Analytical predetermination of radio interference levels of power transmission lines

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ANALYTICAL PREDETERMINATION OF RADIO INTERFERENCE LEVELS OF POWER TRANSMISSION LINES

by

Barry Boccabella

A Thesis submitted to the Faculty of Graduate Studies through the Department of Electrical Engineering in Partial Fulfillment of the requirements for the Degree of Master of Applied Science at The University of Windsor

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ABSTRACT

Three methods of predetermining transverse Radio Interference (RI) magnetic field profiles are described. The first two methods were developed by Clade and Gary (1966), and Moreau and Gary (1972). Both of these methods use oversimplified line models in the calculation of the profiles. The earlier method models the transmission line as being lossless, and lossless line modal analysis together with image currents are utilized in calculating the magnetic fields. The second method uses a transposed lossless line model and employs an offset image to account for the resistive earth effect.

The third method, based on the work of Perz and Raghudev (1974), in contrast to the first two, recognizes the existence of distributed currents in the lossy earth instead of image currents. This method makes use of lossy line modal analysis to calculate the attenuations and modal currents, and analytical expressions for the calculation of magnetic fields. These improvements render the third method the most rigorous available. The three methods are applied to the French Chesnoy-Rousson, 220kV, three-phase horizontal transmission line and the RI profiles are compared.
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h - Height of conductors in meters
H - Magnetic Field Intensity in Amps/meter
I - Current due to Elementary Section of Line in Corona in Amps/meter
i - Current in Amps
(J) - Current Modal Matrix in Amps
J - \( \sqrt{-1} \)
K - Conductor Number
k - Variable in Fourier Transform Space
K - Constant
l - Length in meters
L - Length in meters
(L) - Inductance Matrix in Henry/meter

\[ M_0 = \sqrt{-\mu_0} \]

\[ M_g = \sqrt{\frac{\mu_0}{\mu_r}} - \frac{1}{\mu_r} \]

N - Voltage Modal Transformation Matrix in Volts
n - Mode Number
N - Current Modal Transformation Matrix in Amps
P - Power in Watts
P - Square of Complex Propagation Matrix
Q - Charge in Coulombs
R - Radius in meters
\( \rho = \) Resistance in Ohms
LIST OF SYMBOLS

A - Vector Potential in Volts
BW - Bandwidth in Hz
{C} - Capacitance Coefficient Matrix in Farads/meter
D - Distance in meters
E - Electric Field Intensity in Volts/meter
f - Frequency in Hz
F - Centre Frequency in Hz
{G} - Geometry Matrix
h - Height of conductors in meters
H - Magnetic Field Intensity in Amps/meter
i - Current due to Elementary Section of Line in Corona in Amps/meter^{1/2}
I - Current in Amps
{I} - Current Modal Matrix in Amps
j - \sqrt{-1}
k - Conductor Number
k_{1} - Variable in Fourier Transform Space
K - Constant
l - Length in meters
L - Length in meters
{L} - Inductance Matrix in Henry/meter
m_{a} - \sqrt{2\omega\mu_{0}\varepsilon_{0}}
m_{g} - \sqrt{\omega^{2}\mu_{g}\varepsilon_{g}-\omega^{2}\mu_{0}\varepsilon_{0}-j\omega\mu_{g}\varepsilon_{g}}
{M} - Voltage Modal Transformation Matrix in Volts
n - Mode Number
{N} - Current Modal Transformation Matrix in Amps
P - Power in Watts
{P} - Square of Complex Propagation Matrix
Q - Charge in Coulombs
r - Radius in meters
R - Resistance in Ohms
Re - Real part of
s - Area of Surface in meter$^2$
S - Poynting Vector in Watts/meter$^2$
SD - Spectral Density of
t - Transposed (used as a subscript)
V$\_0$ - Velocity of Light in meters/second
V - Voltage in Volts
\{V\} - Voltage Matrix in Volts
x - Lateral Ground Distance in meters
y - Distance in Vertical Direction in meters
\{y\} - Shunt Admittance Matrix in Mho/meter
z - Distance Along Conductor Direction in meters
\{z\} - Series Impedance Matrix in Ohms/meter
Z - Modal Characteristic Impedance in Ohms
Z$\_c$ - Intrinsic Impedance of Conductor Material in Ohms
Z$\_g$ - Intrinsic Impedance of Ground in Ohms
\{Z\} - Characteristic Impedance Matrix in Ohms
\(\alpha\) - Attenuation in Nepers/meter
\(\beta\) - Phase Constant in Radians/meter
\(\gamma\) - Propagation Constant in meter$^{-1}$ \((=\alpha+j\beta)\)
\(\Gamma\) - Excitation Function in Amps/meter$^{1/2}$
\(\varepsilon\) - Permittivity in Farads/meter
\(\lambda\) - Eigenvalue or wavelength in meters
\(\mu\) - Permeability in Henry/meter
\(\rho_c\) - Resistivity of Conductor in Ohm-meters
\(\rho_e\) - Resistivity of Ground in Ohm-meters
\(\rho_s\) - Resistivity of Sky Wire in Ohm-meters
\(\omega\) - $2\pi f$ in Radians/second
CHAPTER 1
INTRODUCTION

1.1 HISTORICAL REVIEW

Interest in corona related phenomena arose about fifty years ago mainly because of the corona losses associated with the 230kV transmission system. These losses are attributed to corona discharges that occur on the conductors of a line and on line hardware. Corona discharges in turn are due to the partial breakdown of air in the immediate vicinity of the conductor when the applied voltage exceeds a critical value. At this applied voltage the electric field intensity is termed the critical voltage gradient on the conductor surface.

In 1920, Peek (1) formulated empirical expressions for critical voltage gradients and corona loss. These expressions showed the dependence of corona loss on such factors as conductor surface condition, atmospheric pressure, temperature, humidity and air density. The energy needed to sustain this loss is part of the transmitted 60Hz power.

In the 1950's transmission line operating voltages had to be increased in order to meet the increased power demands. The increased operating voltages necessitated the introduction of new concepts in transmission line design, such as bundled conductors, to minimize corona losses. In fact, present day 765kV lines with bundled conductors are operating satisfactorily from the corona loss point of view.
1.2 CORONA TYPE INTERFERENCE

Around 1950 the use of Power Line Carrier (P.L.C.) was greatly expanded for the protection and control of complex power systems. Corona discharges on the lines were found to interfere with proper operations of the P.L.C. system, especially during adverse weather conditions. An interest arose therefore, in the interference effects due to corona discharges.

A single corona discharge on a conductor at a point can be represented by the connection of a current pulse generator at that point. This current pulse propagates in both directions along the line away from the point of injection. There are many such corona sources assumed to be evenly distributed along the conductors. These random sources act independently of each other and are therefore regarded as uncorrelated. Each pulse has a certain power spectrum which can for convenience be considered in terms of a large number of small frequency bands. Each band is specified by its central frequency and the magnitude of its power. At any point on the line the total power within a particular frequency band is the sum of the power in the pulses arriving from all parts of the line in the same frequency band.

The high frequency power from corona discharges is of concern for two important reasons. As mentioned earlier, these currents interfere with P.L.C. operation at 40kHz to 200kHz. Secondly, they have adverse effects on radio reception at 540kHz to 1720kHz near transmission lines; hence the term radio interference or RI.

RI currents flowing on transmission lines produce an
electromagnetic field propagating along the lines as plane waves in the Transverse Electro-Magnetic (T.E.M.) mode. If the RI field is large enough, it will interfere with normal radio broadcast reception to the extent that radio programs become unintelligible.

1.3 RI PROFILES — DETERMINATION

The RI field is usually determined at or close to ground level. A representation of this RI field intensity versus the lateral distance away from the transmission line is called the RI field transverse profile. Until the late 1950's RI profiles could only be determined through measurements conducted under operating lines. Mather and Bailey (2),(3), describe how such an investigation was carried out.

It was not until 1956 that Adams (4) defined some concepts which have enhanced the analytical research of RI predetermination and in particular the prediction of RI levels near EHV lines. Adams described the impulse currents in terms of their power spectral density. He introduced the idea of the generation function, (also known as excitation function) a quantity which expresses the intensity of corona generation per unit length of conductor in corona. This in fact, is proportional to the square root of the average power per unit length due to corona sources and therefore, the unit is \( \mu \text{Amps/meter}^{1/2} \).

In 1959 Adams (5) also introduced a method of resolving 'n' conductor voltages and currents of an 'n'-phase lossless transmission power line into 'n' sets of orthogonal components that propagate independently of each other. These sets are called natural modes. A method of calculating specific modal attenuations was also developed by Adams and Barthold (6). Hence the real transmission line is
replaced by 'n' independent modal lines, each with 'n' uncoupled conductors. This concept of independent modal propagation had already been observed by Bekku\(^{(7)}\) in 1923 and mathematically formulated by Carson and Hoyt\(^{(8)}\) in 1927. The Adams modal method is based on the eigenvalue analysis of the characteristic impedance matrix of lossless power transmission lines.

With the advent of 500kV transmission in the late 1950's and early 1960's power lines were constructed to minimize corona losses. Therefore, attempts were made to predetermine RI profiles for long lines from measurements on short test lines. Reichman and Leslie \(^{(9)}\) postulated that RI levels on long lines were proportional to the geometric mean of the maxima and minima of the standing waves at the centre of a short open-ended test line. Perz \(^{(10)}\) has shown that the equivalent RI current per unit length can be determined from the minima of the standing waves. The latter method has the advantage that because standing wave minima do not depend on the attenuation the actual RI generation on long lines can be determined. In France \(^{(11)}\) "cage" tests were conducted on sections of EHV line conductors. On these conductors the excitation function, \(\Gamma\), could be studied. Apart from the determination of \(\Gamma\), which usually is the main objective of such tests, other useful conclusions were reached, \(^{(12)}, (13)\) such as

a) RI levels are higher during rain than during fine weather.

b) Above a certain rain intensity the RI level remains constant (saturation).

1.4 RI PROFILES — EXISTING METHODS

Commonly excepted methods of RI profile calculations usually follow similar basic steps described in \(^{(14)}, (15), (16)\) and are also
published in a joint French-American survey (17). The calculations are performed at a nominal frequency which represents the centre frequency of the bandwidth of the detection system used. The power of the RI currents in this bandwidth is

\[ P = \int_{F-\frac{1}{2}BW}^{F+\frac{1}{2}BW} SD(i^2) df \]  

- 1.1

where \( SD(i^2) \) is the power spectral density of the RI current, \( F \) is the centre frequency, \( f \) is any frequency and \( BW \) is the bandwidth. Assuming plane wave propagation in the TEM mode and performing modal analysis the modal components of the RI currents flowing in the conductors can be found. The following is an outline of the steps used in this thesis to calculate RI fields.

1) Determination of injected noise currents on a long line from a unit length of one conductor in corona. The injected conductor currents are proportional to \( \Gamma \) and the capacitance coefficient matrix.

2) The injected line currents flowing in one direction are transformed into modal currents. Each set of conductor currents of one mode is propagating with a distinct complex modal propagation constant. Hence, this step necessitates eigenvalue analysis of lossy lines.

3) The cumulative effect of uniformly distributed uncorrelated noise sources is analyzed in the centre of a long line. The resulting modal fields of the transverse profiles are considered to be uncorrelated in the calculation of the total horizontal magnetic or the vertical electric field intensities.
4) These three steps are repeated for the remaining conductors in corona, one at a time. The outbursts of corona noise from each conductor occur near the peak voltage of each conductor and hence, are not coincident in time.

5) The measured transverse profiles of three, separate in time, RI noise signals depend on the time response of the field intensity meter. This is taken into account in the final computation of the lateral profiles.

6) Three methods of RI profiles are discussed in this thesis. The results are compared for the same value of excitation function, \( r = 1.0 \, \mu \text{Amps/meter}^{1.5} \), and identical meter response.

1.5 OBJECTIVES OF THIS WORK

The currently accepted methods use many simplifying assumptions that can neither be regarded as being consistent with each other nor with the real transmission line above lossy ground. For example, induced distributed currents that exist in the earth are replaced by concentrated fictitious image line currents, and only the horizontal magnetic field intensity is considered in the calculation. In addition one of the methods (15) lowers the earth surface by an amount equal to the depth of penetration and then employs the image principle above this new surface of symmetry. This artifice is used to account for the effect of an imperfect earth.

The calculation of the field by use of the perfect image principle is an oversimplification. In fact the magnetic field penetrates the resistive soil. The depth of penetration is 2.25 meters for a frequency of 1.0MHz and an earth resistivity of 20\( \Omega \cdot \text{m} \), whereas for 0.5MHz and 2000\( \Omega \cdot \text{m} \) it is 31.8 meters. There are a
number of other assumptions in each method which are reviewed in this thesis.

In 1974 two papers (18), (19) were published which explained the use of semi-infinite integrals for the calculation of fields, and clearly outlined the assumptions involved in the work of Carson (20). Distributed currents in the ground were considered here rather than fictitious image currents. Through the use of Fourier transforms, expressions for the magnetic field at ground level were derived. These field intensity equations, although complicated, are of practical importance for the calculation of RI fields. The horizontal modal magnetic field components of the RI profiles are combined with those produced by line currents. Hence, the image principle is replaced by a more accurate means of computing the field due to the distributed earth currents. The field expressions are not easily evaluated analytically and are therefore evaluated by use of an appropriate numerical method.

In this thesis two existing methods (14), (15) and the new method (19) have been applied for the calculation of RI profiles on the Chesnoy-Rousson (14) three conductor transmission lines. The frequencies and earth resistivities considered lie between 500KHz to 1MHz and 20-1000 Ohm-meters respectively.

1.6 ORGANIZATION OF THESIS

In chapter two, characteristics of the corona discharge are described and the concept of the excitation function as applied to RI predetermination is introduced. Since existing RI predetermination methods use lossless line models, chapter three deals with the techniques of modal analysis as applied to such lines, and outlines
a method to calculate the modal attenuations. These can then be used to describe and explain two French developed methods of RI predetermination as shown in chapter 4. Modal analysis using the lossy line model is explained in chapter 5 as well as a description of an RI predetermination method developed in Canada using this model. Finally in chapter 6, conclusions are drawn with regard to the objectives of this work.

2.1 CORONA DISCHARGE

Corona discharge, as described by Parrot (21), is a partial breakdown of air caused by an excessive electric field stress in the immediate vicinity of a charged conductor. The electric field intensity or voltage gradient is a result of the voltage (usually AC voltage for transmission lines) impressed on the conductor. If the conductor is perfectly smooth then corona discharges initiate at a well defined critical voltage, Vc, referred to ground, below which there are no corona discharges. However, when the conductor has an uneven surface due to protrusions and contamination deposits then corona discharges occur at a lower voltage. The voltage gradient on the centre phase conductor of a three phase horizontal transmission line is usually about ten percent greater than that on the outside phase conductors. This gives rise to about 6 dB more noise produced by the centre conductors during fair weather.

The generation of corona can be explained by the Townsend mechanism (22). In the case of the conductor with negative applied DC voltage, Trichel pulses are observed. A free electron in the high
CHAPTER 2

CHARACTERISTICS OF CORONA DISCHARGE AND INJECTED CURRENT

In this chapter the corona discharge mechanism is described and the characteristics of the resulting impulse currents injected into the power line conductors are discussed. A section on the concept of the excitation function is included, followed by a discussion on its use in finding the injected current for single and multiconductor systems above a zero-potential ground plane. This chapter concludes with a description of calculating the RI currents on long lines.

2.1 CORONA DISCHARGE

Corona discharge, as described by Perz\(^{(21)}\), is a partial breakdown of air caused by an excessive electric field stress in the immediate vicinity of a charged conductor. The electric field intensity or voltage gradient is a result of the voltage (usually AC voltage for transmission lines) impressed on the conductor. If the conductor is perfectly smooth then corona discharges initiate at a well defined critical voltage, \(V_c\), referred to ground, below which there are no corona discharges. However, when the conductor has an uneven surface due to protrusions and contamination deposits then corona discharges occur at a lower voltage. The voltage gradient on the centre phase conductor of a three phase horizontal transmission line is usually about ten percent greater than that on the outside phase conductors. This gives rise to about 6dB more noise produced by the centre conductors during fair weather.

The generation of corona can be explained by the Townsend mechanism\(^{(22)}\). In the case of the conductor with negative applied DC voltage, Trichel pulses are observed. A free electron in the high
field region near the cathode accelerates and liberates an electron upon impact with a neutral atom. The liberated electron collides with other neutral atoms thus producing more free electrons. Each of these electrons are accelerated in the electric field and take part in the collision process. Hence an avalanche effect takes place. This process is very rapid, of the order of a nano-second. For conductors with positively applied DC voltage, the electrons in the surrounding space, formed mainly by cosmic radiations, move toward the high field region. This establishes electron avalanches which maintain the highly ionized state near the conductor. As a result, bursts of pulses and streamers are observed. For alternating applied voltages, both positive and negative corona discharges will exist and interact. It has been found that the pulses of most interest, in so far as RI is concerned, are due to the positive corona discharges. More detailed explanations on these mechanisms are well documented in the literature (22), (23).

When the liberated electrons reach the conductor surface they inject a current pulse into it. This current pulse divides into two equal parts that propagate in opposite directions away from the point of injection. In the case of a single conductor above ground, it is the ground that forms the return circuit. These pulse currents are the RI currents. In a multiconductor transmission line, the ground and the remaining conductors form the return circuit. When a current pulse is produced on one of the conductors, pulses of the same shape but opposite polarity appear simultaneously on the return circuits.

2.1.1 SINGLE DISCHARGE

To fully appreciate the behaviour of RI currents in the conductors
of a transmission line, it would be beneficial to first consider the characteristics of a single corona discharge and the cumulative effect of uniformly distributed corona discharges on the line.

Each current impulse can be represented by the following equation (24)

\[ I = I_0(e^{-at} - e^{-bt}) \]  

where \( a \) and \( b \) determine the trailing and leading edges of a double exponential impulse. Typical values of \( a \) and \( b \) are 2.2 and 21 microsecond\(^{-1}\) respectively which yields a rise time of 0.1 microseconds and time to half tail of 0.4 microseconds. The long decay time for the tail is accounted for by the slow diffusion rate of the remaining positive ions near the point of injection. A typical corona pulse is shown in figure 2.1. The steep front of this impulse is the major cause for the existence of frequency components in the Radio Frequency range. Through Fourier analysis the frequency spectrum of the impulse can be determined and it is of the form illustrated in figure 2.2.

Figure 2.2 indicates that for low frequencies the amplitude of the RI current in constant. At high frequencies the amplitude decreases as the inverse of the frequency squared. At intermediate frequencies (around 1MHz) the relationship has been approximated by Perz (24) as decreasing inversely with frequency.

2.1.2 MULTIPLE DISCHARGES

On a transmission line corona pulses occur in groups near the positive peak of the voltage when the critical voltage is exceeded. These individual pulses are of random amplitude and occur at
irregular intervals of time and distance along the conductor. Each current pulse has a similar frequency spectrum to all other current pulses, and the average spectrum can be considered as being the frequency spectrum of the current on long lines (29). Therefore, despite the pulses being produced by random uncorrelated sources, the spectrum on any part of the line is similar to that in Figure 2.2.

