A new energy-based method for evaluating the damping properties of cable-damper systems.

Xianshu Jiang

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation


https://scholar.uwindsor.ca/etd/7144

This online database contains the full-text of PhD dissertations and Masters’ theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000ext. 3208.
A New Energy-Based Method for
Evaluating the Damping Properties of Cable-Damper Systems

by
Xianshu Jiang

A Thesis
Submitted to the Faculty of Graduate Studies and Research
Through Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the Degree of
Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

2006

© 2006 Xianshu Jiang
NOTICE:
The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

Canada

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Abstract

External dampers and cross-ties are commonly used to control vibrations of stay cables on cable-stayed bridges. While dampers contribute directly to the structural damping of the attached cables, cross-ties mainly help to increase the stiffness of the connected cable system. The idea of incorporating cross-ties with dampers has also been attempted in a few recent applications to benefit from their combined effect.

A new energy-based method is proposed in the current study to evaluate the damping properties of a cable-damper system. The proposed method overcomes the weaknesses and limitations in the previous studies; and its validation is verified through prior work.

Parametric studies are done to investigate the effect of some parameters on damping properties of a cable-damper system. Also, studies are conducted to investigate the combined effect of a damper and cross-ties.

The conversion method which links the effect of the external damper in a cable-damper system to the equivalent Rayleigh damping of a single cable system is presented, which should be of significant interest to bridge design engineers.
Acknowledgments

I wish to acknowledge my supervisor, Dr. Shaohong Cheng, for her conscientious supervision for this thesis, Dr. Rupp Carriveau for his tremendous support and invaluable guidance throughout my research. It was such a wonderful learning experience and I enjoyed working with them. I also wish to thank my thesis committee, Dr. Nader Zamani and Dr. Barbara Budkowska for their helpful comments on my thesis. Finally, I wish to thank my wife, Junru Zhao for her help in writing and editing.
# Table of Contents

Abstract .....................................................................................................................................iii
Acknowledgments.......................................................................................................................iv
List of Figures............................................................................................................................vii
List of Tables.............................................................................................................................ix
Nomenclature..............................................................................................................................x

CHAPTER 1  Literature Review.................................................................................................1
  1.1 Introduction ..............................................................................................................1
  1.2 Motivations ...............................................................................................................9
  1.3 Objectives ...............................................................................................................13

CHAPTER 2 Energy-Based Evaluation of Damping Properties in Cable-Damper System 16
  2.1 Introduction ............................................................................................................16
  2.2 Energy-Based Damping Evaluation of a Cable-Damper System .......................19
  2.3 Finite Element Model ............................................................................................21
  2.4 Time-History Analysis Approach .........................................................................25
  2.5 Input of Rayleigh Damping in the Finite Element Analysis ..............................28
  2.6 Equivalent Cable Damping Conversion Method .................................................31

CHAPTER 3 Validation of the Energy-Based Damping Evaluation Method .........................35
  3.1 Physical Tests ........................................................................................................35
  3.2 Validation of the Proposed Finite Element Model and Time-History Analysis Approach .............................................................................................................40
  3.3 Validation of the Proposed Energy-Based Damping Evaluation Method 43

CHAPTER 4 Parametric Study ...............................................................................................46
  4.1 Basic Parameters ....................................................................................................46
  4.2 Ranges and Combinations of Parameters .............................................................47
  4.3 Results ....................................................................................................................50

CHAPTER 5 Discussion...........................................................................................................55
  5.1 Effects of Non-dimensional Damping Parameter ................................................55
  5.2 Effect of Non-Dimensional Cable Bending Parameter .......................................59
  5.3 Effect of Damper Location ....................................................................................60
  5.4 Effect of the Stiffness of a Damper Assembly .......................................................67
List of Figures

Figure 1-1 The Ponte de Normandie in Le Havre, France .......................................................... 1
Figure 1-2 Different Types of Aerodynamically Treated Cable Surface ........................................ 4
Figure 1-3 Cross-Tie Configuration of Cape Girardeau Bridge, Missouri, USA ............................... 5
Figure 1-4 Oil Dampers of the Tsurumi Tsubasa Bridge, Japan ...................................................... 5
Figure 1-5 Combined Application of Cross-ties and Damper on Cable-Stayed Bridges ............ 12
Figure 2-1 Kinetic Energy Decay Curves of Different Cable-Damper Systems .............................. 18
Figure 2-2 Configuration of a Cable Damper System .................................................................... 19
Figure 2-3 Numerical Model of the Cable-Damper System Used in the Current Study ............... 22
Figure 2-4 PIPE59 Element in ANSYS10.0 ............................................................................. 22
Figure 2-5 COMBIN14 Element in ANSYS10.0 ........................................................................ 24
Figure 2-6 Finite Element Model for Damper Assembly ................................................................. 24
Figure 2-7 Relationship between Rayleigh Damping Ratio and Frequency ................................ 30
Figure 2-8 Finite Element Models for Equivalent Damping Calculation ...................................... 32
Figure 2-9 Kinetic Decay Sketches for Equivalent Damping Calculation ..................................... 33
Figure 2-10 Equivalent Rayleigh Damping Calculation Sketch .................................................... 34
Figure 3-1 Physical Test Setup in Reference [22] ......................................................................... 36
Figure 3-2 Displacement Time-History at the Middle Point (damping factor = 1680 N·m/s) ......... 42
Figure 3-3 Time-History of Kinetic Energy Decay of the Cable-Damper System ...................... 43
Figure 3-4 Finite Element Models used for Validation Analysis ................................................... 45
Figure 4-1 Time-History of Kinetic Energy Decay Curve (Stiffness = 14.9 N/m) ......................... 54
Figure 5-1 Kinetic Energy Decay Curves (Damper Location Parameter = 0.06L) ......................... 56
Figure 5-2 Kinetic Energy Decay Curves (Damper Location Parameter = 0.2L) ......................... 56
Figure 5-3 Kinetic Energy Decay Curves (Damper Location Parameter = 0.3L) ......................... 57
Figure 5-4 Kinetic Energy Decay Curves (Damper Location Parameter = 0.4L) ......................... 57
Figure 5-5  Kinetic Energy Decay Curves (Damper Location Parameter =0.45L)....................58
Figure 5-6  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 100) ...61
Figure 5-7  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 200) ...62
Figure 5-8  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 300) ...62
Figure 5-9  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 400) ......63
Figure 5-10 Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 500) ....63
Figure 5-11 Cable Damper Connection.....................................................................................66
Figure 5-12 Kinetic Energy Decay Curve (Considering Stiffness of Damper Assembly) ........69
List of Tables

Table 3-1  Cable Properties in Reference [22] ................................................................. 36
Table 3-2  Physical Test Result from Reference [22] .......................................................... 38
Table 3-4  Comparison of Results .................................................................................. 41
Table 3-5  Verification Results ....................................................................................... 45
Table 4-1  Parameters of the Cable-damper System Used in the Current Study .......... 49
Table 4-2  Kinetic Energy Decay Ratio (Damper Location 0.06L) ....................................... 51
Table 4-3  Kinetic Energy Decay Ratio (Damper Location 0.2L) .......................................... 51
Table 4-4  Kinetic Energy Decay Ratio (Damper Location 0.3L) .......................................... 52
Table 4-5  Kinetic Energy Decay Ratio (Damper Location 0.4L) .......................................... 52
Table 4-6  Kinetic Energy Decay Ratio (Damper Location 0.45L) ....................................... 53
Table 4-7  Kinetic Energy Decay vs. Stiffness of Damper Assembly Changes .................. 54
Nomenclature

$\delta_n$ modal damping for nth mode, [0]

$D_n$ modal dissipated energy per cycle, J, $[F \cdot L]$ 

$U_n$ modal potential energy, J, $[F \cdot L]$ 

$L$ total length of the cable, m, $[L]$ 

$d$ kinetic energy decay ration in first vibration second, [0] 

$E_{max}$ maximum kinetic energy of cable-damper system in first vibration period, J, $[F \cdot L]$ 

$E_{ave}$ average kinetic energy of cable-damper system in first one second vibration period, J, $[F \cdot L]$ 

$x,y,z$ local coordinate axis in Cartesian system, m, $[L]$ 

$F,T$ damping force or torque, N or Nm, $[ML/T^2]$ 

$c_v$ damping coefficient, Nm/s, $[ML^2/T^3]$ 

$c_{vl}$ linear damping coefficient in COMBIN14 element, Nm/s, $[ML^2/T^3]$ 

$c_{v2}$ second damping coefficient in COMBIN14 element, Nm/s, $[ML^2/T^3]$ 

$T$ time, s, $[T]$ 

$[M]$ mass matrix 

$[C]$ damping matrix
\[ [K] \quad \text{stiffness matrix} \]

\[ \{u\}, \{\dot{u}(t)\} \quad \text{acceleration vector} \]

\[ \{u\}, \{\ddot{u}(t)\} \quad \text{velocity vector} \]

\[ \{u\}, \{u(t)\} \quad \text{displacement vector} \]

\[ \{F^a\} \quad \text{external force vector} \]

\[ \Delta \quad \text{diagonal matrix, } \Delta = \text{diag}(2\omega_i\xi_i) \]

\[ \omega_i \quad \text{angular frequency of system, rad/s, } [1/T] \]

\[ \xi_i \quad \text{modal damping ratio, } [0] \]

\[ \Omega^2 \quad \text{diagonal matrix listing eigenvalues} \quad \omega_i^2, \omega_2^2, \ldots, \omega_n^2, [0] \]

\[ \delta \cdot t \quad \text{time step of Newmark integration scheme, s, } [T] \]

\[ \alpha, \beta \quad \text{integration parameters of Newmark integration scheme, } [0] \]

\[ C \quad \text{Rayleigh damping, Nm/s, } [ML^2/T^3] \]

\[ a_0, a_1 \quad \text{Rayleigh damping factors, } [0] \]

\[ \psi, \psi_1 \quad \text{non-dimensional damping parameter, } [0] \]

\[ \zeta \quad \text{non-dimensional bending stiffness parameter, } [0] \]

\[ \delta \quad \text{damping ratio, } [0] \]
EA equivalent axial rigidity of cable, N, \( M \cdot L / T^2 \)

