Modelling of air damping effect on the performance of micro-resonators

Ankang Wang

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation

https://scholar.uwindsor.ca/etd/8618
Modelling of air damping effect on the performance of micro-resonators

By

Ankang Wang

A Thesis

Submitted to the Faculty of Graduate Studies
through the Department of Mechanical, Automotive, and Materials Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

©2021 Ankang Wang
Modelling of air damping effect on the performance of micro-resonators

By
Ankang Wang

APPROVED BY:

A. Ahmadi,
Department of Electrical & Computer Engineering

N. Zamani,
Department of Mechanical, Automotive & Materials Engineering

J. Ahamed, Advisor
Department of Mechanical, Automotive & Materials Engineering

March 04, 2021
DECLARATION OF CO-AUTHORSHIP

I. Co-Authorship

I hereby declare that this thesis incorporates material that is the result of joint research of the author and his supervisor Prof. Jalal Ahamed. In all cases, the key ideas, primary contributions, experimental designs, data analysis, interpretation, and writing were performed by the author. The contribution of the professor included providing feedback on the refinement of ideas and editing of the manuscript.

I am aware of the University of Windsor Senate Policy on Authorship and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained written permission from each of the co-author(s) to include the above material(s) in my thesis.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.
II. General

I declare that, to the best of my knowledge, my thesis does not infringe upon anyone’s copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledge in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained written permission from the copyright owners to include such material in my thesis.

I declare that this is a true copy of my thesis, including any final revisions, as Approved by my thesis committee and graduate studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.
ABSTRACT

The dynamic performance of the micro-resonator depends on the loss mechanism. As most of the energy is lost from air damping in the ambient pressure, creating a vacuum condition around the micro-resonator will mitigate the energy loss. This paper presents numerical and analytical modeling techniques to understand the effect of vacuum level on energy loss in micro-resonator. Two vacuum regions were investigated; the first region is in the pressure range of 1-10 Pa, while the second region is in the pressure range of 20-100 Pa. Both were investigated in the medium vacuum pressure regime. Analytical and numerical results were compared at these two pressure regions with previous experimental literature study. The goal was to graphically compare the previous and current work and find similar trend relationships i.e. if the pressure decreases, the measured quantity $Q/Q_{\text{max}}$ exponentially increases. There is a qualitative good agreement between the analytical and numerical model with a vacuum sealed pacakging platform. Both the squeeze film damping and slide film damping are present in the cavity and the squeeze film damping is the dominating energy loss. The air gaps between moving structure and fixed fingers create the squeeze film damping and cause energy loss difference, while the smaller air gaps between them generate large damping force and reduce the performance of the micro-resonator. In contrast, the larger air gaps exist between them generate smaller damping force and less influence on micro-resonator (fewer energy losses from the resonator). The air damping is classified into three damping regimes: viscous damping, molecular damping, and intrinsic damping. The pressure between 1-100 Pa is in the molecular damping regimes (medium vacuum), and air damping is the dominating loss in the low to medium vacuum. Furthermore, intrinsic losses, such as anchor loss and thermoelastic damping loss, are dominating at a high vacuum in the pressure range of 0.1 to $10^{-5}$ Pa.
DEDICATION

To my family, for their continuous support.
I would like to acknowledge the following people/organizations for their assistance in the completion of this thesis:

Dr. Jalal, for your support and guidance which helped me progress in the past two years. Thank you for providing me the opportunity to work with you and always keeping me motivated. Thank you for sacrificed free time for guidance me with logical analysis in this thesis and correcting me all the time based on your experiences.

Committee members Dr. Zamani and Dr. Ahmadi for their time, valuable comments and suggestions.

Dr. Tyler from Teledyne Micralyne, for taking time out, valuable comments, suggestions, and guiding through out the project and in the analytical model.

Dean Spicer from Teledyne Micralyne, for the time, valuable comments, suggestions and guidance in the research.

Teledyne Micralyne- for financial and technical supports.

MITACS- for financial support

CMC- for the support with CAD tools, fabricating the sensor chip, and providing me the necessary equipment and devices in a timely manner.

My research team at the MicroNano laboratory for their input and suggestions. This was a team effort that I was happy to be a part of.
TABLE OF CONTENTS

DECLARATION OF CO-AUTHORSHIP ........................................................................... iii

ABSTRACT ................................................................................................................. v

DEDICATION .............................................................................................................. vi

ACKNOWLEDGEMENTS .............................................................................................. vii

LIST OF FIGURES ..................................................................................................... x

LIST OF TABLES ....................................................................................................... xvi

Chapter 1 .................................................................................................................. 1

1.1 Introduction ........................................................................................................... 1

1.2 Literature Review of Air Damping ......................................................................... 9

1.3 Objective .............................................................................................................. 15

1.4 Motivation ........................................................................................................... 15

Chapter 2 .................................................................................................................. 19

2.1 Numerical Modeling ............................................................................................ 19

2.2 Finite Element Modeling Approach .................................................................... 21

2.3 Parameters Analyzed ......................................................................................... 24

2.4 Finite Element Modeling Results ....................................................................... 26

Chapter 3 .................................................................................................................. 32

3. Analytical Analysis ............................................................................................... 32

3.1 Air damping Framework ..................................................................................... 32
LIST OF FIGURES

Figure 1. A gyroscope device fabricated and packaged in our lab and operated in open-air environment. 4

Figure 2. Schematic diagram of the concept of the integrated encapsulation process [8]. .........................5

Figure 3. Schematic diagram of the direct wafer bonding process for sealing micro-resonator. It does not apply intermediate layer and the cap wafer is prefabricated and bonded on the resonator for sealing [8]. ..5

Figure 4. Schematic diagram of the concept of wafer bonding with intermediate layers [8]. .......................6

Figure 5. MicraSilQ process using the wafer-level fabrication (three silicon wafers bonded together as one device), Adapted from [9]. Getter material deposited in the cavity between the device wafer and the lid wafer. ........................................................................................................................................7

Figure 6. MIDIS process with the device wafer thickness of 30 um thick single crystal silicon wafer that can be vacuum encapsulated under high vacuum pressure of 1.3 Pa. Adapted from [10]..........................8

Figure 7. Capacitor cell of comb finger, the air damping causes the viscous and the stiffness effects together in the air gaps, dominated damping depends on the air gaps between two moving finger and fixed finger [17]. .......................................................................................................................................11

Figure 8. Pressure versus quality factor by analyzing Bao energy transfer model and Christian model with Zook’s experiments results. $Q_{\text{sim}}$ is results without three assumptions (Constant particle velocity, constant change in particle velocity, and constant beam position), Adapted from [18].............................12

Figure 9. Illustrates the relationship between pressure and quality factor of micro-devices. Pressure regions classified into intrinsic damping (Pa<1 Pascal), molecular damping (1 Pascal<Pa<100 Pascal), and viscous damping (near Pa>10^4 or atmospheric pressure) [15]. .................................................................17

Figure 10: Schematic top view the resonator. Symmetric structure for obtaining desired model shape. Red and green colored air gaps exist between moving fingers and fixed fingers. Red colored air gaps is moving plate move upward to fixed comb, green colored air gaps is reverse. Each corner fixed by the
anchors and fold beams allow moving plate to move up and down to sense the changes in the capacitance between fixed fingers and moving fingers.

Figure 11. 3D finite element model of the resonator. Thermoviscous acoustic frequency domain applies in the air domain around the micro-resonator, the pressure acoustic frequency domain applies in the outer frame of the air domain.

Figure 12. Air domains choose for thermoviscous acoustic frequency domain and thermoviscous acoustic-structure boundary.

Figure 13. Folded beam and hinge length and width (spring) on the top and bottom of 2D view resonator in Figure 10.

High vacuum damping region are defined at pressures close to the zero Pascal and air damping does not dominate the energy loss anymore. Therefore, below 1 Pa, the simulated pressure range is depicted in

Figure 14. Some simulated amplitudes are difficult to be detected in the real peak point (resonance frequency) by finite element modeling simulation because of the inaccurate thermoelastic damping. While some low pressures (0.001 Pa, 0.005 Pa, and 0.0001 Pa) failed to estimate Q values at the resonance frequency, a few low-pressure values (0.1 Pa, 0.5 Pa, 0.01 Pa, and 0.0005 Pa) show a similar curve trend as shown in Figure 15 and Figure 16.

Figure 15. Amplitude vs frequency for a pressure range between 10 and 100 Pa. When the pressure decreases, amplitude increase with slight decrease of resonance frequency.

Figure 16. Amplitude vs frequency at different pressure ranges between 1 and 10 Pa. When the pressure decreases, amplitude increase with slight decrease of resonance frequency.

Figure 17. Amplitude vs frequency at different pressure ranges between 0.5 and 0.0005 Pa. A few low-pressure values (0.1 Pa, 0.5 Pa, 0.01 Pa, and 0.0005 Pa) are similar trend with figure 14-15.

Figure 18 depicts that the pressure is extremely low, near 0.01 Pa, the air damping has no effect on the resonator, and the air damping coefficient is minimized.
Figure 19. Combination of Q factor vs varied pressure ranges between 0.0005 and 100 Pa. This FEM plot classified into intrinsic damping region and molecular damping region. .......................................................... 30

Figure 20. Damping due to air gaps between comb fingers. The smaller air gaps generated the larger squeeze-film air damping coefficient, which developed a smaller Q value. In contrast, the larger the air gaps, the smaller squeeze-film air damping coefficient is generated, which developed a higher Q value. 31

Figure 21. Pressure built-up by squeeze-film motion when the moving plate moves toward fixed fingers. Air gap distances determine the magnitude of squeeze film damping between the movable plate and the fixed wall. ........................................................................................................ 33

Figure 22. Common air damping mechanism in MEMS devices: (a) Squeeze-film damping (movable plate moves toward fixed fingers or fixed wall) and (b) Slide-film damping (movable plate slip to adjacent fixed walls or fixed comb). .................................................................................................................. 35

Figure 23. Squeeze and slide film damping exists in air gaps between comb fingers set for the sense mode motion [14] ............................................................................................................................... 36

Figure 24. Comparison of Q-factor varying with pressure range, 0.0005 < Pa < 100, between FEA model and Christian model at intrinsic damping region and molecular damping region. ......................... 43

Figure 25. Q-factor vs pressure range for 1 < Pa < 10. It is proven that the Christian (equation 16) has one order magnitude smaller than the simulated result. ................................................................. 44

Figure 26. Q factor vs pressure range for 0.0005 < Pa < 0.5. The blue curve presents the peak Q values from figure 9 and the Q values below 1 pascal (0.1 Pa, 0.5 Pa, 0.01 Pa, 0.0005 Pa). ......................... 45

Figure 27. Linear proportional relationship between resonance frequency and pressure. Pressures below 1 pascal are 0.1 Pa and 0.5 Pa. As for remaining points, there is a 10 Hz difference. FEM frequency set as fixed value f=794 Hz. ........................................................................................................ 45

Figure 28. Comparison between FEA model and analytical analysis showing relationships between pressure and Q-factor. As the pressure decreases, the Q-factor exponential increase (MicraSilQ platform). ...................................................................................................................... 48
Figure 29. Combination of the Q-factor from finite element model, analytical model, and Bao analysis (MicraSilQ platform).

Figure 30. Numerical analysis from finite element model in the pressure range between 10 and 100 Pa. Q factor is highest at 10 Pa (PolyMUMPS platform).

Figure 31. Numerical analysis from finite element model in the pressure range between 10 and 1 Pa. Pressure range between 1 Pa and 4 Pa is not stable due to thermoelastic damping (PolyMUMPS platform).

Figure 32. An Example of the Gaussian fit method used in solver from excel.

Figure 33. Updated Quality factor by applying Gaussian fit method in the pressure range between 1 Pa and 10 Pa without changing the tolerance, applying Gaussian fit method has proved the trend is linear increase with pressure decrease and Q value exponential increase (PolyMUMPS platform).

Figure 34. FEM Relationship between the Quality factor and pressure range between 1 Pa and 100 Pa. (PolyMUMPS platform).

Figure 35. PolyMUMPS fabrication combining of the Q-factor from finite element model, analytical Q, and Bao analysis. Bao analysis has intersection region between analytical Q and FEA model. Analytical Q is the combination of squeeze film and slide film analysis.

Figure 36. Cross-sectional view depicting the 7 layers of the PolyMUMPS process. Adapted from [31].

Figure 37. Cross-sections depicting the PolyMUMPS fabrication process steps for the proposed micro-resonator.

Figure 38. Cross-sections depicting the PolyMUMPS fabrication process steps for the proposed micro-resonator (cont).

