Investigations of nonlinear p-y piles and pile groups in soft clay subjected to static loading-distributed parameter sensitivity analysis.

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INVESTIGATIONS OF NONLINEAR $p-y$ PIELES AND PILE GROUPS IN SOFT CLAY SUBJECTED TO STATIC LOADING-DISTRIBUTED PARAMETER SENSITIVITY ANALYSIS

BY

MARCIA REGINA MORA

A Thesis

Submitted to Faculty of Graduate Studies and Research through Civil and Environmental Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

Windsor, Ontario, Canada

2006
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ABSTRACT

For the design of a pile foundation, the engineer must take into account the axial loads and overturning moments. Although some structures deal with pile under axial load it is very common to see the utilization of piles subjected to lateral loads, especially in areas suitable for earthquakes or offshore platforms, bridges, high-rise structures, piers and so on.

Laterally loaded piles embedded in soft clay below the water table under static load is presented in this study under the theoretical formulation and the numerical investigation of sensitivity analysis. In this study both single piles and pile groups are analyzed under the sensitivity approach.

A wide range of lengths and a large group of different boundary conditions were applied on single piles under static loadings, addressed to the pile head. In terms of structure, here the single piles are considered as one dimensional beam and the soil supporting selected is a soft clay and is defined by \( p-y \) relationship once under lateral load, deflection of the pile is directly dependent on the soil response and in this case the soil response is a nonlinear function of pile deflection and the depth of the soil below the surface. The \( p-y \) model mentioned above for soft clay was developed by Cox, Reese and Grubbs and has been used widely all over the world and is applied in the neighbourhood of the pile.

Under the broad variation of the boundary conditions the group of piles are also analyzed through the sensitivity approach and in this case the cap is considered as a plate and the pile members are recognized as one dimensional beam. The \( p-y \) relationship once more is the more appropriate model to understand and represent the behaviour of the pile-soil system. For the case of a group of piles, the utilization of a specific modifier factor had to be applied once the group of piles change with respect to spacing of the piles and also the location of the piles inside of the group.

The physical parameters of the soft clay and the stiffness of the pile used for the description of the \( p-y \) relationships are taken as the design variables of the continuous type. They are space dependent.

To be able to analyze the sensitivity of the single pile and the group of pile subjected to lateral loads the adjoint method for nonlinear system is used. The first variation of the
functional of maximum deformations as a result of the changes of the design variables is formulated based on the virtual work principle.

The variations of maximum generalized deflection located at the pile head as a result of the changes of the design variables are determined by sensitivity integrands and the design variables related. The sensitivity integrands are integrated and the numerical assessment of the outcomes are presented and discussed in details.

Although some specific design variable appeared in the sensitivity analysis of the deformation of the pile soil system each one as expected had demonstrated particular differences and significance through the analysis, and theses will be also part of the final discussion.
Dedicated to
my parents Alba and Francisco Mora
I would like to address my sincere gratitude to my advisor Dr. B. B. Budkowska for her relentless support, guidance and important theoretical and technical insights throughout the process to accomplish this thesis.

I would like to express also my profound gratitude to my Committee Members Dr. S. Cheng and Dr. M. Ahmadi for their constructive and valuable suggestions to improve this thesis.

I would like to give a special and huge thank you to my wonderful and dear friends at University of Windsor: Dahlia Hafez, Sharefah Al-Shammari and Sunghan Lee for always being there to help me out with everything. I am forever grateful! Many thanks!

There are also a great number of friends that helped me through this work with their time, their technical support, and most of all, with their encouragement. Thanks to all; Lebing Liu, Zain-Al-Abedin, Andre Bom, Vrushali Trickle, Cindy Kumalas, T.I.M. Nazmur Rahman, Li Li, Wafa Polies, Family Radjul.

In particular, I would like also to express enormous love, appreciation and gratitude to my family members for their loving support, endless encouragement and countless hours that I was away from them; Rubens Wachockier, Kim Wachockier, Alba Mora and Francisco Mora. Many thanks!!!!
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Figure 8.1  Primary structure - Pile head deflection $y_t$ versus lateral static load $P_t$ applied to the pile head – Free head – Length of the pile $L=10T$  

Figure 8.2  Adjoint Structure - Pile head deflection $y_t$ versus lateral static load $P_t$ applied to the pile head – Free head – Length of the pile $L=10T$  

Figure 8.3  The exact sensitivity of lateral deflection $\delta_y^p$ expressed in (m) caused by changes of the design variables $b$ when applied force $P_t$ have values $P_t$  

Figure 8.4  The pile head lateral deflection $y_{top}$ versus the ratio $(c/c_0)$ of design variable $c$ with respect to the initial value of the design variable $c_0$, for the case study with free head pile subjected to a concentrated lateral force $P_t = 270$ kN, where the pile length considered is equal $L = 10T$  

Figure B.1  Lateral deflection of primary structure for free head pile under variable lateral force - Pile length $L = 4T$  

Figure B.2  Distribution of bending moments of primary structure for free head pile under variable lateral force – Pile length $L = 4T$
Figure B.3 Distribution of lateral deflection $y_a$ ($\overline{P}$) of the adjoint structure subjected to $\overline{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.4 Distribution of bending moments $\overline{M}$ ($\overline{P}$) of adjoint structure subject to $\overline{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.5 Distribution of soil reaction $p_a$ of primary structure subjected to $\overline{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\overline{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.7 Distribution of sensitivity operators $C_{EI}^{P}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness EI when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $= 4T$

Figure B.8 Distribution of sensitivity operators $C_{c}^{P}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of cohesion c when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $= 4T$

Figure B.9 Distribution of sensitivity operators $C_{\gamma'}^{P}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $= 4T$

Figure B.10 Distribution of sensitivity operators $C_{b}^{P}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile’s width b when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $= 4T$

Figure B.11 Distribution of sensitivity operators $C_{\varepsilon_{50}}^{P}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $= 4T$
Figure B.12 The quantitative assessment of sensitivity factor $A_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subject to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.13 The quantitative assessment of sensitivity factor $A_{c}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.14 The quantitative assessment of sensitivity factor $A_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.15 The quantitative assessment of sensitivity factor $A_{y}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.16 The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.17 The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.18 The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.19 The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.20 The quantitative assessment (in %) of relative sensitivity factor $F_{y}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$
Figure B.21 The quantitative assessment (in %) of relative sensitivity factor 
$F_{\varepsilon_{50}}^{py}$ affecting the top lateral deflection $y_t$ due to the changes of 
$\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral 
force $P_t$ - Free head pile - Pile length $L = 4T$

Figure C.1 Lateral deflection of primary structure for free head pile under 
variable lateral force Pile length $L = 5T$

Figure C.2 Distribution of bending moments of primary structure for free head 
pile under variable lateral force - Pile length $L = 5T$

Figure C.3 Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure 
subject to $\bar{P} = 1$ when the primary structure is subjected to variable 
lateral force $P_t$ - Free head pile - Pile length $L = 5T$

Figure C.4 Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure 
subject to $\bar{P} = 1$ when the primary structure is subjected to variable 
lateral force $P_t$ - Free head pile - Pile length $L = 5T$

Figure C.5 Distribution of soil reaction $p_a$ of primary structure subject to 
$\bar{P} = 1$ when the primary structure is subjected to variable lateral 
force $P_t$ - Free head pile - Pile length $L = 5T$

Figure C.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ 
when the primary structure is subjected to variable lateral force $P_t$ - 
Free head pile - Pile length $L = 5T$

Figure C.7 Distribution of sensitivity operators $C^{py}_{EI}$ affecting the changes of 
the pile head lateral deflection $\delta_{yt}$ due to the changes of bending 
stiffness $EI$ when the pile structure is subjected to variable 
concentrated lateral force $P_t$ - Free head pile - Pile length $= 5T$

Figure C.8 Distribution of sensitivity operators $C^{py}_{c}$ affecting the changes of 
the pile head lateral deflection $\delta_{yt}$ due to the changes of cohesion $c$ 
when the pile structure is subjected to variable concentrated lateral 
force $P_t$ - Free head pile - Pile length $= 5T$

Figure C.9 Distribution of sensitivity operators $C^{py}_{\gamma'}$ affecting the changes of 
the pile head lateral deflection $\delta_{yt}$ due to the changes of the 
submerged soil unit weight $\gamma'$ when the pile structure is subjected 
to variable concentrated lateral force $P_t$ - Free head pile - Pile 
length $= 5T$
Figure C.10 Distribution of sensitivity operators $C^{P_y}_b$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ — Free head pile — Pile length $= 5T$

Figure C.11 Distribution of sensitivity operators $C^{P_y}_{\varepsilon_{50}}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ — Free head pile — Pile length $= 5T$

Figure C.12 The quantitative assessment of sensitivity factor $A^{P_y}_{EI}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subject to variable concentrated lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.13 The quantitative assessment of sensitivity factor $A^{P_y}_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.14 The quantitative assessment of sensitivity factor $A^{P_y}_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.15 The quantitative assessment of sensitivity factor $A^{P_y}_{\gamma'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.16 The quantitative assessment of sensitivity factor $A^{P_y}_{\varepsilon_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.17 The quantitative assessment (in %) of relative sensitivity factor $F^{P_y}_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 5T$

Figure C.18 The quantitative assessment (in %) of relative sensitivity factor $F^{P_y}_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 5T$

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The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^y$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^y$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\varepsilon_{50}}^y$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 5T$

Lateral deflection of primary structure for free head pile under variable lateral force Pile length $L = 10T$

Distribution of bending moments of primary structure for free head pile under variable lateral force – Pile length $L = 10T$

Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$

Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$

Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$

Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$

Distribution of sensitivity operators $C_{EI}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 10T$
Figure D.8  Distribution of sensitivity operators $C^y_c$ affecting the changes of the pile head lateral deflection $\delta_{y_1}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure D.9  Distribution of sensitivity operators $C^y_y$ affecting the changes of the pile head lateral deflection $\delta_{y_1}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.10  Distribution of sensitivity operators $C^y_b$ affecting the changes of the pile head lateral deflection $\delta_{y_1}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.11  Distribution of sensitivity operators $C^y_{\varepsilon_{50}}$ affecting the changes of the pile head lateral deflection $\delta_{y_1}$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.12  The quantitative assessment of sensitivity factor $A^y_{P_t}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $E_I$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.13  The quantitative assessment of sensitivity factor $A^y_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.14  The quantitative assessment of sensitivity factor $A^y_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.15  The quantitative assessment of sensitivity factor $A^y_{\gamma'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$
Figure D.16 The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^{py}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.17 The quantitative assessment (in %) of relative sensitivity factor $F_b^{py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ when the pile is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.18 The quantitative assessment (in %) of relative sensitivity factor $F_c^{py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.19 The quantitative assessment (in %) of relative sensitivity factor $F_{E_l}^{py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $E_l$ of the pile when the pile is subjected to variable concentrated lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.20 The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma_l}^{py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma$ when the pile is subjected to variable concentrated lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.21 The quantitative assessment (in %) of relative sensitivity factor $F_{E_{50}}^{py}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.22 Distribution of lateral deflections $y_a (\bar{M})$ of the adjoint structure subjected to $\bar{M} = 1$ when the primary structure is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.23 Distribution of bending moments $M (\bar{M})$ of the adjoint structure subjected to $\bar{M} = 1$ when the primary structure is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$

Figure D.24 Distribution of the soil resistance $p_a$ of the adjoint structure subjected to $\bar{M} = 1$ when the primary structure is subjected to variable lateral force $P_l$ - Free head pile - Pile length $L = 10T$
Figure D.25 Distribution of sensitivity operators $C_{E1}^{\delta\theta}$ affecting the changes of the pile top angle of flexural rotation $\delta\theta_t$ due to the changes of the bending stiffness $E1$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.26 Distribution of sensitivity operators $C_{c}^{\delta\theta}$ affecting the changes of the pile top angle of flexural rotation $\delta\theta_t$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.27 Distribution of sensitivity operators $C_{\gamma'}^{\delta\theta}$ affecting the changes of the pile top angle of flexural rotation $\delta\theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.28 Distribution of sensitivity operators $C_{b}^{\delta\theta}$ affecting the changes of the pile top angle of flexural rotation $\delta\theta_t$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.29 Distribution of sensitivity operators $C_{\varepsilon_{50}}^{\delta\theta}$ affecting the changes of the pile top angle of flexural rotation $\delta\theta_t$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.30 The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^{P\delta\theta}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.31 The quantitative assessment of sensitivity factor $A_{c}^{P\delta\theta}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.32 The quantitative assessment of sensitivity factor $A_{\gamma'}^{P\delta\theta}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$
The quantitative assessment of sensitivity factor $A_{b}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A_{EI}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\varepsilon 50}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of $\varepsilon 50$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of the submerged unit weight of the soil $\gamma$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Lateral deflection of primary structure for fixed head pile under variable lateral force Pile length $L = 4T$

Distribution of bending moments of primary structure for fixed head pile under variable lateral force - Pile length $L = 4T$
Figure E.3 Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.4 Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.5 Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.7 Distribution of sensitivity operators $C_{\gamma t}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.8 Distribution of sensitivity operators $C_{c}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.9 Distribution of sensitivity operators $C_{\gamma'}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.10 Distribution of sensitivity operators $C_{b}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$

Figure E.11 Distribution of sensitivity operators $C_{\varepsilon_{50}}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile – Pile length $L = 4T$
The quantitative assessment of sensitivity factor $A_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment of sensitivity factor $A_{c}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment of sensitivity factor $A_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment of sensitivity factor $A_{\gamma'}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_l$ - Fixed head pile - Pile length $L = 4T$
The quantitative assessment (in %) of relative sensitivity factor $F^{py}_{50}$ affecting the top lateral deflection $\gamma_t$ due to the changes of the $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$

Figure F.1 Lateral deflection of primary structure for fixed head pile under variable lateral force Pile length $L = 5T$

Figure F.2 Distribution of bending moments of primary structure for fixed head pile under variable lateral force – Pile length $L = 5T$

Figure F.3 Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.4 Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.5 Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.7 Distribution of sensitivity operators $C^{py}_{EI}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.8 Distribution of sensitivity operators $C^{py}_{c}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$

Figure F.9 Distribution of sensitivity operators $C^{py}_{\gamma'}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Distribution of sensitivity operators $C^{Py}_b$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Distribution of sensitivity operators $C^{Py}_b$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment of sensitivity factor $A^{Py}_{EI}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment of sensitivity factor $A^{Py}_{EI}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment of sensitivity factor $A^y_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment of sensitivity factor $A^y_{\gamma}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment of sensitivity factor $A^y_{\varepsilon_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F^{Py}_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F^{Py}_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
The quantitative assessment (in %) of relative sensitivity factor $F_{EI}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 5T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\varepsilon_{SO}}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{SO}$ when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 5T$

Lateral deflection of primary structure for fixed head pile under variable lateral force $P_t$ - Pile length $L = 10T$

Distribution of bending moments of primary structure for fixed head pile under variable lateral force $P_t$ - Pile length $L = 10T$

Distribution of lateral deflections $y_a$ ($\bar{P}$) of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of bending moments $\bar{M}$ ($\bar{P}$) of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of soil reaction $p_a$ of primary structure subjected to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of sensitivity operators $C_{EI}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$
Distribution of sensitivity operators $C^{R_Y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of sensitivity operators $C^{R_Y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of sensitivity operators $C^{R_Y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of sensitivity operators $C^{R_Y}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of $e^{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A^{P_Y}_{E}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $E$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A^{P_Y}_{c}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A^{P_Y}_{b}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A^{P_Y}_{y'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$
The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^P$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_b^P$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_c^P$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^P$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^P$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{\varepsilon_{50}}^P$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$

Distribution of lateral deflection of primary structure for Free head pile under variable bending moment $M_t$ - Pile length $L = 4T$

Distribution of bending moments of primary structure for Free head pile under variable bending moment $M_t$ - Pile length $L = 4T$

Distribution of lateral deflections $y_a$ ($\bar{M}$) of the adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 4T$

Distribution of bending moments $\bar{M}$ ($\bar{M}$) of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 4T$

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Figure H.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.7 Distribution of sensitivity operators $C^{M_0}_{E_t}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness $E_t$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.8 Distribution of sensitivity operators $C^{M_0}_{c}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta_t$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

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Figure H.10 Distribution of sensitivity operators $C^{M_0}_{b}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta_t$ due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.11 Distribution of sensitivity operators $C^{M_0}_{\varepsilon_{50}}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta_t$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.12 The quantitative assessment of sensitivity factor $A^{M_0}_{E_t}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of bending stiffness $E_t$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$
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Figure H.14 The quantitative assessment of sensitivity factor $A_{b}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.15 The quantitative assessment of sensitivity factor $A_{\gamma}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.16 The quantitative assessment of sensitivity factor $A_{\varepsilon_{so}}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of $\varepsilon_{so}$ when the pile is subjected to variable bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.17 The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.18 The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of cohesion $c$ when the pile is subjected to variable bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.19 The quantitative assessment (in %) of relative sensitivity factor $F_{\varepsilon_{so}}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of bending stiffness $E\varepsilon_{so}$ of the pile when the pile is subjected to variable concentrated bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

Figure H.20 The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^{M_{t}}$ affecting the top flexural angle of rotation $\theta_{t}$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated bending moment $M_{t}$ – Free head pile – Pile length $L = 4T$

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The quantitative assessment (in %) of relative sensitivity factor $F_{59}^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of $\varepsilon_{59}$ when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Distribution of lateral deflection of primary structure for Free head pile under variable bending moment $M_t$ – Pile length $L = 5T$

Distribution of bending moments of primary structure for Free head pile under variable bending moment $M_t$ – Pile length $L = 5T$

Distribution of lateral deflections $v_a (\bar{M})$ of the adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of bending moments $\bar{M} (\bar{M})$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of soil reaction $p_a$ of primary structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subject to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of sensitivity operators $C_{EI}^{M0}$ affecting the changes of the pile top flexural angle of rotation $\delta\theta_t$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of sensitivity operators $C_{c}^{M0}$ affecting the changes of the pile top flexural angle of rotation $\delta\theta_t$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Distribution of sensitivity operators $C_{\gamma'}^{M0}$ affecting the changes of the pile top flexural angle of rotation $\delta\theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$
Figure I.10  Distribution of sensitivity operators $C_b^{M_b}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.11  Distribution of sensitivity operators $C_{\varepsilon_{50}}^{M_0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.12  The quantitative assessment of sensitivity factor $A_{EI}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.13  The quantitative assessment of sensitivity factor $A_c^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.14  The quantitative assessment of sensitivity factor $A_b^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.15  The quantitative assessment of sensitivity factor $A_{\gamma'}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.16  The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.17  The quantitative assessment (in %) of relative sensitivity factor $F_b^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.18  The quantitative assessment (in %) of relative sensitivity factor $F_c^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of cohesion $c$ when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 5T$
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Figure I.20  The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma}^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure I.21  The quantitative assessment (in %) of relative sensitivity factor $F_{\epsilon_{50}}^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of $\epsilon_{50}$ when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

Figure J.1  Distribution of lateral deflection of primary structure for free head pile under variable bending moment $M_t$ – Pile length $L = 10T$

Figure J.2  Distribution of bending moments of primary structure for free head pile under variable bending moment $M_t$ – Pile length $L = 10T$

Figure J.3  Distribution of lateral deflections $y_a (\bar{M})$ of the adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.4  Distribution of bending moments $\bar{M} (\bar{M})$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.7  Distribution of sensitivity operators $C_{EI}^{M0}$ affecting the changes of the pile top flexural angle of rotation $\delta\theta_t$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$
Figure J.8  Distribution of sensitivity operators $C_{M_0}^{0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.9  Distribution of sensitivity operators $C_{\gamma}^{M_0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.10  Distribution of sensitivity operators $C_{b}^{M_0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.11  Distribution of sensitivity operators $C_{\varepsilon_{50}}^{M_0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.12  The quantitative assessment of sensitivity factor $A_{E_l}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of bending stiffness $E_l$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.13  The quantitative assessment of sensitivity factor $A_{c}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.14  The quantitative assessment of sensitivity factor $A_{b}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

Figure J.15  The quantitative assessment of sensitivity factor $A_{\gamma'}^{M_0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$
The quantitative assessment of sensitivity factor $A_{50}^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_b^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_c^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of cohesion $c$ when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_e^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F^\gamma^{M0}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $F_{50}^{M0}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

Distribution of lateral deflections $y_a$ ($P$) of the adjoint structure subjected to $P = 1$ when the primary structure is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

Distribution of bending moments $M$ ($\bar{M}$) of the adjoint structure subjected to $P = 1$ when the primary structure is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

Distribution of the soil resistance $p_a$ of the adjoint structure subjected to $P = 1$ when the primary structure is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$
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Figure J.26  Distribution of sensitivity operators \( C_c^{My} \) affecting the changes of the pile head lateral deflection \( \delta_{y_t} \) due to the changes of cohesion \( c \) when the pile structure is subjected to variable concentrated bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.27  Distribution of sensitivity operators \( C_{\gamma'}^{My} \) affecting the changes of the submerged soil unit weight \( \gamma' \) when the pile structure is subjected to variable concentrated bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.28  Distribution of sensitivity operators \( C_b^{My} \) affecting the changes of the pile head lateral deflection \( \delta_{y_t} \) due to the changes of the pile’s width \( b \) when the pile structure is subjected to variable concentrated bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.29  Distribution of sensitivity operators \( C_{e50}^{My} \) affecting the changes of the pile head lateral deflection \( \delta_{y_t} \) due to the changes of \( e_{50} \) when the pile structure is subjected to variable concentrated bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.30  The quantitative assessment of sensitivity factor \( A_{e50}^{My} \) affecting the top lateral deflection \( y_t \) due to the changes of \( e_{50} \) when the pile is subjected to variable bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.31  The quantitative assessment of sensitivity factor \( A_c^{My} \) affecting the top lateral deflection \( y_t \) due to the changes of cohesion \( c \) when the pile is subjected to variable bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)

Figure J.32  The quantitative assessment of sensitivity factor \( A_{\gamma'}^{My} \) affecting the top lateral deflection \( y_t \) due to the changes of the submerged soil unit weight \( \gamma' \) when the pile is subjected to variable bending moment \( M_t \) – Free head pile – Pile length \( L = 10T \)
The quantitative assessment of sensitivity factor $A^M_{b_y}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment of sensitivity factor $A^M_{Ei}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $Ei$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $\frac{F^{My}_{\varepsilon_{50}}}{E_i}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $\frac{F^{My}_{c}}{E_i}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $\frac{F^{My}_{\gamma}}{E_i}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $\frac{F^{My}_{b}}{E_i}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ - Free head pile - Pile length $L = 10T$

The quantitative assessment (in %) of relative sensitivity factor $\frac{F^{My}_{Ei}}{E_i}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $Ei$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 10T$

Distributions of bending moments $M$ of the primary structure for the pile B (first trailing row) in the 3x3 pile group subjected to variable lateral load $P_g$ of discrete variability - Piles pinned to the cap, and the pile spacing is equal to $5D$ - Pile length $L = 10T$

Distributions of lateral deflections $y$ of the primary structure for the pile B (first trailing row) in the 3x3 pile group subjected to variable lateral load $P_g$ of discrete variability - Piles pinned to the cap, and the pile spacing is equal to $5D$ - Pile length $L = 10T$
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Figure K.4  Distributions of sensitivity operators \( C_{c\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of cohesion \( c \) of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)

Figure K.5  Distributions of sensitivity operators \( C_{c\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of cohesion \( c \) of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)

Figure K.6  Distributions of sensitivity operators \( C_{c\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of cohesion \( c \) of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)

Figure K.7  Distributions of sensitivity operators \( C_{\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of soil submerged unit weight \( \gamma' \) for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)

Figure K.8  Distributions of sensitivity operators \( C_{\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of soil submerged unit weight \( \gamma' \) for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)

Figure K.9  Distributions of sensitivity operators \( C_{\gamma}^{\delta\gamma} \) affecting the changes of the pile head lateral deflection \( \delta y_t \) due to the changes of soil submerged unit weight \( \gamma' \) for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal to 5D – Pile length \( L = 10T \)
Figure K.10 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal $5D$ – Pile length $L = 10T$

Figure K.11 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.12 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.13 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.14 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.15 Distributions of sensitivity operators $C_{\varepsilon_{50}}^{R_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.16 The quantitative assessment of sensitivity factor $A_{c}^{R_y}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of cohesion $c$ of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$
Figure K.17 The quantitative assessment of sensitivity factor $A_{\gamma_{EI}}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of bending stiffness $EI$ of the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.18 The quantitative assessment of sensitivity factor $A_{\gamma'}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the submerged soil unit weight $\gamma'$ of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.19 The quantitative assessment of sensitivity factor $A_{\epsilon_{so}}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the $\epsilon_{so}$ of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.20 Figure K. 20 The quantitative assessment of sensitivity factor $A_{b}\gamma_{EI}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.21 The total quantitative assessment of sensitivity factor $A_{\gamma_{EI}}^{py}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the design variables for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$

Figure K.22 The quantitative assessment of the relative sensitivity factor $F_{\gamma_{EI}}^{py}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the design variables for the pile B (first trailing row) in the 3x3 free head pile group subjected to lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal to $5D$ – Pile length $L = 10T$
CHAPTER 1
INTRODUCTION

1.1 Problem statement

Throughout the last four decades several researchers have been developing more suitable methods of analysis of laterally loaded piles embedded in different types of soils subjected to a lateral load. They developed better soil models that will provide a better representation of the behavior of the soil that will be the support of the pile. Among those who significantly contributed to development of soil models as well used in the laterally loaded pile-soil systems are: Hetenyi (1946), Terzaghi (1955), Broms (1964), Murchinson (1969), Matlock (1970), Reese (1970), Poulos (1971), Cox (1974), Dunnivant (1986) and many others.

Due to fast advancement of personal computers that made possible to conduct the major computational analysis in a very short time, and the extensive improvements of numerical methods in the past decades, the development of methods of analysis of laterally loaded pile-soil system has been substantially accelerated. It is a fact that the majority of the infrastructure systems built during the post-war period approach the designed life-service and the requirements to deal with this situation promotes new challenges, and increase the necessity to develop suitable methods for prediction and assessment of the deterioration of the systems during life-service. The knowledge concerning the possible effects of aging of infrastructure systems can be employed at the design stage to provide a justified rationale for more economically sound and durable design of the systems. The sensitivity analysis of distributed parameters provides solid basis that can be beneficial when used for the design purpose and when employed in the process of rehabilitation of aging structural systems.

This study presents a HSS 508x13 single piles and pile groups subjected to lateral force and bending moments of discrete variability. The single piles and pile groups are embedded in $p-y$ soft clay (Matlock, 1970), below the ground water table. The different
boundary conditions that are applied to the piles are investigated in the framework of sensitivity theory of distributed parameters. Among the boundary conditions, it is important to emphasize that the pile head can be either free or fixed head. Different pile lengths are also an important input, and they will vary from short to long piles.

The pile structure is modeled as a one dimensional beam element. The adjacent soil is described by $p$-$y$ curves that are distributed in continuous fashion along the pile axis. The $p$-$y$ model of soft clay developed by Matlock (1970) relates the soil reaction $p$ and lateral deflection $y$ at arbitrary depth $x$. Characteristically, when the depth $x$ increases, the same deflection $y$ is associated with larger value of soil reaction $p$. The developed $p$-$y$ model although spatially continuous, however, does not provide a transfer of deformation to the neighboring soil. The deformation the $p$-$y$ soil model can develop has a local character. This is due to the fact that $p$-$y$ soil model does not possess shear connections which enables one to shift local deflection to the adjacent vicinity.

The design variables used in this analysis are: bending stiffness ($E I$), width of the pile ($b$), submerged unit weight of soil ($\gamma'$), cohesion of the soil ($c$), and the strain corresponding to one half of the maximum principal strain difference of the soil ($\varepsilon_{50}$). The sensitivity performances investigated in these results are the top lateral deflection and the top angle of flexural rotations. They are important issues in assessment of stability of upper structure behaviour.

1.2 Objectives

The main objectives of this study are specified as follow:

1. To perform the sensitivity analysis of lateral displacement and angle of rotation at the piles head embedded in soft clay located below the water table, subjected to lateral loads or bending moments.
2. To understand the behavior of the laterally loaded piles based on the sensitivity analysis results.
3. To determine the distributions of sensitivity operators along the depth of the piles. They provide information for a designer that is required in the design stage for rehabilitation or for redesign piles or pile groups.

4. To assess the effect of the changes of the sensitivity operators on the lateral deflection and rotations at the pile head.

5. To find out the importance of each physical parameter has in the system.

1.3 Procedures

To study of the sensitivity analysis of single piles and pile groups subjected to different boundary conditions, require complete analysis of these structures. The procedures described below are essentials for conducting the sensitivity analysis. They are specified as follows:

1. Determine: the soil type and properties, free or fixed head pile type and properties, and boundaries conditions such as constraints, types of loads and allowable deflections of the system.

2. Determine the length of the single pile and pile groups employing the relative stiffness factor “T” of the piles. It is done using the Characteristic Load Method introduced by Evans and Duncan (1982).

3. Use the software COM624P version 2.0 to analyze single piles.

4. Use the software FB-Pier version 3.0 to perform analysis on pile groups.

5. Plot curves that present the load applied (Pi and Mi for sensitivity analysis) to the pile head against lateral deflection (y_{top}) of the pile’s top. Take into account different pile length and boundary conditions.

6. Plotting the distributions of lateral deflections y of primary structure, lateral deflections y_{a}, bending moments M of primary structure, bending moments M_{a} of adjoint structure, soil reaction p of primary structure and soil reaction p_{a} of adjoint structure along the length of the pile.

7. Integrate the sensitivity operators to calculate the sensitivity factors using Simpson’s method.
8. Conduct analysis and comparative studies of the results of the sensitivity analysis with different boundary conditions and pile length.
9. Discuss the results and show the conclusions.

1.4 Methodology and significance of Sensitivity Analysis

The method of sensitivity analysis of laterally loaded pile embedded in nonlinear soil is based on virtual work principle. Based on the principle that the virtual work done by the unit load applied to the pile head of the adjoint structure equals to the virtual work done by the internal forces of the pile-soil system. The method considered that the spatial integrations are used to calculate the virtual work done by the internal forces of the pile-soil system.

The sensitivity operators are a spatial function and they are the integrands used to calculate the virtual work. Subsequently the sensitivity operators are integrated into a scalar numbers, which will are called “sensitivity factors”. The sensitivity operators are particularly important once they demonstrate the spatial distribution of the influence that any changes on the design variables affect the generalized pile head deflection. The effects that any changes on the design variables will promote on the generalized lateral pile head deflection, will be numerically described through the sensitivity factors.

The distributions of sensitivity operators and sensitivity factors are viewed in a graph format. The importance of those results in engineering application such as rehabilitation, design, improvement and management of the structures will be addressed in conclusions.

1.5 Study organization

This study is organized in the form of chapters and appendices. Chapter 1 presents the introduction with the problem statement and the description of the procedures to achieve
the objectives. Chapter 2 presents a literature review of laterally loaded single piles, pile groups and review of sensitivity analysis on reference to pile foundations.

The theoretical sensitivity analysis formulation of single piles subjected to lateral loads and bending moments embedded in soft clay below the ground water table is presented in Chapter 3. Chapter 4 presents the theoretical formulation of sensitivity analysis of pile groups under lateral loads and bending moments. The numerical investigations are covered in two chapters. Chapter 5 presents the analysis and results of single piles. The analysis and results for the pile groups are presented in the Chapter 6.

Different softwares were used during this study to analyze single piles and piles groups, and to perform the sensitivity analysis of them. Chapter 7 presents the programming of sensitivity analysis for single piles and for pile groups.

The discussion of the results is present at Chapter 8. The conclusions appear in Chapter 9. Several appendices present the results of the sensitivity analysis performed on single piles and pile groups with different boundary conditions.
CHAPTER 2
LITERATURE REVIEW

2.1 Methods of analysis of laterally loaded single piles

2.1.1 General

Piles are mainly used to transmit axial forces from the structures to the subsoil. However, many structures which require to be constructed on piles are not only subjected to vertical gravity forces, but also to horizontal ones. A vertical pile resists lateral load by mobilizing passive pressure in the soil surrounding it, and the degree of distribution of the soil reaction according to Das (1999) depends on:

- The stiffness of the pile;
- The stiffness of the soil;
- The fixity of the ends of the pile.

Among many variables, the length of the pile, the type of pile head and restrained conditions play very important role. In general, laterally loaded piles can be divided in two major categories, such as:

- Short or rigid piles;
- Long or flexible piles.

In the past 40 years many researches have come up with suggestions for safe allowable lateral loads for the large spectrum of different variables. Some approaches are based on full scale tests and some on theoretical consideration. However, two branches of analysis of soil adjacent to the pile can be identified:

- sub grade reaction approach;
- elastic approach.
2.1.2 Models for use in analysis of a single piles

As mentioned in Reese and Van Impe (2001), a large number of models have been applied to solve problems related to the design of single piles subjected to a lateral loading. Among these, some are just updated versions of the principal models (sub grade reaction model and elastic approach) which lead us to take in consideration the following ones.

2.1.2.1 Sub grade-reaction model

The first attempt to model the soil behaviour as an elastic media was presented by Winkler in 1867. Originally in this model it is proposed that the soil could be represented by a series of unconnected linear-elastic springs. This model, known later as the Winkler Model, introduces the concept of the beam on an elastic foundation, which can deform when the load is applied. In Winkler’s method the idea of an elastic representation of the soil was an important step to the understanding of the response of the soil. The model presented demonstrates a lack of continuity, and is commonly called "Beam on Elastic Foundation Method". This method requires determination of $k_s$ defined as:

\begin{equation}
(2.1) \quad k_s = \frac{q}{\delta}
\end{equation}

$k_s$ = coefficiente of subgrade reaction (force/length$^3$)
$q$ = bearing pressure (force /length$^2$)
$\delta$ = displacement of foundation component

However, the soil is a continuous media and the displacement at a point is influenced by stresses and forces at other points. In state of the deficiency reporting continuity of deformation Winkler model has been used extensively due to its simplicity in carrying on the analysis.

For many years, this method has been used in real problems. This applicability through a period of time brought numerous studies that resulted in applied empirical correlations to determine the subgrade modulus (Equation 2.1). They cover a large group of soils as referred to Poulos and Davis (1980) and Poulos and Hull (1989).
2.1.2.2 Elastic pile and elastic soil

According to Poulos & Davis (1980) and Poulos and Hull (1989) an large number of piles subjected to lateral loading that use an elastic model (and numerous variations of the model) have been studied. The authors above mentioned significantly contributed to the development of analysis of single piles under lateral load. They also investigated interaction of piles with close spacing in a variety of cases. Reese and Van Impe (2001) allude to the elastic solution. However, their approach does not address larger deformation or collapse of the pile foundation in nonlinear soil.

The representation of the soil as a linear elastic continuum is a satisfactory solution once it is taken into account the continuous nature of the soil. Some advantages of continuous method are the following:

- The linear elastic model is an idealized representation of the real soil and can be used to present approximate outcomes for varying modulus with respect to depth in a case with a layered system;
- The elastic approach facilitates the analysis over a pile group under lateral load;
- The elastic model permits consistent analysis of either immediate or total final movements;
- The elastic methodology can be adjusted to allow the soil yield.

However the applicability of this method yet faces the difficulty regarding the determination of the soil modulus.

