SUPPLY CHAIN RISK MANAGEMENT IN AUTOMOTIVE INDUSTRY

Shiping Zhu

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SUPPLY CHAIN RISK MANAGEMENT IN AUTOMOTIVE INDUSTRY

By

Shiping Zhu

A Dissertation

Submitted to the Faculty of Graduate Studies through the Industrial and Manufacturing Systems Engineering Graduate Program in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy at the University of Windsor

Windsor, Ontario, Canada

2018

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Supply Chain Risk Management in Automotive Industry

by

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DECLARATION OF CO-AUTHORSHIP / PREVIOUS PUBLICATION

I hereby certify this thesis incorporates material that is the result of joint research wherein all key ideas, primary contributions, experimental designs, data analysis, and interpretations were performed by the author and Dr. Guoqing Zhang as the advisor.

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<th>Thesis Chapter</th>
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<tr>
<td>Chapter 3</td>
<td>Zhang G. Zhu S. (2012) “Manufacturer Cooperation in Supplier Development Under Risk with Nonlinear Return in Auto Industry”, 2012 INFORMS Annual Conference.</td>
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ABSTRACT

The automotive industry is one of the world's most important economic sectors in terms of revenue and employment. The automotive supply chain is complex owing to the large number of parts in an automobile, the multiple layers of suppliers to supply those parts, and the coordination of materials, information, and financial flows across the supply chain. Many uncertainties and different natural and man-made disasters have repeatedly stricken and disrupted automotive manufacturers and their supply chains. Managing supply chain risk in a complex environment is always a challenge for the automotive industry.

This research first provides a comprehensive literature review of the existing research work on the supply chain risk identification and management, considering, but not limited to, the characteristics of the automotive supply chain, since the literature focusing on automotive supply chain risk management (ASCRM) is limited. The review provides a summary and a classification for the underlying supply chain risk resources in the automotive industry; and state-of-the-art research in the area is discussed, with an emphasis on the quantitative methods and mathematical models currently used. The future research topics in ASCRM are identified.

Then two mathematical models are developed in this research, concentrating on supply chain risk management in the automotive industry. The first model is for optimizing manufacturer cooperation in supply chains. OEMs often invest a large amount of money in supplier development to improve suppliers’ capabilities and performance. Allocating the investment optimally among multiple suppliers to minimize risks while maintaining an acceptable level of return becomes a critical issue for manufacturers. This research develops a new non-linear investment return mathematical model for supplier
development, which is more applicable in reality. The solutions of this new model can assist supply chain management in deciding investment at different levels in addition to making “yes or no” decisions. The new model is validated and verified using numerical examples.

The second model is the optimal contract for new product development with the risk consideration in the automotive industry. More specifically, we investigated how to decide the supplier’s capacity and the manufacturer’s order in the supply contract in order to reduce the risks and maximize their profits when the demand of the new product is highly uncertain. Based on the newsvendor model and Stackelberg game theory, a single period two-stage supply chain model for a product development contract, consisting of a supplier and a manufacturer, is developed. A practical back induction algorithm is conducted to get subgame perfect optimal solutions for the contract model. Extensive model analyses are accomplished for various situations with theoretical results leading to conditions of solution optimality. The model is then applied to a uniform distribution for uncertain demands. Based on a real automotive supply chain case, the numerical experiments and sensitivity analyses are conducted to study the behavior and performance of the proposed model, from which some interesting managerial insights were provided. The proposed solutions provide an effective tool for making the supplier-manufacturer contracts when manufacturers face high uncertain demand.

We believe that the quantitative models and solutions studied in this research have great potentials to be applied in automotive and other industries in developing the efficient supply chains involving advanced and emerging technologies.
DEDICATION

This dissertation is in commemoration of my entering university studies 40 years ago. I dedicate this in memory of my honourable parents Zhu Qingxiu and Zhou Xueling for their kindness, unconditional love and devotion in raising me and their support in cultivating my spirit. They encouraged me during my teenage years to self-learn high school courses when all schools were shut down during the Great Cultural Revolution in China. Thanks to their vision, after working in a factory for 7 years, I succeeded in The National College Entrance Examination in 1978, which changed the course of my life.

I also dedicate this dissertation to my honourable ancestor, Mr. Zhu Bin (朱彬) who was a First-Degree Scholar (举人) and Scholar of Classical Texts (训诂学家). From his youth to his elder years he studied unceasingly, even taking a national-level Imperial Examination at the age of 70 in order to benefit the people. He published several famous books and his three sons (Zhu Shiyan 朱士彦; Zhu Shida 朱士达; Zhu Shilian 朱士廉) all were Imperial Scholars (进士). My ancestors’ spirit of perseverance inspired me to complete this study in the last 10 years and realized PhD dream in my sixties.

值此作者（朱世平）考上大学 40 年之际，谨以此博士论文献给我尊敬的父亲朱庆庥、母亲周学灵。感谢他们的养育之恩，更感谢他们在我青少年时期鼓励我坚持自学，当时正值中国文化大革命，读书无用论盛行，学校不正常授课。正因为他们的远见，我才能于 1978 年中国恢复高考，当工人 7 年后考取大学，改变了自己的命运。

也以此论文献给我尊敬的祖先，我的七世祖朱彬先生（清代举人，训诂学家），他一生好学，著作等身，70 岁高龄时还坚持参加科举考试，经世济民，实现自己的抱负。在他的言传身教下，他的三个儿子朱士彦，朱士达（我的六世祖），朱士廉都高中进士，并有突出的政绩，成就了“朱氏兄弟三进士”的佳话。我祖先的好学精神和顽强毅力激励了我，经过 10 年刻苦学习，我终于在花甲之年圆了博士梦，以实际行动传承家风，希望能用自己所学理论与在汽车行业的实际经验相结合，继续为社会作出贡献。
ACKNOWLEDGEMENTS

This Ph.D. study has enriched my knowledge in the field of Automotive Supply Chain Risk Management. It has given me the opportunity to summarize my 15 years of working experience in the automotive industry and to make new contributions to the area.

First, I would like to express my sincere thanks to Professor Guoqing Zhang for the continuous support of my Ph.D. study and related research, for his patience, motivation, immense knowledge, and encouragement. Over the last 10 years, Dr. Zhang has helped me to overcome many of the unexpected difficulties and provided the excellent supporting efforts and technical knowledge that has guided me throughout the research work and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D. study.

Next, I would like to show my gratitude to all my thesis committee members: Prof. Y. Wang, Prof. W. Abdul-Kader, and Prof. M. Wang for their insightful comments and encouragement, also for their constructive criticism and willingness to positively enrich this report. Thanks also to those fellow graduate students who have offered their valuable suggestions, kind assistance, and friendly cooperation.

Last but not least, I would like to thank my family: my husband, Shaolin Cui, my daughter, Zhongying, and my son, Matthew Jiachen. Without their support and encouragement, this work would not have been possible. My daughter, my son and I studied simultaneously in different programs and encouraged each other to achieve our academic success.
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CHAPTER 1
INTRODUCTION

1.1 Motivation
The automotive industry has been driven to optimize its supply chain performance by its demanding and fiercely competitive business environment. However, many uncertainties and natural and man-made disasters have repeatedly stricken and disrupted automotive OEMs and their supply chains. In order to protect themselves against the harmful effects of the supply chain uncertainties and disruptions, automotive OEMs have turned their attention to supply chain risk management. Unfortunately, the research work and publications have not kept up the same pace that the automotive industry requires. There are limited amounts of the published research papers which directly address the automotive supply chain risk management. To lay a foundation for further research in the area, this research first provides a review of the existing research work on the supply chain risk identification and management, considering the characteristics of the automotive supply chain. Then, the research tries to model the optimal contract for product development with the risk consideration for the automotive industry, and to model optimizing manufacturer cooperation in the supply chain.

1.2 Research Objective and Methodologies
The purpose of this research is to find (1) the underlying supply chain risk management problems in the automotive industry; (2) the theoretical models to help the decision-making of supply chain managers in uncertain situations. The characteristics of the automotive supply chain are discussed in the literature review. It also summarizes the theoretical work and practices related to or applicable to automotive supply chain risk identification and classification, as well as examines the methodologies of supply chain risk assessment. The theoretical models, the qualitative approaches and practical tools for supply chain risk management are reviewed. It is pointed out in this research that implementing an automotive supply chain risk assessment is a complex and challenging task. The mathematical models for real-time supply chain disruption management still need development. More research is needed to model uncertain situations in order to mitigate risk impacts.
This research includes two mathematic models for automotive supply chain risk management:

1.2.1 Model for supplier development and manufacturer cooperation with a nonlinear return to minimize risks.

The automotive supply chain is a multiple layer and complex network, so the relationship between supply chain members is very important for risk management. Supplier development is a long-term, resource-consuming business activity that requires commitment from both manufacturer and suppliers. Automotive manufacturers often invest heavily in supplier development to improve their supplier’s capabilities and performance. How to allocate the investment optimally among multiple suppliers to minimize risk while maintaining an acceptable level of return is a critical issue faced by automotive OEMs. Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation in supplier development under risk. Talluri’s model assumes that the return of investment to the supplier is proportional to the investment. However, in most situations, the return is nonlinear. This research extends Talluri’s work and presents a new mathematical model for supply chain development by revising investment return from linear to non-linear and applies it to the auto industry.

1.2.2 Optimal contract model for product development with risk consideration (penalty and compensation).

As a common ex-ante strategy in risk management, supply chain contracts play an important role for supply chain members, such as OEMs and suppliers, to coordinate, and to share risks arising from various sources of uncertainty. In the automotive industry, when developing new products, e.g., electric cars, the demand is highly uncertain. Generally, the manufacturer forecasts the demand and shares the information with suppliers. At the same time, the manufacturer needs to provide the planned yearly order quantity $O$, and then the supplier needs to decide the capacity $Q$ according to order $O$. As a part of the procurement contract, the manufacturer can claim compensation or penalty to prevent profit loss caused by the supplier’s delivery shortage. We model a single–period make-to-order supply chain consisting of a supplier and a manufacturer with demand uncertainty. The purpose is to determine the optimal contract variables, capacity $Q$ and order $O$. Based on the newsvendor
model and Stackelberg game theory, we develop a mathematical model for the product
development contract, where both demand uncertainty and compensation are considered.
The analytical solution for the situation that the demand follows uniform distribution is
developed, and computational tests, as well as sensitivity analyses, are also reported. The
proposed solution provides an effective tool for supplier-manufacturer contracts when the
manufacturer faces highly uncertain demand.

1.3 Structure of the Dissertation
The dissertation is organized as follows: Chapter 2 is a literature review of supply chain
risk identification, assessment, and management in the automotive industry. Chapter 3
studies manufacturer cooperation in supplier development under risk with nonlinear return.
Chapter 4 studies the optimal contract for product development with risk consideration
(penalty and compensation). Chapter 5 studies the optimization for product development
with risk consideration in uniform demand distribution. Finally, Chapter 6 provides a
conclusion on the research and the future work for automotive supply chain risk
management.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction
The automotive industry is one of the world's most important economic sectors in terms of revenue and employment. According to automotive industry statistical data from OICA (2016), almost 95 million cars and commercial vehicles were produced in the world in 2016, with over 8 million direct jobs in the assembly and manufacture of components, representing over 5% of the world’s industrial employment, and almost five times more indirect jobs (González-Benito et al., 2013).

The automotive supply chain is complex owing to the large number of parts assembled into an automobile, the multiple layers of suppliers to supply those parts, and the coordination of materials, information, and financial flows across the supply chain. Over the past decades, automotive supply chains in the world have been stricken and disrupted repeatedly by natural and human-made disasters, such as earthquakes, tsunamis, hurricanes, strikes, economic crises, SARS, plant fires and explosions, terrorist attacks, and other disruptive events. Such supply chain disruptions can detrimentally impact a firm’s short-term performance. At the same time, the automotive business has become increasingly more complex as globalization has become an industry norm. Other new practices, such as just-in-time (JIT) delivery, lean manufacturing or lean production (Nakashima and Sornmanapong, 2013), and supplier consolidation, are also employed. As a result, automotive supply chains have become increasingly more vulnerable to various risks. Thus, supply chain risk management (SCRM) plays a critical role in the automotive industry.

The automotive industry is well known for its efforts to improve its supply chains based on its demanding business environment and to protect against the harmful effects of supply chain disruptions to the companies through SCRM. However, research work and publications have not caught up with the pace required by the industry. We searched related articles on ASCRM, mainly from Scopus, SCI, SSCI, and ABI/INFORM, and some related materials available on the Internet. A search using “automotive supply chain/logistics risk management” as the keywords generated 111 document results from the Scopus database
by September 2018. Our review is not limited to these articles. Figure 2.1 shows the chronological distribution of papers and a generally increasing trend from 2008. However, there is no review literature devoted to ASCRM.

![Figure 2.1. Chronological distribution of papers from 2000 to Sept.2018 from Scopus](image)

To lay a foundation for further research in the area, this study provides a review of the general research work on supply chain risk identification and management for the automotive supply chain where the literature is available and seeks the answers to two key questions: (1) What are the underlying supply chain risk resources in the automotive industry? (2) How have they been addressed in the current literature, with a focus on quantitative methods and mathematic models? This review is based on 125 papers and websites addressing SCRM for the automotive industry and other related industries.

SCRM is "the process of risk mitigation achieved through collaboration, coordination, and application of risk management tools among the partners, to ensure continuity coupled with long-term profitability of the supply chain" (Faisal et al., 2007). It should be noted that risk cannot be completely eliminated from supply chains, but strategies can be developed to manage these risks if the dynamics between the variables related to risks in a supply chain are understood (Faisal et al., 2006). The main objective of SCRM is to maximize the expected profit or minimize the expected loss when a supply chain disruption occurs (Tang, 2006). Risk management has become an essential tool in addressing risk issues in supply chain management.

In general, the SCRM process consists of four components (Hallikas et al., 2004): risk identification, risk assessment, risk management decisions and implementation
(prioritization of risks), and risk monitoring. In line with these classifications, this review presents a summary and analysis with emphasis on the first three aspects of ASCRM and provides quantitative models and future research directions as well.

The rest of this literature review is organized into six sections. Section 2 discusses the characteristics of the automotive supply chain. Section 3 studies automotive supply chain risk identification and classification. Section 4 examines the methodologies of supply chain risk assessment. Section 5 reviews the theoretical models, the qualitative approaches, and practical tools for risk management. Some future research directions are discussed in Section 6. Finally, Section 7 concludes the literature review.

2.2 Automotive Supply Chains and Literature Distribution on Automotive Supply Chain Risks

The automotive supply chain includes raw material manufacturers, multilayer part suppliers, car manufacturers, dealers, and customers. There are about 20,000 parts in a car, and if even only one of these parts is not available, then the vehicle cannot be assembled or shipped. Typically, there are three to five layers in an automotive supply chain, which comprises thousands of suppliers. Figure 2.2 displays the schematic automotive supply chain (Timothy, 2011).

In the past 30 years, the automotive industry has undergone major changes in its supply chains. Competitive pressure has forced automotive original equipment manufacturers (OEMs) to improve the quality, to strive to reduce the product development time, and to lower the development and manufacturing costs of their products. Many Asian and Eastern European countries, with cheap and skilled labor, offer attractive opportunities for reducing the supply chain costs. However, these globalization and outsourcing opportunities come with significant risks, including the cultural and linguistic differences, foreign exchange rate fluctuation, duty and customs regulations, quality problems, and political and economic stability. The international logistics (inventory management, border-crossing procedures, and transportation delays) involve more challenges that could impact the product availability than domestic logistics.
For decades, the automotive industry has employed the different ways to cut the costs for gaining the competitive advantage. Several common methods include the following:

- adopting just-in-time (JIT) principles to create lean supply chains,
- single-sourcing most subassemblies to maximize scalability,
- outsourcing to emerging countries, and
- globalizing and following the OEMs to their international market.

These practices result in the low inventories, increase the additional dependence on suppliers, add to the network complexity, and increase the supply chain risks (Thun and Hoenig, 2011; Nakashima and Sornmanapong, 2013).

In summary, the automotive industry is characterized by low margins, high volumes, high costs, global supply chains, and multilayer suppliers (Simchi-Levi, 2010; Singhal et al. 2011). Automotive supply chains are deep and broad. Cost reduction efforts such as JIT, single-sourcing, outsourcing, and globalization add to the network complexity and increase the supply chain risks in the automotive industry. The automotive supply chain risks considered here include the different risks that come from sourcing, supply, production, storage, logistics, and distribution in the automotive supply chain.
ASCRM has attracted considerable attention in the past decade. The review reported here is based on 111 articles found by searching “automotive supply chain management/logistics risk,” mainly in the Scopus database. The distributions of those articles and journal impacts are summarized in Table 2.1.

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<td>International Journal of Production Economics</td>
<td>Dubey et al. (2018); Mohammaddust et al. (2017); Hääntsch and Huchzermeier (2016); Matsuo (2015); Pernot and Roodhooft (2014); Sun et al. (2012); Thun and Hoenig (2011a); Doran et al. (2007); Yang et al. (2017)</td>
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<td>International Journal of Production Research</td>
<td>Yoon et al. (2018); Grötsch et al. (2013); Hsu et al. (2011); Thun et al. (2011b); Canbolat et al. (2008); Caux et al. (2006); Singh et al. (2005)</td>
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<td></td>
<td>Journal of Cleaner Production</td>
<td>Schöggl et al. (2016); Govindan et al. (2014); Lee (2011); Zimmer et al. (2017); de Oliveira et al. (2017)</td>
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<td>Management Decision</td>
<td>Neumüller et al. (2016)</td>
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<td>Production Planning and Control</td>
<td>Zhang et al. (2018); Xie et al. (2009)</td>
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<td>Hanenkamp (2013)</td>
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<td>Greener Management International</td>
<td>Oldham and Votta (2003)</td>
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<td>Supply Chain Management</td>
<td>International Journal of Procurement Management</td>
<td>Hellström et al. (2011)</td>
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<td>International Journal of Supply Chain Management</td>
<td>Nakashima et al. (2014)</td>
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<td>International Journal of Information Systems and Supply Chain Management</td>
<td>Sharma et al. (2017)</td>
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<td>Kırılmaz and Erol (2017); Caniëls et al. (2013)</td>
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<td>Blos et al. (2009); Towill et al. (2000)</td>
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<td>The IUP Journal of Supply Chain Management</td>
<td>Sharma and Bhat (2014c)</td>
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<td>Logistics Management</td>
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<td>Fan and Stevenson (2018); Friday et al. (2018); Hohenstein et al. (2015); Durach et al. (2015); Bell et al. (2013); Wieland and Wallenburg (2012); Lin and Zhou (2011); Blackhurst et al. (2008); Lippert and Forman (2006); Lalwani et al. (2006); Svensson (2004)</td>
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<tr>
<td>Automotive</td>
<td>Automotive Industries AI</td>
<td>Barclay (2008); Richardson (2005)</td>
</tr>
<tr>
<td></td>
<td>International Journal of Automotive Technology and Management</td>
<td>Belzowski et al. (2006)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>International Journal of Advanced Manufacturing Technology</td>
<td>Chen et al. (2016); Diabat et al. (2013); Elmaraghy and Majety (2008)</td>
</tr>
<tr>
<td></td>
<td>Journal of Manufacturing Technology Management</td>
<td>Lotfi and Saghiri (2018); Palanisamy and Zubar (2013)</td>
</tr>
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<td></td>
<td>Manufacturing Engineer</td>
<td>Cervi (2007)</td>
</tr>
<tr>
<td></td>
<td>Computers and Industrial Engineering</td>
<td>Ghadge et al. (2017)</td>
</tr>
<tr>
<td></td>
<td>Journal of Global Information Management</td>
<td>Seth et al. (2017)</td>
</tr>
<tr>
<td>Environmental Science, Engineering</td>
<td>Management of Environmental Quality</td>
<td>Munguia et al. (2010); Cebrat et al. (2008)</td>
</tr>
<tr>
<td></td>
<td>Resources, Conservation and Recycling</td>
<td>Naini et al. (2011)</td>
</tr>
<tr>
<td>Business</td>
<td>Benchmarking</td>
<td>Sharma and Bhat (2014b); Datta et al. (2013)</td>
</tr>
<tr>
<td></td>
<td>Global Business Review</td>
<td>Sharma and Bhat (2016)</td>
</tr>
<tr>
<td></td>
<td>Advanced Engineering Informatics</td>
<td>Mo and Cook (2018)</td>
</tr>
</tbody>
</table>
2.3 Automotive Supply Chain Risk Identification and Classification

Risk identification is the first step and a subjective component within the SCRM process. To reduce supply chain risks, firms should understand the universe of risk categories as well as the events and conditions that drive them. We analyzed the resources of supply chain risks existing in the automotive industry, and classified the risks into 10 categories shown in the fishbone diagram in Figure 2.3. Table 2.2 illustrates the automotive industry risk profile, risk events, and references.

**Figure 2.3. Automotive supply chain risk fishbone diagram**

**TABLE 2.2 AUTOMOTIVE INDUSTRY SUPPLY CHAIN RISK CLASSIFICATIONS, EVENTS, AND REFERENCES**

<table>
<thead>
<tr>
<th>Automotive Supply Chain (SC) Risks</th>
<th>Risk Events</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disruption/disaster</td>
<td>Natural disaster: Thailand’s devastating 2011 floods; Great East Japan Earthquake 2011.</td>
<td>Hsieh et al. (2016); Matsuo (2015); MacKenzie et al. (2014); Davarzani et al. (2011); Thun and Hoenig</td>
</tr>
<tr>
<td><strong>Supply</strong></td>
<td>“Upstream” activities in SC: a) purchasing risks, production capacity, supplier relationship, b) supplier dependence risk, single sourcing, c) supply chain transparency risk.</td>
<td>Thun and Hoenig (2011a); Simchi-Levi et al. (2015); Hudin et al. (2015); Hântsch and Huchzermeier (2016); IBM Global Services (2009); Sharma and Bhat (2014a); Davarzani et al. (2011); Kull and Talluri (2008)</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td>“Downstream” activities in SC: demand uncertainty, forecast/planning accuracy, high inventories or capacity risk.</td>
<td>Chopra and Meindl (2010); Simchi-Levi (2010); Sharma and Bhat (2014a); Sharma and Bhat (2014c)</td>
</tr>
<tr>
<td><strong>Logistics</strong></td>
<td>Transportation, delivery problem, delay, border crossing, and customs regulations.</td>
<td>Rice et al. (2003); Xie et al. (2009); Sharma and Bhat (2014a)</td>
</tr>
<tr>
<td><strong>Quality</strong></td>
<td>Recall issues, defects and corrective actions, engineering change, result in customer dissatisfaction, and market share shrinkage.</td>
<td>Sun et al. (2012); Rewilak (2015); Haefele (2014); Thun and Hoenig (2011a, 2011b);</td>
</tr>
<tr>
<td><strong>Globalization</strong></td>
<td>Geographically more diverse, more vulnerable to supply disruption, exchange-rate and energy-price risks, culture and language differences, trade regulations, and political and economic stability.</td>
<td>Thun and Hoenig (2011a); Richardson (2005); Sharma and Bhat (2014b); Canbolat et al. (2008); Zimmer et al. (2017)</td>
</tr>
<tr>
<td><strong>Environmental and social impact</strong></td>
<td>Global production systems have ecological impacts globally both “upstream” and “downstream” of a specific manufacturer or supplier.</td>
<td>O'Rourke and Dara (2014); Sharma and Bhat (2014a); Caiazzo et al. (2013); Schöggel et al. (2016); Hântsch and Huchzermeier (2016); Caniëls et al. (2013); Diabat et al. (2013); Lee and Cheong (2011); Naini et al. (2011); Munguía et al. (2010); Zimmer et al. (2017)</td>
</tr>
<tr>
<td><strong>IT system</strong></td>
<td>IT infrastructure, system breakdown, RFID has “three high problems,” information sharing.</td>
<td>Tinham (2004); Choi et al. (2017); Huang et al. (2012); Lippert (2008); Chopra and Meindl (2010); Strassner and Fleisch (2005); Hellström et al. (2011); Seth et al. (2017)</td>
</tr>
<tr>
<td><strong>Technology changes</strong></td>
<td>Software breakthroughs; new energy sources.</td>
<td>Hill et al. (2015); Cebrat et al. (2008)</td>
</tr>
<tr>
<td><strong>Financial</strong></td>
<td>Financial instability, insolvency or bankruptcy; untimely payment and exchange-rate risk.</td>
<td>Sharma and Bhat (2014a); Isidore (2009); Faisal et al. (2007); Bullis (2012)</td>
</tr>
</tbody>
</table>

### 2.4 Automotive Supply Chain Risk Assessment and Measures

Once risks are identified, their impacts and probabilities must be assessed. Risk assessment involves “a set of logical, systematic, and well-defined activities that provide the decision makers with a sound identification, measurement, quantification, and evaluation of the risk associated with certain natural phenomena or man activities” (Haimes, 2004). In this
The models and methods used in the literature for assessing the automotive supply chain risk are presented.

### 2.4.1 Probability Impact Matrix

The international engineering standard ISO14971 defines and assesses a risk $R$ as the product of probability and the harm of an event $e$: $R = P_e \times S_e$, where $S_e$ and $P_e$ refer to the severity and probability of $e$, respectively (Heckmann et al., 2015; ISO 14971:2007). Supply chain risk assessment aims to estimate the risk probability of occurrences and their adverse effect on the entire supply chain. In practice, the exact quantification of these values is often difficult because a precise assessment of the probability of occurrence and its effect is hardly possible; however, a qualitative method is advisable to evaluate the identified risk. The probability impact matrix is a qualitative risk assessing tool that has two dimensions: “probability” (from low to high) and “impact” (from weak to grave) based on a Likert scale. Through their survey in India, Sharma and Bhat (2014c) concluded that a likelihood/impact matrix is a widely used method of risk assessment in the automotive industry.

### 2.4.2 Fuzzy Assessment Method

Ghadimi et al. (2012) developed a weighted fuzzy assessment method for product sustainability assessment. A case study of an automotive component was conducted to illustrate the efficiency of the developed method. The results show how a simple replacement in the product material can lead a manufacturer toward producing more sustainable products and achieving the ultimate goal of sustainable manufacturing. Zimmer et al. (2017) developed a fuzzy analytical hierarchy process to estimate and assess social risks along global supply chains. Their model was applied to a case study of a German premium car manufacturer and showed a great benefit for practitioners in purchasing functions of focal companies.

Vujović et al. (2017) applied fuzzy logic to classify risk factors in production supply chains with uncertain data from the automotive industry. Palanisamy and Zubar (2013), Datta et al. (2013), and Diabat et al. (2013) also utilized fuzzy logic for their ASCRM research.
2.4.3 Mean-Variance Analysis
Mean-variance analysis introduced by Markowitz (1959) has been a standard tool for risk management. It tries to achieve a balance between the expected return and the specific risk measured by variance (Wu et al., 2010). Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation in supplier development under risk. They presented a set of optimization models that address supplier development undertaken by manufacturing firms to improve their suppliers’ capabilities and performance. The objective function is to minimize the risk of the manufacturer’s investments to suppliers. Zhang and Zhu (2012) extended Talluri’s manufacturer cooperation model to the automotive industry where the return is nonlinear. Many automotive OEMs have implemented supplier development programs to assist suppliers. When cooperating, firms share resources, benefits, as well as cost and risk.

However, some other traditional quantitative risk assessment tools, such as value at risk and conditional value at risk, have rarely been applied to the automotive supply chain.

2.4.4 Bayesian Networks
A Bayesian network is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph. Lockamy and McCormack (2012) and Lockamy (2014) used Bayesian networks to assess supply disaster risks in the automotive industry. The empirical data of design/methodology/approach are from 15 casting suppliers to a major US automotive company. They found that Bayesian networks can be used to develop supplier risk profiles, which can assist managers in making decisions regarding current and prospective suppliers. Abolghasemi et al. (2015) proposed a Bayesian method based on supply chain operations reference (SCOR) metrics. The method can manage supply chain risks and to improve supply chain performance. It was applied to one of the biggest automotive companies in Iran.

2.4.5 Other Risk Assessment Methods
Schöggel et al. (2016) provided a conceptual framework and an aggregation method for supply chain sustainability assessment using quantitative and qualitative indicators. Their results are based on a literature review of sustainability assessment in supply chains as well as on focus group workshops with experts from the European automotive and electronics
industry. Their paper contributes to the theory and practice of sustainability assessment in supply chains. Methods for assessing risk are also addressed in supplier evaluation and selection (Cagnin et al., 2016; Canbolat et al., 2008; Kull and Talluri, 2008).

Sharma and Bhat (2014c) also investigated the tools and techniques used in supply chain risk assessment practices by Indian automobile companies. For risk identification and assessment, scenario planning, a likelihood/impact matrix, and checklists are the most frequently used tools. Other risk assessment tools included failure mode effects and analysis, Six Sigma, simulations, and analytical hierarchy and network processes. Marasova et al. (2017) applied heuristics to risk assessment within the automotive industry supply chain.

A summary of the risk assessment and management methodology reviewed in this paper is given in Table 2.3.

<table>
<thead>
<tr>
<th>SC Risk Assessment Methodology</th>
<th>Reference in Automotive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability/likelihood impact matrix</td>
<td>Thun and Hoenig (2011); Sharma and Bhat (2014b)</td>
</tr>
<tr>
<td>Fuzzy logic</td>
<td>Palanisamy and Zubar (2013); Datta et al. (2013); Ghadimi et al. (2012); Diabat et al. (2013); Zimmer et al. (2017); Vujović et al. (2017)</td>
</tr>
<tr>
<td>Mean–variance</td>
<td>Zhang and Zhu (2012)</td>
</tr>
<tr>
<td>Bayesian network</td>
<td>Lockamy and McCormack (2012); Lockamy (2014); Abolghasemi et al. (2015)</td>
</tr>
<tr>
<td>Other risk assessment methods</td>
<td>Schöggl et al. (2016); Cagnin et al. (2016); Canbolat et al. (2008); Kull and Talluri (2008); Sharma and Bhat (2014b); Marasova et al. (2017)</td>
</tr>
</tbody>
</table>

2.5 Automotive SCRM, Modeling, Methods, and Tools

Motivated by the requirements of real-world practice, SCRM has attracted increasing attention from academia and industry (Tang, 2006; Wu et al., 2011; Simchi-Levi et al., 2014). This section provides a review of SCRM, modeling, methods, and tools for the automotive industry.
2.5.1 Optimization Mathematical Modeling

Optimization modeling is one of the widely used quantitative approaches used to manage automotive supply chain risk. These models include the following:

- linear programming (Kırılmaz and Erol, 2017, Caux et al., 2006), nonlinear programming (Zhang and Zhu, 2012, Cedillo-Campos, et al. 2017), mixed integer programming (Häntsch and Huchzermeier, 2016; Ghadge et al., 2017), mixed integer nonlinear programming (Rezapour et al., 2017; Mohammaddust et al., 2017), and stochastic models (Nakashima et al., 2014)
- multi-objective models (Häntsch and Huchzermeier, 2016);
- game theory (Naini et al., 2011; Swinney and Netessine, 2009); and
- newsvendor models (Nakashima and Sornmanapong, 2013).

Kırılmaz and Erol (2017) developed a linear programming model for a procurement plan by considering the cost criterion as the first priority and the risk criterion as the second priority to mitigate supply-side risks. The proactive approach to SCRM is applied to an international automotive company.

Dal-Mas et al. (2011) presented a multi-echelon mixed integer linear program (MILP) model for strategic design and investment capacity planning of the biofuel ethanol supply chain under price uncertainty. Linear/mixed integer multicriteria optimization models were used by Elmaraghy and Majety (2008) for determining the identified supply chain design parameters in an automotive powertrain supply chain design. Mixed integer nonlinear (MINL) models were used by Rezapour et al. (2017) to find the most profitable resilient network and risk mitigation policies and by Mohammaddust (2017) to evaluate risk mitigation strategies for a four-tier SC in a competitive automotive supply chain.

Häntsch and Huchzermeier (2016) presented a multiperiod, multi-objective optimization model that enables robust production network and location planning during times of increased market uncertainty and risk exposure in the automotive industry.

Nakashima and Sornmanapong (2013) used the newsvendor model to determine the optimal order quantity to maximize the expected profit under different scenarios. Nakashima et al. (2014) studied stochastic inventory control systems with consideration
for the view of second-tier semiconductor suppliers in automotive industries using a simulation approach.

Game theory was used by Naini et al. (2011) to design a mixed performance measurement system for environmental supply chain management that measures and evaluates business operations. The authors applied their proposed method to a case study of the supply chain of one of Iran's biggest automotive companies, SAIPA. Swinney and Netessine (2009) applied game theory to model a contracting game.

2.5.2 Quantitative-Based Strategies

- Supply Chain Coordination and Cooperation
  Because the automotive supply chain is a multilayer complex network, effective coordination between supply chain members is very important for risk management.

  Belzowski et al. (2006) surveyed the difficulties faced by automotive manufacturers and suppliers in managing their supply chains. With thousands of Tier-1 to Tier-N suppliers located across the globe, the external SCM linkages compound the complexity for both manufacturers and suppliers to manage their supply chain. Therefore, it is very critical for automotive companies to develop and execute internal integration and external collaboration strategies to survive challenges and to mitigate supply chain risks.

  Pernet and Roodhooft (2014) conducted a retrospective case study of an automotive supplier relationship and investigated whether the management control system design of supplier relationships is associated with good performance. Matsuo (2015) focused on a case of supply disruption of the automotive microcontroller units after the 2011 Tohoku Earthquake. An SCM framework hierarchy was applied to analyze the issues from the perspective of execution, design, strategy, and sustainability. Montshiwa et al. (2016) conducted a quantitative study that included supply chain cooperation as a term in their business continuity plan. The results of 75 automobile parts markers in disaster-prone regions (Asia and North America) were studied.

