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Parameter Selection for Improved Handling Performance of Automated Vehicles

Sijie Zhang
University of Windsor

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Parameter Selection for Improved Handling Performance of Automated Vehicles

By

ZHANG Sijie

A Thesis
Submitted to the Faculty of Graduates Studies
Through the Department of Mechanical, Automotive, & Materials Engineering
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at the University of Windsor

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Parameter Selection for Improved Handling Performance of Automated Vehicles

by

Sijie Zhang

APPROVED BY

______________________________
A. Emadi
Department of Electrical & Computer Engineering

______________________________
A. Rahimi
Department of Mechanical, Automotive, & Materials Engineering

______________________________
B. Minaker, Advisor
Department of Mechanical, Automotive, & Materials Engineering

May 22, 2020
Declaration of Originality

I hereby certify that I am the sole author of this thesis and that no part of this thesis has been published or submitted for publication.

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Abstract

With the increasing popularity of autonomous vehicles, much research has been done focusing on vehicle vision and tracking ability, but the handling performance of the vehicle itself is still important, especially under emergency circumstances. However, production vehicles are developed with the ease of control for human drivers as one of the criteria. What if this restraint is removed from the vehicle, and then the machine driver is developed? Is the vehicle able to reach a higher limit than the current maximum handling performance? This research attempts to answer these questions, through the application of a curvature based lateral controller to a 10 DOF nonlinear vehicle model. The concept of a driver’s internal vehicle model is used, and is proposed to represent varying skill levels of the driver. The simulation is conducted in Simulink, with integrated MATLAB codes. The results obtained from the one-lap vehicle tracking simulation show only a marginal handling performance increase with the change of handling characteristics and different drivers’ skill level.
To my parents, for their love and support.
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Chapter 1

Introduction

1.1 Motivation

It is said that vehicle handling is an overall measure of the responsiveness of the vehicle-driver combination[1]. Therefore, 'the vehicle plus the human driver equals handling' is a general formula for the automotive engineering community. To increase handling performance, improvement can be either made on the vehicle or the drivers’ skills. Since dramatically raising the average person’s driving skills is hard to realize, automotive manufacturers have traditionally designed and built cars with high controllability, to indirectly make the vehicle safer.

As times change, autonomous vehicles have become a new popular topic among the automotive and software industries. To develop and test new machine drivers for the best handling performance, production vehicles are the current choice for prototyping. However, production vehicles are developed with the ease of control for human drivers as one of the criteria. What if this restraint is removed from the vehicle, and then the machine driver is developed? Is the vehicle able to reach a higher limit than the current maximum handling performance? This research explores the potential to improve the handling of autonomous vehicles through careful selection of the vehicle parameters, in particular, the effect of suspension roll stiffness distribution, with no regard to human controllability. It will use a series of numerical experiments featuring vehicles with varying handling characteristics, and driver models representative of both human and autonomous drivers.

1.2 Literature Review

1.2.1 Vehicle Model

There is no standard for researchers to generate the equations of motion of a vehicle. Based on the assumptions made about the vehicle, the model complexity varies dramatically. For vehicle handling and stability, the simplest and most widely used vehicle model is the yaw plane or ‘bicycle’ model (which has nothing to do with bicycles) that treats the vehicle as having a single-track (i.e., the effect of the left and right side tires are assumed to be identical, so the width is ignored).
This classic two degrees of freedom (DOF) bicycle model includes only the lateral and yaw motions, and uses a simple linear tire model. Such a model is simple enough to be used as a path planner or a plant for controller design[2].

**Bicycle Model**

The bicycle model is discussed in detail by Minkine[3] in his 2019 text. To obtain the equations of motion of the model, both kinetic and kinematic analyses are conducted. A single track model that ignores the track width (across the axle) of the vehicle, as shown in Figure 1.1, is the key assumption. The road surface is assumed to be flat, so that lateral, longitudinal, and yaw motions are the only three motions existing on this model. However, since the lateral force is the only one assumed to change the vehicle’s motion, the model only has two degrees of freedom, and the forward speed is assumed to be held constant.

The model begins with Newton–Euler equations of translation and rotation.

\[
\sum f = m(\dot{\mathbf{v}} + \mathbf{\omega} \times \mathbf{v}) \quad (1.1)
\]

\[
\sum m_G = I_G \mathbf{a} + \mathbf{\omega} \times I_G \mathbf{\omega} \quad (1.2)
\]

where \( \mathbf{v} \), \( \mathbf{\omega} \) and \( \mathbf{a} \) are the linear velocity, angular velocity and angular acceleration vectors of the vehicle, and \( I_G \) is the inertia matrix.

In the field of vehicle dynamics, it is common to use the Society of Automotive Engineers(SAE) standard notation for forces and velocities. In this case, the velocity and angular velocity are \( \mathbf{v} = u\hat{\mathbf{i}} + v\hat{\mathbf{j}} + w\hat{\mathbf{k}} \) and \( \mathbf{\omega} = p\hat{\mathbf{i}} + q\hat{\mathbf{j}} + r\hat{\mathbf{k}} \). The force and moment vectors are \( \mathbf{f} = X\hat{\mathbf{i}} + Y\hat{\mathbf{j}} + Z\hat{\mathbf{k}} \) and \( \mathbf{m} = L\hat{\mathbf{i}} + M\hat{\mathbf{j}} + N\hat{\mathbf{k}} \). The assumptions about the limited motions are substituted to get:

\[
\sum Y = m(\dot{\mathbf{v}} + ru) \quad (1.3)
\]

\[
\sum N = I_{zz}\dot{r} \quad (1.4)
\]

where \( \mathbf{v} \) and \( u \) are the lateral velocity and longitudinal velocity of the vehicle’s centre of mass; \( \dot{\mathbf{r}} \) is the yaw rate (angular velocity around the vertical axis); \( m \) and \( I_{zz} \) are the mass and yaw moment of inertia. The lateral force is \( Y \) and \( N \) is the yaw moment. The sum of the lateral force and yaw
moment are:

\[ \sum Y = Y_f + Y_r \]  \hspace{1cm} (1.5) 

\[ \sum N = aY_f - bY_r \]  \hspace{1cm} (1.6)

where \( Y_f, Y_r \) are the lateral force at front tire and rear tire; \( a \) and \( b \) are the distance from center of gravity to front and rear axle. Assembling these equation gives:

\[ Y_f + Y_r = m(\dot{\alpha} + ru) \]  \hspace{1cm} (1.7)

\[ aY_f - bY_r = I_{zz}\dot{r} \]  \hspace{1cm} (1.8)

Rearranging into vector form:

\[
\begin{bmatrix}
1 & 1 \\
a & -b
\end{bmatrix}
\begin{bmatrix}
Y_f \\
Y_r
\end{bmatrix}
= 
\begin{bmatrix}
m(\dot{\alpha} + ru) \\
I_{zz}\dot{r}
\end{bmatrix}
\]  \hspace{1cm} (1.9)

A linear tire model is often implemented with the yaw plane model to keep the simplicity and linearity of the model, where lateral force is considered to be proportional to the tire slip angle \( \alpha \) only. Because the tire force opposes motion, the direction of the tire force is always opposite to the direction of the slip angle. This gives:

\[ Y_f = -c_f \alpha_f \]  \hspace{1cm} (1.10)

\[ Y_r = -c_r \alpha_r \]  \hspace{1cm} (1.11)

or:

\[
\begin{bmatrix}
Y_f \\
Y_r
\end{bmatrix}
= 
\begin{bmatrix}
c_f & 0 \\
0 & c_r
\end{bmatrix}
\begin{bmatrix}
\alpha_f \\
\alpha_r
\end{bmatrix}
\]  \hspace{1cm} (1.12)

The tire slip angle is defined by the lateral and forward velocities of the tire \( (v_l \text{ and } u_l, \text{ respectively}):\n
\[ \tan \alpha = \frac{v_l}{u_l} \]  \hspace{1cm} (1.13)

or in the case of the front tire, where the tire can steer:

\[ \tan(\alpha_f + \delta) = \frac{v_l}{u_l} \]  \hspace{1cm} (1.14)

where \( \delta \) is the steer angle of the front tire. If one uses the location of the front tire relative to the centre of mass, kinematics requires that:

\[ v_l = v + ra \]  \hspace{1cm} (1.15)
and:

\[ u_t = u + \frac{r^2}{2} \]  \hspace{1cm} (1.16)

where \( t \) is the track width (across the axle). Assuming small slip and steer angles, and noting that \( u \gg r^2 \):

\[ \alpha_f = \frac{v + ra}{u} - \delta \]  \hspace{1cm} (1.17)

and:

\[ \alpha_r = \frac{v - rb}{u} \]  \hspace{1cm} (1.18)

or:

\[
\begin{bmatrix}
\alpha_f \\
\alpha_r
\end{bmatrix} = \frac{1}{u} \begin{bmatrix}
1 & a \\
1 & -b
\end{bmatrix} \begin{bmatrix}
v \\
r
\end{bmatrix} - \begin{bmatrix}
\delta_f \\
0
\end{bmatrix}
\]  \hspace{1cm} (1.19)

then:

\[
\begin{bmatrix}
Y_f \\
Y_r
\end{bmatrix} = -\frac{1}{u} \begin{bmatrix}
c_f & 0 \\
c_r & 0
\end{bmatrix} \begin{bmatrix}
1 & a \\
1 & -b
\end{bmatrix} \begin{bmatrix}
v \\
r
\end{bmatrix} - \begin{bmatrix}
\delta_f \\
0
\end{bmatrix}
\]  \hspace{1cm} (1.20)

Substituting into the kinetic equations gives:

\[
\begin{bmatrix}
m(\dot{v} + ru) \\
I_{zz}\dot{r}
\end{bmatrix} = -\frac{1}{u} \begin{bmatrix}
c_f & 0 \\
c_r & 0
\end{bmatrix} \begin{bmatrix}
1 & a \\
1 & -b
\end{bmatrix} \begin{bmatrix}
v \\
r
\end{bmatrix} - \begin{bmatrix}
\delta_f \\
0
\end{bmatrix}
\]  \hspace{1cm} (1.21)