For the present calculation of the RI transient field, it is necessary to know the time by a unit length of one conductor in corona. For this, use is made of the statistical concept that forms to a mutually uncorrelated stationary random process. That is, over a sufficiently long time the average power expended per unit length by the conductor under corona is constant. Adams (4) was the first to use this statistical concept and he defined the generation function (now commonly termed excitation function) as being the average of the average power expended per unit length. The excitation function is denoted and characterizes the conductor's ability to produce RI currents. Later, Gary (28) gave a very clear and concise explanation of the physical significance of the excitation function. It is agreed that if at a given frequency is a quantity proportional to the average total power expended per unit length of conductor and frequency bandwidth. Therefore, the unit of $I$ is the square root of Ampere/square meter or Ampere/meter. That is, the equation that relates $I$ with the injected current $I$ is...

Fig. 2.1 - Graph of a Typical Corona Pulse

Fig. 2.2 - Graph of Frequency Spectrum for RI Currents
irregular intervals of time and distance along the conductor. Each current pulse has a similar frequency spectrum to all other current pulses, and the average spectrum can be considered as being the frequency spectrum of the RI currents on long lines (25). Therefore, despite the pulses being produced by random uncorrelated sources, the spectrum on any part of the line is similar to that in figure 2.2.

2.2 RI CURRENT INJECTION FROM UNIT LENGTH OF CONDUCTOR

For the predetermination of the RI transversal field profiles, it is necessary to know the RI currents injected into the line by a unit length of one conductor in corona. For this, use is made of the fact that the behaviour of the discharges conforms to a mutually uncorrelated stationary random process. That is, over a sufficiently long time the average power expended per unit length by the conductor under corona is constant. Adams (4) was the first to use this statistical concept and he defined the generation function (now commonly termed excitation function) as being the square root of the average power expended per unit length. The excitation function is denoted by $\Gamma$, and characterizes the conductor's ability to produce RI currents. Later, Gary (26) gave a very clear and concise explanation of the physical significance of the excitation function. It is agreed that $\Gamma$ at a given frequency is a quantity proportional to the average total power expended per unit length of conductor, and frequency bandwidth. Therefore, the unit of $\Gamma$ is the square root of Ampere$^2$/meter, or Ampere/meter$^{1/2}$.

For a single conductor line it is shown (12), (21), (26), (27) that the equation that relates $\Gamma$ with the injected current $i$ is
\[ i = \frac{C}{4\pi \varepsilon_0} \] - 2.2

where \( C \) is the capacitance coefficient in farads/meter between the conductor and ground, \( \varepsilon_0 \) is permittivity of free space, and \( \Gamma \) and \( i \) are expressed in Amps/meter\(^2\).

### 2.3 Capacitance Coefficient Matrix of Power Lines

For multiconductor EHV transmission lines, (2.2) will have to be modified to account for the presence of all the other conductors. The following is the resulting equation for a multiconductor system,

\[ [i] = \frac{1}{4\pi \varepsilon_0} [C][\Gamma] \] - 2.3

where \([i]\) is a column matrix of conductor impulse currents propagating in one direction, \([C]\) is the square capacitance coefficient matrix and \([\Gamma]\) is a column matrix with a single entry for the conductor in corona. Entries in matrix \([C]\) are derived from Maxwell's potential coefficients described next.

Consider a number, '\( n \)', of charged conducting bodies, isolated in space, each carrying a charge \( Q_i \), \( i = 1, 2 \ldots n \), as shown in figure 2.3. Applying the superposition theorem, the potential of each body depends on all the charges of the system by a set of '\( n \)' independent linear equations.

\[ V_i = p_{11}Q_1 + p_{12}Q_2 + \ldots + p_{1n}Q_n \]

\[ V_i = p_{i1}Q_1 + p_{i2}Q_2 + \ldots + p_{in}Q_n \] - 2.4

\[ V_n = p_{n1}Q_1 + p_{n2}Q_2 + \ldots + p_{nn}Q_n \]

\( V_i \) in (2.4) is the potential difference or voltage between the equipotential surface of body \( i \) and zero reference voltage at an infinitely large distance. The coefficients \( p \) are the Maxwell's
potential coefficients and they depend on the geometry (distances) of the system.

(2.4) in matrix form is

\[ \{V\} = \{p\} \{Q\} \]  

where \( \{V\} \) and \( \{Q\} \) are column matrices of order 'n' and \( \{p\} \) is an nxn square symmetric matrix. Premultiplying (2.5) by \( \{p\}^{-1} \)

\[ \{Q\} = \{p\}^{-1} \{V\} \]

\[ = \{C\} \{V\} \]  

Therefore

\[ \{C\} = \{p\}^{-1} \]  

\( \{C\} \) in (2.7) is the capacitance coefficient matrix of the system of conducting bodies isolated in space.

It is fortunate that it is relatively simple to determine the entries of \( \{p\} \) and hence, its inverse \( \{C\} \), for a system of parallel conductors, or power transmission lines. In this system the ground is represented by a perfectly conducting zero potential plane. Therefore the system can be reduced to that of line conductors and their images as shown in figure 2.4. The entries of matrix \( \{p\} \) are (21)
Fig. 2.4 - Three-Phase Transmission Line Above Perfectly Conducting Ground Showing Image Conductors
\[ p_{kk} = \frac{1}{2\pi \varepsilon_0} \ln \frac{2 h_k}{r_k} \quad - 2.8 \]
\[ p_{km} = \frac{1}{2\pi \varepsilon_0} \ln \frac{D_{km}}{d_{km}} \quad - 2.9 \]

where \( \varepsilon_0 \) is the absolute permittivity of free space, \( k \) and \( m \) refer to conductor numbers, \( h_k \) is the height of conductor \( k \), \( r_k \) is the radius of conductor \( k \), \( D_{km} \) is the distance between conductor \( k \) and the image of conductor \( m \), and \( d_{km} \) is the distance between conductors \( k \) and \( m \). If only the logarithm parts of (2.8) and (2.9) are considered and arranged in matrix form, then this square symmetric geometry matrix is denoted by \( \{G\} \) (29). Matrix \( \{G\} \) is symmetric and 60\(\{G\} \) is the key matrix in the modal analysis of lossless lines. The potential coefficient matrix of a line is

\[ \{p\} = \frac{1}{2\pi \varepsilon_0} \{G\} \quad - 2.10 \]

Therefore, from (2.7)

\[ \{C\} = \{p\}^{-1} = 2\pi \varepsilon_0 \{G\}^{-1} \quad - 2.11 \]

Substituting (2.11) into (2.3) yields

\[ \{i\} = i_{G} \{G\}^{-1} \{\Gamma\} \quad - 2.12 \]

(2.3) is the basic equation to calculate the injected corona currents that propagate in one direction on a multiconductor line. \( \Gamma \) refers to a unit length of the conductor in corona. For example, if one phase of a three phase transmission line is under corona then (2.3) is of the form
\[
\begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix} = \frac{1}{4\pi\varepsilon_0}
\begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{12} & C_{22} & C_{23} \\
C_{13} & C_{23} & C_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
0 \\
0
\end{bmatrix}
\]

where the subscript 1 in \( I_1 \) refers to the conductor under corona. In this thesis, as in other analyses of RI currently in use, the excitation function has been set equal to unity. As has been mentioned before the value of the excitation function can be and usually is found experimentally (14), (15), but setting it at unity at this stage and adding the appropriate dB correction at a later point yields the same result.

Since \( I \) depends only on the field intensity at and near the conductor and is hence, a very localized quantity, it does not depend on the large scale geometry of the system. However, the currents injected are proportional to \( \{C\} \) (2.3) which depend only on the geometry of the system of conductors. As a result \( I \) can be measured in the laboratory or on short test lines using conductors from the real line. Also different researches in different laboratories will obtain the same value of \( I \) provided that the same conductor is used as well as keeping the voltage gradient and atmospheric conditions constant.

Experimental determination of the excitation function is impossible during fair weather because of the unstable amplitude levels of corona discharges under these conditions. Tests conducted at Les Renardiers (12), (28) showed that it is possible to obtain reproducible levels of corona under heavy rain. Moreover the corona levels under heavy rain are the maximum that can be generated by a line. For these reasons the excitation function, and therefore the resulting
RI profiles, are determined under conditions of heavy rain. For this purpose cage test setups may be provided with artificial rain producing equipment. However, researches (13) have shown that results obtained under artificial and natural rain are not identical. Therefore, more credence is attached to the excitation function evaluation under natural heavy rain.

2.4 RI CURRENTS ON LONG LINE

2.4.1 SINGLE CONDUCTOR LINE

The RI current at the point of measurement on a long single conductor line due to one elementary section of the conductor being under corona is (24)

$$i = i_0 e^{-az} \text{Amps/meter}^{\frac{1}{2}}$$

where $a$ is the attenuation constant in Np/meter, $z$ is the distance from the point of measurement to the elementary section of conductor under corona and $i_0$ is half the current injected by the elementary section of the conductor in corona. The injected current is expressed in Amps/meter$^{\frac{1}{2}}$.

To find the total RI current in the centre of a line of length $2L$, all the currents, $i$, must be summed. The injected elementary random currents are assumed to be uniformly distributed along the line and they all have the same value of $i_0$. The summation takes the form of quadratic addition which is the law of addition for stationary random phenomena. Therefore, the amplitude of a sinusoidal current $I$, equivalent in average power to that of all the the sources combined is

$$I^2 = \Sigma i^2$$
\[ = \sum_{0}^{2} i_{0}e^{-2az} \]
\[ = \int_{-1}^{1} i_{0}e^{-2az} \, dz \]
\[ = 2\int_{0}^{1} i_{0}e^{-2az} \, dz \]
\[ = \left. \frac{i_{0}^{2} - 2a1}{a} \right|_{0}^{2} \]

where 2l is the length of conductor.

If \( l \) is large then

\[ I^{2} = \frac{i_{0}^{2}}{a} \quad - 2.16 \]

or

\[ I = \frac{i_{0}}{\sqrt{a}} \quad - 2.17 \]

In (2.17), \( i_{0} \) has the units of Amps/meter\(^{1/2}\) and \( a \) is in Np/meter. Therefore the units of \( I \) are Amperes. (2.16) in essence, states that at any frequency the spectral density within a narrow bandwidth of the total current is equal to the average spectral density of the RI current per unit length of conductor divided by the attenuation. As evident from (2.14) the attenuation of the currents propagating along the single conductor line must be known in order to calculate the long line currents.

2.4.2 MULTICONDUCTOR LINES

(2.17) is valid for multiconductor lines by resolving the injected currents into natural modes each characterized by a specific attenuation. Therefore, (2.17) can be written as
where \( k \) refers to the conductor number and \( n \) refers to the mode number. The problem remains to find for multiconductor lines such a resolution of injected conductor currents into sets of independently propagating modal currents each with a unique attenuation. These sets of currents and voltages are named modal sets. Bekku\(^7\) was the first to postulate the fact that there are as many distinct modes of propagation as there are ungrounded conductors in the system. Each of these modes has its own characteristic attenuation and therefore (2.17) or (2.18) can only be used when considering each distinct mode individually.

On any single or multiconductor line there will be some attenuation due to the resistance losses in the ground and in the conductors. There have been suggested methods of determining attenuation such as the one described by Adams and Barthold\(^6\). In this method a completely lossless system is assumed and the tangential magnetic field calculated on the conducting surfaces. The same field is assumed to exist immediately below these surfaces. The attenuation is then found through consideration of the amount of power lost through the conducting surfaces per unit length of line by application of Poyntings theorem. It is also possible to find the attenuation through experimental means.

For the calculation of RI profiles on a multiconductor line, only one conductor is considered to be under corona at any one time. Therefore, the injected currents due to this one conductor being in
corona are resolved into modes independently from the injected currents due to another conductor in corona. (This will be explained in chapter 3).

Having realized the necessity to resolve injected currents into modes, it is now necessary to describe modal analysis (chapter 3).

with the general solution and then its application to a lossless line system. Modal Analysis is then introduced as a means of calculating voltage and current components known as modes, from which further calculations can be made. A discussion on how to calculate attenuations for the so called near-lossless transmission line is given in the final section.

3.1 GENERAL SOLUTION

The fundamental differential equations of a system of 'k' parallel conductors above ground are

\[ \frac{d}{dx}(V) = -(A)(I) \tag{3.1} \]

\[ \frac{d}{dx}(I) = -(Y)(V) \tag{3.2} \]

where \( x \) is the co-ordinate axis parallel to the line conductors. (Note that in most analyses the co-ordinate axis parallel to the conductors is usually denoted 'z', but in this case it could be mistaken for the series impedance). \( V \) and \( I \) are column matrices representing conductor voltages and currents, and \( [A] \) and \( [Y] \) are the square series impedance and shunt admittance matrices per unit length. Differentiating and combining (3.1) and (3.2),
CHAPTER 3

THE HIGH FREQUENCY SIGNAL PROPAGATION ALONG A MULTICONDUCTOR LINE

The following work involves solving the transmission line equations in such a way that propagation of such quantities as voltages and currents can be studied in detail. The first part deals with the general solution and then its application to a lossless line system. Modal Analysis is then introduced as a means of calculating voltage and current components known as modes, from which further calculations can be made. A discussion on how to calculate attenuations for the so called near-lossless transmission line is given in the final section.

3.1 GENERAL SOLUTION

The fundamental differential equations of a system of 'k' parallel conductors above ground are (30)

\[
\begin{align*}
\frac{d}{dx} \{V\} &= -\{z\} \{I\} & \text{ (3.1)} \\
\frac{d}{dx} \{I\} &= -\{y\} \{V\} & \text{ (3.2)}
\end{align*}
\]

where \(x\) is the co-ordinate axis parallel to the line conductors. (Note that in most analysis the co-ordinate axis parallel to the conductors is usually denoted 'z', but in this case it could be mistaken for the series impedance). \(\{V\}\) and \(\{I\}\) are column matrices representing conductor voltages and currents, and \(\{z\}\) and \(\{y\}\) are the square series impedance and shunt admittance matrices per unit length. Differentiating and combining (3.1) and (3.2),

\[
\frac{d^2}{dx^2} \{V\} = \{z\} \{y\} \{V\}
\]
matrix

$$V = \{P\} \{V\}$$

- 3.3

and

$$\frac{d^2}{dx^2} \{I\} = \{y\} \{z\} \{I\}$$

- 3.4

where

$$\{P\} = \{z\} \{y\}$$

- 3.5

$$\{P_t\} = \{y\} \{z\}$$

- 3.6

and $$\{P_t\}$$ is the transpose of the matrix $$\{P\}$$.

The exponential form of solutions of (3.3) and (3.4) for voltage and current waves propagating in the positive direction of 'x' are;

$$\{V\} = e^{-x(P)^{1/2}} \{V_0\}$$

- 3.7

and

$$\{I\} = e^{-x(P_t)^{1/2}} \{I_0\}$$

- 3.8

where the subscript '0' refers to the reference point $$x=0$$. (3.7) and (3.8) contain functions of generally non-symmetric matrices $$\{P\}$$ and $$\{P_t\}$$. The resolution of these equations into a linear form through eigenvalue analysis is the basis of modal analysis of lossy lines. Differentiating (3.7), and from (3.1),

$$\frac{d}{dx}\{V\} = -(P)^{1/2} e^{-x(P)^{1/2}} \{V_0\} = -(P)^{1/2} \{V\}$$

- 3.9

From (3.5) and (3.9) the direct relationship between voltages and currents can now be found to be

$$\{V\} = (P)^{-1/2} \{z\} \{I\}$$

$$= \{(z)\{y\}\}^{-1} \{z\} \{I\}$$

$$= \{Z\} \{I\}$$

- 3.10

where $$\{Z\}$$ is the characteristic impedance
matrix

\[
(Z) = (((z) (y)))^{1/2} \cdot (z)^{-1}
\]

\[
= (((z) (y)))^{1/2} \cdot (z)
\] - 3.11

Another form for the characteristic impedance matrix \( (z) \) is found by differentiating \( (I) \) in (3.8) and substituting the result in (3.2),

\[
\frac{d(I)}{dx} = -(P \cdot \delta^2 (I)) = -(y) (V)
\] - 3.12

Premultiplying (3.12) by \( (y)^{-1} \) and comparing the result with (3.10),

\[
(V) = ((z) (I)) = (y)^{-1} \cdot (P \cdot \delta^2 (V))
\] - 3.13

From (3.6), (3.11) and (3.13),

\[
(Z) = (((z) (y)))^{1/2}
\]

\[
= (y)^{-1} \cdot ((y) (z))^{1/2}
\] - 3.14

The characteristic impedance matrix (3.11) and (3.14) is always symmetric and its complex entries for a lossy line depend on the line geometry, frequency, and the conductor and earth resistivities. For a lossless line the entries of \( (z) \) are real and depend on the line geometry only.

The composition of the \( (z) \) and \( (y) \) matrices depend on the line geometry and whether the system is considered to be lossless or lossy. In general the \( (z) \) and \( (y) \) matrices are

\[
(z) = (R) + j\omega (L)
\] - 3.15

\[
(y) = j\omega (C)
\] - 3.16

where \( (R) \) is a square real symmetric matrix whose entries depend on the system losses, \( (L) \) is a square real matrix corresponding to series, self and mutual inductances, and \( (C) \) is a
square matrix of shunt capacitance coefficients. The shunt losses, and hence conductances are neglected.

For a lossless line the \( \{z\} \) and \( \{y\} \) matrices are purely imaginary, (pure reactance and admittance), due to the resistance and conductance being zero. In the lossy line case certain factors have to be introduced to account for the lossy conductors and lossy ground. In the case of the \( \{y\} \) matrix the shunt losses are neglected and therefore, \( \{y\} \) is the same as for a lossless line. The \( \{z\} \) matrix however, will have to be modified by the use of Carson Correction Factors (C.C.F.) (19),(20). These factors were first derived by J. Carson in 1926 as a means of representing the effect of the lossy ground by series self and mutual complex impedances in the equivalent circuit of the transmission line. This will be dealt with further in a later chapter.

3.2 LOSSLESS LINE

A particular case of (3.1) and (3.2) is that for a lossless line. In this instance the series impedance is purely inductive and both the conductor and ground resistivities are identically zero. In practical lines however, resistive losses do exist, but the lossless analysis is a useful and practical approximation for lines with relatively low losses.

The expressions for \( \{z\} \) and \( \{y\} \) in the lossless case are

\[
\{z\} = j\omega \{L\} \\
\{y\} = j\omega \{C\}
\]

- 3.17

- 3.18

The entries of \( \{L\} \) and \( \{C\} \) for the lossless case can be found using the electromagnetic theory together with the image principle (30).
\( \{L\} = \mu_0 \frac{1}{2\pi} \{G\} \) \hspace{1cm} (3.19)

\( \{C\} = 2\pi \varepsilon_0 \{G\}^{-1} \) \hspace{1cm} (3.20)

\( \{G\} \) is a matrix whose entries depend solely on the line geometry, (2.10), and is therefore named the geometry matrix, \( \mu_0 \) is the permeability of free space and \( \varepsilon_0 \) is the permittivity of free space.

The matrix \( \{P\} \) in the lossless case becomes

\[
\{P\} = \{z\} \{y\} = \{y\} \{z\} = -\omega^2 \varepsilon_0 \mu_0 \{1\} \]

where \( \{1\} \) is the unit matrix.

