Maintenance Strategies Design and Assessment Using a Periodic Complexity Approach

Khaldon Taha Meselhy Ahmed
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Maintenance Strategies Design and Assessment
Using a Periodic Complexity Approach

By
Khaldon Taha Meselhy Ahmed

A Dissertation
Submitted to the Faculty of Graduate Studies
through Industrial and Manufacturing Systems Engineering
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy at the
University of Windsor

Windsor, Ontario, Canada
2008
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ABSTRACT

People become more dependent on various devices, which do deteriorate over time and their operation becomes more complex. This leads to higher unexpected failure chance, which causes inconvenience, cost, time, and even lives. Therefore, an efficient maintenance strategy that reduces complexity should be established to ensure the system performs economically as designed without interruption.

In the current research, a comprehensive novel approach is developed for designing and evaluating maintenance strategies that effectively reduce complexity in a cost efficient way with maximum availability and quality.

A proper maintenance strategy application needs a rigorous failure definition. A new complexity based mathematical definition of failure is introduced that is able to model all failure types. A complexity-based metric, “complication rate”, is introduced to measure functionality degradation and gradual failure.

Maintenance reduces the system complexity by system resetting via introducing periodicity. A metric for measuring the amount of periodicity introduced by maintenance strategy is developed. Developing efficient maintenance strategies that improve system performance criteria, requires developing the mathematical relationships between maintenance and quality, availability, and cost. The first relation relating the product quality to maintenance policy is developed using the virtual age concept. The aging intensity function is then deployed to develop the relation between maintenance and availability. The relation between maintenance and cost is formulated by investigating the maintenance effect on each cost element.

The final step in maintenance policy design is finding the optimum periodicity level. Two approaches are investigated; weighted sum integrated with AHP and a comfort zones approach. “Comfort zones” is a new developed physical programming based optimization heuristic that captures designer preferences and limitations without substantial efforts in tweaking or calculating weights.

A mining truck case study is presented to explain the application of the developed maintenance design approach and compare its results to the traditional reward renewal theory. It is shown that
the developed approach is more capable of designing a maintenance policy that reduces complexity and simultaneously improves some other performance measures. This research explains that considering complexity reduction in maintenance policy design improves system functionality, and it can be achieved by simple industrially applicable approach.
Dedication

To
My Lovely Inspiring Parents
And
My Little Lovely Supporting Family
Acknowledgment

During the last three years, I have worked with great people whose contributions deserve special mention. It is my pleasure that I have now the opportunity to express my gratitude for all of them in my humble acknowledgment.

In the first place I would like to record my gratitude to my supervisor, Prof. Hoda A. ElMaraghy, for her supervision, advice, and guidance from the very early stage of this research as well as giving me extraordinary experiences throughout the work. Above all and the most needed, she provided me with unflinching encouragement and support in various ways. Her truly scientist intuition has made her as a constant oasis of ideas and passions in science, which exceptionally inspired and enriched my growth as a researcher. I am indebted to her more than she knows.

Sharing the first place is my Co-supervisor, Prof. Waguih H. ElMaraghy for his advice, supervision, and crucial contribution, which made him a backbone of this research. His involvement with his originality has triggered and nourished my intellectual maturity. During last three years I have known him as a sympathetic and principle-centered supervisor. His overly enthusiasm and integral view on research and his mission for providing nothing but high-quality work has made a deep impression on me. I owe him lots of gratitude for having me shown this way of research. He could not even realize how much I have learned from him. I am really glad that I have come to get know him in my life.

It is also my pleasure to pay tribute to my committee members, Prof. F Baki, Prof. Z. Pasek and Prof. G. Zhang for their valuable contributions and ideas that enriched the quality of this work. Collective and individual acknowledgments are also owed to my colleagues at IMSE department and IMS center. I wish to especially express my warm and sincere thanks to Zaina Batal for her indispensable help and Jacquie Mummery and Brenda M. Schreiber for their continuous sincere help.

Finally, the words fail me to express my appreciation to my lovely wife, Asmaa and my little wonderful kids, Faroha and Gogo whose dedication, love and persistent confidence in me, has taken the load off my shoulders and supported me all the way till finishing this thesis.
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<td>AI</td>
<td>Aging Intensity</td>
</tr>
<tr>
<td>AHP</td>
<td>Analytical Hierarchy Process</td>
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<tr>
<td>CBM</td>
<td>Condition Based Maintenance</td>
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<td>CI</td>
<td>Consistency Index</td>
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<td>CM</td>
<td>Corrective Maintenance</td>
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<td>DP</td>
<td>Design Parameter</td>
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<td>FR</td>
<td>Functional Requirement</td>
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<td>IR</td>
<td>Inconsistency Ratio</td>
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<tr>
<td>L</td>
<td>Failure Threshold</td>
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<td>LC</td>
<td>Labor Cost</td>
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<tr>
<td>MP</td>
<td>Markov Process</td>
</tr>
<tr>
<td>MTBF</td>
<td>Mean Time Between Failure</td>
</tr>
<tr>
<td>MTTF</td>
<td>Mean Time To Failure</td>
</tr>
<tr>
<td>MTTR</td>
<td>Mean Time To Repair</td>
</tr>
<tr>
<td>NHPP</td>
<td>Non-Homogeneous Poisson Process</td>
</tr>
<tr>
<td>OC</td>
<td>off-time cost</td>
</tr>
<tr>
<td>PM</td>
<td>Preventive Maintenance</td>
</tr>
<tr>
<td>PMF</td>
<td>Preventive Maintenance Frequency</td>
</tr>
<tr>
<td>PML</td>
<td>Preventive Maintenance Level</td>
</tr>
<tr>
<td>Pr</td>
<td>periodicity</td>
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<tr>
<td>RC</td>
<td>maintenance resources cost</td>
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<tr>
<td>RL</td>
<td>repair level</td>
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<tr>
<td>RT</td>
<td>repair time</td>
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<tr>
<td>SMP</td>
<td>Semi-Markov Process</td>
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<td>SMM</td>
<td>Self Maintenance Machine</td>
</tr>
<tr>
<td>TC</td>
<td>total maintenance cost</td>
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NOMENCLATURE

\( \frac{C_p}{f} \) ratio of preventive maintenance cost to the cost of the same level failure

\( C_{pm} \) Maintenance cost

\( C_{pm}^o \) Perfect maintenance cost

\( D(t) \) expected number of maintenance performed by time \( t \)

\( e_{f_k} \) Effort factor \( ef \) of the \( k \)th process task

\( I \) information content

\( K \) cost of one time unit downtime

\( N \) number of failures

\( R \) overhaul (perfect maintenance) cost

\( R_m \) resources cost for minimal repair

\( T \) time between maintenance

\( T_{D} \) time difference=(\( T_{max}-T_{min} \))

\( T_{min} \) time required for minimal maintenance

\( T_{max} \) time required for perfect maintenance

\( t_{f/p} \) ratio between failure repair time and preventive maintenance time of the same level repair

\( V_n \) virtual age after maintenance \( n \)

\( V(t) \) virtual age at time \( t \)

\( w \) weighting factor

\( \chi \) maintenance level

\( \nu \) complication rate

\( \lambda \) sudden failure rate

\( \beta \) Weibull distribution shape parameter

\( \theta \) Weibull distribution scale parameter
1 Introduction

1.1 Motivation and Problem Statement

Our everyday life, personal and business alike, has become more and more dependent on various devices. Airplanes are used for travelling. Manufacturing systems use machines to make products, etc. For some people, such as patients using artificial hearts, their lives literally rely on these devices. These devices do deteriorate over time and their operation become more complex and less reliable which leads to higher chance that they fail unexpectedly, in which case a repair is necessary. Unexpected failure of these devices not only causes inconvenience but also costs time, money, and sometimes injuries or even lives. To reduce the chance of unexpected or premature failures, a maintenance strategy should be established because it ensures that the system performs as designed without interruption.

However, maintenance has other effects on different system performance criteria, especially for a complex system, like cost, quality, and availability. Therefore, an efficient maintenance strategy should reduce operation complexity with minimum cost and minimum operation interruption. Moreover, in cases where devices are critical for maintaining or sustaining life, an efficient maintenance strategy should also reduce the chance of unexpected failures.

Although the main role of maintenance in any operating system is to reset the system functionality in order to reduce the operational complexity, upon reviewing the developed maintenance strategies since Barlow and Hunter (1960) till now, it was found that there is no systematic approach for developing maintenance strategies based on function resetting and complexity reduction. But all the developed strategies consider other maintenance effects like cost, availability or quality. It is realized that maintenance affects these performance parameters, but it is not the key factor as each one of these parameters is primarily managed by its own management system. The lack of such an approach results in designing maintenance policies that target some performance criteria at the expense of system complexity and furthermore this targeted performance criteria may not be
considerably improved when considering the overall system. An example for that is a maintenance policy that targets minimizing cost rate in a manufacturing system; it may reduce maintenance cost but this cost reduction may not be a significant amount when considering the total production costs.

1.2 Objectives and Approach

The main objective of this research is to develop a new approach for designing and evaluating maintenance strategies that effectively reduce complexity in a cost efficient way with maximum availability and quality. In relation to the main objective, it is required to develop a metric that measures the ability of maintenance strategy to reset the system functionality.

These objectives are achieved through the following steps:

1- Developing a failure definition that captures functional failure as well as physical failure: since a great deal of maintenance actions (corrective maintenance) are triggered by a failure event. A new definition is introduced based on the complexity theory.

2- Developing a mathematical metric for periodicity: the main function of maintenance in any system is to introduce periodicity that prevents the system continuous degradation and hence instability. The amount of periodicity introduced by a maintenance strategy into the system should be measured in order to be able to compare the different alternatives or to design a new one.

3- Investigating the relationship between periodicity and product quality: in this step, the main concern is the manufacturing system and the periodicity is expected to have effects on many system performance criteria. Product quality is traditionally one of the most important criteria in today’s manufacturing systems. Therefore, it is important to investigate what is the effect of periodicity on product quality.

4- Investigating the relationship between periodicity and availability: availability directly affects system productivity. Both “under maintenance” and “over maintenance” are expected to negatively affect system availability. Therefore, the
relationship between periodicity amount and availability needs to be investigated in order to determine the right amount of periodicity.

5- Investigating the relationship between periodicity and maintenance related costs: the resetting process needs costs to be carried out. This cost should at least balance the benefits of resetting process. So, the relationship between resetting costs and periodicity is investigated.

6- Determining the right amount of periodicity: after determining the relationships between periodicity and the different criteria, a multi-objective optimization is performed to determining the best periodicity level that reduces system complexity while satisfying the other performance criteria.

1.3 Assumptions and Constraints

The following assumptions and limitations are considered throughout the research:

1) The research is constrained to single unit systems. This may mean physically single unit or a whole system of different units but it is maintained together as a single entity.

2) The research is constrained to time based maintenance strategies. Therefore, condition based maintenance is not included.

3) Unit failure rate is non-decreasing. This limitation means that initial infant mortality periods are not considered.

4) The quality of products (in case of studying machine maintenance) or the quality of performance (in case of studying product maintenance) is deteriorating with age. This age may be in absolute time units or it may be in terms of the number of produced products since last perfect maintenance.

5) It is feasible to carry out the preventive maintenance at any time.

6) The unit is not available upon failure (no self maintenance).

7) It is assumed that the repair/maintenance level is a continuous variable. While in practice, it is discrete because there is definite course of actions for each maintenance level.
1.4 Thesis Outline

The thesis is organized as follows:

Chapter (2) introduces an extensive literature survey about the maintenance main role and the different categorizations and types of maintenance actions. The concept of imperfect maintenance and the different modeling approaches are surveyed. The different maintenance policy structures and the effect of maintenance on system performance criteria are then presented.

Chapter (3) introduces a novel definition for the failure that is equally able to model all types of failures and a new metric, complication rate, is introduced to measure the system performance degradation and gradual failure.

Chapter (4) introduces a novel metric for the periodicity. A mathematical derivation and the physical meaning are explained. The calculation of the periodicity of a maintenance policy is explained theoretically and using an industrial example.

Chapter (5) introduces the derivation of the mathematical relationship between the maintenance policy periodicity and the products quality using the concept of virtual age.

Chapter (6) introduces the relationship between the maintenance policy and the corresponding steady state availability using the concept of aging intensity function.

Chapter (7) introduces the derivation of the relationship between the maintenance policy and the corresponding maintenance related cost. The cost components are detailed and the effect of maintenance policy on each of them is studied.

Chapter (8) introduced different approaches for calculating the optimum periodicity level including weighted sum, and comfort zones. And extending this research to multi-unit case is discussed.

Chapter (9) introduces a case study for mining truck maintenance policy design.

Chapter (10) includes a summary and conclusions.

It is worth noting that although some parts of this research are developed considering the case of manufacturing system and the words machine and unit are used interchangeably throughout the research, the approach and results applicability are not restricted to manufacturing systems.
2 Literature Survey

This chapter introduces a literature survey related to the maintenance management field including the different categorizations of maintenance actions and the developed maintenance policies. Then, the concept of complexity is introduced with explanation of the different types of complexities.

2.1 Maintenance General Role

All components and systems encountered in our daily life experience aging either with time or usage or both. As the system ages, its functionality deteriorates and its operation becomes more complex which may lead to different types of risks according to the system being studied. Therefore, maintenance is performed in order to restore the system functionality to an acceptable condition. Hence, the main role of maintenance is system function restoration. However, in most cases, there are some other performance factors that are affected by the maintenance policy like total production cost, product quality, system availability and reliability as shown in Figure 2-1.

![Diagram](image)

Figure 2-1 Maintenance main role and performance factors affected by maintenance
But all these factors cannot be primary controlled by the maintenance policy, but each has its own management policies. For example, product quality is mainly controlled by the quality control system; cost is managed by cost management system and so on.

### 2.2 Maintenance Policy Design

Maintenance policy is defined by Dekker (1996) as the concept or strategy that describes what events (failure, passing of time, certain item condition) trigger what type of maintenance action. Therefore, the maintenance policy may contain many different types of maintenance action each of them corresponds to a triggering event.

#### 2.2.1 Types of Maintenance Actions

There are two main categorizations of maintenance actions; the first one categorizes the action according to the action trigger and the second one categorizes the maintenance action according to the level of restoration of functions.

The maintenance action categorization according to the trigger is shown in Figure 2-2 (Kaiser 2007, Aurich et al. 2006, and Shaomin and Clements-Croome 2005):

![Figure 2-2 Maintenance actions categorization according to the trigger](image)

Corrective maintenance (CM), according to MIL-STD-721B, means all actions performed as a result of failure, to restore an item/system to a specified condition (Wang and Pham, 2006c). Some researchers refer to CM as repair and both terms would be used alternatively throughout the thesis. This type of maintenance action is not concerned with scheduling inspections or routine services. Generally, system failures seldom, if ever
occur at a convenient time. As a result, scheduling these repairs often constitutes a high priority and likely interferes with operation schedules and other planned activities. In some cases when material, equipment, or skilled maintenance personnel are not available, the problem significantly worsens (Stephens, 2003).

Preventive Maintenance is one of the most popular maintenance actions used in modern maintenance management systems. Preventive maintenance, According to MIL-STD-721B, means all actions performed in an attempt to retain an item in specified condition by providing systematic inspection, detection, and prevention of incipient failures (Wang and Pham, 2006c). Maintenance routines are scheduled by analyzing historical system failure data. Time-based empirical and parametric distributions such as Weibull, Normal, Exponential, and Gamma distributions have been widely used to model the uncertainty in failure times (Kaiser, 2007). However, since PM relies on time-based models, it does not take into account the conditions or degradation characteristics of the individual components, making it nearly impossible to avoid catastrophic random breakdowns. In addition, PM can lead to unneeded maintenance routines being performed, resulting in unnecessary downtime and loss in production capacity. These types of problems have led to the development of predictive maintenance policies that focus on predicting failures.

Predictive maintenance applies various sensing technology like vibration level, the level of metal particles in the lubricant, the temperature or the humidity (Ghasemi et al., 2007a) and analytical tools to measure and monitor various systems and their components. The observed characteristics are compared with established standards and specifications in order to predict failures. Whereas corrective maintenance is applied after the failure and preventive maintenance uses precautionary measures to avert possible problems, predictive maintenance actually evaluates the existing equipment condition and, based on a projected trend of the deterioration process, failures are predicted and appropriate steps are taken (Stephens, 2003). An increasingly popular form of predictive maintenance is condition-based maintenance (CBM) which is performed either assuming the state of the system is known with certainty like Wang (2000) or assuming that state of the equipment is unknown, but can be estimated based on its observed condition as per Ghasemi et al., 2007 (a, b).
On the other hand, maintenance actions categorization according to the level of restoration is depicted in Figure 2-3.

![Diagram of maintenance actions categorization](image)

Figure 2-3 Maintenance actions categorization according to the maintenance level

Perfect maintenance is the action, which restores the system operating condition to 'as good as new' condition. That is, upon a perfect maintenance, a system has the same lifetime distribution and failure rate function as the new one. Examples of perfect maintenance actions are complete overhaul of an aircraft engine and replacement of a failed system by a new one.

Minimal maintenance is action, which restores the system to the same failure rate as it had when it failed. The system operating state after the minimal repair is often called 'as bad as old' in the literature. Changing a flat tire on a car is an example of minimal repair because the overall failure rate of the car is essentially unchanged.

Imperfect maintenance is action, which makes a system not 'as good as new' but better than the state just before the maintenance. Usually, it is assumed that imperfect maintenance restores the system operating state to somewhere between 'as good as new' and 'as bad as old' Clearly, imperfect maintenance is the general case which can include the two extreme cases; minimal and perfect. Engine tune-up is an example of imperfect maintenance.

Worse maintenance is action, which un-deliberately makes the system failure rate or actual age increase but the system does not break down. Thus, upon worse repair a system's operating condition becomes worse than that just prior to its failure.

Worst maintenance is action, which un-deliberately makes the system break down.
The maintenance level is being referred in the literature by different names; Shaomin and Clements-Croome (2005) referred to it as maintenance quality while Pham and Wang (1996) referred to it as maintenance degree. Therefore, all these terms will be used interchangeably during the current dissertation.

The imperfect maintenance has been modeled in the literature using many methods as follows:

**(1) (p,q) method**

Nakagawa (1979) proposed the (p, q) rule to model the imperfect maintenance such that after the maintenance, a unit is restored to the 'as good as new' state with probability p and restored to the 'as bad as old' state with probability q. Such that q = 1 - p. Clearly, p = 1 represents the perfect maintenance case while p = 0 represents the minimal maintenance case. In this sense, minimal and perfect maintenances are special cases of imperfect maintenance and imperfect maintenance is the general case. This model has been used to model imperfect maintenance by many researchers like Brown and Proschan (1983) and Haijun and Shaked (2003). Lim et al. (1998) extend the (p,q) imperfect repair model, and propose a new Bayesian imperfect repair model where the probability of perfect repair, P, is considered to be a random variable.

**(2) p(t), q(t) method**

Block et al. (1985) extend the above (p,q) modeling method to the age-dependent imperfect repair such that an item is perfectly maintained with probability p(t) and minimally maintained by probability q(t) (p(t) = 1 - q(t)) where t is the age of the item (the time since the last perfect maintenance). This method has been used by Block et al. (1988). An alternative method for (p(t), q(t)) is proposed by Makis and Jardine (1992) denoted by p(n, t) in a way that maintenance returns a system to the “as good as new” state with probability p(n, t) or to the “as bad as old” state with probability q(n, t) where t is the age of the system and n is the number of failures since last perfect maintenance.

**(3) Improvement Factor Method**

Malik (1979) introduced the concept of improvement factor in the maintenance scheduling problem. He explained that maintenance changes the system failure rate to some value corresponding to newer age but not all the way to zero. This treatment
method for imperfect maintenance makes the failure rate after maintenance somewhere between the failure rate of the new item and the failure rate of the old item where the degree of improvement in failure rate is called improvement factor. Examples for this method application can be found in Lie and Chun (1986), Jayabalana and Chaudhuri, 1992 (a, b), Chan and Shaw (1993a) and Doyen and Gaudoin (2004).

(4) Virtual Age Method

Kijima et al. (1988) developed an imperfect repair model by using the idea of the virtual age process of a repairable system. The model is described by the following equation:

\[ V_n = V_{n-1} + \chi T_n \]  

Equation 2-1

Where:
- \( V_n \) virtual age after \( n \)th maintenance
- \( V_{n-1} \) virtual age after \( (n-1) \)th maintenance
- \( \chi \) level of maintenance
- \( T_n \) time difference between \( n-1, n \) maintenances

Obviously, \( \chi = 0 \) corresponds to a perfect maintenance while \( \chi = 1 \) corresponds to a minimal maintenance.

(5) Shock Model

Consider a unit, which is subject to shocks occurring randomly in time. At time \( t = 0 \), the damage level of the unit is assumed to be 0. Upon occurrence of a shock, the unit suffers a non-negative random damage. Each damage, at the time of its occurrence, adds to the current damage level of the unit, and between shocks, the damage level stays constant. The unit fails when its accumulated damage first exceeds a specified level. Kijima and Nakagawa (1991) proposed a cumulative damage shock model with imperfect maintenance. The maintenance is imperfect in the sense that each maintenance reduces the damage level by \( 100(1- b)% \), \( 0 \leq b \leq 1 \), of total damage. Note that \( b = 1 \) means minimal maintenance and \( b = 0 \) means perfect maintenance. Examples of shock model application are found in Kijima and Nakagawa (1992) and Finkelstein (1997).

(6) Quasi-renewal Process

Hongzhou and Hoang (1996) treated imperfect maintenance in a way that, upon each maintenance, the lifetime of a system will be reduced to a fraction \( \alpha \) of its immediately
previous one where $0 \leq \alpha \leq 1$ such that the successive lifetimes are defined to constitute a decreasing quasi-renewal process with parameter $\alpha$. Examples of quasi-renewal method can be found in Pham and Wang (2001) and Yang and Lin (2005).

(7) Cost model

Ben-Daya (1999) proposed a cost based model where the level of maintenance is expressed by the ratio $\frac{C_{pm}}{C^0_{pm}}$, where $C_{pm}$ stands for the maintenance cost $C^0_{pm}$ stands for the perfect maintenance cost.

2.2.2 Maintenance Policy

Maintenance policy is completely determined in two steps; first, the policy structure is determined then, the policy parameters are chosen according to the objectives. Therefore, different maintenance policy structures are presented, and then the different methods for determining the policy parameters are presented.