EI equivalent flexural rigidity of cable, \( Nm^2 \), \( M \cdot L^3 / T^2 \)

M mass per unit length of cable, Kg, \( M \)

H axial force acted on cable, N, \( M \cdot L / T^2 \)

\( u_n \) cable middle point displacement at nth cycle

\( u_{n+m} \) cable middle point displacement at (n+m)th cycle

\( \omega \) first undamped angular frequency of cable vibration, rad/s, \( 1 / T \)

\( \omega_d \) first damped angular frequency of cable vibration, rad/s, \( 1 / T \)

\( \Omega \) imaginary part of the eigenvalue

\( \sigma \) real part of the eigenvalue

\( \Gamma_d \) damper location parameter, [0]

\( \omega_{1,s} \) first mode angular frequency of string equivalent to cable without damper, Hz, \( 1 / T \)

\( L_d \) distance of damper to cable one near end, m, \( L \)

\( a_0, a_1 \) Rayleigh damping factors, 1/s, \( 1 / T \)

\( c' \) damper coefficient per unit length of cable, N/s, \( ML / T^3 \)

\( k' \) lateral support stiffness per unit length of cable, N/m\(^2\), \( M / (T^2 L) \)

\( \Delta T \) The selected time duration for calculating \( E_{ave} \), s, \( T \)
CHAPTER 1  Literature Review

1.1  Introduction

The cable-stayed bridge is a modern type of bridge. It became popular during the 1950's. Over the last three decades, the cable-stayed bridge has gained increasing popularity due to its economic advantages and spectacular aesthetic qualities. Recently, due to advances in materials, design and construction technologies, bridge spans have become longer and longer. The current longest span cable-stayed bridge is the Tatara Bridge in Ehime, Japan. It has a main span of 890m. The second longest, the Ponte de Normandie in Le Havre, France, spans 856m, as shown in Figure 1-1. The Sutong Bridge in China, which is designed to span 1088m, is currently under construction. It is expected to be open to the public in 2009.

Figure 1-1  The Ponte de Normandie in Le Havre, France

As a primary structural member of a cable-stayed bridge, the stay cable plays an important role in the overall behaviour of the entire structure. Due to its low
internal damping and high flexibility, the bridge stay cable is sensitive to dynamic excitation. Wind is an important source of external excitation which can cause cable vibration. Wind-induced cable vibration can be classified into several categories according to its excitation mechanisms: a) Vortex induced vibration (or Aeolian oscillation); b) Buffeting due to wind gust; c) Classical galloping (typically observed in iced cables); d) Wake interference (or wake galloping and resonant buffeting); e) Parametric excitation; f) Reynolds number drag instability; g) Rain-wind induced vibration; h) High-speed vortex excitation; and i) Dry inclined galloping [1]. The latter three phenomena are specifically related to inclined bridge stay cables. Hikami & Shiraish [2] first addressed the issue of rain-wind-induced cable vibration on the Meiko-Nishi Bridge in Japan. The observed vibration, with amplitude up to two times the cable diameter and frequency between 1-3 Hz, occurred when moderate rain was accompanied by wind. The excitation mechanisms of high-speed vortex excitation and dry inclined galloping are not yet clear, while some others such as rain-wind-induced vibration are now better understood. Countermeasures have been adopted to various degrees of success in practice.

Frequent and excessive vibration can result in premature cable, connection failure and/or breakdown of cable corrosion protection system, which reduces the life of a cable structure [3]. The cumulative fatigue damage to a cable assembly due to
vibration is another important issue. In addition, large amplitude vibration of cables (on the order of 1-2m) has been observed when moderate rain and wind exist [4]. Such larger amplitude vibration will certainly raise public concern about the safety of the bridge [5].

Two different means have been developed to suppress and prevent cable vibration: a) The aerodynamic countermeasures, i.e. to improve the aerodynamic properties of the cable; and b) The mechanical countermeasures, i.e. to increase the damping of the cable. Various methods have been proposed to improve cable aerodynamic properties. Flamand [6] and Bosdogianni & Olivari [7] proposed to attach helical wire whirling on the cable surface; Virlogeux [8] suggested adopting dimpled surface cable; and Saito et al. [9] advised to use axially protuberant cables. Figure 1-2 shows four types of aerodynamic treatment of the cable surface. Each of these methods has been proven to be effective and successful to some extent. Installing cross-ties in the cable plane or attaching an external damper to the cable near the cable-deck anchorage point are used to increase the damping of the cable system, which can suppress cable vibration effectively, as shown in Figure 1-3. Introduction of transverse elements, cross-ties, in the cable plane reduces the free length of the crossed cable and thus increases the frequency and stiffness of the cable system. This will yield higher critical wind speed and helps to mitigate cable
vibration [10, 11, and 12]. In general, the behaviour of a cable network is characterized by an increase of the frequencies with respect to the longest cable. Installing damping devices on or near the cable-deck anchorage point is also effective. Figure 1-4 shows an oil damper installed near the deck-anchorage point. There are three types of external damper available: passive, active and semi-active. Since cable vibration is often dominated by resonance, very good efficiency can often be achieved by suitable tuning of a passive damper [13].

![Sketch of Helical Wire](image1)

![Dimple Surface Cable of the Tatara Bridge](image2)

![Axially Protuberant Cable](image3)

![U Shape Grooving Cable of Yuge Bridge](image4)

Figure 1-2 Different Types of Aerodynamically Treated Cable Surface
In the last three decades, much research has been carried out to investigate the effects of different types of external dampers on suppressing cable vibration through physical experiments. Using scaled model cables, Xu et al. [14] studied the effects of an oil damper on cable vibration mitigation. The relationship between the increase of the cable modal damping ratio and the damper coefficient was investigated through free vibration tests. Wu [15] made an experimental study on the application of a
magnetorheological (MR) damper to cable vibration. Based on the results, the MR damper was recommended for controlling both free and forced vibration. Gu and Du [16] successfully reproduced rain-wind-induced cable vibration in a wind tunnel. They examined the effect of cable inclination angle, cable frequency and cable damping on cable response.

Generally speaking, experimental tests are expensive. Such tests are commonly used to verify analytical theories and methods. Analytical methods are sometimes more useful to help people understand the vibration mechanisms, and more researchers have devoted their efforts to analytical methods. For example, by using an analytical/numerical hybrid method, Y.L. Xu et al. [17] explored forced vibration of sagged cables with discrete oil dampers under harmonic excitation. Kyu-Sik Park et al. [18] studied the effect of a hybrid control system (lead rubber bearing and hydraulic actuator or magnetorheological fluid damper) on reducing the structural response by numerical simulation. Kovacs et al. [19] studied the effect of viscous-friction dampers and found that they were also effective in increasing system damping. Further, the installation of a Tuned Mass Damper (TMD) on the vertical hanger of an arch bridge was also proven to be effective [20]. It is very difficult to make a comprehensive evaluation of vibration mitigation effects and maintenance.
cost on different types of dampers. Some representative analytical methods are reviewed below:

For the vibration of a taut cable with a concentrated viscous damper, S. Krenk [21] derived a partial differential equation to describe linear oscillations of a cable under the assumption of a virtually unchanged cable force. The solution was expressed in terms of damped complex-valued modes. This leads to a transcendental equation of complex eigenfrequencies. A simple iterative solution to the frequency equation for all complex eigenfrequencies was proposed. A damping ratio of the vibration modes, determined from the argument of the complex eigenfrequency, was derived with high accuracy in two iterations. An accurate asymptotic approximation of the damping ratio of the low mode was given. Depending on its damping parameter, the formula allowed for explicit determination of the optimal location of the viscous damper. In S. Krenk’s work, the cable was considered as a string, thus its bending stiffness was ignored.

However, the work by Tabatabai & Mehrabi [22] showed that the cable bending stiffness is an important parameter and should not be neglected. They proposed a governing differential equation for the vibration of cables with a parameter of cable flexural stiffness. The equation was then converted to a complex eigenvalue problem, which contained a non-dimensional cable parameter. This
equation can be solved using many commercial software packages, such as MATLAB and MathCAD. The estimated values using the method presented were verified by the results from laboratory testing on a scaled cable model. A parametric study was conducted for a wide range of non-dimensional parameters. Among the parameters associated with most stay cables, the influence of cable sag is relatively small, whereas the cable flexural stiffness can have a significant impact on the resulting cable damping ratio. In both S. Krenk and Tabatabai & Mehrabi’s research, the damper has to be located near the cable-deck anchorage point. The distance between the anchorage point and the external damper location is restricted to be within 6% of cable length. This is due to the analytical assumption made in simplifying the governing differential equation.

In the study by J.A. Main and N.P. Jones [23], the damper can be placed at any location along the cable. The free vibration of a taut cable with an attached linear viscous damper was explored using an analytical formulation of a complex eigenvalue problem. They derived an expression for the eigenvalues, which was independent of the damper coefficient. With the expression, for a given damper, the modal damping ratio and the corresponding oscillation frequency of each mode can be obtained without approximation.
Some researchers have attempted to explore the damping problem with an energy-based method. According to the energy-based definition of the damping ratio [24], the modal damping, $\delta_n$, for the nth mode can be evaluated as the ratio between the modal dissipated energy per cycle, $D_n$, and the modal potential energy, $U_n$, i.e.

$$\delta_n = D_n / 4\pi U_n$$

(1-1)

Yamaguchi et al. [25] studied modal damping of structural cables with and without damping treatment. Modal damping, based on the energy definition, was derived in the form of the product of the modal strain energy ratio and the loss factor. The loss factor is one of the damping parameters and is defined as the ratio of the dynamically dissipated energy to the strain energy stored per cycle [26]. Using the finite element method, the ratio of modal strain energy to the total potential energy associated with modal vibration is calculated for both axial and bending deformations. It has been found that a very large contribution of initial cable stress to the total potential energy causes a very small strain energy ratio and lower modal damping in the structural cables.

1.2 Motivations

Some researchers have tried to solve problems related to the cable-damper systems by analytical methods [21, 22]. They typically derived the governing
differential equation first, then made some simplifications based on the assumption that the vibration shape of the cable can be represented by its first mode shape, i.e. a half sine wave. This assumption is reasonable and valid when the damper is located very close to the cable-deck anchorage, typically less than 6% of the cable length. The magnitude of the first modal damping of the cable-damper system can be solved from the simplified governing differential equation.

