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Distribution channel strategies in a mixed market

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Abstract

This paper studies equilibrium channel strategies in a mixed market with a public firm and a private firm. The public firm is concerned with social welfare, while the private firm aims to maximize its own profit. Each firm decides whether to adopt an integrated or a decentralized channel. We examine two standard market competition modes, Bertrand and Cournot. Within each competition mode, we consider two typical vertical contracts, wholesale-price and two-part tariff contracts. Our results suggest that equilibrium channel structures depend on the market competition mode, the vertical contract form, and the level of product substitutability. Specifically, the channel strategy of the private firm depends mainly on the vertical contract form: under a two-part tariff contract, the private firm always chooses decentralization; under a wholesale-price contract, the private firm chooses integration for most scenarios except for highly substitutable products under Bertrand competition (i.e., under very intense competition). The channel strategy of the public firm depends mainly on the competition mode: under Bertrand competition, the public firm always chooses decentralization; under Cournot competition, the public firm always chooses the opposite of the private firm’s strategy.

Keywords: Channel strategy; Mixed market; Bertrand competition; Cournot competition; Vertical contract.

*Corresponding author.
Highlights:

1. We study channel strategies with a public firm competing with a private firm.
2. Two standard market competition modes, Bertrand and Cournot, are considered.
3. We consider both wholesale-price and two-part tariff contracts.
4. Channel strategies of the private firm rely mainly on the vertical contract form.
5. Channel strategies of the public firm depend mainly on the competition mode.
Distribution channel strategies in a mixed market

Abstract This paper studies equilibrium channel strategies in a mixed market with a public firm and a private firm. The public firm is concerned with social welfare, while the private firm aims to maximize its own profit. Each firm decides whether to adopt an integrated or a decentralized channel. We examine two standard market competition modes, Bertrand and Cournot. Within each competition mode, we consider two typical vertical contracts, wholesale-price and two-part tariff contracts. Our results suggest that equilibrium channel structures depend on the market competition mode, the vertical contract form, and the level of product substitutability. Specifically, the channel strategy of the private firm depends mainly on the vertical contract form: under a two-part tariff contract, the private firm always chooses decentralization; under a wholesale-price contract, the private firm chooses integration for most scenarios except for highly substitutable products under Bertrand competition (i.e., under very intense competition). The channel strategy of the public firm depends mainly on the competition mode: under Bertrand competition, the public firm always chooses decentralization; under Cournot competition, the public firm always chooses the opposite of the private firm’s strategy.

Keywords: Channel strategy; Mixed market; Bertrand competition; Cournot competition; Vertical contract.

1. Introduction

Mixed markets with private firms competing against public firms are common in various industries such as healthcare, transportation, steel, electricity and telecommunications (Epstein,
2009; Ghosh and Mitra, 2010; Levin and Tadelis, 2010; Matsumura and Ogawa, 2012; Scrimitore, 2013). For instance, a Canadian crown corporation, VIA Rail, competes against other privately owned passenger railways such as Rocky Mountaineer and bus companies such as Greyhound Lines. Another crown corporation, Canada Post, competes against many other private logistics firms such as UPS. In China, the government-owned FAW Group Corporation competes against private firms such as Geely Automobile Holdings Ltd. Many Chinese public hospitals compete against private hospitals in the healthcare market. In the US, the government-sponsored firm, Federal Home Loan Banks, competes against other private banks. In general, public firms owned by the government seek to maximize social welfare, while private firms are typically profit-driven. This is common in many studies in the mixed oligopoly literature (De Fraja and Delbono, 1990; Anderson et al., 1997; Ghosh and Mitra, 2010; Matsumura and Ogawa, 2012; Scrimitore, 2013, 2014).

Besides horizontal competition, public firms often operate in vertical mixed markets in many countries (Glaeser and Scheinkman, 1996; Bose and Gupta, 2013). Various distribution patterns exist in vertical mixed markets with competing public and private firms. For example, some public healthcare services in the UK are outsourced by the government to private firms such as Sunderland Home Care Associates, which competes against many other integrated private healthcare firms (in this example the public firm chooses to decentralize while the private firms choose to integrate). The federally-owned integrated Canada Post Corporation (also, the US Postal Service) competes against the private logistics firm UPS, which franchises its service to independently-owned stores (in this example the public firm chooses to integrate while the private firm chooses to decentralize). In China, both the government-owned FAW Group Corporation and the private GEELY Automobile Holdings Ltd sell their vehicles through
independent private dealers (in this example the public and the private firms both choose to decentralize). Motivated by these different distribution channel structures in mixed markets with competing public and private firms, this paper attempts to explain when and why these distribution patterns occur in a mixed market. Specifically, our research questions are:

i). Under what circumstances should a public (private) firm employ an integrated or a decentralized channel?

ii). What are the inherent forces driving various equilibrium channel strategies in a mixed market?