Rearranging gives the classic linear first order differential equation form:

\[
\begin{bmatrix}
m & 0 \\
0 & I_{zz}
\end{bmatrix} \begin{bmatrix}
\dot{v} \\
\dot{r}
\end{bmatrix} = \frac{1}{u} \begin{bmatrix}
c_f + c_r & ac_f - bc_r + mu^2 \\
ac_f - bc_r & a^2c_f + b^2c_r
\end{bmatrix} \begin{bmatrix}
v \\
r
\end{bmatrix} = \begin{bmatrix}
c_f \\
ac_f
\end{bmatrix} \begin{bmatrix}
\delta_f \\
0
\end{bmatrix}
\]  \hspace{1cm} (1.22)

Steady State Analysis

Assuming steady state motion indicates the vehicle is cornering at a constant rate, which removes the \( \dot{v} \) and \( \dot{r} \) part from the equation of motion, leaving:

\[
\begin{bmatrix}
m & 0 \\
0 & I_{zz}
\end{bmatrix} \begin{bmatrix}
\dot{v} \\
\dot{r}
\end{bmatrix} = \frac{1}{u} \begin{bmatrix}
c_f + c_r & ac_f - bc_r + mu^2 \\
ac_f - bc_r & a^2c_f + b^2c_r
\end{bmatrix} \begin{bmatrix}
v \\
r
\end{bmatrix} = \begin{bmatrix}
c_f \\
ac_f
\end{bmatrix} \begin{bmatrix}
\delta_f \\
0
\end{bmatrix}
\]  \hspace{1cm} (1.23)

Once can now divide \( v \) and \( r \) by \( \delta_f \), leaving a linear algebraic equation in two unknowns, often referred to as the 'steady state gains'.

\[
\begin{bmatrix}
\frac{\dot{v}}{\delta_f} \\
\frac{\dot{r}}{\delta_f}
\end{bmatrix} = \begin{bmatrix}
\frac{c_f}{c_f} \\
\frac{c_r}{ac_f}
\end{bmatrix}
\]  \hspace{1cm} (1.24)

One can solving the algebraic equation to obtain expressions for the steady state gains:

\[
\frac{\dot{v}}{\delta_f} = \frac{c_f u(c_c b^2 + ab c_r - m a u^2)}{c_f c_a a^2 + 2c_f c_r a b - m c_f a u^2 + c_f c_r b^2 + m c_r m b u^2}
\]  \hspace{1cm} (1.25)
For a vehicle, there is a cornering radius $R$ determined by vehicle path that must obey the following relationship:

$$R = \frac{u}{r}$$  \hspace{1cm} (1.27)

At very low speed, the cornering radius tends toward a limit value, the kinematic cornering radius $R_0$, as shown in Figure 1.2, which is determined by the steer angle. Assuming a small steer angle, geometry gives:

$$R_0 = \frac{a + b}{\delta_f}$$  \hspace{1cm} (1.28)

By combining Equation 1.26, Equation 1.27 and Equation 1.28, one can show:

$$\frac{R}{R_0} = 1 - \frac{mu^2(ac_f - bc_r)}{(a + b)^2c_fc_r}$$  \hspace{1cm} (1.29)

In Equation 1.29, the $ac_f - bc_r$ term becomes the one that dictates the vehicle response. If $ac_f > bc_r$, then $R > R_0$. It indicates that the vehicle corners less than it is steered, which is called understeer. If $ac_f < bc_r$, then $R < R_0$. It indicates that the vehicle corners more than it is steered, which is called oversteer. If $ac_f = bc_r$, then $R = R_0$. It indicates that the vehicle corners exactly as it is steered, which is called neutral steer. Ideally, the neutral steer is the most balanced configuration. Since Equation 1.29 is based only on the steady state linear model, in reality, it is very hard for a production vehicle to perfectly reach a neutral steer configuration. In practice, almost all production vehicles are designed with an understeer configuration, based on stability concerns[3]. An
understeer vehicle is always stable in yaw, where an oversteer vehicle will have a stability limit based on the forward speed.

**High DOF Vehicle Models**

For research focused on not only the yaw motion, but also the roll and pitch motions, the vehicle model can be expanded by considering all translational and rotational motions, as well as adding four degrees of freedom for the wheel rotation dynamics. Even more sophisticated models are widely developed, where four additional degrees of freedom are included to model the motion of the individual suspensions. These models are more frequently used for ride quality studies.

Shyrokau et al.[4] implemented such a vehicle model for simulation while straight-line braking and “sine with dwell” cornering in order to investigate the stability of a proposed controller for a full electric vehicle. In fact, Setiawan et al.[5] simulated and validated a fourteen DOF vehicle model. The conclusion drawn was that such a model is accurate enough to represent actual vehicle dynamic behaviour, with slight differences created by the assumption of body flexibility and movement of the suspension roll center. To further increase the accuracy of the vehicle model, a more detailed suspension was implemented by Venture et al.[6], expanding the vehicle model to thirty-eight DOF, including the toe, camber and kingpin angle (orientation of the wheel carrier), along with the change of the track width and wheelbase. The result showed that a reduced twenty DOF model was sufficient for dynamic parameter identification.

Furthermore, virtual prototyping has been growing in popularity, due to improving computer technology and availability of sophisticated commercial multibody dynamics software, and is widely used in industry to improve the vehicle design process. It is possible to use these virtual prototyping tools to build highly accurate models[7] with potentially hundreds of degrees of freedom. Such virtual prototyping software includes Simpack, ADAMS, and CarSim/TruckSim[8].

**1.2.2 Controller Design**

**Feedforward-Feedback Control**

The vehicle driver is a critical model component and challenging to fully describe, due in part to human cognitive limitations[9]. According to MacAdam[10], human drivers have physical limitations including, but not limited to, the sensory channels: 1) vision, 2) vestibular and kinesthetic, 3) tactile, and 4) auditory. The main effect of the physical limitation is that it adds a delay to the control transport time.

Other than physical limitations, physical attributes are also important when defining a human driver. The physical attributes include: first, preview utilization, which is the ability of a human driver to look ahead and preview the road while driving, and second, adaptive control behaviour, which indicates that the drivers have differing levels of adaptation to control a vehicle. For example, no matter what skill level a driver has, nearly all have the basic ability to drive a car, even if it
is a car they have never driven before, but when different situations arise in the vehicle motions, they may have a different response. In general, this is defined as the different skill level of the driver.

Based on this line of thinking, the concept of the ‘internal vehicle model’ was described with MacAdam’s driver model development structure. The accuracy of the parameters of the internal vehicle model represents a human driver’s understanding of a particular vehicle. This accuracy decides the sensitivity of the driver towards the vehicle status and the reaction of the driver in responding to a situation such as oversteer. By changing the accuracy of the parameters of the internal vehicle model, the simulated driver’s understanding of the vehicle is changed. For example, for a vehicle that has a weight of 1234 kg, a more skilled driver may react as though the vehicle weighs 1230 kg, while a less skilled driver reacts as though the mass is 1000 kg.

In MacAdam’s work, an inverted yaw plane model with a nonlinear tire model is used as the internal vehicle model. It receives the full vehicle status from the external vehicle model, (which is a highly sophisticated model representing a real vehicle that provides accurate vehicle response to the steer input), and outputs the raw steer response as the driver’s reaction to the upcoming event. A predicted path can be derived based on this steer response. Simultaneously, the actual previewed scene has been processed to generate a desired path. Both the desired path and predicted path are handled by the steering controller to minimize the error between them. The final steer output is then obtained, after being modified with a delay to represent a human’s physical limitations. Once the external vehicle model receives the final input, its status will be changed, closing the loop.

A driver’s preview feature can be also included using feedforward control. The feedforward loop usually provides the vehicle a prediction steer input based on a previewed road geometry. When combined with a feedback loop, it reduces the effort of the feedback in handling upcoming disturbances. Based on this feature, Zhou and Peng presented a control system structure including a feedforward controller to preview the road profile and predict a steer response from the driver’s internal empirical vehicle yaw dynamics model. In order to compensate for the remaining deviation, the feedback component was designed based on natural human limitations, with control parameters determined by an optimal search procedure. Later, Kapania used the feedforward-feedback control structure aiming to reduce the vehicle lateral offset to the track, as well as simultaneously maintaining stability. Besides developing a feedback controller to reduce the lookahead error, the planar bicycle model with the steady state sideslip was implemented as a feedforward steering control. Mammar also developed a feedforward controller to preprocess the reference signal input and a feedback controller to satisfy the requirement of the robust stability and damping response for the system with the gain obtained by a looping shape control synthesis.
Lateral Controller Design

MacAdam developed a controller that is proposed to minimize the lateral path offset error, which is quantified as a sum of the squared error of each increment between the predicted and desired paths. The number of increments is decided by the length of the driver preview time. With an additional weighting function that considers the body sideslip angle and body roll angle as components of the performance index, the controller using this cost function is able to calculate a suitable steer response, even under emergency situations such as spin-out and roll-over.

Another widely applied lateral controller was developed by Braghin. There were error functions defined to describe a vehicle’s status in terms of the track. The first one is the heading angle error that determined the difference between the current heading direction of the vehicle and the track heading direction at the current track section. However, even if such an error is eliminated, the vehicle can be driven off the track, but remain parallel to the track. Therefore, a second error function is used to measure the distance from the track to the vehicle’s center of gravity. Initially, the control law is written:

\[
\delta = K_1 e_1 + K_2 e_2
\]  \hspace{1cm} (1.30)

where the first error function \(e_1\) is treated as a feedforward contribution and the second error function \(e_2\) as a feedback. By analyzing the kinematic steer angle, both the proportional gain \(K_1\) and \(K_2\) can be described as a function of visual distance determined by the preview time. Then, both the slip angle and vehicle’s transient behaviour were also taken into account as another two coefficients, \(K_3\) and \(K_{tr}\), due to their effects on the final steer angle calculation. The control law becomes:

\[
\delta = K_{tr} K_3 (K_1 e_1 + K_2 e_2)
\]  \hspace{1cm} (1.31)