2.1.2.3 Elastic pile and finite element for soil

The elastic pile and finite element for soil is similar to the prior method (elastic pile and elastic soil) except that the soil has been modeled through finite elements. According to Van Impe (2001) the elements can be fully three-dimensional and nonlinear in the physical properties. The elements can be selected as linear or nonlinear. In spite of this, some other problems appeared when the elements were selected as linear or nonlinear. Some special procedures are still needed to take into account. Among them are the following:
a) The tensile stress in the soil,

b) Modeling layered soil,

c) Accounting the separation between pile and soil during cyclic loading,

d) The collapse of sand against the back of the pile,

e) Accounting the changes of the soil characteristics associated with different type of loads.

Reese and Van Impe (2001) noted that Yegian and Wright (1973) and Thompson (1977) did interesting studies using 2D finite elements. Thompson investigated a plane stress model and obtained (through his study), soil response curves that agreed well with results at near the ground surface from full scale field tests. In the same track Portugal and Seco e Pinto (1993) used also finite elements based on p-y curves to obtain a good prediction of the observed lateral behaviour of the foundation piles of a Portuguese bridge. The utilizations of 3D finite elements approach to developed p-y curves were described in Kooijman (1989) and Brown et al. (1989).

2.1.2.4 Rigid pile and plastic soil

Reese and Van Impe (2001) had mentioned the rigid pile and plastic soil model presented by Broms; it was used in analysis of a single pile embedded in cohesive soils. Another model for piles embedded in soil was proposed by Broms in 1965.

Broms assumed the pile as a rigid element, and a solution is found by use of the equations of statics for the distribution of ultimate resistance of the soil that puts the pile in equilibrium. The soil resistance considered was for cohesive and cohesionless soil as well. After the ultimate loading is computed for a pile of particular dimensions, Broms suggests that the deflections for the working load may be computed by the equations suggested by the theory. The method presented by Broms employs several simplifying assumptions but still can be useful for the initial selection of piles.

Reese and Van Impe (2001) pointed to the fact that the solutions for the equations will yield the size and length of the pile for the expected loading and the pile can then be
employed at the starting point for the $p$-$y$ method of analysis. Further benefits from the Broms method are:

a) The mechanics of the problem of lateral loading was clarified;

b) The method may be used as a check for some of the results from the $p$-$y$ method of analysis.

2.1.2.5 Characteristic load method

Another model that has been widely used in analysis of single piles was presented by Duncan et al. (1994). The method known by CML - the Characteristic Load Method is based on the earlier work of Evans and Duncan (1982) and states numerous solutions that were made with nonlinear $p$-$y$ curves for a range of soils and pile-head conditions. As noted by Reese and Van Impe (2001) the results were analyzed with the view of obtaining simple equations that could be used for rapid prediction of the response of piles under lateral loading. Dimensionless variables were employed in the prediction equations. The authors assert that the method can be used to solve for:

1. Ground–line deflections due to lateral load for free-head conditions, fixed-head conditions, and the flag–pole condition;

2. Ground–line deflections due to moments applied at the ground line;

3. Maximum moments for the three conditions in case (1);

4. The location of the point of maximum moment along the pile;

The soil may be either clay or sand, both limited to uniform strength with depth. The characteristic shear load $V_c$, and the characteristic moment load, $M_c$, were defined as follows:

\begin{align*}
V_c &= \lambda b^2 E R_L \left( \frac{\sigma_P}{E R_I} \right)^m \left( \varepsilon_{50} \right)^n \\
M_c &= \lambda b^3 E R_L \left( \frac{\sigma_P}{E R_I} \right)^m \left( \varepsilon_{50} \right)^n
\end{align*}

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\[ R_1 = \left( \frac{I}{\pi b^4 / 64} \right) \]

where

\( V_c \) = characteristic shear load,
\( M_c \) = characteristic bending moment,
\( \lambda \) = a dimensionless parameter dependent on the soil's stress-strain behaviour,
\( b \) = diameter of pile,
\( E \) = modulus of elasticity of the pile material,
\( R_1 \) = ratio of moment of inertia (dimensionless),
\( I \) = moment of inertia of pile,
\( \sigma_p \) = representative passive pressure of soil,
\( m \) and \( n \) = exponents factors determined by the soil type and force type,
\( \varepsilon_{50} \) = axial strain at which 50 percent of the soil strength is mobilized.

Equations and nonlinear curves were developed for computing the value of the maximum bending moment and where it occurs along the pile.

According to Van Impe (2001) Duncan and his co-workers were ingenious in developing equations and curves that give useful solutions, where the limitations in the method with respect to applications were noted by the authors. Later Endley et al (1997) began with recommendations for formulating \( p-y \) curves and developed equations similar to those of Duncan et al., emphasizing the prediction of piles in various soils. The Endley equations were designed to deal with piles that penetrated only a short distance into the ground surface as well as with long piles.

Liu (2004) presented as an advantage of this method the fact that the analysis can be obtained quickly and directly. Therefore can also be used to ensure the outcomes from more sophisticated analysis, and used to determine the relative stiffness factor \( T \).

2.1.2.6 Nonlinear approach for pile and \( p-y \) model for soil

The \( p-y \) relationship approach presented in Reese and Van Impe (2001) was developed in the late 1940s and 1950s when energy companies built offshore structures that were
designed to sustain relatively large horizontal loads from waves, and about the same time (1956) offshore structures were built in the USA for military defence.

Back in the 1940s Timoshenko stated the relevant differential equation, and in 1946 Hetenyi presented solutions for beams on elastic foundation with a linear response. Many others in this period had worked over piles under lateral loading and in 1948 Palmer and Thompson presented a numerical solution to the nonlinear differential equation.

The development of the theoretical approach was necessary and became widely available with the advent of the computers, but not sufficient to a complete understanding of the behaviour of the pile and surrounding soil.

Then in 1950 with the large support of the offshore industry several full scale tests of fully instrumented piles were made, and in 1953 in the conference sponsored by the American Society for Testing and Materials Gleser (1953) presented full scale tests for lateral loading piles.

As mentioned in Reese and Van Impe (2001) a matter of historical interest, Terzaghi (1955) wrote – “If the horizontal loading tests are made on flexible tubes or piles – values of soil resistance – can be estimated for any depth, if the tube or pile is equipped with fairly closely spaced strain gauges and if, in addition, provisions are made for measuring the deflections by means of an accurate deflectometer. The strain gauge readings determine the intensity and distribution of the bending moments over the deflected portion of the tube or the pile, and on the basis of the moment diagram the intensity and the distribution of the horizontal loads can be ascertained by an analytical or graphic procedures”... “If the test is repeated for different horizontal loads acting on the upper end of the pile, a curve can be plotted for different depths showing the relationship between p and y”.

With the support of computers and full scale tests, and based on Terzaghi recommendations, Reese and Matlock (1956) conceive an extremely accurate method of measuring the bending moments and formal procedures for interpreting data. Two integrations of the bending moment data yielding accurate values of deflection but special techniques were required for the two differentiations to yield adequate values of soil
resistance. The results presented by them were the first set of comprehensive recommendations for predicting the response of a pile to lateral loading.

According to Reese and Van Impe (2001) the model which represents loading on the pile commonly refers to the two dimensional case; no torsion or out-of-plane bending is assumed. The horizontal lines across the pile are meant to show that it is made up of different sections; for example, steel pipe could be used with the wall thickness varied along the length. The finite difference equation method, employed for the solution of the beam column equation, allows the different values of bending stiffness (EI) to be considered. Further the method proposed by Reese and Matlock allows EI to be nonlinear and a function of the computed values of bending moment. For many solutions it is unnecessary to vary the bending stiffness even though the loading is carried to a point where a plastic hinge is expected to develop. An axial load is indicated and is considered in the solution with respect to its effect on bending and not in regard to computing the required length to support a given axial load.

The Figure (2.1) below presents the model proposed for a pile under lateral loading where the soil around the pile is replaced by a set of mechanisms that merely indicate that the soil resistance $p$ is a nonlinear function of a pile deflection $y$. The mechanisms, and the corresponding curves that represent their behaviour, are widely spaced in the sketch but are considered to be varied continuously with depth. As may be seen, the $p$-$y$ curves are fully variable with respect to distance $x$ along the pile and pile deflection $y$.

![Figure 2.1](image-url)  The model for a pile under lateral load with p-y curves (Reese and Van Impe (2001))
Described as versatile by Reese and Van Impe (2001) the p-y model provides also a practical means for design. The method was suggested over 30 years ago (McClelland and Foch (1958), Reese and Matlock (1956)) and the two developments during the 1950s made the method possible: the digital computers for solving the problem of the nonlinear, fourth order differential equation for the beam column and the remote reading strain gauge for use in obtaining soil-response (p-y) curves from experiment.

Liu (2004) emphasized that the reasons for the widely use of this method and the strong popularity of this model is that is based on full scale field models and employed the commonly used soil strength parameters to simulate the soil resistance deflection relationship. The p-y approach allows to take in account the complex relationship developed between the deflection and the soil resistance. The p-y model also allows the consideration of the different phases, described as an elastic, nonlinear elastic, softening and plastic flow.

2.1.3  Field testing performed over piles under lateral loading in soft clay

Through all those years researchers have been working on the piles subjected to lateral loading, and many theoretical formulations had been presented among several experiments to enable comparisons between them and promote information about accuracy of the analytical methods.

Many tests have been reported in the literature on the results of field testing over full scale piles. In 2001, Reese and Van Impe presented an extensive collection of those tests around the world. Most of the tests collected contain all necessaries information’s about pile, soil and loading and pile head restraint, instrumentation and results. However in few ones some data is missing and estimation can be made to achieve a comparison desired.

Under the case studies for piles installed into cohesive soils with free water above ground surface the authors above mentioned presented Lake Austin, Sabine and Manor cases. The first one, Lake Austin was presented by Matlock (1970) in which the results from lateral load tests employing a steel pipe pile that was 319 mm in diameter, with a wall thickness of 12.7 mm, and a length of 12.8m. The bending moment at which the extreme
fibers would first yield was computed to be 231 kN-m, and the bending moment for the formation of a fully plastic hinge was computed to be 304 kN-m. According to Reese and Van Impe the pile was driven into clays that were slightly overconsolidated by desiccation, slightly fissured, and classified as CH according to the Unified System of Soil Classification.

2.2 Methods of analysis of laterally loaded pile groups

2.2.1 Overview

The presence of the piles installed in groups is very significant and the knowledge of the response of the group of piles to loading still encourages researchers and engineers to continue support on how to approach the problem.

Numerous groups of piles must support loadings that are both axial and lateral; however for the purpose of this study only group of piles subjected a lateral loading and bending moment will be taken in account. Once again the approach to analysis of the behaviour of the piles, here a group of them, has been extensively discussed and several methods of analysis had been addressed until today.

Reese and Van Impe (2001) had mention that the behaviour of a group of piles may be influenced by two forms of interaction:

1. Interaction between piles in close proximity where efficiency is involved:
   - Here the relevant forces are transmitted through the soil.

2. Interaction by distribution of loading to individual piles from the pile cap:
   - In this case the forces are transmitted by the superstructure.

Liu (2004) describe those varieties of methods to analyze the pile group system, and presented them to split into five categories:
1. Simple static methods that ignore the presence of the soil and consider the pile group as a purely structural system.

2. Methods that reduce the pile group to a structural system but take some account to the effect of the soil by determining equivalent freestanding lengths of the piles.

3. Methods in which the soil is assumed to be an elastic continuum and interaction between piles can be fully considered.

4. Group Reduction Method and Group Amplification Method, which is based on single pile analysis with modified modulus.

5. Methods in which the soil is modeled by p-y curve modified according to the interaction of the piles in the group.

The first two methods consider interaction between the piles through the pile cap and not interaction between the soil and piles. However they assumed that once the loads on any pile are known, the deflections of that pile may be calculated from these loads alone.

According to Poulos and Davis (1980) the third method removes the limitation described above and allows consideration of pile interaction with the soil and the deflections of the piles are calculated together in a group. The fourth method considers the piles work jointly through the cap and the interaction between the pile and soil is modeled by modified p-y curves.

2.2.2 Simple static analysis method

The most common way to assume a group of pile foundation behave under lateral loading is to assume that both the structure and the piles are rigid and that only axial resistance of the piles is considered. The pile group is considered as a simple statically determinate system, ignoring the effect of the soil. Based on these assumptions, (Terzaghi 1956) Van Impe 2001, presented a graphical solution of Culmann, but this method can be employed either in an analytical or a graphical way.

A force polygon was used to analyze the equilibrium state of the resultant external load and the axial reaction of each pile in the group. The application of the Culmann’s method
is limited to the case of a foundation with a group made of three similar piles. According to Reese and Van Impe (2001) a supplemental method to this graphical solution was proposed in 1930 by Brennecke and Lohmeyer as referred by Terzaghi 1956, with emphasis that method is restricted to the case where all of the piles tops are on the same level. The vertical component of the resultant load is distributed in a trapezoidal shape in such way that the total area is equal the magnitude of the vertical component, and its center of gravity lies on the line of the vertical component of the resultant load.

The authors also highlighted that the vertical load is distributed to each pile assuming in this way that the trapezoidal load is separated into independent blocks at the top of the piles, except at the end piles.

As mentioned in Liu (2004) this method cannot take into account different conditions of fixity at the pile head, and always assumes zero moment at the head of each pile.

2.2.3 Equivalent-bent method

Among numerous ways to perform analysis of group of piles subjected a lateral, eccentric or inclined loading, this method of analysis is based on a transformation of the actual system in an equivalent one, as shown in the figure below.

![Figure 2.2 Principle of equivalent-bent approach (Liu (2004))](image-url)
The basic system is presented in the Figure 2.2 part a, and the transformed one, considered as equivalent appear in Figure 2.2.part b. The equivalent lengths and areas must be determined, and the equivalent bent may be analyzed by standard structural analysis techniques, and then the deflections, rotations, and pile stress in the system the piles are assumed are determined. In order to simplify the structural analysis, Poulos and Davis (1970) suggested that the pile cap is frequently assumed to be rigid and the piles assumed to behave elastically.

As mentioned in Liu (2004) the authors Saul (1968) and Reese et al. (1970) presented matrix analysis in which the above assumptions are executed in numerical investigations; however Nair et al. (1969) presented a more convenient way to compute this system by hand. In Saul’s paper according to Reese and Van Impe (2001) the torsional loading and dynamic forces are also incorporated.

2.2.4 Elastic continuum analysis of pile behaviour

According to Liu (2004) the major approach of elastic continuum analysis method is to consider a pile, pile cap and elastic soil material to be a system totally determined by the theory of elasticity.

The elastic displacement of pile tops was first considered in 1917 by Westergaard (Karol 1960). As mentioned in Reese and Van Impe (2001), Westergaard assumed linearly elastic displacement of pile heads under a compressive load and developed a method to find center of rotation of a pile cap. With the rotation known, the displacements and stress in each pile could be analyzed as a result. In the early 1920s Nokkentved presented a method similar to that one developed by Westergaard, and the difference was that he defines a point that is dependent only on the geometry of the pile group, so that forces which pass through this point will produce only unit vertical and horizontal translations of the pile cap.

After Nokkentved (as mentioned in Reese and Van Impe (2001) others introduce modifications as Vetter, and Vandepitte ) who applied the concept of the elastic center in
developing of the limit-state-design method which was later formulated by Hansen in 1959, however the last author extended the method to the 3D case.

According to Reese and Van Impe (2001) and Liu (2004) was Hrennikoff (1950) who presented a comprehensive structural treatment for the 2-dimentional case. He considered the axial, transverse, and rotational resistance of piles on the cap. The load-displacement relationship of the pile head was assumed to be linearly elastic. The assumption of this theory was that all piles must have the same load-displacement relationship. The characteristic of the model is that it considered the laterally loaded pile as an elastic beam on an elastic foundation with uniform stiffness. The influence factor of the pile group in calculating displacement was defined as the summation of all the contributions of single piles. The significance of this method is that it resents the potential for the analytical treatment of the soil-pile interaction systems.

Radosavljevic (1957) made a reference in which he considered a laterally loaded pile as an elastic beam in an elastic medium with a uniform medium, and after him Reese and Van Impe (2001) noted that Turzynski (1960) presented a formulation based on the matrix method for cases with 2D approach.

The model presented in the 1960s by Turzynski used models from various researchers and among them was Asplund (1956) model. Asplund (1956) formulated the matrix method for both two-dimensional and three-dimensional cases. Here the author calculated the stiffness matrix of the pile group and employed an elastic center method to treat laterally loaded piles. He also considered the pile arrangement for economical reasons. In this method, laterally loaded piles are merely regarded as elastic beams on an elastic bed with a uniform spring constant.

Also as referred to Reese and Van Impe (2001) there are more papers presented, the most expressive ones are by Francis (1964), Aschenbrenner (1967) and Saul (1968) among several others. Francis, computed the 2D case using the influence coefficient method, and for the other two they had presented the same topic with a variation in using 2D and 3D as a frame for it.
2.2.5 Group reduction factor method

Liu (2004) noted that the group reduction factor method considered the lateral load resistance of the pile group which is determined based on the single pile analysis and modified according to the pile spacing. Prakash (1962) worked with the model using sands, and after Davisson (1970) suggested that the piles would work independently if the pile spacing is more than eight diameters of piles based on their tests of piles embedded in sand.

As mentioned in Priyanto (2002), Davisson (1970) also proposed that the pile resistance in a group would equal 75% of a single pile if the distance between piles were 3 pile diameters. In a case of piles spaced with distance between 3-8 diameters, the outcomes of the soil resistance can be obtained by interpolation.

Prakash and Prakash (1989) showed that Davisson recommendations appear to be somewhat conservative. Arsoy and Prakash (2001) performed 14-full scale tests on piles in sand and analyzed them to re-evaluate the group reduction factor $G_c$. They showed that the group reduction factor, $G_c$, is a function of pile spacing, displacement and relative density. In addition, they concluded that the group action disappears at 6-diameter pile spacing for $2 \times 2$ groups and 7-diameter pile spacing for groups having six piles or less in the direction of loading.

2.2.6 Group amplification method

Ooi and Duncan in 1994 had proposed a different approach to analyze the laterally loaded pile group; it is called the group amplification method. This method applies an amplify coefficient to the lateral deflection bending moment of piles in the group.

According to Priyanto (2002) the deflections and bending moments of pile groups will be greater than those of single piles, so that this procedure tried to determine amplification factors for deflection $C_y$ and an amplification factor for moments $C_m$, that formulated as:
(2.5) \[ y_g = C_y y_s \]

(2.6) \[ M_g = C_m M_s \]

where

- \( C_y \) = deflection amplification factor (dimensionless)
- \( C_m \) = moment amplification factor (dimensionless)
- \( y_g \) = group deflection (L)
- \( y_s \) = single pile deflection under the same load (L)
- \( M_g \) = maximum moment in a pile in the group (F.L);
- \( M_s \) = maximum moment in a single pile under the same load (F.L)

Some considerations about the amplifications factors were made and they are:

- The value of \( C_y \) and \( C_m \) is greater than or equal to 1.0.
- The values of amplification factors (\( C_y \) and \( C_m \)) depend on the soil type, diameter of single pile, spacing of piles, passive earth pressure coefficient, angle of internal friction for sand and undrained shear strength for clay.

The limitations of the method accounted are:

- Can be used for rectangular (non circular) group with uniform or non uniform spacing.
- Can be applied for vertical pile, not the batter piles.
- Cannot determine the distribution of load.
- The arrangement of piles in a group is not taken into account.
- It is applied for long piles and embedded in a uniform, homogenous soil.
As emphasized by the authors, Ooi and Duncan (1994), it has been found that there is a
good agreement between the results of this method and the field load test results

With respect to the two methods described above (2.2.5 and 2.2.6), they both are highly
empirical because they depend on limited test results. But they had provided an effective
way to calculate the pile group behaviours through the analysis of single piles.

2.2.7 The p-multipliers method

The combination of computer technology improvement combined with full-scale field
lateral load test results have made it possible to analyze the laterally loaded piles using
either the finite difference or finite element method. As for single piles the \( p-y \) curve
method is the most commonly used approach for analyzing the lateral response of
laterally loaded pile groups using finite difference or finite element techniques to solve
the beam bending equation. The pile is represented as a beam supported laterally by the
soil, which is modeled using nonlinear load versus deflection curves. The advancement in
the equipment and methodology for measurement of the displacements and stresses in the
full scale tests open the possibility for understanding the behaviour of piles inside of the
group and how much lateral load each pile can carry. Generally speaking, the tests
indicate an unequal distribution of the lateral load among the piles in the group, and this
discrepancy is caused by "shadowing" effect, which is a term used to describe the overlap
of zones of resistance, and the consequential reduction of lateral soil resistance. The main
idea behind the p-multiplier method is that the behaviour of a pile in a group is similar to
the methodology used for analysis of a single pile, except that the soil resistance should
be reduced, which means the p-values are reduced using a p-multiplier.

According to Mokwa and Duncan (2001) the concept of the p-multipliers was described
by Brown et al. (1988) as a way of accounting for pile group effects by adjusting the \( p-y \)
curves. The \( p-y \) curves of single piles are modified to account for the influence of the
interaction between the different piles in the group. As shown in Figure 2.3, the p-
multiplier \( (f_m) \) is the reduction factor of soil resistance \( p \) for the same deflection of \( y \). The
$p$-$y$ curve is compressed in the direction of $p$, so that the soil resistance, $p$, of the piles in the group will be smaller than the soil resistance of single piles.

![Diagram showing p-y curves for single pile and pile in group](image)

**Figure 2.3 The concept of p-multiplier ($f_m$) (Brown et al. 1988)**

According to Liu (2004) the values of p-multiplier proposed by Brown et al. (1988) are the results of an isolated pile embedded in dense sand subjected to cyclic loading and full-scale tests for pile groups. Brown and Shie (1991) also presented the p-multipliers from the results of 3-D finite element analysis.

From 11 experimental studies Mokwa and Duncan (2001a) were able to recommend p-multipliers where the outcomes of the single pile load tests were used. As noted and reported by the authors the piles installed in groups at close spacing deflected more than a single pile subjected to the same lateral load per pile because of group effects. In fact most researchers share the same opinion that the group effect are small when center-to-center pile spacing parallel to the load ($P$) exceed 6 pile diameters, and exceed 3 pile diameters when the spacing is perpendicular to the load ($P$).

Different full-scale tests were performed (centrifuge and others) in distinct types of soils, and from them authors like Cox et al. (1984), Brown and Reese (1985), Morrison and

Mokwa and Duncan (2001b) collected and reviewed over 350 journal articles and other publications pertaining to lateral resistance, testing, and analysis of pile caps, piles, and pile groups, and through combining the research work that has been done before they proposed a way to construct the value of p-multipliers, f_m, for all kinds of soil. The results from their tests were assimilated into tables and charts, from which the trends and similarities can be observed.

The centrifuge model study of laterally loaded pile groups in clay was proposed by Ilyyas et al. (2004), using a series of centrifuge model tests that has been conducted in their research to examine the behaviour of laterally loaded pile groups in normally consolidated and over consolidated kaolin clay. As referred to Liu (2004), the model proposed by Ilyyas at al. (2004) considered that the pile groups have a symmetrical plan layout consisting of 2, 2×2, 2×3, 3×3 and 4×4 piles with a center-to-center spacing of three or five times the pile width. The piles are connected by a solid aluminium pile cap placed just above the ground level. It is established that the pile group efficiency reduces significantly with increasing number of piles in a group. The tests also reveal the shadowing effect phenomenon in which the front piles experience larger load and bending moment than that of the trailing piles. The shadowing effect is most significant for the lead row piles and considerably less significant for subsequent rows of trailing piles. They also pointed out that the approach adopted by many researchers of taking the average performance of piles in the same row is found to be inappropriate for the middle rows, of piles for large pile groups as the outer piles in the row carry significantly more load and experience considerably higher bending moment than those of the inner piles. They also compared their p-multiplier results with those of other researchers as stated in Table 2.1.
<table>
<thead>
<tr>
<th>Author/soil type and shear strength</th>
<th>Size of pile group</th>
<th>Average p-multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Leading row</td>
<td>2\textsuperscript{nd} tracks</td>
</tr>
<tr>
<td>CLAY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ilyas et al. (2004) / normally consolidated clay:</td>
<td>2x1</td>
<td>0.8</td>
</tr>
<tr>
<td>undrained shear strength = 0 - 20kPa</td>
<td>2x2</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>3x3</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>4x4</td>
<td>0.65</td>
</tr>
<tr>
<td>Brown et al. (1987) / over consolidated clay:</td>
<td>3x3</td>
<td>0.7</td>
</tr>
<tr>
<td>strength = 70-180kPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meimom et al. (1986) / silty clay:</td>
<td>2x2</td>
<td>0.9</td>
</tr>
<tr>
<td>strength = 25kPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rollins et al. (1998) / clayed silt :</td>
<td>3x3</td>
<td>0.6</td>
</tr>
<tr>
<td>strength = 50-75kPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAND</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brown et al. (1988) / clean medium sand:</td>
<td>3x3</td>
<td>0.8</td>
</tr>
<tr>
<td>friction angle $\Phi = 38^\circ$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>McVay et al. (1995) / medium dense sand</td>
<td>3x3</td>
<td>0.8</td>
</tr>
<tr>
<td>McVay et al. (1998) / medium dense sand</td>
<td>4x4</td>
<td>0.8</td>
</tr>
<tr>
<td>Ruesta and Townsend (1997) / loose fine sand:</td>
<td>4x4</td>
<td>0.8</td>
</tr>
<tr>
<td>friction angle $\Phi = 32^\circ$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 Comparison of p-multiplier values from various experimental tests and field tests for pile groups with pile spacing equal 3 piles width, from Ilyas et al 2004, in Liu (2004)

Using the same p-multipliers method but with a slight difference in the boundary conditions Mostafa and El Naggar (2002) described an analysis of dynamic lateral response of pile groups. The authors considered the concept of the p-multiplier for the dynamic loading case, and the analysis proposed incorporates the static $p-\gamma$ curve.
approach and the plane strain assumptions to represent the soil reactions within the framework of the Winkler model. The model accounts for the nonlinear behaviour of the soil, the energy dissipation through the soil, and the pile group effect. The model was validated by analyzing the response of pile groups subjected to lateral static/dynamic loading and comparing the results with field measured values. An intensive parametric study was performed employing the proposed analysis, and the results were used to establish dynamic soil reactions for single piles and pile groups for different types of sand and clay under harmonic loading with varying frequencies applied at the pile head. “Dynamic” $p$-multipliers were established to relate the dynamic load transfer curves of a pile in a group to the dynamic load transfer curves for a single pile.

The dynamic $p$-multipliers were found to vary with the spacing between piles, soil type, peak amplitude of loading, and the angle between the line connecting any two piles and the direction of loading. The study conducted by the authors also indicated the effect of pile material and geometry, pile installation method, and pile head conditions on the $p$-multipliers. The results obtained for the calculated $p$-multipliers compared well with $p$-multipliers back-calculated from full scale field tests.

### 2.2.8 Field testing performed over group of piles under lateral loading in soft clay

Gandhi and Selvam (1997) studied the behavior of a pile group under lateral load through laboratory experiments on aluminum pipe piles with outer diameters of 18.2 mm. The piles were driven in medium to fine sand with 60% relative density in different configurations and were subjected to lateral load under fixed head conditions. Some of the piles were instrumented with strain gauges to measure bending moments at varying depths. To quantify the effect of pile driving, the authors considered the behavior of a single driven pile and compared it with that of a bored pile. Test data are analyzed to arrive at the group effect for various spacings, and the results are presented in a nondimensional form and a method for the prediction of field group behavior is illustrated. The predictions are compared with the field results from the literature and are found to be in favorable agreement.

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Rollins et al. (1998) performed a static lateral load test on a full-scale pile group to determine the resulting pile-soil-pile interaction effects. The 3 × 3 pile group at three-diameter spacing was driven into a profile consisting of soft to medium-stiff clays and silts underlain by sand. The piles were instrumented with inclinometers and strain gages. The load carried by each pile was measured. A single pile test was conducted for comparison. The pile group deflected over two times more than the single pile under the same average load, and the group effects significantly reduced load capacity for all rows relative to single pile behaviour. As the authors observed that the trailing rows carried less than the leading row, and middle row piles carried the lowest loads. Maximum moments in the group piles were 50-100% higher than in the single pile. P multipliers were 0.6, 0.38 and 0.43 for the front, middle and back row piles, respectively. The authors also emphasized a good agreement between the measured and computed pile group responses that was obtained using the p-multiplier approach.

Patra and Pise (2001) performed experimental investigations on model pile groups of configuration 1×1, 2×1, 3×1, 2×2, 3×2 for embedment length to diameter ratios L/d =12 and 38, spacing from 3 to 6 pile diameter, and pile friction angles δ = 20° and 31°, subjected to lateral loads were conducted in dry Ennore sand obtained from Chennai, India. The load-displacement response, ultimate resistance, and group efficiency with spacing and number of piles in a group have been quantitatively and qualitatively investigated in the paper. Analytical methods have been proposed by the authors to predict the ultimate lateral capacity of single pile and pile groups, and they also proposed methods to account for pile friction angle, embedment length-to-diameter ratio, the spacing of piles in a group, pile group configuration, and soil properties. These methods used are capable of predicting the lateral capacity of groups reasonably well as noted and substantiated by comparison with the experimental results of the writers and other researchers.

Ng et al. (2001) presented results of full-scale lateral load tests of one single pile and pile groups in Hong Kong. The test piles, which are embedded in superficial deposits and decomposed rocks, are 1.5 m in diameter and approximately 30 m long. The large-diameter bored pile groups consist of one two-pile group at 6D (D = pile diameter)
spacing and one two-pile and one three-pile group at 3D spacing. Their paper aims to investigate the nonlinear response of laterally loaded large-diameter bored pile groups and to study design parameters for large-diameter bored piles associated with $p-y$ method using a 3D finite element program, FLPIER. Predictions using soil parameters based on published correlations and back analysis of the single-pile load test are compared. It is found by the authors that a simple hyperbolic representation of load-deflection curves provides an objective means to determine ultimate lateral load capacity, which is comparable with the calculated values based on Brom’s theory. Lateral deflections of bored pile groups predicted using the values of the constant of horizontal subgrade reaction, suggested by Elson and obtained from back analysis of the single pile load test, are generally in good agreement with the measurements, especially at low loads.

Haung et al. (2001) studied the effect of construction on laterally loaded pile groups. The performed full-scale lateral load tests on groups of bored and a group of driven precast piles. These tests were part of a research project for the proposed high-speed rail system in Taiwan. The effect of construction was obtained by the performing Standard penetration tests, cone penetration tests and Marchetti Dilatometer tests (DMT) before and after pile installation. The authors emphasize the needs of the numerical analyses of the laterally loaded piles and these were conducted using $p-y$ curves derived from preconstruction and post construction DMT and by applying the concept of p-multipliers. They showed also, through comparison of preconstruction and post construction, that the installation of bored piles softened the surrounding soil, while the driven piles caused a densifying effect.

O’Neill and Haung (2003) compared the behavior of bored and driven piles in cohesionless soil. The comparison was performed on 2 piles groups, and showed that the effect of installation was found to reduce the soil stiffness within the bored pile group, making the soil less efficient in resisting lateral pile movements than in the driven pile group. However, structurally, the bored piles were more resistant to flexural loading. The net effect was that the system of bored piles was stiffer than the system of driven displacement piles.
2.3 Literature Review on the sensitivity analysis of pile foundation

According to Saltelli et al (2000) the common definition for sensitivity analysis is the study of the relationships between information flowing in and out of the models. Models, from engineering point of view, are formed to best represent the behaviour of structures, components, systems or the process of them. Numerous models can't be developed using just a simple mathematical approach once the complexity of the structure or process requires something with more variables, inputs, and numerous parameters. In this case a model might be generate aimed to describe the process, as more realistic as possible, and for accomplish this task many mathematical tools, and full scale tests are necessary to provide a large numbers of observations, measurements and also the behaviour of the structures or components during the test.

Researchers and engineers with the outcomes of the tests associated with mathematical tools are able to build a model where the equations, inputs, parameters, variables and outcomes, among others, are accurate as possible to represent the real problem. However the model have to be under evaluation with certain frequency which will provide the adjustments necessary to deal with uncertainties associated with the model and with the process.

Liu (2004) stated that sensitivity analysis was developed to deal simply with uncertainties in the input variables and with model parameters, which is a common sense with Priyanto (2002), Rahman (2004), Abedin (2004) and others.

According to Priyanto (2002) Kleiber et al. (1997) stated that there are two major reasons of doing sensitivity analysis:

➢ The overall computational cost required by such algorithms depends strongly on the efficiency of gradient evaluation, because the gradients of functions describing system behaviour with respect to parameters are essential for system optimization and reliability assessment.

➢ It is now broadly accepted that any realistic large-scale engineering simulation has to be completed by an extensive study on response sensitivity to system parameters just to broaden our understanding of the system behaviour.
The understanding and applicability of sensitivity analysis have been used by different technological segments, and for every single one the approach and outcomes are different. Speaking from the civil engineering point of view, more specific from the structural perspective, the sensitivity analysis is used in terms of the structural optimization. This means that the binomial technical and economical approach will work toward the optimal design. The structure has to be technically and economically effective.

Haftka et al. 1990 suggested that the minimum cost may be the primary consideration in doing structural optimization in civil engineering.

A mathematical model is presented by a series of equations, input factors, parameters, and variables in such manner to characterize the structure investigated. The utilization of sensitivity analysis approach intended to increase the confidence in the model applied and its estimation, by providing a better understanding of how the model response variables will respond to the changes in the inputs, models and so on.

Even among simple models the measurement of certain parameters and how accurate they are it is a hard task to accomplish, and for some parameters the change of values is time related. Among so many changes and uncertainties the utilization of sensitivity analysis allows the designer to predict some changes and determine the level of accuracy will be necessary for analyze each one of the variables of the system.

The sensitivity analysis might be more applicable to provide outcomes about:

- The model accuracy with the structure under studies.
- The optimal boundary conditions.
- The factors that most contribute to the results.
- The quality of the model.
- The interactions between variables.
- The accuracy of the parameters.

According to Liu (2004) there are many ways to perform a sensitivity analysis, and among these the most common one is the sampling–based. This is a specific way to perform an analysis which takes in account the repeatedly number of combinations that it’s possible to perform with the achievable values.
To promote a better understanding of sensitivity analysis some definitions were stated by Kohn (2002), and have been used in recent papers and thesis.

- **Boundary conditions**: constraints that specify the particular solution of the state equations, as an example can be the initial values of the state variables.
- **Independent variable**: the dimension over which the state of the system changes.
- **Parameters**: constants (the opposite of variables) in the state equations.
- **Sensitivity coefficient**: a partial derivative of a state variable with respect to variations in a parameter value, and these quantities might vary with respect to time.
- **State equations**: equations that specify the state variables as function of the independent variable (typically differential equations)
- **State variables**: the quantities that specify the instantaneous state of the system.
- **Steady state**: a solution of the state equations when the time derivates of the state variables are all set to zero. Can be also called stationary state.
- **Transient**: the temporal profile of the state variables after a perturbation in the boundary conditions.
- **Design variables**: a group of parameters that have potential for change in a permissible range in order to improve or optimize a structure (Haftka et al. (1990)).

The examples of the design variables can be physical parameters of the structure (stiffness), strength parameters of the soil model, or geometric parameters (cross-section dimensions, length).