We examine two standard competition modes in the product market, Bertrand and Cournot competition, both common in practice. In general, Bertrand competition applies if capacity and output can be easily adjusted, and Cournot competition applies otherwise. de Jong et al. (2008) showed that both competition modes are common in practice and the application of each competition mode depends on the characteristics of industries. Bertrand competition is more suitable in industries such as steel pipe and tubes, wine and brandy, telegraph apparatus and petroleum refining, while firms engage in Cournot competition in industries like railroad equipment, electro-medical apparatus, steel works and blast furnaces. As will be seen in this paper, the competition mode (Bertrand or Cournot) critically determines the public firm’s channel strategy.

Within each competition mode, we consider two standard vertical contracts, wholesale-price and two-part tariff contracts, both common in practice (Cachon, 2003; Anand et al., 2008). Wholesale-price contracts are popular mainly due to simplicity, but result in low efficiency in supply chains. While two-part tariff contracts can increase channel efficiency, it needs further information about downstream decisions (Cachon, 2003). Wholesale-price contracts apply to
supply chains where firms lack the information to coordinate, while two-part tariff contracts apply to supply chains when sufficient information can be accessed. As pointed out by Anand et al. (2008), two-part tariff contracts can also serve as a form of quantity discount contracts in the operations and marketing literatures. In this paper, as we study the channel strategies of firms, we take the two-part tariff contract as a representative form of nonlinear (non-wholesale-price) contracts, which represents the case that information sharing and channel coordination exists between channel partners and thereby “double marginalization” is no longer the main concern of a firm when it decides to decentralize.

The sequence of decisions is as follows. First, the public and the private firms simultaneously decide whether to adopt an integrated channel or to decentralize by distributing products through an exclusive private retailer. In the second stage, for any firm who decentralizes, the wholesale price is set. In the third stage, product market competition (Bertrand or Cournot) occurs. This sequence has been widely adopted in the literature (McGuire and Staelin, 1983; Lee and Staelin, 1997; Bhardwaj and Balasubramanian, 2005; Piccolo and Reisinger, 2011).

Studying channel strategies in a mixed market yields the following new insights. First, the channel strategy of the private firm depends mainly on the vertical contract form: under a two-part tariff contract, the private firm always chooses decentralization; under a wholesale-price contract, the private firm chooses integration for most scenarios except for highly substitutable products under Bertrand competition (i.e., under very intense competition). Second, the channel strategy of the public firm depends mainly on the competition mode. Specifically, under Bertrand competition, the public firm always chooses decentralization because it can fully adjust the downstream competition to maximize social welfare through subsidy. Under Cournot competition, because changing its wholesale price has conflicting impacts on the two firms’ downstream
quantities, the public firm can no longer fully adjust the downstream competition by decentralization, and thus it does not always choose decentralization but always chooses the opposite of the private firm’s strategy.

To compare our theoretical results with the real-world examples mentioned earlier, we summarize our results of equilibrium channel structures in Table 1 and the real-world examples in Table 2. From Table 1, under wholesale price contract, Bertrand and Cournot competition give the same equilibrium channel structure that the public firm chooses decentralization while the private firm chooses integration (except for highly substitutable products under Bertrand competition). Thus, in Table 2 we give one common example of healthcare mixed market for both Bertrand and Cournot competition under wholesale price contract, noting that both competition modes apply to and appear in healthcare market. For each real-world example in Table 2, we give references/evidences showing why the example could fit a particular combination of competition mode and contract structure. Comparing Table 1 and Table 2, we can see that our theoretical results fit these real-world examples, which implies that our theoretical results could be viewed as one possible explanation of these different channel structures in real-world mixed markets.

Table 1. Summary of theoretical results of equilibrium channel structures

<table>
<thead>
<tr>
<th>Contract Structure</th>
<th>Competition Mode</th>
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<tr>
<td></td>
<td>Bertrand</td>
</tr>
<tr>
<td>Wholesale price contract</td>
<td>Public Firm: decentralization</td>
</tr>
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<td></td>
<td>Private Firm: integration (except for highly substitutable products)</td>
</tr>
<tr>
<td>Two-part tariff contract</td>
<td>Public Firm: decentralization</td>
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<td></td>
<td>Private Firm: decentralization</td>
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Table 2. Summary of real-world examples