Another way to calculate the first modal damping of a cable-damper system is by the logarithmic decrement method. Since the first mode shape of a cable can be assumed to be a half sine wave when the damper location relative to the cable end is less than 6% of cable length, the cable-damper system can be considered as a single degree of freedom system with the displacement at cable midpoint as the general coordinate. Thus once a perturbation is introduced, either force or displacement, at the cable middle point, then the cable is suddenly released, the cable will start to vibrate freely. The displacement time-history of the cable middle point can be recorded. This process can be done by physical tests or numerical simulations. Based on the displacement time-history of the cable middle point, the first modal damping of the cable-damper system can be obtained by using the logarithmic decrement method. However, this approach has a similar limitation like the analytical method in that the damper location should be very close to the cable end, so that the assumption
of the cable vibration shape being a half sine is valid. However, in a cable-damper system, the shape of cable vibration will be a combination of several mode shapes when the damper location to the cable end is beyond 6% of cable length. Moreover, even when the damper is installed within 6% of cable length to the cable end, its effect on reducing the amplitude of cable vibration at the damper location will be significant if the damping coefficient of the damper is relatively large. Consider an extreme case, of which an external damper is installed at 4% of cable length from the cable end, with the cable being fixed at both ends and has a total length of L. If the damping coefficient of the external damper is infinitively large, vibration of the cable in the cable-damper plane will not be allowed at the damper location. This is equivalent to adding an additional rigid constraint to the cable at this location, and the original cable is broken down into two parts, with lengths being 0.04L and 0.96L, respectively. Thus, the vibration shape of such a system cannot be represented by a pure half sine wave.

Moreover, a new idea of incorporating cross-ties with dampers has also been attempted in a few recent applications, as illustrated in Figure 1-5. The cross-ties can increase the stiffness of the cable network, while the damper can contribute to the system damping. However, there is no method available to study the combined effect of cross-ties and damper.
In summary, when considering the combined effect of cross-ties and damper, or the location of the damper is not very close to the cable-deck anchorage, typically, beyond 6% of cable length, or its damping coefficient is relatively large, which has been encountered in practical application, either the analytical method or the logarithmic decrement method will no longer be valid. So, it is necessary to develop a new method which is more general, and is capable of handling more practical scenarios. To eliminate the difficulties that exist in the conventional methods, the new approach should not be based on the assumption that the vibration shape of a cable-damper system can be simply represented by a half sine wave. One possible way to circumvent these challenges is through the use of the energy method, which could relate the variation of the system kinetic energy to its damping property.
Yamaguchi et al. [25] studied the damping property of a single cable by the energy method, with the focus on the internal damping of the cable itself. Very limited research has been done to study the damping property of a cable-damper system using the energy method. In this study, a new energy-based approach will be proposed to study the damping of a cable-damper system. The relation between the kinetic energy decay ratio of the system and its damping property will be investigated. In the proposed method, there are no restrictions in terms of the location and the damping coefficient of the external damper. Further, by considering damper stiffness in the formulation, it is possible to consider the combined effect of cross-ties and external dampers. A parametric study will be carried out for the following non-dimensional parameters to examine their impact on the damping property of the whole system: a) external damper position; b) non-dimensional cable bending stiffness parameter; and c) non-dimensional damping parameter. Finally, a method to convert the kinetic energy decay ratio in a cable-damper system to the equivalent Rayleigh damping of a single cable will be proposed, which should be of significant interest to bridge design engineers.

1.3 Objectives

The objectives of this study are as follows:
1) To derive an energy-based method for the determination of damping of a cable-damper system. For a given cable-damper system, the magnitude of damping would affect decay rate of cable vibration. In other words, it would influence how fast the kinetic energy in the system is dissipated. Therefore, the damping property of a cable-damper system can be related to the decay rate of system kinetic energy.

2) To derive a finite element based numerical model for a cable-damper system. The finite element model will be developed using the commercial software ANSYS.

3) To verify the proposed energy-based method and numerical model. The simulated cable response will be compared with the results obtained from an earlier physical experiment.

4) To carry out a parametric study. A verified time-history analysis will be used to calculate the cable-damper system kinetic energy decay ratio with a combination of different parameters, i.e. external damper position, non-dimensional cable bending stiffness parameter, and non-dimensional damping parameter.

5) To derive a method which can convert the decay ratio of kinetic energy in a cable-damper system to the equivalent Rayleigh damping of a single cable. By doing so, it will be possible to link the damping effect provided by an external damper with the equivalent Rayleigh damping of a single cable.
6) To perform case studies to explore the effect of damper stiffness on cable vibration. When the damper is installed along cross-ties, the effect of cross-ties can be considered as a spring. This effect will be studied in several cases.
CHAPTER 2 Energy-Based Evaluation of Damping Properties in Cable-Damper System

2.1 Introduction

System vibration will only decay in the presence of damping, and the decay of system kinetic energy is also dependent on damping. How fast the system kinetic energy will decay depends on how much damping exists in the system. In other words, the decay rate of system kinetic energy can be interpreted as the system damping. In this study, the decay rate of kinetic energy of a cable-damper system is defined by Eq. (2-1) as follows:

$$\frac{d}{dt} = \frac{E_{\text{max}} - E_{\text{ave}}}{E_{\text{max}}}$$

(2-1)

where $E_{\text{max}}$ is the maximum kinetic energy of the cable-damper system, and $E_{\text{ave}}$ is the average kinetic energy of the system within a certain time period, i.e.

$$E_{\text{ave}} = \frac{E_{\text{total}}}{\Delta T},$$

(2-2)

where $\Delta T$ is the selected time duration for calculating $E_{\text{ave}}$.

Figures 2-1 (a) and (b) show the kinetic energy decay curves for two different cable-damper systems. The maximum kinetic energy, $E_{\text{max}}$, which occurs in the first vibration cycle is assumed to be same for these two cable-damper systems. The shaded areas in the figures represent the total kinetic energy, $E_{\text{total}}$, within a specified
time duration of \((t_2-t_1)\). A FORTRAN program [30] to calculate the total kinetic energy of the system within certain time period is written based on Newton-Cotes formula [35].

Referring to the definition of \(E_{ave}\) given by Eq. (2-2), it can be seen that for the same cable-damper system, the longer the selected time duration \((t_2-t_1)\) is, the less the magnitude of the average kinetic energy \(E_{ave}\) will be. It can be inferred that if the selected time duration approaches to infinity, the average kinetic energy of the system will approach to zero and consequently the variation of \(\bar{d}\) in Eq.(2-1) will close to 1.0 for any cable-damper systems. Thus, the kinetic energy decay ratio cannot be used as an index of damping properties of the cable-damper system.

(a) Kinetic Energy Decay Curve of a Cable-Damper System
In Figures 2-1 (a) and (b), it can also be seen that if two cable-damper systems have different damping properties, they will have different kinetic energy decay ratios within the same time duration. The larger the damping property of the cable-damper system has, the faster the kinetic energy will decay. This relationship is valid for any selected time duration.

Theoretically, any time period can be selected to study the kinetic energy decay ratio of a cable-damper system. However, if the selected time period is too short, it may introduce large numerical error; whereas if it is too long, the system kinetic energy decay ratio $\bar{d}$ will approach one for any cable-damper system, as explained in the previous sections. A reasonable choice of the time duration to calculate $E_{ave}$ and $\bar{d}$ should be related to the dynamic properties of the system, in particular, the free vibration frequencies.
In the current study, the time period is chosen as one second, which can cover at least six free vibration cycles of the cable-damper system.

### 2.2 Energy-Based Damping Evaluation of a Cable-Damper System

For a typical cable damper system, as shown in Figure 2-2, one possible way to determine the system kinetic energy is to use an analytical method, i.e. to derive the governing differential equation, impose the appropriate boundary condition, and solve it.

![Figure 2-2 Configuration of a Cable Damper System](image)

The governing differential equation of a cable-damper system derived by Armin B. Mehrabi and Habib Tababain [31] is:

\[
\frac{\partial^3}{\partial x^2} \left( E I \frac{\partial^2 u}{\partial x^2} \right) - H \frac{\partial^2 u}{\partial x^2} - h \frac{d^2 y}{dx^2} + k \frac{\partial^2 u}{\partial t^2} + c \frac{\partial u}{\partial t} + m \frac{\partial^2 u}{\partial t^2} = 0
\]  

(2-3)

where: \( u \) is the transverse displacement due to vibration; \( x \) is the coordinate along cable chord; \( y \) is the coordinate perpendicular to cable chord; \( H \) is the cable force along cable chord; \( h \) is the additional horizontal force in the cable due to vibration,
\( k' \) is the lateral spring constant per unit length of the cable; \( c' \) is the viscous damping coefficient per unit cable length.; and \( m \) is the weight per unit cable length.

Since in the current study, it is assumed that the cable has no lateral support and sag, so Eq. (2-3) can be simplified as:

\[
\frac{\partial^2}{\partial x^2} \left( E I \frac{\partial^2 u}{\partial x^2} \right) - H \frac{\partial^2 u}{\partial x^2} + c \frac{\partial u}{\partial t} + m \frac{\partial^2 u}{\partial t^2} = 0
\]

(2-4)

where \( c' = c/L \) is the damper coefficient per unit length of cable.

The boundary conditions for Eq. (2-4) are:

i) At \( x = 0 \): \( u = 0, \frac{du}{dx} = 0 \);

ii) At \( x = L \): \( u = 0, \frac{du}{dx} = 0 \);

The solution to Eq. (2-4), can be found analytically, however it requires tremendous effort.