### 2.3.1 Classification of Sensitivity Methods

Among the sensitivity analysis methods most applied, two of them are the most common and have been reported by different authors, as Haftka et al. (1990), Choi and Chang (1992), Mota Soares and Leal (1992). They are:

- **Discrete method**: generally adopted for finite element analysis. According to Haftka et al. 1990 the discrete methods are only applicable to non-structural sensitivity analysis involving systems of linear equations, eigen value problems, and others.
Finite differences: the derivatives of the physical responses with respect to the design variables are computed after has been performed an analysis using the original design variables and their values.

Semi analytical: the derivatives of the physical responses are obtained directly from the modification of the finite element formulations in which the finite difference equation is applied to calculate the derivatives of the stiffness matrix.

Analytical: same approach as semi analytical method except that the first derivative of the stiffness matrix regarding to the design variables is calculated analytically.

Also pointed by the same authors when dealing with structural applications, the discrete method has two disadvantages, which are:

(1) Not all structural analysis solutions methods lead to the type of discretized equation;

(2) Operating on the discretized equations often required access to the source code of the structural analysis that are usually not provided in most structural analysis programs

Variational method: the equations that govern the structure are differentiated (before they are discretized) to carry out the design sensitivity analysis, and the gradients of objective and constraint functions regarding to the design variables are expressed analytically.

In the variational approach two methods are presented.

Direct differentiation method: this method differentiates the system equations with respect to the design variables to obtain the first order equations for displacement sensitivities.

Adjoint method: in this method, a system subjected to initial action, usually deformation, is evaluated through the sensitivity response of the system. According to Priyanto (2002) the Lagrange multipliers allow the evaluation the sensitivity response without explicit calculation of the displacement sensitivity.
2.3.2 Sensitivity Analysis Applied Previously

Sensitivity analysis approach has been used in many scientific areas as was said before, and each one has a particular way to employ this important and useful tool. Numerous papers and extensive work have been done employing sensitivity analysis on structures, rafts foundations, pile foundations and other structural elements.

With respect to piles foundations that have been analysed using sensitivity analysis they can be classified by different aspects, among them might be load, boundary conditions and so on. The following sections will present the sensitivity analysis applied basically to foundations structures.

Raft foundation

A raft foundation on a soil has been analyzed by Valliappan et al. (1997, 1999) through sensitivity analysis. The author and his colleagues proposed an algorithm for the semi-analytical sensitivity method applied to non-linear analysis and a modification of the two-point constraint approximation, named bi-point constraint approximation. Using the structural optimization together with the finite element method Valliappan et al. (1999) applied the two methods on a raft-pile foundation system, looking for the optimal design. For the optimization process, Valliappan et al. (1999) carried out the sensitivity analysis using the approximate semi analytical method while the constraint approximation was obtained from the combination of extended Bi-point and Lagrangian polynomial approximation methods. The design variables defined by the authors were the raft thickness, cross-section, length and number of piles. The constraints selected were the maximum and differential displacement, and the target of the problem was the cost of the foundation.

Pile foundation under axial loading

Many studies have been developed employing piles, however very few has considered the sensitivity analysis advance. Among these Budkowska and Szymczak (1993a, 1994b) presented a simple one-dimensional idealization of the pile in conjunction with the soil model consisting of a continuously distributed system of springs and a spring located on
the pile toe. The Winkler-type model of the soil behaviour was assumed. The considerations based on the virtual work theorems were valid for both linear and nonlinear in the pile material and the soil behaviour.

The authors derived first variations of a vertical displacement and axial force of axially loaded piles due to changes of the design variables. The design parameters in this sensitivity analysis are the pile material and the soil behaviour. The first variations of the displacements and the internal forces enables any researcher to assess the quantities under consideration due to the design variables increment without running full reanalysis of the pile. The authors presented a numerical example that shows the good performance of the accuracy of the approximation of the change of the design variables.

The same authors, Budkowska and Szymczak (1995b), used the same models for the pile and for the soil, but this time they decided to include different design variables. Again they presented numerical examples as a part of their study and the results shows that the accuracy of the approximation of the changes is also good, even for 30% changes of the pile length.

**Piles undergoing torsion**

Other two investigations were done by Budkowska and Szymczak, both dealt with consideration of pile under torsion loadings. On the first one, Budkowska and Szymczak (1993b) derived first order variations of an angle of the pile twist and a torque at a specified cross-section of the pile due to design variable variations. As pointed by Priyanto (2002) some considerations based on some variational theorems of mechanics were restricted to the linear range of the structure behaviour.

In the second paper, Budkowska and Szymczak (1994b) presented the sensitivity analysis for piles under the same consideration, but at this time different design variables were employed, like an increment of the pile length. The authors considered a circular cross-section pile, made of a linear elastic material subjected to torsional loadings. The considerations were based on calculus of variations with moving boundaries. The sensitivity analyses facilitate the calculation of changes in the quantities under
consideration due to the pile end shifts without a full re-analysis of the pile. The authors presented a detailed discussion of a pile embedded in multilayered soil is also presented, and the accuracy of the approximation of the change of the angle of the pile top twist due to the pile length variation was investigated and showed a good accuracy.

**Piles under bending load condition**

Budkowska (1998a) presented the sensitivity analysis of piles subject to bending due to variable length. The pile structure is modeled as a beam element and the soil is simulated by the elastic foundation of Winkler type. The functional of bending and shear energy are defined in the scope of variational calculus based on the principle of virtual energy. Budkowska (1998a) employ the concept of the adjoint structure subject to unit dummy load. The first variations of deformation and static field components are formulated with moving 1 ends. The evaluation of changes of kinematic and static fields utilizes Taylor's mean value theorem, as well as approximate values of variations of deformation field component not defined within the intervals of shift of soil and support conditions. The final forms of sensitivity equations are accompanied with set of equations defining the behaviour of natural boundary conditions for primary and adjoint structure. They form the basis of the numerical investigations of the piles penetrating homogeneous and non-homogeneous soil as well. The obtained sensitivity results are compared with exact solutions and accuracy of the results is examined from the error analysis standpoint.

Employed the same methods but different design variables, Budkowska (1998b) performed the sensitivity analysis of the long piles embedded in homogeneous elastic soils under bending moments. The derived sensitivity equations are valid for arbitrary distribution of the design variable vector. The obtained sensitivity equations resulted in formulation of the under integral sensitivity operators associated with each of the design variables. The determined under integral sensitivity operators enable one to localize the most effective domains where the variations of the design variables affect mostly the changes of the quantity under consideration.
Buckling of piles partially embedded in soil

Budkowska and Szymczak (1996, 1997) presented the first variation of the critical buckling load of a pile partially embedded in a soil. Pile is idealized as one-dimensional column and the soil is represented by Winkler-type elastic foundation. The effect of negative skin friction is neglected. The design variables are the pile material, the soil properties, and the pile ends locations. The accuracy of the approximation of the change of the critical buckling load due to some design variable variations is also investigated and shown that the first-order sensitivity analysis leads to a good approximation of the change of the critical load, due to the design variable variations within broad limits of its changes.

Piles under lateral loading

An approximate procedure for calculation of changes of maximum values of an arbitrary displacement and an internal force of laterally loaded piles due to some increments of the pile cross-section dimensions, the pile material constants and the soil parameters was proposed by Budkowska and Szymczak (1992a, 1992b; 1995). The pile was simulated as one dimensional beam element, and the response of soil was modeled as the Winkler type foundation. The method presented by the authors can be actually applied to both linear and nonlinear behaviour of the pile material and the soil.

The first order variation of the maximum value of quantity under consideration was evaluated with the aid of the adjoint structure concept, and the accuracy of the calculation of the changes of the maximum value of the flexural moment of the pile by means of its first variation was also taken in account. Results of the numerical examples were given, and they dealt with the linear structures to allow the conclusion to be drawn that the approximation of the exact results by means of the first variations are reasonably good (Budkowska and Szymczak, 1992a).

Budkowska and Cean (1995) presented the sensitivity analysis of short piles subjected to lateral load, when the piles were embedded in a homogeneous sandy soil. They employed the same models and methods. The design variables were taken as the stiffness of the pile.
EI, and the modulus of the subgrade reaction of the soil $k$. The authors considered the comparative analysis of the pile loaded by the horizontal force and the bending moment. The effects of changes of the design variables on the components of kinematic field of pile due to both types of loadings case presented. The quantitative results show the same value. The distributions of the sensitivity operators that could be used for engineering practice in design, and rehabilitation here presented (sensitivity due to EI) affects the shape of the pile structure. Therefore the information about the most effective and rational improvement of the pile structure can be obtained from sensitivity analysis. Moreover, the sensitivity of soil due to change of $k$ is important for improvements or modification of soil adjacent to the pile.

The authors adopted the same models of piles and soil, and the result of the sensitivity analysis facilitates one to analyze the effect of changes of the internal forces and generalized displacements of laterally loaded piles for various scenarios of changes of boundaries of the arbitrary soil layer resulting in expansion or shrinkage, as well as translation upwards or downwards.

The numerical investigations presented by Budkowska, Sekulovic, and Saha (1999b) were compared with the exact solutions obtained by means of reanalysis of the problem. Through the paper the accuracy of the sensitivity outcomes has been examined in the framework of the error analysis, and it permits one to determine the acceptable range of variability of the depth of soil layers assuring acceptable error of approximation.

The general formulation of the sensitivity analysis of laterally loaded pile embedded in a homogeneous soil medium was presented by Budkowska (1997a). The author approached the problem taken in consideration by means of the principles of variational calculus. The main part of the analysis was connected with the concept of functional with constraints, which was then transformed into augmented functional without constraints. The second part of the paper, Budkowska (1997b) investigated the short steel piles subjected to a bending type of load. The pile structures were embedded in homogeneous sand, and clayey soil modeled as a one-dimensional structural element supported by Winkler type foundation.

The modulus of subgrade reaction for clayey was considered constant, while that for sandy soil varies linearly. The design variables were taken as the bending stiffness of the
pile structure and modulus of subgrade reaction. Some conclusions of considerable importance for engineering practice were presented based on the distributions of under integral sensitivity operators for short piles embedded in clayey and sandy soils.

Also considering laterally loaded piles Barakat et al. (1999) presented a general approach to the reliability-based analyses and the optimum designs. Behaviour and side constraints specified by standard specifications for piles were taken into account by this approach, and the typical effect of corrosion of piles with time was considered in formulation with limiting state functions. The laterally loaded piles computer program called RELLOP was used to work on the solution of reliability-based. A general reliability based methodology was developed and implemented in the developed computer program for both element and system limit states. Some numerical examples demonstrating the feasibility of considering multiple limit states and system reliability requirements in the design of laterally loaded piles were also presented.

A sensitivity analysis of lateral displacements of long single pile subjected to static horizontal forces applied at the soil surface was presented by Budkowska and Suwarno (2002a). The soil considered by the authors was stiff clay below the water table, and was modeled by means of a \( p-y \) model. The material characteristics of the pile-soil system were taken as the design variables, and the sensitivity functional of a nonlinear pile-soil system is formed with the aid of the adjoint system that demonstrated the nonlinear features.

According to Priyanto (2002) the determination of the first variation of lateral displacement functional due to the changes of the design variables resulted in formulation of the sensitivity integrands associated with each design variables.

Then, taken into account a group of piles, Budkowska and Suwarno (2002b) performed a sensitivity analysis on a pile group subjected to horizontal loading embedded in stiff clay located below the water table. The \( p-y \) relationship used by the author can be viewed in Figure 2.4. They used the same methods for single piles, and take into account the group effect by introducing the \( p \)-multiplier (\( f_m \)) factor. The quantitative assessment of the
locations of the soil phases, integrations of the sensitivity operators, and the relative sensitivity factors are also presented and come to the conclusions of the comparisons of the design variables' importance for each soil phase.

The design variables chosen for investigation were those connected with pile-soil strength parameters, and the pile-soil system was considered as a five parametric sensitivity system. It was analyzed based on the adjoint pile method that has nonlinear features, and the equation of first variation of deformation determined has dependent on the sensitivity operators connected with each of the design variables. Among the conclusions presented

**Figure 2.4** The p-y used by Budkowska and Suwarno (2002).

Working with single piles and group of piles Budkowska and Priyanto (2002a) compared the sensitivity integrands for short piles and long piles embedded in soft clay located below the water table. The type of load considered was cyclic, and the p-y model was employed to simulate the soil effect on the behaviour of the pile-soil system. The model used is presented in Figure 2.5.
by the authors, the sensitivity operators presented were strongly dependent on the magnitude of the applied load.

In their second paper, connected with pile groups, Budkowska and Priyanto (2002b) presented the behaviour of a pile-soil system embedded in the soft clay below the water table under cyclic lateral loading. The p-y relationship considered by the authors is presented in Figure 2.6. Once again the soil was simulated by means of a p-y relationship modified by the p-multipliers ($f_m$), factor that is dependent on spacing and a pile location within a pile group. The design variables were taken as the bending stiffness of the pile and soil strength parameters that appear in the p-y constitutive relationship.

The adjoint structure method was adopted for sensitivity investigations, and special approaches were developed for the purpose of sensitivity analysis to assure that the pile investigated was the nonlinear member of pile group. It assures that the kinematic and
static fields produced by unit-generalized load depended on the magnitude of the load applied to the primary pile group system. The first variation of kinematic functional due to variations of the design variables vector was formed with the aid of variational calculus. The authors concluded that the determination of nonlinear sensitivity integrands that were strongly dependent on the magnitude of the applied load.

![Diagram](image)

**Figure 2.6** The p-y curves for soft clay below the ground water table (Matlock, 1970 model) in laterally loaded pile group subjected to a cyclic load used by Priyanto (2002)

Suwarno (2003) presented his study using sensitivity analysis on laterally loaded single piles and group of piles embedded in stiff clay below the ground water table. The author conducted his research on a soil model shown in the Figure 2.7. The sensitivity analysis performed by Suwarno was focused on the changes of the design variables which were the stiffness of the pile, the cohesion of the soil, the modulus of the subgrade reaction, the
effective unit weight, the strain at which 50% soil strength is mobilized, and the width of
the pile where the soil reaction was developed. The author presented the theoretical
formulation and numerical investigation performed on single piles and pile groups.

\[ p = A_c p_c \left( 1 - \frac{y - 0.45y_p}{0.45y_p} \right)^{2.5} \]

\[ E_{sc} = -\frac{0.085 p_c}{y_{50}} \]

\[ y_p = 4.1A_c y_{50} \]

\[ y_{50} = e_{50} b \]

![Diagram](image)

**Figure 2.7** The p-y curves for stiff clay below the ground water table (Reese et al.
(1975) model) employed in Suwarno (2003) studies

The performance of the pile-soil system embedded in stiff clay above water table was
analyzed by Budkowska and Liu (2004). The authors used adjoint structure method. The
first variation of performance functional due to the changes of the design variables was
determined in the framework of sensitivity analysis theory. The p-y relationship taken in
account by the authors is presented in Figure 2.8. The results presented showed a good
agreement between the actual results done by computer software and the formulated ones.
In his study Abedin (2004) investigated the sensitivity of laterally loaded pile, embedded in sand below water table. The purpose of his study was to determine how sensitive the structural response was to the changes of the design variables. The p-y relationship considered by the author is showed in the Figure 2.9 below. According Abedin (2004) sensitivity analyses also displayed the relative influence of change of each design variables on change of deformation at the top of the pile, for different length of the pile with different boundary conditions.

\[
p_a = \gamma' x \left[ \frac{K_o x \tan \phi \tan \beta}{\tan (\beta - \phi) \tan \alpha} + \frac{\tan \beta}{\tan (\beta - \phi)} \right] \left( b + \tan \beta \tan \alpha \right) + K_o x \tan \beta (\tan \phi \sin \beta - \tan \alpha) - K_o b
\]

\[
p_{sd} = K_o b \gamma' \left( \tan^8 \beta - 1 \right) + k_o \beta \gamma' x \tan \phi \tan \beta
\]

\[
y_k = \left[ \frac{B_s P_k}{k_s} \left( \frac{60}{b} \right)^{0.8} \left( \frac{A_k - l}{B_s} \right) \right]^{1.25} \left( \frac{2.25 A_k}{B_s} \right)
\]

Figure 2.8   The p-y curve used by Liu (2004) in “Sensitivity analysis of laterally loaded piles embedded in stiff clay above the water table”
\[ p_u = \gamma' x \left[ \frac{K_o \tan \phi \tan \beta}{\tan(\beta - \phi) \tan \alpha} + \frac{\tan \beta}{\tan(\beta - \phi)} (b + \tan \beta \tan \alpha) + K_o \tan \beta (\tan \phi \sin \beta - \tan \alpha) - K_a b \right] \]

where:
- \( \gamma' \) = submerged unit weight of sand;
- \( \phi \) = angle of internal friction;
- \( a = \phi / 2 \);
- \( b = 45 + \phi / 2 ; \)
- \( K_a = \tan^2 (45 - \phi /2) \);
- \( b = \) width of pile at depth \( x \);
- \( A_s \) and \( B_s \) = dimensionless coefficients

\[ P_{sd} = K_o b \gamma' \left( \tan^\phi \beta - 1 \right) + k_o \beta \gamma' \times \tan \phi \tan \beta \]

\[ p = kxy \]

\[ p = B_p \left( 60y / b \right)^{0.8 \left( A_s / B_s \right) - 1} \]

\[ p = p _s + 48 (y - b / 60) (A_s - B_s) / b \]

\[ p = p_s A_s \]

Figure 2.9 The p-y relationship investigated by Abedin (2004)
The \( p-y \) equations associated with each \( p-y \) relationship internal are specified below the figures. Rahman (2004) investigated the performance of the pile-soil system embedded in sand below the water table using adjoint structure method. The author considered cyclic lateral load of quasi-static type applied to the system, and a quantitatively assessed of the impact of each change design variables on the change of the maximum deformation and angle of flexural rotation.

**Laterally Loaded Pile Model**

For LE zone: \( p = kxy \)

For NE zone: \( p = B_c p_c \left( \frac{60}{b} y \right)^{0.8} \left( \frac{A_c - 1}{B_c} \right) \)

For LH zone:
\[
p = p_c \left[ B_c + \frac{48}{b} \left( y - \frac{b}{60} \right) (A_c - B_c) \right]
\]

For PF zone: \( p = A_c p_c \)

where \( p_c \) is the ultimate soil resistance which depends on the depth \( x \) and the soil strength parameters.

**Figure 2.10** The \( p-y \) curves for sand below the ground water table subjected to cyclic lateral loadings used by Rahman (2004)
CHAPTER 3
THEORETICAL FORMULATION OF SENSITIVITY ANALYSIS OF DISTRIBUTED PARAMETERS USED IN INVESTIGATIONS OF LATERAL LOADED PILES

3.1 Single Piles

3.1.1 General

The interaction between structures and earth mass from the engineering point of view occurs when the structure itself is in contact with the soil. The soil in a particular situation can be in its own place or can be placed artificially. In all situations any structure will develop a specific “structure-soil” interaction which will be directly related with a kind of structural design and the adjacent media present.

There are many different types of structures and each one will promote a specific response to the adjacent material, and as a result stresses will be generated both in structures and in the soil present. Those stresses developed due the presence of the structure in a continuum medium of an earth material can be considered as a different continuum that promotes many changes in both, structure and soil.

For the purpose of this study the structure analyzed is a steel pile and is considered as a deep foundation. According to Coduto (1994) engineers and contractors have developed many types of deep foundation, each of which is best suited to support a certain loading in different soil conditions. As the author recommends, the term “deep foundation” appears to be the most common in North America for piles and in his definition steel pile is considered as a deep foundation.

The prediction of the load capacity is another important point for consideration, and had been subject of discussion since the late 1920s. According to Davisson (1989) in that specific period of time the engineers only had the “feeling” that deep foundations were more reliable and worked better for higher loads. Designs were based on previous experience and common sense instead of reliable formulas. The uncertainties caused by lack of consistent theoretical basis promoted a necessity to develop ways to understand better the pile behaviour and predict the load capacity to develop reliable technically and economical design.
There are plenty of soil-structure types. Each one will impose a different kind of load to the ground; in which deep foundation is embedded. The typical loads applied to the piles are divided into two categories which are: vertical (or axial) loads and lateral loads. These loads can induce the following internal forces in the pile structure: axial force, bending moment, and shear force.

As mention by Coduto (1994) the axial loads are those that act parallel to the axis of the foundation. The lateral loads are those that act perpendicular to the axis. In general, in order to design the foundations the engineer has to deal with a plenty of input parameters that have to be taken into account. The important part of the design process is to meet the criteria of strength ability and the serviceability required by the local codes.

3.1.1.1 Pile and soft clay
The pile-soil behaviour in soft clay below the ground water table considered here takes into account the pile as a structure capable to carry lateral load of various types (cyclic or static). The pile is embedded in a ground conceived as soft clay.

The pile structure under lateral load gained a significant amount of attention especially after 1950’s when many soil structures like retaining walls, offshore structures, dams, harbour structures and others had to be built being supported by piles. Coduto (1994) emphasized that deep foundations have to consider various sources of lateral loads that can result from single cause or be result of a combination of the following causes:

- Earth pressure (on the back of the retaining walls)
- Wind loads
- Seismic and earthquake loads
- Berthing loads (from ships when they are in contact with piers or different harbour structures)
- Vehicle acceleration and braking forces (on bridges)
- Eccentric vertical loads (on columns)
- Ocean waves forces (on offshore structures)
- River current forces (on bridges piers)
- Cable forces (from electrical transmissions towers)
- Structural loads (on abutments for arch or suspension bridges)
3.1.2  

$p-y$ curve for soft clays

3.1.2.1  

$p-y$ curve theory for soft clay

The analysis of a particular deep foundation laterally loaded appear to be more complicated once some structures cannot resist a large lateral deflection necessary to mobilize the lateral soil capacity. Most of the codes set limits for the allowable lateral deflections which stimulate a different approach in terms of the load deformation behaviour. According to Coduto (1994), some analysis of the load-deformation behaviour is conducted considering the nonlinear relationship between the lateral resistance and deflection. In the last 3 decades it was possible to achieve it since the engineers had access to sufficient amount of experimental data and reliable analysis of nonlinear systems that can be conducted by computers. The most broadly use of nonlinear analysis is $p-y$ method which is based on the work present by McClelland and Foch, as noted by Coduto (1994). They model the soil resistance using a series of nonlinear springs.

This method considers the pile structure as an elastic beam, and as described in Coduto (1994), has been well accepted once it was verified by full scale load tests. Moreover, this method is capable to consider many variables such as:

- Any nonlinear load-deflection curve.
- Variations of the load-deflection curve with depth.
- Variations in the foundation stiffness (EI) with depth.
- Elasto-plastic (i.e., nonlinear) flexural behaviour in the foundation.
- Any required head constraint condition of kinematic or force type (including free, restrained, pure moment, and others).

When the lateral load comes close to the ultimate lateral capacity, the nonlinear $p-y$ curves produce large deflexion, which drives the displacement of the foundation to larger values.

Basically, the most important attribute of the $p-y$ method is to be able to define the lateral load-deflection relationship between the foundation element and the soil. This relationship is expressed as $p-y$ curves, where $p$ is the lateral soil resistance per unit length of the foundation and $y$ is the lateral deflection at arbitrary depth $x$. Numerous researchers
have been working with $p$-$y$ curves for different types for soils, among them there are Matlock (1970), Reese et al. (1974), Dunnvant and O’Neill (1989), and so many others. Matlock (1970) using the principle described started his studies using piles embedded in soft clay below the water table using static and cyclic loads. As described in Priyanto (2002), Matlock had performed lateral load tests employing steel pipe piles with 42 ft long and 12.75 inches in diameter in a ground represented by soft clays near to Lake Austin. According to Matlock (1970) the samples collected at Lake Austin demonstrated that the clay present at the local had shear strength around 800 lb/ft$^2$. After this test the same pile was recovered and had been change for another spot, now at Sabine Pass in Texas. This time the clay in Texas showed shear strength that the average was around 300 lb/ft$^2$ in the upper zone.

The results of full scale tests of Matlock (1970) were presented in the format of developed $p$-$y$ curves. The difference in stress distribution around pile embedded in a soil and then subjected to lateral lading is illustrated in the Figure 3.1.

The figure above shows a plain view of a pile section at arbitrary depth where the soil response is investigated. When there’s no bending acting on the model, then there will be no unbalance forces acting on the pile. In the part (a) of the figure it is show that the model is subjected to a uniform distribution of stresses around the pile before the applied

Figure 3.1 - The $p$-$y$ model
load. After the lateral load has been applied, part (b) on Figure (3.1), the pile will be deflected through a distance $y$, and the stresses will change around the pile as shown in Figure 3.1 (b). The unit stress has decreased on the side where the load has been applied and has been increased on the opposite side. The unbalance force now is $p$, in units of force per unit length along the pile, and as pointed at Liu (2004), can be found by integrating the unit stress.

There is nonlinear relationship between $p$ and $y$. The knowledge of the $p$-$y$ curve had opened new approaches in the field of pile foundation.

### 3.1.2.2  $p$-$y$ curve theory for soft clay below the water table subjected to static loading

The characteristic shape of the $p$-$y$ curve developed for soft clays below water table under static load is presented in Figure 3.2.

![Figure 3.2 Characteristic shape of p-y curve for soft clay below water table subjected a static loading](image)

The physical relationship between the lateral displacement $y$ at arbitrary point of depth $x$ and the soil reaction $p$ is defined bellow:
(3.1) \[ p = p_u \times 0.5 \left( \frac{y}{y_{50}} \right)^{1/3} \]

(3.2) \[ p = p_u \]

Where \( p \) = the soil reaction,
\( p_u \) = ultimate soil’s resistance
\( y \) = lateral displacement,

The value \( p_u \), of ultimate soil resistance, incorporates the strengths parameters of the soft clay investigated. The \( p_u \) varies in a linear fashion with respect with the depth \( x \). The variability of the ultimate soil resistance along the pile axis is shown in Figure 3.3.

**Figure 3.3 Variability of ultimate soil resistance \( p_u \) along the pile axis**

The ultimate soil resistance \( p_u \) for \( x \leq x_r \) is defined as:

(3.3) \[ p_u = \left[ 3 + \frac{y'}{c} x + \frac{0.5}{b} x \right] c b \]
When \( x \geq x_r \), then:

\[
(3.4) \quad p_u = 9\, c\, b
\]

Where \( c \) = the undrained cohesion of the soft clay,
\( y' \) = the soil’s submerged unit weight,
\( b \) = the pile’s width

As noted in Van Impe (2001), Matlock (1970) stated that the value \( J \) was determined experimentally to be 0.50 for soft clays

\[
J = \text{constant, for soft clay } J = 0.50
\]

Based on the continuity of \( p_u \), the depth where the soil resistance is reduced, \( x_r \) is determined from equations (3.3) and (3.4), which result in the following expression:

\[
(3.5) \quad x_r = \frac{6\, c\, b}{y'\, b + J\, c}
\]

Where \( x_r \) = the depth of the reduce resistance

The parameters \( y_{50} \) and \( \varepsilon_{50} \) are related as follows:

\[
(3.6) \quad y_{50} = 2.5\, \varepsilon_{50} \cdot b
\]

Where \( \varepsilon_{50} \) = strain corresponding to 50% of the maximum principal strain difference.

According to Priyanto (2002) the value recommended of \( \varepsilon_{50} \) by Wang and Reese (1993) for soft clays below the ground water table is equal 0.02.

\[
b = \text{the width of the pile where the soil reaction is developed}
\]

A nonlinear elastic relationship between \( p \) and \( y \) will develop when \( x \leq x_r \). This relationship will be transformed into plastic flow. For any applied load the soil response can be localized in one of those two stages. The accurate position is dependent on the value of lateral deflection \( y \) of the pile.

Priyanto (2004) in his investigations of soft clay subjected to cyclic loading of quasi static type presented that the \( p-y \) behaviour of soft clay for \( x \leq x_r \) is defined by a set of
relationships that require the fulfillment some constraints imposed on the lateral
displacement $y$. The set of $p$-$y$ relationships for soft clay subjected to a cyclic loading is
specified below for comparison purpose, as being different when the pile is subjected to
static loading.

For $x \leq x_r$ and $0 \leq y \leq 8 \ast y_{50}$

$$p = p_u \ast 0.5 \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \quad (3.7)$$

For $x \leq x_r$ and $y \geq 8 \ast y_{50}$

$$p = p_u \quad (3.8)$$

For $x \geq x_r$ and $y \leq 8 \ast y_{50}$

$$p_u = 9 \ast c \ast b \quad (3.9)$$

For $x \geq x_r$ and $y \geq 8 \ast y_{50}$

$$p = p_u \quad (3.9)$$

$$p_u = 9 \ast c \ast b \quad (3.9)$$

The graphical representation of the soil model for soft clay below the ground water table
under static load was presented in the Figure 3.2 and the figure shows that the soil can be
at 2 different stages. Those two stages are well known as nonlinear elastic and plastic
flow.

Some important observations can be there considered about each one of those stages.

- In the nonlinear elastic stage, exist the unique relationship between $p$ and $y$ such
  that increase of $p$ implies the increase of $y$;
- In the plastic flow stage there is no unique relationship between $p$ and $y$, which
  means that for one unique value of $p$ there are infinite possible values of $y$.

With the previous knowledge of the equations 3.1 – 3.4 and through the expressions 3.7 -
3.9 it is possible observe that some parameters such as $c$ (cohesion), $\gamma'$ (soil’s submerged
unit weight), $b$ (width of the pile), and $\varepsilon_{50}$ (strain corresponding to 50% of the maximum
principal strain difference) are those whose contribute to the $p$-$y$ relationship.
As pointed by Priyanto (2002) the increase of the cohesion c, submerged unit weight $\gamma'$ and the width of the pile b when the soil is in the nonlinear elastic stage results in an increase of the soil reaction $p$.

Nevertheless, the value of $\varepsilon_{50}$ is known as weakness parameter or deformability parameters once with its increase the soil reaction will decrease.

### 3.1.3 Theoretical formulation of sensitivity analysis of laterally loaded piles embedded in soft clay below the water table

#### 3.1.3.1 Introduction

The sensitivity analysis of laterally loaded piles presented in this study is focused at the development of a theoretical basis to assess a change of the quantity that defines the performance of the pile-soil system caused by the changes of the material parameters that contribute to and decide on the physical behaviour of the system.

The model adopted in this study considers a pile structure as an one dimensional beam element supported by nonlinear springs. The behaviour of springs is represented by the $p-y$ curves, characteristic for the soft clay below the water table under static load. The model is viewed in the Figure 3.4.

![Figure 3.4 A pile element modeled by a beam supported by nonlinear p-y springs](image)

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3.1.3.2 Primary and adjoint structure

The concepts of primary and adjoint structure have been used successfully by many authors in receiving the results as pointed previously on Chapter 2. The model is built using the real structure called primary structure, to which the actual forces have to be applied. Once the load is applied, the pile-soil system takes in account the deformations and the internal forces that will be developed.

The structure called adjoint structure is the virtual structure. In other words it is a structure that carries a unit force applied to the point of interest of a structure being in the state of deformation of the primary structure.

The adjoint structure has the same boundaries conditions as the primary one. The adjoint structure acts basically as a auxiliary structure that has the deformations and internal forces generates by the unit load, that is virtual load.

Figure 3.5 below presents the concept of the primary and the adjoint structure.

Figure 3.5 A primary structure subjected to a lateral force $P_t$ and the corresponding adjoint structure subjected to a lateral unit force $\bar{P} = 1$ applied at the pile top.
It is important to emphasize that \( P = 1 \) is applied to the point of interest in an investigation, in other words, can be any point along the pile axis. In our investigation it is applied at the pile top.

Stated by Liu (2004) that the model considered the pile head as a part connected to the superstructure most of the time, which means that the head of the pile will get the attention due its deflection and deformation. At the time that the sensitivity analysis will be taken in consideration only the external forces applied to the pile head will matters.

Figures 3.6 to 3.8 presents all possible load cases covered on this research.

**Figure 3.6** The primary structure subjected to a lateral force \( P_t \) and the corresponding adjoint structure subjected to a unit bending moment \( M_t = 1 \) applied at the pile top.
Figure 3.7 A primary structure subjected to a bending moment $M_t$ and the corresponding adjoint structure subjected to a unit bending moment $P_t = 1$ applied at the pile top.

Figure 3.8 A primary structure subjected to a bending moment $M_t$ and the corresponding adjoint structure subjected to a unit bending moment $M_t = 1$ applied at the pile top.
3.1.3.3 Sensitivity operators and factors

This study considers a pile embedded in soft clay. The pile is modeled as a one-dimensional beam element and the soil is simulated as nonlinear p-y springs distributed in a continuous media along the pile.

The pile embedded in soil along its length is subjected a bending moment. The behaviour of the pile structure subjected to bending is described by the following relationship:

\[(3.10) \quad EI \, y'' = -M\]

where:
- \(EI\) = the bending stiffness of the pile,
- \(y''\) = second derivative of lateral deflection,
- \(M\) = the bending moment.

The two equations already presented, that is Equation 3.1 and Equation 3.4, represent the soil’s behaviour as nonlinear springs distributed in a continuous way along the pile length.

The interaction of the pile-soil system requires the compatibility of deformation of the pile structure and the adjacent soil. The suitable differential equation that governs the association between the deflection \(y\), the soil reaction \(p\) (that can be viewed as a distributed load) and \(EI\) is given as:

\[(3.11) \quad EI \, y^{(IV)} - p = 0\]

where:
- \(p\) = the soil reaction as a distribute load along pile axis, in kN/m,
- \(y^{(IV)}\) = the fourth derivative of \(y\) with respect to spatial variables.

The capacity of lateral resistance of the system is a function of different physical parameters, which have to be taken in account. It is postulated that parameters of the pile and the soil that affect the performance of the pile-soil system are consider as a design variables.
They are conveniently arranged in the vector of the design variables $S$, that it is given as:

\[(3.12) \quad S = [EI, c, \gamma', b, \varepsilon_{s0}]^T\]

The first basic consideration is that the system is considered on a state of static equilibrium when the deformation analysis is conducted. The Figures 3.5 to 3.8 allows to identify two different systems. The first to which external load is applied is called the primary system and the other one to which a virtual load is applied is called the adjoint system. For the primary system is considered that the pile deflection will be assigned as $y$ and rotations are denoted as $\theta$.

The application of external load to the primary structure at $x = 0$, the values of deflection are $y = y_t$, and rotation will be $\theta = \theta_t$.

There will be also changes in the deflection $y$ and in the rotation $\theta$ if the design variables varied, and the changes will be expresses as $\delta y$ and $\delta \theta$.

Through the Figures 3.5 to 3.8 it’s possible to identify some variables, and they will be presented as a part of two different groups:

- **State variables**: $y, y', y'', y'''$, $y^{(iv)}$, $P_t$, $M_t$, $M$, $p$ (they are components of vector of state variables)
- **Design variables**: $S$ (this is vector that are arranges in vector $\delta S$ having components like $\delta EI$, $\delta c$, $\delta \gamma'$, $\delta b$, $\delta \varepsilon_{s0}$)

The changes of the design variables can be expressed through the vector $\delta S$, which is defined as:

\[(3.13) \quad \delta S = [\delta EI, \delta c, \delta \gamma', \delta b, \delta \varepsilon_{s0}]^T,\]

where the symbol $\delta$ stands for the variation, or for the change of the variable.