<table>
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<tr>
<th>Contract Structure</th>
<th>Competition Mode</th>
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<tr>
<td></td>
<td>Brtrand</td>
<td>Cournot</td>
</tr>
<tr>
<td>Wholesale price contract</td>
<td>Healthcare firms often engage in either Bertrand (Capps et al., 2010; Gowrisankaran et al., 2013) or Cournot competition (Deo and Corbett, 2009).</td>
<td></td>
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<td></td>
<td>Some public healthcare services in the UK are outsourced by the government to private firms such as Sunderland Home Care Associates, which competes against many other integrated private healthcare firms (in this example the public firm chooses to decentralize while the private firms choose to integrate).</td>
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<tr>
<td></td>
<td>The UK healthcare sectors are funded mainly based on performance metrics such as the number of patients treated and length of workload, which resembles the wholesale price contract (British Medical Association, 2014)</td>
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<tr>
<td>Two-part tariff contract</td>
<td>Auto firms often engage in Bertrand competition (Tremblay et al., 2013).</td>
<td>Cournot competition fits well in the network of postal delivery (Crew and Kleindorfer, 2009).</td>
</tr>
<tr>
<td></td>
<td>In China, both the government-owned FAW Group Corporation and the private BYD Company Limited sell their vehicles to independent private dealers through franchising (in this example the public and the private firms both choose to decentralize)</td>
<td>The federally-owned integrated Canada Post Corporation (also, the US Postal Service) competes against the private logistics firm UPS, which franchises its service to independently-owned stores (in this example the public firm chooses to integrate while the private firm chooses to decentralize).</td>
</tr>
<tr>
<td></td>
<td>Franchise distribution is common in the automotive industry, which is a typical two-part tariff contract (Lafontaine and Morton, 2010).</td>
<td>UPS distributes its business through franchising, which is a typical two-part tariff contract (The UPS Store, 2014).</td>
</tr>
</tbody>
</table>

The remainder of the paper is organized as follows. Section 2 surveys the related literature. In Section 3, we examine the channel strategies of a mixed duopoly under Bertrand competition. The channel strategies under Cournot competition are discussed in Section 4. Finally, Section 5 concludes the paper.
2. Literature Review

Two streams of literature are related to our research, the mixed market literature and the channel strategy literature. In the following, we shall review each stream separately.

A vast body of research has been conducted on mixed markets (see, e.g., Fraja and Delbono, 1990). Recent mixed market literature mostly focuses on the following topics: effect of subsidy and privatization (White, 1996; Fjell and Heywood, 2004; Kato and Tomaru, 2007; Tomaru and Saito, 2010; Matsumura and Tomaru, 2013; Scrimitore, 2013, 2014), choice of competition modes (Ghosh and Mitra, 2010; Matsumura and Ogawa, 2012; Scrimitore, 2013, 2014), price and non-price competition (Ishibashi and Matsumura, 2006; Ishibashi and Kaneko, 2008; Zikos, 2010; Matsumura and Sunada, 2013; Lutz and Pezzino, 2014), and effect of entry (Matsumura and Kanda, 2005; Heywood and Ye, 2009; Wang and Chen, 2010; Li and Zhang, 2011; Bennett and La Manna, 2012; Wang et al., 2014). None of these works studied distribution channel strategies. Besides, all the aforementioned researches were conducted with horizontal competition, while we focus on channel strategies where both horizontal and vertical competitions exist, which has not been studied before in the mixed market literature.

A few papers looked into vertical mixed markets. Glaeser and Scheinkman (1996) examined which level should be privatized in a three-tier vertical market with an upstream firm, a downstream firm and a retailer. Similarly, Bose and Gupta (2013) discussed welfare implications of privatization in a bilateral monopoly. Departing from these studies, which focused on the effects of privatization in a single supply chain, we study the channel strategies of two competing supply chains in a mixed market. Overall, to the best of our knowledge, none of the existing literature on mixed markets looked into, in the presence of a private competitor in the market,
whether public goods or services should be distributed through a private partner or directly by a public firm, which is the focus of this paper.

Before reviewing the vast literature on distribution channel strategies, we would like to first highlight in general how this paper makes a contribution by extending this branch of study to a mixed market. To the best of our knowledge, almost all the existing studies on channel strategies focused on competitions between private for-profit firms (one exception being Zhao et al. (2010), which will be discussed in detail later). Introducing a public welfare-oriented firm, this paper establishes new results in equilibrium channel structures. Specifically, we show that various channel structures with the public or private firm choosing integration or decentralization can occur in equilibrium, but the classic equilibrium that both firms choose integration is never an equilibrium in a mixed duopoly. The reason is that the public firm behaves fundamentally different from private firms. In particular, as mentioned earlier, we find that the competition mode (Bertrand or Cournot) critically determines the public firm’s choice of channel strategy, which is never reported in the literature on private firms.