In the present study, a numerical approach will be used to solve this equation. A finite element model is developed which contains both cable and damper elements. Time-history analysis is carried out to simulate the free vibration process of a cable-damper system. A perturbation (displacement) is introduced at the mid-span of the cable finite element model and then released suddenly. The responses of the cable-damper system in terms of nodal displacement, nodal velocity and total kinetic energy are calculated at each time step of integration. The kinetic energy of the
cable-damper system can be obtained at each time step \( t_1, t_2, t_3 \), and so on. The maximum kinetic energy of the cable-damper system can be easily obtained. The total kinetic energy within first second divided by one is the average kinetic energy of the system free vibration within the first second. Newton-Cotes formula [35] is used to calculate the total kinetic energy of the system in first one second and a FORTRAN program is written for this purpose [30]. As a result, the kinetic energy decay rate of a cable-damper system can be determined. Similar analyses can be carried out for different parameter combinations. Moreover, a method to convert the kinetic energy decay ratio of a cable-damper system to the equivalent Rayleigh damping of a single cable will be proposed, which will be more useful in practical design.

2.3 Finite Element Model

The actual physical cable, as illustrated in Figure 2-2, is fixed at both ends. An external damper is installed in the transverse direction of the cable. A finite element model of the cable-damper system is developed by ANSYS10.0, as shown in Figure 2-3.
The cable is simulated with the PIPE59 element, which is a uniaxial element with tension-compression, torsion, and bending capabilities [28]. As shown in Figure 2-4, the element has six degrees-of-freedom at each end, i.e., translation in the x, y, and z direction and rotation about the x, y, and z axis. PIPE59 is similar to general 3D beam element. In addition, it is capable of simulating stress stiffening effect and large deflection behaviours. The initial strain in the axial direction can be easily used to simulate the cable axial force. Other elements, such as BEAM4 in ANSYS, have the same capabilities as PIPE59. The reason why PIPE59 element is selected in the current study is due to its simplicity in data input.
The damper is simulated with the COMBIN14 element. COMBIN14 can model longitudinal or torsional deformation in 1D, 2D, and 3D applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees-of-freedom at each end, i.e., translation in the x, y, and z direction. The longitudinal spring constant should have a unit of "Force/Length". The unit of damping coefficient is "Force*Time/Length". The damping force $F$ or torque $T$ is computed as:

$$F = -c_v \frac{du}{dt}$$

(2-5)

where $c_v$ is the damping coefficient given by $c_v = c_{v1} + c_{v2} \frac{du}{dt}$; $\frac{du}{dt}$ is the velocity determined in the previous substep of time-history analysis; $c_{v1}$ is a constant of linear damping coefficient; and the second damping coefficient $c_{v2}$ is available to simulate the non-linear damping effect that needs to be considered in some fluid environments.

In the present study, the damper is simulated as a 1D COMBIN14 element, as shown in Figure 2-5. The spring or damper property of the element may be removed from the element. Thus, COMBIN14 element can also be used to simulate a single damper or a single spring.
The finite element model presented in Figure 2-3 is used for most analysis (modal analysis and time-history analysis) in the current study. In order to evaluate the combined effect of cross-ties and external damper on suppressing cable vibrations, a more complicated finite element model associated with an external damper attached to a cable, as shown in Figure 2-6, is used. In this model, two COMBIN14 elements are used in series. One is used to simulate the behaviour of an external damper and the other is used to simulate that of the cross-ties. The effect of cross-tie stiffness in a damper cross-ties assembly can thus be examined.

Figure 2-6 Finite Element Model for Damper Assembly
2.4 Time-History Analysis Approach

The equation of equilibrium governing the dynamic response of a structural system is [27]:

\[
[M][\ddot{u}] + [C][\dot{u}] + [K][u] = \{F^a\}
\]  

(2-6)

where \([M]\) is the mass matrix; \([C]\) is the damping matrix; \([K]\) is the stiffness matrix; \(\{\ddot{u}\}\) is the acceleration vector; \(\{\dot{u}\}\) is the velocity vector; \(\{u\}\) is the displacement vector, and \(\{F^a\}\) is the external force vector.

There are two methods in the ANSYS software, which can be employed for the linear solution to Eq. (2-6), i.e. the forward difference time integration method and the Newmark time integration method. The forward difference method is used for explicit transient analysis and the Newmark method is used for implicit transient analysis. The Newmark method will be used in the current study. Eq. (2-6) can be converted to the following form by multiplying the mass orthonormalized eigenvectors shape (free vibration mode shapes):

\[
\{\ddot{u}(t)\} + \Delta\{\dot{u}(t)\} + \Omega^2 \{u(t)\} = \{F^a\}
\]  

(2-7)
where $\Delta$ is a diagonal matrix, $\Delta = \text{diag}(2\omega_i \xi_i)$, $\omega_i$ is the $i^{\text{th}}$ angular frequency of system, $\xi_i$ is the $i^{\text{th}}$ modal damping ratio, and $\Omega^2$ is a diagonal matrix listing eigenvalues $\omega_1^2, \omega_2^2 \ldots \omega_n^2$ for the free vibration equation $[M]\ddot{u} + [K]u = 0$.

A typical row in Eq. (2-7) can be written as:

$$\frac{d^2u}{dt^2} + 2\xi \omega \frac{du}{dt} + \omega^2 u = f$$ (2-8)

In the Newmark integration scheme, the equilibrium of Eq. (2-8) is considered at time instant $t + \delta t$, i.e.

$$\left(\frac{d^2u}{dt^2}\right)_{t+\delta t} + 2\xi \omega \frac{du}{dt}_{t+\delta t} + \omega^2 u_{t+\delta t} = f_{t+\delta t}$$ (2-9)

At time instant $t + \delta t$, we have

$$\frac{du}{dt}_{t+\delta t} = \left(\frac{du}{dt}\right)_t + \left[ \left(1 - \alpha\right) \left(\frac{d^2u}{dt^2}\right)_{t+\delta t} + \alpha \left(\frac{d^2u}{dt^2}\right)_{t+\delta t} \right] \delta t$$ (2-10)

$$u_{t+\delta t} = u_t + \delta t \left(\frac{du}{dt}\right)_t + \left[ \left(1 - \beta\right) \left(\frac{d^2u}{dt^2}\right)_t + \beta \left(\frac{d^2u}{dt^2}\right)_{t+\delta t} \right] \delta t^2$$ (2-11)

where $\alpha$ and $\beta$ are parameters to be chosen to optimize the stability and accuracy of the solution.

The default values in ANSYS 10.0 for $\alpha$ and $\beta$ are 0.5050 and 0.2525, respectively, and this will introduce a certain level of numerical damping. If no other damping is present in the time-history analysis, the lack of numerical damping can be.
undesirable as high frequencies of the structure can produce unacceptable levels of numerical noise [29]. Newmark proposed the constant average acceleration method as an unconditional stable scheme, in which case \( \alpha = 0.5 \) and \( \beta = 0.25 \) [27]. \( \Delta t \) is the increment of the time step, which can be controlled by the input parameter in the ANSYS software.

Generally, analysis starts from \( t = 0 \), and it is assumed that the initial conditions of displacement, velocity, and acceleration are known. By combining Eqs. (2-9)-(2-11), the displacement, velocity, and acceleration of each node at the next time step \( t + \Delta t \) can be obtained. At this point the system kinetic energy can be calculated. Following the same procedure, the response of the cable-damper system and system kinetic energy at any subsequent time point can be determined. The kinetic energy at each time point will be used to calculate the average kinetic energy of the cable-damper system in the first second. To accelerate the rate of convergence during time-history analysis, automatic time step technique is used. However, the maximum time step is kept less than 1/120 of the first vibration period of the cable-damper system to ensure the accuracy of the results.
2.5 Input of Rayleigh Damping in the Finite Element Analysis

There are several forms of damping available in the ANSYS software, including the Rayleigh damping, the material-dependent damping, the constant material damping coefficient, the constant damping ratio, the modal damping, and the element damping. The Rayleigh damping and element damping (COMBIN14 element) are used in the current study. The method to input Rayleigh damping parameters is discussed in the following sections.

It is assumed that the Rayleigh damping is proportional to a combination of the mass and stiffness matrices, which is given by the sum of the two alternative expressions shown in Eq. (2-12).

\[ c = \alpha_0 [M] + \alpha_1 [K] \]  
(2-12)

where \( \alpha_0 \) and \( \alpha_1 \) are constants, they are called Rayleigh damping factors and have units of \( \text{sec}^{-1} \) and \( \text{sec} \), respectively.

Once determined, the parameters \( \alpha_0 \) and \( \alpha_1 \) can be input to the ANSYS software. It is apparent that Rayleigh damping can be converted to the following relation between the damping ratio and the modal frequency, as shown in Eq. (2-13)

\[ \xi_n = \frac{\alpha_0}{2\omega_n} + \frac{\alpha_1 \omega_n}{2} \]  
(2-13)
The two Rayleigh damping factors, \( a_0 \) and \( a_1 \), can be evaluated by the solution of a pair of simultaneous equations if the damping ratios \( \xi_m \) and \( \xi_n \) associated with two specific frequencies \( \omega_m \) and \( \omega_n \) are known. Writing Eq. (2-13) for each of these cases and expressing the two equations in the matrix form yields:

\[
\begin{bmatrix}
\xi_m \\
\xi_n
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
1/\omega_m & \omega_m \\
1/\omega_n & \omega_n
\end{bmatrix} \begin{bmatrix}
a_0 \\
a_1
\end{bmatrix}
\]

(2-14)

The magnitude of Rayleigh damping factors can be expressed as:

\[
\begin{bmatrix}
a_0 \\
a_1
\end{bmatrix} = 2 \cdot \frac{\omega_m \cdot \omega_n}{\omega_m^2 - \omega_n^2} \begin{bmatrix}
\omega_n & -\omega_m \\
-1/\omega_n & 1/\omega_m
\end{bmatrix} \begin{bmatrix}
\xi_m \\
\xi_n
\end{bmatrix}
\]

(2-15)

When the Rayleigh damping factors are obtained, the proportional damping matrix that will give the required values of damping ratio at specified frequencies will be given by the Rayleigh damping according to Eq. (2-12). A simpler method to formulate the proportional damping is to make it proportional to either mass or the stiffness matrix, as shown in Figure 2-7. In the figure, the horizontal axis "\( \omega \)" is the circular frequency of the structure, whereas the vertical axis "\( \xi \)" is the damping ratio.
In practice, when applying this proportional damping matrix derivation procedure, it is recommended that $\omega_m$ be taken as the fundamental frequency of the multiple degree-of-freedom system and $\omega_n$ be set among the frequencies of the higher modes which contribute significantly to the dynamic response. So if $\omega_n$ can be decided, and then the Rayleigh damping factors can be found.