Heilmeier et al. took a different approach, and proposed a curvature based controller. The goal of this controller is to minimize the difference between the current path curvature and the target path curvature, but rather than directly reducing the error by a proportional controller, a corrective curvature is computed under the current vehicle status, i.e., lateral acceleration and forward speed. Since the lateral acceleration can also be regarded as the second derivative of the lateral offset, the equation can be then expressed as a second order system, which offers two more parameters to adjust the resultant corrective curvature. By having the corrective curvature added to the target curvature, the optimized target curvature is obtained, which will enter the proportional controller along with the current curvature. By following his method, the feedback part of the controller can be written:

\[
\delta = K(k_t - k_c - k)
\]  \hspace{1cm} (1.32)

where \(K\) is the proportional gain, \(k\) is the current curvature, \(k_t\) is the target curvature coming from
the curvature profile, and $k_c$ is the corrective curvature calculated from the centripetal acceleration $a_c$, where:

$$a_c = \frac{v^2}{R} = u^2 k_c \quad (1.33)$$

where $u$ is the vehicle’s forward velocity, $k_c$ and $R$ are the curvature and its equivalent radius. Since the centripetal acceleration can be also written as a double derivative of the offset, $\ddot{d}$, the equation becomes:

$$\ddot{d} = u^2 k_c \quad (1.34)$$

For a second-order unforced system, the general form can be written:

$$\ddot{d} + 2\xi \omega \dot{d} + \omega^2 d = 0 \quad (1.35)$$

where $\xi$ is the damping ratio and $\omega$ is the natural frequency. Substituting back into Equation 1.33:

$$u^2 k_c + 2\xi \omega \dot{d} + \omega^2 d = 0 \quad (1.36)$$

Rearrange to get:

$$k_c = -\frac{1}{u^2}(2\xi \omega \dot{d} + \omega^2 d) \quad (1.37)$$

Because differentiation will amplify noise in any numerical simulation, $\dot{d}$ is replaced:

$$\dot{d} = u \sin(\Delta \psi) \quad (1.38)$$

The rate of offset here is considered as the lateral component of the vehicle heading velocity where $\Delta \psi$ is the heading error calculated by taking the difference between the vehicle yaw angle and track trajectory heading. To complete the feedback loop, the vehicle’s current curvature is required. It can be estimated from:

$$k = \frac{r}{u} \quad (1.39)$$

where $r$ the is vehicle yaw rate and $u$ is the forward speed. The linear relationship between steer angle and curvature are applied as the feedforward component to complete the controller.
1.3 Objectives and scope

The primary objective of the research is to find out how different vehicle handling characteristics affect the handling performance while driven by drivers with different skill levels. It explores the question: is it the case that a vehicle design best suited for human control should be modified to be suited for machine control? To reach this goal, a simulation with a proper vehicle model and controller is developed.

1.3.1 Vehicle Model Design

The vehicle model used in this research should be accurate enough to represent a real vehicle’s behaviour. Because the handling performance is the criteria to be observed, high speed cornering should be considered. Therefore, it is important that yaw, pitch, and roll motions are all included in the vehicle model. It should be of sufficiently high fidelity and number of degrees of freedom to capture small changes in the behaviour of interest.

The handling characteristics should be varied, while most of the structure of the suspension remains the same. For example, normally the easiest way to achieve different handling characteristics is to change the distance from the center of gravity to the front axle (denoted $a$) and to the rear axle (denoted $b$), but if a high DOF vehicle model is applied, $a$ and $b$ will be involved in most of the calculation, which will not only change handling characteristics but also all related vehicle motion. In this case, a parameter that has a minimal impact on the suspension properties is required; as such, the roll stiffness will be modified, as practical experience shows that it can change the handling characteristics, by affecting the normal force between the tire and the road.

1.3.2 Controller Design

The purpose of the controller is to choose the vehicle inputs such that it follows closely to the track path. To realize that, there are many controllers that could be chosen based on the literature, but to control the vehicle, as well as allow the assignment of differing driving skills, a control structure of feedforward and feedback seems to be the best suited. This is especially true when one considers how the features of the bicycle model fit perfectly into the feedforward model, i.e., close, but not the same as the vehicle model itself. Overall, the controller, including both lateral and longitudinal motion, is supposed to provide a good track following ability, while minimizing any oscillations.
Chapter 2

Vehicle Dynamics and Modelling

In this chapter, a 10 DOF vehicle model is developed. It is the main vehicle model that is driven by the lateral and longitudinal controller on the track, representing the real vehicle. In this full car model, the roll centre is separated from vehicle body’s mass centre during analysis so that the vehicle roll dynamics are more realistic. This results in a more accurate estimate of lateral load transfer, which has a high contribution to the vehicle handing, especially when cornering at high speed[16]. The nonlinear tire model is also discussed.

2.1 Vehicle Modeling as the Plant

The development of the 10 DOF vehicle model begins with the Newton-Euler equations of motion.

2.1.1 Equations of Motion

The analysis is conducted with two general force and rotation equations:

\[ \sum f = m(\ddot{v} + \omega \times v) \]  \hspace{1cm} (2.1)

\[ \sum m_G = I_G \alpha + \omega \times I_G \omega \]  \hspace{1cm} (2.2)

Substituting the SAE standard notation gives:

\[
\sum \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = m \begin{pmatrix} \ddot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + m \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix}
\]  \hspace{1cm} (2.3)

and:

\[
\sum \begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} + m \begin{pmatrix} p \\ q \\ r \end{pmatrix} \times \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix}
\]  \hspace{1cm} (2.4)
A common assumption, also used here, is that the vehicle is symmetric in the \( xz \) plane (i.e., left to right), eliminating two of the cross products of inertia \( (I_{xy} \text{ and } I_{xz}) \). The third value, although not necessarily zero, is typically small enough that its effect can be safely ignored, unless the vehicle carries its mass much higher above the ground on one axle vs the other (e.g., in a truck with a tall cargo area above the rear axle). That indicates the values in the inertia matrix are all zeros except \( I_{xx}, I_{yy} \text{ and } I_{zz} \). Therefore,

\[
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix} =
\begin{bmatrix}
I_{xx} & 0 & 0 \\
-0 & I_{yy} & 0 \\
-0 & 0 & I_{zz}
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{p}} \\
\dot{\mathbf{q}} \\
\dot{\mathbf{r}}
\end{bmatrix} +
\begin{bmatrix}
\mathbf{p} \\
\mathbf{q} \\
\mathbf{r}
\end{bmatrix} \times
\begin{bmatrix}
I_{xx} & 0 & 0 \\
-0 & I_{yy} & 0 \\
-0 & 0 & I_{zz}
\end{bmatrix}
\begin{bmatrix}
\mathbf{p} \\
\mathbf{q} \\
\mathbf{r}
\end{bmatrix}
\]

(2.5)

For the vector \( \mathbf{\omega} \), there is a skew symmetric matrix, \( \tilde{\mathbf{\omega}} \), that simplifies the cross product calculation, like so:

\[
\mathbf{\omega} \times \mathbf{v} = \tilde{\mathbf{\omega}} \mathbf{v}
\]

(2.6)

where:

\[
\tilde{\mathbf{\omega}} =
\begin{bmatrix}
0 & -r & q \\
r & 0 & -p \\
-q & p & 0
\end{bmatrix}
\]

(2.7)

Eventually, the six equations of motion are:

\[
\sum X = m(\dot{u} - rv + qw)
\]

(2.8)

\[
\sum Y = m(\dot{v} + ru - pw)
\]

(2.9)

\[
\sum Z = m(\dot{w} - qu + pv)
\]

(2.10)

\[
\sum L = I_{xx}\dot{\mathbf{p}} - (I_{yy} - I_{zz})qr
\]

(2.11)

\[
\sum M = I_{yy}\dot{\mathbf{q}} - (I_{zz} - I_{xx})rp
\]

(2.12)

\[
\sum N = I_{zz}\dot{\mathbf{r}} - (I_{xx} - I_{yy})pq
\]

(2.13)

To obtain the sum of forces is quite straightforward; it is simply the sum of the forces on each wheel in the corresponding direction:

\[
\sum X = X_{lf} + X_{lr} + X_{rf} + X_{rr}
\]

(2.14)

\[
\sum Y = Y_{lf} + Y_{lr} + Y_{rf} + Y_{rr}
\]

(2.15)

\[
\sum Z = Z_{lf} + Z_{lr} + Z_{rf} + Z_{rr}
\]

(2.16)

The expressions for the moments are not as straightforward as the forces. By inspection of the FBDs of the vehicle, as shown in Figures 2.1 and 2.2, it is seen that there are moments around
the all three of the yaw, pitch, and roll axes, caused by the forces in the longitudinal, lateral and vertical directions. The moment of these forces can be written as:

\[
\sum L = (Y_{lf} + Y_{rf} + Y_{lr} + Y_{rr})(h_{cg} - h_r) + (Z_{lf} + Z_{rf}) \frac{t}{2} - (Z_{lr} + Z_{rr}) \frac{t}{2}
\]

\[
\sum M = (X_{lf} + X_{lr} - X_{rf} - X_{rr})h_{cg} + a(Z_{lf} + Z_{rf}) - b(Z_{lr} + Z_{rr})
\]

\[
\sum N = (X_{lf} + X_{lr} - X_{rf} - X_{rr}) \frac{t}{2} + a(Y_{lf} + Y_{rf}) - b(Y_{lr} + Y_{rr})
\]

In Figure 2.2, the height of the centre of mass is \(h_{cg}\) and \(h_r\) is the height of the roll centre. The height of the roll centre is obtained by solving a simple geometry problem associated with front roll centre \(h_{sf}\) and rear roll centre \(h_{rr}\), as shown in Figure 2.3. The front and rear roll centres are two points determined from the geometry of the suspension mechanism, where all the suspension linkage forces can be considered to act. It is also the point about which the chassis will roll if no lateral motion of the tires is permitted.

\[
S_{AEC} = S_{ABDC} - S_{ABE} - S_{DCE}
\]

\[
= \frac{1}{2}(h_{rf} + h_{rr})(a + b) - \frac{1}{2}h_{rf}a - \frac{1}{2}h_{rr}b
\]

\[
= \frac{1}{2}(h_{rf}a + h_{rr}b)
\]
And:

\[ S_{AEC} = \frac{1}{2} h_r a + \frac{1}{2} h_r b \]  \hspace{1cm} (2.21)

Substituting gives:

\[ h_r = \frac{h_{rf} a + h_{rr} b}{a + b} \]  \hspace{1cm} (2.22)

An important point when computing the forces and moments is that in order to compute the sum of forces in the longitudinal and lateral directions, the vertical forces are required first, since the tire model applied in this research uses the vertical force to calculate the lateral and longitudinal forces. The vertical force includes both the vehicle weight and the deflection force generated by the spring and damper as well as the anti-roll bar. For vehicle weight distributed on each wheel:

\[ Z_m = \frac{0.5 mg}{a + b} \begin{pmatrix} b \\ a \\ b \\ a \end{pmatrix} \]  \hspace{1cm} (2.23)

