Mathematical modeling and experimental study of squeeze casting of magnesium alloy AM50A and aluminum alloy A356.

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MATHEMATICAL MODELING AND EXPERIMENTAL STUDY OF SQUEEZE CASTING OF MAGNESIUM ALLOY AM50A AND ALUMINUM ALLOY A356

By

Fang Yu

A Dissertation
Submitted to the Faculty of Graduate Studies and Research
Through Engineering Materials
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The Degree of Doctor of Philosophy at the
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ABSTRACT

In recent years, the squeeze casting process has been widely used with various aluminum alloys to manufacture near-net shape automotive components. Preliminary research has also demonstrated technical feasibility potential of squeeze casting for magnesium. A better understanding of squeeze casting process is essential for applying the process for the production of large automotive components, such as engine block, using aluminum and magnesium. Meanwhile, simulation can help to achieve the analysis and optimization of the casting process. Unfortunately, for squeeze casting, no appropriate model is presently available.

In this study, a mathematical model has been developed to simulate the transport phenomena and solidification occurring in squeeze casting process. The model was based on the control-volume finite difference approach and on an enthalpy method.

An experimental system was developed capable of characterizing local in-cavity pressures, determining casting/die interfacial heat transfer, and observing pressurized solidification phenomena taking place in squeeze casting of aluminum and magnesium alloys. It was found that, during squeeze casting process, the local cavity pressure distribution was inhomogeneous.

Experimental correlations of heat transfer coefficient were integrated into the model with local cavity pressures estimated by a force balance approach. Hence, instead of using static boundary condition, a dynamic boundary condition was established in the model. In order to minimize the deviation of calculation, experimental correlations between solidification temperatures and applied pressures were also integrated into the
model. The predicted results, including cooling curves, solidification times, and local pressure cavity pressures, were compared with the experimental measurements and they were found to be in good agreement.

The model was further advanced to predict shrinkage porosity during squeeze casting by a newly proposed criterion based on “burst-feeding” theory. The proposed model is able to predict the occurrence and location of porosity formation under a specified applied pressure and holding time. Comparison of the experimental results with the result of computations, the model not only successfully predicted the occurrence of porosity under certain circumstances, but also indicated the correct locations where porosity formed. Hence, it can be used for the optimization of the squeeze casting process.
To My Family
Yingping Huang
Yue (Lylia) Yu
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### LIST OF ABBREVIATIONS, SYMBOLS, NOMENCLATURE

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<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>AA</td>
<td>Aluminum Association</td>
</tr>
<tr>
<td>Al</td>
<td>Aluminum</td>
</tr>
<tr>
<td>ASTM</td>
<td>America Society of Testing Materials</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon Dioxide</td>
</tr>
<tr>
<td>CSC</td>
<td>Cracking Susceptibility Coefficient</td>
</tr>
<tr>
<td>CSM</td>
<td>Cyclic Steady Heat Balance Method</td>
</tr>
<tr>
<td>DAS</td>
<td>Dendritic Arm Space</td>
</tr>
<tr>
<td>DAS</td>
<td>Data Acquisition System</td>
</tr>
<tr>
<td>DSC</td>
<td>Direct Squeeze Casting</td>
</tr>
<tr>
<td>FDA</td>
<td>Finite Difference Analysis</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite Difference Methods</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Methods</td>
</tr>
<tr>
<td>FVM</td>
<td>Finite Volume Methods</td>
</tr>
<tr>
<td>GTAW</td>
<td>Gas Tungsten Arc-Welding</td>
</tr>
<tr>
<td>HSC</td>
<td>Hot-cracking Sensitivity Criterion</td>
</tr>
<tr>
<td>HTC</td>
<td>Heat Transfer Coefficient</td>
</tr>
<tr>
<td>IHTC</td>
<td>Interfacial Heat Transfer Coefficient</td>
</tr>
<tr>
<td>ISC</td>
<td>Indirect Squeeze Casting</td>
</tr>
<tr>
<td>LCD</td>
<td>Liquid Crystal Display</td>
</tr>
<tr>
<td>LED</td>
<td>Light-Emitting Diode</td>
</tr>
</tbody>
</table>
Mg Magnesium
MCF Metal Compression Forming
PCPC Pressure Counter Pressure Casting
RMS Root Mean Square
SC Squeeze Casting
SF₆ Sulfur Hexafluoride
SVD Partial Squeeze and Vacuum Die Casting Machinery
TM Trade Mark
TMP Technical Machine Products
UL Updated Lagrangian
VSC Indirect Vertical Squeeze Casting

Symbols

\( A \)
  air gap

\( a, b, c, d, e, f \)
  constant

\( c \)
  the velocity of the junction of the moving and stationary fluid or sound speed in fluid.

\( C_{li}, C_{so}, C_{mu} \)
  heat capacities for liquid, solid and mushy phases

\( D \)
  diameter

\( E \)
  fluid modules, Young’s modules

\( F \)
  force

\( f_s \)
  fraction of solid

\( g \)
  gravity
\( G_f \) \hspace{1cm} \text{gradient of } f_s \\
\( Gr \) \hspace{1cm} \text{Grashof number} \\
\( h \) \hspace{1cm} \text{heat transfer coefficient} \\
\( H \) \hspace{1cm} \text{enthalpy} \\
\( H_f \) \hspace{1cm} \text{latent heat} \\
\( k \) \hspace{1cm} \text{thermal conductivity} \\
\( L \) \hspace{1cm} \text{length} \\
\( m \) \hspace{1cm} \text{coefficient of friction} \\
\( Nu \) \hspace{1cm} \text{Nusselt number} \\
\( P \) \hspace{1cm} \text{pressure} \\
\( Pe \) \hspace{1cm} \text{Pelcet number} \\
\( Pr \) \hspace{1cm} \text{Prandtl number} \\
\( q \) \hspace{1cm} \text{heat flux, thermal impedance} \\
\( r, z \) \hspace{1cm} \text{cylindrical co-ordinate direction} \\
\( R \) \hspace{1cm} \text{radius} \\
\( Re \) \hspace{1cm} \text{Reynolds number} \\
\( S \) \hspace{1cm} \text{source term} \\
\( S_n \) \hspace{1cm} \text{compressive stress} \\
\( t \) \hspace{1cm} \text{time} \\
\( t_n \) \hspace{1cm} \text{friction stress} \\
\( T \) \hspace{1cm} \text{temperature} \\
\( \Delta T \) \hspace{1cm} \text{solidification range of temperature} \\
\( V \) \hspace{1cm} \text{volume}
\(W\)  work
\(u, v, w\)  velocities

**Greek Symbols**

- \(\alpha\)  thermal expansion coefficient
- \(\gamma\)  material property
- \(\varepsilon\)  Strain
- \(\sigma\)  stress
- \(\tau\)  shear stress
- \(\nu\)  Poisson’s ratio
- \(\alpha\)  thermal volume expansion coefficient
- \(\lambda\)  liquid fraction
- \(\Gamma\)  diffusivity
- \(\Delta\)  difference
- \(\mu\)  viscosity
- \(\rho\)  density

**Subscripts**

- \(a\)  applied
- \(b\)  bulk
- \(c\)  casting, critical
- \(\text{con.}\)  Convection

xxxii
cond  conduction

d  die

eff  effective value

l  internal

i  instant

int  interface

l  liquid

m  mushy or melting

o  initial

r  radius

rad  radiation

ref  refereed value

s  solid, side

T  top

w  mold wall

1, 2, 3  Principal Stress directions or locations

x, y, z  direction

Superscripts

*  dimensionless
INTRODUCTION

Since the energy crisis in the 70s, in order to increase fuel efficiency and reduce greenhouse gas emissions, the automotive industry has been moving towards making lighter vehicles. This weightlight can be achieved by new designs, downsizing, and lighter materials. Regarding materials, in past decades, there has been a continuous increase in the use of light alloys in automotive components such as aluminum (Al), magnesium (Mg) alloys [1]. In 2004-model vehicles, a typical family vehicle weighed 1,538 kgs with 9% weight of parts made from aluminum and magnesium [2].

Aluminum has increasingly been chosen by automakers to reduce vehicle weight for several reasons [3]:

a. Aluminum’s density is only one-third of steel, which means even if a component made from aluminum is 1.5 times thicker than a steel version, it can still be 50 percent lighter;

b. Aluminum can absorb twice as much energy as steel at the same weight;

c. Aluminum is a corrosion resistant material, unlike steel which must be coated with other metals, like zinc, to improve its corrosion resistance;

d. The light weight and stiffness of aluminum can enhance a vehicle’s acceleration, handling, and reduce its noise, vibration and harshness characteristics; and

e. Aluminum is 100 per cent recyclable. Although it accounts for less than 10 percent of a vehicle’s total weight, it represents 35-50 percent of the total material scrap value at the end of its useful life.
With the above advantages, 116 lb per vehicle now makes aluminum the number three material used in automobiles [4].

Magnesium is an attractive material for automotive applications due to the following reasons [5]:

a. It is lightweight, 36% lighter than Al and 78% lighter than iron (Fe) in terms of density, which is suited for further weight reduction;

b. Mg alloys have the highest strength-to-weight ratio of all the structural metals;

c. Magnesium is abundant and the eighth most common element; seawater, the main source of its supply, contains 0.13% Mg, which represents a virtually unlimited supply; and

d. Magnesium is also recyclable; instituting a recycling system would extend supplies and save energy.

The consumption of magnesium in automotive industry has been quickly increasing with an average annual growth rate over 10% [6].

The growth of consumption in both aluminum and magnesium mainly results from die casting, sand casting, and other conventional processes. Due to process limitations, however, there are problems associated with those casting processes. For example, die cast parts have poor mechanical properties due to the presence of entrapped gas and porosity. Thus they are not suitable for manufacturing large and thick automotive components from which enhanced engineering performance is required.

Compared to other conventional casting processes, the most attractive feature of squeeze casting (SC) is that it can make castings virtually free of porosity. Hence,
squeezing castings usually have excellent as-cast quality, and are heat treatable, which is difficult to achieve with other conventional casting processes. With this unique advantage, squeeze casting technology has been extensively developed and widely used for aluminum alloys since the mid-60s. Application of squeeze cast aluminum automotive components, such as road wheels, steering wheel knuckles, and engine blocks, have been quite common [7]. But, squeeze cast magnesium automotive components in mass production have not been reported in the open literature yet.

Although the metal casting family has a long history [8], squeeze casting is relatively new among varieties of castings. Despite significant research activity on the squeeze casting process, some fundamental questions still need to be answered, and the process must be optimized [9] to expand its application, especially for the emerging use of magnesium.

Mathematical modeling of casting processes started as early as the mid-60s. With the continued advancement and popularity of computers, especially after the 80s, increasing numbers of researchers have been involved in casting simulation through either research and development, or application. Mathematical modeling has changed the way that the casting industry works. Instead of experimental trials followed by corrective action in years past, now simulation helps do the analysis and optimization in advance and undertake preventive action. By shortening development times, eliminating product defects, and reducing cost, simulation not only improves the product quality but also makes the production more efficient [10].

So far, many casting simulation models have been developed, and some of them even are commercialized. Unfortunately, available commercial simulation software only
INTRODUCTION

covers conventional casting processes, e.g. sand and die casting. For squeeze casting, no
appropriate models and source codes are available presently.

1.1 Motivation

In squeeze casting, an external pressure is applied during the whole stage of
solidification, which makes the condition at the casting-die interface different from other
conventional casting processes. It has been presumed that squeeze casting takes place
under hydrostatic pressures [11], which results in a homogeneous distribution of local
pressures in the castings. Due to the limitation of measurement technique, and the
extreme environment in the cavity of squeeze castings resulting from poor thermal,
mechanical and chemical conditions, it was almost impossible to characterize local cavity
pressures in the past. As a result, pressures in squeeze casting were cited only as the
hydraulic pressure. When defining boundary conditions in terms of the heat transfer
coefficient (HTC), a fixed constant value of HTC is usually used. By using these constant
HTCs in simulation, the accuracy of the predicted results certainly is called into question.

Simulation is a very important method for optimizing squeeze casting process. A
proper mathematical model is the primary key. A good model should represent features
of the behaviour of physical phenomena; a good simulation should reflect every aspect of
the object being cast. Unfortunately, although there are several articles on the simulation
of squeeze casting, those models only emphasized certain aspects of the process.
Therefore a comprehensive mathematical model containing information on phase change,
heat transfer, fluid flow, and casting-die interaction for squeeze casting is necessary.

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Since casting is a transient process, not only the metal itself changes its phase from liquid to solid, but also the casting-die interfacial (boundary) condition changes during the process. The changes of these two factors affect each other. Generally, in casting simulation model, minor changes in the boundary conditions can significantly affect the numerical prediction results. Therefore, to obtain reliable and valid prediction through simulation, proper boundary conditions must be imposed; therefore a good understanding of the real situation is critical.

### 1.2 Objectives

The main aim of this project was to build a numerical model for simulating squeeze casting processes for light metals, i.e., aluminum and magnesium alloys. The objectives of this study were:

a. To develop an experimental technique to characterize local cavity pressures in squeeze casting of aluminum and magnesium alloys;

b. To develop an experimental technique to determine heat transfer coefficients at the casting-die interface;

c. To obtain the relationship between the heat transfer coefficient and the local pressure;

d. To investigate pressurized solidification behaviour of castings;

e. To develop a mathematical model coupled with localized boundary conditions, which is capable of predicting transport phenomena taking place during cavity filling and pressurized solidification; and to verify the model with experimental measurements; and

f. To advance the model for prediction of porosity in squeeze castings.
1.3 Outline of This Dissertation

This dissertation begins with an introduction in Chapter 1. An overview of squeeze casting is introduced in Chapter 2 to give both a basic concept of this casting process and a background review of modeling and simulation development in the casting industry. Chapter 3 describes the experimental setup and procedures, as well as materials employed in this study. Intensive content regarding the experiments in squeeze casting magnesium and aluminum alloys are contained in Chapters 4, 5 and 6. Experiments performed to characterize local cavity pressures are stated in Chapter 4; measurements conducted to find out the relationship between local cavity pressure and heat transfer coefficient are introduced in Chapter 5; and the observation of pressurized solidification phenomena is given in Chapter 6.

Chapter 7 summarizes the formulation of a coupled mathematical model. Consideration of heat transfer, fluid flow and phase change formed the basic model; the incorporation of local cavity pressures and heat transfer coefficients formed the coupled model. With the consideration of a new proposed criterion, an advanced model was established for predicting porosity in squeeze casting. Numerical results are presented, and experimental verification of the numerical model is carried out and described in Chapter 8. In Chapter 9, the mathematical model is applied to predict shrinkage porosity in squeeze casting by proposing a new criterion, and allowed to more accurately determine the effect of hydraulic pressures and holding times on porosity formation. The computational and experimental results are compared in Chapter 9. A summary is presented in Chapter 10, which outlines the important findings of this study, and also lists several recommendations for future work.
2 BACKGROUND REVIEW

2.1 Squeeze Casting

Squeeze casting is a hybrid manufacturing process combining in one single operation the desirable features of both casting and forging processes. In this way, it enables to make sound castings [12, 13]. A scheme of the squeeze casting process is shown in Figure 2.1, where:

a. A pre-specified amount of molten metal is poured into a preheated die cavity, located on the bed of a hydraulic press (Figure 2.1 (a));

b. The press is activated to close off the die cavity and to pressurize the liquid metal. This is carried out very quickly, rendering solidification of the molten metal under pressure (Figure 2.1 (b)); and

c. The pressure is held on the metal until solidification completes. This not only increases the rate of heat flow, but also most importantly eliminates macro/micro shrinkage porosity. In addition, since nucleation of gas porosity is pressure-dependent, the porosity formation due to dissolved gases in the molten metal is restricted. (Figure 2.1 (c)) [13, 14].

It is widely believed [12] that the concept of pressurized solidification was first introduced in 1819 via a British patent (No. 4371). The inventor, James Hollingrake, described his invention as “Making and Working a manufacturer for Applying a Method of Casting and Forming Metallic Substances into Various Forms and Shapes with
Improved Closeness and Soundness in Texture”. In 1878, Chernov in Russia suggested in his paper that steam pressure might be applied to molten metal whilst it solidified in a die [15].

![Schematic diagram showing a direct squeeze casting process.](image)

Figure 2.1 Schematic diagram showing a direct squeeze casting process.

Some researchers believe that it was not until 1931 that the first squeeze casting experiments were conducted on Al-Si alloys by Welter [12]. However, others [15] thought it was not until 1937 that the first squeeze casting experiments were conducted for the production of brass and bronze cylinders.

Squeeze casting had been extensively investigated and used in the former Soviet Union, as described by Plyatskii in his book “Extrusion Casting” of 1965 [11]. The process had found application in large batch production at more than 150 plants. Several plants were apparently producing over 200 different types of squeeze cast components using iron, steel, and various nonferrous alloys. Western interest in the process was then
awakened, resulting in active research and/or commercial production in many countries [13, 15-20].

Squeeze casting has been variously referred to as “Liquid pressing”, “Extrusion casting”, “Pressure crystallization”, “Squeeze forming”, and “Liquid forging”.

2.1.1 Squeeze Casting Types

There are two distinct types of squeeze casting processes [21, 22], direct and indirect. However, both versions utilize the essentials of the process as described above.

2.1.1.1 Direct Squeeze Casting

Direct squeeze casting (DSC) as shown in Figure 2.1 is sometimes termed liquid-metal forging since it is accomplished in equipment more akin to that used for forging than to that used for die casting [22]. For the direct squeeze casting, two further modes may be distinguished based on liquid metal displacement initiated by the punch movement: (i) without metal movement, and (ii) with metal movement [14].

2.1.1.2 Indirect Squeeze Casting

Indirect squeeze casting (ISC) was first named in 1980 by Kaneko [23], at that time, it was called indirect pressure solidification.

Indirect squeeze casting, as shown in Figure 2.2, on the other hand [22], is akin to die casting, which is performed in die-cast-like equipment (vertical or horizontal) and tooling.

During indirect squeeze casting, properly prepared melt is poured into the shot sleeve of a horizontal or vertical squeeze casting machine. From there, it is injected into
the die through relatively large gates and at relatively slow velocity (usually under 0.5 m/s). Melt in the die cavity is then solidified under pressures.

Figure 2.2 UBE indirect squeeze casting process [22].

DSC was developed first but is used occasionally to produce simple, symmetrical castings such as pistons, calipers and master cylinders. The use of DSC is very limited due to the features of the process and the die design: the maximum weight of casting is usually no more than 10 kg. The backward process (ISC) is more suitable for industrial needs, because it enables the production of intricately shaped castings such as alloy wheels for the auto industry. The development of indirect vertical squeeze casting (VSC) machines, essentially extended the application of the squeeze casting process. It allowed the fabrication of complex castings such as alloy wheels with practically no internal defects. Die coat is not required and a metal pressure of up to 100 MPa is applied throughout solidification. The maximum casting size is limited by the machine capacity [14, 22].

2.1.2 Advantages and Limitation of Squeeze Casting

Squeeze casting differs from other die casting processes in two major details:
2 BACKGROUND REVIEW

a. the molten metal being cast enters the die cavity without turbulence, either under the force of gravity (vertical operation) or by piston pressure (horizontal operation); and

b. the molten metal thereafter is subjected to an applied pressure and the pressure is maintained until the liquid to solid transformation is complete.

Comparing squeeze casting with other casting processes, the overall advantages of squeeze casting, based on the literatures [13-16, 22, 24, 25], are discussed as follows.

2.1.2.1 Superior Mechanical Properties

Superior mechanical properties obtained from squeeze casting have been reported for both magnesium [26, 27] and aluminum alloys [21, 28].

Regarding magnesium alloys, Luo et al [26] compared, through experiments, the squeeze cast and die cast AZ91D alloy components, the results shows the significantly improved ductility for the squeeze cast samples over the conventional die cast parts. Zhou et al [27] studied magnesium alloy AM50 cast by squeeze casting and die casting processes. Table 2.1 listed the mechanical properties of both the squeeze cast and die cast alloy AM50 specimens. Figure 2.3 shows a representative true stress versus strain curve for squeeze cast and die cast AM50 alloys.

| Table 2.1 | Tensile properties of both the squeeze cast and die cast AM50 alloy [27] |
| --- | --- | --- |
| Casting condition | 0.2% YS MPa | UTS MPa | Elongation (%) |
| Squeeze cast (as-cast) | 75 | 175 | 8.0 |
| Die cast (as-cast) | 80 | 100 | 2.1 |

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As to aluminum alloys, Yue and Chadwick [21] squeeze cast several commercial Al-Si alloys. Improvements on properties were achieved by squeeze casting compared with gravity casting (Table 2.2). Kaneko et al [28] compared the mechanical properties of squeeze cast and gravity cast aluminum wheel made from A356 (T6), where the properties were significantly improved with squeeze casting.

Table 2.2 Comparison of mechanical properties of squeeze cast and gravity cast Al-Si alloy [21]

<table>
<thead>
<tr>
<th>Alloy</th>
<th>0.2% proof stress (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravity cast</td>
<td>110</td>
<td>200</td>
<td>2</td>
</tr>
<tr>
<td>Squeeze cast</td>
<td>126</td>
<td>233</td>
<td>2.7</td>
</tr>
</tbody>
</table>
2.1.2.2 Fine Structure

Yue and Chadwich [21] addressed, in squeeze casting, high pressure increased the heat-transfer coefficients, which in turn leads to alteration in microstructure. In their experiment of squeeze casting commercial Al-Si 7% alloy, there was a clear microstructure refinement (Figure 2.4) as compared with gravity sand casting.

Figure 2.4 Microstructure of Al-7%Si: (a) gravity cast; and (b) direct squeeze-cast at 150 MPa [21].
2 BACKGROUND REVIEW

2.1.2.3 Minimized Porosity

The study by Zhou et al. [27] revealed the porosity distribution in the polished die cast and squeeze cast AM50 alloys through the optical microscopy examination as shown in Figures 2.5 and 2.6, respectively. Representative pores can be easily spotted in the die cast plates of AM50 alloy with a section thickness of 10 mm as indicated in Figure 2.5. However, it is evidently shown in Figure 2.6 that the squeeze cast AM50 with the same section thickness is virtually free of gas and shrinkage porosities. Figure 2.7 presents quantitatively the percentage of the porosity of both the squeeze cast and die cast AM50 alloys, based on the density measurements. In comparison with that (4.00%) of the die casting, the porosity level of the squeeze castings is only 0.12%. The difference in casting soundness in terms of the porosity level between squeeze casting and die casting is evident, which is consistent with the observation of their microstructure.

Figure 2.5 Optical micrographs showing porosity in die cast AM50 alloy [27].

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Figure 2.6 Optical micrographs showing almost porosity-free squeeze cast AM50 alloy [27].

Figure 2.7 Porosity levels of squeeze cast and die cast AM50 [27].

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2 BACKGROUND REVIEW

2.1.2.4 Heat-treatable

Die castings contain inherent porosity due to the practices of using narrow gates and pressure to fill cavity dies. But in squeeze casting, air entrapment problems are avoided by the slow filling of dies under low pressures. Yue and Chadwick [21], in their study of squeeze cast Al-Si 7% alloy, found that both the 0.2% proof stress and UTS were significantly increased from as-cast 126 MPa and 233 MPa to T6 heat treated 330 MPa and 368 MPa, respectively. It has also been demonstrated by Kaneko et al [28] that T6 treatment (solution treatment and age hardening) can be applied to squeeze cast aluminum alloy A356. Kim [29] studied the characteristics of the direct squeeze cast 7075 wrought Al alloy, which was heat treated at 120 °C for 129.6 ks. The results show that the wear loss amount of squeeze cast was decreased.

2.1.2.5 Weldable

Hwang [30] reported a study on the quality of gas tungsten arc-welded (GTAW) squeeze cast A356 alloy, the effect of T6 heat treatment, filler metals, and shielding gas flow rates on the qualities of the weld with or without an anodic coating were fully investigated. Increasing the shielding gas flow rate could reduce the pore numbers, the pore volume and the loss of magnesium in the fusion zone, which in turn improved the mechanical properties of the weld.

2.1.2.6 Good Surface Finish

Lynch et al [31] reported a textile bobbin flange made from A380 aluminum. A critical requirement of this component is surface smoothness of 32RMS (note: surface roughness can be measured by a profilometer, a stylus device used to trace across the surface profile. The results are expressed as RMS, which is the root mean square of the
deviations from the center line). This is because a porosity-free surface ensures that the textile thread is not easy to be damaged during service after the entire surface periphery has been machined. Britnell and Neailey [32] reported that squeeze casting surface finish can reach 0.4-3.2 μm, which is almost the same as high pressure die castings.  

2.1.2.7 High Productivity

Compared to hot forging, the cycle time of squeeze casting is reduced substantially [19]. Squeeze casting can form complex shapes in a single operation at high production rates and low-unit costs.  

2.1.2.8 Composite Fabrication

Porosity, as the most critical microstructure feature often occurs during the production of metal matrix composites, which is attributed to the shrinkage of metals during their solidification. Squeeze casting has a clear advantage over other processes in producing zero-porosity castings [9]. In the study by Hu [33], SiCw/ZK51A magnesium matrix composite was fabricated by a modified squeeze casting process. The composite possesses very high modulus and mechanical strength as compared with the unreinforced matrix alloy, with the increment in modulus and 0.2% offset yield strength of the composite being linearly proportional to the whisker volume fraction.  

2.1.2.9 Casting of Wrought and Special Alloys

It is possible to directly squeeze cast wrought and special alloys because fluidity or castability are not critical in this process. Yue et al [21, 34] successfully squeeze cast several high strength Al-alloys A6061 and A7010. It was found that generally, that the squeeze-cast fully heat treated alloys exhibit mechanical properties lying between those of the longitudinal and short transverse properties of the wrought material.
Like other casting processes, squeeze casting has a few shortcomings or disadvantages: high capital cost, shortened die life, limited shape complexity, difficult to produce thin sections, and limited maximum size and weight of components [14, 22].

2.1.3 Squeeze Casting of Aluminum and Magnesium for Automotive Applications

Almost all alloys used in permanent mould could also be a candidate for squeeze casting [22]. The process has been used for a wide range of metals, ranging from lowest melting point alloys of lead and zinc [17, 35] to very high melting point metals such as iron and nickel [19, 36]. Recently, effort in the automotive industry has been concentrated upon light metal alloys, aluminum and magnesium [12, 14, 37].

2.1.3.1 Squeeze Casting of Aluminum Alloys and Its Applications

For aluminum alloys, A356 is one of the most popular alloys used in squeeze casting, but it is by no means the only applicable alloy. Squeeze casting of both commercial aluminum casting alloys and aluminum wrought alloys have been reported either in production or research [12, 16, 17, 19, 21, 28, 32, 34, 35, 38-56]. Among those aluminum casting alloys, squeeze casting of Al-Si alloys have been widely studied [12, 17, 19, 21, 28, 32, 38, 39, 43-45, 47, 52, 55-64]. Of aluminum wrought alloys, 7000 series have obtained more attention [12, 17, 21, 38, 40, 47, 48, 54, 62].

The range of part configurations, which can be made by squeeze casting, includes solid, hollow and ring configurations, and also bearing, flanges, wheel and gears [37].

The products currently produced by squeeze casting worldwide include cross members, control arms, steering knuckles, pistons, engine blocks, scroll compressors and
wheels [7, 13, 21, 38, 45, 65-67]. Component weights in aluminum have ranged from 0.5 to 45kg [12]. Figure 2.8 shows some of the components made by squeeze casting.

Figure 2.8 Applications of squeeze casting: (a) engine block [67] and (b) steering knuckles [22].
2 BACKGROUND REVIEW

2.1.3.2 Squeeze Casting of Magnesium Alloys

For squeeze casting of magnesium alloys, although there is currently no commercial application reported, lots of research have been done upon different alloys [43, 54, 65, 68-76]. Among them, AZ31, AZ91 and AM50 are the alloys which researchers have paid more attention [26, 43, 54, 65, 75]. Squeeze casting of magnesium alloys with rare earth [68, 70, 77], calcium, zirconium [69, 72] and lithium [71, 73] have also been studied.

It has been reported in reference 18 in Russia that squeeze-expulsion casting was employed to manufacture magnesium panels, 80 inch long and 35 inch wide with a wall thickness averaging 0.010 inch, the application of squeeze castings of magnesium alloys in the automotive industry is rarely reported in the western world.

2.1.4 Recent Development in Squeeze Casting

Although there is only limited variety of squeeze casting processes, researchers never stop developing new techniques for squeeze casting improvement.

2.1.4.1 Metal Compression Forming

Metal compression forming (MCF) [78], is a type of direct squeeze casting with a unique feature of plunger, which prevents the back flow of metal when casting is squeezed by the intensifier. High productivity and near-net-shape processing make MCF an economical production method. There are two different feeding methods: gravity pour and low pressure feed. The prototype product made by this method is an A356 motor mount bracket.
2 BACKGROUND REVIEW

2.1.4.2 Two-step Squeeze Casting

This modified direct squeeze casting process [33] operates in two steps: (1) imposing very low pressure to melt during infiltration, (2) upon the completion of infiltration, high pressures are applied during solidification. The advantages of this technique include the minimization of preform deformation and the prevention of possible entrapping of gas and oxidation.

2.1.4.3 Partial Squeeze and Vacuum Die Casting Machinery (SVD)

SVD [79] is a hybrid of squeeze casting and vacuum die casting. Such a combination of the vacuum effect before injection and partial squeezing after injection provides excellent defect-free die cast products. Detailed information on vacuum assistant squeeze casting can be found in reference 55 and 79.

2.1.4.4 Pressure Counter Pressure Casting (PCPC)

This technique [80] uses a similar mould positioning and sealed furnace like low pressure and vacuum low pressure processes. With the PCPC, the casting cavity is placed entirely inside a pressure chamber. Upon closure of the casting cavity, the casting chamber and furnace chamber are pressurized with equal pressures. The pressure in the casting chamber is slowly exhausted while the pressure in the casting furnace is increased. This allows metal to rise in the filling tubes and into the casting cavity at a controlled and tranquil rate, under a countering pressure. Multiple cavities and multiple fill tubes can be used. Solidification is controlled directionally via sequenced cooling, just as the pressure filling and intensification is monitored to give maximum feeding in shrinkage-prone regions of the casting.
The pressure is maintained upon the metal bath and casting until solidification is complete and then the pressure is released. The casting can be handled without damage when they are extracted from the machine. The metal yield is extremely high, often exceeding 95%.

2.1.5 Squeeze Casting Process Parameters

There are lists of process parameters which may affect the casting quality in any squeeze casting operation. Tremendous research works have been done on the effect of individual parameters.

2.1.5.1 Cleanliness of the Melt

Melt quality and quantity is always the most important process parameter in squeeze casting [13-15]. Cleanliness of the melt refers to removal of the dross or slag prior to pouring. If melt cleanliness was not performed properly, particles of slag plastered themselves to the die wall and became major surface defects in the final casting [81]. Non-metallic inclusions can have an adverse effect on the mechanical properties of castings [12].

2.1.5.2 Holding Time

The importance of holding time can be found from Storchheim’s [81] work on steel castings, where a problem of vertical cracking in casting was overcome by reducing the holding time of a pressure from 20s to 5s.

2.1.5.3 Die Temperature

As described in references 14, 25, 40, and 42, low die temperatures cause premature solidification, thermal fatigue and cold laps, while high die temperature can
result in surface and other defects. Therefore, a balance between the low and high die temperature need to be considered during process design to prevent any detrimental issues. Ghomashchi [14] suggested that the die temperature is usually held at between 200 °C and 300 °C for aluminum and magnesium alloys. For copper alloys, Sriram [25] reported that the die should be generally maintained at 300/350 °C and the punch at 200/250 °C.

2.1.5.4 Pouring Temperature

The previous researchers [35, 40, 42, 68-70, 77] studied the effect of pouring temperature on the casting quality.

For the squeeze casting of the aluminium alloy, the best temperature to use was either 660 or 690 °C. The former would give a better property at the top of the casting while the latter, at the bottom of the casting. Table 2.3 shows the effects of casting temperatures and the levels of applied pressures on the grain sizes of squeeze cast wrought aluminum alloy AA7010 [34]. However, Lee et al. [63] reported that B390 scroll compressor can only achieve sound quality at the melt temperature of 600 °C, whereas many defects were found in the wrap at higher melt temperatures. When the melt temperature was too low, castings exhibited poor mechanical properties.

For the squeeze casting of the magnesium alloys, the highest UTS value was obtained with pouring temperature of 750 °C combining die temperature of 250 °C for magnesium-zinc-rare earth alloy [69, 70].

For the squeeze casting of the zinc alloy, the best temperature was 460 °C [35].
Table 2.3 Squeeze casting conditions and grain size measurements [34, 82]

<table>
<thead>
<tr>
<th>Pouring Temperature</th>
<th>Casting pressure (MPa)</th>
<th>Grain size (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1053K</td>
<td>0.1</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>300</td>
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<td>50</td>
<td>320</td>
</tr>
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<td>903K</td>
<td>0.1</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>70</td>
</tr>
</tbody>
</table>

2.1.5.5 Time Delay Before Applying Pressure

It has been suggested [15, 40, 47, 52] that optimum results are obtained when the pressure is applied near zero fluidity temperature of molten metal. The zero fluidity temperature is defined as the temperature below which the metal loses its fluid flow properties. It is usually midway between the solidus and liquidus temperatures of an alloy [15].

2.1.5.6 Die Coating and Lubricant

The previous studies [14, 17, 24, 40, 41] showed that the most popular lubrication medium, i.e., the die coating, is usually graphite based for aluminum and magnesium alloys during squeeze casting. However, Ramnati et al [17] reported a die coating used for tin-lead alloys was molybdenum disulfide aerosol spray.

2.1.5.7 Cavity Filling Speed

It has been proposed [17, 57, 60, 83] that a cavity filling speed of 0.2 ~ 0.4 m/s for small components and 0.1 m/s for large components usually eliminate air bubbles and
cracks. Above that speed there is a problem of splashing which results in oxide formation and poor surface finish.

2.1.5.8 Pressure Level

The previous research work [14-17, 21, 25, 38, 39, 41, 42, 48, 50, 51, 57, 58, 61, 62, 65, 66, 68-70, 77, 79, 84, 85] concluded that the applied pressure is the most important parameter which distinguishes squeeze casting from other casting processes. The effect of the applied pressure includes changing solidification behaviour, suppressing gas in solution, and reducing dendritic arm space (DAS). It has been suggested that a minimum level of applied pressure should be maintained in order to eliminate casting defects such as porosity and segregation.

Rolland et al. [86] emphasized, in squeeze casting, a critical pressure had to be exceeded to obtain dense castings. Low pressures did not produce fully densified castings nor eliminate all internal porosity [81].

Skolianos [50], through squeeze casting of aluminum AA6061 alloy under different pressure levels, found that the ultimate tensile strength of the as-cast and heat-treated castings increased with increasing pressure levels.

Yue [34] suggests that a pressure of 50MPa is sufficient to produce pore-free aluminum castings of wrought alloy AA7010. Rajagopal [15] reported that a pressure of 70MPa (10 ksi) is generally sufficient to optimize mechanical properties for many alloys including A356 aluminum. Lynch [31] has found that a specific pressure of 35 to 70MPa is required to apply to metal poured at or below the normal casting temperature for the aluminum alloys. Ghomashchi [14] summarized that the applied pressure varies between 50 and 150 MPa for aluminum and magnesium alloys.
However, the above mentioned applied pressure or specific pressure in practice is just hydraulic pressures out of casting machines rather than local pressures in die cavities. In the practical operation of squeeze casting, the cavity pressure is even eliminated as a quality control variable by simply using large machine and providing more than the required press load [15]. The main reason is that the real characteristics of the cavity pressure in squeeze casting had yet to be fully understood.

Due to the extreme hostile environment of squeeze casting cavity (thermal, mechanical and chemical conditions in the die), there are few reliable pressure sensors which can withstand such conditions. Hence, measuring local cavity pressures directly inside the die becomes an extremely challenging task. It is not surprising that, despite the presence of numerous publications on squeeze casting, very few of them are related to the characterization of cavity pressures.

Matsubara et al [52] was probably one of the earliest researchers studied the cavity pressure characteristics in squeeze casting. They measured the difference between the load applied and the load reached at the bottom of cavity in squeeze casting of pure aluminum, aluminum alloys and copper alloys, and estimated the friction force at the casting/die interface.

The transfer ability of molten metal pressure in the squeeze casting of Al-Si alloys was studied by Yasushi et al [59]. They found that the occurrence of shrinkage cavity depends on the duration of the transferred pressure, but not so much as the value of the transferred pressure. Krishna and Pehlke [87] attempted using pressure transducer to directly measure cavity pressure in squeeze casting of aluminum, but with only one measured location at the gate.
With the advent of novel advanced pressure transducers recently, several researchers [88-93] have successfully carried out experimental measurements of cavity pressures in die casting. Probably due to the fact that mechanical and thermal conditions in a squeeze casting cavity are more severe than those in die casting, information on characterization of local cavity pressures in squeeze casting of light metals, in particular magnesium alloys, is very limited in the open literature. To develop a better understanding of the process, local cavity pressures in squeeze casting must be characterized.

2.1.6 Solidification Behavior of Squeeze Casting

In squeeze casting process, an applied high pressure during solidification of melts eliminates air gap at the casting-die interface. The air gap elimination enhances heat transfer from the casting to the die, and consequently increases solidification rates [20, 43, 69, 94-96]. Since solidification is a process of crystal nucleation and growth, which is dictated by a heat extraction rate, the development of microstructure and mechanical properties of castings is in turn influenced by their solidification behavior [96].

The microstructural phase constitution and solidification behavior of magnesium alloys are often predictable with the help of equilibrium phase diagrams. However, in practice, there exists difference between the actual solidified microstructure and that predicted from phase diagrams. This is because solidification taking place in most casting processes is under non-equilibrium conditions due to the application of various solidification rates and/or pressures. The influence of solidification rates on liquidus temperatures of magnesium alloys was demonstrated in the past [16, 97].
BACKGROUND REVIEW

Squeeze casting processes involve the application of external pressures, which causes the melting point of most alloys to increase according to the Clausius-Clapeyron equation [13-15].

\[ \frac{dT}{dP} = \frac{T_m \Delta V}{H_f} \]  

(2-1)

where \( P \) is the applied pressure, \( T_m \) is the equilibrium solidification temperature, \( \Delta V \) is the volume change during solidification, and \( H_f \) is the latent heat of fusion. During solidification, normally both \( \Delta V \) and \( H_f \) are negative due to the shrinkage of metals and heat release, respectively. Thus, \( dT/dP \) is positive, which indicates that an increase in applied pressure leads to a rise in solidification temperatures. Using Equation (2-1), an increase in liquidus and solidus temperatures was determined to be about 0.05-0.07 °C /MPa for pure magnesium [14, 34, 98]. Franklin [13] suggested that the majority of metals show a rise of 2 - 6 °C for every 100 MPa of external pressures applied. For aluminum alloys, Yue [34] observed an upward movement of the liquidus temperature of 18 °C for an Al-7010 alloy squeeze cast at 300MPa. For magnesium alloys, previous studies [14, 43, 98] showed that an applied pressure of 115MPa increased the melting temperature for AZ91 and AZ31 by 7.58 °C and 8.70 °C, respectively. Moreover, it has been reported [7, 50, 58] that the high applied pressure enhances heat transfer and consequently increases solidification rates during squeeze castings of aluminum alloys. However, experimental work on the subject with a systematic approach for magnesium alloys is very limited in the public domain.
2.1.7 Casting/Die Interfacial Heat Transfer Coefficient in Squeeze Casting

To make numerical prediction sufficiently accurate, the dynamic boundary conditions of heat transfer at the interface between casting and die must be established.

Sekhar [99] in his brief article described the sensitivity of the interface heat transfer coefficient (HTC) to pressure, where he mentioned when the pressure is varied in die and squeeze casting operations for aluminum alloys, from ~0.1 to ~100 MPa, the rate of the increase in the heat-transfer coefficient \( (h) \) with pressure \( (p) \) occurs at approximately a rate of \( \frac{dh}{dp} \) of \( 10^{-3} \) to \( 10^{-4} \) m s\(^{-1}\) K\(^{-1}\). The rate nonlinearly decreases with increasing pressure, and has the simple units of velocity divided by the temperature. However, what his discussions did not address is the variation of the interfacial heat transfer coefficient in the casting geometry during a particular process cycle. Since interfacial heat transfer coefficient values may change spatially or with time.

Cho and Hong [100] evaluated the heat transfer coefficient at the casting/die interface in squeeze casting of Al-4.5wt%Cu alloy by using an inverse method. It was found that the interfacial HTC value at the bottom wall is smaller than that at the side wall of the casting, and explained it as the friction force at the side wall of the casting causes an air gap to form between the bottom of the casting and the die bottom wall surface. With an applied pressure of 50 MPa, the interfacial HTC value at the side wall of the casting is approximately 4700 W/m\(^2\)-K, while at the bottom wall of the casting, the value was estimated to vary in the range 1500 ~ 2200 W/m\(^2\)-K.

Sekhar et al [101] carried out a combined theoretical and experimental work to study the effect of applied pressure and die coatings on the heat transfer coefficient at the metal-die interface during solidification of Al-Si eutectic alloy against a H-13 die. It was...
found that the application of a pressure of 196 MPa increases the heat transfer coefficient from about $3.4 \times 10^3$ to $5.25 \times 10^4$ W/m$^2$-K.

Nishida and Matsubara [102] investigated the effect of pressure on heat transfer at the metal mould-casting interface by pouring pure aluminum, obtained the relationship between thermal resistance at the mould-casting interface and time under gravity pouring and applied pressure. Their experiments show that interfacial heat transfer is improved considerably when the metal-mould contact pressure is increased by pressurizing the casting during the early stage of solidification. Also the heat transfer coefficient increases with increasing pressure.

Jain [103] measured the contact HTC between a tool steel die and aluminum alloys (2024-T4, 2024-O, 6061-O, and 1100-O). In his study, MoS$_2$ was used as a lubricant. He concluded: (a) for the lubricated condition, the HTC increases with pressure but tends to level off around the yield strength of the work piece; (b) under zero load, the HTC is one order of magnitude lower than those at higher pressures; (c) as yield strength of the specimen increases, the heat transfer coefficient decreases for a specific load; and (d) up to a certain pressure (less than the yield strengths of alloys), the HTC under dry condition is lower than that under lubricated conditions.

Zhang and Cantor [104], based on their previous work, proposed a linear function which correlated the HTC and applied pressures for squeeze casting for aluminum alloy.

Deng et al [105] through their experiments and measurements for both billets of gallium and tin found that the HTC of the metals to the die are related with contacting pressure as a parabolic and a linear function, respectively.
Drenchev and Sobczak [106] developed an algorithm to estimate the heat transfer coefficient between the casting and the metal die, and applied it to a process of casting aluminum cylinders. The heat transfer coefficients under both air pressure and 220 MPa were obtained and compared as a function of time or the casting surface temperature. The magnitude of HTC under a pressure of 220 MPa is about 800 W/m²·K.

Krishina et al [87] and Pehlke [107] studied the heat transfer of aluminum alloys in squeeze casting within a pressure range of 29-116 MPa. The maximum HTC estimated was about 4500 W/m²·K, and when the applied pressure is above a threshold value no significant improvement on HTC occurs.

### 2.1.8 Porosity in Squeeze Casting

Generally, there are two factors that may cause the formation of porosity in castings: solidification shrinkage, or dissolved gas, or a combination of the two [108, 109], which are often present in aluminum castings. But for magnesium, some researchers concluded that shrinkage was the main reason for the formation of porosity, because the difference in solubility of hydrogen between the solid and liquid phase is relatively small compared to aluminum alloys. Others, through investigation argued that the dissolved hydrogen did contribute to the porosity formation [110] in magnesium alloys. However, this argument is beyond the scope of this study. Thus, this study will focus on the shrinkage porosity.

M. Flemings [109] defined two kinds of shrinkage porosity: i) surface porosity and ii) porosity within a casting. It was suggested that, in castings, porosity is more likely to be formed at the heavier sections where feeding is the poorest.
In squeeze casting, like other conventional castings, shrinkage porosity can be divided into two types: macroscopic and microscopic shrinkage porosities.

The mechanism of formation of macroscopic shrinkage porosity mentioned in [66] is that a “hot spot” formed in liquid area cannot be fed by pressurized metal movements. The last part of the casting to solidify is usually the area most susceptible to the incidence of porosity [13].

Different from macroporosity, microscopic pores can exist throughout the casting [66]. Microporosity can arise from liquid-to-solid shrinkage, evolution of dissolved gases as the melt solidifies, or by combined action of the two.