The higher frequency is decided by the modal coefficient, which is an index of modal contribution to the dynamic response. Depending on the type of excitation, the modal coefficient can be computed in five different ways [28]. Modal coefficient is computed for velocity excitation in this study since the energy dissipation depends on the velocity of cable vibration. The definition of the modal coefficient is given as Eq. (2-16).

\[
\alpha_0 = 0, \xi = \frac{\alpha_1}{2} \omega \\
\alpha_1 = 0, \xi = \frac{\alpha_2}{2\omega}
\]
$A_i = \frac{S_{vi} \gamma_i}{\omega_i}$  \hspace{1cm} (2-16)

where $S_{vi}$ is the velocity spectrum of the $i^{th}$ mode, $\omega_i$ is the natural circular frequency of the $i^{th}$ mode, and $\gamma_i$ is the participation factor of the $i^{th}$ mode.

The modal coefficient is a linear function of the velocity spectrum, and the energy spectrum is a quadratic function of the velocity spectrum. Therefore, the square of modal coefficient can be used to indicate the modal energy distribution. When the square of the modal coefficient of a particular mode is less than 4% of the maximum modal coefficient of the system, the circular frequency of the mode will be considered as $\omega_n$ for determining the Rayleigh damping.

2.6 Equivalent Cable Damping Conversion Method

The present work focuses on studying the variation of $\bar{d}$ with respect to different parameter combinations, using the proposed energy-based method. Since bridge designers are more interested to know the effect of external damper in terms of equivalent cable damping, a detailed method is proposed in this section to convert the kinetic energy decay ratio of a cable-damper system to the equivalent Rayleigh damping of a single cable. To link the effect of an external damper to the equivalent cable damping, two finite element models are develop, as shown in Figure 2-8. The first model is used to simulate a cable-damper system and there is no other source of
damping in this system. Another model is used to simulate the behaviour of a single cable with Rayleigh damping. The cable bending stiffness parameters are identical in both models.

![Diagram of cable-damper model and cable alone model](image)

(a) Cable-damper model  (b) Cable alone model

Figure 2-8  Finite Element Models for Equivalent Damping Calculation

As indicated in Figure 2-8(a), for a given damper location, if the non-dimensional damping parameter has been specified, the kinetic energy decay ratio of the system can be obtained by the proposed energy-based method. By varying the magnitude of the non-dimensional damping parameter, its relation with the system kinetic energy decay ratio at this damper location can be established, as illustrated in Figure 2-9(a). The curve in Figure 2-9(a) is not the exact curve profile, but is used to illustrate the conversion method. For a single cable shown in Figure 2-8(b), the kinetic energy decay ratio could also be obtained through the proposed energy-based method. If the damping within the cable could be represented by a specific Rayleigh damping, a relationship between the kinetic energy decay ratio of the cable and the
cable damping property in terms of Rayleigh damping can be established, as shown in Figure 2-9(b).

![Kinetic Decay Sketches for Equivalent Damping Calculation](image)

The relation between the damping effect provided by an external damper and its equivalent Rayleigh damping can be easily developed by integrating information provided in Figure 2-9.

Here is an example to show the procedure of obtaining equivalent Rayleigh damping of an external damper with non-dimensional damping parameter $\psi_1$:

Step 1: Draw a horizontal line corresponding to the non-dimensional damping parameter $\psi_1$ in Figure 2-9 (a), as shown in Figure 2-10 (a).

Step 2: Draw a vertical line in Figure 2-10 (a) from the intersection of the horizontal line obtained from step 1 and the kinetic energy decay curve. Then find the
kinetic energy decay ratio $\delta_i$ corresponding to the non-dimensional damping parameter $\nu_i$.

Step 3: Draw a horizontal line corresponding to the kinetic energy decay ratio $\delta_i$ obtained at step 2 in Figure 2-9 (b), as shown in Figure 2-10(b).

Step 4: Draw a vertical line in Figure 2-10 (b), from the intersection of the horizontal line obtained at step 3 and the kinetic energy decay curve. Then find the Rayleigh damping $\xi_i$ of the cable.

The Rayleigh damping $\xi_i$ obtained in step 4 has the same capacity to suppress cable vibration as the external damper with a non-dimensional damping parameter. Therefore, it is considered to have the equivalent damping to that of the external damper. Similarly, equivalent Rayleigh damping for external dampers with other damping properties could be obtained following the same procedure.

Figure 2-10 Equivalent Rayleigh Damping Calculation Sketch
CHAPTER 3 Validation of the Energy-Based Damping Evaluation Method

A preliminary analysis is carried out to examine the validity of the proposed energy-based method. The validation focused on two aspects: a) verification of the validity of the proposed finite element model and time-history analysis procedure, and b) verification of the concept of the proposed energy-based damping evaluation method, i.e. an external damper has the same effect to suppress vibration of a cable-damper system as its equivalent Rayleigh damping does in a single cable.

The validation is based on the experimental work by Tabatabai and Mehrabi [22]. Tabatabai and Mehrabi did physical tests using a representative cable. This selected "representative" cable is based on the statistical evaluation from a database of bridge stay cables. And the database was created by Tabatabai et al [33] based on over 1,400 stay cables from 16 cable-stayed bridges.

3.1 Physical Tests

The setup of the physical test done by Tabatabai and Mehrabi is shown in Figure 3-1. A representative cable was selected based on statistical information from a bridge stay cable database. This cable was scaled down by a factor of 6.76, and the model cable was assembled with the scaled properties calculated on the basis of similarity theory. The model cable consisted of a bundle of seven-wire, 6.4mm...
diameter strands encased in a polyvinyl chloride pipe. Cement grout was injected in the encasing pipe based on the common practice in the U.S.A. A tensile force \( (H) \) of 122.1kN was applied on the cable. Table 3-1 lists the cable parameters:

Table 3-1  Cable Properties in Reference [22]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable length ( L )</td>
<td>13.695m</td>
</tr>
<tr>
<td>Equivalent axial rigidity ( EA )</td>
<td>49138 kN (including grout and cover pipe)</td>
</tr>
<tr>
<td>Equivalent flexural rigidity ( EI )</td>
<td>2.28 kN ( \cdot ) m² (including grout and cover pipe)</td>
</tr>
<tr>
<td>Bending stiffness parameter ( \xi )</td>
<td>100</td>
</tr>
<tr>
<td>Mass per unit length ( m )</td>
<td>3.6kg/m</td>
</tr>
</tbody>
</table>

A viscous damper is attached to the cable at a location of 6% cable length to one of the cable end. The damper consists of a cylindrical container and a closely fit moving disk. Oils of various viscosities were filled in the cylinder to adjust the damper coefficient.

Three tests have been carried out on the model, one without damper and the other two with a damper containing an oil of different viscosity. The damper
coefficient in each case was measured in the laboratory and the response of the dampers (variation of force with respect to velocity) was assumed to be linear. For the two tests conducted with dampers, the damper coefficients were taken as $1680 \, N\cdot s/m$ and $15130 \, N\cdot s/m$, respectively.

In each test, the mid-point of the cable was moved transversely from its equilibrium position by applying a certain weight, and then released suddenly. The free vibration response of the first mode at cable mid-point was recorded using an accelerometer attached to that point. The damping ratio was obtained from the displacement time-history curve by logarithmic decrement method (free-vibration decay method) according to the following equation:

$$\ln \frac{u_n}{u_{n+m}} = 2\pi \cdot m \delta \frac{\omega}{\omega_D}$$

(3-1)

where $u_n$ is the displacement at cable middle point in the $n^{th}$ cycle; $u_{n+m}$ is the displacement at cable middle point displacement in the $(n+m)^{th}$ cycle; $\omega$ is the first undamped angular frequency of cable vibration; $\omega_D$ is the first damped angular frequency of cable vibration; and $\delta$ is the damping ratio of the cable-damper system.

The experimentally measured and theoretically calculated damping ratios by Tabatabai and Mehrabi [22] are listed in Table 3-2.
Table 3-2  Physical Test Result from Reference [22]

<table>
<thead>
<tr>
<th>Case number</th>
<th>Damping factor $N \cdot m/s$</th>
<th>Damping parameter $\psi$</th>
<th>Bending stiffness parameter $\xi$</th>
<th>First frequency (Hz)</th>
<th>Damping ratio associated with damper $\delta$ (%)</th>
<th>Calculated damping ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>7.028</td>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1680</td>
<td>2.5</td>
<td>100</td>
<td>7.028</td>
<td>2.7</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>15130</td>
<td>22.8</td>
<td>100</td>
<td>7.320</td>
<td>1.5</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The damping ratios obtained from the physical experiment are presented in two forms. One is the directly measured total damping ratio of the system and the other is the damping associated with the damper alone. The latter is obtained by subtracting the damping ratio of the cable without damper from the directly measured value. In the derivation, it is assumed that the intrinsic damping of the cable (for the cable without damping) remain constant with the addition of the external damper.

A discrepancy between the experimental and the analytical results can be seen in Table 3-2. For Case 2, the damping ratio from analysis is lower than that from experiment. The analytical result is 2.0 whereas the experimental result is 2.4. For Case 3, the damping ratio from analysis is higher than that from the experiment, which are 1.5 and 1.2, respectively. This discrepancy could be caused by
experimental error. Generally speaking, it is possible to determine an appropriate viscous damping property by experiments. The commonly used experimental methods for this propose include: the logarithmic decrement method (free-vibration decay method) [32], the resonant amplification method, the half-power (band-width) method [34], and the resonance energy loss per cycle method. Among them, the logarithmic decrement method is the simplest and the most often used one. Tabatabai and Mehrabi used this method to determine the damping ratio of the system in their study [22]. The main advantage of the logarithmic decrement method is that the requirement of equipment and instrumentation is minimal. The vibration can be initiated by any convenient method and only the relative displacement amplitude needs to be measured. However, the damping ratio obtained by this method is amplitude dependent. For example, "m" consecutive cycles in the earlier portion of the high-amplitude free vibration response will yield a different damping ratio than "m" consecutive cycles in the later stage of a much lower response. It is usually found that in such a case, the damping ratio decreases with decreasing amplitude of free vibration response. For the complexity of damping problem, sometimes it is difficult to obtain an accurate solution of the problem even with very complex and advanced equipment. Therefore, the logarithmic decrement method is still widely used to make approximate estimation of the damping properties of structures due to simplicity.
3.2 Validation of the Proposed Finite Element Model and Time-History Analysis Approach

Both the modal and time-history analysis are carried out by simulating the behaviour of the cable model used in the physical tests described in Section 3.1. The period of the first free vibration of the cable-damper system obtained from the modal analysis is compared with the physical test results given in reference [22]. The time-history of cable nodal displacement is used to calculate the Rayleigh damping of the cable-damper system, which is then compared with the previous study results[22].