Figure 39. Photolithography process is used during exposure of photoresist using a positive tone, where exposed areas are being removed to etch the underlying structural layers.
Figure 40. 2D view of resonator drawn in L-Edit, showing the location of bond pads and electrical connections. Poly0 used for electrical connecting from internal device to bond pads and wire bonding to the outer breadboard. ................................................................. 62

Figure 41. 2D view of resonator drawn in L-Edit, showing double thickness structure (Poly1 and poly2) and electrical connections (poly0) with bond pads. ................................................................. 63

Figure 42. Bonding diagram of the resonator chip with their respective bond pads and connecting electrodes (poly0). Devices uses the wire bonding DIP 40. ................................................................. 65

Figure 43. Frequency response test setup that includes two DC power supplies, a waveform generator, a spectrum analyzer, an oscilloscope, and a breadboard to interface with the chip. ................................................................. 66

Figure 44. Electrical signal received from spectrum analyzer and model shape capture from FEM. It captures resonant frequency in detail by adjusting the start frequency, stop frequency, bandwidth, step size, and zoom in. .................................................................................................................. 67

Figure 45. Moving average filter is a technique to extract the signal from a noisy environment in the frequency domain. Moving average filter is only used in open air condition, not inside the vacuum chamber. ........................................................................................................................ 68

Figure 46. Breadboard front-end electronics classify into three parts: actuation Figure 47, detection Figure 48, and opened- air MEMS resonator (non-vacuum). ................................................................. 70

Figure 47. Actuation circuit where IA = AD620, RG = 500kΩ, R1 = 1KΩ and C1 = 1µF. ................................. 71

Figure 48. Detection circuit where OA = OP177, IA = AD620, RG = 500kΩ, RF = 10kΩ and CF = 10pF. ........................................................................................................................................... 71

Figure 49. The sense-mode system with an amplitude of 1100 and resonant frequency of 10.0 kHz, illustrating a 29% gain drop from a 5 Hz relative shift between the operating frequency and sense-mode resonant frequency [1]. .................................................................................................................. 73

Figure 50. Vacuum chamber setup to estimate Q factor at varying pressure. .................................................. 74
Figure 51. $Q/Q_{max}$ versus pressure resonator in the vacuum chamber. Comparison between FEA modeling, an analytical model of this thesis, Bao Christian model [15] and energy transfer model [30] with previously reported experimental results [5,33], shows good agreement of trend of Q factor change with pressure. .............................................................. 76
### Table 1. Knudsen number depends on the degree of rarefaction. Knudsen number increases as the pressure decrease [18].

Table 2. Various geometry parameters, dimensions, and SI units of the resonator.

Table 3. Material properties of the micro-resonator and ambient air properties.

Table 4. Specific parameters for analytical analysis needs.

Table 5. Summary of different percent error relative to analytical Quality factor.
Chapter 1

1.1 Introduction

Microelectromechanical systems (MEMS) is the technology that combines electrical and mechanical systems into a chip fabricated at a micro-scale (10^{-6} m). A large portion of MEMS inertial based devices are based on vibrating micro-structures suspended above a substrate. The micro-structures include masses, flexures, actuators, detectors, levers, linkages, gears, dampers, and many other functional building blocks [1]. Those components can be combined to build complete sophisticated systems on a chip and allow electrical components to integrate on the same chip as mechanical sensor elements to provide an unmatched integration capability for sensing and actuation applications. Resonators utilizing a small vibrating mass as a sensing element are used mainly for dynamic motion and orientation sensing. MEMS has fast developed and created the capability of integrating micro-devices with low cost, small size, and light weight [1].

Resonators are crucial to determine dynamic motions, such as acceleration, vibration, and orientation. They are also used in radiofrequency or microwave components, such as oscillators and filters, to attain a higher quality frequency reference. There are six different types of resonators: coaxial resonators, dielectric resonators, crystal resonators, ceramic resonators, surface acoustic wave resonators (SAWs), and yttrium iron garnet (YIG) resonators [2], and their main purpose is adjusting broader range of resonance frequency.

MEMS resonators based inertial sensors are used in a wide range of applications in the aerospace, satellite, defense, automotive, consumer electronics, and medical instruments for motion, position
and navigation purposes. The aerospace and satellite industry require high-performance resonators for navigation and guidance systems. MEMS inertial sensor’s application in the automotive industry includes ride stabilization, rollover detection, prevention, brake systems, air bag, and electronic stability control (ESC). In consumer electronics markets, they are largely used for image stabilization in digital cameras and phones, orientation and motion sensing in the phone and gaming consoles, and orientation sensing in virtual reality products. In medical instruments, they are used in micro-robotics, hearing aid sensors, and implantable devices for sensing signal for blood molecules (glucose) [1,3]. In a MEMS gyroscope the resonator mass is driven into vibration, usually at its resonance mode, via an external sinusoidal force and is known as the drive mode. When a rotation is applied orthogonally, the gyroscope displaces, and this is detected through capacitance difference via electrodes and is known as the sense mode. The drive and sense modes typically correspond to the motion on two orthogonal axes. The drive and sense mode operate at the resonance frequency to maximize the displacement and sensitivity. Utilizing matched resonance in both the drive and sense modes can attain the maximum response gain and sensitivity in a mode match gyroscope. Increasing the difference between drive and sense frequencies decreases sensitivity and may result in signal processing errors.

For a gyroscope, the vibrating proof mass needs to be vibrating at its resonance frequency while the device is in operation. Any energy dissipation during vibration will reduce its amplitude of motion and will require additional energy to keep it vibrating. Therefore, the performance of a MEMS resonator depends on the efficiency loss in the drive mode and energy loss via electrical signal in the sense mode. The Quality factor, Q, is a measure of relative energy loss of the resonator and determines how long an oscillator will resonate. It is defined as the ratio of maximum stored energy to the total dissipated energy per cycle [5]. High-quality factor leads to high sensitivity,
high stability, and high resolution. A navigation grade MEMS resonator with a high Q factor, usually in the range of a million, can be considered as state of the art [6]. A low Q-factor could result in the attenuation of the amplitude, making the motion unstable. Q-factor is expressed by the following formula:

\[
Q = \frac{E_{\text{energy, max, stored}}}{E_{\text{energy, total, dissipated}}} = \frac{m\omega}{c} = \frac{k\omega}{c} = \frac{\sqrt{k}m}{c}
\]  \hspace{1cm} (1)

In Equation 1, \(k\) is the spring constant of resonator, \(\omega\) is the radial frequency, \(c\) is the coefficient of damping force, and \(m\) is the effective mass.

To optimize the Quality factor, it is crucial to reduce the damping associated with the system. There are three primary energy dissipation methods: thermoelastic dissipation (TED), anchor loss, and gas damping. Among them, anchor loss and air damping are extrinsic losses, while TED is an intrinsic loss. Extrinsic losses can be reduced via optimizing the resonator’s geometry. The three damping methods are combined into total energy dissipation. The total dissipation represents a direct sum of the energy dissipated term [5],[7].

\[
\frac{1}{Q_{\text{measure}}} = \frac{1}{Q_{\text{pressure}}} + \frac{1}{Q_{\text{anchor}}} + \frac{1}{Q_{\text{TED}}}
\]  \hspace{1cm} (2)

Among different mechanisms that dissipate energy, the air damping is a dominant energy loss for the micro-resonators in the open air or the low vacuum region [8]. Q-factor of a micro-resonator is strongly reduced when operating in a non-vacuum (open-air) environment due to the gas damping. If the resonator is not packaged with a vacuum seal, then it operates in the ambient air and atmospheric pressure as shown in Figure 1. For a low Q-factor sensing system, the long-term testing cause inaccuracy and degradation of signal stability.
Figure 1. A gyroscope device fabricated and packaged in our lab and operated in open-air environment. 

Q-factor is low for devices operating without a vacuum seal, in the order of 10 to 1000 [3]. There are many ways to create a vacuum or sealing process around the resonator. 

One approach is called the integrated encapsulation process (Figure 2), which is mainly used to seal or protect caps for MEMS resonators. The disadvantage of the integrated encapsulation process is the complicated processing compatibility issues. This sealing process aims to avoid contamination from environments (dust material, moisture, and gas molecules). This integrated encapsulation process, as shown in, consists of several steps: depositing the sacrificial layer on the silicon substrate; depositing the micro-resonators above the sacrificial layer; depositing an extra layer of the sacrificial layer around the micro-resonators which is followed by the packaging cap layer deposition and patterning for the formation of encapsulation shell with release etching
channels and holes; removal of the sacrificial layers allowing micro-resonator to vibrate in this cavity. Lastly, sealing the release hole and channels by another layer for protecting the encapsulation shells.

![Diagram of integrated encapsulation process](image1)

**Figure 2.** Schematic diagram of the concept of the integrated encapsulation process [8].

The second approach is the wafer bonding process (**Figure 3**). The cap wafer is prefabricated with microcavities and bonded with the device wafer to provide sealing and protection. Compared with the integrated vacuum sealing process, the wafer bonding process is more straightforward and more versatile.

![Diagram of direct wafer bonding process](image2)

**Figure 3.** Schematic diagram of the direct wafer bonding process for sealing micro-resonator. It does not apply intermediate layer and the cap wafer is prefabricated and bonded on the resonator for sealing [8].
The direct wafer bonding process is classified as silicon-to-silicon fusion and silicon to glass anodic process; both wafer bonding processes are needed to be used if the two bonding surfaces require flat contact. The available roughness is limited from 1 um to 4 nm for good bonding results for silicon-to-silicon fusion and silicon to glass anodic bonding [8].

The third bonding process is called the eutectic bonding, which is the most notable wafer bonding with intermediate layers (Figure 4). This process melts two metal (silicon or gold) to form a stable intermediate compound that facilitates the bond. This approach is commonly used when a lower bonding temperature and strong bonding interface is required but a direct wafer bonding process is not compatible with the device [8]. One advantage of this eutectic bonding approach is filling the gaps between the lid wafer and the device wafer.

Various commercially available wafer-scale vacuum encapsulation processes exist in the market. One such platform is commercially known as the MicraSilQ process developed by Micralyne [9]. As shown in Figure 5, the MicraSilQ process is a typical example that combines direct wafer bonding between the base wafer and device wafer and with an intermediate layer between device wafer and lid wafer. MicraSilQ platform has three wafers: the base wafer is the through-hole
electrical connection, at the center is the device wafer and the top is the lid wafer. The MicraSilQ process combines three silicon wafers bonded together as one device with freely moving structures sandwiched between the base and lid wafer. Getter material is deposited inside the cavity of the lid wafer which is used to keep the vacuum level inside constant. Getter material is able to chemically absorb active gasses inside the cavities of a MEMS device. Hermetic wafer bonding between the lid wafer and the device wafer relies on the formation of Gold- silicon Eutectic above 363 °C to perform sealing. The leak rates less than 1e-17 Pa*m3/s of non-gettable species have been repeatedly achieved for each die on the wafer eutectic bonding process. The result of this vacuum encapsulation is very low noise and extremely sensitive devices [9].

Figure 5. MicraSilQ process using the wafer-level fabrication (three silicon wafers bonded together as one device), Adapted from [9]. Getter material deposited in the cavity between the device wafer and the lid wafer.
Another commercially available vacuum encapsulated method, as shown in **Figure 6**, is called the MIDIS process developed from Teledyne DALSA semiconductor Inc. (TDSI). MIDIS process is based on the high aspect ratio, bulk micromachining of a 30-um thick single crystal silicon wafer that can be encapsulated under high vacuum of 1.3 Pa [10]. Further, the MIDIS process is one of the world's lowest demonstrated total leak rate equivalent from the encapsulated vacuum cavity that varies from one fabrication run to another between 45 mol/s (7.5E-13 atm cm$^3$/s) to 2500 mol/s (4.1667E-11 atm cm$^3$/s) [10]. And the lower leak rate in the MIDIS process slows the degradation of the Q of the encapsulated MEMS resonators. It allows the devices with desired specifications over a longer lifetime [11]. The step-by-step development of the MIDIS process is described in [12].

**Figure 6.** MIDIS process with the device wafer thickness of 30 um thick single crystal silicon wafer that can be vacuum encapsulated under high vacuum pressure of 1.3 Pa. Adapted from [10].

However, several challenges exist with the vacuum encapsulated methods in terms of achieving and ensuring a stable and low-pressure regime over time. Getter material is one of the most effective ways to hold vacuum constant in the lid cavity because of the getter property to chemically absorb active gases under vacuum wafer-level packaging (VWLP) [13]. Use of the getter material reduces both the initial pressure inside the cavity and the effect of any outgassing that occurs from the sealing and device, which is seen to stabilize Q and air pressure. It maintains
a low cavity vacuum with a pressure of 0.5 Pa, and lasts for five or more years [9]. The disadvantages of this approach include the cost of the getter film and unreliable packaging techniques sensitive to shock, which severely limits vibration and long-term usability [13].