Pressurized solidification is probably the most effective action, which distinguish squeeze casting from all others. To reduce or eliminate gases related porosity, pressures are applied to the liquid metal prior to and during solidification which increase the solubility of gases in the melt. Thus, a large concentration of gases can be held in solution, and these gases are not evolved during solidification due to the difficulties in nucleation of bubbles against the pressure [13, 41].

For solidification shrinkage related porosity, the role of “squeeze” pressure in squeeze casting is to force-feed metal [66]. Porosity elimination is achieved by “burst-feeding” liquid or semi-liquid metal through a network of solid skeletons. When there is a completely solid region interrupting this flow of metal which usually happens in the vicinity of the last region to solidify, there has to be some amount of plastic deformation of the solidified crust in order for the applied pressure to force liquid going through the crust to the solidifying region [13, 15, 41].
To attain a pressure level in the last solidifying part of the casting to make a soundness casting, a minimum specific pressure (also called as nominal pressure or critical hydraulic pressure) is required, usually it refers to the press force divided by the plan area of the casting.

The specific pressure suggested by many researchers varied in a wide range. Franklin [13] suggested that applied pressure within the range 30 to 108 MPa is the minimum required to eliminate shrinkage and gas porosity for the majority of ferrous and non-ferrous materials. Yue [34] found that, for wrought alloy AA7010 and cylindrical casting (diameter 0.10 m, maximum height 0.19 m), a pressure of 50 MPa is sufficient to produce pore-free castings. Rajagopal [15] reviewed that a pressure of 70 MPa (10 ksi) is generally sufficient to optimize mechanical properties for many alloys including A356 aluminum. Ha [20], in his experiments, found that the critical pressure for magnesium alloys AZ91D and AZ31 are 100 MPa and 50 MPa respectively, and summarized that, for aluminum alloys, the critical pressure required to produce pore free castings increases with a decrease in the freezing range of the alloy. Hu [111] reported pore free squeeze cast AZ91D with a pressure of 87 MPa. Yong [69] through investigation concluded that the optimum applied pressure range is from 50 to 100 MPa for magnesium alloys RZ5 and RZ5DF. Plyatskii [11] listed specific pressure required for different geometries of casting.

To determine the pressure level required for a specific casting, the following factors may have to be taken into account: (a) alloy strength near its freezing temperature, (b) the growth morphology of the alloy crystallites, and (c) the freezing range of the alloy [15]. Obviously, the differences in alloy characteristics and component geometry make
the prediction of actual pressure levels difficult [15]. Wang through his research derived an equation for critical pressure [112]. Xing [76] proposed a rule for shrinkage prediction.

However, Y. Iwata et al [113] from their study in squeeze casting of Al-Si alloys found that the formation of shrinkage mainly depended on the duration of the pressure transfer, not so much on the magnitude of the transferred pressure. But the duration varied with the Si content of the alloys.

Pressure application (holding) time is decided by alloy type, casting configuration and heat transfer conditions. It is only necessary to apply pressure until solidification is complete, although slight longer time may be used to avoid hot tearing. Prolonged holding times provide little benefit, and may cause wall cracking or problems with punch redrawal, due to thermal contraction of the casting on to the rigid punch [13]. Youn [114] found that the hold time after completely filling affects the mechanical properties of casting.

To achieve the full advantages of squeeze casting in terms of both economic and technical aspects, the process has to be run with optimized parameters. Mathematical modeling as a great tool provides many benefits for process simulation and optimization, by which some primitive and time-consuming procedures in finding the appropriate set of parameters for producing sound castings, could be avoided [9].

### 2.2 Simulation of Casting Processes

Numerical Simulation of casting processes started from the mid 60s, the first publication about this topic was presented by Henzel and Keeverian [115], in which a
study of solidification of castings was conducted by numerical analysis. Since then, the simulation of casting has been an area of intense research activities [116].

The above Henzel and Keverian's paper 115 described then only the application of the “Transient Heat Transfer” program solving heat transfer equations for castings. But nowadays, simulation of casting is able to do not only thermal analysis, but also coupled fluid flow and stress analysis, so that it is capable of predicting the die filling, shrinkage of cavity, macro/micro porosity and micro-segregation, estimating mechanical properties, analyzing deformation and stress distribution, and also hot- and cold-cracking.

Foundry is an industry with a very long history. The foundry industry itself has been developing with time from an art to science. In the current environment, the intensive competition is the challenge that the industry has to face. The key is that it should have the capability to not only make high quality casting components, but also at the same time reduce production cost and shorten development times. It has been proven that, whether or not it is the design of a new component or redesign of existing products, computer aided simulation techniques provide a significant contribution to product quality improvement, lead time and cost reduction [117]. Computer simulation has also been proven to be an effective educational tool in the industry.

As the foundry industry discovered benefits and advantages associated with such techniques, and thank to the popularity of computer, the numerical simulation of casting has been continuously and immensely proliferated from the 90s. Simulation models developed and under developing by various institutes covers almost all conventional casting processes: die casting [118-125], sand casting [126, 127], semi-solid [128], stir casting [129], continuous casting [130-134], permanent mold casting [135-136], lost
foam [137-139], investment casting [140-142], single/twin-roll casting [143-146], wire casting [147] and squeeze casting [104, 148-153]. Besides those models, Table 2.4 lists most commercial software presently available for casting simulation.

The goal of simulations is to accurately model all of the underlying physics of the process so that important process variables may be identified and effectively controlled. For computer-aided modeling to be successfully implemented into the design stage, it should perform a wide variety of tasks [117]. Process simulation must have the capability to accurately model the properties of a wide range of casting alloys. Also process conditions like initial melt temperatures, slow shot to fast shot transitions, piston velocities and preheat of the die must be described just as the operators would on the shop floor. Based on their functionality and capability, the numerical models for simulation of casting presently available in the literature can be classified into three categories:

a. Basic model. This model mainly focuses on strategies of how to solve the governing equations including the thermal analysis and fluid flow of the casting;

b. Coupled model. This model not only solves the governing equations, but also considers the interaction between the casting and the die during the solidification process, and couples it into the whole simulation; and

c. Advanced Model. The last one, based on the previous two models is aiming at widening traditional simulation scope, such as porosity and mechanical properties prediction, etc.
Table 2.4 List of commercial software for casting simulation

<table>
<thead>
<tr>
<th>Software name</th>
<th>Developer</th>
<th>Website</th>
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</thead>
<tbody>
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</table>
2.2.1 Basic Model

Since most casting processes involves metal flow in cavity filling and change of thermal profiles during solidification, modeling not only fluid flow but also energy balance is essential for numerical simulation of the casting processes. Without predicting of flow patterns and temperature distribution, any further simulation of casting processes becomes impossible. Therefore, models capable of predicting metal flow and temperature profiles are referred to as a basic model. Over the last few decades, modeling in this area has been well developed with countless publications. The basis behind modeling in this field is the solution of the energy equation, in its most general form is:

\[
\frac{\partial}{\partial t}(\rho h) + \text{div}(\rho uh) = \text{div}(k\text{grad}(T))
\]  

(2-2)

where \( \rho \) is density, \( h \) is the enthalpy, \( u \) is the velocity, \( k \) is the thermal conductivity.

Since the solution of the energy equation in casting involves phase change during solidification, latent heat release must be counted in the modeling. A variety of techniques have been developed to incorporate the release of latent heat. The strategies include: the apparent heat capacity method, the effective heat capacity method, heat integration method, source-based method, enthalpy methods, the Kirchoff transformation and the heat Integration method. Many of these techniques are discussed in earlier works [154-160]. In Voss and Tsai’s work [161], four commonly used mathematical models of latent heat release (included the linear, quadratic, the lever rule and Scheil’s equation) were employed to study the effect of the rate of latent heat release on alloy casting solidification (temperature distribution, solidification patterns). The maximum temperature discrepancies were found to occur in the mushy zone for aluminum alloy Al-
Cu 4.5%. For any alloy with a wide mushy zone, throughout the solidification process, large differences were noticed between the fluid flow and solidification patterns generated by the four modes of latent heat release.

As mentioned earlier, casting processes always start with filling the molten metal into cavity. Fluid flow effects are important, thus models have been developed to calculate velocity distribution in the liquid portions of molten metal caused by forced convection during the early stages of casting [159, 160, 162, 163] and density difference induced natural convection during solidification [164]. For these cases the momentum equations (n equations for n dimensional flows) and the continuity equation must be solved. The momentum equations are of the form:

\[
\frac{\partial}{\partial t} (\rho u) + \text{div}(\rho u u) = \text{div}(\rho u \nabla u) - \frac{\partial P}{\partial x} \tag{2-3}
\]

\[
\frac{\partial}{\partial t} (\rho v) + \text{div}(\rho u v) = \text{div}(\rho u \nabla v) - \frac{\partial P}{\partial y} \tag{2-4}
\]

\[
\frac{\partial}{\partial t} (\rho w) + \text{div}(\rho u w) = \text{div}(\rho u \nabla w) - \frac{\partial P}{\partial z} - \rho g \tag{2-5}
\]

where \(\rho\) is the density, \(u, v\) and \(w\) are velocities in 3 directions, \(P\) is the pressure, \(g\) is the gravity force.

The continuity equation is:

\[\text{div}(\rho u) = 0 \tag{2-6}\]

The continuity equation and momentum equations are solved in conjunction. Certain assumptions must often be made in solving these equations such as the Boussinesq Approximation (for temperature varying density) [135], and a viscosity
approximation near the solid-liquid interface for pure metals to allow for a gradual change in material viscosity from solid to liquid. For mushy zone in alloys, Darcy’s law was employed [135, 163].

For the filling stage, $K$-$\varepsilon$ model was very popular for turbulent flow [135, 153], and filling free surface tracking [165]. Thorpe et al. [118] developed a technique called SPH (smoothed particle hydrodynamics), a Lagrangian method, for modeling heat and mass flows. Lu and Lee [125] reported a simplified approach for the simulation of metal flow.

Models with fluid flow have appeared in the work of numerous authors [156-158].

### 2.2.2 Coupled Model

The basic model involves the computation of the fluid flow and the temperature distribution only. Besides that, calculations usually just use constant thermal physical properties and assume a fixed boundary condition. However, during the casting process, as the molten metal loses heat and solidifies, the deformation of the casting may not only cause the residual stress, but also change the boundary conditions such as interfacial heat transfer coefficient. In addition to this, the thermophysical properties of the metal also change with temperature. Therefore it is necessary to take into consideration of all these.

In a coupled model, the flow/thermal analysis is carried out first to obtain the temperature change. Then stress/deformation analysis is conducted to attain the new boundary conditions. Hence, the computation is based on the real dynamic boundary condition and metal thermophysical properties.
2 BACKGROUND REVIEW

The sensitivity of thermophysical properties on the simulations was discussed in references 166, 167, and 168. Neves et al [166] indicated that, for sand casting, altered thermophysical properties changed both the solidification time, and the microstructure. The integration of pressure or temperature-dependent materials properties into numerical simulation has been demonstrated in the previous studies [118, 149, 151, 158, 168]. Study of Tadayon et al [149] and Gethin et al [151], considered the effect of pressure on the alloy solidification characteristics. It was shown that increasing the solidification temperature and distorting the alloy phase relationship, the alloy thermophysical properties of enthalpy and heat conductivity were changed accordingly. Cherukuri and Johnson [168] included temperature dependence in the constitutive equations relating strain rate to stress and temperature-dependent thermal conductivity in their simulation modeling.

In casting processes, with decreasing temperature, the alloy changes from initial liquid state, to mushy state, and finally reaches solid state. In liquid state, a metal can be typically treated as a Newtonian viscous fluid, with no accumulating stresses and no fixed deformations. A liquid metal can transmit hydrostatic pressure. Metal in solid state often exhibit elastic and/or plastic behaviors, which depends on the magnitude of applied stress (higher or below yield stress). A metal in mushy state often exhibits a combination of visco-elastic/plastic behavior, which is difficult to characterize. This is because their mechanical behavior is dependent not only on temperature, strain, and strain rate, but also on their processing history, or more specifically, the current morphology and degree of agglomeration of the solid particles in the mushy region. Mushy metals can be described as shear thinning and static thickening materials.
Due to this reason, the models proposed for describing the thermo-mechanical behavior of castings can be categorized into five classes:

a. Comprehensive elastic thermo-mechanical models,
b. Simplified elastic thermo-mechanical models,
c. Elastic/plastic thermo-mechanical models,
d. Comprehensive visco-elastic/plastic thermo-mechanical models, and
e. Simplified visco-elastic/plastic thermo-mechanical models.

With those solution models, numerous simulations have been done on prediction of air gap [130, 153, 169], distortion [121], stress [120, 121, 165, 170-172]. But few of them are really coupled thermo-mechanical models, which are capable of finding out the change of boundary conditions, and applying new boundary conditions into the computation.

In the study by Samonds and Zhu [173], a viscoplasticity material model was used, and a multi-body mechanical contact algorithm was employed to compute the contact and gap formation between the casting and die. The interfacial heat transfer coefficient was adjusted by the contact. If the contact pressure was non-zero, the effective heat transfer was increased linearly with that pressure up to a maximum value. The adjusted heat transfer coefficient had the form,

\[ h_{\text{eff}} = \frac{1}{\frac{1}{h_0} + \frac{1}{(h_{\text{air}} + h_{\text{rad}})}} \tag{2-7} \]

where \( h_0 \) is the initial value of the heat transfer coefficient

\[ h_{\text{air}} = \frac{k_{\text{air}}}{g} \tag{2-8} \]
where \( k_{\text{air}} \) is the conductivity of air or 0 for vacuum, \( g \) is the gravity, and \( h_{\text{rad}} \) is the radiation heat transfer coefficient.

Chiumenti et al. [174] derived a mechanical model for the cast part and the die material from a thermo-elasto-viscoplastic free energy potential which can determine heat transfer in radiation, conduction and convection separately based on the mechanical quantities such as the contact pressure or air-gap induced by actual deformation.

Lewis [116, 175] applied a thermo-elasto-visco-plastic analysis for determining air gap and proposed a modified equation for interfacial heat transfer

\[
h_i = \frac{k_i}{\delta_i + k_i / h_0}
\]

(2.9)

where \( h_0 \) is the initial value of the heat transfer coefficient, \( \delta_i \) is the air gap, and \( k_i \) is the conductivity of the air/gas which occupies the gap.

Further, Lewis and Ransing [176] proposed a correlation based on an empirical equation which can capture any realistic interfacial heat transfer coefficient variation with respect to casting interface temperature.

Trovant and Argyropoulos [177, 178] developed an experimental equation to correlate the interfacial heat transfer coefficient and air gap,

\[
HTC = \frac{1}{k \cdot A + r} + C
\]

(2-10)

where \( k \) is the conductivity of air/gas, \( A \) is the air gap, and \( r \) and \( C \) are constant.

Fackeldey et al. [179] reported that, in their coupled modeling of the solidification process, temperatures, stresses and microstructures were predicted. Their coupled model
employed a simplified relation, in which heat transfer depends only on the contact pressure between the casting and die considering a mean hardness of casting and die.

\[ H_{\text{eff}} = \gamma_1 (\frac{P_{\text{int}}}{H_e})^{\gamma_2} \]  \hspace{1cm} (2-11)

where \( H_{\text{eff}} \) is the heat transfer coefficient, \( \gamma_1 \) and \( \gamma_2 \) are material parameters, \( H_e \) is the Vickers Hardness and \( P_{\text{int}} \) is the contact pressure at the interface.

Menai and Bellet [172] developed a thermo-elasto-visco-plastic model (FEM), in which the thermal and mechanical resolutions are coupled at each increment of computational time. The local heat transfer coefficient is computed as a function of the current local air gap that has been updated by the mechanical computation of the previous time increment.

\section*{2.2.3 Advanced Model}

An advanced model in fact is a post processor of simulation programs. Based on computed results obtained by the Basic-plus-Coupled model, the model uses certain specific criterion function to predict casting defects such as porosity, shrinkage, segregation, hot cracking, and estimate the mechanical properties and microstructure.

To ensure the accuracy of prediction by advanced models, two critical issues must be taken into consideration during modeling. One is the establishment of a right Basic-plus-Coupled model. Through it, a reliable thermal and mechanical field can be gained. The other is to develop correct criterion functions, capable of judging and determining the existence of defects.
2.2.3.1 Shrinkage Porosity

In Nakagawa and Takebayashi’s simulation of solidification of magnesium alloy castings [180], the following two methods were examined for prediction of shrinkage porosity:

a. Iso-$f_s$ time contour. In this method, the fraction of solid ($f_s$) of specified value is computed at each control node and Iso-contours are plotted. Then the shrinkage porosity is assumed to occur in the region where the closed loop of Iso-contour forms. This method is based on the fact that, if the mushy zone is isolated from the gate or the riser, molten metal cannot be fed to this zone.

b. Gradient of $f_s$. In this method, a distribution of the positional gradient of $f_s$ (termed as $G_f$) is plotted when $f_s$ reaches a specified value. Then the shrinkage porosity is assumed to occur in the region where $G_f$ is less than a certain critical value. This method is based on the consideration that when $G_f$ is small the mushy zone becomes large (thus the pressure drop is large) so that the feeding of the molten metal hardly occurs. Region shrinkage porosity usually occurs in the finally solidified regions or in isolated areas of molten metal.

Nakagawa and Takebayashi [175] then concluded that shrinkage porosity can be predicted by using the methods. But the value of $f_s$ used to judge the shrinkage porosity should be carefully chosen.

Trovant and Argyropoulos [181] account for temperature-varying thermophysical properties and determine the shrinkage profile resulting from phase and density change.

Early work by Niyama et al. [182] resulted in the widely known criterion for predicting fine shrinkage (feeding) porosity as below:
where $G$ is the temperature gradient, $R$ is the cooling rate [165, 183], and $C_{crit}$ is a critical value. Figure 2.9 shows how to calculate temperature gradient $G$ and cooling rate $R$, in which 1, 2 and 3 represent different locations, $T$ is temperature, and $T'$ is the temperature after a certain time $\Delta t$.

The critical value ($C_{crit}$) of this function corresponded to a critical value of pressure drop in the feeding channel for shrinkage porosity formation.

The Niyama criterion and its modifications implicitly use the interdendritic flow concept, i.e. Darcy’s law. Therefore they can not predict the amount, size and distribution of shrinkage porosity or hydrogen effects on porosity in aluminum castings [124].

Kubo and Pehlke [184] successfully developed a model reflecting Campell’s [108] porosity formation theory in solidification, which considers that porosity forms.
once the liquid metal pressure drops below the gas pressure. This can be expressed in another criterion:

\[ P_{g} > P_{c} \text{ (critical pressure)} \]  \hspace{1cm} (2-13)

where \( P_{g} \) is the gas pressure, \( P_{c} \) is the critical pressure consisting of the local liquid metal pressure and surface tension.

Li et al [152] reported a modified model which combines the Niyama criterion and critical pressure criteria together for squeeze casting of aluminum alloys.

However, Kim and Kubo in their publication [124] argued that the above models failed to include the entrapped air that might be the initial site of porosity formation. In their model (Kubo model), they use the results of trapped air analysis as the initial sites of porosity. Not only the location and the severity of shrinkage porosity but also amount, size and distribution of shrinkage porosity can be predicted.

Another type model for porosity is called continuum-stochastic model, which can predict the distribution of porosity and maximum pore size, and unfortunately has not incorporated shrinkage [185].

2.2.3.2 Hot Tear/Hot Cracking

Pellini's [186] strain theory of hot tear formation is now widely accepted for predicting hot tearing. The key features of this theory include:

a. Hot tear develops in regions of hot spots;

b. Tearing is a strain controlled phenomenon. For most alloys, it coincides with the liquid fraction range between approximately 0.2 and 0.01; and

c. The strain accumulated within the hot spot, which, in turn, is responsible for hot tears.
Based on Pellini's theory, an approach for prediction of hot tearing was developed by Chandra [187]. It contains two key steps: (1) prediction of grain size and thickness of liquid film around the solid grains at various stages of solidification, and (2) development of a strain based hot tear or fracture criterion.

Investigations on hot tearing have been reported in many previous studies [118, 135, 153, 156-162, 165] and it is clear that the internal crack is generated when stresses prevent the contraction of an alloy at its temperature near the solidus.

Cracks are generated when the alloy has a wide solidification temperature range and a small amount of eutectic. Recently, many methods for predicting hot tearing have been proposed using computer simulation.

Clyne and Davie [188], proposed that hot tearing is due to an opening of the mushy zone in a "vulnerable" region where the dendrite arms can be pulled apart easily. They introduced a hot crack criterion known as cracking susceptibility coefficient (CSC):

\[ \text{CSC} = \frac{t_v}{t_r} \]  

(2-14)

where \( t_v \) is the time spent in the mushy zone in the vulnerable region and \( t_r \) is a normalization time during which stresses in the mushy zone can be relaxed.

Another approach to hot cracking was proposed by Rappaz et al [164], which defines a hot-cracking sensitivity criterion (HSC):

\[ \text{HSC} = \dot{\varepsilon}_{p,\text{max}}^{-1} \]  

(2-15)

where \( \dot{\varepsilon}_{p,\text{max}}^{-1} \) is the maximum strain rate that could be sustained by the mushy zone before creating a fixed-cavity depression.
However, these hot cracking criteria are based on binary alloys and are not appropriate for the prediction of cracking in multicomponent alloys [189].

2.2.3.3 Macrosegregation

Several studies in the literature have examined macrosegregation profiles, in particular the formation of inverse segregation. Flemings [109] proposed a model which assumed the interdendritic liquid driven only by contraction on solidification, the solute diffusion was neglected and the temperature gradients and velocity distributions were measured or assumed. The derived "local solute redistribution equation" was shown to predict successfully the formation of macrosegregation, particularly inverse segregation [190].

In recent publications a number of models for describing the coupled fluid flow and heat and species transport have been proposed, mainly for the case when flow in a casting is caused by natural convection. The derivation of the set of governing equations for the mushy zone based on the mixture theory approach is presented in [156-159]. The derivation of the set of governing equations based on a volume-averaging procedure is presented in references 160-162. The predicted results of macrosegregation based on these models were reported by Bennon and Incropera [159, 160, 163]. They studied macrosegregations mainly associated with a relatively weak natural convection during solidification of a binary metal alloy. However, the study by Kuznestov [191] considered the occurrence of macrosegregation during strip casting of a multicomponent steel, in which strong force convection caused by the change of the height of melt free surface.
2.2.3.4 Microstructure (Grain Number, Interphase Spacing, Structural Transitions, Phase Ratio)

Xiong and Liu [165] developed a model, for S.G iron (ductile iron), which can predict:

a. the size and amount of primary austenite dendrites;

b. The size and number of spheroidal graphite; and

c. the spacing of pearlite.

A coupled model developed by Fackeldey et al [179] considered the main kinetic and thermodynamic aspects that influence microsegregation, such as solid state back diffusion, secondary dendrite arm coarsening, primary tip undercooling. Output data for each node of the enmeshed geometry are local primary and secondary dendrite arm spacing as well as the fraction solid as a function of the solidification path.

2.2.3.5 Mechanical properties

Presently, the prediction of mechanical properties of castings is mainly based on the simulation results and some empirical relationships between microstructure and mechanical properties. The predictable as-cast mechanical properties include hardness, tensile strengths and elongation. Attempts have been made to numerically predict mechanical properties of iron castings [165, 192]. However, it appears that research in this topic is still in infancy.

Simulation of casting has reached a sophistication which already makes it an important aid to foundry industry, and this will be strengthened with increasing the accuracy and speed of computation [193].

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2.2.4 Numerical Methods

Two techniques are available for solving the governing equation(s): finite difference methods including control volume approaches (FDM solutions), and finite element methods (FEM solutions). FEM solutions are generally advantageous in problems with irregular geometries [116, 119, 172, 194], while FDM solutions are conceptually simpler and more widely accepted in computational fluid dynamics modeling [52, 120, 135, 171]. Recently, FDM/FEM integrated for casting simulation has been reported. Detailed review of FEM and FDM can be found in references 155, 154, 165, and 195.

2.2.5 Modeling of Squeeze Casting (SC)

2.2.5.1 Status of SC Models

The available publications show that simulation of squeeze casting did not start until early 90s. Error! Reference source not found. summarized some published works.

Tadayon et al [149] developed a finite element model for squeeze casting process, in which the effect of pressure on thermophysical properties was incorporated. The model has been used for predicting thermal front during squeeze casting of an aluminum (AA 7000 series) vehicle road wheel. Gethin et al [151] continued this work by combining flow and heat transfer in squeeze casting process where the flow model was based on simple mass conservation. In the model, when the punch touched the metal surface (early stage), the heat transfer coefficient at the casting/die interface was assumed to be constant determined by the ferrostatic head at the lower surface of the pool. During the
pressurization stage, the heat transfer coefficient was assumed to linear with applied pressure.

Zhang and Cantor [104] developed a finite difference model to simulate the heat flow during squeeze casting aluminum alloy A356. An equation considering the heat transfer coefficients as a linear function of applied pressure was proposed and used in the model. Computed results showed that solidification and cooling rates during squeeze casting increased with increasing ingot/die heat transfer coefficients under high pressures.

Lee et al. [196] applied a two-dimensional finite element code (TOPAZ2D) for heat transfer analysis in indirectly squeeze cast 5083 wrought aluminum alloy. The effect of die geometry on microstructure was studied. A pressure-dependent heat-transfer coefficient and the equivalent heat capacity method were employed by the model. The cooling behavior during squeeze casting process was simulated and compared with measured results.

In the study by Maeng et al. [64], the effect of processing parameters on the microstructure and mechanical properties of modified B390 alloy in direct squeeze casting was investigated by using commercial finite volume method (FVM) code for heat transfer analysis, and MAGMAsoft for cooling curves. In this model, the heat transfer was considered constant for a specific applied pressure.

Hu and Yu [207] developed a 2-D finite difference model for heat transfer events in squeeze casting of magnesium alloy AZ91D. The model used the heat transfer coefficients as a linear function of applied pressure.
Lee et al [63] simulated melt flow by using the software MAGMAsoft for the direct squeeze casting an orbiting scroll compressor of aluminum alloy B390. In the model, only the melt flow during the pouring step and the solidification rate were calculated.

Youn et al. [57] conducted a die filling and solidification analysis of engine bracket mounting (aluminum alloy A356) in horizontal squeeze casting process by the commercial MAGAMsoft with add-on module high pressure die casting. In the model, constant heat transfer coefficients were employed for die/die and die/casting interfaces.

Li et al [152] proposed a method for numerical prediction of porosity defects in squeeze casting. In this advanced model, Darcy law was employed to simulate the squeeze effect. The cyclic steady heat balance method (CSM) was used for estimation of steady die temperature. A modified Niyama’s criterion was adopted to evaluate the shrinkage porosity.

Recently Postek et al [197] developed a coupled model to evaluate the effects of initial stresses in the pressurized casting process. The problem formulation was done in the Updated Lagrangian (UL) frame. An implicit time integration scheme is applied to solve the transient thermomechanical problem in a staggered form. The constitutive mechanical model is elasto-viscoplastic with hardening incorporated with a Von Mises yield function. In the model, the interfacial heat transfer coefficient was coupled with the contact between die and cast.
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Alloy /Part</th>
<th>Process</th>
<th>Model type</th>
<th>Numerical method</th>
<th>Features &amp; Problems</th>
<th>Publication year</th>
</tr>
</thead>
</table>
| Tadayon et al [122] | AA 7000 series /Vehicle wheel | Direct Squeeze Casting | Basic | FEM | Applied pressure: 55, 110 MPa  
Incorporated the effect of pressure on the thermophysical properties  
Calculate heat transfer only;  
No fluid flow included;  
Constant boundary conditions. | 1991 |
| Gethin et al [151] | AA 7000 series /Vehicle wheel | Direct Squeeze Casting | Basic | FEM | Applied pressure: 55 MPa  
Incorporated the effect of pressure on the thermophysical properties;  
Combined flow and heat transfer;  
Approximated flow calculation;  
Linear relationship between pressure and HTC. | 1992 |
| Zhang and Cantor [104] | A356/Coupon | Direct Squeeze Casting | Basic | FDM | Applied pressure: >50 MPa  
Calculate heat transfer only;  
Linear relationship between pressure and HTC. | 1995 |
| Lee et al [56] | 5083/Coupon | Indirect Squeeze Casting | Basic | FEM (TOPAZ2D) | Pressure range: 25-100 MPa  
Calculate heat transfer only;  
Constant boundary conditions. | 1999 |
| Maeng et al [64] | B390/Coupon | Direct Squeeze Casting | Basic | FVM (MAGMA) | Maximum Pressure 100 MPa  
Validate cooling rate results;  
Constant boundary conditions. | 2000 |

(To be continued)
Table 2.5 Modeling of Squeeze Casting Processes

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Alloy /Part</th>
<th>Process</th>
<th>Model type</th>
<th>Numerical method</th>
<th>Features &amp; Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lee et al [63]</td>
<td>B390 /Scroll compressor</td>
<td>Direct Squeeze Casting</td>
<td>Basic</td>
<td>FVM (MAGMA)</td>
<td>Pressure range: 10-50 MPa; Calculated heat transfer and fluid flow; Boundary condition was not clear.</td>
</tr>
<tr>
<td>Hu and Yu [217]</td>
<td>AZ91D /Coupon</td>
<td>Direct Squeeze Casting</td>
<td>Basic</td>
<td>FDM</td>
<td>Pressure range: 10-200 MPa; Calculated heat transfer only; Linear relationship between pressure and HTC.</td>
</tr>
<tr>
<td>Li et al [152]</td>
<td>Al /Suspension part</td>
<td>Indirect Squeeze Casting</td>
<td>Advanced</td>
<td>FEM (Solid3d)</td>
<td>Applied Pressure: 100 MPa; Modified Niyama's Criteria was proposed; Cyclic Steady Method was used for heat balance.</td>
</tr>
<tr>
<td>Youn et al [57]</td>
<td>A356 /Engine bracket mounting</td>
<td>Horizontal Squeeze casting process</td>
<td>Basic</td>
<td>FVM</td>
<td>Applied Pressure: 70-150 MPa; Included both filling and solidification; Constant boundary conditions.</td>
</tr>
<tr>
<td>Postek et al [197]</td>
<td>LM25 /Coupon</td>
<td>Direct Squeeze Casting</td>
<td>Coupled</td>
<td>FEM</td>
<td>Applied Pressure: 200 MPa; HTC coupled with casting and die contact; Non-linear effect on solidification</td>
</tr>
</tbody>
</table>

Publication year:
- 2000
- 2002
- 2003
- 2004
- 2005
2 BACKGROUND REVIEW

2.3 Summary

The above review in this chapter gives an overall perspective of squeeze casting process, and the development of castings simulation. Despite extensive utilization of computer simulation for various casting process, published work on development of advanced models incorporating dynamic boundary conditions for squeeze casting is limited.

Since the squeeze casting technology is relatively new for light alloys, aluminum and magnesium, more fundamental research is needed for a scientific understanding of the process. In particular, as a cost-effective and resource-efficient tool, advanced mathematical models coupled with dynamic boundary conditions and capabilities of predicting the formation of casting defects have to be fully developed for the optimization of squeeze casting processes. Therefore, the purpose of this study is to investigate the characteristics of cavity pressure in squeeze casting, and establish a generalized relationship between pressure and the casting/die interfacial heat transfer coefficient (HTC), then incorporate local pressure/HTC to build up an advanced numerical model to simulate transport phenomena occurring in squeeze casting of magnesium and aluminum alloys.
3 EXPERIMENTAL

In the first part of this chapter, a description of the experimental squeeze casting system and peripheral equipment is given. Then, the pressure transducer used to characterize local cavity pressure and the measurement procedures are described. Following the description of temperature and casting/die interfacial heat transfer coefficient measurements, casting density measurements are explained, and the data acquisition system developed for this study is introduced. After describing the alloys used, the casting procedures are presented.

3.1 Setup

The experimental squeeze casting system employed consists of a 75-ton laboratory hydraulic press, a die set, a metal melting unit and a data acquisition system. Figure 3.1 gives an overview of the system, which is described in detail in the succeeding sections.

Figure 3.2 schematically shows a portion of the integrated system, which includes the press and die assembly consisting of the die, sleeve and plunger.
Figure 3.1 Light metals casting Lab.
Figure 3.2 Schematic diagram showing the squeeze casting system. 1. Upper platen, 2. Die, 3. Die cavity, 4. Sleeve, 5. Sleeve cavity, 6. Plunger, 7. Moving platen, 8. Bottom platen. For the clarity of presentation, the embedded pressure transducers and thermocouples are not included.
3 EXPERIMENTAL

3.1.1 The Press

Figure 3.3 shows a 75-ton heavy duty hydraulic press made by Technical Machine Products (TMP, Cleveland, Ohio, USA) used in the experimental study. The specifications and features of this press are summarized in Table 3.1.

![TMP 75-Ton hydraulic press](image)

**Figure 3.3** TMP 75-Ton hydraulic press.
During operation, the bottom platen attached to the shot sleeve and main ram, moves upward when the press is activated. Once the die closes, the main ram takes over to push the melt into cavity and applied a pre-set pressure to the melt. The operation can be carried out either automatically or manually. The fully automatic operation was dictated by an embedded programmable logic controller. A touch screen menu in a custom-designed software serves as a graphic user interface (GUI) between users and the controller. Hence, the configurations and programming of an operation become easily achievable.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Heavy duty 4-post</td>
</tr>
<tr>
<td>Max Tonnage</td>
<td>75 Metric, adjustable</td>
</tr>
<tr>
<td>Daylight Opening</td>
<td>0.762m between moving bolster and press head</td>
</tr>
<tr>
<td>Bore</td>
<td>0.203m</td>
</tr>
<tr>
<td>Main cylinder travel (Stroke)</td>
<td>0.203m</td>
</tr>
<tr>
<td>Motor H.P.</td>
<td>L: 7.5</td>
</tr>
<tr>
<td>Power</td>
<td>600V 60Hz 3pH 27FLA</td>
</tr>
<tr>
<td>Watts per platen</td>
<td>14.4KWA</td>
</tr>
<tr>
<td>Max HYD PSI</td>
<td>20.68 MPa</td>
</tr>
<tr>
<td>Work area</td>
<td>0.61m x 0.61m</td>
</tr>
<tr>
<td>Dimension</td>
<td>2.21m x 2.21m x 1.12m</td>
</tr>
<tr>
<td>Control</td>
<td>Manual/Auto switch</td>
</tr>
<tr>
<td>Fast close</td>
<td>0.076 m/s</td>
</tr>
<tr>
<td>Slow close</td>
<td>0.001 m/s</td>
</tr>
<tr>
<td>Fast open</td>
<td>0.076 m/s</td>
</tr>
<tr>
<td>Press head cylinder speed</td>
<td>0.001 m/s</td>
</tr>
<tr>
<td>Press head cylinder capacity</td>
<td>4 tons, adjustable</td>
</tr>
<tr>
<td>Press head cylinder travel</td>
<td>0.101m</td>
</tr>
</tbody>
</table>
3.1.2 The Die

The die set (assembly) manufactured by the Technical Support Center (TSC, University of Windsor) consists of two parts: the top die with an inner diameter of 0.1016 m and a height of 0.0965 m; and the bottom sleeve with an inner diameter of 0.1016 m and a height of 0.1270 m (Figure 3.4).

Table 3.2 lists the materials used for the die and their service conditions. For the upper die and bottom sleeve, both the insert and the plunger head are also changeable. The die combination enables to make a cylindrical coupon with a diameter of 0.101 m and a height of 0.080 m (Figure 3.5).

Figure 3.4 Schematic of die assembly (a) solid model and (b) cross section.
Both the upper die and the bottom sleeve were heated by Acrolab (Acrolab Ltd. in Windsor, Ontario, Canada) ceramic band heaters (Figure 3.6), of which temperatures are separately controlled by Shinko (Shinko Electric Industries Co., Ltd. in Tokyo, Japan) temperature controllers. The size and power of the heaters are 0.2 m diameter × 0.2 m height, 6000 Watts, and 0.15 m diameter × 0.15 m height, 2000 Watts for upper die and bottom sleeve, respectively. To facilitate independent control of the two band heaters, the controller unit features two programmable LED display panels. Two control modes (automatic and manual) are available in the panels for selection during operation. Figure 3.6 shows the die assembly with the band heaters installed in the press.

Table 3.2 The die set specification

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Material</th>
<th>Hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Upper die</td>
<td>H13</td>
<td>48-50R</td>
</tr>
<tr>
<td>2</td>
<td>Upper die insert</td>
<td>P20</td>
<td>32-34R</td>
</tr>
<tr>
<td>3</td>
<td>Upper die insert support</td>
<td>Cold rolled steel</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Bottom sleeve</td>
<td>H13</td>
<td>48-50R</td>
</tr>
<tr>
<td>5</td>
<td>Bottom Plunger head</td>
<td>D2</td>
<td>54-56R</td>
</tr>
<tr>
<td>6</td>
<td>Bottom Plunger ring</td>
<td>Brass</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Plunger</td>
<td>4140</td>
<td>38R</td>
</tr>
</tbody>
</table>
All dimensions in meters
Figure 3.5 Configuration of castings.

Figure 3.6 Top die and bottom sleeve with the band heaters assembled in the press.
3.1.3 Alloy Melting Unit

3.1.3.1 Furnace

The metal melting unit (Figure 3.7) includes a Lindberg/Blue M CF56822C Crucible furnace (Lindberg/Blue M Thermo Electron Corporation, Asheville, North Carolina, USA), a Lindberg/Blue M Controller CC58114C, and crucibles. The furnace temperature can be pre-specified and automatically controlled. Table 3.3 lists the specifications of the resistance furnace employed in this study.

Table 3.3 Specification of Lindberg furnace and controller

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Furnace type</td>
<td>CF56822C</td>
</tr>
<tr>
<td>Maximum temperature</td>
<td>1200 °C</td>
</tr>
<tr>
<td>Electrical supply</td>
<td>208/240V, 50/60 Hz</td>
</tr>
<tr>
<td>Power</td>
<td>2400 Watts</td>
</tr>
<tr>
<td>Exterior dimensions (WxLxH)</td>
<td>20&quot; x 20&quot; x 18&quot;</td>
</tr>
<tr>
<td>Heated zone</td>
<td>Φ8&quot;x12&quot;</td>
</tr>
<tr>
<td>Controller type</td>
<td>Lindberg/Blue M Controller CC58114C</td>
</tr>
<tr>
<td>Electrical supply</td>
<td>208/240V, 50/60 Hz</td>
</tr>
<tr>
<td>Exterior dimensions (WxLxH)</td>
<td>14&quot; x 18&quot; x 10&quot;</td>
</tr>
</tbody>
</table>
3.1.3.2 Crucible

Aluminum alloys were melted in a graphite crucible (SALAMANDER SUPER A4, top diameter 0.114 m, height 0.139 m) with a maximum holding capacity of 2 kgs aluminum. The crucible was supplied by Morganite Crucible Inc. in UK (Figure 3.8a).

Magnesium alloys were melted in a mild steel crucible with a maximum holding capacity of 2 liters (diameter 0.105 m, height 0.240 m), which was made by the Technical Support Center, University of Windsor (Figure 3.8b).
Alloy melting was carried out in individual batches according to experiment design. The temperature of the melt was regularly measured by a digital thermometer Omega HH509 (Omega Engineering Inc., Stamford, Connecticut, USA) during melting. The molten metal was poured directly from the crucible into the sleeve to minimize the melts temperature drop.
3.1.4 Protective Gas

3.1.4.1 Protection for Molten Mg Alloy

To protect the melt from any excessive oxidation or possible burning, protection gas (Sulfur Hexafluoride SF₆ 0.5% + carbon dioxide CO₂) was used during both melting and casting.

The gas protection system used in this study includes a gas cylinder (size of 200, Pressure: 600PSG at 20 °C), a Matheson 81-CF-320 dual stage regulator, and a flow meter combination (Figure 3.9). The output gas pressure and flow rate of the system varies from 0 to 0.34 MPa and 0.5 to 5 slpm, respectively. Steel distribution tubes and quick connectors provided by TSC were used to transport gas from the gas cylinder.

Figure 3.9 81-CF-320 Dual Stage Regulator and Flow meter Combination.

3.1.4.2 Protection for Molten Al alloy

To minimize hydrogen absorption in melt, protection gas argon was used during both melting and casting of aluminum alloy. The gas protection system used for aluminum alloy includes a gas cylinder (size of 200, Pressure: 600PSG at 20 °C), and a Harris 25-100C-580 Single Stage Regulator (Figure 3.10).
Steel distribution tubes and quick connectors provided by TSC were needed to transport gas from the gas cylinder to the crucible.

3.2 **Pressure Measurements**

3.2.1 **Pressure Transducer**

Kistler pressure transducers 6175A2 (Kistler Instrument Corp., Amherst, New York, USA) were used in experiments to measure local cavity pressures. The sensor (type 6175A2, Figure 3.11) consisted of a high temperature quartz sensor built into a rugged adapter. The sensor has a front diameter of 8 mm, was flush with the front of the adapter, separated by a cylindrical gap of $<10\mu m$, and measures the pressure directly. The specification of the pressure transducers is summarized in Table 3.4.
The external force acts on a quartz sensor element, which yields a proportional electrical charge (pC = picocoulomb) that is converted by the Kistler charge amplifier 5039A 111 (Figure 3.12) into a proportional voltage of 0 - 10 V or current 4 - 20 mA. The length of the sensor cable has no influence on the signal transfer.
Table 3.4 Specification of pressure transducer 6175A2

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>bar</td>
<td>0...2000</td>
</tr>
<tr>
<td><strong>Overload</strong></td>
<td>bar</td>
<td>2500</td>
</tr>
<tr>
<td><strong>Uniform sensitivity (at 250°C)</strong></td>
<td>PC/bar</td>
<td>-6.7</td>
</tr>
<tr>
<td><strong>Linearity, all range</strong></td>
<td>% FSO</td>
<td>≤ ±2</td>
</tr>
<tr>
<td><strong>Natural frequency</strong></td>
<td>KHz</td>
<td>= 30</td>
</tr>
<tr>
<td><strong>Acceleration sensitivity</strong></td>
<td>Bar/g</td>
<td>&lt; 0.07</td>
</tr>
</tbody>
</table>

**Operating temperature range**

<table>
<thead>
<tr>
<th></th>
<th>°C</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Connector</strong></td>
<td></td>
<td>0...200</td>
</tr>
<tr>
<td><strong>Die (sensor, cable)</strong></td>
<td></td>
<td>0...300</td>
</tr>
<tr>
<td><strong>Melt (at the front of the sensor)</strong></td>
<td>°C</td>
<td>&lt; 850</td>
</tr>
<tr>
<td><strong>Temperature coefficient of sensitivity</strong></td>
<td>%/°C</td>
<td>±0.01</td>
</tr>
<tr>
<td><strong>Insulation resistance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>at 20 °C</td>
<td>TΩ</td>
<td>&gt; 10</td>
</tr>
<tr>
<td>at 30 °C</td>
<td>TΩ</td>
<td>&gt; 0.1</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>g</td>
<td>280</td>
</tr>
</tbody>
</table>

The temperature of the melt in contact with the front face of the sensor must not exceed 850°C. All parts of the sensor are corrosion-resistant. The high temperature non-detachable cable was protected by steel braiding and has a degree of protection (IP 65, against dust and jet of water). The “Fischer” connector was selflocking and splashproof.
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3.2.2 Measurement Procedures

As shown in Figure 3.13, four pressure transducers (P1, P2, P3 and P4) were inserted into the cavity through the top wall of the die. The sensing tips of the transducers were flush to the inner top die surface. Transducer P1 was positioned at the center of the inner top die surface. In the radial direction, transducers P2, P3 and P4 were arranged 0.020, 0.030 and 0.040 m away from the centerline of the casting, respectively. Figure 3.14 shows the placement of the pressure transducers into the top die insert.

Calibrations were made for both the new and used pressure transducers. Appendix I describes the calibration procedure in detail. Cleaning of the used pressure transducers must follow instruction provided by Kistler (Appendix II).
Figure 3.13 Scheme diagram showing the locations in which pressure transducers are embedded in the system, (a) top view, and (b) cross-section view.