The vector of maximum deformation $\Delta$ of the pile-soil system is related to the top pile point that means to the head of the pile. This vector consists of two components, that is the lateral displacement $y_T$ and the angle of rotation $\theta_T$.

\[(3.14) \quad \Delta = \{y_T, \theta_T\}^T\]
The first order variation of the vector of the pile head deformation is:

\[ \delta \Delta = \{ \delta y_T, \delta \theta_T \}^T \]  

(3.15)

Using the virtual work principle (Wishizu, 1976), for an elastic system, the virtual work, \( \Pi \), done by the unit force in the adjoint structure can be expresses as:

\[ \Pi = \bar{T} \cdot \Delta \]  

(3.16)

For the first order variation of the virtual work the equation will be defined as:

\[ \delta \Pi = \bar{T} \cdot \delta \Delta \]  

(3.17)

The primary structure represented by a pile–soil system subjected to a constant external horizontal force \( P \), or a constant external moment \( M \), is considered as being in a state of deformation. Part of these variations of deformations, \( \delta y_T, \delta \theta_T \), are imposed on the primary structure due to the changes of the design variables.

Equation 3.17 still valid for a minor variation \( \delta \Delta \), once the nonlinear system continues to be considered linear without excessive error in a very small variation increment. With respect to the sensitivity analysis carried out in this study, it will be considered only for small variations of the design variables. This consideration is valid as for as the nonlinear characteristic of the pile-soil system.

Applying the sensitivity analysis results to large increment of design variables variations should be carried out with special attention.

It’s conducted based on the virtual work principle applied with respect to increments of generalized deformation of primary structure. Then:

\[ \bar{T} \delta y_T = - \int_0^L M^p \delta y'' dx + \int_0^L P^p \delta y dx \]  

(3.18)

and

\[ \bar{T} \delta \theta_T = - \int_0^L M^M \delta y'' dx + \int_0^L P^M \delta y dx \]  

(3.19)

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\( \bar{M}_p \) = the bending moment of the adjoint structure subjected to \( \bar{P} = 1 \)
\( \bar{p}_p \) = the soil reaction of the adjoint structure subjected to \( \bar{P} = 1 \)
\( \bar{M}^M \) = the bending moment of the adjoint structure subjected to \( \bar{M} = 1 \)
\( \bar{p}^M \) = the soil reaction of the adjoint structure subjected to concentrated bending moment \( \bar{M} = 1 \)

\( \delta y, \delta y'' \) = variations of deformations imposed on the primary system

The equations (3.18) and (3.19) can be combined in one general equation, which is:

\[
(3.20) \quad \int_0^L \delta M \delta y'' \, dx + \int_0^L \delta p \delta y \, dx = 0
\]

\( \bar{M} \) = the bending moment of the adjoint structure subjected to unit force
\( \bar{p} \) = the soil reaction of the adjoint structure subjected to unit force

Equation (3.12) vector \( \bar{S} \) is presented and it can be combined with equations (3.10) and (3.11), and the increment of internal forces of the pile-soil system can be expressed as:

\[
(3.21) \quad \delta M = - \frac{\partial M}{\partial y''} \delta y'' + \frac{\partial M}{\partial (El)} \delta (El)
\]

\[
(3.22) \quad \delta p = \frac{\partial p}{\partial y} \delta y + \frac{\partial p}{\partial S} \delta S
\]

The variations \( \delta y \) and \( \delta y'' \) are imposed on the primary structure in the presence of constant load, consequently the increments of internal forces are equal to zero. This implies:

\[
(3.23) \quad \delta M = 0
\]

and

\[
(3.24) \quad \delta p = 0
\]

The two unknowns \( \delta y'' \) and \( \delta y \) can be determined from the previous equations (3.21) and (3.22), taking into account the conditions (3.23) and (3.24). Thus:

\[
(3.25) \quad \delta y'' = - \frac{\partial y''}{\partial M} \frac{\partial M}{\partial (El)} \delta (El)
\]

and
(3.26) \[ \delta y = - \frac{\partial y}{\partial p} \frac{\partial p}{\partial S} \delta S \]

To be able to determine \( \delta y \) and \( \delta \theta \) of the primary structure it's possible to utilize the adjoint structure as a helper structure or auxiliary structure, that is adjoint structure.

Applying \( \delta y^* \) and \( \delta y \) to the Equations (3.21) and (3.22) and incorporate on equation (3.13), the equation 3.20 becomes

(3.27) \[
\bar{\delta} \delta A = \int_0^L M \frac{\partial y^*}{\partial M} \frac{\partial M}{\partial (EI)} \delta (EI) \, dx \\
- \int_0^L \left[ \frac{\partial y}{\partial p} \frac{\partial p}{\partial c} \delta c + \frac{\partial y}{\partial p} \frac{\partial p}{\partial \gamma} \delta \gamma + \frac{\partial y}{\partial p} \frac{\partial p}{\partial b} \delta b + \frac{\partial y}{\partial p} \frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} \right] \, dx
\]

Now the equation above can be rewritten in another format:

(3.28) \[
\bar{\delta} \delta A = \int_0^L M \frac{\partial y^*}{\partial M} \frac{\partial M}{\partial (EI)} \frac{\delta (EI)}{EI} \, dx \\
- \int_0^L \left[ \frac{\partial y}{\partial p} \frac{\partial p}{\partial c} \frac{\delta c}{c} + \frac{\partial y}{\partial p} \frac{\partial p}{\partial \gamma} \frac{\delta \gamma}{\gamma} + \frac{\partial y}{\partial p} \frac{\partial p}{\partial b} \frac{\delta b}{b} + \frac{\partial y}{\partial p} \frac{\partial p}{\partial \varepsilon_{50}} \frac{\delta \varepsilon_{50}}{\varepsilon_{50}} \right] \, dx
\]

Thus the Equation (3.30) can be written in a compact form as:

(3.29) \[
\bar{\delta} \delta A = \int_0^L \left[ C_{EI} \frac{\delta (EI)}{EI} \, dx + \int_0^L \left[ C_c \frac{\delta c}{c} + C_\gamma \frac{\delta \gamma}{\gamma} + C_b \frac{\delta b}{b} + C_{\varepsilon_{50}} \frac{\delta \varepsilon_{50}}{\varepsilon_{50}} \right] \, dx \right]
\]

Where \( C_{EI}, C_c, C_\gamma, C_b, C_{\varepsilon_{50}} \) are the normalized sensitivity integrands/operators \( (C_c, \ldots) \) affecting changes of top lateral deflection due to the changes or variations of the bending stiffness EI, the cohesion c, the unit soil weight \( \gamma \), the width of the pile b, the \( \varepsilon_{50} \) respectively.

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The Figure (3.9) demonstrate the physical meaning of the normalized sensitivity integrands/operators $C(\ldots)$, where the value of normalized sensitivity integrands for different value of $x$ (or depth) along the pile axis are plotted versus the location $x$. The $C(\ldots)$ indicates numerically the location of the influences that the design variables have on the deformation on the pile head.

![Diagram showing the physical interpretation of normalized sensitivity integrands/operators and sensitivity factors.]

**Figure 3.9**  Physical interpretation of the normalized sensitivity integrands/operators $C(\ldots)$ and sensitivity factors $A(\ldots)$, after Liu (2004)

The larger is the $C(\ldots)$ value represented along the depth, at any point $x$, the greater will be the influence that the changes on the design variables has on the pile head deformation. As an example pointed in the Figure 3.9, at point $a$, $x = x_a$, the value of $C(\ldots)$ is bigger than at point $b$, $x = x_b$. The meaning of what was written above is that for the same amount of change of a design variable, the change at point $x_a$ will promote more of the pile head deformation change than at point $x_b$. The pile head deflection is more sensitive to the changes of design variables at point $x_a$ than at point $x_b$.  

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For the point \( xc \), the value of the normalized sensitivity integrands/operators is equal to zero. This means that the variation of the design variables at point \( xc \) will not promote any deformation change of the pile head. At the point \( xc \) the lateral deflection at the pile head is insensitive to the changes of a design variable subjected to investigation.

The values \( \frac{\delta EI}{EI}, \frac{\delta c}{c}, \frac{\delta y}{y}, \frac{\delta b}{b}, \frac{\delta e_{50}}{e_{50}} \) are considered scalars, and they can be represented in a fraction or percent format. They also don’t have units to express them with.

Using the equation (3.29) it’s possible to prove that the normalized sensitivity integrand/operator, \( C(e, \ldots) \), has kN unit.

The equations of \( C_{EI}, C_c, C_y, C_b, \) and \( C_{e_{50}} \) were developed in accordance to Figures 3.2 and 3.3 where the values of deflection \( y \) at the depth \( x \) define which range the equations in the \((p-y)\) relationship will be represented. The numerical forms of \( C_{EI}, C_c, C_y, C_b, \) and \( C_{e_{50}} \) are presented in the Equations (3.32) ~ (3.39) and the detail derivation of those are presented in Appendix A.

Thus, when the deflection \( 0 \leq \frac{y}{y_{50}} \leq 8 \) and \( x \leq x_r \)

\[
(3.30) \quad C_{EI} = (-y'' y') EI
\]

\[
(3.31) \quad C_c = \left[-1.5 \left(\frac{y}{y_{50}}\right)^{\frac{1}{3}} y (3b + Jx)\right] c
\]

\[
(3.32) \quad C_y = \left[-1.5 \left(\frac{y}{y_{50}}\right)^{\frac{1}{3}} y (b x)\right] y'
\]

\[
(3.33) \quad C_{e_{50}} = \left[0.5 \left(\frac{y}{y_{50}}\right)^{\frac{1}{3}} y \left[(3b + Jx) c + \gamma b x \left(\frac{1}{e_{50}}\right)\right]\right] e_{50}
\]

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(3.34) \[ C_b = \left( \frac{y_a}{y_{50}} \right)^3 y \left( c + \frac{1}{3} \gamma' x - \frac{J x c}{6} \right) \]

For \( \frac{y}{y_{50}} \geq 8 \) and \( x \leq x_r \):

(3.35) \[ C_{El} = (-y_a y'')EI \]

(3.36) \[ C_c = C_b = C_{e_{50}} = C_{\gamma'} = 0 \]

When \( 0 \leq \frac{y}{y_{50}} \leq 8 \) and \( x \geq x_r \):

(3.37) \[ C_{El} = (-y_a y'')EI \]

(3.38) \[ C_c = -13.5 \left( \frac{y_a}{y_{50}} \right)^{1/3} y b \]

(3.39) \[ C_{e_{50}} = 4.5 \left( \frac{y_a}{y_{50}} \right) yc b \left( \frac{1}{e_{50}} \right) e_{50} \]

(3.40) \[ C_b = 9 \left( \frac{y_a}{y_{50}} \right)^{1/3} yc b \]

(3.41) \[ C_{\gamma'} = 0 \]
for $\frac{y}{y_{50}} \geq 8$ and $x \geq x_r$;

\begin{equation}
(3.42) \quad C_{EI} = (-y_a y'')EI
\end{equation}

\begin{equation}
(3.43) \quad C_c = C_b = C_{\varepsilon_{50}} = C_{\gamma} = 0
\end{equation}

The sensitivity operators for single piles and pile groups are present in the Appendix B ~ J for single piles, and in the Appendix K for pile groups. The graphical representation of the sensitivity operators is essential in the sensitivity analysis and some relevant reasons are the following:

✓ The distribution of the sensitivity operators give to the user a clear understanding of which design parameter will promote more changes.

✓ The sensitivity operators are critical to perform an assessment of the effect of each design variable on the changes of the top pile lateral deflection.

The sensitivity operators are fundamental part of the sensitivity analysis, and with the aid of these operators some engineering applications will be addressed in the Chapter 9. As an example of the aid of the sensitivity operators in the sensitivity analysis, a free head pile subjected to a concentrated lateral load is analysed through its segments in its axis. The pile segments will resist the lateral load and will be efficient if the values of the sensitivity operators will appear different than zero. In other words, if the values of the sensitivity operators are equal to zero these segments are not being used to resist the lateral load, which leads a non economical structure, once some segments are not being used.

The distributions of the sensitivity operators of the primary and adjoint structure are presented in the Appendices, however it is important to explain the notation used in them. To promote a differentiation of the primary and adjoint structures presented previously in this chapter in the Figures 3.5, 3.6, 3.7 and 3.8 the notations used will follow the notations proposed by Liu(2004). The notation used is presented in the following figure:
Load type applied to the primary structure. "M" for bending moment. "P" for lateral load.


The design variable that changes

Figure 3.10 Convention for the notation of sensitivity operators/integrands and sensitivity factors, after Liu (2004)

The values $E_I$, $c$, $\gamma$, $b$ and $\varepsilon_{50}$ are considered constants, and the value of variation, $\delta E_I$, $\delta c$, $\delta \gamma$, $\delta b$ and $\delta \varepsilon_{50}$ are also considered constants. If it is assumed the previous values are constants, it’s appropriate to consider $\frac{\delta E_I}{E_I}$, $\frac{\delta c}{c}$, $\frac{\delta \gamma}{\gamma}$, $\frac{\delta b}{b}$ and $\frac{\delta \varepsilon_{50}}{\varepsilon_{50}}$ constants.

Consequently the equation (3.29) can be rewritten in another format:

\[
\overline{1} \delta \Delta = \frac{\delta (E_I)}{E_I} \int_0^L C_{EI} dx + \frac{\delta c}{c} \int_0^L C_C dx + \frac{\delta \gamma}{\gamma} \int_0^L C_\gamma dx + \frac{\delta b}{b} \int_0^L C_b dx + \frac{\delta \varepsilon_{50}}{\varepsilon_{50}} \int_0^L C_{\varepsilon_{50}} dx
\]

Once the required integrations are solved, the equation (3.44) can be expressed as:

\[
\overline{1} \delta \Delta = \frac{A_{EI}}{E_I} \frac{\delta (E_I)}{E_I} + A_c \frac{\delta c}{c} + A_\gamma \frac{\delta \gamma}{\gamma} + A_b \frac{\delta b}{b} + A_{\varepsilon_{50}} \frac{\delta \varepsilon_{50}}{\varepsilon_{50}}
\]
where \( A_{EI}, A_c, A_r, A_b, \) and \( A_{eo} \) are known as the sensitivity factors \( A_{(\cdots)} \) affecting the changes of the pile head deformation due to the changes of the design variables.

\[
(3.46) \quad A_{EI} = \int_0^L C_{EI} \, dx
\]

\[
(3.47) \quad A_c = \int_0^L C_c \, dx
\]

\[
(3.48) \quad A_r = \int_0^L C_r \, dx
\]

\[
(3.49) \quad A_b = \int_0^L C_b \, dx
\]

\[
(3.50) \quad A_{eo} = \int_0^L C_{eo} \, dx
\]

Figure (3.9) provides also a clear understanding of the physical meaning of the sensitivity factors \( A_{(\cdots)} \), which can be viewed through the area hatched. The area hatched on the figure above is exactly the value of the sensitivity factor \( A_{(\cdots)} \).

Equation (3.45) actually shows that the sensitivity factors define the relationship between the changes of the design variables and the variations of the pile head deformations. Once the design variables are defined, the variations on the pile head deformations are certainly easy to obtain.

The values of \( A_{(\cdots)} \) imply the magnitude of the influence of the design variables on the performance of the pile head deformations. Repeating values of \( A_{(\cdots)} \), they imply that the larger is the value of the sensitivity factor, the more important is the effect that the design variable has on the variation of the pile head deformation.
The sensitivity factors $A_{(\cdot \cdot \cdot)}$ can be expressed in terms of unit of a bending moment, that is (kN \cdot m).

To perform the numerical investigation, the integration of the equations (3.43) to (3.47) is solved utilizing the Simpson's method. The idea behind Simpson's method is to evaluate the function at three points within each little interval, and then calculate the area underneath a parabola fitted to those three points.

The Figure (3.11) shows how to explain this method. In order to perform a integration of a function $y = f(x)$, within an interval $[a, b]$, the interval described is divided into $n$ parts, or subintervals. Where $n$ is a even number. So the length ($h$) of each interval will be:

$$h = \frac{b - a}{n}$$

Using the Simpson's rule, for a random odd number $i$, the area hatched, can be calculated as:

$$A_r = \int_{x_{i-1}}^{x_{i+1}} f(x) \, dx = \frac{h}{3} \left[ f(x_{i-1}) + 4f(x_i) + f(x_{i+1}) \right]$$
The integration of the entire interval \([a, b]\) can be obtained by summing all the discrete intervals like the hatched part in the center at \(x_i\) (where \(i\) is an odd number between 0 and \(n\)). Then, the numerical integration of the function \(f(x)\) in the interval given as \([a, b]\) is expressed as:

\[
\int_a^b f(x)\,dx = \frac{h}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + \ldots + 2f(x_{x-2}) + 4f(x_{x-1}) + f(x_n) \right]
\]

In order to analyze the influence of each design variable compared to other design variables requires to compare it with total sensitivity of a system that is defined by Equation (3.54). Then:

\[
A_{\text{total}} = |A_{E_1}| + |A_c| + |A_{y_1}| + |A_b| + |A_{E_{50}}|
\]

The next step is defining the relative sensitivity factors \((F_{E_1}, F_c, F_y, F_b, F_{E_{50}})\); these factors are responsible for the determination which design variable is more critical to the performance of the pile. The relative factors will show (in percent) the contribution of the specified design variables in total change of the deformation investigated.

\[
F_{E_1} = \frac{|A_{E_1}|}{A_{\text{total}}}
\]

\[
F_c = \frac{|A_c|}{A_{\text{total}}}
\]

\[
F_y = \frac{|A_{y_1}|}{A_{\text{total}}}
\]

\[
F_b = \frac{|A_b|}{A_{\text{total}}}
\]

\[
F_{E_{50}} = \frac{|A_{E_{50}}|}{A_{\text{total}}}
\]

Through the relative sensitivity factors it is possible to establish which design variable, among the ones used in this study, play the most important or critical role in the performance of the pile. If some changes are promoted by a specific design variable the question arises at this point is the following: How beneficial is sensitivity analysis for the performance of the pile-soil system?
CHAPTER 4
THEORETICAL FORMULATION OF PILE GROUP

4.1 Overview

Among the numerous methods of analysis of piles subjected to lateral loads the finite element method is the most common and the most used one. As had been explained before in the Chapter 2, the finite element approach takes into account the beam model associated with the non linear soil model response. The theoretical formulations that have been used for single piles are also the most pertinent for the pile groups analysis. The important difference between the utilization of this method in a pile group analyze is that the deflection of a pile in a group is different the deflection for of isolated pile.

Introducing the p-multipliers concept into the p-y curve of the pile group will be explained in this Chapter for a pile group embedded in a soft clay bellow the water table. A typical view of the pile group structure that will be considered in this work is shown in Figure 4.1.

![Figure 4.1 A typical view of the pile group system under a lateral load Pg](image)

Figure 4.1 A typical view of the pile group system under a lateral load Pg
4.2 Laterally loaded pile group

Using large scale tests for pile groups and isolated piles embedded in dense sand subjected a cyclic load researchers had observed through the measurement of deflections and stresses that each pile in a group carries unequal lateral loads, depending on their position in the group arrangement and the spacing between them.

The deflections and bending moments of a pile in a pile group under lateral load is expected to be different than the deflections of the same pile in a single arrangement. In other words, for a certain assembly where piles are closely spaced in pile groups they will behave differently than the single isolated pile. For the same load applied it can be said, that the deflection of a pile inside of the group will be grater than the deflection of the single isolate pile. Therefore, inside of the group a loss of efficiency of the piles occurred. The cause of this loss is related to the “shadowing phenomenon”, which in terms of soil resistance means that the loss of soil resistance in the trailing rows is smaller than in the leading row.

The concept of the $p-y$ multiplier was proposed by Brown et al., in (1988). It indicates that the $p-y$ curve proposed for a single pile embedded in a soil should be modified when the analysis of a pile groups embedded in soil are conducted. The concept takes in account the introduction of $p$-multiplier ($f_m$) that account for the loss of soil resistance. Once more, the idea is based on the values of the $p-y$ curves for single piles and then modifies them with respect to soil reaction $p$ to obtain the $p-y$ curve for a group of piles.

Mokwa and Duncan (2001b) formulated equation and proposed design curves for the $p$-multiplier, $f_m$, for all kinds of soil based on the analysis of the state-of-the-art soil values. They are used in investigation and design of laterally loaded pile groups using the popular $p-y$ method. They present data graphs that show $p$-multipliers as functions of pile spacing, in a useful way for engineering design practice.

The nomenclature used for describing the locations of piles in the pile group is presented in Figure 4.2.
To analyze a pile in a pile group, the lateral load resistance ($P_{gp}$) of a pile is equal to the lateral load resistance of a single pile ($P_{sp}$) multiplied by a p-multiplier ($f_m$):

\[(4.1) \quad P_{gp} = f_m P_{sp}\]

Where
- $P_{gp}$ = the lateral load resistance of a pile in a group
- $P_{sp}$ = the lateral load resistance of a single pile

The results of the curves proposed by Mokwa and Duncan (2001b) are presented in the chart form that they are shown in Figure 4.3. In this figure the relationships between the p-multipliers, pile spacing and pile locations are illustrated.
Pile distance s (D)

Figure 4.3 The p-multiplier design curves proposed by Mokwa and Duncan (2001b)

Although there is a variation among the number of rows in the arrangements of group of piles, Brown and Reese (1985), Morrison and Reese (1986), and McVay et al (1955), had found that little variation exists among the response of piles in a given row. So the current state of practice is to associate the value of the p-multiplier \( f_m \) with the row and to use the value of \( f_m \) for all piles in the same row.

Another important observation with respect to the bending moments is that for the corner piles in the front row, the bending moment should be adjusted when the piles are spaced \( (S<3D) \) very closely. It is generally assumed that p-multipliers are constant with depth, even when there are variations in the soil properties with depth.
Mokwa and Duncan (2001b) also proposed a relationship between the pile group efficiency \( G_e \) and p-multiplier \( f_m \) as shown below.

\[
G_e = \frac{(Q_u)_g}{n(Q_u)_s}
\]

\[
G_e = \frac{\sum_{i=1}^{N_r} f_{mi}}{N_r}
\]

Where

- \( f_m \) = p-multiplier
- \( G_e \) = pile group efficiency
- \( (Q_u)_g \) = ultimate lateral load capacity of the group
- \( (Q_u)_s \) = ultimate lateral load capacity of a single pile
- \( n \) = the number of the piles in the group
- \( N_r \) = the number of the rows
- \( f_{mi} \) = the p-multiplier for row \( i \)

According Priyanto (2002) the value of the pile group efficiency \( G_e \) obtained by using the Equation (4.2) can be different from using the Equation (4.3). However, the value of \( G_e \) obtained from both equations might result in the same trends that can be considered, from the practical point of view, as a same value.
CHAPTER 5
NUMERICAL INVESTIGATIONS OF SINGLE PILE

5.1 General

The sensitivity analysis already presented in the previous chapters will be better understood once a numerical investigation of it is performed. The design variables discussed previously and others important factors that take part in this sensitivity analysis (and its importance through the process) will be presented in the further chapters once the necessary parameters and variables are defined.

The sensitivity analysis will be carry out for single piles and for group of piles. However, at this point, in this chapters our attention will be focused at single piles only.

5.2 Load and constraints type

The sensitivity analysis of single piles, which is conducted in this study, is aimed at the following objectives:

1. Conducting the sensitivity analysis of pile head lateral deflection $y_t$ for free head pile subjected to lateral force $P_t$ applied to the pile head.

2. Conducting the sensitivity analysis of pile head angle of flexural rotation $\theta_t$ for free head pile subjected to lateral forces $P_t$ acting at the pile head.

3. Conducting the sensitivity analysis of pile head lateral deflection $y_t$ for fixed head pile subjected to lateral forces $P_t$ applied to the pile head.

4. Conducting the sensitivity analysis of pile head lateral deflection $y_t$ for free head pile subjected to bending moments $M_t$ being applied to the pile head.

5. Conducting the sensitivity analysis of pile head angle of flexural rotation $\theta_t$ for free head pile subjected to bending moments $M_t$ acting at pile head.

The pile lengths of 2T, 3T, 4T, 4.5T, 5T, 6T, 7T, 8T, 9T and 10T are employed in this analysis. This study performed the sensitivity analysis of laterally loaded piles under static loading.
5.3 Determination of the design parameters:

5.3.1 Soil properties

The soil defined for this study is a \((p-y)\) soft clay, below the water table. The soil can be considered as a homogeneous, and this model adopted one single layer. In other words, the depth is considered as an infinite. Although many authors presented typical values for some parameters related to soft clays, Terzaghi (1967) suggested some traditional parameters for soft clays located in glacial areas, which are specified as following:

- Porosity \((n)\) - 55%;
- Void ratio \((e)\) - 1.2
- Water content \((w)\) - 45%
- Saturated unit weight \((\gamma_s)\) - 17 - 18 kN/m\(^3\)

One of the conditions for this study is to consider the soil located below the water table. This means, that the water table is located at the ground level.

The unit weight for this study was considered to be \(\gamma_s = 17.66\) kN/m\(^3\), and the effective unit weight is determined as the following:

\[
\gamma' = \gamma_s - \gamma_w
\]

Where:

- \(\gamma_w = 9.81\) kN/m\(^3\) unit weight of water
- \(\gamma_s = 17.66\) kN/m\(^3\) unit weight of the soft clay
- \(\gamma'\) = effective unit weight of the soft clay

\[
(5.2) \quad \gamma' = 17.66 - 9.81 = 7.85\text{ kN/m}^3
\]
The values of the unconfined compressive strength ($q_u$) of the soft clay was proposed by several authors, however according to Das (1999) the value of $q_u$ can be expected to be between 24 and 48 kPa. The study presented considered the value of $q_u = 47.88 \sim 48$ kPa.

The undrained cohesion ($c_u$) or well known as shear strength ($s_u$) of the soil is defined as:

\[
 c_u = s_u = \frac{q_u}{2} = \frac{47.88}{2} = 23.94 \text{ kPa}
\]

The $\varepsilon_{50}$ is the strain corresponding to one-half the maximum principal stress difference. The typical values, which are presented in the manual of the software COM624P, are given in the table below:

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\varepsilon_{50}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft Clay</td>
<td>0.02</td>
</tr>
<tr>
<td>Medium Clay</td>
<td>0.01</td>
</tr>
<tr>
<td>Stiff Clay</td>
<td>0.01</td>
</tr>
<tr>
<td>Very Stiff Clay</td>
<td>0.005</td>
</tr>
<tr>
<td>Hard Clay</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Table 5.1 The typical value of $\varepsilon_{50}$ (after Wang and Reese 1993).

Based on Table 5.1, the typical value of $\varepsilon_{50}$ for soft clay is 0.02.

5.3.2 Pile's physical properties

The pile used in this study is a standard hollow steel pile HSS 508x13 defined by "Hollow Structural Sections to ASTM A 500 Grade C", is issued by Canadian Institute of Steel Construction 2000.

The section property of the pile is calculated in the following steps and presented in Figure 5.1:
5.4 Determination of the piles length

According to Lee (2005) a laterally loaded pile-soil system is considered as reaching a point of failure when the full passive resistance is mobilized along the entire length of the pile. The length of the pile can be considered as a short, intermediate or long, and the behavior the piles subjected to a lateral loads are strongly related to the length of the pile. The determination of the pile-soil system of a linear elastic type is easily obtained through the linear theory of subgrade reaction. In linear theory, (Priyanto (2002)) the soil response is modeled by means of Winkler type foundation that uses the coefficient of subgrade reaction \( k \).

The soil considered in this study is a soft clay. Thus taking into account the type of soil and Terzaghi’s (1956) recommendation, the values of \( k \) for references can be taken as a constant.

According to Das (1999), Davisson and Gill (1963) proposed the determination of the length of piles subjected to a lateral load being a product of a scalar and the characteristic length \( \lambda_c \).

---

Figure 5.1 Pile’s properties used in the sensitivity analysis

<table>
<thead>
<tr>
<th>Pile Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
</tr>
<tr>
<td><strong>Weight</strong></td>
</tr>
<tr>
<td><strong>d₁</strong></td>
</tr>
<tr>
<td><strong>d₂</strong></td>
</tr>
<tr>
<td><strong>w (design wall thickness)</strong></td>
</tr>
<tr>
<td><strong>Iₓ = Iᵧ</strong></td>
</tr>
<tr>
<td><strong>Allowable stress</strong></td>
</tr>
<tr>
<td><strong>Modulus of elasticity E</strong></td>
</tr>
<tr>
<td><strong>Stiffness El</strong></td>
</tr>
<tr>
<td><strong>Yield moment</strong></td>
</tr>
<tr>
<td><strong>Plastic moment</strong></td>
</tr>
<tr>
<td><strong>Area A</strong></td>
</tr>
</tbody>
</table>

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where
\( EI \) stands for the bending stiffness,
\( b \) is the width of the pile,
\( k \) means the coefficient of subgrade reaction

Considering cohesionless soils there are other factors that were proposed by Matlock and Reese (1960) and pointed by Das (1999). However, when the pile is embedded in a soft clay, located below the ground water table, and assumed a \( p-y \) relationship, other approaches for determined how the pile length are more applicable.

Matlock and Reese (1960) proposed the relative stiffness factor \( T \), that is used to determine the length of the piles subjected a lateral load in a soil that can be described by a \( p-y \) curve.

### 5.4.1 Determination of relative stiffness factor \( T \)

In order to establish the concept of relative stiffness factor \( T \), there is a need to introduce the concept of the **characteristic shear load** and **characteristic moment load**. Evans and Duncan (1982) developed the concept of **characteristic shear load**, \( V_c \), and **characteristic moment load**, \( M_c \), which led to the following formulas:

\[
(5.5) \quad H_c = \lambda B^2 E R I \left( \frac{\sigma_p}{E R I} \right)^m \left( \varepsilon_{50} \right)^n
\]

\[
(5.6) \quad M_c = \lambda B^2 E R I \left( \frac{\sigma_p}{E R I} \right)^m \left( \varepsilon_{50} \right)^n
\]

\[
(5.7) \quad R_I = \frac{1}{\pi b^4/64}
\]
\[ R_I = 1.00 \] for solid circular cross sections,
\[ R_I = 1.70 \] for solid square cross sections.

For soft clay behavior:

\[ \lambda = 1.00 \] for \( H_C \)

\[ \lambda = 1.00 \] for \( M_C \)

For cohesive soils, as an example, clay:

\[ \sigma_p = 4.2 s_u \]

Where:

- \( H_C \) = characteristic shear load,
- \( M_C \) = characteristic moment load,
- \( \lambda \) = a dimensionless parameter on the soil's stress-strain behaviour,
- \( b \) = diameter of the pile,
- \( E \) = modulus of elasticity of pile (200 GPa for steel),
- \( R_I \) = dimensionless relative moment of inertia of the pile section,
- \( \sigma_p \) = representative passive pressure of soil,
- \( m, n \) = exponents from Table 5.2,
- \( I \) = moment of inertia of pile,
- \( s_u \) = undrained shear strength of soil, in this study, \( s_u = c \).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>For ( H_C )</th>
<th>For ( M_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesive</td>
<td>( 0.683 )</td>
<td>( -0.22 )</td>
</tr>
<tr>
<td>Non Cohesive</td>
<td>( 0.57 )</td>
<td>( -0.22 )</td>
</tr>
</tbody>
</table>

Table 5.2 Values of exponents \( m \) and \( n \) (Evans and Duncan, 1982)

Evans and Duncan (1982) developed a method to express the lateral load deflection behavior in the chart form. In this method, the lateral loads \( H_t \) or \( M_t \) vs. the pile head deflections \( y_t \) are plotted in the form of charts. They are shown in the following Figure 5.2 - Figure 5.5.
Then for cohesive soils:

(5.10) \[ \sigma_p = 4.2c_u = 4.2 \times 23.94 = 100.55 \text{ kPa} \]

\[ I = 550 \times 10^{-6} \text{ m}^4 \]

\[ B = 0.508 \text{ m} \]

\[ E = 2 \times 10^8 \text{ kPa} \]

\[ \varepsilon_{50} = 0.02 \]

\[ \lambda = 1.0 \]

\[ m \text{ and } n \]

From Table 5.2

\[ m = 0.68 \text{ and } n = -0.22 \] horizontal force

\[ m = 0.46 \text{ and } n = -0.15 \] horizontal moment

\[ (5.7) \quad R_i = \frac{I}{\pi b^4/64} = \frac{550 \times 10^6}{\pi \times 0.508^4/64} = 0.168 \approx 0.17 \]

\[ (5.5) \quad H_c = \lambda B^2 E R_i \left( \frac{\sigma_p}{E R_i} \right)^m \left( \varepsilon_{50} \right)^n \]

\[ (5.5) \quad H_c = 1 \times 0.508^2 \times 2 \times 10^8 \times 0.17 \times \left( \frac{100.55}{2 \times 10^8 \times 0.17} \right)^{0.68} \times (0.02)^{-0.22} \]

\[ H_c = 3,607 \text{ kN} \]

\[ (5.6) \quad M_c = \lambda B^3 E R_i \left( \frac{\sigma_p}{E R_i} \right)^m \left( \varepsilon_{50} \right)^n \]

\[ (5.6) \quad M_c = 1 \times 0.508^3 \times 2 \times 10^8 \times 0.17 \times \left( \frac{100.55}{2 \times 10^8 \times 0.17} \right)^{0.46} \times (0.02)^{-0.15} \]

\[ M_c = 22,937 \text{ kN} \]
Figure 5.2  Load-deformation curves for Free Head Pile in clay – static loading
(after Evans and Duncan 1982)
Figure 5.3  Load-deformation curves for Fixed Head Pile in clay – static loading (after Evans and Duncan 1982)
Figure 5.4  Moment-deformation curves for Free Head Pile in clay – static loading (after Evans and Duncan 1982)
Figure 5.5  Load-moment relationships - curves for Free Head Pile in clay – static loading (after Evans and Duncan 1982)
Figure 5.6  Load-moment curves for Fixed Head Pile in clay – static loading (after Evans and Duncan 1982)
The characteristic shear load $H_c$ and characteristic bending moment $M_c$ are determined for plastic behavior as:

For plastic behavior, assuming $\frac{y_t}{b} = 0.05$ from Figure (5.2) $\Rightarrow \varepsilon = 0.02$

\begin{equation}
(5.11) \quad y_t = 0.05 \times 0.508 = 0.0254
\end{equation}

\begin{equation}
(5.12) \quad \frac{H_t}{H_c} = 0.02168
\end{equation}

$$H_t = 0.02168 \times H_c = 78.21 \text{ kN}$$

\begin{equation}
(5.13) \quad \frac{M_{\text{max}}}{M_c} = 0.0064
\end{equation}

$$M_{\text{max}} = 0.0064 \times M_c = 147.11 \text{ kNm}$$

For piles with free head

$A_y = 2.43$

$B_y = 1.62$

\begin{equation}
(5.14) \quad T = 3 \frac{y_t EI}{A_y H_t}
\end{equation}

\[
T = 3 \sqrt[3]{\frac{0.0254 \times 2 \times 10^8 \times 550^{-6}}{2.43 \times 78.21}} = 2.45 \text{ m}
\]

\begin{equation}
(5.15) \quad T_m = 2 \frac{y_t EI}{B_y M_t}
\end{equation}

\[
T_m = 2 \sqrt[3]{\frac{0.0254 \times 2 \times 10^8 \times 550^{-6}}{1.62 \times 313.61}} = 2.34 \text{ m}
\]

\begin{equation}
(5.16) \quad \frac{M_t}{M_c} = 0.013672
\end{equation}

$$M_t = 0.013672 \times M_c = 313.61 \text{ kNm}$$

For piles with fixed head
Ay = 0.93 given data

\[ \frac{H_t}{H_c} = 0.045572 \]

\[ H_t = 0.045572 \times H_c = 164.38 \text{ kN} \]

\[ \frac{M_t}{M_c} = 0.01760 \]

\[ M_t = 0.01760 \times M_c = 403.70 \text{ kNm} \]

\[ T = \frac{y_t EI}{A_y H_t} \]

\[ T = \frac{0.0254 \times 2 \times 10^8 \times 550^{-6}}{0.93 \times 164.38} = 2.63 \text{ m} \]

To be consistent with Evans and Duncan method the letter Ht was kept until now. From this point on, the notation Pt will be used for Ht in this study. The meaning of Pt is exactly the same as Ht, that is, it represents a lateral load applied to the pile-soil system.