The same finite element model is used for both the modal and the time-history analysis. The cable is simulated by the PIPE59 element and the damper is simulated by one COMBIN14 element. The input parameters of cable properties and cable pretension in the finite element model are computed based on the given physical model in Reference [22]. They are listed in Table 3-3.
Table 3-3  Input Parameters Used in the Finite Element Model

<table>
<thead>
<tr>
<th>Cable property</th>
<th>Elastic modulus (N/m²)</th>
<th>8.489E10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Radius (m)</td>
<td>1.360E-2</td>
</tr>
<tr>
<td></td>
<td>Cross sectional area (m²)</td>
<td>5.810E-4</td>
</tr>
<tr>
<td></td>
<td>Weight density (kg/m³)</td>
<td>6196.740</td>
</tr>
<tr>
<td></td>
<td>Initial strain in the axial direction</td>
<td>2.476E-3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Damper property</th>
<th>Damping coefficient (Nm/s)</th>
<th>1680, 15130</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Damper stiffness (N/m)</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Damper location</td>
<td>0.06L</td>
</tr>
</tbody>
</table>

Using the data provided in Table 3-3, both the modal analysis and time-history analysis are carried out. The modal analysis results are shown in Table 3-4. The analysis results match the experimental results very well and the maximum error is about 2%.

Two separate time-history analyses are carried out for a damper coefficient of 1680N*m/s and 15130 N*m/s, respectively. The results of damping ratio of the cable-damper system are given in Table 3-4.

Table 3-4  Comparison of Results

<table>
<thead>
<tr>
<th>Test number</th>
<th>Damping factor c N·m/s</th>
<th>Damping parameter (\psi)</th>
<th>Bending stiffness parameter (\xi)</th>
<th>Fundamental frequency (Hz)</th>
<th>Damping ratio associated with damper (\delta) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reference results [22]</td>
<td>Current study</td>
<td>Reference [22] results</td>
<td>Current study</td>
<td></td>
</tr>
<tr>
<td>Test</td>
<td>FEA</td>
<td>Test</td>
<td>Analytical</td>
<td>FEA</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>100</td>
<td>7.028</td>
<td>6.875</td>
</tr>
<tr>
<td>2</td>
<td>1680</td>
<td>2.5</td>
<td>100</td>
<td>7.028</td>
<td>6.917</td>
</tr>
<tr>
<td>3</td>
<td>15130</td>
<td>22.8</td>
<td>100</td>
<td>7.320</td>
<td>7.255</td>
</tr>
</tbody>
</table>
The displacement time-history of the cable middle point is illustrated in Figure 3-2, based on which the system damping can be determined by the logarithmic decrement method. Figure 3-3 shows the time-history of the kinetic energy decay rate of the cable-damper system. The system kinetic energy ratio as defined by Eq. (2-1) can therefore be computed.

![Displacement Time-History at the Middle Point(damping factor=1680 Nm/s)](image)

Figure 3-2  Displacement Time-History at the Middle Point(damping factor=1680 N·m/s)
The results in Table 3-4 revealed that the fundamental frequency and the damping ratio of the cable-damper system simulated by the proposed finite element model and analyzed by the proposed time-history analysis approach agree reasonably well with that by Tabatabai et al [22].

3.3 Validation of the Proposed Energy-Based Damping Evaluation Method

In Section 3.2 the validity of the proposed finite element model and the time-history analysis procedure have been verified. Another series of finite element analysis will be carried out in this section to testify the validity of the proposed
energy-based method. The proposed method assumes that the damping provided by the external damper in a cable-damper system has the same damping effect to dissipate kinetic energy and suppress cable vibration as the equivalent Rayleigh damping in a single cable.

Tabatabai and Mehrabi [22] obtained the equivalent Rayleigh damping of a cable-damper system through physical experiments and numerical simulation. The cable properties and damper parameters are given in Table 3-1 and Table 3-2, respectively. These parameters are used in the current study to verify the validity of the proposed energy-based method. Two finite element models as shown in Figure 3-4 are developed in this study. Model A is a cable-damper system with no Rayleigh damping in the cable, whereas model B is a single cable with the Rayleigh damping obtained in the physical tests by Tabatabai and Mehrabi [22]. The input of Rayleigh damping to the finite element model is according to the method discussed in Chapter 2. The properties of the cables in these two models are identical. A transverse displacement is introduced at the cable middle point and then released suddenly. This process is simulated numerically by the finite element method. Time-history analysis is carried out for each model. The kinetic energy decay ratios of the two models are obtained, which are presented in Table 3-5. Results show that the kinetic energy decay ratio due to the existence of an external damper in Model A is very close to the
kinetic energy decay ratio due to the equivalent Rayleigh damping in Model B. Thus, the idea of using energy-based method to evaluate the damping property of a cable-damper system is proved to be valid.

Model A: Cable-damper system without Rayleigh damping
Model B: Single cable with Rayleigh damping

Figure 3-4  Finite Element Models used for Validation Analysis

Table 3-5  Verification Results

<table>
<thead>
<tr>
<th></th>
<th>Kinetic energy decay ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model A</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Case I (damper coefficient 1680Nm/s)</td>
<td>0.76</td>
</tr>
<tr>
<td>Case II (damper coefficient 15130Nm/s)</td>
<td>0.72</td>
</tr>
</tbody>
</table>
4.1 Basic Parameters

The parameters that affect the damping properties of a cable-damper system include the external damper location, the damping coefficient of the external damper, the cable bending stiffness, the cable sag extensibility, and the stiffness of the damper assembly. Non-dimensional parameters are used in the current study for the damper location, the damping coefficient of the damper, and the cable bending stiffness. Since for more than 95% of cables in the stay cable database [22], the mechanical damper attached to the cable is not very sensitive to the sag extensibility parameter, this parameter is not considered in the current study. So far, the knowledge of the effect of stiffness of a damper assembly on a cable-damper system is very limited. Also, no available measurements can be found to be used to study the effect. The current study attempts to examine this effect by using the proposed energy-based damping evaluation method. Several cases are chosen in this study to illustrate the capabilities of the proposed energy-based method, and to explore the impact of damper assembly stiffness on the damping properties of a cable-damper system. The interactive effect among the damper location, the damping coefficient, and the cable bending stiffness on the damping properties of a cable-damper system is also studied.
The definition of the non-dimensional damper location parameter is expressed as  \( \Gamma_d = L_d / L \), where \( L_d \) is the distance from the damper to the cable deck-anchorage point and \( L \) is the total length of the cable [22]. The cable non-dimensional bending stiffness parameter is defined in terms of \( \xi = L \sqrt{H / EI} \) [22], where \( H \) is the cable tension along the chord and \( EI \) is the cable bending stiffness. It is clear that the larger the cable non-dimensional bending stiffness parameter is, the more flexible the real cable would be. The non-dimensional damping parameter is defined as \( \psi = \pi^* c / mL \omega_{1s} \) [22], where \( c \) is the damping coefficient of the mechanical viscous damper; \( m \) is the mass per unit length of the cable; and \( \omega_{1s} \) is the first modal angular frequency of string (no bending stiffness) equivalent to the cable without damper.

4.2 Ranges and Combinations of Parameters

A database of over 1,400 stay cables from 16 cable-stayed bridges was created by Tabatabai et al [33]. This database includes several cable parameters such as the sag-extensibility parameter and the non-dimensional cable bending stiffness parameter. The variation ranges of these parameters are also presented in the database.
Most of the non-dimensional cable bending stiffness parameter are within the range of 10 to 600, and five values within this range are selected to be used in this study: 100, 200, 300, 400, and 500.

For the non-dimensional damping parameter, its magnitude depends on the range of damping coefficient as well as $\sqrt{Hm}$, which varies from 3 to $43 \text{kN} \cdot \text{s/m}$, with a mean of $21 \text{kN} \cdot \text{s/m}$. It is assumed the damping coefficient varies between $0-200 \text{kN} \cdot \text{s/m}$, the non-dimensional damping parameter is approximately within the range of 0 to 67. Therefore, a range of 0 to 60 is assumed for the non-dimensional damping parameter. Ten values were taken: 0.2, 0.5, 1.0, 2.0, 4.0, 6.0, 10, 20, 30, and 60.

The damper location is taken as within 6% of the cable length to its anchorage point in most of the previous studies. Using the proposed energy-based method, the damper location can be beyond this limitation. Five damper locations are chosen: 0.06L, 0.2L, 0.3L, 0.4L, and 0.45L.

A new idea of incorporating cross-ties with dampers has also been attempted in a few recent applications, as shown in Figure 1-5. The cross-ties can increase the stiffness of the cable network, while the damper can contribute to the system damping. The combined effect of the damper and cross-ties is examined by using the proposed finite element model shown in Figure 2-6. However, no available numerical
or physical testing results can be used to verify the current analysis on the stiffness of damper assembly. Thus, only a few cases corresponding to different damper assembly stiffness are analyzed in the present study to demonstrate qualitatively the feasibility of the application of the proposed energy-based damping evaluation method to the damper-cross-ties combination.