The current at any point \( x \) is related to the current at \( x=0 \) by the exponential function of the square root of the matrix \( \{P_t\} \), as shown in (3.8). By expanding into a series, the term \( \exp(-x \{P_t\}^{1/2}) \) for this lossless case is

\[
e^{-x \{P_t\}^{1/2}} = e^{-x(-\omega^2 \varepsilon_0 \mu_0 \{1\}^{1/2})} = e^{-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2} \{1\}^{1/2}} \]

\[
= e^{-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2} \{1\}} \frac{(-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2})^2}{(1) + (1)(2)} + \ldots \} \{1\} \]

\[
= e^{-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2} \{1\}} \]

(3.22)

Therefore, by substituting (3.22) in (3.8),

\[
\{I\} = e^{-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2} \{1\}} \{I_0\} = e^{-x(-\omega^2 \varepsilon_0 \mu_0)^{1/2} \{1\}} \{I_0\} \]
\[ e^{-x\left(-\omega^2 \varepsilon_0 \mu_0 \right)^{1/2}} I_0 \]

\[ = e^{-j\frac{2\pi f x}{v_0}} I_0 = e^{-j\frac{2\pi x}{\lambda}} I_0 \]

where \( v_0 \) is the velocity of light, \( \lambda \) is the wavelength and \( f \) is the frequency. Similar expressions may be obtained for the voltages by replacing \( \{I\} \) and \( \{I_0\} \) with \( \{V\} \) and \( \{V_0\} \). The attenuation in this case is zero and the wave is guided along the line at the velocity of light. (3.23) therefore, demonstrates that in the lossless case there will only be a phase shift in the propagating wave and no attenuation.

### 3.3 Modal Analysis

In a multiconductor system the propagation of any disturbance introduced on one conductor is affected by the presence of the other conductors and ground. This fact complicates the analysis of such systems, such as in regard to RI studies. An effective tool for dealing with this problem is afforded by modal analysis. Through such an analysis the original \( k \)-conductor line is replaced by '\( k \)' independent conductor modal systems, along which a particular mode propagates. Each of these modal systems describe the values of the propagating modal quantities at any point of the real line. Since there is no coupling between the modal lines, the solution to propagation problems is greatly facilitated.

The modes are derived from the eigenvalue analysis of a square matrix and its transpose. With each eigenvalue there is associated an eigenvector direction. These directions define a new system of linearly independant co-ordinate axes. For a symmetric matrix in lossless line
analysis the eigenvectors or current and voltage modes form a single orthogonal system. Their projections on the cartesian reference system of co-ordinates are the modal components. The analysis of lossy lines is associated with eigenvalue analysis of functions of non-symmetric matrices with complex entries. This is employed in chapter 5

3.4 MODAL ANALYSIS AS APPLIED TO THE SPECIAL CASE OF A LOSSLESS SYSTEM

3.4.1 CHARACTERISTIC IMPEDANCE MATRIX AS A BASIS FOR MODAL ANALYSIS

In modal analysis of a general transmission line, the matrix from which all calculations originate is matrix \( P \), and its transpose \( P^t \). However, a near lossless system approximated by a lossless system gives rise to a \( P \) matrix which is diagonal with identical real values (3.21). Hence, the characteristic impedance matrix \( \{Z\} \), (3.11) and (3.14), is diagonalized through eigenvalue analysis, leading to mutually orthogonal modes of voltage and current since \( \{Z\} \) is real and symmetric.

Referring to (3.14) and (3.17) to (3.21),

\[
\{Z\} = (P)^{-1} \{z\} = \{(z)\{y\}\}^{-1}_{\mu^2 \frac{\mu_0}{\varepsilon_0}}
\]

\[
= \left(-\omega^2 \varepsilon_0 \mu_0 (1)\right)^{-1}_\omega \frac{\mu_0}{\varepsilon_0} \{G\}
\]

\[
= \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} \{G\}
\]

\[
= 60\{G\}
\]

where \( \sqrt{\frac{\mu_0}{\varepsilon_0}} \) is the intrinsic impedance of free space \( \sqrt{\frac{\mu_0}{\varepsilon_0}} = 2\pi \cdot 60 = 377 \), and \( \{G\} \) is the square symmetric geometry matrix, (2.8), (2.9) and (2.10). Since the characteristic impedance matrix \( \{Z\} \) for a lossless system is only a function of the line geometry, then it is totally independent of frequency.
The voltages and currents are related through the \( \{Z\} \) matrix in the same manner as indicated in (3.10) and (3.13). By applying eigenvalue analysis to matrix \( \{Z\} \), it is now possible through modal analysis to obtain a set of modal components for the voltages and currents of each conductor.

3.4.2 CHARACTERISTIC IMPEDANCE MATRIX FOR THE CASE OF A THREE PHASE HORIZONTAL LINE WITH SKY WIRES

If a symmetrical three conductor transmission line system is considered, (3.10) in expanded form is

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
A & B & C & I_1 \\
B & D & B & I_2 \\
C & B & A & I_3
\end{bmatrix}
\]

where \( A, B, C \) and \( D \) are the real entries of the characteristic impedance matrix, and the subscripts in the voltage and current matrices refer to the conductor numbers. Should the characteristic impedance matrix include the effect of grounded sky wires it would be necessary to perform the following operations so as to have an equation in the form of (3.25).

For a symmetrical three conductor line system with two sky wires, (3.25) becomes

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
A' & B' & C' & F' & G' & I_1 \\
B' & D' & B' & H' & H' & I_2 \\
C' & B' & A' & G' & F' & I_3 \\
0 & F' & H' & G' & S' & I_4 \\
0 & G' & H' & F' & L' & S' & I_5
\end{bmatrix}
\]

or

\[
\{V\} = \{Z'\} \{I\}
\]

where \( A', B', C' \) and \( D' \) are the entries of the \( \{Z'\} \) matrix which depend on the phase conductors only, \( F', G' \)
and \( H' \) are the entries due to both a conductor and a sky wire, \( L' \) and \( S' \) are entries which depend on sky wires only, and \( \{Z'\} \) is the characteristic impedance of the system including sky wires. \( V_4 \) and \( V_5 \) are zero because the sky wires are considered continuously grounded.

From (3.26) five simultaneous equations can be obtained.

\[
V_1 = A'I_1 + B'I_2 + C'I_3 + E'I_4 + F'I_5 \tag{3.28}
\]

\[
V_2 = B'I_1 + D'I_2 + B'I_3 + H'I_4 + H'I_5 \tag{3.29}
\]

\[
V_3 = C'I_1 + B'I_2 + A'I_3 + F'I_4 + E'I_5 \tag{3.30}
\]

\[
O = E'I_1 + H'I_2 + F'I_3 + S'I_4 + L'I_5 \tag{3.31}
\]

\[
O = F'I_1 + H'I_2 + E'I_3 + L'I_4 + S'I_5 \tag{3.32}
\]

From (3.31) and (3.32), \( I_4 \) and \( I_5 \) can be expressed in terms of \( I_1 \), \( I_2 \) and \( I_3 \). Substituting \( I_4 \) and \( I_5 \) into (3.28), (3.29) and (3.30)

\[
V_1 = AI_1 + BI_2 + CI_3 \tag{3.33}
\]

\[
V_1 = BI_1 + DI_2 + BI_3 \tag{3.34}
\]

\[
V_1 = CI_1 + BI_2 + AI_3 \tag{3.35}
\]

or

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
A & B & C \\
B & D & B \\
C & B & A
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} \tag{3.36}
\]

This equation is identical to (3.25) except that \( \{Z\} \) in this case is the characteristic impedance of the system with eliminated sky wires.

### 3.4.3 Characteristic Impedance Matrix for the Case of a Transposed Transmission Line

A representation of one repeated element of continuously transposed transmission line is illustrated in figure 3.1. The impedance matrix for each section is found using (3.24) and (3.25), thus
Fig. 3.1 - Representation of a Three-Phase Transmission Line Including Three Transposition Sections A, B and C of Equal Length
Section A

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{12} & Z_{22} & Z_{23} \\
Z_{13} & Z_{23} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

- 3.37

Section B

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{22} & Z_{23} & Z_{12} \\
Z_{23} & Z_{33} & Z_{13} \\
Z_{12} & Z_{13} & Z_{11}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

- 3.38

Section C

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{33} & Z_{13} & Z_{23} \\
Z_{13} & Z_{11} & Z_{12} \\
Z_{23} & Z_{12} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

- 3.39

Taking the average of the three sections of equal length,

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\frac{1}{3}
\begin{bmatrix}
Z_{11} + Z_{22} + Z_{33} & Z_{12} + Z_{23} + Z_{13} & Z_{12} + Z_{23} + Z_{13} \\
Z_{12} + Z_{23} + Z_{13} & Z_{11} + Z_{22} + Z_{33} & Z_{12} + Z_{23} + Z_{13} \\
Z_{12} + Z_{23} + Z_{13} & Z_{12} + Z_{23} + Z_{13} & Z_{11} + Z_{22} + Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

- 3.40

Therefore, the characteristic impedance matrix for a continuously transposed transmission line is

\[
\{Z\} =
\begin{bmatrix}
s & m & m \\
m & s & m \\
m & m & s
\end{bmatrix}
\]

- 3.41

where

\[
s = \frac{Z_{11} + Z_{22} + Z_{33}}{3}
\]

- 3.42

and

\[
m = \frac{Z_{12} + Z_{23} + Z_{13}}{3}
\]

- 3.43

3.5 MODAL TRANSFORMATION MATRIX AND MODAL COMPONENTS

3.5.1 TRANSPOSED TRANSMISSION LINE

Consider for the case of the transposed line the basic relation

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
\{Z\}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

- 3.49
\[
\begin{bmatrix}
\mathbf{v}(n) \\
\mathbf{v}_2(n) \\
\mathbf{v}_3(n)
\end{bmatrix} = (Z) \begin{bmatrix}
\mathbf{i}(n) \\
\mathbf{i}_2(n) \\
\mathbf{i}_3(n)
\end{bmatrix} - 3.45
\]

Therefore, for the case when modal components are considered. For each value of \(n\) a value for \(\lambda(n)\) has to be found so as to satisfy the relation

\[
\begin{bmatrix}
\mathbf{v}(n) \\
\mathbf{v}_2(n) \\
\mathbf{v}_3(n)
\end{bmatrix} = \lambda(n) \begin{bmatrix}
\mathbf{i}(n) \\
\mathbf{i}_2(n) \\
\mathbf{i}_3(n)
\end{bmatrix} - 3.46
\]

Therefore, combining (3.45) and (3.46),

\[
\begin{bmatrix}
\mathbf{i}(n) \\
\mathbf{i}_2(n) \\
\mathbf{i}_3(n)
\end{bmatrix} = 0
\]

\[
\lambda(n) = \{Z\}^{-1} \mathbf{i}(n) - 3.47
\]

or

\[
\begin{bmatrix}
\mathbf{i}(n) \\
\mathbf{i}_2(n) \\
\mathbf{i}_3(n)
\end{bmatrix} = 0
\]

This matrix equation will yield three homogeneous equations, and to solve for the values of \(\lambda(n)\) then the determinant of the first matrix in (3.48) must be found and equated to zero thus
The characteristic equation obtained from (3.49) is

\[(s-\lambda(n))(s-\lambda(n)^2-2m^2(s-\lambda(n))m)=0\]  

Therefore,

\[\lambda(1)=s-m=Z(1)\]  

\[\lambda(2)=\lambda(3)=s+2m=Z(2)=Z(3)\]  

Substituting (3.51) and (3.52) into (3.48) will yield

\[I_1^{(1)}+I_2^{(1)}+I_3^{(1)}=0\]  

\[I_1^{(2)}+I_2^{(2)}+I_3^{(2)}=0\]  

\[I_1^{(3)}-2I_2^{(3)}+I_3^{(3)}=0\]  

(3.53) and (3.54) are the same but their solutions must be orthogonal because each was derived for a different mode. A solution to (3.53), (3.54) and (3.55) is

\[I_1^{(1)}=1, I_2^{(1)}=-2, I_3^{(1)}=1\]  

\[I_1^{(2)}=1, I_2^{(2)}=0, I_3^{(2)}=-1\]  

\[I_1^{(3)}=1, I_2^{(3)}=1, I_3^{(3)}=1\]  

It can be seen by inspection that each modal vector, or eigenvector, is orthogonal to all other eigenvectors. If the eigenvectors are arranged in matrix form then

\[\{N\}=[1 \ 1 \ 1]
-2 \ 0 \ 1
1 \ -1 \ 1\]  

where \(\{N\}\) is known as the modal transformation matrix. A similar method may be performed using
voltages but it will be found that, because the lossless line is considered, the voltage modal matrix is the same as that for currents.

It is possible to obtain modal currents by using the phase currents and the modal matrix in the following way.

\[
\begin{bmatrix}
I_1^{(1)} \\
I_2^{(1)} \\
I_3^{(1)}
\end{bmatrix} = (N)^{-1} \begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
I_1^{(1)} & I_1^{(2)} & I_1^{(3)} \\
I_2^{(1)} & I_2^{(2)} & I_2^{(3)} \\
I_3^{(1)} & I_3^{(2)} & I_3^{(3)}
\end{bmatrix} = (N) \begin{bmatrix}
I_1^{(1)} & 0 & 0 \\
0 & I_2^{(1)} & 0 \\
0 & 0 & I_3^{(1)}
\end{bmatrix}
\]

The method of RI calculation described in Chapter 4 will require the use of normalized modal transformation matrices with eigenvectors of unit length. Therefore, the length of each eigenvector in (3.59) is \(\sqrt{5}, \sqrt{2}\) and \(\sqrt{3}\) respectively, and the normalized modal matrix is

\[
(N_n) = \begin{bmatrix}
1/\sqrt{5} & 1/\sqrt{2} & 1/\sqrt{3} \\
-2/\sqrt{6} & 0 & 1/\sqrt{3} \\
1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3}
\end{bmatrix}
\]

and

\[
(N_n)^{-1} = (N_{nt})
\]

where n denotes normalized and t denotes transposed.

3.5.2 GENERAL UNTRANSPOSED TRANSMISSION LINE

The above calculations were made for the particular case of the transposed line.

However, in general the characteristic impedance matrix is of the form given in (3.25). The following is the derivation of the modal matrix for the general case.
Taking into account the basic relation of (3.25), any power line conductor voltage is a function of all conductor currents. For example,

$$V_1 = AI_1 + BI_2 + CI_3$$  \[3.64\]

We look for such voltage and current components which satisfy the following simple equation,

$$\begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \end{bmatrix} = \begin{bmatrix} f_1(n) \\ f_2(n) \\ f_3(n) \end{bmatrix}$$  \[3.65\]

so that any conductor voltage component is proportional to the same conductor current component through the impedance $Z(n)$ only. For example,

$$v_1(n) = Z(n) I_1(n)$$  \[3.66\]

These components have also to satisfy the relation

$$\begin{bmatrix} v_1(n) \\ v_2(n) \\ v_3(n) \end{bmatrix} = \{Z\} \begin{bmatrix} I_1(n) \\ I_2(n) \\ I_3(n) \end{bmatrix}$$  \[3.67\]

Hence, from (3.65) and (3.67),

$$\begin{bmatrix} I_1(n) \\ I_2(n) \\ I_3(n) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$  \[3.68\]

which for the case of the symmetrical three phase transmission line can be written in the form
\[
\begin{bmatrix}
A-Z(n) & B & C \\
B & D-Z(n) & B \\
C & B & A-Z(n)
\end{bmatrix}
\begin{bmatrix}
I_{1}^{(n)} \\
I_{2}^{(n)} \\
I_{3}^{(n)}
\end{bmatrix} = 0
\]

This homogeneous equation has a non-trivial solution if the determinant of the square matrix is equal to zero for a non-zero column matrix \(I\).

The equation
\[
\begin{bmatrix}
A-Z(n) & B & C \\
B & D-Z(n) & B \\
C & B & A-Z(n)
\end{bmatrix} = 0
\]
leads to the solution of a third degree equation which is of the form
\[
(A-Z(n))((D-Z(n))(A-Z(n))-B^2)-B^2(A-Z(n)-C)+C(B^2-C(D-Z(n)))=0
\]

The roots of (3.71) are
\[
Z(1) = \frac{1}{2} \bigg((A+C+D) - \sqrt{(A+C+D)^2 + 8B^2}\bigg)
\]
\[
Z(2) = A-C
\]
\[
Z(3) = \frac{1}{2} \bigg((A+C+D) + \sqrt{(A+C+D)^2 + 8B^2}\bigg)
\]

where
\[
H = \sqrt{(A+C-D)^2 + 8B^2}
\]
and are termed the modal characteristic impedances. Successive substitution of the modal characteristic impedances in (3.69) will yield the following ratios.
\[
\frac{I_2^{(1)}}{I_2^{(1)}} - \frac{I_1^{(1)}}{I_1^{(1)}} = -p
\]
\[ I_2^{(3)} = \frac{I_2^{(1)}}{I_1^{(3)}} = q \]
\[ I_1^{(1)} = 0 \]
\[ I_2^{(2)} \]
\[ I_3^{(2)} = -1 \]
\[ I_3^{(1)} \]

where

\[ p = \frac{2B}{Z^{(1)} - D} \]
\[ q = \frac{2B}{Z^{(3)} - D} \]

The current modal components must add up to the actual conductor currents. Therefore,

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \begin{bmatrix}
I_1^{(1)} \\
I_2^{(1)} \\
I_3^{(1)}
\end{bmatrix} + \begin{bmatrix}
I_1^{(2)} \\
I_2^{(2)} \\
I_3^{(2)}
\end{bmatrix} + \begin{bmatrix}
I_1^{(3)} \\
I_2^{(3)} \\
I_3^{(3)}
\end{bmatrix}
\]

can be written in the form

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} q
\]

where

\[
\begin{bmatrix}
1 \\
p + I_1^{(2)} \\
-1
\end{bmatrix}
\]
This finally provides the relation between actual conductor currents and the modal conductor components in terms of phase 1 components. Therefore,

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \{N\}
\begin{bmatrix}
I_1^{(1)} \\
I_1^{(2)} \\
I_1^{(3)}
\end{bmatrix}
\]

where the current modal transformation matrix \(\{N\}\) for a horizontal line is

\[
\begin{bmatrix}
N_{11} & N_{12} & N_{13} \\
N_{21} & N_{22} & N_{23} \\
N_{31} & N_{32} & N_{33}
\end{bmatrix}
\]

This provides the relation between actual conductor voltages and voltage modal components as

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 \\
p & 0 & q \\
1 & -1 & 1
\end{bmatrix}
\]

Similarly the voltage modal components, determined from (3.65), must add up to the actual conductor voltages thus,

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix} = \begin{bmatrix}
v_1^{(1)} \\
v_2^{(1)} \\
v_3^{(1)}
\end{bmatrix} + \begin{bmatrix}
v_1^{(2)} \\
v_2^{(2)} \\
v_3^{(2)}
\end{bmatrix} + \begin{bmatrix}
v_1^{(3)} \\
v_2^{(3)} \\
v_3^{(3)}
\end{bmatrix}
\]

This can be written in the form

\[
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix} = v_1^{(1)} \begin{bmatrix}
1 \\
v_2^{(1)} \\
v_3^{(1)}
\end{bmatrix} + v_1^{(2)} \begin{bmatrix}
1 \\
v_2^{(2)} \\
v_3^{(2)}
\end{bmatrix} + v_1^{(3)} \begin{bmatrix}
1 \\
v_2^{(3)} \\
v_3^{(3)}
\end{bmatrix}
\]

This should be noted that since the modal transformation matrices are cyclic and the line is lossless, the \(\{N\}\) and \(\{V\}\) matrices are related and the modal components for phase 1 are required and the modal transformation matrix can be written.