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>Constraint</th>
<th>Free head</th>
<th>Fixed head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short pile</td>
<td>Load type</td>
<td>P_t</td>
<td>M_t</td>
</tr>
<tr>
<td>T</td>
<td>2.45</td>
<td>2.35</td>
<td>2.63</td>
</tr>
<tr>
<td>2T</td>
<td>4.90</td>
<td>4.69</td>
<td>5.27</td>
</tr>
<tr>
<td>3T</td>
<td>7.35</td>
<td>7.04</td>
<td>7.90</td>
</tr>
<tr>
<td>4T</td>
<td>9.80</td>
<td>9.38</td>
<td>10.54</td>
</tr>
<tr>
<td>5T</td>
<td>12.25</td>
<td>11.73</td>
<td>13.17</td>
</tr>
<tr>
<td>Long pile</td>
<td>6T</td>
<td>14.70</td>
<td>14.07</td>
</tr>
<tr>
<td>7T</td>
<td>17.15</td>
<td>16.42</td>
<td>18.44</td>
</tr>
<tr>
<td>8T</td>
<td>19.60</td>
<td>18.76</td>
<td>21.07</td>
</tr>
<tr>
<td>9T</td>
<td>22.05</td>
<td>21.11</td>
<td>23.71</td>
</tr>
<tr>
<td>10T</td>
<td>24.50</td>
<td>23.45</td>
<td>26.34</td>
</tr>
</tbody>
</table>

** Intermediate Pile

<table>
<thead>
<tr>
<th>**</th>
<th>**</th>
<th>**</th>
</tr>
</thead>
<tbody>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
</tr>
</tbody>
</table>

Table 5.3 The lengths of piles used in the sensitivity analysis

89
The relative stiffness factors $T$ determined are shown in Table (5.4).

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Relative stiffness factor $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free head pile under lateral force $P_t$</td>
<td>2.45 m</td>
</tr>
<tr>
<td>Fixed head pile under lateral force $P_t$</td>
<td>2.63 m</td>
</tr>
<tr>
<td>Free head pile under bending moment $M_t$</td>
<td>2.35 m</td>
</tr>
</tbody>
</table>

Table 5.4 The relative stiffness factor $T$ for different boundary conditions

5.5 Long and short piles - Considerations

The determination of the relative stiffness factor $T$ is an important fact, once the piles can be considered short, intermediate or long piles. According to Evans and Duncan (1982) a pile can be considered as short if its embedded length is less than $5T$, and "long pile" will be called the pile which has more than $5T$ embedded in a soil.

As pointed out by many authors the "long pile" and the "short pile" performs differently under the lateral load, and there is only small deflection at the bottom of long pile, so the long pile is considered as fixed at the bottom. The short pile keeps almost straight shape when the load is applied, and it rotates along a certain point located at the pile axis.

Other criteria are also important to be taken into account when the pile length is considered. In case of long relatively flexible piles, such as timber piles, this corresponds that the piles having length of at least 20 diameters. On the other hand, if the long piles are relatively stiff, such as those made of steel or concrete, the length considered must be at least 35 diameters.

5.6 Load-deflection relationship

For assessment of the nonlinearity of the pile-soil system subjected to a lateral load it is appropriate to utilize the load-deflection relationship, shown in charts for different values of $P_t$ and $M_t$. In order to give a brief idea about the nonlinearity of the pile-soil system for each type of constraints, some charts are presented.
The pile head load-deflection relationship for all cases covered in this study for different pile lengths, pile loads, pile constraints are included in the thesis and presented in the attached CD.

Figure 5.7 Pile head deflection $y$, versus lateral force $P$, applied to the top of the pile head for a free head pile embedded in a soft clay below water table. Pile length $L$ varies from 3T to 10 T.
Figure 5.8 Pile head deflection $y_t$ versus lateral force $P_t$ applied to the top of the pile head for a fixed head pile embedded in a soft clay below water table. Pile length $L=$ varies from $3T$ to $10T$.

Figure 5.9 Pile head deflection $y_t$ versus bending moment $M_t$ applied to the top of the pile head for a free head pile embedded in a soft clay below water table. Pile length $L=$ varies from $3T$ to $10T$. 

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5.7 The adjoint structure concept – internal forces and deformations

As pointed previously the consideration of the adjoint structure is critical for the solution of the model considered. The adjoint structure being considered as a deformed primary structure, that is subjected to unit load. It is made of the same material, having the same boundary conditions (and so on). The only difference between them is the virtual unit force applied to the structure.

To be able to carry out the solution, various steps for every type of constraints considered are presented on the Figure (5.10).

In the Figure (5.10) the system (p) denotes the primary structure subjected to a real loading, and the system (a) is the adjoint structure while the system (b) and (c) are the temporary systems used to calculate the internal forces and deformations of the adjoint structure for better accuracy. The relationship between (a), (b) and (c) results in the following outcome:

\[
(a) = \frac{(b) - (c)}{2}
\]

where the letters (a), (b) and (c) in parenthesis represent the adjoint pile-soil system, temporary over - loaded system and temporary under - loaded system shown in Figure 5.10, respectively.

The steps presented in Figure (5.10) take into account temporary systems. They are important once they are used to achieve more accurate results for the analysis of the adjoint structure shown in (a).

During all steps, the internal forces and deformations of the temporary systems (b) and (c) can readily be calculated through the COM624P program. Consequently, the difference of the system (b) and (c) can be determined by subtracting them, including lateral deflection, rotation, bending moment and soil reaction etc., in system (c) from the corresponding result in system (b).

The components of the adjoint system (a) are obtained by means of the equation (5.19). According to Liu (2004) to ensure the result gained from (b) and (c) produces the same physical status as primary structure, the force \( P_t \) or \( M_t \) was also applied in addition to the unit force in the temporary system (b) and (c).
Figure 5.10 The method used to calculate various components of internal forces of the adjoint structure when subjected to unit generalized load according to Liu (2004)
Figure 5.10 shows how the stresses and deformations are calculated when a lateral unit force is applied to the pile head of part i. Both primary and adjoint structure are subjected to a lateral loading. To be able to reach the solution for the adjoint structure, the system (b) and (c) are temporarily over-loaded and under-loaded.

In the other situations such as ii, iii, and iv of Figure 5.10, the steps (b) and (c) are always presented in a certain way to calculate the internal forces and deformations of the adjoint structure while the primary and adjoint structure are subjected to generalized unit load force and bending moment, bending moment and lateral load or bending moment and bending moment respectively.

The deflection of the adjoint structure $y_a$ are determined as:

\[
y_a = \frac{y_{(b)} - y_{(c)}}{2}
\]

the bending moment $M_a$ and

\[
M_a = \frac{M_{(b)} - M_{(c)}}{2}
\]

the soil resistance $p_a$ are determined in the formula below:

\[
p_a = \frac{p_{(b)} - p_{(c)}}{2}
\]

5.8 COM624P – Computer Program

Wang and Reese (1993) developed a computer program called COM624P for use in the analysis of stresses and deflection of piles or drilled shafts under lateral loads. The basic program presented by the authors was developed for the purpose of highway construction and that required application of microcomputers. The program solves the equations giving pile deflection, rotation, bending moment, and shear by using iterative procedures because of nonlinearity of the $p-y$ soil response. The beam-column-soil equations are solved by finite difference method.

According to Priyanto (2002) and Liu (2004) the program provides a user-friendly/menu-driven input and a graphics output in microcomputer environment. The version of the
program COM624P for the microcomputer was developed in 1989. Several new features were included in the program such as: generating \( p-y \) curves for rock, capability of analysis of piles embedded in sloping soil profiles. The COM624P of version 2.0 is used to conduct numerical research on sensitivity analysis of laterally loaded piles.

5.9 Results of sensitivity analysis of laterally loaded pile

The results of the sensitivity analysis are presented in the form of diagrams saved in the attached CD, whose content is shown in details in APPENDIX B ~ J. A typical example of the results of the sensitivity analysis for various types of bending load, geometry and boundary conditions are specify as follows:

- free head pile embedded in the soft clay below water table, subjected to lateral concentrated load, with pile length \( L = 4T, 5T \) and \( 10T \) m (APEENDIX B, C and D).
- free head pile embedded in the soft clay below water table, subject to bending moment, with pile length \( L = 4T, 5T \) and \( 10T \) m (APEENDIX E, F and G).
- fixed head pile embedded in the soft clay below water table, subjected to lateral concentrated load, with pile length \( L = 4T, 5T \), and \( 10T \) m (APEENDIX H, I and J).

The discussions and conclusions on the results of the sensitivity analysis are presented in Chapters 8 and 9.

5.10 Results of single piles – Method of verification

The numerical results obtained with COM624P and the results achieved through the sensitivity analysis equations should be verified and compared. The results are checked by introducing certain variations in the design variables. Through the equation (3.45) the relationship between the changes of the lateral deflection \( \Delta \) and the design variables are established.
The sensitivity factors $A(\ldots)$ are determined in the sensitivity analysis, and in order to check the accuracy of the sensitivity factors, a certain variation can be introduced in one of the design variables whereas the other design variables were left unchanged.

Then the change of the lateral deflection $\delta A$, obtained from the sensitivity analysis, due to variation of the design variable can be easily determined in accordance with Equation (3.45). On the other hand, the variation of the design variable can be introduced directly to the input file of the primary structure.

Utilizing COM624P it is possible to analyze the new deflection of the pile head under lateral load, related with the changed variable.

As an example, the boundary condition selected take into account a free head pile, and the original parameters which are given as follows: pile length $L=10\,\text{m}$, lateral load applied at pile head $F = 270\,\text{kN}$, the design variable $c = 23.94\,\text{kN/m}^2$ ($\sim 24\,\text{kN/m}^2$).

Based on the sensitivity analysis described, the sensitivity factor due to the changes of the design variable $c$ result in $A_c = -4.87921799038\,\text{kN/m}$ and the lateral deflection of primary structure was given as $y_{\text{top}} = 0.07613\,\text{m}$.

To be able to verify the variation of the design variable we apply $1/1000$ of deviation to the design variable $c$, that is:

\begin{equation}
\delta c = \frac{1}{1000} c = \frac{1}{1000} 24 = 0.024\,\text{kN/m}^2
\end{equation}

the deflection change $\delta A_1$ based on Equation (3.45) can be calculated as follows:

\begin{equation}
\delta A_1 = A_c \cdot \frac{\delta c}{c \cdot 1} = -4.87921799038 \frac{0.024}{24 \times 1} = -0.004879218\,\text{m}^2
\end{equation}

we can also change the parameter of the input file of laterally loaded pile analysis software such as FB-Pier to a new value by increasing $1/1000c$. The new value will be:
Equation (5.25) \[ c = 24 + \frac{1}{1000} \times 24 = 24.024 \text{ kN/m}^2 \]

The input of cohesion \( c \) shown in Equation (5.25) when considered as the input of file of FB-Pier results in the new deflection (due to the changed parameter) given as \( y_t = 0.3611 \) m.

The accuracy of the deflection change based on the FB-Pier calculation was obtained as follows:

\[ \delta \Delta_2 = y_{\text{top, changed parameter}} - y_{\text{top, primary structure (without changes)}} \]

\[ \delta \Delta_2 = 0.3611 - 0.3624 = -0.0013 \text{ m} \]

The relative error was given as follows:

\[ \text{relative error} = \frac{(\delta \Delta_t - \delta \Delta_2)}{\delta \Delta_2} \times 100\% \]

\[ \text{relative error} = \frac{\left( (-0.004879218) - (-0.0013) \right)}{(-0.0013)} \times 100\% = 2.75\% \]
CHAPTER 6
PILE GROUP – NUMERICAL INVESTIGATION

6.1 Introduction

The previous chapter took in account the numerical investigation of single piles. For pile groups the assumptions of the physical properties of the piles are exactly the same as previously considered for the single piles. However the behaviour of a pile in a group arrangement will be different than a single pile. The numerical analysis will be carry out on pile groups as it is showed in Figure 6.1. The figure shows the model adopted that consists of 9 piles with length equal 10T in a group configuration 3x3, and with a cap on the top. The p-multipliers are introduced into the soil-piles reaction model, and the software adopted in a pile group is the FB-Pier.

6.2 Loads and constraints

The cases concerned in this study take in account lateral force and bending moment applied on the pile-soil system. The piles in various pile groups are characterized by different spacing, that varies from 2D to 5D. The type of pile constraints used in the pile groups are: pinned or fixed.

➢ Study 1 Sensitivity analysis of top lateral deflection $\delta_{y_i}$ for pile groups with the piles pinned to the cap subjected to lateral concentrated force. The analysis is carried out for the piles located in the center of the leading row, first trailing row and second trailing row. The pile spacing “s” is equal to 2D, 3D, 4D and 5D. The pile length L is equal to 10T (24.5 m).

➢ Study 2 Sensitivity analysis of top lateral deflection $\delta_{y_i}$ for the pile groups with the piles fixed to the cap subjected to lateral concentrated force. The analysis is conducted for the piles located in center of the leading row, first trailing row

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and second trailing row. The pile spacing “s” is equal to 2D, 3D, 4D and 5D. The pile length L is equal to 10T (26.3 m).

**Study 3** Sensitivity analysis of top lateral deflection $\delta_y$ for pile groups with the piles pinned to the cap subjected to bending moment at the pile head. The analysis is carried out for the piles located in the center of the leading row, first trailing row and second trailing row. The pile distance “s” is equal to 2D, 3D, 4D and 5D. The pile length L is equal to 10T (23.5 m).

![Diagram of pile group geometry](image)

Figure 6.1 Pile group geometry used in the pile group analysis
6.3 The determination of lateral forces $P_g$ and $P_{gi}$—Lateral forces used in case study 1 and 2

6.3.1 Primary structure of pile group when the system is subjected to lateral force

The force $P_g$, is the force applied to the cap of a primary pile group. The basic concept of analysis of pile group when subjected to lateral load is based on the principle that the pile group will produce the same deflection under the force $P_g$ as the single pile under lateral force $P_t$.

In order to compare the sensitivity analysis of single piles with those obtained from sensitivity analysis of the piles in a group, the previous concept has to be incorporated. To be able to determine the force $P_g$ this study took into account the utilization of the values already calculated for single piles, and utilizing the MATLAB to run the FB-Pier program for the pile groups, and then combining the both lateral deflections and lateral forces results. The results of lateral deflections and lateral load were plotted together in order to obtain the $P_g$ required in the pile group to carry out the same deflection produced by the force $P_t$ employed in the analysis of the single piles. Figure (6.2) shows (as an example) a single free head pile subjected to lateral concentrated force applied to the pile head, and the piles in the pile group are pinned to the pile cap. The pile group is subjected to lateral concentrated force at the pile cap. The better understanding of the force applied and the model itself can be viewed on the Figure (6.7).

On the Figure (6.2) the ordinate values represents the external force $P_g$ that the pile group is subjected, and also the external lateral force $P_t$ applied to the pile head of single pile. The values represented by the abscissa correspond to the lateral deflections produced by the application of external force $P_g$ or $P_t$.

It is also possible to obtain the $P_g$ versus $y_{top}$ relationships for pile groups by using the FB-Pier program to calculate the deflection corresponding to force $P_g$ series and then plot the result in the same coordinate system with the single pile.

Figure (6.2) shows that it is possible to determine the force $P_g$ required to acquire the same deflection produced by the force $P_t$ employed in sensitivity analysis of single isolated pile.
The determination of $P_g$ is based on the principle that the pile group will produce the same deflection under the force $P_g$ as the single pile lateral force $P_t$.

**Figure 6.2** Determination of the force $P_g$ applied to the cap of the piles pinned to the cap (with variable spacing) subjected to lateral concentrated force. Pile group spacing “$s$” is equal to 2D, 3D, 4D and 5D. The pile length $L$ is equal to 10T (24.5 m).
6.3.2 Adjoint structure of pile group when the pile group is loaded by lateral force

In order to perform a numerical investigations and further sensitivity analysis on the piles inside of the group of piles a model consisted of the primary and adjoint structure should be used. The concept is very close to the model adopted in the analysis of the single piles previously presented in this study.

Once the force $P_g$ was determined as explained before, the system for the primary structure is completely determined. However, the system requires an analysis of adjoint structure, and to carry out this analysis is imperative that we determine the values of lateral forces $P_{gi}$ applied at the pile cap that will result in application of the unit force to the pile member under investigation.

The distribution of the load applied to the cap to the pile members is not even, which means that the values of $P_{gi}$ will vary according to type of row (leading or trailing) that each pile stay in a group. The utilization of the $f_m$ multipliers, already described in Chapter 4, will be applied in the analysis. The unit lateral force $P_{gi}$ related to the pile member under study, is the force applied to the pile cap that will result in the shear force reaction of the head of the pile member under study equal to unit force 1.

Figure (6.7) presents the model adopted, and the utilization of the adjoint structure can be better visualized. The necessity of the two steps that are described as a “temporary over-loaded structure” and “temporary under-loaded structure” are particularly important to determine the internal forces of the adjoint structure. They are shown in the Figure (6.7) as (d) and (e).

This model of analysis of adjoint pile group system was used previously by different authors, but the most clear explanation was outlined in Liu (2004) and Rahman (2004). They both considered that after the application of the force $P_g$, there will be shear forces. Those shear forces were indicated in the Figure (6.7) as $V_1$-$V_9$, and they were produced at each pile head.

Once this statement is valid the following equation is valid for the shear forces:

\[
P_g = \sum_{i=1}^{9} V_i
\]

(6.1)
As an example to demonstrate the understanding of the model, the pile number 5 inside of the pile group will be investigated and analyzed in order to perform the sensitivity analysis. The pile chosen to analysis appears to be inside of the shaded area in the Figure 6.7(c). According to the model adopted, an adjoint structure utilizing $P_{g_1}$ should be part of the analyze. The model itself requires to find a force $P_{g_1}$ applied to the pile cap of the adjoint structure that will produce unit shear force at the pile head of the shaded pile. In that case the force $P_{g_1}$ can be calculated as follows:

\[
P_{g_1} = \frac{\sum_{i=1}^{9} V_i \cdot \bar{I}}{V_5 \cdot \bar{I}} = \frac{P_g \cdot \bar{I}}{V_5 \cdot \bar{I}}
\]

Where:

$P_{g_1}$ = the lateral force applied to the adjoint structure pile cap that will produce unit shear force at the pile head under analysis;

$V_i$ = the shear force produced at the pile head number $i$ by the application of the force $P_g$ to the pile cap of the primary structure;

$V_5$ = the shear force produced at the head of the pile number 5 in the shaded area (called in this study as Pile B) by the application of the force $P_g$ to the pile cap of the primary structure.

To be able to perform the analysis of each pile, the force $P_{g_1}$ applied to a pile subjected to analysis can be determined through the following equation:

\[
P_{g_1} = \frac{\sum_{i=1}^{9} V_i \cdot \bar{I}}{V_j \cdot \bar{I}} = \frac{P_g \cdot \bar{I}}{V_j \cdot \bar{I}}
\]

\[
\frac{P_{g_1}}{I} = \frac{P_g}{V_j}
\]

Where

$V_j$ = the shear force produced at the top of pile under consideration by the application of the force $P_g$ to the pile cap of the primary structure.

Some results of $P_{g_1}$ for different $P_g$ applied are presented in the Figures (6.3) through (6.6). The figures present also the values of $P_{g_1}$ versus $P_g$ that will be consider in this analysis. To determine the value of $P_{g_1}$, as pointed before, the system requires the
utilization of other two "helpers" structures, that will be acting as a temporary ones. Again the Figure (6.7) describes in details the model and its components. The equation below should be applied in order to determine the internal forces of adjoint structure described by the model;

\[
(6.5) \quad (c) = \frac{1}{2} [(d) - (e)]
\]

Where

\( (c) \) = method of determination of internal forces of the adjoint structure with \( P_{gi} \) applied at the pile cap,

\( (d) \) = adjoint system that over-load the temporary structure that has the same physical properties as the primary structure but loaded by the force \( (P_g + P_{gi}) \) applied to the pile cap since the sensitivity analysis is conducted in the vicinity of load \( P_g \),

\( (e) \) = adjoint system that under-loaded temporary structure that has the same physical properties as the primary structure but under loaded by the force \( (P_g - P_{gi}) \) applied at the pile cap since the sensitivity analysis is conducted in the vicinity of load \( P_g \).

![Figure 6.3](image-url)  

**Figure 6.3** Force \( P_{gi} \) of Pile A (2nd trailing row), Pile B (1st trailing row), Pile C (leading row) of group of 3x3 piles with the spacing 2D and the length \( L=10T \) versus the applied lateral concentrated force \( P_g \) when the piles are pinned to the cap.
Figure 6.4  Force $P_{g1}$ of Pile A (2$^{nd}$ trailing row), Pile B (1$^{st}$ trailing row), Pile C (leading row) of group of 3x3 piles with the spacing 3D and the length $L=10T$ versus the applied lateral concentrated force $P_{g}$ when the piles are pinned to the cap.
Figure 6.5  Force $Pg_i$ of Pile A (2nd trailing row), Pile B (1st trailing row), Pile C (leading row) of group of 3x3 piles with the spacing 4D and the length $L=10T$ versus the applied lateral concentrated force $Pg$ when the piles are pinned to the cap.

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Figure 6.6  Force $P_{g1}$ of Pile A (2\textsuperscript{nd} trailing row), Pile B (1\textsuperscript{st} trailing row), Pile C (leading row) of group of 3x3 piles with the spacing 5D and the length $L=10T$ versus the applied lateral concentrated force $P_g$ when the piles are pinned to the cap.
6.4 The determination of bending moments \( M_g \) and \( M_{g1} \)—Bending moments used in case study 3

6.4.1 Primary structure when the pile groups is subjected to bending moment

The bending moment denoted as \( M_g \) is the bending moment applied to the pile head of each pile in the pile group. The concept is based on the principle that the pile group will
produce the same pile head deflection under the bending moment $M_g$, which is applied to the pile heads directly inside the pile group, as the single pile under bending moment $M_t$.

To be able to compare the sensitivity analysis of single piles with the sensitivity analysis to the pile pinned connection that is located below the rigid pile group cap of each pile in a pile group, the previous concept has to be guaranteed.

To determine the bending moment $M_g$, the study took into account the utilization of the values already calculated for single piles, and employing the MATLAB to run the FB-Pier Program for the pile groups. Then the analysis is focused on combining both lateral deflections and bending moments results. The results of lateral deflections and bending moment are plotted together in order to obtain the $M_g$ used in the pile group to carry out the same deflection produced by the bending moment $M_t$ employed in the analysis of the single piles.

The Figure (6.8) (shows as an example) the comparison of the bending moment of a single free head pile subjected to bending moment at the pile head, and the pile group with the piles in that are pinned to the pile group cap. The pile group of Figure (6.8) is subjected to bending moment. The analysis of the bending moment applied and the pile group model ($M_g$) that is distributed equally to the piles pinned is shown in Figure (6.9).

In the Figure (6.8) the ordinate values represent the bending moment $M_g$ that the pile group is subjected to, and the bending moment $M_t$ applied to the pile head of a single pile. The values represented by the abscissa correspond to the lateral deflections produced by the application of bending moment $M_g$ or $M_t$.

The relationship of $M_g$ versus $y_{top}$ for pile groups it is also possible to obtain by using the FB-Pier program. It allows to calculate the deflection corresponding to bending moment $M_g$ and then to plot the result in the same coordinate system together with the single pile. By means of Figure (6.8) it is possible to determine the bending moment $M_g$ required to acquire the same deflection produced by the bending moment $M_t$ employed in sensitivity analysis of single isolated pile.
Figure 6.8  Determination of the total bending moment Mg applied to the pile group cap of the piles pinned to the cap subjected to bending moment at the pile head. Pile group spacing “s” is equal to 2D, 3D, 4D and 5D. The pile length L is equal to 10T (23.5 m)
6.4.2 Adjoint structure when pile group is subjected to bending moment

In order to perform a numerical investigation and further sensitivity analysis of the piles inside of the pile group a model consisted of primary and adjoint structure should be applied.

Previously the bending moment \( M_g \) was determined as explained before. Thus, the system of the primary structure is completely determined. However, the system requires an analysis of adjoint structure. To carry out the sensitivity analysis it is an imperative that we determine the values of lateral forces \( P_{gi} \) applied at the pile cap that will result in unit shear force reaction that is applied to the pile member under investigation.

This model was also used previously by different authors, such as Liu (2004) and Rahman (2004). In order to find out the force \( P_{gi} \), it is necessary to introduce a pile group load shown in Figure (6. 9 (c)), in which a force \( \overline{9} \) (the force necessary to determine lateral displacement of pile group when the primary structure is subjected either lateral force or bending moment) is applied in addition to the primary structure shown in Figure (6. 9 (b)). The force \( \overline{9} \) is the multiple of \( \overline{1} \) and the number of pile members (9) in the pile group arrangement.

Figure (6.9) presents in graphical way the methodology used for the determination of the adjoint load applied to the adjoint structure when the primary pile group is subjected to bending moment \( M_g \).

The final value of the force \( P_{gi} \) can be reached by subtracting the shear forces produced at the pile heads of structure shown in Figure (6. 9 (b)) from the shear forces produced at the pile heads of structure shown in Figure (6. 9 (c)).

The resultant of the operation "(c)-(b)" is indicated in Figure (6. 9 (d)), in which the shear force differences \( \Delta V_i - \Delta V_9 \) exist at the heads of each pile in the pile group.

\[
\bar{9} = \sum_{i=1}^{9} \Delta V_i
\]

As an example to demonstrate the understanding of the methodology of the load application to the adjoint pile group system, the pile number 5 is investigated and analyzed in order to perform the sensitivity analysis. The pile chosen appears inside of the shaded area in the Figure 6.9(e), and according to the methodology developed, the adjoint
structure utilizing $P_{gi}$ should be part of the analysis. The model itself requires to find a force $P_{gi}$ applied to the pile cap of the adjoint structure that will produce unit shear force at the pile head of the shaded pile. In that case the force $P_{gi}$ can be calculated as follows:

\[
(P_{gi}) = \frac{9}{\Delta V_i} \sum_{i=1}^{9} \frac{\Delta V_i}{\Delta V_i} \cdot \bar{I} = \frac{9}{\Delta V_i}
\]

The force $P_{gi}$ for the pile group (3x3) consists of 9 piles that can be obtained by means of the following equation:

\[
(P_{gi}) = \frac{9}{\Delta V_j} \sum_{i=1}^{9} \frac{\Delta V_i}{\Delta V_j} = \frac{9}{\Delta V_j}
\]

where
- $P_{gi}$ = the lateral force applied to the adjoint pile group cap that will produce unit shear force at the pile head under analysis,
- $\Delta V_i$ = the shear forces difference produced at the pile head by subtracting the shear forces produced at the pile heads of pile group system shown in Figure (6.9 (b)) from the shear forces produced at the pile heads of pile group system shown in Figure (6.9 (c)),
- $\Delta V_j$ = the shear force difference produced at the top of the pile subjected to analysis by subtracting the shear forces produced at the pile head j of pile group system shown in Figure (6.9 (b)) from the shear forces produced at the pile head j of pile group system shown in Figure (6.9 (c)).

As stated in the previous model, in order to increase the accuracy of the analysis, the additional steps using "helpers" structures are required. They are described as a "temporarily over-loaded structure" and "temporarily under-loaded structure". They are shown in the Figure (6.9) as (f) and (g).

Once those procedures are implemented the following equation is used for determination of the adjoint shear forces:

\[
(6.9) \quad (e) = \frac{1}{2} [(f) - (g)]
\]
Where

(e) = the adjoint structure with \( P_{gi} \) applied at the pile cap,

(f) = the temporally over-loaded pile group system which has the same physical properties as the primary structure and subjected to the lateral force \( P_{gi} \) applied to the pile cap in addition to the \( M_g \) acting at the pile heads in a pile group,

(g) = the temporally under-loaded pile group system structure that has the same physical properties as the primary structure and with force \(-P_{gi}\) applied at the pile cap in addition to the \( M_g \) acting at the pile head of piles in a pile group.

Figure 6.9 Method used to determine load \( P_{gi} \) applied to the adjoint pile group when the primary pile groups is loaded by bending moment \( M_g \) applied to the pile head of members in a pile group
6.5 Results of the sensitivity analysis of laterally loaded pile groups

The results as an example of the sensitivity analysis for pile B (First trailing row) in a pile group embedded in the soft clay below water table, subjected to lateral concentrated force \( P_g \) applied to the pile cap, with the pile members pinned to the pile group cap, having spacing \( S = 2D \), with pile length \( L = 10T = 24.5 \text{m} \) and are shown in the APPENDIX K.
CHAPTER 7

COMPUTER PROGRAMS USED TO PERFORM THE SENSITIVITY ANALYSIS ON SINGLE PILES AND PILE GROUPS

7.1 Overview

The computer programs used to perform sensitivity analysis of laterally loaded piles subject to lateral concentrated load and bending moment are COM624P, FB-Pier, MATLAB, and Microsoft Windows Excel. The theoretical formulation of sensitivity analysis developed is presented in Chapter 3. To investigate the sensitivity performance of single piles and piles in a pile group subjected to different types of load requires the initial structural analysis of the piles embedded in soft clay below the water table. The distribution of bending moments, deflections and soil reaction along the pile length are the basic data information on the structure that are necessary and essentials to carry out the sensitivity analysis of the pile system.

The basic two programs that are fundamental in this study are COM624P version 2.0 and FB-Pier version 3.0. Each of them takes into account the $p-y$ curve model developed by Matlock in the 70's and applicable to the type of soil that surrounds the piles. The COM624P performs the analysis of primary and adjoint structure of single piles, using finite difference method for spatial discretization. It determines the soil reactions, bending moments, lateral deflections, and shearing forces along the pile length using an iterative process that considers a non-linear response of the soft clay below the water table where the pile is embedded. The COM624P program can be used to analyse a great variety of elements with different boundaries conditions. FB-Pier is another program of the great importance in this study. It refers not only structural aspects but also to numerous geotechnical characteristics, as well as the soil-structure design possibilities. The program was used to perform the analysis of the piles in a pile group using a non-linear Finite Element Method (FEM) to investigate the $p-y$ soil-pile system. The flow chart of the sensitivity analysis of performed piles is presented in Figures (7.1) and (7.2).
Figure 7.1 Flow chart of sensitivity analysis of the laterally loaded piles (Part 1)
Figure 7.2 Flow chart of sensitivity analysis of the laterally loaded piles (Part 2)
7.2 Performance of sensitivity analysis on single piles

The sensitivity analysis of the single piles that is to be performed requires taking into account different boundaries conditions described previously. In order to generate numerous input files Liu (2004) developed (using MATLAB) a series of modules that aim at the preparation of the input files for COM624P. Their purpose is to determine the internal forces of the primary and adjoint structures with different boundary conditions (kinematics), pile length, type and magnitude of the forces applied, constraint types, and properties. The referred author conducted such analysis using a different model of \( p-y \) curve, for the soil utilized. The referred researcher employed stiff clay, and the geometrical characteristics were entirely different than those used in this study.

Based on Liu (2004) MATLAB modules a new sequence was developed in order to perform the analysis using other conditions that are appropriate for this study, and the soil present.

The sequence of MATLAB modulus that are necessary to create the input files using COM624P program aims at perform once the analysis of laterally loaded piles, them to calculate and plot the results as follows:

1. Step 1: preparing and input file generating the module

MATLAB files – gendirectory1.m
geninput1.m

```matlab
drive='C:\';
maindir='SINGLE';
for SupportStyle=1:1:3
    for PileLength=0:1:9
        for PercentClay=1:1:11
            sSS=num2str(SupportStyle);
            sPL=num2str(PileLength);
            if PercentClay<10
                sPC=strcat('0',num2str(PercentClay));
            else
                sPC=num2str(PercentClay);
            end
            mkdir(drive,strcat(maindir,\',',sSS,\',',sPL,\',',sPC));
        end
    end
end
```

Figure 7.3 MATLAB file “gendirectory.m”

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2. Step 2: calculating module

MATLAB files – calculate1.m

```matlab
OriginalInputFileName=strcat(sSS,sPL,sPC,sForceVector,'0');
MinusInputFileName=strcat(sSS,sPL,sPC,sForceVector,'1');
PlusInputFileName=strcat(sSS,sPL,sPC,sForceVector,'2');
AdjointOtherLessInput=strcat(sSS,sPL,sPC,sForceVector,'3');
AdjointOtherMoreInput=strcat(sSS,sPL,sPC,sForceVector,'4');

dos([compath ' ' strcat(OriginalInputFileName, '.inp') ' ' strcat(OriginalInputFileName, '.out') ' ' strcat(OriginalInputFileName, '.grh')]);
dos([compath ' ' strcat(MinusInputFileName, '.inp') ' ' strcat(MinusInputFileName, '.out') ' ' strcat(MinusInputFileName, '.grh')]);
dos([compath ' ' strcat(PlusInputFileName, '.inp') ' ' strcat(PlusInputFileName, '.out') ' ' strcat(PlusInputFileName, '.grh')]);
dos([compath ' ' strcat(AdjointOtherLessInput, '.inp') ' ' strcat(AdjointOtherLessInput, '.out') ' ' strcat(AdjointOtherLessInput, '.grh')]);
dos([compath ' ' strcat(AdjointOtherMoreInput, '.inp') ' ' strcat(AdjointOtherMoreInput, '.out') ' ' strcat(AdjointOtherMoreInput, '.grh')]);

ResultData=readdatal(strcat(OriginalInputFileName,'.grh'));
```

Figure 7.4 Part of the MATLAB file called “calculate1.m” where a function is created to read the data from COM624P analysis performed on primary and adjoint structure

3. Step 2A: creation of sub function to read the data

MATLAB file – readdatal1.m

```matlab
function ResultData=readdatal(DataFileName);
nNodes=101;
hfDataFile=fopen(DataFileName,'r t');
if hfDataFile==-1
    hfDataFile
    pause;
end
for u=1:1:3
    sLine=fgetl(hfDataFile);
end
ResultData=fscanf(hfDataFile,'%f',[5,nNodes]);
ResultData=ResultData1;
fclose(hfDataFile);
```

Figure 7.5 MATLAB sub function called “readdatal1.m”

4. Step 2B: creation of sub function to plot the results in a graph format

MATLAB file – plotfig4ppp1.m

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Figure 7.6 Part of the MATLAB file called “plotfig4pppl.m” where a function is created to plot the data that are read from COM624P analysis performed for primary and adjoint structure

5. Step 3: sensitivity analysis integrands module

MATLAB file – chartplot1.m

6. Step 4: sensitivity analysis module

MATLAB file – sanalysis1.m
load('ymc.dat','-mat');

ForceVectorLength=size(CEIMatrix,2);

AEI=simpsonquad(CEIMatrix,RealLength);
Ac=simpsonquad(CcMatrix,RealLength);
Agamac=simpsonquad(CgamacMatrix,RealLength);
Ab=simpsonquad(CbMatrix,RealLength);
Ae50=simpsonquad(Ce50Matrix,RealLength);
ATotal=abs(AEI)+abs(Ac)+abs(Ab)+abs(Agamac)+abs(Ae50);
FEI=abs(AEI)./ATotal.*100;
Fc=abs(Ac)./ATotal.*100;
Fb=abs(Ab)./ATotal.*100;
F50=abs(Ae50)./ATotal.*100;
if SupportStyle~=2
    AEIo=simpsonquad(CEIOtherMatrix,RealLength);
    Aco=simpsonquad(CcOtherMatrix,RealLength);
    Agamaco=simpsonquad(CgamacOtherMatrix,RealLength);
    Abo=simpsonquad(CbOtherMatrix,RealLength);
    Ae50o=simpsonquad(Ce50OtherMatrix,RealLength);
    ATotalo=abs(AEIo)+abs(Aco)+abs(Abo)+abs(Agamaco);
    FEIo=abs(AEIo)./ATotalo.*100;
    Fco=abs(Aco)./ATotalo.*100;
    Fgmaco=abs(Agamaco)./ATotalo.*100;
    Fbo=abs(Abo)./ATotalo.*100;
    Fe50o=abs(Ae50o)./ATotalo.*100;
end
if PercentClay<10
    sPercentClay=strcat(num2str(10*PercentClay) ,'%');
elseif PercentClay ==10
    sPercentClay='100% clay where pile is resting on sand'
else
    sPercentClay='100% clay where pile is resting on clay';
end
save('SAResult.dat', 'AEI', 'Ac', 'Ab', ...
    'FEI', 'Fc', 'Fb', ...
    'FEIo', 'Fco', 'Fbo', ...
    'PercentClay = ' sPercentClay ]); sXLabel=[FType,' (',FUnit,')1; sTitle={['Fig. ',num2str(PageNumber),' Quantitative assessment of...
7. Step 4A: plot sub function
MATLAB file - plotbar.m
8. **STEP 4B: numerical integration using Simpson’s rule**

**MATLAB file – simpsonquad.m**

```matlab
function QuadResult=simpsonquad(Matrix,L)
%L=Length of pile
[nRow,nColumn]=size(Matrix);
if rem(nRow,2)==0
    disp('There should be even number segments');
    return;
end
QuadResult=[];
for i=1:nColumn
    QuadColumn=0;
    for j=2:2:(nRow-1)
        QuadLength=2*L/(nRow-1);
        QuadColumn=QuadColumn+QuadLength*(Matrix(j-1,i)+4*Matrix(j,i)+Matrix(j+1,i))/6;
    end
    QuadResult(i,1)=QuadColumn;
end
```

**Figure 7.9 MATLAB sub function called “simpsonquad.m”**

A characteristic input file of the COM624P can be viewed in the Appendix L, section L.1. For more details of the input file, the acceptance refer to the COM624P manual is recommended.
7.3  **Performance of sensitivity analysis of pile groups**

In order to generate numerous input files for pile groups Liu (2004) using MATLAB developed series of modules that prepare the input files for FB-Pier to calculate the primary and adjoint structures with different input parameters, in terms of the pile length, type and magnitude of the forces applied, constraint types, and properties. To perform the sensitivity analysis of pile groups the referred author conducted his analysis using a different model of p-y curve than those used in current studies (soft clay). Moreover the parameters used in previous studies were entirely different than those used in this study. Based on Liu (2004) MATLAB modules a new sequence was created in order to perform the sensitivity analysis using the parameters that are appropriate for this study and the soil model employed.