- Dual Sourcing
  Dual sourcing is an important strategy to mitigate supply chain risk. Thun and Hoenig (2011a) pointed out that building up redundancies is an important way to create a resilient supply chain. As in other industries, automotive OEMs often use dual sourcing or multiple
sourcing to create redundancies. They statistically analyzed survey data from 67 automotive manufacturing plants in Germany and concluded that dual sourcing or multiple sourcing is a valid and reliable factor in SCRM. Davarzani et al. (2011) studied strategies of single, dual, and multiple sourcing to handle potential disruptions. They proposed a sourcing model and demonstrated it in the decision-making process for a supply chain in the automotive industry.

- **Supply Contracts**

Supply chain contracts are used to coordinate supply chain members, OEMs, and suppliers to align their interests with those of the supply chain system and to achieve optimal supply chain efficiency. Supply chain contracts also play an important role in supply chain members sharing risks arising from various sources of uncertainty, such as demand, price, and product quality. Considerable research work has focused on pricing strategies and order allocation in multi-supplier systems.

Ghadge et al. (2017) studied a supply chain risk-sharing contract to mitigate demand uncertainty and price-volatility-related risks in a globalized business environment. They developed and analyzed an integer programming model followed with an automotive case study to get insights into a buyer–supplier relationships while considering multiple buyer-supplier power and dependence scenarios.

Selviaridis and Norrman (2014) studied performance-based contracting (PBC) in service supply chains. Based on agency theory, they studied two cases of logistics service supply chains, one of which is in the automotive industry, and identified the key influencing factors. Swinney and Netessine (2009) investigated short-term contracts, long-term contracts, and dynamic contracts under the threat of supplier default, because contracting with suppliers prone to default is an increasingly common problem in some industries, particularly automotive manufacturing. Game theory was applied to model a two-period contracting game with two identical suppliers, a single buyer, deterministic demand, and uncertain production costs. They concluded that the possibility of supplier default offers a new reason to prefer long-term contracts over short-term contracts.

- **Supplier Selection**
Supplier selection strategies have been identified as vital for risk mitigation in automotive companies (Chen et al., 2016). Multiple-criteria decision-making techniques such as fuzzy quality function deployment (FQFD), the mathematical modeling and analytical hierarchy process (AHP) (Kull and Talluri, 2008; de Oliveira et al. 2017), and the analytical network process (ANP) (Palanisamy and Zubar, 2013) are popular approaches used to evaluate as well as to select suppliers in the automotive industry.

Chen et al. (2016) presented an automotive company case study and evaluated the results through weighted goal programming (WGP) and preemptive goal programming (PGP) methods.

Cagnin et al. (2016) evaluated supplier selection methods in the automotive industry compared with the identified models in the literature.

- Contingency Strategies

Risk management strategies in supply chains can be divided into two categories: mitigation strategies and contingency strategies. The former focuses on taking precautions in advance of risk occurrence through strategic inventory and dual sourcing. The latter refers to the set of actions taken in post-disaster conditions, such as contingency rerouting and revenue management (Tomlin, 2006). Contingency rerouting is a cost-effective risk management strategy for major disruptions such as natural disasters.

Grötsch et al. (2013) investigated antecedents that support proactive SCRM implementation from a contingency theory perspective. The hypotheses were developed and tested via content analysis in 63 interviews with representatives from the automotive industry. They found that a mechanistic management control system, a rational cognitive style, and relational buyer-supplier relationships have positive impacts on proactively managing supplier insolvency risks. Svensson (2004) examined key areas, causes, and contingency planning of corporate vulnerability in supply chains for a subcontractor in the automotive industry.

MacKenzie et al. (2014) modeled a severe supply chain disruption and post-disaster decision-making process and applied the model to a simulation based on the 2011 Japanese earthquake and tsunami, which caused closure of several facilities of key suppliers in the
automobile industry and subsequently supply difficulties for both Japanese and US automakers.

Giard and Sali (2014) studied optimal stock-out risk for a component in an automotive supply chain. The strategy is to trigger the use of an emergency supply before its occurrence to prevent stock-out propagation along the downstream part of the supply chain. Their model considers the cost of producing and maintaining a safety stock and the costs incurred by the emergency supply. Two alternatives emergency policies were compared in their analytical solutions.

In addition to the theoretical models, the simulation approach is used in automotive supply chain disruption research (Lalwani et al., 2006; Canbolat et al., 2008; MacKenzie et al., 2014).

### 2.5.3 Qualitative Approaches

- **Empirical Approach**

Most research in automotive SCRM employs the empirical approach to analyze the risk and assess the risk management strategies qualitatively. González-Benito et al. (2013) concluded that the empirical approach includes three types: case studies, surveys, and secondary sources. They found that ~60% of research on automotive supply chain risk employed the empirical approach, some of which combined different methods, such as case studies and surveys or case studies and mathematical modeling.

Thun and Hoenig (2011a) conducted an empirical analysis of SCRM practices. The analysis was based on a survey of 67 manufacturing plants conducted in the German automotive industry. After investigating the vulnerability of supply chains in general and examining key drivers of supply chain risks, the study identified supply chain risks by analyzing their likelihood to occur and their potential impact on the supply chain.

Thun et al. (2011b) used the same empirical approach and the same data as Thun and Hoenig (2011a) to analyze SCRM in small and medium-sized enterprises (SMEs). The analysis shows that SMEs predominantly focus on reactive instruments that absorb risks through the creation of redundancies instead of preventing risks.
Some researchers used empirical approaches for conducting regional ASCRM studies (e.g., Ceryno et al. (2015) for the Brazilian automotive industry, Sharma and Bhat (2014a, 2014b, 2016) and Sharma et al. (2017) for the Indian automobile industry, Doran et al. (2007) for the French automobile industry, Towill et al. (2000) for the European automotive supply chain “health check” procedure, Singh et al. (2005) for the Australian automotive manufacturing industry, Blos et al. (2009) for the Brazilian automotive and electronic industries, Lin and Zhou (2011), Xie et al. (2009), and Barclay (2008) for the Chinese automotive industry supply chain, and Shimizu et al. (2013) and Chino et al. (2017) for the Japanese automotive industry). Their research offered an initial profile and revealed insight into the regional automotive industry SCRM and helped to improve it.

Also, there is some research (e.g., Lippert and Forman (2006)) employing the empirical approach to study tiers of ASCRM. Lippert (2008) tested theoretical models through a survey of hundreds of supply chain members using an information technology innovation for part-level visibility and logistics operation along the entire first tier of a major US automotive supply chain.

Davarzani et al. (2015) used the case study method to study the influence of economic and political risks (EPRs) to supply chains. They interviewed SC professionals for three cases from an automotive SC. Sroufe and Curkovic (2008) utilized case-based research for a sample of firms in the automotive industry to examine the ISO 9000:2000 standard and supply chain quality assurance. de Oliveira et al. (2017) verified that ISO 31000:2009 can be used as a standardized method to perform SCRM. They developed a pathway to apply ISO 31000:2009 risk assessment tools and techniques to integrate a procedure for SCRM based on AHP and provided an automotive supply chain example.

- Risk Mitigation Strategies

  - Build a Resilient Supply Chain

A resilient supply chain is critical to the success of an enterprise (Hsieh et al., 2016). Determining how to build a resilient supply chain to mitigate uncertainty is a priority for automotive companies (Chen et al. 2016). Recently, supply chain managers have changed their main focus from reducing costs to giving importance to supply chain continuity and resiliency (Kırlımaçı and Erol, 2017). Rezapour et al. (2017) studied resilient supply chain
network design under competition using a real-life case study in a highly competitive automobile supply chain. They recommended three policies used to mitigate disruption risk: (1) keeping emergency stock at the retailers, (2) reserving backup capacity at the suppliers, and (3) multiple sourcing.

- **Increase Flexibility**

Flexibility is commonly associated with the ability to change or react. Owing to the importance of flexibility for achieving a competitive advantage and mitigating risks, researchers increasingly study how entire supply chains can deliver flexibility to their customers. Flexibility includes production diversification, geographic diversification, increased overall flexibility, flexible input sourcing (e.g., dual sourcing), backup suppliers, localized sourcing, flexible supply contracts, flexible manufacturing, and flexible distribution (Ceryno et al., 2015; Chopra and Sodhi, 2004; Tomlin, 2006; Chopra and Meindl, 2010; Thun and Hoenig, 2011a).

Thomé et al. (2014) studied supply chain flexibility in the automotive industry based on empirical research on three Brazilian automotive supply chains. A multiple case study was designed for the research with internal and external validity checks, within-case analysis, and cross-case comparisons.

- **Rethink the Global Supply Chain**

During recent years, offshoring and outsourcing have transformed automotive sectors into global networks of design, production, and distribution across the global value chains coordinated by the major automotive OEMs (Bailey and De Propris, 2014). As manufacturing activities tended to be shifted to low-labor-cost locations in Asia, Africa, and Latin America, high-end design, research and development, and product development have stayed and been anchored mostly to high-cost and high-knowledge-intensive home economy locations. However, very recently the weaknesses and risks inherent in such global value chains have been exposed, triggering attempts to rethink their nature and also raising possibilities to insource some manufacturing activities to home countries. Even Science Magazine had a special section entitled “Rethink the Global Supply Chain” in its June 2014 publication. Bailey and De Propris (2014) studied reshoring for opportunities and limits for manufacturing in the UK automotive sector. Thun and Hoenig (2011a),
Richardson (2005), Sharma and Bhat (2014b), Canbolat et al. (2008), and Zimmer et al. (2017) identified globalization risks in the automotive industry.

Implement Green Supply Chain Management

As the public becomes more aware of environmental issues and global warming, the environmental and social impact of supply chains has attracted considerable research attention. Green supply chain management (GSCM) has emerged as an important organizational philosophy and a proactive approach to reduce environmental risks. To ease the increasing pressures resulting from globalization and stricter regulations, address increased community and consumer pressures, and cope with developing countries' aims to enter the World Trade Organization (WTO), automotive supply chain managers have been considering and implementing GSCM practices to improve both their economic and environmental performances (Diabat et al. 2013).

There has been considerable research on GSCM for the automotive industry. Lee and Cheong (2011) and Lee (2011) used qualitative methods of interviews and document analysis to collect data on Hyundai Motor Co. and its key first-tier supplier in the Korean automobile industry. They developed a carbon footprint measurement and evaluation program in the supply chain and provided a track record to improve carbon and energy efficiency. Diabat et al. (2013) explored green supply chain practices and performances in the automotive industry using a fuzzy multiple-criteria decision-making method. Govindan et al. (2014) used an empirical approach to investigate the impact of lean, resilient, and GSCM practices on supply chain sustainability. They simultaneously studied the three dimensions of sustainability (environmental, social, and economic) and the lean, resilient, and GSCM paradigms considered strategic for supply chain competitiveness. Munguía et al. (2010) identified pollution prevention opportunities in the Mexican automotive refinishing industry to improve environmental and occupational conditions in developing countries.

Other researchers, such as Azevedo et al. (2012) and Caniëls et al. (2013), also proposed some theoretical frameworks and empirical approaches for the analysis of the influence of green and lean upstream supply chain management practices on business sustainability.
2.5.4 Practical Tools in ASCRM

Proactive SCRM can lead to greater customer satisfaction, lower total cost, improved delivery performance, and higher quality outcomes. Some practical tools have been developed for SCRM that have shown promising application results in automotive industries.

- Risk-Exposure Model
  Simchi-Levi et al. (2014, 2015) developed a new risk-exposure model to identify risks and mitigate disruptions in the automotive supply chain quantitatively. Unlike traditional SCRM methods, which rely on knowing the likelihood of occurrence and the magnitude of impact for every potential event that could materially disrupt a firm’s operation, they developed a mathematical model that focuses on the impact of potential failures at nodes along the supply chain, rather than the cause of the disruption. In their model, a supply chain network was created first. Each node stands for a supplier facility, a distribution center, or a transportation hub. Three parameters need to be determined for each node: (1) time to recovery (TTR), (2) performance impact (PI), and (3) the risk exposure index (REI). TTR is the time it would take for a particular node to be restored to full functionality after a disruption. Because accurate TTR information is not available in many cases, Simchi-Levi et al. (2015) introduced the time-to-survive (TTS) concept. TTS means the maximum amount of time that the system can function without performance loss if a particular node is disrupted. PI is a measure of the disruption at the node during TTR. REI measures how severe the risk exposure of each node is, and its value ranges between 0 and 1 (least to largest). Simchi-Levi et al.'s model allow companies to identify areas of hidden risk in the supply chain effectively.

- Failure Mode and Effect Analysis
  Failure mode and effect analysis (FMEA) has been suggested as an SCRM tool (Canbolat et al., 2008; Curkovic et al. 2013; Rewilak, 2015; Sharma and Bhat 2014b; Pandey and Sharma, 2017). FMEA is commonly used in the automotive industry to collect information related to risk management decisions in an engineering capacity, but not typically in a supply chain capacity.
Based on FMEA, Canbolat et al. (2008) developed a risk assessment and management method for sourcing components and subsystems to emerging markets for automotive OEMs. They used a process failure mode effect analysis (PFMEA) structure to characterize the risks and developed a simulation model to quantify risk factors so that an automotive OEM can evaluate risk mitigation strategies.

Curkovic et al. (2013) surveyed 67 industrial companies, including four automotive OEMs, to identify how companies manage risks through supplier assessment and selection and whether FMEA plays a role in the process. They found that some companies used the FMEA model to select and assess suppliers.

Pandey and Sharma (2017) applied an FMEA-based interpretive structural modeling approach to model automotive supply chain risk. A tractor manufacturing company was studied as an example. First, 17 potential modes of failures or risk sources were identified through the literature and weighted risk priority numbers (WRPNs) were calculated and then 11 failure models were selected as key learning aspects based on higher WRPN values. A further interpretive structural modeling approach was used to model the structural relationship among these key risks.

- Multiple-Criteria Decision-Making Models

In the literature, we found that multiple-criteria decision making (MCDM) or multiple-criteria decision analysis (MCDA) is used for risk assessment in the automotive industry, such as in supplier ranking and selection (Kull and Talluri, 2008; Palanisamy and Zubar, 2013; Datta et al., 2013; Neumüller et al., 2016).

Blackhurst et al. (2008) developed a multicriteria scoring procedure (also called a factor weighting procedure) to create risk indices for parts and suppliers in the automotive industry. The procedure has three steps: (1) Assign a weight to parts and suppliers; the weights are based on the probability of each category of disruption occurring and its impact. (the weights are based on the probability of occurrence and the impact of each category of disruption) (2) Calculate part- and supplier-specific risk indices to form a heat graph. (3) Track risk indices over time to identify trends toward higher risk levels.

Diabat et al. (2013) explored green supply chain practices and performances in the automotive industry using a fuzzy multiple-criteria decision-making method. Datta et al.
(2013) utilized fuzzy logic in an MCDM process for evaluation and selection of third-party reverse logistics providers for a reputed Indian automobile part manufacturing company.

Other ASCRM research topics include managing risks in JIT and sequence supply networks (Wagner and Silveira-Camargos, 2012), lean process supply chains (Cervi, 2007; Azevedo et al., 2012), optimized programming by resource management (Hanenkamp, 2013), entrepreneurial SCM competence and performance of manufacturing SMEs (Hsu et al., 2011), natural hedging as a risk prophylaxis and supplier financing instrument in automotive supply chains (Hofmann, 2011), and an economic P-chart model considering due date and quality risks to mitigate quality risks (Sun et al., 2012).

Other practical tools in ASCRM include a strategic materials positioning matrix (SMPM) (Saueressig et al., 2017). Using a case study method, SMPM was applied to two families of items (bolts and plastic finishing) purchased by an automotive industry company in southern Brazil. The materials were organized by SMPM into four classes: noncritical, strategic, risk, and competitive. The result of the analysis showed a significant reduction of shortages in the assembly line and storage facility units required for warehousing.

2.5.5 Summary
A summary of the literature reviewed for risk management in this paper is shown in Figure 2.4. The SCRM research methodologies reviewed are given in Table 4.
Figure 2.4. Summary diagram of the literature reviewed for risk management in this study

**TABLE 2.4: SUPPLY CHAIN RISK MANAGEMENT (SCRM) MODEL, STRATEGY, AND TOOLS**

<table>
<thead>
<tr>
<th>SC Risk Research Methodology</th>
<th>Reference in Automotive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Modeling</td>
<td></td>
</tr>
<tr>
<td>Linear programming</td>
<td>Kırılmaz and Erol (2017); Caux et al. 2006</td>
</tr>
<tr>
<td>Nonlinear programming</td>
<td>Zhang and Zhu (2012)</td>
</tr>
<tr>
<td>Mixed integer linear programming</td>
<td>Dal-Mas et al. (2011); Elmaraghy and Majety (2008)</td>
</tr>
<tr>
<td>Mixed integer nonlinear programming</td>
<td>Rezapour et al. (2017); Mohammaddust et al. (2017)</td>
</tr>
<tr>
<td>Stochastic model</td>
<td>Nakashima et al. (2014)</td>
</tr>
<tr>
<td>Game theory</td>
<td>Naini et al. (2011); Swinney and Netessine (2009), Yang et al. (2017)</td>
</tr>
<tr>
<td>Newsvendor</td>
<td>Nakashima and Sornmanapong (2013)</td>
</tr>
<tr>
<td>Simulation</td>
<td>Lalwani et al. (2006); Canbolat et al. (2008); MacKenzie et al. (2014); Nakashima and Sornmanapong (2013)</td>
</tr>
<tr>
<td>Quantitative-based Strategy</td>
<td></td>
</tr>
<tr>
<td>Supply chain coordination and cooperation</td>
<td>Belzowski et al. (2006); Zhang and Zhu (2012); Matsuo (2015); Montshiwa et al. (2016)</td>
</tr>
<tr>
<td>Dual sourcing</td>
<td>Davarzani et al. (2011)</td>
</tr>
</tbody>
</table>
2.6 Conclusion and Future Research

This study reviewed the existing research work on ASCRM. We first classified the risks into ten categories and discussed risk assessment methods, and then focused on summarizing the research on ASCRM’s mathematic modeling, quantitative approaches, qualitative approaches, and practical tools.

In the modeling approach, optimization models are widely used; these include linear and nonlinear programming, mixed integer programing, and multi-objective models. Newsvendor models are used for formulating the problem with uncertain demands, and game theory are also employed for supply chain coordination and supply contracts.
Qualitative approaches include empirical approaches and risk mitigation strategies. The empirical approach is a popular method in ASCRM research. About 60% of the research employs the empirical approach to analyze risk and to assess risk management strategies qualitatively in the automotive industry (González-Benito et al., 2013). Also, some new research trends in risk mitigation strategies can be applied to the automotive supply chain; these include building a resilient supply chain and addressing the environmental impact of supply chain risks. Some practical tools such as multiple-criteria decision-making methods, FMEA, and PFMEA simulation models, and risk-exposure models have been utilized to mitigate risk in automotive supply chains.

The major conclusions from this study of ASCRM are as follows: (1) Research on ASCRM has been increasing in recent years. (2) Most papers are published in the following journals: International Journal of Production Economics, International Journal of Production Research, Journal of Cleaner Production, International Journal of Physical Distribution and Logistics Management, Industrial Management and Data Systems, and International Journal of Advanced Manufacturing Technology. (3) The main mathematical models include optimization models, such as linear and nonlinear programming, mixed integer programming, multi-objective models, newsvendor models, and game theory; tools such as value at risk and conditional value at risk have rarely been used. (4) The automotive industry recognizes the importance of ASCRM and expends considerable effort on its investigation, but academic research is thus far insufficient. (5) The automotive industry has a very complex, multitier SCM and the risk sources of automotive supply chains are widespread. (6) There are some practical tools and risk mitigation strategies, such as MCDM and FMEA, but there is a lack of real-time risk alarm systems and tools for supply chain resilience.

Based on the reviewed papers in ASCRM, we propose a few important future research directions as follows:

1). Study and development of systematic methods and systems to analyze ASCRM by integrating the different risk sources: The automotive supply chain is a huge multitier suppliers network. A typical automotive OEM has up to 10 tiers between itself and its raw materials. For example, Ford has ~1,400 Tier-1 suppliers across 4,400 manufacturing sites.
and hundreds of thousands of lower-tier suppliers (Simchi-Levi et al., 2015). Most research work in the automotive industry is based on specific points of views, i.e., the supplier’s or manufacturer’s. There is no systematic method to analyze and integrate different ASCRM strategies, such as how to choose supply locations, transportation, to optimize the objectives for both manufacturers and suppliers as a system and to reduce geographic or political risks in the automotive supply chain in the global environment. The existing research on the impact on the automotive network resulting from supply chain risk has not been sufficient. This area requires more related research to be conducted in the future.

2). Use of data- and big-data-based ASCRM: One of the major automotive supply chain risks is nontransparency resulting from the multiple layers in the supply chain. Typically, OEMs have substantial data about Tier-1 suppliers, but they lack data from Tier-2 to Tier-N suppliers. Nontransparency makes it very difficult to monitor risks and issue warnings. Big-data analytics can provide a basis for transparency in automotive supply chains. With the help of real-time big data availability, OEMs and suppliers can improve their supply chain transparency, monitor the occurrence of risks, provide early warnings and responses, and enable managers to use the developed risk mitigation strategy to prevent the risks. Big-data-based sense-and-respond systems for ASCRM are worth further research.

3). Study of the downstream risks to the automotive supply chain: There is not enough research on automotive-industry-specific models. The special aspect of the automotive supply chain is its complexity owing to the huge number of multitier suppliers and globalized network. Further study is required on downstream risks to the automotive supply chain besides demand uncertainty, such as call-back risk and how to build a resilient network.

4). Implementation of more quantitative models: Through a literature review, we found that the majority of research on automotive supply chain risk employs empirical approaches. There is a real demand in the automotive industry to use quantitative models to evaluate supplier risks in terms of different tiers and different type of suppliers. Some quantitative models just address simple two-layer supply chains owing to the lack of deep downstream suppliers’ information. Various mathematical models have been developed to assist in planning under uncertainties with simplified real-life situations (Singhal et al.,
Proposed future work includes quantitative model development for complex ASCRM and improvement of the mathematical models to cope with real-life situations. Traditional risk modeling, including the use of the utility function, variance, standard deviation, mean–variance, value at risk, and conditional value at risk, has rarely been applied to ASCRM. These modeling methods can be applied to ASCRM.

5). Addressing advanced technology challenges: In recent decades, revolutions in information technology and telecommunications has brought about dramatic changes in our daily lives and the automotive industry as well. Automakers continuously offer new high-technology features in their products (e.g., GPS, telematics, various sensors, ADAS, RFID, etc.). These high-technology features present many technological challenges in the automotive supply chain. One of these challenges is the risk posed to vehicle design, production, quality, and after-sales services by the short product development cycle and the long useful life of vehicles. Automotive manufacturers must mitigate risk through their component suppliers. Future research needs to be done on car manufacturers' selection of proper suppliers and on improving coordination and cooperation among supply chain vendors.

6). Research on autonomous cars and car-sharing services: In recent years, autonomous cars have emerged as the future of the automotive industry. Experts have predicted that fully autonomous cars will arrive at the market by 2025 to 2030 (Liuima, 2016). Car-sharing services using autonomous vehicles could be attractive for many private buyers as well. It is suggested that new-car sales in the US could be eroded by as much as 40%. Like any new product, autonomous cars will have demand uncertainty because of many obstacles, such as adoption rate, technological challenges, liability disputes, laws, and regulations. Demand uncertainty implies overcapacity risk or under capacity risk. Future work includes improving forecast accuracy to optimize contracts and production capacity and to reduce supply chain risk.

This research tries to fill the research gap that lacks quantitative models for ASCRM. In the following chapters, we will implement two mathematic models for ASCRM research.
(1) Model for supplier development and manufacturer cooperation with a nonlinear return to minimize risk. The model and numerical experiment will be illustrated in Chapter 3.

(2) Optimal contract model for product development with risk consideration (penalty and compensating). Chapter 4 will develop the mathematical model. Chapter 5 will apply the model to a uniform distribution case for numerical experiment and sensitivity analysis for a real-world ASCRM case.
CHAPTER 3
OPTIMIZING MANUFACTURER COOPERATION IN SUPPLIER DEVELOPMENT UNDER RISK

3.1 Introduction
An automotive supply chain is a multiple layered, complex network, so the relationship between supply chain members is very important for risk management. Supplier development is a long-term, resource-consuming business activity that requires commitment from both manufacturers and suppliers. Manufacturers often spend a great amount of investment in supplier development to improve their suppliers’ capabilities and performances. The automotive industry is no exception. All of the automotive OEMs, such as Toyota, Honda, and the Big Three U.S. automakers, Chrysler, Ford, and General Motors, have implemented supplier development programs to assist suppliers (Liker & Choi, 2004), which have resulted in quality improvement and cost reduction. However, there is a risk due to the uncertainty of returns in supplier development. For example, for the same investment of resources, the return from each supplier can vary from the manufacturer’s expectation. Furthermore, supplier development poses potential opportunistic behavior on the part of the supplier (Wagner, 2006), which may lead to total failure or termination of the relationship earlier than expected.

How to perform risk management in new product development and supplier development, or how to allocate the investment optimally among multiple suppliers to minimize risk while maintaining an acceptable level of return is a critical issue faced by manufacturers or automotive OEMs. This research tries to address these problems. This chapter is organized as follows. First, a literature review about risk management in new product and supplier development is provided in section 3.2. In section 3.3, Markowitz’s mean-variance portfolio theory and Talluri’s manufacturer investment portfolio model are introduced. The proposed non-linear return model in both SMMS and TMMS are developed in section 3.4. Moreover, numerical experiments and results are shown in section 3.5. Finally, a summary of this chapter is provided in section 3.6.
3.2 Literature Review

We first review the available papers related to the keywords “risk management in new product and supplier development” searching from Scopus.

Quigley, et al. 2018 studied supplier development decisions for prime manufacturers with extensive supply bases producing complex, highly engineered products. They proposed a novel modeling approach, a Poisson–Gamma model within a Bayesian framework, which can help/allow supply chain managers to decide the optimal level of investment and improve quality performance under uncertainty. The model helps to understand the relationship between the degree of epistemic uncertainty, the effectiveness of development programs, and the levels of investment. It was found that the optimal level of investment does not have a monotonic relationship with the rate of effectiveness. The expected optimal investment monotonically decreases if an investment decision was deferred until the epistemic uncertainty is removed, because the prior variance increases, but only if the prior mean is above a critical threshold. Several methods were developed to apply the model to industrial decisions in practice, which enables the model to be used with typical data available to major companies, and also with computationally efficient approximations that can be implemented easily. The model was applied to a real industry example. The results support the planning decisions (already in practice?): to learn more about supplier quality and to invest in improving supplier capability.

Recently, Mizgier et al. (2017) published their work on multi-objective capital allocation for supplier development under risk. Due to the complexity of today’s supply chains and the globalization of businesses, the importance of supplier development has been increased significantly. Manufacturers need to make an informed decision to choose and develop only a fraction of the suppliers since their resources are limited. Furthermore, when choosing suppliers for a development program, manufacturers have the risk of uncertain returns from this investment. The authors proposed a multi-objective model for capital allocation for supplier development under risk. Their model was applied to an example of a global car manufacturer and supports the decision-making process about which suppliers are selected for the development program. A supplier’s performance is assessed by stock market returns and the cost of capital of suppliers. An informed decision about the tradeoffs between risk and cost of a supplier development program can be reached based on their
multi-objective model. In the paper, they demonstrated the different allocation schemes for supplier development depending on the risk averse-ness of the manufacturer.

Proch et al. (2017) studied supplier development in a decentralized supply chain with a single manufacturer and a single supplier. Supplier development usually requires relationship-specific investments. It is crucial for the participating firms to decide the allocation of investment. The effects of relationship-specific investments on the efficiency and effectiveness of supplier development were investigated in the study referencing the relational view. Then they formulated and solved a continuous time optimal control model. The results characterized the decision to invest in supplier development and showed that the supplier's incentive to participate in supplier development depends critically on the manufacturer's share of investment costs. Through the numerical analysis, they also found that although the subsidy can lead to significant improvement in supply chain performance, subsidizing a constant share of investment costs is not always economically reasonable from the manufacturer's point of view. Therefore, they proposed a negotiation-based algorithm that assists the manufacturing firm in gradually increasing the share of investment costs to ensure an efficient level of subsidy, resulting in both perfect supply chain coordination and a win-win situation.

Hosseini et al. (2015) introduced a two-phase supplier selection procedure for selecting a supplier portfolio based on value, development, and risk consideration. In most of the existing supplier selection research, the supplier selection decisions are based on supplier eligibility at the time of the decision making. The proposed two-phase method in this paper is based on the long-term trend of value, stability, and relationship of potential suppliers. In the first phase, a set of criteria are used to evaluate suppliers and assign a comparable value to them. In the second phase, this value is analyzed for the long term, using a multi-objective portfolio optimization model. A supplier portfolio is determined by maximizing the expected value for the development of suppliers and minimizing their correlated risk. Their procedure provides a new view of the supplier selection problem. Numerical tests using the proposed approach showed promising results by selecting for higher value suppliers with a lower risk of failure.
Chiang and Wu, (2016), studied early supplier involvement as well as contract design during new product development. Using upfront supplier resources and expertise, and allowing for risk sharing with suppliers have become vital for the new product development. This is due to the intricacy of interfirm collaboration while dealing with unproven technology and market uncertainty. However, it remains difficult to achieve a successful implementation of early supplier involvement (ESI) in a new product development phase. In this paper, the game theoretical contracting strategies are proposed to achieve manufacturer objectives, such as predictable design timelines, sufficient supplier commitment, and radical in-process innovations. Based on real options analysis, an compatible incentive mechanism was designed to suggest which project stage to engage the supplier best, while considering various project factors, such as revenue forecast, technical uncertainty, market competition, and team capability. On the other side, the supplier can use the analysis to determine whether to participate and if so, the appropriate level of resource commitment. The equilibrium analysis provides managerial insights into how to best balance the time-to-market mandate with the need for accruing significant innovations through supply chain partnerships.

Choi et al. (2008) studied mean-variance analysis of the newsvendor problem for inventory management with stochastic demand. The objective of a typical newsvendor problem is to either minimize the expected cost or maximize the expected profit. However, using the expected values to measure the performance alone is not sufficient for decision makers, because it ignores their risk preference. Authors conducted a mean-variance analysis of the newsvendor problem and developed analytical models and investigated the problem's structural properties. They also discussed a case with a stockout penalty cost and a safety-first objective. The proposed solution schemes can help to identify the optimal solutions.

Wei and Choi, (2010), extended the research work of Choi et al. 2008 on mean-variance analysis to supply chains under wholesale pricing and profit sharing schemes. The research explored the use of wholesale pricing and profit sharing scheme (WPPS) for coordinating supply chains under the mean-variance (MV) decision framework. First, the necessary and sufficient conditions for coordinating the centralized supply chain by WPPS were established analytically. Then they found that there exists a unique equilibrium of the Stackelberg game with WPPS in the decentralized case. They also discussed that in the
asymmetric information case, the retailer could pretend to be more risk-averse and benefit from it. As a result, a new measure was proposed for the manufacturer to prevent this cheating from happening.

Grochowski and Ohlhausen, (2015), studied cooperation models as the success factor for interdisciplinary, inter-organizational research and development (R&D) in the automotive industry. The automotive industry has been challenged by the shortage of fossil fuels, the politics of global warming and rising competition from new markets nowadays. The stable success of automotive companies depends on the development of more efficient and alternative fuel vehicles with new technologies and materials that meet the individual requirements of the customers. The development of automobiles is so complex that it requires the skills of various engineering and science disciplines, which are spread all over the supply chain. Hence the only way to stay successful in the automotive industry is by cooperation and collaborative innovation. Therefore, cooperation models for interdisciplinary and interorganizational development are in high demand and are very critical in the automotive industry. This paper used a case study (research campus ARENA2036 in Germany) to analyze and evaluate the cooperation models according to the applicability to interdisciplinary, interorganizational development projects in the automotive industry. ARENA2036 is the largest and leading research platform for mobility in Germany, housing automobile manufacturers, suppliers, research establishments and university institutes. Based on interviews with the partners and the preceding analyses of cooperation models, suggestions for implementation are given to ARENA2036 for investments, agreements, communications, and flexible adaption tasks.

Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation in supplier development under risk. The authors presented a set of optimization models that address supplier development undertaken by manufacturing firms to improve their suppliers’ capabilities and performances. They considered two scenarios: the single manufacturer and multiple suppliers (SMMS) case and the two manufacturers and multiple suppliers (TMMS) case. In the SMMS case, authors suggested optimal investments in various suppliers by effectively considering risk and return. In the TMMS case, they investigated whether manufacturers with differing capabilities could gain risk reduction benefits from cooperating with each other in supplier development.
From the literature review above, we can see the recent research trend in the area of risk management in new product and supplier development. Due to the complexity of modern supply chains and the globalization of businesses, the importance of supplier development has increased significantly recently (Mizgier et al. 2017). Also due to the resource limits and uncertainty in the supply chain, how to conduct risk management in new product and supplier development, or how to select, coordinate, cooperate with suppliers, and allocate the investment optimally among multiple suppliers to minimize risk while maintaining an acceptable level of return is getting more attention in academia and industry.

In summary, the research topics for risk management in new product and supplier development are: optimal level of investment or capital allocation for supplier development under risk (Mizgier, et al. 2017, Quigley, et al. 2018); supplier selection (Hosseininasab, et al. 2015), early supplier involvement and contract design during new product development, with risk sharing with suppliers (Chiang and Wu, 2016); and cooperation for supplier development investments (Grochowski and Ohlhausen, 2015, Talluri et al. 2010). The mathematical models include Poisson–Gamma model within a Bayesian framework (Quigley, et al. 2018), game theory (Chiang and Wu, 2016), newsvendor (Choi et al. 2008), mean-variance analysis (Choi et al. 2008, Wei and Choi, 2010), Markowitz’s model (Talluri et al. 2010). Table 3.1 shows a summary of the literature review. During the literature review, we found that there is limited research that deals with optimizing manufacturer cooperation for supplier development under risk in the automotive industry. This research is conducted in order to fill the gap.

Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation for supplier development under risk. Talluri’s model assumed that the return of investment to suppliers is proportional to the investment. Actually, in most situations the return is nonlinear. This research revised the above manufacturer cooperation model with a nonlinear return and intended to apply it to the auto industry.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Subject</th>
<th>Supply Chain Structure</th>
<th>Model</th>
<th>Solution Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quigley, et al. (2018)</td>
<td>Supplier quality improvement: The value of information under uncertainty</td>
<td>prime manufacturers with extensive supply bases producing complex, highly engineered products</td>
<td>a Poisson–Gamma model within a Bayesian framework, which can support supply chain managers to decide the optimal level of investment and improve quality performance under uncertainty</td>
<td>the model is used with typical data available to major companies, and with computationally efficient approximations that are implemented easily</td>
</tr>
<tr>
<td>Mizgier, et al. (2017)</td>
<td>Multi-objective capital allocation for supplier development under risk</td>
<td>Manufacturers need to decide to choose and develop some fraction of suppliers</td>
<td>A multi-objective model for capital allocation for supplier development under risk</td>
<td>The model was applied to a global car manufacturer and supported the decision-making process to select which suppliers for the development program. A supplier’s performance is assessed by stock market returns and cost of capital of suppliers</td>
</tr>
<tr>
<td>Proch, et al. (2017)</td>
<td>A negotiation-based algorithm to coordinate supplier development in decentralized supply chains</td>
<td>a decentralized supply chain with a single manufacturer and a single supplier</td>
<td>a continuous time optimal control model</td>
<td>Numerical analysis, and proposed a negotiation-based algorithm to assists the manufacturing firm in gradually increasing the share of investment costs to ensure a supply chain coordination and a win-win situation</td>
</tr>
<tr>
<td>Hosseininasab, et al. (2015)</td>
<td>Selecting a supplier portfolio with value, development, and risk consideration</td>
<td>a two-phase supplier selection procedure for selecting a supplier portfolio using value, development, and risk consideration</td>
<td>a multi-objective portfolio optimization model</td>
<td>A supplier portfolio is determined by maximizing the expected value and development of suppliers and minimizing their correlated risk</td>
</tr>
<tr>
<td>Chiang and Wu, (2016)</td>
<td>Supplier Involvement and Contract Design during New</td>
<td>Manufacturer and early supplier involvement for the new product development</td>
<td>game theoretical contracting strategies</td>
<td>The equilibrium analysis provides managerial insights into how to best balance the time-to-market mandate</td>
</tr>
<tr>
<td>Product Development</td>
<td>Choi et al. (2008)</td>
<td>Mean-Variance Analysis for the Newsvendor Problem</td>
<td>inventory management with stochastic demand.</td>
<td>with the need for accruing significant innovations through supply chain partnerships</td>
</tr>
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<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Wei and Choi, (2010)</td>
<td>Mean-variance analysis of supply chains under wholesale pricing and profit sharing schemes</td>
<td>Manufacturer and suppliers in centralized, decentralized supply chain</td>
<td>extended the research work of Choi, et al. 2008 on mean-variance analysis to supply chains under wholesale pricing and profit sharing schemes</td>
<td>found that there exists a unique equilibrium of the Stackelberg game with wholesale pricing and profit sharing scheme (WPPS) in the decentralized case</td>
</tr>
<tr>
<td>Grochowski and Ohlhausen, (2015)</td>
<td>Cooperation models as a success factor for interdisciplinary, inter-organizational research and development in the automotive industry</td>
<td>automotive industry supply chain cooperation and collaborative innovation</td>
<td>a case study to analyze and evaluate the cooperation models according to the applicability to interdisciplinary, interorganizational development projects in the automotive industry</td>
<td>Based on interviews with partners and the preceding analyses of cooperation models, suggestions for implementation are given to ARENA2036 for investments, agreements, communications, and flexible adaption tasks</td>
</tr>
<tr>
<td>Talluri et al. (2010)</td>
<td>Manufacturer cooperation in supplier development under risk</td>
<td>SMMS - single manufacturer and multiple suppliers TMMS - two manufacturers and multiple suppliers</td>
<td>Applied Markowitz’s model to manufacturer cooperation in supplier development under risk; the objective function is to minimize the risk while achieving the target return when the manufacturer allocates the investment amounts</td>
<td>the mean-variance method with linear return</td>
</tr>
</tbody>
</table>
3.3 Mathematic Model

3.3.1 Markowitz Model
Markowitz model, or mean-variance analysis, also known as the modern portfolio theory (MPT), was introduced by economist Harry Markowitz in 1950’s, for which the author was later awarded a Nobel Prize in economics [1]. The model assembles a portfolio of assets mathematically to maximize the expected return for a given level of risk, defined as variance. The key point of the model is that an asset's risk and return should be assessed by how it contributes to a portfolio's overall risk and return. Markowitz model is used as risk measurement. Mossin (1973), Choi et al. (2008), and Wei and Choi (2010) studied problems in inventory and supply chain systematically using the mean-variance method.

3.3.2 Talluri’s Model – Linear Return Model
Talluri et al. (2010) applied Markowitz’s model to the manufacturer investment portfolio. Figure 3.1 shows the case where a single manufacturer engages in supplier development efforts with multiple suppliers (SMMS).

![Diagram showing a single manufacturer and multiple suppliers](image)

- \( x_j \): amount manufacturer invests in supplier \( j \);
- \( R_j \): return rate from investing in supplier \( j \) from M1

Figure 3.1. Single manufacturer and multiple supplier case (Talluri et al, 2010)
In SMMS, the objective function is to minimize the risk while achieving the target return when the manufacturer allocates the investment amounts. Risk is affected by variability of returns from suppliers and the amounts invested.

We minimize this objective function:

\[
\text{Var}\left( \sum_{j=1}^{n} x_j R_j \right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left( \sum_{j=1}^{n} (r_{jt} - r_j) x_j \right)^2
\]

Subject to these constraints:

\[
\sum_{j=1}^{n} x_j = X, \quad \text{budget constraint}
\]

\[
\sum_{j=1}^{n} r_j x_j \geq \rho X \quad \text{return expectation constraint}
\]

\[
l_j \leq x_j \leq \mu_j, \quad \forall j = 1 \sim n \quad \text{manufacturer investment constraint}
\]

Where \( j \) is the \( j^{th} \) supplier;

\( n \) the total number of suppliers;

\( x_j \) is the amount the manufacturer invests in supplier \( j \);

\( X \) is the total budget;

\( \mu_j \) is the maximum amount that can be invested in supplier \( j \);

\( l_j \) is the minimum amount that needs to be invested in supplier \( j \);

\( R_j \) is the random variable representing “rate of return” from investing in supplier \( j \);

\( \rho \) is the minimum overall expected rate of return required by manufacturer.

### 3.4 Proposed Nonlinear Return Model

Zhang and Zhu (2012) updated/replaced the above manufacturer cooperation model with a nonlinear return model and applied it to the auto industry. Talluri’s model assumes that the return of investment to a supplier is proportional to the investment. In fact, in most situations the return is nonlinear. For example, to improve the quality of products, a new technique should be used or new equipment with high performance should be purchased.

This study extends the work by Talluri et al. (2010) by developing a new investment scheme and examining how Markowitz’s model can be used to help firms optimally allocate resources under nonlinear returns and then comparing the results to the linear case.
One example where the return $r_j$ and the investment $x_j$ follows a nonlinear relationship is shown below in Figure 3.2 (Zhang, 2010):

![Figure 3.2. A nonlinear relationship between investment $x_j$ and return $r_j$](image)

3.4.1 The SMMS Model

First, we extended Talluri’s SMMS model to replace the linear relationship with a nonlinear relationship between the return rate $r_j$ and the investment $x_j$ to. The goal of the manufacturer is to allocate investment amounts so that a target return is achieved at minimum risk. The risk is affected by the variability of returns from suppliers and amounts invested. The objective function is to minimize the variance of the supplier development investment portfolio:

$$Var \left( \sum_{j=1}^{n} x_j r_j \right)$$  \hspace{1cm} (3.1)

Subject to:

$$\sum_{j=1}^{n} x_j = X$$  \hspace{1cm} (3.2)

$$\sum_{j=1}^{n} \sum_{i=1}^{k_j} r_{ij} x_{ij} \geq \rho X$$  \hspace{1cm} (3.3)

$$x_{ij} \geq r_{ij}^l y_{ij} \hspace{1cm} \forall i,j$$  \hspace{1cm} (3.4)

$$x_{ij} \leq r_{ij}^u y_{ij} \hspace{1cm} \forall i,j$$  \hspace{1cm} (3.5)
\[ y_{ij} = \begin{cases} 1, & \text{if investor } j \text{ invests in level } i \\ 0, & \text{otherwise} \end{cases} \quad \forall i, j \] (3.6)

\[ \sum_{i=1}^{k_j} y_{ij} = 1 \quad \forall j \] (3.7)

\[ \sum_{i=1}^{k_j} x_{ij} = x_j \quad \forall j \] (3.8)

\[ l_j \leq x_j \leq u_j \quad \forall j \] (3.9)

\[ x_j, x_{ij} \geq 0 \quad \forall i, j \] (3.10)

Where \( R \) is the return rate;

\( X \) is the total available budget for the next period;

\( i \) is the \( i \)th investment level;

\( \rho \) is the average return from \( n \) suppliers;

\( r_{ij}^l \) is the return lower limit;

\( r_{ij}^u \) is the return upper limit;

\( y_{ij} = 1 \) if supplier \( j \) invests \( i \)th level, \( y_{ij} = 0 \) otherwise.

To evaluate the objective function, (as Talluri et al. 2010, page 168), we have:

\[ \text{Var} \left( \sum_{j=1}^{n} x_j R_j \right) = \left( 1/T \right) \sum_{t=1}^{T} \left[ \sum_{j=1}^{n} E_{jt} x_j \right]^2 = \left( 1/T \right) \sum_{t=1}^{T} \left[ \sum_{j=1}^{n} \sum_{i=1}^{k_j} (r_{ijt} - r_{ij}) x_{ij} \right]^2 \] (3.11)

where

\[ E_{jt} = \sum_{i=1}^{k_j} y_{ij}(r_{ijt} - r_{ij}) \] (3.12)

\[ \sum_{j=1}^{n} E_{jt} x_j = \sum_{j=1}^{n} \left\{ \sum_{i=1}^{k_j} y_{ij}(r_{ijt} - r_{ij}) \sum_{i=1}^{k_j} x_{ij} \right\} \]

\[ = \sum_{j=1}^{n} \left\{ [y_{1j}(r_{i1t} - r_{1j}) + y_{2j}(r_{i2t} - r_{2j})][x_{1j} + x_{2j}] \right\} \]

\[ = \sum_{j=1}^{n} \left\{ [x_{1j}(r_{i1t} - r_{1j}) + x_{2j}(r_{i2t} - r_{2j})] \right\} \]

(3.13)
3.4.2 The TMMS Model

Secondly, we apply a non-linear relationship between return rate \( r_j \) and investment \( x_j \) to Talluri’s TMMS model.

We assume cooperation, and that the return rate depends on the sum of investment of two manufacturers. We wish to minimize

\[
\text{Var} \left( \sum_{j=1}^{n} x_j R_j + \sum_{j=1}^{n} x'_j R'_j \right) \tag{3.14}
\]

Subject to:

\[
\sum_{j=1}^{n} x_j = X \tag{3.15}
\]

\[
\sum_{j=1}^{n} x'_j = X' \tag{3.16}
\]

\[
\sum_{j=1}^{n} \sum_{i=1}^{k_j} r_{ij} (x_{ij} + x'_{ij}) \geq \rho X \tag{3.17}
\]

Figure 3.3. TMMS model (Talluri et al, 2010)

\( x_j \): amount manufacturer invests in supplier \( j \);

\( x'_j \): amount manufacturer 2 invests in supplier \( j \);

\( R_j \): return rate from investing in supplier \( j \) from M1;

\( R'_j \): return rate from investing in supplier \( j \) from M2.
\[
\sum_{j=1}^{n} \sum_{i=1}^{k_j} r_{ij} (x_{ij} + x_{ij}') \geq \rho' X' \quad (3.18)
\]
\[
\sum_{j=1}^{n} \sum_{i=1}^{k_j} \max[r_{ij}, r_{ij}'] (x_{ij} + x_{ij}') \geq \max[\rho, \rho'] (X + X') \quad (3.19)
\]
\[
x_{ij} + x_{ij}' \geq r_{ij} y_{ij} \quad \forall i, j \quad (3.20)
\]
Constraint (3.20) is used in the single manufacturer case. In the two-manufacturer case, it becomes (3.21)
\[
x_{ij} + x_{ij}' \geq r_{ij} y_{ij} \quad \forall i, j \quad (3.21)
\]
\[
x_{ij} + x_{ij}' \leq r_{ij} y_{ij} \quad \forall i, j \quad (3.22)
\]
Constraint (3.22) is used in the single manufacturer case. In the two-manufacturer case, it becomes (3.23)
\[
x_{ij} + x_{ij}' \leq r_{ij} (y_{ij} + y_{ij}') \quad \forall i, j \quad (3.23)
\]
\[
\sum_{j=1}^{k_i} y_{ij} = 1 \quad \forall i \quad (3.24)
\]
\[
\sum_{i=1}^{k_j} x_{ij} = x_j \quad \forall j \quad (3.25)
\]
\[
\sum_{i=1}^{k_j} x_{ij}' = x_j' \quad \forall j \quad (3.26)
\]
\[
l_j \leq x_j \leq \mu_j \quad \forall j \quad (3.27)
\]
\[
l_j' \leq x_j' \leq \mu_j' \quad \forall j \quad (3.28)
\]
\[
x_j, x_{ij} \geq 0 \quad \forall i, j \quad (3.29)
\]
\[
y_{ij} = \{0, 1\} \quad \forall i, j \quad (3.30)
\]
To evaluate the objective function, (as Talluri et al. 2010, page 168), we have:
\[
\text{Var} \left( \sum_{j=1}^{n} x_j R_j \right) = \left( \frac{1}{T} \right) \sum_{t=1}^{T} \left[ \sum_{j=1}^{n} E_{jt} x_j + \sum_{j=1}^{n} E_{jt}' x_j' \right]^2 = \left( \frac{1}{T} \right) \sum_{t=1}^{T} \sum_{j=1}^{n} \left[ x_{ij} + \sum_{i=1}^{k_j} (r_{ij} - r_{ij}') x_{ij}' \right]^2
\]
\[
(3.31)
\]
since
\[
E_{jt} = \sum_{i=1}^{k_j} x_{ij} (r_{ijt} - r_{ij}) \quad \text{and} \quad E'_{jt} = \sum_{i=1}^{k_j} x'_{ij} (r'_{ijt} - r'_{ij})
\] (3.32)

3.5 Numerical Experiments and Results

We use the data from Talluri et al. (2010)’s paper for our numerical experiments. Manufacturer 1’s historical return data in Table 3.2 are randomly generated from four normal distributions, N(0.15, 0.0225), N(0.2, 0.04), N(0.25, 0.0625), and N(0.3, 0.09), for suppliers 1, 2, 3 and 4, respectively. The average return for manufacturer 1 from the four suppliers is \( \rho = 0.225 \). Manufacturer 1’s total investment budget is assumed to be \( X = \$100,000 \). The maximum investment limit for every supplier is \$50,000.

Manufacturer 2’s historical return data in Table 3.3 are randomly generated for suppliers 1, 2, 3, and 4 with Normal distributions, N(0.2, 0.04), N(0.25, 0.0625), and N(0.3, 0.09), and N(0.35, 0.1225), respectively, with all coefficient of variation (CV) controlled at 1 (see Table 3.3). Manufacturer 2’s budget is also assumed to be \$100,000. The average return for manufacturer 2 from the four suppliers is \( \rho = 0.275 \).

**Table 3.2. Manufacturer 1’s expected returns and actual historical returns by the supplier**

<table>
<thead>
<tr>
<th>Actual returns</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
<th>( r_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>0.1725</td>
<td>0.1248</td>
<td>0.1706</td>
<td>0.1882</td>
<td>0.1229</td>
<td>0.1482</td>
<td>0.1521</td>
<td>0.1380</td>
<td>0.1301</td>
<td>0.1295</td>
<td>0.1500</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>0.2098</td>
<td>0.2053</td>
<td>0.2014</td>
<td>0.1815</td>
<td>0.2151</td>
<td>0.2140</td>
<td>0.2512</td>
<td>0.2611</td>
<td>0.2126</td>
<td>0.2586</td>
<td>0.2000</td>
</tr>
<tr>
<td>Supplier 3</td>
<td>0.1132</td>
<td>0.2169</td>
<td>0.2103</td>
<td>0.2438</td>
<td>0.2549</td>
<td>0.2655</td>
<td>0.2272</td>
<td>0.2059</td>
<td>0.2188</td>
<td>0.1846</td>
<td>0.2500</td>
</tr>
<tr>
<td>Supplier 4</td>
<td>0.2893</td>
<td>0.2385</td>
<td>0.2131</td>
<td>0.1262</td>
<td>0.2080</td>
<td>0.4015</td>
<td>0.3539</td>
<td>0.2041</td>
<td>0.2612</td>
<td>0.3659</td>
<td>0.3000</td>
</tr>
</tbody>
</table>

Unit: \$/per dollar invested.
Expected return: \( r_j \).

**Table 3.3: Manufacturer 2’s expected returns and actual historical returns by the supplier**

<table>
<thead>
<tr>
<th>Actual returns</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
<th>( r_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier 1</td>
<td>0.2301</td>
<td>0.1911</td>
<td>0.2237</td>
<td>0.2163</td>
<td>0.1606</td>
<td>0.2421</td>
<td>0.2283</td>
<td>0.1553</td>
<td>0.2783</td>
<td>0.2331</td>
<td>0.2000</td>
</tr>
<tr>
<td>Supplier 2</td>
<td>0.2578</td>
<td>0.3305</td>
<td>0.2412</td>
<td>0.2137</td>
<td>0.2860</td>
<td>0.3473</td>
<td>0.2654</td>
<td>0.2584</td>
<td>0.2522</td>
<td>0.2211</td>
<td>0.2500</td>
</tr>
<tr>
<td>Supplier 3</td>
<td>0.3918</td>
<td>0.2606</td>
<td>0.3446</td>
<td>0.3401</td>
<td>0.1493</td>
<td>0.3325</td>
<td>0.4027</td>
<td>0.1415</td>
<td>0.2428</td>
<td>0.2583</td>
<td>0.3000</td>
</tr>
<tr>
<td>Supplier 4</td>
<td>0.4882</td>
<td>0.3894</td>
<td>0.3991</td>
<td>0.5917</td>
<td>0.3724</td>
<td>0.3304</td>
<td>0.3304</td>
<td>0.1658</td>
<td>0.2380</td>
<td>0.3422</td>
<td>0.3500</td>
</tr>
</tbody>
</table>

Unit: \$/per dollar invested.
Manufacturer 2 is assumed to have the better capability in implementing supplier development initiatives than manufacturer 1 for every candidate supplier.

In the following numerical experiment, we use a GAMS algorithm to model every case. The experiment has two steps. In the first step, the linear return model, or Talluri’s original model is studied to check if our GAMS algorithm is correct. In the second step, the nonlinear return model, or our proposed model is studied. Then a comparison is made between the results of both models.

### 3.5.1 Results for SMMS Case

Using GAMS on the model and the data from Table 3.2, we get the following SMMS results in Figure 3.4 below

- Linear return, which is the same as a result of the Talluri et al. (2010) paper. It verifies our GAMS algorithm.

![Figure 3.4. Supplier allocation vs overall expected return (ρ) with investment limits](image)

- Nonlinear return: both expected return \( r_{ij} \) and actual return \( r_{ijt} \) are multiplied by \([0.9, 1.0, 1.1]\). Using GAMS to our proposed nonlinear return SMMS models, the nonlinear returns are given in Table 3.4 and Table 3.5.
Table 3.4 Nonlinear return investment for SMMS.

<table>
<thead>
<tr>
<th>Investment range</th>
<th>Nonlinear returns (multiplier)</th>
<th>Investment level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 18788.0</td>
<td>0.9</td>
<td>1</td>
</tr>
<tr>
<td>18788.1 - 34608.1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>34608.2 - 50000.0</td>
<td>1.1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3.5 Nonlinear return solutions for investment in SMMS case with $\rho = 0.27$

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Manufacturer's investment allocation</th>
<th>Investment level</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>S2</td>
<td>34608.2</td>
<td>3</td>
</tr>
<tr>
<td>S3</td>
<td>27413.7</td>
<td>2</td>
</tr>
<tr>
<td>S4</td>
<td>37978.1</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 3.5. Supplier allocation vs overall expected return ($\rho$) with investment limit and nonlinear return

Figure 3.4 and 3.5 show investments across the four suppliers at different levels of $\rho$ when investment is restricted to a maximum of $50,000 for each supplier. It is evident from Figure 3.4 that at low levels of $\rho$ the manufacturer needs to consider investing more in suppliers 1 and 2 and to a lesser degree in suppliers 3 and 4. As the $\rho$ value increases the manufacturer must consider investing more in suppliers 3 and 4 and less in suppliers 1 and 2. The managerial implication is that when high overall expected return is risky to achieve or is infeasible, the manufacturer may lower the expectation or adjust the investment
allocation based on the analysis from Figure 3.4. Nonlinear return in Figure 3.5 has a similar trend to linear return in Figure 3.4, but the performance is delayed for $\rho$. For example, in linear return, when $\rho = 0.15 \sim 0.18$, supplier allocation is kept the same, while in nonlinear, this happens when $\rho = 0.15 \sim 0.19$. In linear return, when $\rho \geq 0.23$, supplier 1 gets 0 investment, while in the nonlinear case, this happens when $\rho \geq 0.25$. When $\rho = 0.27$, supplier allocation is more even in nonlinear return than linear return. It means that in high overall expected return $\rho$ situation, nonlinear return solution is less risky than that of linear return.

### 3.5.2 Results for TMMS Case

Using GAMS on our proposed nonlinear return TMMS models, one example of the nonlinear solutions for investment is given in Table 3.6 and Table 3.7.

**Table 3.6 Nonlinear return investment for TMMS.**

<table>
<thead>
<tr>
<th>M1 investment range</th>
<th>M2 investment range</th>
<th>Nonlinear returns (multiplier)</th>
<th>Investment level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-18788.0</td>
<td>0-18788.0</td>
<td>0.9</td>
<td>1</td>
</tr>
<tr>
<td>18788.1-34608.1</td>
<td>18788.1-34608.1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>34608.2-50000.0</td>
<td>34608.2-50000.0</td>
<td>1.1</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 3.7 Nonlinear return solutions for investment in TMMS case with $\rho = 0.29$.**

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Manufacturer 1</th>
<th>Investment level</th>
<th>Manufacturer 2</th>
<th>Investment level</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>0</td>
<td>34608.2</td>
<td>3</td>
</tr>
<tr>
<td>S2</td>
<td>49700.7</td>
<td>3</td>
<td>37036.8</td>
<td>3</td>
</tr>
<tr>
<td>S3</td>
<td>46844.9</td>
<td>3</td>
<td>2935.9</td>
<td>3</td>
</tr>
<tr>
<td>S4</td>
<td>3454.3</td>
<td>2</td>
<td>25419.1</td>
<td>2</td>
</tr>
</tbody>
</table>

From Table 3.6, we see that manufacturer 1 (M1) and manufacturer 2 (M2) have the same investment range in a nonlinear return situation. Table 3.7 shows the nonlinear return solutions for investment in TMMS case with $\rho = 0.29$.

Using GAMS on the models and the data from Table 3.2 and Table 3.3, both TMMS results with linear and nonlinear return rates are shown in the following figures: Figure 3.7, Figure 3.8, and Figure 3.9 (corresponding to Fig.4, Fig. 5, Fig. 6 from the original paper).

In the original paper, Fig. 4 compares the case of cooperation and non-cooperation between two manufacturers with the linear return.
Figure 3.7a. (original Fig.4) with linear return Cooperation vs. non-cooperation: TEB/risk vs. $\rho$. (with linear return)

The vertical axis is TEB/Risk. Where TEB is Total Expect Benefit, which can be obtained from:

For manufacturer 1 with non-cooperation,  
$$\text{TEB} = \sum_{j=1}^{n} r_j x_j$$

For manufacturer 2 with non-cooperation,  
$$\text{TEB}' = \sum_{j=1}^{n} x'_j$$

For M1 and M2 with cooperation  
$$\text{TEB}'' = \sum_{j=1}^{n} \max[r_j, r'_j](x_j + x'_j)$$

The Risk in formula TEB/Risk can be obtained from $\text{Risk} = \sqrt{\text{Var}}$ for each case

The figure below shows the nonlinear return case:

Figure 3.7b. Cooperation vs. non-cooperation: TEB/risk vs. $\rho$. (with nonlinear return)
Similarly, for nonlinear return, we have:

For manufacturer 1 with non-cooperation,  
\[
TEB = \sum_{j=1}^{n} \sum_{i=1}^{k_j} r_{ij} x_{ij} 
\]
(3.33)

For manufacturer 2 with non-cooperation,  
\[
TEB' = \sum_{j=1}^{n} \sum_{i=1}^{k_j} r'_{ij} x'_{ij} 
\]
(3.34)

For M1 and M2 with cooperation,  
\[
TEB'' = \sum_{j=1}^{n} \sum_{i=1}^{k_j} \max[r_{ij}, r'_{ij}](x_{ij} + x'_{ij}) 
\]
(3.35)

Moreover, Risk = \sqrt{\text{Var}} for each case.

Figure 3.7a compares the cases of cooperation and non-cooperation between the two manufacturers. It is evident from this figure that when manufacturer 1’s expectations of returns increase while manufacturer 2’s is held constant at 0.275, cooperating with manufacturer 2 is always beneficial to manufacturer 1 because the TEB/risk ratio is higher. High TEB to risk ratio indicates high benefit per unit of risk, which is preferred by manufacturers.

The results with the linear return are the same as the results from the Talluri et al. (2010) paper. It verifies our GAMS algorithm for TMMS model. Nonlinear return in Figure 3.7b has a similar trend to the linear return in Figure 3.7a.

In the original paper, Fig.5 examined the impact of various levels of expected returns on the TEB/Risk ratio from manufacturer 1’s perspective. The current expected returns for manufacturer 2 are (0.2, 0.25, 0.3, and 0.35). The expected return (ER) ratio indicates an increase or decrease of expected returns from these base level values at which the ER ratio is 1. The actual performance data listed in Table 1 and Table 2 is left unchanged when varying the ER ratio. Manufacturer 2’s ER ratio was varied from 0.8 to 1.5 while manufacturer 1’s ER ratio was set at 1.0 and \( \rho \) at 0.225. Using GAMS to model, we get the following TMMS results for Figure 3.8 corresponding to Fig. 5 in the original paper.
From Figure 3.8, we can see that it is not always beneficial for manufacturer 1 to cooperate with the manufacturer 2. When the ER ratio of manufacturer 2 is $\geq 1.35$ (original paper 1.42), or $\leq 0.89$ (original paper 0.92) manufacturer 1 should choose to not cooperate with manufacturer 2, because manufacturer 2 is relatively under-performing when compared to their expectations. In other words, in the linear return case, only if the manufacturer 2 ER ratio is from 0.89 to 1.35, should manufacturer 1 choose to cooperate with manufacturer 2. In the nonlinear return case, the cooperation region is smaller, which is from 0.94 to 1.28.

Figure 3.9 provided similar analysis and strategies that must be pursued by manufacturer 2. In this case, manufacturer 1’s ER ratio varied from 0.8 to 1.5, while manufacturer 2’s ER ratio was set at 1 and $\rho$ at 0.275. The actual performance data listed in Table 1 and Table 2 is left unchanged when varying M1’s ER ratio. Using GAMS to model, we get the following TMMS results for Figure 3.9.
Figure 3.9 shows that when manufacturer 2 is a better-performing manufacturer, manufacturer 2 is likely to benefit from cooperating with manufacturer 1, unless manufacturer 1 is highly under-performing or over-performing. The results in Figure 3.9a (linear return) and Figure 3.9b (nonlinear return) show TEB/risk in the nonlinear situation is larger than that in a linear situation.

3.6 Summary

Since the automotive supply chain has multiple layers and is a complex network, the relationship between supply chain members is very important for risk management. Automotive OEMs often invest heavily in supplier development to assist suppliers, which have resulted in quality improvement and cost reduction. However, there is a risk for the manufacturer to implement supplier development. Manufacturers need to allocate the investment optimally among multiple suppliers to minimize risk while maintaining an acceptable level of return. Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation in supplier development under risk. Talluri’s model assumes that the return of investment from the supplier is proportional to the investment. Actually, in most situations the return is nonlinear. We revised the above manufacturer cooperation model with nonlinear returns and applied it to the automotive industry. The non-linear relationship between the return $r_j$ and investment $x_j$ is closer to a real situation. Using the same data in
Talluri’s study and GAMS program, we did a numerical experiment for our nonlinear return model. The revised nonlinear return models were subject to the same analyses as Talluri’s model, such as, investigate the SMMS and TMMS cases, study the collaborative supplier development, and identify if cooperation or non-cooperation are beneficial for manufacturers. Also, we compared the results of the numerical experiment for the linear model and the nonlinear model. The numerical experiments show that non-linear returns algorithm has better results than linear return, such as less risk in high return situation for SMMS and higher TEB/risk in TMMS.

The nonlinear model can be applied to auto OEMs to decide how to optimally allocate their supplier development investments among multiple suppliers to minimize risk while maintaining an acceptable level of return and corporate profitability. It also provides a novel method to assess the benefits of cooperating with other firms in supplier development efforts. Future work includes the application of the non-linear return supplier development model to the automotive industry, such as more suppliers (30 ~ 100), and more total investment amount.
4.1 Introduction

Automotive supply chains are complex due to the large number of parts assembled into an automobile, the multiple layers of suppliers to supply those parts, and the coordination of materials, information, and financial flows across the supply chain. There are many types of supply chain risks in the automotive industry. As a common ex-ante strategy in risk management, supply chain contracts play an important role for supply chain members, such as OEMs and suppliers, when coordinating and sharing the risks that arise from various sources of uncertainty, including demand, price, and product quality.

Automotive product development has a long lead time from concept design to launch that requires more than 30 months as shown in Figure 4.1 (Hill, 2007) below.

![The Automotive Product Development Timeline](image)

Figure 4.1. The automotive product development timeline (Source: Center for Automotive Research)

There are many uncertainties and risks during the product development time frame (Zhu and Zhang, 2017). First, new product development necessitates significant investments from both OEMs and suppliers. For example, the suppliers of innovative parts need to build
new factories or assembly lines. However, the demand forecast for a new or innovative product, such as an electric car, is highly inaccurate due to the long lead time and uncertainty of customer’s preference for the new products. Thus, deciding upon a suitable production capacity for a new facility is challenging for both the manufacturer and supplier. This process involves a large investment, some workers, and facility/equipment building. It is of interest for both the manufacturer and the supplier to engage in a collaborative effort to reduce the risk. It is common practice that both partners sign a contract after the supplier is selected to provide the new parts (Asian and Nie, 2014). However, there is limited research on the capacity decision for product development with risk consideration in such kind of contracts, especially for the automotive industry (Zhang, 2015, Zhu and Zhang, 2017).

This study is based on a real case in the automotive industry. An OEM plans to develop a new vehicle model, namely an electric vehicle (EV). The demand is still highly uncertain, even though the various forecasting methods are utilized to estimate the demand for this new vehicle model. There are many new and innovative components and sub-systems required for a new EV, and the rechargeable battery pack is one of the most expensive and investment intensive sub-systems. The total investment for developing the battery pack and establishing its production capacity is in the range of multi-millions of dollars. The battery pack supply contract signed between OEM and supplier normally sets up a planned annual volume based on the uncertain demand forecast. Thus, the OEM and the supplier need to decide a suitable capacity for the new battery pack production lines collaboratively for two reasons: to reduce the risk of over-capacity or under-capacity, and to maximize the profits for both sides.

The objective of the research in this chapter is to design a supply contract when developing a new product in order to reduce the risks and maximize profits under uncertain demands. To reduce the risks and maximize their profits, this study investigates how to decide the supplier’s capacity and the manufacturer’s order in the supply contract when the demand of the new product is highly uncertain. Based on the newsvendor model and Stackelberg game theory, we developed a mathematical model for the product development contract where both demand uncertainty and compensation are considered. The analytical solution for the demand situation for uniform distribution is developed, and computational tests are
also reported. The proposed solution provides an effective tool for supplier-manufacturer contracts when the manufacturer faces highly uncertain demand.

The remainder of this chapter is organized as follows: a literature review is presented in section 4.2; a proposed model based on Newsvendor model and Stackelberg game theory is developed in section 4.3; the constraint optimization is discussed in section 4.4; lastly, concluding remarks including a summary of the main results and some future work direction is given in section 4.5. Proofs of all propositions appear in section 4.6 Appendix.

4.2 Literature Review

Automotive supply chains are multi-layered and complex networks. Supply chain contracts are used to coordinate supply chain members, OEMs and suppliers, in order to make their interests align with that of the supply chain system and to achieve optimal supply chain efficiency. Supply chain contracts also play an important role because they allow supply chain members to share risks arising from various sources of uncertainty, such as demand, price, and product quality. There are many studies on supply contracts under uncertainty (Chen et al. 2014, and Ghadge et al. 2017). However, to the best of our knowledge, there is limited research on optimal contract design specifically for product development in the automotive industry.

In the following paragraphs, we will review the available papers related to the keywords “contract, demand uncertainty, product development” from Scopus in the period from 2001 to 2018.

