the cold flow as well as the hot flow temperature measurements will be presented in this research. The objective of the second part of the experiment is to investigate the operating characteristics of the micro-scale vortex tube utilizing the same tube geometry for the first part of the test at a higher inlet supply pressure and by varying the cold air mass fraction. The inlet pressures considered here are 200, 300 and 400 kPa. Details of the experimental results are presented in Appendix D. The cold air mass ratio is systematically varied from approximately 0.05 to 0.95 by means of the control valve located at the hot end exit. The cold and hot air flow dimensionless temperatures are presented in Figures 26 to 34. The cold flow dimensionless temperature is represented by the lower curves while the upper curves for the hot flow dimensionless temperature.

For the smallest \( L/D \) ratio, the dimensionless hot and cold temperature as a function of the cold air mass ratio and the inlet pressure as a parameter are shown in Figures 26 to 28. The symbols in these particular sets of curves represent data points collected at different cold air mass ratio and different inlet pressures and the lines are interpolating splines. The best fit curve is found not to be a good choice to represent the variation in the cold and the hot flow temperatures for this particular tube geometry. It can be clearly seen for \( L/D = 10 \) and for the three different \( dc/D \) ratios, for the value of the cold mass fraction of approximately 0.05, which represents the lowest cold flow rate leaving the orifice opening, the temperature measured is lower than the inlet air temperature due to the effect of the energy separation. From Figure 26 at 200 kPa inlet pressure, when the value of \( y_c \) is increased, the temperature of the cold flow drops until it reaches its lowest value at \( y_c = 0.38 \). With the slight increase of \( y_c \), a sharp increase in the cold flow temperature is observed (which is more obvious with the 300 and 400 kPa inlet pressure) is also associated with a slight drop in the hot flow temperature. The sharp increase occurs at larger values of the cold air mass fraction as the inlet pressure increases. This is associated with an increase in fluctuations in the measured pressures and flow rates. It is believed that the flow
instability phenomenon which occurs at a certain value of $y_c$ causes radial mixing between the cold and hot flow temperatures. The instability needs further investigation for proper explanation. This case is only observed with the shortest tube length of $L/D$ equal to 10. By further increasing the value of $y_c$, the cold flow dimensionless temperature increases and approaches zero as $y_c$ approaches 1.

Figure 26  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.25$
Figure 27  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.4$

Figure 28  Vortex tube performance for $L/D = 10$ and $d_c/D = 0.55$
The results obtained for larger $L/D$ ratios (Figures 29 to 34) show a smooth variation of the cold and the hot flow temperature for different inlet pressures and different values of $y_c$. No sharp increase in the cold flow temperature is observed. As a general observation, an increase in the inlet pressure is seen to cause the values of the dimensionless cold temperature difference to increase over the whole range of the cold air mass fraction. Furthermore, it is found that the cold air mass ratio corresponding to the lowest cold air stream temperature decreases with the increasing of the supply pressure for similar tube geometry. Except for $d_c/D = 0.25$, it is not expected for $y_c$ to reach the value of 0.3 as observed in conventional vortex tubes [41 and 42]. When the smallest orifice diameter is used, however, the $y_c$ value obtained is found to be in the range of 0.2 to 0.4. This difference in the value of $y_c$ appears to be related to the relative size of the inlet nozzle hydraulic diameter to the size of the orifice diameter used. Similarly, the maximum hot air temperatures seen to be at values of cold mass ratio different than those for the conventional devices.