(a) Pressure transducer and its support
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(b) Top die insert

(c) Top insert with pressure transducer and thermocouple

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Along the inner side die surface, three transducers (P5, P6 and P7) were placed 0.010, 0.040 and 0.070 m downward from the inner top die surface. Due to the contour shape of the inner side die surface, an extension pin with a diameter of 0.011 m and a length of 0.025 m was placed in front of the tip of each transducer for preventing potential damage resulting from melt cavity filling. The front surfaces of the pins were machined to accommodate the contour shape of the inner side die surface for the accuracy of pressure measurements. Figure 3.15 shows how the pressure transducers were mounted in the die along the inner side surface. Real-time pressure data were

(d) Whole assembly
Figure 3.14 Assembly of pressure transducers and the top die insert: (a) pressure transducer and its support; (b) top die insert; (c) top insert with pressure transducer and thermocouple and (d) assembly.
recorded at a regular interval of 100 ms over the entire user-defined period of measurement during the pressurized solidification stage of squeeze casting.

![Diagram of pressure transducer and support](image)

(a) Pressure transducer

![Diagram of side assembly](image)

(b) Side assembly

Figure 3.15 Assembly of side surface pressure transducers (a) pressure transducer, support and dummy and (b) side sensing assemblies.

### 3.3 Temperature and HTC Measurements

#### 3.3.1 Thermocouples

The thermocouples used in experiments were KTSS-116U-12 and KTSS-116U-24 (Omega Engineering Inc., Stamford, Connecticut, USA, Figure 3.16). The KTSS-116U thermocouple assemblies with transition joints of molded glass-filled nylon provide an economical yet durable thermocouple probe for a variety of sensing applications. The
measuring temperature is limited only by thermocouple type and sheath material. Stainless steel sheaths have a maximum temperature of 1650°F (900 °C).

Figure 3.16 KTSS-116U-12 thermocouple.

3.3.2 Temperature Measurement With or Without Applied Pressure

As shown in Figure 3.17, thermocouples T1, T2, T3 and T4 were placed at the top interface between the casting and die, and 0, 0.020, 0.030 and 0.040 m away from the centerline of the casting. Thermocouples T5, T6, and T7 were positioned at the side interface between the casting and die, 0.010, 0.040 and 0.070 m downward from the inner top die surface. Thermocouple T9 was placed 0.040 m down from the inner top die surface and 0.020 m from the casting central line. Thermocouple T10 was embedded in the casting center. Addition five thermocouples were placed along the casting center line 0.010, 0.020, 0.030, 0.040 and 0.050 m downward from the inner top die surface. Figures 3.18 and 3.19 show the thermocouple supports for both the top and side surfaces of the die. The placement of the thermocouples assembly is shown in Figures 3.14 (c) and 3.15 (b) for the top and side surfaces, respectively. Calibration results of thermocouples are listed in Appendix III.
Figure 3.17 Scheme diagram showing the locations in which thermal couples were embedded in the system.
In an effort to observe the solidification behaviour of both magnesium alloy AM50A and aluminum alloy A356 under the atmospheric pressure (with no applied pressure, i.e., at 0.1 MPa) and for the purpose of comparison with those under specific applied pressures, about 140 grams of melt sample was taken from the well-stirred alloy melt at 695 °C into a small steel crucible with a dimension of 0.05 m in diameter and 0.04 m in height (Figure 3.20). A chromel-alumel (K-type) thermocouple protected by a thin...
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steal sheath was positioned at a distance of 0.02 m from the bottom of the crucible center, and was connected to the data acquisition system to measure the temperature variation.

(b) Real setup

Figure 3.20 Apparatus for observing solidification behavior at 0.1 MPa pressure: (a) schematic diagram and (b) real setup.

3.3.3 Heat Transfer Coefficient Determination

To determine heat transfer coefficients at the interface between the casting and die, thermocouples Tmt1 and Tmt2 as shown in Figure 3.21 were positioned 0.002 and
0.005 m away from the interface at the top of the die insert, respectively. Meanwhile on the side, Tms1 and Tms2 were 0.010 m and 0.020 m away from the side interface, respectively. To measure casting temperatures, thermocouples T1 and T6 were placed inside the casting 0.0008 m away from the top surface center, and the middle of side surface, respectively.

Figure 3.21 Experimental setup for determining heat transfer coefficients at the interface between the casting and die.
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3.4 Data Acquisition System

A data acquisition system (DAS, Figure 3.22) made by the TSC was used in this study. It contains a National Instruments (National Instruments Corporation, Austin, Texas, USA) PCI-6033E processor with eight temperature (K type) channels and eight sensor channels. The specification of PCI-6033E processor is described in Table 3.5. During experiment, the analogue signals from thermocouples and sensors were fed to the data acquisition system through a National Instrument SCB-100 connector, and ultimately saved in a computer drive for permanent storage. A customized software was developed based on Lab View (a graphical development software by National Instruments Corporation, Austin, Texas, USA), which enables users to program and monitor the casting process.

Table 3.5 Specification of PCI-6033E processor

<table>
<thead>
<tr>
<th>Family</th>
<th>Bus</th>
<th>Analog Inputs</th>
<th>Input Resolution</th>
<th>Max Sampling Rate</th>
<th>Input Range</th>
<th>Analog Outputs</th>
<th>Digital I/O</th>
<th>Counter/Timers</th>
<th>Triggers</th>
</tr>
</thead>
<tbody>
<tr>
<td>NI 6033E</td>
<td>PCI 64 SE/32 DI</td>
<td>16 bits</td>
<td>100 kS/s</td>
<td>±0.1 to ±10</td>
<td>2</td>
<td>8</td>
<td>2, 24-bit</td>
<td>Analog, digital</td>
<td></td>
</tr>
</tbody>
</table>

Real-time pressure data were recorded at regular intervals of 100 ms over the entire user-defined period of measurement during the pressurized solidification stage of squeeze casting.
Figure 3.22 Data Acquisition System (a) interface box including PCI-6033 processor and (b) the integrated system.
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3.5 Alloys

The testing alloys selected for this study were magnesium alloy AM50A provided by Nanjing Welbow Metals Co., Ltd. and Norsk Hydro and aluminum alloy A356 supplied by Ford Motor Company. These alloys have wide usage in automotive industry because of their very good castability and mechanical properties. Table 3.6 lists the chemical composition of AM50A and A356 alloys.

Table 3.6 Chemical composition of magnesium alloy AM50A and Aluminum alloy A356

<table>
<thead>
<tr>
<th></th>
<th>Al</th>
<th>Mg</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Zn</th>
<th>Ni</th>
<th>Ti</th>
<th>Sr</th>
<th>Si</th>
<th>Be</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM50A</td>
<td>4.87 Balance 0.0047 0.0008 0.28 0.0037 0.0005 / / 0.026 0.0006 0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A356</td>
<td>Balance 0.50 0.099 0.01 / 0.003 0.021 0.075 0.014 7.15 0.01 0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Raw materials received in ingot form (12kg/ingot for magnesium & 20 kg/ingot for aluminum) were cut into small pieces and then put into crucible for melting (Figure 3.23).
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4

(b) Mg alloy ingots and sectioned pieces

Figure 3.23 Raw materials: (a) Al alloy ingots and sectioned pieces and (b) Mg alloy ingots and sectioned pieces.

3.6 Density Measurements of Castings

To evaluate the densification effect due to squeeze casting, samples for density measurements were taken from specified locations in cast coupons as shown in Figure 3.24. Those samples labeled A - F correspond to pressure transducers location P1, P2, P4, P5, P6, and P7 respectively. G is the casting center. Each sample was a cube of 0.01x0.01x0.01 m³. Following the measurement of sample weight in air and distilled water, the actual density (D_a) of each sample was determined using Archimedes principle based on ASTM Designation D3800:

\[ D_a = \frac{W_o D_w}{W_o - W_w} \]

where \( W_o \) and \( W_w \) are the weights of samples in air and in water respectively, and \( D_w \) the density of water.
Figure 3.24 Schematic diagram showing the locations where samples for density measurement were sectioned.

Figure 3.25 Density measurement setup.

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3.7 Squeeze Casting Procedure

The squeeze casting operation procedures (shown in Figure 3.26) are summarized as follows: (a) preheating and the graphite-lubricated die to 275 °C, and selecting the operating parameters (applied pressure and holding time); (b) pouring molten magnesium alloy AM50A or aluminum alloy A356 into the shot sleeve until they reach 695 °C or 725 °C respectively; (c) injecting the melt into the die cavity by the plunger with an upward velocity of 0.055 m/s once the die was closed by moving up the lower platen with the sleeve; (d) applying a predetermined pressure to the solidifying melt for a preset dwell time upon the completion of solidification; and (e) retracting the plunger and the lower platen to its original position, and ejecting the cast coupon.

(a) Preheating and lubricating tooling
(b) Filling sleeve
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Figure 3.26 Schematic diagram showing squeeze casting operation. (a) Preheating and lubricating tooling, (b) Filling sleeve, (c) Filling die cavity, (d) Solidifying melt under pressure and (e) take out the casting coupon. For the clarity of presentation, the embedded pressure transducers and thermocouples are not included.

Primarily, the process includes two steps: (1) cavity filling, and (2) pressurized solidification which includes the stages of pressure built up and holding. Based on the variation of applied pressure with time, one cycle of squeeze casting processes can often be categorized into a few stages. The squeeze casting process employed in this study was divided into four stages in terms of the history of applied pressures (Figure 3.27).

a. filling stage, during which the pressure plunger pushes the metal into the cavity;
b. stage of pressure buildup, during which the applied pressure increases from the initial value to a preset point;
c. holding stage, during which the press applies the preset pressure to the casting coupon; and
d. pressure release stage, during which the plunger is retracted and the pressure is released once the set holding time is reached.

![Figure 3.27 Typical history curve of an applied pressure of 30 MPa in one cycle of the squeeze casting operation.](image)
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For safety reasons, a transparent polycarbonate shield was placed in front of the press during the casting operation. The usage of the transparent shield prevents molten metal from splashing during squeeze casting, which may result in physical injury to operators.
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZE CASTING

As reviewed in Chapter 2, the cavity pressure during the squeeze casting process is one of the critical factors for making sound castings. However, in most practical operations of squeeze casting, the cavity pressure usually is not listed as the first process parameter. Instead, the hydraulic pressure is usually measured and used as specific pressure, and the cavity pressure is often eliminated as a quality control variable by simply using large machine which provides a press load higher than that required. The main reason is that the real characteristics of the cavity pressure in squeeze casting had yet to be fully investigated.

Thus, the principal objective of this chapter is to elucidate a fundamental understanding of pressure distribution in die cavities during squeeze casting of magnesium alloy AM50A and aluminum alloy A356. Particular emphasis has been placed on measuring the history of local pressures in real-time with advanced piezoelectric quartz pressure transducers in parallel to temperature measurements. The inhomogeneous distribution of in-cavity pressures is highlighted and its impact on heat transfer between the die and casting is discussed.

4.1 Results and Discussion of Magnesium Alloy AM50A

4.1.1 Local Pressure and Temperature Change with Time

Figure 4.1 presents the typical local in-cavity pressure change with time at four
locations along the top surface of the casting during the pressurized solidification step of squeeze casting when the external pressure was applied. P1, P2, P3, and P4 represent the pressure histories at the transducer-embedded locations shown in Figure 3.13. It is evident from Figure 4.1 that the local cavity pressures at all the sampling locations built up very rapidly after the cavity filling completed once the external force was applied. Then, at each location, the local pressure reached its own peak before starting to decrease. It took about 5 seconds for P1, P2, P3 and P4 increased to their peak values of 34, 38, 43 and 47 MPa from 0 MPa, respectively. Along the top surface, the peak values of the local pressures increased as the distance of the sampling locations away from the centerline of the casting extended. Around 25 seconds after the applied pressure set in, the individual local pressures decreased to their own stable plateau.

Figure 4.1 Typical experimental results of local pressure measurements on the top surface of a cylindrical squeeze casting of magnesium alloy AM50A under an applied pressure of 60 MPa.
CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZE CASTING

The variation of the local pressure with time along the side surface of the casting is shown in Figure 4.2. As can be seen from Figure 4.2, the local pressures at locations P5, P6 and P7 increased quickly to the highest points, 23, 37 and 50 MPa respectively after the external force was applied. This indicates that the individual local pressure along the side surface decreased when the casting height increased. All the local pressures decreased slowly afterward.

Figure 4.2 Typical experimental results of local pressure measurements on the side surface of a cylindrical squeeze casting of magnesium alloy AM50A under an applied pressure of 60 MPa.

Figure 4.3 (a) & (b) presents the variation of the local pressures with time along both the top and side surfaces of the casting under applied pressure of 30MPa. The variation of the local pressures with time along both the top and side surfaces of the casting under applied pressure of 90MPa are shown in Figure 4.4 (a) & (b). These curves exhibit the trends similar to those under applied pressure of 60MPa. However, it appears
that there are differences in local pressure measurements between various applied pressures, which will be addressed in the following sections.

Figure 4.3 Local pressure change with time on (a) the casting top and (b) side surfaces under an applied pressure of 30 MPa.
Figure 4.4 Local Pressure change with time on (a) the casting top and (b) side surfaces under an applied pressure of 90 MPa.
4.1.2 Variation of Pressure Transfer Rate and Temperature

Examination of Figures 4.1 through 4.4 reveals that, at the beginning of the external force application, the local pressure increased up to 95% of its peak value along the top surface at a much faster pace than those at the side surface of the casting despite that the local pressures along the top surface decreased considerably faster than those at the side surface after a certain period of time.

To compare pressure change rates under different applied pressures, first derivatives of the pressure curves from Figures 4.1 through 4.4 were made by using the moving average method (Appendix IV). The dP/dt vs time curves are shown in Figures 4.5 - 4.10 for both the top and side surfaces of the casting under applied pressures of 30, 60 and 90 MPa, respectively. From these curves, the observations on the initial pressure change rate (dP/dt) for the three levels of the applied pressures can be described as follows:

a. The dP/dt curves for all applied pressure are very similar. All dP/dt curves started from a peak value and dropped quickly. After about 5 seconds, they became almost constant which indicated a steady pressure change;

b. On the top surface, the dP/dt increased from the center to side, which implied that differences in local material desification were presented;

c. On the side surface, the dP/dt decreased from the bottom to top, which also implied local densification varied with the casting height;

d. The dP/dt on the top surface was greater than that on the side surface, which indicates that pressures gradient in the directions parallel to applied pressure were different from those in perpendicular to the pressure; and
e. The peak dP/dt value on both top and side surfaces increased with applied pressures.

For the clarification of discussion, the increasing rate of the local pressures up to 95% of its peak value is defined as an initial local pressure transfer rate thereafter in the text.

Figure 4.5 Initial local pressure change rates on the top surface under applied pressure of 30 MPa.
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZE CASTING

Figure 4.6 Initial local pressure change rates on the side surface under applied pressure of 30 MPa.

Figure 4.7 Initial local pressure change rates on the top surface under applied pressure of 60 MPa.
Figure 4.8 Initial local pressure change rates on the side surface under applied pressure of 60 MPa.

Figure 4.9 Initial local pressure change rates on the top surfaces under applied pressure of 90 MPa.
Figure 4.10 Initial local pressure change rates on the side surface under applied pressure of 90 MPa.

Figure 4.11 compares the initial pressure transfer rates on the top surface to those at the side under applied pressure of 30, 60 and 90 MPa respectively. The average initial pressure transfer rate on the top surface varies from 8 to 24 MPa/s under the three applied pressures. But, at the side surface, the range of average initial pressure transfer rate is only 4-17 MPa/s. This observation suggests that the pressure transfer in a direction perpendicular to the external force is faster than that in a parallel direction.

Figure 4.12 presents the temperature measurements from the experiment. The temperature at the side interface between the casting and die is represented by line “T6” while line “T3” characterizes the temperature at the top interface between the casting and die. Line “T10” gives the temperature at the casting center. It can be seen from Figure 4.12 that the temperatures at all the three locations (T3, T6 and T10) increased once the
die cavity was completely filled. At the beginning of the external pressure application, T6, T3, and T10 immediately increased to their highest points of 434, 546 and 626 °C respectively from the die temperature of 275 °C. It is well documented [198] that the liquidus and solidus temperatures of magnesium alloy AM50A are 620 and 435 °C respectively. The recorded temperature measurements indicate that, even at the very early stage of the process, a completely-solidified shell was formed around the side of the casting upon the completion of cavity filling since the highest temperature at the side interface was only 434 °C below the solidus temperature of 435 °C. The top portion with a temperature of 550 °C between the liquidus and solidus temperatures was semi-solid. The formation of a solid shell is attributed primarily to a relatively slow filling velocity, which was employed for minimizing melt flow turbulence during cavity filling. However, the temperature of 626 °C at the casting center was above the liquidus temperature, which indicates that the casting center was fully liquid. Even under the high external pressure of 60 MPa, however, latent heat release and slow heat transfer resulting from the considerably thick section (over 0.05 m) of the casting maintained the central temperature (T3) at 626 °C for about 15 seconds before it started decreasing. It is interesting to see that temperatures T3 and T6 increased relatively faster than T10 at the beginning. This is because the thermal conductivity of magnesium alloy AM50A decreases with increasing temperature. The thermal conductivity of liquid AM50A is 79 W/m*K at 626 °C lower than that of the semi-solid or solid alloy (95 W/m*K at 546 °C, and 98 W/m*K at 434 °C) [199]. After reaching their highest points, both temperatures T6 and T3 decreased with time. It is worthwhile mentioning that temperature T3 decreased by 50 °C from 547 to 497 °C in 3 seconds significantly faster than that by 34 °C at T6 from 434 to 400 °C.
Figure 4.11 Initial local pressure transfer rates on the top and side surfaces of a cylindrical squeeze casting of magnesium alloy AM50A under applied pressure of 30, 60 and 90 MPa.
within the first second. This observation indicates that higher heat transfer took place at T3 than that at T6. The high heat transfer is, at least in part, due to the occurrence of rapid pressure transfer at P3 (28 MPa/s) compared with that at P6 (7 MPa/s) as shown in Figure 4.12. It should be noted that locations P3 and P6 geometrically corresponds to the positions of T3 and T6, respectively.

This is because that the fast local pressures transfer could enable the casting to firmly contact the die surface as quickly as possible during cooling. The close contact between the casting and die enhances localized heat transfer, and consequently increases the solidification rate of the casting. The high initial pressure transfer may play a critical role in the squeeze casting of some potential magnesium automotive components such as knuckles and engine blocks, which have a section thickness even much thinner than the experimental casting used in this study.

![Graph](image)

*Figure 4.12 Typical experimental results of local temperature measurements in a cylindrical squeeze casting of magnesium alloy AM50A under an applied pressure of 60 MPa.*

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Once a firm contact between the die and casting is established by the local pressure, local heat transfer from the casting to the die becomes primarily influenced by the temperature difference present at the casing/die interface. As can be seen from Figure 4.12, both temperatures T6 and T3 declined gradually as time increased. In particular for T6, sluggish temperature decreasing took place due to the relatively low temperature difference present at the side interface than that at the top interface.

### 4.1.3 Pressure Distribution on Casting Surfaces

A pressure distribution along the top and side surfaces of the cylindrical casting 5 seconds after the completion of the cavity filling under the applied pressures of 30, 60, 90 MPa is shown in Figure 4.13 (a), (b) and (c), respectively. It can be seen from Figure 4.13 (a) that there was an increase in the pressure from 34 to 47 MPa as the distance away from the center increases on the top surface. All the measured pressures were much lower than the original applied pressure of 60 MPa. Along the casting side, the pressure decreased considerably to 23 MPa from the original applied pressure of 60 MPa with increasing the height from the bottom to the top of the casting. It is evident that the pressure distribution is inhomogeneous on either the side or top surfaces of the casting. This finding contradicts the existing viewpoint about squeeze casting, i.e., “molten metal is solidified under applied hydrostatic pressure” [16].

The early examination of the thermal histories at three sampling locations in the casting, as shown in Figure 4.12, indicates that, during the cavity filling prior to the commencement of the pressure application, solidification took place in a certain outer portion of the casting, where a semisolid and/or solid shell has already formed. Once the
external pressure was applied upon the completion of the cavity filling, with the presence of the semisolid and/or solid material of a certain yield strength in the casting, it is almost impossible for the applied pressure to be transferred in a hydrostatic manner through the entire casting.

(a) 60 MPa

(b) 30 MPa
Figure 4.13 Initial pressure distributions 5 seconds after an external pressure of (a) 60 MPa, (b) 30 MPa, and (c) 90 MPa being applied.

(a) Thermal isocontours upon filling completion

(b) Deformation of metal flow under pressure
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZE CASTING

To account for the pressure distributions along the top and side surfaces of the casting, a pressure transfer mechanism is proposed as schematically illustrated in Figures 4.14 (a) - (c). Despite its continuous cooling during pressurized solidification, temperatures on the side surface of the casting increase with increasing the height, which has been predicted by a developed mathematical model to be presented in Chapter 7. The high alloy temperature lowers its strengths, which makes the upper side of the casting easier to be densified and deformed either plastically or elastically than the lower side. Meanwhile, the thermal contraction of the casting also takes place due to the continuous cooling under the constantly applied pressure. The high temperature results in less contraction to occur at the upper side of casting than that at the lower side. As a result of the presence of the constant applied pressure, alloy densification, deformation, thermal contraction, and friction forms along the side interface between the casting and die. The
high densification and deformation, and low contraction generates more friction at the upper side than that at the lower side. The variation of the friction along the side surface of the casting gives rise to the increasing loss of the applied pressure with its height. Therefore, the decreasing trend of the local pressure along the side surface should be attributed to the occurrence of the friction between the casting and the die, which is also responsible for degrading the applied pressure transferred to the top surface from the bottom.

Figure 4.15 Experimental observation on the geometry of a casting upon filling completion before an external pressure being applied, in which a shallow crater formed at the top of the casting.

It can also be found from Figures 4.1, 4.3 and 4.4 that the measured pressure at the top center is lower than those at the rest area of the top surface. This pressure distribution might be interpreted as follows. Due to the relatively long cavity filling period, the occurrence of early solidification in certain areas induced thermal contraction
to take place in the casting. Under the action of gravity, a shallow crater formed around the center of the top surface of the casting as shown in Figure 4.15. The presence of the crater could provide certain space for the cast material to move towards the center of the top surface via alloy densification and deformation under the external applied pressure. Since the metal in the upper central portion of the casting was relatively hot and had low yield strength compared to the material in the outer portion, it is expected that heavy densification and deformation occurred along the centerline rather than along the side of the casting. The non-uniform densification and deformation could lead to the formation of friction internally and externally. The heavier the densification and deformation, the higher the friction loss present along the centerline of the casting. As a result, the retained pressure at the center was lower than those at the rest area of the top surface due to the high friction along the centerline. Moreover, the friction should also be responsible for the overall pressure loss during transfer in both of the directions perpendicular and parallel to the pressure applying direction.

To verify the above proposed mechanism, density measurements were conducted on samples taken from different locations of the casting as shown in Figure 3.24. The results (Figures 4.16 – 4.18) indicate the density did vary with locations within the casting. Compared to the other sample locations, the density at the top center and the upper side of the casting are somewhat high. This observation evidently indicates that these two areas experienced heavier densification.
Figure 4.16 Density distribution under an applied pressure of 30 MPa.

Figure 4.17 Density distribution under an applied pressure of 60 MPa.
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZE CASTING

4.2 Results and Discussion of Aluminum Alloy A356

4.2.1 Local Pressure Change with Time

Figure 4.19 presents the typical local in-cavity pressure change for aluminum alloy A356 with time at four locations along the top surface of the casting during the pressurized solidification step of squeeze casting when the external pressure was applied. P1, P2, P3, and P4 represent the pressure histories at the transducer-embedded locations. It is evident from Figure 4.19 that the local cavity pressures at all the sampling locations built up very rapidly after the cavity filling completed once the external force was applied. Then, at each location, the local pressure reached its own peak before starting to decrease. It took about 5 seconds for P1, P2, P3 and P4 increased to their peak values of
34, 37, 38 and 41 MPa from 0 MPa, respectively. Along the top surface, the peak values of the local pressures increased as the distance of the sampling locations away from the centerline of the casting extended.

![Pressure-time graph](image)

Figure 4.19 Typical experimental results of local pressure measurements on the top surface of a cylindrical squeeze casting of aluminum alloy A356 under an applied pressure of 60 MPa.

The variation of the local pressure with time along the side surface of the casting is shown in Figure 4.20. As can be seen from Figure 4.20, the local pressures at locations P5, P6 and P7 increased quickly to the highest points, 22, 32 and 39 MPa respectively after the external force was applied. This indicates that the individual local pressure along the side surface decreases when the casting height increases. All the local pressures decreased slowly afterward.
Figure 4.20 Typical experimental results of local pressure measurements on the side surface of a cylindrical squeeze casting of aluminum alloy A356 under an applied pressure of 60 MPa.

Figure 4.21 (a) & (b) show the variation of the local pressures with time along both the top and side surfaces of the casting under applied pressure of 30MPa. The variation of the local pressures with time along both the top and side surfaces of the casting under an applied pressure of 90MPa are shown in Figure 4.22 (a) & (b). Those curves exhibit the trends similar to those under an applied pressure of 60MPa. However, it appears that there are differences in local pressure measurements between various applied pressures, which will be addressed in the following sections.
Figure 4.21 Local Pressure change with time on (a) the casting top and (b) side surface under an applied pressure of 30 MPa.

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Figure 4.22 Local Pressure change with time on (a) the casting top and (b) side surfaces under an applied pressure of 90 MPa.
4.2.2 Variation of Pressure Transfer Rates and Temperatures

Examination of Figures 4.19 - 4.22 manifests that, at the beginning of the external force application, the local pressures increased up to 95% of its peak value along the top surface in a much faster pace than those at the side surface of the casting despite that the local pressures along the top surface decreased considerably faster than those at the side surface after a certain period of time.

To compare pressure change rates under different applied pressures, the first derivatives of the initial part of the pressure curves given in Figures 4.19 ~ 4.22 were calculated by using the moving average method (Appendix IV). The dP/dt vs time curves are shown in Figures 4.23 ~ 4.28 for both the top and side surfaces under applied pressures of 30, 60 and 90 MPa respectively. From those dP/dt curves, the following observations on the initial pressure change rate dP/dt can be obtained:

a. the dP/dt values for all applied pressures decreased with increasing time;
b. on the top surface, the dP/dt increased from the center to side;
c. on the side surface, the dP/dt increased from the bottom to top;
d. the dP/dt on top surface was greater than those on side surface; and
e. the maximum dP/dt on both the top and side surfaces increased with increasing applied pressures.

For the clarification of discussion, the increasing rate of the local pressures up to 95% of its peak value for aluminum alloy A356 is also defined as an initial local pressure transfer rate.
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZ CASTING

Figure 4.23 Initial local pressure change rates on the top surfaces under applied pressure of 30 MPa.

Figure 4.24 Initial local pressure change rates on the side surface under applied pressure of 30 MPa.
Figure 4.25 Initial local pressure change rates on the top surfaces under applied pressure of 60 MPa.

Figure 4.26 Initial local pressure change rates on the side surface under applied pressure of 60 MPa.
4 CHARACTERIZATION OF LOCAL CAVITY PRESSURE IN SQUEEZ CASTING

Figure 4.27 Initial local pressure change rates on the top surfaces under applied pressure of 90 MPa.

Figure 4.28 Initial local pressure change rates on the side surface under applied pressure of 90 MPa.
Figure 4.29 compares the initial pressure transfer rates on the top surface to those at the side under applied pressure of 30, 60 and 90 MPa respectively. The average initial pressure transfer rate on the top surface varies from 10 to 23 MPa/s under the three applied pressures. But, on the side surface, the range of average initial pressure transfer rate is only 3-14 MPa/s. This observation suggests that the pressure transfer in a direction perpendicular to the external force is faster than that in a parallel direction.

Figure 4.30 presents the temperature measurements from the experiment with an applied pressure of 90 MPa. The temperature at the side interface between the casting and die is represented by line “T6” while line “T2” characterizes the temperature at the top interface between the casting and die. Line “T10” gives the temperature at the casting center. It can be seen from Figure 4.30 that the temperatures at all the three locations (T2, T6 and T10) increased once the die cavity was completely filled. At the beginning of the external pressure application, T2, T6 and T10 immediately increased to the highest points of 550, 570 and 616 °C from the die temperature of 275 °C, respectively. It is well documented [198] that the liquidus and solidus temperatures of aluminum alloy A356 are 613 and 555 °C respectively. The recorded temperature measurements indicate that, even at the very early stage of the process, a mushy solid mixed shell was formed around the side of the casting upon the completion of cavity filling since the highest temperature at the side interface was only 550 °C below the solidus temperature of 555 °C. The top portion with a temperature of 570 °C between the liquidus and solidus temperatures was semi-solid. The formation of a solid shell is attributed primarily to a relatively slow filling velocity, which is employed for minimizing melt flow turbulence during cavity filling. However, the temperature of 616 °C at the casting center was above the liquidus
Figure 4.29 Initial local pressure transfer rates on the top and side surfaces of a cylindrical squeeze casting of aluminum alloy A356 under applied pressure of 30, 60 and 90 MPa.
temperature, which indicates that the casting center was fully liquid. Even under the high external pressure of 60 MPa, however, latent heat release and slow heat transfer resulting from the considerably thick section (over 0.05 m) of the casting maintained the central temperature (T10) at 616 °C for about 20 seconds before it started decreasing.

![Temperature Graph](image)

Figure 4.30 Typical experimental results of local temperature measurements in a cylindrical squeeze casting of aluminum alloy A356 under an applied pressure of 90 MPa.

### 4.2.3 Pressure Distribution on Casting Surfaces

A pressure distribution along the top and side surfaces of the cylindrical casting 5 seconds after the completion of the cavity filling under the applied pressures of 30, 60, 90 MPa is shown in Figure 4.31. From Figure 4.31 (a), there was an increase in the local pressures from 33 to 40 MPa as the distance away from the center increased on the top
surface. All the measured pressures were much lower than the hydraulic applied pressure of 60 MPa. Along the side of the casting, the pressure decreased considerably to 21 MPa from the original applied pressure of 60 MPa with increasing the height from the bottom to the top of the casting. It is evident that the pressure distribution is inhomogeneous on either the side or top surfaces of the casting. The pressure transfer mechanism proposed in the previous section (4.1.3) for magnesium alloy AM50A can also explain the pressure distributions along the top and side surfaces of the casting of aluminum alloy A356.

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To verify the above proposed mechanism, density measurements were conducted on samples taken from different locations in the casting as shown in Figure 3.24. The results (Figures 4.32 ~4.34) indicate the density did vary from one location to another within the casting. Compared to the other sample locations, the densities at the top center and the upper side of the casting were somewhat higher. This observation evidently indicates that these two areas experienced heavier densification than other locations during pressurized solidification.
Figure 4.32 Density distribution under an applied pressure of 30 MPa.

Figure 4.33 Density distribution under an applied pressure of 60 MPa.
4.3 **Summary**

The transient in-cavity local pressures behavior in squeeze casting of magnesium alloy AM50A and aluminum alloy A356 has been investigated. The in-cavity pressures were directly measured in parallel with the acquisition of thermal histories for both magnesium alloy AM50A and aluminum alloy A356 under different applied pressure of 30, 60 and 90 MPa. Local cavity pressures at various locations change with time in the duration of casting solidification and cooling. The pressure transfer rate varies considerably within the cylindrical geometry of the casting. The pressure transfer in a direction perpendicular to the external force is faster than that in a parallel direction. A pressure transfer mechanism is proposed to explain the inhomogeneous distribution of local cavity pressures in the cavity, which was verified by density measurement.
To make the simulation of squeeze casting physically realistic, it is essential to obtain reliable and accurate values of heat-transfer coefficients (HTC) at the casting/die interface. In Chapter 2, it was reviewed that several numerical approaches have been developed in recent years to predict casting/die air gap width and then calculate interfacial HTC (IHTC). However, those proposed mechanisms do not seem match the scenario for squeeze casting.

As revealed in previous Chapter 4 through investigation, the local pressure distribution is not homogeneous in cavity during solidification of squeeze casting. As a result, questions arise:

a. What happens to the heat-transfer coefficient (HTC) at the casting/die interface?

and,

b. What is the relationship between local cavity pressures and the HTC?

The heat balance method has been employed in this study to estimate the heat flux and HTC based on the direct measurements of local temperatures in the die and casting. The effect of applied pressure on the HTC values for both magnesium alloy AM50A and aluminum alloy A356 was investigated. The correlation between HTC and local pressures was established and will be integrated into the mathematical model.
5.1 Determination of HTC

There are two techniques often being used to calculate heat transfer coefficients during casting. The first one traditionally is known as the inverse method [100], which involves measuring temperatures in the casting and die at various internal points, and using an inverse technique to estimate the surface temperatures and temperature gradient.

The second one is so called energy balance technique, in which attempts are made to monitor temperatures at the inner and outer surfaces of the interface. With known short distance between temperature measuring surfaces, temperature gradient can be determined [177, 226].

The governing equation for one dimensional heat conduction in the die can be expressed:

\[
\rho_d C_{pd} \frac{\partial T}{\partial t} = k_d \frac{\partial^2 T}{\partial z^2} \quad \text{(for the top)} \tag{5-1}
\]

\[
\rho_d C_{pd} \frac{\partial T}{\partial t} = k_d \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \quad \text{(for the side)} \tag{5-2}
\]

where \(k_d\), \(C_{pd}\), and \(\rho_d\) are the heat conductivity, specific heat, and density of the die respectively. The boundary condition at the casting/die interface is

\[
q = h(T_{\text{casting}} - T_{\text{die}}) \tag{5-3}
\]

where \(q\) and \(h\) are the heat flux and heat transfer coefficient at the casting/die interface, \(T_{\text{casting}}\) and \(T_{\text{die}}\) are the casting and die surface temperatures respectively.

With the experimental setup for determining HTC as described in 3.3.3, in this study, the HTC will be determined by performing an approximate energy balance based on the above equations (5-1) to (5-3).
For the top:

\[ h(T_{\text{casting}} - T_{\text{die}}) = k_{\text{die}} \left( \frac{T_{\text{die}} - T_{\text{die1}}}{\Delta Z_1} \right) + \frac{\Delta T_{\text{die1}} \Delta Z_1 C_{pd} \rho_d}{\Delta t} \]  
\[ k_{\text{die}} \left( \frac{T_{\text{die}} - T_{\text{die1}}}{\Delta Z_1} \right) = k_{\text{die}} \left( \frac{T_{\text{die1}} - T_{\text{die2}}}{\Delta Z_2} \right) + \frac{\Delta T_{\text{die2}} \Delta Z_2 C_{pd} \rho_d}{\Delta t} \]  

where \( T_{\text{die1}} \) and \( T_{\text{die2}} \) are the local die temperature at location 1 and 2 respectively (Figure 5.1), \( \Delta t \) is the time interval step of testing, \( \Delta T_{\text{die1}} \) and \( \Delta T_{\text{die2}} \) are the die temperature change for a time interval, \( \Delta Z_1 \) and \( \Delta Z_2 \) are distances between locations.

Figure 5.1 Temperature distribution at the top interface between the casting and die.

For the side:

\[ 2hr_0(T_{\text{casting}} - T_{\text{die}}) = 2k_{\text{die}}(T_{\text{die}} - T_{\text{die1}}) \ln \left( \frac{r_1}{r_0} \right) + \frac{\Delta T_{\text{die1}} C_{pd} \rho_d (r_1^2 - r_0^2)}{\Delta t} \]  
\[ \frac{2k_{\text{die}}(T_{\text{die}} - T_{\text{die1}})}{\ln \left( \frac{r_1}{r_0} \right)} = \frac{2k_{\text{die}}(T_{\text{die1}} - T_{\text{die2}})}{\ln \left( \frac{r_2}{r_1} \right)} + \frac{\Delta T_{\text{die2}} C_{pd} \rho_d (r_2^2 - r_1^2)}{\Delta t} \]  

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where \( r_0, r_1, \) and \( r_2 \) are the radius for side die/casting interface, location 1 and 2 respectively (Figure 5.2).

![Diagram of temperature distribution at the side interface between the casting and die.]

Figure 5.2 Temperature distribution at the side interface between the casting and die.

Table 5.1 lists the thermophysical properties of magnesium alloy AM50A, aluminum alloy A356 and die steel H13.

Table 5.1 Thermophysical properties of magnesium alloy AM50A, aluminum alloy A356 and die steel H13 [199]

<table>
<thead>
<tr>
<th>Properties</th>
<th>Mg Alloy AM50</th>
<th>Al Alloy A356</th>
<th>H13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Conductivity (W/m ( \cdot ) K)</td>
<td>80</td>
<td>159</td>
<td>25</td>
</tr>
<tr>
<td>Specific Heat (J/kg ( \cdot ) K)</td>
<td>1220</td>
<td>963</td>
<td>461</td>
</tr>
<tr>
<td>Density (kg/m(^3))</td>
<td>1710</td>
<td>2685</td>
<td>7761</td>
</tr>
<tr>
<td>Latent Heat (J/kg)</td>
<td>3.7(\times)10(^5)</td>
<td>4.3(\times)10(^5)</td>
<td></td>
</tr>
<tr>
<td>Liquidus Temperature (°C)</td>
<td>625</td>
<td>613</td>
<td></td>
</tr>
<tr>
<td>Solidus Temperature (°C)</td>
<td>427</td>
<td>542</td>
<td></td>
</tr>
</tbody>
</table>
In this study, experimental temperature curves in both the casting and the die were recorded continuously during the process. The temperature and gradients at the interface were estimated accordingly. The heat transfer coefficients can be calculated based on Equations (5-4) to (5-7).

5.2 Results and Discussion of Magnesium Alloy AM50A

Figures 5.3 and 5.4 show the simultaneous changes of temperatures and applied pressures with time in the casting and die during the entire period of the squeeze casting including cavity filling and pressurized solidification. P is the applied hydraulic pressure referring to Figure 3.17. T1, T6 and T10 represent the temperatures at the casting top surface center, the middle of the side surface, and the casting center, respectively. The die temperatures near the top and side interface between the casting and die as shown in Figure 5.1 and 5.2 are denoted by Tmt1, Tmt2, Tms1 and Tms2, respectively. Referring to Figure 3.13, P1 and P6 stand for the local pressures at the casting top surface center and the middle of the side surface, respectively. To ease the result presentation and discussion given in the succeeding sections, time zero was intentionally set at the instant when the cavity filling was completed and the applied pressure set in.
Figure 5.3 Simultaneous changes of casting temperature and applied pressure.

Figure 5.4 Typical experimental measurements for heat transfer analysis for an applied pressure of 60 MPa
5.2.1 Heat Transfer on the Top Surface

Figures 5.5 - 5.7 show the recorded temperatures histories at the casting top surface center (T1) and in the top wall of the die (Tmt1 and Tmt2) for the applied pressures of 30, 60 and 90 MPa, respectively. As observed from Figures 5.5 - 5.7, the temperature responses to the three different applied pressures are very similar as measuring time extends.

However, under a specific applied pressure, the temperatures at locations T1, Tmt1 and Tmt2 varied considerably with time. Upon the completion of the cavity filling (time = 0), T1 instantly increased to about 523 °C for applied pressures of 30, 60 and 90 MPa, due to the direct contact between thermocouples T1 and the top layer of the casting. Then, the casting top surface temperature (T1) kept unchanged for about 6 seconds. Meanwhile, the die temperature (Tmt1 and Tmt2) increased by over 100 °C. After that, the casting top surface temperature dropped rapidly. But, the die temperature still remained increasing for about another 10 seconds before it began to decrease. The temperature results of the casting top surface show that, at the beginning of the pressurized solidification, it took about 6 seconds for the applied pressure to force the casting top surface in full and firm contact with the die.
Figure 5.5 Temperature measurements for HTC calculation adjacent to the top interface between the casting and die under an applied pressure of 30 MPa.

Figure 5.6 Temperature measurements for HTC calculation adjacent to the top interface between the casting and die under an applied pressure of 60 MPa.
As discussed in Chapter 4, a shrinkage crater was present at the center of the top surface of the casting at the end of the cavity filling before the application of the external pressures. The establishment of the full and firm contact between the casting top surface and the die indicates that the shrinkage crater was almost completely eliminated, in which air gap was present as a thermal resistance. The alloy densification and deformation resulting from pressurization should be responsible for the elimination of the crater since it took place in the temperature range (525°C) in which magnesium alloy AM50A is at its semisolid state.

In addition, slow increases in the die temperatures right after the exertion of the applied pressures also indicate that there was a gap between the casting top surface and die due to the presence of the shrinkage crater (Figure 5.8). As the application of the pressures proceeded, the established firm contact between the casting and die kept
increasing the mold temperature. The continuous heat transfer to the mold from the casting maintained the die temperature at high temperatures, and resulted in a decrease in $T_1$.

(a) End of filling

(b) Densification and deformation after pressure applied
Figures 5.9-5.11 show the instantaneous heat flux across and heat transfer coefficient at the interface center (location 1) between the casting top surface and the top wall of the die as a function of time for applied pressure of 30, 60 and 90 MPa, respectively.
Figure 5.9 Heat flux across and heat transfer coefficient at the top interface between the casting and the die under an applied pressure of 30 MPa.

Figure 5.10 Heat flux across and heat transfer coefficient at the top interface between the casting and the die under an applied pressure of 60 MPa.
The heat flux increase reached a peak in about 4 seconds and then gradually decreased as the process continued. A significant increase in the heat flux is attributed to the thermal contact at the casting/die interface and the large temperature difference at the beginning, while a gradual decrease in heat flux is considered to be caused by a reduction in the thermal gradient.

However, the heat transfer coefficient did not increase to its maximum until about 22-25 seconds after the pressure was applied. It should be pointed out that, as the applied pressure increased from 30 to 90 MPa, the peak value of the HTC rose from 8,000 to 10,000 W/m² K. As the process continued, the HTC decreased slowly. A significant increase in the heat transfer coefficients should be attributed to the establishment of firm
thermal contact between the casting and the die as described previously. A slowly drop of heat transfer coefficients for the rest of the process is considered to be caused by losing good casting/die contact, which can be explained by thermal impedance:

Because real surfaces are never truly flat or smooth, the contact plane between a surface and a material can also produce a resistance to the flow of heat. This contact plane is depicted in Figure 5.12 [200]. Actual contact occurs at the high points, leaving air-filled voids where the valleys align. The air voids resist the flow of heat and force more of the heat to flow through the contact points. This constriction resistance is referred to as surface contact resistance and can be a factor at all contacting surfaces. The thermal impedance, \( R \), of a material is defined as the sum of its thermal resistance and any contact resistance between it and the contacting surfaces as defined in Equation (5-6).

\[
R = R_{\text{material}} + R_{\text{contact}} \tag{5-8}
\]

where \( R \) is the thermal impedance, \( R_{\text{material}} \) is the thermal resistance, and \( R_{\text{contact}} \) is the contact resistance.

![Figure 5.12 Schematic representation of two surfaces in contact and heat flow across the interface [200].](image)

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By checking the local temperature (Figures 5.5-5.7) and the materials mechanical properties (yield strength, Figure 5.13), it was found that the initial casting surface temperatures were high and the local surface yield strengths were low. Therefore, good surface contact between casting and die was quickly established by the applied pressures, which led to an increase in heat transfer coefficient significantly because of a decrease in thermal impedance.

At the moment when the heat transfer coefficient reached its peak value, the temperature curves (Figures 5.4-5.6) indicated that the casting surface has dropped to solidus temperature. Meanwhile, from Figure 5.13, it was found that when the temperature is below solidus temperature (435 °C), the corresponding yield strength increases linearly with decreasing temperature. The increased yield strength made plastic deformation of the casting difficult, which minimized compensation for the thermal contraction of the casting. This should weaken the established contact between the casting and die. Hence, the heat transfer coefficient at the top interface gradually decreased.
5.2.2 Heat Transfer at the Side Interface

Figures 5.14 - 5.16 present typical measurements of temperatures at the middle of the casting side surface (T6) and in the side wall of the die (Tms1 and Tms2) for the applied pressures of 30, 60 and 90 MPa, respectively. As can be seen from Figures 5.14 - 5.16 in comparison with the temperature results at the casting top surface, the temperature at the casting side surface dropped very rapidly upon the application of pressures. This is because the applied pressure eliminated the air gap between the casting and the side wall formed during the cavity filling, and consequently enhanced heat transfer between the casting and die. About 2 seconds after its rapid drop, the decreasing rate of the casting temperature was reduced. The reason for this is that the continuous
heat release from the casting to the mold increased the mold temperature and reduced the temperature gradient at the casting and die interface as pressurized solidification proceeded. Meanwhile, because of a certain distance of $\text{Tms}_1$ (0.010 m) and $\text{Tms}_2$ (0.020 m) away from the casting/die interface, there was several seconds of time delay before the die temperatures in the side wall began to arise quickly from the original die temperature about 275 °C. As the process proceeded, the difference between temperature $\text{Tms}_1$ and $\text{Tms}_2$ became evident, which indicates that heat was extracted from the casting by the side wall of the die.