(1) Age-dependent PM Policy

Under this policy, developed by Barlow and Hunter (1960), a unit is always preventively maintained at its age $T$ or failure, whichever occurs first, where $T$ is a constant. Later, as the concept of imperfect maintenance was established, various extensions and modifications of the age replacement policy have been proposed. Therefore, the PM at $T$ and the CM at failure might be minimal, imperfect, or perfect. If $T$ is a random variable, the policy is referred to as the random age dependent maintenance policy that is in force when it is impractical to maintain a unit in a strictly periodic fashion. Many policies in the literature lie under this category like age replacement policy (Barlow and Hunter, 1960), repair replacement policy (Block et al., 1993), (T-N) policy (Nakagawa, 1984), (T,t) policy (Sheu et al., 1993), and (T,n) policy (Sheu et al., 1995).
(2) Periodic PM Policy

In the periodic PM policy, a unit is preventively maintained at fixed time intervals $kT$ ($k = 1, 2, ...$) independent of the failure history of the unit, and repaired at intervening failures where $T$ is a constant. In some early research, the block replacement policy was examined in which a unit is replaced at pre-arranged times $kT$ and at its failures [Haijun (2005)]. The block replacement policy derives its name from the commonly employed practice of replacing a block or group of units in a system at prescribed times $kT$ independent of the failure history of the system and is often used for multi-unit systems. Examples of the policies that belong to this category are periodic replacement with minimal repair (Barlow and Hunter, 1960), overhaul and minimal repair policy (Xiao-Gao et al., 1995), $(T_0, T^*)$ policy (Nakagawa 1981, a and b), $(n, T)$ policy (Nakagawa, 1986)

(3) Failure Limit Policy

Under the failure limit policy, PM is performed only when the failure rate or other reliability indices of a unit reach a predetermined level and intervening failures are corrected by repairs. This PM policy makes a unit work at or above the minimum acceptable level of reliability. Examples of these policies are the cost policy (Lie and Chun, 1986) where the preventive maintenance is performed whenever the unit reaches a predetermined failure rate and intervening failures are corrected by minimal repairs. And the failure limit policy (Bergman, 1978) in which replacement (perfect maintenance) is performed based on measurement of some increasing state variable.

(4) Sequential PM Policy

Unlike the periodic PM policy, a unit is preventively maintained at unequal time intervals under the sequential PM policy. Usually, the time intervals become shorter and shorter as time passes, considering that most units need more frequent maintenance with increased ages. Some examples of these policies are found in Nguyen and Murthy (1981) and Nakagawa (1986, 1988)
(5) Repair Limit Policy
There are two general types of repair limit policies: repair cost limit policy and repair time limit policy. For the repair cost limit policy, when a unit fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit; otherwise, the unit is replaced (Kapur et al. 1989, Yun and Bai 1987).

The repair time limit policy is proposed by Nakagawa and Osaki (1974) in which a unit is repaired at failure: if the repair is not completed within a specified time T, it is replaced by a new one; otherwise the repaired unit is put into operation again, where T is called repair time limit.

(6) Repair Number Counting and Reference Time Policy
Repair number counting policy is introduced by Morimura and Makabe (1963a) where a unit is replaced at the kth failure and the first (k - 1) failures are removed by minimal repair. Upon replacement, the process repeats. Later, Morimura (1970) extends this policy by introducing another policy variable T, critical reference time, which is a positive number. Under this extended policy, all failures before the kth failure are corrected only with minimal repair. If the kth failure occurs before an accumulated operating time T, it is corrected by minimal repair and the next failure induces replacement. But if the kth failure occurs after T, it induces replacement of the unit.

(7) Group Maintenance Policy
The main idea of group maintenance policy is to group the system components/modules in categories of units such that each category is maintained together either upon the failure of one of the category elements or at predetermined time intervals, kT. Examples of group maintenance policies can be found in Gia-Shie (2008) and Shey-Huei and Jhy-Ping (1997).

(8) Opportunistic Maintenance Policies
Maintenance of a multi-component system differs from that of a single-unit system because there exist different dependence types between the components in the systems. It
may be economic, failure or structural. Therefore, when repairing or maintaining a component. It is an opportunity to maintain another dependent component. Different alternatives for opportunistic maintenance are found in Saranga (2004), Lirong and Haijun (2006), and Xiaojun et al. (2006).

A detailed description of the different types of maintenance policies can be found in Wang (2002). All the previously explained maintenance policies show that each policy has its own decision control parameters that completely determine it. Therefore, the final step in determining the maintenance policy, after choosing the maintenance policy, is finding the policy parameters. The literature includes many methods to be applied in this step; some of them are explained as follows:

**Semi-Markov processes (SMP)**

Semi-Markov processes (SMP) take a standard Markov process to another level by incorporating more parameters, namely the amount of time the equipment spends in a particular state (Tomasevicz and Asgarpoor, 2006). SMP is often used when equipment has a finite number of states and has specified holding times or sojourn times in each state. Although a standard Markov process is useful for the purpose of simplicity, Markov processes neglect the sojourn times of each state (Ge et al., 2007). This method is used by Chan and Downs (1978) to determine the parameters of age dependent preventive maintenance policy.

**Non-homogeneous Poisson Process (NHPP)**

A non-homogeneous Poisson process is a Poisson process with rate parameter $\lambda(t)$ that is a function of time. In maintenance context, the non-homogeneous Poisson process model (NHPP) represents the number of failures experienced up to time $t$ $\{N(t), t \geq 0\}$. The NHPP method has been applied by Shey-Huei and Griffith (1996) to determine the parameters of repair number counting policy and by Sheu and Chang (2002) to determine the parameters of the sequential maintenance policy and by Chien et al. (2006) to determining the parameters of generalized replacement policy with imperfect repairs.
**Stochastic process**

The stochastic process method main idea is finding the appropriate stochastic distribution that describes the system failure behavior. For example, Shaomin and Clements-Croome (2006) developed an extended Poisson process (EPP) probability distribution to describe bathtub failure shape. Then, they used it to derive the optimum parameters for corrective only maintenance policy and periodic maintenance policy.

**Quasi-renewal process methods**

Quasi-renewal process can be defined as follows: Let \( \{N(t), t \geq 0\} \) be a counting process and let \( X_n \) be the operating time between the \( (n-1)^{th} \) and \( n^{th} \) event of the process. Then, if \( \{X_1, X_2, \ldots, X_n\} \) observe a sequence of non-negative random variables such that \( X_1 = Z_1, X_2 = \alpha Z_2, X_3 = \alpha^2 Z_3 \) and \( Z_i \) are independent identically distributed random variables, then the \( \{N(t), t \geq 0\} \) is said to be quasi renewal process with parameter \( \alpha \) (Hongzhou and Hoang, 1996). The application of quasi renewal process in finding the maintenance policy parameters has been explained by Yang and Lin (2005) and Wang and Pham (2006a) used the quasi renewal process to find the optimum parameters for the maintenance policy of a series system.

There are some other techniques for determining the maintenance policy parameters in the literature like neural networks (Fontaine et al., 1996), goal programming (Oke and Ayomoh, 2005) and evolutionary algorithm (Samrout et al., 2007).

Therefore, the complete determination of maintenance policy can be explained as shown in Figure 2-4.
2.3 Maintenance Policy Objectives

As explained earlier, maintenance has a main rule and some other corresponding effects. To the author knowledge, there is no reported research in the literature that defines the relationship between maintenance policy and the corresponding system resetting. But the relationships between maintenance and the other effects (cost, quality and availability) have been studied.

2.3.1 Maintenance Effect on Cost

Optimal maintenance policies aim to provide maximum system performance at the lowest possible maintenance cost. Wang and Pham (2006c) defined two maintenance cost models being used in the literature:

- Assuming $d_a$ is the loss cost per unit of system down time because the system is not available, and $d_i$ is the maintenance cost per unit time which includes materials and labor costs, and can be estimated from historical repair data and $D(t)$ is the expected number of maintenances (corrective and predictive) till time $t$. then, the maintenance cost rate can be expressed as $\lim_{t \to \infty} (d_a + d_i) \frac{D(t)}{t}$ an example of this cost model application can be found in Zhao (1994)
- Assuming that it costs $d_{\text{min}}$ dollars to perform minimal maintenance and $d_{\text{max}}$ dollars to perform perfect maintenance, and assuming that both $d_{\text{min}}$ and $d_{\text{max}}$ include system loss cost, materials cost, and labor cost, then, the maintenance cost rate can be expressed as:

\[
\text{Cost Rate} = \lim_{t \to \infty} \left( N(t)/t + \left( d_{\text{max}} - d_{\text{min}} \right) \left[ N(t)/k \right] / t \right)
\]

Equation 2-2

where $N(t)$ expresses the total number of maintenances till time $t$ and $k$ represents the number of imperfect maintenances between replacements (perfect maintenance).

It is noted that both models ignore the effect of maintenance level on the different maintenance cost components.

2.3.2 Maintenance Effect on Availability

Repairable system availability is defined as $\frac{MTBF}{MTBF + MTTR}$ (Smith and Hinchcliffe, 2006). Therefore, it can be noticed that the maintenance policy affects all the terms involved in availability definition (Smith, 1992). But, the relation between maintenance and MTBF (mean time between failure) received much more attention in the literature than the relation between maintenance and MTTR (mean time to repair). For example, Amari (2006) developed the bounds of MTBF in terms of the applied periodic maintenance policy parameters as $\frac{T \cdot R(T)}{1 - R(T)} \leq MTBF \leq \frac{T}{1 - R(T)}$ and Mondro (2002) developed an approximate relation describing MTBF when a system has periodic maintenance as

\[
MTBF_{\text{approximate}} = \frac{T}{\log(R_{\text{approximate}}(T))}
\]

Equation 2-3

On the other hand, there is no detailed research in the literature relating the maintenance to MTTR except some researches that assumed correlation between the maintenance time and the next time to failure like Wang and Pham (2006b) and Goel et al. (1992) who
assumed that the time to failure of each component is correlated with the corresponding repair time and the correlation is modeled by a bi-variate distribution.

### 2.3.3 Maintenance Effect on Quality

The relationship between Maintenance and quality has been addressed in many literature like Ben-Daya and Duffuaa (1995), Aurich et al. (2006), Chen et al. (2006), Madu (1999), Ollila and Malmipuro (1999), Yong et al. (2001), and Yong and Jionghua (2005). Nevertheless, this relation has not been adequately investigated in the literature and there are no adequate models relating them as explained by Ben-Daya and Rahim (2000) and Cassady et al. (2000). Some of the developed models in the literature are:

- Chen et al. (2006) used the response model and considered that the quality measure Y as function of the process variables \((X, z)\) Where X is the vector of adjustable process variables. And z is the noise process variables. The maintenance affects the values of the adjustable process variables and consequently affects the quality characteristic.

- Wang et al. (1996b) assigned a variable \(q\) \((0 < q < 1)\) that represents the machine's condition. When \(q = 1\), the machine is in perfect condition and no defective products will be made. When \(q = 0\), the machine is in the breakdown condition and makes only defective products. For \(0 < q < 1\), it makes a good product with probability \(q\) and a defective one with probability \((1 - q)\), which is called the quality level. Furthermore, the value of the variable \(q\) decreases as more products are made. Then, they tried to find the relationship between \(q\) and the number of produced products since last machine perfect maintenance.

- Aurich et al. (2006) presented a data-based system for quality oriented productive maintenance (QPM). The QPM directly connects tool and machine conditions with relevant product quality parameter via cause and effect coherences within the whole manufacturing process chain.
2.4 Complexity

Complexity can be simply described as the difficulty in dealing with the system under consideration. The previous sections showed that the main maintenance role is to reset the system in order to reduce the operation complexity. Therefore, this section reviews the different definitions and classifications of complexity and how it is measured and its effect on maintenance.

Many complexity discussions are centered on this basic notion of difficulty and efforts are focused on characterizing and quantifying this difficulty. There is no consensus on a single definition of complexity so far but three main types of complexity can be distinguished in the literature (Lee, 2003):

- **Probabilistic**: The central idea of this approach is that the more disordered a system is, the more information is needed to describe it and thus the system is more complex (Gell-Mann and Lloyd, 1996).

- **Algorithmic (Kolmogorov complexity)**: is defined for a string of symbols, $x$, as the length of the shortest program that instructs a computer to produce output $x$ and then halt (Becher and Figueira, 2005).

- **Computational**: is the amount of time, memory, or other resources required for solving a computational problem with respect to the size of a problem (Goldreich et al., 1998).

In the current research, the main focus would be on the first complexity type, probabilistic complexity, since it is the most relevant one to our industrial engineering research scope. In this category, the most recent and widely accepted complexity definition, in the functional domain, is presented by Suh (2005b) in the axiomatic design and complexity theory. Suh assumed that the design world consists of four domains; customer, functional, physical and process, and the design process is inter-mapping between these domains. The main focus in the current research is on the functional domain where the system functions are described by independent functional requirements (FRs). Lee (2003) defined the complexity as the difficulty in achieving the functional requirements as a consequence of the uncertainty of satisfying them.” The complexity is
an abstract notion and cannot be measured. But, assuming that the complexity is directly proportional to the uncertainty, the uncertainty can be used as a measure for the complexity (Lee, 2003). According to this definition, the complexity is categorized according to two main criteria; time dependent and time independent.

2.4.1 Complexity Categorization

Figure 2-5 explains the categorization of complexity accepted from the axiomatic design and complexity theory (Suh 2005 a, b). These different types of complexity are explained in the following sections:

Figure 2-5 Complexity categorization in axiomatic design and complexity theory

2.4.2 Real Complexity

The real complexity is that part of complexity that cannot be avoided or eliminated after the design process because it exists inherently in the design. It is expressed by the amount of uncertainty that the value of FR will lie within the design range. This probability is represented by the shaded area in Figure 2-6. Therefore, if this event is denoted by \( i \), then:

Real complexity \( C_R = \) uncertainty of event \( i \)

\[
= \log_2 1 / P_i = - \log_2 (\text{common range area})
\]

\[
= \int f(FR) dFR
\]

Equation 2-4
Where the design range represents the limits of FR that was predetermined by the designer, while the system range is the real probability distribution function of the FR.

2.4.3 Imaginary Complexity

The imaginary complexity, on the other hand, is not really inherent in the design. But it is an added complexity due to uncertainty resulting from lack of knowledge or understanding (Suh, 2005b). Lee (2003) defined three sources of ignorance or lack of understanding:

- ignorance of FRs which is related to the failure to properly understand and define the design task
- lack of knowledge required to synthesize or identify proper DPs (design parameters)
- Ignorance of the design matrix structure, which causes iterations in the design process to reach a set of target values of response (Suh, 2001).

Imaginary complexity is defined by the following relationship (Lee, 2003):

\[
\text{Imaginary complexity } C_I = \log_2 \frac{1}{P(\text{Selecting a correct sequence})}
\]

Equation 2-5

Where \( P(\text{selecting a correct sequence}) \) represents the probability that the right design sequence (determined by the design matrix) is chosen. Therefore, CI depends on the amount of existing knowledge about the design matrix. CI ranges from 0 (complete
knowledge) to $\log_2 \frac{n!}{z}$ (complete lack of knowledge), where $z$ is the number of valid sequences and $n$ is the total number of functional requirements

### 2.4.4 Combinatorial Complexity

Suh (2005a) defined the combinatorial complexity as the complexity that indefinitely increases as a function of time due to a continued expansion in the number of possible combinations, which may eventually lead to chaotic state or a system failure. Therefore, Suh (2005a) recommended interrupting this indefinite complexity increase by introducing re-initialization or resetting of functional requirements. In the long run, as the FRs are being reset after each period of working time; this means that the system has a functional periodicity characteristic. Otherwise, the system would not be stable (Suh, 2004). This requirement led to the definition of the periodic complexity.

### 2.4.5 Periodic Complexity

Lee (2003) defined the Periodic complexity as the type of uncertainty that stops increasing at some point and returns to initial (or near initial) acceptable level of uncertainty. Suh (2005a) indicated that this point is a functional point at which, the functional requirements reach a certain value. This concept is explained in Figure 2-7. This definition of periodic complexity leads to the definition of the periodicity term, which expresses the periodic change.
2.5 Periodicity

The periodicity in the axiomatic design context has not been defined explicitly. But, it has been explained using many examples like Suh (2001, 2005a, 2005b), Lee (2003), and Takata et al. (2004). These examples show that the periodicity is a feature in the system that enables it to re-initialize its FRs as they reach certain values or at periodic intervals. This periodicity feature may exist in the system naturally like the biological cells (Lee, 2003) or it can be introduced into the system through maintenance, which resets the system functionality (Suh 2001, 2005a). Suh (2005a) explained that the functional periodicity can be introduced into systems by various means according to the system. The relationship between complexity and periodicity in the axiomatic design and complexity theory is very essential as Lee (2003) explained that introducing periodicity leads to more predictability, which is a prerequisite for reducing the uncertainty and hence reducing the complexity. Therefore, the solution for reducing system complexity is introducing more periodicity. Although this relationship is not defined mathematically, all the real life examples stated in Lee (2003) and Suh (2005a, b) prove it. Since the main motivation of this research is to reduce system complexity, and given the periodicity-complexity
relationship, then this objective can be achieved by introducing more periodicity. This method of approaching the problem avoids the ambiguity of complexity mathematical definition (because it will not be used) as the main concern will be introducing more periodicity instead of measuring the complexity. The main challenge in applying this approach is that no measure for the periodicity exists in the literature.

2.6 Literature Review Outcome

This literature survey indicates the following gaps:

- The first step in maintenance policy design is selecting the maintenance policy. There are many policies developed in the literature but there is no systematic methodology developed to guide the decision maker how to select the most appropriate one for the considered system.

- The main role of maintenance, system resetting, is not incorporated nor considered in the maintenance policy design process by any mean. All current maintenance literature just considers one or many of the performance factors to optimize the maintenance policy parameters according to it/them.

- The system ability to being reset is characterized by the periodicity feature which was developed within the context of axiomatic design and complexity theory. Yet, there is no metric defined in the literature to measure the amount of periodicity.

In the current research, the last two gaps would be covered. The periodic preventive maintenance policy structure will be considered throughout the research since it is the most popular one that exists in many real life applications. The first gap will be considered in the future research.
3 Functional Failure from Complexity Perspective

3.1 Introduction

Many maintenance activities are triggered by failure events. This failure is normally interpreted as physical failure, which is easily visible. But, in many cases, the system fails to perform its intended function without a visible physical failure. In the current research, a complexity based functional failure definition is developed.

Generally, any component/system has two main modes of failure, sudden and gradual. In the sudden mode, a system switches from operating state directly to the failure state. But in the gradual mode, the system passes by many in-between states before failure. Nevertheless, most of the reliability and maintenance related research use a failure rate model, which is based on the two states assumptions. This assumption neglects the actual system failure nature, which leads to ineffective maintenance strategies.

3.1.1 Failure Literature Review

The term “failure” is widely used in daily life and in the branch of reliability and maintainability engineering. From a manufacturing perspective, machine failure is the trigger for corrective maintenance. Therefore, it is extremely important to detect failures as, or even before, they occur. Thus, modern manufacturing systems need reliable failure detection mechanisms. This fact has been emphasized by including the diagnosis ability as one of the key characteristics of new types of manufacturing systems such as flexible or reconfigurable manufacturing systems (Koren et al., 1999). The effect of the used failure detection mechanism on the system performance depends on the adopted failure definition (Fashandi and Umberg, 2003): “A common element that is vastly ignored but is rather critical to a sound reliability specification is the definition of equipment failure.
Even the most vigorous reliability testing program is of little use if the equipment being tested has poorly defined failure parameters.

There are physical and operational approaches for failure definition found in both academic literature and industrial practice. The physical approach in defining failure has been widely accepted, where failure is defined as "an undesirable and unplanned change in an object attribute or structure" (Umeda et al., 1994a). Therefore, failure is synonymous with breakdown (Hajji et al., 2004). The breakdown is characterized by a physical change in any of the modules or the parameters such that the system is totally unable to continue performing its function. A breakdown of any of the machine tool modules (heads, controls, etc.) is an example of this failure type.

The second approach in defining failure is based on the system operation. Fashandi and Umberg (2003) defined failure as: "Any unplanned interruption or variance from the specifications of equipment operation" An example of the application of this failure definition is used in the quality control charts where it is indicated that a system is in need for repair if the process carried out by that system is out of control (Jensen et al., 2006). Some researches consider operational failure as a symptom of physical failure, such as Umeda et al. (1994a), who defined a failure symptom as the function that has not been performed due to a failure.

Physical failures normally lead to operational/functional failures; however, the reverse is not necessarily true. Operational failure can happen without being preceded by physical failure. For example, a cutting tool breakage (physical failure) would certainly lead to machine functional failure, while deterioration of machining precision to a level below specifications (functional failure) can happen without any physical failure in the machine or with the tool.

This concept of functionality versus physical state has been considered by Umeda et al. (1994b). They developed a new concept of maintenance where maintaining the system functionality is emphasized instead of its physical state. The main idea is to keep the system working as long as its functional requirements can be satisfactorily satisfied even if one of the components breaks down. This approach to maintenance allows the system
to continue functioning even with failed modules, which improves the system fault
tolerance. Based on this concept of maintaining system functionality, Umeda et al. (1992)
and (1995) developed the Self-Maintenance Machine (SMM) that keeps performing its
basic functions even during periods of physical failure, which prevents the occurrence of
hard failures. They achieved SMMs through control algorithms or functional redundancy.
For control type SMM, repairs are accomplished by controlling parameters without
changing or re-organizing the machine structure. While for functional redundancy
SMMs, the potential functions of machine modules are used in a slightly different way
from the original design in order to perform the function(s) of the failed part(s).

It is clear from this discussion that functional failure of any module is the triggering event
for either functional delegation or control action. Nevertheless, a precise definition of the
functional failure is still needed.

The previous review shows that there is no unified and precise definition of physical and
operational failure. This ambiguity about failure may lead to ineffective fault detection
and hence loss of production capacity.