![Figure 29](image)

**Figure 29**  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.25

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Figure 30  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.4$

Figure 31  Vortex tube performance for $L/D = 30$ and $d_c/D = 0.55$
Figure 32  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.25$

Figure 33  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.4$
Figure 34  Vortex tube performance for $L/D = 50$ and $d_c/D = 0.55$

The optimum values of the cold air mass fraction, $y_c$, correspond to the lowest cold flow temperature and are presented in Figures 35 through 37. For $L/D = 10$ and $d_c/D = 0.25$ and 1.1, the trend of the optimum cold air mass fraction change is similar (Figure 35). For $d_c/D = 0.4$, the value of the optimum cold air mass fraction is within the error bar of that for $d_c/D = 1.1$ at 200 and 300 kPa inlet pressure, however, is different at an inlet pressure of 400 kPa. This may be due to the instabilities mentioned previously.
Figure 35  Optimum cold air mass fraction vs. inlet pressure for $L/D = 10$

For $L/D = 30$, the optimum cold air mass fraction value changes in an approximately linear manner with the inlet pressure (Figure 36). The cases corresponding to $L/D = 50$ are shown in Figure 37. The trends are also approximately linear with the slope approaching zero for large $d_c/D$ values.
Figure 36  Optimum cold air mass fraction vs. inlet pressure for $L/D = 30$

Figure 37  Optimum cold air mass fraction vs. inlet pressure for $L/D = 50$
4.3 Effect of Orifice Diameter

Figures 38 to 40 show the optimum dimensionless cold temperature performance of the micro-scale vortex tube for different sizes of the orifice diameter. In all cases the dimensionless cold temperature decreases with increasing $d_c/D$ ratio reaching constant values at $d_c/D$ in the range of 0.5 to 0.55. All the results obtained in optimizing the orifice diameter, $d_c$, are in good agreement with the conventional vortex tube [10, 23].

Figure 38 Optimum conditions vs. dimensionless orifice diameter for $L/D = 10$
Inlet pressure

$\frac{L}{D} = 30$

Figure 39  Optimum conditions vs. dimensionless orifice diameter
for $L/D = 30$

$\frac{T_e - T_o}{T_o}$

$rac{L}{D} = 50$

Figure 40  Optimum conditions vs. dimensionless orifice diameter
for $L/D = 50$
4.4 Effect of Tube Length

The effects the $L/D$ ratio on optimum dimensionless cold temperature are shown in Figures 41 to 43. Except for a vortex tube with $d_c/D = 0.25$ operated at 400 kPa inlet pressure, it can be generally observed that the maximum $L/D$ ratio of 50 gives the maximum cold temperature drop. Although the curves do not indicate a mathematical optimum tube length, by looking to curve’s trend, it can be inferred that increasing the $L/D$ ratio beyond 50 will not be of great advantage. The results of the $L/D$ value obtained are in a close agreement to that reported by Dyskin and Kramarenko [19] who also conducted their experiment on a micro-scale vortex tube.

![Graph showing effect of tube length on cold temperature drop](image)

**Figure 41** Optimum conditions vs. dimensionless tube length for $d_c/D = 0.25$

The cold temperature drop obtained at 400 kPa inlet pressure, 0.25 $d_c/D$ and 10 $L/D$ ratio was lower than that obtained for a larger $L/D$ ratio. This inconsistency with the other results obtained at different tube’s geometry may be attributed to the higher level of fluctuation taking place with that tube geometry as mentioned previously.
Figure 42  Optimum conditions vs. dimensionless tube length for $d_c/D = 0.40$

Figure 43  Optimum conditions vs. dimensionless tube length for $d_c/D = 0.55$
CHAPTER 5 – CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The performance characteristics of a micro-scale vortex tube are presented over a wide range of working pressure, different cold air mass ratio as well as different vortex tube length and orifice diameter. The following conclusions may be drawn:

1. Experiments conducted on the micro-scale vortex tube with fixed geometry at low Reynolds numbers have shown that:

   • Dimensionless temperature increases in both the cold and the hot air flows as the Reynolds number increases from zero and reaches maximum values before a Reynolds number of approximately 500 and 800 for the hot and cold flow respectively.

   • The cold outlet dimensionless temperature decreases steadily after the maximum to become negative at a Reynolds number in the order of 2000. This implies that the cooling effect occurs at inlet Reynolds numbers consistent with turbulent flow.