Due to the deflection of the springs and dampers that caused by motions of roll and pitch, the
vertical force on each wheel can be obtained by summing the deflection force and the distributed weight on each wheel. The deflection length can be obtained by:

\[
z = z_g + a \frac{t}{2} \phi \begin{pmatrix}
1 \\
1 \\
-1 \\
-1
\end{pmatrix} + \theta \begin{pmatrix}
a \\
b \\
a \\
b
\end{pmatrix}
\]

(2.24)

The deflection rate can be obtained by:

\[
\dot{z} = \dot{z}_g + \frac{t}{2} p \begin{pmatrix}
1 \\
1 \\
-1 \\
-1
\end{pmatrix} + q \begin{pmatrix}
a \\
b \\
a \\
b
\end{pmatrix}
\]

(2.25)

The deformation of anti-roll bar can be obtained by:

\[
z_{ar} = \frac{t}{2} \phi \begin{pmatrix}
1 \\
1 \\
-1 \\
-1
\end{pmatrix}
\]

(2.26)

These forces are linear approximations of the true force generated by the suspension mechanism. In order to include the nonlinearities, a much more sophisticated suspension model would be required. In most cases, the vehicle will be expected to remain in the small deflection region where the linear approximation is adequate. The sum of normal force then becomes:

\[
\sum Z = Z_m - \begin{pmatrix}
k_f & 0 & 0 & 0 \\
0 & k_r & 0 & 0 \\
0 & 0 & k_f & 0 \\
0 & 0 & 0 & k_r
\end{pmatrix} z - \begin{pmatrix}
c_f & 0 & 0 & 0 \\
0 & c_r & 0 & 0 \\
0 & 0 & c_f & 0 \\
0 & 0 & 0 & c_r
\end{pmatrix} \dot{z} - \begin{pmatrix}
k_{rf} \\
k_{rr} \\
k_{rf} \\
k_{rr}
\end{pmatrix} \phi \frac{t}{2}
\]

(2.27)

It should be noted that the vertical forces are also affected by an effect called ‘jacking force’[3]. When one considers a tire while cornering, the road applies a force on the tire pointing toward the roll centre. As a result, the lateral force is not the only force that acts on the tire, but there is also a vertical force, a component of the forces acting through the suspension linkages, which is called jacking force, as shown in Figure 2.4. The jacking force must balance the moment around the roll centre caused by lateral force. For the left rear tire, it can be written:

\[
\frac{t}{2} h_{lr} = h_{rr} Y_{lr}
\]

(2.28)
This indicates that either an increase in the lateral force or roll centre height will increase the effect of the jacking force. The jacking force tends to lift up the chassis on the outer side of a corner, and press down the inner side. When the value is significantly high, positive tire camber will often result, which dramatically reduces the contact area between the tire and road, and will end up reducing the grip of the tire in the plane of the road.

### 2.2 Tire Model

By studying the free body diagram (FBD) for each tire, the equations for wheel dynamics can be added (neglecting the rolling resistance):

\[
I_w \alpha_w = \tau_{axle} - \tau_{brake} - X R_w \tag{2.29}
\]

where \( \tau_{axle} \) is the torque acting at the axles at a certain engine speed and gear, \( \tau_{brake} \) is the braking torque applied on axles based on the braking bias, and \( \alpha_w \) is the angular acceleration of the wheel. The traction force \( X \) is determined by the selected tire model.

The choice of the tire model becomes critical since it is the only contact to the road, and plays a key role in vehicle handling performance. Moreover, due to the complexity of the tire, a linear tire model is usually insufficient to use for more realistic vehicle dynamics analysis. Fortunately, many sophisticated tire models have been developed for the past half century[17] to serve for different situations.
research purposes. The tire models can vary from empirical to theoretical. A fully empirical model can have very high accuracy for a certain tire condition and very low computational efforts at the same time, but when the conditions change, this model doesn’t fit anymore. On the other hand, a fully theoretical model is able to capture as much detail as possible, but at the cost of computational speed[2]. Among all these tire models, the ‘Magic Formula’ tire model has been selected for tire longitudinal and lateral force calculation. The Magic Formula is widely used, and nicknamed so because at first inspection, it is not obvious how or why it will do a good job at predicting tire behaviour, yet it is surprisingly good. As a semi-empirical model, the Magic Formula tire model features some structures and strategies of theoretical models, but keeps the computational advantage for simulation[17]. The general form of the formula is:

\[ y = D \sin(C \arctan(B(1 - E)(x + S_h) + E \arctan(B(x + S_h)))) + S_v \]  \hspace{1cm} (2.30)

where:
\[ B = \text{Stiffness Factor} \]
\[ C = \text{Shape Factor} \]
\[ D = \text{Peak Factor} \]
\[ E = \text{Curvature Factor} \]
\[ S_h = \text{Horizontal Shift} \]
\[ S_v = \text{Vertical Shift} \]

The input \( x \) represents longitudinal, lateral or combined slip ratio and \( y \) represents tire force in the longitudinal or lateral direction. The factors \( A - E, S_h, \) and \( S_v \) are based on normal load and camber angle, and are determined using tire data obtained from experimental measurements. Since it is very hard and expensive to obtain the tire data, the example data provided in Genta’s book[18] is used for calculation. By following Genta’s example, the equations are written as functions of coefficients \( a_i \) representing for the data during side slip and \( b_i \) representing for the data during longitudinal slip. For pure longitudinal slip:

\[ X = D \sin(C \arctan(B(1 - E)(\sigma) + E \arctan(B\sigma)))) \]  \hspace{1cm} (2.31)
\[ C = b_0 \]  \hspace{1cm} (2.32)
\[ D = \mu_p Z \]  \hspace{1cm} (2.33)
\[ \mu_p = b_1Z + b_2 \]  \hspace{1cm} (2.34)
\[ BCD = (b_3Z^2 + b_4Z) \cdot e^{-b_5Z} \]  \hspace{1cm} (2.35)
\[ E = b_6Z^2 + b_7Z + b_8 \]  \hspace{1cm} (2.36)
\[ S_h = b_9Z + b_{10} \]  \hspace{1cm} (2.37)
\[ S_v = 0 \]  \hspace{1cm} (2.38)
For pure side slip:

\[
Y = D \sin(C \arctan(B(1 - E)(\alpha) + E \arctan(B\alpha)))
\]  
(2.39)

\[
C = a_0
\]  
(2.40)

\[
D = \mu_{\gamma_p}Z
\]  
(2.41)

\[
\mu_{\gamma_p} = b_1 Z + b_2
\]  
(2.42)

\[
BCD = a_3 \sin(2 \arctan(\frac{Z}{a_4}))(1 - a_5|\gamma|)
\]  
(2.43)

\[
E = a_6 Z + a_7
\]  
(2.44)

\[
S_h = a_8 \gamma + a_9 Z + 10
\]  
(2.45)

\[
S_v = a_{11} \gamma Z + a_{12} Z + a_{13}
\]  
(2.46)

Due to the inclusion of jacking forces, the load \(Z\) in the Magic Formula cannot be found in a straightforward fashion. Because the calculation of the jacking force requires the value of lateral force, which is found from the tire model, the entire relationship between normal force and lateral force becomes a loop. To solve the loop during simulation, the jacking force is set with an initial value of 0 N to start the loop. The final value will be decided after 10 iterations, when the difference between each iteration is insignificant.

### 2.3 Rotation Matrices

There are three coordinate systems used in the simulation, including the road surface, the vehicle chassis, and the front tires. To minimize the number of state variables and still be able to switch between these systems, the application of rotation matrices becomes essential.

#### 2.3.1 Rotation Matrix Between the Vehicle and the Track

If the coordinate system rotates around the \(x\) axis by the roll angle \(\phi\), by projecting the new coordinates onto the original axes, the relationship can be written as:

\[
\mathbf{v}_2 = R_x \mathbf{v}_1
\]  
(2.47)

where:

\[
R_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\]  
(2.48)
Rotation around the $y$ axis is similar; the pitch angle is $\theta$:

$$
R_y = \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
$$  \hfill (2.49)

And rotation around the $z$ axis:

$$
R_z = \begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
$$  \hfill (2.50)

$$
R_{xyz} = R_y R_z
$$  \hfill (2.51)

The rotation matrix $R_{xyz}$ is computed by taking the product for all three matrices in the sequence shown. It should be noted that different orders in the combination of rotations does change the resulting rotation matrix. The ‘roll-pitch-yaw’ sequence is a common choice in the automotive industry.

### 2.3.2 Rotation Matrix Between the Tire and the Vehicle

The tire model is set up as a separate body attached to the vehicle chassis; this separation only affects the front wheels since the steering is the reason that the coordinate system rotates. Because there is only one rotation and the tires are assumed to only have longitudinal and lateral motions, the matrix can be simplified into 2-D:

$$
R_5 = \begin{bmatrix}
\cos \delta & \sin \delta \\
-\sin \delta & \cos \delta
\end{bmatrix}
$$  \hfill (2.52)

The wheel velocity used to calculate the tire slip angle and ratio can now be obtained by multiplication of the rotation matrix and the vector of the wheel velocity computed in the frame of vehicle chassis. Once longitudinal and lateral force at the tire are computed using the tire model, this matrix is required once again to transform the forces back to the vehicle frame. Because the rotation matrix is orthogonal, its inverse is simply its transpose, therefore:

$$
f_v = R^T f_t
$$  \hfill (2.53)

### 2.3.3 Angular velocity conversion

Following the rotation sequence $xyz$, the coordinates rotate around $x$ axis first, then the new coordinates system $x_1y_1z_1$ starts to rotate around $y_1$ to obtain the $x_1y_1z_2$, and finally, the third rotation around $z_2$ gives the $x_2y_2z_2$. Therefore, the direction of $\phi$ is $\vec{x}$, where the projection of $\vec{x}$
on the x axis is: 1
on the y axis is: 0
on the z axis is: 0

The direction of \( \dot{\theta} \) is \( \overrightarrow{y_1} \), where the projection of \( \overrightarrow{y_1} \)
on the x axis is: 0
on the y axis is: \( \cos \theta \)
on the z axis is: \( -\sin \theta \)

The direction of \( \dot{\psi} \) is \( \overrightarrow{z_2} \), where the projection of \( \overrightarrow{z_2} \)
on the x axis is: \( -\sin \theta \)
on the y axis is: \( \cos \theta \sin \phi \)
on the z axis is: \( \cos \theta \cos \phi \)

As a result, the angular velocity can be written in terms of the rates of change of the rotation
angles, as a differential equation. These are known as the kinematic differential equations.

\[
\begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 & -\sin \theta \\
    0 & \cos \phi & \cos \theta \sin \phi \\
    0 & -\sin \phi & \cos \theta \cos \phi
\end{bmatrix}
\begin{bmatrix}
    \dot{\phi} \\
    \dot{\theta} \\
    \dot{\psi}
\end{bmatrix}
\tag{2.54}
\]

The inverse relationship is:

\[
\begin{bmatrix}
    \dot{\phi} \\
    \dot{\theta} \\
    \dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
    1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
    0 & \cos \phi & -\sin \phi \\
    0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta
\end{bmatrix}
\begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix}
\tag{2.55}
\]

Because the conversion between angular velocity and Euler rate is not straightforward, for more
details see[19].
Chapter 3

Controller Design

This chapter introduces multiple analytical controller designs with the feedforward and feedback structure described in the literature review in Chapter 1. Learning from these previous works, the controller in this research is divided into a lateral controller and a decoupled longitudinal controller. The lateral control is based on curvature error minimization as the feedback, with a yaw plane model inversion as the feedforward. The longitudinal controller is a hyperbolic tangent function that provides different sensitivity of the throttle and brake in different situations.