Erkoyuncu et al. (2011) conducted a literature review and found that the current research did not consider cost uncertainty for an industrial product–service system. They analyzed the drivers of uncertainty, factors of the sustainability of a contract and the activities dealing with service delivery. They reviewed suitable methods for considering service uncertainty in cost estimation and proposed to substitute the traditional probability theory with Fuzzy set theory, taking advantage of its capability to assign probabilities to ambiguous events or vague knowledge, which suits whole life cycle applications.
Liu and Özer (2010) studied channel incentives in sharing new product demand information, which is often highly uncertain, as well as robust contracts. Distribution channels with a manufacturer and a retailer are considered in their paper. The authors investigated the impact of improved demand information for centralized and decentralized channels and modeled both channels using the newsvendor model. They found that the manufacturer's incentive to share its improved demand information depends on the supply contract signed with the retailer. Furthermore, mandating the manufacturer to disclose its improved demand information can reduce the total channel profit. Three types of widely used contract forms, price-only contracts, quantity flexibility contracts, and buyback contracts, are analyzed for their robustness under an unanticipated demand information update observed by the manufacturer. The results show that the quantity flexibility contract with a high return rate is not robust. The buyback contract is robust and always achieves information sharing while preserving channel performance.

Kim and Netessine (2013) investigated collaborative cost reduction and component procurement by manufacturers and suppliers during the development of an innovative product under information asymmetry. They focused on two stages: the product development stage and the production stage. A game-theoretic model was developed to capture the incentive dynamics that arise when a manufacturer and a supplier exert collaborative efforts to reduce the unit cost of a critical component; however, the supplier may be unwilling to share its private cost information. They investigated how information asymmetry and procurement contracting strategies interact to influence the supply chain parties' incentives to collaborate. The main model is to maximize profit. The authors considered different procurement contracting strategies and identified the expected margin commitment (EMC) as a simple and effective strategy to promote collaboration.

Reimann and Schiltknecht (2009) studied the interdependence of contractual and operational flexibilities from a manufacturers' point of view. Manufacturers in the market of specialty chemicals are exposed to high uncertainty and financial risk since their customers are granted a large degree of freedom concerning demand quantity and time. To deal with changing customer requirements, manufacturers can exploit their operational flexibility, i.e., the capability to adapt planning and production or place emphasis on the contractual flexibility represented by the capability to select the product portfolio or the
possibility proactively to obtain the advance demand information from customers. The authors used a two-stage stochastic program based on the newsvendor concept to quantify the effect of this contractual flexibility and relate it to the manufacturing flexibility concerning capacity allocation. The first stage occurs before the reveal-date where besides the demand distribution no precise information is available, and the second stage occurs after the reveal-date when the required amount of the product is known. To maximize the manufacturer’s expected profit, they combined the two-stage models into a stochastic mixed integer linear programming framework. Through the valuation of these flexibilities and a case study, the paper provides the first insights for the manufacturer about which customer requests to accept, how to set up the associated contracts with the customers and how to allocate capacity for a given portfolio of products.

Chen et al. (2014) researched stable and coordinating contracts for a decentralized supply chain with a single retailer and multiple suppliers where the agents are risk-averse. CVaR is used as the objective function for each agent to capture the behavior of managerial decision making better. The Pareto optimality concept, which is equivalent to maximizing the sum of objectives of all agents, was used to solve supply chain optimality. They showed that the supply chain is coordinated only when the least risk-averse agent bears the entire risk and the lowest-cost supplier handles all production. Coordinating contracts allow flexible objective sharing among all the agents, but competition makes certain contracts unstable. The concepts of contract core and contract equilibrium were introduced to study the stability of the coordinating contract. Contract core reflects the agents’ “bargaining power” and restricts the set of coordinating contracts to a subset which is “credible,” while contract equilibrium helps to characterize contracts that are immune to opportunistic renegotiation. Their research showed that the contract core concept imposes conditions on the share of profit among different agents, and the contract equilibrium concept imposes conditions on how the payment changes with the order quantity.

Asian and Nie (2014) studied coordination in supply chains with uncertain demand and disruption risks using supply contracts. A supply chain problem is investigated where a buyer sources a short life-cycle product from two suppliers: a cheap but unreliable main supplier and a perfectly reliable but expensive backup supplier. The buyer wants to sign an option contract with the backup supplier to remedy supply and demand uncertainty. To
improve supply chain coordination, the option contract is reconstructed for optimization problems, which is modeled with the newsvendor concept. Results revealed under demand uncertainty and supply disruptions, the proposed mechanism led the backup supplier to choose an underproduction policy and provided an insight on the effectiveness of contract-based mitigation strategies that enable firms to ensure responsive backup capacity. The idea of win-win coordination under demand uncertainty is analogous to our research, but the life-cycle and development phase in the automotive industry are quite different from the industry in Asian and Nie (2014). Therefore, the supply chain structures are quite different.

The above contract optimization papers are not directly related to the automotive industry, however. We searched for further literature using the keywords “automotive supply chain/logistics risk management”, and found the following papers related to contracts in the automotive industry.

In order to understand the buyer-supplier power and their dependence, Ghadge et al. (2017) developed a supply chain risk-sharing contract in a globalized business environment. The authors conducted an automotive case study with demand uncertainty and price volatility risks. The objective function is to minimize total purchase cost for the buyer and to maximize the commitment quantity for the supplier. To reflect the possible leverages involved in the decision-making, multiple buyer-supplier power and dependence scenarios are considered. Their risk-sharing contract model provides a relational perspective on the dynamics of supply chain design and collaboration which also potentially contributes a novel perspective on current theory in buyer-supplier power and dependence. Comparing with our research, their supplier’s objective function and modeling methodology are quite different.

Selviaridis and Norrman (2014) studied Performance-Based Contracting (PBC) in service supply chains based on agency theory. The authors studied two cases of logistics service supply chains, one of which is in the automotive industry, and collected data through semi-structured interviews with 30 managers of providers and sub-contractors and reviewed 35 documents especially including contracts and target letters. The paper identified the factors that influence the provider’s willingness to bear the financial risk induced by PBC in
service supply chains. The authors’ research methodology is different from ours, as they focus on performance contract based on agency theory and use a qualitative method.

Swinney and Netessine (2009) investigated the issues of contracting with suppliers prone to default, since it has become an increasingly common problem, particularly in automotive manufacturing. Game theory was applied to model a two-period contracting game with two identical suppliers, a single buyer, deterministic demand, and uncertain production costs. They found that the buyer prefers short-term contracts when a supplier’s failure is not possible or, on the other hand, the buyer prefers long-term contracts when a supplier’s failure is possible. They also found that dynamic long-term contracts allow the buyer to coordinate the supply chain in the presence of default risk. The authors concluded that the possibility of supplier default offers a new reason to prefer long-term contracts over short-term contracts. Their research differs from our research, as they focus on contracting with suppliers prone to default rather than finding the buyer’s optimal order and supplier’s optimal production capacity under demand uncertainty.

Yang et al. (2017) researched capacity investment strategy under cost sharing contracts for various industries including the automotive industry. Two capacity sharing contracts were proposed: the full capacity cost sharing contract (FCCSC) and the partial capacity cost sharing contract (PCCSC). In FCCSC, a retailer shares an agreed upon percentage of the capacity cost with the manufacturer. In contrast, a retailer shares capacity cost in PCCSC only when the manufacturer's capacity level exceeds a certain threshold. Their research found that the retailer would share more cost but a lower capacity in PCCSC than that in FCCSC. They also found the threshold of capacity level would decide the choices of FCCSC or PCCSC by the retailer and manufacturer, and only in a certain interval would both players choose the PCCSC.

Table 4.1 summarizes our literature review on the topic of supply chain contracts.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Subject</th>
<th>Supply Chain Structure</th>
<th>Model</th>
<th>Solution Approach</th>
<th>Auto-specific</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erkoyuncu et al. (2011)</td>
<td>Understanding service uncertainties in industrial product–service system cost estimation</td>
<td>Product–service system (PSS), service supply chain</td>
<td>Literature review</td>
<td>Fishbone diagram, diagrams, tables</td>
<td>No</td>
</tr>
<tr>
<td>Selviaridis and Norrman (2014)</td>
<td>Performance-based contracting in service supply chains: A service provider risk perspective</td>
<td>Performance-Based Contracting (PBC) in service supply chains</td>
<td>Data were collected through semi-structured interviews with 30 managers and review of 35 documents.</td>
<td>Identified the factors that influence the provider willingness to bear the financial risk induced by PBC</td>
<td>Yes</td>
</tr>
<tr>
<td>Ghadge et al. (2017)</td>
<td>Using risk sharing contracts for supply chain risk mitigation: A buyer-supplier power and dependence perspective.</td>
<td>Multiple buyer-supplier power and dependence scenarios</td>
<td>Risk sharing contract model. The objective function is to minimize total purchase cost for the buyer and maximize the commitment quantity for the supplier</td>
<td>Integer programming model</td>
<td>Yes</td>
</tr>
<tr>
<td>Liu and Özer (2010)</td>
<td>Channel incentives in sharing new product demand information and robust contracts</td>
<td>A distribution channel with a manufacturer and a retailer</td>
<td>Convex ordering and Newsvendor model. Expected channel profit in the centralized channel; The retailer's optimal order quantity, its expected profit, and the manufacturer's profit in the decentralized channel.</td>
<td>Three widely used contract forms: price-only contracts, quantity flexibility contracts, and buyback contracts are analyzed in the decentralized channel. Mathematical proof, numerical experiments</td>
<td>No</td>
</tr>
<tr>
<td>Authors and Year</td>
<td>Description</td>
<td>Participants</td>
<td>Methodology</td>
<td>Findings</td>
<td>Conclusion</td>
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<tr>
<td>Kim and Netessine (2013)</td>
<td>Collaborative cost reduction and component procurement under information asymmetry</td>
<td>a manufacturer and a supplier</td>
<td>Game –theoretic model with an objective function and constraint functions</td>
<td>Expected margin commitment (EMC), screening contract. Mathematical proof, numerical experiments</td>
<td>No</td>
</tr>
<tr>
<td>Swinney and Netessine (2009)</td>
<td>Long-term contracts under the threat of supplier default</td>
<td>A two-period contracting game with two identical suppliers, a single buyer, deterministic demand, and uncertain production costs</td>
<td>Game theory; A benchmark model without failure; short-term contracts, long-term contracts, and dynamic contracts under the threat of supplier default</td>
<td>Mathematical proof. Concluded that the possibility of supplier default offers a new reason to prefer long-term contracts over short-term contracts</td>
<td>Yes</td>
</tr>
<tr>
<td>Reimann and Schiltknecht (2009)</td>
<td>The interdependence of contractual and operational flexibilities in the market of specialty chemicals</td>
<td>A manufacturer who can process multiple products, and multiple customers with different demands</td>
<td>Two stage stochastic program based on the newsvendor concept. The objective is to maximize the expected profit with respect to the probability distribution of demands, subject to several constraints. Modeled flexibility</td>
<td>The model is transferred to stochastic mixed integer linear program. Numerical analysis, case study</td>
<td>Yes</td>
</tr>
<tr>
<td>Chen et al. (2014)</td>
<td>Stable and Coordinating Contracts for a Supply Chain with Multiple Risk-Averse Suppliers</td>
<td>multiple suppliers, single retailer with uncertain demand. All agents are risk-averse</td>
<td>CVaR objective function for both retailer and suppliers as a risk measure</td>
<td>Newsvendor Pareto optimality Mathematical proof</td>
<td>No</td>
</tr>
<tr>
<td>Asian and Nie (2014)</td>
<td>Coordination in supply chains with uncertain demand and disruption risks: Existence, analysis, and insights</td>
<td>A buyer who sources a short life-cycle from two suppliers, a cheap but unreliable main supplier and a reliable but expensive backup supplier</td>
<td>Centralized Benchmark Model, Decentralized Benchmark Model, the option contract, win-win coordination mechanism</td>
<td>Newsvendor, theorem, computational results, and analysis</td>
<td>No</td>
</tr>
</tbody>
</table>
In summary, contracts for product development under demand uncertainty can be modeled using game theory, newsvendor, and stochastic theory. Supply chains include a manufacturer with one or more upstream members and one or more downstream members. The objective functions are usually to maximize the expected profit of the entire supply chain or separate supply chain members. Uncertain demand can be modeled by the probability distribution of demands with known mean and variation or known probability density $f_{D_i}$ and cumulative density function $F_{D_i}$. The models can be solved by mathematical proof, stochastic mixed integer program, and numerical analysis in a case study. The works listed in Table 4.1 dealt with different kinds of supply chains and supply chain contracts in several industries: short-cycle product supply chains (Asian and Nie 2014), service supply chains (Erkoyuncu et al. 2011, Selviaridis and Norrman 2014), and supply chains in the chemical industry (Reimann and Schiltknecht 2009). Very few research dealt with supply chain contract models in the automotive industry. Swinney and Netessine (2009) studied long-term contracts under the threat of supplier default for auto industry supply chains, but this is out of the scope of our research. Limited research exists in contract modeling considering characteristics of the automotive industry, especially for how to decide a suitable production capacity of a new facility, which is a challenging problem faced by both the manufacturers and the suppliers. The purpose of our research is to try to fill the research gap to meet the demand of automotive industry supply chain risk management. Our research mainly focuses on contract models for new product development with uncertain demand in the automotive industry, to reduce the risk of over-capacity or under-capacity and also to maximize the profits for both manufacturers and suppliers.

Zhang (2015) formulated the first mathematical model for this problem but does not provide a solution nor numerical analysis. This work verifies the model, provides a solution approach and conducts numerical analysis. This chapter will focus on optimal production development for new products in the automotive industry for which the demand is uncertain.
4.3 The Model

4.3.1 Problem Statement
As mentioned previously, this study is motivated by a real case in the automotive industry. This problem also exists in many similar manufacturing situations, and can be stated as follows.

Consider a two-stage supply chain that consists of one supplier and one manufacturer. Generally, the manufacturer forecasts the yearly demand of the new product, most of which is the planned order quantity, denoted as $O$, then shares this information with the supplier. Because the auto industry uses a make-to-order policy, the actual order quantity, denoted $q$, may be far different from the planned order quantity, depending on the realized demand. Considering the demand uncertainty, the supplier then needs to decide the capacity $Q$ according to order $O$. As a part of the procurement contract; the manufacturer can claim compensation or penalty to prevent the profit loss caused by the supplier’s delivery shortage.

The problem here is to determine the following two variables to reach an optimal contract under demand uncertainty:

- For the supplier: the number of units the supplier can produce, i.e., the capacity of the part $Q$
- For the manufacturer: the planned yearly order quantity $O$ (as a reference for the capacity)

4.3.2 Assumptions
Based on the real-world problem in the auto industry, we have the following assumptions:

- The demand for the finished product is uncertain but the distribution of the demand can be estimated. Our model is independent of the distribution functions.
- The information about the demand and capacity is shared between the supplier and OEM.
- The supplier needs to invest heavily to develop the production process to produce the component; building the facility with the designed capacity takes a long time.
There is a penalty if the supplier cannot supply the amount of the component in the contract.

There is a compensation if the supplier can supply 20% above the contract amount if the actual demand is higher than expected (according to an automotive OEM supply chain expert).

### 4.3.3 Model Structure and Notations

We formulate the single period make–to-order supply chain contract consisting of a supplier and a manufacturer with demand uncertainty by using a Newsvendor model and Stackelberg game theory. The newsvendor model is a classical mathematical model in operations management and applied economics, used to determine optimal inventory levels under demand uncertainty. It is characterized by fixed prices and uncertain demand for a (perishable) product. We applied the newsvendor model to determine the supplier’s optimal capacity and the manufacturer’s optimal order.

The Stackelberg game is a strategic game in which the leader moves first and then the follower moves sequentially, and they compete on quantity. The Stackelberg game can be applied in economics to consider the idea of a “Stackelberg equilibrium” in a duopoly (Stackelberg, 2011). The Stackelberg model can be solved to find the subgame perfect Nash equilibrium, i.e. the strategy profile that serves each player best, given the strategies of the other player; this means every player plays in a Nash equilibrium in every subgame. The Nash equilibrium is a solution concept of a non-cooperative game involving two or more players in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy (Osborne and Rubinstein, 1994). In the Stackelberg game, backward induction is used to calculate the subgame perfect Nash equilibrium, i.e. we must first calculate the best response functions of the follower then consider the best response function of the leader [1][2], (Stackelberg, 2011, Fudenberg and Tirole, 1993).

Applying the Stackelberg game to our study, the manufacturer as the leader decides yearly order quantity \( O \), then the supplier as the follower determines its production capacity \( Q \) according to the order \( O \). In Stackelberg games, each player makes its own decision, to maximize its individual profit or minimize the cost. In our model, the manufacturer’s
objective is to minimize the total cost related to that component that satisfies the demand, and the supplier tries to maximize the profit. We have these objective functions:

- Maximize the supplier’s expected profit
- Minimize the manufacturer’s expected cost

For any given order from the manufacturer, the supplier has two options regarding its capacity: $Q \leq O$, or $Q > O$. If the demand greatly exceeds expectations, i.e. the demand $> 1.2O$ and the supplier can provide this extra quantity, it receives a compensation. Thus, it has two sub-options for $Q > O$. Similarly, there are two options for $Q \leq O$. The structure of the subgame perfect Nash equilibrium is given in Figure 4.2.

![Figure 4.2. The structure of the Stackelberg model for the contract](image)

Based on backward induction for determining the subgame perfect Nash equilibrium, we first calculate the supplier’s response for each of the manufacturer’s sub-options, and then find the best response for each option; next we compare the supplier’s response for each option to find the overall best response. However, it may be difficult to compare the performance of each option (or sub-option) because the performances depend on the parameters and distributions. Thus, we can compare the performances based on given demand distribution and given parameters.

**Notations**

The notation used in our model is shown below:
Parameters

$D$  Annual demand of the product at OEM
$c_s$  Supplier's unit variable cost, including raw materials, purchased components, labour cost, variable overhead cost and transportation
$c_0$  Supplier’s unit fixed cost invested in equipment maintenance, supervisory costs and insurance and plant administration
$c_{sb}$  Supplier’s fixed cost for equipment at the beginning of development
$g_0$  Manufacturer’s unit fixed cost invested for the equipment
$g_b$  Manufacturer’s fixed cost for equipment at the beginning of development
$p$  Unit price of the component the manufacturer pays to the supplier
$T$  Planning horizon, $t = 1, 2, \ldots, T$, in this model $T = 1$
$h_1$  Unit compensating cost that manufacturer pays to supplier, if $O >> D_t$
$e_1$  Unit penalty that supplier pays to manufacturer, if $Q < O$ and $D_t > Q$
$s_1$  Unit shortage cost of manufacturer if $D_t > Q$
$m_1$  Unit high production compensation M pays to S, if $Q > O$ and $D_t > 1.2O$
$c_m$  Manufacturer's unit variable cost, including labor cost and transportation
$\pi$  Supplier’s profit

Decision Variables

$O$  The manufacturer's planned yearly order quantity, as a reference for the supplier's capacity
$Q$  The number of units the supplier can produce, i.e., the capacity of the component

4.3.4 The Supplier's Objective Function

The objective function of the supplier is to maximize its expected profit. For a given demand, the supplier’s profit can be formulated as follows:

$Supplier's\ profit = min\ (demand,\ production\ amount) \times (unit\ price - variable\ cost) - fixed\ cost - penalty + Compensation.$

Before formulating the model, we introduce the related revenue and each cost component. To simplify the written mathematics, we represent the demand $D$ with $x$. Assume that the demand is a continuous nonnegative random variable with density function $f(x)$ and
cumulative distribution function $F(x)$. If $Q \leq x$, all $Q$ units are sold and a profit of $Q(p-c)$ results. On the other hand, if $Q > x$, only $x$ units are sold and a profit of $x(p-c)$ results.

The production cost of the supplier consists of two parts, variable cost $c_s$ and fixed cost $c_{sf}(Q): c_s$ is the supplier's unit variable cost, including raw materials, purchased components, labor cost, overhead cost and transportation; $c_{sf}(Q)$ is the fixed cost of the supplier invested for the equipment, which is related to the supplier's production capacity. The fixed investment cost is a yearly amount based on the total investment at the beginning of development.

$$c_{sf}(Q) = c_{sb} + c_0 \times Q$$

Because the demand for the new product is highly uncertain, the actual demand is usually different from the manufacturer's planned order quantity. The supplier's production capacity depends on the manufacturer's contracted order quantity. In practice, it is acceptable for the supplier if the difference between the actual demand and the contracted order quantity is less than 20%. However, the supplier may request some compensation due to investment lost if the actual demand is much lower than the contracted order quantity (more than 20%). The low demand compensation $h(D, O, Q)$ is formulated as the following:

$$h(D, O, Q) = h_1 \times (\min(O, Q) - D - 0.2 \times \min(O, Q))^+ = h_1 \times (0.8 \min(O, Q) - D)^+$$

On the other hand, the supplier may decide to build the facility with a smaller capacity than the contracted order quantity from the OEM, which may result in paying the penalty $e(Q, O)$ where

$$e(Q, O) = e_1 \times (D - Q)^+$$

Thus, we have the supplier's profit

$$\pi = \min(Q, x) \times (p - c_s) - c_{sf}(Q) - e(Q, O) + h(x, O, Q)$$

The supplier has two options (or strategies): set the capacity less than the contracted order quantity, or set the capacity greater than the order quantity. Next, we will discuss the two cases respectively.

**Case 1: $Q < O$**
In this case, the supplier’s capacity is less than the contracted order quantity. Thus, the supplier has the risk of paying the penalty if the demand is more than the capacity. On the other hand, if the demand is far less than the capacity, the supplier will get the low demand compensation \( h(D, O, Q) \) from the manufacturer, which is simplified below:

\[
h(D, O, Q) = h_1 \times (0.8 \min(O, Q) - D)^+ = h_1 \times (0.8Q - D)^+
\]

The supplier’s expected profit can then be formulated as follows:

\[
E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_Q^{\inf} Q(p - c_s)f(x)dx - c_s f(Q)
- \int_Q^0 e_1(x - Q)f(x)dx + \int_Q^{0.8Q} h_1(0.8Q - x)f(x)dx
\] (4.1)

As stated before, to calculate the subgame perfect Nash equilibrium with backward induction, we first calculate the follower’s (supplier’s) objective function to get their optimal solution, then consider the leader’s (manufacturer’s) objective function.

To determine the value of \( Q \) that maximizes the supplier’s expected profit \( E(\pi) \), we apply Leibniz’s rule to equation (4.1) (see appendix A). We get:

\[
\frac{dE(\pi)}{dQ} = Q(p - c_s)f(Q) + (p - c_s)[1 - F(Q)] - Q(p - c_s)f(Q) - c_0 + e_1[F(O) - F(Q)] + 0.8h_1 F(0.8Q)
- (p - c_s + e_1)F(Q) + p - c_s - c_0 + e_1 F(O) + 0.8h_1 F(0.8Q)
\]

Setting \( \frac{dE(\pi)}{dQ} = 0 \), we get the supplier’s optimal production capability \( Q^* \)

\[
F(Q^*) = \frac{p - c - c_0 + e_1 F(O) + 0.8h_1 F(0.8Q^*)}{p - c + e_1}
\] (4.2)

To verify the solution, we check the second derivative of \( E(\pi) \). If \( \frac{d^2E(\pi)}{dQ^2} \leq 0 \), the function \( E(\pi) \) has the max value in the optimal solution \( Q \). Since

\[
\frac{d^2E(\pi)}{dQ^2} = -(p - c_s + e_1)f(Q) + 0.64h_1 f(0.8Q)
\] (4.3)

It is difficult to check if the second derivative of \( E(\pi) \) is negative at \( Q^* \). In Appendix A, we proved that in a uniform distribution, \( \frac{d^2E(\pi)}{dQ^2} \leq 0 \) is true. Thus, function (4.1) is concave in uniform distribution and has the max value in the optimal solution \( Q^* \).
It is noted that we require \( Q \leq O \), which is the constraint of Case 1. We will discuss the constraint solution later.

To solve the manufacturer’s decision variable \( O \), we assume the optimal solution is \( Q^* = R(O) \).

### 4.3.5 The Manufacturer’s Objective Function

The manufacturer’s objective is to minimize the expected cost (Zhang, 2015). The manufacturer’s cost can be formulated as the follows.

\[
\text{Manufacturer’s cost} = \min \left( \text{Demand, production amount} \right) \times (\text{unit price} + \text{variable cost}) - \text{fixed cost} - \text{penalty} + \text{Compensation} + \text{shortage cost}
\]

\[
= (p + c_m) \times \min(Q, x) + g(O) - e(Q, O) + h(x, O, Q) + \text{shortage cost}
\]

Where: \( g(O) = g_b + g_0O \) which is the manufacturer’s fixed cost.

The manufacturer’s shortage cost is same to the supplier’s penalty cost (Nahmias, 2012, pp. 269), except the distribution regions are different.

As we stated before, the supplier may have two options: \( Q < O \) or \( Q > O \). Accordingly, the manufacturer also has these 2 cases.

**Case 1: \( Q \leq O \)**

In Case 1, the supplier’s capacity is less than the contracted order quantity. If the demand is more than the capacity, the manufacturer may have the chance to get the penalty \( e(Q, O) \) from the supplier, which reduces manufacturer’s cost. On the other hand, if the demand is far less than the capacity, the manufacturer will pay the supplier low demand compensation \( h(D, O, Q) \), which increases the manufacturer’s cost. The manufacturer’s expected cost can be formulated as follows:

\[
E(\text{cost}) = \int_0^Q (p + c_m)xf(x)dx + \int_Q^{\inf} (p + c_m)Qf(x)dx + g(O) - \int_Q^0 e(x-Q)f(x)dx + \int_0^{0.8Q} h_1(0.8Q-x)f(x)dx + \int_Q^{\inf} s(x-Q)f(x)dx
\]

(4.4)

Based on the Stackelberg model procedure, we substitute the supplier’s optimal solution \( Q^* = R(O) \) to (4.4),
\[ E(\text{cost}) = \int_0^{R(O)} (p + c_m)xf(x)dx + \int_{R(O)}^{\infty} (p + c_m)R(O)f(x)dx + g_b + g_0\cdot 0 \]

\[ - \int_0^{R(O)} e_1(x - R(O))f(x)dx + \int_0^{0.8R(O)} h_1(0.8R(O) - x)f(x)dx \]

\[ + \int_{R(O)}^{\infty} s(x - Q)f(x)dx \]

To determine the value of \( O \) that minimizes the manufacturer’s expected cost, we have

\[ \frac{dE(\text{cost})}{dO} = 0. \]

Apply Leibniz’s rule to equation (4.5), and we get the manufacturer’s optimal order quantity \( O = O^* \), which is shown in Appendix B.

\[ R'(O)(p + c_m + e_1F(O) - s) + R'(O) \cdot F(R(O)) \cdot (s - p - c_m - e_1) \]

\[ + e_1 \cdot f(O)(R(O) - O) + 0.8h_1 \cdot R'(O) \cdot F(0.8R(O)) + g_0 = 0 \]

(4.6)

### 4.3.6 Model of Case 2 Q > O

In Case 2, the supplier’s capacity is more than the contracted order quantity. The supplier does not have a risk to pay the penalty \( e(Q, O) \) even if the demand is more than the capacity, which means penalty \( e(Q, O) = 0 \). As it is in Case 1, if the demand is far less than the capacity, the manufacturer will pay the supplier low demand compensation \( h(D, O, Q) \).

Since \( Q > O \), low demand compensation becomes:

\[ h(D, O, Q) = h_1 \times (0.8 \min(O, Q) - D_t)^+ = h_1 \times (0.8O - D_t)^+ \]

On the other hand, we also need to consider high production compensation. According to the assumptions, there is compensation if the supplier can supply 20% above the contract amount when the actual demand is very high, which means when demand \( x > 1.2O \) and \( x < Q \), the manufacturer will give high production compensation to the supplier. In emergency orders, some manufacturers pay three times more than the normal price for emergency orders. High production compensation can be formulated by

\[ m(O, Q) = \int_{1.2O}^{Q} m_1(x - 1.2O)f(x)dx \]

where \( m_1 \) is high production compensation per unit.

Case 2 also has two different subcases. Case 2a: \( Q > 1.2O \); and Case 2b: \( O < Q < 1.2O \). They are discussed separately as follows.

- Case 2a: when \( Q > 1.2O \)
In this situation, the supplier’s capacity is more than the contracted order quantity by at least 20%. Thus the supplier may have chance to get the high production compensation, or \( m(O, Q) > 0 \).

The supplier’s objective functions for Case 2a is:

\[
E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_{Q}^{\inf} Q(p - c_s)f(x)dx - c_{sf}(Q) \\
+ \int_{1.20}^{Q} m_1(x - 1.20)f(x)dx + \int_{0}^{0.80} h_1(0.80 - x)f(x)dx
\]

(4.7)

Using the same steps in Case 1, apply Leibniz’s rule to equation (4.7), set \( \frac{dE(\pi)}{dQ} = 0 \), we get the supplier’s optimal production capability \( Q^* \): (see Appendix A.)

\[
F(Q^*) = 1 - \frac{1}{p - c_s} [c_o - m_1(Q^* - 1.20) \cdot f(Q^*)]
\]

(4.8)

Next, the second derivative is examined. (See Appendix A)

\[
\frac{dE^2(\pi)}{dQ^2} = f(Q) \cdot (m_1 - p + c_s) + m_1 \cdot Q \cdot f'(Q)
\]

(4.9)

As proved in Appendix A, in uniform distribution \( U(a, b) \), and \( m_1 \) satisfies \( m_1 < p - cs = p (1 - cs/p) \), we have \( \frac{dE^2(\pi)}{dQ^2} < 0 \), so \( E(\pi) \) has the maximum value at \( Q^* \).

For manufacturer in Case 2a.

If the manufacturer invests extra for more than \( O \) is assumed, manufacturer’s cost needs to add the item of \( \int_{0}^{Q} g_0(x - O)f(x)dx \).

\[
E(\text{cost}) = \int_0^Q (p + c_m)x f(x)dx + \int_{Q}^{\inf} (p + c_m)Qf(x)dx + g(O) \\
+ \int_{0}^{Q} g_0(x - O)f(x)dx + \int_{1.20}^{Q} m_1(x - 1.20)f(x)dx \\
+ \int_{0}^{0.80} h_1(0.80 - x)f(x)dx + \int_{Q}^{\inf} s(x - Q)f(x)dx
\]

(4.10)

Substitute \( Q = R(O) \) to (4.10) and apply Leibniz’s rule to it to determine the value of \( O \) that minimizes the manufacturer’s expected profit \( E(\text{cost}) \),

\[
\frac{dE(\text{cost})}{dO} = (p + c_m - s)R'(O) \left( 1 - F(R(O)) \right) + g_o + 0.8h_1 F(0.80) \\
+ g_0 F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1 F(1.20) \\
+ f(R(O)) \cdot R(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)]
\]
set $\frac{dE(\text{cost})}{dO} = 0$, the optimal solution $O^*$ is obtained from

$$
(p + c_m - s)R'(O) \left(1 - F(R(O))\right) + g_0 + 0.8h_1 F(0.8O) + g_0 F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1 F(1.2O) + f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)] = 0
$$

(4.11)

$\frac{d^2E(\text{cost})}{dO^2}$ will be examined later.

- **Case 2b: when $O < Q \leq 1.2O$**

In this situation, supplier’s capacity is more than manufacturer’s order, but does not exceed 20%. According to the assumption, the manufacturer will not give the high production compensation to the supplier. So $m(O, Q) = 0$.

- The supplier’s objective function is:

$$
E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_0^{\inf} Q(p - c_s)f(x)dx - c_{sf}(Q) + \int_0^{0.8O} h_1(0.8O - x)f(x)dx
$$

(4.12)

To determine the value of $Q$ that maximizes the supplier’s expected profit $E(\pi)$, we apply Leibniz’s rule to equation (4.12), and set $\frac{dE(\pi)}{dQ} = 0$ the supplier’s optimal production capability $Q^*$ is

$$
F(Q^*) = 1 - \frac{c_0}{p - c_s}
$$

(4.13)

This result can also be found from (4.8) when setting $m_1 = 0$. As shown in Appendix A, we always have $\frac{dE^2(\pi)}{dQ^2} < 0$ in Case 2b, so $E(\pi)$ has a maximum at $Q^*$.

From (4.13), we can see that $Q^*$ is a constant in Case 2b and it not related to $O$, which means $Q \neq R(O)$. $R'(O) = 0$.

- The manufacturer’s objective function is:

$$
E(\text{cost}) = \int_0^Q (p + c_m)xf(x)dx + \int_0^{\inf} (p + c_m)Qf(x)dx + g(O) + \int_0^Q g_0(x - O)f(x)dx + \int_0^{0.8O} h_1(0.8O - x)f(x)dx + \int_0^{\inf} s(x - Q)f(x)dx
$$

(4.14)
To determine the value of \( O \) that minimizes the manufacturer’s expected cost, we apply Leibniz’s rule to equation (4.12) and set \( \frac{dE(\text{cost})}{dO} = 0 \). But as shown in Appendix B, there is no \( O \) solution to satisfy \( \frac{dE(\text{cost})}{dO} = 0 \). Actually, \( \frac{dE(\text{cost})}{dO} > 0 \) in this case.

- Mathematically, since the first derivative is always > 0, and the target function is to minimize the cost, the optimal solution is at the smallest \( O \), i.e., \( O = 0 \). For Case 2b \( O < Q \leq 1.2O \), boundary or constraint \( O = Q/1.2 \) is the smallest \( O \).

So the optimal solution or Case 2b is a constraint solution:

\[
F(Q^*) = 1 - \frac{c_0}{p - c_s} \quad (4.13)
\]

\[
O^* = 1/1.2 \cdot Q^* \quad (4.17)
\]

This is a constraint optimization problem. We will discuss it in the next section.

### 4.4 Constraint Optimization

From above sections, the optimization solutions for subgame Case 1 \( Q < O \) and Case 2 \( Q > O \) are developed. In some subcases there are no optimal solutions, such as Case 2b, in which a boundary solution needs to be considered. In other cases, the solutions may not satisfy the assumptions. The constraint optimization solutions are required to make the subgame perfect, since the contract mathematic model is based on some constraints, such as \( Q < O \) for Case 1, and \( Q > O \) for Case 2. To determine the subgame perfect equilibria, the backward induction method is applied to solve the constraint optimization problem [2].