   • Except for low Reynolds numbers (i.e.: less than 2000) the cold mass fraction is approximately constant as the Reynolds number increases.

2. The experiment conducted to determine the performance curves of the micro-scale vortex tube at high inlet pressure indicate:

   • The optimum cold air mass fraction, \( y_c \), is not constant and tends to be higher when compared with the conventional vortex tube.
• The effects of $L/D$ and $d_o/D$ ratios are similar to that in the conventional devices.

• Unstable operation is observed at small $L/D$ and high inlet pressure.

5.2 Recommendations

A further investigation of the flow instability noticed at small $L/D$ and high inlet pressure is recommended in order to obtain more information on its nature of the flow.

It would be useful to instrument the micro-scale vortex tube with pressure transducer capable of measuring the pressure fluctuations that occur during that instability.
REFERENCES


[26] **J. P. Hartnett and E. R. Eckert.** Experimental study of the velocity and temperature distribution in a high velocity vortex type flow, Publication of the Heat Transfer Laboratory, University of Minnesota, Minneapolis, Minn., May 1957, pp. 751-758.


57

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Appendix A - Micro-Scale Vortex Tube Design Drawing

General Assembly Drawing
Note: 1. Male-female guide pin to be provided for every piece
2. All dimensions are in mm

Item number | 1 | 2 | 3 | 4 | 5
---|---|---|---|---|---
Material of Construction | Aluminum | Phenolic | Aluminum | Aluminum |
Number of pieces | 3 | 1 | 3 | 2 | 1

Elevation

Micro-Scale Vortex Tube

Date: January 15, 2006
E-mail: elubaidi@gmail.com

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Item number | 1  
---|---
Material of Construction | Aluminum
Number of pieces | 3
Orifice opening (X) | 0.7 mm
| 0.8 mm
| 0.9 mm

Note: 1. Male-female guide pin to be provided for every piece
2. All dimensions are in mm

Material of Construction | Plexiglas
Number of pieces | 1

Note: 1. Male-female guide pin to be provided for every piece
2. All dimensions are in mm

Mechanical Engineering Department

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Appendix B – Experimental Uncertainty Analysis Calculations

All major factors affecting the measurement accuracy are discussed in this section. With the precision of the measuring devices for each variable such as temperature, pressure and flow rates are known; the measurements uncertainty associated with each variable can be estimated. This method is based on the theoretical relationship of each variable.

The uncertainties of the instruments used are calculated using the formula given by Figiola and Beasley [44]:

\[ u_d = \pm \sqrt{u_o^2 + u_c^2} \] (11)

where \( u_c \) is the summation of errors in the instrument and are calculated using the following formula:

\[ u_c = \pm \sqrt{e_1^2 + e_2^2 + \ldots + e_k^2} \] (12)

The method of Kline and McKlintock [45] is used to determine the propagation of uncertainties.

1. Pressure Instruments

Three pressures gages having different ranges are used in this experiment. The low range (0 – 17.5 kPa) has an uncertainty of ± 0.01 kPa, the medium range (0 - 206.0 kPa), an uncertainty of ± 1.7 kPa and the high range pressure gage (0 – 413 kPa), an uncertainty of ± 3.4 kPa. The uncertainty of the barometric pressure gage is ± 0.07 kPa.
2. Temperature Instruments

The temperatures of the inlet, cold and hot air are measured using type - T (Copper Constantan) thermocouple probes. The increment and the accuracy of these devices are ± 0.1 °C and ± 0.4°C respectively. The resolution, $u_o$, is calculated from:

$$u_o = \pm \frac{1}{2}\text{ increment}$$  \hspace{1cm} (13)