Micro-resonators for navigation grade applications are fabricated under the vacuum MEMS packaging technology to ensure the device's maximum dynamic performance (high and stable Q values). Depending on the vacuum level, micro-resonators may experience air damping against the moving part. Which film damping dominates and which film damping experiences less influence of vacuum condition inside the vacuum encapsulated environment on micro-resonators has not been studied extensively and will be addressed in this thesis by performing a detailed analysis of air damping.

1.2 Literature Review of Air Damping

There has been extensive research conducted in the past on air damping mechanisms, which consists of analytical models. Sen Ren et al. have studied a micromachined pressure sensor with an integrated resonator operating at atmospheric pressure. Sen et al [15] proposed the design of a low-cost resonant pressure sensor without vacuum encapsulation. While a variety of technologies (glass tube evacuation technology, silicon direct fusion bonding technologies, selective anisotropic etching and hydrogen evacuation technology, and adhesive vacuum bonding and getter materials) [15] can be exploited for vacuum of resonant pressure sensors for a high Q factor in the order of $10^4$, vacuum packaging of resonant pressure sensors is complicated and relatively costly. Sen et al’s goal was to obtain the Q factor as high as possible at atmospheric pressure (Q above 1000). Their preliminary measurements show that the resonator pressure sensor's fundamental frequency is approximately 34.55 kHz with a pressure sensitivity of 20.77 Hz/kPa. Over the full-scale
pressure range of 100-400Kpa and the whole temperature range of -20 to +60 ° C, Quality factors ranging from 1,146 to 1,772 were obtained [15].

Bao and yang [16] investigated the squeeze-film effect in a MEMS resonator. Using the Christian model, the energy transfer model, and Boltzmann's transportation equation [16], the authors investigate the results of the analytical model and experiment, validated by Zook [34], of the air damping effect on the performance of micro-resonators in low and high vacuum region. Each method will be addressed later in the analytical analysis.

Squeeze film damping (defined in section 3.1.1) due to the capacitive gaps between the moving and fixed electrodes was studied in [17]. In this paper, researchers focused on the squeeze film damping on comb drives. Wu Zhou concluded that the viscous effect dominates in the low-frequency zone, while the stiffening effect (resist deformation) is dominated in the higher frequency zone (Figure 7). Therefore, the air damping causes the viscous and the stiffness effects together in the air gaps.

The capacitor gaps influence the stiffening level. The damping coefficients vary by according to the air gaps between two capacitors (moving fingers and fixed fingers). The narrower air gap that exists between two capacitors has higher damping coefficients. When capacitor gaps change from 1.5 µm to 4µm in the comb resonator, the coefficient varies from 64.86 µN. s/m to 3.92 µN. s/m in half of the sensor model [17]. During the fabrication process, the larger error of the etched angle in the DRIE (deep-reaction-ion-etching) process may cause the damping coefficient difference. For example, the slight reduction with a small etch angle (0° -1°) affects a larger damping coefficient. The distance of air gaps (g) between the capacitor plates also changes the dynamic behaviour in a gap-closing design. The damping coefficient is different when the micro-resonator is in a closing and opening direction under the acceleration input. In the close direction, the
damping coefficient increases with increasing acceleration. In the opening direction, the damping coefficient decreases with increasing acceleration (accelerometer). Ten sensors were tested with 3 µm air gaps to conclude that the damping coefficient is 21.88 µN.s/m, which is higher than the designed value (18.53 µN s/m) half sensor.

Figure 7. Capacitor cell of comb finger, the air damping causes the viscous and the stiffness effects together in the air gaps, dominated damping depends on the air gaps between two moving finger and fixed finger [17].

The viscous effect and stiffness depend on the structural dimension and inner gas state. Packaging micro-resonator is one of the most critical and challenging technology areas. Q-factor enhancement for MEMS devices with getter film is reported in [13], which investigated the effects of total pressure, different gas composition, and presence of the getter film improvement on the performance of micro-resonators (high Q, the stability of sensor signal, lifetime and removal danger gases like H₂ and H₂O). This article compared the quality factor differences between a MEMS resonator's theoretical evaluation within a getter film and conventional measurement results. Both data are based on the presence and removal of the getter film and results in the
difference in residual gas composition analysis with a pressure regime of 100 Pa to 1E+6 Pa. To conclude, although a getter film is expensive, the Q-factor is of high value with high stability of sensor signal and stable lifetime. The quality factor of sample 1 without getter film and backfilling is 3000 at 100 Pa. The Q-factor of sample 2 with getter film (without backfilling) is equal to 5000 at 41 Pa [13]. The reason for this result is because the outgassing (H₂, CH₄, and CO) of the internal surface is eliminated by the getter film materials.

Early works of micro-resonator and squeeze-film damping were reported in [18]. Firstly, Sarne Hutcherson reviewed the Christian's free molecular model derived from the gas damping of a vane or fiber that can swing like a pendulum in the low vacuum [26]. Christian developed a Maxwell-Boltzmann velocity distribution model used for gases and compared the relationship between the theoretical value of Quality factor with Zook's experimental results [18].

![Figure 8](image)

**Figure 8.** Pressure versus quality factor by analyzing Bao energy transfer model and Christian model with Zook’s experiments results. $Q_{sim}$ is results without three assumptions (Constant particle velocity, constant change in particle velocity, and constant beam position), Adapted from [18].
**Figure 8** illustrates results from Christian’s theoretical model which has an order magnitude higher than testing results from Zook’s experiments [16]. Bao’s theoretical results most closely approximate Zook’s experimental values in the pressure between 0.08 and 3 Torr [18].

However, Christian free molecule’s calculated Quality factor has a high limitation that is only valid if the vane is oscillating in an "unbounded" space (no nearby walls). In other words, Christian's model calculated several molecules striking the microbeam working in the control volume. Most of MEMS application has fixed walls like substrate, fixed comb fingers, and anchors, so the Christian model is no longer valid because of the nearby walls. Therefore, Kadar et al. found the Christian model modified by the Maxwell-Boltzmann (MB) inappropriately describing the velocity distribution of molecules in a gas as a whole instead of the distribution of actual numbers of molecules colliding with the beam. Hence, Kadar et al. modified Christian's model by applying the Maxwellian-Stream (MS) distribution of molecules that strike a surface [25].

Kadar followed the same process of Christian's model and found that the Quality factor calculated by the Maxwellian-Stream reduces Christian's result by a factor of $\pi$. Later, Li proposed that the Maxwellian-Stream distribution used in Kadar et al.'s paper does not consider the difference in number and velocity distribution of molecules impacting on two sides of a vibrating beam [18]. Li updated Kadar's work and replaced the MS with a relative velocity of molecules distributions (MU) for molecules striking the beam's front and back. Li found that the resulting Quality factor reduces Kadar's results by another factor of 1.5.

Due to the model's modification, the authors (Sarne et al.) proposed a direct approach called the energy transfer model from Bao et al. [18]. This approach considers the effect of damping on the nearby walls. Since Bao et al. only assumed the elastic collisions between gas molecules with micro-resonators structure and ignoring the intermolecular collision, they used conservation of
linear momentum and conservation of kinetic energy laws to calculate the velocity change before and after each collision.

To make reasonable limitation of the molecules, Bao et al. have made three major assumptions: constant particle velocity, constant change in particle velocity, and constant beam position, which reveals Quality factor results that better fit the pressure of around 10 Torr, which corresponds to a Knudsen (\(k_n\) determine the degree of rarefaction in Table 1) number of 4.5. In this range, gases are in the transition regime compared with Zook's experimental values. Analyzing Bao's three assumptions can achieve further improvement. By removing assumptions (\(Q_{\text{sim,NA}}\) in Figure 8), the calculated values agree well with Zook's experimental values in the pressure range between 0.08 to 3 Torr (Knudsen number is between 561 and 15)[18]. Lastly, they investigated parameter relationships among the Stokes number (\(\beta\)), Knudsen number (\(K_n\)), gaps to amplitude (d0/A0) with dissipated energy, and Quality factor. The Quality factor increased with increasing Stokes number [18]. Damping and Quality factor linearly depend on frequency (increased damping as frequency increases). Knudsen number and Quality factor increase with a decreasing gap to amplitude. The large gap to amplitude means more collisions between molecules and micro-beams in the close direction (in which gas molecules gain energy) or fewer collisions in the open direction (in which gas molecules lose energy).

Several types of damping such as squeeze film, slide film and gas drag damping can reduce the Quality factor. Q factor highly depends on the air gap distance between two capacitors. Air dominated damping depends on the direction of motion [28]. The viscous effect and stiffness effect coexist in the air gaps, but the viscous effect dominates in the low-frequency zone. Detailed understanding of the effect of air damping inside vacuum-sealed resonator has not been done in
the literature. This research is investigating the detailed analysis of the air damping effect on sealed micro-resonator at various vacuum regions.

1.3 Objective

The objective of this thesis is to perform analysis of the air damping effect on sealed micro-resonator at various vacuum regions to provide a model to predict the damping loss and utilize it to design best resonator with low damping. It is critical to present the developing analytical and numerical modelling techniques to provide a complete understanding of air damping in MEMS resonators. Analytical modelling is based on utilizing various formulations available in the literature as a baseline. Next, a finite element analysis (FEA) based numerical modelling of a vacuum-sealed MEMS resonator using simulation tool COMSOL Multiphysics 5.5 is developed to investigate damping and its relationship with resonator design and cavity pressure parameters by recreating the actual operating environment of the resonator.

Comparison of Q factors at different vacuum regions, the analytical analysis and finite element-based modelling approach, and critical results outlining air damping and quality factor at various pressure regimes are then conducted.

Utilizing model results, a MEMS resonator is designed for maximizing Q-factor and minimizing air damping for vacuum-sealed packaging to mitigate damping losses. Finally, experimental validation is done to compare the accuracy of the theoretical modelling approach.

1.4 Motivation

This thesis aims to analyze the Quality factor at low-pressure regions (molecular vacuum) and compare this analytical result and FEA modeling results with the previous experimental
measurements from literature, to show demonstrate the capability of the model tool. In addition to understand the influence of air damping on the dynamic performance of the micro-resonators. Instead of analyzing the other mechanism, the paper focuses on air damping because air pressure dominates in the low to medium vacuum regime. But in the high vacuum, material damping and loss through the anchor dominate the damping mechanism. Though understanding of the relationship between the quality factor and the pressure varies, the literature review supports the fundamental relationships between quality factor and pressure difference. The air damping category includes three regions: intrinsic damping, molecular damping, and viscous damping [15] (Figure 9). The characteristic of viscous damping in the continuum model (viscous flow) [18], which is highly limited on micro-resonator operated only in open-air with near atmospheric pressure. Some of the devices such as accelerometers and gyroscope use continuum theory with failed predictions [15]. Therefore, this study investigated the feature of air in those devices and found that gas rarefaction effects (pressure decrease) are so significant in between the interaction of movable structure and each molecule is considered individually to obtain an accurate prediction of fluid (air) impact on oscillating devices.[15]. Knudsen number, $K_n$, stands for the ratio of mean free path of the gas molecules to the characteristic length of the flow, where $K_n$ could determine the pressure regimes and degree of validity of the continuum model. For example, when the Knudsen number is greater than 10, the velocity distribution is in the free molecule regime (Table 1).
Figure 9. Illustrates the relationship between pressure and quality factor of micro-devices. Pressure regions classified into intrinsic damping (Pa<1 Pascal), molecular damping (1 Pascal<Pa<100 Pascal), and viscous damping (near Pa>10^4 or atmospheric pressure) [15].

Figure 9 shows the relationship between pressures and Quality factor at the resonant frequency. If the device exposes to the lower ambient pressure, the quality factor can be increased. Generally, a more reasonable assumption is that air acts like a viscous fluid and resistor against micro-resonators' motion. Viscous damping analyzes the pressure around the atmosphere (10^5 pa), where the Q value of air damping is small (Q approximately equal to several hundred or thousands). Air damping plays an essential role in improving MEMS device’s performance at a low-pressure regime to acquire accurate prediction during the design stage. Researchers investigating the low-vacuum region found that the squeeze film damping dominates at low pressure (about 1 kPa and
below) [18]. In the molecular damping, molecules striking the micro-resonator (intermolecular instances are neglected), air pressure is defined in the pressure range of $1 < p_a < 100$. In this region, the comb fingers collide with air molecules during the micro-resonator's motion. The Quality factor increases exponentially when the pressure decreases ($Q = \frac{\sqrt{km}}{c}$). Intrinsic damping remains constant and becomes dominant ($Q > 10^5$, $K_n$ is high) at a high vacuum and highly dependent on the material and design.

**Table 1.** Knudsen number depends on the degree of rarefaction. Knudsen number increases as the pressure decrease [18].