Figures 5.17 - 5.19 gives the instantaneous heat flux across and heat transfer coefficient at the side interface between the casting and die for the applied pressures of 30, 60 and 90 MPa, respectively.

![Figure 5.14 Temperature measurements for HTC calculation adjacent to the side interface between the casting and die under an applied pressure of 30 MPa.](image-url)
Figure 5.15 Temperature measurements for HTC calculation adjacent to the side interface between the casting and die under an applied pressure of 60 MPa.

Figure 5.16 Temperature measurements for HTC calculation adjacent to the side interface between the casting and die under an applied pressure of 90 MPa.

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Figure 5.17 Heat flux & Heat transfer coefficient at the side interface under applied pressure of 30 MPa.

Figure 5.18 Heat flux & Heat transfer coefficient at the side interface under applied pressure of 60 MPa.
Figure 5.19 Heat flux & Heat transfer coefficient at the side interface under applied pressure of 90 MPa.

The heat flux increased with increasing time and reached a peak in about 2.5 seconds and then gradually decreased as the process continued. A significant increase in the heat flux is attributed to the thermal contact at the casting/die interface and the large temperature difference at the beginning, while a gradual decrease of heat flux is considered to be caused by a reduction in the thermal gradient, and an increase in thermal impedance of the die.

Variation of the heat flux at the side interface is very similar to those at the top interface. But, at the side interface, it reached its peak value fast than at the top interface. This is because the air gap between the casting and the side wall was eliminated even during the cavity filling, and consequently enhanced heat transfer between the casting and die.
As to the heat transfer coefficient, however, it arose from a relatively high value and in a relatively fast pace compared with those at the top interface. Its initial heat transfer coefficient is about 2000-2500 W/m² K (compared to less than 500 W/m² K on the top). It took about 15 seconds for the heat transfer coefficient to reach its peak value (22-25 seconds for top). This is because of the contact between the casting and the side wall formed even during the cavity filling. The relatively low side surface temperature of the casting should be responsible for a quick rise to the peak value of heat transfer coefficient. This is because lower temperature resulted in an increase in yield strength of casting, which promotes the thermal impedance at the interface.

5.2.3 Heat Transfer and Local Pressure

For the purpose of comparison, the heat transfer coefficients for the three applied pressures of 30, 60 and 90MPa are plotted in Figures 5.20 and 5.21 for the top and side interfaces, respectively. It appears in Figures 5.20 and 5.21 that within the first 5 seconds, the heat transfer coefficients had almost the same value up to 5000 W/m²·K. However, as the process went on, the heat transfer coefficients became dependent on local pressures. Table 5.2 lists and compares some of the HTC values.

During the initial 5 seconds, the heat transfer coefficients are almost the same under different applied pressures. This can be explained with the help of local pressure measurements as shown in Figures 4.5 - 4.9, where it indicated that local pressure change was not stable until 5 seconds. The initial heat transfer coefficient of about 500 W/m² K for top is close to Cho and Hong’s results [100]. But there exists differences in the initial HTCs between the top and side interfaces, because the side contact was improved by pressure during cavity filling as explained in the previous section (5.2.2).
Figure 5.20 Heat transfer coefficients at the top interface.

Figure 5.21 Heat transfer coefficients at the side interface.
Table 5.2 Comparison of heat transfer coefficient under different applied pressures on both top and side interfaces

<table>
<thead>
<tr>
<th>Applied pressure</th>
<th>Top interface</th>
<th>Side interface</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum h (W/m² K)</td>
<td>Local Maximum Pressure (MPa)</td>
</tr>
<tr>
<td>30 MPa</td>
<td>8400</td>
<td>14</td>
</tr>
<tr>
<td>60 MPa</td>
<td>9400</td>
<td>34</td>
</tr>
<tr>
<td>90 MPa</td>
<td>10090</td>
<td>59</td>
</tr>
</tbody>
</table>

After the initial 5 seconds, although the developing trends of heat transfer coefficients are similar for all the applied pressures, their magnitudes are different from each other. Figures 5.20 and 5.21 manifested that, the higher the local pressure, the higher the local heat transfer coefficient. This should be attributed to the fact that the high pressures improve surface contact. Moreover, under the same applied pressure, the magnitudes of local heat transfer coefficients at the top interface were higher than that at the side. This HTC difference may result from the relatively low surface temperature present at the side compared to the top which increased material strength, consequently weakened surface contact and increased the thermal impedance.

Upon reaching their peak value, all the HTCs started to decrease. However, the decrease of the HTCs at the top interface was slower than those at the side interface. The peak values of heat transfer coefficients obtained under different applied pressures are within the range reported in Nishida and Matsubara’s work [102].
To do regression analysis of the above results, it might be reasonable to include both local pressure and temperature in a formula as below:

\[ h = f(P, T) \]  

(5-9)

where \( h \) is the local heat transfer coefficient, \( P \) and \( T \) are the local pressure and temperature, respectively.

Based on the above experimental data, regression analysis was carried out.

For top surface, a relation was obtained as:

\[ A(q)h(t) = B(q)T(t) + C(q)P(t) + D(q)e(t) \]  

(5-10)

where \( h(t) \) is the time-dependent local heat transfer coefficient, \( P(t) \) and \( T(t) \) are local pressure and temperature respectively, \( e(t) \) is the time-dependent system noise. Factors \( A(q) \), \( B(q) \), \( C(q) \) and \( D(q) \) can be expressed as:

\[
A(q) = 1 - 0.9333q^{-1} - 0.05694q^{-2} \\
B(q) = -66.34 + 66.47q^{-1} \\
C(q) = -9.157 + 9.54q^{-1} \\
D(q) = 1 - 0.6362q^{-1}
\]

where \( q^{-1} \) is the delay operator.

For side surface, the discrete time model could be simulated as

\[ h(t) = \frac{B(q)}{F(q)}u(t) + e(t) \]  

(5-11)

where:

\[
u(t) = \begin{bmatrix} p(t) \\ T(t) \end{bmatrix} \]

\[
B(q) = \begin{bmatrix} B_1(q) \\ B_2(q) \end{bmatrix} \\
F(q) = \begin{bmatrix} F_1(q) & F_2(q) \end{bmatrix} \\
B_1(q) = 1.856q^{-1} - 1.819q^{-2}
\]
Equation (5-10) and (5-11) correlate the heat transfer coefficient as a function of both the local pressure and temperature, which gives evidence that the interface contact is a combined result of the deformation caused by local pressures and contraction due to temperature decrease of die. The delay operators reflect the fact that the change of local pressure and temperature also play an important role in determining the interface contact. Figure 5.22 and 5.23 compare the regression curves, shown as a solid line computed from Equation (5-10) and (5-11), with the original sampled data plotted as symbols. For all the tests, the correlation coefficients are found to a value over 0.90, which provides certain confidence for potential applications.

Figure 5.24 and 5.25 displays a three-dimensional (3-D) plot for the purpose of demonstrating the combining effect of the local pressure and die temperature on the heat transfer coefficient, which is predicted by Equation (5-10) and (5-11).
Figure 5.22 Experimental and simulated heat transfer coefficients at the top interface.

Figure 5.23 Experimental and simulated heat transfer coefficients at the side interface.
Figure 5.24 Experimental and simulated heat transfer coefficients at the top interface.

Figure 5.25 Simulated heat transfer coefficients at the side interface.
5.3 Results and Discussion of Aluminum Alloy A356

5.3.1 Heat Transfer on the Top Surface

Figures 5.26 - 5.28 shows the recorded temperatures histories at the casting top surface center (T1) and in the top wall of the die (Tmt1 and Tmt2) for the applied pressures of 30, 60 and 90 MPa, respectively. As observed from Figures 5.26 - 5.28, the temperature responses are very similar to those for magnesium alloy AM50A described in the previous section.

![Temperature measurements for HTC calculation at the top interface under an applied pressure of 30 MPa.](image-url)
Figure 5.27 Temperature measurements for HTC calculation at the top interface under an applied pressure of 60 MPa.

Figure 5.28 Temperature measurements for HTC calculation at the top interface under an applied pressure of 90 MPa.
Figure 5.29 - 5.31 show the instantaneous heat flux across and heat transfer coefficient at the interface (location 1) between the casting top surface and the top wall of the die as a function of time for the applied pressure of 30, 60 and 90 MPa, respectively.

The heat flux increased to a peak in about 4 seconds and then gradually decreased as the process continued. A significant increase in the heat flux is attributed to the thermal contact at the casting/die interface and the large temperature difference at the beginning, while a gradual decrease of heat flux is considered to be caused by a reduction in the thermal gradient.

![Graph showing heat flux and heat transfer coefficient](image)

Figure 5.29 Heat flux across & Heat transfer coefficient at the top interface between the casting and the die under an applied pressure of 30 MPa.
Figure 5.30  Heat flux across & Heat transfer coefficient at the top interface between the casting and the die under an applied pressure of 60 MPa.

Figure 5.31  Heat flux across & Heat transfer coefficient at the top interface between the casting and the die under an applied pressure of 90 MPa.
It can also be seen from Figures 5.28 - 5.30 that the heat transfer coefficient increased quickly during the first 7 seconds after the pressure was applied. Then it increased slowly as time increased. A significant increase in the heat transfer coefficient at the beginning is attributed to the establishment of a firm thermal contact between the casting and the die as described previously. After 7 seconds, the heat transfer coefficient increase rate slowed down, this is considered to be caused by losing good casting/die contact, which can be explained by thermal impedance (section 5.2.1). A continuous rise of the HTC occurred after the pressure was applied. This observation may be due to the fact that the volume contraction of the casting was further offset by casting deformation resulting from the applied pressure, which improved surface contact between the casting and die.

By examining the variation of the local temperature (Figures 5.25 - 5.27) and the material's yield strength (Figure 5.32), it was found that, at the beginning, the casting surface temperatures were high, the local surface yield strengths were low, therefore good surface contact between casting and die was quickly established by applied pressure, which lead to an increase in heat transfer coefficient significantly because of the decrease of thermal impedance.

As time moves on, the temperature curves (Figures 5.25 - 5.27) indicated that the casting surface has dropped to solidus temperature. Meanwhile, from Figure 5.32, it was found that when the temperature is below solidus temperature (545 °C), the correspondent yield strength increases linearly with decreasing temperature. The increased yield strength directly weakens the established contact between the casting and die. Therefore the heat transfer coefficient increase rate at the top interface slowed down.
5.3.2 Heat Transfer and Local Pressure

To do regression analysis of the heat transfer coefficient for aluminum alloy A356, it is reasonable to also include both local pressure and temperature in a formula just like what expressed in equation (5-9).

Based on the above experimental data, regression analysis was carried out by MatLab software using differential method. The discrete time model could be simulated as

\[ A(q)h(t) = B(q)T(t) + C(q)P(t) + D(q)e(t) \]  

(5-12)

where \( h(t) \) is the time-dependent local heat transfer coefficient, \( P(t) \) and \( T(t) \) are local pressure and temperature respectively, \( e(t) \) is the time-dependent system noise. Factors \( A(q), B(q), C(q) \) and \( D(q) \) can be expressed as:

Figure 5.32 Yield strength of magnesium alloy A356 vs temperature [199].
$A(q) = 1 - 1.983q^{-1} + 0.9828q^{-2}$

$B(q) = -13.49 + 27.26q^{-1} - 13.77q^{-2}$

$C(q) = -181.8 + 363q^{-1} - 181.2q^{-2}$

$D(q) = 1 - 1.789q^{-1} + 0.8187q^{-2}$

$q^{-1} = \text{Delay Operator}$

Figure 5.33 compares the regression curves, shown as a solid line computed from Equation (5-12), with the original sampled data plotted as symbols. For all the tests, the correlation coefficients are found to a value over 0.90, which provides certain confidence for potential applications.

Figure 5.34 displays a three-dimensional (3-D) plot for the purpose of demonstrating the combining effect of the local pressure and temperature on the heat transfer coefficient, which is predicted by Equation (5-12).
5.4 **Summary**

An experimental investigation on heat transfer behavior of squeeze casting of both magnesium alloy AM50A and aluminum alloy A356 has been carried out. The temperature curves of casting and die were directly recorded and used for estimating local heat flux and heat transfer coefficient for both magnesium alloy AM50A and aluminum alloy A356 under applied pressure of 30, 60 and 90 MPa. The application of external pressure on the solidifying both magnesium alloy AM50A and aluminum alloy A356 affects the heat transfer. Local heat flux and heat transfer coefficients rise as
applied pressures increase, because high pressures enhance surface contact and reduce the thermal impedance.

The characteristics of heat transfer at the top and side interfaces are different, which should be attributed to the elimination of air gap at the side interface by the applied pressure during cavity filling. The relatively low side surface temperature of the casting should be responsible for a quick rise to the peak value of heat transfer coefficient. This is because lower temperature resulted in an increase in yield strength of casting, which promotes the thermal impedance at the interface.

The present work demonstrates that the heat transfer coefficient can be derived from the measurement of local pressures and temperatures during squeeze casting. Regression equations are established for determination of heat transfer coefficients during squeeze casting of both magnesium alloy AM50A and aluminum alloy A356 in this study. The established correlation among the heat transfer coefficient, local pressures and temperatures provides an open avenue for designing a strategy for numerical simulation of squeeze casting processes.
In this chapter, an experimental investigation on the non-equilibrium solidification phenomena of magnesium alloy AM50A and aluminum alloy A356 under various applied pressures has been carried out. Thermal and pressure sensors were employed to measure temperature and pressure variation with time and their distribution in a cylindrical squeeze casting. A computer-based thermal analysis system was used to generate cooling curves and their derivatives. Detailed information about pressure influence on liquidus temperatures and solidification rates is presented. Theoretical calculation of non-equilibrium liquidus and solidus temperatures of magnesium alloy AM50A and aluminum alloy A356 was also conducted. The comparison of the calculated results and experimental measurements implies that local pressures, instead of nominal pressures (hydraulic pressure out of the casting machine), play an important role in determining solidification temperatures of magnesium alloys AM50A and aluminum alloy A356. The effect of pressure levels on solidification temperature increases and rates are discussed.

6.1 Results and Discussion of Magnesium Alloy AM50A

6.1.1 Liquidus and Solidus Temperatures Under 0.1 MPa Pressure

Figure 6.1 shows a typical cooling curve of magnesium alloy AM50A under 0.1 MPa pressure. As can be seen from Figure 6.1(a), the temperature at the center of the
casting began to drop upon reaching 683.5 °C which was slightly below the melt temperature of 695 °C due to superheat loss during melt transfer. 50.8 seconds after pouring, the temperature appears decreasing to a plateau. However, the enlarged view of the plateau area given in Figure 6.1(b) reveals that the central temperature dropped to the bottom of a valley, i.e., 624.4 °C. Before further decreasing, the temperature actually rose to 625.1 at 59.6 seconds after pouring. This observation indicates that an undercooling of 0.7 °C took place once solidification commenced from 624.4 °C. It has been suggested [20] that the release of latent heat should be responsible for the temperature rise (625.1), which can be considered as the liquidus temperature of AM50A under 0.1 MPa applied pressure. As cooling further proceeded to 427.8 °C, the cooling curve experienced a slope change at 392.1 seconds as illustrated in Figure 6.1(c). The occurrence of the slope change should be attributed to the formation of eutectic phases which ended the solidification process.

As discussed above, examination of the cooling curve illustrated in Figure 6.1 identifies two distinguished stages during solidification of AM50A alloy under no applied pressure. The nucleation of primary magnesium α-phase happened in stage I, from which the nonequilibrium liquid temperature was recorded as 624.4-625.1 °C. Stage II is the occurrence of the eutectic reaction, i.e., L → Mg (α) + Mg17Al12, where the nonequilibrium solidus temperature was determined as 427.8 °C. Compared to the values reported by other researchers [198, 199, 201-204], the measured nonequilibrium liquidus (625.1 °C) and solidus (427.8) temperatures are in good agreement with the previous data of liquidus (620-627 °C) and solidus (428-435 °C) temperatures listed in Table 6.1.
Temperature (°C)

Experiential Observation on Pressurized Solidification

(a) Whole curve

(b) Liquidus temperature

Temperature (°C)

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Figure 6.1 Cooling curve of AM50A under 0.1 MPa pressure: (a) whole curve, (b) liquidus temperature and (c) solidus temperature.

Table 6.1 Liquidus and solidus temperature of Mg alloy AM50A

<table>
<thead>
<tr>
<th>Source</th>
<th>Liquidus Temperature (°C)</th>
<th>Solidus Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norsk Hydro [198]</td>
<td>620</td>
<td>435</td>
</tr>
<tr>
<td>ASM handbook [201]</td>
<td>620</td>
<td>435</td>
</tr>
<tr>
<td>Magma Database [199]</td>
<td>624</td>
<td>435</td>
</tr>
<tr>
<td>Hu [204]</td>
<td>623</td>
<td>428</td>
</tr>
<tr>
<td>Riddle [202, 203]</td>
<td>627.6</td>
<td>430.2</td>
</tr>
<tr>
<td>This study</td>
<td>625.1</td>
<td>427.8</td>
</tr>
</tbody>
</table>

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6.1.2 Local Cooling Behavior

Representative cooling curves measured in different locations inside a cylindrical squeeze casting of magnesium alloy AM50A solidified under an applied pressure of 60 MPa are shown in Figure 6.2. The temperature at the side interface between the casting and die is represented by line “T6” while line “T3” characterizes the temperature at the top interface between the casting and die. Line “T10” gives the temperature at the casting center. It can be seen from Figure 6.2 that the temperatures at all the three locations (T3, T6 and T10) increased once the thermocouples touched the melts. During period AB, T10 increased to 687.1 °C immediately once the melt was poured into the sleeve cavity. The cavity filling represented by segment BC of Line “T10” commenced at point B. As soon as the cavity filling completed at point C, the desired hydraulic pressure started being applied. As a result, an increase in T10 from 625.1 to 630.5 °C took place. Meanwhile, T6 and T3 increased instantly to the highest points of 434.2 and 547.1 °C from the die temperature of 275 °C, respectively. The recorded temperature measurements indicate that, even at the very early stage of the process, a completely-solidified shell was formed around the side of the casting upon the completion of cavity filling since the highest temperature at the side interface was 434.2 °C close to the solidus temperature of 427.8 °C. The top portion with a temperature of 547.1 °C between the liquidus and solidus temperatures was semi-solid. The formation of a semi-solid shell at the early stage of solidification should be attributed primarily to a relatively slow filling velocity, which was employed for minimizing melt flow turbulence during cavity filling. However, the temperature of 627.8 °C at the casting center was above the liquidus temperature, which indicates that the casting center was fully liquid upon the application...
of the external pressure. Even under the high external pressure of 60 MPa, however, latent heat release and slow heat transfer resulting from the considerably thick section (over 0.05 m) of the casting maintained the central temperature (T10) around 627.8 °C for about 15 seconds before it started decreasing. The exhibition of a plateau on cooling curve T10 implies that the formation of primary –Mg phase took place around 627.8 °C when the latent heat of melt started evolving at the liquidus temperature [205]. Therefore, it can be considered that 627.8 °C is the liquidus temperature of magnesium alloy AM50A which solidifies under an external pressure of 60 MPa.

![Diagram](image-url)

Figure 6.2 Typical experimental results of local temperature measurements in a cylindrical casting of magnesium alloy AM50A solidified under applied pressure of 60MPa.

In addition, it is interesting to see that temperatures T3 and T6 increased relatively faster than T10 at the beginning. This may be primarily because the thermal conductivity
of magnesium alloy AM50A decreases with increasing temperature. The thermal conductivity of liquid AM50A is 79 W/m*K at 627 °C lower than that of the semi-solid or solid alloy (95 W/m*K at 546 °C, and 98 W/m*K at 434 °C) [199]. After reaching their highest points, both temperatures T3 and T6 decreased with time. It is worthwhile mentioning that temperature T3 decreased by 50 °C from 547 to 497 °C significantly faster than that by 34 °C at T6 from 434 to 400 °C within the first 3 seconds. This observation indicates that higher heat transfer took place at T3 than that at T6 due to the presence of a temperature gradient at T3 much steeper than that at T6.

6.1.3 Effect of Applied Pressures on Liquidus Temperature

Figure 6.3 shows typical cooling curves measured in the center of the casting solidified under different the applied pressures of 30, 60 and 90 MPa. It can be seen for all three cases that, once the pressure was applied, the temperature increased rapidly until reaching a plateau, i.e., the liquidus temperature. Then, the temperature was kept almost constant for a certain period of time before starting to decrease. However, the variation of applied pressures influenced the attainable level and the holding duration of the liquidus temperature plateau. The liquidus temperature of magnesium alloy AM50A can be considered 627.0, 627.8 and 630.5 °C for the applied pressures of 30MPa, 60MPa and 90MPa respectively. Compared with the non-equilibrium liquidus temperature of 625.1 °C at 0 MPa, this experimental observation indicates that the liquidus temperature of the alloy increased by 1.9, 2.6 and 5.4 °C under the applied pressure levels of 30MPa, 60MPa and 90MPa respectively.
Figure 6.3 Typical experimental results of temperature measurements at the center of a cylindrical casting of magnesium alloy AM50A solidified under applied pressure of 30MPa, 60MPa and 90MPa.

The integration of Clausius-Clapeyron Equation (2-1) generates an exponential function

\[ T_p = T_m \exp \left( \frac{\Delta V}{H_f} P \right) \]  \hspace{1cm} (6-1)

where \( T_p \) is the solidification temperature under an applied pressure, \( T_m \) is the equilibrium solidification temperature, \( P \) is the applied pressure, \( \Delta V \) is the volume change during solidification, and \( H_f \) is the latent heat of fusion. From Equation (6-1), the liquidus temperature increase (\( \Delta T = T_p - T_m \)) due to applied pressures can be determined with the thermophysical properties of magnesium alloy AM50A listed in Table 5.1. The calculated increases in liquidus temperatures of the alloy is 3.2, 6.5 and 9.7 °C for the...
applied pressures of 30MPa, 60MPa and 90MPa respectively. The comparison of the calculated and experimental results indicated that there is a deviation in terms of liquidus temperatures as illustrated in Figure 6.4. The deviation between the calculated and experimental results increases with increasing applied pressure levels. The deviation may be due to the fact that the hydraulic applied pressures were used in calculation since it is almost impossible to directly measure local pressures at the central location inside the casting where liquid metal was present and the temperatures were measured. This could give rise to an overestimation of local pressures exerted on specific areas in the casting, where temperature measurements were performed as the applied pressure increased.

![Figure 6.4 Change of liquidus temperatures of magnesium alloy AM50A with applied pressures.](image-url)
The theoretical expression (Equation 6-1) indicates explicitly that solidification temperatures of materials should vary exponentially with applied pressures. However, the regression analysis of the calculated results (Figure 6.4) reveals that a linear relation between the liquidus temperatures and applied pressures prevails in the current range of the pressures (0 ~ 90 MPa) which are relatively low:

\[ T_l = a \times P + T_m \]  

(6-2)

where \( T_l \) is the liquidus temperature of magnesium alloy AM50A under applied pressures, \( a \) is a constant equal to 0.092, \( P \) is the applied pressure, \( T_m \) is the nonequilibrium solidification temperature (625.1 °C) at 0 MPa. For Equation 6-2, the regression coefficient is found to be 1.00. Instead of using a complex exponential function for computation of liquidus temperature of alloy AM50A under applied pressures, the linear relation (Equation 6-2) provides a reasonable estimation of the liquidus temperatures.

To evaluate the direct response of the liquidus temperature to the variation of applied pressures, it is essential to measure melt temperature and pressures simultaneously during the initial period of solidification. A representative cooling curve of magnesium alloy AM50A solidified under various pressure levels is presented in Figure 6.5. When the melt was considered to be still at its liquid state at the very early stage of solidification, three different levels of pressures, i.e., 3, 30 and 60 MPa were applied to the melt. Figure 6.6 gives the enlarged region of the cooling curve to reveal the instantaneous response of the liquidus temperature in the center of the casting to the change of applied pressures. At the beginning of solidification, a very low pressure of 3MPa resulted in the liquidus temperature to reach 625.3 °C. The temperature had almost
Figure 6.6 Enlarged curves showing the effect of applied pressures on the liquidus temperature of magnesium alloy AM50A.

### 6.1.4 Pressure Loss during Solidification

Figure 6.7 show the variation of local in-cavity pressures with time at the top center of the casting solidified under different nominal applied pressures. Three lines in Figure 6.7 represent the pressure histories for the applied pressures of 90, 60 and 30 MPa, respectively. It can be seen from Figure 6.7 that the local cavity pressures at all the three pressure levels built up very rapidly once the once the external force was applied to the casting. Then, at each location, the local pressures reached their high levels before starting to decrease. It took around 5 seconds for the local pressures increased to their peak values of 62, 37, and 14 MPa from 0 MPa for the applied pressures of 90, 60 and 30 MPa, respectively. This pressure difference between the instantaneous experimental measurements and the hydraulic applied pressures indicates that there was a pressure loss
during the application of external pressures. Mathematically, the pressure loss \( P_L \) can be deduced from the following formula:

\[
P_L = \left( \frac{P - P_i}{P} \right) \times 100\%
\]  

(6-3)

where \( P_L \) is the pressure loss, \( P \) is the applied pressure, and \( P_i \) is the instantaneous local pressure which is experimentally measured after the pressure peak. The comparison of the pressure losses at the different pressure levels is illustrated in Figure 6.8. It can be seen from Figure 6.8 that the pressure loss decreased initially and then increased with time for all the three applied pressure levels. A large percentage of the pressure loss occurred at a relatively low level of the applied pressure of 30 MPa compared to those of 90 and 60 MPa. The pressure loss rose significantly as the applied pressure decreased. The alloy densification and deformation induced by the applied pressure may primarily responsible for the behavior of the pressure loss during pressurized solidification. As discussed in the previous section (Figure 6.2), the casting was relatively hot and has low yield strength at the early stage of pressurized solidification. Since temperature variation inside the geometry of the casting varied yield strengths of the alloy significantly from one location to another, non-uniform densification and deformation took place in the casting once the applied pressure set in. The non-uniform densification of the alloy and deformation of the casting could lead to the formation of friction internally and externally, respectively. At a low pressure level (30MPa), a large percentage of the applied pressure was probably needed for alloy densification. However, the percentage of the pressure required for densification decreased as the applied pressure increased. As a result, the retained portion of applied pressures, i.e., local pressures, in the casting during pressurized solidification decreased with increasing applied pressures. This implies that,
instead of using the local pressure, the direct employment of the applied pressure to calculate liquidus temperatures increases the deviation between the calculated results and experimental measurements as illustrated in Figure 6.4. As pressurized solidification proceeds, an increase in pressure loss may be due to the presence of high friction between the casting and die since either plastic or elastic deformation of the casting resulting from high applied pressures could be more predominant than its thermal contraction.

Figure 6.7 Typical pressure histories during the solidification of magnesium alloy AM50A under the different applied pressures of 30, 60, and 90 MPa.
6 EXPERIMENTAL OBSERVATION ON PRESSURIZED SOLIDIFICATION

Figure 6.8 Pressure losses during the solidification of magnesium alloy AM50A under the different applied pressures of 30, 60, and 90 MPa.

6.1.5 Determination of Solidus Temperature

Although the cooling curves in Figure 6.3 markedly show the liquidus temperatures of alloy AM50A solidified under different applied pressures, it is very difficult to determine the solidus temperature, i.e., the end of the solidification process, directly from the measured temperature-time curves. It has been indicated [205-207] that the solidus temperature resulting from the eutectic reaction in thermal analysis becomes much less distinct as the analysis is performed at relatively high cooling rates over 3.5 °C/s. It should be noted that the cooling rate employed in this study is around 6.3 °C/s. To facilitate the evaluation of solidus temperatures of alloy AM50A solidified under different applied pressures, the measured temperature-time curves in Figure 6.3 were differentiated to obtain the first-derivative curves. This is because, in general, the
derivative curve shows distinctly a discontinuity when the heat of solidification or phase transformation evolves.

Figures 6.9 – 6.11 present representative curves in both the temperature-time and first derivative formats for magnesium alloy AM50A solidified under applied pressures of 30, 60 and 90 MPa, respectively. It has been observed that the solidification sequence of AM50A, like other magnesium alloys, starts with the nucleation of primary magnesium (α-Mg), and ends with solidification reactions involve the formation of eutectic phases [27, 110, 208, 209]. Equipped with this knowledge, the last discontinuity (S) present in the $dT/dt$ curves shown in Figures 6.9 – 6.11 is identified as the solidus temperature of AM50A under applied pressures, at which the $\beta$-Al$_{12}$Mg$_{17}$ eutectic transformation occurred. The experimentally determined solidus temperatures under the applied pressures of 30, 60 and 90MPa are 428.5, 429.6 and 430.7, respectively. Compared with the non-equilibrium solidus temperature of 427.8 °C at 0 MPa, this experimental observation indicates that the solidus temperature of the alloy increased by 0.8, 1.9 and 2.9 °C under the applied pressure levels of 30, 60 and 90 MPa, respectively.

Similar to the determination of liquidus temperature, the theoretical solidus temperatures are also calculated by using Equation (6-1) with the thermophysical properties of magnesium alloy AM50A listed in Table 5.1.
Figure 6.9 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 30 MPa.

Figure 6.10 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 60 MPa.
Figure 6.11 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 90 MPa.

Figure 6.12 shows the effect of applied pressures on the theoretically-calculated solidus temperatures of the alloy. The regression analysis of the calculated results reveals that a linear relation between the solidus temperatures and applied pressures is also available in the current range of the pressures (0 - 90 MPa) which are relatively low,

\[ T_s = b \times P + T_{s,m} \]  \hspace{1cm} (6-4)

where \( T_s \) is the solidus temperature of magnesium alloy AM50A under applied pressures, \( b \) is a constant equal to 0.072, \( P \) is the applied pressure, \( T_{s,m} \) is the solidus temperature (427.8 °C) at 0 MPa. For Equation 6-4, the regression coefficient is found to be 1.00. Instead of using a complex exponential function for computation of solidus temperature of alloy AM50A under applied pressures, the linear relation (Equation 6-4) provides a reasonable estimation of the solidus temperatures.
The calculated increases in solidus temperature of the alloy is 2.5, 5.0 and 7.6 °C for applied pressure of 30MPa, 60MPa and 90MPa respectively. The comparison of the calculated and experimental results indicated that there is a deviation in terms of solidus temperatures as illustrated in Figure 6.12. The deviation between the calculated and experimental results increases with increasing applied pressure levels. The deviation may be attributed to the fact that the hydraulic applied pressures was used in calculation since it is almost impossible to directly measure local pressures at the central location inside the casting where the temperatures were measured. This could give an overestimation of local pressures exerted on specific areas in the casting, where temperature measurements were preformed, as the applied pressures increased.

Figure 6.12 Change of solidus temperature of magnesium alloy AM50A with applied pressure.
6.1.6 Effect of Applied Pressures on Solidification Times, Rates and Temperatures Increases

Figure 6.13 shows the variation of solidification times and rates at the center of the cylindrical casting AM50A with applied pressures. As indicated in Figure 6.13, an increase in applied pressures from 3 to 60 MPa significantly reduces the solidification time and consequently enhances the solidification rates. This phenomenon could be attributed to not only solidification temperature increases by applied pressures but also the effect of air gap between the casting and die on heat transfer coefficients at the casting/die interface. The study by Trovant and Argyropoulos [177] indicates that minimizing the size of air gap enhances heat transfer from the casting to the die. The elimination of the air gap becomes possible with the help of applied pressures. Due to the removal of the air gap, the applied pressures reduce the thermal resistance substantially, and consequently increase heat transfer rates at the interface [9, 20, 217].

![Figure 6.13 Variation of solidification times and rates with applied pressures.](image-url)
It appears from Figure 6.13 that the change in the solidification time and rate levels off as the applied pressure approaches 60 MPa. Further increase in the pressure beyond the value of 60 MPa has a minor influence on the solidification time and rate. The reason for this is probably because, once the air gap being eliminated, further increase in the pressure could lead to no further improvement in heat transfer at the casting/die interface.

The effect of applied pressures on the measured increases in both the liquidus and solidus temperatures are illustrated in Figure 6.14. The solidification temperature increases of AM50A rise with increasing applied pressures.
From the measured results, the following correlations were deduced based on the regression analysis:

\[ \Delta T_i = 0.0566 \times P - 0.078 \]  \hspace{1cm} (6-5)

\[ \Delta T_s = 0.0329 \times P - 0.099 \]  \hspace{1cm} (6-6)

where \( \Delta T_i \) is the increase in liquidus temperature, \( \Delta T_s \) is the increase in solidus temperature, \( P \) is the applied pressure. Equations 6-5 and 6-6, with the regression coefficient 0.954 and 0.993 respectively, suggest that the solidification temperature increase is linearly proportional to the applied pressure when the pressure varies from 0 to 90 MPa. The magnitude of the slope of Equation 6-5, i.e., the increasing rate of liquidus temperature (0.0566 °C/MPa), is somewhat higher than that (0.0329 °C/MPa) of the solidus temperature. This indicates that the applied pressure influences the liquidus temperature more significantly than the solidus temperature. The finding of the increasing rate of the liquidus temperature of AM50A with the applied pressure is very close to the value of 0.06 °C/MPa for magnesium reported in reference 13, 34 and 98.

6.2 Results and Discussion of Aluminum Alloy A356

6.2.1 Liquidus and Solidus Temperature Under 0.1 MPa Pressure

Figure 6.15 shows typical cooling curves of aluminum alloy A356 under 0.1 MPa pressure in both the temperature-time and derivative formats. Examination of the cooling curves illustrated in Figure 6.15 identifies following reactions:

a. The temperature at the center of the casting began to drop upon reaching 719.5 °C. 71.5 seconds after pouring, the temperature appeared decreasing to a plateau.

However, the enlarged view of area a given in Figure 6.15 (b) reveals that the

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central temperature dropped to the bottom of the temperature valley, i.e., 613.2 °C. Before further decreasing, the temperature actually rose to 613.5 at 80.8 seconds after pouring. This observation indicates that an undercooling of 0.3 °C took place once solidification commenced from 613.2 °C. It has been suggested [21] that the release of latent heat should be responsible for the temperature rise (613.5), which can be considered as the liquidus temperature of A356 under 0.1 MPa applied pressure. The formation of primary aluminum dendrites termed reaction a began at this stage.

b. As cooling further proceeded to 567.5 °C, the cooling curve experienced a slope change at 272.5 seconds as illustrated in Figure 6.15 (c). The occurrence of the slope change should be attributed to the formation of main silicon eutectic phases (reaction b).

c. A final eutectic reaction $L \rightarrow Al + Si + Mg_2Si$, named reaction c, started at 558.4 °C and ended at 540.5 °C (Figure 6.15 (d)), which was determined as the nonequilibrium solidus temperature. Compared to the values reported in references 198, 210, 211 and 212, the measured nonequilibrium liquidus (613.5 °C) and the solidus (554.4 °C) temperatures are in good agreement with the previous data of liquidus (613-615 °C) and solidus (542-555 °C) temperatures listed in Table 6.2.
6 EXPERIMENTAL OBSERVATION ON PRESSURIZED SOLIDIFICATION

(a) Whole cooling curve

(b) Reaction a

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Figure 6.15 Cooling curve of A356 under 0 MPa pressure: (a) whole curve, (b) reaction a, (c) reaction b and (d) reaction c.
Usually, after the main silicon-forming eutectic reaction, a posteutectic reaction is evident in nonstrontium-treated alloys. Since the alloy employed in this study was modified by strontium, this reaction was obscured by lowering the eutectic associated with modification [213].

Table 6.2 Reported liquidus and solidus temperatures of Al alloy A356

<table>
<thead>
<tr>
<th></th>
<th>Liquidus Temperature (°C)</th>
<th>Solidus Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASM handbook [210]</td>
<td>615</td>
<td>555</td>
</tr>
<tr>
<td>Magma Database [199]</td>
<td>613</td>
<td>542</td>
</tr>
<tr>
<td>This study</td>
<td>613.5</td>
<td>554.4</td>
</tr>
</tbody>
</table>

6.2.2 Local Cooling Behavior

Representative cooling curves measured in different locations inside a cylindrical squeeze casting of aluminum alloy A356 solidified under an applied pressure of 90 MPa are shown in Figure 6.16.
Figure 6.16 Typical experimental results of temperature measurements at the center of a cylindrical casting of aluminum alloy A356 solidified under applied pressure of 90MPa.

The temperature at the side interface between the casting and die is represented by line "T6" while line "T2" characterizes the temperature at the top interface between the casting and die. Line "T10" gives the temperature at the casting center. It can be seen from Figure 6.16 that the temperatures at all the three locations (T2, T6 and T10) increased once the thermocouples touched the melts. During period AB, T10 increased to 694.5 °C immediately once the melt was poured into the sleeve cavity. The cavity filling represented by segment BC of Line "T10" commenced at point B. As soon as the cavity filling completed at point C, the desired hydraulic pressure started being applied. As a result, an increase in T10 from 613.5 to 617.7 °C took place. Meanwhile, T6 and T2 increased instantly to the highest points of 550 and 570 °C from the die temperature of...
275 °C, respectively. The temperatures on the top and side surfaces of the casting indicated that a semi-solid shell was formed. The formation of a semi-solid shell at the early stage of solidification should be attributed primarily to a relatively slow filling velocity, which was employed for minimizing melt flow turbulence during cavity filling. However, the temperature of 617.7 °C at the casting center was above the liquidus temperature, which indicates that the casting center was fully liquid upon the application of the external pressure. Even under the high external pressure of 90 MPa, however, latent heat release and slow heat transfer resulting from the considerably thick section (over 0.05 m) of the casting maintained the central temperature (T10) around 617.7 °C for about 12.6 seconds before it started decreasing. The exhibition of a plateau on cooling curve T10 implies that the formation of primary-Al phase took place around 617.7 °C when the latent heat of melt started evolving at the liquidus temperature [213]. Therefore, it can be considered that 617.7 °C is the liquidus temperature of aluminum alloy A356 which solidified under an external pressure of 90 MPa.

6.2.3 Effect of Applied Pressures on Liquidus Temperature

Figure 6.17 shows typical cooling curves measured in the center of the casting solidified under different applied pressures of 30, 60 and 90 MPa. It can be seen for all three cases that, once the pressure was applied, the temperature increased rapidly until reaching a plateau, i.e., the liquidus temperature. Then, the temperature was kept almost constant for a certain period of time before starting to decrease. However, the variation of applied pressures influenced the attainable level and the holding duration of the liquidus temperature plateau. The liquidus temperature of aluminum alloy A356 is 614.4, 615.8 and 617.7 °C for applied pressure of 30, 60 and 90 MPa respectively. Compared with the
non-equilibrium liquidus temperature of 613.5 °C at 0.1 MPa, this experimental observation indicates that the liquidus temperature of the alloy increased by 0.9, 2.1 and 4.1 °C under the applied pressure levels of 30, 60 and 90 MPa respectively.

Figure 6.17 Typical experimental results of temperature measurements at the center of a cylindrical casting of aluminum alloy A356 solidified under applied pressure of 30, 60 and 90 MPa.

The integration of Clausius-Clapeyron Equation (2-1) generates an exponential function

\[ T_p = T_m \exp \left( \frac{\Delta V}{H_f} P \right) \]  

(6-7)

where \( T_p \) is the solidification temperature under an applied pressure, \( T_m \) is the equilibrium solidification temperature, \( P \) is the applied pressure, \( \Delta V \) is the volume change during solidification, and \( H_f \) is the latent heat of fusion. From Equation (6-7), the liquidus
temperature increase \((\Delta T = T_p - T_m)\) due to applied pressures can be determined with the thermophysical properties of aluminum alloy A356 listed in Table 5.1. The calculated increases in liquidus temperatures of the alloy is 1.5, 3.0 and 4.5 °C for applied pressures of 30, 60 and 90 MPa respectively. The comparison of the calculated and experimental results indicated that there is a deviation in terms of liquidus temperatures as illustrated in Figure 6.18. The deviation between the calculated and experimental results increases with increasing applied pressure levels. The deviation may be attributed to the fact that the hydraulic applied pressures was used in calculation since it is almost impossible to directly measure local pressures at the central location inside the casting where liquid metal was present and the temperatures were measured. This could give an overestimation of local pressures exerted on specific areas in the casting, where temperature measurements were preformed as the applied pressure increases.

The theoretical expression (Equation 6-7) indicates explicitly that solidification temperatures of materials should vary exponentially with applied pressures. However, the regression analysis of the calculated results (Figure 6.18) reveals that a linear relation between the liquidus temperatures and applied pressures prevails in the current range of the pressures \((0 \sim 90 \text{ MPa})\) which are relatively low:

\[
T_l = a \times P + T_m
\]  

(6-8)

where \(T_l\) is the liquidus temperature of aluminum alloy A356 under applied pressures, \(a\) is a constant to be 0.045, \(P\) is the applied pressure, \(T_m\) is the non-equilibrium solidification temperature \((613.3 \text{ °C})\) at 0 MPa. For Equation 6-8, the regression coefficient is found to be 0.977. Instead of using a complex exponential function for computation of liquidus temperatures of alloy A356 under an applied pressure between 0-
90 MPa, the linear relation (Equation 6-8) provides a reasonable estimation of the liquidus temperatures.

![Graph](image)

Figure 6.18 Change of liquidus temperatures of aluminum alloy A356 with applied pressures.

### 6.2.4 Determination of Solidus Temperature

Although the cooling curves in Figure 6.17 markedly show the liquidus temperatures of alloy A356 solidified under different applied pressures, it is very difficult to determine the solidus temperature, i.e., the end of the solidification process, directly from the measured temperature-time curves. To facilitate the evaluation of solidus temperatures of alloy A356 solidified under different applied pressures, the measured temperature-time curves in Figure 6.17 were differentiated to obtain the first-derivative curves. This is because, in general, the derivative curve shows distinctly a discontinuity when the heat of solidification or phase transformation evolves.
Figures 6.19-6.21 present representative curves in both the temperature-time and first derivative formats for aluminum alloy A356 solidified under applied pressures of 30, 60 and 90 MPa, respectively. It has been observed that the curves for three different levels of applied pressures are very similar. The solidification sequence of A356 starts with the nucleation of primary aluminum (reaction a), then the main silicon eutectic phases (reaction b), and ends with the final eutectic reaction c. But, different from the curves in Figure 6.15 obtained under 0 MPa, for reaction c, only starting temperature can be identified. This may because that the applied pressure altered the solidification condition, consequently, changed the solidification behavior. Figures 6.22 and 6.23 displays the microstructures of aluminum alloy A356 solidified under the applied pressure of 0 and 60 MPa respectively. As can be seen from Figure 6.22, primary Al, eutectic silicon, Chinese script α-AlFeSi and AlMgCuSi phases are present in the microstructure of alloy A356 solidified without the applied pressure. However, only fine eutectic silicon and α-AlFeSi phases plus primary Al phases can be identified in the alloy solidified under 60 MPa pressure (Figure 6.23). This metallographic observation suggests that reaction c for forming AlMgCuSi was suppressed during pressurized solidification, in which the alloy experienced a relatively fast cooling. The suppression of reaction c was also observed by Tenekedjieva et al [213] when cooling rates increased from 0.8 to 3.5 °C/s.

Because of the suppression of reaction c, discontinuity b present in the dT/dt curves shown in Figure 6.20-6.22 may be considered as the solidus temperature of the alloy solidified under applied pressures. Based on this consideration, the solidus temperature of alloy A356 for the applied pressures of 30, 60 and 90 MPa are 555.3, 556.0
and 557.1 °C, respectively. Compared with the non-equilibrium reaction temperature of 554.4 °C for reaction b, the experimental observation indicates that the solidus temperature of the alloy increased by 0.9, 1.6 and 2.7 °C for the pressures of 30, 60 and 90 MPa, respectively.

Figure 6.19 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 30 MPa.
Figure 6.20 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 60 MPa.

Figure 6.21 Cooling curve and their 1st derivative (dT/dt) curve under pressure of 90 MPa.
Figure 6.22 The microstructure for Al alloy A356 cast under a pressure of 0 MPa shows 1) eutectic silicon, 2) Al-Fe-Si phases and 3) Al-Mg-Cu-Si plates.

Figure 6.23 The microstructure for Al alloy A356 cast under a pressure of 60 MPa shows 1) eutectic silicon and 2) Al-Fe-Si phase.
The effect of applied pressures on the measured increases in both the liquidus and solidus temperatures of aluminum A356 are illustrated in Figure 6.24. The solidification temperature increases of A356 rise with increasing applied pressures.