Therefore, the uncertainty for each of these devices is:

$$u_d = \pm \sqrt{0.05^2 + 0.4^2} = \pm 0.4 \, ^\circ \text{C}$$

3. Uncertainty in Dimensionless Temperature

In order to estimate the uncertainty in dimensionless cold flow temperature, $\frac{T_c - T_0}{T_o}$, the Kline and McClintock method [45] is used:

$$U_{Uc} = \sqrt{\left(\frac{\partial U_c}{\partial T_c} u_{Tc}\right)^2 + \left(\frac{\partial U_c}{\partial T_o} u_{T0}\right)^2}$$  \hspace{1cm} (14)

$$U_{Uc} = \sqrt{\left(\frac{u_{Tc}}{T_o}\right)^2 + \left(\frac{T_c x u_{T0}}{T_o^2}\right)^2}$$  \hspace{1cm} (15)

where $U_c = \frac{T_c - T_0}{T_o} = \frac{T_c}{T_o} - 1$, and $\frac{\partial U_c}{\partial T_c} = \frac{1}{T_o}$

Equation 23 can also be used to calculate the uncertainty in the hot flow dimensionless temperature. The estimated uncertainty values for the hot and cold flow dimensionless temperature are presented in Appendix B and C. They are also
indicated in the graphs for the low and high pressure tests in forms of error bars. For the low and high pressure tests, however, the uncertainty in the cold and hot dimensionless temperature is ± 0.0019.

4. Uncertainty in Flow Rate Measurements

Four rotameters are used in measuring the hot and cold flow rate. The uncertainties in each of them are:

- Rotameter 1 = ± 8.33 x 10^-7 m³/sec
- Rotameter 2 = ± 8.33 x 10^-7 m³/sec
- Rotameter 3 = ± 1.67 x 10^-6 m³/sec
- Rotameter 4 = ± 5.83 x 10^-6 m³/sec

Rotameters 1 and 2 were used together to measure the total flow rate entering the micro-scale device in the low pressure tests. Therefore, the uncertainty is:

\[ u_d = \pm \sqrt{(8.33 \times 10^{-7})^2 + (8.33 \times 10^{-7})^2} \]

\[ = \pm 1.178 \times 10^{-6} \text{ m}^3/\text{sec} \]

For the high pressure tests, rotameters 3 and 4 are used together to measure the total flow rate. Therefore, the uncertainty is:

\[ u_d = \pm \sqrt{(1.67 \times 10^{-6})^2 + (5.83 \times 10^{-6})^2} \]

\[ = \pm 6.06 \times 10^{-6} \text{ m}^3/\text{sec} \]

5. Uncertainty in Tube Geometry Measurements

The micro-scale vortex tube, manufactured at the University of Windsor, has a tolerance of ± 0.013 mm. Therefore, the uncertainty in \( d/D \) ratio is:
The uncertainty in the \( \frac{d_c}{D} \) ratio is found to be ± 0.007. They are not indicated in Figures 37 through 39 because they are within the size of the symbol of the \( \frac{d_c}{D} \). Applying the same above method, the uncertainty in the \( \frac{L}{D} \) ratio is:

\[
U_{\frac{L}{D}} = \sqrt{\left( \frac{U_{\frac{d_c}{D}}}{d_c} \right)^2 + \left( \frac{U_D}{D} \right)^2}
\]  

The uncertainties in the \( \frac{L}{D} \) ratio are found to be in the range of ± 0.065 to 0.325 mm. There is also no indication of the uncertainties in Figures 40 through 42 because the error bars for such small values are smaller than the symbol representing the \( \frac{L}{D} \) values.

6. Uncertainty in Air Viscosity Calculations

From viscosity data taken from Moran et al. [43], the variation of viscosity with temperatures was found to have a slope of \( 5 \times 10^{-8} \) kg/m.s. K in the temperature range of these experiments. Since the temperature of the inlet air varied within 4 K, then the uncertainty in viscosity is:

\[
4 \times 5 \times 10^{-8} = \pm 2 \times 10^{-7} \text{ kg/m.s}
\]

7. Uncertainty in Air Density Calculations

The uncertainty in the inlet air density, \( \rho = \frac{P_o}{R T_o} \), is calculated as follow:

\[
U_\rho = \sqrt{\left( \frac{\partial \rho}{\partial P} x U_P \right)^2 + \left( \frac{\partial \rho}{\partial T} x U_T \right)^2}
\]  

(18)
Assuming that the uncertainty in the gas constant, \( R \), is negligible this gives an uncertainty in density of \( \pm 2.3 \times 10^{-3} \text{ kg/ m}^3 \).