### 3.2 Failure Definition

Moubray (1997) defined the functional failure as the inability of any asset to fulfill a
function to a standard of performance, which is acceptable to the user. A similar
definition is developed by Grall et al. (2006). They assumed that the deterioration
condition of any device can be modeled by a stochastic ageing process such that when the
system is new, the ageing variable equals zero and when the ageing variable reaches a
predetermined level, called failure level (L), the system is deemed to have failed. This
model is shown in Figure 3-1, where the vertical axis represents the system state variable
and the horizontal axis represents time. The dots represent the system state variable at
different time instances and show that the system state variable increases with time till it
reaches the failure threshold (L). In this model, failure is defined precisely by a threshold
level of a system state variable beyond which, the system is considered failed even if it is
still working.
Figure 3-1 Failure threshold definition

However, Grall et al. (2006) did not specify the system state variable on which the failure threshold should be based on. Hence, their model is not considered complete. On the other hand, the definition of Moubray (1997) is limited to one function while most systems in real life are required to fulfill many functional requirements. The choice of a suitable system state variable is a core issue in failure definition. From the literature survey, it can be concluded that both types of failures, physical and functional, lead to the same result, which is loss of system functionality. Therefore, system functionality is a suitable system state variable for defining failure.

The concept of system functionality is modeled in the Axiomatic Design and Complexity theory, introduced by Suh (2001). The design world is assumed to consist of four domains; customer, functional, physical and process domains, such that the design process interplays between those domains and the design is described in each domain by certain parameters. They are respectively customer wants, functional requirements, design parameters, and process parameters whereas system functionality is described by the Function Requirements (FRs). Suh (2005b) defined the information content of the system as:
\[ I_{sys} = -\sum_{i=1}^{m} \log_2 P_i \]  

Equation 3-1

where:

\[ I_{sys} \]  
information content of the system

\[ P_i \]  
probability that FRi is satisfied

\[ m \]  
number of FRs

Therefore, the information content is a direct measure of the uncertainty of satisfying the function requirements. This uncertainty is therefore a measure for the system complexity (Lee, 2003). The complexity is categorized according to two perspectives; real (time independent) and temporal (time dependent) behavior. From the time independent perspective, complexity may be real, which is defined by the equation of information content, or imaginary which arises due to lack of information about the design. From the time dependent perspective, complexity may be combinatorial or periodic.

The complexity is represented graphically by the intersection between the design range (the accepted range of FR) and the system range, defined by the actual FR probability density function, as shown by the shaded area in Figure 2-6 (Suh, 1998).

Complexity is expressed mathematically by the Equation 2-4. Accordingly, as complexity increases, the uncertainty of satisfying the functional requirements also increases. Thus, complexity is a measure of system functionality. Therefore, it is proposed to use the complexity as the system state variable in the failure model and consequently, the definition of functional failure can be stated as: "The system fails to perform its intended function(s) when its Complexity reaches a predetermined threshold level F". The determination of the failure threshold level F is a strategic management decision. There are many factors to be traded off in this decision including cost, product quality, and system availability. This definition is shown in Figure 3-2, which shows the complexity change for a typical system as it increases with time.
As long as the complexity is less than the failure threshold, the system is considered functional and good for operation. As the complexity exceeds the failure threshold (the shaded region), the system is deemed to have failed.

The application of the proposed failure definition can be explained using the example presented by ElMaraghy et al. (2005). Assume that the functional requirement of a machine is to satisfy a specified production demand. Hence, the design range lies between the two extremes of the expected demand. When the machine is new, the availability distribution lies completely within the functional design range, hence the demand would certainly be satisfied and the complexity would be zero. As the machine ages, the failure rate increases, the availability distribution shifts away from the design range and the certainty of fulfilling the demand decreases and, hence, the complexity increases. Assuming the minimum acceptable demand satisfaction certainty is 90%, then, the failure threshold = \(-\log_2 0.9 = 0.152\). Therefore, when the availability complexity reaches 0.152, the machine is considered to be functionally in a failure state even if it still working.

Although the developed failure model relies on an uncertainty-based complexity measure, the model can also be applied to other complexity definitions. For example, ElMaraghy and Urbanic (2004) defined the main factors affecting operational complexity as:

(i) The number and diversity of features to be manufactured, assembled or tested
(ii) The number, type and effort of the tasks to produce the features. They derived the following relations for process complexity factor:

\[
P_{cd} = \frac{\sum_{k=1}^{K} e_{f_k}}{K}
\]

Equation 3-2

where:

\( K \) Number of process tasks

\( e_{f_k} \) Effort factor \( e_f \) for the \( k \)\textsuperscript{th} process task

such that:

\[
e_f = \frac{P_N \cdot P_D + C_N \cdot C_D}{P_N + C_N}
\]

Equation 3-3

where:

\( P_N \) quantity of physical tasks

\( C_N \) quantity of cognitive tasks

\( P_D \) physical effort factor

\( C_D \) cognitive effort factor

In this case, the process complexity factor can be considered as the system parameter to use for defining the system functional failure. This concept applies to both manual and automated systems. In the case of manufacturing system for example, in manual processes, as the machine ages and its functionality deteriorates, the physical and cognitive effort required by the worker increases in order to maintain the production quality and volume. Therefore, \( P_D(t) \) and \( C_D(t) \) are increasing functions in time and so the process complexity factor will be. In this case, a complexity factor threshold can be defined such that when it is exceeded by the required effort, the machine should be repaired. For automated processes, ideally, there should be no or minimal human
involvement. However, as the machine ages and its functionality deteriorate, there would be a need for human interference to maintain acceptable machine functionality. Hence, a threshold of human interference level can be defined such that the machine would need repair if the required human interference overpasses this predefined threshold.

3.3 Failure Forms

Two failure forms have been identified in the literature; sudden and gradual. Blache and Shrivastava (1994) illustrated failure categorization as they studied a foam spray machine in a car assembly plant. Machine failure was categorized as:

1. Catastrophic failure, which causes an immediate inability of a system to achieve its function.

2. Performance degradation failures.

Typically, sudden failure occurs randomly and its time of occurrence is modeled by an exponential distribution, where the mean of which denotes the failure rate (Ebling, 1997). The basic assumption in this model is that the system has two discrete states; operation and failure (Kenne and Nkeungoue, 2008). This assumption applies to sudden failure but it is inapplicable to gradual failure where the system gradually experiences many states between operation and failure. Therefore, it is suggested to model the gradual failure by a performance parameter whose value at any time represents the system functional status. Since the complexity as explained is a measure of system functionality, it is proposed to model the gradual failure by the rate of complexity change, which is named “Complication Rate”. This metric quantifies the item / system functionality deterioration per unit time. Assume that the complexity at time t is denoted as \( C(t) \), then

\[
\text{complication rate, } \nu(t) = \frac{dC(t)}{dt}
\]

Equation 3-4

Hence, failure occurs when \( C(t) = \int_{0}^{t} \nu(t) \, dt = F \)
The simplest form of complexity change is linear and it is used in the current research, although other non-linear forms can be investigated similarly. The linear complexity change is shown in Figure 3-3:

![Figure 3-3 Linear gradual failure](image)

Assuming that complexity is zero when the system is new, then

\[ T = \frac{F}{\nu} \]  

Equation 3-5

where \( T \) denotes the time duration till the onset of gradual failure. Hence, the gradual failure rate would be expressed as follows:

\[
\text{Gradual Failure Rate} = \frac{1}{T} = \frac{\nu}{F}
\]  

Equation 3-6

Therefore, the total system failure rate is a function of both sudden failure rate (\( \lambda \)), the complication rate (\( \nu \)) and the failure threshold (\( F \)). The total failure rate is not expected to be simply the summation of the sudden and gradual failure rates because in most real cases these two failure modes are dependent. This dependency is due to the effect of system functional state on the sudden failure rate (\( \lambda \)). Therefore; the total failure rate \( \lambda_T \) would generally be expressed by the following relationship:
The exact relationship is case-specific and its determination requires a lot of historical failure and performance data.

This proposed new failure rate relationship captures all failure modes and therefore, is more realistic than the traditional definition of failure rate.

The complexity changes in sudden and gradual failures are illustrated in Figure 3-4 and Figure 3-5 respectively. Figure 3-4 shows the case of sudden failure where complexity increases gradually with time till a random failure suddenly occurs. This causes a significant complexity increase to surpass the failure threshold. This type of failure is well modeled by the rate of failure occurrence or simply the failure rate ($\lambda$).

![Figure 3-4 Complexity change due to sudden failure](image)

Figure 3-5 explains the case of gradual failure where the complexity gradually increases until it surpasses the failure threshold, which causes system functional failure.
3.4 Case Study

Ott et al. (2005) introduced a case study of producing an “air-receiver magnetic assembly” Samples of size 5 were taken from the production line every shift. The depths of cut of 25 samples were collected. According to the customer wants analysis, it was determined that the producing machine has one functional requirement, which is low depth cutting deviation with a deviation design range of [-1, +1] mm.

Traditionally, such a problem is analyzed using quality control charts like $\bar{X}$ and $R$ control charts. But, in the current research, the application of complexity-based approach will be explained.

From samples data (Shown in Appendix A), the mean and standard deviation at each sampling point can be determined as follows:

Sample $k$ mean:  
\[
\bar{X}_k = \frac{\sum_{i=1}^{n} X_{ik}}{n}, \quad k = 1, \ldots, 25
\]  
Equation 3-8

Sample $k$ Standard Deviation:
Where \( n \) is the sample size (5). Assuming that the samples are drawn from a normally distributed population and since the sample size is relatively small (5), the samples readings follow the Student distribution. For the sake of representing the change of system range with time, it will be represented at each sample point by a line segment from \( \overline{X} - 3S \) to \( \overline{X} + 3S \) as shown by the vertical thick lines in Figure 3-6. The design range is represented in Figure 3-6 by the shaded area in the deviation range \([-1, 1]\).

Figure 3-6 illustrates the system range changes with time relative to the design range. Therefore, it is much more informative than traditional control charts because it completely illustrates the change in system range distribution with time. It shows any changes in either distribution mean or dispersion, which helps the decision maker to understand the change in machine/ process functionality and whether it is due to mean.
width dispersion or both. Using the results of system range calculations, the machine complexity at each sampling point can be calculated using the following steps:

Step 1. calculate t values of upper and lower design range limits:

\[ t_{u_i} = \frac{1 - \bar{X}_i}{S_i} \quad i = 1, \ldots, 25 \tag{Equation 3-10} \]

\[ t_{l_i} = \frac{-1 - \bar{X}_i}{S_i} \quad i = 1, \ldots, 25 \]

Step 2: calculate the probability associated with the design range:

\[ P_i = F(t_{u_i}) - F(t_{l_i}) \tag{Equation 3-11} \]

where \( F(t) \) is the student t distribution cumulative function

Step 3: calculate the machine functional complexity as follows:

\[ C_i = -\log_2 P_i \tag{Equation 3-12} \]

The results of these calculations are shown in Figure 3-7. A linear regression analysis is performed to construct a complexity trend line as shown by the straight line in Figure 3-7 with coefficient of determination, \( R^2 \), of 0.2579 (this low value of \( R^2 \) can be attributed to the high dispersion of data as exhibited in the figure). The regression analysis indicates that the complexity can be modeled by the following equation:

\[ C_n = 0.01n + 0.023 \tag{Equation 3-13} \]

where:

- \( n \) the sample number (indication of sample time)
- \( C_n \) complexity at the time of sample \( n \)
Since the samples are drawn from the production line at the beginning of each shift, then, the complication rate of this machine is 0.01 per shift. Therefore, assuming the machine failure threshold is set to be 0.3, then, the machine complexity is expected to exceed the pre-defined threshold at \( \frac{0.3 - 0.023}{0.0103} \approx 26.9 \) shifts. Therefore, a preventive maintenance should be planned before that time. Therefore, if this machine has a multi levels maintenance strategy, the duration between any two successive preventive maintenances should be less than 26 shifts. If the machine is in a plant that operates 2 shifts per day, 5 days per week, then the least preventive maintenance frequency is every 13 days \( \approx 2.6 \) week.

### 3.5 Summary and Conclusions

A new failure definition has been presented and modeled based on the complexity theory. Its main advantages are that it is mathematically defined and that it is applicable to both types of failures; functional and physical. The proposed model uses the system complexity as a measure of functionality and determines a failure threshold for complexity. This threshold is problem-specific determined by experienced decision makers and represents a trade-off between cost, quality and availability.
The "complication rate" term is introduced to measure system functionality deterioration and gradual failure. It represents the rate of change of complexity. The complication rate combined with the failure rate completely defines the system failure behavior.

This new approach for failure modeling captures and reveals the behavior of selected system functionalities. It can be used to enhance preventive maintenance planning in order to keep desired system functionalities above a certain pre-determined level/threshold.

The mathematical formulation of the proposed novel complexity-based functional failure metric utilizes readily available data and it is easy to use by managers and maintenance planners.
4  A Periodicity-Based Metric for Assessing Maintenance Strategies

4.1 Introduction

As explained earlier, the main role of the maintenance is to reset the system to keep it in an acceptable condition and keeps it functioning within the designed range. Therefore, the maintenance function introduces periodicity into the system. This periodicity is not quantified yet in the literature. But it is just expressed as a condition to keep system stability. In order to effectively evaluate maintenance strategies, it is necessary to have a metric for the amount of periodicity introduced by that strategy.

In this chapter, the traditional performance criteria used in the literature to evaluate maintenance strategies will be discussed and the lack of effective criteria to measure the capability of maintenance strategy to restore the system functionality will be highlighted. Then, the periodicity concept is explained from the point of view of the complexity theory. And since the developed periodicity metric would be applied to maintenance strategies, a new approach is developed and explained to model any multi-class age-dependent maintenance policy. A novel metric for periodicity is then proposed and mathematically derived for a single resetting plan. The case of independent multi resetting plans is then investigated and applied to a maintenance case.

The application of the new periodicity metric is explained using a maintenance case study from the auto manufacturing.

4.2 Maintenance Strategies Evaluation Criteria

Maintenance systems have to provide the required reliability, availability, efficiency and capability (Dekker, 1996). This research focuses on the administrative maintenance actions at the policy level rather than its detailed technical aspects at the component level.
Therefore, the proposed approaches and metric resulting from this research are not restricted to certain type of systems or industry.

There are numerous maintenance strategies introduced in the literature as explained in the literature survey. This numerous and diverse maintenance policies/strategies need a reliable evaluation method to compare their effectiveness. Different criteria have been used in the literature to assess maintenance strategies, which are summarized as follows:

- Cost: the cost of implementing a given maintenance policy or the cost of production when a maintenance strategy is applied have drawn the largest attention as an evaluation criteria in order to design maintenance strategies with minimum cost (Morel et al. 2002, Moore and Starr 2006, and Amari et al. 2006).

  Availability: the adoption of availability as an evaluation criterion for maintenance strategies is appropriate when considering maintenance of many systems like manufacturing systems because the system availability is a direct measure of its ability to fully utilize its present capacity and meet the required production rate and due dates. Therefore, the maintenance strategy would be designed to minimize the total system downtime resulting from failure, inspection, repair or regular preventive maintenance action (Ceschini and Saccardi 2002, Naikan and Rao 2005, and Wang and Pham 2006b).

- Reliability: the maintenance strategy in the reliability centered maintenance approach focuses on maximizing the reliability of the most important functions of the system, and avoiding or removing maintenance actions that are not strictly necessary for enhancing the system reliability (Rausand, 1998).

- Quality: the importance of the resulting product quality as an evaluation criterion for maintenance strategy has been emphasized by many researchers like Ben-Daya and Duffuaa (1995). Nevertheless, the developed models in this area are very scarce and most of the work uses the quality inspection data for maintenance planning (Ben-Daya, 1999).

This brief overview of the existing maintenance evaluation methods and criteria highlights the need for a new criterion to evaluate the main role of maintenance strategies, which defines the required and sufficient frequency and extent of the maintenance actions.
4.3 Maintenance Strategies and Periodic System Complexity

The main task of maintenance is to periodically reset the system either by repairing failures or by preventive maintenance. This resetting should be defined in terms of specific parameters that may be related to functionality such as production rate or available capacity, or physical parameters such as machine tool power efficiency. The notion of periodic resetting in the functional domain has been defined by Suh (2005a) and Suh (2004) as a mechanism to reduce complexity and restore the desired state of operation. The role of periodicity in transforming combinatorial complexity into periodic complexity has been explained in the literature survey and the information content and is defined as explained by Equation 3-1.

The existence of ‘periodicity’ causes the deteriorating specified functions to exhibit a cyclic behavior that restores their desired characteristics periodically. Therefore, periodicity re-initializes the system functionality to a “like new” state, which assures a high degree of functional certainty. Hence, introducing periodicity reduces, if not eliminates, uncertainty and consequently decreases the complexity associated with combinatorial complexity. Furthermore, a system with an infinite time-dependent combinatorial complexity cannot be sustained because the uncertainty associated with its future events would be very large and the system becomes risky and unreliable due to its uncertain chaotic performance. Lee (2003) introduced many examples of periodicity in systems from different fields including manufacturing systems. The deterioration of manufacturing systems performance is characterized by a time varying system range and may be considered time dependent complexity. Hence, carrying out maintenance actions would serve to re-set the system performance characteristics.

The machine available capacity will be used in this research to illustrate the maintenance role in machine functional resetting. When the machine is new, it has certain failure rate, which influences its available capacity. In a manufacturing system, the production schedules and parts/machines assignment is planned to satisfy the production demands based on the availability of such resources. Availability ($\text{Av}$) is defined by Naikan and Rao (2005):
Assuming that the machine failure rate increases with time (Chan and Shaw, 1993b), then the downtime increases, which leads to decreased availability and hence reduced probability of meeting the demand. In order to ensure meeting the demand; preventive maintenance should then be carried out to reset the machine failure rate and consequently the machine available capacity to some value near the new state. This cycle of deterioration and resetting is shown in Figure 4-1.

It is clear from this discussion that periodicity is important for the long-term system functionality.

However, how often a system should be reset? to what extent the design parameters should be re-set? What is a desirable level of periodicity? and at what cost? Remain unanswered questions. Another important question is how much periodicity does exist given a certain maintenance regime and how much periodicity is needed to achieve the desired
functionality goals? Therefore, a metric to quantify the amount of periodicity is needed to help design new and effective maintenance strategies and evaluate existing ones.

### 4.4 Maintenance Modeling

In order to develop a periodicity metric for evaluating the maintenance strategies, it is necessary to define a model of the maintenance strategies. The developed policies are currently described in a textual non-mathematical manner. Therefore, there is a need to develop a standard mathematical methodology for modeling maintenance strategies.

The time-based maintenance strategies are the main focus of the current research. They are currently the most commonly used in industry due to their ease of scheduling and integration with the production schedules, and the simplicity of their application because they do not require the use of sophisticated condition monitoring technologies. The developed maintenance policies, reported in the literature, and the maintenance strategies applied in industry, indicate that there are two sub-strategies for any maintenance policy:

- The failure repair sub-strategy describes when to repair the failure and the level of repair.
- The preventive maintenance sub-strategy describes the number of preventive maintenance classes and their levels. Where maintenance class represents certain preventive maintenance hierarchy level like monthly maintenance or bi-weekly, etc.

Therefore, the whole maintenance strategy can be fully determined by defining the five criteria shown in Table 4-1. The first two criteria determine the failure repair sub-strategy and the last three determine the preventive maintenance sub-strategy. Most of the developed and applied maintenance strategies agree that the failure should be repaired when it happens (assuming that a perfect failure detection system is in place). Therefore, this criterion can be excluded from the model as it does not define a specific maintenance policy parameter. The maintenance strategy is then defined by the remaining four parameters as shown in Table 4-1.

In this model, the repair level/preventive maintenance level is represented by a continuous real variable in [0 - 1] range. It represents the imperfection level of the
repair/maintenance course of action, where 0 means restoring the system to its state just before failure and 1 means restoring it to the original new state (for further explanation of imperfect maintenance, refer to Pham and Wang (1996)). Both extremes are theoretical. In reality, the adopted repair/maintenance level is normally somewhere in-between.

Table 4-1 Maintenance strategy parameters

<table>
<thead>
<tr>
<th>Maintenance Strategies Sub-categories</th>
<th>Defining Criterion</th>
<th>Maintenance Policy Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure Repair</td>
<td>When to repair a failure?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Repair Level</td>
<td>RL</td>
</tr>
<tr>
<td>Preventive Maintenance</td>
<td>Number of preventive maintenance classes</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Frequency of carrying out each preventive maintenance class</td>
<td>PMF1,..., ,...,PMFN</td>
</tr>
<tr>
<td></td>
<td>Level of each preventive maintenance class</td>
<td>PML1,..., ,...,PMLN</td>
</tr>
</tbody>
</table>

The proposed maintenance modeling approach agrees with the statement made by Anderson (2007) that the "Preventive Maintenance has two features, an activity to be performed, and a frequency at which the activity is performed". However, the proposed model is more generic because it models the whole maintenance strategy including the failure repair, and considers the cases of multi-preventive maintenance classes, which makes it more realistic.

It is important to notice that the maintenance strategy parameters may remain constant throughout the system life such that the maintenance strategy does not change with time. Alternatively, the maintenance strategy may be dynamic where its parameters change with time. An example of a dynamic maintenance strategy is the sequential preventive maintenance policy (Kim et al., 2007) where the preventive maintenance frequency increases with system age to overcome the faster deterioration.
4.5 Periodicity Modeling

The periodicity is a result of a resetting plan. Each resetting action re-initializes the system functionality, and if the resetting actions are recurring according to a certain pattern (plan), then the plan introduces periodicity into the system. A system here may mean a single unit or a whole system but the main focus in the current research will be on the case of a single unit, therefore, the resetting plan means the maintenance policy. The two words unit and system will be used interchangeably in the remainder of this article. First, the case of a single resetting plan is introduced and the formula for the resulting periodicity is developed. Then, the periodicity resulting from multiple resetting plans is investigated.

4.5.1 Periodicity of Single Resetting Plan

A periodicity has two essential dimensions that completely define it; frequency of resetting or time between resetting and extent of resetting which expresses the level of re-initialization. These two aspects are explained by Figure 4-2.