<table>
<thead>
<tr>
<th>Regime</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuum regime (Navier-Stokes equations</td>
<td>$K_n \leq 10^{-3}$</td>
</tr>
<tr>
<td>without slip boundary conditions)</td>
<td></td>
</tr>
<tr>
<td>Slip regime (Navier-Stokes equations with</td>
<td>$10^{-3} \leq K_n \leq 10^{-1}$</td>
</tr>
<tr>
<td>slip boundary conditions)</td>
<td></td>
</tr>
<tr>
<td>Transition regime (molecular approach)</td>
<td>$10^{-1} \leq K_n \leq 10$</td>
</tr>
<tr>
<td>Free-molecule regime (molecular approach)</td>
<td>$K_n &gt; 10$</td>
</tr>
</tbody>
</table>
Chapter 2

2.1 Numerical Modeling

Numerical simulation is preferred prior to fabricating a MEMS device to allow flexibility in design iteration to maximize performance (Q-factor). While experimental design iteration is very expensive due to the fabrication complexities and time, however, numerical simulation gives us the opportunities to adjust many design variations for MEMS devices’ better dynamic performance and computationally perform experimentation on design parameters. Finite element simulation techniques are therefore, widely used to perform virtual experimentation of micro-scale devices.

In our design of the resonator, the finite element simulation needs to re-create the surrounding air inside the sealed cavity and allow simulating the device actuation, damping and dynamic response. The problem involves solving solid mechanics and viscous damping, therefore, creating a Multiphysics problem. A finite element model (FEM) was developed for the micro-resonator to simulate its operation at various pressures using simulation tool COMSOL Multiphysics which allows solving coupled Multiphysics problems.
Figure 10: Schematic top view the resonator. Symmetric structure for obtaining desired model shape. Red and green colored air gaps exist between moving fingers and fixed fingers. Red colored air gaps is moving plate move upward to fixed comb, green colored air gaps is reverse. Each corner fixed by the anchors and fold beams allow moving plate to move up and down to sense the changes in the capacitance between fixed fingers and moving fingers.

A 2D top view of the resonator structure is shown in Figure 10, where the entire system is symmetric to obtaining desired model shape. The device consists of the movable plate (proof mass) and four folded beams (spring) at the top and the bottom of the structure. Each of the folded beams is fixed by the anchor at each corner. The external sinusoidal force (1.56E-6 N in the y-axis) will be applied to actuate the resonator to oscillate in the motion direction (drive mode) at the resonant frequency. Air gaps exist between the movable comb finger and the fixed comb, allowing the
movable plate to sense the motion. When the fingers are displaced, it will cause a change in capacitance by providing high DC voltage to the device. The electro-mechanical system can sense the changes in capacitance using detection circuitry. In proportion to the amount the micro-resonator has displaced.

2.2 Finite Element Modeling Approach

Finite element modeling simulation replaces the use of experiments and gives a thorough understanding of the design. Compared to running experimental measurement, finite element modeling simulation allows for quicker and often more efficient optimization of the design. In addition, finite element modeling simulation gives the audience a visual feeling. Finite element modeling simulation also allows for an iterative approach in finding the optimal design.

The COMSOL Multiphysics platform has many sets of core physics interfaces for fields such as solid mechanics, acoustics, fluid flow, heat transfer, chemical species transport, and electromagnetics [36]. The acoustic module allows for the adjustment of the sound and pressure inside of a volume. Under the acoustic module, there are two options used for the model: thermoviscous acoustic and pressure acoustic. The thermoviscous acoustic defines a volume around the resonator, which allows to change the background equilibrium temperature, equilibrium pressure, and material properties of the air fluids such as dynamic viscosity, thermal conductivity, and heat capacity as the model development needs [36]. It could solve all sets of linearized equations from a compressible flow: Navier-Stokes (momentum conservation), continuity (mass conservation), and energy conservation equation. This also needs to work with acoustic modules to accurately model the air pressure. Acoustic modules are only available for adjusting the acoustic
pressure in the outer frame air domain (ambient pressure) [36], which is considered as atmospheric pressure outside of the vacuum chamber.

The first step is to define the geometry shown in Figure 10. The geometry is enclosed by the air with the dimension of 4000 um *4000 um *920 um (Figure 11). The cavity distance from the top surface of the resonator to the top layer of air and cavity distance from the bottom surface of the resonator to the bottom layer of air depends on the fabrication platform. As for the MicraSilQ platform, there is a cavity in between lid wafer and device wafer and cavity in between the base wafer and the device wafer, as shown in Figure 5. The cavities provide a space to allow resonator vibration. For material selection, Polysilicon was used for the resonator device air properties to specify an air domain around the resonator devices (Figure 11).

![Figure 11. 3D finite element model of the resonator. Thermoviscous acoustic frequency domain applies in the air domain around the micro-resonator, the pressure acoustic frequency domain applies in the outer frame of the air domain.](image-url)
The air domain also exists in gaps between four comb fingers and gaps between top and bottom spring hinges. In selecting solid mechanics, the device is anchored at the edge of the four corners and inner fixed comb fingers. A body load node applies to the entire structure with a total force of 1.56E-6 N in the y-direction.

Multiphysics is added to the model which includes the thermoviscous acoustic and pressure acoustics domain. The thermoviscous acoustic node applies the air domain around the resonator (Figure 12). The pressure acoustics node utilizes the outer frame of the air domain (Figure 11).

Figure 12. Air domains choose for thermoviscous acoustic frequency domain and thermoviscous acoustic-structure boundary.

Thermoviscous acoustic structure boundary 1 is automatically selected in the setting (edge of each air gap of the resonator). Fine mesh size is selected to obtain accurate results (maximum mesh size: 120 um, minimum mesh size: 0.45 um) [37]. Setting the frequency domain range was around 800
24 Hz (analytical calculation of the resonant frequency) in the study. Then the simulation is run to predict the Q values.

2.3 Parameters Analyzed

Various parameters were investigated including the pressure, proof mass size, comb size, spring hinges, and spring fold. For example, proof mass size of 2430 um*2323.6*60 um. Table 2 shows the length, width and thickness of the comb fingers and overlap area. The dimension of hinge length, hinge width, fold width, and fold height are 1118um, 8.9um, 32um, and 24.8um, respectively, as shown in Table 2 and Figure 13. All parameter variables from finite element modeling simulation have to be consistent with analytical analysis for the comparison. There are three tables below (Table 2-4) that show the micro-resonator's dimension, material properties of silicon and air, and the relative parameter of the equations. Those parameters are converted to SI units for better computation analysis later.

Table 2. Various geometry parameters, dimensions, and SI units of the resonator.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model symbol</th>
<th>Dimension(µm)</th>
<th>SI unit (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Device length</td>
<td>L</td>
<td>2430</td>
<td>2.43e-3</td>
</tr>
<tr>
<td>Device width</td>
<td>W</td>
<td>2323.6</td>
<td>2.32e-3</td>
</tr>
<tr>
<td>Structure layer thickness</td>
<td>h</td>
<td>60</td>
<td>6e-5</td>
</tr>
<tr>
<td>Comb-finger length</td>
<td>lcf</td>
<td>240</td>
<td>2.4e-4</td>
</tr>
<tr>
<td>Comb-finger width</td>
<td>wcf</td>
<td>18.7</td>
<td>1.87e-5</td>
</tr>
<tr>
<td>comb-finger thickness</td>
<td>tcf</td>
<td>60</td>
<td>6e-5</td>
</tr>
<tr>
<td>Comb-finger overlap area (um^2)</td>
<td>Aoverlap</td>
<td>1,066,648</td>
<td>1.0666e-6</td>
</tr>
<tr>
<td>moving mass comb finger to fixed comb finger gap</td>
<td>g</td>
<td>5</td>
<td>5e-6</td>
</tr>
<tr>
<td>Hinge length</td>
<td>Hl</td>
<td>1118</td>
<td>1.118e-3</td>
</tr>
<tr>
<td>Hinge width</td>
<td>Hw</td>
<td>8.9</td>
<td>8.9e-6</td>
</tr>
<tr>
<td>Fold width Wc2</td>
<td>Wc2</td>
<td>32</td>
<td>3.2e-5</td>
</tr>
<tr>
<td>Fold height Lc2</td>
<td>Lc2</td>
<td>24.8</td>
<td>2.48e-5</td>
</tr>
</tbody>
</table>
Figure 13. Folded beam and hinge length and width (spring) on the top and bottom of 2D view resonator in Figure 10.

The folded beam (Figure 13) functions as a spring against the motion. It allows the resonator to vibrate vertically. The resonator model consists of four pieces of the same dimension as the folded beam used in each corner (Figure 10).

Table 3. Material properties of the micro-resonator and ambient air properties

<table>
<thead>
<tr>
<th>Silicon properties</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson ratio</td>
<td>v</td>
<td>0.28</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>ρ</td>
<td>2330</td>
</tr>
<tr>
<td>Young's modulus (Pa)</td>
<td>E</td>
<td>1.70E+11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Air properties</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Viscosity (kg/ms) at 15 °C</td>
<td>μ</td>
<td>1.83E-05</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>ρ_air</td>
<td>1.204</td>
</tr>
</tbody>
</table>

The silicon and air properties, in Table 3, are from COMSOL Multiphysics. As for convenience, specific parameters have been calculated (Table 4), such as moving area (368000 μm²), volume (2.21e+8 m³), radial frequency (4988.849 Hz), mass (5.1493E-7 kg), spring constant (12.8366...
N/m), gap cavity between lid wafer and device wafer (20 µm), and gap cavity between the base wafer and device wafer (50 µm). All those parameters from FEM simulation will be used in the analytical analysis for comparisons.

**Table 4. Specific parameters for analytical analysis needs**

<table>
<thead>
<tr>
<th>Devices</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moving area</td>
<td>A</td>
<td>3,680,000 (µm²)</td>
</tr>
<tr>
<td>Volume</td>
<td>V</td>
<td>2.21e+8 (m³)</td>
</tr>
<tr>
<td>Frequency</td>
<td>ω</td>
<td>4988.849 (radian)</td>
</tr>
<tr>
<td>Mass</td>
<td>M</td>
<td>5.1493e-7 (ρV)</td>
</tr>
<tr>
<td>Spring constant</td>
<td>K</td>
<td>12.8366 (N/m)</td>
</tr>
<tr>
<td>Gap between the top surface of</td>
<td>dpₜ</td>
<td>20 (µm)</td>
</tr>
<tr>
<td>the device to the lid wafer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gap between the bottom surface</td>
<td>dpᵦ</td>
<td>50 (µm)</td>
</tr>
<tr>
<td>of device to substrate (base</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wafer)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.4 Finite Element Modeling Results

The goal of the finite element modeling (FEM) is to capture the varying Quality factor as the air pressure varies. Decreasing pressure to minimize the air damping effect will cause the Q-factor of the resonator to increase.

The equilibrium pressure (ambient pressure) at the thermoviscous acoustics is changed to estimate the amplitude in the molecular damping regime (1<Pa<100). Most of the estimated resonant frequencies are in the range between 770 Hz and 2200 Hz. It has a few shifts of resonance frequency when the pressure varies, as shown in figure 15-17. This study evaluated the pressure difference between 100 pascals and 0.0005 pascals to investigate the air pressure in three damping regimes, and mainly studied the medium and high vacuum region (to optimize Q factor). In the medium pressure region, there is a relationship between frequency versus the amplitude found in Figure 15. If the pressure declines with resonance frequency, the amplitude will increase. The
highest amplitude (53092) occurs at a pressure of 10 pascals, depicted in Figure 15, with a resonance frequency of 995 Hz. Every peak represents a resonance frequency. This resonance frequency uses minimum effort to generate the greatest amplitude \( W_n = W_d \). If all the peak values line up as one curve, they stand for a part of the molecular regime in Figure 19. This approves the theoretical validation in Figure 9. After this, the main tasks are to investigate much lower pressure at 10 pascals on the resonator’s performance. The molecular region at the pressure range between 1 pascal and 10 pascals simulates with the corresponding amplitude, shown in Figure 16, which has a similar curve trend as Figure 15. The lower pressure of 1 Pa with the frequency of 804 Hz receives the highest Q value of 6.14E+5.

The amplitude in the pressure range of 5 to 10 Pa is more reliable than the amplitude in the pressure range of 1 to 4 Pa. The numerical model experiences instability at high vacuum region. The frequency steps in the range of 5 to 10 Pa is 1 Hz different, which is smaller than the pressure range between 1 Pa and 4 Pa (4 Hz frequency steps difference). The reason of this problem is that some surrounding frequency around resonance frequency needs a higher tolerance to detect the amplitude. Thermoelastic damping problem and anchor loss would be dominant at these pressures but were not included in the model and finite element modeling simulation presented a relative error greater than the relative tolerance. An in-depth analysis is done separately to investigate the thermoelastic damping (currently the FEA model results are accepted due to the peak clearly shown). Alternatively, this error message can be neglected by changing the tolerance limit to the higher value but keeping in mind that this would cause a less accurate result. Therefore, it is not an ideal option because the tolerance of the amplitude changes sharply.