8. Reynolds Number calculations, \( Re \)

The Reynolds number is defined as:

\[
Re = \frac{\dot{m} d_n}{4 A \mu}
\]

where \( d_n \) is the hydraulic diameter of the inlet nozzle and \( A \) is the cross-sectional area of the inlet nozzle. The propagation of uncertainties to Reynolds number is calculated using the method of Kline and McKlintock [45]:

\[
\frac{U_{Re}}{Re} = \sqrt{\left(\frac{U_p}{\rho}\right)^2 + \left(\frac{U_{\dot{Q}}}{Q}\right)^2 + \left(\frac{U_{dn}}{dn}\right)^2 + \left(\frac{U_{\mu}}{\mu}\right)^2}
\]

The uncertainties to Reynolds number are presented in Appendix C and D. It is also indicated as an error bars in the graphs of the Reynolds number versus dimensionless temperature, inlet pressure and the cold air mass fraction. They are found in the range of \( \pm 74 \) to 196.

9. Uncertainty in Cold Air Mass Fraction

The uncertainty in the cold mass flow rate, \( \dot{m}_c \), as a function of the air flow rate, \( \dot{Q}_c \), and the gas density, \( \rho \) is:
For the low pressure tests, the value is ± 10^{-6} kg/s and for the high pressure tests it is ± 2 \times 10^{-6} kg/s.

The uncertainty in the inlet mass flow rate, \( \dot{m}_o \), is:

\[
U_{\dot{m}_o} = \sqrt{\left( \frac{\partial \dot{m}_o}{\partial \dot{Q}_o} U_{\dot{Q}_o} \right)^2 + \left( \frac{\partial \dot{m}_o}{\partial \rho_o} U_{\rho_o} \right)^2}
\]

(22)

It is found to be ± 1.4 \times 10^{-6} kg/s for the low pressure tests and ± 7.3 \times 10^{-6} kg/s for the high pressure tests. Therefore, for the cold air mass fraction, \( y_c \), it is:

\[
U_{y_c} = \sqrt{\left( \frac{\partial y_c}{\partial \dot{m}_c} U_{\dot{m}_c} \right)^2 + \left( \frac{\partial y_c}{\partial \dot{m}_o} U_{\dot{m}_o} \right)^2}
\]

(23)

Since \( y_c = \frac{\dot{m}_c}{\dot{m}_o} \), then the above equation can be written in

\[
U_{y_c} = \sqrt{\left( \frac{U_{\dot{m}_c}}{\partial \dot{m}_o} U_{\dot{m}_o} \right)^2 + \left( \frac{\partial \dot{m}_c}{\partial \dot{m}_o} U_{\dot{m}_o} \right)^2}
\]

(24)

The uncertainties in the cold air mass fraction for the low and high pressure tests are presented in Appendix C and D respectively. The values for the low pressure tests are within ± 0.01 to 0.13 while for the high pressure tests are within ± 0.007 to 0.046.
Table C.1  Low pressure test for tube length, $L = 20$ mm and tube orifice diameter, $d_c = 0.5$ mm

<table>
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<tr>
<th>Inlet condition</th>
<th>Cold exit</th>
<th>Hot exit</th>
<th>Uncertainty results</th>
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<tr>
<td>$P_o$ (kPa)</td>
<td>$T_o$ (°K)</td>
<td>Flow (kg/s) x 10^5</td>
<td>$P_c$ (kPa)</td>
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<tr>
<td>2.8 298.1 1.04 522</td>
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<td>34.5 298.1 4.40 2215</td>
<td>2.83 0.35 297.9 -0.0004</td>
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Table C.2  Low pressure test for tube length, $L = 20$ mm and tube orifice diameter, $d_c = 0.8$ mm