If the modal components for phase 1 are not required and the actual conductor voltages in terms of the inverse modal transformation can be written.
The ratios of modal voltage components are equal to the ratios of current modal components as the result of (3.65). Therefore,

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} = V_1^{(1)} \begin{bmatrix}
1 \\
\frac{2}{1} \\
\frac{3}{1}
\end{bmatrix} + V_1^{(2)} \begin{bmatrix}
1 \\
\frac{2}{1} \\
\frac{3}{1}
\end{bmatrix} + V_1^{(3)} \begin{bmatrix}
1 \\
\frac{2}{1} \\
\frac{3}{1}
\end{bmatrix}
\]

\[
= V_1^{(1)} \begin{bmatrix}
1 \\
p \\
1
\end{bmatrix} + V_1^{(2)} \begin{bmatrix}
0 \\
0 \\
q
\end{bmatrix} + V_1^{(3)} \begin{bmatrix}
1 \\
-1 \\
1
\end{bmatrix}
\]

This provides the relation between actual conductor voltages and voltage modal components of phase number 1 as

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} = \{M\} \begin{bmatrix}
V_1^{(1)} \\
V_1^{(2)} \\
V_1^{(3)}
\end{bmatrix}
\]

\[
= \{N\} \begin{bmatrix}
V_1^{(1)} \\
V_1^{(2)} \\
V_1^{(3)}
\end{bmatrix}
\]

where \(\{M\}\) is the voltage modal transformation matrix. Therefore, it should be noted that since the characteristic impedance matrix is symmetric and the line is lossless, the \(\{M\}\) and \(\{N\}\) matrices are identical.

If the modal components of phase number 1 are required and the actual conductor quantities are known, then use is made of the inverse modal transformation matrix thus:
\[
\begin{bmatrix}
I_1^{(1)} \\
I_1^{(2)} \\
I_1^{(3)}
\end{bmatrix} = \{N\}^{-1}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} ;
\begin{bmatrix}
V_1^{(1)} \\
V_1^{(2)} \\
V_1^{(3)}
\end{bmatrix} = \{M\}^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} - 3.90
\]

where

\[
\{N\}^{-1} = \frac{1}{2(q-p)}
\begin{bmatrix}
q & -2 & q \\
q-p & 0 & p-q \\
-p & 2 & -p
\end{bmatrix} - 3.91
\]

To find the normalized modal matrix the eigenvectors have to be of unit length. For horizontal lines the eigenvector lengths in \{N\} are \(\sqrt{p^2+2}\), \(\sqrt{2}\) and \(\sqrt{q^2+2}\) respectively. Hence, from (3.83)

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = I_1^{(1)} \frac{1}{\sqrt{p^2+2}} + I_1^{(2)} \frac{1}{\sqrt{q^2+2}} + I_1^{(3)} \frac{1}{\sqrt{q^2+2}} - 3.92
\]

and this equation illustrates the unit length eigenvectors. It may be expressed as

\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \frac{1}{\sqrt{p^2+2}} \begin{bmatrix} 1 \\ \frac{p}{\sqrt{p^2+2}} \\ 0 \\
\end{bmatrix} + \frac{1}{\sqrt{q^2+2}} \begin{bmatrix} 1 \\ -1 \\ \frac{q}{\sqrt{q^2+2}} \\
\end{bmatrix} - 3.93
\]

or it may be expressed in the form
\[
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix} = \begin{bmatrix}
\{N_n\}'(1) \\
\{N_n\}'(2) \\
\{N_n\}'(3)
\end{bmatrix}
\]

where \(\{N_n\}\) is the normalized modal transformation matrix,

\[
\{N_n\} = \begin{bmatrix}
\frac{1}{\sqrt{p^2+2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{q^2+2}} \\
\frac{p}{\sqrt{p^2+2}} & 0 & \frac{q}{\sqrt{q^2+2}} \\
\frac{1}{\sqrt{p^2+2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{q^2+2}}
\end{bmatrix}
\]

\[
\{N_n\}^{-1} = \{N_n\}^{-1}
\]

and

\[
\begin{bmatrix}
I_1'(1) \\
I_1'(2) \\
I_1'(3)
\end{bmatrix} = \begin{bmatrix}
I_1(1)\sqrt{p^2+2} \\
I_1(2)\sqrt{2} \\
I_1(3)\sqrt{q^2+2}
\end{bmatrix}
\]

It should be noted that \(I_1'(1)\), \(I_1'(2)\) and \(I_1'(3)\) have no physical interpretation and hence, cannot be measured.

**3.6 MODAL ATTENUATION OF NEAR-LOSSLESS LINES**

On a lossless transmission line the attenuation of all modal components is zero and their velocity of propagation is that of the speed of light. However, for a near-lossless line, which is the actual case and for which the preceding analysis is performed, an attenuation exists and must be determined.

One method of obtaining modal attenuations is by measuring it on an actual line. For each mode the line has voltages applied to it of the same ratio as the corresponding column in the modal matrix. The modal voltages are recorded at distance \(z\) down the line and the
Transmission Line Running in Direction of z axis:-
The x-z Plane is the Ground Surface

\[ I_k \]

\[ j \]

\[ E_k \]

\[ H_k \]

\[ z \]

\[ z + dz \]

\[ dx \]

\[ ds \]

\[ S_k = E_k \times H_k \]

Fig. 3.2 - Demonstration of Direction of Power Loss Through Earth Surface due to E and H Fields

Attenuation is calculated using

\[ V(z) = e^{-\alpha(n)z}V(0) \]  

- 3.98

where \( V \) is the voltage, \( z \) is the distance travelled and \( \alpha(n) \) is the attenuation of mode \( n \). Another method has been suggested by Adams and Barthold (6) which approaches the problem from the standpoint of power losses. The following is a simplified explanation of it.

The power loss in a line due to the losses in the ground is the result of the active power flow into the ground. This power flow is connected with the existence of the Poynting vector component, perpendicular to the ground surface (figure 3.2)
S = ExH

defining the instantaneous power flow density into the ground. E and H are instantaneous vectors of electric and magnetic field intensity respectively. The power flow through the elementary surface ds = j \cdot dx \cdot dz for a single conductor k is

\[ d^2P = -(E_k \cdot xH_k) \cdot ds \]
\[ = -(kE_k \cdot xH_k) \cdot (-jdxdz) \]
\[ = -E_k H_k dxdz \]

The power flow through the belt dz wide (-\infty < x < +\infty) is

\[ dP = \int_{-\infty}^{+\infty} E_k H_k dx \cdot dz \]

Then the first derivative of instantaneous power loss in the line with respect to z (z is the direction of signal propagation along the line) is

\[ \frac{dP}{dz} = \int_{-\infty}^{+\infty} E_k H_k dx \]

In the case of a complex Poynting vector:

\[ S_k = E_k^* xH_k \]

where \( E_k \) and \( H_k \) are complex vectors of electric and magnetic field intensity respectively.

The real part of the complex Poynting vector represents the time-average power flow density. Therefore, using the same procedure as was shown above and making use of the fact that the complex values of vectors \( E_k \) and \( H_k \) are related by intrinsic impedance of the ground \( Z_g \) by

\[ E_k = Z_g H_k \]

we can find the first derivative of real
power loss in the line as

\[
\frac{dP}{dz} = \text{Re} \int_{-\infty}^{+\infty} -EH \, dx
\]

\[= \text{Re}(Z_g) \int_{-\infty}^{+\infty} -H^2 \, dx \quad \text{ - 3.105} \]

where

\[
\text{Re}(Z_g) = \text{Re}\left(\sqrt{j \omega \mu_0 \rho_e}\right)
\]

\[= \sqrt{\frac{\mu_0 \rho_e}{\pi f}} \quad \text{ - 3.106} \]

and \(\rho_e\) is earth resistivity. \(H\) is the rms value of magnetic field intensity vector on the earth surface which can be found by making use of the Ampere's Circuital Law and the image principle. For the case of a one conductor system, (figure 3.3) \(H_{con}\) and \(H_{im}\) represent magnetic field intensity vectors at the lateral distance \(x\) due to the conductor and its image currents respectively. Their vertical components cancel out but horizontal ones add up to the resultant vector \(H\). Hence, for a single conductor \(k\)

\[H_k = 2H_{con} \cos \phi \quad \text{ - 3.107} \]

where

\[\cos \phi = \frac{h_k}{r_k}
\]

\[= \frac{h_k}{\sqrt{h_k^2 + x^2}} \quad \text{ - 3.108} \]

and

\[H_{con} = \frac{I_k}{2\pi r_k} \quad \text{ - 3.109} \]

Therefore,
Fig. 3.3 - Demonstration of Resultant Magnetic Field due to an Infinitely Long Conductor Above Perfectly Conducting Ground by means of an Image Conductor and Ampere's Circuit Law.
The integral in (3.114) is solved by parts or taken from integral tables. The attenuation of current flowing in only one conductor the attenuation is proportional to the square root of frequency $f$ and specific resistance $\rho$. $Z_0$ in the denominator is that of a single conductor above ground. The conductor height $h_k$.

Since $I_k$ in the last equation is a function of $z$ only, (3.105) can be written in the following form.

$$ \frac{dP}{dz} = -\text{Re}(Z_g) \sum_{k=1}^{\infty} \frac{h_k^2}{2 \pi} \int_{-\infty}^{\infty} \frac{1}{(x^2 + h_k^2)^2} dx $$  

$$ - \text{3.111} $$

d$P/\text{dz}$ can also be obtained by the derivation of power $P$ determined as:

$$ P = VI $$

$$ = Z_0 I_1^2 e^{-\alpha z} $$

$$ = Z_0 (1_0 e^{-az})^2 $$

$$ = Z_0 1_0^2 e^{-2az} $$  

$$ - \text{3.112} $$

where $Z_0$ is the characteristic impedance of the line. Therefore,

$$ \frac{dP}{dz} = -2az_0 I_0^2 e^{-2az} $$

$$ = -2az_0 I_0^2 $$  

$$ - \text{3.113} $$

Comparison of the right hand sides of (3.111) and (3.113) provides the attenuation constant, and from (3.106)

simplifies to

$$ \alpha = \frac{\text{Re}(Z_g)}{Z_0} \sum_{k=1}^{\infty} \frac{1}{2 \pi \sqrt{(x^2 + h_k^2)^2} dx} $$

From (3.106),

$$ \frac{\text{Re}(Z_g)}{Z_0} \cdot \frac{1}{4\pi h_k} = \frac{\sqrt{\pi H_0 e}}{4\pi h_k Z_0} $$  

$$ - \text{3.114} $$
The integral in (3.114) is solved by parts or taken from integral tables.

The attenuation due to earth losses, (3.114), indicates that for a current flowing in only one conductor the attenuation is proportional to the square root of the product of frequency \( f \) and specific earth resistivity \( \rho_e \). \( Z_0 \) in the denominator is that of a single conductor above lossless ground and, hence, it is a pure resistance defined by the conductor height \( h_k \) and radius \( r_k \); \( Z_0 = 60 \ln(2h_k/r_k) \).

In a multiconductor line, (3.98) to (3.114) are applied to each of the uncoupled modal lines. The current modes and the modal impedances are those for a lossless line. For each set of normalized modal current components, (3.95), the magnetic field intensity \( H(n)(x) \) is found. The modal fields are described in detail in chapter 4. (3.105) is changed to

\[
\frac{dp(n)}{dz} = \text{Re}(Zg) \int_{-\infty}^{+\infty} (\sum_{n=1}^{3} H(n))^2 dx
\]

- 3.115

For a three phase horizontal line, (3.95) and (3.110), the resulting modal field \( H(n) \) is

\[
H(n) = \frac{h_1 N_1(n)}{\pi(x^2+h_1^2)} + \frac{h_2 N_2(n)}{\pi(x^2+h_2^2)} + \frac{h_3 N_3(n)}{\pi(x^2+h_3^2)}
\]

- 3.116

where \( N_k(n) \) are the k-th entries of column 'n' in (3.95). Since \( \sum_{k=1}^{3} (N_k(n))^2 = 1.0 \), then (3.113) simplifies to

\[
\frac{dp(n)}{dz} = -2\alpha_e Z(n)
\]

- 3.117

From (3.106), (3.115) and (3.116), after integrating and
comparing the result with (3.117), the contribution of earth losses to modal attenuation \( a_e^{(n)} \) is found to be,

\[
a_e^{(n)} = \sqrt{\frac{\mu_0 \rho_e \times N_1^{(n)}}{4\pi Z(n) h_1}} + \frac{N_2^{(n)}}{h_2} + \frac{N_3^{(n)}}{h_3}
\]  

- 3.118

Next, the contribution of phase conductor resistivities is found from the Poynting vector at the surface of conductors with specific resistivity \( \rho_c \) (14) and the resulting attenuation is

\[
a_c^{(n)} = \frac{\sqrt{\pi f \mu_0 \rho_c}}{4\pi r_c b Z(n)}
\]  

- 3.119

where \( r_c \) is conductor radius in a bundle of 'b' conductors.

Similarly, the attenuation due to sky wires is

\[
a_s^{(n)} = \frac{\sqrt{\pi f \mu_s}}{4\pi r_s Z(n)} \mathcal{Z}(i_s^{(n)})^2
\]  

- 3.120

In (3.120), \( r_s \) is the radius of the sky wire conductor, \( \mu \) is the permeability and \( \rho_s \) is the specific resistivity. \( i_s^{(n)} \) is the current induced in the sky wires (3.31) and (3.32) for normalized modal current components in the phase conductors. In a horizontal line the absolute value of each sky wire current, \( |i_s^{(n)}| \), is the same for one mode, and the number of sky wires is one or two. It should be noted that \( |i_s^{(n)}| << 1.0 \) and, hence, the effect of \( i_s^{(n)} \) on the magnetic field at the phase conductor and earth surface is neglected in (3.119) and (3.118) respectively.

The total modal attenuation is the sum of the three separate attenuations in (3.118), (3.119) and (3.120) and is
\[ \alpha(n) = \alpha_e(n) + \alpha_c(n) + \alpha_s(n) \] - 3.121

Examination of (3.118) to (3.121) reveals that for any power line the total attenuation is proportional to the square root of frequency,

\[ \alpha(n) \propto \sqrt{f} \] - 3.122

The effect of earth resistivity \( \rho_e \), is affecting only \( \alpha_e(n) \) and it can be stated that in general \( \alpha_e(n) \) increases less than \( \sqrt{\rho_e} \).

The Adams and Barthold method for computing attenuation is based on lossless line modal analysis and all losses calculated from the Poynting vector. Hence, the modal impedances and modal current components are all real, and mutually orthogonal. The approximating of the nearly lossless system seems to be justified for low values of \( \rho_e \) and relatively high frequencies.

The next two sections deal with explaining the methods of transverse RI magnetic field profile calculations derived by Cline and Gary, and Moreau and Gary. The modal analysis described in Chapter 3 is utilized by both of these methods. Each is explained with the help of an example. These examples are from applying each method to the French Chesnoy-Houston, 220 kV, three phase horizontal transmission line. The line geometry is illustrated in figure 9.1. A flow chart and some representative modal fields are also shown.

Finally, numerical computations are conducted and profiles of the two methods compared for frequencies of 1 MHz and 500 kHz, earth resistivities of 20, 200, and 2,000 Ohm-meters, and distances from the center conductor ranging from zero to 300 meters.

4.3 TWO METHODS OF DERIVING TRANSVERSAL FIELD PROFILES:

Both methods of calculating the horizontal components of the
CHAPTER 4

TRANSVERSAL RI FIELDS PROFILES USING IMAGE PRINCIPLES

The work in this chapter will concentrate on describing two methods of calculating transversal RI magnetic field profiles. The first section deals with the fields derived using Ampere’s Circuitual Law applied to each conductor current and its image; similar to (3.107) to (3.110). The calculations are made only of the field component in the x-direction and at ground level since measurements are usually made at this location. The method used for such calculations is to find the modal field component at any point at ground level due to each modal current component on each conductor of a long line. These fields are then added vectorially via the superposition theorem to yield a modal field.

The next two sections deal with explaining the methods of transverse RI magnetic field profile calculations derived by Clade and Gary, and Moreau and Gary. The modal analysis described in Chapter 3 is utilized by both of these methods. Each is explained with the help of an example. These examples are from applying each method to the French Chesnoy-Rousson, 220 kV, three phase horizontal transmission line; the line geometry is illustrated in figure 4.1. A flow chart and some representative modal fields are also shown.

Finally, numerical computations are conducted and profiles of the two methods compared for frequencies of 1MHz and 500kHz, earth resistivities of 20, 200, and 2,000 Ohm-meters, and distances from the centre conductor ranging from zero to 300 meters.

4.1 TWO METHODS OF DERIVING TRANSVERSAL FIELD PROFILES

Both methods of calculating the horizontal components of the
transversal RI magnetic field profiles described in sections 4.2 and 4.3 use an image principle. The following subsections will be devoted to explaining the difference in these two versions of the image principle and their application in finding the modal fields.

4.1.1 TRANSVERSAL FIELD PROFILES - PERFECT IMAGE

The application of the perfect image principle in the calculation of transverse magnetic field is shown in figure 4.2. Here, a mode 1 set of currents, $I_1^{(1)}$, $I_2^{(1)}$ and $I_3^{(1)}$ is flowing in the conductors. The following equations illustrate the calculation of the transverse modal field at any lateral distance, point A, due to the mode 1 current, $I_1^{(1)}$, in conductor 1, figure 4.2.

Using the perfect image principle, the modal field at any lateral distance $x_1$, point A, due to a mode 1 current in conductor
with reference to figure 4.2 is

\[ H_{x}^{\text{icon}}, H_{y}^{\text{con}} = \begin{cases} H_{x}^{\text{icon}}, H_{y}^{\text{con}} & \text{due to image current} \\ H_{x}^{\text{lim}}, H_{y}^{\text{lim}} & \text{due to conductor current} \end{cases} \]

Image Conductors

As can be seen the symmetrical location of the image about the earth's surface causes the \( \hat{e}_{x} \) and \( \hat{e}_{y} \) induction fields to cancel. The horizontal magnetic field is orientated in the \( \hat{e}_{x} \) direction.

The polyphase network proposed to arrive at (4.1) is again used for calculating the transverse magnetic fields due to the conductor components of conductors 1, 2, and 3. These fields are calculated at the points A, B, C.