The sequence of MATLAB modulus that required to create the input files using FB-Pier program that generates the input files to perform the sensitivity analysis of laterally loaded piles.

The input file name convention presented in Figure (7.10) was created by Liu (2004) and was utilized in this study. The file name proposed by Liu (2004) covers numerous parameters with respect to the pile distance, pile length, load type, constraints and others. The sequence of MATLAB modulus that is required to create the input files using FB-Pier is the following;

1. **Step 1: preparing and input file generating the module.**

   This module is used to generate the directory system in which the input and result data are saved. This file generates two files, ‘C:\G’ and ‘C:\GSA’, where the first one holds the input data and the second one is used to hold the sensitivity analysis result data. However, this study at this point, just will utilize the MATLAB files to generate the input files. The sensitivity analysis performed for the pile groups will be executed with the aid of Excel.

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2. Step 2: input file generating the module -

MATLAB file – MMgroupgeninputproducemdata.m

The input files are generated producing the force applied versus the lateral deflection at the top of the pile graphics. Along with the data generated, the module is used to localize the lateral force Pg and the bending moment Mg presented in the Figure 6.2 and Figure 6.8.

```
clear all;
drive='C:\';
maindir='G';
PileDiameter=0.508;
NumberPileNode=51;
fmultifier=[0.44,0.56,0.76;
          0.58,0.67,0.82;
          0.72,0.78,0.88;
          0.86,0.89,0.94];
for SupportStyle=1:1:3
   switch SupportStyle
      case 1
         sSupportStyle='O';
         Force=150:150:2700;
         T=2.45;
      case 2
         sSupportStyle='1';
         Force=200:200:3000;
         T=2.63;
      case 3
         sSupportStyle='O';
         Force=50:50:800;
         T=2.35;
   end
for PileDistance=2:1:5
   switch PileDistance
      case 2
         fMu=fmultifier(1,:);
   end
end
```

Figure 7.11 Part of the MATLAB file called “MMgroupgeninputproducemdata.m” that generates the input files resulting in Pg, and Mg data versus lateral deflection
3. Step 3: this module calculates the input files

MATLAB file – MMgroupcalculate.m

The files generated by step 2 are here calculated. The MATLAB command alone the FB-Pier program to calculate of the laterally loaded piles can be viewed in the figure below.

```matlab
function ForceVector = MMgroupcalculate()
    case 3
        ForceVector = [0,0,0,0,Force(ForceStep),0];
        nForceVector = ForceVector(5);
        SupportName = 'Free Head Bending Moment';
        if (nForceVector<100)
            sForceVector = strcat('00',num2str(nForceVector));
        elseif (nForceVector<1000)
            sForceVector = strcat('0',num2str(nForceVector));
        else
            sForceVector = num2str(nForceVector);
        end
        end
    end
    InputFileName = strcat(sDrivePath,sSS,sPD,sLC,sForceVector,'0','in');
    OutputFileName = strcat(sDrivePath,sSS,sPD,sLC,sForceVector,'0','out');
    FBEXE = 'C:\Program Files\FBPier eng.exe m:256';
    dos([FBEXE, 'I;',InputFileName, 'O:', OutputFileName, '1']);
    InputFileName = strcat(sDrivePath,sSS,sPD,sLC,sForceVector,'2','in');
    OutputFileName = strcat(sDrivePath,sSS,sPD,sLC,sForceVector,'2','out');
    dos([FBEXE, 'I:',InputFileName, 'O:', OutputFileName, '1']);
    ForceStep = ForceStep + 1;
    if (RowofForce>ColumnofForce)
        break;
    end
end
```

Figure 7.12 Part of the MATLAB file called “MMgroupcalculate.m” that calculates the input files produced by “MMgroupgeninputproducepdelta.m”

4. Step 4: calculate the force Pg and the bending moment Mg determined previously in Chapter 6, Sections 6.3 and 6.4, and described also in Figures (6.7) and (6.9).

MATLAB file – MMgroupplotdelta.m

The lateral forces Pg and bending moments Mg calculated are plotted and presented in graphical format in the Figures (6.2) and (6.8).
case 3
    ForceVector=[0,0,0,0,Force(ForceStep),0];
    nForceVector=ForceVector(5);
    SupportName='Free Head Bending Moment';
    if (nForceVector<100)
        sForceVector=strcat('00',num2str(nForceVector));
    elseif (nForceVector<1000)
        sForceVector=strcat('0',num2str(nForceVector));
    else
        sForceVector=num2str(nForceVector);
    end
    DataFileName=strcat(sDrivePath,sSS,sPD,sLC,sForceVector,10 ');
    [ResultDataM,
    ResultDatay]=groupreaddata(DataFileName,NumberPile,NumberPileNode);
    switch SupportStyle
    case {1,2}
        Delta(PileDistance-1,ForceStep)=ResultDatay(((4)*NumberPileNode+l) , 2) ;
    case 3
        Delta(PileDistance-1,ForceStep)=
    ResultDatay(((4)*NumberPileNode+l),2);
    end
    ForceStep=ForceStep+1;
    [RowofForce,ColumnofForce]=size(Force);
    if (ForceStep>ColumnofForce)
        break;
    end
    end
    switch SupportStyle
    case 1
        ForcePoly=150:150:2700;
        ForceUnit='kN ';
    case 2
        ForcePoly=200:200:3000;
        ForceUnit='kN ';
end

Figure 7.13 Part of the MATLAB file called “MMgroupplotdelta.m” that calculates the Pg and Mg and presents them in a graphic format

5. Step 4A: read data sub function

MATLAB file – groupreaddata.m

function [ResultDataM, ResultDatay,ResultDataP]=groupreaddata(DataFileName,
NumberPile,NumberPileNode)
    hfDataFile=fopen([DataFileName, '.VMW'],'r ');
    if hfDataFile=-1
        hfDataFile
        pause;
    end
    ResultDataM=fread(hfDataFile, [15,inf], 'float32');
    ResultDataM=ResultDataM';
    fclose(hfDataFile);
    hfDataFile=fopen([DataFileName, '.pil'],'r ');
    if hfDataFile=-1
        Message='error in open source file'
        pause;
end

Figure 7.14 Part of the MATLAB file called “groupreaddata.m” that is a sub function that reads the data of “MMgroupplotdelta.m”

From this point, the sensitivity analysis of the pile groups performed is conducted using Excel program.
CHAPTER 8
DISCUSSION

8.1 Introduction

A general discussion on the sensitivity analysis of lateral deflection that occurred at the top of the pile embedded in soft clay below the ground water will be addressed in this Chapter. The case selected to be discussed is the case study that takes in account the pile of length \( L = 10T \) subjected to a lateral static load \( P_t \). Although the other cases are equally important they will be considered and discussed in the next Chapter.

In the Appendices B ~ J the results of pile lateral deflections, bending moments, soil reactions, sensitivity operators are plotted against the depth of the pile embedded in soft clay below the ground water table.

The first eleven figures presented in the Appendices B ~ J showed that the abscissas are associated with the lateral deflection, bending moments, soil reactions and sensitivity operators distributed along the depth of the pile embedded in the soft clay. In those figures the values of the ordinates show the depth \( x \), which is given using units of meters and relative stiffness factors, \( T \). The different concentrated static loads applied can be viewed in the legend.

8.2 General discussion on the sensitivity analysis results

The general discussion will be addressed to the case of the sensitivity analysis of lateral deflection at the top of the free head pile embedded in a soft clay below the ground water table having length \( L = 10T \) subjected to concentrated lateral forces \( P_t \). The other two cases will be discussed in the next Chapter. This section discusses each of the sensitivity operators and describes also physical interpretations.

All the results of sensitivity analysis of the free head pile that will be discussed are present in the Appendix D, free head pile subjected to lateral load with length equal 10T.
The pile results in terms of the distributions of lateral deflections, bending moments and soil reactions along the length of the pile for both primary and adjoint structure are also presented in Appendix D.

8.2.1 Discussion on the lateral deflections and bending moments of the primary structure

The lateral deflection of primary structure for a free head pile subjected to a lateral static load $P_t$ is presented in Appendix D but also can be viewed on Figure (8.1), below. The relationship between lateral forces applied and lateral deflections is nonlinear, and the deflections increase as the load applied increase.

![Figure 8.1](image)

**Figure 8.1** Primary structure - Pile head deflection $y_t$ versus lateral static load $P_t$ applied to the pile head – Free head – Length of the pile $L=10T$

The distribution of the bending moments of primary structure for a free head pile subjected to variable static lateral force $P_t$ can be viewed in Figure D2, and it shows that the maximum bending moment for all loads applied occurs at the depth around top $2T$. Also, a non-linear relationship exist between increasing load applies and the maximum bending moment.

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8.2.2 Discussion on the lateral deflections and bending moments of the adjoint structure

The distribution of the lateral deflections $y_a$ (m) of the adjoint structures subjected to $P = 1$ when the primary structure is subjected to the lateral static load $P_t$ shows that the relationship between lateral deflection $y_a$ and load applied $P_t$ is almost linear as it is presented in the Figure (8.2.)

![Graph showing the relationship between lateral deflection $y_t$ and load applied $P_t$.]

**Figure 8.2** Adjoint Structure - Pile head deflection $y_t$ versus lateral static load $P_t$ applied to the primary pile head – Free head – Length of the pile $L=10T$

The distribution of the lateral deflection shown in Figure D3 demonstrates that the major factors that promote changes in the lateral deflections of the adjoint structure subjected to horizontal force $\bar{P} = 1$ being in the state of deformation of the primary structure is the magnitude of the lateral force applied to the primary structure.

Figure D6 presents the distribution of the soil resistance $p_a$ of the adjoint structure subjected to $\bar{P} = 1$ when the primary structure is subjected to the lateral force $P_t$. It is possible to observe that the soil resistance increases as the depth becomes larger, and
when the maximum value of $p_a$ is reached then, the soil resistance started to decrease and it became very small. Also the value of the soil resistance $p_a$ is equal zero at the depth when the lateral deflection $y_a (\bar{P})$ is equal zero. It changes the sign when the lateral deflection $y_a (\bar{P})$ changes sign.

When larger load is applied, the lateral deflection increase, and the soil resistance become weaker. Thus, for the adjoint structure, subjected to a constant unit force, the larger the force applied at the primary structure, the larger the lateral deflection will be.

8.2.3 Discussion of the sensitivity operators $C_{\{\bullet,\bullet,\bullet\}}^P$

The sensitivity operators that will be discussed are presented in the Appendix D in Figures D6 ~D11. They show the consequences of the changes of the design parameters on the horizontal displacement of the top of the laterally loaded piles.

The effects of the change of the design parameters on the lateral displacement at the top of the piles can be investigated through the sensitivity operators $C_{\{\bullet,\bullet,\bullet\}}^P$. These operators are presented in kN, when they are related to lateral concentrated force applied to the pile, or in kN.m when the system faces a bending moment applied to it. The design parameters that represent the strength of the pile is the bending stiffness ($E I$), and those who characterize the strength of the soil such as $c$, $\gamma'$, and $b$. Several authors stated that the increase of those four parameters can reduce the displacement due to the same loading, and the results of the analysis of the piles performed in this study shared the same view. However, the weakness of the soil is demonstrated through the parameter $e_{50}$.

The observations on the distribution of sensitivity operators are provided in Figures D7, D8, D9 and D10 where the distributions of the sensitivity operators $C_{\{\bullet,\bullet,\bullet\}}^P$, $C_{c}^P$, $C_{\gamma'}^P$, and $C_{b}^P$ presented have negative sign, which means that the increase of these design parameters under constant load results in the decrease of $\delta_{y_1}$ or $\delta_{\theta_1}$. 

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The distribution of first operator $C_{EI}^{P_Y}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of bending stiffness EI when the pile structure is subjected to a static lateral concentrated load will be discussed with the aid of Figure D6. The design parameter EI represents the strength of the pile, and the increase of this parameter might reduce the displacement developed at the pile head under the same values of load. Review of $C_{EI}^{P_Y}$ shown in Figure D6 allows identifying the nonlinear relationship between the operators with respect to depth and load. The distribution and shape of the $C_{EI}^{P_Y}$ is identified as similar to the bending moment at the primary and adjoint structure. Although the p-y model relationship used in this study is not complex, the $C_{EI}^{P_Y}$ it is just affected by the pile itself.

The negative values in Figure D7 developed in the top half of the pile length show that the increase of EI design parameter under constant load will result in the decrease of $\delta_{yt}$. For long piles the change of EI will be more pronounced in the top half of the pile, that is, between $1T$ and $4T$. Then, the bending stiffness changes will affect the pile head deflection more than the changes of EI localized below the depth $5T$. Between depth of $4T$ and $5T$ the value of the sensitivity operator $C_{EI}^{P_Y}$ will decrease and reaches zero at the depth $z = 5T$.

The next sensitivity operator that will be discussed is $C_{c}^{P_Y}$. It is related to the design parameter $c$ (cohesion), and it represents together with others, the strength of the soil in which the pile is embedded. As pointed previously, the increase of this design parameter can decrease the displacement at the pile head under the same load. Figure D8 presents the distribution of sensitivity operators $C_{c}^{P_Y}$ affecting the changes of the pile head lateral deflection due to the changes of the cohesion $c$ when the pile is subjected to a lateral static load. The values of sensitivity operators are larger at the top of the pile. The negative sign demonstrates that the design parameter $c$ will affect the lateral deflection at
the pile head under constant load. As pointed to EI, once we increase the design parameter c the value of $\delta y_t$ will decreased. For loads between 30 and 90 kN it is possible to recognize that the increase of load promotes an increase in the dependence of the lateral deflection. Based on Figure D8 it can be concluded that basically up to the depth of the reduced resistance ($x_r$), which is equal 4.57m, the curves have a slightly discontinuity. Below depth of 4T the changes on the design parameter c have no influence on the change of the lateral deflection of the pile head. The results of the sensitivity operator between 0 and 3T have minus sign, thus it implies that it is more economical to strengthen the depths where the values are different than zero having negative values. Those results just occurred in the first one third of the pile length, close to the ground.

Figure D9 shows the distribution of sensitivity operators $C_{\gamma'}^{P_y}$ affecting the changes of the pile head lateral deflection due to the changes of the submerged soil unit weight when the pile structure is subjected to a static lateral load. The figure shows that for a range of loads between 30 kN and 270 kN the value of the sensitivity operator $C_{\gamma'}^{P_y}$ has zero value at the pile head, (ground level), and shows a rapidly increase until the depth of 1T to return to zero at the depth 2T. When the load is bigger, around 300 kN the behaviour of the $C_{\gamma'}^{P_y}$ is slightly different and maintains values equal to zero between soil surface and the depth of 0.5T. Bellow the depth of 0.5T, $C_{\gamma'}^{P_y}$ increases quickly to the depth of 1T and than it follows the same pattern as in the case of the smaller load application. Once more the presence of the negative values of $C_{\gamma'}^{P_y}$ indicates that the design variable $\gamma'$ will affect the changes of the lateral deflection at the pile head as the previous design parameters EI and c do. The examination of the position of the negative values of $C_{\gamma'}^{P_y}$ against depth shows that it just occurs between the ground level and the depth $x_r$, after
that $C_{\gamma}^{P_{y}}$ has zero value. This means that the changes of the $\gamma'$ below the depth $x_r$ do not affect the changes of the lateral deflection at the pile head.

From the Figure D10 it is possible to observe the distribution of the sensitivity operator $C_{b}^{P_{y}}$ affecting the changes of the pile head lateral deflection due to the changes of the pile's width $b$. The negative values, as pointed formerly, lead to conclusion that between the ground level to the depth of $4T$ ($\sim 9.80$ m) the changes of the width of the pile will promote changes of the lateral deflection at the pile head. However, the curves showed a gap at the depth $x_r$ where the resistance of the soil is reduced. Bellow the depth of $5T$ all values of the sensitivity operators are equal zero, which means that $\delta b$ is independent of $\delta y_t$.

The major differences between the distributions of $C_{b}^{P_{y}}$ and $C_{c}^{P_{y}}$ are the magnitude and the shape of the diagrams, especially for higher loads. Comparing the effects of loading (30, 60 and 90) kN on the magnitude of $C_{b}^{P_{y}}$ and $C_{c}^{P_{y}}$ it can be noticed, that they have about the same values. However, the application of 120 kN load produces difference between $C_{b}^{P_{y}}$ and $C_{c}^{P_{y}}$. The differences increase together with increase of load until around 270 kN. Then, the difference increases mostly reaching roughly 70%. This fact shows that the effect of $\delta b$ on $\delta y_t$ at the depths between $1T$ and $3T$ is much greater than $\delta c$. Bellow the depth $3T$ which is close to the reduced resistance $x_r$, the value of $C_{b}^{P_{y}}$ is also equal zero.

Figure D11 presents the distribution of the sensitivity operators $C_{\varepsilon_{50}}^{P_{y}}$ affecting the changes of the pile head lateral deflection due to the changes of the strain corresponding to 50% of maximum deviator in triaxial test, $\varepsilon_{50}$. In contrast to the distribution of $C_{b}^{P_{y}}$,
and $C_{c, p}$ the values of the sensitivity operator $C_{c, p}$ are positives and the values of $C_{c, p}$ related to small loads show continuity, even at the proximity of $x_r$. The effects of the changes of design variable $\varepsilon_{50}$ are not affected by the presence of depth equal to $x_r$. The higher loads values introduce a discontinuity in $C_{c, p}$ close to ground surface, where the sensitivity operators are equal to zero. The maximum value of $C_{c, p}$ occur close to the ground, at approximately at the depth equal to 0.5T, and it is related to the highest value of load.

The sensitivity operators shown in the Figures D5 ~ D11 reveal some similarities in patterns and locations. The distribution of $C_{c, p}$ follows the distribution of the bending moments of the both structures. The sensitivity operators $C_{c, p}$, $C_{b, p}$, $C_{y, p}$, and $C_{c, p}$ show an unique dependence on the soil type. Some of them are affected by the depth of the reduced resistance, $x_r$.

The distribution of sensitivity operators $C_{c, p}$, $C_{b, p}$ and $C_{c, p}$ show that they have non-zero values starting at the ground surface until the depth of 4T. Below that depth the values of the sensitivity operators remain zero. In the case of $C_{y, p}$ the largest value occur around the depth of 1T. Below the depth 2T all values of the sensitivity operators remain zero.

### 8.2.4 Discussion on the lateral deflections and bending moments of the adjoint structure subjected to bending moment $\bar{M} = \bar{I}$

The assessment of sensitivity of the maximum angle of flexural rotation $\theta_t$ due to the changes of the pile-soil strength parameters when the pile is subjected to a concentrated lateral force $P_l$ applied on the pile head is presented in a graphic format in Appendix D,
Figures D22 through Figure D29. Figure D22 presents the distribution of lateral deflections $y_a(M)$ of the adjoint structure subjected to $M = 1$ when the primary structure is subjected to the lateral force $P_t$. There is a significant similarity between the Figure D22 and the Figure D3, and this relationship between the diagrams can be viewed not only through the shape of the distribution, but also with respect to the magnitude of the values of the lateral deflection.

Figure D23 presents the distribution of the bending moments $M(M)$ of the adjoint structure subjected to $M = 1$ when the primary structure is subjected to the lateral force $P_t$. The characteristic point in this figure is the value of bending moment equal to one, that appears at the head of the pile.

The distributions of the soil resistance $p_a$ of the adjoint structure, subjected to $M = 1$ when the primary structure is subjected to the lateral force $P_t$ is presented in Figure D24, and some close similarities can be noticed. The soil reaction $p_a$ of the adjoint structure subjected to $M = 1$ results in values around 10 times smaller than the values obtained in Figure D6 when the adjoint structure is subjected to a $P = 1$. Other common factors are the depth of the distribution and the shape of the distribution of the soil resistance.

8.2.5 Discussion of the sensitivity operators $C_{\theta\cdot\cdot\cdot}^P$

The results of the distribution of sensitivity operators $C_{\theta\cdot\cdot\cdot}^P$ affecting the top of flexural rotation $\theta_t$ of the pile due the changes of the design variables when the pile structure is subjected to a concentrated lateral force $P_t$ are presented in Figures D25 to D29.

The first design variable that will be addressed is the bending stiffness $E_I$, and its sensitivity operator $C_{E_I}^P \theta$. The distributions of the sensitivity operator $C_{E_I}^P \theta$ are shown in the Figure D25. It is important to point that the shape of the distributions in Figure D25 is
very similar with the shape of $C_{EI}^{P_y}$ presented in the Figure D7. However, the numerical values of $C_{EI}^{P_y}$ when compared with $C_{EI}^{P_\theta}$ are different. The highest value of $C_{EI}^{P_y}$ is 4 times larger than $C_{EI}^{P_\theta}$.

Figure D28 presents the distributions of the sensitivity operator $C_{b}^{P_\theta}$ affecting the changes of the pile head angle of flexural rotation $\delta\theta_t$ due to the changes of the pile's width $b$. Figures D28 and D10 reveal the same shape, similar discontinuities at the same depth, and have only small differences in values, with $C_{b}^{P_\theta}$ being smaller than $C_{b}^{P_y}$.

The sensitivity operators $C_{c}^{P_\theta}$ affecting the changes of the pile head angle of flexural rotation $\delta\theta_t$ due to the changes of cohesion $c$ look similarly to the distributions of the sensitivity operator $C_{c}^{P_y}$. Through the Figures D26 and D8 it is possible to recognize that the shapes and the depth of the discontinuities are similar. However, the numerical values are quite different. The highest value of $C_{c}^{P_y}$ is twice larger than the highest value of $C_{c}^{P_\theta}$.

The distribution of the sensitivity operators $C_{\gamma_1}^{P_\theta}$ affecting the top angle of flexural rotation $\theta_t$ of a pile of length $L=10T$ due to the changes of the submerged soil unit weight when the pile structure is subjected to a concentrated lateral force $P_t$ is presented in Figure D27. The $C_{\gamma_1}^{P_\theta}$ sensitivity operator shows the same pattern as the other sensitivity operators before. This means that there is a great number of similarities between results of Figure D27 and Figure D9. This refers to the shape of the distributions and the depths where the sensitivity operators are equal to zero. The numerical values are not the same, but they are very close.
The last sensitivity operators described here is the sensitivity operator presented in the Figure D29, that is the $C_{e_{s0}}$. The conclusions are the same as described previously in the discussion on the sensitivity operators $C_{Ei}^P$, $C_{C}^P$, and $C_{y}^P$. The same pattern of shape is observed. However the values of $C_{e_{s0}}^P$ are twice higher than the values of $C_{e_{s0}}^P$.

8.2.6 Discussion on the quantitative assessment of the sensitivity factors

The numerical integration of values of sensitivity operators $C(...)$ has as a result the sensitivity factors $A$. Figures D12, D13, D14, D15, D16, D30, D31, D32, D33 and D34, present the relationship between the sensitivity factors and the lateral force $P_t$ applied. The ordinates represent the sensitivity factors expressed in kN.m and the abscissas carry values of the applied force in kN.

As shown in the Figure D12 the quantitative assessment of $A_{ei}^{P_y}$ in the nonlinear elastic stage increases gradually until the load is equal to 270 kN. Then, for loads higher than 270 kN the values just started increased considerably.

All figures show that the trend of the sensitivity factors is very similar. They all share the same pattern that demonstrated that values of $A$ increase fast in nonlinear distribution when the values of the pile head force applied increase. Figures D12 through D16 can be conclude that once the load values increase the sensitivity factors will also increase. The only exception is the sensitivity factors $A_{b}^{P_y}$ shown in Figure D14 that presents a highest value of $A_{b}^{P_y}$ when the load applied is equal to 270 kN, and after that the values of $A_{b}^{P_y}$ start to decrease.

Figures D30 through D34 presents the quantitative assessment of sensitivity factors $A_{ei}^{P_0}$, $A_{c}^{P_0}$, $A_{y}^{P_0}$, $A_{b}^{P_0}$, $A_{e_{s0}}^{P_0}$ affecting the top angle of flexural rotation $\theta_t$ due to changes

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of the bending stiffness $E_I$, the cohesion $c$, the pile width $b$, the submerged unit weight of the soil $\gamma'$, and the $\varepsilon_{50}$ when the pile structure is subjected to lateral force $P_t$. The sensitivity factors observed in the figures quoted above present the same type of distribution as the sensitivity factors described in the Figures D12 through D16. However, the values of $A_{\gamma'}^{P’}$ are 50% smaller than the values of the sensitivity factors $A_{\gamma'}^{P_0}$.

### 8.2.7 Discussion on the relative sensitivity factors $F$

The Equations (3.55) through (3.59) present the $F$ factors that were introduced and defined in Chapter 3. These factors allow to determine which design variable is more critical to the performance of the pile; they enable one to compare the relative significance of the sensitivity factors $A$. In Figures D17, D18, D19, D20, D21, D35, D36, D37, D38 and D39 the relative sensitivity factors are presented in percentage (%) along the ordinate axis, and the lateral forces applied $P_t$ in (kN) are viewed along the abscissa axis.

The quantitative assessment (in %) of relative factor $F_{E_I}^{P_0}$ affecting the top lateral deflection $y_t$ due to the changes of the bending stiffness $E_I$ of the pile when the pile is subjected to a concentrated lateral force $P_t$ is presented in Figure D19. The values of the loads increasing between 30 kN and 300 kN imply the decrease of the values of the relative sensitivity factors decrease from 1.8% to 0.5% for 270 kN. The distribution is visibly nonlinear. The application of lateral load larger than 270 kN results in a slightly increase of $F_{E_I}^{P_0}$, to reach 0.6% when $P_t$ is equal to 300 kN.

Figure D20 presents the quantitative assessment (in %) of relative factor $F_{\gamma'}^{P_0}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to a concentrated lateral force $P_t$. Now, the distribution of the relative factors is completely different than those associated with the previously factors.
The values of \( F_{py} \) range between 4.2% for 30 KN to 6.2% for 300 KN. However, for the most of the loads for 90 KN to 270 KN the values remain around 5.2%.

In Figure D17 the quantitative assessment (in %) of relative factor \( F_{by} \) affecting the top lateral deflection \( y_t \) due to the changes of the pile width \( b \) when the pile is subjected to a concentrated lateral force \( P_t \) is presented. In the Figure cited the values of the relative sensitivity factors varied between 32% for 30kN to 27% for 300 kN. The distribution of the \( F_{by} \) values is also non linear without any distinguished pattern. The \( F_{by} \) values start with 32% for the lowest load and then practically remain constant around 29 and 30% for 270 kN. When the load reaches 300kN the value of \( F_{by} \) dropped to 27%.

The quantitative assessment (in %) of the relative factor \( F_{cy} \) affecting the top lateral deflection \( y_t \) due to the changes of the cohesion of the soil \( c \) when the pile is subjected to a concentrated lateral force \( P_t \) is presented in Figure D18. The lack of the distinguishable pattern, is similar to the case of \( F_{by} \), with values of \( F_{cy} \) varied from 67% to 71% when the loads increase from 30 kN to 300 kN. The average of \( F_{cy} \) of 69% occurred for most of the loads applied.

The quantitative assessment (in %) of relative factor \( F_{cysq} \) affecting the top lateral deflection \( y_t \) due to the changes of \( \epsilon_{50} \) of the pile when the pile is subjected to a concentrated lateral force \( P_t \) is presented in Figure D21. The distribution is similar to the \( F_{cysq} \), However the values of \( F_{cysq} \) range between 18% and 20% remaining in the average for the most of the loads applied.

The quantitative assessment (in %) of relative factors \( F_{ps} \), \( F_{c} \), \( F_{py} \), \( F_{by} \), and \( F_{psEi} \) affecting the top angle of flexural rotation \( \theta_t \) due to the changes of the design
variables $E_l$, $b$, $c$, $\gamma^*$ when the pile is subjected to a concentrated lateral force $P_t$ is presented in Figures D35 through D39. The average values of the relative sensitivity factors $F_{E_0}$, $F_{C_0}$, $F_{\gamma}$, $F_b$, and $F_{E_l}$ are 24%, 65%, 6.5%, 27.5% and 0.5%, respectively.

The different pattern of distributions of $F_{E_l}^{P_y}$ and $F_{E_l}^{P_0}$ shows the importance of results of relative sensitivity factors. The design parameter $E_l$ is slightly more important in $F_{E_l}^{P_y}$ than in $F_{E_l}^{P_0}$ (0.82%). The difference exists, although it is very small.

Comparison of the results of $F_{E_l}^{P_y}$ and $F_{E_l}^{P_0}$ shows that the higher values appear at the top of the pile. Then they decreased when the load reached 270 kN. After that they started increase again. The effect of the design variable $E_l$ decreases when the load applied increases. When the load increases above 300 kN the soil starts to become weaker. From this point the effect of changes of $E_l$ on $\gamma_t$ and $\theta_t$ becomes larger. This situation corresponding to the soil plastic flow phase.

The results related with the design parameter $c$ showed that the relative sensitivity factors increase when the load applied increases. For most of the loads between 60 and 270 kN the values of $F_c$ practically remain the same. However, they started to increase significantly when the load reaches value of 300 kN.

The relative sensitivity factors with respect to the design variable $b$ show that the values of $F_b$ decrease when the applied loads increase. It is just a slightly decrease. The explanation of this fact could be that the soil shows a tendency to become weaker in terms of its strength.
The relative sensitivity factor $F_{\theta}^{P_y}$ basically remains the same within all ranges of applied loads. Regarding $F_{\theta}^{P_y}$, the values of $F_{\theta}^{P_y}$ increase as the load increases. This means that the soil still has a very significant contribution in the analysis.

The relative sensitivity factors $F_{\theta}^{P_0}$, $F_{c}^{P_0}$ have basically the same value for all range of loads, without significant changes. The relative sensitivity factors $F_{b}^{P_0}$ keep the same pattern and values when compared with the distribution of $F_{b}^{P_y}$. They decrease when the applied loads increase. The pattern of the distribution of the values of the relative sensitivity factor $F_{\theta}^{P_0}$ follows the distribution of the relative sensitivity factor $F_{\theta}^{P_y}$.

8.3 Error analysis of sensitivity of pile head in the vicinity of the applied load caused by the changes of the design variables

The numerous sensitivity analyses performed in this study require a verification and correlation of the results. This is connected with the nonlinear character of the investigated system for which the sensitivity analysis is conducted in the vicinity of the applied load.

Also through the entire process of calculation, the errors are introduced. They are caused by the type of software used to analyse the single piles, COM624P, that provided all output results with only 4 digits. Thus, they are not very accurate and require to perform the error analysis. Another source of errors is related to the number of nodes used along the pile to analyze deformations and internal forces. The number of possible nodes in this software is not sufficient for all cases. Some specific cases require more nodes to be able to describe better the behaviour of the pile during the analysis of a system. A different source of errors can be expected caused by the fact that the value of the load that should be applied to the adjoint structure must be considerably small whereas the adjoint structure itself is in the state of advanced nonlinear deformation. This cause of error is
also revealed in Liu (2004) and Rahman (2004). Both authors used FB-Pier with COM624P programs.

To perform verification of the results of the sensitivity analysis Rahman (2004) stated that a certain amount of change of the design variables is appropriate. In his study the author did compare the deflection changes based on the sensitivity analysis with the changes included. This type of approach was performed also by Liu (2004). Both authors utilized the MATLAB program to carry out the error analysis. This study used MATLAB – version 7 to perform the error analysis on the single piles and partially of the error analysis of pile groups. To execute the error analysis of single piles the program COM624P was used together with MATLAB to obtain a better analysis in terms of accuracy. The piles group are investigated using the FB-Pier for verification process. However the error analysis incorporated Microsoft Excel program.

The methodology used to assess the errors of sensitivity analysis was described already on Chapter 5. It has presented there that the errors of the sensitivity factors A can be easily locate by performing some steps. It is shown in Rahman (2004) that in order to calculate the actual change of any design variable it is important to determine the top deflection for the initial input design variable. Then, the new design variable must be increased or decreased by some percentage of the design variable selected. At the same time, the other variables maintain the same values. The new results of the lateral deflection contain the changes of the lateral deflection due the changes in a specific design variable by certain amount previously selected.