Figure 4-2 (a) represents the reference resetting policy with time between resetting $T$ and full resetting. Figure 4-2 (b) explains the effect of decreasing the resetting frequency by increasing the time between resetting to $2T$ while keeping the resetting level (full resetting). The resetting policy represented by Figure 4-2 (b) has less periodicity than the one represented by Figure 4-2 (a), which leads to a noticeable increase in the average complexity level. The effect of resetting level is demonstrated in Figure 4-2 (c) where different resetting levels are shown while keeping the time between resetting at $T$. The resetting level is shown in the figure by the ratio $\frac{a}{L}$ where $L$ is the total system complexity before the resetting process and $a$ is the amount of complexity actually decreased by the resetting process. Therefore, the resetting policy represented by Figure 4-2 (c) has less periodicity than the one represented by Figure 4-2 (a) which causes an increase in the average complexity level.
Complexity

(a) Full resetting at time intervals $T$

(b) Full resting at time intervals $2T$

(c) Resetting at time intervals $T$ with different resetting levels

Figure 4-2 Effect of Periodicity aspects on complexity

The resetting frequency represents the number of resettings per unit time. It assumes real values in the range $[0, \infty]$ where 0 means no resetting at all and $\infty$ means system resetting at infinitesimal time intervals. The resetting extent is a mean of quantifying the amount of resetting and it can be expressed by the following relationship:
Resetting Extent = \frac{\text{amount of resetting}}{\text{amount of full re-initialization}} \quad \text{Equation 4-2}

Where the amount of resetting for any functional parameter (such as production rate, availability,...etc) is defined as the difference between the parameter's current value and its value after resetting. For example, consider the availability functional requirement as shown in Figure 4-1. The resetting amount is the difference between the availability after and the availability before resetting. Furthermore, to make the resetting extent more generic and dimensionless, it is expressed in terms of the uncertainty of fulfilling the functional requirement, which represents the complexity (Suh, 2005a). Therefore, assuming the complexity of any new system to be zero (i.e. designed system fulfills the functional requirement), then the resetting extent is expressed as:

\text{Resetting Extent} = \frac{\text{complexity before resetting} - \text{complexity after resetting}}{\text{complexity before resetting}} = 1 - \frac{\text{complexity after resetting}}{\text{complexity before resetting}} = \frac{a}{L} \quad \text{Equation 4-3}

For simplicity, it is assumed that the complexity increases linearly with rate \( v \). The complexity change in the presence of a resetting plan with time between resetting \( T \) and resetting extent \( \chi \) is shown in Figure 4-3.
When there is no periodicity (pr = 0, combinatorial complexity case), the complexity continues to increase without resetting as represented by the dashed line (line of zero periodicity). The other theoretical extreme is when the system is fully reset at infinitesimal time periods such that it stays always at zero complexity (pr = ∞, system functionality is perfectly maintained). This case is represented by the line of full periodicity. Therefore, as the periodicity increases, area B increases and area A decreases. The periodicity is, therefore, expressed as:

$$ pr = \lim_{r \to \infty} \frac{\text{Area } B}{\text{Area } A \times t} $$

Equation 4-4

Where $t$ is the time elapsed since the system is new.

The area $A$ can be expressed by the following relationship:

$$ A = \sum_{i=1}^{n} T \chi' C_{i-1} + \frac{\nu T^2}{2} $$

Equation 4-5

Where:

$C_i$ = complexity at time $iT$ where $i$ represents the number of resettings

$\chi'$ = $(1 - \chi)$
The complexity at any time $iT$ is described by the following relationship:

$$C_i = \chi'C_{i-1} + \nu T$$  \hspace{1cm} \text{Equation 4-6}

Where:

$T$ \hspace{1cm} \text{time between resetting}

$\nu$ \hspace{1cm} \text{Complication rate}

The complication rate is a new term introduced in this research to expresses the rate of increase of complexity per unit time. It is a property of each system that depends on the rate of functionality deterioration. It is represented in Figure 4-3 by the slope of the complexity line.

From Equation 4-5, the area $A$ can be calculated as follows:

$$A = 0 + \frac{\nu T^2}{2} + (\chi'\nu T^2 + \frac{\nu T^2}{2}) + \chi'(\chi'\nu T + \nu T) T + \frac{\nu T^2}{2} + \ldots \ldots .$$

$$= \nu T^2 \left( \frac{1}{2} \right) + \nu T^2 \left( \frac{1}{2} + \chi' \right) + \nu T^2 \left( \frac{1}{2} + \chi' + \chi'^2 \right) + \nu T^2 \left( \frac{1}{2} + \chi' + \chi'^2 + \chi'^3 \right) + \ldots \ldots .$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} \nu T^2 \left( \frac{1}{2} + \sum_{k=1}^{i} \chi'^{k-1} \right)$$  \hspace{1cm} \text{Equation 4-7}

From Equation 4-4 and Equation 4-7, the periodicity relationship can be formulated as follows:
\[ pr = \lim_{n \to \infty} \frac{\frac{1}{2} vn^2T^2 - \sum_{i=1}^{n} \nu T^2 \left( \frac{1}{2} + \sum_{k=1}^{i} x^{k-1} \right)}{\sum_{i=1}^{n} \nu T^2 \left( \frac{1}{2} + \sum_{k=1}^{i} x^{k-1} \right)} \times \frac{1}{nT} \]

\[ = \lim_{n \to \infty} \left( \frac{\frac{1}{2} n}{T \sum_{i=1}^{n} \nu T^2 \left( \frac{1}{2} + \sum_{k=1}^{i} x^{k-1} \right)} - \frac{1}{nT} \right) = \lim_{n \to \infty} \frac{\frac{n}{2T \sum_{i=1}^{n} \left( \frac{1}{2} + \sum_{k=1}^{i} x^{k-1} \right)}}{\frac{n}{2T \left( \frac{1}{2} + \sum_{i=1}^{n} \sum_{k=1}^{i} x^{k-1} \right)}} \]

Equation 4-8

\[ = \lim_{n \to \infty} \frac{n}{2T \left( \frac{1}{2} + \lim_{n \to \infty} \left( \frac{n-1}{n} x' + \frac{n-2}{n} x'^2 + \ldots + \frac{1}{n} x^{n-1} \right) \right)} \]

Given the mathematical fact that \( \lim_{k \to \infty} \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \), \(|x| \leq 1\), then, with approximation, the periodicity relationship can be expressed as follows:

\[ pr = \frac{1}{2T \left( \frac{1}{2} + \frac{1}{1-x'} - 1 \right)} = \frac{1}{T \left( \frac{2}{1-x'} - 1 \right)} \]

Equation 4-9

The periodicity relationship is plotted in Figure 4-4 where each curve represents the relationship between resetting extent versus the corresponding time between resetting at certain periodicity level. It is clear from Figure 4-4 that a slight change in the resetting frequency at low resetting extent levels considerably affects the periodicity. For example, if a machine tool has a maintenance policy including daily preventive maintenance at low level, then increasing this rate to a “between shifts” frequency would dramatically increase the resulting periodicity due to the change in maintenance policy.
It is also noted that for each periodicity level, there is a maximum time between resetting, which is realized at the maximum resetting extent (1). Therefore, its value is described by the following relationship:

\[
T_{\max} = \frac{1}{pr(2 - 1)} = \frac{1}{pr}
\]

Equation 4-10

It is important for the maintenance decision maker to know this relationship to be aware of the maintenance policy design limits.

### 4.6 Multiple Resetting Plans Periodicity

The discussion in section 4.5 applies to a system affected by one resetting plan. However, in most real life cases, the systems are reset by multiple resetting plans. For example, there could be two plans for machine tool resetting; where the machine is reset by failure...
repair plan and by preventive maintenance plan. These plans can be independent or dependent. An example of a dependent plan is the maintenance policy developed by Nakagawa (1986), where the resetting level of each preventive maintenance depends on the number of preceding failure repairs (resetting). While the Periodic preventive maintenance policy described by Xiao-Gao et al. (1995) represents an example of independent plan because the preventive maintenance and failure repair parameters are independent.

In the current research, the focus will be on the case of independent resetting plans. The independency condition allows the total periodicity to be expressed as the summation of all periodicity elements. Therefore,

$$ p_r = \sum_{i=1}^{n} p_{r_i} = \sum_{i=1}^{n} \frac{1}{T_i \left(\frac{2}{\lambda_i} - 1\right)} $$

Equation 4-11

4.7 Periodicity-Based Maintenance Policy Evaluation

The periodicity due to maintenance programs is introduced by two independent sources; the Failure Repair and the Pre-planned Preventive Maintenance. Therefore, the periodicity resulting from each source would be calculated separately and the total periodicity is determined using Equation 4-11.

4.7.1 Failure Repair Periodicity

In this step, the periodicity due to failure repair is calculated assuming the system has a known failure rate $\lambda$. The system is assumed to be repaired (functionality reset) as it fails, which is the common practice in industry. Therefore, the resetting rate is the same as the failure rate $\lambda$.

From Equation 4-3, it can be stated that:

$$ \text{complexity after resetting} = (1 - \text{Resetting Extent}) \times \text{complexity before rerestting} $$

Equation 4-12
The resetting extent in the maintenance context is represented by the repair level in the case of failure repair and by the maintenance level in the case of preventive maintenance. This can be explained by reviewing the different imperfect maintenance modeling approaches such as (p, q), improvement factor, or virtual age (Pham and Wang, 1996). Nakagawa (1988) used the improvement factor approach to calculate the hazard rate after maintenance operations as follows:

\[ h(t)_{\text{after maintenance}} = a_k h(t)_{\text{before maintenance}} \]  

Equation 4-13

Where \( a_k \) represents the improvement factor or the maintenance level, where \( a_k = 0 \) for perfect maintenance and \( a_k = 1 \) for minimal maintenance and \( h(t) \) represents the hazard rate at time \( t \). The similarity between Equation 4-12 and Equation 4-13 indicates that in the case of maintenance, the resetting extent is expressed by the repair level/maintenance level. Therefore, the failure repair periodicity can be expressed by the following equation:

\[ pr_{rep} = \frac{\lambda}{2} \left( \frac{1}{RL} - 1 \right) \]  

Equation 4-14

4.7.2 Preventive Maintenance Periodicity

The second source of periodicity is the preventive maintenance. This is the main source of periodicity when the maintenance strategy calls for minimal repair of failures. The system is reset with each preventive maintenance, therefore, the resetting rate is the same value as preventive maintenance frequency; PMF. And the periodicity extent is expressed by the level of preventive maintenance; PML. Therefore, the periodicity resulting from a preventive maintenance of \( n \) classes can be described by the following relationship:
From Equation 4-14 and Equation 4-15, the total system periodicity resulting from a given maintenance policy can be expressed as:

\[ pr_{PM} = \sum_{i=1}^{n} \frac{PMF_i}{2} \left( \frac{2}{PML_i} - 1 \right) \]

Equation 4-15

### 4.7.3 Total Maintenance Policy Periodicity

From Equation 4-14 and Equation 4-15, the total system periodicity resulting from a given maintenance policy can be expressed as:

\[ pr = \frac{\lambda}{2RL} + \sum_{i=1}^{n} \frac{PMF_i}{2} \left( \frac{2}{PML_i} - 1 \right) \]

Equation 4-16

Therefore, given the maintenance policy parameters, \( RL, PML_i \), and \( PMF_i \) and the failure rate \( \lambda \), the maintenance policy periodicity can be calculated. This calculated periodicity; \( pr \) is a measure of the ability of the maintenance strategy to reset the system functionality. It is quite clear that in manufacturing system case, all the maintenance policy parameters included in the periodicity formula affect not only the maintenance cost but also the cost of production as well as the system availability and its associated ability to meet demand requirements and production quality. Therefore, making a sound decision regarding the parameters of the maintenance policy would involve trade-offs between all the relevant criteria.

### 4.8 Illustrating Example

In the following example, the maintenance policy used at an assembly plant of an auto manufacturer in North America is used to illustrate the application of the proposed new approach for maintenance strategy modeling. The periodicity introduced by this maintenance strategy is then calculated. This assembly plant builds cars belonging to two different vehicle platforms. The plant maintenance program is performed by highly trained maintenance workers. During production shift, these maintenance workers are on
call to deal with any failure. In addition, the preventive maintenance is carried out during the plant shutdown.

The plant maintenance policy is described as follows:

- When a failure happens, it is instantaneously minimally repaired to quickly restore the production
- The preventive maintenance policy comprises four classes:
  - Between shifts preventive maintenance
  - Weekly preventive maintenance
  - Semi-annual preventive maintenance
  - Annual preventive maintenance

Each one of these preventive maintenance classes has associated courses of action for each machine, which are described in detail in their maintenance manuals. The first three classes of preventive maintenance are carried out by the plant maintenance workers, while the annual most comprehensive (highest class) is carried out by the machine manufactures during the Christmas vacation shutdown.

The exact determination of maintenance level for each class requires a lot of measurements and data. Nevertheless, based on the courses of action in each maintenance class, the maintenance level can be estimated to a high degree of accuracy. For example, the annual preventive maintenance includes testing all the machine parameters and restoring them to the near-original values consistent with the machine specifications. Therefore, assuming 95% task efficiency, the maintenance level for this class can be estimated as 0.95.

The following table lists the maintenance level for each class:

<table>
<thead>
<tr>
<th>Preventive Maintenance Class</th>
<th>Maintenance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between shifts</td>
<td>0.05</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.3</td>
</tr>
<tr>
<td>Mid-Annual</td>
<td>0.6</td>
</tr>
<tr>
<td>Annual</td>
<td>0.95</td>
</tr>
</tbody>
</table>
The plant operates two shifts per day, five days per week. One shift is considered the time unit. Using the proposed maintenance modeling approach, the plant maintenance strategy can be fully described by the following parameters:

<table>
<thead>
<tr>
<th>Repair Level</th>
<th>RL</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Preventive Maintenance Classes</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Preventive Maintenance Frequency</td>
<td>PMFi</td>
<td>1.0 Between shifts</td>
</tr>
<tr>
<td>Preventive Maintenance Level</td>
<td>PMLi</td>
<td>0.05</td>
</tr>
</tbody>
</table>

This maintenance strategy applies to every resource/machine in the plant. One of these resources is a frame-welding robot. This robot experiences random failures with an average of one failure/week.

By applying Equation 4-16, the amount of functional periodicity introduced by the maintenance strategy can be calculated as follows:

\[
p_r = \frac{1}{10(\frac{2}{0} - 1)} + \frac{1}{10(\frac{2}{0.05} - 1)} + \frac{1}{240(\frac{2}{0.3} - 1)} + \frac{1}{480(\frac{2}{0.6} - 1)} + \frac{1}{480(\frac{2}{0.95} - 1)}
\]

\[= 0.047\]

This calculated periodicity measures the relative ability of the maintenance strategy to reinitialize the robot functionality. This measure is relative because it has no physical embodiment. But it is useful when used to compare different maintenance strategy alternatives.

The company is considering a new alternative maintenance strategy described by the following parameters:
<table>
<thead>
<tr>
<th>Repair Level</th>
<th>RL</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Preventive Maintenance Classes</td>
<td>N</td>
<td>4</td>
</tr>
<tr>
<td>Preventive Maintenance Frequency</td>
<td>PMFi</td>
<td>0.5</td>
</tr>
<tr>
<td>Preventive Maintenance Level</td>
<td>PMLi</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The maintenance manager wants to compare the performance of these two maintenance strategies in terms of machine functional resetting. The periodicity of the new maintenance strategy is calculated as follows:

\[
P_{\text{new}}^p = \frac{1}{10(\frac{2}{0} - 1)} + \frac{1}{2(\frac{2}{0.1} - 1)} + \frac{1}{40(\frac{2}{0.3} - 1)} + \frac{1}{120(\frac{2}{0.5} - 1)} + \frac{1}{480(\frac{2}{0.95} - 1)}
\]

\[
= 0.065
\]

It is clear from the values of periodicity for the two maintenance strategies that the proposed new maintenance policy provides more periodicity of resetting the machine(s) functionality than the original policy. Hence, the performance of the second maintenance policy is superior to the old one.

### 4.9 Discussion and Conclusions

Maintenance introduces periodicity into the systems, which is required to keep the system functional stability throughout its life. A novel general metric for quantifying the periodicity has been presented and developed. A formula for calculating the periodicity introduced by a maintenance policy is derived. A new term called complication rate has been introduced to measure functional deterioration. The proposed periodicity metric can be used to quantitatively compare the resetting ability of different maintenance policies, which vastly enhances decision-making in selecting appropriate maintenance strategies.

It is important to note that the periodicity is not the only decision factor; there are many others such as the cost of the selected maintenance policy. In addition, the resulting
products quality (in case of manufacturing systems) and system availability, and hence ability to meet production schedules and avoid lost revenue, late delivery penalties and missed opportunity cost are all affected by the choice of the maintenance policy and should be considered in the selection decision. The relationships between periodicity and these factors have not been investigated yet in the literature; it is currently being considered by the authors and will be presented in future work.

It has been shown that the calculation of periodicity introduced by a maintenance strategy, using the proposed model and formulation, is quite simple and makes it practically applicable in industry. Although the application of the developed periodicity metric has been discussed in the context of the maintenance field; nevertheless, the method is general enough and can also be applied in any application that involves system resetting such as natural and socio-technical systems just to name a few.
5 Maintenance Strategy Periodicity and Product Quality

5.1 Introduction

Product quality is one of the most priorities of today’s manufacturing systems. Although product quality is affected by many factors such as product design, quality control systems, continuous improvement,...etc., but the machine maintenance still represents the most important factor affecting product quality in the manufacturing phase. The relationship between machine maintenance and products quality has been observed by academic researchers and industrial experts. In this chapter, a mathematical relationship between a maintenance strategy and the corresponding average product quality will be developed using the periodicity metric. This relationship will be developed in two steps; first, the relationship between maintenance strategy and the machine average virtual age will be developed. Then, this relation will be utilized to determine the average product quality. These two steps are explained by Figure 5-1.

![Figure 5-1 Effect relationship between maintenance policy and products quality](image)

5.2 Virtual Age and Quality Deterioration

The relationship between machine state and the quality of produced products has been emphasized by Ross (1971) who indicated that the quality of the produced products is function of the machine state. Wang et al. (1996a) used this assumption and assigned a
variable \( q \) (0 ≤ \( q \) ≤ 1) that represents the machine quality condition where \( q \) is the probability of the machine producing defective parts. They indicated that the variable \( q \) is a function of the number of products produced since the last maintenance. This definition is based on the assumption that all maintenances are perfect. But this is not always the case. In the current research, this definition will be extended to include the general maintenance case. It will be stated that the variable \( q \) is a function in the machine virtual age. The term virtual age has been introduced by Kijima et al. (1988) to model the imperfect maintenance. except that, in the current research, the virtual age definition of Pham and Wang (1996) will be adapted where the virtual age is defined as the accumulated age, such that after repair (maintenance), the lifetime of a system will be reduced to a fraction \( \chi \) of the one immediately preceding maintenance where \( \chi \) represents the repair (maintenance) level. Therefore,

\[
q(t) = f(V(t))
\]

Equation 5-1

Where \( q(t) \) is the machine quality at time \( t \) and \( V(t) \) is the machine virtual age at time \( t \) and \( f \) is a non-decreasing function (according to the assumption of Wang et al. (1996a)) that the probability of producing defective parts is increasing with the number of produced parts). In the current research, it will be assumed that at \( V(0)=0 \) and \( q(0)=0 \), i.e. the probability of a new machine producing defective parts is zero.

Therefore, for a resetting plan of parameters \( \chi, T \) as shown in Figure 5-2, the machine starts with \( V=0 \) and \( V \) keep increasing till it reaches \( T \). Whenever, the machine is reset by level \( \chi \). Assuming that the machine complexity is directly proportional to its virtual age (this assumption is pretty logic because it is normal that the uncertainty and hence the complexity increases with age) and given Equation 4-3, so \( V \) reaches to \( (1-\chi) T \). Then \( V \) increases with time till it reaches to \( (1-\chi) T+ T \) at time \( 2T \) and so on. Therefore, at steady state, the \( V \) will be ranging from \( \lim_{n \to \infty}(T \sum_{i=1}^{n}(1-\chi)^i) \) and \( \lim_{n \to \infty}(T \sum_{i=0}^{n}(1-\chi)^i) \) as explained in Figure 5-2.
Therefore, eventually the $V$ will be ranging from $T\left(\frac{1}{\lambda} - 1\right)$ and $T\left(\frac{1}{\lambda}\right)$. Therefore the average $V$ would be:

\[
V_{\text{average}} = T \frac{\left(\frac{1}{\lambda} - 1\right) + \left(\frac{1}{\lambda}\right)}{2}
\]

Equation 5-2

\[
V_{\text{average}} = T\left(\frac{\frac{1}{\lambda} - \frac{1}{2}}{2}\right)
\]

It is noticed here that:

\[
V_{\text{average}} = \frac{1}{2pr}
\]

Equation 5-3

Equation 5-3 represents an important observation because it explains that the machine average virtual age is inversely proportional to applied maintenance plan periodicity and it determines the exact formula of this relationship. Therefore, using this easy but powerful relationship, the maintenance decision maker can predict the machine performance (in terms of the average virtual age) given the maintenance policy parameters, which represents a considerable enhancement in maintenance decision
making. Therefore, assuming that the product quality is function in machine virtual age, the average product quality would be described as:

\[ q_{\text{average}} = q(V_{\text{average}}) \]

\[ = q(T\left(\frac{1}{\chi} - \frac{1}{2}\right)) \]  

\text{Equation 5-4}

There is no unique relation that can be defined for the calculation of the function \( q \), but it considerably depends on the machine under consideration. This diversity in defining the relation between machine aging and the quality of produced products can be due to the diversity and specialty of the literature in that field; the aging in manufacturing machine tools is different from electronics machines and different from the chemical machines.

Nevertheless, there are many models in the literature trying to develop general formulas for this relation:

1- Trindade \textit{et al.} (2007) presented the quality deterioration function \( \psi(t) \) which is a non-decreasing function such that if \( t_2 > t_1 \) then \( \psi(t_2) > \psi(t_1) \). The function \( \psi \) expresses the probability of producing defective parts. So it assumes real values in the range \([0, 1]\). Trindade \textit{et al.} (2007) defined the following form for the quality deterioration function:

\[ \psi(t) = 1 - ke^{-ct} \]  

\text{Equation 5-5}

Where \( k, c \) are constant

2- Al-Fawzan and Rahim (2001) defined the quality as the drift of the quality characteristic from its target value. They defined a linear random quality function \( x(t) \) as follows:

\[ x(t) = \xi + \theta t \]  

\text{Equation 5-6}

Where \( \xi \) denotes the quality characteristic mean and \( \theta \) is a random variable representing the coefficient of the linear relationship.

3- Wang \textit{et al.} (1996a) defined the function \( q(n) \) which expresses the probability that the \( n \)th job completed on the machine since last maintenance is non-defective. They assumed that the function \( q \) is a non-increasing function in \( n \). That is, as more jobs are
completed, the probability of producing defective products increases. They assume that \( q(n) \) is described by the following formula:

\[
q(n) = \sum_{k=1}^{K} c_k r_k^n
\]

Equation 5-7

Where \( 0 < r_k \leq 1 \) and \( c_k \) are parameters that need to be determined from historical data, and \( K \) is an integer, it is chosen according to the desired degree of accuracy.