High vacuum damping region are defined at pressures close to the zero Pascal and air damping does not dominate the energy loss anymore. Therefore, below 1 Pa, the simulated pressure range
is depicted in Figure 14. Some simulated amplitudes are difficult to be detected in the real peak point (resonance frequency) by finite element modeling simulation because of the inaccurate thermoelastic damping. While some low pressures (0.001 Pa, 0.005 Pa, and 0.0001 Pa) failed to estimate Q values at the resonance frequency, a few low-pressure values (0.1 Pa, 0.5 Pa, 0.01 Pa, and 0.0005 Pa) show a similar curve trend as shown in Figure 15 and Figure 16.

Figure 19 is derived from the combination of Figure 14, Figure 15, and Figure 16. Figure 17 gives a direct relationship between pressure versus Quality factor. If all the peak values line up as one curve from Figure 14 to Figure 16, we get the same plot as Figure 17. This curve of the Quality factor agrees with the theoretical analysis in the molecular region in Figure 9. One can see the curve of the Quality factor exponentially increased with decreasing pressure.

![Figure 15. Amplitude vs frequency for a pressure range between 10 and 100 Pa. When the pressure decreases, amplitude increase with slight decrease of resonance frequency.](image-url)

Figure 15 depicts when the pressure decreases, the resonance frequency slightly decreases; the amplitude will increase (Air damping coefficient decrease). When the pressure decreases much
lower, the resonance frequency slightly decreases, and the expected amplitude is higher (less effect of air damping).

Figure 16. Amplitude vs frequency at different pressure ranges between 1 and 10 Pa. When the pressure decreases, amplitude increase with slight decrease of resonance frequency.

Figure 17. Amplitude vs frequency at different pressure ranges between 0.5 and 0.0005 Pa. A few low-pressure values (0.1 Pa, 0.5 Pa, 0.01 Pa, and 0.0005 Pa) are similar trend with figure 14-15.
Figure 18 depicts that the pressure is extremely low, near 0.01 Pa, the air damping has no effect on the resonator, and the air damping coefficient is minimized.

Figure 19. Combination of Q factor vs varied pressure ranges between 0.0005 and 100 Pa. This FEM plot classified into intrinsic damping region and molecular damping region.

Meanwhile, the pressure is largely changing some parameter values. The effective viscosity of air, squeeze film damping, and slide-film damping decreases with the pressure decreasing.
Figure 20. Damping due to air gaps between comb fingers. The smaller air gaps generated the larger squeeze-film air damping coefficient, which developed a smaller $Q$ value. In contrast, the larger the air gaps, the smaller squeeze-film air damping coefficient is generated, which developed a higher $Q$ value.

This FEM investigated the air gaps between moving fingers and fixed fingers are different; the Quality factor changes as well, which is proven in the literature [17]. Smaller air gap in-between comb fingers result in more squeeze film damping. The larger the air gaps, the smaller squeeze-film air damping coefficient is generated, which developed a higher $Q$ value (gap, $g=10e^{-6}$ m, Figure 18). On the other hand, the smaller air gaps generated the larger squeeze-film air damping coefficient, which developed a smaller $Q$ value ($g=2.6e^{-6}$ m, Figure 18). The Quality factor is in direct proportion to the gap distance but inversely proportional to the air pressure.
3. Analytical Analysis

The numerical simulation results in the previous section showed how the pressure inside the cavity will impact the device performance. It provided predictive Q-factor for an optimal geometry and also showed how to develop the finite element model and the relationship between the amplitude versus pressure and quality factor versus pressure at pressure range between 100 Pa and 10 Pa (medium pressure regime), pressure range between 10 Pa and 1 Pa (medium to low pressure regime), and pressure range below 1 Pa (low pressure regime). It also combines the different pressure ranges into one plot, as shown in Figure 19. Then, some findings have been discussed in the last section, such as the pressure decrease causing the change in parameters and smaller air gaps generating large damping force, thereby dissipating more energy from the resonator (Figure 20).

However, the FEA analysis at near high vacuum was not stable as with very few molecules and less impact from viscous damping, the intrinsic damping starts to dominate. To understand the characteristics at near high vacuum, analysis was carried out to capture Q-factor change. This analysis is used to compare the finite element model results and the analytical results. Lastly, it helps in calibrating the fabricated device and comparing the results during the experiments in the lab.

3.1 Air damping Framework

The force on the moving plate is caused by the built-up pressure and is always against the plate's movement. During the moving plate movement, the air's viscous flow dissipated some of the work
and transferred it into heat (Figure 21). In other words, the air acts as a damper, and damping is called squeeze-film damping. The damping force of squeeze film is strongly dependent on the air gap distance. The larger the gap, the smaller the damping force generated, and the developed pressure is negligible. However, when the moving plate displaces closer to the fixed finger, smaller air gaps are generated. The pressure build-up in between finger and moving plate has a larger damping force and causes the decreased Quality factor. The squeeze-film damping has crucial effects on the dynamic behavior of a micro-structure. As for the fabrication of the micro-structure design, the squeeze film- air damping needs to be controlled to an expected level. If the micro-resonators are designed to be packaged at atmospheric pressure, then the gas viscous damping is classified into three categories: slide-film gas damping, squeeze film damping, and gas drag damping.

Figure 21. Pressure built-up by squeeze-film motion when the moving plate moves toward fixed fingers. Air gap distances determine the magnitude of squeeze film damping between the movable plate and the fixed wall.
3.1.1 Squeeze-Film Damping

When the surface is parallel to a nearby wall (electrode or substrate) and its motion is toward the fixed wall, the phenomenon of squeeze-film damping can occur. Squeeze-film damping results when a pressure difference develops between the gap and the environment. When the movable plate moves close to the fixed wall, the air film in the gap squeezes. A positive differential pressure develops in the air gap as the gas pressure increases, and the gas is squeezed out of the gap. When the movable plate moves away from the fixed wall, the gas is drawn into the gap, decreasing the gas pressure in the gap and causing a negative differential pressure. For one-dimensional flow, the damping is inversely proportional to the gap size, i.e., the smaller the gap, the greater the resulting damping force (Figure 22a).
Figure 22. Common air damping mechanism in MEMS devices: (a) Squeeze-film damping (movable plate moves toward fixed fingers or fixed wall) and (b) Slide-film damping (movable plate slip to adjacent fixed walls or fixed comb).

The moving plate squeezes closely to fixed fingers to reject the airflows out of gaps (Figure 22a). Air pressure develops in the gaps increasing slide film damping around the combs. When the moving plate moves away from the fixed fingers, the pressure in the gap is reduced to keep the air flowing into the gap. This only considers air gaps in the device wafer; however, the air gaps also exist in the cavity above and below the device structure in Figure 5.

3.1.2 Slide-Film Damping

Slide film damping occurs between the movable plate's adjacent wall and fixed comb (Figure 22b) [19]. The resulting flow from this motion is like Couette flow (Figure 22b) in most cases.
3.2 Viscous Air Damping

In relatively high pressure or near atmosphere, where air behaves as a continuum, viscous damping between the moving structure and substrate is often the dominant energy loss mechanism [19],[20]. There are five main damping force components and each one can be related to a specific Q-factor. Q1 and Q2 represent slide-film gas damping above and underneath the movable plates. Q3 represents the squeeze-film gas damping of the movable plate, stationary electrodes (fixed fingers), and suspended flexural spring (top and bottom of structure) (in Figure 10). Q4 represents slide-film gas damping on the sidewalls of the movable fingers and squeeze-film gas damping between the tips and base of the fingers. Q5 represents gas drag damping [14, 15].

![Diagram of damping forces](image)

*Figure 23. Squeeze and slide film damping exists in air gaps between comb fingers set for the sense mode motion [14].*

The air gap between combs is a few micrometers, and the diagram in Figure 23 can represent its cell. The air gaps are shown in between the moving plate and fixed fingers.
The overall Q factor resulting from viscous damping of the surrounding gas is combined from equations (5-6, 9-11).

\[
\frac{1}{Q_{overall}} = \frac{1}{Q_1} + \frac{1}{Q_2} + \frac{1}{Q_3} + \frac{1}{Q_4} + \frac{1}{Q_5} \tag{3}
\]

Penetration depth, \(\delta\), is used to determine the slide-film gas model as Couette flow or Stoke flow. Penetration depth \(\delta\) is defined as [14]

\[
\delta = \sqrt{\frac{2\mu}{\rho_{air}\omega}} \tag{4}
\]

Where dynamic viscosity \(\mu\) is the viscosity of movable plate, \(\rho_{air}\) is the air density, \((\omega = 2\pi f)\) is the radial frequency.

The assumption is that the gas undergoes Couette flow because \(d_p\) is less than \(\delta\). For slide film gas damping above the movable plate, \(Q_1\) can be expressed as [15],[22]

\[
Q_1 = \frac{m\omega d_p t}{\mu A} \tag{5}
\]

where \(A\) is the surface area of device, \(\mu\) is the coefficient of viscosity and \(d_p t\) is the gap between the top surface of the movable device and the base wafer.

For slide-film gas damping underneath the movable plate, \(Q_2\) can be expressed as

\[
Q_2 = \frac{m\omega d_p b}{\mu A} \tag{6}
\]

where gap distance \(d_p t\) is the gap between the bottom surface of the movable device and the lid wafer.
The coefficient of squeeze-film gas damping associated with viscous damping force can be written as [23].

\[
C = \frac{64\sigma PLW}{\pi^8 \omega g} \sum_{m,n \text{ odd}} \frac{m^2 + \left(\frac{n}{p}\right)^2}{(mn)^2 \left\{m^2 + \left(\frac{n}{p}\right)^2 + \frac{\sigma^2}{\pi^4}\right\}}
\] (7)

where p is the ambient pressure, L is the length of movable fingers, W is the width of movable finger, and \(\sigma\) is referred to as the squeeze number [24]

\[
\sigma = \frac{12u_{\text{eff}}W^2\omega}{ph^2}
\] (8)

where p is the ambient pressure, h is the thickness of the structure, and W is the width of movable fingers.

Q₃ mainly focuses on spring, which can be computed by the expression

\[
Q_3 = \frac{m\omega}{C_3} = \frac{m\omega}{C_{ss} + C_{ps} + C_{pe}}
\] (9)

where \(C_{ss}\), \(C_{ps}\) and \(C_{pe}\) represent the squeeze-film damping coefficient within the suspended flexural spring, between the movable plate and the suspended flexural spring, and between the movable plates and stationary electrodes (fixed fingers), respectively.

Q₄ is the study of the comb fingers. The gas damping consists of slide film gas damping on the sidewalls of the movable fingers and the squeeze-film gas damping between the tip and base of the fingers. Q₄ can be expressed as

\[
Q_4 = \frac{1}{Q_{\text{sidewall}}} + \frac{1}{Q_{\text{tip}}} = \frac{\mu A_{\text{overlap}}}{m\omega g} + \frac{C_{\text{tb}}}{m\omega}
\] (10)

where g is the comb finger gap, \(A_{\text{overlap}}\) is the total area of the comb finger overlaps, and \(C_{\text{tb}}\) is the squeeze-film coefficient between the tip and base of the movable fingers and stationary fingers.
Due to a velocity gradient that exists from the boundary layer to a more distant point in the surrounding gas, $Q_s$ acts on the movable plates and is written by [14]

$$Q_5 = \frac{m\omega}{10.7\mu l_{drag}} \quad (11)$$

where $l_{drag}$ is the characteristic dimension of the movable plates, and $l_{drag}$ is $\frac{1}{2}W$ of the movable plate.

This can also be written as

$$\frac{1}{Q_{overall}} = \sum_{i=1}^{5} \frac{1}{Q_i} = \frac{\mu}{m\omega} \left( \frac{A}{dp_t} + \frac{A}{dp_b} + \frac{A_{overlaps}}{g} + \frac{C_{ss}+C_{ps}+C_{pe}+C_{tb}}{\mu} + 10.7l_{drag} \right) \quad (12)$$

However, in the continuum regime, energy dissipation is usually too high for most applications where theoretical $Q$ values are only several hundred to thousands.

### 3.3 Molecular Damping Region

Most of the MEMS resonators operate in a much lower pressure regime. The air damping force on a microstructure reduces significantly when air pressure is reduced to about 1 kPa and below. Gas molecules are so far apart in low pressure that interaction between them can be neglected; therefore, a free molecule model can be used [1].

In the second article from the literature, the Christian model proposed a free molecular model for damping at low vacuum [25, 26].

$$Q_{chr} = \frac{H\rho\omega}{4} \sqrt{\frac{\pi}{2}} \sqrt{\frac{RT}{M_m}} \frac{1}{P} \quad (13)$$

where $H$ is the thickness of the plate, $\rho$ is the mass density of the plate, $M_m$ is the molar mass of the gas, $P$ is the ambient pressure, and $R (=8.31kg.m^2/sec^2.k)$ is the universal molar gas constant.
Though the Christian model can qualitatively explain the damping effect in a low vacuum, the damping effect is underestimated by an order of magnitude when the calculated Quality factor is compared with experimental data by Zook [25, 26].