Table 4-1 Parameters of the Cable-damper System Used in the Current Study

<table>
<thead>
<tr>
<th>Damper location $\Gamma_d = x / L$</th>
<th>0.06</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-dimensional cable bending stiffness parameter $\xi = L \sqrt{H / EI}$</td>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Non-dimensional damping parameter $\psi = \pi \cdot c / mL\omega$</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Stiffness of damper assembly</td>
<td>1.49E1</td>
<td>1.49E4</td>
<td>1.49E5</td>
<td>1.49E6</td>
<td>1.49E7</td>
</tr>
</tbody>
</table>

The combination of parameters consists of two groups: 1) the stiffness of the damper assembly is not considered; 2) the stiffness of the damper assembly is considered with the damper location at 0.06L, non-dimensional damping parameter as 6, and non-dimensional bending stiffness parameter as 200.
In the first group, all possible combinations of the damper location, the non-dimensional cable bending stiffness parameter, and the non-dimensional damping parameter are examined with a total number of 250 cases, for each combination case, both modal analysis and time-history analysis are conducted. The fundamental frequency of the cable-damper system is obtained from modal analysis. This is used to determine the maximum time step used in the time-history analysis. The results from the time-history analysis are used to obtain the kinetic energy decay rate curves, which are then used to determine the kinetic energy decay ratio of the cable-damper system.

In the second group, the damper location parameter is 0.06, the non-dimensional cable bending stiffness parameter is 200, and the non-dimensional damping parameter is 6. The stiffness of the damper assembly is taken as 14.9, 1.49E4, 1.49E5, 1.49E6, and 1.49E7, respectively.

4.3 Results

The results are presented in two sections corresponding to the two parameter combination groups. Tables 4-2 to Table 4-6 show the kinetic energy decay ratio of the cable-damper system at five different damper locations. Theses results correspond to the first group of parameter combinations.
Table 4-2  Kinetic Energy Decay Ratio (Damper Location 0.06L)

<table>
<thead>
<tr>
<th>Non-dimensional damping parameter</th>
<th>Non-dimensional cable bending stiffness parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>100  200  300  400  500</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5571  0.5294  0.5203  0.5266  0.4868</td>
</tr>
<tr>
<td>1</td>
<td>0.6086  0.5644  0.5467  0.5471  0.5032</td>
</tr>
<tr>
<td>2</td>
<td>0.6679  0.6063  0.5766  0.5672  0.5217</td>
</tr>
<tr>
<td>4</td>
<td>0.7374  0.6578  0.6086  0.5884  0.5414</td>
</tr>
<tr>
<td>6</td>
<td>0.8011  0.6997  0.6328  0.6020  0.5564</td>
</tr>
<tr>
<td>10</td>
<td>0.8154  0.7022  0.6329  0.5988  0.5562</td>
</tr>
<tr>
<td>20</td>
<td>0.8011  0.6783  0.6120  0.5796  0.5444</td>
</tr>
<tr>
<td>30</td>
<td>0.7321  0.6178  0.5636  0.5408  0.5168</td>
</tr>
<tr>
<td>60</td>
<td>0.6800  0.5829  0.5366  0.5202  0.5013</td>
</tr>
</tbody>
</table>

Table 4-3  Kinetic Energy Decay Ratio (Damper Location 0.2L)

<table>
<thead>
<tr>
<th>Non-dimensional damping parameter</th>
<th>Non-dimensional cable bending stiffness parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>100  200  300  400  500</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7532  0.6494  0.5992  0.5778  0.5336</td>
</tr>
<tr>
<td>1</td>
<td>0.8603  0.7595  0.6897  0.6459  0.5967</td>
</tr>
<tr>
<td>2</td>
<td>0.9003  0.8239  0.7575  0.7016  0.6511</td>
</tr>
<tr>
<td>4</td>
<td>0.9138  0.8426  0.7811  0.7206  0.6726</td>
</tr>
<tr>
<td>6</td>
<td>0.8971  0.8056  0.7346  0.6754  0.6315</td>
</tr>
<tr>
<td>10</td>
<td>0.8693  0.7577  0.6812  0.6282  0.5877</td>
</tr>
<tr>
<td>20</td>
<td>0.8102  0.6843  0.6039  0.5654  0.5311</td>
</tr>
<tr>
<td>30</td>
<td>0.6981  0.5754  0.5133  0.4993  0.4743</td>
</tr>
<tr>
<td>60</td>
<td>0.6294  0.5247  0.4738  0.4708  0.4510</td>
</tr>
</tbody>
</table>
### Table 4-4  Kinetic Energy Decay Ratio (Damper Location 0.3L)

<table>
<thead>
<tr>
<th>Non-dimensional damping parameter</th>
<th>Non-dimensional cable bending stiffness parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8139</td>
</tr>
<tr>
<td>0.5</td>
<td>0.8926</td>
</tr>
<tr>
<td>1</td>
<td>0.9231</td>
</tr>
<tr>
<td>2</td>
<td>0.9318</td>
</tr>
<tr>
<td>4</td>
<td>0.9138</td>
</tr>
<tr>
<td>6</td>
<td>0.8848</td>
</tr>
<tr>
<td>10</td>
<td>0.8222</td>
</tr>
<tr>
<td>20</td>
<td>0.6951</td>
</tr>
<tr>
<td>30</td>
<td>0.6172</td>
</tr>
<tr>
<td>60</td>
<td>0.4953</td>
</tr>
</tbody>
</table>

### Table 4-5  Kinetic Energy Decay Ratio (Damper Location 0.4L)

<table>
<thead>
<tr>
<th>Non-dimensional damping parameter</th>
<th>Non-dimensional cable bending stiffness parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8471</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9064</td>
</tr>
<tr>
<td>1</td>
<td>0.9346</td>
</tr>
<tr>
<td>2</td>
<td>0.9112</td>
</tr>
<tr>
<td>4</td>
<td>0.8780</td>
</tr>
<tr>
<td>6</td>
<td>0.8431</td>
</tr>
<tr>
<td>10</td>
<td>0.7771</td>
</tr>
<tr>
<td>20</td>
<td>0.6431</td>
</tr>
<tr>
<td>30</td>
<td>0.5517</td>
</tr>
<tr>
<td>60</td>
<td>0.4088</td>
</tr>
</tbody>
</table>
Results corresponding to the second group of parameter combinations show the combined effect of the cross-ties and the external damper. To study this combined effect, the finite element model in Figure 2-6 is used.

When the damper is attached to the cable through a very flexible spring, the force transferred to the external damper by the spring will be very small and the external damper can be considered as being isolated from the cable. Therefore, very limited energy will be dissipated through the external damper. Figure 4-1 shows the time-history of kinetic energy decay of a cable-damper system when the stiffness of the damper assembly is 14.9 N/m. The peak kinetic energy in each cycle almost remains as a constant.
The results in Table 4-7 show the effect of the damper assembly stiffness on the kinetic energy decay ratio of the cable-damper system. It can be seen that the effect of the damper assembly in suppressing cable vibration increases monotonically with the increase of its stiffness.

Table 4-7  Kinetic Energy Decay vs. Stiffness of Damper Assembly Changes

<table>
<thead>
<tr>
<th>Stiffness of damper assembly (N/m)</th>
<th>Damper fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.49E1</td>
<td>1.49E4</td>
</tr>
<tr>
<td>Kinetic energy decay ratio</td>
<td>0.488</td>
</tr>
</tbody>
</table>
CHAPTER 5 Discussion

In the current study, the effect of an external damper in suppressing cable vibration of a cable-damper system is expressed in terms of kinetic energy decay ratio. Also, it has been proved that such an effect can be interpreted as the equivalent Rayleigh damping of a single cable. In other words, a higher kinetic energy decay ratio in a cable-damper system means a higher equivalent Rayleigh damping in a single cable. Thus, in the following section, the term 'Rayleigh damping' is used directly to indicate the damping effect of the external damper in a cable-damper system.

5.1 Effects of Non-dimensional Damping Parameter

The relation between the non-dimensional damping parameter and the kinetic energy decay ratio corresponding to the variation of non-dimensional cable bending stiffness parameter (B.P.) is plotted in Figure 5-1 to Figure 5-5 based on numerical results presented in Table 4-2 to Table 4-6. Each figure corresponds to a specific damper location.
Figure 5-1  Kinetic Energy Decay Curves (Damper Location Parameter =0.06L)

Figure 5-2  Kinetic Energy Decay Curves (Damper Location Parameter =0.2L)
Figure 5-3  Kinetic Energy Decay Curves (Damper Location Parameter =0.3L)

Figure 5-4  Kinetic Energy Decay Curves (Damper Location Parameter =0.4L)
Two conclusions can be drawn from these five figures:

1) The suppression effect represented by the kinetic energy decay ratio increases as a function of non-dimensional damping parameter up to an optimum level. Further increase of the non-dimensional damping parameter would result in the reduction of energy dissipation. One possible reason could be that, for lower non-dimensional damping parameters, the effect of an external damper is relatively small and it has no considerable influence on the vibration shape of the cable-damper system. On the other hand, for higher non-dimensional damping parameters, the external damper acts more like as a rigid support and thus the vibration shape of the cable changes. As a result, at the point where the damper is
attached to the cable, the amplitude of vibration is relatively small and therefore less kinetic energy is dissipated through the external damper.

2) The optimum value of the non-dimensional damping parameter varies with the damper location. However, at a specific damper location, the optimum non-dimensional damping parameter is almost independent of the non-dimensional cable bending parameter. For example, for a damper located at 0.06L to the cable-deck-anchorage point, the optimum non-dimensional damping parameter is approximately 6, and this result is consist with what Tabatabai et. al have found[22]. For the damper location of 0.2L, 0.3L, 0.4L, and 0.45L, the optimum non-dimensional damping parameters are approximately 2, 2, 1 and 1, respectively. The trend revealed by this set of results suggests that the optimum non-dimensional damping parameter may decrease as the damper location moves towards the center of the cable span. This means that on a real cable-stayed bridge, the optimum damping coefficient of the external damper decreases when it is moved further away from the cable end.