The three horizontal fields \( H_{x}^{\text{icon}}, H_{x}^{\text{con}}, \) and \( H_{x}^{\text{lim}} \) are all in the same direction, \( \hat{e}_{x} \) and therefore, can be vectorially summed to yield the total horizontal field,

\[ H_{x} = \begin{cases} H_{x}^{\text{icon}}, H_{x}^{\text{con}} & \text{due to conductor current} \\ H_{x}^{\text{lim}} & \text{due to image current} \end{cases} \]

A point of measurement

Fig. 4.2 - Magnetic Field Components due to Mode 1 Current on Conductor 1 Using Perfect Image Principle
1 with reference to figure 4.2 is

\[ H_1^{(1)} = H_1^{(1)} \hat{a}_x + H_1^{(1)} \hat{a}_y \]

\[ = (h_{1\text{con}} + H_1^{(1)}) \hat{a}_x + (y_{1\text{con}} + H_1^{(1)}) \hat{a}_y \]

\[ = (\frac{I_1^{(1)}}{2\pi r_1} \frac{1}{r_1^2} \hat{a}_x + \frac{I_1^{(1)}}{2\pi r_1} \frac{x}{r_1^2} \hat{a}_y + \frac{I_1^{(1)}}{2\pi r_1} \frac{h_1}{r_1} \hat{a}_x + \frac{I_1^{(1)}}{2\pi r_1} \frac{h_1}{r_1} \hat{a}_y) \]

\[ = (\frac{I_1^{(1)}}{2\pi r_1} \frac{1}{r_1^2} \hat{a}_x + \frac{I_1^{(1)}}{2\pi r_1} \frac{x}{r_1^2} \hat{a}_y) \]

\[ = (\frac{I_1^{(1)}}{\pi} \frac{h_1}{r_1^2} \hat{a}_x + \frac{I_1^{(1)}}{\pi} \frac{h_1}{r_1^2} \hat{a}_y) \]

where \( \hat{a}_x \) and \( \hat{a}_y \) are the unit vectors in the \( x \) and \( y \) directions respectively. As can be seen the symmetrical location of the image about the earth's surface causes the \( \hat{a}_y \) direction fields to cancel. The total magnetic field therefore, is oriented in the \( \hat{a}_x \) direction.

The method proposed to arrive at (4.1) is again used for calculating the transverse magnetic fields due to mode 1 components of conductors 2 and 3. These fields, calculated at the same point \( A \), are

\[ H_2^{(1)} = \frac{I_2^{(1)}}{\pi (x_2^2 + h_2^2)} \hat{a}_x \]  

\[ H_3^{(1)} = \frac{I_3^{(1)}}{\pi (x_3^2 + h_3^2)} \hat{a}_x \]

The three horizontal fields \( H_1^{(1)} \), \( H_2^{(1)} \) and \( H_3^{(1)} \) are all in the same direction, \( \hat{a}_x \), and therefore, can be vectorially summed to yield the modal field thus,
\[ H^{(1)} = I^{(1)}_1 h_1 + I^{(1)}_2 h_2 + I^{(1)}_3 h_3 \]
\[ = \frac{I^{(1)}_1 h_1}{\pi(x_1^2 + h_1^2)} + \frac{I^{(1)}_2 h_2}{\pi(x_2^2 + h_2^2)} + \frac{I^{(1)}_3 h_3}{\pi(x_3^2 + h_3^2)} \]
\[ = \left( \frac{I^{(1)}_1 h_1}{\pi(x_1^2 + h_1^2)} + \frac{I^{(1)}_2 h_2}{\pi(x_2^2 + h_2^2)} + \frac{I^{(1)}_3 h_3}{\pi(x_3^2 + h_3^2)} \right) \hat{a}_x \quad - \quad 4.4 \]

Expressions for mode 2 and 3 magnetic fields are arrived in the same manner, and they are,

\[ H^{(2)} = \left( \frac{I^{(2)}_1 h_1}{\pi(x_1^2 + h_1^2)} + \frac{I^{(2)}_2 h_2}{\pi(x_2^2 + h_2^2)} + \frac{I^{(2)}_3 h_3}{\pi(x_3^2 + h_3^2)} \right) \hat{a}_x \quad - \quad 4.5 \]

\[ H^{(3)} = \left( \frac{I^{(3)}_1 h_1}{\pi(x_1^2 + h_1^2)} + \frac{I^{(3)}_2 h_2}{\pi(x_2^2 + h_2^2)} + \frac{I^{(3)}_3 h_3}{\pi(x_3^2 + h_3^2)} \right) \hat{a}_x \quad - \quad 4.6 \]

4.1.2 TRANSVERSAL FIELD PROFILES - OFFSET IMAGE

The application of the offset image principle in the calculation of transverse magnetic field is shown in figure 4.3. The image in this case is located at a depth of \( h+2\delta \) below the true ground surface, where \( h \) is the height of the conductor above the true ground surface and \( \delta \) is the depth of penetration. As in the perfect image case, a set of mode 1 currents, \( I^{(1)}_1, I^{(1)}_2 \) and \( I^{(1)}_3 \) is flowing in the conductors. The following equations illustrate the calculation of the transverse magnetic field at any lateral distance \( x_1 \), point A, due to mode 1 current, \( I^{(1)}_1 \), in conductor 1, figure 4.3.

When using the 'offset' image the modal field at any lateral ground distance \( x_1 \), due to a mode 1 current on conductor 1, with reference to figure 4.3, is
Fig. 4.3 - Magnetic Field Components due to Mode 1 Current on Conductor 1 Using Offset Image Principle
\[ H_1^{(1)} = H_1^{(1)} \hat{a}_x + H_1^{(1)} \hat{a}_y \]
\[ = \left( \frac{I(1)}{2\pi \rho_1} \frac{1}{r_1} \right) \hat{a}_x + \left( \frac{I(1)}{2\pi \rho_1} \frac{x_1}{r_1} \right) \hat{a}_y \]
\[ = \frac{I(1)}{2\pi} \left( \frac{h_1}{x_1^2 + h_1^2} + \frac{h_1 + 2\delta}{x_1^2 + (h_1 + 2\delta)^2} \right) \hat{a}_x + \frac{I(1)}{2\pi} \left( \frac{x_1}{x_1^2 + h_1^2} - \frac{x_1}{x_1^2 + (h_1 + 2\delta)^2} \right) \hat{a}_y \] - 4.7

\[ \delta \] is the depth of penetration given by
\[ \delta = \sqrt{\frac{\rho}{\pi f}} \] - 4.8

\[ \rho \] is the resistivity of earth, \( f \) is the frequency and \( \mu \) is the earth permeability. Inspection of (4.7) shows that the field in the \( \hat{a}_y \) direction is now non-zero. Therefore, the total field will not be oriented along the \( \hat{a}_x \) only, but rather it will be inclined to it, that is, in the x-y plane. However, the RI magnetic fields are usually measured with a loop antenna which is oriented so that only the horizontal component is detected. Therefore, (4.7) becomes
\[ H_1^{(1)} = \frac{I(1)}{2\pi} \left( \frac{h_1}{x_1^2 + h_1^2} + \frac{h_1 + 2\delta}{x_1^2 + (h_1 + 2\delta)^2} \right) \hat{a}_x \] - 4.9

Similar expressions may be obtained for the fields due to mode 1 currents on the other two conductors and at the same point of measurement. They are
\[ H_2^{(1)} = \frac{I(1)}{2\pi} \left( \frac{h_2}{x_2^2 + h_2^2} + \frac{h_2 + 2\delta}{x_2^2 + (h_2 + 2\delta)^2} \right) \hat{a}_x \] - 4.10
\[ H_3^{(1)} = \frac{1}{3} \left( \frac{h_3}{x_3^2 + h_3^2} + \frac{h_3 + 2\delta}{x_3^2 + (h_3 + 2\delta)^2} \right) \hat{a}_x \] - 4.11

As in the case when the perfect image is used, the fields \( H_1^{(1)} \), \( H_2^{(1)} \) and \( H_3^{(1)} \) are all in the same direction and therefore, using the superposition principle the mode 1 field in this case is

\[ H^{(1)} = H_1^{(1)} + H_2^{(1)} + H_3^{(1)} \]

\[ I_1^{(1)} = \frac{1}{2\pi} \left( \frac{h_1}{x_1^2 + h_1^2} + \frac{h_1 + 2\delta}{x_1^2 + (h_1 + 2\delta)^2} \right) \hat{a}_x \]

\[ I_2^{(1)} = \frac{1}{2\pi} \left( \frac{h_2}{x_2^2 + h_2^2} + \frac{h_2 + 2\delta}{x_2^2 + (h_2 + 2\delta)^2} \right) \hat{a}_x \]

\[ I_3^{(1)} = \frac{1}{2\pi} \left( \frac{h_3}{x_3^2 + h_3^2} + \frac{h_3 + 2\delta}{x_3^2 + (h_3 + 2\delta)^2} \right) \hat{a}_x \] - 4.12

Once again, the method used to arrive at (4.12) may be used to find the mode 2 and 3 fields thus

\[ H_2^{(2)} = \frac{1}{2\pi} \left( \frac{I_1^{(2)} h_1}{x_1^2 + h_1^2} + \frac{I_2^{(2)} h_2}{x_2^2 + h_2^2} + \frac{I_3^{(2)} h_3}{x_3^2 + h_3^2} + \frac{I_1^{(2)} (h_1 + 2\delta)}{x_1^2 + (h_1 + 2\delta)^2} \right) \hat{a}_x \]

\[ I_2^{(2)} = \frac{1}{2\pi} \left( \frac{I_2^{(2)} (h_2 + 2\delta)}{x_2^2 + (h_2 + 2\delta)^2} + \frac{I_3^{(2)} (h_3 + 2\delta)}{x_3^2 + (h_3 + 2\delta)^2} \right) \hat{a}_x \] - 4.13
\[ H(3) = \frac{1}{2\pi} \left( \frac{I_1^{(3)} h_1}{x_1^2 + h_1^2} + \frac{I_2^{(3)} h_2}{x_2^2 + h_2^2} + \frac{I_3^{(3)} h_3}{x_3^2 + h_3^2} + \frac{I_1^{(3)} (h_1 + 2\delta)}{x_1^2 + (h_1 + 2\delta)^2} \right) \]

(4.4), (4.5), (4.6), (4.12), (4.13) and (4.14) are the basic equations used in the methods of RI profile predetermination which follow.

4.2 FIELD PROFILE CALCULATIONS - CLADE AND GARY METHOD, METHOD A

In 1966 Clade and Gary (15) published a method of RI field calculation that will now be discussed. (This method will hereafter be designated as Method A). In this method, the modal currents are derived from eigenvalue analysis of the corresponding lossless system. However, in order to find the cumulative effect of the random injected modal currents, the modal attenuations are necessary. These are calculated using the simplified method derived by Adams and Barthold (6) (Chapter 3, section 5). Transversal modal magnetic fields are calculated using the perfect image principle (4.4, 4.5 and 4.6) and total fields obtained by assuming the modes to be uncorrelated.

4.2.1 BASIC STEPS

The basic steps involved in the calculation of the RI field are as follows.

Step 1. - Calculate geometry matrix \( \{G\} \), (2.8), (2.9) and (2.10) and the characteristic impedance matrix, \( \{Z\} \) (3.24)

Step 2. - Calculate the normalized current modal matrix, \( \{N\} \) using the method outlined in (3.64) to (3.96)

Step 3. - Calculate the injected currents from a unit length of one
conductor in corona making use of the capacitance coefficient matrix \( \{C\} \), (2.3), (2.11) and (3.20), or (2.12)

Step 4. - Calculate the modal attenuations using the approach of Adams and Barthold (6) (3.118), (3.119) and (3.121)

Step 5. - Calculate the modal currents in the centre of a long line, due to all elementary sections of the conductor in corona (2.18), (3.60), (3.61) and (3.90) to (3.97).

Step 6. - Use Ampere's Circuital Law, the perfect image principle and the superposition principle to obtain the modal fields (4.4), (4.5) and (4.6).

Step 7. - Combine the modal fields to give what is called a monophase field.

Step 8. - The monophase fields are due to only one of the conductors in corona. Similarly, monophase fields due to other conductors in corona are calculated. These fields are combined in a manner depending on the meter response to yield the total measured three phase field.

Figure 4.4 shows a flow chart of the above steps as applied to a computer program.

4.2.2 EXPLANATION OF STEPS

The following is a description of Method A applied to the Chesnoy-Rousson three phase transmission line shown in figure 4.1.

First, the geometry matrix \( \{G\} \) and the characteristic impedance matrix \( \{Z\} \) are calculated using (3.24). The entries of the \( \{Z\} \) matrix are real and

\[
\{Z\} = 6\{G\}
\]
Fig. 4.4 - Flow Chart of Method A
\[
\begin{bmatrix}
7.76 & 1.31 & 0.71 \\
1.31 & 7.76 & 1.31 \\
0.71 & 1.31 & 7.76
\end{bmatrix}
\]

= \begin{bmatrix}
466 & 78.6 & 42.6 \\
78.6 & 466 & 78.6 \\
42.6 & 78.6 & 466
\end{bmatrix}

The columns of the current modal matrix are the eigenvectors of the matrix \( \{Z\} \). To obtain these eigenvectors, an eigenvalue analysis has to be performed on the matrix \( \{Z\} \), (3.64) to (3.96). These eigenvalues of \( \{Z\} \) are the modal characteristic impedances and their values for the Chesnoy-Rousson line are

\[
\begin{align*}
Z^{(1)} &= 374 \text{ Ohm} \\
Z^{(2)} &= 423 \text{ Ohm} \\
Z^{(3)} &= 600 \text{ Ohm}
\end{align*}
\]

and the normalized current modal matrix in this case is

\[
\begin{bmatrix}
0.45 & 0.71 & 0.55 \\
-0.77 & 0 & 0.64 \\
0.45 & -0.71 & 0.55
\end{bmatrix}
\]

This completes step 2.

In step 3, the injected currents due to a unit length of one phase conductor of the line in corona is given by (2.3) and is

\[
\{I\} = \frac{1}{4\pi\varepsilon_0} (G\{\Gamma\})
\]

\[
= \frac{1}{2} (G^{-1}\{\Gamma\})
\]

\[
= \frac{1}{2} \begin{bmatrix}
0.133 & -0.021 & -0.0087 \\
-0.021 & 0.136 & -0.021 \\
-0.0087 & -0.021 & 0.133
\end{bmatrix}
\]
This concludes step 4.

for the case of the conductor number
1 (outer conductor) being in corona. It should be noted that the
matrix of excitation functions of order 3x1, is so arranged that

\[
\{\Gamma\} = \begin{bmatrix}
\Gamma_1 \\
0 \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}\]

If, for example, the second (centre) conductor is considered to be
under corona then the \{\Gamma\} matrix would be

\[
\{\Gamma\} = \begin{bmatrix}
0 \\
\Gamma_2 \\
0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}\]

As pointed out earlier the fields in this thesis are calculated for
\(\Gamma=1\mu\text{Amp/meter}^{1/2}\).

To calculate the equivalent injected modal currents due to all the
uncorrelated sources along the phase conductor, knowledge of the modal
attenuations is necessary. The method for the derivation of modal
attenuations, \(\alpha^{(n)}\), is outlined in chapter 3, section 5. For a
frequency of 500kHz and earth resistivity of 100 Ohm-meters the modal
attenuations for the line in figure 4.1 are

\[
\alpha^{(1)}=0.00000785 \text{ Np/meter}
\]
\[ a^{(2)} = 0.0000443 \text{ Np/meter} \quad - 4.24 \]

\[ a^{(3)} = 0.000332 \text{ Np/meter} \quad - 4.25 \]

This concludes step 4.

In order to find the total modal currents, step 5, at the point of observation due to all elementary lengths of one conductor in corona, the following operations, explained in chapter 3, are performed. First the phase current matrix is premultiplied by the inverse modal matrix thus

\[ \{I_e\} = (N)^{-1}\{I\} \quad - 4.26 \]

where \((N)^{-1} = (N_t)\) in this lossless case and the entries of \(\{I_e\}\) are shown in (3.92) to (3.94), and

\[ \{I_{tot}\} = \{1/\sqrt{\alpha}\}\{I_e\} \quad - 4.27 \]

where \(\{1/\sqrt{\alpha}\}\) is a diagonal matrix whose entries are made up as follows;

\[
\{1/\sqrt{\alpha}\} = \begin{bmatrix}
\frac{1}{\sqrt{\alpha}(1)} & 0 & 0 \\
0 & \frac{1}{\sqrt{\alpha}(2)} & 0 \\
0 & 0 & \frac{1}{\sqrt{\alpha}(3)}
\end{bmatrix} \quad - 4.28
\]

The matrix \(\{I\}\) whose entries are the modal components of each conductor is

\[ \{I\} = (N)\{I_{tot}\}_{\text{diag}} \quad - 4.29 \]

where \(\{I_{tot}\}_{\text{diag}}\) is matrix \(\{I_{tot}\}\) arranged in diagonal form.

For the Chesnoy-Rousson line the numerical values are
\{I_e\} = (N)^{-1} \{I\} \\
\rightarrow (N_t) \{I\} \\
\begin{bmatrix}
0.45 & -0.77 & 0.45 \\
0.71 & 0 & -0.71 \\
0.55 & 0.64 & 0.55
\end{bmatrix}
\begin{bmatrix}
0.0667 \\
-0.0105 \\
-0.0044
\end{bmatrix} \\
\rightarrow 0.0361 \\
0.0505 \\
0.0275 \\
\{I_{tot}\} = (1/\sqrt{\pi}) \{I_e\} \\
\begin{bmatrix}
357 & 0 & 0 \\
0 & 150 & 0 \\
0 & 0 & 55
\end{bmatrix}
\begin{bmatrix}
0.0361 \\
0.0505 \\
0.0275
\end{bmatrix} \\
= \begin{bmatrix}
12.90 \\
7.58 \\
1.51
\end{bmatrix} \\
\{I\} = (N) \{I_{tot}^{\text{diag}}\} \\
\begin{bmatrix}
0.45 & 0.71 & 0.55 \\
-0.77 & 0 & 0.64 \\
0.45 & -0.71 & 0.55
\end{bmatrix}
\begin{bmatrix}
12.90 & 0 & 0 \\
0 & 7.58 & 0 \\
0 & 0 & 1.51
\end{bmatrix} \\
= \begin{bmatrix}
5.81 & 5.38 & 0.83 \\
-9.93 & 0 & 0.97 \\
5.81 & -5.38 & 0.83
\end{bmatrix} \\
\rightarrow 4.32 \\
\begin{aligned}
\text{The modal fields, step 6, due to the modal currents described in} \\
(4.32) \text{are found using (4.4), (4.5) and (4.6).}
\end{aligned} \\
\begin{aligned}
The modal fields must now be combined into a total monophasic \\
field, (that is the total field due to one conductor in corona). This \\
is performed using quadratic addition, which is the method of combining 
uncorrelated stationary random quantities (15). That is, the modes are
\end{aligned}
assumed to be uncorrelated. Therefore, the monophase field in step 7 is

\[ H = \sqrt{\sum_{n} (H(n))^2} \]  

- 4.33

where \( H \) is the monophase field. The units of \( H \) are \( \mu \text{amp/meter} \). The \( H \) field can be expressed in dB by using the simple relation,

\[ H(\text{dB}) = 20 \log \frac{H}{1 \mu\text{Amp/meter}} \]  

- 4.34

Having converted the \( H \) field to dB it is now possible to add a term which takes into account the actual excitation function rather than the assumed value of \( 1 \mu\text{amp/meter} \). That is

\[ H = 20 \log \frac{H}{1 \mu\text{Amp/meter}} + 20 \log \frac{\Gamma}{1 \mu\text{Amp/meter}^{1/2}} \]  

- 4.35

Figures 4.5 and 4.6 show a sample of the modal fields obtained by using Method A.