![Figure 6.24 Change of solidification temperatures of A356 with applied pressures](image)

From the measured results, the following correlations were deduced based on the regression analysis:

\[
\Delta T_l = 0.0444 \times P - 0.237 \quad (6-9)
\]

\[
\Delta T_s = 0.0297 \times P - 0.099 \quad (6-10)
\]

where \(\Delta T_l\) is the increase in liquidus temperature, \(\Delta T_s\) is the increase in solidus temperature, \(P\) is the applied pressure. Equations 6-9 and 6-10, with the regression coefficient 0.972 and 0.991 respectively, suggest that the solidification temperature...
increase is linearly proportional to the applied pressure when the pressure varies from 0 to 90 MPa. The magnitude of the slope of Equation 6-5, i.e., the increasing rate of liquidus temperature (0.0444 °C/MPa), is somewhat higher than that (0.0297 °C/MPa) of the solidus temperature. This indicates that the applied pressure influences the liquidus temperature more significantly than the solidus temperature.

6.3 Summary

The application of external pressures affects the solidification behaviour of magnesium alloy AM50A and aluminum alloy A356.

For magnesium alloy AM50A, both the liquidus and solidus temperatures rise as applied pressures increase. The experimental observation indicates that the liquidus temperature of AM50A alloy increased by 1.9, 2.6, and 5.4 °C and the solidus temperature by 0.8, 1.9, and 2.9 °C under the applied pressure levels of 30, 60 and 90 MPa, respectively. For aluminum alloy A356, the experimental observation indicates that the liquidus temperature of A356 alloy increased by 0.9, 2.1 and 4.1 °C under the applied pressure levels of 30, 60 and 90 MPa, respectively. Despite the difficulty in their experimental measurement, the instantaneous local pressures are responsible for the solidification temperature increases. The direct employment of the hydraulic applied pressure to calculate liquidus temperatures by the Clausius-Clapeyron equation gives rise to an overestimation of both the solidus and liquidus temperatures. The deviation between the calculated results and experimental measurements increases with increasing applied pressures. This deviation may be attributed to the decreasing extent of local
pressures required for alloy densification during solidification, once the applied hydraulic pressures increases beyond a certain level.

The experimental results reveal that the increasing rate of liquidus temperature (0.0566 °C/MPa), is somewhat higher than that of the solidus temperature (0.0329 °C/MPa) for magnesium alloy AM50A. This indicates that the applied pressure influences the liquidus temperature more significantly than the solidus temperature. Also, an increase in the applied pressure reduces solidification times and increases solidification rates of the alloy due to enhanced heat transfer at the casting/die interface. It seems that an applied pressure of 60 MPa is sufficient to maximize the solidification rate. Any further increase in the pressure beyond the value of 60 MPa has only a minor influence on the solidification time and rate.
7 MATHEMATICAL MODELING OF SQUEEZE CASTING

This study deals with the development of a complex mathematical model to simulate a squeeze casting process. The model not only solves the momentum and the energy equations for heat transfer and fluid flow, but also takes the dynamic boundary conditions at the casting/die interface into consideration. Due to the complexity of governing equations which describe the squeeze casting process, numerical techniques are necessary for solving the problem. In this study, a numerical technique based on the primitive variable approach and the control volume finite difference method known as the SIMPLER algorithm [214] was employed.

7.1 Mathematical Formulation for Heat Transfer and Fluid Flow

The analysis of transport phenomena involved in squeeze casting as shown in Figure 3.26 indicates that the process consists of two primary steps, which are (1) cavity filling and (2) pressurized solidification of melt in a die cavity. Although heat transfer involves in both the steps, forced convection takes place only during cavity filling while natural convection is present in the step of pressurized solidification [9, 60].

Due to the symmetric nature of the squeeze cast coupon, only half of the squeeze casting system was required for the computation, and can be considered by a two-dimensional cylindrical model (Figure 7.1).
The assumptions for this model include the following:

- The flow is incompressible;
- The fluid is Newtonian and the physical properties such as density, thermal conductivity, and viscosity are temperature dependent.
- The viscous dissipation terms in the energy equation are neglected because the fluid is incompressible. Hence the velocity gradients in the system are small.

7.1.1 Governing Equations

Based on the above assumptions, the governing equations in cylindrical coordinates can be written as follows:

For casting process, the energy equation in two dimension cylindrical system can be expressed as:
The governing equations for laminar and incompressible flow with no viscous dissipation in cylindrical coordinate systems can be expressed as following:

**Continuity equation:**

\[
\frac{\partial \rho}{\partial t} + \rho \left( \frac{\partial u_r}{\partial r} + \frac{u_z}{r} \frac{\partial u_r}{\partial z} \right) + \frac{\partial (\rho u_z)}{\partial z} = 0
\]  

(7-2)

**Momentum equation:**

**R component**

\[
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{\partial P}{\partial r} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) + \frac{\partial^2 u_r}{\partial z^2} \right] + \rho g_r
\]  

(7-3)

**Z component**

\[
\rho \left( \frac{\partial u_z}{\partial t} + u_z \frac{\partial u_z}{\partial r} + u_r \frac{\partial u_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho g_z
\]  

(7-4)

7.1.2 Enthalpy Technique

Since the energy equation consists of both transient and conductive terms in which two dependent variables are included, temperature \( T \) and enthalpy \( H \), an enthalpy technique [217-219] which accounts for the phase change is used to convert the energy equation into a non-linear equation with only one dependent variable, enthalpy. Thus, the existing SIMPLER [214] algorithm can easily be implemented. Temperature and enthalpy are related via the following state equation based on thermodynamics:

\[
\frac{dH}{dT} = C(T)
\]  

(7-5)
If a constant specific heat for each phase is considered and $H = 0$ is chosen to correspond to alloy at its solidus temperature, the relation between temperature and enthalpy becomes

$$T = \begin{cases} 
\frac{T_s + H}{C_s} & ; H \leq 0 \\
T_s + H \frac{\Delta T}{(C_m \Delta T + H_f)} & ; 0 < H < H_f + C_m \Delta T \\
T_s + \frac{H - H_f - (C_m - C_l) \Delta T}{C_l} & ; H \geq H_f + C_m \Delta T
\end{cases}$$

(7-6)

where $\Delta T = T_i - T_s$ is the solidification range.

The energy equation can be rewritten as follows.

$$\frac{\partial (\rho H)}{\partial t} + u_r \frac{\partial (\rho H)}{\partial r} + u_z \frac{\partial (\rho H)}{\partial z} = \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial H}{\partial r} \right) + \frac{\partial H}{\partial z} \frac{\partial H}{\partial z} \right] + \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial S}{\partial r} \right) + \frac{\partial^2 S}{\partial z^2} \right]$$

(7-7)

$$\begin{cases} 
\text{Solid phase} & H \leq 0 \gamma = \frac{k_s}{C_s} S = 0 \\
\text{Mushy phase} & 0 < H < H_f + C_m \Delta T \gamma = \frac{k_m \Delta T}{(C_m \Delta T + H_f)} S = 0 \\
\text{Liquid phase} & H \geq H_f + C_m \Delta T \gamma = 1, S = \frac{k_l}{C_l} [H_f + (C_m - C_l) \Delta T]
\end{cases}$$

where

$$k_m = \lambda k_l + (1 - \lambda) k_s, \quad C_m = \lambda C_l + (1 - \lambda) C_s, \quad \rho_m = \lambda \rho_l + (1 - \lambda) \rho_s, \quad \lambda = \frac{H}{H_f}$$

As shown, Equation (7-7) is expressed only in terms of a single dependent variable, $H$, and one source term $S$. The transformation of the energy equation simplifies the numerical solution of the present case and makes it possible to use the existing algorithm.
7.1.3 Forced Convection During Cavity Filling

It should be pointed out that laminar behavior (Re < 2300) is a dominating flow mechanism in most squeeze casting processes. During cavity filling, it was essential to take forced convection into consideration for modeling the events of heat transfer between the cavity and melt. Detailed description of forced convection will be given in section 7.3.2.2.

7.1.4 Melt Velocity Upon Filling Completion

In pressurized casting processes, the phenomena of plunger actuation caused by forcing the molten metal into the die have been called water hammer [215]. The water hammer mechanism describes that completely stopping the melt takes place at the end of the die cavity once the melt contacts the end surface. Farther away upstream, the melt is still moving so that the end-contacting melt is compressed with an increase in its pressure and density. The interface between the stationary and moving melt could travel in the opposite direction of filling. The velocity of this interface movement can be expressed as:

\[ c = \frac{E}{\sqrt{\rho}} \]  

(7-8)

where \( E \) is the bulk modulus of the fluid media and \( \rho \) is the fluid density, and \( c \) is the speed of sound in a substance. Once the interface moves through the entire melt, the melt velocity can be assumed to be zero [216]. The water hammer mechanism was adopted in this study to determine the velocity transition of melt flow when the die cavity filling was completed and before a desired pressure was applied.
Since E is about 500 MPa and ρ is about 1650 kg/m³ for AM50A melt [199], the sonic speed calculated using Equation 7-8 is about $5.5 \times 10^2$ m/s. This indicates that the melt flow would stop immediately upon reaching the internal top surface of the die cavity, because the traveling time for the melt flow bouncing back from the surface over the entire cavity was only 0.0144 ms. In this model, it was assumed that the velocity of the melt became zero instantaneously after the melt contacted the internal top surface of the die cavity. During the computation, this assumption was employed as one of the boundary conditions.

### 7.1.5 Natural Convection

In the momentum equations, it was assumed that the fluid follows the Boussinesq approximation. Therefore, the density $\rho$ is regarded as temperature dependent on the body forces but it is constant elsewhere in the equations. The relationship between the density and temperature can be written as follows:

$$\rho = \rho_{\text{ref}} \left[1 - \beta(T - T_{\text{ref}})\right] \quad (7-9)$$

where $\rho_{\text{ref}}$ is the constant value of density used throughout the momentum and continuity equations, $T_{\text{ref}}$ is reference temperature (typically the melting temperature of the material is chosen), and $\beta$ is the volumetric coefficient of thermal expansion of the material. By defining the following effective pressure, $P'$:

$$P' = P + \rho_{\text{ref}} g \quad (7-10)$$

By substituting it into the momentum equations, it results in the following expression for the gravity term:

$$g\rho_{\text{ref}} \beta(T - T_{\text{ref}}) \quad (7-11)$$
Hence, the constant density term has been replaced by a temperature varying relation which allows convection effects to be incorporated based on the temperature difference \((T - T_{\text{ref}})\).

In order to solve the problem in a more general way, it is convenient to non-dimensionalize the governing equation. This can be done by defining dimensionless independent variables of the form [217-219]:

\[
\begin{align*}
    z^* &= \frac{z}{L}; r^* = \frac{r}{L}; u_z^* = \frac{u_z L}{\alpha}; u_r^* = \frac{u_r L}{\alpha}; H^* = \frac{H}{H_f + C_m \Delta T}; t^* = \frac{t \alpha_1}{L^2};
    \\
    \rho^* &= \frac{\rho_1}{\rho_{\text{ref}}}; P^* = \frac{P}{\rho_1 \alpha_1 / L^2}; S^* = \frac{S c_l}{k_1 (H_f + C_m \Delta T)}
\end{align*}
\]

The dimensionless governing equations are:

\[
\frac{\partial H^*}{\partial t^*} = \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \Gamma_\mu \frac{\partial H^*}{\partial r^*} \right) + \frac{\partial}{\partial z^*} \left( \Gamma_\mu \frac{\partial H^*}{\partial z^*} \right) \right] + \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* S^* \right) + \frac{\partial^2 S^*}{\partial z^*} \right] \quad (7-12)
\]

\[
S_E = \left[ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* S^* \right) + \frac{\partial^2 S^*}{\partial z^*} \right] \quad (7-13)
\]

\[
\frac{1}{r^*} \frac{\partial (r^* u_r^*)}{\partial r^*} + \frac{\partial u_z^*}{\partial z^*} = 0 \quad (7-14)
\]

\[
\frac{\partial u_{z1}^*}{\partial t^*} + u_r^* \frac{\partial u_{z1}^*}{\partial r^*} + u_z^* \frac{\partial u_{z1}^*}{\partial z^*} = - \frac{\partial P^*}{\partial r^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \Gamma_\mu \frac{\partial u_{z1}^*}{\partial r^*} \right) + \frac{\partial}{\partial z^*} \left( \Gamma_\mu \frac{\partial u_{z1}^*}{\partial z^*} \right) - \left( \frac{\partial}{\partial r^*} \left( \Gamma_\mu \frac{\partial u_{z1}^*}{\partial r^*} \right) \right) \quad (7-15)
\]

\[
\frac{\partial u_{z2}^*}{\partial t^*} + u_r^* \frac{\partial u_{z2}^*}{\partial r^*} + u_z^* \frac{\partial u_{z2}^*}{\partial z^*} = - \frac{\partial P^*}{\partial z^*} + \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \Gamma_\mu \frac{\partial u_{z2}^*}{\partial r^*} \right) + \frac{\partial}{\partial z^*} \left( \Gamma_\mu \frac{\partial u_{z2}^*}{\partial z^*} \right) + \text{Gr}_{\text{mT}} (H^* - 1) \quad (7-16)
\]

where:

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7 MATHERMATICAL MODELING IN SQUEEZE CASTING

\[
\begin{aligned}
\text{Solid Phase } & \quad h^* \leq 0; \quad \Gamma_a = \alpha_s / \alpha_i; \quad \Gamma_\mu = \infty; \quad S^* = 0; \\
\text{Mushy Phase } & \quad 0 < h^* < 1; \quad \Gamma_a = \frac{k_m \Delta T}{\rho_m \alpha_i (C_m \Delta T + H_m)}; \\
& \quad \Gamma_\mu = Pr; \quad S^* = 0; \\
\text{Liquid Phase } & \quad h^* \geq 1; \quad \Gamma_a = 1; \quad \Gamma_\mu = Pr; \\
& \quad S^* = -1 + \frac{C_i \Delta T}{H_f + C_m \Delta T};
\end{aligned}
\]

Pr = ν / α is the Prandtl number; Gr = gβL^3 H_f / α^2 ρ^* C is a modified Grashof number.

7.2 Mathematical Formulation for Casting/Die Contact

As described in the previous chapters, during the solidification of squeeze casting, the local pressures between casting and die not only change with locations but also time. It is critical to take this variation into account, which makes the simulation results more reliable.

7.2.1 Theoretical Consideration of Casting Deformation Under Pressure

The magnesium alloy solidification process involves multiphase transforming from liquid to semi-solid, and then to solid (Figure 7.2). Different from most of other casting processing, while high pressure is applied in the entire solidification step of squeeze casting, casting deformation happens due to the application of the external pressure (stress). Therefore, it is necessary to analyze deformation mechanisms involved in solidification.
7.2.1.1 Thermal Contraction

During the solidification of squeeze casting, the temperature of molten metal decreases, and consequently it results in a volume contraction $\Delta V_s$, which can simply be expressed as:

$$\Delta V_s = \alpha \Delta T V$$  \hspace{1cm} (7-17)

where $\alpha$ is the coefficient of thermal expansion, $\Delta T$ is the change in temperature, and $V$ is the original volume. The rate of volume contraction ($v_s = \frac{\Delta V_s}{t}$) depends on solidification rate. In squeeze casting, pressure buildup has to be faster than solidification, which enables applied pressure to densify casting and minimize defect formation [76].
7.2.1.2 Plastic Deformation

As the melt continues to cool down, it first becomes mushy state and then solid from outside to inside. Since the yield stress of the alloys at high temperature is very low, plastic deformation must be considered when it is within a high temperature range.

Generally, the yield stress at high temperature can be expressed as:

\[ \sigma = \sigma_m \exp[b(T_m - T)] \]  \hspace{1cm} (7-18)

where \( \sigma_m \) is the yield stress at melting temperature \( T_m \), \( b \) is a temperature factor, and \( T \) is the temperature of the material [76].

Levy-Mises Equations (Ideal Plastic Solid) describe the plastic deformation as following [220, 221]:

\[ d\varepsilon_i = \frac{d\bar{\varepsilon}}{\sigma} \left[ \sigma_i - \nu(\sigma_2 + \sigma_3) \right] \]  \hspace{1cm} (7-19)

\[ d\varepsilon_2 = \frac{d\bar{\varepsilon}}{\sigma} \left[ \sigma_2 - \nu(\sigma_1 + \sigma_3) \right] \]  \hspace{1cm} (7-20)

\[ d\varepsilon_3 = \frac{d\bar{\varepsilon}}{\sigma} \left[ \sigma_3 - \nu(\sigma_2 + \sigma_1) \right] \]  \hspace{1cm} (7-21)

where \( \varepsilon_1, \varepsilon_2, \) and \( \varepsilon_3 \) are the principal stresses, \( \nu \) is Poisson’s ratio. The Poisson’s ratio of magnesium alloy AM50A is 0.35 [197]. \( e, \sigma \) are the effective strain and stress.

The yield criteria for ductile metals (Von Mises’s criteria) is:

\[ \sigma_0 = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \]  \hspace{1cm} (7-22)

where \( \sigma_0 \) is the yield stress in uniaxial tension.
7.2.1.3 Elastic Deformation

When the temperature of a solidified part is low enough where its yield strength becomes higher than the pressure applied, only elastic deformation occurs. For elastic deformation, Hook’s law is valid [220, 221]:

\[
\frac{\sigma}{\varepsilon} = E \tag{7-23}
\]

where \(E\) is the modulus of elasticity in tension or compression. The stress-strain relations are:

\[
\varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu (\sigma_x + \sigma_y) \right] \tag{7-24}
\]

\[
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu (\sigma_x + \sigma_y) \right] \tag{7-25}
\]

\[
\varepsilon_z = \frac{1}{E} \left[ \sigma_z - \nu (\sigma_x + \sigma_y) \right] \tag{7-26}
\]

\[
\varepsilon_x = \varepsilon_y = -\nu \varepsilon_x \tag{7-27}
\]

For two-dimensional problem in elasticity, Airy’s stress function combines both equilibrium and compatibility equations together into an integration [222]:

\[
\frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = 0 \quad \text{or} \quad \nabla^4 \Phi = 0 \tag{7-28}
\]

where \(\Phi\) is the stress function, i.e. \(\sigma_x = \frac{\partial^2 \Phi}{\partial y^2}, \sigma_y = \frac{\partial^2 \Phi}{\partial x^2}, \tau_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y}\)

The consequence of temperature change over the elastic body can be considered by:

\[
\nabla^4 \Phi + \alpha E \nabla^2 T = 0 \tag{7-29}
\]
7.2.1.4 Friction between Casting and the Die

a. Coulomb Friction.

The Coulomb Friction is proportional to the stress normal to the surface of the workpiece. The proportionality is defined by a coefficient of friction which is assumed to be constant throughout the metal forming operation.

\[ t_f = m s_n \]  

(7-30)

where \( t_f \) is the friction stress tangential to the surface, \( m \) is the coefficient of friction, and \( s_n \) is the compressive stress normal to the surface [223, 224].

This law is not applicable to bulk metal forming because high contact pressures are involved. At high contact pressures the Coulomb law predicts friction stresses greater than the shear strength of the metal and sticking rather than sliding is modelled at the interface. Coulomb friction is more applicable to sheet metal forming where surface pressures are lower [224].

b. Tresca Friction.

The Tresca Friction stress at the contact surface is equal to a fraction of the shear yield stress of the workpiece material. It is calculated using a constant friction factor.

\[ t_f = m \tau \]  

(7-31)

where, \( m \) is the friction factor, and \( \tau \) is the shear yield strength of the workpiece [223, 224].

This law is applicable to bulk metal forming because, unlike Coulomb friction, the amount of friction is independent of the normal stress at the surface [224]. Therefore, in this model, the Tresca law was employed.
c. Viscoplastic Friction.

The Viscoplastic Friction is rate dependent. It is calculated from a constant friction factor, the workpiece velocity relative to the tool velocity, a power index and the consistency of the workpiece material.

\[ t_f = -akvp \]  
\[ (7-32) \]

where \( a \) is the friction factor, \( v \) is the relative velocity, and \( p \) is the sliding velocity sensitivity index [224].

### 7.2.2 Mathematical Formulation for Predicting Local Cavity Pressures

#### 7.2.2.1 Force Balance during Squeeze Casting

For the present squeeze casting system, a force balance (Figure 7.3) can be written as:

\[ F_a = F_T + F_s + F_f \]  
\[ (7-33) \]

where \( F_a, F_T, F_s, \) and \( F_f \) are applied force, force transferred to top surface, frictional force along the side casting/die interface and force for internal deformation respectively, which will be detailed below.

a. **Applied hydraulic force, which can also be expressed as**

\[ F_a = P_a \times \pi r^2 \]  
\[ (7-34) \]

where \( P_a \) is the so-called specific pressure or hydraulic pressure caused by the applied force, and \( r \) is the radius of the cylinder.
Figure 7.3 Schematics illustrating force balance

b. $F_T$ is the force transferred to the top, which is

$$F_T = \int_0^r P_t(r)2\pi dr$$

(7-35)

where $P_t$ is the local pressure on the top surface.

c. $F_S$ is the casting side/die interfacial friction force, which is

$$F_S = \int_0^h F_s(h)2\pi dh$$

(7-36)

where $h$ is the height of the cylinder, and $F_s$ is the local friction stress. According to Tresca Friction law applicable to bulk metal forming, friction stress at the contact surface is equal to a fraction of the shear yield stress of the workpiece material. It is calculated using a constant friction factor.

$$F_s = m \times \tau_{ys}$$

(7-37)
where \( m \) is the friction factor, \( \tau_{ys} \) is the shear yield strength of the casting contact surface, and \( \tau_{ys} = \sigma_y / 2 \) (\( \sigma_y \) is the tensile yield strength). The characteristics of \( \sigma_y \) of AM50A is shown in Figure 5.13. The relationship between \( \sigma_y \) (unit: MPa) and temperature can be written as:

\[
\sigma_y = \begin{cases} 
1 & (T \geq 624 \degree C) \\
-0.01T + 7.6 & (435 \degree C \leq T < 624 \degree C) \\
-0.28T + 125 & (1 < T < 435 \degree C)
\end{cases}
\]  

(7-38)

For this side surface, the local shear yield strength then can be obtained by substituting the side surface temperature \( T_s \) into equation 7-38. For local friction factors, when local temperatures are higher than solidus temperature, it is assumed as under sticking condition, where the friction stress is equal to the flow shear stress of the alloy [221]. Therefore \( m \) here is set as 1. When the local temperature is below the solidus temperature, the following equation was proposed for mathematical modeling.

\[
m = \frac{T}{T_s}
\]

(7-39)

where \( T \) is the local casting surface temperature, \( T_s \) is the solidus temperature.

Meanwhile, with an assumption of ideal plastic/elastic solid for the casting, the following are valid for calculating pressure transfer in radius direction:

\[
\sigma_1 - \sigma_3 = \sigma_y = 2\tau
\]

(7-40)

\[
\varepsilon = \frac{\sigma}{E}
\]

(7-41)

where \( \sigma_1 \) and \( \sigma_3 \) are principal stresses, \( \sigma_y \) is yield strength, and \( \tau \) is shear yield strength.

d. \( F_i \) is the force acting on the internal of the casting
If the element is oriented so that its edge are parallel to the principal stress directions, then the increment of work during a time period (dt) for deformation $de_1$, $de_2$, and $de_3$ is given by [225]

$$\text{d}W = (\sigma_1 de_1 + \sigma_2 de_2 + \sigma_3 de_3)V \quad (7-42)$$

Where $\sigma_1$, $\sigma_2$ and $\sigma_3$ are principal stresses, $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are correspondent strains. $W$ is the work, and $V$ is the volume.

If this is divided by time $dt$, one obtains an expression for the instantaneous power,

$$\frac{dW}{dt} = (\sigma_1 \dot{\varepsilon}_1 + \sigma_2 \dot{\varepsilon}_2 + \sigma_3 \dot{\varepsilon}_3)V \quad (7-43)$$

where $\dot{\varepsilon}_1$, $\dot{\varepsilon}_2$ and $\dot{\varepsilon}_3$ are correspondent strain rates.

For a time period (dt), the force $F_I$ is calculated by:

$$F_I = \frac{dW}{de} \quad (7-44)$$

For the casting and die contact model built based on the above discussion, the grid control volumes used to solve the energy equation were chosen as the domain for the deformation model. Deformations were solved for each control volume. The force transferred in the direction of applied pressure then can be calculated as:

$$F_T = F_a - (F_S + F_I) \quad (7-45)$$

Figure 7.4 shows schematically a flow chart of the calculation procedure.
7.3 Initial and Boundary Conditions

7.3.1 Initial Conditions

In the domain of calculation, the initial temperatures of the melt were 695 °C for AM50A and 725 °C for A356; and the die temperature was 275 °C.
The initial velocity of molten metal in the sleeve was set as zero.

Due to water hammer phenomena discussed in the preceding section 7.2.4, the velocity of the filling instantly decreased to zero once the cavity filling was completed.

7.3.2 Boundary Conditions

Newton’s law of cooling was applied at boundaries between the casting and the die, which was of the form

\[ q'' = -k \frac{\partial T}{\partial z} = h_{c/d} (T_c - T_d), \text{ or } q'' = -k \frac{\partial T}{\partial z} = h_{c/d} (T_c - T_d) \]  

(7-46)

where \( q'' \) is the heat flux across the casting/die interface, the \( k \) is the thermal conductivity of the alloys, \( h_{c/d} \) is the heat transfer coefficient across the casting/die interface, \( T_c \) is the casting temperature at the casting/die interface, and \( T_d \) is the die temperature at the casting/die interface. The punch temperature is kept constant (275 °C).

7.3.2.1 Heat Transfer Coefficient While the Melt Is In Sleeve

In the initial stage, after the molten alloy was poured into the sleeve, it took about 20 seconds for the machine to move the sleeve, close the die and then fill the cavity. Hence, during this early 20 seconds, the top of the melt was experiencing natural convection with the cover gas, while its side and bottom were releasing heat to the sleeve and the plunger by conduction, respectively.

For the top surface free convection, the average heat transfer coefficients can be presented in the following expression [227]:

\[ \bar{Nu}_f = C(Gr_f Pr_f)^m \]  

(7-47)
where $Nu_f$ is the Nusselt number, $Gr$ and $Pr$ are Grashof and Prandtl numbers, and $C$ and $m$ are constant. Subscripts $f$ indicates that the properties in the dimensionless groups are evaluated at the interface temperature.

For the side and bottom surfaces, the heat transfer coefficient proposed by Cho and Hong [100], and Kron and Fredriksson [226] was employed, which is expressed as:

$$h = at + b$$

(7-48)

where $h$ is the heat transfer coefficient, $a$ and $b$ are the constants, and $t$ is time.

**7.3.2.2 Heat Transfer Coefficient during Filling**

In squeeze casting processes [60], laminar behavior with slow cavity filling velocities is a dominating flow mechanism in an attempt to minimize turbulence for air entrapment. However, slow cavity filling velocities could result in large heat loss of melt. Transitional flow was often used aiming at balancing the tradeoff between the extent of flow turbulence and the melt heat loss. In this study, a filling velocity of 0.055 m/s of the melt flow in the cylindrical cavity was selected, which results in a transitional flow with $Re = 6140$. It was essential to take forced convection into consideration for modeling the events of heat transfer between the cavity and the melt.

For transitional flow, the correlation for Nusselt number is [227]:

$$\langle Nu \rangle_{D,H}^{10} = \langle Nu_{D,I} \rangle^{10} + e^{(2200 - Re_{D,H})/365} \left[ \left( \frac{1}{\langle Nu_{D,I} \rangle^2} + \langle Nu_{D,I} \rangle^2 \right)^{-5} \right]$$

(7-49)

where $Nu_{D,I}$ and $Nu_{D,t}$ are the laminar and turbulent Nusselt number correlations. Since the melt flow was not fully developed, the entrance effect of the melt flow into the cavity played an important role in dictating heat flow.
For laminar flow, the entrance length ($L_\delta$) of the melt flow was determined based on the formula ($L/D > 0.05Re$ and $L/D = 0.06RePr = 0.06PeD$) suggested in references 227 and 228. Within the entrance length, for forced flow in cylindrical cross sections, where hydraulic diameter $D_h = D$, and $T_s$ is uniform, the correlations for Nusselt number are [228]:

\[
Nu_{D,j} = 2.409 \left( \frac{L/D_h}{Pe_{D,j}} \right)^{-1/3} - 0.7 \quad 0 < \frac{L/D_h}{Pe_{D,j}} \leq 0.03 \quad (7-50)
\]

\[
Nu_{D,j} = 3.66 + \frac{0.0499}{(L/D_h)Pe_{D,j}} \quad 0.03 \leq \frac{L/D_h}{Pe_{D,j}} \quad (7-51)
\]

where $L$ is the entrance length, $D$ is the diameter, and $Pe$ is the Peclet number, $Pe=RePr$.

For turbulent flows of liquid metals ($Pr < 0.1$)

\[
Nu_{D,j} = 4.8 + 0.0156 Re_{D,j}^{0.85} Pr_{j}^{0.93} \quad \text{Uniform } T_s \quad (7-52)
\]

\[
Nu_{D,j} = 6.3 + 0.0167 Re_{D,j}^{0.85} Pr_{j}^{0.93} \quad \text{Uniform } q_s \quad (7-53)
\]

where $Re$ is the Reynolds number and $Pr$ is the Prandtl number.

The heat transfer coefficient calculate from equations 7-47 - 7-53 was the average value over the entire length of the cylindrical cavity. The average heat transfer coefficient was determined by the following expression:

\[
h = \frac{Nu k}{D} \quad (7-54)
\]

where $h$ is average heat transfer coefficient, $Nu$ is Nusselt number, $k$ is heat conductivity and $D$ is the diameter.

For the melt flow in the cylindrical cavity, the local convective heat flux was determined by
where \( T_w \) is the wall temperature, and \( T_b \) is the bulk temperature, i.e., the energy-average fluid temperature across the cavity, which was calculated from:

\[
T_b = \frac{\int_0^\infty \rho 2\pi r dr \cdot u \cdot C_p T}{\int_0^\infty \rho 2\pi r dr \cdot u}
\]  
(7-56)

where \( \rho \) is the density, \( u \) is the velocity, \( C_p \) is specific heat, \( T \) is temperature.

### 7.3.2.3 Heat transfer coefficient during pressurized solidification

The experimental results in Chapter 5 indicate that HTC changes with local cavity pressures during solidification. Therefore, heat transfer coefficients during pressurized solidification of squeeze casting of the alloys can be calculated by using the empirical equations (5-10), (5-11) and (5-12) proposed in Chapter 5 which correlated the HTCs to local cavity pressures and temperature (except HTC at side interface for A356, a constant HTC value from reference 100 was used instead). During the computation, the local pressure is calculated as described at every time step as described in section 7.2.2.

### 7.4 Pressure Effect on Solidification Temperatures

The experimental results presented in Chapter 6 indicated that the solidification temperature increase is linearly proportional to the applied pressure when the pressure varies from 0 to 90 MPa. The correlations (Equations 6-5, 6-6, 6-9 and 6-10) deduced based on the regression analysis were employed in numerical simulation.
7.5 **Thermal and Mechanical Properties**

7.5.1 **Mechanical and Thermophysical Properties of AM50A**

In the computational process, the density, specific heat and conductivity were considered to vary with temperature. Temperature-dependent thermal and mechanical properties were listed in Appendix V.

7.5.2 **Mechanical and Thermophysical properties of A356**

Temperature-dependent thermal and mechanical properties were used as shown in Appendix VI. In the computational process, the density, specific heat, conductivity and etc. were considered to vary with temperature.

7.6 **Numerical Solution Procedure**

7.6.1 **Numerical Technique**

Patankar [214] developed a FDM/control volume approach known as SIMPLER (Semi Implicit method for Pressure Linked Equations Revised) algorithm. The SIMPLER algorithm is able to solve the energy, continuity, and momentum equation, and any equations which can be expressed in the following differential form:

\[
\frac{\partial}{\partial t} (\rho \phi) + \nabla (\rho u \phi) = \nabla ( \Gamma \nabla [\phi] ) + \text{SourceTerm} \quad (7-57)
\]

where \( \phi \) can be any variables of interest. This equation is a very general expression characterizing many governing phenomena in nature. The equation has four distinct terms defined from left to right as: the unsteady term, the convective term, the diffusive term, and the source term. The energy equation and the continuity and momentum equations
can easily be expressed in this form, and thus the SIMPLER algorithm can be used to solve these problems.

### 7.6.2 Discretization of the Governing Equations

The discretization of the governing equations is preformed on an orthogonal grid of fixed size, consisting of nodal points, control volumes and control volume faces as depicted in Figure 7.5. In solving the energy equation, each control volume’s temperature is defined at the nodal point, and solving for the continuity and momentum equations, a staggered grid approach is adopted, whereby, velocities are defined at control volume interfaces. In this study, the grid size was optimized by comparing the results calculated using different grid sizes.

![Figure 7.5 Structure of the grid system.](image)

The partial differential equations are integrated over each control volume and over an arbitrary time interval $\Delta t$, and a set of governing equations is obtained in which the temperature or velocity of a particular control volume or face is expressed in terms of the temperature or velocities of the surrounding control volumes or faces. If $\phi$ represents a general unknown dependent variable, then the algebraic equations can be expressed in the general form:
where the subscripts $E$, $W$, $N$ and $S$ denote the values of the unknown dependent variable immediately to the east, west, north and south of the control volume of interest, respectively. For example, the resulting values obtained for the $a$ and $b$ coefficients for the energy equation in a two dimensional convective-diffusion problem in Cartesian coordinates are given in Table 7.1.

Some restrictions apply to the values that $a$ and $b$ coefficients can have. The Scarborough criterion, for instance, specifies that in order to avoid unrealistic results, all coefficients of equation (7-57) must be of the same sign. In this model, all terms are chosen to keep positive. It is easy to see how allowing one of the coefficients to become negative can cause erroneous output. For example when calculating temperature ($\phi = T$), if the coefficient $a_E$ were negative, then an increase in the temperature $T_E$ would induce a decrease in the temperature of the node in question $T_P$. When discretizing the convective term of the governing equations by using simple central difference approximations, introducing negative coefficients becomes a possibility. For this reason, various schemes have been used to preserve the Scarborough criterion, including: the upwind scheme, the hybrid scheme, and the power law scheme. Of these, the power law scheme gives very accurate results with minimal computational effort.
Table 7.1 Coefficients $a$ and $b$ for the governing equations in a two dimensional convective-diffusion problem in Cartesian coordinates

<table>
<thead>
<tr>
<th>Momentum Equations</th>
<th>Energy Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_p \phi_p = a_E \phi_E + a_w \phi_w + a_N \phi_N + a_S \phi_S + b$</td>
<td>$a_p \phi_p = a_E \phi_E + a_w \phi_w + a_N \phi_N + a_S \phi_S + b$</td>
</tr>
<tr>
<td>$a_E = D_e A(</td>
<td>P</td>
</tr>
<tr>
<td>$a_w = D_w A(</td>
<td>P</td>
</tr>
<tr>
<td>$a_N = D_s A(</td>
<td>P</td>
</tr>
<tr>
<td>$a_S = D_s A(</td>
<td>P</td>
</tr>
<tr>
<td>$a^0_p = (\rho_p^0 \Delta x \Delta y / \Delta t)$</td>
<td>$a^0_p = (\rho_p^0 \Delta x \Delta y / \Delta t)$</td>
</tr>
<tr>
<td>$b = S_c \Delta x \Delta y + a_p^0 \phi^0_p$</td>
<td>$b = a_p^0 \phi^0_p + D_e (S_{e1} - S_{p1}) - D_w (S_{p1} - S_{w1}) + D_n (S_{n1} - S_{p1}) - D_s (S_{p1} - S_{s1}) + D_e (S_{e2} - S_{p2}) - D_w (S_{p2} - S_{w2}) + D_n (S_{n2} - S_{p2}) - D_s (S_{p2} - S_{s2})$</td>
</tr>
<tr>
<td>$a_p = a_E + a_w + a_N + a_S + a_p^0 - S_p \Delta x \Delta y$</td>
<td>$a_p = a_E + a_w + a_N + a_S + a_p^0 - S_p \Delta x \Delta y$</td>
</tr>
<tr>
<td>$F_e = (\rho u)_e \Delta y \Delta z$</td>
<td>$D_e = \Gamma_e \Delta y \Delta z / (\delta x)_e$</td>
</tr>
<tr>
<td>$F_w = (\rho u)_w \Delta y \Delta z$</td>
<td>$D_w = \Gamma_w \Delta y \Delta z / (\delta x)_w$</td>
</tr>
<tr>
<td>$F_n = (\rho u)_n \Delta x \Delta z$</td>
<td>$D_n = \Gamma_n \Delta x \Delta z / (\delta y)_n$</td>
</tr>
<tr>
<td>$F_s = (\rho u)_s \Delta x \Delta z$</td>
<td>$D_s = \Gamma_s \Delta x \Delta z / (\delta y)_s$</td>
</tr>
<tr>
<td>Where $A(</td>
<td>P</td>
</tr>
</tbody>
</table>

Finally, to improve the realism of the model, when the diffusion coefficients (i.e. conductivity and viscosity) change between adjacent control volumes, a harmonic mean is used to evaluate an equivalent diffusion value. For example, the equivalent conductivity between the nodal points of two adjacent equal sized control volumes would be calculated by

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The use of equation (7-58) results in more accurate output especially in cases where the conductivity of one control volume is much greater than the conductivity of the other.

7.6.3 Solution of the resulting algebraic equations

The SIMPLER algorithm adopts a combination of the Gauss-Seidel method and TDMA (Tri-Diagonal Matrix Algorithm). The algebraic equations generated from the discretization of a governing relation are cast into a tri-diagonal matrix and solved via back substitution. The dependent variables are solved along a horizontal line from one end of the domain to the other via the TDMA. The process is repeated for every line in the domain until the entire domain has been updated. At every solution step, the immediate calculated values for every node are used in the next subsequent calculation. This technique is known as the line by line method.

The SIMPLER algorithm adopts a trial and error technique whereby velocity and pressure correction equation are used to obtain convergence. Full details of the algorithm can be found in the text of Patankar [214]. The main elements of the algorithm can be summarized as follows:

(a) Guess a velocity field for the entire domain

(b) Calculate pseudovelocities using a velocity/pressure relation derived from the momentum equations

(c) Obtain the corresponding pressure field from the continuity equation

(d) Solve the momentum equations to obtain the updated velocity values

(e) Solve the pressure correction equation

\[ k_e = \frac{2k_pe}{k_p + k_e} \]  

(7-59)
(f) Use the velocity correction equations to update the velocity field

(g) Solve for all other governing equations with $\phi$’s, if necessary

(h) Repeat steps 2-8 until convergence is met.

The convergence for each iteration is assured with the dimensionless time step under $2.5 \times 10^{-6}$ according to the stability criterion ($\Delta t < \Delta t^2 / 4\alpha$).

7.7 Summary

A mathematical model has been developed to simulate heat transfer and solidification phenomena of magnesium alloy AM50A and aluminum alloy A356 occurring in squeeze casting process.

The model based on the enthalpy method to solve the solidification problem. The entrance effect, water hammer mechanism and forced convection have been taken into consideration to model fluid flow and heat transfer phenomena taking place in the cavity filling step of squeeze casting. The proposed force (energy) balance scheme enables the model to predict local cavity pressures in the pressurized solidification step of squeeze casting. The integration of a correlation between heat transfer coefficients and predicted local cavity pressures into the model results in the establishment of a dynamic boundary condition at the interface between the casting and die. The established dynamic condition ensures the accuracy and reliability of numerical prediction.
8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

The mathematical model developed in this study was aimed at predicting temperature profiles, fluid flow, movement of solidification fronts and local cavity pressures in squeeze casting of both magnesium alloy AM50A and aluminum alloy A356. In this chapter, first, the model will be used to examine the effect of entry flow, natural convection, and localized heat transfer on solidification behavior of a cylindrical casting in terms of the computed results. After that, the predictions from the mathematical model will be compared with the experimental results.

8.1 Numerical Results and Discussion of Magnesium Alloy AM50A

8.1.1 Effect of Natural Convection on Melt Flow And Cooling Behavior

Figures 8.1 and 8.2 show the predicted velocity fields at different times during the cavity filling and pressurized solidification stages of squeeze casting with an applied pressure of 60 MPa through a sequence of vector plots. To be consistent with the time set in the previous chapters, time zero was set at the moment when pressure was applied.

It can be observed that a marked downward flow started in the region adjacent to the side wall of the domain immediately after the melt was poured into the sleeve (Figure 8.1 (a)). The side melt flow then was deflected as it approached the bottom portion of the
casting. The deflection resulted in the flow moving toward the casting centerline. Meanwhile, the melt flow in the vicinity of the casting center was upward vertically. Once moving toward the top free surface, the central melt flow was redirected horizontally to the liquid/mush interface. Accordingly, the deflection of two opposite flows at the free surface and the casting bottom led to a circulating zone in the right side part of the domain. Due to the fact that during the initial stage, the top of the casting was experiencing natural convection with the cover gas, while its side and bottom were releasing heat to the sleeve through convection and the plunger by conduction, heat transfer on both the side and bottom of the sleeve were much stronger than the top. Therefore density induced flow was developed quickly in the lower right corner (Figure 8.1 (b)). The occurrence of solidification in the area close to the casting/die interface forced the natural convection moved towards the upper center (Figure 8.1 (c) & (d)). Once the cavity filling was completed, the velocity was assumed to zero according to the water hammer phenomena discussed previously.

Natural convection was restored upon the completion of cavity filling and the commencement of pressurized solidification. The computed velocity fields illustrated in Figure 8.2 show that a circulating flow initiated in the upper right corner instead of the lower right corner during cavity filling. This is because, during the pressurized solidification stage, high heat transfer occurred from the top surface of the casting to the die. As the solidification further proceeds, the advancement of the liquid/mush front pushed the circulating flow to the center of the casting. The circulating zone tended to become smaller and weaker progressively, and disappeared at around 15s after the pressurized solidification began.
8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(a) -21 seconds

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8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(b) -11 seconds

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NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

-6 seconds

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Figure 8.1 Predicted velocity field during the cavity filling stage: (a) -21s, (b) -11s, (c) -6s, and (d) -1s.
8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(a) 1 second

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8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(b) 5 seconds

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8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(c) 10 seconds

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Figure 8.2 Predicted velocity field under an applied pressure of 60MPa: (a) 1s, (b) 5s, (c) 10s and (d) 15s.

The aforementioned behavior might be further interpreted as follow. At the beginning of solidification, rapid cooling through the interfaces between the casting and die wall triggered temperature gradients in the casting coupon. Due to the dependence of density on temperature, the presence of temperature gradients resulted in the formation of
decreasing density profiles from the interfaces towards the center of the coupon. As a result, buoyancy forces generated by the density difference present in the casting induced upward fluid motion in its center and downward flow in the vicinity of the moving interface between the liquid and mush phases. The thermally induced density gradients from the up, bottom and side casting which were either perpendicular or parallel to the gravitational body force together generated a melt circularity in the casting, which was predicted by the model.

Figure 8.3 shows the comparison of predicted cooling curves at the center of the casting with and without considering the effect of natural convection during the squeeze casting process under an applied pressure of 60 MPa. For the clarification of discussion, the two primary steps, i.e., cavity filling and pressurized solidification, are specified in Figure 8.3. It can be seen from Figure 8.3 that natural convection significantly influences the cooling behavior of the casting during the cavity filling and at the early stage of pressurized solidification. With natural convection, the casting center reaches its liquidus temperature even before the pressurized solidification, and was able to stay at that temperature for about 12 seconds. This may be because the quick temperature drop resulting from natural convection at the early stage of solidification minimized temperature gradients in the liquid phase of the casting. But, it took over 25 seconds for the casting center to be at the liquidus temperature, and dropped to a temperature below the liquidus line just 5 seconds later when no natural convection was considered by the model. The difference in cooling behavior between the two cases with and without natural convection indicates that the presence of natural convection not only accelerated temperature drop to the liquidus temperature but extended the early stage of
solidification. It has been pointed out [229] that an extended early stage of solidification would result in enhanced and persistent nucleation during solidification, which benefits the formation of fine microstructures. Once the temperatures dropped below the liquidus temperature, the effect of natural convection was reduced considerably, and heat transfer between the casting and die is primarily dominated by conduction. As a result, the casting was cooled at the almost same solidification rates for both the cases.