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<th>Inlet condition</th>
<th>Cold exit</th>
<th>Hot exit</th>
<th>$Y_c$</th>
<th>Uncertainty results</th>
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<td>$T_o$ ($^\circ$K)</td>
<td>Flow (kg/s) x 10$^5$</td>
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<td>$T_c$ ($^\circ$K)</td>
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Table C.3  Low pressure test for tube length, $L = 20$ mm and tube orifice diameter, $d_c = 1.1$ mm

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Table C.5  Low pressure test for tube length, \( L = 60 \) mm and tube orifice diameter, \( d_c = 0.8 \) mm

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Table C.6  Low pressure test for tube length, \( L = 60 \) mm and tube orifice diameter, \( d_c = 1.1 \) mm

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Table C.8  Low pressure test for tube length, $L = 100$ mm and tube orifice diameter, $d_c = 0.8$ mm

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Table C.9  Low pressure test for tube length, $L = 100$ mm and tube orifice diameter, $d_c = 1.1$ mm

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Table D.1 High pressure test for tube length, \( L = 20 \text{ mm} \) and tube orifice diameter, \( d_c = 0.5 \text{ mm} \)

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Table D.2 High pressure test for tube length, $L = 20$ mm and tube orifice diameter, $d_c = 0.8$ mm

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<td>$P_c$ (kPa) x 10^5</td>
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**Table D.3** High pressure test for tube length, L = 20 mm and tube orifice diameter, d_c = 1.1 mm
Table D.4  High pressure test for tube length, $L = 60$ mm and tube orifice diameter, $d_c = 0.5$ mm

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<th>Uncertainty calculations</th>
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</thead>
<tbody>
<tr>
<td>$P_o$ (Pa)</td>
<td>$T_o$ (K)</td>
<td>Flow (kg/s) x 10$^4$</td>
<td>$Re$</td>
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<tr>
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<td>-----------</td>
<td>-----------</td>
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### Table D.5 High pressure test for tube length, L = 60 mm and tube orifice diameter, d_c = 0.8 mm

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<th>Uncertainty calculations</th>
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<td>T_c - T_o</td>
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**Notes:**
- Flow (kg/s) and Hot exit values are given in units of 10^4.
- Uncertainty calculations involve standard deviation and expected error margins.
Table D.6 High pressure test for tube length, $L = 60$ mm and tube orifice diameter, $d_c = 1.1$ mm

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<th>Uncertainty calculations</th>
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<td>(kg/s)</td>
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<td>$Re$</td>
<td>$T_c$ (°C)</td>
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Table D.7 High pressure test for tube length, \( L = 100 \) mm and tube orifice diameter, \( d_c = 0.5 \) mm

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<th>Uncertainty calculations</th>
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<td>( T_o ) (°K)</td>
<td>Flow ( \times 10^4 ) (kg/s)</td>
<td>( P_c ) (kPa)</td>
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<td>Flow (kg/s) $\times 10^3$</td>
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Table D.8: High pressure test for tube length, $L = 100$ mm and tube orifice diameter, $d_c = 0.8$ mm
### Table D.9 High pressure test for tube length, $L = 100 \text{ mm}$ and tube orifice diameter, $d_t = 1.1 \text{ mm}$

<table>
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<tr>
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<th>Hot exit</th>
<th>Uncertainty calculations</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$T_o$ (°K)</td>
<td>Flow (kg/s) x 10^3</td>
<td>$T_c$ (°K)</td>
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**Note:** The table continues with similar entries for various inlet conditions and pressure levels.
VITA AUCTORIS

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1983 Received Bachelor of Science from the University of Baghdad, Baghdad, Iraq in June.

1987 Mechanical and Electrical Contractors – Self Employed, Baghdad-Iraq.

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