An energy transfer model for squeeze-film damping is also available [15].

\[ Q_{E,sq} = (2\pi)^{\frac{3}{2}} \rho H \omega \left( \frac{d_0}{L} \right) \sqrt{\frac{RT}{M_p P}} \]  \hspace{1cm} (14)

If the Christian model is an order of magnitude larger than the experiment result, you could apply the equation measured by Zook et al. [25]

\[ Q_{E,sq} = \frac{1}{8.7} Q_{chr} \]  \hspace{1cm} (15)

If the Knudsen number \((K_n)\) is not large enough, the actual Quality factor might be smaller than that given in equation (14), due to longer stay time under the plate for the incident molecules.

There is an expression conversion between \(Q_{chr}\) and \(Q_{E,sq}\) (energy transfer model).[15]

\[ Q_{E,sq} = 16\pi \left( \frac{d_0}{L} \right) Q_{chr} \]  \hspace{1cm} (16)

Alternatively, the Quality factor for squeeze-film damping by energy transfer model in a low vacuum is [2]

\[ Q_{E,sq} = \frac{2\pi E_p}{\Delta E_{cycle}} = \frac{16M_p d_0 \omega}{\sqrt{8\pi \rho V L}} \]  \hspace{1cm} (17)

where the gap distance \(d_0=g\) (5µm), \(E_p\) is the energy of the plate, peripheral length of the gap is \(L\) (60µm), \(M_p\) is the mass of the plate \((M_p = A H \rho)\), \(v\) is the average velocity of the molecules, \(m\) is the mass of the gas molecules, and \(L\) is the distance the molecule travels under the beam.

\[ V = \sqrt{\frac{8kT}{\pi m}} \]  \hspace{1cm} (18)

It is also expressed by the atmospheric pressure with some conversion
\[ Q_{e,sq} = \frac{16M_pd_0\omega P_{atm}}{\rho_{atm}V_L^2 \frac{1}{P}} \]  

(19)

Where \( P_{atm} \) is the atmospheric pressure, \( \rho_{atm} \) is the mass density of gas at atmospheric pressure, and \( P \) is the desired ambient pressure.

3.4 Intrinsic Damping Region

At the low-pressure condition, squeeze film damping remains effective in rarefied air, the effective viscosity \( (\mu_{eff}) \) would replace the coefficient of viscosity \( \mu \). Due to pressure variation, the effective viscosity is not a constant value. Based on the analytical work using Boltzmann’s transportation equation by Fukui and Kaneko [14], a simple empirical approximation is obtained for the effective coefficient of viscosity.

\[ \mu_{eff} = \frac{\mu}{1 + 9.658k_n^{1.159}} \]  

(20)

where \( K_n \) is Knudsen number and \( g \) is the gap between substrate and plate [14]. (\( g \) is 6e-5 m in resonator).

\[ K_n = \frac{\lambda}{g} \]  

(21)

At atmosphere pressure (1*10^5 Pa), using the expression of mean free path [27].

\[ \lambda = \sqrt{\frac{\pi * \mu * 1}{8 * u * \sqrt{\rho P}}} \]  

(22)

Where \( \mu \) is the viscosity of air (1.83E-5) at 15 °C, \( \rho \) is the density of air, \( P \) is the atmospheric pressure and \( u \) is the numerical factor equal to 0.4987445.

According to the ideal gas law, the pressure change will cause the mean free path to vary at the same time. Therefore, the mean free path is inversely proportional to pressure [15]

\[ \lambda = \frac{P_0}{P} \lambda_0 \]  

(23)
And $\sigma$ is referred to as the squeeze number [24].

$$\sigma = \frac{12u_{eff}W^2\omega}{ph^2}$$  \hspace{1cm} (24)

C is the viscous damping coefficient [24]

$$C = \frac{64\sigma Plw}{\pi^5 \omega g} \sum_{m,n \text{ odd}} \frac{m^2 + (\frac{n}{\beta})^2}{(mn)^2 \left( [m^2 + (\frac{n}{\beta})^2]^2 + \frac{\sigma^2}{\alpha^2} \right)}$$  \hspace{1cm} (25)

Where l is the gap length in between the moving finger and fixed fingers and the w is the gap width in between moving fingers and fixed fingers.

As micro-structures are made from silicon, the Quality factor at high vacuum, $Q_a$, is in the range of $10^5$ [15]. The exact value is dependent on the geometric design of the structure. The expression of $Q_a$ is found through experimental measurement of the coefficient of damping force [24].

$Q_a$ caused by internal friction and the support losses at high vacuum [24].

$$Q_a = \frac{M_p \omega}{C}$$  \hspace{1cm} (26)

where $M_p$ is the mass of the oscillating plate.

### 3.5 Analytical Result

Collecting and graphically displaying all data from the viscous regime [15], molecular regime (Christian model, energy transfer model [15]), and intrinsic regime from Boltzmann’s transportation equation by Fukui and Kaneko [15], analytical computation is performed on Excel. It illustrates the relationship between analytical results and finite element model simulation, shown in Figure 24.
Both Quality factors have an intersection region in the intrinsic damping regimes and molecular damping regimes. Looking at figure 22, it can be seen that the Christian model has a higher Quality factor than the finite element model within the pressure range of 80 – 100 Pa. However, Christian model’s Quality factors have lower values than finite element model in the pressure range between 70 Pa and 1 Pa. Figure 25 observes this Quality factor difference over the pressures. Figure 26 shows the finite element model and the analytical results are a horizontal line at extremely low pressure. The percentage error between 70 Pa and 100 Pa is 13.5% and the percentage below 1 Pa is 71.2%. The relationship between pressure and frequency is linearly proportional in a pressure range between 0.1 Pa and 100 pascals (Figure 27). The lower pressures below 1 pascal are 0.1 Pa and 0.5 Pa. The difference between computational analysis results and simulated results have some impact: simulated results have unstable Q values at high vacuum regions due to the influence of the resonator’s thermoelastic damping. The Q values of the Christian model results are higher.
than the finite element model results. According to the sixth article in the literature [18], the Christian model is not valid for our model because it only validates without the fixed wall and anchor. However, the model used in this study has fixed fingers and fixed anchors. Therefore, a more thorough study needs to be conducted to prove that the finite element model works well.

![Figure 25. Q-factor vs pressure range for 1 < Pa < 10. It is proven that the Christian (equation 16) has one order magnitude smaller than the simulated result.](image)

Figure 25. Q-factor vs pressure range for $1 < Pa < 10$. It is proven that the Christian (equation 16) has one order magnitude smaller than the simulated result.
3.6 Direct Approach For Analytical Study

Instead of using equations from three different regions, there is another method from literature which redoes the equation between the Quality factor and various pressure and is in good
agreement [15]. The direct approach of analytical study for air damping effect on comb fingers according to paper [28], considering the slide-film damping (Q1 and Q2) between the proof mass and package cavity and between proof mass and substrate, the cavity distance is very large, and squeeze film damping effect are dominated in the (sensing) moving direction. In the resonator geometry design, the squeeze film damping dominates because comb fingers are used for sensing the capacitance change.

Slide-film coefficient can be expressed by the Navier stokes equation,

$$C_{\text{Slide}} = \mu_{\text{eff}} \frac{A}{d}$$  \hspace{1cm} (27)

where C is the slide film damping coefficient, A is the actual overlapping plate area, d is the air gaps between moving and fixed plates on the comb fingers, and $\mu = 1.75 \times 10^{-5}$ Pa is the air viscosity constant at room temperature (20 °C) and standard atmosphere pressure [29].

Squeeze film air damping can be expressed by the semi-theoretical and semi-empirical expression:

$$C_{\text{squeeze}} = \mu \frac{L_y L_z^3}{d^3} \beta \left( \frac{L_z}{L_x} \right)$$  \hspace{1cm} (28)

Beta function $\beta \left( \frac{L_z}{L_x} \right)$ is represented by:

$$\beta \left( \frac{L_z}{L_x} \right) = \left\{ 1 - \frac{192}{\pi^5} \left( \frac{L_z}{L_x} \right) \sum_{n=1,3,5}^\infty \tanh \left( \frac{n\pi L_x}{L_z} \right) \right\}$$  \hspace{1cm} (29)

Where $L_x, L_y, L_z$ are geometry sizes of the plate in the three dimensions. Beta function $\beta \left( \frac{L_z}{L_x} \right)$ is assumed to be close to 1.

In the dynamic systems, the Quality factor can be expressed as

$$Q = \frac{1}{2\xi}$$  \hspace{1cm} (30)
where the $\xi = C/2\sqrt{km}$ is the damping ratio. But the $\mu$ is needed to be replaced by the $\mu_{eff} = \mu/(1 + 6K_n)$, and $K_n$ is the Knudsen number relevant with cavity pressure, temperature, and critical dimension.

Figure 28 compares graphically the FEA and analytical models. The black dashed curve in 

**Figure 28** shows the simulated Q values in the peak point of figure 14-16 or **Figure 19** at the resonant frequency, which is exerting minimum force to induce the motion. There is a red curve showing in

**Figure 28**, which represents the Quality factor versus pressure in the molecular damping (100<Pa<1) and pressure in the intrinsic damping (Pa<1). When the pressure decreases, the Q value exponentially increases until it levels off (it means air is acting as a vacuum condition). The red curve is an analytical study from equation (28, 29 and 30). This analytical Q is a combination of squeeze film (blue circle) and slide film damping (green). This analytical analysis studies the Q values with a pressure range of the molecular region (1<Pa<100). The high vacuum or lower pressure estimates the intrinsic damping region; both regions combine to integrate as red curve represents the computational analysis from the MATLAB code separately. This has similar trends as a black dashed curve from finite element model. Both methods are in good agreement for the Quality factor with a percentage error of 30% in pressure between 1 and 10 Pa and 19.5% in pressure between 20 Pa and 100 Pa.
Figure 28. Comparison between FEA model and analytical analysis showing relationships between pressure and Q-factor. As the pressure decreases, the Q-factor exponential increase (MicraSilQ platform).

Figure 28 shows the comparison relationship between the Q from the finite element model and Q from the analytical analysis. The dashed line, standing for the Q of the FEA model from the finite element modeling simulation, is consistent with the analytical Q (red line) prediction. The analytical Q is summing up the squeeze film and slide film damping. The dominant damping is squeeze film damping because it closely touches with analytical Q. In the other words, slide film damping has less influence on the overall Q factor, so the Q values are high in

Figure 28. Hence, the damping is dominated by the squeeze film damping in the air damping at the low and medium vacuum region. Lastly, Q factor from FEA and the analytical is in good agreement with the percentage error of 30% (1 <Pa<10) and 19.5%(20<Pa<100).
Zhou [17] has discussed the early works of micro-resonator on squeeze-film damping, Bao proposed a direct method, called as Energy transfer model, to study squeeze-film damping in a low vacuum [29]. The resonator (accelerometer) is one axis motion in the vertical direction. Therefore, assuming the velocity is in the x-direction at end of the plate becomes [30]

\[ V_x = V_{x0} + \Delta N \times 2 \dot{x} = V_{x0} + \frac{\nu V_{x0}}{(d_0 - x)V_{yz0}} \dot{x} \quad (31) \]

where \( \dot{x} \) presents velocity of air molecules, \( d_0 \) is the original distance between the movable finger and fixed fingers, \( \Delta N \) is numbers of molecules strike on movable plates, and \( V_{x0} \) and \( V_{yz0} \) are velocity in the x-direction and the yz plane [30].

The kinetic energy of the molecule leaving the plate area underneath at the end is:

\[ e_{k,in} = \frac{1}{2} m [v_{yz0}^2 + v_{xo}^2] \quad (32) \]

Extra energy by the molecules:

\[ \Delta e_k = \frac{1}{2} m \left[ \frac{2lv_{xo}^2}{(d_0 - x)V_{yz0}} \dot{x} + \frac{l^2v_{xo}^2}{(d_0 - x)^2v_{yz0}^2\dot{x}^2} \right] \quad (33) \]

Average energy loss of the plate in one vibration cycle:

\[ \Delta E_{cycle} = \frac{1}{4} \rho_0 \bar{v}L \int_0^{2\pi} \frac{ml^2v_{xo}^2}{(d_0 - x)^2v_{yz0}^2} A_0^2 \omega^2 \cos^2 \omega t d(\omega t) = \frac{\pi l^2 A_0^2 \omega}{16} \rho_0 \bar{v} \frac{L}{d_0} \quad (34) \]

Where \( L \) is peripheral length \( L=2 \ (Lx+Lz) \), and \( l \) is the molecule lateral traveling distance between movable fingers and fixed fingers. It means the energy loss from the vibrating plate is absorbed by the air molecules. Therefore, the Quality factor for squeeze-film damping in the energy transfer model in low vacuum is

\[ Q_{E,Sq} = \frac{16M_p d_0 \omega P_{atm}}{\rho_{atm} \bar{v} L l^2} * \frac{1}{P} \quad (35) \]
Figure 29. Combination of the Q-factor from finite element model, analytical model, and Bao analysis (MicraSilQ platform).