![Diagram of predicted cooling curves with and without natural convection](image)

**Figure 8.3** Predicted cooling curves with and without considering natural convection.

### 8.1.2 Temperature Distribution and Solidification Front

Figure 8.4 shows the temperature isocontours at different instants of time during cavity filling. Ten seconds after the cavity filling, the temperature distribution of the melt became vertically symmetric resulting from pre-solidification at the bottom and along the periphery of the melt. This is because, in the sleeve cavity, heat conduction-dominated rapid chill took place at the bottom and along the side surfaces of the melt, and the top
surface was slowly cooled by natural convection. During the filling, the temperature profile tended to move upward and toward the melt interior. Once the die cavity was filled, a firm contact between the top surface of the melt and the die surface was formed under the applied pressure of 60 MPa. This resulted in a rapid cooling to the top surface of the melt through heat conduction to the die.

Figure 8.4 Temperature profiles in the casting at different times during the filling: (a) -12s, (b) -1s.
Figure 8.5 Temperature profiles in the casting at different times during the pressurized solidification stage under an applied pressure of 60 MPa: (a) 1s, (b) 13s, (c) 23s and (d) 30s.
8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

The temperature profiles in the casting at different times during the stage of pressurized solidification are given in Figure 8.5. As the pressurized solidification proceeded, the rapid cooling from the top surface moved the hot area downward, which distorted the vertically symmetric temperature profile formed at the early stage of the process. The distorted temperature profiles are mainly attributed to the differences in the initial heat transfer conditions between the top, side and bottom surfaces of the casting. The relatively low heat transfer owing primarily to natural convection on the top surface of the melt during filling extracts less amount of heat from the top surface than the periphery and bottom surface.

The transient progression of the solidification fronts 10 sec and 22 sec after the commencement of the casting process is illustrated in Figure 8.6. The phase front appeared vertically symmetric at the early stage due to the cooling conditions surrounding the melt discussed above. As filling continued, the solidification at the top surface of the melt set in despite slow convection cooling, which altered the vertically symmetric shape of the phase front.
Figure 8.6 Temporal development of solidification front during the filling: (a) 10s, (b) 22s.

Figure 8.7 displays the evolution of the solidification front during the stage of pressurized solidification. Upon the completion of filling, the solidification of the top portion of the melt was accelerated and dictated by heat conduction between the top surface and die surface under the applied pressure instead of natural convection. The conduction-controlled solidification of the top portion tended to move the phase front downward. Meanwhile, the ongoing solidification at the bottom and side surface of the casting pushed the front upward and inward, respectively. As a result, the upper, central portion about 0.03m down from the top surface of the casting became the location of last solidification.
Figure 8.7 Temporal development of solidification front in the casting during the pressurized solidification stage under an applied pressure of 60 MPa: (a) 1s, (b) 13s, and (c) 30s.
8.1.3 Effect of Pressure On Solidification Temperatures And Time

Figure 8.8 shows predicted cooling curves of solidification at the casting center under applied pressures of 30MPa, 60MPa and 90MPa. Prior to applying pressures during cavity filling (up to 22 sec in Figure 8.6), the cooling rates for all three melts are the same. Upon the application of pressures, however, the prediction clearly indicates that, compared with 625.1 °C under 0 MPa, the liquidus temperatures of magnesium alloy AM50A increased by 1.9, 2.6 and 5.4 °C under applied pressures of 30 MPa, 60 MPa and 90 MPa, respectively. Meanwhile, the pressures of 30, 60 and 90 MPa increased the solidus temperature of 427.8 °C of the alloy under 0 MPa to 428.5, 429.7 and 430.8 °C, respectively.

![Diagram showing cooling curves under different pressures](image)
Figure 8.8 Predicted cooling curves of solidification at casting center under different applied pressure, (a) entire curve and (b) enlarged view of liquidus regions.

Figure 8.9 illustrates the effect of applied pressures on the solidification time at the center of the casting. As indicated in Figure 8.9, the higher level of applied pressures, the lower the solidification times, and consequently the higher solidification rates. This can be explained that applied pressures kept the casting and die in close contact when the casting contraction took place during solidification. The improved surface contact between the casting and die reduced thermal resistance substantially, and consequently enhanced heat transfer at the casting/die interface significantly.
8.2 Comparison of Numerical Prediction with Experimental Results of Magnesium Alloy AM50A

In this section, the numerical predictions given in the preceding section will be compared with experimental results.

8.2.1 Cooling Curve and Liquidus Temperature

Figure 8.10 presents typical experimental results of temperature measurements while the coupon was cast under an applied pressure of 60MPa with a melt temperature of 695 °C and a die temperature of 275 °C. The temperature at the center of the cast coupon is represented by line “T10”, while line “T9” characterizes the temperature at the location 0.040 m down from the casting top surface and 0.025 m away from the
centerline of the casting. The symbols employed for the lines in Figure 8.10 and the succeeding diagrams of this subsection are consistent with the system used for numbering the thermocouples given in the previous chapter and sections. During experiments, it took about 22 seconds for the melt to be filled into the cavity since a relatively slow filling velocity was employed to satisfy the flow requirement of squeeze casting processes as discussed in the preceding section. After the commencement of the process, the temperature rise at T10 was recorded instantaneously by segment AB of line T10. The temperature rise at location T9 was recorded by segment FG of line T9 till the melt was filled into cavity. Before the filling was completed, the center of the casting as the last solidification area was cooled at a slower rate than its outer portion. The temperature of 627 °C just above the liquidus temperature (625.1 °C) at the center was able to maintain at the liquidus temperature up to 28 seconds, as shown by segment CD of line T10. Although the detected temperature indicated by G was somewhat around the liquidus temperature, it dropped quickly. This indicates that pre-solidification occurred in area T9 of the casting adjacent to the casting/die interface during filling. The relatively slow filling causing a loss of almost all superheat content of the melt was responsible for the phenomenon. As solidification time further increased, the temperature at location T9 decreased toward the solidus temperature considerably about 4 seconds earlier than that at location T10. This observation suggests that solidification in the outer portion of the casting finished somewhat earlier than that of the center.
In order to compare the model prediction with the experimental data, the predicted temperature variation with time at both locations T10 and T9 where the experimental measurements were conducted is presented in Figure 8.11 (a) and (b). During the stage of pressurized solidification, the predicted and measured temperature histories are in good agreement. However, there is some minor deviation between the prediction and experimental data, especially after the casting temperature dropped below the liquidus temperature. The inaccuracy in the determination of HTC should be, at least partly, responsible for the deviation. Also, this may result from the fact that the inhomogeneous distribution of temperatures took place even during the early stage of melting pouring into the sleeve cavity. But, this simulation assumed that the initial temperature was homogeneous in the casting at the very beginning of the computation because of the difficulty of temperature determination during pouring.
Figure 8.11 Comparison of computational and experimental cooling curves under an applied pressure of 60 MPa at locations (a) T10 and (b) T9.
8.2.2 Last solidified location

To determine the last solidified location in the casting, the total solidification times during which the casting was cooled from the liquidus to solidus temperatures were determined based on the temperature measurements at a depth of 0.01 m, 0.02 m, 0.03 m, 0.04 m and 0.05 m away downward from the top surface center of the casting along the central line of the casting coupon. Figure 8.12 presents the representative experimental measurements of temperatures at different locations along the central line of the casting.

![Temperature vs. Time Graph](image)

Figure 8.12 Experimental measurement of temperatures at different locations along the casting centerline under an applied pressure of 60 MPa.

Figure 8.13 compares the predicted and measured solidification times at the desired locations. The filled squares in Figure 8.12 represent the experimental measurements. The location 0.03 m away from the top surface has the longest solidification time of 39.9 seconds among the five measurements. This infers that the
upper, central portion of the casting was the last solidified area since the total height of the casting is 0.08 m. Compared with the predicted solidification times at various distances from the top surface, the computational and experimental results are in close agreement.

Figure 8.13 Predicted and measured solidification times vs. depth from the casting top along its central line under an applied pressure of 60 MPa.

8.2.3 Local Cavity Pressure

As discussed earlier, the proposed force balance scheme enables the developed model to predict local cavity pressure in squeeze casting. To verify predictions, Figure 8.14 compares the predicted local pressures with the experimental measurements at various locations (P1, P2, P3 and P4) during squeeze casting of magnesium alloy AM50A under an applied pressure of 60MPa. They are basically in fair agreement.
The prediction of the local pressure at the casting center further indicates that a difference between the hydraulic applied pressure (60 MPa) and local pressure are present during squeeze casting. Since the magnitude of the hydraulic pressure is much greater than that of the local pressure, the direct application of the hydraulic pressure is theoretical calculations would lead to an overestimation of solidification temperatures under pressures.

Figure 8.14 Comparison of computational and experimental local cavity pressure at locations P1, P2, P3 and P4 under applied pressure of 60MPa.
8.3 Numerical Results and Discussion of Aluminum Alloy A356

8.3.1 Effect Of Natural Convection On Melt Flow And Cooling Behavior

Figure 8.15 and 8.16 show the predicted velocity fields at different times during the cavity filling and pressurized solidification through a sequence of vector plots. To be consistent with the time set in the previous chapters, time zero was also set at the moment when pressure was applied.

It can be observed that a marked downward flow started in the region adjacent to the side wall of sleeve immediately after the melt was poured into the sleeve (Figure 8.15 (a)). The side melt flow then was deflected as it approached the bottom portion of the casting. The deflection resulted in the flow moving toward the casting centerline. Meanwhile, the melt flow in the vicinity of the casting center was upward vertically. Once moving toward the top free surface, the central melt flow was redirected horizontally along the top surface. Accordingly, the deflection of two opposite flowed at the free surface and the casting bottom led to the development of a circulating flow in the computational domain. Due to the fact that during the initial stage, the top of the casting was experiencing natural convection with the cover gas, while its side and bottom were releasing heat to the sleeve through convection and the plunger by conduction. Heat transfer on both the side and bottom of the sleeve were much faster than that at the top surface of the melt. Consequently, large temperature gradients generated quickly in the right corner of the domain, where a circulating zone of density induced flow was developed due to natural convection (Figure 8.15 (b)). Once solidification in the area close to the casting/die interface set in, the moving solidification front forced the circulating flow to move upward towards the inner melt (Figure 8.15 (c)).
During the filling stage, the plunger moving speed (0.05 m/s) is predominant. Once the cavity filling was completed, the velocity was assumed to be zero according to the water hammer phenomena discussed previously.

During the pressurized solidification stage, the natural convection was restored as shown in Figure 8.16. Upon the completion of cavity filling, high heat transfer took place at the top of casting compared to its bottom. This was due to the establishment of firm contact at the interface between the top inner wall of the die and the top surface of the casting and large difference in temperature between the top inner wall of the die and the casting. As a result, a steep temperature gradient was present in the top part of casting. Also, because of continuous heat transfer to the side wall of the die from the casting, a circulating zone of melt flow formed in the upper right portion of the domain (Figure 8.16 (a)). As the solidification further proceeded, the advancement of the liquid/mush front pushed the circulating flow to the upper center of the casting (Figure 8.16 (b) and (c)). The circulating zone tended to become smaller and weaker progressively, and disappeared at around 25s.
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(b) -11 seconds
Figure 8.15 Predicted velocity field during the initial stage: (a) -21s, (b) -11s, and (c) -1s.
NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

![Diagram with vectors and axes labeled RADIUS (m) and HEIGHT (m). A marker indicates a velocity of 0.05 m/s.]

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8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

(c) 15 seconds

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The aforementioned behavior might be interpreted as follow. At the beginning of solidification, rapid cooling through the interfaces between the casting and die wall triggered temperature gradients in the casting coupon. Due to the dependence of density on temperature, the presence of temperature gradients resulted in the formation of...
reducing density profiles from the interfaces towards the center of the coupon. As a result, buoyancy forces generated by the density difference present in the casting induced upward fluid motion in its center and downward flow in vicinity of the moving interface between the liquid and mush phases. The thermally induced density gradients from the up, bottom and side casting which were either perpendicular or parallel to the gravitational body force together generates a melt circularity in the casting, which was predicted by the model.

Figure 8.17 shows the comparison of predicted cooling curves at the center of the casting with and without considering the effect of natural convection during the squeeze casting process under an applied pressure of 60 MPa. For the clarification of discussion, the two primary steps, i.e., cavity filling and pressurized solidification, are specified in Figure 8.17. It can be seen from Figure 8.17 that natural convection influences the cooling behavior of the casting during the cavity filling and at the early stage of pressurized solidification. With natural convection, the casting center reached its liquidus temperature even before the pressurized solidification, and was able to stay at that temperature for about 20 seconds. This may be because the quick temperature drop resulting from natural convection at the early stage of solidification minimized temperature gradients in the liquid phase of the casting. But, it took over 25 seconds for the casting center to remain at the liquidus temperature, and dropped to a temperature below the liquidus line just 5 seconds later when no natural convection was considered by the model. The difference in cooling behavior between the two cases with and without natural convection indicates that the presence of natural convection not only accelerates temperature drop to the liquidus temperature but extends the early stage of solidification.
It has been pointed out [229] that an extended early stage of solidification would result in enhanced and persistent nucleation during solidification, which benefits the formation of fine microstructures. Once the temperatures dropped below the liquidus temperature, the effect of natural convection was reduced considerably, and heat transfer between the casting and die was primarily dominated by conduction. As a result, the casting was cooled with an almost the same solidification rate for both the cases.

![Figure 8.17 Predicted cooling curves with and without considering natural convection.](image)

**8.3.2 Temperature Distribution and Solidification Front**

Figure 8.18 shows the temperature isocontours at different instants of time during cavity filling. Twelve seconds after the cavity filling, the temperature distribution of the melt became vertically symmetric resulting from pre-solidification at the bottom and along the periphery of the melt. This is because, in the sleeve cavity, heat conduction-
dominated rapid chill took place at the bottom and along the side surfaces of the melt, and the top surface was slowly cooled by natural convection. During cavity filling, the temperature profile tended to move upward and toward the melt interior.

The temperature profiles in the casting at different times during the stage of pressurized solidification are given in Figure 8.19. As the pressurized solidification proceeded, the rapid cooling from the top surface moved the hot area downward (Figure 8.19 (a)). The movement of the hot area distorted the vertically symmetric temperature profile formed at the early stage of the process. It became not only vertically but also more or less horizontally symmetric as shown in Figure 8.19 (b) and (c). The distorted temperature profiles are mainly attributed to the differences in the heat transfer conditions

Figure 8.18 Temperature profiles in the casting at different times during the filling: (a) -12s, (b) -1s.
between the top, side and bottom surfaces of the casting during the process. The relatively low heat transfer owing primarily to natural convection on the top surface of the melt during filling extracts less amount of heat from the top surface than the periphery and bottom surface. Once the die cavity was filled, the firm contact between the top surface of the melt and the die surface was formed under the applied pressure. This resulted in a rapid cooling to the top surface of the melt through heat conduction to the die.
Figure 8.19 Temperature profiles in the casting at different times during the stage of pressurized solidification: (a) 1s, (b) 10s, (c) 25s and (d) 35s.

Figure 8.20 Temporal development of solidification front during the filling: (a) -12s, (b) -1s.
The transient progression of the solidification fronts 10 sec and 21.9 sec after the commencement of the casting process is illustrated in Figure 8.20. The phase front appears vertically symmetric at the early stage due to the cooling conditions surrounding the melt discussed above. As filling continued, the solidification at the top surface of the melt set in despite of slow convection cooling, which altered the vertically symmetric shape of the phase front.

Figure 8.21 displays the evolution of the solidification front during the stage of pressurized solidification. Upon the completion of filling, the solidification of the top portion of the melt was accelerated and dictated by heat conduction at the interface between the top casting surface and the top die wall under the applied pressure instead of natural convection. The conduction-controlled solidification of the top portion tended to move the phase front downward. Meanwhile, the ongoing solidification at the bottom and side surface of the casting pushed the front upward and inward, respectively. As a result, the upper, central portion about 0.03m down from the top surface of the casting became the location of last solidification.
Figure 8.21 Temporal development of solidification front in the casting after the completion of cavity filling: (a) 1s, (b) 20s, and (c) 45s.
8.3.3 Effect of Pressure On Solidification Temperatures And Time

Figure 8.22 shows predicted cooling curves of solidification at the casting center under applied pressures of 30MPa, 60MPa and 90MPa. Prior to applying pressures during cavity filling (up to 22 sec in Figure 8.22), the cooling rates for all the three melts were almost the same. Upon the application of pressures, however, the prediction clearly indicates that, compared with 613.5 °C under 0 MPa, the liquidus temperatures of magnesium alloy AM50A increased by 0.9, 2.2 and 4.1 °C under applied pressures of 30 MPa, 60 MPa and 90 MPa, respectively (Figure 8.22(b)).
Figure 8.22 Predicted cooling curves of solidification at casting center under different applied pressure, (a) entire curve and (b) enlarged view of liquidus regions.

Figure 8.23 illustrates the effect of applied pressures on the solidification time at the center of the casting. As indicated in this figure, the higher level of applied pressures, the lower the solidification time, and consequently the higher solidification rate. This can be explained that applied pressures kept the casting and die in close contact when the casting contraction took place during solidification. The improved surface contact between the casting and die reduced thermal resistance substantially, and consequently enhanced heat transfer at the casting/die interface significantly.
8 NUMERICAL RESULTS AND EXPERIMENTAL VERIFICATION

Figure 8.23 Variation of start and end solidification times and rates under different applied pressure.

8.4 Comparison of Numerical with Prediction Experimental Results of Aluminum Alloy A356

In this section, the numerical predictions for alloy A356 given in the preceding section will be compared with experimental results.

8.4.1 Cooling Curve and Liquidus Temperature

A typical experimental result of the temperature measurement while the coupon was cast under an applied pressure of 60MPa with a melt temperature of 725 °C and a die temperature of 275 °C is presented in Figure 8.24. The temperature at the center of the cast coupon is represented by line “T10 computational”. During experiments, it took about 22 seconds for the melt to be filled into the cavity since a relatively slow filling
velocity was employed to satisfy the flow requirement of squeeze casting processes as discussed in the preceding section. After the commencement of the process, the temperature rise at the casting center was recorded instantaneously by segment AB of line T10 experimental. During cavity filling, the temperature at the casting center remained above the liquidus temperature (613.5 °C) despite its continuous decrease as given by segment BC of line T10. After the pressure was applied, segment CD of line T10 indicates that the casting center was able to maintain at the liquidus temperature up to 28 seconds before further decreasing. This is due to the geometrical nature of the casting, of which the cross section was relatively thick.

In order to compare the model prediction with the experimental data, the predicted temperature variation with time at location T10 where the experimental measurements was conducted is also presented in Figure 8.24. In general, the predicted and measured temperature histories are in reasonable agreement. However, there is some minor deviation between the prediction and experimental data, especially below the liquidus temperature. The inaccuracy in the determination of HTC should be, at least partly, responsible for the deviation. Also, this may be because that the inhomogeneous distribution of temperature started even during the early stage of melt pouring into the sleeve cavity since cooling happened to liquid metal during melt transfer to the die cavity from a crucible. But, in this computation, the temperature was assumed homogeneous in the casting at the very beginning of the computation because of difficulty in determining heat loss during melt transfer.
Figure 8.24 Comparison of computational and experimental cooling curves at casting center under an applied pressure of 60 MPa.

8.5 Summary

A two-dimensional mathematical model has been developed based on the enthalpy method to simulate both convective and conductive heat transfer, and solidification phenomena of both magnesium alloy AM50A and aluminum alloy A356 occurring in the two primary steps, which are cavity filling and pressurized solidification during squeeze casting. The model predicted the characteristics of the convection-related melt flow, the temperature distributions, the cooling curves, the shape and position of the solidification front, and total solidification times and rates of a relatively thick cylindrical squeeze casting.
The computed results indicate that the natural convection affects the melt flow in the casting during the pressurized solidification, which consequently determines the last solidified location. The cooling behavior of the squeeze cast coupon is also influenced by the natural convection. The effect extends the early stage of pressurized solidification, which may result in enhanced and persistent nucleation in the casting. The prediction also reveals that the applied pressure increases the solidification temperatures and rates of both magnesium alloy AM50A and aluminum alloy A356, but decreases their solidification times resulting from high heat transfer across the casting/die interface. To verify prediction made by the model, temperature measurements at various locations inside an experimental squeeze casting were performed. Comparisons of the numerical results with the experimental measurements show close agreement.

The general agreement between computated local cavity pressures and experimental measurements for magnesium alloy AM50A suggests that the proposed force balance scheme used in the model gives reasonable pressure prediction.
APPLICATION OF THE MATHEMATICAL MODEL IN POROSITY PREDICTION

Certain research work on numerical prediction of porosity in casting, as reviewed in Chapter 2 (section 2.2.3.6), has been carried out. Several prediction criteria have been proposed, such as critical temperature-gradient in last solidified regions [180], Niyama criterion [182], critical pressure [184], modified Niyama method [152]. However, by applying those criteria to castings with very thick section, i.e., the cylindrical coupon employed in this study, it was found that, no matter what magnitude of pressures being applied, closed contours of the solidifications front are present in the casting with thick cross-sections. Although the last solidification area can be located by the closed contours, it is almost impossible to determine the occurrence of porosity in the thick squeeze casting.

In this chapter, the “burst-feeding” theory was integrated into the developed mathematical model presented in Chapter 7. Numerical prediction of porosity occurrence in squeeze cast cylindrical coupons of magnesium alloy AM50A was carried out. The effect of pressure levels and holding time on the porosity formation was discussed. The numerical prediction was compared with experimental observation for the purpose of verification.
9.1 Proposed Criteria for Porosity Prediction in Squeeze Casting

9.1.1 Proposed Criteria

As reviewed in Chapter 2, for solidification shrinkage related porosity, the role of "squeeze" pressure in squeeze casting is to force-feed metal [66] as illustrated in Figure 9.1. Porosity elimination is achieved by "burst-feeding" liquid or semi-liquid metal through a network of solid skeletons. When there is a completely solid region interrupting this flow of metal—which usually happens in the vicinity of the last region to solidify—there has to be certain amount of plastic deformation of the solidified crust in order for applied pressures to be transferred through the crust to the solidifying region [13, 15, 41].

![Figure 9.1 Burst-feeding to eliminate porosity.](image)

As illustrated in Figure 8.7 for very thick squeeze castings, there exist closed contours of the solidification front, which subsequently forms a solid "shell" during solidification. If the applied pressure is high enough to keep deforming the solid shell plastically before the last location is solidified, no shrinkage porosity forms in the
casting. Mathematically, this can be considered by the yield criterion for ductile metals (Von mise’s criterion) as:

\[
\sigma = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \geq \sigma_0 \quad (9-1)
\]

where \(\sigma_0\) is the local temperature dependent yield strength of materials in uniaxial tension. To facilitate the integration of this yield criterion into the developed model, a normalized stress is defined by \(P_n = \sigma / \sigma_0\).

To achieve porosity free, the following two conditions must be satisfied:

a. \(\sigma / \sigma_0\) is larger than 1 in the area adjacent to the last solidification zone (Figure 9.2a), and

b. Contours of \(\sigma / \sigma_0 > 1\) are open to applied pressures (Figure 9.2b).

When the applied pressure is not able to make any plastic deformation, i.e. \(\sigma / \sigma_0 < 1\), to compensate the contraction of the last solidifying area, shrinkage porosity will form (Figure 9.3).
Figure 9.2 Schematic diagram illustrating two conditions for Burst-feeding to eliminate porosity: (a) $\sigma/\sigma_0 > 1$ is larger than 1 in the area adjacent to the last solidification zone and (b) contours of $\sigma/\sigma_0 > 1$ are open to applied pressures.

Figure 9.3 Schematic diagram illustrating no Burst-feeding available to eliminate porosity as $\sigma/\sigma_0 < 1$ is less than 1 in the area adjacent to the last solidification zone.
In the case that the applied pressure is released prior to the completion of solidification, the porosity formation will be decided by the development of local pressures even if the local normalized stress around the last solidification area is larger than 1. When local residual stresses, at the instant of applied pressures released, can compensate negative pressures caused by the solidifying zone’s contraction, there will be no porosity. Otherwise, the porosity will form.

9.1.2 Calculation Procedure

Figure 9.4 gives a flow chart of calculation procedures for porosity prediction. The simulation was performed on the same domain as shown in Figure 7.1. The calculation started with input of initial condition (melts temperature, mold temperature, applied pressure, holding time, etc.) as introduced in Chapter 7. In addition to calculation of temperature field and dynamic boundary conditions, a subroutine was added to determine the solidified zone and calculate local normalized pressure in every time step. Then, the criterion proposed in section 9.1.1 was employed to determine the occurrence \( \sigma/\sigma_0 \) is larger or less than 1, if the \( \sigma/\sigma_0 > 1 \) is open to applied pressures) and location of porosity (last solidification zone).
Figure 9.4 Flow chart for porosity prediction.
9.2 Numerical Predictions

Figures 9.5 and 9.6 show the temperature isocontours and solidification front at the final stage of solidification under an applied pressure of 60 MPa. As can be seen from the predictions, the upper, central portion (0.03m down from top) of the casting becomes the location of last solidification. The last solidification region is usually susceptible to porosity formation. But, it is almost impossible to predict whether porosity occurs only based on the iso-contours of temperatures and solidification fronts. Attempts of adopting the Iso-fs method and the Niyama criterion were made to predict the porosity formats for the thick squeeze casting solidification under an applied pressure of 60 MPa. The predicted results are displayed in Figures 9.7 and 9.8. Both the methods predict that porosity should form in the casting because of the presence of the closed loop of iso-contours. However, the experimental results given in Figure 9.9 reveals that no porosity is present in the cylindrical coupon squeeze cast under 60 MPa. The prediction from the Iso-fs method and Niyama criterion is obviously contradictive to the presented experimental observation.

Figure 9.10 displays the transient normalized stresses at the last stage of the pressurized solidification process. It is clear from Figure 9.10 that, even at the final stage, the local pressure can still make necessary deformation for feeding because of the presence of normalized stresses larger than 1 in the open solid shell. Therefore, there should be no porosity for the casting.
Figure 9.5 Temperature profiles in the casting at the last stage of solidification under an applied pressure of 60 MPa.

Figure 9.6 Solidification front in the casting at the last stage of solidification under an applied pressure of 60 MPa.
Figure 9.7 Prediction from the Iso-fs method for the casting under an applied pressure of 60 MPa at the last stage of solidification.

Figure 9.8 Prediction from the Niyama criterion for a casting under an applied pressure of 60 MPa at the last stage of solidification.
9 APPLICATION OF THE MATHEMATICAL MODEL IN POROSITY PREDICTION

Figure 9.9 Cross sections of the casting squeeze cast under an applied pressures of 60 MPa.

Figure 9.10 Distribution of normalized stresses under an applied pressure of 60 MPa at the last stage of solidification.
Figure 9.11 shows the computed normalized pressures under a relatively lower applied pressure of 18 MPa. The profile indicates that during the final stage of the process, the pressure is unable to make necessary deformation for shrinkage compensation because of the normalized stress is less than 1 in the solid region open to the applied pressure. Hence, the formation of porosity in the squeeze casting is unavoidable.

![Figure 9.11 Distribution of normalized stresses under an applied pressure of 18 MPa at the last stage of solidification.](image)
9.3 Comparison of Numerical Prediction with Experimental Results

9.3.1 Magnitude of Applied Pressures

Figures 9.12-9.13 show the cross sections of the cylindrical coupon of magnesium alloy AM50A squeeze cast under the applied pressures of 30, and 18 MPa with a fixed holding time of 60 seconds, respectively. The cross section of the cylindrical coupon cast under 60 MPa with the same holding time has already been shown in Figure 9.9. Table 9.1 summarizes the porosity observation in the casting for five different applied pressures of 6, 18, 30, 60, and 90 MPa.

It can be seen from Figures 9.9, 9.12 and 9.13 that no obvious porosity is found in the casting center for the applied pressures of 60 and 30 MPa, respectively. As the pressure decreased to 18 MPa, without change of its holding time, porosity formed in the upper center of the casting. The experimental results evidently reveal that the magnitude of applied pressures influences the porosity formation in the squeeze casting of magnesium alloy AM50A. It seems that an applied pressure of 30 MPa is sufficient to eliminate porosity in a relatively thick cylindrical squeeze casting prepared in this study. This indicates that the porosity prediction given in the previous section (Figure 9.10 and Figure 9.11) is consistent with the experimental observation.
Figure 9.12 Cross sections of the casting squeeze cast under an applied pressures of 30 MPa.

Figure 9.13 Porosity in a casting under applied pressure of 18 MPa.
Table 9.1 Porosity observation in the casting for five different applied pressures

<table>
<thead>
<tr>
<th>Applied pressure (MPa)</th>
<th>6</th>
<th>18</th>
<th>30</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

The effect of applied pressure levels on the formation of porosity might be explained as follows: M. Flemings [109] mentioned that the formation of pores within a solidifying casting is much more difficult than at the surface because the pressure drop must be sufficient to counter-balance the atmospheric pressure acting on the riser during sand casting.

But, for castings under pressures, the situation might be different. It was found from Figure 4.12 that, when the filling was completed, the side of the casting has solidified (T6), and the top surface was in mushy state (T3). Several seconds after applying pressure, the top surface also solidified, which means a closed solid shell formed.

By assuming that the applied pressure is not high enough to make any deformation on the shell, the solidification happens inside the casting by only "nature" force. As the solidification continues from surfaces towards the center, the bonding force of solid continuously attracts the liquid/mushy metal, which promotes the formation of solidification shrinkage. Meanwhile, the external pressure is not sufficient to have any influence on the development of internal contraction. This is because sufficient pressure drop develops inside casting, which leads to the generation of a negative pressure [230,
Under such a circumstance, the formation of void in the casting center due to a volume contraction (about 5% for magnesium alloy) seems unavoidable.

By measuring the local pressure on the casting surface, it was found that the pressure decreased with time, and its distribution was not uniform during the solidification as shown in Figures 4.1-4.4. If pressures in certain areas are high enough to locally deform the casting during solidification, the deformation compensates the formation of void resulting from contraction, and consequently eliminate the porosity.

The magnitude selection of applied pressures for squeeze casting should consider the following factors. First, the applied pressure must be high enough so that the casting would have a good contact with the die for rapidly forming a solid shell. Meanwhile, the strength of the solid shell must be strong enough to withstand the combined external applied and internal negative pressures mentioned above, and maintain the shape of the casting without sinks. Secondly, the applied pressure must be large enough to make plastic deformation, fill the pore forcedly, and eliminate the porosity. Accompanying the solidification process, with continuous temperature drop, material properties vary considerably. Therefore, application of external pressures may be considered to vary with the proceeding solidification process during squeeze casting, more specifically with time. With the assistance of numerical simulation, the optimization of applied pressure profiles with time becomes achievable. Figure 9.14 shows the predicted normalized stress distributions for the casting solidified under 30 MPa and 28 MPa. The prediction (Figure 9.14a) clearly indicates that maintaining the current level of the pressure (30 MPa) toward the end of solidification eliminate shrinkage porosity. Meanwhile, an applied
pressure of 28 MPa is unable to make necessary deformation for last solidification. Therefore, shrinkage porosity exists.

Figure 9.14 Normalized pressure under applied pressure of (a) 30 MPa and (b) 28 MPa.
It is interesting to note that the casting solidified under very low pressure turned out not having internal porosity. As shown in Figure 9.15, instead of having internal porosity, a shallow crater formed at the top of the casting when the applied pressure was so small (< 2MPa) that the force even was unable to push the casting top surface to have a good contact with the die. This can be explained that when applied pressures were too low, no firm contact between the top surface of the casting and the die could be established. The temperature measurements (Figure 9.16) indicate that the top surface of the casting stayed at the mushy state for a certain period of time. This evidences that poor contact was present between the top surface of casting and the die, which led to the formation of a top-open solid shell in the casting. Once solidification completed, an open surface sink (crater) formed in the top part of the casting.

Figure 9.15 Cross section of a coupon showing a sallow crater at the casting top.

Flemings [109] described that when a sufficiently solid skin forms that is not punctured by the atmospheric pressure, it can deform the casting surface, pushing it inward to form defects known to foundrymen as draw, or sinks. Ohsasa et al [230, 231]
measured directly the negative pressure developed in solidifying aluminum ingots and observed the external shrinkage pore. Therefore, the observation on the formation of surface sinks in this study is consistent with the published results.

Figure 9.16 Top surface temperature under 0 MPa applied pressure.

9.3.2 Pressure Holding Time

Figure 9.17 shows the influence of pressure holding time on the formation of porosity during squeeze casting for magnesium alloy AM50A solidified under an applied pressure of 30 MPa. As can be seen from Figure 9.17, a holding time of 15 seconds or less led to the formation of porosity in the casting center for 30 MPa. The porosity disappeared as the holding time increased to twenty-one seconds.

The porosity evolution with holding times may be attributed to the fact that certain time is needed for solidification shrinkage to be compensated by pressurized
feeding. Figure 9.18 presents the variation of thermal expansion coefficients of AM50A with temperatures.
Figure 9.17 Evolution of porosity with different holding time under an applied pressure of 30 MPa: (a) 10s, (b) 15s, and (c) 21s.

Figure 9.18 Thermal expansion coefficient of AM50A [199].

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It is revealed in Figure 9.18 that the thermal expansion coefficients above 560 °C up to the liquidus temperature is one or several orders higher than these below 560 °C. In other words, most shrinkage takes place in a temperature range between the liquidus temperature and 560 °C during solidification of magnesium alloy AM50A. Referring to the cooling curve shown in Figure 4.12, it is evident that the casting center needs around 20 seconds to drop its temperature below 560 °C under the applied pressure of 30 MPa. This implies that large amount of shrinkage occurs in the casting over the time period of 20 seconds. Therefore, it is essential for the applied pressure to be held long enough so that solidification shrinkage is compensated by the pressurized feeding. Numerical prediction (Figure 9.19) indicates that the pressure still can make plastic deformation for feeding the solidification area at the instant of 21s. As a result, the porosity in the casting was eliminated.

Figure 9.19 Normalized stress profile at instant of 21s holding time under pressure of 30 MPa.
For the applied pressure at a relatively low level of 18 MPa, Figure 9.20 reveals porosity evolution and pore movement in the casting with different pressure holding times. As shown in Figure 9.20, an applied pressure of 18 MPa was unable to eliminate porosity in the casting even with extended holding times up to 60 seconds. Also, it is interesting to note that the shrinkage pore tended to move upward along the central line from the lower to the upper portion of the casting. The phenomenon should be attributed to the change of heat transfer condition at the bottom surface of the casting. To release pressure application, the plunger was moved downward away from the bottom surface of the casting after the applied pressure was held for a desired period of time. The plunger removal led to the occurrence of natural convection between the casting bottom surface and the surrounding air. The natural convection slowed down heat extraction from the casting bottom compared to heat conduction between the plunger and casting during pressurization. Meanwhile, the top and side surfaces of casting were still in contact with the die, which resulted in relatively fast heat transfer. When the holding time was short, the lower portion of the casting became the last solidification area due to slow heat transfer and low solidification rates taking place at the casting bottom. With increasing holding times, the last solidification occurred in the upper part of the casting. This is because the applied pressure persistently kept the casting bottom surface in firm contact with the plunger, which led to rapid heat transfer and high solidification rates present at interface between the casting bottom and the plunger. Figure 9.21 compared the experimental results with numerical predictions, which shows a very good agreement.
Figure 9.20 Evolution of porosity with different holding time under pressure of 18 MPa.
9 APPLICATION OF THE MATHEMATICAL MODEL IN POROSITY PREDICTION

(a) 10s

(b) 20s

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Figure 9.21 Comparison of numerical with experimental results for castings solidified under 18 MPa with different holding times (a) 10, (b) 20, (c) 30 and (d) 60s.
9.4 Summary

Based on the mathematical model developed, a new criterion has been proposed for numerical prediction of porosity taking place in squeeze castings of magnesium alloy AM50A. The computational results have been compared with experimental data. It is found that a pressure of 30 MPa plus a pressure holding time of 21 seconds are the minimum requirement for eliminating internal porosity in a relatively thick cylindrical squeeze casting of magnesium alloy AM50A employed in this study. Under an appropriate pressure level, a porosity free casting can be obtained with a holding time less than the total solidification time.

The porosity location in squeeze castings is determined by heat transfer conditions surrounding the casting. Changes in the boundary conditions of heat transfer around the casting result in the movement of location of porosity inside the casting.

For exploring the mechanism of porosity formation, both internal and external, and for understanding the pressure characteristics in cavity, numerical predictions made by the proposed criterion were in good agreement with experimental results.
In order to fulfill the objectives stated in Chapter 1, an experimental apparatus was developed: i) to characterize local in-cavity pressures, ii) to determine casting/die interfacial heat transfer coefficient, and iii) to observe pressurized solidification phenomena taking place in squeeze casting of aluminum alloy A356 and magnesium alloy AM50A. Based on the control-volume finite difference approach plus an enthalpy method, a mathematical model was then developed to simulate the squeeze casting processes. The mathematical model was validated by the experimental measurements. The main conclusions can be summarized as follows:

a) An experimental apparatus was developed to characterize local cavity pressures in real-time during squeeze casting. Pressure transducers at different locations of the casting/die interface were employed, which enables to obtain a good understanding of transient in-cavity pressure behaviour in squeeze casting.

b) The in-cavity pressures were measured simultaneously with the acquisition of thermal histories. The results show that the local cavity pressures at various locations change with time in the duration of casting solidification and cooling. The distribution of local cavity pressures is found to be inhomogeneous in the cavity under the constant applied pressure.

c) An experimental technique was developed to determine heat transfer coefficients at the casting/die interface using a heat balance method. Experiments have been
10 SUMMARY AND CONCLUSIONS

conducted to obtain the local heat transfer coefficients at the casting/die interface under different applied pressures for both magnesium alloy AM50A and aluminum alloy A356.

d) The relationships between local heat transfer coefficients, local cavity pressures and local temperatures were established in forms of empirical equations for both magnesium alloy AM50A and aluminum alloy A356.

e) Experiments have also been carried out to study the effect of pressure on solidification of squeeze cast magnesium alloy AM50A and aluminum alloy A356. The influence of applied pressure on solidification behaviours has been observed.

f) For squeeze casting of magnesium alloy AM50A, an increase in the applied pressure level from 3 to 60 MPa reduces solidification times from 53 to 32 seconds with increasing solidification rates from 4.2 to 6.4 °C/s. But, an increase in applied pressure levels does not significantly change the shape of cooling curves.

g) The experimental observation indicates that the liquidus temperature of the alloy AM50A increased by 1.9, 2.6 and 5.4 °C and solidus by 0.8, 1.9 and 2.9 °C under the applied pressure levels of 30, 60 and 90 MPa, respectively. For aluminum alloy A356, the liquidus increased by 0.9, 2.1 and 4.1 °C and solidus by 0.9, 1.6 and 2.7 °C, respectively. The investigation on the effect of applied pressures on solidification temperatures shows the increasing rate of liquidus temperature (0.0566 °C/MPa for AM50A and 0.0444 °C/MPa for A356), is somewhat higher

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than that (0.0329 °C/MPa for AM50A and 0.0297 °C/MPa) of the solidus temperature. However, the response of experimental solidification temperature increases to a certain applied pressures is lower than those resulting from theoretical calculation.

h) A mathematical model coupled with localized boundary conditions was developed to predict transport phenomena taking place during cavity filling and pressurized solidification in squeeze casting. The model was based on the enthalpy method to solve the solidification problem.

i) By taking into consideration the entrance effect, water hammer phenomena, and natural convection, the model becomes capable of predicting melt flow phenomena in squeeze casting.

j) With the inclusion of a force (energy) balance scheme into the model, local cavity pressures can be estimated.

k) Correlating heat transfer coefficient with local pressure and temperature led to the establishment of a dynamic casting/die boundary condition, which ensures the accuracy and reliability of numerical prediction.

l) Linear empirical correlations between solidification temperature increases and applied pressures were determined and integrated into the model for accurate prediction of solidification temperatures.

m) The validity of the mathematical model has been proven by the good agreement between the numerical prediction and experimental results, including cooling
curves, solidification times, and local pressure cavity pressures, for both magnesium alloy AM50A and aluminum alloy A356.

n) The model has further been developed to investigate the evolution of shrinkage porosity in squeeze casting of magnesium alloy AM50A. A new criterion was proposed based on “burst-feeding” theory, which indicates that porosity can only be eliminated if a certain amount of plastic deformation occurs in the surrounding solid capable of feeding liquid or semi-solid metal into the last solidifying region.

o) The advanced model is able to predict the occurrence and location of porosity formation under a specified applied pressure and holding time. It is found that a pressure of 30 MPa plus a pressure holding time of 21 seconds are the minimum requirement for eliminating internal porosity in a relatively thick cylindrical squeeze casting of magnesium alloy AM50A employed in this study. In comparing the experimental results with the computation, the model not only successfully predicted the occurrence of porosity under certain circumstances, but also indicated the correct location where porosity formed.

In conclusion, the mathematical model developed in this study is valid in terms of predicting melt flow, solidification shrinkage phenomena for the squeeze casting process. It can be used to optimize the process by the best combination of process parameters.
11 FUTURE WORK

Suggested future work for the squeeze casting process may include the following:

a) Improving the local in-cavity pressure model by extending the experimental study of in-cavity pressures to cavities with complex geometries;

b) Developing a computational method, such as the inverse method, to accurately calculate the local heat transfer coefficient during the squeeze casting process;

c) Upgrading the mathematical model to simulate the casting internal pressure distribution so that it can be used to make local pressure prediction more accurately during solidification of squeeze casting;

d) Extending the application of the mathematical model to predict defects such as micro-porosity; and

e) Studying the microstructural evolution of magnesium and aluminum alloys during pressurized solidification.
STATEMENT OF ORIGINALITY

Aspects of this work constitute, in the author's opinion, new and distinct contributions to knowledge. These include:

1. Establishment of a technique involving both experiment and computation to determine the characteristics of local cavity pressures during squeeze casting of aluminum and magnesium alloys;

2. Casting/die interfacial heat transfer coefficients were correlated with local cavity pressures during solidification of squeeze casting;

3. Through thermal analysis, it was found that, during squeeze casting, solidification temperatures were overestimated by directly using hydraulic pressures instead of local cavity pressures.

4. A mathematical model has been developed to simulate the heat transfer and fluid flow, and phase change of squeeze casting process. In the model, a force balance approach was employed to estimate local in-cavity pressure, so that the boundary conditions were dynamic instead of being constant;

5. A new criterion based on “burst-feeding” theory was proposed for predicting shrinkage porosity formation in squeeze cast part with thick cross section; and

6. The fundamental principle of water hammer phenomena were innovatively integrated into the model to determine velocities in the melts upon the completion of cavity filling in squeeze casting.
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REFERENCES


REFERENCES


REFERENCES


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Appendix I Pressure Transducer Calibration

To make sure that pressure transducer used in experiment are accurate, calibrations had been made for both new and used pressure transducers. A special mount assembly was designed and made so that it can be integrated into the Instron machine (Figure A1.1).

During calibration, pre-set force was continuously applied to the transducer sensing area through the upper side pin, the response of the pressure was recorded by data acquisition system.

(a) Instron machine and data acquisition system
Figure A1.1 Pressure transducer calibration setup, (a) Instron machine and data acquisition system and (b) pressure transducer mounting assembly

Figure A1.2 shows calibration results of a new pressure transducer, an even, linear increase and decrease response step is in agreement with pre-set applied force step of 1000N, which indicates that the transducer is in good shape.

Figure A1.3 shows calibration result of a used but cleaned transducer, which apparently is also in good shape.

Figure A1.4 shows calibration result of a used and unclean transducer, obviously, the response of the transducer to the 1000N per step increase and decrease applied force is neither linear nor symmetry.
Figure A1.2 Calibration curve for new pressure transducers

Figure A1.3 Calibration result for used and cleaned pressure transducer
Figure A1.4 Calibration curve for used and unclean pressure transducers
Appendix II Annotated 6175A Cleaning Instructions

Mount the 15mm shoulder of the transducer in a collet. Loosen and unscrew the M8 hex nut. Use a brass rod (approx 6...7mm diameter) and a small hammer (no more than say 250gm) to gently tap the Measuring element out of the back side (nut side) of the adapter. Place the brass rod against the center of the measuring element and tap the other end of the brass rod with the hammer.

WARNING: NEVER use a steel pin/rod or screwdriver in place of the brass rod to remove the measuring element!

WARNING: NEVER attempt cleaning the adapter bore or measuring element by mechanical means; no metallic brushes, metal dental picks, sand paper, etc.

Inspect the adapter bore and diaphragm of the measuring element for metallic flash or other contamination.

CAUTION: The following step uses sodium hydroxide! Sodium hydroxide is poisonous and can react violently and boil over / splash back when mixing it with water.

WARNING: NEVER add sodium hydroxide to hot water.

WARNING: ALWAYS wear protective glasses and gloves.