In the current research, the approach of Trindade et al. (2007) will be adopted to model the quality deterioration.

From Equations Equation 5-2 and Equation 5-5, the average percent defective can be expressed as follows:

\[
\psi_{\text{average}} = 1 - k e^{-c \psi_{\text{average}}}
\]

\[
= 1 - k e^{-c T \left( \frac{1}{\lambda} - \frac{1}{2} \right)}
\]

Equation 5-8

Assuming that the quality is described by the probability of producing acceptable part, then, the average quality can be expressed as follows:

\[
q_{\text{average}} = k e^{\frac{c}{2 pr}}
\]

Equation 5-9

It is clear from Equation 5-9 that the average products quality is solely function in periodicity. Therefore, different maintenance policies with the same periodicity level would lead to the same average product quality. The relationship between periodicity level and the average product quality level for different values of parameter \( c \) is explained in Figure 5-3.
Figure 5-3 shows that as the periodicity increases the average product quality increases exponentially. Then, with further periodicity increase, the quality improvement slowly increases. This figure explains that increasing the periodicity to certain extent directly affects the product quality and any further periodicity increase is considered waste of resources because it would not considerably enhance product quality. It is also noted from the figure that the product quality is affected by the value of the parameter $c$. The physical significance of parameter $c$ can be understood from analyzing the derivative of Equation 5-5:

$$\frac{d\psi(t)}{dt} = -k \frac{d}{dt} e^{-ct} = kce^{-ct}$$

Equation 5-10

Equation 5-10 shows that as $c$ increases, the rate of increasing the percentage of defective parts with time increases, i.e., faster machine functionality deterioration which means higher complication rate. Therefore, the parameter $c$ and the complication rate $v$ are strongly correlated.
5.3 Summary and Conclusion

The mathematical relation describing the average products quality as function in maintenance policy parameters is developed using the periodicity metric for the maintenance policy. A simple equation, describing the average machine virtual age as a function of the periodicity of the applied maintenance policy, is derived. This equation can considerably help the maintenance decision maker in predicting the steady state machine performance under any maintenance policy. It has been found that the average product quality is affected by both the periodicity of the applied machine maintenance policy and the machine complication rate.
6  Maintenance Strategy Periodicity and System Availability

6.1  Introduction

Manufacturing systems are normally designed with certain capacity level that economically satisfies the demand. Therefore, any capacity losses (traditionally due to machine failure) affect manufacturing system profitability as well as other negative effects and the same discussion applies to many other systems as well. Therefore, a maintenance policy is applied both to repair the failures and to preventively maintain the machines in order to reduce the probability of future failures. But these maintenance actions have a negative effect on availability as they increase the machine off-time. Therefore, finding the optimum maintenance policy is a tradeoff between these two contradicting effects which requires a mathematical model relating the system availability to the maintenance policy. In this chapter, the relationship between the machine availability and the applied maintenance policy will be developed.

Machine availability is defined as:

\[
availability = \frac{UP \text{ Time}}{Total \text{ Time}} = \frac{Total \text{ Time} - Down \text{ Time}}{Total \text{ Time}} = 1 - \frac{Down \text{ Time}}{Total \text{ Time}} \quad \text{Equation 6-1}
\]

Assuming that any failure will be repaired instantaneously as it happens, then, the down time in Equation 6-1 would express only the time needed for machine resetting, hence, the availability could be expressed as

\[
availability = 1 - \frac{Resetting \text{ Time}}{Total \text{ Time}} \quad \text{Equation 6-2}
\]

The machine is reset by two ways, either failure repair or preventive maintenance as explained in Figure 6-1
Therefore, the resetting time can be expressed as:

\[ \text{Resetting Time} = \text{Failure Repair Time} + \text{Preventive Maintenance Time} \quad \text{Equation 6-3} \]

For normalization purpose, Equation 6-3 will be reformulated to express resetting time per unit time; RT as follows:

\[ RT = \lambda \times \text{Single Failure Repair Time} \]
\[ + \text{PMF} \times \text{Single Preventive Maintenance Time} \quad \text{Equation 6-4} \]

Where:

- \( \lambda \) Machine Failure Rate
- \( \text{PMF} \) Preventive Maintenance Frequency

Assuming that the repair level is \( x_r \), the time required for minimal repair is \( T_{mn} \) and the preventive maintenance level is \( x_p \), then

\[ RT = \lambda (T_{mn} + x_r T_D) + \text{PMF} (T_{mn} + x_p T_D) \quad \text{Equation 6-5} \]

In Equation 6-5, it is assumed that the maintenance level is the only parameter that determines the maintenance time irrespective of the type of maintenance either it is repair or a preplanned maintenance. But in real applications, the failure repair needs more time than the preplanned maintenance of the same level due to the time spent on fault diagnosis and securing any resources required for repairs. This feature can be incorporated in the model by adding a factor \( t_{fr} \), ratio of failure repair time to the same level preventive maintenance, to represent the difference between the time required for
repairing a sudden failure and the time required for performing a preventive maintenance of the same level. The value of $t_{flp}$ factor depends on many system parameters that relate to the ability of the maintenance management system to deal with sudden failures like availability of spare parts, availability of maintenance personnel, etc. therefore, the determination of the $t_{flp}$ value can be done in two ways:

- Objective: collecting historical data from maintenance records and calculating the time ratio
- Subjective: the maintenance decision maker determines the factor value from his experience about the system.

It is expected that $t_{flp} \geq 1$ because the preplanned maintenance cannot take more time than the same level sudden failure repair. Therefore, Resetting time equation can be rewritten as:

$$RT = \lambda (T_{min} + \zeta_T D) t_{flp} + PMF (T_{min} + \zeta_p T_D)$$  \hspace{1cm} \text{Equation 6-6}$$

### 6.2 Derivation of Availability Relationship

In the current research, the main focus will be on maintenance strategies with minimal failure repair because they are the most popular ones in the literature and the real application. The reason for that may be attributed to that failures normally occur during the production time. Therefore, the failure repair causes unplanned system shutdown or at least machine shutdown, which incurs extremely high cost (it can reach up to $8000 per minute in some auto manufacturing systems) in addition to other negative consequences like missed schedules and missed delivery. Therefore, the resetting time is expressed as:

$$RT = \lambda T_{min} t_{flp} + PMF (T_{min} + \zeta_p T_D)$$  \hspace{1cm} \text{Equation 6-7}$$

The failure rate expresses the expected number of system failures per unit time. There are two failure rate trends experienced by a system; constant failure rate (CFR) and time dependent failure rate. The constant failure rate represents the case of completely random
or chance failures, so the machine is memory-less. This failure type should dominate during the useful life of the machine. However, due to the aging factor (which may be due to the absolute age or the usage age), the failure rate seldom follows exactly the CFR model. The general case is that the failure rate is time dependent. One of the most useful probability distributions that may be used to model both the increasing and decreasing failure rates is the Weibull distribution. It is characterized by a failure rate function of the form:

\[ \lambda = at^b \]  

Equation 6-8

The function \( \lambda(t) \) is increasing for \( a > 0, b > 0 \) and it is decreasing for \( a > 0, b < 0 \).

For mathematical convenience, Ebling (1997) expressed \( \lambda(t) \) for the Weibull distribution in the following form:

\[ \lambda(t) = \frac{\beta}{\theta} \left( \frac{t}{\theta} \right)^{\beta-1} \]  

Equation 6-9

Where:

- \( \beta \) Weibull distribution shape parameter
- \( \theta \) Weibull distribution scale parameter

Where the shape parameter \( \beta \) determines the failure rate trend as shown in Table 6-1. This result conforms to the non-homogeneous Poisson process (NHPP) power law equation described by Pham (2003). A very large number of numerical applications show the adequacy of the power law process in describing the failure pattern of mechanical equipment experiencing degradation (Pham, 2003).
Table 6-1 Relationship between Weibull shape parameters and failure rate trend

<table>
<thead>
<tr>
<th>$\beta$ Value</th>
<th>Failure rate Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \beta &lt; 1$</td>
<td>Decreasing Failure Rate</td>
</tr>
<tr>
<td>$\beta = 1$</td>
<td>Constant Failure Rate, Exponential Distribution</td>
</tr>
<tr>
<td>$1 &lt; \beta &lt; 2$</td>
<td>Increasing Failure rate (IFR) Concave</td>
</tr>
<tr>
<td>$\beta = 2$</td>
<td>IFR, Rayleigh distribution</td>
</tr>
<tr>
<td>$2 &lt; \beta &lt; 3$</td>
<td>IFR convex</td>
</tr>
<tr>
<td>$3 &lt; \beta &lt; 4$</td>
<td>IFR approaches normal distribution</td>
</tr>
</tbody>
</table>

Another derivation of the failure rate formula for a degrading machine is introduced by Jiang et al. (2003). He introduced the aging intensity (AI) function $L(t)$ to quantitatively evaluate the aging property of products or systems. It is defined as follows:

$$ L(t) = \frac{\lambda(t)}{\int_0^t \lambda(t) dt} - \frac{t \lambda(t)}{\int_0^t \lambda(t) dt} $$

Equation 6-10

Jiang et al. (2003) indicated that $L(t) = 1$ if the failure rate is constant and $L(t) > 1$ if the failure rate is increasing and $L(t) < 1$ if the failure rate is decreasing. The physical meaning of AI function is that as its value increases, the tendency of aging becomes stronger. The interesting mathematical feature of AI function is that for Weibull distribution, $L(t) = \beta$ as indicated by Jiang et al. (2003) and Nanda et al. (2004).

Assuming that:

1. the brand new machine has a failure rate $\lambda_0$
2. and the time between resetting is $T$
3. the resetting level is $\chi_p$ (which is the preventive maintenance level)

Then, using the analysis of section 5.2 and Equation 6-10, the steady state average failure rate of a machine under maintenance policy of periodicity $p$ is
\[ \lambda_{\text{average}} = \lambda(V_{\text{average}}) = \frac{\beta}{\theta} \left( \frac{V_{\text{average}}}{\theta} \right)^{\beta - 1} \]
\[ = \frac{\beta}{\theta} \left( \frac{1}{2 \cdot pr \cdot \theta} \right)^{\beta - 1} \]
\[ = \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{-\beta} \]  
\text{Equation 6-11} 

Substituting for \( \lambda \) in Equation 6-7 by \( \lambda_{\text{average}} \) from Equation 6-11 and standing for \( \chi_p \) by \( \chi \) for simplifying, then the resetting time per unit time would be expressed as:

\[ RT = \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{-\beta} T_{\text{mn}} t_{f/p} + \frac{1}{T} \left( T_{\text{mn}} + \chi T_D \right) \]  
\text{Equation 6-12} 

From Equation 6-2 and Equation 6-12, the availability would be expressed as follows:

\[ \text{availability} = 1 - \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{-\beta} T_{\text{mn}} t_{f/p} - \frac{1}{T} \left( T_{\text{mn}} + \chi T_D \right) \]  
\text{Equation 6-13} 

Substituting for \( \frac{1}{T} \) in Equation 6-13 from Equation 4-9, the availability would be expressed as:

\[ \text{availability} = 1 - \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{-\beta} T_{\text{mn}} t_{f/p} - pr \left( \frac{2}{\chi} - 1 \right) \left( T_{\text{mn}} + \chi T_D \right) \]  
\text{Equation 6-14} 

Given that the value of \( T_{\text{mn}} \) is normally much less than the value of \( \chi T_D \) for preventive maintenance. Then, Equation 6-14 can be approximated as follows:

\[ \text{availability} = 1 - \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{-\beta} T_{\text{mn}} t_{f/p} - pr \left( \frac{2}{\chi} - 1 \right) \chi T_D \]  
\text{Equation 6-15} 

Equation 6-15 determines the relationship between machine availability and the maintenance policy parameters as well as other machine and system parameters. It shows that the increasing periodicity has two contradicting effects on the availability; one is positive due to reducing the average failure rate and consequently reducing failure repair.
time and the other is negative due to the increased preventive resetting (maintenance) time.

Equation 6-15 shows also that the machine availability under any maintenance policy with periodicity level \( pr \) would lie in the range:

\[
\text{Equation 6-15} = \left[ \lim_{x \to 0} \left( 1 - \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{1-\beta} T_{min} t_{f/p} - (2-x) pr T_D \right), 1 - \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{1-\beta} T_{min} t_{f/p} - pr T_D \right],
\]

the minimum availability corresponds to low \( \chi \) values (low maintenance level) while the maximum availability corresponds to \( (\chi=1) \), i.e., perfect maintenance. This conclusion means that from the availability perspective, it is better to perform perfect maintenances at long intervals than performing low-level maintenances at frequent intervals.

The relation between availability and periodicity for different \( t_{f/p} \) values is shown in Figure 6-2.

![Figure 6-2 Effect of periodicity on machine availability for different \( t_{f/p} \) levels](image)

The figure shows that for any \( t_{f/p} \) value, the availability first increases with increasing the periodicity level till it reaches a peak point. The value of the peak availability and its
corresponding periodicity depends on the value of $t_{f/p}$. Then with further increase of the periodicity level, the availability decreases. This relationship between machine availability and maintenance policy periodicity may be due to the fact that at low periodicity values, any increase in periodicity causes improvement in the steady state machine failure rate and hence less failure repair time and consequently better availability. But at high periodicity levels, any increase in periodicity level increases the preventive maintenance time without a balancing reduction in failure repair time, which causes availability decrease.

It is noted that as the $t_{f/p}$ value increases, the availability decreases which is due to the increased failure repair time.

### 6.3 Summary and Conclusion

Maintenance is important to keep the designed capacity available at an acceptable level most of the time. In this chapter, the relation between the maintenance policy and the corresponding system availability is developed using the aging intensity function. This relation indicated that there is an optimum periodicity level that leads to the maximum availability, this level and the corresponding maximum availability depend on the value of the ratio of failure repair time to the time of the same level preventive maintenance. The developed relationship will be used in formulating the multi-objective optimization problem to find the optimum periodicity level.
7 Maintenance strategy Cost Analysis

7.1 Introduction

In recent years, many businesses have focused on the cost in order to become world-class companies because it is strongly believed that this would make their businesses more competitive at the international market, thus, leading to improved profit generation (Oke and Charles-Owaba, 2006). For large organizations, the focus is usually on improving the maintenance activities such that minimum amounts of funds are expended (Anily et al., 1999). This has motivated the study of maintenance costs and development of many maintenance cost optimization models. Nevertheless, Most of these models did not pay enough attention to the details, components and structure of maintenance cost on the task level, which may lead to results considerably far from reality.

In the current research, maintenance cost components would be studied in detail in the case of manufacturing systems to derive the relationships describing all maintenance cost elements in terms of resetting plan parameters. These cost elements will then be used to derive the total maintenance cost formula.

7.2 Cost of Machine Resetting

The cost of machine resetting can be divided into 3 main elements as shown in Figure 7-1; cost of resources, labor cost, and off time cost. In the following subsections, each one of these elements will be studied in detail.
7.2.1 Resources Cost

Machine resetting needs resources to be performed. Here, resources mean all the physical resources required for maintenance/repair other than labor. These may include spare parts, oils, lubricants, tools, etc. assuming that the cost of resources for full resetting is $R$ and resources cost for minimal resetting is $R_m$, and assuming all failure repairs are minimal. Then, resources cost of failure repair is $R_m$ and assuming that the minimal repair maintenance resources cost is considerably small compared to the overhaul (full resetting) cost, then, according to the model presented by Pongpech and Murthy (2006), the resources cost per resetting of preventive maintenance is:

$$RC_{PM} = R^* \chi^a$$

Equation 7-1

Where:

- $\chi$: Preventive Maintenance Resetting Extent
- $RC_{PM}$: Preventive Maintenance Resources Cost
- $a$: adjustment factor

The factor $a$ is used to make Equation 7-1 general and adjustable to each individual case. But in the rest of this research, the relationship would be considered linear and $a=1$.

Minimal resetting is interpreted similar to minimal repair in the maintenance context (Pham and Wang, 1996). It means that the system is reset after sudden failure to its state just before the failure occurrence.

The resetting extent is represented differently according to the type of resetting operation. In the case of failure repair, the failure repair level represents the resetting extent while in case of preventive maintenance; the maintenance level represents the resetting extent.
Therefore, for a resetting plan of parameters $T$ and $\chi$ applied to a system with failure rate $\lambda$, the maintenance resources cost per unit time would be expressed as:

$$RC = \lambda R_m + \frac{1}{T} R \chi^a$$  \hspace{1cm} \text{Equation 7-2}$$

Substituting for $\lambda$ by steady state $\lambda_{\text{average}}$ which has been derived in chapter 6, then:

$$RC = \lambda_{\text{average}} R_m + \frac{1}{T} R \chi^a$$  \hspace{1cm} \text{Equation 7-3}$$

$$= \frac{\beta}{\theta} (2 * pr * \theta)^{1-\beta} R_m + \frac{1}{T} R \chi^a$$

### 7.2.2 Machine Off-Time Cost

The resetting process is performed while the machine is not operating. There are two different models used in the industry in this aspect. The first model assumes that the machines are working in shifts and they are turned off between the shifts and during vacations. In this type of manufacturing, the preventive maintenance is normally scheduled to be performed during the machine off time and in this case, there is no off time cost associated with preventive maintenance. But failures occur while the machine is operating; hence, the failure repair will be associated with off time cost. This scenario is prevalent in manual and semi-automated manufacturing systems.

The second type of manufacturing systems is that where machines work continuously without shutting down except for failure repair or preventive maintenance. In this case, any machine shut down would be associated with off time cost. This type of manufacturing systems is prevalent in automated manufacturing systems.

In the current research, the second type of manufacturing systems where any machine shut down is associated with off time cost is considered. It is noticed that preventive maintenance is pre-planned either during the machine off time as in the first case or it is scheduled according to the machine production schedule as in the second case. But the failure repair cannot be preplanned. This difference between sudden failure repair and preplanned maintenance has been traditionally incorporated in the developed literature.
models using a ratio $C_{p/f}$, cost ratio of preventive maintenance to the same level failure repair. It is logical that this ratio assumes values in the range [0,1] where 0 means that preventive maintenance does not incur any off time cost (the first earlier mentioned manufacturing systems type) and 1 means that off time cost does not differ between sudden failure repair case and preplanned preventive maintenance case. It is expected that not all the maintenance cost elements are necessarily different between failure repair and preventive maintenance. An example for that is the maintenance resources cost which depends solely on the maintenance level. This consideration was not possible to be taken in the developed models in the literature because, as mentioned earlier, the maintenance cost was not studied on the task level. But, due to the detailed approach of the current research, the ratio $C_{p/f}$ can be correctly incorporated (applied to the relevant cost components only) in the model.

Assuming that an unscheduled downtime unit costs $K$, then, a preplanned downtime unit cost is $C_{p/f}K$. The determination of the value of $C_{p/f}$ may be subjective in most cases because it can include many subjective factors like missed due dates or the moral effects on labor because of unplanned production stoppage.

To minimize the effects of unscheduled failure repair, it will be assumed that all failures are minimally repaired, then the off time cost (OC) associated with machine resetting is:

$$OC = \begin{cases} 
K * C_{p/f} (T_m + bT_d) & \text{for Preventive Maintenance of Level } X \\
K * t_{f/p} * T_m & \text{for Failure Repair} 
\end{cases}$$

Equation 7-4

Where:

- $T_m$ time required for minimal resetting
- $T_d$ the difference between the times required for full resetting and for minimal resetting.
- $b$ adjustment factor

The values of $T_m$ and $T_d$ are normally determined by the machine manufacturer and can be found in operation and maintenance manuals. Therefore, the total off time costs per unit time can be expressed as follows:
\[ OC = K[\lambda t_{f/p} T_m + \frac{1}{T} C_{p/f} (T_m + \lambda^b T_d)] \]  
Equation 7-5

Substituting for \( \lambda \) by \( \lambda_{\text{average}} \) from Equation 6-11:

\[ OC = K\left[\frac{\beta}{\theta} (2*pr*\theta)^{-\beta} t_{f/p} T_m + \frac{1}{T} C_{p/f} (T_m + \lambda^b T_d)\right] \]  
Equation 7-6

7.2.3 Labor Cost

The resetting process is always done manually even in the most automated manufacturing systems. Therefore, the resetting process is always associated with labor costs, which has been addressed by Chan and Asgarpoor (2006) as one of the main components in maintenance costs. The resetting labor cost depends on the time spent by the workers in the resetting process and their hourly wage. The time required for the resetting process has been derived earlier in Chapter 6. Concerning the labor wage, there are different models in literature and industry that determine labor wage structure. In the current research, the typical manufacturing systems labor wage structure explained by Park (1997) will be adopted where there is a machine operator that runs and monitors the machine and performs only the minimal repairs and in case of high levels/extensive repair or maintenance, the maintenance operator is called to do it. In the current research, a third resetting level will be suggested to represent maintenance Outsourcing because maintenance outsourcing to third-party contractors has become an increasingly popular option for manufacturers to achieve tactical and/or strategic objectives (Ye, 2007). This wage structure is shown in Figure 7-2.
A real example of this structure is Daimler Chrystler assembly plant in Ontario, where the yearly machines overhaul is performed by machine suppliers, while all other maintenance tasks are performed by the plant workers either machine labor or maintenance labor.

From complexity perspective, this maintenance hierarchy can be described as follows: the low complexity maintenance tasks are performed by the less trained labor and as the complexity of the task increases, it needs more skillful labor to perform it which means that the complexity is interlinked to the labor skills (ElMaraghy and Urbanic, 2004), then, it would be convenient to include the maintenance task complexity as a parameter for determining the required maintenance labor skills level and hence the labor wage.