To determine the actual change in the lateral deflection due to the change of the design parameter, it is observed according to the relationship shown below, as an example.

\[
\text{Actual change in the lateral deflection on the pile head due the change of the design variable (c) cohesion} = \frac{\text{Lateral deflection on the top of the pile corresponding to } c + \delta c}{\% \text{ of } c} - \text{Lateral deflection corresponding to the initial input } c
\]
The amount of the changes of the specific design variable requires that a certain amount of percentage be applied to the design selected and applied to the $y_t$ of sensitivity analysis.

Change in the lateral deflection at the top of the pile due to the changes of the design variable $c$ cohesion obtained based on sensitivity analysis

\[
\frac{\delta c}{c} = \text{Percentage of the change of the design variable} \times \text{Sensitivity factor of the initial input } A_c
\]

The relative error will be the following:

\[
\text{relative error} = \frac{\delta \Delta_1 - \delta \Delta_2}{\delta \Delta_2}
\]

Where:

$\delta \Delta_1$ = The change of the lateral deflection at the top of the pile due to the changes of the design variable obtained from sensitivity analysis

$\delta \Delta_2$ = Actual change in the lateral deflection of the pile head due the change of the design variable

As an example the results of the analysis of relative errors described in Chapter 5, the sensitivity factors $A$ of a free head single pile subjected to lateral concentrated load embedded in a soft clay below the water table, (of length = 10 T (24.50m)) are presented in the Table 8.1 below. The analysis of the errors were conducted using the normalized variation $\frac{\delta E}{E'I}$, $\frac{\delta c}{c'}$, and $\frac{\delta b}{b}$ of the design variables as a 0.10% to perform the calculations presented below.
<table>
<thead>
<tr>
<th>Lateral Forces $P_t$ (kN)</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{Py}}^{\text{EI}}$</td>
<td>-0.5</td>
<td>-0.3</td>
<td>-0.6</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.6</td>
<td>-0.4</td>
<td>-0.5</td>
<td></td>
</tr>
<tr>
<td>Change of design variables c, b and $\text{EI}$</td>
<td>17</td>
<td>40</td>
<td>27</td>
<td>37</td>
<td>49</td>
<td>57</td>
<td>46</td>
<td>64</td>
<td>86</td>
<td>59</td>
</tr>
<tr>
<td>$A_{\text{c}}^{\text{Py}}$</td>
<td>33</td>
<td>-</td>
<td>23</td>
<td>32</td>
<td>56</td>
<td>53</td>
<td>39</td>
<td>55</td>
<td>73</td>
<td>69</td>
</tr>
<tr>
<td>$A_{\text{b}}^{\text{Py}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.1 Results of the sensitivity factors $A$ when occurred changes on the design variables ($\text{EI}$, $c$ and $b$) for the sensitivity analysis of top lateral deflection of a free head single pile subjected to lateral concentrated loads embedded in a soft clay below the water table, and length = 10 T (24.50m)

8.4 Quantitative estimation of the sensitivity factors $A$

Based on the definitions and equations presented in Chapter 3, the results of the sensitivity factors are calculated and are present in the Figures D12, D13, D14, D15, D16, D30, D31, D32, D33 and D34.

The quantitative assessment of the sensitivity analysis of $\delta_y^P$ for a free head pile subjected to a concentrated lateral force $P_t$ with pile length $L=10T$, embedded in a soft clay below the ground water table, allows discussion on some specific characteristics of some results. It is of considerable importance for an engineering application.

From this point it is assumed that $P_t = P_1$. Then:

\[
(8.2) \quad \overline{I} \delta_y^P = \frac{\delta(\text{EI})}{\text{EI}} \int_0^L C_{\text{EI}}^\text{py} \, dx + \frac{\delta c}{\gamma_0} \int_0^L C_{c}^\text{py} \, dx + \frac{\delta b}{\gamma_0} \int_0^L C_{b}^\text{py} \, dx + \frac{\delta c_{\text{py}}}{\gamma_0} \int_0^L C_{\text{py}}^\text{py} \, dx + \frac{\delta c_{\text{py}}}{\gamma_0} \int_0^L C_{\text{py}}^\text{py} \, dx
\]

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The integration of the sensitivity operators, gives the sensitivity factors $A$ that are involved in $\delta_y^P$ as shown below:

\begin{equation}
\delta_y^P = A_{EI}^P \frac{\delta(EI)}{EI} + A_c^p \frac{\delta C}{c} + A_y^p \frac{\delta y'}{y'} + A_b^p \frac{\delta b}{b} + A_{\delta_{50}}^p \frac{\delta \delta_{50}}{\varepsilon_{50}}
\end{equation}

The design variables variations are expressed as scalar factors, and expressed in %. Equation (8.3) allows us to change independently each design variable associated with each term specified by Equation (8.3). Thus, effect of change of each design variable on $\delta_y^P$ can be accepted independently as follows:

\begin{align}
\delta_y^P &= A_{EI}^P \frac{\delta(EI)}{EI} \\
\delta_y^C &= A_c^p \frac{\delta C}{c} \\
\delta_y^y &= A_y^p \frac{\delta y'}{y'} \\
\delta_y^b &= A_b^p \frac{\delta b}{b} \\
\delta_y^{\delta_{50}} &= A_{\delta_{50}}^p \frac{\delta \delta_{50}}{\varepsilon_{50}}
\end{align}

The analysis of the units of the Equation (8.4) gives the following:

\begin{align}
[\text{kN}] \cdot [\text{m}] \cdot [\text{m}] &= A_{EI}^P [\text{m}] \frac{\delta(\text{EI})}{\text{EI}} [\%]
\end{align}

Consequently, taking for $\bar{I} = 1 \text{kN}$, the Equation (8.9) can formally be written for arbitrary force $P_i$ having value of $P_i$ as:

\begin{align}
\delta_y^P [\text{m}] &= A_{EI}^P [\text{m}] \frac{\delta(\text{EI})}{\text{EI}} [\%]
\end{align}
Equation (8.10) enable one to state that the sensitivity of lateral deflection expressed here in [m] caused by the changes of the design variable such as bending stiffness EI, when the concentrated lateral force $P_i$ is applied represents in fact the same value as $P_i$.

This interpretation enables one for modifications of Figures D12, D13, D14, D15, D16, D30, D31, D32, D33 and D34. The equation in Figures (8.3) has the objective through an example to show a typical example the sensitivity of lateral deflection $\delta_y^P$ expressed in meters cause by the changes of the design variables b, when applied force $P_i$ have values $P_i$.

\[
(8.11) \quad \delta_y^P [\text{m}] = -0.0001 \times 10^{-3} (P_k)^2 + 0.0042 \times 10^{-3} (P_k) + 0.3995 \times 10^{-3}
\]

Where;

$P_k$ = arbitrary lateral load applied to the pile head

![Graph of Equation (8.11)](image)

Figure 8.3 The exact sensitivity of lateral deflection $\delta_y^P$ expressed in (m) caused by changes of the design variables b when applied force $P_i$ have values $P_i$.
Another way to conduct the assessment of the sensitivity of lateral deflection due to the changes of each design variable is by using the ratios of \( \frac{P_i}{P_1} \) and \( \frac{\delta y_{E1}}{\delta y_{E1}} \). The first term is defined as:

\[
(8.12) \quad I_P = \frac{P_i}{P_1}
\]

Where:

\( P_i \) = the value of lateral force applied to the pile head used in investigation;

\( P_1 \) = the initial lateral force employed in investigation that it is applied to the adopted structure;

Presented the equation 8.12 and stated \( P_i \) and \( P_1 \) the following equation is stated as an example, using the design variable \( c \):

\[
(8.13) \quad I_c^{Py} = \frac{\frac{P_i}{P_1}}{\frac{\delta y_c}{\delta y_c^1}}
\]

Where:

\( \frac{P_i}{P_1} \) = the sensitivity of pile head lateral deflection due to the changes of cohesion \( c \) when the pile-soil system is subjected to force \( P_i \);

\( \frac{\delta y_c}{\delta y_c^1} \) = the sensitivity of the pile head deflection due to the changes of cohesion \( c \) when the pile-soil system is subjected to force \( P_1 \);

The \( I_P \) and \( I_c^{Py} \) are called the relative load factor and relative deflection sensitivity factor correspondingly.
The relationship between $I_c^{Py}$ and $I^p$ shows how sensitive is the lateral deflection of the pile caused by the changes of the soil cohesion $c$. Here soil cohesion is taken as an example, when the applied load acting to the pile head increases linearly.

The same approach can be valid to the others design variables. These equations are specified as follows:

$$I_{El}^{Py} = \frac{P_i}{\delta y_{El}}$$  \hspace{1cm} (8.14)

$$I_{\gamma}^{Py} = \frac{P_i}{\delta y_{\gamma}}$$  \hspace{1cm} (8.15)

$$I_{b}^{Py} = \frac{P_i}{\delta y_{b}}$$  \hspace{1cm} (8.16)

$$I_{50}^{Py} = \frac{P_i}{\delta y_{50}}$$  \hspace{1cm} (8.17)

They are called relative deflection sensitivity factor due the changes of the design variables $EI$, $b$, $\gamma'$, and $\varepsilon_{50}$ respectively.

8.5 **Assessment of error of lateral deflection based on comparative analysis of exact solution and sensitivity analysis solution**

The assessment of error of lateral deflection based on comparative analysis of exact solution and sensitivity analysis solution is presented as another way to perform an evaluation on the results obtained. Then the error of analysis then will generate a value of
maximum acceptable variation of the design variable analysed, in which the sensitivity analysis result can predict acceptable change of the pile head deflection. Figure 8.5 below presents the basis error analysis for the free head pile subjected to a concentrated lateral force $P_t = 270$ kN, with the pile length equal to 10T.

**Figure 8.5** The pile head lateral deflection $y_{top}$ versus the ratio $(c/c_0)$ of design variable $c$ with respect to the initial value of the design variable $c_0$, for the case study with free head pile subjected to a concentrated lateral force $P_t = 270$ kN, where the pile length considered is equal $L=10T$. 

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The Figure (8.5) is used with the design variable \( c \) (soil cohesion) as an example for discussion of error analysis of sensitivity of the top lateral deflection.

The exact lateral pile head deflection \( y_{\text{top}}^{\text{Exact}} \) that was showed in the Figure (8.6) can be determined by taking as an input the changed design variable to \( (c_0 + \delta c) \) into the input file of COM624P or FB-Pier programs. The approximate lateral pile head deflection \( y_{\text{top}}^{\text{App}} \) based on the sensitivity analysis is determined through the following equation:

\[
(8.18) \quad y_{\text{top}}^{\text{App}} = y_{\text{top}}^{\text{Exact}}(c_0) + A_c \delta c
\]

Where:

\[
y_{\text{top}}^{\text{Exact}}(c_0) = \text{the exact lateral pile head deflection while the soil cohesion } c \text{ is equal to the initial value } c_0;
\]

Then, the error of the predicted lateral pile head deflection will be defined as:

\[
(8.19) \quad ER_{\text{top}}^P = \left| \frac{y_{\text{top}}^{\text{App}} - y_{\text{top}}^{\text{Exact}}}{y_{\text{top}}^{\text{Exact}}} \right|
\]
CHAPTER 9
CONCLUSIONS AND PROPOSAL FOR FUTURE RESEARCH

9.1 Conclusion

This Chapter presents the conclusions based on the sensitivity analysis results for the
laterally loaded pile embedded in soft clay below the ground water table subjected to a
lateral concentrated force or bending moment, with pile’s length varying between short
and long piles. The conclusions will be shown on the free head short and long piles
subjected to lateral concentrated load and bending moment having variable values of
discrete type, and for fixed head short and long pile under lateral concentrated load of
variable values of discrete type.
The conclusions are:

- **Conclusions on the sensitivity operators C**
  Among the five design variables used in this study, the design variable \( EI \) is the one that
  represents the strength of the pile, and the others \( c, \gamma', b, \) and \( \varepsilon_{50} \) represent the strength of
  the soil. Through the distribution of the sensitivity operators some of the design variables
  have an effect on the pile soil system, and that effect can be observed once the results of
  the sensitivity operators show negative values along the pile depth.
  Once the sensitivity operators are integrated they produce as a result the sensitivity
  factors \( A_{EI}, A_c, A_{\gamma'}, A_b \), and all of them had negative values. This means that the
  increase of the design variable \( EI, c, \gamma' \) and \( b \) will cause the reduction of the pile head
deflection.
  On the other hand, the design variable \( \varepsilon_{50} \) shows to have an opposite effects in stopping
  additional pile deflection. The distribution of the outcomes of sensitivity operator \( C_{\varepsilon_{50}} \)
  demonstrates to be on the positive side of the axis. Consequently their integrations that
give the sensitivity factors \( A_{\varepsilon_{50}} \) also have positive values. The meaning of these results
as the positive ones is that any increase of the design variable \( \varepsilon_{50} \) will promote the
increase of the pile head deflection.
The case when the bending moments are applied to the pile-soil system, the values of the sensitivity operators are the most significant for the bending stiffness, EI. As pointed previously the sensitivity operator \( C_{EI} \) is just affected by the strength of the pile.

For all case studies the pattern of the distributions of sensitivity operators \( C \) with respect to the load applied to the structure is that the design variables increased their value as the load applied to the pile head increases.

The depth \( x_r \) called the depth of the reduced resistance is also an important factor affecting the sensitivity operator distribution outcomes. The distribution of the sensitivity operators at this point may change a sign (positive into negative and vice versa) or become equal to zero, which is the case of the sensitivity operator \( C_y \).

The results of sensitivity operators of short piles, show that they are distributed all the way along the pile length. As the length of the pile increases, the distribution with positive or negative values are developed to the top \( 4T \)'s of length.

Most of the results of the sensitivity operators within the depth \( OT \) and \( 4T \). This means that for the specific study, of a pile embedded in a soft clay below the ground water table under a static concentrated lateral loads, the most economical pile length can be set as \( 4T \).

The sensitivity operators \( C \) of a pile in the pile group have the same distribution pattern as the single pile of the same length and boundary conditions. For a pile in a pile group and a single pile with the same pile head lateral deflection, the sensitivity operators \( C_{EI} \), present the same small values for both the pile in a pile group, and for a single pile. For the other operators, \( C_{e_0}, C_y', C_{b}, C_{\varepsilon_{50}} \) all presented values are at least twice greater for a single pile than for a pile in a pile group.

> **Conclusions on the sensitivity factors \( A \) and relative sensitivity factors \( F \)**

The sensitivity factors \( A \) increase for short piles and for long piles as the load values applied to the pile head increase. However, the sensitivity factors \( \frac{P_y}{A_{\varepsilon_{50}}} \) affecting the top lateral deflections due the changes of \( \varepsilon_{50} \) when the free head short pile is subjected to a lateral force \( P_t \) present different pattern of distributions. In other words, when the load reaches the value of 240 kN the sensitivity values started to decrease. For very high values of loads, the value of \( A \) is almost equal zero.
For the free head short piles, subjected to a lateral load, the relative sensitivity factor $F_{ei}$ decrease rapidly as the load applied to primary structure increases, and when the highest load of value 300KN is applied, the results jumps to a maximum value.

For the other two cases that is for the, fixed head short pile subjected to a lateral load and free short pile subjected to a bending moment the values of the relative sensitivity factors are almost the same for the applied load values. They decrease by a very small amount as the load applied increases.

For the long piles, the relative sensitivity factor $F_{ei}$ behaves in the different way. That is, the values of $F_{ei}$ increase gradually as the load applied to primary structure increases. The explanation of the behavior that causes the increase of relative sensitivity factor $F_{ei}$, can be also clarified using the energy viewpoint. When the load applied increases allows the pile bending stiffness allows the development of deflection at larger depth. Consequently, the soil located at larger depth is involved into deformation. This process allows the pile to absorb more portion of energy. Consequently it plays a bigger role in the effect of changing of the lateral pile head deflection of the pile.

The average value of the relative sensitivity factors $F$ for the free head piles subjected to lateral concentrated force, fixed head piles subjected to lateral concentrated force, free head piles subjected to bending moment and all the piles subjected to both lateral load and bending moment are shown in Table 9.1 to Table 9.4 respectively.

These percents are based on the average values of the relative sensitivity factors corresponding to all the load series applied to the pile head of single piles.

<table>
<thead>
<tr>
<th>Sensitivity factors</th>
<th>Short pile</th>
<th>Long pile</th>
<th>Average between short and long pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_c$</td>
<td>65%</td>
<td>67%</td>
<td>66%</td>
</tr>
<tr>
<td>$F_b$</td>
<td>26%</td>
<td>28%</td>
<td>27%</td>
</tr>
</tbody>
</table>

Table 9.1 The average of the two largest values of the relative sensitivity factors $F$ for the free head piles subjected to the lateral concentrated load embedded in soft clay below the ground water table
<table>
<thead>
<tr>
<th>Sensitivity factors</th>
<th>Short pile</th>
<th>Long pile</th>
<th>Average between short and long pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_c$</td>
<td>68%</td>
<td>69%</td>
<td>68.5%</td>
</tr>
<tr>
<td>$F_b$</td>
<td>27%</td>
<td>28%</td>
<td>27.5%</td>
</tr>
</tbody>
</table>

Table 9.2 The average of the two largest values of the relative sensitivity factors $F$ for the fixed head pile subjected to the lateral concentrated load embedded in soft clay below the ground water table.

<table>
<thead>
<tr>
<th>Sensitivity factors</th>
<th>Short pile</th>
<th>Long pile</th>
<th>Average between short and long pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_c$</td>
<td>64%</td>
<td>49%</td>
<td>56.5%</td>
</tr>
<tr>
<td>$F_b$</td>
<td>30%</td>
<td>24%</td>
<td>27%</td>
</tr>
</tbody>
</table>

Table 9.3 The average of the two largest values of the relative sensitivity factors $F$ for the free head pile subjected to the bending moment embedded in soft clay below the ground water table.

<table>
<thead>
<tr>
<th>Sensitivity factors</th>
<th>Short pile</th>
<th>Long pile</th>
<th>Average of short and long pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_c$</td>
<td>66%</td>
<td>62%</td>
<td>64%</td>
</tr>
<tr>
<td>$F_b$</td>
<td>28%</td>
<td>27%</td>
<td>27%</td>
</tr>
</tbody>
</table>

Table 9.4 The average of the two largest values of the relative sensitivity factors $F$ for all the piles subjected to both the lateral concentrated load and bending moment embedded in soft clay below the ground water table.

Through the Tables above it is simple to find out that the design variable $c$ present the dominant effect in changing the pile head deflection of the laterally loaded pile embedded in soft clay below the ground water table.

The design variable $b$ is the second important variable in affecting the lateral deflection change of the laterally loaded pile embedded in soft clay. The design variable $E_I$ and $\gamma'$ only have a small portion of the influence to change the lateral deflection of the pile.

The relative sensitivity factors $F$ varies slightly with different support type and different pile length.
Differences between short and long piles
The short piles fail primarily by the failure of the soil, while the long piles generally fail by reaching the bearing capacity of the pile section.
The changes of the design variables affecting the top deformations in short piles are distributed along the entire length of the pile.
The distributions of sensitivity operators for long piles are generally located at the depth between 0 and 4T.

Differences among the types of constraint
The main difference of the free head pile and fixed head pile is demonstrated in the distributions of the sensitivity operators $C_{EI}$. For fixed head pile, the maximum value of $C_{EI}$ occurs at the top of the pile, while for free head pile $C_{EI}$, has zero value at the pile head.
The maximum value of $C_{EI}$ is located at the depth 2T where the bending moments of the primary and adjoint structure have maximum value. The design variable $EI$ has more significant effects on the changes of the lateral deflection of the laterally loaded pile in the fixed head than with free head.
The sensitivity operators of other design variables $C_c$, $C_{y'}$, $C_b$, and $C_{e50}$ show some differences, however none of them promote great differences in the characteristics of free head piles and fixed head piles. Thus they are basically pretty much similar.

Differences between $\delta y_t$ and $\delta \theta_t$
For the free head pile subjected to bending moment to the sensitivity operator $C_{EI}^{M\theta}$ used in the analysis of $\delta \theta_t$, has the maximum value at the top of the pile. It has maximum value around the depth equal 2T of the pile for the sensitivity analysis of $\delta \theta_t$.
The sensitivity operators $C$ for the other design variables ($c$, $y$, $b$, and $e_{50}$) of $\delta y_t$ and $\delta \theta_t$ only vary in magnitude but have similar characteristics. The values of the sensitivity operators $C$ affecting the changes of lateral deflections $\delta y_t$ are usually twice higher than the sensitivity operators $C$ affecting the changes of the pile head rotation $\delta \theta_t$.

Conclusions about pile groups results
Although the results presented in the Appendix K are just related with the pile B, in the first trailing row, other results were gathered, and they are part of the CD attached. The analysis performed for the piles pinned to pile group cap under lateral load in a group,
subjected to a lateral force show that largest values of bending moments are for the piles located in the center of leading row, not in the first or second trailing rows. Next to the leading row, the first trailing row is that where the largest values of bending moment is developed in the center pile of the group.

The lateral deflection developed in the primary structure has the same distribution of the single pile at the conditions, however the values are significantly smaller in the pile group when compared with single pile. The lateral displacement at the top of the piles of the primary pile groups is basically the same for each row due to the constraints imposed on the pile cap, however the distributions of the lateral displacements of the adjoint structure vary for each row.

The value and distribution of the sensitivity operator $C_{EI}$, and the sensitivity factor $A_{EI}$ of each row corresponding to the same lateral load applied are relatively similar, once they both are basically related with the design variable $EI$. The distributions of $C_c$, $C_{Y'}$, $C_b$, and $C_{e50}$ are closely likely, and the piles located in the leading row present the largest values.

The behaviour of the sensitivity factors $A_c$, $A_{Y'}$, $A_b$, and $A_{e50}$ is very similar for the piles in different rows. The justification for this is that the load applied on the piles inside the group has very similar values. Regarding the distribution of the sensitivity factors the same conclusions are drawn. In other words, all they have very similar values, however the largest are developed in the leading row. Consequently the leading row is the more sensitive to the changes of the design variables related to soil properties connected with strength.

In practical terms, it can be concluded that in the piles in a pile group located in the leading row are those that require more attention when it is necessary to improve the soil design variables.

The relative sensitivity factors $F$ appear to be relatively constant and not affected by the locations of the pile in the pile group.

**Spacing of piles in a pile group**

When the spacing between piles increases the system requires a larger load to be able to the pile group to generate the same amount of displacement as the single isolated pile displacement. The distribution of bending moments at the primary structure increases
when the spacing increase. The piles in a group when compared with single piles with the same conditions, presented values of bending moment at the primary structure relatively smaller.

The spacing between piles affects fm-multipliers, that have greater values for the leading row. Once the spacing between piles increases the values of the fm-multipliers also increased.

The distributions and values of sensitivity operators $C_{EI}$, and sensitivity factors $A_{EI}$ for piles in the pile group with different spacing are relatively similar to each other. The same pattern is observed in sensitivity investigations of corresponding laterally loaded single piles.

For the pile spacing equal to 2D of the pile group the outcomes of sensitivity operators $C_c, C_{\gamma'}, C_b$ and sensitivity factor $A_c, A_{\gamma'}, A_b$ and $A_{\varepsilon_{50}}$, determined are smaller than the corresponding sensitivity characteristic of piles located in pile groups with spacing equal to 5D. The spacing also affects the values of the sensitivity operators and sensitivity factors. However the relative sensitivity factors $F$ show to be independent of the spacing between piles.

9.2 The application of this study

The sensitivity analysis of the laterally loaded piles embedded in soft clay below water table can be applied to the following engineering problems:

✓ To determine (among all design parameters) which design variable is the most effective in reduction of the deflection on the pile head;

✓ To support decisions on maintenances, repairs, rehabilitations, renovations and replacements of the infrastructures supported by the laterally loaded piles based on distribution of sensitivity operators;

✓ Planning the service life of the structure maintenances, repairs, rehabilitations, renovations and replacements of the infrastructures supported by the laterally loaded piles based on the information obtained from the sensitivity operators;

✓ Evaluate the impact of each change of material properties on the changes of maximum deformations of laterally loaded piles based on sensitivity operators;
Evaluating the ageing process of the infrastructure system supported by the laterally loaded piles based on the deterioration rates of the design variables involved in the infrastructure system.

9.3 Recommendation for future research

The design, assessment and evaluation of the design parameters of laterally loaded deep foundation is still an important field to improve. However other future research is suggested:

1. A future research on applicability of the sensitivity analysis to the piles embedded in non-homogenous soils subjected to combinations of different types of loads, such as cyclic and static.
2. Developing a friendly computer software to deal with the sensitivity analysis of all types of soils, combinations of layered soils, load conditions and boundary conditions.
3. Develop a friendly computer software to be able to gather information from COM624P and FB-Pier and create an interface easily understandable that can analyze all different types of geometry with a different types of constraints and use all the information for the sensitivity analysis.
4. Combining the sensitivity analysis results of the laterally loaded pile with the economic considerations.
5. Work on a methodology to assess and investigate the infrastructure systems supported by the laterally loaded piles.
6. After conducting the sensitivity analysis performed for all types of soils and variety of boundary conditions, a future research could create a manual that could contain all results together and present examples applicable for every case.
7. To determine economical lengths of laterally loaded piles for all different types of soils, boundary conditions and load types, hence recommending the economical pile lengths for laterally loaded piles to the engineering society.
REFERENCES


APPENDIX A

Sensitivity analysis of the top lateral displacement due lateral load on piles embedded in nonlinear medium
APPENDIX A

Sensitivity analysis of the top lateral displacement due lateral load on piles embedded in non-linear medium.

The adjoint and primary structures showed in Chapter 3 (Figures 3.6 through 3.9) are described and derived in more detail.

The pile element is embedded in soft clay below the water table and the p-y model that describe this specific soil and its conditions were given in Equations (3.1) ~ (3.6).

Using the adjoint structure principle the $\delta_{y_{TOP}}^{FORCE(P)}$ can be determined using the Equation (3.26) and presented as:

\[
\delta_{y_{TOP}}^{FORCE(P)} = - \int_{0}^{L} M \delta y'' \, dx + \int_{0}^{L} P \delta y \, dx,
\]

which can be rewritten as:

\[
\delta_{y_{TOP}}^{FORCE(P)} = - \int_{0}^{a} M \delta y'' \, dx + \int_{0}^{a} p \delta y \, dx + \int_{a}^{L} P \delta y \, dx.
\]

Where:

$M$ and $p$ are dependable on the state variables vector and the design variables $\bar{S}$

\[
\bar{S} = (s_1, s_1, s_1, \ldots, s_n)
\]

$M_a$ = Distribution of bending moment of the adjoint structure subjected to unit force.

$p_a$ = Distribution of soil reaction of the adjoint structure subjected to unit force.

From equation:

\[
M = - E I y''
\]
it is possible to relate that the bending moment $M$ is defined by the bending stiffness $EI$ and $y\prime\prime$.

The soil resistance $p$ is a function of the parameters $c, \gamma', \varepsilon_{50}, b$ and deflection $y$ as stated in Equations (3.1) ~ (3.9). The vector $S$ can be introduced as:

(A.3) \[ S = \{c, \gamma', \varepsilon_{50}, b\}^T \]

The variations of the two variables $\delta M$ and $\delta p$ can be defined as:

(A.4) \[ \delta M = M, y\prime\prime \delta y\prime + M, E I \delta (EI) \]

(A.5) \[ \delta p = p, y \delta y + p, S \delta S \]

Where we know the second product $(p, s \delta S)$ in Equation (A.5) might be expressed as:

(A.6) \[ p, S \delta S = p, c \delta c + p, y \delta \gamma' + p, \varepsilon_{50} \delta \varepsilon_{50} + p, b \delta b \]

And we also know that:

(A.7) \[ p = pu\cdot \frac{1}{2} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \]

(A.8) \[ \frac{\delta p}{\delta y} = pu\cdot \frac{1}{2} \cdot \frac{1}{3} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \cdot \frac{1}{y_{50}} \cdot \delta y \]

(A.9) \[ \frac{\delta p}{\delta y} = pu\cdot \frac{1}{6} \left( \frac{y}{y_{50}} \right)^{\frac{2}{3}} \cdot \frac{1}{y_{50}} \cdot \delta y \]

(A.10) \[ \delta p = pu\cdot \frac{1}{6} \left( y^2 \cdot y_{50} \right)^{\frac{1}{3}} \delta y \]
The changes of internal forces are equal to zero since the primary structure is subjected to a constant load and remain in equilibrium, $\delta M = 0$ and $\delta p = 0$.

The Equations (3.10) and (3.12) become:

(A.12) \[ \delta M = 0 \]

(A.13) \[ \delta p = 0 \]

The variations of the displacement field $\delta y^*$ and $\delta y$ are imposed on the primary structure and are required by Equation (A.1), can be determined from Equations (A.5) and (A.11) using Equations (A.12) and (A.13):

(A.14) \[ \delta y'' = -\frac{M_{EI} \delta (EI)}{M_{y^*}} \]

(A.15) \[ \delta y = -\frac{pS \delta S}{p_y} \]

The denominator of equation (A.14) turned to be

(A.16) \[ M_{y^*} = -EI \]

And the numerator;

(A.17) \[ M_{EI} \delta (EI) = -y'' \delta (EI) \]

The final form of the Equation (A.14) will be:

(A.18) \[ \delta y'' = -\frac{y'' \delta (EI)}{EI} \]
The Equation (A.18) will be applied into the first integral of the Equation (3.26)

\[ (A.19) \quad - \int_{0}^{L} \bar{M} \delta y'' \, dx = \int_{0}^{L} M y'' \delta EI \, dx \]

As postulated previously the adjoint structure has the same material as the primary structure, which means the relationship stated in the Equation (3.10) is also valid to the adjoint structure. Than we can state:

\[ (A.20) \quad -\bar{M} = EI y'' \]

The Equation (A.20) can be applied into Equation (A.19), and:

\[ (A.21) \quad - \int_{0}^{L} \bar{M} \delta y'' \, dx = - \int_{0}^{L} y'' \delta EI \, dx \]

The Equation (A.11) can be solved once we already stated Equations (A.12) and (A.13)

\[ (A.22) \quad p_{u} \cdot \frac{1}{6} (y^{2} y_{50})^{\frac{1}{3}} \cdot \delta y = p_{s} \delta S \]

\[ (A.23) \quad \delta y = - \frac{6 (y^{2} y_{50})^{\frac{1}{3}}}{p_{u}} \cdot p_{s} \delta S \]

Substitution equation (A.21), and (A.23) into equation (A.1) and integrated, we have the first integral of Equation (A.1):

\[ (A.24) \quad \delta y_{\text{force}} = \int_{0}^{L} y'' \delta EI \, dx - \int_{0}^{a} p_{u} \frac{6 (y^{2} y_{50})^{\frac{1}{3}}}{p_{u}} \cdot p_{s} \delta S \, dx - \int_{a}^{x_{r}} p_{u} \frac{6 (y^{2} y_{50})^{\frac{1}{3}}}{p_{u}} \cdot p_{s} \delta S \, dx \]

Equation (A.24) is valid for \( y \leq 8y_{50} \) in the range \( x \leq x_{r} \)
Therefore, the second integral (II) of equation (A.24) will be:

For $y \leq 8y_{50}$ and $x \leq x_r$

(A.25) \[ p_a = p_u \frac{1}{2} \left( \frac{y_{a}}{y_{50}} \right)^{\frac{1}{3}} \]

(A.26) \[ p_a \left( \frac{y^2 \cdot y_{50}}{p_u} \right)^{\frac{1}{3}} = 3 \left( \frac{y_{a} \cdot y^2}{y_{50}} \right)^{\frac{1}{3}} \]

(A.7) \[ p = p_u \frac{1}{2} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \]

(3.3) \[ p_u = \left[ 3 + \frac{y^t}{c} x + \frac{y}{b} x \right] cb \]

Applying Equation (3.3) into Equation (A.7)

(A.27) \[ p = 0.5 \left[ 3 + \frac{y^t}{c} x + \frac{y}{b} x \right] cb \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \]

(A.28) \[ p = \left[ \left( \frac{(3b+Jx)c+y'b}{} \right) + \gamma bx \right] \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \]

From Equation (A.6) we know:

(A.6) \[ p, s \delta S = p, c \delta c + p, \gamma' \delta \gamma' + p, \epsilon_{50} \delta \epsilon_{50} + p, b \delta b \]

and

(A.29) \[ \frac{\partial p}{\partial s} \delta S = \frac{\partial p}{\partial c} \delta c + \frac{\partial p}{\partial \gamma} \delta \gamma' + \frac{\partial p}{\partial \epsilon_{50}} \delta \epsilon_{50} + \frac{\partial p}{\partial b} \delta b \]

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(A.30) \[ \frac{\partial p}{\partial c} \delta c = \frac{1}{2} (3b + Jx) \left( \frac{y}{y_{50}} \right)^3 \delta c \]

(A.31) \[ \frac{\partial p}{\partial \gamma'} \delta \gamma' = \frac{1}{2} (bx) \left( \frac{y}{y_{50}} \right)^3 \delta \gamma' \]

(A.32) \[ \frac{\partial p}{\partial \epsilon_{50}} \delta \epsilon_{50} = \frac{1}{6} [(3b + Jx)c + \gamma'bx] \left( \frac{y}{y_{50} \cdot \epsilon_{50}} \right) \left( \frac{1}{\sqrt[3]{y_{50}}} \right) \delta \epsilon_{50} \]

\[ = \frac{\partial p}{\partial \epsilon_{50}} \delta \epsilon_{50} = \frac{1}{6} [(3b + Jx)c + \gamma'bx] \left( \frac{y}{y_{50}} \right)^3 \left( \frac{1}{\epsilon_{50}} \right) \delta \epsilon_{50} \]

(A.32) \[ \frac{\partial p}{\partial b} \delta b = 0.5 [(3c + \gamma'x)] \left( \frac{y}{2.5 \epsilon_{50} b} \right)^3 - \frac{1}{2} [(3c + \gamma'x)b + Jxc] \frac{1}{3} \left( \frac{y}{2.5 \epsilon_{50} b} \right) (-1) \frac{y}{2.5 \epsilon_{50} b^2} \delta b \]

\[ \frac{\partial p}{\partial b} \delta b = \frac{1}{2} [(3c + \gamma'x)] \left( \frac{y}{y_{50}} \right)^3 \frac{1}{b} - \frac{1}{6} [(3c + \gamma'x)b + Jxc] \frac{1}{b} \left( \frac{y}{y_{50}} \right) \delta b \]

\[ \frac{\partial p}{\partial b} \delta b = \frac{1}{2} [(3c + \gamma'x)] \left( \frac{y}{y_{50}} \right)^3 \frac{1}{b} - \frac{1}{6} [(3c + \gamma'x)b + Jxc] \frac{1}{b} \left( \frac{y}{y_{50}} \right)^3 \delta b \]

\[ \frac{\partial p}{\partial b} \delta b = \left( c + \frac{1}{3} \gamma'x \right) - \frac{Jxc}{6b} \left( \frac{y}{y_{50}} \right)^3 \delta b \]