4.2.3 RI FIELD PROFILES FOR ALL PHASE CONDUCTORS IN CORONA

The field in (4.35) is due to only one conductor under corona. The total field due to all conductors being under corona depends on the response of the measuring instrument. For example in the case of a normal three phase 60Hz transmission line, the voltage peaks of each phase are shifted by \( 120^\circ \) or 5.5 milliseconds from one another. This means that as long as the voltage of one phase conductor is above the corona inception voltage for a short time interval near the peak, the RI line currents due to this conductor being under corona will die away before RI currents due to the next phase conductor under corona begin to flow. The response time of measuring instruments is much larger
Fig. 4.5 - Graph of Transverse Modal Magnetic Field Intensity Versus Distance for Centre Phase Excitation Using Method A. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
Fig. 4.6 - Graph of Transverse Modal Magnetic Field Intensity Versus Distance for Outer Phase Excitation Using Method A. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
than 5.5 milliseconds. Usually the RI field measuring instrument has the quasi-peak response. A method of calculating the decibel level of all the components has to be found. In step 8 the CISPR procedure is used and it is described as follows.

Consider a three phase system whose three fields due to $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ are, say 10, 12 and 11dB above an arbitrary reference field value. Beginning with the first phase field, its value is compared to the other two fields. If it is less than the other two fields then 3dB is added to it, that is the first field is now 13dB. However, if only one of the other fields is larger then 1dB would be added, and if neither of the other fields are greater, then the 10dB value remains unchanged. The same procedure is followed again using the 12dB and finally the 11dB field, making sure that only the original 10, 12 and 11dB field values are used when comparisons are made and not the adjusted one. After all the adjustments are made, the highest adjusted dB value, in this case 13dB, is considered to be the measured RI field for this transmission line.

4.3 FIELD PROFILE CALCULATIONS - MOREAU AND GARY METHOD, METHOD B

In 1972 Moreau and Gary published a method (15) (hereafter known as Method B) in which the lossy transmission line is modelled as a completely transposed line. The effect of the lossy earth is taken into account by offsetting the image as explained in section 4.1. On doing so the perfect image principle is applied about a fictitious ground plane lowered from the actual ground plane by a distance equal to the depth of penetration. The modal attenuation values are necessary in order to find the total modal injected currents at the point of observation due to all the random corona sources. In Method
B, the attenuation values are available from experimental data compiled by Moreau and Gary (15) from measurements. These experiments showed that modal attenuations for a given frequency and earth resistivity differ very little, especially if all transmission lines are classified into a horizontal and a triangular configuration.

4.3.1 BASIC STEPS

A brief step by step outline of this method follows.

Step 1. - Since this method assumes the line to be completely transposed the normalized current modal matrix is readily available in the form of the Clarke's components matrix (3.59).

Step 2. - Calculate the injected currents from unit length of one conductor in corona (2.3).

Step 3. - Calculate the modal currents at the point of observation due to all elementary sections of the conductor in corona. This involves the use of the modal attenuations.

Step 4. - Use Ampere's Circuital Law, the 'offset' image principle and the superposition principle to obtain the modal horizontal fields (4.12), (4.13) and (4.14).

Step 5. - Same as Step 7 and Step 8 in method A.

Figure 4.6 shows the flow chart used in programming the above steps in Method B.

4.3.2 EXPLANATION OF STEPS

The preceding outline of Method B will now be elaborated on with the use of numerical values obtained from the use of the Chesnoy-Rousson line shown in figure 4.1.

The normalized current modal matrix, Step 1, is the well known Clarke's matrix (3.59) and, therefore, need not be calculated. This
Fig. 4.7 - Flow Chart of Method B
affords considerable simplicity to this method.

The calculation of the modal currents, Step 2 and Step 3, proceeds in the same manner as described in (4.20), (4.21), (4.22), (4.26), (4.27), (4.28) and (4.29). It will be noted that modal attenuation values are needed in these calculations. Table 4.1 shows the values (15) of the modal attenuations for frequency of 500kHz and earth resistivity of 100 Ohm-meters. The values are given for both a triangular and horizontal configuration. If modal attenuations are required for other frequencies and earth resistivities then the equation

$$a(n)(f) = a(n)(f_0, \rho_0)(f) 0.8(\rho/\rho_0)^{0.5}$$

- 4.36

is used, where $f_0$ and $\rho_0$ are 500kHz and 100 Ohm-meters respectively.

It should be noted that the only difference between Methods A and B will be the values of the entries in the $\{1/\sqrt{\pi}\}$ and $\{N\}^{-1}$ matrices. Otherwise the calculations which will now be performed would yield the

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Attenuation in Nepers/meters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Triangular Configuration</td>
</tr>
<tr>
<td>1</td>
<td>$21.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>2</td>
<td>$21.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>3</td>
<td>$342 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4.1 - Modal Attenuations for a Three Phase Transmission Line with Frequency at 500kHz and Earth Resistivity at 100 Ohm-meters
same values as those in Method A. The injected current values for Method B with conductor 1 in corona are

\[
(I) = \frac{1}{4\pi e_0} (C)(\Gamma)
\]

\[
= \frac{1}{4\pi e_0} G^{-1} \Gamma
\]

In Step 4 the modal currents described in (4.40) are found using

\[
\begin{bmatrix}
0.133 & -0.021 & -0.0087 \\
-0.021 & 0.136 & -0.021 \\
-0.0087 & -0.021 & 0.133
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.0667 \\
-0.0105 \\
-0.0044
\end{bmatrix}
\]

and (4.35).

Figures 4.8 and 4.9 show examples of modal field profiles using Method B.

4.3.3 RI FIELD PROFILES FOR ALL PHASE CONDUCTORS IN CORONA

The total transmission of the three-phase field due to all conductors under corona is explained in Method A, or a later section of the book. Here the fields are first converted to dB above the reference value. Then if any monophase is 3dB above the remaining fields then this is considered the final field. If however the difference between any two monophase is less than 3dB then the procedure is that the total three-phase field is

\[
\{I_{tot}\} = \left\{\sqrt{3}\right\} \{I_e\}
\]

\[
\begin{bmatrix}
300 & 0 & 0 \\
0 & 136 & 0 \\
0 & 0 & 54
\end{bmatrix}
\]

\[
\begin{bmatrix}
10.2 \\
6.84 \\
1.62
\end{bmatrix}
\]

\[
(I) = (N)\{I_{tot}\}_{\text{diag}}
\]
\[
\begin{bmatrix}
0.408 & 0.707 & 0.577 \\
-0.816 & 0 & 0.577 \\
0.408 & -0.707 & 0.577
\end{bmatrix}
\begin{bmatrix}
10.2 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
-0.816 & 0 & 0.577 \\
0.408 & -0.707 & 0.577
\end{bmatrix}
\begin{bmatrix}
0 \\
6.84 \\
0
\end{bmatrix}
= \begin{bmatrix}
4.16 & 4.84 & 0.935 \\
-8.32 & 0 & 0.935 \\
4.16 & -4.84 & 0.935
\end{bmatrix}
= -4.40
\]

In Step 4 the modal fields due to the modal currents described in (4.40) are found using (4.12), (4.13) and (4.14).

The monophase fields in Step 5 are found using (4.33), (4.34) and (4.35):

Figures 4.8 and 4.9 show some examples of modal field profiles using Method B.

4.3.3 RI FIELD PROFILES FOR ALL PHASE CONDUCTORS IN CORONA

The total transmission line three-phase field due to all conductors under corona can be found using CISPR addition explained in Method A, or a later version of the CISPR addition explained in Moreau and Gary's publication (15). Here the fields are first converted to dB above a reference value. Then if any monophase field is 3dB above the remaining two fields then this is considered the final field. If however the difference between any two monophase is less than 3dB then the proposed equation for the total three-phase RI field is

\[
H_{\text{eff}} = \frac{H_1 + H_2}{2} + 1.5\text{dB}
\]

- 4.41

where \(H_1\) and \(H_2\) are the two monophase fields.

4.4 COMPARISON OF METHODS A AND B ON THE CHESNOY-ROUSSON POWER LINE

In the previous sections two methods of predetermining the
Fig 4.8 - Graph of Transverse Modal Magnetic Field Intensity Versus Distance for Centre Phase Excitation Using Method B. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters
Fig. 4.9 - Graph of Transverse Modal Magnetic Field Intensity Versus Distance for Outer Phase Excitation Using Method B. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters
transversal RI magnetic field profiles of a transmission line using the image principles were described. This section however, will compare the modal and monophase fields obtained by applying these methods to the Chesnoy-Rousson transmission line.

4.4.1 COMPARISON OF MODAL FIELDS

The first two graphs, figures 4.5 and 4.8 show the modal components of the RI transversal magnetic field intensity due to the centre phase conductor in corona, found using Method A and B respectively. These graphs are valid for a frequency of 500kHz and an earth resistivity of 100 Ohm-meters. It is seen that no mode 2 field exists. This can be explained by consideration of the modal currents in each conductor, calculated using modal analysis (3.64) to (3.97) for Method A and (4.44) to (4.63) for Method B. If the central phase is in corona the modal currents for a symmetrical line (like the Chesnoy-Rousson Line) would be in the form

\[
\begin{bmatrix}
-a & 0 & c \\
2ak_1 & 0 & ck_2 \\
-a & 0 & c \\
\end{bmatrix}.
\]

The centre column represents the mode 2 components and because they are all zero there will be no mode 2 fields. It can also be seen from figures 4.5 and 4.8 that the modal fields are symmetrical about the y axis.

Figures 4.6 and 4.9 show modal fields as in figures 4.5 and 4.8 except that the outer phase conductor (that is conductor number 1 of figure 4.1) is now in corona. The modal currents in this case are of the form
\[
\{I\} = \begin{bmatrix}
  a & -b & c \\
  -2ak_1 & 0 & ck_2 \\
  a & b & c
\end{bmatrix}, (k_1 \text{ and } k_2 = 1.0)
\]

In this case mode 2 current components exist and, therefore, mode 2 fields appear. They are, however, assymetrical since the mode 2 currents on the outer conductors are out of phase. (There is no mode 2 current on the centre conductor).

Though the graphs have been drawn using results obtained from the implementation of RI calculation methods on the Chesnoy-Rousson line, it should be stated that similar results would be obtained on any three phase horizontal line.

4.4.2 COMPARISONS OF MONOPHASE FIELDS

Having obtained the modal fields of each excitation phase, the phase fields are now obtained using the quadratic addition rule, (4.33). Figures 4.10, 4.11 and 4.12 show the results obtained from quadratic addition of the modal fields for centre and outer phase excitations, using Methods A and B, for earth resistivities of 20, 100, and 1,000 Ohm-meters and for a frequency of 1MHz.

When considering the effect of earth resistivity on RI profiles, it can be seen that the attenuation increases with an increase in earth resistivity, and thus a decrease in RI level should be observed for both methods: (4.27) and (4.39). For Method B, an increase in earth resistivity also increases the depth of penetration (4.8) and thus with the image currents being further away from the earth surface, the RI profiles should decrease. An examination of figures 4.10 to 4.12 reveals that the above mentioned behavior does indeed happen.

In arriving at the profiles for outer phase excitation in figures
Fig. 4.10 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1MHz and Earth Resistivity is 20 Ohm-meters.
Fig. 4.11 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1MHz and Earth Resistivity is 100 Ohm-meters.
Fig. 4.12 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1MHz and Earth Resistivity is 1,000 Ohm-meters
Fig. 4.13 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 500kHz and Earth Resistivity is 20 Ohm-meters
MONOPHASE FIELD IN DB (H)

Fig. 4.14 - Graph of Monophasic Magnetic Field Intensity Versus Lateral Distance. Frequency is 500 kHz and Earth Resistivity is 100 Ohm-meters.

LATERAL DISTANCE FROM CENTRE PHASE IN METERS (x)

A = Method A
B = Method B
0 = Outer Excitation Phase
I = Centre Excitation Phase
Fig. 4.15 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 500kHz and Earth Resistivity is 1,000 Ohm-meters.
Fig. 4.16 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance (Concentrating on Large Distances). Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
4.10 to 4.12, conductor number 1 (figure 4.1) was assumed to be in corona. One could also have selected conductor number 3 to be in corona when considering outer phase excitation. However, regardless of which outer conductor is chosen (number 1 or number 3 in figure 4.1) the phase field will be identical and the justification of this is as follows. The modal currents due to corona discharges on conductor number 3 are of the form

\[
\begin{bmatrix}
  a & b & c \\
  -2ak_1 & 0 & ck_2 \\
  a & -b & c
\end{bmatrix}
\]

\(k_1 \text{ and } k_2 = 1.0\)

As can be seen (4.43) and (4.44) are identical except for the polarity of the mode 2 currents (that is column 2 of (4.44)). This means that the mode 2 fields shown in figures 4.6 and 4.9 will be of the same magnitude but of opposite sign. However, when quadratic addition is performed, the squaring of the modal fields eliminates the need to consider their signs. Therefore, having identical magnitudes the phase fields due to either of the outer conductors in corona are identical.

Figures 4.13, 4.14 and 4.15 show the RI profiles for conditions identical to those in 4.10, 4.11 and 4.12 except that the frequency is 500kHz. The most noticeable difference in comparing the profiles corresponding to 1MHz and 500kHz is that the RI fields are higher in the 500kHz case because of the decrease in modal attenuation for both methods. It should be noted that according to (4.8) a decrease in frequency would yield an increase in depth of penetration which in turn should decrease the RI level. However, from the profiles it can be concluded that the decrease in modal attenuation has a greater
effect than does the increase in depth of penetration. Table 4.2 gives the modal attenuations that were used in the calculation of fields shown in the graphs. As can be seen the attenuation in all cases decrease with decreasing frequency and earth resistivity. Modal currents flowing in the conductors are inversely proportional to the square root of the corresponding attenuation, hence the fields bear a similar relationship to the attenuation at any lateral distance. This is consistent with conclusions drawn from figures 4.10 to 4.15.

Figure 4.16 shows the "far fields" profile for both centre (conductor 2) and outer (conductor 1) phase excitation for a frequency of 500kHz and earth resistivity of 100 Ohm-meters. A close inspection of figure 4.15 shows that the rate of change of field with respect to distance for distances greater than about thirty meters is practically independent of the method used and the phase conductor in corona. The same is true for other values of frequency and earth resistivities.

With regard to the differences in the two methods used, it can be seen from careful study of figures 4.10 to 4.15 that RI fields calculated using Method B are generally larger very close to and under the line, compared to using Method A. However for large distances from the line Method A results in higher fields. To explain this, the figures in table 4.2 are to be examined. In almost all cases the attenuations for Method B are larger than those in Method A. This would suggest that if the two methods are similar in every other respect then Method A should give higher fields than Method B. This is the case for large distances but not so for fields near the line.

With regard to ease of programming, there is no doubt that Method B is superior due to two reasons. First, Method B employs the well
<table>
<thead>
<tr>
<th>Earth Resistivity in Ohm-meters</th>
<th>Attenuation of Mode 1 in Np/meter</th>
<th>Attenuation of Mode 2 in Np/meter</th>
<th>Attenuation of Mode 3 in Np/meter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency of 1MHz</td>
<td>Method A</td>
<td>Frequency of 500kHz</td>
</tr>
<tr>
<td>20</td>
<td>7.95x10⁻⁶</td>
<td>Method A</td>
<td>11.1x10⁻⁶</td>
</tr>
<tr>
<td>100</td>
<td>11.1x10⁻⁶</td>
<td>Method A</td>
<td>19.3x10⁻⁶</td>
</tr>
<tr>
<td>1,000</td>
<td>23.5x10⁻⁶</td>
<td>Method A</td>
<td>61.1x10⁻⁶</td>
</tr>
</tbody>
</table>

Table 4.2 - Modal Attenuations for the Chesnoy-Rousson Line for Different Frequencies, Earth Resistivities and for Methods A and B
known Clarke's modal matrix, whereas Method A has to use modal analysis to compute its modal matrix. Also, the attenuation in Method B is known ab initio, whereas Method A has to calculate each attenuation. Due to these two points Method B involves less programming and less computer time to arrive at its results, a very important factor when many calculations have to be made.

The effect of the offset image in Method B complicates the comparison of results. The offset image tends to increase the H field under and near the line as compared with that of a perfect image. This effect will decrease with increasing lateral distances.

The two methods of predetermining transverse RI magnetic field just discussed were originally published in Europe but have gained acceptance in North America. Method B in particular is widely used by several utilities due to its simple computer programming. However, these methods have many simplifying assumptions. To assess the accuracy of results from Methods A and B a new method of RI field calculations will be considered. This method, described in the next chapter is as rigorous as the state of the art permits.

The series self and mutual impedances are defined in (3.17) for a lossless line. However, in calculating the characteristics of electromagnetic waves propagating along the conductors of a lossy transmission line, it is important to consider the effects of the resistance of earth on propagation. In the previous chapter.
CHAPTER 5

TRANSVERSAL RI FIELD PREDETERMINATION WITHOUT THE ASSUMPTION OF IMAGES

In the previous chapter two methods of RI field calculations were described, in both of which the artifice of the ideal or offset image is used. A method of RI field calculation will now be outlined which neither assumes near lossless conditions nor the existence of an image. The total magnetic field above the lossy ground is the vector sum of the field produced by the currents in the conductors, $H^0$, and the field due to the earth currents distributed throughout the soil, $H^1$. The method of calculating this total field above ground is also described in (19). It was found that the Electro-Magnetic field is elliptically polarized. In this thesis the total magnetic field at ground level in the horizontal direction only is considered. No image concept has been used in this method.

To estimate the RI field profiles the above method is utilized. In this thesis the method is simplified by assuming the modal fields in the centre of a very long line are random.

Since this method uses a lossy line approach the lossless modal analysis is no longer available. Modal analysis has to be performed on a lossy line and this is discussed in the following section.

5.1 LOSSY LINE MODAL ANALYSIS

5.1.1 CARSON CORRECTION FACTORS

The series self and mutual impedances are defined in (3.17) for a lossless line. However, in calculating the characteristics of electromagnetic waves propagating along the conductors of a lossy transmission line, it is important to consider the effects of the resistivity of earth on propagation. In the previous chapter,
\[ R_c = \text{conductor resistance per unit length} \]
\[ L_c = \text{conductor inductance per unit length} \]
\[ R_e = \text{resistance due to earth resistivity} \]
\[ L_e = \text{inductance due to earth resistivity} \]
\[ C = \text{shunt capacitance} \]

Fig. 5.1 - Equivalent Circuit for a Single-Conductor Transmission Line of Length \( dx \)

Simplified approaches were described that took into account the effect of lossy earth. In 1926, J. Carson \(^{(20)}\), published a paper in which he postulated that the effect of earth resistivity could be taken into account by additional series self and mutual impedances, \( R_e + jL_e \) as illustrated in figure 5.1. In this paper there were many inherent assumptions that were not delineated properly. Also the actual evaluation of the additional impedances, known as Carson's Correction Factors, is itself rather complicated. The final solution, which was in the form of infinite integrals, was attributed to a Mr. R.M. Foster, the details of which were never published.