ElMaraghy and Urbanic (2004) proposed a complexity factor $P_D$ for both product and process. In the current research, the process complexity factor $P_{CD}$ will be used to measure the maintenance task complexity. ElMaraghy and Urbanic (2004) applied their model to the manufacturing process but their idea can be used with other processes like maintenance. They defined process complexity factor as follows:

$$P_{CD} = \frac{\sum_{i=1}^{K} P_{N_i} \times P_{D_i} + C_{N_i} \times C_{D_i}}{P_{N_i} + C_{N_i} \times K}$$

Equation 7-7

Where:

- $K$ Number of process tasks
Therefore, when applying this notion to the maintenance process, the tasks are the machine maintenance steps, which are traditionally described by the machine manufacturer in the machine maintenance manual. The determination of the values of $P_{D_i}$ and $C_{D_i}$ is subjective but their values should be determined based on the perception of machine worker such that the complex maintenance tasks from the point of view of the machine worker would be delegated to a higher maintenance skill labor. Therefore, the maintenance task complexity determines the operator that will perform the maintenance; the machine worker, the maintenance labor, or outsourcing. Traditionally, the maintenance task complexity is a function in maintenance level because every maintenance level corresponds to a set of tasks and hence a certain degree of complexity. Normally the task complexity increases with increasing the maintenance level. This increase may be due to physical complexity increase or cognitive complexity increase or both. Based on this discussion, the relationship between maintenance level and labor wage is constructed as shown in Figure 7-3. This figure determines the maintenance labor wage in two steps; first, the task complexity of the maintenance is determined from the maintenance level (the lower chart). Then this task complexity is used to find the required maintenance skill and hence the maintenance labor wage for that maintenance task.
Therefore, and based on the labor cost model developed by Kennedy (1993), the resetting process labor cost can be described by the following relationship:

\[
LC = T_{off} \times W = W(T_m + x^bT_d)
\]

Equation 7-8

Where:
- \(W\) maintenance labor wage per unit time
- \(T_{off}\) the down time elapsed in the maintenance operations

Assuming the resetting task complexity is directly proportional to the resetting extent. Then, the labor wage can be calculated in the following steps:

1- Calculating the task complexity given the maintenance/repair level

\[
C_T = aX^b + c
\]

Equation 7-9

Where:
- \(C_T\) Task Complexity

Figure 7-3 Relation between maintenance labor wage and maintenance level
adjustment factors

c task complexity for zero resetting extent process (task Complexity of
minimal repair process)

2- Calculating the labor wage given the maintenance task complexity

Figure 7-3 shows that labor wage is a step function in task complexity. But for the sake of
modeling simplicity, it will be assumed that it is a continuous function of the form

\[ W = L \times C^a_r = L(a \chi^b + c)^a \]  

Equation 7-10

Where \( L \) represents a constant. For example, in case of linear relationship between labor
wage and task complexity, \( L \) would represent the line slope.

By neglecting the high order terms in Equation 7-10,

\[ W = L(a^a \chi^{bn} + c^n) \]

\[ = Lc^n + La^a \chi^{bn} \]

\[ = W_{mn} + L' \chi^{b'} \]  

Equation 7-11

Where \( W_{mn} \) stands for the minimum labor wage, which normally represents the wage of
the machine labor who traditionally performs the minimal repair tasks. Hence, for a
resetting plan of parameters \( \chi \), \( T \) and minimal repair at failure, the labor cost per unit time
would be expressed as follows:

\[ LC = \lambda W_{mn} t_{f/p} T_{mn} + \frac{1}{T} (W_{mn} + L' \chi^{b'}) (T_n + \chi^b T_d) \]  

Equation 7-12

Substituting for \( \lambda \) by steady state average \( \lambda \) from Equation 6-11, then:

\[ LC = \frac{\beta}{\theta} (2 * pr * \Theta)^{1-\beta} W_{mn} t_{f/p} T_{mn} + \frac{1}{T} (W_{mn} + L' \chi^{b'}) (T_n + \chi^b T_d) \]  

Equation 7-13

7.2.4 Total Periodicity Cost

For simplicity, in the current research, linear relationships will be assumed. In reality, these relations are not necessarily linear and the determination of the real relationship
needs more investigations and data collection. Therefore, from Equation 7-3, Equation 7-6 and Equation 7-13, the simplified total resetting cost per unit time is:

\[ TC = RC + OC + LC \]

\[ = \left[ \frac{1}{T} \frac{1}{\bar{X} R} \right] \]

\[ + K \left[ \frac{1}{T} \frac{1}{C_{pj} (T_{mn} + X T_D)} \right] \]

\[ + \left[ \frac{1}{T} (T_{mn} + X T_D) (W_{mn} + L X^b) \right] \]

Equation 7-14

In manufacturing systems maintenance, it is normal that the value of \( X T_D \) is considerably larger than \( T_{mn} \), therefore, Equation 7-14 can be simplified as follows:

\[ TC = \left[ \frac{1}{T} \frac{1}{\bar{X} R} \right] \]

\[ + K \left[ \frac{1}{T} \frac{1}{C_{pj} X T_D} \right] \]

\[ + \left[ \frac{1}{T} X T_D (W_{mn} + L X^b) \right] \]

\[ = \lambda_{average} (R_m + K t_{pj} T_{mn} + W_{mn} t_{pj} T_{mn}) + \frac{X}{T} (R + C_{pj} K T_D + T_D (W_{mn} + L X^b)) \]

Equation 7-15

Rearranging Equation Equation 7-15, the total cost can be expressed as:

\[ TC = \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{1-\beta} (R_m + K t_{pj} T_{mn} + W_{mn} t_{pj} T_{mn}) \]

\[ + pr(2 - \chi)(R + C_{pj} K T_D + T_D (W_{mn} + L X^b)) \]

Equation 7-16

Based on industrial experience, maintenance labor wage is not the main component in total maintenance costs as its value is relatively small compared to either maintenance resources cost or off time cost. Therefore, at any periodicity level, \( pr \), the total maintenance cost lies in the range:

\[ \left[ \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{1-\beta} (R_m + K t_{pj} T_{mn} + W_{mn} t_{pj} T_{mn}) + pr(R + C_{pj} K T_D + T_D W_{max}), \right. \]

\[ \frac{\beta}{\theta} (2 \cdot pr \cdot \theta)^{1-\beta} (R_m + K t_{pj} T_{mn} + W_{mn} t_{pj} T_{mn}) + 2 pr(R + C_{pj} K T_D + T_D W_{mn})] \]
The maintenance cost for different $C_{p/f}$ values is depicted in Figure 7-4.

It is noticed in this figure that maintenance costs first decreases with increasing periodicity, which is due to the decrease in failure rate resulting from machine resetting. Then, with further periodicity increase, the maintenance costs performance depends on the value of $C_{p/f}$. In case $C_{p/f} = 0$, i.e. all the preventive maintenance is carried out in the machine downtime which eliminates the off time cost component. Therefore, increasing the periodicity would reduce sudden failures and their associated costs with a little costs increase (labor and resources). It is worth here to notify that the very right hand side of the $C_{p/f}=0$ curve in Figure 7-4 (at large values of periodicity) is not practical because the maintenance costs cannot be kept at low level with indefinite increase in periodicity. But logically, when the periodicity increases over certain limit, the time needed for preventive maintenance would be more than the regular machine downtime, and off time costs will start to be incurred and therefore, maintenance costs increase. The reason why this
phenomenon does not appear in $C_{pof}=0$ curve in Figure 7-4 is that the limit of regular machine downtime is not incorporated in the developed model.

The other extreme case is $C_{pof}=1$, i.e., there is no machine regular downtime and any machine stoppage either for planned maintenance or sudden failure repair would incur off-time costs. In this case, there would be optimum periodicity level, which satisfies minimum cost and any increase in the periodicity causes cost increase. This increase is steeper as the value of factor $C_{pof}$ increases. Hence, using this chart, the optimum periodicity level that satisfies the balance between costs of preventive maintenance and savings in failure repairs can be determined.

### 7.3 Conclusion

Maintenance cost components; maintenance resources cost, off-time cost, and labor cost, have been investigated. A mathematical formula describing each component is derived. It is shown that the maintenance cost elements can be expressed as function in the maintenance resetting periodicity, pr. The total maintenance cost relationship shows that there is an optimum periodicity level that satisfies minimum maintenance cost. The derived cost relationship can support maintenance decision making because it easily explains the possible changes in incurred cost corresponding to maintenance plan changes.

The maintenance cost is one of the objectives the maintenance decision is based on and the determination of the optimum periodicity level considering all the contributing objectives is the subject of the next chapter.
8 Optimal Maintenance Strategies

8.1 Introduction

The determination of the optimum periodicity level is a trade-off between reducing system complexity on one side and improving quality, availability and cost on the other side. Traditionally, these are conflicting objectives as is the case in many real engineering problems. These types of problems are solved using multi-objective optimization techniques. There are two general approaches to deal with multi-objective optimization. The first approach produces a single solution. There are two methods under this approach. This first method is to combine all the individual objective functions into a single composite function using methods such as utility theory, weighted sum, etc. The main drawback of this approach lies in the difficulty of making proper selection of the weights or utility functions to characterize the decision-makers preferences. In practice, it is normally very difficult to precisely and accurately select these weights even for someone familiar with the problem domain. Compounding this drawback is that scaling amongst objectives is needed and small perturbations in the weights can sometimes lead to quite different solutions. The second method is to consider all but one objective as constraints. The problem here is how to move objectives to the constraint set, a constraining value must be established for each of these former objectives. This can be rather arbitrary. In both cases, an optimization method would return a single solution rather than a set of solutions that can be examined for trade-offs.

The second multi-objective optimization approach is to determine an entire Pareto optimal solution set or a representative subset. A Pareto optimal set is a set of solutions that are non-dominated with respect to each other. While moving from one Pareto solution to another, there is always a certain amount of sacrifice in one objective(s) to achieve a certain amount of gain in the other(s).

In the current research, the first multi-objective optimization approach will be adapted.
8.2 Maintenance Optimization Problem Formulation

The original maintenance optimization problem can be formulated as follows:

Objective (1): Maximize \( \text{periodicity} = pr \)

Objective (2): Maximize \( q_{\text{average}} = ke^{-2pr} \)

Objective (3): Maximize \( \text{availability} = 1 - \frac{\beta}{\theta} (2 * pr * \theta)^{-\beta} T_{\text{min}} t_{f/p} - pr(2 - \chi)T_D \)

Objective (4): Minimize \( TC = \frac{\beta}{\theta} (2 * pr * \theta)^{-\beta} (R_m + Kt_{f/p} T_{\text{min}} + W_{\text{min}} t_{f/p} T_{\text{min}}) + pr(2 - \chi)(R + C_{p/f} KT_D + T_D(W_{\text{min}} + L X^{b})) \)

Subject to:

\[
\frac{1}{pr(\frac{2}{\chi} - 1)} \leq \frac{F}{\nu} \quad \text{Constraint (1)}
\]

\( pr \geq 0 \quad \text{Constraint (2)} \)

\( 0 \leq \chi \leq 1 \quad \text{Constraint (3)} \)

The objectives are respectively to minimize system complexity, maximize quality, maximize availability, and minimize total maintenance costs. The decision variables in this group of objectives are the resetting parameters, \( pr \) and \( \chi \). All other parameters included in the objective equations are either unit or system parameters that should be determined a priori. The first constraint expresses the condition that the resetting should be carried out before the system experiences a gradual failure. The relation stated in constraint (1) can be derived from Equation 3-5 and Equation 4-9 where \( T \) is replaced with \( \frac{1}{pr(\frac{2}{\chi} - 1)} \).

Therefore, the maintenance optimization requires solving a nonlinear multi-objective optimization problem. As stated in Section 8.1, there are many approaches for solving such a problem. In the current research, the weighted sum method will be adopted since it is a most widely used method (Kim and De Weck, 2004). The weighted sum method transforms multiple objectives into an aggregated scalar objective function by
multiplying each objective function by a weighting factor and summing up all contributors:

\[ J_{\text{weighted sum}} = w_1J_1 + w_2J_2 + \ldots + w_nJ_n \]  

Equation 8-1

Where \( w_i \) represents the weight of the objective \( J_i \). If \( \sum_{i=1}^{n} w_i = 1 \) and \( 0 \leq w_i \leq 1 \), then the weighted sum is said to be a convex combination of objectives. This formulation has the disadvantage that in many real cases, the units of the different objectives are different as in this case; the cost is measured in $ in the range \([0, \infty]\), the quality is measured by the probability of not producing defective items and lies in the range \([0,1]\) and the availability is measured by the percentage of uptime with respect the total time and lies in the range \([0,1]\). This problem can be resolved by the normalization of objectives. The aim of the normalization process is to make all the normalized objectives lie in the same range such that the optimization results would not be affected by the scales of the various objectives. Let us denote the normalized objectives as \( J' \) Then, the global objective would be written as:

\[ J_{\text{weighted sum}} = w_1J'_1 + w_2J'_2 + \ldots + w_nJ'_n \]  

Equation 8-2

Where:

\[ J' = \frac{J}{J_{\text{max}} - J_{\text{min}}} \]  

Equation 8-3

For the quality and availability objectives, \( J_{\text{max}} - J_{\text{min}} = 1 \), therefore, \( J' = J \) But for the periodicity and cost objectives, \( J_{\text{max}} = \infty \) To resolve this problem, an expected interval of interest of periodicity is determined and the minimum and maximum values of the cost are calculated by solving a single objective (cost) optimization problem. Therefore, the objective of the aggregate optimization problem is formulated as follows:
Minimize:

\[ J = (1 - pr^c) + w_1(1 - ke^{2pr}) + w_2 \left( \frac{\beta}{\theta} (2 * pr * \theta)^{1-\beta} T_{mn} t_{f/p} + pr(2 - \chi)T_D \right) \]

\[ + \frac{\beta}{\theta} (2 * pr * \theta)^{1-\beta} \left( R_m + Kt_{f/p}T_{mn} + W_{mn} t_{f/p}T_{mn} \right) + pr(2 - \chi)(R + C_{p,f}K T_D + T_D(W_{mn} + L \chi^d)) \]

Equation 8-4

It is noted in Equation 8-4 that the objectives are considered two groups; the first one includes reducing system complexity (maximizing periodicity) while the second group includes the other performance factors (quality, availability and cost) and the ‘reducing complexity’ objective is given a weighting factor equal to the whole other group weight to reflect the fact that reducing complexity is the main maintenance role. While the relative importance of the other performance criteria would be reflected on the choice of \( w_1, w_2, \) and \( w_3 \) factors.

This model would be solved by determining the different weights and solving the optimization problem to determine the optimum periodicity level and resetting extent and hence determine the optimum maintenance policy for the assigned weights.

### 8.3 Weighted Sum Approach

In the current section, the optimization problem would be solved by determining the weighting factors for the different objectives and solving the resulting optimization problem to determine the best periodicity level. But this approach has been traditionally criticized in the literature due to the dependence of the solution on the choice of the weighting factors, which is considered a subjective decision and needs a lot of experience and knowledge about the system. In the current research, a comprehensive approach for determining the weights is introduced that can be applied to utilize qualitative as well as quantitative information about the objectives preference. Applying this approach considerably mitigates the drawback of the weighting sum method. This approach utilizes the first step of Analytical Hierarchy Process (AHP) introduced by Saaty (1979). AHP
presents a structured procedure for determining the weights of the objectives through constructing a matrix of pair-wise comparisons. The detailed method of determining the weights factors using AHP is explained in appendix (B) and its application is explained by the following numerical example

8.3.1 Numerical example for weighted sum periodicity Optimization

The application of AHP approach for determining weights factors is explained using the following numerical example. Assuming the following preferences are determined by the maintenance decision maker based on experience and knowledge about the system.

- Cost minimization is strongly favored over due date fulfillment.
- Product quality maximization is strongly favored over due date fulfillment.
- Product quality maximization is a little bit favored over cost minimization

From these statements, the objectives comparison matrix can be constructed as follows:

\[
A = \begin{bmatrix}
\text{cost} & \text{avail.} & \text{quality} \\
\text{cost} & 1 & 5 & 0.5 \\
\text{avail.} & 0.2 & 1 & 0.2 \\
\text{quality} & 2 & 5 & 1
\end{bmatrix}
\]

Equation 8-5

Note that reducing complexity is not included in the preference matrix as it is already assumed that its importance is equivalent to the sum of all the other objectives.

Since the consistency of this matrix is unknown, Equation B-2 cannot be used, but according to Saaty and Hu (1998) statement that the normalized row average can be used as an approximation for weights, the objectives weights can be calculated as follows:

\[
w = \begin{bmatrix}
\text{cost} & 0.354 \\
\text{avail.} & 0.09 \\
\text{quality} & 0.556
\end{bmatrix}
\]

The next step is to check the comparison matrix consistency ratio by calculating \(\lambda_{\text{max}}\) and applying Equations B-4 and B-5 as follows:

\[
\lambda = \begin{bmatrix}
3.063 \\
3.014 \\
3.085
\end{bmatrix}
\]
Therefore, \( \lambda_{\text{max}} = 3.085 \). Substituting in Equation B-4;

\[
CI = \frac{3.085 - 3}{2} = 0.043
\]

Therefore, inconsistency ratio \( IR = \frac{0.043}{0.58} = 0.073 \), i.e., \( IR < 0.1 \). Therefore, the comparison matrix is acceptable and the weighted sum objective function can be formulated as follows:

Minimize

\[
J = \frac{1 - pr}{0.05} + 0.556(1 - ke^{3pr}) + 0.09\left(\frac{pr}{\theta}(2 * pr * \theta)^{-\beta} T_{\text{min}}^{f/p} + pr(2 - \chi)T_D\right) + 0.354
\]

\[
= \frac{(\lambda_o + \frac{pr}{\theta}(2 * pr * \theta)^{-\beta})(R_m + K_t^{f/p} T_{\text{min}} + W_{\text{min}}^{f/p} T_{\text{min}}) + pr(2 - \chi)(R + C_p R K T_D + T_D(W_{\text{min}} + L \chi^{\beta}))}{38700}
\]

This is a nonlinear optimization problem. Using the GAMS solver (Pinter, 2007) and using the parameters of the example used in chapter (4) listed below:

- Failure distribution shape parameter (\( \beta \)) = 2.8
- Failure distribution scale parameter (\( \theta \)) = 50
- Minimal maintenance/repair time (\( T_{\text{min}} \)) = 0.5
- Perfect maintenance/repair time (\( T_{\text{min}} + T_D \)) = 10
- Cost of minimal repair/maintenance resources (\( R_m \)) = 200
- Cost of overhaul (perfect maintenance) resources (\( R \)) = 8000
- Cost of unit offtime (\( K \)) = 50000
- Minimum labor wage per unit time (\( W_{\text{min}} \)) = 200
- Maximum labor wage per unit time (\( W_{\text{max}} \)) = 800
- Complexity failure threshold (\( F \)) = 0.3
- Complication rate (\( u \)) = 0.01

The detailed GAMS program code and results report are described in Appendix C. The optimization solution indicates that the optimum periodicity level is 0.05 and the resetting extent is 1.0. Therefore, the optimal maintenance strategy calls for fully resetting the system each 20 time units.
The optimization results are expected to be affected considerably by the values of all the system and unit parameters included in objective function. The determination of this effect requires a comprehensive sensitivity analysis. But, in the current research, a sensitivity analysis for only two parameters, $t_{fl/p}$ and $C_{pf}$, would be considered because these two parameters are expected to have a relatively high uncertainty in their determination due to the large experience and information needed to calculate them precisely which makes their determination in practice is relatively subjective.

8.3.2 Sensitivity Analysis

A sensitivity analysis is performed on the previously solved example to investigate the effect of changing the parameters $t_{fl/p}$ and $C_{pf}$ on the optimization solution. The investigated range for $C_{pf}$ is the whole range; i.e. $[0,1]$ while for $t_{fl/p}$, only the expected range in real applications would be investigated which is assumed to be $[1, 10]$. Therefore, the optimization problem described in section 8.3.1 would be solved for discrete combinations of the $(t_{fl/p}, C_{pf})$ in the described ranges. The GAMS optimization solutions are represented in Figure 8-1.
The sensitivity analysis shows that the optimum periodicity is slightly affected by the change in $t_{flp}$ value while it is considerably affected by the value of $C_{plf}$ and this effect decreases as $C_{plf}$ approaches to 1.0.

These results show that the determination of failure repair to equal level preventive maintenance time ratio is not an effective factor in determining the optimum periodicity level. While the value of $C_{plf}$ and consequently the type of the system considerably affects the optimum periodicity level. Figure 8-1 shows that at low values of $C_{plf}$, the optimum periodicity level is relatively high. But as the value of $C_{plf}$ increases, the optimum periodicity level decreases. This result can be attributed to the fact that low $C_{plf}$ values mean that preventive maintenance is less costly than failure repair, therefore, it is better to increase the preventive maintenance (system resetting) to avoid the expensive Failures. But at large $C_{plf}$ values, the preventive maintenances and failure repairs of the same level have approximately the same cost. Therefore, performing
excessive preventive maintenance would add costs without balancing savings in less failure repairs. So, the optimum periodicity level is relatively small.

8.4 Comfort Zones Method

In the current research, a new methodology is proposed, using the comfort zones for determining the suitable periodicity and hence the maintenance policy. The comfort zones method relies on the idea of regions in physical programming developed by Messac (1996). The comfort zones method provides the means for direct expression of the maintenance decision maker preference and limitations, which fundamentally impacts the maintenance policy design. Rather than expending substantial efforts tweaking or calculating weights and re-optimizing until a given set of preferences is achieved. Hence, the decision maker is allowed to concentrate more on the physical problem at hand and less on the art of converging to satisfactory weights.

The main idea in the comfort zones method is that the maintenance decision maker defines the following four ranges/zones for each criterion:

- Desirable Zone
- Tolerable Zone
- Undesirable Zone
- Unacceptable zone

Therefore, the designer has to provide a scalar value to define each zone boundary.

A step class function is then defined for each zone to reflect the decision maker perception about the differences between the different zones. In the current research, the following class function definition is proposed:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Class Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desirable</td>
<td>1.0</td>
</tr>
<tr>
<td>Tolerable</td>
<td>0.8</td>
</tr>
<tr>
<td>Undesirable</td>
<td>0.5</td>
</tr>
<tr>
<td>Unacceptable</td>
<td>0</td>
</tr>
</tbody>
</table>
The class function can be defined in endless ways; however, the proposed function would be followed in the current research for the purpose of explaining the methodology.

Next, the step class function is plotted for each criterion and finally, the class function summation for all the included criteria is plotted and the maximum point is located. This maximum point expresses the most comprehensively desirable solution.

The application of the comfort zones methodology to the example studied in the previous section is explained here in detail as follows:

Assuming the maintenance decision maker defined the following comfort zones boundaries:

<table>
<thead>
<tr>
<th>Zone</th>
<th>Periodicity</th>
<th>Quality</th>
<th>Availability</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desirable</td>
<td>&gt;=0.035</td>
<td>&gt;=0.95</td>
<td>&gt;=0.75</td>
<td>&lt;= 10,000</td>
</tr>
<tr>
<td>Tolerable</td>
<td>0.025-0.035</td>
<td>0.9-0.95</td>
<td>0.65-0.75</td>
<td>10,000- 20,000</td>
</tr>
<tr>
<td>Undesirable</td>
<td>0.01 -0.025</td>
<td>0.85-0.9</td>
<td>0.55-0.65</td>
<td>20,000- 25,000</td>
</tr>
<tr>
<td>Unacceptable</td>
<td>&lt;0.01</td>
<td>&lt;0.85</td>
<td>&lt;0.55</td>
<td>&gt; 25,000</td>
</tr>
</tbody>
</table>

The most challenging task in constructing this matrix is determining the periodicity comfort zones. Because of the intimate relationship between maintenance strategy periodicity and system complexity, the limits of these zones reflect the decision maker preferences about the system complexity and its determination needs a lot of experience about the system and historical data about previous maintenance policies and the corresponding system performance.