3.1 Driver Model Overview

![Figure 3.1: Overview of control scheme]

Figure 3.1 gives an overview of the control scheme. The initial inputs of the steering and throttle/brake enter the nonlinear vehicle model, and a set of vehicle states is computed by the solver. Then, a preview system uses these states to generate the virtual preview points. The comparison between these points and the reference track provide the driver model with target velocity, lateral offset and heading error. To minimize the errors and reach the target velocity, the driver model is developed with a decoupled longitudinal and lateral controller. The steering and throttle/brake output are the results, and used as the control variables for the vehicle.
3.2 Lateral Controller Design

The feedforward controller is based on the bicycle model under steady state conditions. After the solving the equation of motion under steady state, by manipulating Equation 3.1, the following relation can be formed between the steer angle and the trajectory:

\[ \delta_f = \frac{a + b}{R} + k_{us} \frac{u^2}{gR} \]  

(3.1)

where the cornering radius \( R \) is defined as:

\[ R = \frac{u}{r} \]  

(3.2)

and the understeer gradient \( k_{us} \) is defined as:

\[ k_{us} = \frac{mg}{a + b} \left( \frac{b}{c_f} - \frac{a}{c_r} \right) \]  

(3.3)

The lateral controller is inspired by the work of Heilmeier et al.\[15]. To realize the feedforward function, a steady state yaw plane model inversion is implemented at the beginning to predict an approximate value of the steer angle, which is then followed by a proportional controller as the feedback part to check and minimize the curvature difference. It can be clearly seen in Equation 3.1 that the steer calculation consists two parts. The first part is the linear relationship between steer angle and cornering radius, which accounts for the steer angle of the vehicle under low speed where the tires don’t generate large lateral forces. The high speed condition is expressed in the second part, where the influence of lateral acceleration is seen. Since the relationship between lateral acceleration and curvature can also be written as:

\[ a_y = \frac{u^2}{R} = ur \]  

(3.4)

Therefore:

\[ \delta_f = \frac{a + b}{R} + k_{us} \frac{a_y}{g} \]  

(3.5)

Now, recalling the concept of Heilmeier et al., the desired damped second order system expression for the offset is written as:

\[ \ddot{d} + 2\xi\omega \dot{d} + \omega^2 d = 0 \]  

(3.6)

Knowing that the rate of offset is the one of the components of vehicle forward speed gives:

\[ \dot{d} = u \tan(\Delta \psi) \]  

(3.7)

Since the lateral deviation and heading error are defined as the vectors with the direction from current point to target point, the offset and heading error are both positive as shown in Figure 3.2.
In this case, in order to correct the vehicle tracking direction, the vehicle must turn left, e.g. positive steer input. Following this rule, the equation is modified, taking \( \dot{\delta} \) as a substitute for lateral acceleration:

\[
\delta_f = \frac{a + b}{R} + \frac{k_{us}}{g} (2\xi \omega u \tan(\Delta \psi) + \omega^2 \dot{d})
\]  

(3.8)

Even though the yaw plane model is accurate enough to interpret the vehicle’s motion on the track, the nonlinearity from the full car model, especially the inclusion of the roll motion, increases the difficulty to obtain error minimization in all cases. To improve the performance of the controller and fill the gap between the feedforward and the target, a proportional controller is implemented by taking the difference between the target curvature of the track, and the current curvature.

\[
\delta_f = K_p (k_t - k_c)
\]  

(3.9)

With current curvature being estimated by yaw rate divided by forward speed, a yaw controller is formed to help vehicle countersteer at large body slip angles. Finally, the overall control law is

\[
\delta_f = \frac{a + b}{R} + \frac{k_{us}}{g} (2\xi \omega u \tan(\Delta \psi) + \omega^2 \dot{d}) + K_p (k_t - \frac{r}{u})
\]  

(3.10)

### 3.3 Longitudinal Controller Design

The goal of the longitudinal controller is to send the throttle/brake command according to the target velocity provided by the preview system. A hyperbolic tangent function is selected to meet the requirement since it limits the range of output from -1 to 1, with the positive output representing throttle and the negative output representing the brake. The function is written:

\[
t = \tanh(0.2e_u^3)
\]  

(3.11)

As shown in Figure 3.2, the cubic function characterizes the vehicle’s throttle with a high sensitivity when the error is high and low sensitivity when the error approaches zero. The thrust and brake works as a binary system, that don’t operate at the same time. When the output is negative, the brake system takes the command and distributes the intended brake torque to each axle by multiplying the maximum brake torque with the absolute value of the output. During this period, the throttle is set to zero. Similarly for the thrust system, to obtain the engine torque applied to
the axle, it is necessary to search for the current gear by considering the gear ratios available, the maximum engine speed, and vehicle’s velocity. A torque-rpm engine map is used to fit the current engine speed and find the corresponding torque. The throttle is then the multiplier of this torque to compute the true thrust torque.
The output from longitudinal control is -1 to 1 which decides the percentage of the maximum torque applied to the axle (brake or thrust). The total brake torque is maximized when the controller output is -1. The amount of torque distributed on front and rear axle is determined by front:rear brake ratio (brake bias). The demanded engine torque requires vehicle’s current gear to calculate the maximum torque at current engine speed. The current gear can be found from the current vehicle velocity, the maximum engine speed, and the range of ratios available. The relationship between vehicle velocity and engine speed is obtained by the gear ratio.
Chapter 4

Controller Implementation

4.1 Software Implementation

Originally, the simulation environment of a vehicle running on a track is based on the linear equation of motion generator code EoM, and vehicle simulator developed by University of Windsor Vehicle Dynamics and Control research group[20][21]. To satisfy the research purposes of this project, the software package has been heavily modified to ensure sufficient model fidelity, while improving the simplicity of implementing various types of steering controllers. The modifications include:

1. Vehicle Modelling
   The vehicle model is reduced from 14 DOF to 10 DOF by replacing the entire dedicated multi-body suspension system with a lumped suspension, while maintaining the nonlinear tire model. The equations of motion are recoded by hand rather than using any equation of motion generator. Neither rolling resistance and drag coefficient are considered anymore.

2. Controller
   To explore the possibility of improving lateral control and vehicle response on the track, the lateral controller becomes the main part of the modification in this project. However, because the controller structure is decoupled, the original longitudinal controller was maintained for its reasonable performance.

3. Platform
   Simulink has been chosen as the new platform for the simulation, due to its ability to easily accommodate controllers of varying architecture, its visualization capability, and convenience when it comes to the iterative tuning of the parameters, when comparison of outputs between different trials is required. The original standalone script to run the differential equations solver and initialize the simulation are now integrated functions in Simulink.

From the previous software implementation, there are two essential parts to ensure the feasibility of the simulation: Track pre-processing and vehicle preview[20][21].
4.2 Track Pre-processing

The dataset of the track course comes with three coordinates including $x,y$ and $z$. Since this research doesn’t take elevation into consideration, $z$ is not used. A pre-processing function will extract the dataset and generate the expected information as a reference input, in order to obtain the error, e.g., lateral deviation, speed error, etc. The track dataset is plotted in Figure 4.1.

![Track profile](image)

**Figure 4.1: Track profile**

Figure 4.2 shows the process of the algorithm to generate all the reference profiles. They are shown in Figures 4.3–4.5. The value of maximum lateral acceleration, maximum brake acceleration and maximum drive acceleration are chosen manually in advance. The $ds$ value is the distance between each track point, which is approximately 5 meters for the track in this work. It should be noted that the velocity profile is first generated based on a maximum lateral acceleration criteria, and then requires two filters to check if the velocity data is reasonable in terms of maximum braking deceleration and driving acceleration. This is an iterative process that won’t stop until the velocity profile satisfies both braking and driving accelerations. The equation is written as:

$$v^2_f - v^2_0 = 2ds a_{max/min}$$  \hspace{1cm} (4.1)

The algorithm first checks the brake acceleration by taking $v_t$ as the velocity at the current point
Chapter 4. Controller Implementation

\[
\tan \theta = \frac{dy}{dx}
\]

\[
k = \frac{d\theta}{dx}
\]

\[
u = \sqrt{\frac{\mu_{\text{max}} \cdot k}{\gamma}}
\]

**Figure 4.2:** Algorithm to generate reference profiles

**Figure 4.3:** Heading angle profile
and substituting the maximum braking deceleration into the equation, to compute the maximum velocity at the previous point. If the resulting velocity, in this case $v_0$, is smaller than the commanded previous velocity point in the profile, then that means this previous point is not reasonable.
and the profile exceeds the maximum brake acceleration. To correct this issue, that previous velocity point will be replaced by $v_0$. Similarly, for the driving acceleration check, $v_0$ now is the current point, and $v_1$ will be calculated and compared to the next point to see if the point exceeds the maximum drive acceleration. If it is too high, it will be replaced by the velocity computed using maximum longitudinal acceleration.