#### 4.4.1 Case 1 \( Q < O \)

For Case 1, we assume \( Q < O \). For any given parameters, assuming \( Q^* = R(O) \), \( Q^* \) and \( O^* \) can be calculated by the formula (4.2) and (4.6).

where

\[
F(Q^*) = \frac{p - c - c_0 + e_1 F(O) + 0.8 h_1 F(0.8Q^*)}{p - c + e_1} \quad (4.2)
\]

\[
R'(O)(p + c_m + e_1 F(O) - s) + R'(O) \cdot F(R(O)) \cdot (s - p - c_m - e_1) + e_1 \cdot f(O) (R(O) - O) + 0.8 h_1 \cdot R'(O) \cdot F(0.8 R(O)) + g_0 = 0 \quad (4.6)
\]
Here demand distribution can be any type of distribution.

Checking the results $Q^*$ and $O$, if the result is not within the constraints $Q < O$, the constraint solution is required to adjust the result.

- If $Q^* > O^*$, let $Q^* = R(O) = O$; which means $R'(O) = 1$,

Substituting them to formula (4.6), when $Q^* = O$, the optimal $O^*$ is obtained from (4.18) below:

$$ (p + c_m - s) + F(O) \cdot (s - p - c_m) + 0.8h_1 \cdot F(0.8O) + g_0 = 0 $$  \hspace{1cm} (4.18)

$Q^* = O$ is the constraint solution if $Q^* > O$ in Case 1.

4.4.2 Case 2 $Q > O$

From Figure 4.2, Case 2 can be divided to 2 subcases: Case 2a $Q > 1.2O$, and Case 2b $O < Q \leq 1.2O$.

1) For Case 2a $Q > 1.2O$,

For any given parameters, assuming $Q^* = R(O)$, $Q^*$ and $O^*$ can be calculated by (4.8) and (4.11) below.

$$ F(Q^*) = 1 - \frac{1}{p - c_s} [c_0 - m_1(Q^* - 1.2O) \cdot f(Q^*)] $$  \hspace{1cm} (4.8)

$$ (p + c_m - s)R'(O) \left( 1 - F(R(O)) \right) + g_0 + 0.8h_1F(0.8O) + g_0F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1F(1.2O) + f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)] = 0 $$  \hspace{1cm} (4.11)

Here the type of demand distribution is not specialized, which can be a uniform distribution or a normal distribution.

The results $Q^*$ and $O$ need to be checked to see if the result is within the constraints $Q > 1.2O$ or not. If not, the solutions need to be adjusted to meet the constraints. That means:

- If $Q^* > 1.2O^*$, $Q^*$ is a solution from formula (4.8) and (4.11).
- If $Q^* < 1.2O^*$ from (4.11), we take $Q^* = 1.2O + 1$ since it is concave, so formula (4.8) and (4.11) is till the solution.

2) For Case 2b $O < Q \leq 1.2O$,
In this situation, \( m(O, Q) = 0 \), supplier’s expected profit becomes

\[
E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_Q^{\inf} Q(p - c_s)f(x)dx - c_{sf}(Q) \\
+ \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx
\]

The supplier’s optimal solution from (4.12) is

\[
F(Q^*) = 1 - \frac{c_0}{p - c_s}
\]  

(4.13)

\( Q^* \) is not related to manufacturer’s production order, or \( Q^* \neq R(O) \). On the other hand, according to (4.16), the manufacturer’s objective function has no valid solution. The constraints are considered for the solution.

- Constraint solution is: (from \( Q^* \) to \( O \))

\[
F(Q^*) = 1 - \frac{c_0}{p - c_s}
\]

\[ O = 1/1.2 \cdot Q^* \]

Based on the constraint solution, the profit of the supplier and cost to the manufacturer can be calculated. The final solution can be determined by comparing the subcase results to get the subgame perfect equilibria. Table 4.2 summarized supplier’s profit model \( E(\pi) \), manufacturer’s cost model \( E(\text{cost}) \), Leibniz’s rule application, and constraint optimization solution for Case 1 and Case 2 and their subcases.

**Table 4.2 Summary of supplier’s profit, manufacturer’s cost, Leibniz’s rule application, and constraint optimization solution for Case 1 and Case 2 and their subcases.**

| Case 1: Assume \( Q \leq O \) | \[
\begin{align*}
S. E(\pi) & = \int_0^Q x(p - c_s)f(x)dx + \int_Q^{\inf} Q(p - c_s)f(x)dx - c_{sf}(Q) \\
& - \int_Q^0 e_1(x - Q)f(x)dx + \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx \\
\end{align*}
\]  

(4.1) |
|-----------------|-----------------|
| M.E(cost) | \[
\begin{align*}
& = \int_0^Q (p + c_m)x f(x)dx + \int_Q^{\inf} (p + c_m)Qf(x)dx + g(O) \\
& - \int_Q^0 e_1(x - Q)f(x)dx + \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx + \int_0^{\inf} s(x - Q)f(x)dx \\
\end{align*}
\]  

(4.4) |
Leibniz's
\[
\frac{dE}{dq} = 0
\]
\[
F(Q^*) = \frac{p - c - c_0 + e_1 F(O) + 0.8h_1 F(0.8Q^*)}{p - c + e_1}
\]  
(4.2)

\[
\text{Q} = R(O)
\]

Leibniz's
\[
\frac{dE}{do} = 0
\]
\[
R'(O)(p + c_m + e_1 F(O) - s) + R'(O) \cdot F(R(O)) \cdot (s - p - c_m - e_1)
+ e_1 \cdot f(O)(R(O) - O) + 0.8h_1 \cdot R'(O) \cdot F(0.8R(O)) + g_0 = 0
\]  
(4.6)

Case 1

Optimal Solution
If \( Q \leq O \) \( Q^* \) form (4.2), \( O^* \) from (4.6)

Case 1

Constraint Solution
If \( Q^* > O \), let \( Q^* = R(O) = O^* \)

Case 2: Assume \( Q > O \)

Case 2a: \( Q > 1.2\ O \)

\[
m(O, Q) \neq 0
\]

S.
E(π)
\[
\int_0^Q x(p - c_s)f(x) dx + \int_Q^{\inf} Q(p - c_s)f(x) dx + \int_{1.2O}^Q m_1(x - 1.2O)f(x) dx + \int_0^{0.8O} h_1(0.8O)x dx
\]
\[
+ \int_0^{0.8O} h_1(0.8O - x)f(x) dx
\]
(4.7)

M.
E(cost)
\[
\int_0^Q (p + c_m)xf(x) dx + \int_Q^{\inf} (p + c_m)Qf(x) dx + \int_{1.2O}^Q m_1(x - 1.2O)f(x) dx + \int_0^{0.8O} h_1(0.8O - x)f(x) dx + \int_0^{0.8O} h_1(0.8O - x)f(x) dx + \int_0^{\inf} s(x - Q)f(x) dx
\]
(4.10)

Leibniz's
\[
\frac{dE}{dq} = 0
\]
\[
F(Q^*) = 1 - \frac{1}{p - c_s} [c_0 - m_1(Q^* - 1.2O) f(Q^*)]
\]  
(4.8)

Case 2b: \( O < Q \leq 1.2\ O \)

\[
m(O, Q) = 0
\]

\[
\int_0^Q x(p - c_s)f(x) dx + \int_Q^{\inf} Q(p - c_s)f(x) dx + \int_{1.2O}^Q m_1(x - 1.2O)f(x) dx + \int_0^{0.8O} h_1(0.8O)x dx
\]
\[
+ \int_0^{0.8O} h_1(0.8O - x)f(x) dx
\]
\[
+ \int_0^{\inf} g_0(x - O)f(x) dx + \int_0^{\inf} g_0(x - O)f(x) dx + \int_0^{\inf} h_1(0.8O - x)f(x) dx + \int_0^{\inf} s(x - Q)f(x) dx
\]
(4.12)

\[
\int_0^Q (p + c_m)xf(x) dx + \int_Q^{\inf} (p + c_m)Qf(x) dx + \int_{1.2O}^Q m_1(x - 1.2O)f(x) dx + \int_0^{0.8O} h_1(0.8O - x)f(x) dx + \int_0^{0.8O} h_1(0.8O - x)f(x) dx + \int_0^{\inf} s(x - Q)f(x) dx
\]
(4.14)

Leibniz's
\[
\frac{dE}{dq} = 0
\]
\[
F(Q^*) = 1 - \frac{c_0}{p - c_s}
\]
(4.13)
$Q = R(O)$  
Leibniz's  
$$
\frac{dE}{dO} = 0
$$

$$
(p + c_m - s)R'(O) \left( 1 - F(R(O)) \right) + g_0 + 0.8h_1F(0.8O) + g_0F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1F + f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + m_1)]
\]
(4.11)

$$
\frac{dE(\text{cost})}{dO} = g_0 + 0.8h_1F(0.8O) + g_0F(O) - g_0F(Q) > 0
$$
Q \neq R(O). \text{ No valid solution.}

Need to calculate constraint solutions.

<table>
<thead>
<tr>
<th>Case 2</th>
<th>Optimal Solution</th>
<th>If $Q^\ast &gt; 1.2O$, this is the solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>O* is from (4.11).</td>
<td></td>
<td>F($Q^\ast$) = 1 - $\frac{c_0}{p - c_s}$</td>
</tr>
<tr>
<td>O* = 1/1.2 $\cdot$ Q*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>Constrain Solution</th>
<th>If $Q^\ast &lt; 1.2O$, let $Q^\ast = 1.2O + 1$,</th>
</tr>
</thead>
<tbody>
<tr>
<td>O* is from (4.11).</td>
<td></td>
<td>calculate the objectives</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.5 Remarks

It is common that the demand for any newly developed products is highly uncertain. How to reduce the uncertain demand risk faced by both the manufacturer and the supplier is a challenging problem. This research investigates the new product development in the automotive industry and designs a contract to reduce the risk. We developed the models to determine the manufacturer’s order quantity and the supplier’s planned production capacity so that both the supplier and the manufacturer can reach the optimal decisions in terms of supplier’s maximum profit and manufacturer’s minimum cost while taking into account the risk of market uncertainty.

The Newsvendor model and Stackelberg game theory are applied to formulate the supplier’s profit and the manufacturer’s cost objective function. The models are based on a single supplier, a single manufacturer, and a single period. Two cases are considered, Case 1 $Q \leq O$, and Case 2 $Q > O$, which are described with Nash subgame perfect equilibrium.

Our model shows that:

- Case 2 can be divided to 2 subcases: Case 2a $Q > 1.2O$, and Case 2b $O < Q \leq 1.2O$.  

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• Each case has its mathematical model, which is a constraint optimal problem. The constraint solution needs to be considered if the solution conflicts with the assumption. The final solution can be determined by comparing the subcase results to get the subgame perfect equilibria.

4.6 Appendix

Appendix A

Supplier’s objective function:

Case 1: Q < O

Supplier’s objective function is to maximize its expected profit. The supplier’s expected profit can be expressed by

\[ E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_Q^{\infty} Q(p - c_s)f(x)dx - c_s f(Q) \]
\[ - \int_0^Q e_1(x - Q)f(x)dx + \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx \]  

(4.1)

Apply Leibniz’s rule (Nahmias, S. 2009) to equation (1) to determine the derivative of

\[ dE(\pi) \]
\[ dQ \]

set \[ \frac{dE(\pi)}{dQ} = 0 \]. We get the supplier’s optimal production capability \( Q^* \)

\[ F(Q^*) = \frac{p - c - c_0 + e_1F(O) + 0.8h_1F(0.8Q^*)}{p - c + e_1} \]  

(4.2)

Check if the second derivative is negative to ensure that the total expected profit is maximized at \( Q^* \).

\[ \frac{d^2 E(\pi)}{dQ^2} = -(p - c_s + e_1)f(Q) + 0.64h_1f(0.8Q) \]  

(4.3)

It is difficult to check if \[ \frac{d^2 E(\pi)}{dQ^2} \leq 0 \], or (4.3) < 0. For uniform distribution,
\[ f(x) = \frac{1}{b-a} = f(Q) = f(0.8Q), \text{ so (4.3) becomes:} \]
\[
\frac{d^2 E(\pi)}{dQ^2} = \left[-(p - c_s + e_1) + 0.64h_1 \right] \frac{1}{(b-a)}
\]

Since \( \frac{1}{b-a} \geq 0 \), considering real parameters value, we have

\[-(p - c_s + e_1) \ll 0 \quad \text{Or:} \quad -(p - c_s + e_1) \ll 0.64h_1, \text{ which results } \frac{d^2 E(\pi)}{dQ^2} \leq 0. \text{ It means function } E(\pi) \text{ is a concave function in a uniform distribution, and has the max value in the optimal solution } Q^*.\]

**Case 2: Q > O (include Case 2a: Q > 1.2O and Case 2b: O < Q < 1.2O)**

**Case 2a: when Q > 1.2O, m(O, Q) > 0.**

\[
E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_Q^{\text{inf}} Q(p - c_s)f(x)dx - c_s f(Q) \\
+ \int_{1.2O}^Q m_1(x - 1.2O)f(x)dx + \int_0^{0.8O} h_1(0.8O - x)f(x)dx
\]

(4.7)

To determine the value of Q that maximizes the supplier’s expected profit \( E(\pi) \), we apply Leibniz’s rule (Nahmias, S. 2009) to equation (4.7) to determine the derivative of \( \frac{d E(\pi)}{dQ} \).

\[
\frac{d E(\pi)}{dQ} = (p - c_s)[1 - F(Q)] - c_0 + m_1(Q - 1.2O) \cdot f(Q)
\]

Set \( \frac{d E(\pi)}{dQ} = 0 \), we get the supplier’s optimal production capability \( Q^* \)

\[
(p - c_s)[1 - F(Q)] - c_0 + m_1(Q - 1.2O) \cdot f(Q) = 0
\]

(4.8)

Next, the second derivative is examined.

\[
\frac{d E^2(\pi)}{dQ^2} = -(p - c_s) \cdot F'(Q) + m_1 \cdot f(Q) + m_1 \cdot Q \cdot f'(Q)
\]
\[
= -(p - c_s) \cdot f(Q) + m_1 \cdot f(Q) + m_1 \cdot Q \cdot f'(Q)
\]
\[
= f(Q) \cdot (m_1 - p + c_s) + m_1 \cdot Q \cdot f'(Q)
\]

(4.9)

For uniform distribution \( U(a, b) \),
\[ f(x) = \frac{1}{b - a}, \quad f'(x) = 0 \]

\[ \frac{dE^2(\pi)}{dQ^2} = \frac{1}{b - a} \cdot (m_1 - p + c_s) \]

Since the denominator > 0, we need to analyze nominator. Only if nominator \((m_1 + c_s - p) < 0\), or \(m_1 < p - c_s\), \(E(\pi)\) has the maximum value at \(Q^*\), or \(E(\pi)\) is a convex function is this situation.

If \(m_1 > p - c_s\), \(E(\pi)\) has the minimum value at \(Q^*\) which is not our objective.

eg 1: If \(p = 9300\), \(c_s = 6000\), \(m_1 < p - c_s = 9300 - 6000 = p - 0.645p = 0.355p\).

eg 2: If \(p = 9300\), \(c_s = 8000\), \(m_1 < p - c_s = p (1 - c_s/p) = p (1- 0.86) = 0.14p\).

So choose \(m_1 = p, 2p\), or \(m_1 > p (1 - c_s/p)\) are not valid, which results \(\frac{dE^2(\pi)}{dQ^2} > 0\).

\(E(\pi)\) becomes minimum at \(Q^*\).

**Case 2b: 0 < Q < 1.2 O**

The manufacturer will not give the supplier with high production compensation. So

\(m(O, Q) = 0\)

\[ E(\pi) = \int_0^Q x(p - c_s)f(x)dx + \int_{Q}^{\inf} Q(p - c_s)f(x)dx - c_s f(Q) \]

\[ + \int_0^{0.8O} h_1(0.8O - x)f(x)dx \]

Apply Leibniz’s rule (Nahmias, S. 2009) to equation (4.12) to determine the derivative of \(\frac{dE(\pi)}{dQ}\). Set \(\frac{dE(\pi)}{dQ} = 0\)

We get the supplier’s optimal production capability \(Q^*\)

\[ F(Q^*) = 1 - \frac{c_0}{p - c_s} \]

This result can also be found from (4.8) when setting \(m_1 = 0\).

To examine the second derivative of (4.12), we just simply substitute \(m_1 = 0\) to (4.9)
\[
\frac{dE^2(\pi)}{dQ^2} = f(Q) \cdot (m_1 - p + c_s) + m_1 \cdot Q \cdot f'(Q) = f(Q) \cdot (-p + c_s)
\]

Since \( p > c_s \), we always have \( \frac{dE^2(\pi)}{dQ^2} < 0 \), which proves that function (4.12) is a concave function, and \( E(\pi) \) has a maximum at \( Q^* \). The formula means \( Q^* \neq R(O^*) \).

**Appendix B**

**Manufacturer’s cost function:**

**Case 1:** \( Q < O \)

\[
E(\text{cost}) = \int_0^Q (p + c_m)xf(x)dx + \int_{Q}^{\inf} (p + c_m)Qf(x)dx + g(O) - \int_Q^O e_1(x - Q)f(x)dx
\]

\[
+ \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx + \int_{Q}^{\inf} s(x - Q)f(x)dx
\]

(4.4)

Assume: \( Q = R(O) \)

\[
E(\text{cost}) = \int_0^{R(O)} (p + c_m)xf(x)dx + \int_{R(O)}^{\inf} (p + c_m)R(O)f(x)dx + g_1 + g_0 \cdot 0
\]

\[
- \int_{R(O)}^{O} e_1(x - R(O))f(x)dx + \int_0^{0.8R(O)} h_1(0.8R(O) - x)f(x)dx + \int_{R(O)}^{\inf} s(x - Q)f(x)dx
\]

(4.5)

Apply Leibniz’s rule (Nahmias, S. 2009) to equation (4.5) to calculate \( \frac{dE(\text{cost})}{dO} \).

\[
\frac{dE(\text{cost})}{dO} = \frac{d\text{VI}}{dO} - \frac{d\text{VII}}{dO} + \frac{d\text{VIII}}{dO} + \frac{d\text{IX}}{dO} + \frac{d\text{X}}{dO} + \frac{d\text{XI}}{dO}
\]

\[
= (p + c_m) \cdot R(O) \cdot R'(O) \cdot f(R(O)) + (p + c_m) \cdot R'(O) \left( 1 - F(R(O)) \right) - (p)
\]

\[
+ c_m \cdot R'(O) \cdot R(O) \cdot f(R(O)) + g_0 + e_1R'(O) \left( F(O) - F(R(O)) \right) - e_1O \cdot f(O)
\]

\[
+ e_1R(O)f(O) + 0.8h_1R'(O)F(0.8R(O)) + sR'(O)(F(R(O)) - 1)
\]

\[
\frac{dE(\text{cost})}{dO} = R'(O) \left( p + c_m + e_1F(O) - s \right) + R'(O) \cdot F(R(O)) \left( s - p - c_m - e_1 \right)
\]

\[
+ e_1 \cdot f(O)(R(O) - O) + 0.8 \cdot h_1 \cdot R'(O) \cdot F(0.8 \cdot R(O)) + g_0
\]
Set \( \frac{dE(\text{cost})}{dO} = 0 \), we have

\[
R'(O)(p + c_m + e_1 F(O) - s) + R'(O) \cdot F(R(O)) \cdot (s - p - c_m - e_1) + e_1 \cdot f(O)(R(O) - O) + 0.8h_1 \cdot R'(O) \cdot F(0.8R(O)) + g_0 = 0
\]  
(4.6)

Check if the second derivative of \( E(\text{cost}) \) is positive to ensure that the total expected cost is minimized at \( O^* \).

It is too difficult to calculate \( \frac{d^2 E(\text{cost})}{dO^2} \) in general form. We will discuss it in uniform distribution later.

**Case 2: Q > O**

Since penalty = 0, we have \( \int_{R(O)}^Q e_1(x - R(O))f(x)dx=0 \).

- Case 2a: when \( Q > 1.2O \), add \( m(O, Q) \) and \( g(O, Q) \)

\[
E(\text{cost}) = \int_0^Q (p + c_m)x f(x)dx + \int_Q^\infty (p + c_m)Q f(x)dx + g(O)
\]
\[
+ \int_0^Q g_0(x - O)f(x)dx + \int_{1.2O}^Q m_1(x - 1.2O)f(x)dx
\]
\[
+ \int_{0.8O}^{0.8O} h_1(0.8O - x)f(x)dx + \int_Q^\infty s(x - Q)f(x)dx
\]

But add 2 new items: \( \int_{1.2O}^Q m_1(x - 1.2O)f(x)dx \) and \( \int_0^Q g_0(x - O)f(x)dx \)

Substitute \( Q = R(O) \),

\[
E(\text{cost}) = \int_0^{R(O)} (p + c_m)x f(x)dx + \int_{R(O)}^\infty (p + c_m)R(O)f(x)dx + g(O) + \int_{1.2O}^{R(O)} m_1(x - 1.2O)f(x)dx
\]
\[
+ \int_{0.8O}^{0.8O} h_1(0.8O - x)f(x)dx + \int_{R(O)}^\infty s(x - R(O))f(x)dx + \int_0^{R(O)} g_0(x - O)f(x)dx
\]
\[
(4.10')
\]

Set \( \frac{dE(\text{cost})}{dO} = 0 \), we have

\[
\frac{dE(\text{cost})}{dO} = (p + c_m - s)R'(O) \left( 1 - F(R(O)) \right) + g_0 + 0.8h_1F(0.8O)
\]
\[
+ g_0F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1F(1.2O)
\]
\[
+ f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)]
\]
\[
(4.11)
\]
Next step is to check if the second derivative of $E(\text{cost})$ of Case 2a is positive to ensure that the total expected cost is minimized at $O^*$. However, it is very difficult to calculate \( \frac{dE^2(\text{cost})}{dO^2} \) in general form. We will discuss it in uniform distribution later.

- **Case 2b**: $O < Q < 1.2O$.

$m(O, Q) = 0$.

\[
E(\text{cost}) = \int_0^Q (p + c_m)xf(x)dx + \int_{Q}^{\inf} (p + c_m)Qf(x)dx + g(O)
+ \int_0^Q g_0(x - O)f(x)dx
+ \int_0^{0.8O} h_1(0.8O - x)f(x)dx + \int_{Q}^{\inf} s(x - Q)f(x)dx
\]

To determine the value of $O$ that minimizes the manufacturer’s expected profit $E(\text{cost})$, use

\[
\frac{dE(\text{cost})}{dO} = 0
\]

Case 2b can be considered as a special case of Case 2a, (4.14) can be solved by setting $m(O, Q) = 0$, $Q \neq R(O)$. $R'(O) = 0$ in (4.11) , we get:

\[
\frac{dE(\text{cost})}{dO} = g_0 + 0.8h_1F(0.8O) + g_0F(O) - g_0F(Q)
\]

Set \( \frac{dE(\text{cost})}{dO} = 0 \)

\[
g_0 + 0.8h_1F(0.8O) + g_0F(O) - g_0F(Q) = 0
\]

\[
0.8h_1F(0.8O) + g_0F(O) = -g_0(1 - F(Q))
\]

Analysis:

Since $F(Q) \leq 1$, the right side of the equation (4.16) is $\leq 0$, i.e., $RS = -g_0(1 - F(Q)) \leq 0$.

But the left side of the equation (4.16) is $\geq 0$, i.e. $LS = 0.8h_1F(0.8O) + g_0F(O) \geq 0$. So there is no $O$ solution to satisfy \( \frac{dE(\text{cost})}{dO} = 0 \). Actually, \( \frac{dE(\text{cost})}{dO} > 0 \) in this case.

Mathematically, since the first derivative is always $> 0$, and the target function is to minimize the cost, the optimal solution is at the smallest $O$, i.e, $O = 0$. This is a constraint optimization problem, constraint $Q=1.2O$ needs to be considered to solve the problem.

Then the optimal $O^* > 1/1.2 \cdot Q^*$. 

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So the constraint solution 1 for Case 2b is:

\[
F(Q^*) = 1 - \frac{c_0}{p - c_s} \quad (4.13)
\]

\[
O^* = \frac{1}{1.2} \cdot Q^* \quad (4.17)
\]

For example, substituting the parameter values \((c_0 = 500, p = 9300, c_s = 8000)\) to the formula (4.8) and (4.17), then \(F(Q^*) = 1 - \frac{500}{9300 - 8000} = 0.6154\).

- For normal distribution \(N(7500, 2166.67^2)\), \(Q^* = 8136\) from the inverse of \(F(Q^*) = 0.6154\). So the optimal solution \(O^* = 0.833Q^* = 6780\).

- For uniform distribution \(U(1000, 14000)\), \(F(Q) = \frac{Q - a}{b - a}\), then \(Q = F(Q) (b-a) + a\). Since \(Q^* = 9000\), the optimal solution \(O^* = \frac{1}{1.2} \cdot Q^* = 7500\).
5.1 Introduction
In Chapter 4, we developed a new product optimal contract model. It is assumed that the demand for new products is uncertain in the coming years, but the distribution can be estimated. The solution of the model is independent of the distribution functions. Based on the discussion of the optimal contract models and solutions in the last chapter, we will provide the solution with the uniform demand distribution and conduct some numerical experiments in this chapter. Uniform distribution is a simplest stochastic distribution with equal probability density function (pdf), and the closed form solution can be obtained in real-world application. In the new product development cases, the demand distribution of new products is unknown. The pdf of new product demand can be assumed equally and uniformly distributed within a certain range. Other stochastic demand distribution, such as normal distribution, requires the historical data to calculate the variance about the mean. Based on a real automotive supply chain case, the numerical experiments are also conducted to study the behavior and performance of the proposed model under the uniform distribution in this chapter. Also, some parameter sensitivity analyses are performed in order to obtain the managerial insights into the contract model.

The remainder of this chapter is organized as follows: in Section 5.2, the uniform distribution background is introduced; Section 5.3 provides the contract model solution for the uniform distribution; the constraint solution is discussed in Section 5.4; Section 5.5 illustrates a numerical experiment to get the subgame perfect optimal solution. The sensitive analyses on varying several parameters are conducted in Section 5.6. Finally, the remarks including a summary of the main results and some future work direction are given in Section 5.7. Proofs of all propositions appear in Appendix 5.8.

5.2 Uniform Distribution
For uniform distribution, its probability density function (pdf) is:

\[ f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{otherwise} \end{cases} \]
The cumulative distribution function (CDF) is:

\[
f(x) = \begin{cases} 
0 & \text{for } x < 0 \\
\frac{x-a}{b-a} & \text{for } a \leq x \leq b \\
1 & \text{for } x \geq b 
\end{cases}
\]

For uniform distribution of demand \(x\), assume: \(a \leq x \leq b\), its pdf is shown in the Figure 5.1.

![Figure 5.1. Uniform distribution of the demand. (Source: [4])](image)

### 5.3 Solution for Uniform Distribution

As shown in Figure 5.2, there are two calculation methods to solve the contract model in uniform distribution. The method I applies Leibniz’s rule to supplier’s profit function \(E(\pi)\).

By setting the derivative \(\frac{dE(\pi)}{dQ} = 0\), the supplier’s optimal production capability \(Q^*\) can be expressed as a function of the manufacturer’s order quantity \(O\), i.e. \(Q^* = R(O)\). For uniform distribution, assume \(Q^* = R(O) = \alpha + \beta O\). Substituting \(Q^* = \alpha + \beta O\) to manufacturer’s cost function \(E(\text{cost})\), and setting \(\frac{dE(\text{cost})}{dO} = 0\), the manufacturer’s optimal order \(O^*\) can be found, and \(Q^*\) can be calculated. Substituting \(O^*\) and \(Q^*\) to the integrals of \(E(\pi)\) and \(E(\text{cost})\), \(E^*(\pi)\) and \(E^*(\text{cost})\) can be obtained by conducting an integral operation.

In Method II, the first step is to take the integral operation for \(E(\pi)\) and \(E(\text{cost})\), and they can be expressed in algebra functions. The second step is to take the derivatives \(\frac{dE(\pi)}{dQ} = 0\) and \(\frac{dE(\text{cost})}{dO} = 0\). Then \(Q^*\) and \(O^*\) are obtained. The optimal solutions \(Q^*\) and \(O^*\) can
be verified by checking the secondary derivatives if \( \frac{d^2E(\pi)}{dQ^2} \leq 0 \) and \( \frac{d^2E(\text{cost})}{dO^2} \geq 0 \). Then \( E^*(\pi) \) and \( E^*(\text{cost}) \) can be calculated from the algebra functions.

Both methods have the same results. Note that Method II has the benefit to express objective function \( E(\pi) \) and \( E(\text{cost}) \) as an algebra function of the variables. Therefore they can be easily calculated and analyzed. Also it has the benefit to calculate the second order derivative to prove whether the objective function is a concave or convex function. (See Appendix A for details).

Two methods can be summarized in the flowchart as shown in Figure 5.2.

![Flowchart](image)

Figure 5.2. The flow chart of two methods

In the rest of this chapter, we will discuss the solutions of Case 1 and Case 2 in uniform distribution separately.

- For Case 1: \( Q < O \),

Based on (4.2),

\[
F(Q^*) = \frac{p - c - c_0 + e_1 F(O) + 0.8h_1 F(0.8Q^*)}{p - c + e_1}
\]
From uniform distribution, for \( a \leq Q < b, a \leq O < b, \)

\[
F(Q^*) = \frac{Q^* - a}{b - a}, \quad F(0.8Q^*) = \frac{0.8Q^* - a}{b - a}, \quad F(O) = \frac{O - a}{b - a}
\]

The solution of (4.2) is:

\[
Q^* = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)a + e_1 O}{p - c_s + e_1 - 0.64h_1} = \alpha + \beta O
\]

\[= R(O) \quad (5.1)\]

Where

\[
\alpha = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)a}{p - c_s + e_1 - 0.64h_1} \quad (5.2)
\]

\[
\beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1} \quad (5.3)
\]

\(Q^*\) is the supplier’s optimal production capability.

Based on (4.6), (consider manufacturer’s shortage cost)

\[
R'(O)(p + c_m + e_1 F(O) - s) + R'(O) \cdot F(R(O)) \cdot (s - p - c_m - e_1)
+ e_1 \cdot f(O)(R(O) - O) + 0.8h_1 \cdot R'(O) \cdot F(0.8R(O)) + g_0 = 0
\]

(4.6)

For uniform distribution \( a \leq x < b, \) from (5.1),

\[
Q^* = R(O) = \alpha + \beta O, \quad R'(O) = \beta,
\]

Also from a uniform distribution,

\[
f(O) = \frac{1}{b - a}, \quad F(O) = \frac{O - a}{b - a}, \quad F(R(O)) = \frac{R(O) - a}{b - a},
\]

\[
F(0.8R(O)) = \frac{0.8R(O) - a}{b - a}
\]

Substituting them to (4.6), the solution \( O^* \) is:

\[
O^* = \frac{\alpha \beta (p + c_m + e_1 - s - 0.64h_1) - e_1 \alpha + (0.8h_1 a + sb - pb - bc_m) \beta - g_0 (b - a)}{(s - p - c_m - e_1 + 0.64h_1) \beta^2 + 2e_1 \beta - e_1}
\]

(5.4)
Apply Method II in Figure 5.2, we can prove \( \frac{dE^2(\text{cost})}{dO^2} > 0 \) at \( O^* \), which means manufacturer’s expected cost is the minimum at \( O^* \). So \( O^* \) is the manufacturer’s optimal order in Case 1.

- For Case 2: \( Q > O \), based on (4.8),

\[
F(Q^*) = 1 - \frac{1}{p - c_s}[c_0 - m_1(Q^* - 1.2O) \cdot f(Q^*)]
\]

From uniform distribution, substituting \( F(Q) = \frac{Q-a}{b-a} \) and \( f(Q) = \frac{1}{b-a} \) to (4.8), the solution is

\[
Q^* = \frac{c_0(b - a) + b(c_s - p) + 1.2m_1O}{c_s - p + m_1} = \zeta + \eta O = R(O)
\] (5.5)

where

\[
\zeta = \frac{c_0(b - a) + b(c_s - p)}{c_s - p + m_1} \quad (5.6)
\]

\[
\eta = \frac{1.2m_1}{c_s - p + m_1} \quad (5.7)
\]

Based on (4.11),

\[
(p + c_m - s)R'(O) \left(1 - F(R(O))\right) + g_0 + 0.8h_1F(0.8O) + g_0F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1F(1.2O) + f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)] = 0
\]

For uniform distribution \( a \leq x < b \), from (5-5), \( Q^* \) can be found:

\[
Q^* = R(O) = \zeta + \eta O, \quad R'(O) = \eta,
\]

Also from a uniform distribution, we have:

\[
f(O) = \frac{1}{b-a}, \quad f(R(O)) = \frac{1}{b-a}, \quad F(O) = \frac{O - a}{b - a}, \quad F(1.2O) = \frac{1.2O - a}{b - a},
\]

\[
F(0.8O) = \frac{0.8O - a}{b - a}, \quad F(R(O)) = \frac{R(O) - a}{b - a}, \quad F(0.8R(O)) = \frac{0.8R(O) - a}{b - a}
\]

Substituting them to (4.11), the optimal solution for \( O \) is:
\[ O^* = \frac{\zeta[\eta(p + c_m - s - g_0 - m_1) + g_0 + 1.2m_1] - (p + c_m - s)\eta b - g_0(b - a) + 0.8h_1a}{\eta^2(s - p - c_m + g_0 + m_1) - 2(\eta g_0 + 1.2m_1) + g_0 + 0.64h_1 + 1.44m_1} \]  

(5.8)

where \( O^* \) is the manufacturer’s optimal order.