Figure 28 explains another analysis that has been taken into account to compare with them, which is known as Bao analysis. The full name is the Energy transfer model for squeeze-film air damping in low vacuum [29]. Bao Q-factor has an order magnitude that is less than Q from finite element model for pressures less than 40 Pa. The expected experiment results will validate and touch among the three results.
Chapter 4

4.1 Fabrication Platform 1: Unsealed Device

To fabricate the devices designed in the previous section, there are two avenues: one is the device open to air to test its frequency response to capture resonance frequencies and the other is to fabricate using a vacuum-sealed environment. For the first option, the device will be placed inside an external controlled air chamber and will be tested for various pressure with the Q-factor being monitored. Commercially available is the PolyMUMPS fabrication method is selected for its suitability with MEMS and possibility to fabricate suspended and vibrating structures [31] operating in air, which has been used for this study. The result of this PolyMUMPS has highly limited the energy loss by the air damping. It eliminates the air damping effect on the MEMS device by encapsulating it into a vacuum condition. Once validated and experimented with different vacuum conditions, the next step is to use a more advanced vacuum-sealed fabrication process; for example, MicraSiQ [9]. However, in those platforms, the vacuum is fixed and cannot be probed for various pressure levels. The MicraSiQ fabrication is a commercially available platform from Teledyne Micralyne. It hermetically encapsulated the MEMS devices to prevent contamination. The device layer thickness and device layer material between the PolyMumps and MicraSiQ are different, therefore a simulation is performed to understand the differences between these two in device frequency and expected Q-factor.

Hence, PolyMUMPS numerical result is finding the relationship between the Quality factor and pressure variation. It gives result of the analytical analysis if the finite element model thickness is changed from 60 um (MicraSiQ) to 3.5 um (PolyMUMPS) and with different materials. Our analysis can fit into a new geometry structure.
**Figure 30** and **Figure 31** presents a similar plot curve trend as **Figure 14-16**, except for different structure thickness. The pressure below 4 Pa (in **Figure 31**) has some point missing near the peak due to tolerance issues. Therefore, the Gaussian fit method would solve this tolerance issue by fitting the best curve from a given data (**Figure 32**).

**Figure 30.** *Numerical analysis from finite element model in the pressure range between 10 and 100 Pa. Q factor is highest at 10 Pa (PolyMUMPS platform).*
**Figure 31.** Numerical analysis from finite element model in the pressure range between 10 and 1 Pa.

Pressure range between 1 Pa and 4 Pa is not stable due to thermoelastic damping (PolyMUMPS platform).

Because the pressure below 4 Pa is missing the peak, (resonance frequency), it needs to be figure out by the Gaussian fit method. There is an example in the **Figure 32**, it shows pressure at 4 Pa with frequency range between 731Hz-742 Hz.
Figure 32. An Example of the Gaussian fit method used in solver from excel.

Figure 33. Updated Quality factor by applying Gaussian fit method in the pressure range between 1 Pa and 10 Pa without changing the tolerance, applying Gaussian fit method has proved the trend is linear increase with pressure decrease and Q value exponential increase (PolyMUMPS platform).
Figure 33, for PolyMUMPs fabrication process, uses a similar plot as Figure 16, except for different structure thickness and materials. This plot for analytical Q is higher than analytical Q from figure 15. Figure 34 (same as Figure 19) is the combination of Figure 30 and Figure 33.

Figure 34. FEM Relationship between the Quality factor and pressure range between 1 Pa and 100 Pa. (PolyMUMPS platform).
Figure 35. PolyMUMPS fabrication combining of the Q-factor from finite element model, analytical Q, and Bao analysis. Bao analysis has intersection region between analytical Q and FEA model. Analytical Q is the combination of squeeze film and slide film analysis.

Figure 35 depicts a relationship between pressure and Quality factor. Q from the finite element model is an order magnitude lower than analytical results. Bao analysis is the intersection between Q from finite element model and analytical Q. Overall, this plot is similar to Figure 29 except for different fabrication processes.
4.2 PolyMUMPS Introduction

After the resonator design was defined and modelled, PolyMUMPS (poly-crystalline silicon multi-user MEMS process) was the chosen fabrication method. PolyMUMPS is a commercial process and design handbook is freely available for all the users, which provides guidance on how to design layer and gives design limitation.

PolyMUMPS is the three-layer polysilicon surface micromachining process. Following the design rules and specific tolerance is essential to successfully fabricate the MEMS structure. Rules include complying with each layer's thickness, the spacing between layers, and the overall size of its geometry. The shape of the layers and length of the components are available to change in L-edit.

![Cross-sectional view of PolyMUMPS process](image)

**Figure 36.** Cross-sectional view depicting the 7 layers of the PolyMUMPS process. Adapted from [31].

**Figure 36** is a cross-section view of the three-layer polysilicon PolyMUMPS process [31]. This process has a standard surface micromachining process from these features and is a bottom build-
The first layer deposited on the wafer is silicon nitride, which uses as a hard mask. It protects the silicon from etching away and is used to electrically isolate the MEMS device from the silicon substrate. The second layer deposited above silicon nitride is called Poly0, which is the first polysilicon mechanical layer. This layer uses an electrical ground plane. One can apply different voltages to this layer for actuation purposes. The absence of this layer may cause the MEMS device to stick to the nitride layer. The third layer deposited above poly0 is called the first oxide (sacrificial layer). This layer remains at several air gaps between two layers (poly0 and poly1) after this layer is released at the end of the process. The fourth layer deposited above the first oxide is called the poly1 layer which is the second polysilicon mechanical layer. This is the space where the moving structure and the fixed structure will generate. The fifth layer deposited above poly1 is called the second oxide layer, another sacrificial layer working the same as the first oxide except with different thicknesses. The sixth layer deposited above the second oxide layer is called Poly2, the third mechanical layer. The moving structure and fixed structure will coming together with different thicknesses. The last layer in the topology is called the metal layer, used for probing, bonding, and electrical routing connection to the bond pads [31].

4.3 PolyMUMPS Fabrication Process

This surface micromachining process designs as general as possible to support different designs on a single silicon wafer [31]. This process starts with 150 mm n-type silicon wafers of 1-2 ohm-cm resistivity [31]. The surface is heavily doped with phosphorus in a diffusion furnace, preventing charge feedthrough from the electrostatic devices on the surface. A low-stress LPCVD (low-pressure chemical vapor deposition) silicon nitride is deposited on the wafer after that. Silicon nitride is followed by the deposition of a 500mm LPCVD of the poly0 layer, which is the
photolithography pattern [31]. Above the poly0, the photoresist is spun on and then patterned, as shown in Figure 37a.

Figure 37. Cross-sections depicting the PolyMUMPS fabrication process steps for the proposed micro-resonator.

Figure 38. Cross-sections depicting the PolyMUMPS fabrication process steps for the proposed micro-resonator (cont).
A silicon mask deposit on the top of the photoresist and ultraviolet light (UV) is shined through the exposed parts, as shown in Figure 39. UV light will dissolve the exposed photoresist, and the area covered by the silicon mask is remained to imagine above the nitride layer. This process is known as a positive resist, where the mask directly transfers the pattern from photoresist to poly0 by using photolithography.

Figure 39. Photolithography process is used during exposure of photoresist using a positive tone, where exposed areas are being removed to etch the underlying structural layers.

After the photoresist is exposed, it becomes acidic and puts in an acid and base solution, which removes the exposed photoresist (Figure 37b). After patterning the photoresist, the parts of the Poly0 layer etch away by a plasma etch system in the uncoated photoresist area (Figure 37c). After the etching, the remaining photoresist chemically stripes in a solvent bath. Next, a 2.0 um layer of polysilicon sacrificial, known as the first oxide, is deposited on the wafer by the low-pressure chemical vapor deposition method (LPCVD) (Figure 37d). This oxide layer is a Lithographically
patterned with dimples using a dimple mask in the RIE system (Figure 37e) [22]. The purpose of the dimple is to avoid stiction when the two-layer close to each other or are almost touching (as intermolecular forces and van der Waals forces are very strong in a short distance). It also prevents the poly0 from falling when the oxide releases at the end of the process. The first oxide is a pattern with an anchor1 mask, and reactive ion etched unwanted oxide by lithographically pattern and remove the photoresist. After etching, 2.0 μm thickness of the first structure of polysilicon (Poly0) is deposited (Figure 37f) [31].

A layer of phosphosilicate glass (PSG) is deposited over the poly0 and lithographically patterned using a mask for the poly1 layer. After patterning, the polysilicon (poly1) is etched, and the remaining PSG stripes are using RIE (Figure 38g). Then, the second oxide sacrificial layer is deposited and is patterned using different etch masks. As the request for the double thickness of micro-resonator, the poly1_poly2_via level is used as this provides etch holes in the second oxide down to poly1 layer so that poly1 and poly2 could be double stacked as one whole part structure. The third polysilicon structure, poly2 at a thickness of 1.5 um, is deposited over the poly1 layer (Figure 38h). The photoresist was also deposited [31]. Then, the layer was lithographically patterned with the seventh mask and etched by the RIE process. The photoresist is then stripped in a solution (Figure 38i). The final deposited layer is the metal layer with a thickness of 0.5 um—this layer is used for electrical connection and bonding. The wafer is also lithographically patterned using an etch mask, metal deposits, and lift-off (Figure 38j). Finally, there is the release of sacrificial oxide (Figure 38k). The last step is to immerse the chip in the bath of 49% hydrofluoric acid (HF) at room temperature for 1.5-2 minutes [31]. The chip is immersed in DI water and then alcohol to reduce the stiction. Lastly, the chip needs to bake for 10 minutes at 110 °C.
4.4 Design Layout

MicraSilQ process is encapsulated in vacuum condition, which result has higher Q while the PolyMUMPS is open to air result to lower Q.

The resonator mask CAD file was drawn using a commercial Layout Editor CAD tool called L-Edit (version 2018). The entire chip size for this design was 4.75mm*4.75mm. Figure 40 shows the poly0 for supplying different voltages for actuation and detection purposes and anchor1 to ensure the fixed comb set do not move with vibrating proof mass.

Figure 40. 2D view of resonator drawn in L-Edit, showing the location of bond pads and electrical connections. Poly0 used for electrical connecting from internal device to bond pads and wire bonding to the outer breadboard.
Figure 41 shows the resonator design, consisting of one moving structure, eight fixed comb sets, two anchors, and four springs, ten bond pads and ten connections from electrodes to each bond pad. The electrodes draw on poly0 as a fixed layer, and one can apply electrical signals to it. The connections, bond pads, and anchors drawn on poly0. The poly0 layer electrodes use to actuate the device and detect the capacitance change. The proof masses are double-stacked structure on poly1 and poly2 layers (with a thickness of 2.0 um, 1.5 um individually). For illustrative purposes, the proof mass of poly2 is only presented here. Because the proof mass of poly1 is overlapped by the poly2, the dimension is the same except for the different thicknesses. But the vibrating structure of the resonator is a double-stacked structure on poly1 and poly2 combined. According to the design rule, the poly1 layer encloses the poly2 layer by the distance of 27 um. The poly1_poly2_via layer encloses the poly1 layer by another 10 um. The poly1_poly2_via layer strips off the second oxide (not shown in the figure). The design structure does not need a metal layer (metal layer
supplies electrical signals to electrodes). The overall structure is big geometry, which etched 2795 holes with a thickness of 5um*5um (anti-stiction). The etch hole separation in poly1 and poly2 is 30 um, making sure the subsequent release of poly1 structure. A dimple layer is added at the last step for anti-stiction. The double thickness structure combined poly1 and poly2 has 3.5 um thickness.

After completing the design in L-Edit, it was sent to CMC for fabrication. Figure 42 shows the final bonding diagram of the resonator. The whole structure includes one design. The design's perimeter contains 40 bond pads, which provides connection between the outer electrodes and bond pads from the resonator. The connection is done to make the design packageable and uses a 40-pin grid array (PGA). There are only ten bond pads used for wire bonding connection.
Figure 42. Bonding diagram of the resonator chip with their respective bond pads and connecting electrodes (poly0). Devices uses the wire bonding DIP 40.
Chapter 5

5.1 Experimental Analysis

Experimental characterization and testing of the prototype micro-resonator were performed to capture the dynamic response of the resonator. The testing verified the theoretical results of the Quality factor. The testing focuses on resonator frequency, quality factor, and frequency response. The electrostatic drive and sensing setup consisted of a waveform generator (DG4102), spectrum analyzer (RSA507A), two DC power supplies (E3630A, STP300005H), solderless breadboard, and oscilloscope (DS10054) for debugging as shown in Figure 43.