### 4.3 Vehicle Positioning and Preview

To simulate the vehicle moving on the track and obtaining the information on its position, a position searching algorithm is applied. The algorithm takes the distance the vehicle has travelled from the starting point, and divides by the distance between each point of the track. With a rounding algorithm, this gives the point on the track closest to the vehicle\(^{[20]}\)\(^{[21]}\). By taking the one point behind and a certain number of points ahead, the algorithm is able to construct an array of points that describe the location and the upcoming path. The number of points is determined by:

$$\text{number of points look ahead} = \frac{\text{look ahead distance}}{\text{distance of each segment}}$$

where the distance of each segment is 5 m, and

$$\text{look ahead distance} = 5 + 0.5u$$

where $u$ is the vehicle forward velocity in reference frame. By fitting these track index based points and the corresponding parameters that have been generated previously during the track processing into polynomial curves, the algorithm is able to find the desired previewed parameters (e.g., lateral offset, heading angle, etc) in terms of a set of unevenly distributed preview points.

**Figure 4.6:** Curve fit to obtain parameters corresponding to the preview points with desired distribution
These points are defined as:

\[ p = \text{look ahead distance}[0 0.1 0.2 0.3 0.4 0.6 0.8 1] \]  \hspace{1cm} (4.4)

Before the heading error and lateral offset enter the controller, different weightings are assigned to these previewed parameters, since the closer the points are to the current position, the more emphasis they should have on the error. The weighting factors are obtained from \cite{22}.

\[ k_0 = \frac{1}{29.01}[10 10 6 2 0.8 0.16 0.04 0.01] \]  \hspace{1cm} (4.5)

### 4.4 System Setup

There are plenty of predefined system blocks in Simulink for users to set up their system easily. However, most of these built-in blocks only serve for linear systems. Considering the vehicle model to be nonlinear, a proper user-defined block is required to fulfill the purpose. Among these user-defined function blocks, the level-2 MATLAB S-Function is the one with highest customizability and no restriction on input and output type. The function itself is written in MATLAB and can be called by the corresponding block in Simulink. To write a level-2 MATLAB S-Function, it is important to understand the callback methods that have been defined to meet the general purpose and establish the entire level-2-MATLAB S-Function.

1. **Setup Method**

   This is the method where the properties of the input and output ports, sampling time, and any dialog parameters are defined.

   ```matlab
   function setup(block)
   % Register number of ports
   block.NumInputPorts = 2;
   block.NumOutputPorts = 18;

   % Setup port properties to be inherited or dynamic
   block.SetPreCompInpPortInfoToDynamic;
   block.SetPreCompOutPortInfoToDynamic;

   % Override input port properties
   block.InputPort(1).Dimensions = 1;
   block.InputPort(1).DatatypeID = 0; % double
   block.InputPort(1).Complexity = 'Real';
   block.InputPort(1).DirectFeedthrough = true;
   block.InputPort(1).SamplingMode = 'Sample';
   block.InputPort(2).Dimensions = 1;
   block.InputPort(2).DatatypeID = 0; % double
   block.InputPort(2).Complexity = 'Real';
   block.InputPort(2).DirectFeedthrough = true;
   ```
As shown in Listing 4.1, it is intuitive to define the input and output port properties. There are two more parameters that are registered in this case, which are `NumDialogPrms` and `NumContStates`. The dialog parameter is the parameter that shows up in the dialog in the Simulink interface. It provides the convenience of changing the value of the desired parameters in Simulink directly instead of doing the modifications in a MATLAB file. The `NumContStates` indicates the number of state variables involved in the process. The setup method is also the place to register all other methods required to support the block working properly. All methods registered in the setup are required to be defined later.

2. Method to initialize the block

This is the method where all the initial conditions are defined, including, but not limited to, the initial values used to solve the ordinary differential equations (ODE).

```
function InitializeConditions(block)
    % Load global param
    my_specs();
    vehicle_specs();
    prelim();

    for i=1:18
        block.ContStates.Data(i)=0;
    end
```
The "S-function name" represents the name of the file that defines the Level-2 MATLAB S-Function block; in this case, it is called 'vehiclemodelblock'. The values below represent the initial values of forward velocity and angular velocity of each wheel.

```matlab
block.ContStates.Data(7) = block.DialogPrm(1).Data;
block.ContStates.Data(13) = block.DialogPrm(2).Data;
block.ContStates.Data(14) = block.DialogPrm(3).Data;
block.ContStates.Data(16) = block.DialogPrm(5).Data;
```

**LISTING 4.2: Initial condition setup**

In Listing 4.2, besides setting the initial value for each state variable, the parameters that are implemented in the vehicle model are also loaded. The dialog parameters defined here are intended to adjust the initial values for certain state variables. In this case, `block.ContStates.Data(7)` is forward velocity and the rest represent the angular velocity for each wheel.

3. **Method to define output**

Listing 4.3 lists the states that should also be recorded as outputs.

```matlab
function Outputs(block)
    for i = 1:18
        block.OutputPort(i).Data = block.ContStates.Data(i);
    end
```

**LISTING 4.3: Output setup**

4. **Method to define derivatives**

Listing 4.4 is the key section to define the system. If the system is discrete, the update method can be used instead of the continuous method. Since all the equations of motion in this system are defined in a standalone .m file, that function can be directly called as the derivative method to simplify the process; otherwise, one should specify and define the relationship between the derivatives and the other variables.
5. Method to terminate the block

```matlab
function Terminate(block)
```

Listing 4.5: Block termination

Listing 4.5 shows that the termination method in this case is effectively the same as the `end` at the end of a loop. Some simulations may require a specific conditions to be met to end the block; these can be defined here if needed.

### 4.5 Block Diagram

As shown in Figure 4.8, not only is the ‘vehiclemodelblock’ implemented by Level-2 MATLAB S-function, but also the ‘preprocessblock’. This block is linked to the track preview function, with the track pre-processing being loaded whenever the block is activated. Note that two inputs that vehicle model block takes can only be computed by gathering the information from the outputs of the vehicle model block, which can cause an algebraic loop, i.e., a ‘chicken and egg’ problem that Simulink struggles to solve. Thus, ‘memory’ blocks are placed to add one integration step delay to help the solver start with the initial value, and then take the feedback inputs.

The lateral controller shown in Figures 4.9–4.11 is implemented without using a Level-2 MATLAB S-function, because the structure of the controller is not as complicated as the others. Constructing the controller using a normal block diagram approach makes controller tuning more flexible, by easily monitoring all its elements.
Chapter 4. Controller Implementation

The block diagram implemented in the Simulink

**Figure 4.8:** The block diagram implemented in the Simulink
Chapter 4. Controller Implementation

**Figure 4.9:** Subsystem of the lateral controller

**Figure 4.10:** Subsystem of the feedforward controller
Chapter 4. Controller Implementation

Figure 4.11: Subsystem of the feedback controller
4.6 Controller Tuning

There are multiple adjustable variables in the controller. Thus, the choice of value for each of them is critical to allow the controller to operate at its desired condition. In this research, an iterative method is used for determination of the parameters. However, a large set of iterations will take excessive time. To accelerate the process, disassembling the evaluation parameter or the outputs to select the weighting of the components is an effective strategy.

In this case, the Data Inspector is a good Simulink tool to accelerate the process of tuning. Figures 4.12–4.14 are an example screen shot of the Data Inspector. The advantage of the tool is that it automatically records and saves the selected signals every run, and these simulation runs can be selected for comparison any time when they are required.

In Figure 4.12, the trend of the steer angle (green) is similar to the signals from feed-forward output (blue) and low speed steer output (wheelbase; dark red), while the signal of the proportional gain is approximately always opposite to the others. Then combined with Figure 4.13, it can be found that the peak of the offset (red) usually corresponds to the peak of the steer angle. Therefore, reducing the amplitude of the steer angle increases the chance to flatten the offset curve. In connection to the adjustable variables in the feedback controller, an initial guess can be conducted that a higher gain may be expected, as well as that a lower feed-forward output from the high speed part is more desirable.

When it comes to the signal distribution of the feed-forward value in Figure 4.14, the outputs from Product2:1 and Product3:1 share nearly opposite trends, with Product2:1 taking more weighting in the calculation. From Equation 3.10, it can be seen that the natural frequency is the adjustable variable to control the output from Product2, as well as it can be considered as the gain of the lateral offset. For Product3, both natural frequency and damping ratio are the part of the signal; it will be helpful to see the change of the result in response to change of the damping ratio.

In order to select a proper value, a careful comparison has been done, since the goal of tuning is always to seek the balance of performance and oscillations. By first doing the iteration method on the damping ratio, the resultant offsets and steer angles are shown in Figure 4.15 and Figure 4.16. By reduce the damping ratio, the tracking ability is obviously increased without many oscillations added to the steer angle. Then, the natural frequency is adjusted with the results shown in Figure 4.17 and Figure 4.18. A high frequency definitely assists the vehicle on path tracking, but when it reaches a certain value, in this case, $\omega = 2.0$, the steer angle has been pushed too far, with plenty of oscillations resulting at a certain point. A similar result also showed up when adjusting $K_p$; tracking performance increases with lower gain but as a cost of more oscillations in the steer response.

Table 4.1 is built as a second approach for comparison, as the offset difference for different configurations may vary for different corners. In case that the measurements of max, mean, and min are not able to adequately describe the entire curve, root mean square error is considered as
the final offset score. Overall, the fine tuned controller has the $\xi$, $\omega$, and $K_p$ set at 0.3, 1.6, and 0.8 respectively, regarding both optimal tracking performance and minimal oscillation.

<table>
<thead>
<tr>
<th>variables</th>
<th>Lateral offset [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td>0.3</td>
<td>1.0</td>
</tr>
<tr>
<td>0.3</td>
<td>1.2</td>
</tr>
<tr>
<td>0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>0.3</td>
<td>1.6</td>
</tr>
<tr>
<td>0.3</td>
<td>1.8</td>
</tr>
<tr>
<td>0.3</td>
<td>1.6</td>
</tr>
<tr>
<td>0.3</td>
<td>1.6</td>
</tr>
<tr>
<td>0.3</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 4.4: Tuning of variables with lateral offset as measuring criteria during the iterative tuning.