In calculating the \( [p] \) matrix, the lossy line values of \( \{z\} \) and \( \{y\} \) must be used and they are \(^{(30)}\):
\( \{z\} = (R) + j\omega(L) + \{CCF\} \)

\[ \omega u_0 \frac{R}{Z} + j \frac{\omega}{2\pi} \{G\} + \{CCF\} - 5.1 \]

and

\( \{y\} = j\omega(C) \)

\[ = j\omega 2\pi \varepsilon_0 \{G\}^{-1} - 5.2 \]

where \(\{R\}\) is a diagonal matrix of conductor resistances, \(\{L\}\) is the lossless self and mutual inductance matrix, \(\{G\}\) is the geometry matrix, \(\{C\}\) is the capacitance coefficient matrix and \(\{CCF\}\) is a square matrix of Carson Correction Factors. The expressions used to evaluate \(\{CCF\}\) are given in Appendix II. It will be noted that the \(\{y\}\) matrix is the same for lossy line and lossless line; only the \(\{z\}\) matrix differs.

### 5.1.2 CALCULATION OF MODAL MATRIX

The resulting \(\{P\}\) or \(\{P_t\}\) matrix in the lossy case will be a non-symmetrical matrix with complex entries.

In order to solve (3.8) the eigenvalues of \(\{P_t\}\) must be obtained.

From (3.4), the general current matrix wave equation is

\[ \frac{d^2}{dx^2} \{I\} = \{P_t\} \{I\} - 5.3 \]

where for a symmetrical horizontal three-phase transmission line (5.3) is in the form

\[ \frac{d^2}{dx^2} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} A & D & C \\ B & E & B \\ C & D & A \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} - 5.4 \]

The entries of \(\{P_t\}\) are not the same as those of the characteristic impedance matrix in (3.25).
The solution of (5.3) for currents propagating in one direction of the line at distance $x$ is

$$ \{I(x)\} = e^{-x\{P_t\}^{1/2}} \{I(0)\} \quad - 5.5 $$

In order to express the exponential term in (5.5) in a diagonal form, the function of matrices identity is used thus (31),

$$ e^{-x\{P_t\}^{1/2}} = \{N\} \{e^{-x(\lambda(n))^{1/2}}\} \{N\}^{-1} \quad - 5.6 $$

where $\{N\}$ is the current modal transformation matrix, $\{\exp(-x(\lambda(n))^{1/2})\}$ is a diagonal matrix and $\lambda(n)$ are the eigenvalues of $\{P_t\}$.

There exists a set of modal currents associated with the unique complex propagation constants, $\gamma(n)$, defined as

$$ \gamma(n) = \sqrt{\lambda(n)} = \alpha(n) + j\beta(n) \quad - 5.7 $$

where $\alpha(n)$ is the modal attenuation and $\beta(n)$ is the modal phase constant. For the case of a three phase transmission line the three sets of modal current solutions are

$$ \begin{bmatrix} I_1^{(1)} \\ I_2^{(1)} \\ I_3^{(1)} \end{bmatrix} = e^{-\gamma^{(1)}x} \begin{bmatrix} I_1^{(1)}(0) \\ I_2^{(1)}(0) \\ I_3^{(1)}(0) \end{bmatrix} \quad - 5.8 $$

$$ \begin{bmatrix} I_1^{(2)} \\ I_2^{(2)} \\ I_3^{(2)} \end{bmatrix} = e^{-\gamma^{(2)}x} \begin{bmatrix} I_1^{(2)}(0) \\ I_2^{(2)}(0) \\ I_3^{(2)}(0) \end{bmatrix} \quad - 5.9 $$

$$ \begin{bmatrix} I_1^{(3)} \\ I_2^{(3)} \\ I_3^{(3)} \end{bmatrix} = e^{-\gamma^{(3)}x} \begin{bmatrix} I_1^{(3)}(0) \\ I_2^{(3)}(0) \\ I_3^{(3)}(0) \end{bmatrix} \quad - 5.10 $$
and \( I_k^{(n)}(0) \) is the current at \( x=0 \).

To find \( \lambda^{(n)} \), the eigenvalues of \( \{P_t \} \), the equation

\[
\begin{bmatrix}
I_1^{(n)} \\
I_2^{(n)} \\
I_3^{(n)}
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

is used. This equation is arrived at in a similar method to that of (3.68). To solve (5.11) then

\[
|\{P_t \}-\lambda^{(n)}\{1\}|=0
\]

- 5.12

In the case of a three conductor horizontal line, (5.12) will be

\[
\begin{vmatrix}
A-\lambda^{(n)} & D & C \\
B & E-\lambda^{(n)} & B \\
C & D & A-\lambda^{(n)}
\end{vmatrix}=0
\]

- 5.13

It should be noted the entries in (5.13) are complex values. The solution to (5.13) is (29)

\[
\lambda^{(1)}=\frac{1}{2}(A+C+E)-\frac{1}{2}H
\]

- 5.14

\[
\lambda^{(2)}=A-C
\]

- 5.15

\[
\lambda^{(3)}=\frac{1}{2}(A+C+E)+\frac{1}{2}H
\]

- 5.16

where

\[
H=\sqrt{(A-C-E)^2+8BD}
\]

- 5.17

Substitution of (5.14), (5.15) and (5.16) into (5.11) yields the following ratios,

\[
\frac{I_2^{(1)}}{I_2^{(1)}}=\frac{I_2^{(1)}}{I_2^{(1)}}=p_c
\]

- 5.18
\[ I_2^{(3)} I_2^{(3)} \]
\[ \frac{I_2^{(3)}}{I_1^{(3)}} = q_c \]

\[ 5.19 \]

\[ I_2^{(2)} \]
\[ \frac{I_2^{(1)}}{I_1^{(2)}} = 0 \]

\[ 5.20 \]

\[ I_3^{(2)} \]
\[ \frac{I_3^{(1)}}{I_1^{(2)}} = -1 \]

\[ 5.21 \]

where in this case

\[ p_c = \frac{2B}{\lambda^{(1)} - E} \]

\[ 5.22 \]

\[ q_c = \frac{2B}{\lambda^{(3)} - E} \]

\[ 5.23 \]

The current modal currents must sum to the actual conductor currents.

Therefore

\[ \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ I_2^{(1)} \\ I_3^{(1)} \end{bmatrix} + \begin{bmatrix} 1 \\ I_2^{(2)} \\ I_3^{(2)} \end{bmatrix} + \begin{bmatrix} 1 \\ I_2^{(3)} \\ I_3^{(3)} \end{bmatrix} \]

\[ = I_1^{(1)} \begin{bmatrix} 1 \\ I_2^{(1)} \\ I_3^{(1)} \end{bmatrix} + I_1^{(2)} \begin{bmatrix} 1 \\ I_2^{(2)} \\ I_3^{(2)} \end{bmatrix} + I_1^{(3)} \begin{bmatrix} 1 \\ I_2^{(3)} \\ I_3^{(3)} \end{bmatrix} \]

\[ = I_1^{(1)} \begin{bmatrix} 1 \\ p_c \\ 1 \end{bmatrix} + I_1^{(2)} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + I_1^{(3)} \begin{bmatrix} q_c \\ -1 \end{bmatrix} \]
\[
\begin{bmatrix}
I_1^{(1)} \\
I_1^{(2)} \\
I_1^{(3)} \\
I_1
\end{bmatrix}
= (N)
\begin{bmatrix}
I_1 \\
I_1 \\
I_1 \\
I_1
\end{bmatrix}
\]

\[
(N) \text{ is the current modal transformation matrix. To find the normalized modal matrix the length of each eigenvector is made unity, thus}
\]

\[
(N_n) = \begin{bmatrix}
\frac{1}{{q_c q_c^*}+2} & \frac{1}{{q_c q_c^*}+2} & \frac{1}{{q_c q_c^*}+2} \\
\frac{p}{{p_c p_c^*}+2} & 0 & \frac{q}{{q_c q_c^*}+2} \\
\frac{1}{{p_c p_c^*}+2} & -1 & \frac{1}{{q_c q_c^*}+2} \\
\frac{1}{{p_c p_c^*}+2} & 2 & \frac{1}{{q_c q_c^*}+2}
\end{bmatrix}
\]

It was shown in Chapter 3 that the voltage and current modal transformation matrices are the same in the lossless line case. However, it can be seen that in solving (3.4), matrix \( (P_t) \) is used in modal analysis calculations, and in the lossy line case \( (P) \) is not equal to \( (P_t) \). This then would yield a voltage modal transformation matrix, denoted by \( (M) \), whose entries differ from those of \( (N) \).

5.2 DESCRIPTION OF THE PERZ AND RAGHUVEER SIMPLIFIED METHOD FOR TRANSVERSAL RI FIELD CALCULATIONS

In 1974 Perz and Raghuveer published papers (18), (19), that dealt with the calculation of series self and mutual impedances of a lossy transmission line. Fourier Transforms were used to arrive at an end result identical to that of J. Carson. These papers also highlighted the assumptions inherent in the analysis. A numerical
technique was also suggested for the solution of the semi-infinite integrals which appear in the final expressions for the Carson Correction Factors.

In the same publication, expressions were derived for the magnetic fields in the air and in the ground due to the current flowing in the conductors and the distributed induced current flowing in the ground. These field equations are of importance in this work because they can be applied to RI currents flowing in the transmission line. Both the magnetic field equations and the Carson Correction Factors are expressed in the form of semi-infinite integrals (Appendix II). This method of RI field calculation will hereafter be denoted as Method C.

A point worth emphasizing is that the use of the semi-infinite integral expressions for the evaluation of the magnetic fields in the air does not involve the use of the image principle. Rather, the contribution to the magnetic field in the air by the ground current is given due consideration by taking into account its distributed nature. It is expected that the calculation of the fields in this manner results in the predetermination of RI profiles that are as accurate as the present state of the art permits.

5.2.1 BASIC STEPS

The following is an outline of Method C.

Step 1) Find the geometry matrix \( \{G\} \) (2.8) to (2.10).

Step 2) Find the series impedance and shunt admittance matrices \( \{z\} \) and \( \{y\} \) (5.1) and (5.2). Note that the \( \{z\} \) matrix contains Carson's Correction Factors (5.1) and Appendix II.

Step 3) Find the propagation matrix \( \{P\} \) and its transpose \( \{P_t\} \) (3.5) and (3.6).
Step 4) Through eigenvalue analysis of \( P_t \) the current modal transformation matrix and the modal attenuations are found (5.3) to (5.25).

Step 5) Calculate modal currents \( \{I\} \) (4.37) to (4.40).

Step 6) Calculate modal fields (Appendix II).

Step 7) Calculate phase fields using quadratic addition (4.33).

Step 8) Use CISPR addition to find the total three phase RI field.

Figure 5.2 shows a flow chart for Method C used in computer programming.

5.2.2 EXPLANATION OF STEPS

The above RI profile calculation steps are applied to the French Chesnoy-Rousson transmission line.

The geometry matrix whose entries depend on line geometry only, has been calculated for the case of the lossless line in chapter 4.

\[
\begin{bmatrix}
7.76 & 1.31 & 0.71 \\
1.31 & 7.76 & 1.31 \\
0.71 & 1.31 & 7.76 \\
\end{bmatrix}
\]

- 5.26

The second step involves calculating the \( \{z\} \) and \( \{y\} \) matrices (5.1) and (5.2). At a frequency of 500kHz and for earth resistivity of 100 Ohm-meters,

\[
\{z\} = \{R\} + j \frac{\omega \mu_0}{Z\pi} \{G\} + \{CCF\}
\]

\[
= \begin{bmatrix}
0.0000261 & 0 & 0 \\
0 & 0.0000261 & 0 \\
0 & 0 & 0.0000261 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
7.76 & 1.31 & 0.71 \\
1.31 & 7.76 & 1.31 \\
0.71 & 1.31 & 7.76 \\
\end{bmatrix}
\]

\[+ j0.628 \begin{bmatrix}
1.31 & 7.76 & 1.31 \\
0.71 & 1.31 & 7.76 \\
\end{bmatrix}\]
Fig. 5.2 - Flow Chart of Method C
and \[ \{y\} = j\omega 2\pi e_0 \{G\}^{-1} \]

\[
\begin{bmatrix}
0.133 & -0.021 & -0.0087 \\
-0.000197 & 0.136 & -0.021 \\
-0.0087 & -0.021 & 0.133 \\
-0.0000262 & -0.0000041 & -0.0000017 \\
-0.0000041 & 0.0000268 & -0.0000017 \\
-0.0000017 & -0.0000041 & 0.0000262
\end{bmatrix}
\]

Having obtained the values of \(\{z\}\) and \(\{y\}\), the third step is to find the matrix \(\{P\}\) using (3.5) thus

\[ \{P\} = \{z\}\{y\} \]

\[
\begin{bmatrix}
0.115 & 0.110 & 0.0966 \\
0.110 & 0.115 & 0.110 \\
0.0966 & 0.110 & 0.115
\end{bmatrix}
\frac{5.01}{j} \begin{bmatrix} 0.955 & 0.555 \end{bmatrix}
\]

\[
\begin{bmatrix}
0.0000262 & -0.0000041 & -0.0000017 \\
-0.0000041 & 0.0000268 & -0.0000017 \\
-0.0000017 & -0.0000041 & 0.0000262
\end{bmatrix}
\]

\[ xj \]

\[
\begin{bmatrix}
-1.12 + j0.02 & -0.0278 + j0.0208 & -0.0208 + j0.024 \\
-0.0285 + j0.0222 & -1.14 + j0.0125 & -0.0284 + j0.0222 \\
-0.0206 + j0.024 & -0.0278 + j0.0208 & -1.12 + j0.02
\end{bmatrix}
\times 10^{-4}
\]

The transpose of \(\{P\}\) is

\[ \{P_t\} = \]

\[
\begin{bmatrix}
-1.12 + j0.02 & -0.0285 + j0.0222 & -0.0206 + j0.024 \\
-0.0278 + j0.0208 & -1.14 + j0.0125 & -0.0278 + j0.0208 \\
-0.0208 + j0.024 & -0.0284 + j0.0222 & -1.12 + j0.02
\end{bmatrix}
\times 10^{-4}
\]

As can be seen the entries are complex.
To calculate the current modal transformation matrix the eigenvalues of \( \{ P_t \} \) are found from (5.14)* to (5.17),

\[
\lambda^{(1)} = -0.00011 - j0.000000096 \\
\lambda^{(2)} = -0.00011 - j0.0000044 \\
\lambda^{(3)} = -0.00017 - j0.0000084
\]

The modal matrix given in (5.24) would then be

\[
\{ N \} = \begin{bmatrix}
1 & 1 & 1 \\
-2.06+j0.461 & 0 & 1.08-j0.183 \\
1 & 1 & 1
\end{bmatrix}
\]

and the normalized current modal transformation matrix is

\[
\{ N_n \} = \begin{bmatrix}
0.4 & 0.707 & 0.559 \\
-0.824+j0.184 & 0 & 0.604-j0.102 \\
0.4 & -0.707 & 0.559
\end{bmatrix}
\]

The modal propagation constants, (5.7), are

\[
\gamma^{(1)} = 0.00000472 - j0.0105 \\
\gamma^{(2)} = 0.0000252 - j0.0105 \\
\gamma^{(3)} = 0.000266 - j0.0108
\]

This completes step 4.

Step 5 is identical to that followed in (4.37) to (4.40), except that the values of the modal currents are complex. Therefore, for outer phase excitation:

\[
\{ I \} = \frac{1}{4\pi\varepsilon_0} \{ C \} \{ \Gamma \}
\]

\[
= i\varepsilon \{ G \}^{-1} \{ \Gamma \}
\]
\[
\begin{bmatrix}
0.133 & -0.021 & -0.0087 \\
-0.021 & 0.136 & -0.021 \\
-0.0087 & -0.021 & 0.133
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

\[
= \frac{1}{2}
\begin{bmatrix}
0.0667 \\
-0.0105 \\
-0.0044
\end{bmatrix}
\]

\[
\{I_e\} = (N)^{-1}\{I\}
\]

\[
\begin{bmatrix}
0.4 & -0.824-j0.184 & 0.4 \\
0.707 & 0 & -0.707 \\
0.559 & 0.604+j0.102 & 0.559
\end{bmatrix}
\begin{bmatrix}
0.0667 \\
-0.0105 \\
-0.0044
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.00336+j0.0014 \\
0.0503 \\
0.0285-j0.00107
\end{bmatrix}
\]

\[
\{I_{tot}\} = \left\{ \frac{1}{\sqrt{3}} \right\}\{I_e\}
\]

\[
\begin{bmatrix}
460 & 0 & 0 \\
0 & 199 & 0 \\
0 & 0 & 61.3
\end{bmatrix}
\begin{bmatrix}
0.00336+j0.0014 \\
0.0503 \\
0.0285-j0.00107
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1.55+j0.644 \\
10 \\
1.26-j0.0656
\end{bmatrix}
\]

\[
\{I\} = (N)\{I_{tot}\}_{\text{diag}}
\]

\[
\begin{bmatrix}
0.4 & 0.707 & 0.559 \\
-0.824-j0.184 & 0.604+j0.102 \\
0.4 & -0.707 & 0.559
\end{bmatrix}
\begin{bmatrix}
1.55+j0.694 & 0 & 0 \\
0 & 10 & 0 \\
0 & 0 & 1.26-j0.0656
\end{bmatrix}
\]
In order to calculate the modal fields (step 6) first consider figure 5.3. It shows the field at point A due to mode 1 current on conductor 1 and its corresponding induced ground current. The field due to the current on the conductor, \( H_1^{0(1)} \), can be found simply by application of the Ampere’s Circuital Law thus

\[
H_1^{0(1)} = \frac{I_1^{(1)}}{2\pi r_1}
\]

where the superscript ‘0’ refers to conductor modal current. The \( x \) component is

\[
H_{x1}^{0(1)} = H_1^{0(1)} \cos \theta
\]

where the subscript \( x \) refers to the \( x \) component of the field. The \( y \) component is of no concern, due to the loop antenna of the measuring instrument being oriented so that only \( x \) direction (horizontal) field components are detected.

The \( x \) component of the field at point A due to the induced
Fig. 5.3 - Representation of Field due to Mode 1 Current on Conductor 1 Including Induced Distributed Ground Currents.