Apply method II in Figure 5.2; we can prove that \( \frac{dE^2(\text{cost})}{dO^2} > 0 \), so \( E(\text{cost}) \) is convex function and has the minimum value at \( O^* \) in Case 2 with a uniform distribution (See Appendix A).

From the study above, we can see that in uniform demand distribution, the closed form solutions of optimal \( O^*, Q^*, E(\pi) \) and \( E(\text{cost}) \) can be found. The summary of solutions for both Case 1 and Case 2 in uniform demand distribution is given in Tables 5.1.

### 5.4 Constraint Solution in Uniform Distribution

As we explained in Chapter 4, Case 1 is for \( Q \leq O \); and Case 2 is for \( Q > O \), which consists of subgame Case 2a with \( Q > 1.2O \), and Case 2b with \( O < Q \leq 1.2O \). The solutions of subgame need to be checked to see if they are within the constraints. If not, the solutions need to be adjusted. In order to find the constraint optimization solution, the formulas of uniform distribution is substituted into the equations in Table 4.2. The details are shown in Appendix B.

Case 1, Case 2, their subcases, and the constraint solutions are summarized below.

- **Case 1** \( Q^* \leq O^* \),

\( Q^*, O^* \) is calculated from formula (5.1) ~ (5.4).

1) Case 1 constraint: If \( Q^* > O^* \), let \( Q^* = R(O) = O \); which is defined as Case 1c.

After getting the optimal solution \( O^*, Q^* \), the supplier objective function \( E(\pi)(O, Q) \) can be calculated by the formula (5.14), and the manufacturer objective function \( E(\text{cost})(O, Q) \) can be calculated by the formula (5.12) for all Case 1.

- **Case 2** \( Q^* > O^* \)

1) Case 2a: \( Q^* > 1.2O \), \( Q^*, O^* \) is calculated from formula (5.5) ~ (5.8).

2) Case 2b: \( O < Q \leq 1.2O \)
\[ F(Q^*) = 1 - \frac{c_0}{p - c_s} \]  

(4.13)

In a uniform distribution, (4.13) becomes

\[ Q^* = (1 - \frac{c_0}{p - c_s})(b - a) + a \]  

(5.11)

\[ O^* = 1/1.2 \cdot Q^* \]  

(4.17)

3) Case 2a constraint: If the result \( Q^* < 1.2O \), let \( Q^* = 1.2O + 1 \), which is defined as Case 2c.

After getting the optimal solution \( O^* \), \( Q^* \), formula (5.15) is used to calculate the supplier objective function \( E(\pi)(O, Q) \), and formula (5.13) is used to calculate manufacturer objective function \( E(cost)(O, Q) \) for all Case 2.

Table 5.1 and Table 5.2 summarize the optimal solutions, constraint solutions, and objective functions for Case 1, subcase Case 1c, Case 2, subcase Case 2a, Case 2b, and Case 2c in uniform distribution respectively.

### Table 5.1 Summary of the optimal solutions and constraint solutions, and objective functions for Case 1 and their subcases in a uniform distribution.

<table>
<thead>
<tr>
<th>Case 1: Assume ( Q \leq O )</th>
<th>Case 1 constraint: Case 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td>( Q^* = R(O) = \alpha + \beta O, ; e_1 = 0 )</td>
</tr>
</tbody>
</table>

Optimal Solution

\[ \alpha = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)\alpha}{p - c_s + e_1 - 0.64h_1} \]  

(5.2)

\[ \beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1} \]  

(5.3)

\[ O^* = \frac{\alpha p + c_m + e_1 - s - 0.64h_1 - e_0(\alpha + (0.8b - b_0 - b_0) - g_0(b - a))}{(x - p - c_m + 0.64h_1)\beta + g_0(\beta - e_0)} \]  

(5.4)

\[ O^* = \frac{b(p + c_m - s + g_0) - \alpha(g_0 + 0.8h_1)}{p + c_m - s - 0.64h_1} \]  

(5.10)
Table 5.2 Summary of the optimal solutions and constraint solutions, and objective functions for Case 2 and their subcases in a uniform distribution.

<table>
<thead>
<tr>
<th>Case 2: Assume ( Q &gt; O )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniform Distribution</strong></td>
</tr>
<tr>
<td><strong>Optimal Solution</strong></td>
</tr>
</tbody>
</table>

**Case 2a:** \( Q > 1.2 \ O \)

- \( \zeta = \frac{c_0(b-a)+b(c_2-p)}{c_2-p+m_1} \) (5.6)
- \( \eta = \frac{1.2m_1}{c_2-p+m_1} \) (5.7)
- \( O^* = \frac{\zeta[p+\zeta - s - g_1 - m_1] + g_2 + 12m_1 - \zeta(p+\zeta - s)g_2 + g_3 + 0.8\zeta\delta}{\eta'(s-p+\zeta + g_1 + m_1 - 2\zeta g_2 + 12m_1) + g_3 + 0.64\eta + 1.44m_1} \) (5.8)

**Case 2b:** \( O < Q \leq 1.2O, m_1 = 0 \)

- \( \zeta = 0 \)
- \( \eta = 1.2 \)

**TABLE 5.2** Summary of the optimal solutions and constraint solutions, and objective functions for Case 2 and their subcases in a uniform distribution.
\[ Q^* = R(O) = \zeta + \eta O \quad (5.5) \]
\[ Q^* = (1 - \frac{\epsilon_0}{p-cs})(b-a) + a = Q_{2b} \quad (5.11) \]

| Supplier | \[ \frac{1}{2(b-a)}(c_i-p+m_i)Q^2 + \left[ \frac{1}{b-a}b(p-c_i)-c_i \right]Q \right] 
| Objective | \[ \frac{1}{2(b-a)}(c_i-p)Q^2 + \left[ \frac{1}{b-a}b(p-c_i)-c_i \right]Q \]
| Function | \[ + \frac{1}{2(b-a)}(0.64h_i+1.44m_i)Q^2 \]
| \[ - \frac{0.8a_i}{b-a}O + \frac{1}{2(b-a)}a^2(c_i-p+h_i)-c_{is} \] | \[ - \frac{0.8a_i}{b-a}O + \frac{1}{2(b-a)}a^2(c_i-p+h_i)-c_{is} \]
| \[ E(\pi)(O, Q) \] | \[ \text{or (5.15) when } m_1 = 0 \]

| Manufacturer | \[ \left( -p-c_e+m_p+s+g_s \right) \frac{2}{b-a} + \frac{1}{b-a}(p+c_e-s)b \cdot Q \]
| Objective | \[ \left( -p-c_e+m_p+s+g_s \right) \frac{2}{b-a} + \frac{1}{b-a}(p+c_e-s)b \cdot Q \]
| Function | \[ - \frac{1.2m_s+g_s}{b-a} \cdot O \cdot Q + \frac{(1.44m_s+0.64h_s+g_s)}{2(b-a)}O^2 \]
| \[ + \left( \frac{0.8a_h}{b-a}O + \frac{1}{2(b-a)}a^2h_h+b^2s-(p+c_e)a^2 \right) \] | \[ + \left( \frac{0.8a_h}{b-a}O + \frac{1}{2(b-a)}a^2h_h+b^2s-(p+c_e)a^2 \right) \]
| \[ E(\text{cost})(O, Q) \] | \[ \text{or (5.13) when } m_1 = 0 \]

### Case 2c

Constrain

Solution

If \( Q^* < 1.2O \), let \( Q^* = 1.2O+1 \),

\( O^* \) is from (5.8), \( Q^* = 1.2O+1 \),

Calculate \( E(\pi)(O, Q) \) from (5.15),

\( E(\text{cost})(O, Q) \) from (5.13)

Since the boundary constrains are always satisfied, no need to calculate boundary conditions

---

5.5 Numerical Experiment

In this section, the model and solutions are to be verified with numerical experiments. The hypothetical experiment data are given in Table 5.3. The data refer to an automotive OEM, which develops a new electric vehicle (EV), and a major component supplier, which
develops, produces and supplies the EV battery. The market demand for the new EV is highly uncertain. The supplier needs to invest significantly on equipment or new production line. Therefore, the key issue is to decide the production capacity of new equipment or facility under the uncertain demand.

Parameter \( c_{sb} \) is supplier’s fixed cost for equipment at the beginning. It can be defined as follows: assuming that the supplier’s initial investment is $5,000,000 and the program lifetime is five years. Therefore the annual depreciation is $1,000,000, which is defined as \( c_{sb} \). \( g_b \) is manufacturer’s fixed cost for equipment at the beginning. It is defined in the same way as \( c_{sb} \).

**Table 5.3 Numerical Experiment Parameter Data Set**

<table>
<thead>
<tr>
<th>( c_s )</th>
<th>( c_0 )</th>
<th>( c_{sb} )</th>
<th>( g_0 )</th>
<th>( g_b )</th>
<th>( c_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8,000</td>
<td>$500</td>
<td>$1,000,000</td>
<td>$500</td>
<td>$2,000,000</td>
<td>$465</td>
</tr>
</tbody>
</table>

**5.5.1 An Example of the Contract Model in Uniform Distribution U(1000, 14000)**

In this section, an example of solving the optimal contract model and objective function in a uniform distribution U(1000, 14000) will be summarized in Table 5.4. Following the calculation steps in Table 5.1 and Table 5.2, substituting the parameters in Table 5.3 to the formulas for Case 1 and Case 2 and setting \( a = 1000 \), \( b = 14000 \), we get the optimal solution and constraint solution and their objective function in algebra format for Case 1 and Case 2 as shown in Table 5.4.

**Table 5.4. A Numerical Example of Calculation for Case 1, Case 2 Optimal Solution O*, Q* and Objective Functions E(π)(O, Q), E(cost) with U(1000, 14000), Parameters Data in Table 5.3**

<table>
<thead>
<tr>
<th>Case 1: Assume ( Q \leq O )</th>
<th>Case 1: Assume ( Q &lt; O )</th>
<th>Case 1 constraint: Case 1c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Assume ( Q &lt; O )</td>
<td>Case 1 constraint: Case 1c</td>
<td></td>
</tr>
<tr>
<td>If ( Q &gt; O ), let ( Q = O )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td></td>
</tr>
<tr>
<td>Uniform Distribution Optimal Solution</td>
<td>( \alpha = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)\alpha}{p - c_s + e_1 - 0.64h_1} )</td>
<td>( \alpha = 0 )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>( \beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1} )</td>
<td>( \beta = 1 )</td>
</tr>
<tr>
<td>( O^* )</td>
<td>( O^* = 8964 )</td>
<td>( O^* = 9823 )</td>
</tr>
<tr>
<td>( Q^* )</td>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td>( Q^* = O^* )</td>
</tr>
<tr>
<td></td>
<td>( Q^* = 9920 )</td>
<td>( Q^* = 9823 )</td>
</tr>
<tr>
<td>Supplier Objective Function</td>
<td>( E(\pi)(O, Q) )</td>
<td>( E(cost)(O, Q) )</td>
</tr>
<tr>
<td>From (5.14)</td>
<td>From (5.14)</td>
<td>From (5.12)</td>
</tr>
<tr>
<td>(- 0.558 \cdot Q^2 + 728 \cdot Q + 1.1538462 \cdot O \cdot Q)</td>
<td>(- 0.558 \cdot Q^2 + 728 \cdot Q + 1.1538462 \cdot O \cdot Q)</td>
<td>(- 0.30665 \cdot Q^2 - 5810 \cdot Q + 1.1538462 \cdot O \cdot Q)</td>
</tr>
<tr>
<td>(- 0.576823 \cdot O^2 - 942307.692)</td>
<td>(- 0.576823 \cdot O^2 - 942307.692)</td>
<td>(- 0.30665 \cdot Q^2 - 5810 \cdot Q + 1.1538462 \cdot O \cdot Q)</td>
</tr>
<tr>
<td>=7612107</td>
<td>=8032317</td>
<td>=88727545</td>
</tr>
<tr>
<td>Manufacturer Objective Function</td>
<td>( E(cost)(O, Q) )</td>
<td>Case 2: Assume ( Q &gt; O )</td>
</tr>
<tr>
<td>From (5.12)</td>
<td>From (5.12)</td>
<td>Case 2a: ( Q &gt; 1.2O )</td>
</tr>
<tr>
<td>(- 0.30665 \cdot Q^2 - 5810 \cdot Q + 1.1538462 \cdot O \cdot Q) - ( 0.576823 \cdot O^2 + 500 \cdot O + 114809038)</td>
<td>(- 0.30665 \cdot Q^2 - 5810 \cdot Q + 1.1538462 \cdot O \cdot Q) - ( 0.576823 \cdot O^2 + 500 \cdot O + 114809038)</td>
<td>( Q^* = R(O) = \zeta + \eta O )</td>
</tr>
<tr>
<td>( = 87724721 )</td>
<td>( = 88727545 )</td>
<td>( Q^* = R(O) = \zeta + \frac{O}{\zeta} )</td>
</tr>
</tbody>
</table>

Case 2a: \( Q > 1.2O \)  

Case 2b: \( O < Q \leq 1.2O \)  

Uniform Distribution

\( \zeta = \frac{c_0 (b - \alpha) + b (c_s - p)}{c_s - p + m_1} \)  

\( \zeta = 0 \)
Optimal Solution

\[
\eta = \frac{1.2m_i}{c_i - P + m_i} = -154.8
\]

\[
O^* = \frac{\eta \left( p + c_n - s - g_i - m_i \right) + \frac{12m_i}{\eta} - \left( p + c_n - s \right) \left( p + c_n - s \right) - 2 \left( p + c_n + g_i \right) - g_i + 0.6h + 1.444}{\eta}\]

\[
O^* = 7476
\]

\[
Q^* = \frac{R(O) = \zeta + \eta O}{Q^* = 12175}
\]

Supplier Objective Function \(E(\pi)(O, Q)\)

From (5.15)

\[
\frac{1}{2(b-a)}(s - p - c_m - e_1 + 0.64h_1) \cdot Q^2 + \frac{1}{b-a}(pb + c_m b - c_m b - 0.8a h_1 - sb) \\
\cdot \frac{e_1}{b-a}Q + \frac{1}{2(b-a)}O \cdot Q - \frac{e_1}{2(b-a)}O^2 + g_1 \cdot O + \frac{1}{2(b-a)}[a^2(-p - c_m + h_1) + s b^2] + g_1
\]

\[
= 5677020 \quad (5.22)
\]

Manufacturer Objective Function \(E(\text{cost})(O, Q)\)

From (5.13)

\[
\frac{1}{2(b-a)}(s - p - c_n - e_1 + 0.64h) \cdot Q^2 + \frac{1}{b-a}(pb + c_n b - 0.8a h - sb) \cdot \frac{e_1}{b-a}Q \\
+ \frac{e_1}{2(b-a)}O + \frac{1}{2(b-a)}[c^2(-p - c_n + h_1) + s b^2] + g_1
\]

\[
= 83202890 \quad (5.23)
\]

Case 2c Constrain Solution

If \(Q^*<1.2O\), let \(Q^*=1.2O+1\),

\(O^*\) is from (5.8), \(Q^*=1.2O+1\),

Calculate \(E(\pi)(O, Q)\) from (5.15),

\(E(\text{cost})(O, Q)\) from (5.13)
Through the numerical calculation, we notice that Formula (5.14) (Case 1) and (5.16) (Case 1c) have the same result $E(\pi)$; Formula (5.12) (Case 1) and (5.17) (Case 1c) have the same result $E(\text{cost})$ when $Q = O$. So (5.14) and (5.12) are appropriate for all subcases of Case 1. Also, it is shown that Case 2a formulas are appropriate for both Case 2b and Case 2c. The result of Formula (5.18) (Case 2b) is the same as Formula (5.22) (Case 2a) when $Q = 1.2O$.

The numerical results for the above optimal solutions are summarized in Table 5.5.

**Table 5.5 Comparison of Case 1 and Case 2 solutions and their constraint solutions with U(1000, 14000) and parameter data from Table 5.3**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Solution</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$O^*$</th>
<th>$E(\pi)$</th>
<th>$E(\text{cost})$</th>
<th>$Q^<em>/O^</em>$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \leq O$</td>
<td>Case 1: $Q &lt; O$</td>
<td>$Q^* = R(O) = \alpha + \beta O$</td>
<td>652.054</td>
<td>1.0339</td>
<td>8964</td>
<td>7611281</td>
<td>87724789</td>
<td>1.11</td>
</tr>
<tr>
<td>Case 1c: $Q = O^*$</td>
<td>$Q^* = O$</td>
<td>0</td>
<td>1</td>
<td>9823</td>
<td>9823</td>
<td>8031726</td>
<td>88727545</td>
<td>1.00</td>
</tr>
<tr>
<td>Case 2</td>
<td>Solution</td>
<td>$\zeta$</td>
<td>$\eta$</td>
<td>$O^*$</td>
<td>$Q^*$</td>
<td>$E(\pi)$</td>
<td>$E(\text{cost})$</td>
<td>$Q^<em>/O^</em>$</td>
</tr>
<tr>
<td>$Q &gt; O$</td>
<td>Case 2a: $Q^* &gt; 1.2O$</td>
<td>$Q^* = R(O) = \zeta + \eta O$</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>5677020</td>
<td>83202878</td>
</tr>
<tr>
<td>Case 2b: $O &lt; Q \leq 1.2O$</td>
<td>$O = 1/1.2 Q^*$</td>
<td>0</td>
<td>1.2</td>
<td>7500</td>
<td>9001</td>
<td>5692307</td>
<td>86756730</td>
<td>1.20</td>
</tr>
<tr>
<td>Case 2c: if $Q &lt; 1.2O$</td>
<td>$Q^* = 1.2O + 1$</td>
<td>1</td>
<td>1.2</td>
<td>7500</td>
<td>9001</td>
<td>5692307</td>
<td>86754775</td>
<td>1.20</td>
</tr>
</tbody>
</table>

We need to examine the results in Table 5.5 if the solutions are within the constraints. The optimal solution of Case 1 is $Q^* > O^*$, which is contradicted with Case 1 assumption $Q < O$. Therefore the optimal solution of Case 1, $Q^* = R(O) = \alpha + \beta O$, is invalid. The constraint solution of Case 1c, $Q^* = O^*$, needs to be considered. Considering Case 2a, $Q^* = \zeta + \eta O^*$, the optimal solution for $Q^* > 1.2O^*$ is satisfied with the assumptions of Case 2a. Comparing the optimal solutions of all subcases in Table 5.5, Case 2a has the minimum expected cost $E(\text{cost})$. So the manufacturer, as a leader, will choose Case 2a strategy. In the next section, we will verify the solution of Case 2a ($O^* = 7476$ and $Q^* = 12,715$) is the best response among the three possible options, which will maximize the supplier’s expected profit.

### 5.5.2 Optimal Response Function $E(\pi)$

Since the optimal solutions in Table 5.5 have the different values of variable $O$, it is difficult to compare the supplier’s response functions $E(\pi)$ directly. In this section, we will verify the optimal response function with regard to manufacturer’s order quantity $O$ as $E(\pi)(O)$.

In the example of uniform distribution U(1000, 14000) above, given an $O$, substitute $Q = R(O)$ to the related response function $E(\pi)(O, Q)$ for each subcase shown in Table 5.4.
$E(\pi)(O, Q)$ becomes $E(\pi)(O)$, which is a single variable quadratic function. The following are response functions $E(\pi)(O)$ in each subcase:

- Case 1:

  $Q^* = R(O) = \alpha + \beta O$

  $$\alpha = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)a}{p - c_s + e_1 - 0.64h_1} = 652.05$$

  $$\beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1} = 1.034$$

  Then we have $Q = R(O) = \alpha + \beta O = 652.05 + 1.034O$. Because Case 1 is effective only when $Q \leq O$. The case is invalid for this instance. Then we need to check Case 1c.

- Case 1c: $Q = O$

  $\alpha = 0$, $\beta = 1$, substitute $Q = O$ to Formula (5.20)

  $$E(\pi) = -0.558\cdot O^2 + 727.6923O + 1.15348462\cdot O^2 - 0.5769231\cdot O^2 - 942307.692$$

  $$= 0.0189231\cdot O^2 + 727.6923\cdot O - 942308$$

- Case 2a:

  $Q^* = R(O) = \zeta + \eta O$

  $$\zeta = \frac{c_0(b - a) + b(c_s - p)}{c_s - p + m_1}, \quad \eta = \frac{1.2m_1}{c_s - p + m_1}$$

  $\zeta = 1170000$, $\eta = -154.8$

  Substitute $Q^* = 1170000 -154.8\cdot O$ to Formula (5.22)

  $$E(\pi)$$

  $$= \frac{1}{2(b - a)}(s - p - c_m - e_1 + 0.64h_1) \cdot Q^2$$

  $$+ \frac{1}{b - a}(pb + c_mb - c_mb - 0.8ah_1 - sb) \cdot Q + \frac{e_1}{b - a}O \cdot Q - \frac{e_1}{2(b - a)}O^2$$

  $$+ g_0 \cdot O + \frac{1}{2(b - a)}[a^2(-p - c_m + h_1) + sb^2] + g_1$$

  (5.22)
\[ E(\pi) = -0.000385 \cdot (1170000 - 154.8 \cdot O)^2 + 900 \cdot (1170000 - 154.8 \cdot O) \]
\[-0.119077 \cdot (1170000 - 154.8 \cdot O) + 0.140369 \cdot O^2 - 172.30769 \cdot O - 942307.69 \]
\[= 9.34772 \cdot O^2 - 139353 \cdot O + 525031192 \]

Note that, for Case 2a, \( Q^* > 1.2O \) needs to be satisfied. Thus, from \( Q^* = 1170000 - 154.8 \cdot O > 1.2O \), we have \( O < 7500 \).

- **Case 2b:** \( Q \) is not related \( O, Q \neq R(O) \)

\[ Q^* = \left(1 - \frac{c_0}{p - c_s}\right)(b - a) + a = 9000 \]

For this case \( O < Q^* \leq 1.2O \), we have \( 7500 \leq O < 9000 \).

Substituting \( Q^* = 9000 \) to Formula (5.22), we get the following:

\[ E(\pi) = 0.068923077 \cdot O^2 - 172.30769 \cdot O + 3107692.31 \]

- **Case 2c:** if \( Q \leq 1.2O \) (for \( O \geq 7500 \) for this instance)

Let \( Q = 1.2O + 1 \), substitute \( Q = 1.2O + 1 \) to Formula (5.22), we have the following:

\[ E(\pi) = -0.000385 \cdot (1.2O + 1)^2 + 900 \cdot (1.2O + 1) - 0.119077 \cdot O \cdot (1.2O + 1) + 0.140369 \cdot O^2 \]
\[-172.30769 \cdot O - 942307.69 \]
\[= -0.0030778 \cdot O^2 + 907.572 \cdot O - 941408 \]

In summary, for the given parameters, we have the supplier actions, objective functions and valid ranges in Table 5.6.

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>( Q^* ) function (O)</th>
<th>Objective function(O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>652.05 + 1.034O</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>O</td>
<td>0.0189231 \cdot O^2 + 727.6923 \cdot O - 942308</td>
<td>Any O</td>
</tr>
</tbody>
</table>
To compare the different actions, we use the graph to show the different objective functions and compare the objective functions directly. For uniform distribution U(1000, 14000), \( O \in (1000, 14000) \), the response objective functions \( E(\pi)(O) \) for each subcase are shown in Figure 5.3. Since Case 1 result is not valid, it is not plotted.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>1170000 – 154.8( O )</td>
<td>9.34772( O^2 ) – 139353( O ) + 525031192</td>
<td>( O &lt; 7500 )</td>
</tr>
<tr>
<td>2b</td>
<td>( O &lt; Q \leq 1.2O )</td>
<td>9000</td>
<td>0.068923077( O^2 ) – 172.30769( O ) + 3107692.31</td>
<td>( 7500 \leq O &lt; 9000 )</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>1.2( O ) + 1</td>
<td>– 0.0030778( O^2 ) + 907.572( O ) – 941408</td>
<td>( O \geq 7500 )</td>
</tr>
</tbody>
</table>

Figure 5.3. \( E(\pi)(O) \) for Case 1c, Case 2a, Case 2b, Case 2c and their valid range (base)

From Figure 5.3a, we can see the supplier’s response functions or actions \( E(\pi)(O) \) for the different subcases. The supplier’s best action in each range is different. The stars * in Figure 5.3 represent the optimal solution for each subcase shown in Table 5.5.

Based on the comparison, we obtain the supplier’s best actions and effective ranges as shown in Table 5.7.
Comparing Table 5.7 and 5.5, we find the optimal solutions in Table 5.5 for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this situation, and the optimal solution is given in the following table.

### Table 5.8 The Final Optimal Solution (Base Example)

<table>
<thead>
<tr>
<th>O*</th>
<th>Q*</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>7476</td>
<td>12725</td>
<td>5.68E+06</td>
<td>8.32E+07</td>
</tr>
</tbody>
</table>

#### 5.5.3 Subcase’s Effective Range

From the example above, we can see that each subcase has its effective range. The further details are discussed below.

- Case 1c, \( O \in (a, b) \).
- Case 2a:

\[
Q^* = R(O) = \zeta + \eta O > 1.2O
\]

\[
O < \frac{\zeta}{(1.2 - \eta)}
\]  \hspace{1cm} (5.28)

Where

\[
\zeta = \frac{c_0(b - a) + b(c_s - p)}{c_s - p + m_1}, \quad \eta = \frac{1.2m_1}{c_s - p + m_1}
\]

Substitute \( \zeta, \eta \) to (5.28), we have

\[
O < \frac{\zeta}{(1.2 - \eta)} = \frac{\left(\frac{c_0(b - a) + b(c_s - p)}{c_s - p + m_1}\right)}{\left(1.2 - \frac{1.2m_1}{c_s - p + m_1}\right)}
\]

\[
= \frac{c_0(b - a) + b(c_s - p)}{1.2(c_s - p + m_1) - 1.2m_1}
\]
\[ O < \frac{c_0(b-a) + b(c_s-p)}{1.2(c_s-p)} \]  

(5.29)

(5.28) and (5.29) represent the upper limit of \( O \) for Case 2a.

Therefore in Case 2a, \( O \in (a, \frac{c_0(b-a) + b(c_s-p)}{1.2(c_s-p)}) \)

Note that in Formula (5.29), there is no term \( m_1 \), although \( \zeta \) and \( \eta \) include \( m_1 \). So changing \( m_1 \) will not affect the upper limit of \( O \) for Case 2a.

- **Case 2b, \( O \leq 1.2O \)**

From (5.11)

\[ Q^* = \left(1 - \frac{c_0}{p - c_s}\right)(b - a) + a = \frac{c_0(b-a) + b(c_s-p)}{(c_s-p)} \]

\[ Q^*/1.2 \leq O < Q^* \]

Case 2b’s upper limit of \( O \) is: \( Q^* \)

The lower limit of \( O \)

\[ O > \frac{Q^*}{1.2} = \frac{c_0(b-a) + b(c_s-p)}{1.2(c_s-p)} \]  

(5.30)

So in Case 2b, \( O \in (Q^*/1.2, Q^*) \),

where

\[ Q^* = \frac{c_0(b-a) + b(c_s-p)}{(c_s-p)} \]

From (5.29) and (5.30), we can see that the upper limit of \( O \) for Case 2a equals the lower limit of \( O \) for Case 2b.

- **Case 2c, if \( Q \leq 1.2O \),**

\[ O \geq \frac{c_0(b-a) + b(c_s-p)}{1.2(c_s-p)} \]

Case 2c \( O \in \left(\frac{c_0(b-a) + b(c_s-p)}{1.2(c_s-p)}, b\right) \)

**5.5.4 The Backward Induction Method Algorithm**

The backward induction method above can be summarized in Figure 5.4. It determines subgame perfect equilibria.
Step 1. Apply the formulas in Table 5.1 and Table 5.2 to find the optimal solution and constraint solution, objective function for Case 1, Case 2 and their subcases in uniform distribution. (e.g. Table 5.5)

Step 2. Construct the supplier’s response function $E(\pi)(O)$ for each subcase. Draw the graph for all subcase response function $E(\pi)(O)$ (e.g. Figure 5.3)

Step 3. Find supplier’s best action $E(\pi)(O)$ and effective ranges (e.g. Table 5.6)

Step 4. Verify if the optimal solution from Step 1 is within the valid range. Find manufacturer’s optimal solution $O^*$ and min $E(\text{cost})$.

Step 5. Conclude the subgame perfect optimal solution: $O^*$, $Q^*$, $E(\pi)(O^*, Q^*)$, $E(\text{cost})(O^*, Q^*)$. (e.g. Case 2a in this situation. Table 5.7)

Figure 5.4. The backward induction method algorithm flow chart.

In the next section, we will use this backward induction method algorithm flow to perform the sensitivity analysis.

5.6 Sensitivity Analysis
In this section, the parameter effects are studied, including the effects of different lower and upper limits ($a, b$) for uniform distribution, penalty cost $e_1$ and shortage cost $s_1$ effects, high production compensation $m_1$, and low demand compensation $h$. The examples in the above sections with parameters in Table 5.3 will be used as a base model for comparison.
5.6.1 Effect of Different (a, b)

Table 5.5 shows all the solutions for Case 1, Case 2 and subcases with \( U(1000, 14000) \). In this section, we will test the different lower limits \( a \) and upper limits \( b \), i.e., \( U(5000, 10000) \), \( U(3000, 12000) \) with the same mean, \( \mu = 7500 \), of uniform distributions.

- \( O \in U(5000, 10000) \)

Table 5.9a shows the performance of Case 1, Case 2 and their subcases in \( U(5000, 10000) \). For a supplier to decide the response when \( (a, b) \) change, we can follow the backward induction method algorithm flow shown in Figure 5.4 to determine the subgame perfect equilibria. Table 5.9b summarizes the supplier actions, objective functions and valid ranges for the given parameters in \( U(5000,10000) \).

**Table 5.9a Performance of case 1, case 2 and subcases for \( U(5000, 10000), \mu =7500, b − a = 5000 \)**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>( Q \leq O )</th>
<th>Solution</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( E(\pi) )</th>
<th>( E(\text{cost}) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: ( Q &lt; O )</td>
<td>( Q^* = \min(O, \alpha + \beta O) )</td>
<td>-48.2492</td>
<td>1.0339</td>
<td>8477</td>
<td>8716</td>
<td>5181736</td>
<td>81343133</td>
<td>1.03</td>
<td>( Q^* &gt; O, \text{ invalid} )</td>
<td></td>
</tr>
<tr>
<td>Case 1c: ( Q^* = O^* )</td>
<td>( Q^* = O )</td>
<td>0</td>
<td>1</td>
<td>8687</td>
<td>8687</td>
<td>5246647</td>
<td>81547761</td>
<td>1.00</td>
<td>( Q^* = O )</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.9b Comparison of the supplier actions, objective functions and valid ranges for \( U(5000, 10000) \)**

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>( Q^* ) function (( O ))</th>
<th>Objective function (( O ))</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>( -48.249 + 1.034O )</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>( O )</td>
<td>( 0.0492 \cdot O^2 - 140 \cdot O + 2750000 )</td>
<td>Any ( O )</td>
</tr>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>( 1050000 - 154.8O )</td>
<td>( 24.328 \cdot O^2 - 327320 \cdot O + 1105250000 )</td>
<td>( O &lt; 6730 )</td>
</tr>
<tr>
<td>2b</td>
<td>( O &lt; Q \leq 1.2O )</td>
<td>8077</td>
<td>( 0.1792 \cdot O^2 - 2240 \cdot O + 11230769 )</td>
<td>( 6730 \leq O &lt; 8077 )</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>( 1.2O + 1 )</td>
<td>( -0.008 \cdot O^2 + 279.688 \cdot O + 2752099.999 )</td>
<td>( O \geq 6730 )</td>
</tr>
</tbody>
</table>
Figure 5.5 graphically shows the results in Table 5.9a and 5.9b, or the performance of $E(\pi)(O)$ for $U(5000, 10000)$. Figure 5.5(b) is the zoomed-in picture of Figure 5.5(a) for a better view purpose. The stars represent the optimal solution $O^*$ and response $E(\pi)(O^*)$ of each subcase.

![Graph showing $E(\pi)(O)$ for Case 1c, Case 2a, Case2b, Case2c and their valid range when $U(5000, 10000)$](image)

**Table 5.9c** The Supplier’s Best Action and Effective Ranges for $U(5000, 10000)$

<table>
<thead>
<tr>
<th>Effective range for $O$</th>
<th>[0, 6730]</th>
<th>[6730, 8077]</th>
<th>[6730, 10000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Based on the comparison, we obtain the supplier’s best actions and effective ranges as in Table 5.9c.

Comparing Table 5.9c and 5.9c, we can find that the optimal solutions in Table 5.9a for Case 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this scenario $U(5000, 10000)$ and the optimal solution is given in the following table.
Table 5.9d The final optimal solution for the instance U(5000, 10000)

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>6722</td>
<td>9434</td>
<td>4272549</td>
<td>79415148</td>
</tr>
</tbody>
</table>

- \( O \in \text{U}(3000, 12000) \)

Table 5.10a \( \text{U}(3000, 12000) \), \( \mu = 7500 \), \( b - a = 9000 \) Performance of Case 1, Case 2 and Subcases

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>( Q^* ) function (O)</th>
<th>Objective function(O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>301.9024 + 1.0339 ( O )</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>0 1</td>
<td>9255</td>
<td>9255</td>
</tr>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>1110000 - 154.8 ( O )</td>
<td>Any O</td>
<td></td>
</tr>
<tr>
<td>2b</td>
<td>( O &lt; Q \leq 1.2O )</td>
<td>8538</td>
<td>0.099556 2 ( O ) - 746.6667 2 ( O ) + 5015384</td>
<td>7115 ( O \leq 8538 )</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>1.2O + 1</td>
<td>-0.004444 2 ( O ) + 733.16 2 ( O ) - 248766.667</td>
<td>( O \geq 7115 )</td>
</tr>
</tbody>
</table>

Figure 5.6 illustrates the performance of \( E(\pi)(O) \) in Table 5.10a and 5.10b for \( \text{U}(3000, 12000) \). Figure 5.6(b) is the zoomed-in graph of Figure 5.6(a) for the better view purpose. The stars represent the optimal solution \( O^* \) and response \( E(\pi)(O^*) \) of each subcase.
Figure 5.6. $E(\pi)(O)$ for Case 1c, Case 2a, Case 2b, Case 2c and their valid range when $U(3000, 12000)$

Based on the comparison, we obtain the supplier’s best actions and effective ranges as in the following table.