Figure 4.12: An example of the time history of selected signals in feedback controller presented in Data Inspector. In this run, the natural frequency $\omega$, damping ratio $\xi$ and proportional gain are 1, 0.8 and 1 respectively. Four signals were logged in this run as show in left side of the interface with ticks. The distribution of these signals for controller tuning can be found in Figure 4.10 and Figure 4.11.

During the tuning session, an oscillation can be always found at 86 seconds. By inspecting both the velocity result at the same time period in Figure 4.21(left), it can be found that the vehicle slows down at 86 seconds and accelerate later at 88 seconds. In theory, this should be a corner that velocity decreases when entering the corner and then increases at the exit. However, by checking the location at 86 seconds in Figure 4.21(right), and then locating the coordinate in the
The time history of offset and steer angle in first run for controller tuning. The direction of the set value is the opposite of the steer angle. Both of offset and steer angle reach their peak at same time. This indicates flattening the steer angle may decrease the offset value.

track profile in Figure 4.22, the vehicle is cornering at 86 seconds. The point (2.58,283) must have been previewed as an obstacle in this case, indicating the track is not completely smooth. The oscillation at 86 seconds can be also found in all figures in Chapter 5, and has been considered as one of future improvements in Chapter 6.
Chapter 4.  Controller Implementation

The time history of offset of selected signals in feedforward controller of initial run. Both the signal from Product2:1 and Product3:1 share the same direction and opposite to Feedforward steer:1. However, the value from Product3:1 is much larger than Product2:1, which means it is the main source of the feedforward signal. Therefore, the tuning strategy is to reduce the value from Product3:1 and amplify Product2:1.

**Figure 4.14:** The time history of offset of selected signals in feedforward controller of initial run.

Iterative tuning: comparison of lateral offset with damping ratio of 0.3, 0.4, 0.5 while both natural frequency and proportional gain remain at 1. The offset decreases with increasing damping ratio.

**Figure 4.15:** Iterative tuning: comparison of lateral offset with damping ratio of 0.3, 0.4, 0.5 while both natural frequency and proportional gain remain at 1. The offset decreases with increasing damping ratio.
**Figure 4.16**: Iterative tuning: comparison of the steer response with damping ratio of 0.3, 0.4, 0.5 while both natural frequency and proportional gain remain at 1. The steer angle decreases with increasing damping ratio.

**Figure 4.17**: Iterative tuning: comparison of lateral offset with natural frequency of 1.6, 1.8, 2.0 while damping ratio and proportional gain remain at 0.3 and 1, respectively. The lateral offset decreases with increasing natural frequency, but more oscillations can be found at 124 seconds for $\omega = 2.0$. 

\[\text{time \[s\]} \quad \text{Steer angle \[rad\]} \quad \begin{array}{c} \xi = 0.5 \\ \xi = 0.4 \\ \xi = 0.3 \end{array} \]

\[\text{time \[s\]} \quad \text{Lateral offset \[m\]} \quad \begin{array}{c} \psi = 1.6 \\ \psi = 1.8 \\ \psi = 2.0 \end{array} \]
FiguRe 4.18: Iterative tuning: comparison of steer response with natural frequency of 1.6, 1.8, 2.0 while damping ratio and proportional gain remain at 0.3 and 1, respectively. The steer angle doesn’t change too much with the change of natural frequency.

FiguRe 4.19: Iterative tuning: comparison of lateral offset with gain of 0, 0.5, 1.0 while natural frequency and damping ratio remain at 1.6 and 0.3, respectively. In some sections (76 second and 100 seconds), a higher gain reduces the offset, while in some sections (62 seconds and 112 seconds), a low gain reduces offset.
Figure 4.20: Iterative tuning: comparison of steer response with gain of 0, 0.5, 1.0 while natural frequency and damping ratio remain at 1.6 and 0.3, respectively. There is insignificant difference in steer angle with the change of proportional gain.

Figure 4.21: The velocity decreases at 86 seconds and then increased again. The location of vehicle at time 86 seconds can be then found in the right figure showing $x = 2.58$ and $y = 283$. 
The vehicle at 86 seconds is not located at any corner, but the velocity decreases. It indicates that there is a point here that is previewed as an obstacle by the vehicle. The result could be caused by the track not being smooth enough.
Chapter 5

Results and Discussion

In this chapter, multiple cases are exhibited to compare the results with varied test conditions, including different vehicle handling characteristics and internal vehicle model accuracy. The vehicle handling characteristic is decided by roll stiffness distribution: higher front and lower rear roll stiffness makes vehicle tend more toward understeer behaviour, while lower front and higher roll stiffness tends toward oversteer [23].

The Table 5.1 below is the default vehicle parameters that are implemented during the simulation. The default parameters are set as an understeer configuration. The simulation has a time history of more than 140 seconds. However, to display the maximum detail while maintaining the resolution of the multiple plots, the time history has been trimmed to the result from 60 seconds to 140 seconds.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>$m$</td>
<td>1500 kg</td>
</tr>
<tr>
<td>CG to front axle</td>
<td>$a$</td>
<td>1.2 m</td>
</tr>
<tr>
<td>CG to rear axle</td>
<td>$b$</td>
<td>1.4 m</td>
</tr>
<tr>
<td>Track width</td>
<td>$t$</td>
<td>1.6 m</td>
</tr>
<tr>
<td>Roll inertia</td>
<td>$l_{xx}$</td>
<td>800 kg m$^2$</td>
</tr>
<tr>
<td>Pitch inertia</td>
<td>$l_{yy}$</td>
<td>1600 kg m$^2$</td>
</tr>
<tr>
<td>Yaw inertia</td>
<td>$l_{zz}$</td>
<td>1800 kg m$^2$</td>
</tr>
<tr>
<td>Tire inertia</td>
<td>$l_w$</td>
<td>2.0 kg m$^2$</td>
</tr>
<tr>
<td>Front roll stiffness</td>
<td>$k_{rf}$</td>
<td>5000 N/m</td>
</tr>
<tr>
<td>Rear rear stiffness</td>
<td>$k_{rr}$</td>
<td>5000 N/m</td>
</tr>
<tr>
<td>Front roll center height</td>
<td>$h_{rf}$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Rear roll center height</td>
<td>$h_{rr}$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Center of gravity height</td>
<td>$h_g$</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Front suspension stiffness</td>
<td>$k_f$</td>
<td>20000 N/m</td>
</tr>
<tr>
<td>Rear suspension stiffness</td>
<td>$k_r$</td>
<td>22000 N/m</td>
</tr>
<tr>
<td>Front suspension damping</td>
<td>$c_f$</td>
<td>2000 Ns/m</td>
</tr>
<tr>
<td>Rear suspension damping</td>
<td>$c_r$</td>
<td>2000 Ns/m</td>
</tr>
</tbody>
</table>

Table 5.1: Default vehicle model parameter values [18].
5.1 Case 1: Understeer configuration with different internal vehicle model parameter multiplier

In case 1, the accuracy of the internal vehicle model is changed to represent the driver’s understanding, and the accuracy (multiplier) is set to 100%, 125% and 150%. The relevant vehicle parameters are multiplied by the accuracy; in this case, they are the vehicle mass and wheelbase parameters used in the feedforward controller. It should be noted that not only can the driver can overestimate the parameters, but also underestimate them. The multipliers chosen here are calibrated, in a sense, to human driver behaviour, i.e., better drivers are supposed to follow the lane better. However, somewhat surprisingly, during the process of choosing the parameters, it was found that a vehicle with only 75% driver accuracy, i.e., an underestimate of the vehicle parameters, reached the local minimum overall offset. However, the overestimate condition follows a trend of decreasing controller performance with increasing parameter estimation error. As such, overestimates were then chosen as the means of modelling decreasing driver skill level.

![Comparison of lateral offset with accuracy of 150%, 125%, 100% for the internal vehicle model with understeer handling characteristic.](image)

**Figure 5.1**: Comparison of lateral offset with accuracy of 150%, 125%, 100% for the internal vehicle model with understeer handling characteristic.

In Figure 5.1, it is shown that the vehicle has more offset when the controller has a higher multiplier, but the same trends remain. In the other figures, the vehicle with the multipliers of 1.5 (blue) and 1.25 (red) don’t share the same level of difference as the one between 1.25 (red) and 1.0 (orange) as the blue curve is much harder to be observed, being covered by other two curves. However, when it comes to the oscillatory sections (90 seconds 100 seconds and 120 seconds 130 seconds), the blue has a much higher amplitude. Moreover, such oscillations happen at almost same time set point in the same section in Figures 5.2–5.5 as they are interconnected. When the
steer is fed with a step input, the vehicle will dynamically respond to it: there will be lateral force generated on the tire, which results in the lateral acceleration when the force transfers to the vehicle chassis. Because the tire lateral force is perpendicular to tire pointing direction, it generates torque on vehicle chassis to help the cornering. By observing the figures related to lateral and yaw parameters, this phenomenon is well exhibited.

Despite that, it is worth noticing that in Figure 5.4, the higher the multiplier, the more the corresponding curves shift backward. Thus, the orange curve has a highest lateral acceleration when lateral acceleration is negative and lowest when it goes above zero. At the sections where the oscillations happen in Figure 5.4, they are more pronounced with the higher multiplier, e.g., the oscillations at around 98 seconds and 115 seconds. Normally the vehicle with a higher lateral acceleration while cornering is considered as having better handling performance. Combined with the high offset and sometimes more steer angle and lateral velocity at the same section (98 seconds and 115 seconds), it can be seen that the multiplier doesn’t have much effect on handling performance when cornering, but results in more slip when the vehicle behaviour is oscillatory.
Figure 5.3: Comparison of lateral velocity with accuracy of 150%, 125%, and 100% for the internal vehicle model with understeer handling characteristic.

Figure 5.4: Comparison of lateral acceleration with accuracy of 150%, 125%, and 100% for the internal vehicle model with understeer handling characteristics.
Figure 5.5: Comparison of yaw rate with accuracy of 150%, 125%, and 100% for the internal vehicle model with understeer handling characteristic.
5.2 Case 2: Oversteer configuration with different internal vehicle model parameter multiplier

In case 2, the vehicle configuration has been changed into an oversteer behaviour by setting a lower front roll stiffness and a higher rear roll stiffness, as 3000 N/m and 8000 N/m (5000 N/m and 5000 N/m in Case 1), and the other parameters remain the same. The resulting figures don’t show much difference when compared to Case 1. Different multipliers affect the tracking ability of the vehicle and enlarge the amplitude of steering oscillation in Figure 5.6, which results in the same trends in lateral and yaw parameters. However, the blue (150%) curve seems to be relatively more visible in the Figure 5.7, Figure 5.8 and Figure 5.9. It can be concluded that with an oversteer handling characteristic, the effect of the difference of multiplier (1.25 to 1.50) toward the lateral motion has been amplified when compared to the understeer circumstance: the blue (150%) curve at 98 seconds has the highest amplitude in Figure 5.9 instead of red (100%) in Figure 5.4; the blue (150%) curve at entire section of 110 seconds to 120 seconds has more amplitude. In other words, the curves shift further with the change of the multiplier with oversteer. More details of the comparison between two configurations are discussed in the next two cases.