The total mode 1 field due to mode 1 current on conductor 1 is the sum of the currents in the lossy ground and the conductors. The currents in the lossy ground are represented by the dashed line. The field due to these currents is calculated and added to the field due to the currents in the conductors. The resulting field is the total mode 1 field due to mode 1 current on conductor 1 including induced distributed ground currents.
ground currents (19) is

\[ H_1'(1) = \frac{1}{j\omega \mu_0} \int_0^\infty N(k_1) \frac{k_1^2 - m_a^2}{k_1^2 - m_g^2} \cos(x_1 k_1) dk_1 \]  

- 5.45

In (5.45)

\[ N(k_1) = \frac{I_1(1)}{2\pi} \frac{j\omega \mu_0 \sqrt{k_1^2 - m_g^2 - \nu_{rg} k_1^2} e^{-\gamma_1 k_1}}{k_1 \sqrt{k_1^2 - m_g^2 + \nu_{rg} k_1^2}} \]  

- 5.46

\[ m_a^2 = \omega^2 \mu_0^2 \gamma(1)^2 \]  

- 5.47

\[ m_g^2 = \omega^2 \mu_g^2 + j\omega \mu_g \sigma_g + \gamma(1)^2 \]  

- 5.48

\[ \nu_{rg} = \frac{\mu_g}{\mu_0^2} \]  

- 5.49

\[ \nu = 2\pi f \]  

- 5.50

\[ k_1 \] is a variable in Fourier transformed space and the dash in \( H_1'(1) \) refers to the induced ground current. These two fields can be summed as they both are oriented in the same direction. Therefore, the total field due to mode 1 currents in conductor number 1 is the sum of the fields in the x direction due to the modal current in the conductor and the modal induced current in the ground. Thus

\[ H_{11} = H_{01} + H_1'(1) \]  

- 5.51

Similarly, expressions for the field due to mode 1 current on conductors 2 and 3 are

\[ H_{22} = H_{02} + H_1'(1) \]  

- 5.52

\[ H_{33} = H_{03} + H_1'(1) \]  

- 5.53

The total mode 1 field due to mode 1 currents on all conductors at
point A is

\[ H_x^{(1)} = H_x^{(1)} + H_x^{(2)} + H_x^{(3)} \] - 5.54

Mode 2 and 3 fields are found using the same procedure as just described, and they are

\[ H_x^{(2)} = H_x^{(2)} + H_x^{(3)} + H_x^{(3)} \] - 5.55

\[ H_x^{(3)} = H_x^{(3)} + H_x^{(3)} + H_x^{(3)} \] - 5.56

\( H_x^{(1)}, H_x^{(2)} \) and \( H_x^{(3)} \) are named the modal fields.

It should be noted that these modal fields are complex, that is, they have a magnitude and a phase angle. The modes are assumed to be uncorrelated quantities and therefore, quadratic addition is used to obtain the phase field. The phase field in step 7 thus becomes,

\[ H\text{(monophase)} = \sqrt{\sum_{n=1}^{3} |H_x^{(n)}|^2} \] - 5.57

The field in (5.57) is the monophase field because it is due to only one of the phase conductors being in corona. The whole procedure can be repeated for the other two phase conductors in corona, one at a time, to yield three monophase fields which can be converted to dB by use of (4.34). (4.35) is employed if the actual excitation function is available.

The total three phase RI transverse magnetic field, step 8, can be found using the CISPR law explained in Chapter 4.

5.3 APPLICATION OF METHOD C TO THE CHESNOY-ROUSSON TRANSMISSION LINE

The above method of transverse RI magnetic field profile predetermination is now applied to the Chesnoy-Rousson transmission line and figures 5.4 to 5.12 show the results. In these figures,
Method C results have been plotted together with Method B results.

5.3.1 COMPARISON OF MODAL FIELDS

Figures 5.4 and 5.5 show the modal fields for centre and outer phase excitation respectively. The frequency is 500kHz and the earth resistivity is 100 Ohm-meters. For either centre or outer conductor in excitation the modal profiles obtained by use of Method C are basically similar in shape to those obtained by use of Methods A and B.

5.3.2 COMPARISON OF MONOPHASE FIELDS

Figures 5.6 to 5.8 show the transverse RI field profiles of Method C compared to Method B for a frequency of 1MHz and earth resistivity of 20, 100 and 1000 Ohm-meters. It will be noted that for low earth resistivities the far field values are higher for Method B, whereas, near the line, Method C field values are higher. However, at high earth resistivities (greater than 1000 Ohm-meters) Method C is higher practically everywhere. Therefore, apart from the high earth resistivity profiles, the shape of Method C profiles are similar to those obtained by Method A. The latter has been described in Chapter 4.

It was found in (29) that modal attenuations reached a maximum at a certain earth resistivity (usually around 1000 Ohm-meters). Therefore, at these higher earth resistivities the modal attenuations from Method C are lower than those obtained in the other two methods and, hence, the RI fields for Method C are higher. Therefore, the effect of the attenuations predominates over all other differences between the methods. Table 5.1 shows Method C attenuations.

Figures 5.9 to 5.11 show RI profiles for conditions identical with those of figures 5.6 to 5.8 except that the frequency is 500kHz.
Fig. 5.4 - Graph of Transverse Modal Magnetic Field Intensity Versus Lateral Distance for Centre Phase Excitation Using Method C. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters
Fig. 5.5 - Graph of Transverse Modal Magnetic Field Intensity Versus Lateral Distance for Outer Phase Excitation Using Method C. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
Fig. 5.6 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1 MHz and Earth Resistivity is 20 Ohm-meters
Fig. 5.7 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1MHz and Earth Resistivity is 100 Ohm-meters
Fig. 5.8 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 1MHz and Earth Resistivity is 1,000 Ohm-meters.
Fig. 5.9 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 500kHz and Earth Resistivity is 20 Ohm-meters
Fig. 5.10 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
Fig. 5.11 - Graph of Monophase Transverse Magnetic Field Intensity Versus Lateral Distance. Frequency is 500kHz and Earth Resistivity is 1,000 Ohm-meters
Fig. 5.12 - Graph of Monophasic Transverse Magnetic Field Intensity Versus Lateral Distance (Concentrating on Large Distances). Frequency is 500kHz and Earth Resistivity is 100 Ohm-meters.
<table>
<thead>
<tr>
<th>Earth Resistivity in Ohm-meters</th>
<th>Attenuation of Mode 1 in Np/meter</th>
<th></th>
<th>ATTENUATION OF MODE 2 IN Np/METER</th>
<th></th>
<th>ATTENUATION OF MODE 3 IN Np/meter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency of 1MHz</td>
<td>Frequency of 500kHz</td>
<td>Method B</td>
<td>Method C</td>
<td>Method B</td>
<td>Method C</td>
</tr>
<tr>
<td>20</td>
<td>8.64x10^{-6}</td>
<td>6.69x10^{-6}</td>
<td>4.96x10^{-6}</td>
<td>4.60x10^{-6}</td>
<td>24.1x10^{-6}</td>
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<tr>
<td>100</td>
<td>19.3x10^{-6}</td>
<td>7.22x10^{-6}</td>
<td>11.1x10^{-6}</td>
<td>4.70x10^{-6}</td>
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<tr>
<td>1,000</td>
<td>61.1x10^{-6}</td>
<td>6.60x10^{-6}</td>
<td>35.1x10^{-6}</td>
<td>4.25x10^{-6}</td>
<td>171x10^{-6}</td>
<td>24.0x10^{-6}</td>
</tr>
</tbody>
</table>

Table 5.1 - Modal Attenuations for the Chesnoy-Rousson Line for Different Frequencies, Earth Resistivities and for Methods B and C
Again, as noted in chapter 4, and for the same reasons, the RI profiles are of higher value in the 500kHz case for all methods.

For field profiles at large lateral distances, shown in figure 5.12, once again, as noted in chapter 4, the rate of change of field with lateral distance is practically identical, regardless of the method being used.

In comparing Method C with Method B, the basic differences are that Method C uses distributed currents in the ground, the Carson Correction Factors account for the resistive earth, and lossy line modal analysis to calculate the modal attenuations. It is very difficult and too involved to pinpoint just to what extent each difference will affect the final values. However, in general it can be said that for low earth resistivities Method C results in higher values for the field at large distances and lower values for the field at small distances. For high earth resistivities Method C profiles are always higher.

A major drawback of Method C is that it takes a far greater time to program and involves longer computing time. This last point is due to Carson's Correction Factors, and RI fields due to ground currents, being expressed in terms of infinite integrals and, therefore, numerical methods have to be utilized. It is possible however to minimize a lot of calculating by pre-evaluating these values for different frequencies, earth resistivities and line dimensions and then storing them in a look-up table. If this is practical then computer time could be reduced substantially but it still will be longer than in Method B.
CHAPTER 6

DISCUSSIONS AND CONCLUSIONS

In this thesis three methods of predetermining the transverse RI magnetic fields profiles are described. The first two, Method A (Clade and Gary, 1966) and Method B (Moreau and Gary, 1972) utilize the concept of ideal and offset images respectively. Method C (based on the work of Perz and Raghuveer, 1974) uses the distributed earth currents in its calculations.

Unless exact solutions are derivable, assumptions are necessary to facilitate the development of a particular method. The three methods described in this thesis have a set of common assumptions, as well as their own unique assumptions. Their common assumptions are;

1) All conductors of a transmission line are horizontal, parallel to each other and to an infinite ground plane of uniform resistivity.

2) Only the magnetic field in the x (lateral) direction is considered. This is appropriate if the loop antenna on the measuring instrument is oriented with its axis parallel to the earth's surface and perpendicular to the line.

3) Modal field components at the centre of a long line are combined using the quadratic addition principle.

4) Corona discharges are considered to be uncorrelated stationary random processes and are evenly distributed along the conductor.

5) The response of the measuring instrument is not critical as long as it is common to all methods. This thesis has used the CISPR method of evaluating the response.
6) All waves propagating along the line are practically in the TEM mode.

Each method has apart from the above mentioned assumptions a set of its own unique assumptions. These have been tabulated in table 6.1. Inspection of this table shows that Method A uses a lossless system and its associated image currents. However, in order to calculate the attenuation, the losses are accounted for by using the skin effect theory. In Method B the current eigenvectors are obtained by considering the line to be transposed; the concept of offset image is used to account for the effect of resistive earth. The modal attenuation values are available from a large number of experimental results. Method C models the equivalent circuit of the lossy line by utilizing Carson's Correction Factors and the attenuation is then calculated through eigenvalue analysis. The ground currents are considered to be of a distributed nature and the fields calculated using infinite integrals (20). As far as initial assumptions are concerned, Method C would therefore be expected to yield results that are more accurate than those of Methods A or B.

In comparing transverse RI magnetic fields profiles (figures 4.6 to 4.16, and 5.4 to 5.12) there are two categories that can be studied, near field profiles and far field profiles. In this work the near field is considered to be within 20 meters of the centre conductor and the far field profile is the region outside 20 meters. Most countries that have regulations on allowable transverse RI magnetic fields from a transmission line, provide for a limit on this field value at a certain lateral distance from the centre conductor. This lateral distance is sufficiently far removed from the outer phase
<table>
<thead>
<tr>
<th>Method A</th>
<th>Method B</th>
<th>Method C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clade and Gary</td>
<td>Moreau and Gary</td>
<td>Perz and Raghuveer</td>
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</table>

<table>
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<tr>
<th>Line Model</th>
<th>Transposed Lossless Line</th>
<th>Lossy Line</th>
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<td>Clarke's Matrix</td>
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<td>Experimental Values</td>
<td>Eigenvalue Analysis of Matrix (P)</td>
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<td>Perfect Image</td>
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<td>Ground Current Model</td>
<td>Offset Image</td>
<td>Distributed Currents</td>
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<td>Carson Correction Factors and Magnetic Field Expressions</td>
<td></td>
</tr>
<tr>
<td>Attenuation</td>
<td>Attenuation and Offset Image</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1 - Characteristic Assumptions of Methods A, B and C
conductor, and therefore would fall in the far field region. If a transmission line were to be built within specified mandatory RI field limits then the method that yields the higher RI field values at the lateral distance specified should be used to minimize the risk of exceeding these. It can be seen from the graphs and the results shown in Appendix I, that Method C leads to the highest field values at about the 20 to 50 meter lateral distance range, particularly for large earth resistivities. As the lateral distance is increased into the hundreds of meters, Method A profiles predominate at very low earth resistivities, but Method C profiles are highest at higher earth resistivities.

As has been explained in earlier chapters, it was found that Method B is easier to program than either Method A or C. Moreover, Method B is the simplest and most universal. If many calculations have to be made and if computer time is at a premium, then Method B offers an economically practical calculating method. If, however, conditions are such that computer time is not severely limited, then the advantage of Method B is lost.

In conclusion it can be said that a new method of predetermining transverse RI magnetic field profiles was proposed that contained fewer assumptions and better transmission line modelling than the established methods.

Though it has been shown in this thesis that Method C has many improvements over established methods, there are still many areas in this subject that need further studying. Some of them are:

1) Checking the validity of quadratic addition for combining modal fields.
2) Better determination of attenuation values, both experimentally and theoretically.

3) Deep study of the validity in calculating earth resistivity effect, particularly of the air-earth boundary simplifications.

Method A - Top
Method B - Centre
Method C - Bottom

Normalized and total fields are expressed in dB with the reference point being 0.125 microamps/meter (or 1 microvolt/meter). The model fields are all expressed in microamps/meter. All field values have been calculated based on an excitation function of 1 microamp/meter.
APPENDIX I

The following tables show the results obtained from applying methods A, B and C to the Chesnoy-Rousson transmission line. For each value of frequency, earth resistivity, lateral distance and phase excitation, the field values are arranged thus:

Method A - Top
Method B - Centre
Method C - Bottom

Monophase and total fields are expressed in dB with the reference point being 1/377 μAmps/meter (or 1μ Volt/meter). The modal fields are all expressed in μAmps/meter. All field values have been calculated based on an excitation function of 1μAmp/meter¹⁄₂.
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**TABLE 7**

Magnetic field strength as a function of lateral distance versus resistivity.

**FREQUENCY IS CONSTANT AT 0.00050 H.**
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**Note:** Frequency is constant at 53037.9 MHz.
APPENDIX II

EQUATIONS FOR LOSSY LINE TRANSVERSE RI MAGNETIC FIELD INTENSITY

TOGETHER WITH CARSON CORRECTION FACTORS

The transverse magnetic field at any lateral distance from a lossy transmission line is due to two different currents. The first is the current flowing in the conductor and the second is the induced current flowing in the ground. Perz and Raghuveer (18) showed that the magnetic field in the x direction due to a current on a conductor (say mode n current on conductor k) is

\[ H_{xk}^{(n)} = \frac{I_k(n)}{2\pi} \int_0^\infty e^{-(y_k-h_k)k_1} \cos x_k k_1 dk_1 \]  - A.2.1

By use of simple mathematical tables this integral can be simplified to

\[ H_{xk}^{(n)} = \frac{I_k(n)h_k-y_k}{2\pi(h_k-y_k)^2+x_k^2} \]  - A.2.2

which is the equivalent result to applying Ampere's Circuital Law. Magnetic fields due to ground currents are however, more involved.

Through the use of Fourier Transforms and applying boundary conditions (19), the expression for the electric field above the ground surface and in the direction of the transmission line (z direction) due to the ground current corresponding to mode n on conductor k is

\[ E_{zk}^{(n)} = -\int_0^\infty N(k_1) e^{-\sqrt{k_1^2-m^2} z} \cos x_k k_1 dk_1 \]  - A.2.3

where

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\[ N(k_1) = \frac{j\omega_0 \sqrt{k_1^2 - m_2^2}}{\omega_1 - \mu \omega_1} e^{-h_1 k_1} \]  

- A.2.4

With the use of Maxwell's equations the magnetic field in the x direction due to \( E'_{zk}(n) \) is

\[ H'_{zk}(n) = -\frac{1}{j\omega_0} \frac{a E'_{zk}(n)}{ay} \]

\[ = \frac{1}{j\omega_0} \int_0^\infty N(k_1) \sqrt{k_1^2 - m_2^2} e^{-y_k} \sqrt{k_1^2 - m_2^2} \cos x_k k_1 dk_1 \]  

- A.2.5

(A.2.2) and (A.2.5) give the magnetic fields in the x direction for any value of y above the ground. In this thesis all calculations are performed at ground level, and therefore, y is zero. The particular expressions at y=0 for \( H^0_{zk}(n) \) and \( H'_{zk}(n) \) are

\[ H^0_{zk}(n) = \frac{I_k(n)}{2\pi} \frac{h_k}{h_k^2 + x_k^2} \]  

- A.2.6

\[ H'_{zk}(n) = \frac{1}{j\omega_0} \int_0^\infty N(k_1) \sqrt{k_1^2 - m_2^2} \cos x_k k_1 dk_1 \]  

- A.2.7

In order to derive expressions for the Carson Correction Factors, Perz and Raghuveer (19) have used a method similar to that of Carson (20). The electric field is expressed in terms of its vector and scalar potential thus (19),(20)

\[ E'_{zk}(n) = -\frac{a}{\partial t} A_z - \frac{3}{\partial z} V \]

\[ = -j\omega A_z - \frac{3}{\partial z} V \]  

- A.2.8

where \( A_z \) is the vector potential.
The electric field at a point \((x,y)\) with ground as the reference is
\[
E'_z(x,y) - E'_z(x,0) = -\left( j\omega (A_z(x,y) - A_z(x,0)) \right) \frac{\partial V(x,y)}{\partial z}
\]
\[
= -j\omega \int_0^\infty k_H(x,y) dy - \frac{\partial V(x,y)}{\partial z} \tag{A.2.9}
\]

Using (A.2.6) and (A.2.7) it is possible to derive the following expression (19).
\[
E'_z(x,y) = -\frac{\partial V(x,y)}{\partial z} \frac{j\omega g_1^2 \mu_k}{\pi} \frac{1}{\ln \frac{x_k^2 + (h_k + y_k)^2}{x_k^2 + (h_k - y_k)^2}}
\]
\[
\frac{j\omega g_1^2 \mu_k}{\pi} \int_0^\infty e^{-\frac{(h_k + y_k) k_1}{k_1 + m_1 g}} \cos \frac{k_1}{k_1 + m_1 g} dk_1 \tag{A.2.10}
\]

If the point \((x,y)\) is on the surface of the conductor then the electric field in the direction of the conductor can be expressed in the manner
\[
E'_z(x,y) = z_c I_k(n) \tag{A.2.11}
\]

where \(z_c\) is the internal impedance of the conductor. Using (A.2.10) and (A.2.11)
\[
\frac{\partial V(x,y)}{\partial z} = I_k(n) \left( z_c + \frac{j\omega g_1^2}{2\pi} \frac{2h_k}{r_k} \right) \nabla \cos \frac{k_1}{k_1 + m_1 g}
\]
\[
+ \frac{j\omega g_1^2}{\pi} \int_0^\infty e^{-\frac{2h_k k_1}{k_1 + m_1 g}} \cos \frac{k_1}{k_1 + m_1 g} dk_1 \tag{A.2.12}
\]

where the expression in the major brackets is the self impedance per unit length of conductor \(k\) above a lossy ground. The first two terms in this bracket are due to the conductor above a lossless ground only. This leaves
the last term to be the self impedance due to the resistive earth.

The Carson Correction Factor for conductor $k$ above a lossy ground then becomes

$$CCF(k,k)=z(k,k)$$

for lossy line and

$$z(k,k)$$

for lossless line

$$=\Delta z(k,k)$$

$$=\frac{j\omega g}{\pi} \int_{0}^{\infty} \frac{e^{-2k_{1}h_{k}}}{\mu_{r}g_{1}^{2}+k_{1}^{2}-m_{g}^{2}} dk_{1}$$

- A.2.13

A similar derivation can be used to obtain the expression for mutual series impedances. For conductors $k$ and $m$ above a lossy earth, the mutual series impedance is

$$CCF(k,m)=\Delta z(k,m)$$

$$=\frac{j\omega g}{\pi} \int_{0}^{\infty} \frac{e^{-2k_{1}h_{m}}}{\mu_{r}g_{1}^{2}+k_{1}^{2}-m_{g}^{2}} dk_{1}$$

- A.2.14
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