**TABLE 5.10c** The supplier’s best action and effective ranges for $U(3000, 12000)$

<table>
<thead>
<tr>
<th>Effective range for $O$</th>
<th>[0, 7115]</th>
<th>[7115, 8538]</th>
<th>[7115, 12000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Comparing Table 5.10c and 5.10a, we can find that the optimal solutions in Table 5.10a for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this situation $U(3000, 12000)$ and the optimal solution is given in the following table.

**TABLE 5.10d** The final optimal solution for $U(3000, 12000)$

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7099</td>
<td>11074</td>
<td>4735581</td>
<td>81059799</td>
</tr>
</tbody>
</table>

- Summary of the performance for different $(a, b)$ sets

Comparing the performance of 3 different $(b-a)$ uniform distribution: $U(5000, 10000)$, $U(3000, 12000)$, and $U(1000, 14000)$, we see that three of $E(\pi)(O)$ have similar
characteristics, such as, Case 1 is an invalid solution; Case 2a is the subgame perfect optimal solution for all 3 scenarios. We will only study the performance of Case 2a to analyze the effect of \((b-a)\).

**Table 5.11 The performance of Case 2a for different \((a, b)\) sets with the same \(\mu=7500\)**

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(b-a)</th>
<th>(\mu)</th>
<th>(\zeta)</th>
<th>(\eta)</th>
<th>(O^*)</th>
<th>(Q^*)</th>
<th>(E(\pi))</th>
<th>(E(\text{cost}))</th>
<th>(Q^<em>/O^</em>)</th>
<th>(2a) range</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>10000</td>
<td>5000</td>
<td>7500</td>
<td>1050000</td>
<td>-154.8</td>
<td>6722</td>
<td>9434</td>
<td>4272549</td>
<td>79415148</td>
<td>1.40</td>
<td>(O&lt;6730)</td>
</tr>
<tr>
<td>3000</td>
<td>12000</td>
<td>9000</td>
<td>7500</td>
<td>1110000</td>
<td>-154.8</td>
<td>7099</td>
<td>11074</td>
<td>4735581</td>
<td>81059799</td>
<td>1.56</td>
<td>(O&lt;7115)</td>
</tr>
<tr>
<td>1000</td>
<td>14000</td>
<td>13000</td>
<td>7500</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>5677020</td>
<td>83202878</td>
<td>1.70</td>
<td>(O&lt;7500)</td>
</tr>
</tbody>
</table>

Figure 5.7. The performance of Case 2a for different \((a, b)\) sets with the same \(\mu=7500\)

Table 5.11 and Figure 5.7 summarize the performance, \(O^*\), \(Q^*\), \(E(\pi)\), and \(E(\text{cost})\) of Case 2a for the different \((a, b)\) set with the same \(\mu=7500\) from the above experiments. From Table 5.11, we see that when \((a, b)\) changes, the effective ranges of Case 2a change, which can be calculated from Formula (5.29). All the optimal solutions \(O^*\) of Case 2a are valid since they are within the effective ranges. We also see that when \((b-a)\) increases, \(\zeta\) increases and \(\eta\) keeps the same; \(O^*\) and \(Q^*\) increase as well. The ratio of \(Q^*/O^*\) becomes large. Both supplier’s profit and manufacturer’s \(E(\text{cost})\) increase. It implies that in order to reduce the manufacturer’s cost, the supply chain should forecast the demand more accurately as \((b-a)\) reduces.
5.6.2 Effects of High Production Compensation $m_1$

$m_1$ is unit compensation of high volume production that the manufacturer pays to the supplier, when $Q > O$ and $D_t > 1.2O$. It is only included in Case 2a and Case 2c contract models. In Section 4.6 Appendix A, we derived that only if $(m_1 + c_s - p) < 0$, or $m_1 < p - c_s$, $E(\pi)$ has the maximum value at $Q^*$. Since Case 1c and Case 2b do not include $m_1$, changing $m_1$ will not affect the performance of them. The performances of Case 1c and Case 2b are the same as in Table 5.5. Changing $m_1$ mainly affects Case 2a and 2c, which are the optimal solutions among other subcases in Table 5.5. Also from the discussion of Section 5.5.2, we conclude that changing $m_1$ will not affect Case 2a’s upper limit of $O$, therefore, not affect the effective range of each subcase. Based on the backward induction method algorithm flow in Figure 5.4, we tested the performance of $m_1 = 800$, and $m_1 = 300$, and compared them to the base model result of $m_1 = 1290$ as shown in Table 5.5.

- $m_1 = 800$

### Table 5.12a The solutions of Case 1, Case 2 and subcases for $m_1 = 800, U(1000, 14000)$

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Solution</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$O^*$</th>
<th>$Q^*$</th>
<th>$E(\pi)$</th>
<th>$E(cost)$</th>
<th>$Q^<em>/O^</em>$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \leq O$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1: $Q &lt; O$</td>
<td>$Q^* = R(O) = \alpha + \beta O$</td>
<td>652.0540</td>
<td>1.0339</td>
<td>8964</td>
<td>9920</td>
<td>7611281</td>
<td>87724789</td>
<td>1.11</td>
<td>$Q &gt; O$, invalid</td>
</tr>
<tr>
<td>Case 1c: $Q^* = O^*$</td>
<td>$Q^* = O$</td>
<td>0</td>
<td>1</td>
<td>9823</td>
<td>9823</td>
<td>8031726</td>
<td>88727545</td>
<td>1.00</td>
<td>$Q = O$</td>
</tr>
<tr>
<td>Case 2</td>
<td>Solution</td>
<td>$\zeta$</td>
<td>$\eta$</td>
<td>$O^*$</td>
<td>$Q^*$</td>
<td>$E(\pi)$</td>
<td>$E(cost)$</td>
<td>$Q^<em>/O^</em>$</td>
<td></td>
</tr>
<tr>
<td>$Q &gt; O$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2a: $Q &gt; 1.2O$</td>
<td>$Q^* = R(O) = \zeta + \eta O$</td>
<td>23400</td>
<td>-1.92</td>
<td>5515</td>
<td>12811</td>
<td>4707640</td>
<td>81737158</td>
<td>2.32</td>
<td>$Q &gt; 1.2O$</td>
</tr>
<tr>
<td>Case 2b: $O &lt; Q \leq 1.20$</td>
<td>$O = 1/1.2 Q^*$</td>
<td>0</td>
<td>1.2</td>
<td>7500</td>
<td>9000</td>
<td>5672307</td>
<td>86756730</td>
<td>1.20</td>
<td>$Q = 1.2O$</td>
</tr>
<tr>
<td>Case 2c: if $Q &lt; 1.2O$</td>
<td>$Q^* = 1.2O + 1$</td>
<td>1</td>
<td>1.2</td>
<td>7500</td>
<td>9001</td>
<td>5672307</td>
<td>86754775</td>
<td>1.20</td>
<td>$Q = 1.2O + 1$</td>
</tr>
</tbody>
</table>

Table 5.12c is the same as Table 5.7 where $m_1 = 1290$. This verifies the conclusion from section 5.4.3, i.e. different $m_1$ does not change the effective ranges of subcases. Comparing Table 5.12c and 5.12a, we can find that the optimal solutions in Table 5.12a for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a when $m_1 = 800$. The optimal solution is given in the following table.
### Table 5.12b: The Supplier Actions, Objective Functions, and Valid Ranges for $m_1 = 800$

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>$Q^*$ function (O)</th>
<th>Objective function (O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>$Q \leq O$</td>
<td>$652.054 + 1.034O$</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>$Q = O$</td>
<td>$O$</td>
<td>$0.0189231 \cdot O^2 + 727.6923 \cdot O - 942308$</td>
<td>Any O</td>
</tr>
<tr>
<td>2a</td>
<td>$Q &gt; 1.2O$</td>
<td>$23400 - 1.92O$</td>
<td>$0.184123 \cdot O^2 - 1900.30769 \cdot O + 9587692.307$</td>
<td>$O &lt; 7500$</td>
</tr>
<tr>
<td>2b</td>
<td>$O &lt; Q \leq 1.2O$</td>
<td>$9000$</td>
<td>$0.068923 \cdot O^2 - 172.30769 \cdot O + 3107692.31$</td>
<td>$7500 \leq O &lt; 9000$</td>
</tr>
<tr>
<td>2c</td>
<td>$Q = 1.2O + 1$</td>
<td>$1.2O + 1$</td>
<td>$-0.003077 \cdot O^2 + 907.572 \cdot O - 941408$</td>
<td>$O \geq 7500$</td>
</tr>
</tbody>
</table>

![Graph](image)

- a. $O \in (1000, 14000)$
- b. Zoomed-in graph

**Figure 5.8.** $E(\pi)(O)$ for Case 1c, Case 2a, Case 2b, Case 2c when $m_1 = 800$.

Based on the comparison, we obtain the supplier’s best actions and effective ranges as the following table.

### Table 5.12c: The Supplier’s Best Action and Effective Ranges when $m_1 = 800$

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>$[0, 7500]$</th>
<th>$[7500, 8994]$</th>
<th>$[8995, b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>
### Table 5.12d The final optimal solution when \( m_1 = 800 \)

<table>
<thead>
<tr>
<th>( O )</th>
<th>( Q )</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5515</td>
<td>12811</td>
<td>4707640</td>
<td>81737158</td>
</tr>
</tbody>
</table>

### \( m_1 = 300 \)

**Table 5.13a The solutions of Case 1, Case 2 and subcases for \( m_1 = 300 \), \( U(1000, 14000) \)**

<table>
<thead>
<tr>
<th>Case 1 ( Q \leq O )</th>
<th>Solution</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( \pi(O) )</th>
<th>( \pi(cost) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: ( Q &lt; O )</td>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td>652.0540</td>
<td>1.0339</td>
<td>8964</td>
<td>9920</td>
<td>7611281</td>
<td>87724789</td>
<td>1.11</td>
<td>( Q^* &gt; O ), invalid</td>
</tr>
<tr>
<td>Case 1c: ( Q^* = O^* )</td>
<td>( Q^* = O )</td>
<td>0</td>
<td>1</td>
<td>9823</td>
<td>9823</td>
<td>8031726</td>
<td>88727545</td>
<td>1.00</td>
<td>( Q^* = O )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2 ( Q &gt; O )</th>
<th>Solution</th>
<th>( \zeta )</th>
<th>( \eta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( \pi(O) )</th>
<th>( \pi(cost) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a: ( Q &gt; 1.2O )</td>
<td>( Q^* = R(O) = \zeta + \eta O )</td>
<td>1170000</td>
<td>-0.36</td>
<td>1172</td>
<td>11278</td>
<td>3865360</td>
<td>80404218</td>
<td>9.62</td>
<td>( Q^* &gt; 1.2O ) *</td>
</tr>
<tr>
<td>Case 2b: ( O &lt; Q \leq 1.2O )</td>
<td>( O = \frac{1}{1.2} \cdot Q^* )</td>
<td>0</td>
<td>1.2</td>
<td>7500</td>
<td>9000</td>
<td>5692307</td>
<td>86756730</td>
<td>1.20</td>
<td>( Q^* = 1.2O ) *</td>
</tr>
<tr>
<td>Case 2c: if ( Q &lt; 1.2O )</td>
<td>( Q^* = 1.2O + 1 )</td>
<td>1</td>
<td>1.2</td>
<td>7500</td>
<td>9001</td>
<td>5692307</td>
<td>86754775</td>
<td>1.20</td>
<td>( Q^* = 1.2O + 1 )</td>
</tr>
</tbody>
</table>

**Table 5.13b The supplier actions, objective functions and valid ranges for \( m_1 = 300 \)**

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>( Q^* ) function (O)</th>
<th>Objective function(O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>652.054 + 1.034O</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>O</td>
<td>0.0189231 \cdot O^2 + 727.6923 \cdot O - 942308</td>
<td>Any O</td>
</tr>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>11700 - 0.360 \cdot O</td>
<td>0.090523 \cdot O^2 - 496.307692 \cdot O + 3865360.7424</td>
<td>( O &lt; 7500 )</td>
</tr>
<tr>
<td>2b</td>
<td>( O &lt; Q \leq 1.2O )</td>
<td>9000</td>
<td>0.068923 \cdot O^2 - 172.30769 \cdot O + 3107692.31</td>
<td>7500 \leq O &lt; 9000</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>1.2 \cdot O + 1</td>
<td>-0.003077 \cdot O^2 + 907.572 \cdot O - 941408</td>
<td>( O \geq 7500 )</td>
</tr>
</tbody>
</table>
Figure 5.9. E(π)(O) for Case 1c, Case 2a, Case 2b, and Case 2c when m₁ = 300

Based on the comparison, we obtain the supplier’s best actions and effective ranges as in the following table.

**TABLE 5.13c THE SUPPLIER’S BEST ACTION AND EFFECTIVE RANGES WHEN m₁ = 300**

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>[0, 7500]</th>
<th>[7500, 8994]</th>
<th>[8995, )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Comparing Table 5.13a and 5.13c, we can find the optimal solutions in Table 5.13a for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this scenario and the optimal solution is given in the following table.

**TABLE 5.13d THE FINAL OPTIMAL SOLUTION WHEN m₁ = 300**

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1172</td>
<td>11278</td>
<td>3865360</td>
<td>80404218</td>
</tr>
</tbody>
</table>

Comparing the performance graphs of E(π)(O) when m₁ = 300 (Figure 5.9), m₁ = 800 (Figure 5.8), and m₁ = 1290 (Figure 5.3), we find that when m₁ decrease, E(π)(O) of Case 2a becomes flatter, but is still better than that of other cases in its effective region (a, 7500).
Moreover, Case 2a optimal solution $O^*$ becomes smaller. Comparing the supplier actions, the objective functions $E(\pi)(O)$ and valid ranges for three $m_1$ values (Table 5.6 for $m_1 = 1290$, Table 5.12b for $m_1 = 800$ and Table 5.13b for $m_1 = 300$), we found that when $m_1$ changes, only $E(\pi)(O)$ of Case 2a changes. Other objective functions of Case 1, 1c, 2b keep the same in 3 different $m_1$ values since Case 1, 1c, and 2b do not include the $m_1$ item. In the following discussion, we will only discuss Case 2a performance regarding $m_1$ effect. The effect of different $m_1$ values to Case 2a are summarized in Table 5.14 and displayed in Figure 9.

**Table 5.14 Effect of $m_1$ for Case 2a**

<table>
<thead>
<tr>
<th>Case 2a Solution</th>
<th>$m_1$</th>
<th>$\zeta$</th>
<th>$\eta$</th>
<th>$O^*$</th>
<th>$Q^*$</th>
<th>$E(\pi)$</th>
<th>$E(cost)$</th>
<th>$Q^<em>/O^</em>$</th>
<th>$Q^*/\mu$</th>
<th>$O^*/\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a: $Q &gt; 1.2O^*$</td>
<td>300</td>
<td>11700</td>
<td>-0.36</td>
<td>1172</td>
<td>11278</td>
<td>3865360</td>
<td>80404218</td>
<td>9.62</td>
<td>1.50</td>
<td>0.16</td>
</tr>
<tr>
<td>$Q^* = R(O) = \zeta + \eta O$</td>
<td>800</td>
<td>23400</td>
<td>-1.92</td>
<td>5515</td>
<td>12811</td>
<td>4707640</td>
<td>81737158</td>
<td>2.32</td>
<td>1.71</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>1290</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>5677020</td>
<td>83202878</td>
<td>1.70</td>
<td>1.70</td>
<td>1.00</td>
</tr>
</tbody>
</table>

![Figure 5.10. $m_1$ effect for Case 2a performance](image)

- **a.** $m_1$ effect on $Q^*$, $O^*$
- **b.** $m_1$ effect on $E(\pi)$ and $E(cost)$

Figure 5.10. $m_1$ effect for Case 2a performance

From Table 5.14 and Figure 5.10, we can see that:

a) In Case 2a, $O^*$ is less than the demand means $\mu$, even when $m_1 = 1290$, which is the upper limit of $m_1$. When $m_1$ decreases, $O^*$ will decrease very fast, while $Q^*$ decreases slower, and the ratio of $Q^*/O^*$ becomes larger. For example, when $m_1 = 300$, $Q^*$ is almost 10 times more than $O^*$, which is an unreasonable solution. So $m_1$ not only needs an upper limit $m_1 < p - c_s$, also needs a low limit in order to keep the ratio of $Q^*/O^*$ within a range.
b) The minimum $Q^*$ is 9000 in the parameter set when $m_1 = 0$. It is the same result of Case 2b, in which $m_1 = 0$. $Q$ has no relationship with $O$, or $Q \neq R(O)$.

c) From Figure 5.8b, we can see that when $m_1$ changes, $E(\pi)$ and $E(\text{cost})$ do not change that much since $Q^*$ only changes little. From the above analysis, it can be concluded that the $m_1$ effect to optimal capacity $Q^*$ is not significant.

5.6.3 Effects of Penalty Cost $e_1$ and Shortage Cost $s_1$

$e_1$ is the unit penalty cost that supplier pays to the manufacturer, if $Q < O$ and $D_t > Q$. And $s_1$ is manufacturer’s unit shortage cost if $D_t > Q$. According to (Nahmias, S. 2012 pp 207), “The penalty cost, also known as the shortage cost or the stock-out cost, is the cost of not having sufficient stock on hand to satisfy a demand when it occurs.” In fact, they are the same cost.

In this section, $e_1$ and $s_1$ effects will be tested in Case 1 and Case 2 based on the backward induction method algorithm flow in Figure 5.4.

- $e_1 = s_1 = 13500$

<table>
<thead>
<tr>
<th>Case 1</th>
<th>$Q \leq O$</th>
<th>Solution</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$O^*$</th>
<th>$Q^*$</th>
<th>$E(\pi)$</th>
<th>$E(\text{cost})$</th>
<th>$Q^<em>/O^</em>$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: $Q &lt; O$</td>
<td>$Q^* = R(O) = \alpha + \beta O$</td>
<td>727.24477</td>
<td>1.0378</td>
<td>7801</td>
<td>8823</td>
<td>6408866</td>
<td>86398546</td>
<td>1.13</td>
<td>$Q &gt; O$, invalid</td>
<td></td>
</tr>
<tr>
<td>Case 1c: $Q^* = O^*$</td>
<td>$Q^* = O$</td>
<td>0</td>
<td>1</td>
<td>8690</td>
<td>8690</td>
<td>6810335</td>
<td>87448118</td>
<td>1.00</td>
<td>$Q^* = O$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2</th>
<th>$Q &gt; O$</th>
<th>Solution</th>
<th>$\zeta$</th>
<th>$\eta$</th>
<th>$O^*$</th>
<th>$Q^*$</th>
<th>$E(\pi)$</th>
<th>$E(\text{cost})$</th>
<th>$Q^<em>/O^</em>$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a: $Q &gt; 1.2O$</td>
<td>$Q^* = R(O) = \zeta + \eta O$</td>
<td>1170000</td>
<td>-154.8</td>
<td>7479</td>
<td>12250</td>
<td>5678341</td>
<td>83061145</td>
<td>1.64</td>
<td>$Q^* &gt; 1.2O^*$</td>
<td></td>
</tr>
<tr>
<td>Case 2b: $O &lt; Q \leq 1.2O$</td>
<td>$O = 1/1.2 Q^*$</td>
<td>0</td>
<td>1.2</td>
<td>7500</td>
<td>9000</td>
<td>5692307</td>
<td>85314423</td>
<td>1.20</td>
<td>$Q^* = 1.2O^*$</td>
<td></td>
</tr>
<tr>
<td>Case 2c: if $Q &lt; 1.2O$</td>
<td>$Q^* = 1.2O + 1$</td>
<td>1</td>
<td>1.2</td>
<td>7500</td>
<td>9000</td>
<td>5692307</td>
<td>85313044</td>
<td>1.20</td>
<td>$Q^* = 1.2O^* + 1$</td>
<td></td>
</tr>
</tbody>
</table>

It is noticed that Table 5.15c is the same as Table 5.7 for the base example because there is no $e_1$ or $s_1$ item in Formula (5.29) and (5.30), which determine the effective range of Case 2a and Case 2b. So changing $e_1$ or $s_1$ does not change the effective range of $E(\pi)(O)$.

Comparing Table 5.15a and 5.15c, we can find the optimal solutions in Table 5.15a for Cases 2a, 2b, and 1c are valid. Case 2a gives the minimum cost for the manufacturer. So the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect
optimal solution is Case 2a in this scenario and the optimal solution is given in the following table.

**TABLE 5.15b THE SUPPLIER ACTIONS, OBJECTIVE FUNCTIONS AND VALID RANGES WHEN e₁ = s₁ = 13500**

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>Q* function (O)</th>
<th>Objective function(O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Q ≤ O</td>
<td>727.2447 + 1.034O</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>Q = O</td>
<td>O</td>
<td>0.0189231·O² + 727.6923·O – 942308</td>
<td>Any O</td>
</tr>
<tr>
<td>2a</td>
<td>Q &gt; 1.2O</td>
<td>1170000 – 154.8·O</td>
<td>9.356923·O² – 139492.30769·O + 525557692.30769</td>
<td>O &lt; 7500</td>
</tr>
<tr>
<td>2b</td>
<td>O &lt; Q ≤ 1.2O</td>
<td>9000</td>
<td>0.068923·O² – 172.30769·O + 3107692.31</td>
<td>7500 ≤ O &lt; 9000</td>
</tr>
<tr>
<td>2c</td>
<td>Q = 1.2O + 1</td>
<td>1.2O + 1</td>
<td>– 0.003077·O² + 907.572·O – 941408</td>
<td>O ≥ 7500</td>
</tr>
</tbody>
</table>

![Graph](image1)

- a. O e (1000, 14000)
- b. Zoomed-in graph

Figure 5.11. E(π)(O) for Case 1c, Case 2a, Case2b, and Case2c with e₁ = s₁ = 13500

Based on the comparison, we obtain the supplier’s best actions and effective ranges as in the following table.

**TABLE 5.15c THE SUPPLIER’S BEST ACTION AND EFFECTIVE RANGES WHEN e₁ = s₁ = 13500**

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>0, 7500</th>
<th>[7500, 8994]</th>
<th>[8995, )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>
Table 5.15d The final optimal solution for the instance when \( e_1 = s_1 = 13500 \)

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>7479</td>
<td>12250</td>
<td>5678341</td>
<td>83061145</td>
</tr>
</tbody>
</table>

- \( e_1 = s_1 = 11000 \)

Table 5.16a Results of Case 1, Case 2 and subcase optimal solutions with \( e_1 = s_1 = 11000 \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Relation</th>
<th>Solution</th>
<th>( a )</th>
<th>( \beta )</th>
<th>( O^* )</th>
<th>( E(\pi) )</th>
<th>( E(cost) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q &lt; O )</td>
<td>( Q^* = R(O) = a + \beta O )</td>
<td>900.2665</td>
<td>1.04682</td>
<td>3509</td>
<td>4573</td>
<td>2302191</td>
<td>81495353</td>
<td>1.30</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>( Q^* = O^* )</td>
<td>0</td>
<td>1</td>
<td>4304</td>
<td>4304</td>
<td>2540218</td>
<td>82497929</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 5.16b Comparisons of the supplier actions, objective functions and valid ranges with \( e_1 = s_1 = 11000 \)

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relation</th>
<th>( Q^* ) function (O)</th>
<th>Objective function (O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>( 900.2665 + 1.04682 O )</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>( O )</td>
<td>( 0.0189231 \cdot O^2 + 727.6923 \cdot O - 942308 )</td>
<td>Any O</td>
</tr>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>( 1170000 - 154.8 \cdot O )</td>
<td>( 9.356923 \cdot O^2 - 139492.30769 \cdot O + 525557692.30769 )</td>
<td>( O &lt; 7500 )</td>
</tr>
<tr>
<td>2b</td>
<td>( O \leq Q \leq 1.2O )</td>
<td>( 9000 )</td>
<td>( 0.068923 \cdot O^2 - 172.30769 \cdot O + 3107692.31 )</td>
<td>( 7500 \leq O &lt; 9000 )</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>( 1.2O + 1 )</td>
<td>( -0.003077 \cdot O^2 + 907.572 \cdot O - 941408 )</td>
<td>( O \geq 7500 )</td>
</tr>
</tbody>
</table>
Based on the comparison, we obtain the supplier’s best actions and effective ranges as in the following table.

**Table 5.16c The supplier’s best action and effective ranges when \( e_1 = s_1 = 11000 \)**

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>[0, 7500]</th>
<th>[7500, 8994]</th>
<th>[8995, b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Comparing Table 5.16a and 5.16c, we can find the optimal solutions in Table 5.16a for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this scenario and the optimal solution is given in the following table.

**Table 5.16d The final optimal solution when \( e_1 = s_1 = 11000 \)**

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>7488</td>
<td>10857</td>
<td>5683316</td>
<td>82524737</td>
</tr>
</tbody>
</table>

From the performance of the tested sets, \( e_1 = s_1 = (11000, 13500, 15000) \), we can see the following facts:
(a) For all the tested \(e_1\) and \(s_1\), Case 1 results show the same pattern, \(Q^* > O^*\), which is an invalid solution since it violates Case 1 assumption \(Q < O\). The constraint solution Case 1c \(Q = O\) is considered as the optimal solution for Case 1 when \(e_1\) changes. Actually, when \(Q = O\), penalty \(e_1 = 0\). That means the supplier cannot choose Case 1 \(Q < O\) strategy.

(b) Comparing the solutions in Table 5.15a, Table 5.16a and Table 5.5 (base example), we can find that \(\zeta\) and \(\eta\) in Case 2a and Case 2b solutions are the same for all the different \(e_1\) or \(s_1\). Note that from Formula (5.6) and (5.7) for Case 2a (copied here),

\[
\zeta = \frac{c_o(b-a) + b(c_s - p)}{c_s - p + m_1} \tag{5.6}
\]

\[
\eta = \frac{1.2m_1}{c_s - p + m_1} \tag{5.7}
\]

\(\zeta\) and \(\eta\) do not include \(e_1\). So \(e_1\) changes do not affect \(\zeta\) and \(\eta\). This verifies our model assumption as of the penalty cost \(e(O, Q) = 0\) in Case 2.

For Case 2b, the solution for uniform distribution is:

\[
Q^* = \left(1 - \frac{c_0}{p - c_s}\right)(b - a) + a \tag{5.11}
\]

\[
O^* = 1/1.2 \cdot Q^*
\]

There is no \(e_1\) or \(s_1\) item in Case 2b solution, so \(e_1\) changes do not affect the performance of Case 2b. In Case 2, only item \(s\) affects the calculation of \(O^*\) as shown in Formula (5.8) below.

\[
O^* = \frac{\zeta[\eta(p + c_m - s - g_0 - m_1) + g_0 + 1.2m_1] - (p + c_m - s)\eta b - g_0(b - a) + 0.8h_1a}{\eta^2(s - p - c_m + g_0 + m_1) - 2\eta(g_0 + 1.2m_1) + g_0 + 0.64h_1 + 1.44m_1} \tag{5.8}
\]

(c) From the backward induction algorithm above, we conclude that Case 2a is the subgame perfect optimal solution for 3 different \(e_1\) or \(s_1\) situations. Here we only analyze the effect of \(e_1\) or \(s_1\) to Case 2a. Following the backward induction method algorithm in Figure 5.4, we tested \(e_1\) or \(s_1\) with the different values from 10000 to 18000. Table 5.17 summarizes \(e_1\) or \(s_1\) effects on Case 2a performance from the test.

**Table 5.17 Summary of Varied \(e_1\) or \(s_1\) Effect on Case 2a Performance**
Table 5.17 and Figure 5.13 illustrate the effects of $e_1$ or $s_1$ on Case 2a performance. We can see that the effects of $e_1$ or $s_1$ to $O^*$ and $E(\pi)$ are small, but to $Q^*$ is large. When $s_1$ increases, the optimal solution $O^*$ decreases slightly, but $Q^*$ increases a lot. The ratio of $Q^*/O^*$ becomes larger. At the same time, $E(\pi)$ decreases, and $E(\text{cost})$ increases. So a large $s_1$ is not desirable for both manufacturer and supplier in Case 2a when $Q > O$.

Comparing the performance of Case 1 and Case 2 under different $e_1$ and $s_1$, some insights for managers to choose the optimal strategy can be provided. For example, the supplier should choose Case 2a strategy, i.e., $Q > 1.2O$, to avoid the penalty and get the optimal
profit \( E(\pi) \). Also, the study about \( e_1 \) and \( s_1 \) helps managers to decide the optimal value of \( e_1 \) or \( s_1 \) in the contract. \( e_1 \) or \( s_1 \) cannot be too small or too big. When \( e_1 \) is between $12500 and $15000, \( Q^* \) and \( O^* \) are more reasonable.

### 5.6.4 Effects of Compensation \( h \)

The low demand compensation \( h \) usually includes the supplier’s fixed cost and some handling cost. \( h_1 \) is set to be $2800 per unit in the base experiment above. In this section, tests will be performed on the different compensation values (from $800 to $2800) to analyze the effects of \( h \).

- \( h = 1800 \)

#### Table 5.18a Results of Case 1, Case 2 and Subcase Optimal Solutions with \( h_1 = 1800 \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Solution</th>
<th>( a )</th>
<th>( \beta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( E(\pi) )</th>
<th>( E(\text{cost}) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( Q &lt; O )</td>
<td>( Q^* = R(O) = a + \beta O )</td>
<td>677.3171</td>
<td>0.99023</td>
<td>10080</td>
<td>10659</td>
<td>6591505</td>
<td>86254102</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>Case 1c: ( Q^* = O^* )</td>
<td>( Q^* = O )</td>
<td>0</td>
<td>1</td>
<td>10682</td>
<td>10682</td>
<td>6800272</td>
<td>86736873</td>
<td>1.00</td>
</tr>
<tr>
<td>Case 2</td>
<td>( Q &gt; O )</td>
<td>( Q^* = R(O) = \zeta + \eta O )</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>4722852</td>
<td>82248710</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>Case 2a: ( Q &gt; 1.2O )</td>
<td>( O = 1/1.2 ) ( Q^* )</td>
<td>0</td>
<td>1.2</td>
<td>7500</td>
<td>9000</td>
<td>4730769</td>
<td>85795192</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>Case 2b: ( O &lt; Q &lt; 1.2O )</td>
<td>( Q^* = 1.2O + 1 )</td>
<td>1</td>
<td>1.2</td>
<td>7500</td>
<td>9001</td>
<td>4730769</td>
<td>85793236</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Figure 5.14. \( E(\pi)(O) \) for Case 1c, Case 2a, Case2b, Case2c with \( h_1 = 1800 \)

- a. \( O \in (1000, 14000) \).
- b. Zoomed-in graph
Based on the comparison, we obtain the supplier’s best actions and effective ranges as in Table 5.18c below.

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>[0, 7500]</th>
<th>[7500, 9000)</th>
<th>[9000, )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Comparing Table 5.18c and 5.18a, we can see that the optimal solutions in Table 5.18a for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this situation, and the optimal solution is given in the following table.
**Table 5.18d** The final optimal solution for the instance with \( h_1 = 1800 \)

<table>
<thead>
<tr>
<th>( O )</th>
<th>( Q )</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>7476</td>
<td>12715</td>
<td>4722852</td>
<td>82248710</td>
</tr>
</tbody>
</table>

- \( h = 800 \)

**Table 5.19a** Results of Case 1, Case 2 and subcase optimal solutions with \( h_1 = 800 \)

<table>
<thead>
<tr>
<th>Case 1 ( Q \leq O )</th>
<th>Solution</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( E(\pi) )</th>
<th>( E(cost) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: ( Q &lt; O )</td>
<td>( Q^* = R(O) = \alpha + \beta O )</td>
<td>700.5321</td>
<td>0.95009</td>
<td>11531</td>
<td>11656</td>
<td>4770647</td>
<td>8423267</td>
<td>1.01</td>
<td>( Q &gt; O ). invalid</td>
</tr>
<tr>
<td>Case 1c: ( Q^* = O^* )</td>
<td>( Q^* = O )</td>
<td>0</td>
<td>1</td>
<td>11733</td>
<td>11733</td>
<td>4790588</td>
<td>84302828</td>
<td>1.00</td>
<td>( Q = O )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2 ( Q &gt; O )</th>
<th>Solution</th>
<th>( \xi )</th>
<th>( \eta )</th>
<th>( O^* )</th>
<th>( Q^* )</th>
<th>( E(\pi) )</th>
<th>( E(cost) )</th>
<th>( Q^<em>/O^</em> )</th>
<th>Notes</th>
</tr>
</thead>
</table>
| Case 2a: \( Q > 1.2O \) | \( Q^* = R(O) = \xi + \eta O \) | 1170000   | -154.8   | 7476   | 12715  | 3768648  | 81294542 | 1.7      | \( Q > 1.2O \) *
| Case 2b: \( O < Q \leq 1.2O \) | \( O = 1/1.2 Q^* \) | 0          | 1.2      | 7500   | 9000   | 3769230  | 84833653 | 1.20     | \( Q = 1/1.2O \) *
| Case 2c: if \( Q < 1.2O \) | \( Q^* = 1.2O + 1 \) | 1          | 1.2      | 7500   | 9001   | 3769230  | 84831698 | 1.20     | \( Q = 1.2O + 1 \) *

**Table 5.19b** Comparison of the supplier actions, objective functions and valid ranges with \( h_1 = 800 \)

<table>
<thead>
<tr>
<th>Case (action)</th>
<th>Relations</th>
<th>Q* function (O)</th>
<th>Objective function(O)</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( Q \leq O )</td>
<td>700.53205+ 0.950089</td>
<td>Invalid</td>
<td>Invalid</td>
</tr>
<tr>
<td>1c</td>
<td>( Q = O )</td>
<td>( -0.030308 \cdot O^2 + 850.769231 \cdot O - 1019230 )</td>
<td>Any O</td>
<td>Any O</td>
</tr>
<tr>
<td>2a</td>
<td>( Q &gt; 1.2O )</td>
<td>1170000 – 154.8 \cdot O</td>
<td>9.307692 \cdot O^2 – 139369.2307 \cdot O + 525480769.23076</td>
<td>O &lt; 7500</td>
</tr>
<tr>
<td>2b</td>
<td>( O &lt; Q \leq 1.2O )</td>
<td>9000</td>
<td>0.019692 \cdot O^2 – 49.230769 \cdot O + 3030769.230769</td>
<td>7500 \leq O &lt; 9000</td>
</tr>
<tr>
<td>2c</td>
<td>( Q = 1.2O + 1 )</td>
<td>1.2O + 1</td>
<td>( -0.052308 \cdot O^2 + 1030.649231 \cdot O - 1018330.769615 )</td>
<td>O \geq 7500</td>
</tr>
</tbody>
</table>
Figure 5.15. $E(\pi)(O)$ for Case 1c, Case 2a, Case2b, Case2c with $h_1 = 800$

TABLE 5.19c The supplier’s best action and effective ranges with $h_1 = 800$

<table>
<thead>
<tr>
<th>Effective range for O</th>
<th>[0, 7500]</th>
<th>[7500, 9000]</th>
<th>[9000, b/1.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best action</td>
<td>2a</td>
<td>2b</td>
<td>1c</td>
</tr>
</tbody>
</table>

Comparing Table 5.5 and 5.3, we can see that the optimal solutions in Table 5.3 for Cases 2a, 2b, and 1c are valid. Because Case 2a gives the minimum cost of the manufacturer, then the manufacturer would choose Case 2a. Thus, we conclude that the subgame perfect optimal solution is Case 2a in this situation, and the optimal solution is given in the following table.