![Figure 43](image_url)

*Figure 43. Frequency response test setup that includes two DC power supplies, a waveform generator, a spectrum analyzer, an oscilloscope, and a breadboard to interface with the chip.*

A waveform generator transmits a signal over a certain range of frequencies and measuring the input-output of the system. The standard waveform consists of sine, square, pulse, ramp, and harmonics. The spectrum analyzer display receives the signal’s amplitude as it varies by frequency. The spectrum analyzer shows data in the frequency domain, which is useful when viewing large and small signals on the same scale. The range of the spectrum analyzer is in the dynamic range
between -1600 to 20dBm. A Lock-in amplifier is a digital instrument capable of measuring and extracting signals from a noisy environment. The optimal solution for missing lock-in amplifier would be measuring the moving average filter to eliminate the signal from a noisy environment (Figure 45). Oscilloscopes are electronic instruments that graphically display electrical signals in the time domain. Alternatively, the electrical signal for the micro-resonator is captured by the spectrum analyzer. Therefore, the software from the laptop graphically display electrical signals in the time domain. The software also zoom in the specific range of frequency to visualize resonant amplitude and locate the position, frequency of the desire model shape by comparing the electrical signal (Figure 44).

![Figure 44. Electrical signal received from spectrum analyzer and model shape capture from FEM. It captures resonant frequency in detail by adjusting the start frequency, stop frequency, bandwidth, step size, and zoom in.](image-url)
Figure 45. Moving average filter is a technique to extract the signal from a noisy environment in the frequency domain. Moving average filter is only used in open air condition, not inside the vacuum chamber.

Moving average filter is a technique to extract noise in the signal from a noisy environment in the time domain.

5.3 Breadboard Electronics

The actuation and detection were accomplished using a circuit implemented on a breadboard. The breadboard included an actuation and detection circuit. The circuits provided voltages to the electrodes on the resonator chip and simultaneously detected the motion of the proof mass. To build the circuit on the breadboard, the following electrical and electronic components were used [37]:

(a) Operational amplifiers (OP177 x 2)

(b) Instrumentation amplifiers (AD620 x 3)
(c) Resistors (500 kΩ x 3, 10 kΩ x 2, 1 kΩ x 2)

(d) Capacitors (1 µF x 2, 10 pF x2)

(e) Socket for interfacing chip to breadboard

(f) Jumper wires, scope probes, BNC cables, banana plugs, a BNC splitter, and an RF connector.

6.4 Measurement Results Opened To Air:

After the WLP fabrication, the next step is testing the micro-resonators functionality by verifying the x / y-axis component's resonant frequency response. To receive the signal response, the breadboard front-end electronics has been built. The most functional components classify into three parts: actuation, detection, and opened-air (non-vacuum) MEMS resonator as shown in Figure 46.

The purpose of the actuation is that applied 15 voltage (recommendation from literature [37]) creates a force on the resonant structure, potentially driving the micro-resonator into resonance.
Figure 46. Breadboard front-end electronics classify into three parts: actuation Figure 47, detection Figure 48, and opened-air MEMS resonator (non-vacuum).

Those of applied voltage is accomplished by mixing AC and DC voltage on the breadboard, delivered by independent power sources. The packaged MEMS resonator provided two of four sense electrodes used for actuation (V1 and V2 in Figure 47). The other two connect to the network analyzer's input channel (V1 and V2 in Figure 48). As for detection, the AC signal stands as a carrier signal (Vi). This carrier signal carries the information of the micro-resonators motion when detected. After this, carrier signal is converted by amplifiers to the final signal (Vo), which is connected to a spectrum analyzer for detection to change the AC actuation frequency.
Figure 47. Actuation circuit where \( IA = AD620, \ RG = 500k\Omega, \ R1 = 1K\Omega \) and \( C1 = 1\mu F \).

The actuation circuit consists of two instrumentation amplifiers (AD620), two 500 kΩ and 1 kΩ resistors and two capacitors (1 µF). A schematic diagram of the circuit is shown in Figure 47.

Figure 48. Detection circuit where \( OA = OP177, \ IA = AD620, \ RG = 500k\Omega, \ RF = 10k\Omega \) and \( CF = 10pF \).
The detection circuit on the breadboard consists of two operational amplifiers (OP177), one instrumentation amplifier (AD620), a 500 kΩ and two 10 kΩ resisters, and two 10 pF capacitors. A schematic diagram of the detection circuit is shown in Figure 48.

After detecting the electrical signal in the setup process, the half-power bandwidth measures the Quality factor in the MATLAB. The half-power bandwidth means output power has dropped to half of its peak value. It is commonly used for the cutoff frequency (Figure 49).

For example:

$$BW \approx \frac{w_n}{Q}$$  \hspace{1cm} (equation 2.52) [33]

Where the half-power bandwidth of the system is defined as the difference between the frequencies where the power is half of the resonance power.

$$BW = \Delta f = f_2 - f_1 = \frac{f_c}{Q}$$

Where high band edge $f_2 = f_c + \frac{\Delta f}{2}$, low band edge $f_1 = f_c - \frac{\Delta f}{2}$, $f_c$ is the center frequency (resonant frequency), (Figure 47).
Figure 49. The sense-mode system with an amplitude of 1100 and resonant frequency of 10.0 kHz, illustrating a 29% gain drop from a 5 Hz relative shift between the operating frequency and sense-mode resonant frequency [1].

\[ Q = \frac{f_c}{f_2 - f_1} = \frac{800}{808 - 792} = 50 \]

A general equation is presented above for the Quality factor. Then the electrical signal is obtained from the spectrum analyzer and post-processed in MATLAB.

5.5 Measurement Results In The Vacuum Chamber

The micro-resonator would require replacing the vacuum chamber with the same circuit as open to air. The pressure condition varies around the resonator in the vacuum chamber, which allows adjusting much lower pressures, so that the air damping is minimized (Figure 50).
5.6 Comparison of FEA Results With Previous Experiments

The experimental validation is essential to show that their measurement results are relatively close to our model. Therefore, a literature search was performed to find relevant MEMS resonators Q-factor experiments relative to the vacuum measurement for the air damping effect. Relevant papers [5, 33] have presented the experiment measurement with the relationship between Q-factor and air pressure. The results were normalized by $\frac{Q}{Q_{\text{max}}}$ in order to compare with the FEA model results.

The experimental setup in [5,33] is similar to the one presented in this thesis, allowing Q-factor measurement relative to vacuum chamber testing for the resonator response. It adjusts to the lower pressure for testing the air damping effect in the vacuum chamber. The author, N. Candler,
establishes two resonator designs: A and B. Design models A and B are the same models with single-anchored, double-end tuning fork structures. The resonant frequency of designs A and B are near 150 kHz, and 130 kHz, respectively, and the quality factor of the sealed encapsulated is around 33,000 and 50,000, respectively. Ghaffari [5] tested the experimental Q between 0.1-100 Pa and the highest Q was around 150,000 at 0.6 Pa.

Device wafer encapsulates with sealed cap by single wafer bonding packaging. The encapsulation was intentionally removed from the design A resonator inside the vacuum chamber. (is this repeating from lit review?) The resonators measure at several different pressures in the vacuum chamber; the air as the ambient gas exists in the vacuum chamber. Other results are derived from finite element model from Multiphysics (in Figure 19 or Figure 28). Finite element model simulation converts Q versus pressure to \( \frac{Q}{Q_{max}} \) versus pressure in order to do the comparison with measurement results. The analytical analysis from Bao’s energy transfer model in Figure 29 and Bao’s Christian model in Figure 24 are added into Figure 51 for comparison results with our finite element analysis (Figure 19) and analytical analysis (Figure 28). Any changes by the pressure would result in a corresponding change in \( \frac{Q}{Q_{max}} \). This figure has validated the pressure decrease and the measured quantity \( \frac{Q}{Q_{max}} \) exponentially increases. The result from both the data located in the molecular damping regimes and intrinsic damping regimes are shown. Q factor from Cander [33] has overlapping data points with analytical solution. Cander’s experiment from literature [33] is similar to the what the study intends to run. Cander’s experiment included vacuum chamber where the pressure was varied between 0.01<Pa<10. Ghaffari’s [5] experiment has higher Quality factor compare to this study because the author is using the different resonator, (dual ring resonator).
Figure 51. \( \frac{Q}{Q_{\text{max}}} \) versus pressure resonator in the vacuum chamber. Comparison between FEA modeling, an analytical model of this thesis, Bao Christian model [15] and energy transfer model [30] with previously reported experimental results [5,33], shows good agreement of trend of Q factor change with pressure.

For the reason why the results (Q/Q_{\text{max}}, Pa >0.1) are diverse is because Q factor is influence by total Q (TED, anchor loss, and air damping). each of the design geometry from literature are different, it could influence TED damping. Pressure below 0.1 Pa is level off, energy loss is not from air damping anymore, TED and anchor dominate energy loss.
5.7 Conclusion

This thesis explored the design, simulation, and analytical analysis of Q-factor in three vacuum regimes, direct approach for analysis, and established an FEA modeling technique to allow designing MEMS resonators to mitigate air damping. FEM models were developed for two different fabrication processes. The focus of this research is investigating high Q-factor to enhance MEMS devices with high stability. The Quality factor is strongly influenced by the encapsulation process (internal environment, getter material) of the MEMS resonator and total pressure. To lower air damping influence on the device, the molecular distance is kept far away, and the molecule reaction is less likely to happen. The conditions inside the vacuum chamber provides a better environment to test Q-factor values with medium and high vacuum region. In this study, the comparison between finite element model simulation and analytical results are in good agreement at these two pressure regimes, such that the percent error in the pressure range between 1 Pa and 10 Pa and in the pressure range between 20 Pa and 100 Pa is 30% and 19.5% (Figure 29), respectively, for MicraSilQ platform. Bao analysis is an analysis used to compare with analytical results. The percentage error of Bao analysis compared with analytical results is 84.9% and 70.5% at these two regimes (Table 5).

As for the PolyMUMPS platform shown in Figure 35, the comparison between finite element model simulation and analytical results shows a percent error of 83.6% and 89.6% in the pressure range of 1 Pa to 10 Pa and 20 Pa to 100 Pa, respectively. The percent error between finite element model analysis and Bao analysis is in the pressure ranges of 1 Pa to 10 Pa and 20 Pa to 100 Pa is 93.8% and 39.2%, respectively (Table 5).
For both fabrication platforms, the finite element model simulation, analytical analysis, and Bao’s analysis found that the relationship between these three types of curves is that the resonance frequency declines as the Q-factor value exponentially increases with decreased air pressure. Both have similar relationships in the different pressure regions.

Table 5. Summary of different percent error relative to analytical Quality factor.

<table>
<thead>
<tr>
<th>Pressure Relative to analytical</th>
<th>MicraSilQ Percent error (%)</th>
<th>PolyMUMPS Percent error (%)</th>
<th>Energy transfer model Percent error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;Pa&lt;10</td>
<td>30%</td>
<td>83.6%</td>
<td>93.8%</td>
</tr>
<tr>
<td>20&lt;Pa&lt;100</td>
<td>19.5%</td>
<td>89.6%</td>
<td>39.2%</td>
</tr>
</tbody>
</table>

Lastly, a comparison of the relationship between Q/Qmax and pressure change with previous literature was studied. It can be concluded that the air damping dominates on the molecular damping regimes at pressures between 1 Pa and 100 Pa, and the Q/Qmax sharply decreases with pressures increase.

5.8 Future Works

There are various methods of improving the performance of micro-mechanical devices. One can add a getter film to hermetically seal the MEMS device and compare the dynamic performance of the micro-resonator in the vacuum chamber. Getter film chemically absorbs active gases under a vacuum. Changing the gas composition to an inert gas, such as nitrogen, xenon, argon, can change the air pressure to a much lower region around the resonator. The next step is the experimental comparison between FEA simulation and experiments in the lab, and then measure the frequency response with and without getter film material. The FEM can be iterated and refined to try to make
a more accurate predictions for a variety of devices. Furthermore, the FEM model has the potential for augmenting the simulation of the getter and types of getter to observe long-term effects of the device’s performance. One can compare the result of the difference of Quality factor against different ambient gasses. Instead of testing air pressure, the vacuum chamber also could adjust for the acceleration and temperature. Lastly, finding the leak rate of the micro-resonator with different gasses and comparing it to industry standards would give an indication of the device’s performance.
References


Vita Auctoris

NAME: Ankang Wang

PLACE OF BIRTH: Anhui province, China

YEAR OF BIRTH: 1995

EDUCATION:
- Wuhan Maple Leaf International School, Wuhan, Hubei, China, 2015
- University of Windsor, B.Sc., Windsor, ON, 2019
- University of Windsor, M.Sc., Windsor, ON, 2021