5.2 Effect of Non-Dimensional Cable Bending Parameter

As shown in the Table 4-2 to Table 4-6 in Section 4.3, the non-dimensional cable bending parameter does have substantial influences on the Rayleigh damping.
For the cable-damper systems having the same external damper located at the same location, the smaller the non-dimensional cable bending parameter, the larger the equivalent Rayleigh damping of the system will be. For example, if the non-dimensional damping parameter is 6 and the damper is located at 6% of cable length from the cable deck-anchorage, the kinetic energy decay ratio is respectively 0.8154, 0.7022, 0.6329, 0.5988, and 0.5562 for the non-dimensional cable bending parameter of 100, 200, 300, 400, and 500. Based on the definition of the non-dimensional cable bending parameter, the smaller the non-dimensional cable bending parameter is, the higher the bending stiffness the cable has. For a stiffer cable, the damper would affect the motion of a longer cable segment and thus dissipate more kinetic energy; whereas for a more flexible cable, the damper would affect motion of a shorter portion and thus dissipates less kinetic energy. Theoretically, for the extreme case of a cable with zero bending stiffness, the damper would only affect the cable motion at the point where the damper is attached to the cable.

5.3 Effect of Damper Location

The relation between the damper location and the kinetic energy decay ratio corresponding to the variation of non-dimensional damping parameter (D.P.) is
plotted in Figures 5-6 to 5-10. Each figure shows the kinetic energy decay ratio of a cable-damper system having a specific non-dimensional cable bending stiffness parameter.

![Kinetic energy decay ratio vs. damper location parameter](image)

Figure 5-6   Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 100)
Figure 5-7  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 200)

Figure 5-8  Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 300)
Figure 5-9 Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 400)

Figure 5-10 Kinetic Energy Decay Curves (Non-Dimensional Cable Bending Stiffness: 500)
It can be observed from these five figures that when the external damper is moved towards the center of the cable, the vibration suppressing effect provided by the external damper does not increase monotonically. Three types of trends can be seen in the figures:

Type 1: When the damping parameters are relatively small, the kinetic energy decay ratio increases as the damper is moved towards the cable middle span. 

Type 2: When the damping parameters are relatively large, the kinetic energy decay ratio decreases as the damper is moved towards the cable middle span.

Type 3: When the damping parameters are moderate, the kinetic energy decay ratio decreases first and then increases as the damper is moved towards the cable middle span.

These three types of curves are related to the amount of system kinetic energy dissipated through the external damper. Based on Eq. (2-1), the more the kinetic energy is dissipated through the external damper, the smaller the kinetic energy decay ratio will be. The external damper is fixed on the bridge deck at one end and connects to the cable at the other end. The amount of kinetic energy dissipated through external damper is determined by the motion of the damper-cable connecting point. The dissipated kinetic energy through the external damper can be expressed as:
\[ W = \int_{0}^{T} F \cdot ds \]  

(5-1)

where \( F = -c_v \frac{du}{dt} \) is the damping force, \( T \) is the calculation time period, and \( ds = \frac{du}{dt} \) is the displacement of the damper-cable connecting point.

Therefore, the Eq. (5-1) can be rewritten as:

\[ W = \int_{0}^{T} c_v \left( \frac{du}{dt} \right) \cdot \left( \frac{du}{dt} \right) dt \]  

(5-2)

In the current study, only linear damping is taken into accounted and \( c_v \) is a constant, the velocity \( \left( \frac{du}{dt} \right) \) is a function of the damper location (\( L_d \)) and the damping coefficient \( c_v \), i.e. \( \frac{du}{dt} = u(L_d, c_v) \). Consider the work done by the damper in one second, Eq. (5-2) can be written as

\[ W = c_v \cdot \int_{0}^{1} \left( \frac{du}{dt} \right) (L_d, c_v) \cdot dt \]  

(5-3)

When the perturbation is imposed at the cable mid-point, the shape of the cable will take the form as shown in Figure 5-12. One end of the external damper is connected with the cable at point “A” and another end is fixed at point “B” on the bridge deck. In general, the energy dissipated through the external damper depends on the relative movement between points “A” and “B”. In the cable-damper system shown in Figure 5-12, since point “B” is fixed, the energy dissipated by the external damper only depends on the motion of point “A”. Without considering the damper
effect, the velocity of point “A” will increase when the damper is moved from the
cable end to the middle span. The reason is that within the same time period, the
distance travelled by the point close to the cable middle span are longer than that by
the point close to cable end. When the external damper is considered, it would
decelerate the motion at point “A”. This effect will become greater as the velocity at
point “A” increases. Therefore, for the system shown in Figure 5-12, the velocity at
point “A” depends on not only the damper location but also the damping coefficient.
This indicates that the energy dissipated through the external damper and the kinetic
energy decay ratio is the functions of the location and damping coefficient of the
external damper.

![Figure 5-11 Cable Damper Connection](image)

In Type 1 curves, the damping parameter is very small, and thus the damper
has minor influence on the velocity at point “A” regardless of the damper location.
However, the velocity at point “A” will increase substantially when the damper is
moved towards the cable middle span. Therefore, the velocity at point “A” will increase due to the combined effect of the damper damping coefficient and the damper location. As a result, more energy will be dissipated and the kinetic energy decay ratio will increase when the external damper is moved towards the center span. The Type 2 curves can be explained similarly. The Type 3 curves are more complicated than the other two types of curves. The velocity at cable-damper connecting point will decrease first and then increase as the damper is moved towards the cable middle span due to combined effect of damping coefficient and location of the external damper. Therefore, the kinetic energy decay ratio decrease first and then increase.

5.4 Effect of the Stiffness of a Damper Assembly

A new idea of incorporating cross-ties with dampers to suppress bridge stay cable vibration has also been attempted in a few recent applications, and the configuration of this combined application is shown in Figure 1-5. The cross-ties would increase the stiffness of the cable network system, whereas the damper would contribute to the system damping.

In the finite element model proposed in the current study, two COMBIN14 elements are used in series to simulate the combined effect of cross-ties and damper.
As shown in Figure 2-6, one COMBIN14 element is used to simulate the behaviour of an external damper and another is used to simulate that of the cross-ties.

Analyses have been done to investigate the combined effect of cross-ties and external damper on suppressing cable vibration. The relation between the system kinetic energy decay ratio and the stiffness of a damper assembly is presented in Figure 5-13, which shows clearly that the kinetic energy decay ratio increases monotonically with the increase of the damper assembly stiffness. The kinetic energy dissipated through an external damper depends on the relative movement between the two ends of the damper. For a given cable-damper assembly system shown in Figure 2-6, when the stiffness of the damper assembly is very small, i.e. the cross-ties are very flexible, the damper can be considered as being isolated from the cable. The damper is equivalent as being fixed to the deck at one end and connecting to the cable directly at the other end when the stiffness of damper assembly is very large. Thus, for the first case, less kinetic energy is dissipated through the external damper and the kinetic energy decay ratio is high; while for the second case, more kinetic energy is dissipated and the kinetic energy decay ratio is low.
Figure 5-12 Kinetic Energy Decay Curve (Considering Stiffness of Damper Assembly)
CHAPTER 6  Conclusions and Recommendations

6.1 General Conclusions

A new energy-based method has been proposed to study the damping properties of a cable-damper system and associated finite element models were developed by application of ANSYS10.0. First, the validity of the proposed method, finite element models and time-history procedure have been verified by using the experimental results obtained from previous research. Then, parametric studies were carried out by examining different combinations of non-dimensional damper location, non-dimensional cable bending stiffness parameter, and non-dimensional damping parameter. Also, an analysis has been completed to examine the stiffness effect of a damper assembly on the damping properties of a cable-damper system. The main findings from the current study are concluded in the following sections.

Compared to the existing methods, the advantages of the proposed energy-based damping evaluation method are:

1) The proposed energy-based damping evaluation method can be used to determine the damping properties of a cable-damper system. The associated finite element model consisting of the PIPE59 and COMBIN14 elements developed by ANSYS10.0 can be applied to simulate the behaviour of a cable-damper system conveniently.
2) There is no restriction on the location of the external damper.

3) The bending stiffness of the cable is considered in the formulation.

4) The stiffness effect of the damper assembly is included in the modeling, which allows for numerical simulation of the effect of combined application of cross-ties and dampers on suppressing cable vibration.

The important findings concluded from the parametric studies are:

1) In a given cable-damper system, one optimum non-dimensional damping parameter exists and its magnitude is independent of the non-dimensional cable bending stiffness parameter. The magnitude of the optimum non-dimensional damping parameter decreases as the external damper is moved towards the cable middle span. On the real bridge, it can be inferred that the optimum damper coefficient decrease as the external damper is moved towards the cable middle span.

2) The non-dimensional cable bending stiffness parameter affects the damping properties of the cable-damper system. The larger this parameter is, the smaller this impact will be. This means that the external damper has larger effect on the damping properties of a rigid cable than that of a more flexible cable.
3) Moving the external damper towards the cable middle span may not improve the capacity of the external damper for suppressing cable vibration. The effect of the external damper to suppress cable vibration depends on the combination of the non-dimensional damping parameter and the location of the damper.

4) For the damper assembly system that consists of a damper and cross-ties, the tension stiffness of cross-ties will affect the damping properties of a cable-damper system. The capacity of a damper assembly to suppress cable vibration will increase with its increasing stiffness.

6.2 Recommendations for Future Research

In the current study, the proposed energy-based damping evaluation method has been applied to study the damping properties of a cable-damper system, which contains one cable and one external damper. However, on real cable-stayed bridges, the cable system consists of dozens, or even hundreds of cables, especially when the cross-ties are installed to suppress the cable vibration. In this case, the cable system is like a network. The damping properties of the cable network are obviously different from that of a single cable-damper system. The applicability of the proposed energy-based damping evaluation method to such a cable network system needs to be verified.
In the current study, both ends of the cable are fixed, whereas on the real bridge, the cable is anchored to the bridge deck at one end and the bridge tower at the other end. The motion of the bridge deck and the bridge tower would affect and excite the cable vibration, as well. Therefore, in future study, the interaction between the motion of the bridge deck and tower with that of the cable needs to be considered in the modeling.
REFERENCES


75


VITA AUCTORIS

NAME: Xianshu Jiang
PLACE OF BIRTH: Hebei, China
YEAR OF BIRTH: 1972
EDUCATION: Ningjin High School, Hebei, China
1987-1990
Shanxi Mine College, Taiyuan, Shanxi, China
1990-1994
Lanzhou Railway University, Lanzhou, Gansu, China
1995-1998