![Figure 5.6: Comparison of lateral offset with accuracy of 150%, 125%, 100% for the internal vehicle model with oversteer handling characteristic.](image-url)
Figure 5.7: Comparison of steer angle with accuracy of 150%, 125%, and 100% for the internal vehicle model with oversteer handling characteristic.

Figure 5.8: Comparison of lateral velocity with accuracy of 150%, 125%, and 100% for the internal vehicle model with oversteer handling characteristic.
Chapter 5. Results and Discussion

**Figure 5.9**: Comparison of lateral acceleration with accuracy of 150%, 125%, and 100% for the internal vehicle model with oversteer handling characteristic.

**Figure 5.10**: Comparison of yaw rate with accuracy of 150%, 125%, and 100% for the internal vehicle model with oversteer handling characteristic.
5.3 Case 3: Different vehicle configurations with multiplier of 1.0

To further observe the impacts of the different handling characteristics, a comparison has been done when the multiplier is fixed at 1.0 (100%) in Case 3. The resulting offset comparison in Figure 5.11 shows slight difference on the vehicle tracking performance, which implies that the controller is robust toward the change of roll stiffness. Despite that, at the time around 73 seconds, the peak of the offset spikes out compared to the understeer configuration, and the amplitude becomes lower at 115 seconds. By checking the same time set points in Figure 5.12 and Figure 5.15, no significant difference can be found. However, in Figure 5.13, it shows higher frequencies at both points, but the one at 115 second has a higher amplitude when compared to the understeer curve, which is caused by the sideslip at the wheel and results in the decreasing of the lateral acceleration in Figure 5.14.

A tire utilization map is plotted in the Figure 5.16 by taking the lateral force and longitudinal force divided by normal force at that instant. The $u_x$ represents longitudinal tire utilization and $u_y$ for lateral tire utilization. The plot shows the scatter points at the front tires are distributed mostly around and under the $u_y$ axis while for rear tire, the points are located more above the same axis. The reason for such phenomenon is that the vehicle has been set with a front-to-rear braking ratio (0.65), which generates more braking force at front tire than the rear tire, but since the vehicle is defined as rear wheel drive, more axle drive force is acting on the rear tire. When looking at the overall point distribution, it can be seen that the utilization of the oversteer configuration (red) is slightly wider than the understeer configuration (blue). This indicates that when the vehicle is configured as oversteer, the tire has been marginally better utilized when cornering while braking or accelerating.
Figure 5.11: Comparison between lateral offset with different handling characteristics when the multiplier remains the same.

Figure 5.12: Comparison between steer response with different handling characteristics when the multiplier remains the same.
Figure 5.13: Comparison between lateral velocity with different handling characteristics when the multiplier remains the same.

Figure 5.14: Comparison between lateral acceleration with different handling characteristics when the multiplier remains the same.
Figure 5.15: Comparison between yaw rate with different handling characteristics when the multiplier remains the same.
Figure 5.16: Tire utilization status for a lap of track. The plots at the top are front left and right tires, while the plots at bottom are rear left and right tires. Blue and red scatter points are understeer and oversteer with multiplier of 1.00.
5.4 Case 4: Different vehicle configurations with multiplier of 1.5

In Case 4, the multiplier has been switched to 1.5 to see the results from different handling characteristics. Similar to the previous case, there is insignificant difference in lateral offset in the Figure 5.17, but much more oscillation can be seen in Figure 5.20 at around 87 and 113 seconds. In Figure 5.19, they both reach the higher value than the same one with understeer at same time instant. However, the lateral acceleration in Figure 5.20 at 87 seconds has been amplified while the one at 113 second almost remains the same. Combined with yaw rate plot in Figure 5.21, the oversteer vehicle cornered quite hard and handles better at 87 seconds so that it reached a high lateral velocity and acceleration with less yaw rate, but the side slip at 113 second caused higher velocity yet similar lateral acceleration.

The tire utilization in this case shows the difference from the one in case 3 that there is insignificant difference in the overall distributions between two configurations. However, there are a couple of points decentralized from the cluster that can be caused either by high lateral/longitudinal force or low normal force. Combined with the Figure 5.20, for the points in same configuration, the points shown in front tire are supposed to be caused by high lateral force because they all located at same area (around \( u_r = -0.4 \) and 0.4) in both front left and front right tires, but for the points in rear tire plots, they can only found at rear right tire (around \( u_r = 0.2 \) to 0.4) which should be the result of low normal force. The red points also locate further than the blue points as a result of higher roll stiffness of the rear anti-roll bar generating more lifting force when cornering.

![Comparison between lateral offset with different handling characteristics when the multiplier remains the same.](image)

**Figure 5.17**: Comparison between lateral offset with different handling characteristics when the multiplier remains the same.
**Figure 5.18:** Comparison between steer response with different handling characteristics when the multiplier remains the same.

**Figure 5.19:** Comparison between lateral velocity with different handling characteristics when the multiplier remains the same.
\textbf{Figure 5.20}: Comparison between lateral acceleration with different handling characteristics when the multiplier remains the same.

\textbf{Figure 5.21}: Comparison between yaw rate with different handling characteristics when the multiplier remains the same.
Figure 5.22: Tire utilization status for a lap of the track. The plots at the top are front left and right tires, while the plots at bottom are rear left and right tires. Blue and red scatter points are understeer and oversteer respectively, with multiplier of 1.5.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions

In this thesis, a 10 DOF vehicle model and a curvature based lateral controller have been developed. The effect of varying vehicle parameters that influence handling characteristics on the combined handling performance for both machine and human drivers has been explored. Various driver skill levels have been realized by setting different accuracies of the vehicle model parameters in the internal feedforward control model. To minimize changes in the vehicle parameter set, the stiffnesses of the front and rear anti-roll bars were chosen to change the vehicle handling characteristic. The simulation of the track lapping vehicle has been implemented in Simulink so that the results can be easily observed.

The following conclusions can be drawn from the results of the numerical experiments:

- There was a very modest effect on handling performance and tracking ability as the simulated driver skill was varied. It is postulated that the feedback portion of the lateral controller may be so effective as to compensate for errors in the feedforward portion of the controller. Thus, using reduced accuracy in the feedforward portion of the controller may not be a suitable choice for representation of decreasing human driver skill.

- For different driver skills that are represented by various model parameter accuracy, the controller becomes more sensitive toward the change of handling characteristic, generating more oscillations in the lateral offset results. Both results from machine and human drivers show some sideslip at the same corner, indicating the reduced stability of the oversteer configuration. However, counter to expectations, when using the simulated human driver, the oversteer vehicle actually shows a better lateral performance when compared to understeer, which further confirms the hypothesis that the controller robustness can affect the visibility of the change of handling performance.

- Subsequent investigations into the vehicle model performance using open loop tests (vehicle circling at 15 m/s with constant steer angle of 1 degree) for both handling configurations, resulted in very minor changes in the calculated understeer gradient. It is postulated that
because the nonlinear tire behaviour depends on load transfer through the anti-roll bar, that
the changes in vehicle handling behaviour may not be sufficiently apparent at low lateral
accelerations. As such, selecting the roll stiffness distribution as the parameter to vary to
induce changes in handling performance may not be sufficient to test the limits of controller
performance.

- Because the understeer gradient expression derived from the bicycle model is not directly
  affected by parameters like the anti-roll bar stiffness, the understeer gradient is a challenging
  metric to use to quantify changes in the handling behavior in the nonlinear region. Further,
because of the nonlinearity in the tire model, the understeer or oversteer properties may
vary over the range of lateral acceleration, making the understeer gradient a function of the
requested trajectory, and thus even harder to accurately quantify.

- Slightly increasing tire utilization when the vehicle has the oversteer configuration was
  observed, indicating a potential for improved handling that could be explored in future work.

- Several occurrences of oscillations observed in the vehicle response appear to be caused by
  the discretization of track. The track map used in this research is built by assembling multiple
  real world tracks together which means when these tracks are put together, the connecting
  points between them may not match perfectly. The result is that when the the vehicle is
  running on a straight line, at one point in time, it may start braking and then immediately
  accelerate again. In this case, the connecting points have been identified by the controller
  as an obstacle in the road, and the vehicle will try to avoid it while previewing. This won’t
  influence the handling performance too much, but it will generate some oscillations in some
  results.

6.2 Recommendations

Enhanced vehicle model Even though the current 10 DOF is enough to represent vehicle’s be-
aviour, an entire dedicated multi-body suspension can certainly help by having more vari-
ables being recorded and observed, especially when differences are difficult to detect using
the current state variables. The analysis of the motions of the suspension may provide more
information.

The lateral controller As mentioned before, the lateral controller is quite robust on vehicle path
tracking, which makes it very difficult to analyze the results. Therefore, a milder controller
could be taken into consideration so that it loosens the restrictions on the lateral offset and
heading error, and allows the vehicle to be driven more aggressively on the track.

Handling behaviour metrics In this research, the handling characteristics is more of a relative
term that doesn’t have a criteria to quantify understeer and oversteer. Of course, the roll
stiffness changes the vehicle configuration, and a high rear roll stiffness and low front roll stiffness makes the vehicle handling behaviour trend away from understeer and toward oversteer. Normally, the understeer gradient is the parameter used to quantify the difference in handling. Other metrics should be explored to quantify the handling in the nonlinear region.

**Improved track map** A smoother track map may need to be designed in the future, or the test can be split into multiple short tracks as different scenarios.
References


References


Vita Auctoris

Name: Sijie Zhang
Born: Shanghai, China, 1995
Education: 2014-2018 Honours Bachelor of Applied Science with Automotive Option, Mechanical Engineering, University of Windsor, Windsor, Ontario, Canada