WARNING: ALWAYS observe all printed warnings and handling precautions on the sodium hydroxide container.

Prepare a caustic solution consisting of 15ml (15CC) of pure sodium hydroxide (e.g. Red Devil Lye) in 100ml of COLD distilled water in a 150ml beaker. SLOWLY stir in the sodium hydroxide, about 5ml at a time. Place the beaker on a hot plate and bring the temperature of the solution up to 80-90 °C.

Soak the measuring element (diaphragm end only) and adapter case (bore side down) for approximately 1/2h in the hot caustic solution. Remove and rinse the measuring element and adapter in distilled water and inspect them. If necessary repeat the soaking step a couple of times.
Check the fit of the measuring element in the adapter. The measuring element should be a slip-fit in the adapter and there should not be any binding of the measuring element to the adapter. If there is some binding a very thin layer of metallic flash may still exist on the outside measuring element diaphragm. Gently twist the adapter back and forth while inserting the measuring element into the adapter, which should remove the residual flash from the measuring element.

Thoroughly rinse the measuring element and adapter in distilled water to remove all crystallized sodium hydroxide deposits. A natural fiber bristled brush (artist's brush) or cotton "Q-Tips" can be used to speed up the process.

Clean the parts with 99% isopropyl alcohol to remove any finger oil and/or particulate contamination. Allow the alcohol to evaporate, re-inspect the parts for contamination and check the measuring element to adapter fit. (e.d. Rectified Benzine was called out in the original cleaning instructions as the degreaser. This is a rather mild grease and oil solvent and is used by painters as a turpentine substitute. It also known as Pure VMP Naphtha, NOT to be confused with "benzene" a suspected carcinogen. My personal preference is to use the 99% isopropyl alcohol, which is less toxic.)

Reassemble the measuring element in the adapter. Mount the adapter in a collet and tighten the M8 hex nut to 10 Nm.
Appendix III Thermocouple Calibration

Thermal couples calibration was carried out by the supplier OMEGA. The following is a certificated provided by OMEGA.

<table>
<thead>
<tr>
<th>Nominal Temperature</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>°C</td>
</tr>
<tr>
<td>-320</td>
<td>-196</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
</tr>
<tr>
<td>32</td>
<td>0</td>
</tr>
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<td>212</td>
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<td>100</td>
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<tr>
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</tr>
<tr>
<td>787</td>
<td>419</td>
</tr>
<tr>
<td>419</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Certified By: 

Note: To calculate actual temperature
- Add departure when negative (-)
- Subtract departure when positive (+)

EXAMPLE: Nominal Temperature = 419°C
Departure Temperature = -2°C
Actual Temperature = 417°C

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Appendix IV Moving Average

Given a sequence \( \{a_i\}_{i=1}^N \), an \( n \)-moving average is a new sequence \( \{s_i\}_{i=n+1}^N \) defined from the \( a_i \) by taking the average of subsequences of \( n \) terms,

\[
s_i = \frac{1}{n} \sum_{j=i}^{i+n-1} a_j.
\]

So the sequences \( S_n \) giving \( n \)-moving averages are

\[
S_2 = \frac{1}{2} (a_1 + a_2 + a_3 + \ldots + a_n)
\]

\[
S_3 = \frac{1}{3} (a_2 + a_3 + a_4 + \ldots + a_{n-1} + a_n)
\]

and so on. The plot above shows the 2- (red), 4- (yellow), 6- (green), and 8- (blue) moving averages for a set of 100 data points.

Moving averages are implemented as \texttt{MovingAverage[data, n]} in the \texttt{Mathematica} add-on package \texttt{Statistics`DataSmoothing`} (which can be loaded with the command \texttt{<<Statistics`}).

REFERENCES:

A4.3 http://mathworld.wolfram.com/MovingAverage.html
Appendix V Temperature dependent thermal and mechanical properties of magnesium alloy AM50A [199]

Figure A5.1 Specific heat of AM50A

Figure A5.2 Density of AM50A
Figure A5.3 Thermal conductivity of AM50A

Figure A5.4 Yield strength of AM50A

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Figure A5.5 Young’s modulus of AM50A

Figure A5.6 Thermal expansion coefficient of AM50A

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Appendix VI Temperature dependent thermal and mechanical properties of Aluminum alloy A356 [199]

Figure A6.1 Young's modulus of A356.

Figure A6.2 Yield strength of A356.
Figure A6.3 Thermal expansion coefficient of A356.

Figure A6.4 Density of A356.
Figure A6.5 Thermal conductivity of A356

Figure A6.6 Specific heat of A356

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Appendix VII Source Code of the Squeeze Casting Model

C THIS IS THE SIMPLER PROGRAM.
C***********************************************************************
LOGICAL LSTOP
COMMON/CNTL/LSTOP
C***********************************************************************
C -------------------------
C ------- FILE 5 IS OUTPUT OF GENERAL RESULTS
C 5 changed to 45 on July 12, 2000
OPEN (UNIT=45,STATUS='UNKNOWN',FILE='24TRYEN1.R')
C-------------------------
C ------- FILE 7 FOR TEMP CONTOUR PLOT BY SURFER ---------------
open(unit=7,status='unknown',file='TEM&DIST.dat')
C -------------------------
C ------- FILES 8 & 11 FOR TEMP VS. TIME AT DIFFERENT LOCATIONS
C PLOTTED BY QUTRO PRO OR LOTUS123 -----------
OPEN(UNIT=8,STATUS='UNKNOWN',FILE='PRESSURE&TIME.DAT')
open(unit=11,status='unknown',file='TEM&TIME.DAT')
C -------------------------
C --- FILE 22 FOR COLOR CONTOUR PLOT BY DELTA GRAPH ----
OPEN(UNIT=22,STATUS='UNKNOWN',FILE='VELO&DIST.DAT')
C -------------------------
C ---- FILES 31, 32, 33 & 34 ARE OUTPUT OF U, T, CR & STREAM
C
OPEN(UNIT=31,STATUS='UNKNOWN',FILE='VELODIF.DAT')
OPEN(UNIT=33,STATUS='UNKNOWN',FILE='CONCEN.DAT')
OPEN(UNIT=34,STATUS='UNKNOWN',FILE='STREAM.DAT')
OPEN(UNIT=32,STATUS='UNKNOWN',FILE='TEMPERAT.DAT')
C ---- RESULT FILES FOR INTERFACE POSITION, ADDED on Jan 18, 1995---
OPEN (UNIT=50, STATUS='UNKNOWN',FILE='TEMPDIF.DAT')
CALL DEFAULT
CALL GRID
CALL SETUP1
CALL START
10 CALL DENSE
   CALL BOUND
   CALL OUTPUT
   IF(LSTOP) GO TO 12
   CALL SETUP2
   GO TO 10
12 Continue
CLOSE (45)
close(7)
CLOSE(8)
close(11)
CLOSE(22)
c
CLOSE(31)
CLOSE(32)
CLOSE(33)
CLOSE(34)
CLOSE(50)
STOP
END

SUBROUTINE DIFLOW
C*******************************************************************************
COMMON/COEF/FLOW,DIFF,ACOF
C*******************************************************************************
ACOF=DIFF
IF(FLOW.EQ.0.) RETURN
TEMP=DIFF-ABS(FLOW)*0.1
ACOF=0.
IF(TEMP.LE.0.) RETURN
TEMP=TEMP/DIFF
ACOF=DIFF*TEMP**5
RETURN
END

SUBROUTINE SOLVE
C*******************************************************************************
LOGICAL LSOLVE, LPRINT, LBLK, LSTOP
CHARACTER*8 TITLE
COMMON F(500,500,10), RHO(500,500), GAM(500,500), CON(500,500),
  1 AIP(500,500), AIM(500,500), AJP(500,500), AJM(500,500), AP(500,500),
  2 BCON1(500,500), BCON2(500,500), BCOF1(500,500,5), BCOF2(500,500,5),
  3 BCOF3(500,500,5), X(500), XU(500), XDIF(500), XCV(500), XCVS(500),
  4 Y(500), YV(500), YDIF(500), YCV(500), YCVS(500),
  5 YCVR(500), YCVRS(500), ARX(500), ARXJ(500), ARXJP(500),
  6 R(500), RMN(500), SX(500), SXMN(500), XCVI(500), XCVIP(500),
  7 EM(500,500), ALP(500,500), DLX(500,500), DLY(500,500), DLLX(500),
  8 DLLY(500), REM(500,500), DELY(500), DELX(500), TT(500,500),
  9 DDLX(500), DDLY(500), DDELY(500), DDELX(500), SETA(500,500),
  + YILD(500,500), HPURE(500), RPURE(500,500), BPURE(500,500), MM(500)
COMMON DU(500,500), DV(500,500), FV(500), FVP(500),
  1 FX(500), FXM(500), FY(500), FYM(500), PT(500), QT(500)
COMMON/INDX/NF,NFMAX, NRHO, NGAM, L1, L2, L3, M1, M2, M3,
  1IST, IST, ITER, LAST, TITLE(12), RELAX(12), TIME, DT, XL, YL,
2IPREF, JPREF, LSOLVE(10), LPRINT(12), LBLK(10), MODE, NTIMES(10), RHOCON
3NUSS1, NUSS2, NUSS
DIMENSION D(500), VAR(500), VARM(500), VARP(500), PHIBAR(500)
C*******************************************************************************
ISTF=IST-1
JSTF=JST-1
IT1=L2+IST
IT2=L3+IST
JT1=M2+JST
JT2=M3+JST
C*******************************************************************************
NTIMER=NTIMES(NF)
DO 999 NT=1, NTIMER
DO 391 N=NF, NF
IF(NOT. LBLK(NF)) GO TO 60
COME HERE TO DO BLOCK CORRECTION
C*******************************************************************************
C SUMMING IN I DIRECTION
C*******************************************************************************
DO 22 J=JST, M2
VAR(J)=0.
VARP(J)=0.
VARM(J)=0.
D(J)=0.
DO 33 I=IST, L2
VAR(J)=VAR(J)+AP(I, J)
IF(I.NE. IST) VAR(J)=VAR(J)-AIM(I, J)
IF(I.NE. L2) VAR(J)=VAR(J)-AIP(I, J)
VARM(J)=VARM(J)+AJM(I, J)
VARP(J)=VARP(J)+AJP(I, J)
D(J)=D(J)+CON(I, J)+AIP(I, J)*F(I+1, J, N)+AIM(I, J)*
1F(I-1, J, N)+AJP(I, J)*F(I, J+1, N)+AJM(I, J)*F(I, J-1, N) -
2AP(I, J)*F(I, J, N)
33 CONTINUE
22 CONTINUE
IF((NF.EQ.3).OR. (NF.EQ.4)) VAR(4)=1.
IF((NF.EQ.3).OR. (NF.EQ.4)) VARP(4)=0.
IF((NF.EQ.3).OR. (NF.EQ.4)) VARM(4)=0.
IF((NF.EQ.3).OR. (NF.EQ.4)) D(4)=0.
PHIBAR(M1)=0.
PHIBAR(JSTF)=0.
PT(JSTF)=0.
QT(JSTF)=PHIBAR(JSTF)
DO 44 J=JST, M2
DENOM=VAR(J)-PT(J-1)*VARM(J)+1.E-30
PT(J) = VARP(J)/DENOM
TEMP = D(J)
QT(J) = (TEMP + VARM(J) * QT(J-1))/DENOM
44 CONTINUE
DO 45 JJ = JST, M2
J = JT1 - JJ
45 PHIBAR(J) = PHIBAR(J+1) * PT(J) + QT(J)
DO 47 I = IST, L2
DO 47 J = JST, M2
47 F(I, J, N) = F(I, J, N) + PHIBAR(J)
C---------------------------------------------
C SUMMING IN J DIRECTION
C---------------------------------------------
C
DO 51 I = IST, L2
VAR(I) = 0.
VARP(I) = 0.
VARM(I) = 0.
D(I) = 0.
DO 53 J = JST, M2
VAR(I) = VAR(I) + AP(I, J)
IF(J .NE. JST) VAR(I) = VAR(I) - AJM(I, J)
IF(J .NE. M2) VAR(I) = VAR(I) - AJP(I, J)
VARP(I) = VARP(I) + AIP(I, J)
VARM(I) = VARM(I) + AIM(I, J)
D(I) = D(I) + CON(I, J) + AIP(I, J) * F(I+1, J, N) +
1AJM(I, J)*F(I-1, J, N) + AJP(I, J) * F(I, J+1, N) + AJM(I, J)*
2F(I, J-1, N) - AP(I, J) * F(I, J, N)
53 CONTINUE
51 CONTINUE
IF((NF.EQ.3).OR.(NF.EQ.4)) VAR(4) = 1.
IF((NF.EQ.3).OR.(NF.EQ.4)) VARP(4) = 0.
IF((NF.EQ.3).OR.(NF.EQ.4)) VARM(4) = 0.
IF((NF.EQ.3).OR.(NF.EQ.4)) D(4) = 0.
PHIBAR(L1) = 0.
PHIBAR(ISTF) = 0.
PT(ISTF) = 0.
QT(ISTF) = PHIBAR(ISTF)
DO 57 I = IST, L2
DENOM = VAR(I) - PT(I-1) * VARM(I)
PT(I) = VARP(I)/DENOM
TEMP = D(I)
QT(I) = (TEMP + QT(I-1) * VARM(I))/DENOM
57 CONTINUE
DO 58 II = IST, L2
I = IT1 - II

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58 PHIBAR(I) = PHIBAR(I+1)*PT(I)+QT(I)
   DO 59 I = IST, L2
   DO 59 J = JST, M2
59 F(I,J,N) = F(I,J,N) + PHIBAR(I)
C******************************************************************************
60 CONTINUE
   DO 90 J = JST, M2
   PT(ISTF) = 0.
   QT(ISTF) = F(ISTF, J, N)
   DO 70 I = IST, L2
   DENOM = AP(I, J) - PT(I-1) * AIM(I, J)
   PT(I) = AIP(I, J) / DENOM
   TEMP = CON(I, J) + AJP(I, J) * F(I, J+1, N) + AJM(I, J) * F(I, J-1, N)
   QT(I) = (TEMP + AIM(I, J) * QT(I-1)) / DENOM
   CONTINUE
   DO 80 II = IST, L2
   I = IT1 - II
80 F(I, J, N) = F(I+1, J, N) * PT(I) + QT(I)
90 CONTINUE
C******************************************************************************
   DO 190 J = JST, M3
   J = JT2 - JJ
   PT(ISTF) = 0.
   QT(ISTF) = F(ISTF, J, N)
   DO 170 I = IST, L2
   DENOM = AP(I, J) - PT(I-1) * AIM(I, J)
   PT(I) = AIP(I, J) / DENOM
   TEMP = CON(I, J) + AJP(I, J) * F(I, J+1, N) + AJM(I, J) * F(I, J-1, N)
   QT(I) = (TEMP + AIM(I, J) * QT(I-1)) / DENOM
170 CONTINUE
   DO 180 II = IST, L2
   I = IT1 - II
180 F(I, J, N) = F(I+1, J, N) * PT(I) + QT(I)
190 CONTINUE
C******************************************************************************
   DO 290 I = IST, L2
   PT(ISTF) = 0.
   QT(ISTF) = F(I, JSTF, N)
   DO 270 J = JST, M2
   DENOM = AP(I, J) - PT(J-1) * AJM(I, J)
   PT(J) = AJP(I, J) / DENOM
   TEMP = CON(I, J) + AIP(I, J) * F(I+1, J, N) + AIM(I, J) * F(I-1, J, N)
   QT(J) = (TEMP + AJM(I, J) * QT(J-1)) / DENOM
270 CONTINUE
   DO 280 JJ = JST, M2
   J = JT1 - JJ
280 F(I, J, N) = F(I+1, J, N) * PT(I) + QT(I)
290 CONTINUE
280 F(I,J,N) = F(I,J+1,N)*PT(J) + QT(J)
290 CONTINUE

C----------------------------------------------------------------------------------
DO 390 II = IST,L3
I = IT2-II
PT(JSTF) = 0.
QT(JSTF) = F(I,JSTF,N)
DO 370 J = JST,M2
DENOM = AP(I,J) - PT(J-1)*AMJ(I,J)
PT(J) = AJP(I,J)/DENOM
TEMP = CON(I,J) + AIP(I,J)*F(I+1,J,N) + AIM(I,J)*F(I-1,J,N)
QT(J) = (TEMP + AMJ(I,J)*QT(J-1))/DENOM
370 CONTINUE
DO 380 JJ = JST,M2
J = JT1-JJ
380 F(I,J,N) = F(I,J+1,N)*PT(J) + QT(J)
390 CONTINUE
391 CONTINUE
C************************************************************
999 CONTINUE
ENTRY RESET
DO 400 J = 2,M2
DO 400 I = 2,L2
CON(I,J) = 0.
AP(I,J) = 0.
400 CONTINUE
RETURN
END

SUBROUTINE SETUP
C******************************************************************************
LOGICAL LSOLVE,LPRINT,LBLK,LSTOP
CHARACTER*8 TITLE
COMMON F(500,500,10),RHO(500,500),GAM(500,500),CON(500,500),
  1 AIP(500,500),AIM(500,500),AP(500,500),AMJ(500,500),
  2 BCON1(500,500),BCON2(500,500),BCOF1(500,500,5),BCOF2(500,500,5),
  3 BCOF3(500,500,5),X(500),XU(500),XDIF(500),XCV(500),XCVS(500),
  4 Y(500),YV(500),YDF(500),YCV(500),YCVS(500),
  5 YCVR(500),YCVRS(500),ARX(500),ARXJ(500),ARXJP(500),
  6 R(500),RMN(500),SX(500),SXMN(500),XCVI(500),XCVIP(500),
  7 EM(500,500),ALP(500,500),DLX(500,500),DLY(500,500),DLLX(500),
  8 DLLY(500),REM(500,500),DEL(500),DELX(500),TT(500,500),
  9 DDLX(500),DDLX(500),DDELX(500),DDELX(500),SETA(500,500),
  + YILD(500,500),HPURE(500),RPURE(500,500),BPURE(500,500),MM(500)
COMMON DU(500,500),DV(500,500),FV(500),FVP(500),

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1 FX(500),FXM(500),FY(500),FYM(500),PT(500),QT(500)
COMMON/INDEX/NF,NFMAX,NRHO,NGAM,L1,L2,L3,M1,M2,M3,
1IST,JST,ITER,LAST,TITLE(12),RELAX(12),TIME,DT,XL,YL,

2IPREF,JPREF,LSOLVE(10),LPRINT(12),LBLK(10),MODE,NTIMES(10),RHOCON
3NUSS1,NUSS2,NUSS
COMMON/CNTL/LSTOP
COMMON/SORC/SMAX,SSUM,IMAX,JMAX
COMMON/COEF/FLOW,DIFF,ACOF
DIMENSION U(500,500),V(500,500),PC(500,500),P(500,500)
   EQUIVALENCE(F(1,1,1),U(1,1)),(F(1,1,2),V(1,1)),(F(1,1,3),PC(1,1))
1   ,(F(1,1,4),P(1,1))
DIMENSION COF(500,500,5)
   EQUIVALENCE(COF(1,1,1),AIP(1,1))
   EQUIVALENCE(COF(1,1,2),AIM(1,1)),(COF(1,1,3),AJP(1,1))
   EQUIVALENCE(COF(1,1,4),AJM(1,1)),(COF(1,1,5),AP(1,1))
C***********************************************************************
1 FORMAT(////,15X,'COMPUTATION IN CARTESIAN COORDINATES')
2 FORMAT(////,15X,'COMPUTATION FOR AXISYMMETRIC SITUATION')
3 FORMAT(////,15X,'COMPUTATION IN POLAR COORDINATES')
4 FORMAT(14X,40(1H*),//)
C----------------------------------------------------------------------------------
ENTRY DEFAULT
NFMAX=10
NRHO=11
NGAM=12
LSTOP=.FALSE.
DO 876 IL=1,10
   LSOLVE(IL)=.FALSE.
876   LPRINT(IL)=.FALSE.
       LPRINT(11)=.FALSE.
       LPRINT(12)=.FALSE.
       LPRINT(12)=.FALSE.
       MODE=1
       LAST=5
       TIME=0.
       ITER=0
       DO 877 IL=1,NGAM
877    RELAX(IL)=1.
            DO 878 IL=1,NFMAX
878     NTIMES(IL)=1
            DO 879 IL=1,NFMAX
879    LBLK(IL)=.TRUE.
           DT=1.E10
           IPREF=1
           JPREF=1

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RHOCON=1.
RETURN
C--------------------------------------------------------------------------
ENTRY SETUP1
L2=L1-1
L3=L2-1
M2=M1-1
M3=M2-1
X(1)=XU(2)
DO 5 1=2,L2
  5 X(I)=0.5*(XU(I+1)+XU(I))
X(L1)=XU(L1)
Y(1)=YV(2)
DO 10 J=2,M2
  10 Y(J)=0.5*(YV(J+1)+YV(J))
Y(M1)=YV(M1)
DO 15 I=2,L1
  15 XDIF(I)=X(I)-X(I-1)
DO 18 1=2,L2
  18 XCV(I)=XU(I+1)-XU(I)
DO 20 I=3,L2
  20 XCVS(I)=XDIF(I)
  22 XCVI(I)=0.5*XCV(I)
  22 XCVIP(I)=XCVI(I)
  22 XCVS(L2)=XCVS(L2)+XDIF(L1)
XCVI(L2)=XCV(L2)
DO 35 J=2,M1
  35 YDIF(J)=Y(J)-Y(J-1)
DO 40 J=2,M2
  40 YCV(J)=YV(J+1)-YV(J)
DO 45 J=3,M2
  45 YCVS(J)=YDIF(J)
  45 YCVS(3)=YCVS(3)+YDIF(2)
  45 YCVS(M2)=YCVS(M2)+YDIF(M1)
IF(MODE. NE. 1) GO TO 55
DO 52 J=1,M1
  52 RMN(J)=1.0
52 R(J)=1.0
GO TO 56
55 DO 50 J=2,M1
  50 R(J)=R(J-1)+YDIF(J)
  50 R(J)=R(J)+YDIF(J)
RMN(2)=R(1)
DO 60 J=3,M2
60 RMN(J)=RMN(J-1)+YCV(J-1)
   RMN(M1)=R(M1)
56 CONTINUE
DO 57 J=1,M1
   SX(J)=1.
   SXMN(J)=1.
   IF(MODE.NE.3) GO TO 57
   SX(J)=R(J)
   IF(J.NE.1) SXMN(J)=RMN(J)
57 CONTINUE
DO 62 J=2,M2
   YCVR(J)=R(J)*YCV(J)
   ARX(J)=YCVR(J)
   IF(MODE.NE.3) GO TO 62
   ARX(J)=YCV(J)
62 CONTINUE
DO 64 J=4,M3
64 YCVRS(J)=0.5*(R(J)+R(J-1))*YDIF(J)
   YCVRS(3)=0.5*(R(3)+R(1))*YCVS(3)
   YCVRS(M2)=0.5*(R(M1)+R(M3))*YCVS(M2)
   IF(MODE.NE.2) GO TO 67
   DO 65 J=3,M3
      ARXJ(J)=0.25*(1.+RMN(J)/R(J))*ARX(J)
   65 ARXJP(J)=ARX(J)-ARXJ(J)
   GO TO 68
67 DO 66 J=3,M3
   ARXJ(J)=0.5*ARX(J)
66 ARXJP(J)=ARXJ(J)
68 ARXJP(2)=ARX(2)
   ARXJ(M2)=ARX(M2)
   DO 70 J=3,M3
      FV(J)=ARXJP(J)/ARX(J)
   70 FVP(J)=1.-FV(J)
   DO 85 I=3,L2
      FX(I)=0.5*XCV(I-1)/XDIF(I)
   85 FXM(I)=1.-FX(I)
   FX(2)=0.
   FXM(2)=1.
   FX(L1)=1.
   FXM(L1)=0.
   DO 90 J=3,M2
      FY(J)=0.5*YCV(J-1)/YDIF(J)
90 FYM(J)=1.-FY(J)
   FY(2)=0.
   FYM(2)=1.
FY(M1)=1.
FY(M1)=0.
CON, AP, U, V, RHO, PC AND P ARRAYS ARE INITIALIZED HERE

DO 95 J=1,M1
DO 95 I=1,L1
PC(I,J)=0.
U(I,J)=0.
V(I,J)=0.
CON(I,J)=0.
AP(I,J)=0.
RHO(I,J)=RHOCON
P(I,J)=0.
95 CONTINUE
IF(MODE.EQ.1) THEN
PRINT 1
WRITE (45,1)
END IF
IF(MODE.EQ.2) THEN
PRINT 2
WRITE (45,2)
END IF
IF(MODE.EQ.3) THEN
PRINT 3
WRITE (45,3)
END IF
PRINT 4
WRITE (45,4)
RETURN

C-------------------------------------------------------------------------
ENTRY SETUP2

COEFFICIENTS FOR THE U EQUATION

CALL RESET
NF=1
IF(.NOT.LSOLVE(NF)) GO TO 100
IST=3
JST=2
CALL GAMSOR
REL=1.-RELAX(NF)
DO 102 I=3,L2
FL=XCVI(I)*V(I,2)*RHO(I,1)
FLM=XCVIP(I-1)*V(I-1,2)*RHO(I-1,1)
FLOW=R(1)*(FL+FLM)
DIFF=R(1)*(XCVI(I)*GAM(I,1)+XCVIP(I-1)*GAM(I-1,1))/YDIF(2)
CALL DIFLOW
102 AJM(I,2)=ACOF+AMAX1(0.,FLOW)
DO 103 J=2,M2

C-------------------------------------------------------------------------
FLOW=ARX(J)*U(2,J)*RHO(1,J)
DIFF=ARX(J)*GAM(1,J)/(XCV(2)*SX(J))
CALL DIFLOW
AIM(3,J)=ACOF+AMAX1(0.,FLOW)
DO 103 I=3,L2
IF(I.EQ.L2) GO TO 104
FL=U(I,J)*(FX(I)*RHO(I,J)+FXM(I)*RHO(I-1,J))
FLP=U(I+1,J)*(FX(I+1)*RHO(I+1,J)+FXM(I+1)*RHO(I,J))
FLOW=ARX(J)*0.5*(FL+FLP)
DIFF=ARX(J)*GAM(I,J)/(XCV(I)*SX(J))
GO TO 105
104 FLOW=ARX(J)*U(L1,J)*RHO(L1,J)
DIFF=ARX(J)*GAM(L1,J)/(XCV(L2)*SX(J))
105 CALL DIFLOW
AIM(I+1,J)=ACOF+AMAX1(0.,FLOW)
AJP(I,J)=AIM(I+1,J)-FLOW
IF(I.EQ.M2) GO TO 106
FL=XCV(I)*V(I,J+1)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J))
FLM=XCVIP(I)*V(I-1,J+1)*(FY(J+1)*RHO(I-1,J+1)+FYM(J+1)*
1 RHO(I-1,J))
GM=GAM(I,J)*GAM(I,J+1)/(YCV(I)*GAM(I,J+1)+YCV(J)*GAM(I,J)+
1 1.0E-30)*XCVI(I)
GMM=GAM(I-1,J)*GAM(I-1,J+1)/(YCV(J)*GAM(I-1,J+1)+YCV(J)*
1 GAM(I-1,J)+1.0E-30)*XCVIP(I-1)
DIFF=R(M1)*2.*(GM+GMM)
GO TO 107
106 FL=XCVI(I)*V(I,M1)*RHO(I,M1)
FLM=XCVIP(I)*V(I-1,M1)*RHO(I-1,M1)
DIFF=R(M1)*XCVI(I)*GAM(I,M1)+XCVIP(I)*GAM(I-1,M1))/YDFM(M1)
107 FLOW=RMN(J+1)*(FL+FLM)
CALL DIFLOW
AIM(I,J)=ACOF+AMAX1(0.,FLOW)
AJP(I,J)=AIM(I,J+1)-FLOW
VOL=YCVR(J)*XCVS(I)
APT=(RHO(I,J)*XCVI(I)+RHO(I-1,J)*XCVIP(I-1))/
1/(XCVS(I)*DT)
AP(I,J)=AP(I,J)-APT
CON(I,J)=CON(I,J)+APT*U(I,J)
AP(I,J)=AP(I,J)-VOL*AJP(I,J)+AIM(I,J)+AJP(I,J)+AJM(I,J)
1/RELAX(NF)
CON(I,J)=CON(I,J)+VOL+REL*AP(I,J)*U(I,J)
DU(I,J)=VOL/(XDIF(I)*SX(J))
DU(I,J)=DU(I,J)/AP(I,J)
103 CONTINUE
DO 1001 I=1,L1
DO 1001 J=1,M1
DO 1101 K = 1,5
1101 BCOFI(I,J,K) = COF(I,J,K)
1001 BCON1(I,J) = CON(I,J)
C______________________TEMPORARY USE OF PC(I,J) TO STORE
UHAT_____________________
DO 151 J = 2, M2
DO 151 I = 3, L2
151 PC(I,J) = (AIP(I,J)*U(I+1,J) + AIM(I,J)*U(I-1,J) + AJP(I,J)*U(I,J+1) +
 1AJM(I,J)*U(I,J-1) + CON(I,J))/AP(I,J)
100 CONTINUE

COEFFICIENTS FOR THE V EQUATION---------------------------------------
CALL RESET
NF = 2
IF (.NOT. LSOLVE(NF)) GO TO 200
IST = 2
JST = 3
CALL GAMSOR
REL = 1 - RELAX(NF)
DO 202 I = 2, L2
AREA = R(I)*XCV(I)
FLOW = AREA*V(I,2)*RHO(I,1)
DIFF = AREA*GAM(I,1)/YCV(2)
CALL DIFLOW
202 AJM(I,3) = ACOF + AMAX1(0., FLOW)
DO 203 J = 3, M2
FL = ARXJ(J)*U(2, J)*RHO(1, J)
FLM = ARXJP(J-1)*U(2, J-1)*RHO(1, J-1)
FLOW = FL + FLM
DIFF = (ARXJ(J)*GAM(1, J) + ARXJP(J-1)*GAM(1, J-1))/(XDIF(2)*SXMN(J))
CALL DIFLOW
AIM(2, J) = ACOF + AMAX1(0., FLOW)
DO 203 I = 2, L2
IF (LEQ.L2) GO TO 204
FL = ARXJ(I)*U(I+1, J)*(FX(I+1)*RHO(I+1, J) + FXM(I+1)*RHO(I, J))
FLM = ARXJP(J-1)*U(I+1, J-1)*(FX(I+1)*RHO(I+1, J-1) + FXM(I+1)*
 1 RHO(I, J-1))
GM = GAM(I, J)*GAM(I+1, J)/(XCV(I)*GAM(I+1, J) + XCV(I+1)*GAM(I, J) +
 1 1.0E-30)*ARXJ(J)
GMM = GAM(I, J-1)*GAM(I+1, J-1)/(XCV(I)*GAM(I+1, J-1) + XCV(I+1)*
 1 GAM(I, J-1)+1.0E-30)*ARXJP(J-1)
DIFF = 2.*(GM+GMM)/SXMN(J)
GO TO 205
204 FL = ARXJ(J)*U(L1, J)*RHO(L1, J)
FLM = ARXJP(J-1)*U(L1, J-1)*RHO(L1, J-1)
DIFF = (ARXJ(J)*GAM(L1, J) + ARXJP(J-1)*GAM(L1, J-1))/(XDIF(L1)*SXMN(J))
205 FLOW = FL + FLM
CALL DIFLOW
AIM(I+1,J)=ACOF+AMAX1(0.,FLOW)
AIP(I,J)=AIM(I+1,J)-FLOW
IF(J.EQ.M2) GO TO 206
AREA=R(J)*XCV(I)
FL=V(I,J)*(FYI)*RH0(I,J)+FYM(J)*RH0(I,J-1)*RMN(J)
FLP=V(I,J+1)*(FY(J+1)*RH0(I,J+1)+FYM(J+1)*RH0(I,J))*RMN(J+1)
FLOW=(FY(J)*FL+FVP(J)*FLP)*XCV(I)
DIFF=AREA*GAM(I,J)/YCV(J)
GO TO 207
206 AREA=R(M1)*XCV(I)
FLOW=AREA*V(I,M1)*RHO(I,M1)
DIFF=AREA*GAM(I,M1)/YCV(M2)
207 CALL DIFLOW
AJM(I,J+1)=ACOF+AMAX1(0.,FLOW)
AJP(I,J)=AJM(I,J+1)-FLOW
VOL=YCVR(S(J)*XCV(I)
APT=(ARXJ(J)*RHO(I,J)*0.5*(SX(J)+SXMNJ)+ARXJP(J-1)*RHO(I,J-1)*
10.5*(SX(J-1)+SXMNJ)/(YCVR(J)*DT)
AP(I,J)=AP(I,J)-APT
CON(I,J)=CON(I,J)+APT*V(I,J)
AP(I,J)=(-AP(I,J)*VOL+AIP(I,J)+AIM(I,J)+AJP(I,J)+AJM(I,J))
1/RELAX(NF)
CON(I,J)=CON(I,J)*VOL+REL*AP(I,J)*V(I,J)
DV(I,J)=VOL/YDIF(J)
DV(I,J)=DV(I,J)/AP(I,J)
203 CONTINUE
DO 1002 I=1,L1
DO 1002 J=1,M1
DO 1102 K=1,5
1102 BC0F2(I,J,K)=COF(I,J,K)
1002 BCON2(I,J)=CON(I,J)
200 CONTINUE
COEFFICIENTS FOR THE PRESSURE EQUATION

NF=4
IF(.NOT.LSOLVE(NF)) GO TO 500
IST=2
JST=2
DO 402 I=2,L2
ARHO=R(I)*XCV(J)*RHO(I,1)
CON(I,2)=ARHO*V(I,2)
402 AJM(I,2)=0.
DO 403 J=2,M2
ARHO=ARX(J)*RHO(I,1)
CON(2,J)=CON(2,J)+ARHO*U(2,J)
AIM(2,J)=0.
DO 403 I=2,L2
IF(I.EQ.L2) GO TO 404
ARHO=ARX(J)*(FX(I+1)*RHO(I+1,J)+FXM(I+1)*RHO(I,J))
FLOW=ARHO*PC(I+1,J)
CON(I,J)=CON(I,J)-FLOW
CON(I+1,J)=CON(I+1,J)+FLOW
AIP(I,J)=ARHO*DU(I+1,J)
AIM(I+1,J)=AIP(I,J)
GO TO 405
404 ARHO=ARX(J)*RHO(L1,J)
CON(I,J)=CON(I,J)-ARHO*U(L1,J)
AIP(I,J)=0.
405 IF(I.EQ.M2) GO TO 406
ARHO=RMN(J+1)*XCV(I)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J))
VHAT=AIP(I+1,J)*V(I+1,J+1)+AIM(I,J+1)*V(I,J+1)+AJP(I,J+1)*
IV(I,J+2)+AIM(I,J+1)*V(I,J)+CON(I,J+1)/AP(I,J+1)
FLOW=ARHO*VHAT
CON(I,J)=CON(I,J)-FLOW
CON(I,J+1)=FLOW
AJP(I,J)=ARHO*DV(I,J+1)
AIM(I,J+1)=AJP(I,J)
GO TO 407
406 ARHO=RMN(M1)*XCV(I)*RHO(I,M1)
CON(I,J)=CON(I,J)-ARHO*V(I,M1)
AIP(I,J)=0.
407 AP(I,J)=AIP(I,J)+AIM(I,J)+AJP(I,J)+AJM(I,J)
403 CONTINUE
C WRITE(1) COF
DO 1003 I=1,L1
DO 1003 J=1,M1
DO 1003 K=1,5
1003 BCOF3(I,J,K)=COF(I,J,K)
IF(ITER.LE.1) GO TO 409
DO 408 J=2,M2
DO 408 I=2,L2
AP(I,J)=AP(I,J)/RELAX(4)
CON(I,J)=CON(I,J)+(1.-RELAX(4))*AP(I,J)*P(I,J)
408 CONTINUE
409 CONTINUE
CALL SOLVE
NF=1
IST=3
JST=2
DO 910 I=3,L2
DO 910 J=2,M2
910 F(I,J,9)=F(I,J,NF)
DO 1004 I=1,L1
DO 1004 J=1,M1
DO 1104 K=1,5
1104 COF(I,J,K)=BCOF1(I,J,K)
1004 CON(I,J)=BCON1(I,J)
DO 413 J=2,M2
DO 413 I=3,L2
413 CON(I,J)=CON(I,J)+DU(I,J)*AP(I,J)*(P(I-1,J)-P(I,J))
CALL SOLVE
NF=2
IST=2
JST=3
DO 912 I=2,L2
DO 912 J=3,M2
912 F(I,J,10)=F(I,J,NF)
DO 1005 I=1,L1
DO 1005 J=1,M1
DO 1105 K=1,5
1105 COF(I,J,K)=BCOF2(I,J,K)
1005 CON(I,J)=BCON2(I,J)
DO 414 J=3,M2
DO 414 I=2,L2
414 CON(I,J)=CON(I,J)+DV(I,J)*AP(I,J)*(P(I,J-1)-P(I,J))
CALL SOLVE
COEFFICIENTS FOR THE PRESSURE CORRECTION EQUATION-------------
C READ(1) COF
DO 1006 I=1,L1
DO 1006 J=1,M1
DO 1006 K=1,5
1006 COF(I,J,K)=BCOF3(I,J,K)
NF=3
IF(.NOT.LSOLVE(NF)) GO TO 500
IST=2
JST=2
CALL GAMSOR
SMAX=0.
SSUM=0.
DO 410 J=2,M2
DO 410 I=2,L2
VOL=YCVR(J)*XCV(I)
410 CON(I,J)=CON(I,J)*VOL
DO 474 I=2,L2
ARHO=R(1)*XCV(I)*RHO(I,1)
474 CON(I,2)=CON(I,2)+ARHO*V(I,2)
DO 475 J=2,M2
ARHO=ARX(J)*RHO(1,J)
CON(2,J)=CON(2,J)+ARHO*U(2,J)
DO 475 I=2,L2
IF(I.EQ.L2) GO TO 476
ARHO=ARX(J)*(FX(I+1)*RHO(I+1,J)+FXM(I+1)*RHO(I,J))
FLOW=ARHO*U(I+1,J)
CON(I,J)=CON(I,J)-FLOW
CON(I+1,J)=CON(I+1,J)+FLOW
GO TO 477

476 ARHO=ARX(J)*RHO(L1,J)
CON(I,J)=CON(I,J)-ARHO*U(L1,J)
477 IF(J.EQ.M2) GO TO 478
ARHO=RMN(J+1)*XCV(I)*(FY(J+1)*RHO(I,J+1)+FYM(J+1)*RHO(I,J))
FLOW=ARHO*V(I,J+1)
CON(I,J)=CON(I,J)-FLOW
CON(I,J+1)=CON(I,J+1)+FLOW
GO TO 479

478 ARHO=RMN(M1)*XCV(J)*RHO(I,M1)
CON(I,J)=CON(I,J)-ARHO*V(I,M1)

479 PC(I,J)=0.
SMAXO=SMAX
SMAX=AMAX1(SMAX,ABS(CON(I,J)))
IF(SMAX.NE.SMAXO) IMAX=I
IF(SMAX.NE.SMAXO) JMAX=J
SSUM=SSUM+CON(I,J)

475 CONTINUE
CALL SOLVE

COME HERE TO CORRECT THE VELOCITIES-----------------------
DO 501 J=2,M2
DO 501 I=2,L2
IF(I.NE.2) U(I,J)=U(I,J)+DU(I,J)*(PC(I-1,J)-PC(I,J))
IF(J.NE.2) V(I,J)=V(I,J)+DV(I,J)*(PC(I,J-1)-PC(I,J))
501 CONTINUE

500 CONTINUE

COEFFICIENTS FOR OTHER EQUATIONS---------------------------------------------
IST=2
JST=2
DO 600 NF=5,NFMAX
IF(.NOT.LSOLVE(NF)) GO TO 600

DO 914 I=2,L2
DO 914 J=2,M2
914 F(I,J,9)=F(I,J,NF)
CALL GAMSOR
REL=1.-RELAX(NF)
DO 602 I=2,L2
AREA=R(1)*XCV(I)
FLOW=AREA*V(I,2)*RHO(I,1)
364

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DIFF = AREA * GAM(I,1) / YDIF(2)
CALL DIFLOW
602 AJM(I,2) = ACOF + AMAX1(0., FLOW)
DO 603 J = 2, M2
  FLOW = ARX(J) * U(2,J) * RHO(1,J)
  DIFF = ARX(J) * GAM(1,J) / (XDIF(2) * SX(J))
  CALL DIFLOW
  AIM(2,J) = ACOF + AMAX1(0., FLOW)
DO 603 I = 2, L2
  IF (I.EQ.2) GO TO 604
  FLOW = ARX(J) * U(I+1,J) * (FX(I+1) * RHO(I+1,J) + FXM(I+1) * RHO(I,J))
  DIFF = ARX(J) * 2. * GAM(I,J) * GAM(I+1,J) / ((XCV(I) * GAM(I+1,J) +
  1 XCV(I+1) * GAM(I,J) + 1.0E-30) * SX(J))
  GO TO 605
604 FLOW = ARX(J) * U(L1,J) * RHO(L1,J)
  DIFF = ARX(J) * GAM(L1,J) / (XDIF(L1) * SX(J))
605 CALL DIFLOW
  AIM(I+1,J) = ACOF + AMAX1(0., FLOW)
  AIP(I,J) = AIM(I+1,J) - FLOW
  AREA = RMN(I+1) * XCV(I)
  IF (I.EQ.1) GO TO 606
  FLOW = AREA * V(I+1,J) * (FY(I+1) * RHO(I+1,J) + FYM(I+1) * RHO(I,J))
  DIFF = AREA * 2. * GAM(I,J) * GAM(I+1,J) / (YCV(J) * GAM(I,J) +
  1 YCV(I) * GAM(I,J) + 1.0E-30)
  GO TO 607
606 FLOW = AREA * V(I,M1) * RHO(I,M1)
  DIFF = AREA * GAM(I,M1) / YDIF(M1)
607 CALL DIFLOW
  AJM(I,J+1) = ACOF + AMAX1(0., FLOW)
  AJP(I,J) = AJM(I,J+1) - FLOW
  VOL = YCVR(J) * XCV(I)
  APT = RHO(I,J) / DT
  AP(I,J) = AJP(I,J) - APT
  CON(I,J) = CON(I,J) + APT * F(I,J,NF)
  F(I,J) = (-AP(I,J) * VOL + AIP(I,J) + AIM(I,J) + AJP(I,J) + AJM(I,J))
  1/RELAX(NF)
  CON(I,J) = CON(I,J) + REL * AP(I,J) * F(I,J,NF)
603 CONTINUE
  CALL SOLVE
600 CONTINUE
TIME = TIME + DT
ITER = ITER + 1
  IF (ITER .GE. LAST) LSTOP = .TRUE.
  IF (F(I,J,5) .EQ. 1) LSTOP = .TRUE.
RETURN
END
SUBROUTINE SUPPLY

C******************************************************

LOGICAL LSOLVE,LPRINT,LBLK,LSTOP

CHARACTER*8 TITLE

COMMON F(500,500,10),RHO(500,500),GAM(500,500),CON(500,500),
1 AIP(500,500),AIM(500,500),AJP(500,500),AJM(500,500),AP(500,500),
2 BCON1(500,500),BCON2(500,500),BCOF1(500,500,5),BCOF2(500,500,5),
3 BCOF3(500,500,5),X(500),XU(500),XDIF(500),XCV(500),XCVS(500),
4 Y(500),YV(500),YDF(500),YCV(500),YCVS(500),
5 YCVR(500),YCVRS(500),ARX(500),ARXJ(500),ARXJP(500),
6 R(500),RMN(500),SX(500),SXMN(500),XCVI(500),XCVIP(500),
7 EM(500,500),ALP(500,500),DLX(500,500),DLY(500,500),DLLLX(500),
8 DLYY(500),REM(500,500),DELY(500),DELX(500),TT(500,500),
9 DDLX(500),DDLX(500),DELX(500),DELX(500),SETA(500,500),
+ YILD(500,500),HPURE(500),RPURE(500,500),BPURE(500,500),MM(500)

COMMON DU(500,500),DV(500,500),FV(500),FVP(500),
1 FX(500),FXM(500),FY(500),FYM(500),PT(500),QT(500)

COMMON /INDX/NF,NMAX,NRHO,NGAM,L1,L2,L3,M1,M2,M3,
1IST,JST,ITER,LAST,TITLE(12),RELAX(12),TIME,DT,XL,YL,

2IPREF,JPREF,LSOLVE(10),LPRINT(12),LBLK(10),MODE,NTIMES(10),RHOCON
3NUSS1,NUSS2,NUSS

DIMENSION U(500,500),V(500,500),PC(500,500),P(500,500)

EQUIVALENCE(F(1,1,1),U(1,1)),(F(1,1,2),V(1,1)),(F(1,1,3),PC(1,1))
1 , (F(1,1,4),P(1,1))

C******************************************************

10 FORMAT(26(1H*),3X,A10,3X,26(1H*))
20 FORMAT(1X,4H I =,I6,6I9)
30 FORMAT(1X,1HJ)
40 FORMAT(1X,12,3X,1P7E9.2)
50 FORMAT(1X,1H)
51 FORMAT(1X,'I =',2X,7(I4,5X))
52 FORMAT(1X,'X =',1P7E9.2)
53 FORMAT('TH =',1P7E9.2)
54 FORMAT(1X,'J =',2X,7(I4,5X))
55 FORMAT(1X,'Y =',1P7E9.2)

C******************************************************

ENTRY UGRID

XU(2)=0.
DX=XL/FLOAT(L1-2)
DO 1 I=3,L1
1 XU(I)=XU(I-1)+DX
YV(2)=0.