Then, the following steps show the heuristic of deriving the optimum periodicity level:

Step (1): the four comfort zones are plotted on periodicity, quality, availability and cost figures as depicted in Figure 8-2 to Figure 8-5.
Figure 8-2 Availability comfort zones

Figure 8-3 Comfort zones for product quality
Step (2): For each objective and based on its comfort zones chart, its class function is calculated and plotted as shown in Figure 8-6.

Step (3): the total class function for all the objectives is calculated and plotted as depicted in Figure 8-6, and the point of maximum total class function is located.
Step (4): the periodicity corresponding to the determined optimum point is located on the periodicity axis as shown in Figure 8-6.

Therefore, for the shown example, the optimum periodicity level corresponding to the stated comfort zones is 0.035.

Finally, it is important to mention that previously showed figures and solutions in this chapter are case specific and they depend on the problem data. Nevertheless, the trends of the objectives and the explained steps and solution procedures are quite general and can be applied in any maintenance policy design case. The effect of the problem data on objective trends and optimization solutions needs a comprehensive sensitivity analysis, which would be considered in future work.

Another important note is that the outcome of the optimization problem solution is an optimum periodicity level. This periodicity level can be materialized through many maintenance strategies because the relation between maintenance strategies and periodicity level is a many to one relationship. But this group of maintenance strategies,
associated with a periodicity level, contains one single class maintenance strategy and (theoretically) infinite number of multi-class strategies. As explained in Chapters 5, 6 and 7 respectively, all these strategies would lead to the same product quality level but the availability and maintenance costs would be different and their ranges are described in Chapters 6 and 7. Therefore, in case the decision maker designs a single class maintenance policy like Jamali et al. (2005), Pongpech and Murthy (2006), and Wen-Jinn (2007), the maintenance interval is calculated directly from the optimization result as follows:

$$T_{opt} = \frac{1}{p_{opt} \left( \frac{2}{N_{opt}} - 1 \right)}$$  

Equation 8-6

But, in case of designing a multiple class maintenance policy, there should be a further heuristic to choose the most suitable maintenance policy from the policies associated with the optimum periodicity level. This heuristic is expected to have two modules; the first one is a filtering module that determines the technically feasible multi-class maintenance policies associated with the optimum periodicity level. The second module would have some selection mechanism to choose the best policy. The development of this heuristic is planned in the future work.

### 8.5 Application to Multi-unit Cases

The proposed approach is derived for designing a maintenance policy for single unit. Nevertheless, it is considered a major step in designing a maintenance policy for a whole system like a production line or manufacturing system. The maintenance policy for a multi-unit cases is not simply a multi maintenance policy, i.e. the policy is just each unit is separately maintained according to its own maintenance policy. But due to the different types of dependencies, economical, failure, structural (Wang, 2002) and operational, the optimal maintenance action for a given unit at any time point depends on the states of all other units in the system. Economic dependency means that performing maintenance on several units jointly costs less money and/or time than performing maintenance on each...
subsystem separately. For example, the failure of one unit results in the possible opportunity to perform maintenance on other units (opportunistic maintenance). Failure dependence means that failure distributions of several units are stochastically dependent. The structural dependency means that dismantling of a module requires that other modules be dismantled first due to structural constraint. The operational dependency means that the production layout of certain product links many machines in a certain way that shutting down one machine affects the whole production line.

Traditionally, there were two main approaches for dealing with multi-unit systems maintenance; maintenance grouping and opportunistic maintenance. Group maintenance is convenient for cases where the maintenance cost is composed of fixed (setup) costs and variable costs which depend on the items involved in the maintenance (Wildeman et al., 1997). For example, the setup cost can consist of the downtime cost due to production loss if the system cannot operate during maintenance. Therefore, the first problem in group maintenance is establishing maintenance group activities in the most economic way. Examples of this problem solution are dynamic grouping (Wildeman et al., 1997) and variance reduction grouping (Wilson, 1996). The next problem is designing a group maintenance policy. There are many group maintenance policies developed in the literature like Vergin and Scriabin (1977), Assaf and Shanthikumar (1987), Ritchken and Wilson (1990), Sheu and Jhang (1997), and Wildeman et al. (1997). These policies are quite similar in the idea. An example explaining them is Ritchken and Wilson (1990), (m, T) group replacement policy. It calls for a group perfect maintenance when the system is of age T, or when m failures within the group have occurred, whichever comes first. The (m, T) group replacement policy requires inspection at either the fixed age T or the time when m units have failed, whichever comes first. At an inspection, all failed units are replaced with new ones and all functioning units are serviced so that they become as good as new. The policy decision variables are m and T.

The second multi-item maintenance policy design approach is opportunistic maintenance. It basically refers to the scheme in which preventive maintenance is carried out at opportunities (Cui and Li, 2006). The opportunities for preventive maintenance are traditionally generated by the failure epochs of individual components. At each failure
epoch, the failed components are correctly repaired and other components that are still operational are preventively serviced so that all the components are maintained and restored to certain conditions. An advantage of the opportunistic maintenance is that corrective repair combined with preventive repair can be used to save set-up costs. Opportunistic maintenance is effective when corrective repair on some components requires dismantling of the entire system or corrective repair of some machine requires shutting the whole production line (operational dependence). A representative example for the application of this approach is the serial production line where shutting down a machine shuts down the whole line. Therefore, a corrective repair on these components combined with preventive repair on other or neighboring components might be worthwhile. Another instance is when a certain corrective repair on failed machine can be delayed until the next scheduled preventive maintenance. The drawback of opportunistic preventive maintenance is that by combining both types of repair, the planner may not know in advance which repair actions should be taken, and thus sacrifices the plannable feature of preventive maintenance. Due to the very complex structure of the optimization problem of opportunistic maintenance, research in this area has been confined, for the most part, to two-dimensional control limit policies such as \((n;N)\) policies, where \(n\) represents opportunistic maintenance age and \(N\) stands for the preventive maintenance age in the absence of an opportunity (Rao and Bhadury, 2000). Different approaches have been proposed in the literature for solving this problem like Savic et al. (1995), Mohamed-Salah et al. (1999), Degbotse and Nachlas (2003), Saranga (2004), Zhou et al. (2006), and Iung et al. (2007).

For the future research, some ideas are suggested for the application of the developed periodicity based maintenance policy design approach in multi-machine (unit) case. It is suggested to incorporate the single machine maintenance policy design approach as an initial step in both maintenance grouping and opportunistic maintenance. For maintenance grouping approach, there are no suggested modifications to the first stage; grouping. But, using the periodicity based single machine maintenance design approach is suggested in determining the group replacement/maintenance age, \(T\) in case the group contains identical machines, then the application of the periodicity based maintenance
model is quite direct. But in case the group contains different machines (the general case), then there would be a need for an extension to derive the optimum periodicity for the group. This extension is proposed in the future work.

For opportunistic maintenance, our derived periodicity based maintenance policy design approach can be used to derive the values of $n$, $N$ for each machine according to its parameters. The value of optimum periodicity level calculated by our new approach can help in calculating $n$ such that $n$ lies in a reasonable range around $pr^*$, the calculation of that range is a subject of a proposed future research. The value of $N$ can be determined using the complication rate data as explained by the case study in Section 3.5.

8.6 Application to Different Fields

The complexity based maintenance design approach is suitable for all the applications that can use time-based maintenance i.e., the failures are not catastrophic like manufacturing systems, aircraft non-critical modules, automobiles as well as many other applications. But it is not suitable for the applications that have to use condition-based maintenance.

It is worth noting that the application of the derived approach in different applications may slightly differ according to the application and the objectives. The relations derived in the current research are comprehensive because they consider all the possible objectives. Therefore, these relations need to be tuned with each application. For example, the total maintenance cost equation derived in Chapter (7) considers the downtime cost of the preplanned preventive maintenance while some real life cases states for carrying out the planned preventive maintenance in the between shifts down time. Therefore, there would not be downtime costs accompanying the preventive maintenance and the cost equation needs to be tuned for this specific application. Another example for a different application is the maintenance of non-critical aircraft electrical modules where the module age does not affect the performance quality as the electric component has just two functional states, working or broken down. Therefore, the quality equation should not be included in the model. These examples show that the applicability of the derived approach is quite general.
8.7 Conclusion

This chapter presents the last step in designing the maintenance policy based on finding the optimum periodicity level. Chapters 5, 6, and 7 presented the relationships between the periodicity as well as other unit and system parameters on different system objectives. It has been shown that some of these objectives are conflicting. Therefore, this chapter presented different approaches for finding the best maintenance policy that satisfy the decision maker preferences. First, the simplest and most common approach; weighted sum, was presented with the suggestion and explanation of using AHP objectives weighting method to determine the weights in order to mitigate the drawbacks of weighted sum method. Then, a comfort zones, a new multi-objective optimization approach, was proposed and presented to capture the perception preferences, and limitations of maintenance decision maker and calculate the optimum periodicity level that ultimately maximize decision maker satisfaction.

This chapter presented a relatively easy and simple approach for maintenance policy design based on multi-objectives rather than the traditional mathematically complicated single objectives approaches.

Finally the applications to multi-unit systems and to different fields are discussed and some suggestions for extending this research to the multi-unit system are presented.
9 Case Study

9.1 Introduction

This chapter explains the application of the developed complexity based maintenance design approach for ore trucks. The objectives of this chapter are many folds:

- Explain in detail the steps of applying the developed periodicity based approach to real life example.
- Explain the simplicity of multi-objective periodicity based approach application in comparison to the single objective traditional maintenance policy design approaches.
- Explain the adaptability of the periodicity-based approach to field other than manufacturing systems.

Blischke and Murthy (2003) introduced a case study of an ore loader used in underground metalliferous mining. In the mine, ore is broken up by blasting to fall on the floor of a tunnel. The loader then moves the ore to a chute or conveyor, from which it proceeds to a crusher and ultimately to the mine surface for milling and refining. The loader operates in extreme hot, humid, and dusty conditions and it experiences a high level of vibration as it drives over a rough floor. It also operates on a short cycle of loading and dumping of material and continually handles heavy loads with a lift capacity of 7 tons. Under these tough operating conditions, loaders reliability is less than ideal. A failure study was carried. The full scope of the study included the following aspects:

- Pareto analysis to identify the most frequent causes of failure and to rank failure modes on a cost basis
- Tests to determine whether there was a trend in overall failure rate relative to calendar time
- Weibull analysis of various failure modes to identify burn-in, random, and wear out patterns
- Preventive replacement analysis for components subject to wear out.
The results showed that, for the loader as a whole, most failure modes were of a minor nature, and these generally showed no wear out pattern, and they were tackled by addressing issues in procedures of maintenance operations. But, for some other components, a distinct wear out pattern was identified, and for these components, the optimal preventive replacement policy was examined. Blischke and Murthy (2003) selected the oscillating axle bushing to perform their study because it illustrates features that are relevant in comparable situations across many industries. Figure 9-1 explain the two types of bushings in the ore truck; front axle and rear axle.

![Front Axle Bushing and Rear Axle Bushing](image)

**Figure 9-1 Front and rear axle bushings**

### 9.2 Failure and Maintenance Data

Maintenance activities in the mine site were recorded using a computerized maintenance management system. The various major components of the loaders and other equipment had been coded using the concept of position numbers. The fleet contains six loaders; the failure date for the axle bushing on every one is recorded in a table similar to Table 9-1 (Blischke and Murthy, 2003).
Table 9-1 Failure data for axle bushing for a single vehicle

<table>
<thead>
<tr>
<th>Row</th>
<th>Date</th>
<th>Event</th>
<th>Vehicle Operating Hours</th>
<th>Axle Bushing Operating Hours</th>
<th>Failure (F) or Suspension (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>July, 1990</td>
<td>Ore loader commissioning</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>May, 1991</td>
<td>Axle bushing failure</td>
<td>2662</td>
<td>2662</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>July, 1991</td>
<td>Axle bushing failure</td>
<td>3114</td>
<td>452</td>
<td>F</td>
</tr>
<tr>
<td>4</td>
<td>Feb., 1993</td>
<td>Vehicle overhaul including bushing replacement</td>
<td>5366</td>
<td>2252</td>
<td>S</td>
</tr>
<tr>
<td>5</td>
<td>Aug., 1993</td>
<td>Axle bushing failure</td>
<td>8942</td>
<td>3576</td>
<td>F</td>
</tr>
<tr>
<td>6</td>
<td>Dec., 1994</td>
<td>Current bushing running</td>
<td>13396</td>
<td>4454</td>
<td>S</td>
</tr>
</tbody>
</table>

Where S stands for suspension, which means the replacement of the axle bushing for a reason other than failure. An example for the suspension is the preventive replacement which is carried out because the vehicle is in the maintenance shop and it is a systematic maintenance step to replace the bushing while other work is being done or the whole linkage is replaced with a new one with the old bushing being removed with the old linkage.

Blischke and Murthy (2003) amalgamated the failure data for all axle bushings across the loaders. The resulting data are shown in Appendix (D).

### 9.3 Failure Modeling

To determine the failure trend, the failure data should be analyzed. The first step is to separate the failures from the suspensions. It can be noticed that failure records contains 14 suspensions and 11 failures. Then, the cumulative failure density function can be constructed using the most commonly used method; median rank (Warrington and Jones, 2003).
2005). In the current research, the consistent median rank function proposed by Jacquelin (1993) will be adopted. It is described by the following equation:

\[
F = \frac{i - 0.3175}{n + 0.365}
\]

Equation 9-1

Where:
- \( F \) estimator of the cumulative density function
- \( i \) failure order number
- \( n \) number of samples

assuming that the failure data follows a Weibull distribution (the most typical case), then the Weibull parameters can be derived using the Weibull parameters estimation method explained by Faucher and Tyson (1988).

For Weibull distribution, \( F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^\beta} \), then, using the following mathematical manipulation:

\[
1 - F(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}
\]

\[
\therefore \ln(1 - F(t)) = -\left(\frac{t}{\theta}\right)^\beta
\]

\[
\therefore \ln\left(\frac{1}{1 - F(t)}\right) = \left(\frac{t}{\theta}\right)^\beta
\]

\[
\therefore \ln[\ln(\frac{1}{1 - F(t)})] = \beta \ln\left(\frac{t}{\theta}\right)
\]

Therefore, the following linear form equation can be concluded:

\[
\ln[\ln(\frac{1}{1 - F(t)})] = \beta \ln t - \beta \ln \theta
\]

Equation 9-2

Equation 9-2 indicates that if the failure data follows a Weibull distribution, the \( \ln[\ln(\frac{1}{1 - F(t)})], \ln t \) points would theoretically lie on a straight line of slope \( \beta \) and intercept \( \beta \ln \theta \). Figure 9-2 explains the \( \ln[\ln(\frac{1}{1 - F(t)})], \ln t \) plot. From this figure, it is clear that the failure data follows a five parameters bi-Weibull distribution (Murthy et al., 2004).
This indicates that the failure follows two successive or overlapping Weibull failure patterns where the fitted curves in Figure 9-2 can be used to derive the parameters of each Weibull distribution. First, the failure follows an approximately constant failure rate Weibull distribution of shape parameter $\beta = 1.061$ and scale parameter $\theta = e^{-9.528} = 8011.868 \text{ hr.}$ Then, at age $\ln 8.23$, i.e. 3800 hr., the failure distribution changes to increasing failure rate Weibull distribution with parameters $\beta = 2.552$ and $\theta = e^{-16.524} = 649.987 \text{ hr.}$ The failure rate of the axle bushing is therefore described by the following Equation:
\( \lambda(t) = \begin{cases} 
\frac{1.06}{8011.868} \left( \frac{t}{8011.868} \right)^{0.6-1} & t \leq 3800 \\
\frac{1.06}{8011.868} \left( \frac{t}{8011.868} \right)^{0.6-1} + \frac{2.552}{649.987} \left( \frac{t}{649.987} \right)^{2.552-1} & t > 3800 
\end{cases} \)  \hspace{1cm} \text{Equation 9-3}

This equation is graphed in Figure 9-3, which explains that the axle bushing experiences an approximately constant failure rate 0.00012 till age 3800 hr. then, it experiences an increasing failure rate (wear-out).

![Failure rate for the axle bushing failure bi-weibull distribution](image)

**Figure 9-3** Failure rate for the axle bushing failure bi-weibull distribution

### 9.4 Maintenance Policy Design

In the current section, the design of a maintenance policy for the axle bushing is presented. First, the procedures and the results of the renewal reward theorem are presented and then the application of the periodicity-based approach is explained and a results comparison is shown.

In the current case study, the mine works two shifts per day, but maintenance workers can work outside the normal operating shifts. Loss of ore movement due to breakdown of loaders affects the mine output. An estimate of the value of ore moved per hour when a
loader is working is $15,000. However, the concept of "failure" of the axle bushing covers a range of circumstances. In some cases there is a catastrophic failure, which necessitates dismantling of the loader and repairing it in situ. This can result in two to three hours lost production. In the worst case, there can also be secondary damage to the hydraulic system and to the mechanical linkages.

On the other hand, in most cases, deterioration of the axle bushing is detected in time and the loader is moved to the maintenance bay. If a failure occurs towards the end of the working day, the maintenance crew working in the third shift can repair the loader. Therefore, on balance, the average cost $C_f$ of failure replacement is assessed as $5000. This amount includes the following:

- Cost of the replacement component
  Cost of labor and related overheads, including an allowance for a percentage of overtime working
- Cost of lost production in an average case

For analysis purposes the following assumptions are made.

- The operators became aware of a failure as soon as it occurred.
- The time interval of interest (or horizon) is infinite.
- A spare is available when needed.
- The repair time mean is 2.5 hr.

On the other hand, the cost $C_p$ of preventive replacement is relatively straightforward to estimate. Preventive replacements are carried out at preplanned time, usually in conjunction with routine servicing when the loader is out of service. Thus the cost consists of:

- Cost of the replacement component
- Cost of labor and related overheads

In the present case, this was estimated at $500. Thus the requirements for the steady-state optimality of a preventive replacement policy, namely, the presence of wear-out (increasing failure rate) and $C_f > C_p$, are fulfilled.
9.4.1 Renewal Reward Theorem Approaches

Blischke and Murthy (2003) presented a comparison between two maintenance approaches; age based preventive replacement policy and block preventive replacement policy using the renewal reward theorem. More details about reward renewal theory can be found in Suyono (2003).

For age based preventive replacement policy, the optimum replacement age (from cost point of view) is calculated by optimizing the average cost per unit time described by the following equation:

\[ J(t_p) = \frac{C_f F(t_p) + C_p[1-F(t_p)]}{\int_0^{t_p} [1-F(t_p)] \, dt} \]

Equation 9-4

Where \( C_f \) and \( C_p \) are the costs of replacement in cases of failure and preventive replacement respectively. The minimum of Equation 9-4 is found at replacement age 4476.52 hr. with corresponding cost rate 0.6526 $/hr.

For block replacement policy, Pham (2003) explained that the optimum replacement interval (from cost point of view) is calculated by optimizing the cost per unit time described by the following equation:

\[ G(t) = \frac{C_p + C_f M(t)}{t} \]

Equation 9-5

where the function \( M(t) = \sum_{k=1}^{\infty} F^{(k)}(t) \) denotes the mean number of failures during the time period [0, t] (the renewal function) and \( F^{(k)}(t) \) is the k-fold convolution of the lifetime distribution (a detailed procedure for calculating the K-fold convolution for a distribution function can be found in Aydogdu (2005)). The problem is to derive the optimal block replacement time, \( t^* \), that minimizes \( G(t) \). The optimum block replacement interval is founded at approximately 4000 hr. with corresponding cost rate 0.6542 $/hr.
9.4.2 Evaluation of the Renewal Reward Approach for Maintenance Policy Design

As most of the developed maintenance policy design techniques, the earlier explained technique targets only the cost minimization criterion without considering any other performance criterion. Therefore, this section would comprehensively evaluate the developed solutions to investigate the approach performance with respect to other criteria; availability and quality. First, the periodicity of each technique is calculated using Equation 4-9, then the quality and availability are calculated using Equation 5-9 and Equation 6-15 respectively.

Quality in the current case study context has a different meaning other than the traditional one in manufacturing context. Quality here stands for quality of performance or efficiency. Blischke and Murthy (2003) did not study the effect of axle bushing wear-out on the truck performance. But since the axle bushing main function is to support the axle which transmits the power from the engine to the wheels and some other accessories, then bushing wear out would cause increased power losses and hence less quality/efficiency of power transmission.

The calculations and results for both the age based preventive replacement and block preventive replacement policies are shown in Table 9-2:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Age based preventive replacement</th>
<th>Block preventive replacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>periodicity (pr)</td>
<td>$\frac{1}{4476.52(\frac{2}{1} - 1)} = 0.2234E - 3$</td>
<td>$\frac{1}{4000(\frac{2}{1} - 1)} = 0.25E - 3$</td>
</tr>
<tr>
<td>Availability</td>
<td>$1 - \frac{2.552}{649.987} \times 2.5 \times 649.987^{-0.0002234 \times 2.5} = 0.933$</td>
<td>$1 - \frac{2.552}{649.987} \times 2.5 \times 649.987^{-0.0002234 \times 2.5} = 0.944$</td>
</tr>
<tr>
<td>Quality</td>
<td>$q_{average} = e^{-2 \times 0.2234E - 3} = 0.799$</td>
<td>$q_{average} = e^{-2 \times 0.25E - 3} = 0.819$</td>
</tr>
</tbody>
</table>
These results indicate that in case of adopting age based preventive replacement policy and to minimize the cost, the truck will be available 93.3% of the time and the long run performance quality would be 79.9%. While in case of adopting a block preventive replacement policy, the truck would be available 94.4% and the long run performance quality would be about 82%. It can be noticed that both approaches perform quite similar in this case study.