TABLE 5.19d The final optimal solution for the instance with $h_1 = 800$

<table>
<thead>
<tr>
<th>O</th>
<th>Q</th>
<th>The supplier’s profit</th>
<th>The manufacturer’s cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>7476</td>
<td>12715</td>
<td>3768684</td>
<td>81294542</td>
</tr>
</tbody>
</table>

From the performance of the tested sets of $h_1 = (2800, 1800, 800)$, we can see the following facts:

(a) Case 1 results for all the tested $h_1$ show the same pattern, i.e., $Q^* > O^*$. They are the invalid solutions since they violate Case 1 assumption $Q < O$. 

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(b) From the backward induction algorithm above, we conclude that Case 2a is the subgame perfect optimal solution for 3 different of $h_1$ situations. Here we only analyzed the effects of $h_1$ to Case 2a. Following the backward induction method algorithm in Figure 5.4, we tested different values of $h_1$ from 800 to 3300. Table 5.20 summarizes $h_1$ effects on Case 2a performance from the test.

(3) Study the performance of Case 2a over varied $h_1$.

**Table 5.20 Compensation $h_1$ effect in case 2a**

<table>
<thead>
<tr>
<th>Case 2a Solution</th>
<th>$h_1$</th>
<th>$\zeta$</th>
<th>$\eta$</th>
<th>$O^*$</th>
<th>$Q^*$</th>
<th>$E(\pi)$</th>
<th>$E(\text{cost})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2a: $Q&gt;1.2O$</td>
<td>800</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>3768684</td>
<td>81294543</td>
</tr>
<tr>
<td></td>
<td>1300</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>4245768</td>
<td>81771627</td>
</tr>
<tr>
<td></td>
<td>1800</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>4722852</td>
<td>82248711</td>
</tr>
<tr>
<td>$Q^*=R(O)=\zeta+\eta O$</td>
<td>2300</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>5199936</td>
<td>82725795</td>
</tr>
<tr>
<td></td>
<td>2800</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>5677020</td>
<td>83202879</td>
</tr>
<tr>
<td></td>
<td>3300</td>
<td>1170000</td>
<td>-154.8</td>
<td>7476</td>
<td>12715</td>
<td>6154104</td>
<td>83679963</td>
</tr>
</tbody>
</table>

Figure 5.11. Compensation $h_1$ effect in case 2a
From Table 5.13 and Figure 5.11, we can see that when $h_1$ changes, $\zeta$ and $\eta$ stay the same, since Formula (5.6) and (5.7) do not include $h_1$. It is interesting that when $h_1$ varies, $O^*$ and $Q^*$ almost do not change due to the small effect of $h_1$ as shown in Formula (5.8). It means that varying $h_1$ almost has no effect on order $O$ and capacity $Q$ in the experiment. When $h_1$ increases, the supplier’s profit, and manufacturer’s cost increase. So increasing $h_1$ is good for the supplier, but not good for the manufacturer. The manager should decide the appropriate $h_1$ to balance the manufacturer’s total cost and supplier’s total profit.

### 5.7 Remarks

Based on the study for the model of an optimal contract for product development with risk consideration in Chapter 4, we conducted numerical experiments in uniform demand distribution. The hypothetical experiment data simulate an automotive OEM data when a new electric vehicle (EV) was developed, and also a major component supplier of producing and supplying the EV battery. The market demand for the new EV is highly uncertain. The Newsvendor model and Stackelberg game theory are applied to formulate the objective functions of the supplier’s profit and the manufacturer’s cost. The models are based on a single supplier, single manufacturer, and a single period. Two cases are considered, Case 1 ($Q \leq O$), which includes 2 subcases, Case 1 ($Q < O$) and Case 1 constraint Case 1c ($Q = O$); and Case 2 ($Q > O$), which includes 3 subcases, Case 2a ($Q > 1.2O$), Case 2b ($O < Q \leq 1.2O$) and Case 2c ($Q = 1.2O + 1$). We applied the backward induction method to find Nash subgame perfect equilibrium solution. The calculation procedures are: 1) calculate the optimal solutions $O^*$, $Q^*$, $E(\pi)$, $E(\text{cost})$ of Case 1, Case 2 and subcases using related formulas, which are derived from Leibniz rule; 2) construct supplier’s response function $E(\pi)(O)$ for each subcase; 3) find the supplier’s best action $E(\pi)(O)$ and effective ranges; 4) verify if the optimal solution from Step 1 is within the valid range; 5) find the manufacturer’s optimal solution $O^*$ and objective function to conclude the subgame perfect optimal solution; 6) use the backward induction to determine subgame perfect equilibria: i.e. to decide the best supplier’s action for any given order quantity $O$. Rationally, the supplier will take the action that provides the maximum expected profit. We use numerical examples to illustrate the solution approach and provide the optimal capacity solution.
Our numerical experiments show that:

- Uniform distribution is appropriate for new product development case. With a uniform distribution, all models have the closed form optimal solutions.

- In Case 1, the optimal solution conflicts with the assumption $Q < O$ for the tested examples. So the constraint solution of Case 1c, $Q^* = O^*$, is used in Case 1. It means that the supplier’s planned capacity should be at least equal to the manufacturer’s planned order quantity. Supplier’s capacity strategy of being less than the manufacturer’s planned order quantity is not in the best interest of the supplier. This result provides the managerial or operational decision implication. To get the optimal results, the manager should choose the strategy $Q^* \geq O^*$ in any case.

- There is an effective range for each subcase. For a given $O$, the supplier needs to compare the response actions to decide the best action that provides the maximum expected profit. In most of the tested examples, the response function $E(\pi)(O)$ of subcase Case 2a $Q > 1.2O$ is superior to that of all other subcases in the effective regions.

- The parameter sensitivities were also studied through the numerical experiments.
  - Effects of uniform distribution lower limit $a$ and upper limit $b$: we tested different sets of $(a, b)$ with the same $\mu (7500)$. When $(b-a)$ increases, $O^*$ and $Q^*$ increases too. The ratio of $Q^*/O^*$ becomes large. Both supplier’s profit and manufacturer’s $E(\text{cost})$ increase. It implies that in order to reduce the manufacturer’s cost, supply chain should forecast demand more accurately by reducing $(b-a)$.
  - Effects of high production compensation $m_1$: since Case 1 and Case 2b do not include item $m_1$, only Case 2a, and Case 2c are affected by the $m_1$ change. Comparing the response function $E(\pi)(O)$ for all subcases, Case 2a is the optimal strategy when $m_1$ varies. We also found that the effect of $m_1$ on optimal capacity $Q^*$ is not significant.
  - Effects of the penalty cost $e_1$ and shortage cost $s_1$: these two costs are the same cost. When $e_1$ or $s_1$ increases, the optimal order $O^*$ will decrease slightly, capacity $Q^*$ will increase a lot, supplier’s profit will decrease, and manufacture cost will
increase. So large $s_1$ is not good for both manufacturer and supplier. Comparing the performance of Case 1 and Case 2 under different $e_1$ and $s_1$ will provide the insights for managers to choose the optimal strategy. The supplier should choose Case 2a strategy $Q > 1.2O$ to avoid penalty and get optimal profit $E(\pi)$. Also, the study about $e_1$ and $s_1$ helps managers to decide the optimal value of $e_1$ or $s_1$ in the contract. $e_1$ or $s_1$ cannot be too small or too big. When $e_1$ is between $12500$ and $15000$, $Q^*$ and $O^*$ are more reasonable. Otherwise, the penalty has no meaningful effect on the supplier’s capacity decision.

- Effect of low demand compensation $h$: when $h$ decreases, $Q^*$ and $O^*$ increase, and supplier’s unit benefit and manufacturer’s cost increase too. The manager should decide appropriate $h_1$ to balance $O^*$ and $Q^*$, as well as manufacturer’s cost $E(\text{cost})$ and supplier’s profit $E(\pi)$.

The numerical experiments provide the interesting managerial insights on some critical parameters in the contract model. Future work would include: conducting more sensitivity analyses to parameters, such as supplier’s product price $p$, variable cost $c_s$, and fixed cost $c_0$; also conducting the multiple parameter covariance analyses under different subcases; changing the compensation with different level (20% in this research) for different industries; and treating some key parameters as variables to get optimal solutions. Also for the future work, more analyses can be done with the different stochastic demand distributions, such as Poisson distribution, exponential distribution or normal distribution. At the certain situation, such as normal distribution, it is hard to find a closed form solution. So comparing the two cases to find the implied insights is also important for applying this type of frameworks in practice.

5.8 Appendix

Appendix A: To prove $E(\text{cost})$ in uniform distribution is a convex function

- Case 1: $Q < O$ Manufacturer’s cost function:

$$E(\text{cost}) = \int_0^Q (p + c_m)x f(x)dx + \int_Q^{\inf} (p + c_m)Q f(x)dx + g(O) - \int_Q^O e_1(x - Q)f(x)dx$$
$$+ \int_0^{0.8Q} h_1(0.8Q - x)f(x)dx + \int_Q^{\inf} s(x - Q)f(x)dx$$

(4.4)
\[ \frac{dE(\text{cost})}{dO} = R'(O) \left( p + C_m + e_1F(O) - s \right) + R'(O) \cdot F(R(O)) \left( s - p - C_m - e_1 \right) \]
\[ + e_1 \cdot f(O)(R(O) - O) + 0.8 \cdot h_1 \cdot R'(O) \cdot F(0.8 \cdot R(O)) + g_0 \]

We need to check if the second derivative of \( E(\text{cost}) \) of Case 2a is positive to ensure that the total expected cost is minimized at \( O^* \) or to prove Case 2a \( E(\text{cost}) \) is a convex function.

It is difficult to calculate \( \frac{dE^2(\text{cost})}{dO^2} \). We apply method II in Figure 5.2 to calculate it in a uniform distribution. The step is that first to do integral to \( E(\text{cost}) \), which becomes to an algebra function. Secondly, perform first order derivative to algebra function to find solution \( Q^* \). Thirdly perform second order derivative to algebra function to test \( \frac{dE^2(\text{cost})}{dO^2} \).

Step 1. Substitute uniform distribution pdf \( f(x) = \frac{1}{b-a} \) to manufacturer’s objective function (4.4) and do the integral calculation.

\[ E(\text{cost}) = \frac{1}{2(b-a)}(s - p - c_m - e_1 + 0.64h_1) \cdot Q^2 + \frac{1}{b-a}((p + c_m)b - 0.8ah_1 - sb) \cdot Q + \frac{e_1}{b-a}O \cdot Q \]
\[ - \frac{e_1}{2(b-a)}O^2 + g_0 \cdot O + \frac{1}{2(b-a)}[a^2(-p - c_m + h_1) + sb^2] + g_1 \]  
(5.12)

Step 2. Do the derivative \( \frac{dE(\text{cost})}{dO} \), substitute \( Q^* = \beta O, \frac{dQ^*}{dO} = \beta \) to Formula (5.12)

\[ \frac{dE(\text{cost})}{dO} = \frac{1}{(b-a)}(s - p - c_m - e_1 + 0.64h_1) \cdot Q \cdot \frac{dQ^*}{dO} + \frac{1}{b-a}[(p + c_m)b - 0.8ah_1 - sb] \cdot \frac{dQ^*}{dO} \]
\[ = \frac{1}{(b-a)}[\beta^2(s - p - c_m - e_1 + 0.64h_1) + 2\alpha \beta - e_1] \cdot O \]
\[ + \frac{\beta}{b-a}((p + c_m)b - 0.8ah_1 - sb) \cdot \frac{\alpha \beta}{b-a}(s - p - c_m - e_1 + 0.64h_1) + \frac{\alpha e_1}{b-a} + g_0 \]

Set \( \frac{dE(\text{cost})}{dO} = 0 \), we get the same solution as (5.4):

\[ O^* = \frac{\alpha \beta (p + c_m + e_1 - s - 0.64h_1) - e_1 \alpha + (0.8h_1 a + sb - pb - b c_m) \beta - g_0 (b - a)}{(s - p - c_m - e_1 + 0.64h_1)\beta^2 + 2\alpha \beta - e_1} \]  
(5.4)

Step 3: Do the second derivative test for \( O^* \). Since \( e_1 = s_1 \)

\[ \frac{dE^2(\text{cost})}{dO^2} = \frac{1}{b-a}[\beta^2(s - p - c_m - e_1 + 0.64h_1) + 2\alpha \beta - e_1] \]
\[ = \frac{1}{b-a}[\beta^2(-p - c_m + 0.64h_1) + 2\alpha \beta - e_1] \]  
(5.7)

Where
\[ \beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1} \] (5.3)

Substitute numerical example to (5.3), and (5-7), we have

\[ \frac{dE^2(\text{cost})}{dO^2} > 0 \] so E(cost) is convex function and has the minimum value at O* in Case 1 with uniform distribution.

**Case 2: Q > O**

Since penalty = 0, we have

\[ \int (p + c_m) f(x) dx + \int Q f(x) dx + g(O) \]

\[ + \int g_0 (x - O) f(x) dx + \int m_1 (x - 1.2O) f(x) dx \]

\[ + \int h_1 (0.8O - x) f(x) dx + \int s(x - Q) f(x) dx \] (4.10)

\[ \frac{dE(\text{cost})}{dO} = (p + c_m - s) R'(O) \left( 1 - F(R(O)) \right) + g_0 + 0.8h_1 F(0.8O) \]

\[ + g_0 F(O) - F(R(O)) (g_0 + 1.2m_1) + 1.2m_1 F(1.2O) \]

\[ + f(R(O)) \cdot R'(O) \cdot \left[ (g_0 + m_1) - (g_0 + 1.2m_1) \right] \] (4.11)

We need to check if the second derivative of E(cost) of Case 2a is positive to ensure that the total expected cost is minimized at O*, or to prove Case 2a E(cost) is a convex function.

It is difficult to prove \( \frac{dE^2(\text{cost})}{dO^2} > 0 \). We apply method II in Figure 5.2 to calculate it in uniform distribution. The proof procedure is the same as the one used in Case 1.

Step 1. Substitute uniform distribution pdf \( f(x) = \frac{1}{b - a} \) to manufacturer’s objective function (4.10) and do integral calculation first.

\[ E(\text{cost}) = \frac{(-p - c_m + m_1 + s + g_0)}{2(b - a)} \cdot Q^2 \]

\[ + \frac{1}{b - a} \cdot (p + c_m - s) \cdot Q \cdot \frac{1.2m_1 + g_0}{b - a} \cdot O \cdot Q \]

\[ + \frac{(1.44m_1 + 0.64h_1 + g_0)}{2(b - a)} \cdot Q^2 \]

\[ + \frac{g_0 - 0.8ah_1}{b - a} \cdot O + g_b \]

\[ + \frac{1}{2(b - a)} \cdot \left( a^2h_1 + b^2s - (p + c_m)a^2 \right) \] (5.13)

Step 2: Perform a derivative \( \frac{dE(\text{cost})}{dO} \), substitute \( Q^* = \zeta + \eta O \), \( \frac{dQ^*}{dO} = \eta \) to (5.13)
\[ \frac{dE(\text{cost})}{dO} = \frac{1}{(b-a)}(-p - c_m + m_1 + s + g_1) \cdot Q \cdot \eta + \frac{1}{b-a} (p-s) b \cdot \eta \\
- \frac{1.2m_1 + g_0}{b-a} \cdot (Q + 0\eta) + \frac{1}{(b-a)} (1.2^2 m_1 + 0.8^2 h_1 + g_0) \cdot O + g_0 - \frac{0.8ah_1}{b-a} \]

Set \( \frac{dE(\text{cost})}{dO} = 0 \), we get the same solution as (5.8):

\[ O^* = \frac{\zeta \eta (p + c_m - s - g_0 - m_1) + g_0 + 1.2m_1 - (p + c_m - s) \eta b - g_0 (b-a) + 0.8h_1 a}{\eta^2 (s-p - c_m + g_0 + m_1) - 2\eta (g_0 + 1.2m_1) + g_0 + 0.64h_1 + 1.44m_1} \]

(5.8)

Step 3: Do the second derivative test for \( O^* \),

\[ \frac{dE^2(\text{cost})}{dO^2} = \frac{1}{b-a} [(-p - c_m + m_1 + s + g_0) \cdot \eta^2 - 2(1.2m_1 + g_0) \eta + (1.2^2 m_1 + 0.8^2 h_1 + g_0)] \]

where

\[ \eta = \frac{1.2m_1}{c_s - p + m_1} \]

(5.7)

Substitute numerical example to (5.3), and (5.7), we have \( \eta < 0, s > p \), \( \frac{dE^2(\text{cost})}{dO^2} > 0 \), so E(cost) is convex function and has the minimum value at \( O^* \) in Case 2 with uniform distribution.

- Case 2b: \( O < Q < 1.2O \).

\[ m(O, Q) = 0. \]

\[ E(\text{cost}) = \int_0^Q (p + c_m) x f(x) dx + \int_Q^{\text{inf}} (p + c_m) Q f(x) dx + g(O) \]
\[ + \int_0^Q g_0(x - O) f(x) dx \]
\[ + \int_0^{0.8} h_1(0.8O - x) f(x) dx + \int_Q^{\text{inf}} s(x - Q) f(x) dx \]

(4.14)

\[ \frac{dE(\text{cost})}{dO} = (p + c_m - s) R'(O) \left(1 - F(R(O))\right) + g_0 + 0.8h_1 F(0.8O) \]
\[ + g_0 F(O) - F(R(O))(g_0 + 1.2m_1) + 1.2m_1 F(1.2O) \]
\[ + f(R(O)) \cdot R'(O) \cdot [R(O)(g_0 + m_1) - O(g_0 + 1.2m_1)] \]

(4.11)

\[ \frac{dE(\text{cost})}{dO} = g_0 + 0.8h_1 F(0.8O) + g_0 F(O) - g_0 F(Q) \]
As we discussed in Chapter 4, there is no solution to satisfy \( \frac{dE(\text{cost})}{dO} = 0 \).

So the constraint solution for Case 2b is:

\[
F(Q^*) = 1 - \frac{c_0}{p - c_s}
\]  \hspace{1cm} (4.13)

\[
O^* = \frac{1}{1.2} \cdot Q^*.
\]  \hspace{1cm} (4.17)

For example, substituting the parameter values \( (c_0 = 500, p = 9300, c_s = 8000) \) to Formula (4.8) and (4.17), then \( F(Q^*) = 1 - \frac{500}{9300 - 8000} = 0.6154 \).

- For normal distribution \( N(7500, 2166.67^2) \), \( Q^* = 8136 \) from the inverse of \( F(Q^*) = 0.6154 \). So the optimal solution \( O^* = 0.833Q^* = 6780 \).

- For uniform distribution \( U(1000, 14000) \), \( F(Q) = \frac{Q - a}{b - a} \), then \( Q = F(Q) \cdot (b - a) + a \).

Since \( Q^* = 9000 \), so the optimal solution \( O^* = \frac{1}{1.2} \cdot Q^* = 7500 \).

**Appendix B: Constraint optimization solution in uniform distribution:**

Case 1 \( Q \leq O \)

1) For Case 1

\[
O^* = \frac{\alpha \beta (p + c_m + e_1 - s - 0.64h_1) - e_1 \alpha + (0.8h_1a + s b - pb - bc_m)\beta - g_0(b - a)}{(s - p - c_m - e_1 + 0.64h_1)\beta^2 + 2e_1\beta - e_1}
\]  \hspace{1cm} (5.4)

\[
O^* = R(O) = \alpha + \beta O
\]  \hspace{1cm} (5.1)

Where

\[
\alpha = \frac{(p - c_s - c_0)b + (c_0 - 0.8h_1)a}{p - c_s + e_1 - 0.64h_1}
\]  \hspace{1cm} (5.2)

\[
\beta = \frac{e_1}{p - c_s + e_1 - 0.64h_1}
\]  \hspace{1cm} (5.3)

2) For Case 1c constraint, if \( Q^* = R(O) = O \), which means \( Q^* = R(O) = O \), \( R'(O) = 1 \),

We have solution formula (5.9) for any type of distribution.
\[(p + c_m - s) + F(O) \cdot (s - p - c_m) + 0.8h_1 \cdot F(0.80) + g_0 = 0 \quad (5.9)\]

For uniform distribution, we have:

\[O^* = \frac{b(p + c_m - s + g_0) - a(g_0 + 0.8h_1)}{p + c_m - s - 0.64h_1} \quad (5.10)\]

\(O^*\) from formula (5.10) and \(Q^* = O\) is the constraint solution if \(Q^* > O\). Also if we substitute \(\alpha = 0, \beta = 1\) to (5.4), we can get the same result as (5.10).

3) For Case 2a \(Q > 1.2O\),
• If \(Q^* > 1.2O\), this is the normal situation.

\[O^* = \frac{\zeta[p + c_m - s - g_0 - m_1] + g_0 + 1.2m_1] - (p + c_m - s)\eta b - g_0(b - a) + 0.8h_1a}{\eta^2(s - p - c_m + g_0 + m_1) - 2\eta(g_0 + 1.2m_1) + g_0 + 0.64h_1 + 1.44m_1} \quad (5.8)\]

\[Q^* = R(O) = \zeta + \eta O \quad (5.5)\]

\[\zeta = \frac{c_0(b - a) + b(c_s - p)}{c_s - p + m_1} \quad (5.6)\]

\[\eta = \frac{1.2m_1}{c_s - p + m_1} \quad (5.7)\]

Formula (5.5) ~ (5.8) is the solution when \(Q^* > 1.2O\).

4) For Case 2b \(O < Q \leq 1.2O\),

We have constraint solution 1: (from \(Q^*\) to \(O\))

\[F(Q^*) = 1 - \frac{c_0}{p - c_s} \quad (4.13)\]

\[O = 1/1.2 \cdot Q^*\]

For uniform distribution, we have

\[F(Q^*) = \frac{Q^* - a}{b - a}\]

So \(Q^*\) can be obtained by

\[Q^* = (1 - \frac{c_0}{p - c_s})(b - a) + a \quad (5.11)\]
Calculate the profit of supplier and the cost of manufacture for constraint solution 1 and 2, and compare the results then choose the solution.
6.1 Conclusion

The automotive industry plays an important role in the global economy. The automotive supply chain is complex due to the large number of parts assembled into an automobile, the multiple layers of suppliers for those parts, and the coordination of the material, information, and finance flows across the supply chains. Many uncertainties and natural and man-made disasters have repeatedly stricken and disrupted automotive OEMs and their supply chains. The purpose of this research is (1) to find the underlying supply chain risk management problems in the automotive industry; (2) to develop the theoretical models to help the decision-making of supply chain managers in these uncertain scenarios.

The major contributions of this study are:

1. Provided a literature review of the existing research work on the supply chain risk identification and management, considering (but not limited to) the characteristics of the automotive supply chain, since the literature focusing on automotive supply chain risk is limited. The review provides a summary and a classification for the underlying supply chain risk resources in the automotive industry; and an overview of the current research on automotive SCRM, with an emphasis on the quantitative methods and mathematical models, currently used. Through the literature review, it was found that auto organizations implementing supply chain risk management can gain many benefits, such as an improved focus on risk and more effective risk mitigation. Other benefits include the elimination of potential and unexpected costs, reduced disruption, and decreased recovery time. Therefore, there are improvements in the overall supply chain performance. It is important for auto organizations to understand the risk assessment and management along the supply chain and to develop more theoretical models and practical risk mitigation methods to guide the process in the future.

2. Two mathematical models are developed focusing on the supply chain risk management in the automotive industry. The first model is for optimizing manufacturer cooperation in supply chains. Since the automotive supply chain is a multiple layer and complex network, the relationship between supply chain members is very important for risk management.
Supplier development is a long-term and resource-consuming business activity that requires commitment from both the manufacturer and the suppliers. OEMs often invest a large amount of money in supplier development to improve their suppliers’ capabilities and performance. How to allocate the investment optimally among multiple suppliers to minimize risk while maintaining an acceptable level of return becomes a critical issue faced by manufacturers or automotive OEMs. Talluri et al. (2010) applied Markowitz’s model to manufacturer cooperation in supplier development under risk. Talluri’s model assumes that the return of investment to the supplier is proportional to the investment. Actually, in most situations the return is nonlinear. This research revised Talluri’s manufacturer cooperation model with the nonlinear return and intended to apply it to the auto industry.

(3) The second one is a mathematical model for an optimal contract for product development with risk consideration (penalty and compensating) and demand uncertainty, especially for the automotive supply chain. As a common ex-ante strategy in risk management, the supply chain contracts play an important role for supply chain members, such as OEMs and suppliers, to coordinate and to share risks arising from various sources of uncertainty. The objective of this part of the research is to design a supply contract when developing a new product in order to reduce the risks and maximize profits under uncertainty demands. More specifically, we investigated how to decide the supplier’s capacity and the manufacturer’s order in the supply contract when the demand for the new product is highly uncertain. Based on the newsvendor model and Stackelberg game theory, we developed a single period supply chain model for a product development contract consisting of a supplier and a manufacturer with demand uncertainty. Two cases are considered, Case 1 \( Q \leq O \), and Case 2 \( Q > O \), which included two subcases, Case 2a \( Q > 1.2O \) and Case 2b \( O < Q \leq 1.2O \). The subgames are solved by Nash subgame perfect equilibrium.

(4) The optimal solutions of the models with consideration for demand uncertainty were discussed, and the numerical experiments, which simulate an automotive OEM data when a new electric vehicle (EV), is developed. The analytical solutions are studied for the situation where the demand follows a uniform distribution, and the computational tests are reported. With a uniform distribution, all models have closed form optimal solutions. Based on Nash subgame perfect equilibrium, comparing the performance in the subcases of Case
and Case 2, the optimal solution is chosen. The comparing criteria are based on the value of the manufacturer’s total cost $E(\text{cost})$, since the manufacturer is the leader in the Stackelberg game. The smaller the $E(\text{cost})$, the better the solution. The sensitivity analyses are performed for different parameters under uniform distributions; for the single period decentralized case. The numerical experiment provides the interesting managerial insights on some critical parameters in the contract model. The proposed solution provides an effective tool for making the supplier-manufacturer contracts when the manufacturer faces highly uncertain demand.

6.2 Future Research

The optimal supply chain contract model developed in this research is for a single period. The future work can improve the model for two or more periods. In the single period contract scenario, the forecast accuracy is very critical. If the real demand is far less than expected demand, the supplier will be over-capacitated, and the manufacturer will be requested to pay the compensation to suppliers, which would be very costly. A good example is the plug-in electrical vehicle (EV). Many auto OEMs and EV start-ups forecast a market demand is booming for plug-in EVs in the last few years. They made the production schedules and requested the supplier capacities based on their forecast. However, the actual market demand was far below that. It has posed detrimental risks to the OEMs and suppliers. Some EV start-ups and battery suppliers, such as Fisker and A-123, even went bankrupt. Some OEMs have to compensate the supplier’s cost up to multimillion dollars each year for unused supplier capacity.

In the two period supply chain contract model, half of the full year order quantity is contracted in the first period, and hence the supplier will prepare its capacity accordingly. Based on Product Life Cycle as shown in Figure 6.1, in the introduction stage, the product sales is about 1/3 of the sales in the peak time. From the sales in the introduction stage or the first year, the manufacturer can understand the trend of product lifecycle more. Then the manufacturer and the supplier can adjust the capacity through the second-period contract. If the sales trend is far less than the forecasted demand, the capacity does not need to be increased, so the second-period contract is not considered. Also, both the manufacturer and the supplier can reduce the cost. If the sales trend is equal or larger than
forecasted demand, the second-period contract can catch up the capacity gap. Since the
development of the second-period capacity takes less time, the production quantity can
meet the demand of growth period of product lifecycle in a fairly short time. The timeline
for two – the period contract is illustrated as Figure. 6.2.

A two-period supply chain contract can reduce the risk of going over or under capacity.

Based on this research, another possible future work is to apply non-linear return supplier
development models to the automotive industry, such as more suppliers (30 ~ 100), or more
investment amount. In addition, for a new product development contract model, the future
work would include: conducting more parameter sensitivity analyses for different subcase
response function \( E(\pi)(O) \); changing the compensation for different levels (20% in this
research) and also for different industries; selecting and treating some key parameter as
variables to get optimal solution; developing nonlinear pricing contract model; extending
the contract model to a large and more complex risk game; investigating the performance
of two game players in information asymmetry situations; extending risk sharing model to
three players; studying the contract model in different stochastic demand distributions,
such as Poisson distribution, exponential distribution or normal distribution to gain the managerial insights.

Also based on the literature review in ASCRM, the following research work could be done in the future:

1. The study and development of systematic methods and systems to analyze ASCRM by integrating the different risk sources. Most research work in the automotive industry is based on specific points of views, i.e., the suppliers or the manufacturers. There is no systematic method of the system to analyze and integrate different ASCRM strategies, such as how to choose supply locations, transportation, to optimize the objectives for both the manufacturers and the suppliers as a system and to reduce geographic or political risks in the automotive supply chain in the global environment. The existing research on the impact on the automotive network resulting from supply chain risk has not been sufficient. This area requires more related research to be conducted in the future.

2. The use of data- and big-data-based ASCRM: One of the major automotive supply chain risks is nontransparency resulting from the multiple layers in the supply chain. Typically, OEMs have substantial data about Tier-1 suppliers, but they lack data from Tier-2 to Tier-N suppliers. Nontransparency makes it very difficult to monitor risks and issue warnings. Big-data analytics can provide a basis for transparency in automotive supply chains. With the help of real-time big data availability, OEMs and suppliers can improve their supply chain transparency, monitor the occurrence of risks, provide early warnings and responses, and enable managers to develop risk mitigation strategy to prevent the risks. Big-data-based sense-and-respond systems for ASCRM are worth further research.

3. The study of the downstream risks to the automotive supply chain: There is not enough research on automotive-industry-specific models. The special aspect of the automotive supply chain is its complexity owing to the huge number of multitier suppliers and globalized network. Further study is required on downstream risks to the automotive supply chain besides demand uncertainty, such as call-back risk and how to build a resilient network.

4. The implementation of more quantitative models: Through a literature review, we found that the majority of research on automotive supply chain risk employs empirical
approaches. There is a real demand in the automotive industry to use quantitative models to evaluate supplier risks regarding different tiers and different type of suppliers. Some quantitative models just address simple two-layer supply chains owing to the lack of deep downstream suppliers’ information. Proposed future work includes quantitative model development for complex ASCRM and improvement of the mathematical models to cope with real-life situations. Traditional risk modeling, including the use of the utility function, variance, standard deviation, mean–variance, value at risk, and conditional value at risk, has rarely been applied to ASCRM. These modeling methods can be applied to ASCRM.

5. Addressing advanced technology challenges: In recent decades, revolutions in information technology and telecommunications have brought about dramatic changes in our daily lives and the automotive industry as well. Automakers continuously offer new high-technology features in their products (e.g., GPS, telematics, various sensors, ADAS, and RFID.). These high-technology features present many technological challenges in the automotive supply chain. One of these challenges is the risk posed to vehicle design, production, quality, and after-sales services by the short product development cycle and the long useful life of vehicles. Automotive manufacturers must mitigate risk through their component suppliers. Future research needs to be done on car manufacturers' selection of proper suppliers and on improving coordination and cooperation among supply chain vendors.

6. The research on autonomous cars and car-sharing services: In recent years, autonomous cars have emerged as the future of the automotive industry. Experts have predicted that fully autonomous cars will arrive at the market by 2025 to 2030 (Liuima, 2016). Car-sharing services using autonomous vehicles could be attractive for many private buyers as well. It is suggested that new-car sales in the US could be eroded by as much as 40%. Like any new product, autonomous cars will have demand uncertainty because of many obstacles, such as adoption rate, technological challenges, liability disputes, laws, and regulations. Demand uncertainty implies overcapacity risk or under capacity risk. Future work includes improving forecast accuracy to optimize contracts and production capacity and to reduce supply chain risk.
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