366
DY = YL/FLOAT(M1-2)
DO 2 J=3,M1
2 YV(J)=YV(J-1)+DY
RETURN

C**************************************************************************
ENTRY PRINT
IF (.NOT. LPRINT(3)) GO TO 80

CALCULATE THE STREAM FUNCTION---------------------------------------------
F(2,2,3)=0.
DO 82 I=2,L1
IF (I.NE.2) F(I,2,3)=F(I-1,2,3)-RHO(I-1,1)*V(I-1,2)
* R(1)*XCV(I-1)
DO 82 J=3,M1
RHM=F(I,1)*RHO(I,J-1)+FXM(I)*RHO(I-1,J-1)
82 F(I,J,3)=F(I,J-1,3)+RHM*U(I,J-1)*ARX(J-1)
80 CONTINUE
 IF (.NOT. LPRINT(4)) GO TO 90

CONSTRUCT BOUNDARY PRESSURES BY EXTRAPOLATION
DO 91 J=2,M2
P(1,J)=(P(2,J)*XCVS(3)-P(3,J)*XDIF(2))/XDIF(3)
91 P(L1,J)=(P(L2,J)*XCVS(L2)-P(L3,J)*XDIF(L1))/XDIF(L2)
DO 92 J=1,L2
P(I,1)=(P(I,2)*YCVS(3)-P(I,3)*YDIF(2))/YDIF(3)
92 P(I,M1)=(P(I,M2)*YCVS(M2)-P(I,M3)*YDIF(M1))/YDIF(M2)
P(1,1)=P(2,1)+P(1,2)-P(2,2)
P(L1,1)=P(L2,1)+P(L1,2)-P(L2,2)
P(1,M1)=P(2,M1)+P(1,M2)-P(2,M2)
P(L1,M1)=P(L2,M1)+P(L1,M2)-P(L2,M2)
PREF=P(IPREF,JPREF)
DO 93 J=2,L1
93 P(I,J)=P(I,J)-PREF
90 CONTINUE
PRINT 50
WRITE (45,50)
C ** SEPT 5, 1993 ********
WRITE(31,50)
write(32,50)
WRITE(33,50)
WRITE(34,50)
C**************************************************************************
IEND=0
301 IF(IEND.EQ.L1) GO TO 310
IBEG=IEND+1
IEND=IEND+7
IEND=MIN0(IEND,L1)
PRINT 50
WRITE (45,50)
C ********************************************SEPT 5, 1993 ************
WRITE (31,50)
WRITE (32,50)
WRITE (33,50)
WRITE (34,50)
C ********************************************
PRINT 51,(I,I=IBEG,IEND)
WRITE (45,51) (I,I=IBEG,IEND)
C ********************************************SEPT 5, 1993 ************
WRITE(31,51) (I,I=IBEG,IEND)
WRITE(32,51) (i,i=ibeg,iend)
WRITE(33,51) (I,I=IBEG,IEND)
WRITE(34,51) (I,I=IBEG,IEND)
C ********************************************
    IF(MODE.EQ.3) GO TO 302
    PRINT 52, (X(I),I=IBEG,IEND)
    WRITE (45,52) (X(I),I=IBEG,IEND)
C ********************************************SEPT 5, 1993 ************
    WRITE (31,52) (X(I),I=IBEG,IEND)
    write(32,52) (x(i),i=ibeg,iend)
    WRITE (33,52) (X(I),I=IBEG,IEND)
    WRITE (34,52) (X(I),I=IBEG,IEND)
C ********************************************
    GO TO 303
302 PRINT 53, (X(I),I=IBEG,IEND)
    WRITE (45,53) (X(I),I=IBEG,IEND)
C ********************************************SEPT 5, 1993 ************
    WRITE(31,53) (X(I),I=IBEG,IEND)
    write(32,53) (x(i),i=ibeg,iend)
    WRITE(33,53) (X(I),I=IBEG,IEND)
    WRITE(34,53) (X(I),I=IBEG,IEND)
C ********************************************
    GO TO 301
303 GO TO 301
310 JEND=0
    PRINT 50
    WRITE (45,50)
C ********************************************SEPT 5, 1993 ************
    WRITE (31,50)
    write (32,50)
    WRITE (33,50)
    WRITE (34,50)
C ********************************************
311 IF(JEND.EQ.M1) GO TO 320
    JBEG=JEND+1
JEND=JEND+7
JEND=MIN0(JEND,M1)
PRINT 50
WRITE (45,50)
C ******************************************SEPT 5, 1993 **************
WRITE (31,50)
write (32,50)
WRITE (33,50)
WRITE (34,50)
C ******************************************SEPT 5, 1993 **************
PRINT 54, (J,J=JBEG,JEND)
WRITE (45,54) (J,J=JBEG,JEND)
C ******************************************SEPT 5, 1993 **************
WRITE (31,54) (J,J=JBEG,JEND)
write (32,54) (j,j=jbeg,jend)
WRITE (33,54) (J,J=JBEG,JEND)
WRITE (34,54) (J,J=JBEG,JEND)
C ******************************************SEPT 5, 1993 **************
PRINT 55, (Y(J),J=JBEG,JEND)
WRITE (45,55) (Y(J),J=JBEG,JEND)
C ******************************************SEPT 5, 1993 **************
WRITE (31,55) (Y(J),J=JBEG,JEND)
write (32,55) (y(j),j=jbeg,jend)
WRITE (33,55) (Y(J),J=JBEG,JEND)
WRITE (34,55) (Y(J),J=JBEG,JEND)
C ******************************************SEPT 5, 1993 **************
GO TO 311
320 CONTINUE
DO 999 NF=1,NGAM
   IF (.NOT. LPRINT(NF)) GO TO 999
PRINT 50
C--------SEPT 5,1993------ WRITE (45,50)
PRINT 10, TITLE(NF)
C-------- SEPT 5,1993------ WRITE (45,10) TITLE(NF)
   IFST=1
   JFST=1
   IF(NF.EQ.1.OR.NF.EQ.3) IFST=2
   IF(NF.EQ.2.OR.NF.EQ.3) JFST=2
   IBEG=IFST-7
110 CONTINUE
   IBEG=IBEG+7
   IEND=IBEG+6
   IEND=MIN0(IEND,L1)
PRINT 50
C-------- SEPT 5,1993------ WRITE (45,50)
   PRINT 20, (I,I=IBEG,IEND)
C------ SEPT 5,1993------ WRITE (45,20) (I,I=IBEG,IEND)
   PRINT 30
C------ SEPT 5,1993------ WRITE (45,30)
   JFL=JFST+M1
   DO 115 JJ=JFST,M1
   J=JFL-JJ
   PRINT 40, J,(F(I,J,NF),I=IBEG,IEND)
C------ SEPT 5,1993------ WRITE (45,40) J,(F(I,J,NF),I=IBEG,IEND)
115 CONTINUE
   IF(IEND.LT.L1) GO TO 110
999 CONTINUE

C ********************************************SEPT 5, 1993 **************
cc if(nf.ne.8.or.6) goto 1999
WRITE (31,50)
WRITE (32,50)
WRITE (33,50)
WRITE (34,50)
WRITE (31,10) TITLE(8)
WRITE (32,10) TITLE(6)
WRITE (33,10) TITLE(7)
WRITE (34,10) TITLE(10)
C ********************************************SEPT 5, 1993 ***************
   IFST=I
   JFST=I
   IF(NF.EQ.1.OR.NF.EQ.3) IFST=2
   IF(NF.EQ.2.OR.NF.EQ.3) JFST=2
   IBEG=IFST-7
1110 CONTINUE
   IBEG=IBEG+7
   IEND=IBEG+6
   IEND=MIN0(IEND,L1)
C ********************************************SEPT 5, 1993 ***************
WRITE (31,50)
WRITE (32,50)
WRITE (33,50)
WRITE (34,50)
WRITE (31,20) (I,I=IBEG,IEND)
WRITE (32,20) (I,I=IBEG,IEND)
WRITE (33,20) (I,I=IBEG,IEND)
WRITE (34,20) (I,I=IBEG,IEND)
WRITE (31,30)
WRITE (32,30)
WRITE (33,30)
WRITE (34,30)
C ********************************************SEPT 5, 1993 ***************

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JFL = JFST + M1
DO 1115 JJ = JFST, M1
J = JFL - JJ
C ****************************************** SEPT 5, 1993 ***************
WRITE (31, 40) J, (F(I, J, 8), I = IBEG, IEND)
WRITE (32, 40) J, (F(I, J, 6), I = IBEG, IEND)
WRITE (33, 40) J, (F(I, J, 7), I = IBEG, IEND)
WRITE (34, 40) J, (F(I, J, 10), I = IBEG, IEND)
C ***********************************************
1115 CONTINUE
IF (IEND .LT .L1) GO TO 1110
1999 CONTINUE
RETURN
END

C PROGRAM FOR SQUEEZE CASTING
C
SUBROUTINE USER
C
----------------------------------------------------------------------------------
LOGICAL LSOLVE, LPRINT, LBLK, LSTOP
CHARACTER*8 TITLE
COMMON F(500,500,10), RHO(500,500), GAM(500,500), CON(500,500),
1 AIP(500,500), AIM(500,500), AJP(500,500), AJM(500,500), AP(500,500),
2 BCON1(500,500), BCON2(500,500), BCOFI(500,500,5), BCOF2(500,500,5),
3 BCOF3(500,500,5), X(500), XU(500), XDIF(500), XCV(500), XCVS(500),
4 Y(500), YV(500), YDIFF(500), YCV(500), YCVS(500),
5 YCVR(500), YCVRS(500), ARX(500), ARXI(500), ARXJP(500),
6 R(500), RMN(500), SX(500), SXMN(500), XCV(500), XCVPI(500),
7 EM(500,500), ALP(500,500), DLX(500,500), DLY(500,500), DLLX(500),
8 DLLY(500), REM(500,500), DELY(500), DELX(500), TT(500,500),
9 DDLX(500), DDLY(500), DDELY(500), DDELX(500), SETA(500,500),
+ YILD(500,500), HPURE(500), RPURE(500,500), BPURE(500,500), MM(500)
COMMON DU(500,500), DV(500,500), FV(500), FVP(500),
1 FX(500), FXM(500), FY(500), FYM(500), PT(500), QT(500)
COMMON/INDEX/NF, NFMAX, NRHO, NGAM, L1, L2, L3, M1, M2, M3,
1 IST, JST, ITER, LAST, TITLE(12), RELAX(12), TIME, DT, XL, YL,
2 IPREF, JPREF, LSOLVE(10), LPRINT(12), LBLK(10), MODE, NTIMES(10), RHOCON
3 NUSS1, NUSS2, NUSS
COMMON/CNTL/LSTOP
COMMON/SORC/SMAX, SSUM, IMAX, JMAX
COMMON/COEF/FLOW, DIFF, ACOF
DIMENSION U(500,500), V(500,500), PC(500,500), P(500,500)
EQUIVALENCE(F(1,1,1),U(1,1)), (F(1,1,2), V(1,1)), (F(1,1,3), PC(1,1))
1, (F(1,1,4), P(1,1))
C-------------------------------------------------------------------------------------------
DIMENSION E(500,500),T(500,500),SE(500,500),UUU(500,500)
    1 ,V(V500,500),XY(500,500),KCOND(500,500),CCP(500,500)
    2 ,G1(500,500),G2(500,500),G3(500,500),G4(500,500),RPPURE(500,500),
    3 G5(500,500),G6(500,500),G7(500,500),G8(500,500),GG(500,500),
    4 SHX(500,500),SHY(500,500)
EQUIVALENCE (F(1,1,5),T(1,1)),(F(1,1,6),E(1,1))
C-------------------------------------------------------------------------------------------
ENTRY GRID
MODE=2
XL=0.08
YL=0.0508
R(1)=0.
L1=79
M1=49
    CALL UGRID
    RETURN
ENTRY START
PRINT*, 'ENTER LASTTIME (SEC) = '
READ *, LAST
WRITE(45,*), 'ENTER TIME (SEC) = ', LAST
    MMM=220
    NNN=200
    DIAM=0.05
    VELO=0.01
    MIU=6.97E-07
DT=0.1
C   RT=0.005
APURE=70
TPOUR=690
    TFT1=200
    TFT=275
    TFR=275
    TFB=275
    YYYZYY=695
C-------------------------------------------------------------------------------------------
    DO 90 K=1,4
    LSOLVE(K)=.TRUE.
    CLPRINT(K)=.TRUE.
90   CONTINUE
    LSOLVE(5)=.TRUE.
    CLPRINT(5)=.TRUE.
    CLPRINT(6)=.TRUE.
    TITLE(5)= 'TEMPERATURE'
    TITLE(6)= 'E'
    RELAX(1)=0.9

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RELAX(2)=0.9
RELAX(5)=0.95
C-----------------------------------------------------------------------------------
C***** PROPERTIES OF SOLID AM50A***************************************************************************
RHOS=1710.
CPS=1220.
CONDS=80.
C-----------------------------------------------------------------------------------
C***** PROPERTIES OF LIQUID AM50A***************************************************************************
TREF=620
BETTA=0.0007
RHOL=1650.
CPL=1320.
CONDL=100.
C-----------------------------------------------------------------------------------
£ <  *****  PROPERTIES OF MUSHY AM50A***************************************************************************
LH=370000.
TL=893.*EXP(2.486e-05*APURE*1E06/LH)-273.
WRITE(45,*) 'LIQUIDUS TEMPERATURE = ', TL
TS=708.*EXP(2.486e-05*APURE*1E06/LH)-273.
WRITE(45,*) 'SOLIDUS TEMPERATURE = ', TS
RHOM=(RHOS+RHOL)/2.
CPM=(CPS+CPL)/2.
CONDM=(CONDS+CONDL)/2.
DELTAT= TL-TS
C-----------------------------------------------------------------------------------
C***** HEAT TRANSFER COEFFICIENTS***************************************************************************
C AT TOP SURFACE (PUNCH/CASTING) __________________
   HTCT1=25.6
C AT RIGHT SURFACE (DIE/CASTING) __________________
   CHTCR=5000
   HTCR=1990.5+94.8*APURE
C AT BOTTOM SURFACE (BOTTOM/CASTING) ______________
   CHTCB=6000
   HTCB=1990.5+94.8*APURE
C-----------------------------------------------------------------------------------
C************* INITIAL TEMPERATURE OF CASTING ***************
DO 100 J=1,M1
DO 100 I=1,L1
T(I,J)= TPOUR
   UUU(I,J)=TL
100 CONTINUE
C----------- INITIAL VELOCITIES AND DENSITY-------------------
   DO 105 J=1,M1
DO 105 I=1,L1
U(I,J)=0
V(I,J)=0
c    UUU(I,J)=0
c    VVV(I,J)=0
105 CONTINUE

C-----------------------------------------------
C *************** INITIAL ENTHALPY OF CASTING ***************
CDO 110 J=1,M1
CDO 110 I=1,L1
CE(I,J)= 1 + (T(I,J)- DELTAT - TS)*CPL/(LH+CPM*DELTAT)
C110 CONTINUE
RETURN
ENTRY DENSE
C CALCULATE DENSITY AS A FUNCTION OF TEMPERATURE, ---
DO 150 J=1,M1
DO 150 I=1,L1
CCP(I,J)=0.4836*T(I,J)+1006.7
KCOND(I,J)=-0.0003*T(I,J)*T(I,J)+0.29*T(I,J)+35
IF(T(I,J).GT.624) THEN
RHO(I,J)=(-0.2818*T(I,J) + 1820.2)
END IF
IF (T(I,J).LE.430) THEN
RHO(I,J)=1.01304*10*(I,J) + 1770.1
END IF
IF (T(I,J).LE.435.AND.T(I,J).GT.430) THEN
RHO(I,J)=-0.1304*T(I,J) + 1770.1
END IF
IF (T(I,J).LE.435.AND.T(I,J).GT.430) THEN
RHO(I,J)=(-0.282*T(I,J) + 125)
END IF
IF (T(I,J).LE.435.AND.T(I,J).GT.430) THEN
YILD(I,J)=2
END IF
EM(I,J)= -0.0546*T(I,J)*T(I,J)-33.857*T(I,J) + 44140
IF (T(I,J).LE.399.75) THEN
ALP(I,J) =0.0167*T(I,J)+31
END IF
YILD(I,J)=1.0167*T(I,J)+31
IF (T(I,J).LE.400.25.AND.T(I,J).GT.399.25) THEN
ALP(I,J) = 1.75E-06*(T(I,J)-399.5)+2.50E-05
END IF
IF (T(I,J).GT.400.25.AND.T(I,J).LE.429.75) THEN
ALP(I,J) = 2.59E-05
END IF
IF (T(I,J).LE.430.25.AND.T(I,J).GT.429.75) THEN
ALP(I,J) = 3.3788E-04*(T(I,J)-429.75) + 2.59E-05
END IF
IF (T(I,J).GT.430.25.AND.T(I,J).LE.434.75) THEN
ALP(I,J) = 1.95E-04
END IF
IF (T(I,J).GT.434.75.AND.T(I,J).LE.435.25) THEN
ALP(I,J) = -3.897E-04*(T(I,J)-434.75) + 1.95E-04
END IF
IF (T(I,J).GT.435.25.AND.T(I,J).LE.439.75) THEN
ALP(I,J) = 0
END IF
IF (T(I,J).GT.439.75.AND.T(I,J).LE.440.25) THEN
ALP(I,J) = 6.5E-06*(T(I,J)-439.75)
END IF
IF (T(I,J).GT.440.25.AND.T(I,J).LE.559.75) THEN
ALP(I,J) = 3.25E-06
END IF
IF (T(I,J).GT.559.75.AND.T(I,J).LE.560.25) THEN
ALP(I,J) = 7.16624E-05*(T(I,J)-559.75)+3.25E-06
END IF
IF (T(I,J).LE.560.25.AND.T(I,J).GT.569.75) THEN
ALP(I,J) = 3.91IE-05
END IF
IF (T(I,J).GT.569.75.AND.T(I,J).LE.570.25) THEN
ALP(I,J) = 7.86E-05*(T(I,J)-569.75) + 3.91IE-05
END IF
IF (T(I,J).GT.570.25.AND.T(I,J).LE.584.75) THEN
ALP(I,J) = 7.84E-05
END IF
IF (T(I,J).LE.584.75.AND.T(I,J).GT.585.25) THEN
ALP(I,J) = 1.1863E-04*(T(I,J)-584.75)+7.84E-05
END IF
IF (T(I,J).GT.585.25.AND.T(I,J).LE.594.75) THEN
ALP(I,J) = 1.38E-04
END IF
IF (T(I,J).GT.594.75.AND.T(I,J).LE.595.25) THEN
ALP(I,J) = 2.082E-05*(T(I,J)-594.75)+1.38E-04
END IF
IF (T(I,J).GT.595.25.AND.T(I,J).LE.602.75) THEN
ALP(I,J) = 1.48E-04
END IF
IF (T(I,J).GT.602.75.AND.T(I,J).LE.603.25) THEN
ALP(I,J) = 1.0028E-04*(T(I,J)-602.25) + 1.48E-04
END IF
IF (T(I,J).GT.603.25.AND.T(I,J).LE.609.75) THEN
ALP(I,J) = 1.98E-04
END IF
IF (T(I,J).GT.609.75.AND.T(I,J).LE.610.25) THEN
ALP(I,J) = 1.98E-04
END IF
IF (T(I,J).GT.610.25.AND.T(I,J).LE.614.75) THEN
ALP(I,J) = 2.39E-04
END IF
IF (T(I,J).GT.614.75.AND.T(I,J).LE.615.25) THEN
ALP(I,J) = 4.544E-04*(T(I,J)-614.75)+ 2.39E-04
END IF
IF (T(I,J).GT.615.25.AND.T(I,J).LE.617.75) THEN
ALP(I,J) = 4.66E-04
END IF
IF (T(I,J).GT.617.75.AND.T(I,J).LE.618.25) THEN
ALP(I,J) = 7.02E-04
END IF
ALP(I,J) = 1.21E-03
END IF
IF (T(I,J).GT.620.25.AND.T(I,J).LE.621.25) THEN
ALP(I,J) = -1.47328E-04*(T(I,J)-620.75)+ 1.21E-03
END IF
IF (T(I,J).GT.621.25.AND.T(I,J).LE.623.75) THEN
ALP(I,J) = 4.72E-04
END IF
IF (T(I,J).GT.623.75.AND.T(I,J).LE.624.25) THEN
ALP(I,J) = -7.716E-04*(T(I,J)-623.75)+ 4.72E-04
END IF
IF (T(I,J).GT.624.25.AND.T(I,J).LE.649.75) THEN
ALP(I,J) = 8.61E-05
END IF
IF (T(I,J).GT.649.75.AND.T(I,J).LE.650.25) THEN
ALP(I,J) = -6.158E-05*(T(I,J)-649.75) + 8.61E-05
E N D IF
IF (T(I,J).GT.650.25.AND.T(I,J).LE.799.75) THEN
ALP(I,J) = 5.53E-05
END IF
150 CONTINUE
RETURN
ENTRY BOUND
IF (ITER.GT.NNN.AND.ITER.LT.MMM) THEN
DO 155 J=1,M1
DO 155 I=1,L1
U(I,J)=-0.055
V(I,J)=0
c UUU(I,J)=0
c VVV(I,J)=0
155 CONTINUE
ENDIF
IF (ITER.EQ.MMM) THEN
DO 165 J=1,M1
DO 165 I=1,L1
U(I,J)=0
V(I,J)=0
c UUU(I,J)=0
c VVV(I,J)=0
165 CONTINUE
ENDIF
C----------------------------------------------------------------------
C ************ AXISYMMETRICAL LINE **********************
DO 200 I=2,L2
CE(I,1)=E(I,2)
T(I,1)=T(I,2)
U(I,1)=U(I,2)
V(I,1)=V(I,2)
200 CONTINUE
C----------------------------------------------------------------------
C AT TOP SURFACE (TOP DIE/CASTING) _____________
XXDIF=XDIF(2)
C  HTCT1=10.0*(ITER+1)
DO 250 J=2,M2
V(1,J)=0
c U(1,J)=0
T(1,J) = (KCOND(1,J)*T(2,J)/XXDIF+HTCT1*TFT)/(KCOND(1,J)/XXDIF
1+HTCT1)
IF (ITER.GT.(MMM)) THEN
TFT=275+0.2*(ITER-MMM)
HTCT=1990.5+94.8*BPURE(1,J)
T(1,J) = (KCOND(1,J)*T(2,J)/XXDIF+HTCT*TFT)/(KCOND(1,J)/XXDIF
1+HTCT1)
ELSE
TFT=275+0.2*(ITER-MMM)
HTCT=1990.5+94.8*BPURE(1,J)
T(1,J) = (KCOND(1,J)*T(2,J)/XXDIF+HTCT*TFT)/(KCOND(1,J)/XXDIF
1+HTCT1)
ENDIF
250 CONTINUE
250 CONTINUE
C AT RIGHT SURFACE (DIE/CASTING) ______________

YYDIF=YDIF(M1)
C HTCR1=200.0*(ITER/20+1)
HTCR1=2000
DO 280 I=2,L2
V(I,M1)=0
C U(I,M1)=0
C T(I,1)=T(I,2)
T(I,M1)=(KCOND(I,M1)*T(I,M2)/YYDIF+HTCR1*TFR)/(KCOND(I,M1)/YYDIF
1+HTCR1)
280 CONTINUE
IF (ITER.GT.NNN.AND.ITER.LT.MMM) THEN
ITERR=L1-5*ITER
DO 290 I=2,ITERR
T(I,M1)=(KCOND(I,M1)*T(I,M2)/YYDIF+HTCR1*TFR)/(KCOND(I,M1)/YYDIF
1+HTCR1)
290 CONTINUE
DO 300 I=ITERR+1, L2
HTCRR=6000
T(I,M1)=(KCOND(I,M1)*T(I,M2)/YYDIF+HTCRR*TFR)/(KCOND(I,M1)/YYDIF
1+HTCRR)
300 CONTINUE
ENDIF
C PRINT 427,HTCR
C427 FORMAT(1X,1P6E12.3)
ENDIF
C AT BOTTOM SURFACE (BOTTOM/CASTING) ______________

BXDIF=XDIF(L1)
C HTCB1=200.0*(ITER/20+1)
HTCB1=2000
DO 350 J=2,M2
C V(L1,J)=0
U(L1,J)=0
T(L1,J)=(KCOND(L1,J)*T(L2,J)/BXDIF+HTCB1*TFR)/(KCOND(L1,J)/
1 BXDIF+HTCB1)
IF (ITER.GT.MMM) THEN
TFR=275+0.2*(ITER-MMM)
HTCR=1990.5+94.8*RPURE(I,49)
T(L1,J)=(KCOND(L1,J)*T(L2,J)/BXDIF+HTCB1*TFR)/(KCOND(L1,J)/BXDIF
1+HTCB1)
ENDIF
C PRINT 427,HTCB
C427 FORMAT(1X,1P6E12.3)
ENDIF
1+HTCB)
END IF
CIF(E(L1,J),GE.1.)
C 1 E(L1,J)= 1 + (T(L1,J)- DELTAT - TS)*CPL/(LH+CPM*DELTAT)
CIF(E(L1,J),GT.0. AND.E(L1,J).LT. 1.) E(L1,J)= (T (L1,J)-TS)/DELTAT
CIF(E(L1,J),LE.0.) E(L1,J)= (T(L1,J)-TS)*CPS/(LH+CPM*DELTAT)
350 CONTINUE
C-----------CHECK SOLIDIFICATION-----------
RETURN
ENTRY OUTPUT
IF(ITER.NE.0) GOTO 400
PRINT 401
WRITE(45,401)
401 FORMAT('ITER',4X,'ITER*RT',4X,'E(l 1,1)',
1 4X,'T(11,1)',4X,'E(11,2)',4X,'T(11,2)',4X,'T( 11,11)')
C ------------------------------------
C ************ CONVERT ENTHALPY TO TEMPERATURE**********
400 DO 370 I=2,L2
  DO 370 J=2,M2
  IF(T(I,J).LE.TS) THEN
    IF(T(I,J).LE.TS.OR.ITER.LE.MMM) THEN
      F(I,J,1)=0
      F(I,J,2)=0
    ENDIF
    IF(ABS(F(I,J,1)).LT.1E-4) THEN
      F(I,J,1)=0
    ENDIF
    IF(ABS(F(I,J,2)).LT.1E-4) THEN
      F(I,J,2)=0
    ENDIF
  370 CONTINUE
 402 IF(ITER.LT.MMM) GOTO 444
C---------UNIT THERMAL CONTRACTION ----------------
  DO 412 J=1,M1
    DO 412 I=1,L1
      IF (ITER.EQ.0) THEN
        DLX(I,J)=XL/(L1)
        DLY(I,J)=YL/(M1)
      END IF
      DLX(I,J)=(XL/L1)*ALP(I,J)*(T(I,J)-TT(I,J))
      DDLY(I,J)=DDLY(I,J)+DLY(I,J)
      DLY(I,J)=(YL/M1)*ALP(I,J)*(T(I,J)-TT(I,J))
    412 CONTINUE
 402 IF (ITER.LT.MMM) GOTO 444
C---------UNIT THERMAL CONTRACTION---------------------
c--------LENGTH AFTER contraction in radius direction--------
    DO 415 I=1,L1
    DLLY(I)=0
    DO 415 J=1,M1
    DLLY(I)=DLLY(I)+DLY(I,J)
415  CONTINUE

c--------HEIGHT AFTER contraction in height direction--------
    DO 416 J=1,M1
    DLLX(J)=0
    DO 416 I=1,L1
    DLLX(J)=DLLX(J)+DLX(I,J)
416  CONTINUE

C-------------Stress change in Height direction
C417  FFACTOR=0.2+0.01*(ITER-MMM)
C    IF (ITER.GT.350) THEN
C    FFACTOR=0.4
C    END IF
    DO 418 I=1,L1
        Q=L1-I
        FRICT=0
        DO 417 K=1,Q
            YXY(K,49)=1.5
            IF (T(K,49).GT.TS) THEN
                YXY(K,49)=2
            END IF
        c YXY(K,49)=T(K,49)/TL
        FRICT=FRICT+4*YXY(K,49)*YL*YILD(K,49)/(L1*0.1016)
        C FRICT=FRICT+0.3*YL*APURE/(L1*0.1016)
417 CONTINUE
    HPURE(I)=APURE-FRICT
    IF(HPURE(I).LE.O) HPURE(I)=0
418  CONTINUE

C PRINT 422,YILD(25,49)
C422  FORMAT(1X,1P6E12.3)

C------------DEFORMATION & PRESSURE IN RADIUS DIRECTION-----
    DO 420 J=1,M1
        DELY(J)=0
    DO 420 I=1,L1
        DELY(J)=DELY(J)+YILD(I,J)
420  CONTINUE

    DO 422 I=1,L1
        DELX(I)=0
        MM(I)=0
    DO 422 J=1,M1
        IF (YILD(I,J).LE.10) GOTO 422
        DELX(I)=DELX(I)+YILD(I,J)
422  CONTINUE

380
MM(I)=MM(I)+1
422CONTINUE
DO 423 J=1,M1
   DO 423 I=1,L1
      C RPURE(I)=(DELY(I)-(YL-DLLY(I)))*EM(I,M1)/YL
      RPURE(I,J)=HPURE(I)-YILD(I,J)
      if (HPURE(I).LT.YILD(I,J)) then
         RPURE(I,J)=RRPURE(I,J)
      end if
      RRPURE(I,J)=RPURE(I,J)
   423 CONTINUE
C DEFORMATION AND PRESSURE IN HEIGHT DIRECTION
   DO 425 J=1,M1
      DO 425 I=1,L1
         BPURE(I,J)=HPURE(I)*EXP(16*2*0.3*YL*J/M1)/EXP(16*2*0.3*YL)
      c BPURE(I,J)=HPURE(I)+DLLX(J)*EM(I,J)/XL
      C BPURE(I,J)=(YILD(I,J)+RPURE(I,J))
      C IF (RPURE(I,J).LE.YILD(I,J)) THEN
      C BPURE(I,J)=RPURE(I,J)
      C END IF
      C BPURE(I,J)=RPURE(I,J)/0.35
   425 CONTINUE
C local von mise stress-------------------
   DO 430 J=1,M1
      DO 430 I=1,L1
         SETA(I,J)=(BPURE(I,J)**2+RPURE(I,J)**2+BPURE(I,J)*RPURE(I,J)
            )**0.5/YILD(I,J)
   430 CONTINUE
C temperature gradient------------------------
   DO 432 J=2,M2
      DO 432 I=2,L2
         G1(I,J)=ABS((T(I-1,J)-T(I,J))/XDIF(I))
         G2(I,J)=ABS((T(I-1,J+1)-T(I,J))/YDIF(J))
         G3(I,J)=ABS((T(I+1,J+1)-T(I,J))/XDIF(J))
         G4(I,J)=ABS((T(I+1,J+1)-T(I,J))/XDIF(J))
         G5(I,J)=ABS((T(I+1,J) - T(I,J))/XDIF(J))
         G6(I,J)=ABS((T(I+1,J-1)-T(I,J))/YDIF(J))
         G7(I,J)=ABS((T(I-1,J) - T(I,J))/YDIF(J))
         G8(I,J)=ABS((T(I-1,J-1) - T(I,J))/YDIF(J))
         GG(I,J)=MAX(G1(I,J),G2(I,J),G3(I,J),G4(I,J),
                  G5(I,J),G6(I,J),G7(I,J),G8(I,J))
         GG(I,J)=0.001*GG(I,J)/((TT(I,J)-T(I,J))*0.5)
   432 CONTINUE
C last location pressure------------------------
   SXMI=0
   SXMA=0

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SYM I=0
SYMA=0
SHX2=0
SHY2=0
PPX=0
PPY=0

C unsolidified area: x direction
DO 436 I=2, LI
IF (T(I,2).LT.TS) GOTO 436
SHX(I,2)=I
IF (PPX.EQ.0) THEN
SXM I=SHX(I,2)
ENDIF
IF (SHX(I,2).GT.SHX2) THEN
SXM A=SHX(I,2)
PPX=1
ENDIF
SHX2=SXMA
436 CONTINUE

C unsolidified area: y direction
DO 438 J=2, Ml
IF (T(40,J).LT.TS) GOTO 438
SHY(40,J)=J
IF (PPY.EQ.0) THEN
SYM I=SHY(40,J)
ENDIF
IF (SHY(40,J).GT.SHY2) THEN
SYM A=SHY(40,J)
PPY=1
ENDIF
SHY2=SYM A
438 CONTINUE

C check if P can make plastic deformation
TRIPK=0
DO 440 I=SXM I, SXMA
IF (HPURE(I).GT.YILD(I,49)) TRIPK=TRIPK+1
440 CONTINUE

C check the P change due to thermal contraction
DO 441 J=SYM I,SYM A
DO 441 I=SXM I,SXMA
DAREA=DAREA+(XL*YL/(L1*M1))*(1-(1+ALP(I,J))*(T(I,J)-TT(I,J)))*
*(1+ALP(I,J)*(T(I,J)-TT(I,J)))
EMM=EMM+EM(I,J)
CONTINUE
DIFFX=SXMA-SXMI
DIFFY=SYM-A-SYM1
DTTP=3-(DAREA*EMM*L1*M1)/((DIFFX**2)*(DIFFY**2)*X*L*Y*L)
IF (FFF.GT.1) THEN
DTTP=TTTP-(DAREA*EMM*L1*M1)/((DIFFX**2)*(DIFFY**2)*X*L*Y*L)
FFF=FFF+1
END IF
TTTP=DTTP
END IF
DO 448 J=1,M1
  DO 448 1=1,L1
    IF (T(I,J).LE.TL.AND.T(I,J).GE.TS) THEN
      EWW=EWXWW+(TT(I,J)-T(I,J))*(X*L/L1)*(Y*L/M1)*
        CCP(I,J)*RHO(I,J)+0.5*258000*(1-(TL-T(I,J))/(TL-TS))*
        2(X*L/L1)*2(Y*L/M1)*RHO(I,J)
    ENDIF
    IF (T(I,J).LT.TS.AND.TT(I,J).GE.TL) THEN
      EWW=EWXWW+(TT(I,J)-T(I,J))*(X*L/L1)*(Y*L/M1)*
        CCP(I,J)*RHO(I,J)+0.5*516000*
        2(X*L/L1)*(Y*L/M1)*RHO(I,J)
    ENDIF
      EWW=EWXWW+(TT(I,J)-T(I,J))*(X*L/L1)*(Y*L/M1)*
        CCP(I,J)*RHO(I,J)+0.5*258000*(1-(UUU(I,J)-TS)/(TL-TS))*
        2(X*L/L1)*(Y*L/M1)*RHO(I,J)
    ENDIF
    IF (T(I,J).LT.TS.AND.TT(I,J).LT.TS) THEN
      EWW=EWXWW+(TT(I,J)-T(I,J))*(X*L/L1)*(Y*L/M1)*
        CCP(I,J)*RHO(I,J)
    END IF
  END DO 448
DO 434 J=1,M1
  DO 434 1=1,L1
    TT(I,J)=T(I,J)
  END DO 434
444 PRINT 450,ITER,ITER*DT,T(50,2),T(50,22),T(50,45)
450 FORMAT(I6,1P6E12.3)

C -------------------------------
C WRITE TEMPERATURES AT DIFFERENT LOCATIONS VS TIME

383
C MODITER=100*100/500
C IF(MOD(ITER,MODITER).EQ.0) THEN
  c
  WRITE (11,*) ITER*DT,T(2,2),T(10,2),T(20,2),T(30,2),T(40,2),
  c  T(50,2),T(60,2),T(70,2),T(78,2)
C WRITE (11,*) ITER*DT,T(40,2),T(40,25),T(40,47)
WRITE (11,*) ITER*DT,T(40,2),T(60,2)

WRITE (8,*) ITER*DT,BPURE(3,3),BPURE(3,25),BPURE(3,47),
  1  RPURE(3,49),T(3,49),RPURE(40,49),T(40,49),RPURE(77,49),T(77,49)
C WRITE (11,*) ITER*DT,U(40,20),V(40,20),F(40,20,1)
C
WRITE (8,*) ITER*DT,T(49,49),WWXX/(49*49),XXWXX/(49*49)
C WRITE (50,*) ITER*DT,U(49,49),V(49,49),F(49,49)

IF((ITER*DT).GE.last) GOTO 480
IF(T(30,L).LE.ts) GOTO 480
RETURN
480 LSTOP=.TRUE.
CALL PRINT
    WRITE (7,*) 'VARlABLES="X","Y","TEMP"
    WRITE (7,*) 'ZONE I=I,L1, J=M1, F=POINT'
DO 490 J=1,M1
    DO 490 I=1,L1
C ----------- DATA FILE FOR ISOCONTOUR PLOT VIA SURFER -----------
WRITE(7,*) Y(J),(XL-X(I)),T(I,J)
490 CONTINUE
C-----------DATA FOR SHRINKAGE PREDICTION-----------------
WRITE (50,*) 'VARlABLES="X","Y","G"
WRITE (50,*) 'ZONE I=I,L1, J=M1, F=POINT'
DO 492 J=1,M1
    DO 492 I=1,L1
C ----------- DATA FILE FOR ISOCONTOUR PLOT VIA SURFER -----------
WRITE(50,*) Y(J),(XL-X(I)),GG(I,J)
492 CONTINUE
C ----------- DATA FILE FOR ISOCONTOUR PLOT VIA SURFER -----------
WRITE(22,*) X(I),Y(J),
  1 F(I,J,1), F(I,J,2)
WRITE(22,*) Y(J),XL-X(I),
  1 F(I,J,2), -F(I,J,1)
CONTINUE
RETURN
C
ENTRY GAMSOR
C set GAM for energy equation
IF (NF.EQ.5) THEN
DO 500 J=1,M1
DO 500 I=1,L1
CCP(I,J)=0.4836*T(I,J)+1006.7
KCOND(I,J)=-0.0003*T(I,J)*T(I,J)+0.29*T(I,J)+35
TC=T(I,J)
IF(T(I,J).GT.624) THEN
RHO(I,J)=(-0.2818*T(I,J) + 1820.2)
END IF
IF (T(I,J).LE.430) THEN
RHO(I,J)=-0.1304*T(I,J) + 1770.1
END IF
IF (T(I,J).LE.435.AND.T(I,J).GT.430) THEN
RHO(I,J)=-T(I,J)+ 2144
ENDIF
IF (T(I,J).LE.440.AND.T(I,J).GT.435) THEN
RHO(I,J)=1709
ENDIF
IF (T(I,J).LE.560.AND.T(I,J).GT.440) THEN
RHO(I,J)=-0.0167*T(I,J) + 1716.3
ENDIF
IF (T(I,J).LE.624.AND.T(I,J).GT.560) THEN
RHO(I,J)=-0.0188*T(I,J)*T(I,J) + 21.407*T(I,J) - 4390
END IF
GAM(I,J)=KCOND(I,J)/(CCP(I,J))
500 CONTINUE
DO 910 J=2,M2
DO 910 I=2,L2
TL=893.*EXP(2.486e-05*APURE*1E06/LH)-273.
TS=708.*EXP(2.486e-05*APURE*1E06/LH)-273.
IF (ITER.LE.MMM) THEN
TL=620
TS=435
ENDIF
IF(T(I,J).GT.TL) THEN
CON(I,J)=0
END IF
IF(T(I,J).GT.TL) THEN
CON(I,J)=1.5*258000*(1-(TL-T(I,J))/(TL-TS))
CON(I,J)=0.5*258000*(1-(TL-T(I,J))/(TL-TS))
UUU(I,J)=T(I,J)
ENDIF
IF (T(I,J).LT.TS.AND.UUU(I,J).GE.TL) THEN
    CON(I,J)=1.5*5160000
    CON(I,J)=0.5*5160000
    UUU(I,J)=T(I,J)
ENDIF
    CON(I,J)=1.5*258000*(1-(UUU(I,J)-TS)/(TL-TS))
    UUU(I,J)=T(I,J)
ENDIF
IF (T(I,J).LT.TS.AND.UUU(I,J).LT.TS) THEN
    CON(I,J)=0
    AP(I,J)=0.
    UUU(I,J)=T(I,J)
ENDIF
VW(I,J)=VW(I,J)+CON(I,J)
C IF (VW(I,J).GE.1.5*5160000) CON(I,J)=Cl
C IF (VW(I,J).GT.0.5*5160000) CON(I,J)=0
910 CONTINUE

C SET MOMENTUM VALUES FOR GAMMA
C
C IF (ITER.LE.MMM)GOTO 995
C IF (ITER.GT.200)GOTO 995
IF (NF.EQ.1.OR.NF.EQ.2) THEN
    DO 800 J=1,M1
    DO 800 I=1,L1
    IF (T(I,J).GT.624) THEN
        RHO(I,J)=(-0.2818*T(I,J) + 1820.2)
    END IF
    IF (T(I,J).LE.430) THEN
        RHO(I,J)=-0.1304*T(I,J) + 1770.1
    END IF
    IF (T(I,J).LE.435.AND.T(I,J).GT.430) THEN
        RHO(I,J)=-T(I,J)+ 2144
    ENDIF
    IF (T(I,J).LE.440.AND.T(I,J).GT.435) THEN
        RHO(I,J)=1709
    ENDIF
    IF (T(I,J).LE.560.AND.T(I,J).GT.440) THEN
        RHO(I,J)=-0.0167*T(I,J) + 1716.3
    ENDIF
    IF (T(I,J).LE.624.AND.T(I,J).GT.560) THEN
        RHO(I,J)=-0.0188*T(I,J)*T(I,J) + 21.407*T(I,J) - 4390
    ENDIF
    VV(I,J)=VV(I,J)+CON(I,J)
C IF (VVV(I,J).GE.1.5*5160000) CON(I,J)=Cl
C IF (VVV(I,J).GT.0.5*5160000) CON(I,J)=0
995 CONTINUE

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CRHO(I,J)=1650
END IF
IF (T(I,J).LT.TL) GAM(I,J)=(1E+27*EXP(-0.1102*T(I,J)))
C IF (T(I,J).LT.550) GAM(I,J)=1E+27
IF (T(I,J).GE.TL) GAM(I,J)=(0.0133*EXP(-0.0035*T(I,J)))
GOTO 800
C740 F(I,J,1)=0
C F(I,J,2)=0
800 CONTINUE
END IF
C-----SET SOURCE TERM FOR NATURAL CONVECTION---------------------
IF (NF.EQ.1) THEN
DO 900 J=2,M2
DO 900 I=2,L2
TASTER=0.5*(T(I,J)+T(I-1,J))
IF (T(I,J).GE.TL) THEN
BETTA = -2E-11*T(I,J)**3-3E-9*T(I,J)**2+2E-05*T(I,J)-0.0053
GRN=9.806*BETTA*RHOL
C CON(I,J)=-9.806*RHO(I,J)
CON(I,J)=GRN*(TASTER-TREF)
CON(I,J)=CON(I,J)-GRN*(TASTER-TREF)
END IF
900 CONTINUE
END IF
C-----SET SOURCE TERM ASSOCIATED WITH CYLINDRICAL COORDS-------
IF (NF.EQ.2) THEN
IF (T(I,J).GE.TL) THEN
DO 950 J=2,M2
DO 950 I=2,L2
PRLA=(0.0133*EXP(-0.0035*T(I,J)))
CON(I,J)=0
AP(I,J)=PRLA/(RMN(J)**2)
950 CONTINUE
END IF
END IF
995 RETURN
END
### Vita Auctoris

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