**9.4.3 Application of the Complexity-Based Approach**

After explaining the traditional maintenance policy design approaches, the application of the new developed complexity-based approach would be explained. The developed maintenance design approach in the current research is developed while manufacturing system is considered as the main application in many steps and that is why the application of the derived approach to a manufacturing system is quite direct. This fact is explained by the examples shown in the previous chapters. But, the data format given in the current case study is considerably different from the standard format assumed during equations derivations all over the previous chapters. Therefore, some equations modifications would be required.

The quality is described by Equation 5-9. But the availability equation needs to be modified to adapt to the case study where there are two conditions in the case study different from the standard assumptions made earlier during derivation of availability equation:

1- The preplanned preventive maintenance is performed in the loader off time. So, it does not affect loader availability. Therefore, the loader availability is affected only by failures.

2- The failure rate of the axle bushing is not increasing all the time. But it stays approximately constant till age 3800 hr. then it increases as depicted in Figure 9-3. Therefore, increasing the periodicity over ($\frac{1}{3800} = 2.63e-4$) would not enhance the failure performance.

The availability in this case is described by the following equation
\[ \text{availability} = \begin{cases} 1 - 0.00012T_{\text{failure repair}} & pr \geq 2.63e-4 \\ 1 - (0.00012 + \frac{\beta}{\theta} (2 \times pr \times \theta)^{1-\beta})T_{\text{failure repair}} & pr < 2.63e-4 \end{cases} \quad \text{Equation 9-6} \]

The cost equation needs to be modified accordingly as follows:

\[ TC = \begin{cases} 0.00012C_f + pr \times C_p & pr \geq 2.63e-4 \\ (0.00012 + \frac{\beta}{\theta} (2 \times pr \times \theta)^{1-\beta})C_f + pr \times C_p & pr < 2.63e-4 \end{cases} \quad \text{Equation 9-7} \]

It is noted that the factor \( \chi \), the maintenance level, in Equation 9-7 assumes the value 1 because the axle bushing has a single feasible maintenance alternative, which is replacement.

The optimization problem is then formulated as follows:

Minimize \( w_1 \cdot \text{quality} + w_2 \cdot \text{availability} + w_3 \cdot \text{total cost} \)

Due to the complex structure of the availability and total cost equations (different definition in different ranges), the use of optimization software like GAMS would be complicated. And because of the exclusion of maintenance level (periodicity extent) from the decision variables due to the unfeasibility of different maintenance levels for the axle bushing, then, the graphical approach will be adopted to solve the optimization problem.

Figure 9-4 depicts the relationship between the quality, availability and cost rate and replacement interval. This Figure is helpful for the maintenance decision maker as it explains the effect of choosing the replacement interval \((1/\text{periodicity})\) on all the criteria simultaneously which considerably helps in performing a comprehensive tradeoff between the included criteria.

The following can be noticed from Figure 9-4:

- The availability remains constant at approximately 99% as the replacement interval is \( \leq 3800 \text{ hr} \) due to the constant failure rate and due to that the planned replacements are carried out in the truck down time and do not affect the operation. Then, the availability decreases as the replacement interval increases due to the increasing failure rate.
The performance quality decreases as the replacement interval increases because of the axle bushing ageing.

- The cost rate first decreases considerably with increasing replacement interval in the range of low intervals. Then, the cost rate slightly decreases as the replacement interval increases till age 3800 hr. This cost performance is due to that the replacements in this range are only due to constant rate random failures. But, after 3800 hr., the cost considerably increases due to the failure rate increase ($\beta=2.552$).

![Figure 9-4 Quality, availability and cost versus replacement interval](image)

Figure 9-4 and the previous discussion explains that decreasing the replacement interval to an approximate range of 1000 hr. to 2000 hr. does not affect the availability but it slightly increases the cost rate while considerably improving the quality. Figure 9-5 explains a comparison between the results of renewal reward traditional optimization of the block replacement policy and the developed complexity based approach in case of adopting a replacement interval of 1000 hr.
Figure 9-5 indicates that using the complexity based developed approach improves the availability by about 6% and the Quality by about 9% and the periodicity is improved (increased) about 400% which means considerably less operational complexity while the cost rate increase by less than € 4/hr. i.e., the complexity based approach performs better than the renewal reward approach in all the considered performance criteria except the cost. This result may be attributed to that the renewal reward approach for maintenance policy design considers only the cost criteria at the contrast of the complexity based approach which considers all the criteria. Therefore, it can be concluded that the complexity based approach is more capable of maintenance policy design than traditional approaches because it considers simultaneously multi-objectives and it is application is more simple.

It is worth here to note that the complexity-based approach is compared to the block replacement policy only because of the following reasons:

- Both of them rely on replacing the axle bushing at fixed time intervals. But in age replacement policy, the replacement is carried out at fixed ages, not fixed time intervals.
- The results of block replacement policy and age based replacement policy are quite near as explained by the calculations in section 9.4.2.

9.5 Conclusion

This chapter explains the application of the complexity-based approach for maintenance policy design. A mining loader truck maintenance policy is studied. A comparison between the traditional renewal reward theorem for maintenance optimization and the developed complexity based approach is presented. It has been shown that the complexity base approach is more capable to design maintenance policies considering multi-objectives.

The application on mining loader case explained the generality and flexibility of the developed complexity based approach such that it can be applied to fields other than manufacturing systems.
10 Conclusions and Future Work

10.1 Research Contribution

This research represents the first step in designing maintenance strategies aiming to reduce system complexity. This research presents a comprehensive maintenance strategy design approach that considers reducing the system complexity as the main objective and considers improving the other performance criteria like quality, availability and cost as well. In the way of developing this novel approach, new definitions and metrics are developed; a new rigorous complexity based mathematical definition for the failure is introduced that is able to model all failure types, a new metric called “complication rate” is introduced to measure the system functionality degradation and gradual failure in terms of complexity. Finally and most importantly, a measure for the periodicity is developed that can quantitatively assesses the amount of resetting the maintenance strategy can present to the system.

The new periodicity metric is then used to develop the mathematical relationship relating product quality to the maintenance policy in manufacturing systems and it is used to develop relationships for availability and cost as well. These relationships make it very easy to estimate any performance criteria in terms of the applied maintenance policy.

Finally, a new optimization heuristic called “comfort zones” is developed to calculate the optimum amount of periodicity (maintenance) that the system needs to reduce the operational complexity while keeping the other performance criteria in acceptable ranges. The comfort zones approach is considerably easy to apply and does not need any sophisticated algorithms or calculations.

10.2 Contributions

- A new mathematical definition for the failure is developed. This definition is able to model physical failure as well as functional failure and sudden failure as well as gradual failure because it is based on the common feature of failure; loosing
functionality. Related to this definition, a new measure for gradual failure is introduced. This measure combined with the failure rate can completely describe the system failure.

- A novel metric for measuring the amount of periodicity is developed. This metric can easily compare different maintenance policies from the aspect of reducing the system complexity.

- In case of manufacturing systems, it has been found that the average products quality can be formulated as function in the periodicity of the applied maintenance policy. A mathematical relationships between maintenance policy parameters and both system availability and maintenance related cost are developed.

- Two optimization approaches are presented to calculate the optimum periodicity level; weighted sum and comfort zones. The integration of AHP model with the weighted sum is presented in order to mitigate the subjectivity of the weights determination.

- The comfort zones approach for determining the optimum periodicity level is developed. It is explained that it can capture designer preferences and limitations while being an easy approach compared to any traditional optimization technique.

### 10.3 Conclusions

The following concluding remarks can be pointed out from the current research:

- The application of the developed complexity based maintenance design approach enables designing maintenance policies that reduce the system complexity while improving the other performance criteria

- The new failure definition and the complication rate metric are very useful in modeling all types of failures

- The periodicity metric, while not the only decision criteria, it can easily and effectively enhance the maintenance decision making

- Product quality in manufacturing systems is function in system complication rate and periodicity of applied maintenance policy
- The application of comfort zones method, while not able to find the exact optimum, is easy applicable and useful for finding a near optimal periodicity level.

- The derived complexity based maintenance design approach is applicable to different fields. Generally, all the fields that can apply time based maintenance.

10.4 Recommendations for Future Work

- Extending the model to the multi-unit case considering the different types of dependence and considering the two main multi-unit maintenance approaches; group maintenance and opportunistic maintenance.

- Expanding the model to deal with the cases of dependent resetting plans.

- Considering the case of non-linear complication where the complication rate is not constant all over the life cycle

- Extending the model to the cases of limited life-time systems.

- Considering the discrete nature of the repair/maintenance level in the model where there are definite repair/maintenance courses of procedures.

- Extending the model to the multi-class maintenance polices, where the system is maintained by different maintenance levels with different frequencies.

Conduct a comparison study between the developed maintenance design approach and a condition based maintenance (CBM) approach to investigate the effect on the system complexity as well as the different system performance criteria.
References


Messac, Achille. (1996). *Physical programming: effective optimization for computational design,* Salt Lake City, UT, USA. 962-974


Appendix (A) Depth of Cut Samples Readings

(Ott et al., 2005)

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Appendix (B) AHP approach for objectives weights determination

The Analytical Hierarchy Process (AHP) is a widely-used technique for comparing alternatives with respect to many objectives. The AHP is based on the natural human ability to make sound judgments about problems. It facilitates decision-making by organizing perceptions, feelings, judgments and memories into a framework that exhibits the forces influencing a decision. AHP has many applications in various areas which include systems engineering, operations research and management science, conflict management, capital budgeting, strategic business planning and marketing, and resource planning Vaidya and Kumar (2006). The AHP relies on the ability of the decision maker to decompose the main problem into a hierarchy of smaller decision problems, which consist of different objective and subjective factors that work together to influence the overall goal. The overall result of using the AHP is a priority vector that provides a ranking of the different alternatives under consideration.

The application of AHP involves three major steps. The first step is related to selecting the evaluation criteria and constructing the hierarchy. Secondly, the relative importance (priority) of criteria/alternatives is identified through pair-wise comparisons. Finally, these priorities are synthesized to obtain each alternative overall priority and the one with the highest priority is selected. In the current research, the main interest is concerned with the second step of determining the overall priority of the criteria.

The analytic hierarchy process relies on pair-wise comparisons to evaluate the importance of the criteria, sub-criteria, and alternatives. Saaty (1980) pointed out that making judgments based on pair-wise comparisons enhances the formulation of the problem so that it can be handled more easily. In this step, the decision maker has to construct a matrix of pair-wise comparisons of elements where the entries indicate the strengths with which one element dominates another. These entries may be determined directly from quantitative information about the relative importance of the criteria/objectives if available or they may be determined using a method for scaling of
weights as explained by the following table Bhushan and Rai (2004), Saaty (1986) and Wind and Saaty (1980):

<table>
<thead>
<tr>
<th>Comparison matrix entry (intensity of relative importance)</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two objectives have equal importance to the decision maker</td>
</tr>
<tr>
<td>3</td>
<td>Moderate Importance</td>
<td>Experience and judgment slightly favor one objective over another</td>
</tr>
<tr>
<td>5</td>
<td>Strong Importance</td>
<td>Experience and judgment strongly favor one objective over another</td>
</tr>
<tr>
<td>7</td>
<td>Very strong Importance</td>
<td>An objective is favored very strongly over another, its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Extremely strong Importance</td>
<td>The evidence favoring one objective over another is of the highest possible order of affirmation</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values to reflect fuzzy inputs</td>
<td>Used when there is no good word to describe it</td>
</tr>
<tr>
<td>Reciprocals</td>
<td>Reflecting dominance of second alternative compared with the first</td>
<td>Used when the activity is less important than the other one</td>
</tr>
</tbody>
</table>

A typical pair-wise comparison matrix of order 3 can be expressed as follows:
Where \( a_{ij} \) is the decision maker's judgment on the relative importance of objective \( i \) to objective \( j \) and it can be noticed in Equation B-1 that for all \( i, j \), \( a_{ij} = \frac{1}{a_{ji}} \) which will later be explained in detail.

The goal of AHP is to use the pair-wise comparison matrices to establish the values for the weights of the criteria and alternatives. In this context, the concept of consistency of the comparison matrix should be explained. A matrix is said to be consistent if all its elements \( a_{ij} \) satisfy the transitivity and reciprocity rules. The transitivity rule is satisfied if \( a_{ij} \times a_{jk} = a_{ik} \), for all \( i, j, k \). The reciprocity rule is satisfied if \( a_{ij} = \frac{1}{a_{ji}} \) for all \( i, j \).

Assuming the weight vector is represented as \( \mathbf{w} = [w_1, w_2, w_3]^T \), and assuming a perfect consistent comparison matrix \( \mathbf{A} \), then the weights can be calculated by solving the matrix equation:

\[
\mathbf{A} \mathbf{w} = \mathbf{n} \mathbf{w}
\]

Equation B-2

Where \( \mathbf{n} \) represents the matrix size. But, in typical practice, the decision maker is not perfectly consistent in making pair-wise comparisons. Therefore, the AHP allows a small amount of inconsistency in making comparisons. Inconsistencies take place when \( a_{ik} \times a_{ij} \neq a_{ij} \). The presence of inconsistencies implies that each \( (i, j) \) entry of \( \mathbf{A} \) is actually an approximation of the real weight. Thus, \( \mathbf{A} \) is no longer of rank one. And more than one non-zero eigen-value might exist. In cases associated with inconsistencies, Saaty and Hu (1998) proposed determining the weight vector by solving the following eigen-value problem:

\[
\mathbf{A} \times \mathbf{w} = \lambda_{\text{max}} \times \mathbf{w}
\]

Equation B-3
where \( \tilde{w} \) is an approximation to the underlying exact priority vector, and \( \lambda_{\text{max}} \) is the maximum eigen-value of \( A \). Saaty and Hu (1998) explained that \( \lambda_{\text{max}} \geq n \), with equality holding in the perfectly consistent case. Therefore, it is critical to assess the level of inconsistency in the pair-wise comparison matrix. To do so, the following terms can be defined and calculated according to Saaty (1980) as follows:

Coefficient of Inconsistency (CI): it represents the deviation from the perfect consistency, and it can be calculated as follows.

\[
CI = \frac{\lambda_{\text{max}} - n}{n-1}
\]

Equation B-4

Coefficient of Random Inconsistency (CRI): it is the average CI for randomly generated reciprocal matrices. The CRI values for different order of the matrices are established by Saaty and Mariano (1979) as follows:

<table>
<thead>
<tr>
<th>Matrix Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
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<tbody>
<tr>
<td>CRI</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.9</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Inconsistency Ratio (IR): it is the ratio of the Coefficient of Inconsistency (CI) to the Coefficient of Random Inconsistency (CRI). It expresses the degree of inconsistency in the comparison matrix and it is described by the following Equation.

\[
IR = \frac{CI}{CRI}
\]

Equation B-5

Cao *et al.* (2008) explained that if \( IR > 0.1 \), the decision-maker judgments are required to be revised until an acceptable level of consistency is reached.
Appendix (C) GAMS Program Code and Solution Report

1 _GAMS Rev 148 x86/MS Windows 10/14/08 20:30:42 Page »1
2 General Algebraic Modeling System
3 Compilation
4
5 1 variables
6 2 util utility
7 3 pr periodicity
8 4 x resetting extent;
10 5 positive variable pr ;
11 6 positive variable x;
12 7 x.up =1 ,
13 8
14 9
15 10
16 11 scalar b beta /2.8/ ;
17 12 scalar tm minimal maintenance time /0.5/ ;
18 13 scalar td time difference /9.5/ ;
19 14 scalar c ceta /50/ ;
20 15 scalar Rm cost of minimal repair resources /200/ ;
21 16 scalar R cost of overhall /8000/ ;
22 17 scalar K cost of unit offtime /50000/ ,
23 18 scalar Wm minimum labour wage per unit time /200/ ;
24 19 scalar Wx maximum labour wage per unit time /800/ ;
25 20 scalar l labour wage curve slope;
26 21 scalar w1 quality weight /0.566/ ;
27 22 scalar w2 availability weight /0.09/ ;
28 23 scalar w3 cost weight /0.354/ ;
29 24 scalar kq constat of quality equation /1.2/ ;
30 25 scalar cq constant of quality equation /0.02/ ;
31 26 scalar f failure threshold /0.3/ ;
32 27 scalar lc complication rate /0.01/ ;
33 28 scalar cpf cost of preventive maintenance to failure off time /1.0/ ;
34 29 scalar tfp ratio of failure repair time to preventive maintenance /10.0/ ;
35 30
36 31
37 32
38 33 l=Wx-Wm;
39 34
4 0 35 4 1 36 pr.l=0.01 ; 4 2 37 x.l=0.5; 4 3 38 4 4 39 4 5 40 4 6 41 equations 4 7 42 4 8 43 utility utility function equation 4 9 44 failure gradual failure equation; 5 0 45 5 1 46 utility..util =e= ((1-pr)/0.05)+ w1*(1-(kq*exp(-1*cq/(2*pr))))+ w2*((0.01+ 5 2 (b/c)*((2*pr*C)**((1-b))))*tm*tfp + pr*(2-x)*td)+(w3/38700)* ((0.01+(b/c)*(( 5 3 2*pr*C)**((1-b))))*(Rm+(K*tfp*tm)+(Wm*tfp*tm))+ pr*(2-x)*(R+(K*cpf*td)+(td*( 5 4 Wm+l*x)))- 1300); 5 5 47 failure..pr*((2/x)-l)=g= lc/f; 5 6 48 5 7 49 model weighted_sum /all/ ; 5 8 50 5 9 51 solve weighted_sum using nlp minimizing util; 6 0 52 6 1 53 display pr.l, x.l, util.l; 6 2 6 3 6 4 COMPILATION TIME = 0.000 SECONDS 2 Mb WIN225-148 May 29, 2007 6 5 _GAMS Rev 148 x86/MS Windows 10/14/08 20:30:42 Page »2
6 6 General Algebraic Modeling System 6 7 Equation Listing SOLVE weighted_sum Using NLP From line 51 6 8 6 9 7 0 ---- utility =E= utility function equation 7 1 7 2 utility.. util + (64.7092962955271)*pr + (0.0523837984496124)*x =E= 7 3 20.5815865116279 ; (LHS = 0.216238166056044, INFES = 20.3653483455719 ***) 7 4 7 5 7 6 ---- failure =G= gradual failure equation 7 7 7 8 failure.. (3)*pr - (0.08)*x =G= 0.033333333333333 ; 7 9 8 0 (LHS = 0.03, INFES = 0.003333333333333 ***) 8 1 8 2 _GAMS Rev 148 x86/MS Windows 10/14/08 20:30:42 Page »3
8 3 General Algebraic Modeling System 8 4 Column Listing SOLVE weighted_sum Using NLP From line 51
util utility
0 (.LO, .L, .UP = -INF, 0, +INF)
utility

pr (.LO, .L, .UP = 0, 0.01, +INF)
(64.7093) utility
(3) failure

x resetting extent
.x (.LO..L,.UP = 0,0.5,1)
(0.0524) utility
(-0.08) failure

MODEL STATISTICS

BLOCKS OF EQUATIONS 2 SINGLE EQUATIONS 2
BLOCKS OF VARIABLES 3 SINGLE VARIABLES 3
NON ZERO ELEMENTS 5 NON LINEAR N-Z 4
DERIVATIVE POOL 12 CONSTANT POOL 30
CODE LENGTH 117

GENERATION TIME = 0.016 SECONDS 3 Mb WIN25-148 May 29, 2007
EXECUTION TIME = 0.016 SECONDS 3 Mb WIN25-148 May 29, 2007
SOLVESUMMARY

MODEL weighted_sum OBJECTIVE util
TYPE NLP DIRECTION MINIMIZE
SOLVER CONOPT FROM LINE 51

**** SOLVER STATUS 1 NORMAL COMPLETION
***** MODEL STATUS 3 UNBOUNDED
***** OBJECTIVE VALUE -10000000000.0000

RESOURCE USAGE, LIMIT 0.063 1000.000
ITERATION COUNT, LIMIT 7 10000
EVALUATION ERRORS 0 0

The model has 3 variables and 2 constraints
with 5 Jacobian elements, 4 of which are nonlinear.
The Hessian of the Lagrangian has 2 elements on the diagonal,
1 elements below the diagonal, and 2 nonlinear variables.
util: The variable has reached 'infinity'

** Unbounded solution. A variable has reached 'Infinity'
Largest legal value (Rtmaxv) is 1.00E+10

The allowable range can be changed with option:
Rtmaxv=x xx e + xx
util: The variable is unbounded

CONOPT time Total 0.062 seconds
of which: Function evaluations 0.000 = 0.0%
1st Derivative evaluations 0.000 = 0.0%
Workspace = 0.03 Mbytes
Estimate = 0.03 Mbytes
Max used = 0.01 Mbytes

LOWER LEVEL UPPER MARGINAL

--- EQU utility 20.582 20.582 20.582 1.000
--- EQU failure 0.033 6.8225E+8 +INF

utility utility function equation
failure gradual failure equation

LOWER LEVEL UPPER MARGINAL

--- VAR util -INF -1.00E+10 +INF UNBND
--- VAR pr 0.05 +INF -14.657 NOPT
--- VAR x 1.000 1.000 -191.133

util utility
pr periodicity
x resetting extent

**** REPORT SUMMARY : 1 NONOPT (NOPT)
INFEASIBLE
UNBOUNDED (UNBND)
0 ERRORS

_GAMS Rev 148 x86/MS Windows 10/14/08 20:30:42 Page »

General Algebraic Modeling System
Execution

-- 53 VARIABLE pr.L = 0.05 periodicity
-- VARIABLE x.L = 1.000 resetting extent
-- VARIABLE util.L = -1.0000E+10 utility

EXECUTION TIME = 0.000 SECONDS 2 Mb WIN225-148 May 29, 2007

USER: Onur Kuzgunkaya G061116:1900AP-WIN
University of Windsor, Industrial and Manufacturing SystemsDC5799
License for teaching and research at degree granting institutions
219 **** FILE SUMMARY
220
221 Input C:\Documents and Settings\Khaldon\My Documents\gmsdir\projdir\utilit
222 y function.gms
223 Output C:\Documents and Settings\Khaldon\My Documents\gmsdir\projdir\utilit
224 y function.lst
Appendix (D) Oscillating Axle Bushing Failure Data

(Blischke and Murthy, 2003)

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</table>
VITA AUCTORIS

NAME: Khaldon Ahmed
PLACE OF BIRTH: Cairo, Egypt
YEAR OF BIRTH: 1972
EDUCATION: - Military technical College, Cairo, Egypt
           1990-1995, B. Sc., Aeronautical Engineering
           - Military technical College, Cairo, Egypt
           1997-2000, M. Sc., Mechanical Engineering