A + B + C + D
Thus the second integration (II) of Equation (A.24) will be

\[
\int_0^{x_r} \left[ \frac{6y^2}{50} \right] \frac{1}{3} \frac{\partial p}{\partial \delta s} dx = \int_0^{x_r} \left[ \frac{y^2}{a} \right] \frac{1}{3} \frac{1}{2} (3b + Jx) \left\{ \frac{v}{y_{50}} \right\}^3 \delta c \cdot dx + \\
+ \int_0^{x_r} \left[ \frac{y^2}{a} \right] \frac{1}{3} \left( c + \frac{1}{3} \gamma' x \right) - \frac{1}{6} \delta b \cdot dx \\
- \int_0^{x_r} \left[ \frac{y^2}{a} \right] \frac{1}{3} \frac{1}{6} \left[ (3b + Jx) \gamma' \left\{ \frac{v}{y_{50}} \right\}^3 \left\{ \frac{1}{y_{50}} \right\}^2 \delta \varepsilon_{50} \cdot dx \\
+ \int_0^{x_r} \left[ \frac{y^2}{a} \right] \frac{1}{3} \left( c + \frac{1}{3} \gamma' x \right) - \frac{1}{6} \delta b \cdot dx
\]

The third integral in equation (A.24) will be

for \( y \leq 8y_{50} \) and \( x_r \leq x \leq L \)

\[(3.1)\quad p = pu \quad \frac{1}{2} \left( \frac{v}{y_{50}} \right)^3 \]

\[(3.4)\quad pu = 9cb \]

\[(A.35)\quad Pa = \frac{pu}{2} \left( \frac{ya}{y_{50}} \right) \frac{1}{3} \]

\[
Pa \left( \frac{y^2}{y_{50}} \right)^{\frac{1}{3}} = \frac{pu}{2} \left( \frac{ya}{y_{50}} \right)^{\frac{1}{3}} \frac{6(y^2 y_{50})}{pu} = 3\left( ya y^2 \right)^{\frac{1}{3}}
\]

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(A.36) \[ \{S\}^T = \{c, \varepsilon_{50}, b\} \]

(A.37) \[ \frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} = \frac{\partial p}{\partial c} \delta c + \frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} + \frac{\partial p}{\partial b} \delta b \]

(3.7) \[ \frac{p}{P_u} = \frac{1}{2} \left( \frac{y}{y_{50}} \right)^3 \]

(3.9) \[ P_u = 9cb \]

Then;

(A.38) \[ p = 9cb \frac{1}{2} \left( \frac{y}{y_{50}} \right)^3 = 4.5cb \left( \frac{y}{y_{50}} \right)^3 \]

And we know that \( p \) is independent of \( y' \). Then;

(A.39) \[ \frac{\partial p}{\partial y'} \delta y' = 0 \]

(A.40) \[ E \frac{\partial p}{\partial c} = 4.5b \left( \frac{y}{y_{50}} \right)^3 \delta c \]

(A.41) \[ F \frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} \]

(3.6) \[ y_{50} = 2.5 \varepsilon_{50} b \]

(A.42) \[ p = 4.5cb \left( \frac{y}{2.5\varepsilon_{50}b} \right)^3 \]
(A.43) \[
\frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} = 4.5 \, c \, b \left( \frac{y}{2.5 \, \varepsilon_{50} \, b} \right)^{\frac{2}{3}} \left( -1 \right) \frac{y}{2.5 \, b} \left( \varepsilon_{50} \right)^2 \delta \varepsilon_{50}
\]

\[
\frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} = 1.5 \, b \, c \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \left( - \frac{1}{\varepsilon_{50}} \right) \delta \varepsilon_{50}
\]

\[
\frac{\partial p}{\partial \varepsilon_{50}} \delta \varepsilon_{50} = -1.5 \, b \, c \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \frac{1}{\varepsilon_{50}} \delta \varepsilon_{50}
\]

(A.44) \[
G \frac{\partial p}{\partial b} \delta b = 4.5 \left( \frac{y}{2.5 \, \varepsilon_{50} \, b} \right)^{\frac{1}{3}} + 4.5 \left( \frac{y}{2.5 \, \varepsilon_{50} \, b} \right)^{\frac{2}{3}} \left( -1 \right) \left( \frac{y}{2.5 \, \varepsilon_{50} \, \left( b \right)^2} \right) \delta b
\]

\[
\frac{\partial p}{\partial b} \delta b = 4.5 \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} - 1.5 \left( \frac{2.5 \, \varepsilon_{50} \, b}{y} \right)^{\frac{2}{3}} \left( \frac{y}{y_{50}} \right)^{\frac{2}{3}} \delta b
\]

\[
\frac{\partial p}{\partial b} \delta b = 4.5 \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} - 1.5 \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \delta b
\]

\[
\frac{\partial p}{\partial b} \delta b = 3 \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}} \delta b
\]

Than: \[ E + F + G \]

(A.45) \[
L \int_{x_{pa}}^{p_a} \frac{6 \left( y \cdot y_{50} \right)}{p_{u}} \frac{1}{3} \delta S \, dx = L \int_{x_{r}}^{13.5} \left( \frac{y}{a} \right)^{\frac{1}{3}} \left( y \frac{b}{y_{50}} \right) \delta c \, dx
\]

\[
- \int_{x_r}^{4.5} \left( \frac{y}{a} \right) \frac{y \cdot c \cdot b}{\varepsilon_{50}} \frac{1}{\varepsilon_{50}} \delta e \, dx + L \int_{x_{r}}^{9} \left( \frac{y}{a} \right)^{\frac{1}{3}} \delta c \, dx
\]

And the final equation will be:
\[(A.46) \quad \delta_y = - \int_{y_{top}}^{y} y' \delta E_1 \, dx - \int_{0}^{x} \left( \frac{\delta}{\frac{1}{2} (3 b + J x) \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) \, dx \]

\[= \int_{0}^{x} \left( \frac{\delta}{\frac{1}{2} (b x) \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) \, dx \]

\[+ \int_{0}^{x} \left( \frac{\delta}{\frac{1}{6} \left( 3 b + J x \right) c + y' b x \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) \, dx \]

\[= \int_{0}^{x} \left( \frac{\delta}{\frac{1}{3} \left( c + \frac{1}{3} y' x - \frac{J x c}{6 b} \right) \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) \, dx \]

\[= \int_{L}^{13.5} \left( \frac{\delta}{\frac{1}{3} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) b \, dx \]

\[= \int_{L}^{4.5} \left( \frac{\delta}{\frac{1}{3} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) c b \left( \frac{1}{\delta_e} \right) \, dx \]

\[= \int_{L}^{9} \left( \frac{\delta}{\frac{1}{3} \left( \frac{y}{y_{50}} \right)^{\frac{1}{3}}} \right) c \, dx \]
APPENDIX B

Sensitivity analysis of top lateral displacement $\delta y_1$ for single free head piles with length $L=4T$ subjected to lateral concentrated forces.
Figure B.1  Lateral deflection of primary structure for free head pile under variable lateral force. Pile length $L = 4T$.

Figure B.2  Distribution of bending moments of primary structure for free head pile under variable lateral force – Pile length $L = 4T$.
Figure B.3  Distribution of lateral deflection $y_\alpha (\bar{P})$ of the adjoint structure subjected to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_1$ – Free head pile – Pile length $L = 4T$

Figure B.4  Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_1$ – Free head pile – Pile length $L = 4T$
Figure B.5 Distribution of soil reaction $p_a$ of primary structure subjected to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.6 Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$
Figure B.7  Distribution of sensitivity operators $C_{EI}^{py}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length = 4T

Figure B.8  Distribution of sensitivity operators $C_{c}^{py}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length = 4T
Figure B.9  Distribution of sensitivity operators $C_{\gamma}^{Py}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length = 4T

Figure B.10  Distribution of sensitivity operators $C_{b}^{Py}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length = 4T

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Figure B.11  Distribution of sensitivity operators $C_{e_{50}}^{P_{y}}$ affecting the changes of the pile head lateral deflection $\delta_{y_{t}}$ due to the changes of $e_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_{t}$ – Free head pile – Pile length = 4T

Figure B.12  The quantitative assessment of sensitivity factor $A_{EI}^{P_{y}}$ affecting the top lateral deflection $y_{t}$ due to the changes of bending stiffness $EI$ of the pile when the pile is subject to variable concentrated lateral force $P_{t}$ – Free head pile – Pile length $L = 4T$
Figure B.13  The quantitative assessment of sensitivity factor $A^y_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 4T$

Figure B.14  The quantitative assessment of sensitivity factor $A^y_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 4T$
Figure B.15  The quantitative assessment of sensitivity factor $A^{Py}_y$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$.

Figure B.16  The quantitative assessment of sensitivity factor $A^{Py}_{\varepsilon_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$.
Figure B.17  The quantitative assessment (in %) of relative sensitivity factor $F^y_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.18  The quantitative assessment (in %) of relative sensitivity factor $F^y_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 4T$
Figure B.19  The quantitative assessment (in %) of relative sensitivity factor $F_{EI}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 4T$

Figure B.20  The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 4T$
Figure B.21 The quantitative assessment (in %) of relative sensitivity factor $P_{e_{50}}^p$ affecting the top lateral deflection $y_t$ due to the changes of $e_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 4T$
APPENDIX C

Sensitivity analysis of top lateral displacement $\delta y_t$ for single free head piles with length $L=5T$ subjected to lateral concentrated forces.
Figure C.1  Lateral deflection of primary structure for free head pile under variable lateral force
Pile length L= 5T

Figure C.2  Distribution of bending moments of primary structure for free head pile under variable lateral force – Pile length L= 5T
Figure C.3  Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$

- Free head pile – Pile length $L = 5T$

Figure C.4  Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 5T$
Figure C.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 5T$

Figure C.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 5T$
Figure C.7  Distribution of sensitivity operators $C_{EI}^{Py}$ affecting the changes of the pile head lateral deflection $\delta_Y$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length = 5T

Figure C.8  Distribution of sensitivity operators $C_c^{Py}$ affecting the changes of the pile head lateral deflection $\delta_Y$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length = 5T
Figure C.9  Distribution of sensitivity operators $C^p_y$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ –
Free head pile – Pile length = 5T

Figure C.10  Distribution of sensitivity operators $C^p_b$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile –
Pile length = 5T
Figure C.11  Distribution of sensitivity operators $C_{e50}^{Py}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of $e_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length = 5T

Figure C.12  The quantitative assessment of sensitivity factor $A_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subject to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 5T$
Figure C.13  The quantitative assessment of sensitivity factor $\alpha_c^{py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 5T$

Figure C.14  The quantitative assessment of sensitivity factor $\alpha_b^{py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 5T$
Figure C.15  The quantitative assessment of sensitivity factor $A_{\gamma_1}^{P_1}$ affecting the top lateral deflection $y_1$ due to the changes of the submerged soil unit weight $\gamma_1$ when the pile is subjected to variable lateral force $P_1$ – Free head pile – Pile length $L = 5T$

Figure C.16  The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}^{P_1}$ affecting the top lateral deflection $y_1$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_1$ – Free head pile – Pile length $L = 5T$
Figure C.17 The quantitative assessment (in %) of relative sensitivity factor $F_{py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 5T$

Figure C.18 The quantitative assessment (in %) of relative sensitivity factor $F_{cy}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 5T$
Figure C.19 The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$. Free head pile – Pile length $L = 5T$

Figure C.20 The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$. Free head pile – Pile length $L = 5T$
Figure C.21 The quantitative assessment (in %) of relative sensitivity factor $\frac{\varepsilon_{50}}{P}$ affecting the top lateral deflection $\gamma_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 5T$
APPENDIX D

Sensitivity analysis of top lateral displacement $\delta y_1$ for single free head piles with length $L=10T$ subjected to lateral concentrated forces.
Figure D.1  Lateral deflection of primary structure for free head pile under variable lateral force
Pile length L= 10T

Figure D.2  Distribution of bending moments of primary structure for free head pile under variable lateral force – Pile length L= 10T
Figure D.3 Distribution of lateral deflections $y_a(\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$.
- Free head pile – Pile length $L = 10T$

Figure D.4 Distribution of bending moments $\bar{M}(\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$
Figure D.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile
- Pile length $L = 10T$

Figure D.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Free head pile
- Pile length $L = 10T$
Figure D.7  Distribution of sensitivity operators $C_{EI}^{py}$ affecting the changes of the pile head lateral deflection $\delta_{yt}$ due to the changes of bending stiffness EI when the pile structure is subjected to variable concentrated lateral force $P_t$ – Free head pile – Pile length $L = 10T$

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Figure D.12 The quantitative assessment of sensitivity factor $A_{E_{50}}^{P_y}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $E_l$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$. 

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Figure D.13  The quantitative assessment of sensitivity factor $A_c^{py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

Figure D.14  The quantitative assessment of sensitivity factor $A_b^{py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$
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Figure D.16  The quantitative assessment of sensitivity factor $A_{\varepsilon_{50}}$ affecting the top lateral deflection $\gamma_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ — Free head pile — Pile length $L = 10T$
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Figure D.18 The quantitative assessment (in %) of relative sensitivity factor $f_{C}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_l$ – Free head pile – Pile length $L = 10T$. 

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Figure D.20  The quantitative assessment (in %) of relative sensitivity factor $F_{y}^{Fy}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$
Figure D.21  The quantitative assessment (in %) of relative sensitivity factor $\varepsilon_{50}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ - Free head pile - Pile length $L = 10T$

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Figure D.36  The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ - Free head pile - Pile length $L = 10T$
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Figure D.38  The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{P0}$ affecting the top angle of flexural rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Free head pile – Pile length $L = 10T$
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APPENDIX E

Sensitivity analysis of top lateral displacement $\delta y_0$ for single fixed head piles with length $L=4T$ subjected to lateral concentrated forces.
Figure E.1  Lateral deflection of primary structure for fixed head pile under variable lateral force
Pile length L = 4T

Figure E.2  Distribution of bending moments of primary structure for fixed head pile under variable lateral force – Pile length L = 4T
Figure E.3  Distribution of lateral deflections $y_a (\bar{P})$ of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to the lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$

Figure E.4  Distribution of bending moments $\bar{M} (\bar{P})$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$
Figure E.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$

Figure E.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$
Figure E.7 Distribution of sensitivity operators $C_{EI}^{Py}$ affecting the changes of the pile head lateral deflection $\delta_{y_i}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 4T$

Figure E.8 Distribution of sensitivity operators $C_c^{Py}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 4T$
Figure E.9  Distribution of sensitivity operators $C^p_y$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ –

Fixed head pile – Pile length $L = 4T$

Figure E.10  Distribution of sensitivity operators $C^p_y$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile –

Pile length $L = 4T$
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Figure E.12 The quantitative assessment of sensitivity factor $\lambda_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 4T$
Figure E.13 The quantitative assessment of sensitivity factor $A_c^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_l$ – Fixed head pile – Pile length $L = 4T$

Figure E.14 The quantitative assessment of sensitivity factor $A_b^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_l$ – Fixed head pile – Pile length $L = 4T$
Figure E.15  The quantitative assessment of sensitivity factor $A_{\gamma^1}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma^1$ when the pile is subjected to variable lateral force $P_l$ – Fixed head pile – Pile length $L = 4T$

Figure E.16  The quantitative assessment of sensitivity factor $A_{\gamma_50}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_l$ – Fixed head pile – Pile length $L = 4T$
Figure E.17  The quantitative assessment (in %) of relative sensitivity factor $F^y_b$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$

Figure E.18  The quantitative assessment (in %) of relative sensitivity factor $F^y_c$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$
Figure E.19  The quantitative assessment (in %) of relative sensitivity factor
$F_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$
of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed
head pile – Pile length $L = 4T$

Figure E.20  The quantitative assessment (in %) of relative sensitivity factor
$F_{\gamma'}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit
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$P_t$ – Fixed head pile – Pile length $L = 4T$
Figure E.21  The quantitative assessment (in %) of relative sensitivity factor $F^{Py}_{\varepsilon_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of the $\varepsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 4T$
APPENDIX F

Sensitivity analysis of top lateral displacement $\delta y_0$ for single fixed head piles with length $L=5T$ subjected to lateral concentrated forces.
Figure F.1  Lateral deflection of primary structure for fixed head pile under variable lateral force
Pile length $L = 5T$

Figure F.2  Distribution of bending moments of primary structure for fixed head pile under variable lateral force – Pile length $L = 5T$
Figure F.3  Distribution of lateral deflections $y_a$ ($\bar{P}$) of the adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.4  Distribution of bending moments $\bar{M}$ ($\bar{P}$) of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{P} = 1$ when the primary structure is subject to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.7 Distribution of sensitivity operators $C_{EI}^{P_{y}}$ affecting the changes of the pile head lateral deflection $\delta_{y_{t}}$ due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated lateral force $P_{t}$ – Fixed head pile – Pile length $L = 5T$.

Figure F.8 Distribution of sensitivity operators $C_{c}^{P_{y}}$ affecting the changes of the pile head lateral deflection $\delta_{y_{t}}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_{t}$ – Fixed head pile – Pile length $L = 4T$.
Figure F.9  Distribution of sensitivity operators $C_{py}^P$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.10  Distribution of sensitivity operators $C_{by}^P$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.11  Distribution of sensitivity operators $C_{b}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta_y$ due to the changes of $c_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.12  The quantitative assessment of sensitivity factor $A_{EI}^{P_y}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness EI of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.13  The quantitative assessment of sensitivity factor $A_{E1}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.14  The quantitative assessment of sensitivity factor $A_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.15  The quantitative assessment of sensitivity factor $A_{\gamma^{'}}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_l$ – Fixed head pile – Pile length $L = 5T$

Figure F.16  The quantitative assessment of sensitivity factor $A_{e_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of $e_{50}$ when the pile is subjected to variable lateral force $P_l$ – Fixed head pile – Pile length $L = 5T$

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Figure F.17 The quantitative assessment (in %) of relative sensitivity factor $F_{b}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$

Figure F.18 The quantitative assessment (in %) of relative sensitivity factor $F_{c}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 5T$
Figure F.19 The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 5T$

Figure F.20 The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma'}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 5T$
Figure F.21 The quantitative assessment (in %) of relative sensitivity factor $\frac{\Delta P_y}{\Delta \epsilon_{50}}$ affecting the top lateral deflection $y_t$ due to the changes of $\epsilon_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile

Pile length $L = 5T$
APPENDIX G

Sensitivity analysis of top lateral displacement $\delta y_0$ for single fixed head piles with length $L=10T$ subjected to lateral concentrated forces.
Figure G.1  Lateral deflection of primary structure for fixed head pile under variable lateral force
Pile length $L = 10T$

Figure G.2  Distribution of bending moments of primary structure for fixed head pile under variable lateral force – Pile length $L = 10T$
Figure G.3  Distribution of lateral deflections $y_a(\overline{P})$ of the adjoint structure subject to $\overline{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.4  Distribution of bending moments $\overline{M}(\overline{P})$ of adjoint structure subject to $\overline{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.5  Distribution of soil reaction $p_a$ of primary structure subjected to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{P} = 1$ when the primary structure is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.7  Distribution of sensitivity operators $C_{Ei}^{py}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of bending stiffness $Ei$ when the pile structure is subjected to variable concentrated lateral force $P_t$ — Fixed head pile — Pile length $L = 10T$

Figure G.8  Distribution of sensitivity operators $C_{c}^{py}$ affecting the changes of the pile head lateral deflection $\delta_{y_t}$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated lateral force $P_t$ — Fixed head pile — Pile length $L = 10T$
Figure G.9 Distribution of sensitivity operators $C_{\gamma}^{\gamma'}$ affecting the changes of the pile head lateral deflection $\delta_{\gamma_1}$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.10 Distribution of sensitivity operators $C_{b}^{\gamma'}$ affecting the changes of the pile head lateral deflection $\delta_{\gamma_1}$ due to the changes of the pile's width $b$ when the pile structure is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.11  Distribution of sensitivity operators $C_{P_y}^{P_y}$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ when the pile structure is subjected to variable concentrated lateral force $P_t$. Fixed head pile - Pile length $L = 10T$.

Figure G.12  The quantitative assessment of sensitivity factor $A_{E, EI}^{P_y}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$. Fixed head pile - Pile length $L = 10T$. 

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Figure G.13 The quantitative assessment of sensitivity factor $A_c^p$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$.

Figure G.14 The quantitative assessment of sensitivity factor $A_b^p$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ - Fixed head pile - Pile length $L = 10T$.

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Figure G.15  The quantitative assessment of sensitivity factor $A_{\gamma'}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.16  The quantitative assessment of sensitivity factor $\Lambda_{\varepsilon_{50}}^{P_y}$ affecting the top lateral deflection $y_t$ due to the changes of $\varepsilon_{50}$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.17  The quantitative assessment (in %) of relative sensitivity factor $F_b^P$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.18  The quantitative assessment (in %) of relative sensitivity factor $F_c^P$ affecting the top lateral deflection $y_t$ due to the changes of cohesion $c$ when the pile is subjected to variable lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.19  The quantitative assessment (in %) of relative sensitivity factor $F_{EI}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$

Figure G.20  The quantitative assessment (in %) of relative sensitivity factor $F_{P_t}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of the submerged unit weight of the soil $\gamma'$ when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$
Figure G.21 The quantitative assessment (in %) of relative sensitivity factor $F_{e_{50}}^{Py}$ affecting the top lateral deflection $y_t$ due to the changes of $e_{50}$ when the pile is subjected to variable concentrated lateral force $P_t$ – Fixed head pile – Pile length $L = 10T$. 

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APPENDIX H

Sensitivity analysis of top lateral displacement $\delta y_0$ for single free head piles with length $L=4T$ under bending moment.
Figure H.1 Distribution of lateral deflection of primary structure for free head pile under variable bending moment - Pile length $L = 4T$

Figure H.2 Distribution of bending moments of primary structure for free head pile under variable bending moment – Pile length $L = 4T$
Figure H.3  Distribution of lateral deflections $y_a (\bar{M})$ of the adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$ 

Figure H.4  Distribution of bending moments $\bar{M} (\bar{M})$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$
Figure H.5  Distribution of soil reaction $p_a$ of primary structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.6  Distribution of soil reaction $p_a$ of adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$
Figure H.7  Distribution of sensitivity operators $C^M_{EI}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta$, due to the changes of bending stiffness $EI$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.8  Distribution of sensitivity operators $C^M_{c}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta$, due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$
Figure H.9  Distribution of sensitivity operators $C_{\gamma}^{M_{\theta}}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta$, due to the changes of the submerged soil unit weight $\gamma'$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$

Figure H.10  Distribution of sensitivity operators $C_{b}^{M_{\theta}}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta$, due to the changes of the pile’s width $b$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$
Figure H.11  Distribution of sensitivity operators $C_{e50}^{M0}$ affecting the changes of the pile head top flexural angle of rotation $\delta \theta_1$ due to the changes of $e_{50}$ when the pile structure is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 4T$

Figure H.12  The quantitative assessment of sensitivity factor $A_{EI}^{M0}$ affecting the top flexural angle of rotation $\theta_1$ due to the changes of bending stiffness $EI$ of the pile when the pile is subjected to variable concentrated bending moment $M_t$ - Free head pile - Pile length $L = 4T$
Figure H.13  The quantitative assessment of sensitivity factor $A_c^{M\theta}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of cohesion $c$ when the pile structure are subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 4T$.

Figure H.14  The quantitative assessment of sensitivity factor $A_b^{M\theta}$ affecting the top flexural angle of rotation $\theta_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$. 

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Figure H.15  The quantitative assessment of sensitivity factor $A_{\theta}^{M}$ affecting the top flexural angle of rotation $\theta$, due to the changes of the submerged soil unit weight $\gamma'$ when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 4T$

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APPENDIX I

Sensitivity analysis of top lateral displacement $\delta y_0$ for single free head piles with length $L=5T$ under bending moment.
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Figure I.8  Distribution of sensitivity operators $C_{c}^{M_0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ of the pile length $L=5T$ due to the changes of cohesion $c$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$
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Figure I.11  Distribution of sensitivity operators $C_{e50}^{M0}$ affecting the changes of the pile top flexural angle of rotation $\delta \theta_t$ of the pile length $L=5T$ due to the changes of $e_{50}$ when the pile structure is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 5T$

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Figure J.2 Distribution of bending moments of primary structure for free head pile under variable bending moment - Pile length $L = 10T$
Figure J.3  Distribution of lateral deflections $y_a (\bar{M})$ of the adjoint structure subject to $\bar{M} = 1$ when the primary structure is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$

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- Pile length $L = 10T$

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Figure J.38  The quantitative assessment (in %) of relative sensitivity factor $F_{\gamma y}$ affecting the top lateral deflection $y_t$ due to the changes of width $b$ of the pile when the pile is subjected to variable bending moment $M_t$ – Free head pile – Pile length $L = 10T$.

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Figure J.39  The quantitative assessment (in %) of relative sensitivity factor $\Gamma^M_{b}$ affecting the top lateral deflection $\gamma_t$ due to the changes of bending stiffness EI of the pile when the pile is subjected to variable concentrated bending moment $M_t$ – Free head pile – Pile length $L = 10T$
APPENDIX K

Sensitivity analysis for Pile B (1st tailing row) in a pile group embedded in the soft clay below the ground water table subjected to a lateral concentrated force \( P_g \) applied on the cap of the pile group, with pile members pinned to the pile cap, and with pile spacing between piles equal 2D. Length of the pile equal to 10T.
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Figure K. 6  Distributions of sensitivity operators $C$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of cohesion $c$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal $5D$ – Pile length = $10T$.
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Figure K. 10  Distributions of sensitivity operators $C$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $e_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T.
Figure K. 11 Distributions of sensitivity operators $C$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

Figure K. 12 Distributions of sensitivity operators $C$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of $\varepsilon_{50}$ of the soil for the pile B (first trailing row) in the 3x3 pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T
Figure K. 13  Distributions of sensitivity operators $C$ affecting the changes of the pile head lateral deflection $\delta y_t$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

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Figure K. 16 The quantitative assessment of sensitivity factor $A_{c}^{py}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of cohesion c of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load $P_g$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal $5D$ – Pile length = 10T
Figure K. 17 The quantitative assessment of sensitivity factor $A_{EI}^{Py}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of bending stiffness $EI$ of the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load $Pg$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

Figure K. 18 The quantitative assessment of sensitivity factor $A_{\gamma'}^{Py}$ affecting the top lateral deflection lateral $\delta y_t$ due to the changes of the submerged soil unit weight $\gamma'$ of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load $Pg$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

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Figure K. 19 The quantitative assessment of sensitivity factor $A_{P_{y}^{P_{y}}}$ affecting the top lateral deflection lateral $\delta y_{t}$ due to the changes of the $e_{50}$ of the soil for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load $P_{g}$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

Figure K. 20 The quantitative assessment of sensitivity factor $A_{b}^{P_{y}}$ A affecting the top lateral deflection lateral $\delta y_{t}$ due to the changes of the pile width $b$ for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load $P_{g}$ of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T
Figure K. 21 The total quantitative assessment of sensitivity factor \( A_{\text{Py}}^{\text{total}} \) affecting the top lateral deflection lateral \( \delta y_t \) due to the changes of the design variables for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

Figure K. 22 The quantitative assessment of the relative sensitivity factor \( F_{\text{Py}}^{\text{total}} \) affecting the top lateral deflection lateral \( \delta y_t \) due to the changes of the design variables for the pile B (first trailing row) in the 3x3 free head pile group subjected to a lateral load \( P_g \) of discrete variability – Piles pinned to the cap, and the pile spacing is equal 5D – Pile length = 10T

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APPENDIX L

Input and output data for laterally loaded pile analysis
Input and output data for laterally loaded pile analysis

L.1 Typical example of input data and output data for single free head single pile subjected to a lateral concentrated load embedded in soft clay below the ground water table

L.1.1 Input file from COM624P – version 2.0

Free Head Lateral Force F=030 L=24.5

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</table>

L.1.2 Output file from COM624P – version 2.0

******************************************************************************
PILE DEFLECTION, BENDING MOMENT, SHEAR & SOIL RESISTANCE
******************************************************************************

OUTPUT INFORMATION
******************************************************************************

PILE LOADING CONDITION
LATERAL LOAD AT PILE HEAD = .280E+02 KN
APPLIED MOMENT AT PILE HEAD = .000E+00 M-KN
AXIAL LOAD AT PILE HEAD = .000E+00 KN
X DEFLECTION MOMENT TOTAL SHEAR SOIL FLEXURAL
STRESS RESIST RIGIDITY
M M M-KN LBS/M**2 KN LBS/M KN-M**2
******************************************************************************

307

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<td>.110E+06</td>
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| 11.51 | -.391E-17  | -.231E-08 | .107E-05  | -.201E-05 | .110E+06 |
| 11.76 | .949E-19   | -.751E-11 | .347E-08  | .380E-07  | .110E+06 |
| 12.00 | .325E-21   | .173E-12  | .797E-10  | .153E-10  | .110E+06 |
| 12.25 | -.709E-23  | .622E-15  | .287E-12  | -.283E-11 | .110E+06 |
| 12.49 | -.268E-25  | -.129E-16 | .596E-14  | -.127E-14 | .110E+06 |
| 13.47 | -.178E-33  | -.751E-11 | .347E-08  | .471E-08  | .110E+06 |
| 13.72 | .949E-19   | -.751E-11 | .347E-08  | .380E-07  | .110E+06 |
| 13.96 | .325E-21   | .173E-12  | .797E-10  | .153E-10  | .110E+06 |
L.2.2 Typical example of input data and output data for pile group with the piles pinned to the cap subjected to a lateral concentrated

L.2.2.1 Input file from FB-Pier – version 3

PROBLEM
Default Pier Structure
Units are KN and Meters
University of Windsor
Free Head Lateral Force F=0750 D=5 N=0
Dr. Budkowska
27/09/06
Marcia Mora

PRINT
L=0 M=0 D=1 O=1 S=1 P=1 T=0 F=0 C=0 B=0 I=1 R=0 N=0 X=0

CONTROL
1 U= 1 D= 0 S= 0 R= 0 N= 51 Z= 0 P= 0 V=1.0 : NUMLC
S= 0 T= 0 P= 1 F= 0
I= 500 T= 1e-006

PILE
NSET= 1 S= 0 M= 0 NSEG= 1
C Custom
C T=1 D=0 U=1 : PreCast - linear
L=24.5 E=2.0e+008 I=0.0006,0.0006 J=0.0006 G=8.33e+007 \
A=0.0178 D=0.508 S=1.5500 K=1
E= 0 H= 0 A= 1 S= 1 G= 0 C= 0
5 5 : NPX, NPY
1.016 2.54 2.54 0.812
1.016 2.54 2.54 0.812
0.86 0.89 0.94
0.86 0.89 0.94

MISSING
16 : number of missing piles
1 1
2 2
3 1
4 1
5 1
1 2
5 2
1 3
5 3
1 4
5 4
1 5
2 5
3 5
4 5
5 5
:
SOIL
NSET= 1 L= 1 R= 1 C= 0, W= 0 O= 0 S= 0 : Nlayers,kcyc
32 40715 7.85 24 0.01 24 24132 0.3 55.2 20
6 1 1 1 0 0 0 0 0
E=0,-40 B=-40 S=1
32 40715 7.85 24 0.01 24 24132 0.3 55.2
24132 0.35 1333 1 : Soil set 1 tip info
:
CAP
E= 2.8e+007 U= 0.2 T= 1.5 S= 25
:
LOAD
5 L= 1 F= 755 0 0 0 0 0
:
SWFACT
1 F= 0 0
:

L.2.2 Output file from FB-Pier – version 3 (just part)

The output from FB-Pier has more than 100 pages, and will be out of purpose to present all those pages in this study.

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29 Page 1

Project client : University of Windsor
Project name : Free Head Lateral Force F=0750 D=5 N=0
Project manager : Dr. Budkowska
Computed by: Marcia Mora

Project description:

Analysis Units Specified are: Metric using Meters and Kilo-Newtonns

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29 Page 2

*** PY MULTIPLIERS WILL BE USED ***

AXIAL PILE EFFICIENCY (on all piles) AXEFF = 1.000

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29 Page 3

******************************
ANALYSIS OPTIONS
******************************

TYPE OF ANALYSIS IS = STATIC ANALYSIS

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29 Page 4

PY CURVE DATA:

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<tr>
<th>PHI (DEG)</th>
<th>K kN/M^3</th>
<th>GAMMA' kN/M^3</th>
<th>CU E50 kN/M^2</th>
<th>E100 M/M</th>
</tr>
</thead>
</table>

**PILE NUMBER: 1, SOIL SET NUMBER: 1**

1 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00
2 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00
3 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00
4 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00
5 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00
6 32.00 0.1629E+05 7.850 24.00 0.2000E-01 24.00

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29 Page 12

PILE SET AND CORRESPONDING PILE NUMBER

Pile Set  Piles
1  1, 2, 3, 4, 5, 6, 7, 8, 9,

TOTAL PILE LENGTH FOR EACH PILE SET

Pile Set  Length
INPUT FOR STRUCTURAL ANALYSIS

Input File = "15007500 " Analysis Run on 7-28-2003 at 18:29

NUMBER OF JOINTS = 81
NUMBER OF DIFFERENT ELEMENT TYPES = 3
NUMBER OF LOAD CONDITIONS = 1

Forming General Loads

*** Summary of Load Cases ***

NODE LOAD X Y Z XX YY ZZ
(KN) (KN) (KN) (KN-M) (KN-M) (KN-M)
5 1 750.00 0.00 0.00 0.00 0.00 0.00

LOAD CASE #= 1

PY Multipliers are applied lead to trail row based on the actual displacement. If no displacement occurs in a lateral direction, they are defaulted to 1.0. This can happen in axial load and one direction lateral load cases.

PY MULTIPLIERS APPLIED TO PILE GROUP

PILE#   X-PYM   Y-PYM
1  0.860E+00  0.940E+00
2  0.890E+00  0.940E+00
3  0.940E+00  0.940E+00
4  0.860E+00  0.890E+00
5  0.890E+00  0.890E+00
6  0.940E+00  0.890E+00
7  0.860E+00  0.860E+00
8  0.890E+00  0.860E+00
9  0.940E+00  0.860E+00

**** CONVERGENCE ACHIEVED FOR PIER ANALYSIS ****
THE SOLUTION CONVERGED IN 54 ITERATIONS

SUMMARY OF DISPLACEMENTS AT PILE HEADS ONLY:

NODE X Y Z
(M) (M) (M)
1 0.232E-01 -0.215E-05 0.000E+00
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<th>X</th>
<th>Y</th>
<th>Z</th>
<th>XX</th>
<th>YY</th>
<th>ZZ</th>
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<td>(M)</td>
<td>(M)</td>
<td>(M)</td>
<td>(rad)</td>
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*** Pile Displacements ***

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<tr>
<td>82 0.201E-01 -0.835E-06 0.000E+00 -0.219E-05 -0.631E-02 0.101E-06</td>
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SUMMARY OF ABS MAXIMUM OUT-OF-BALANCE FORCES:

- FZZ = 0.25016E-08 kN
- FXX = 0.55335E-08 kN
- FYY = 0.10941E-08 kN
- MXX = 0.61263E-08 kN-M
- MYY = 0.00000 kN-M
- MZZ = 0.20018E-08 kN-M

OUT OF BALANCE FORCES:

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<th>MXX</th>
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VITA AUCTORIS

Name: Marcia Regina Mora
Place of Birth: São Paulo, Brazil
Date of Birth: December 31, 1959
Education
Universidade Mackenzie
São Paulo, Brazil
Bachelor of Science in Civil Engineering
1985

Universidade de São Paulo
São Carlos Campus, Brazil
Master of Science in Civil Engineering
1993

University of Windsor
Windsor, Ontario, Canada
Candidate for Master of Applied Science in Civil Engineering
2006