Initially, the literature on channel structure revealed that intermediaries can act as competition buffers, i.e., decentralization can mitigate competition (McGuire and Staelin, 1983; Moorthy, 1988; Trivedi, 1998). One key point from these studies is that, when choosing channel strategies, manufacturers should trade off between the benefit from competition buffering and the loss from double marginalization. The equilibrium channel strategies in these papers are as follows. First, the channel structure where both firms integrate is always one equilibrium channel structure. Second, the decentralized channel structure where both firms decentralize occurs as another equilibrium for highly substitutable products. Unlike these studies, we study channel strategies in a different market structure, i.e., a mixed market, and find that various channel structures can
occur in equilibrium. In particular, a big difference of this paper’s results is that both firms integrating is never an equilibrium in a mixed duopoly.

Many researchers extended the above mentioned studies by adding other dimensions such as managerial incentives (Bhardwaj and Balasubramanian, 2005), non-price competition (Zhao et al., 2009, Liu and Tyagi, 2011; Wang et al., 2011; Zhang et al., 2012; Zhou and Cao, 2014), risk aversion (Xiao and Choi, 2009), number of players (Anderson and Bao, 2010), demand uncertainty (Cao et al., 2010). Considering managerial incentives, Bhardwaj and Balasubramanian (2005) demonstrated that an asymmetric channel structure with one integrated manufacturer and the other decentralized manufacturer arises in equilibrium. Zhao et al. (2009) discussed manufacturers’ channel strategies with both price competition and quality competition and found that both manufacturers always integrate in equilibrium. Liu and Tyagi (2011) showed that both retailers always decentralize in equilibrium with both price and location competition.

Unlike the studies above, we examine channel strategies in a mixed market and show that various channel structures can occur in equilibrium. Xiao and Choi (2009) and Zhou and Cao (2014) examined endogenous channel structures with risk aversion and displayed quantity competition, respectively, and they found various channel structures can arise in equilibrium. We depart from their studies by considering endogenous channel strategies in a mixed duopoly, while they consider channel strategies of private firms. Comparisons between integrated and decentralized channels were conducted by Anderson and Bao (2010), Wang et al. (2011) and Zhang et al. (2012), with focuses on the effect of number of players, persuasive advertising and informative advertising, respectively, but they did not study endogenous channel strategies.

A few papers studied channel strategies from different aspects. Considering durable goods in a multi-period single-channel setting, Desai et al. (2004) and Arya and Mittendorf (2006) found
that manufacturers could benefit from channel decentralization even without outside competitors. With tacit collusion, Piccolo and Reisinger (2011) concluded that various channel structures can arise in equilibrium. They examined the competitive channel strategies of two private firms, while we discuss channel strategies of two firms with different ownerships, i.e., a public firm competing against a private firm.

There are also studies on dual-channel supply chain design. Balasubramanian (1998), Chiang et al. (2003) and Cattani et al. (2006) discussed the strategic interaction between a manufacturer-owned direct channel and an independent retail channel. Ordover and Shaffer (2007) and Xu et al. (2010) examined the incentive of a component manufacturer to supply core components to an independent original equipment manufacturer. Khouja et al. (2010) analyzed channel and price strategies of a manufacturer with retail-captive consumers. Considering both price and delivery lead time decisions, Xu et al. (2012) extended the work of Chiang et al. (2003) by analyzing how these decisions jointly affect the manufacturer's dual-channel design. With complements, Xia et al. (2013) and Pun (2013) investigated channel strategies for two competing firms and a monopolist firm. Our research differs from the aforementioned works in that we concentrate on competitive channel strategies in a mixed duopoly, i.e., a public firm and a private firm compete against each other in selecting channel strategies.

The work most closely related to our research is Zhao et al. (2010), which examined channel strategies for a non-profit firm competing against a for-profit firm. The key differences are as follows. First, they assumed the non-profit firm has an objective of maximizing demand, while the public firm in this paper pursues to maximize social welfare. Second, they did not consider endogenous channel strategy of the private firm. Third, they only considered Bertrand competition, while we examine both Bertrand and Cournot competition and it turns out that the
competition mode critically determines the public firm’s choice. Fourth, within each competition mode, we discuss two types of vertical contracts, wholesale-price and two-part tariff contracts, while they considered wholesale-price contract only.

3. Channel strategies under Bertrand competition

Consider a mixed market with one public firm (firm 0) and one private firm (firm 1), manufacturing product 0 and product 1, respectively. As in Scrimitore (2013) and other research, we assume that the public firm maximizes social welfare while the private firm pursues profit maximization. Following Singh and Vives (1984), we express the standard consumer surplus as

\[
CS = \left[ \alpha \left( q_i + q_j \right) - \frac{1}{2} \left( q_i^2 + q_j^2 \right) - \beta q_i q_j \right] - (p_i q_i + p_j q_j), \quad i, j = 0, 1; i \neq j
\]  

(1)

where \( CS \) stands for the consumer surplus, and \( p_i \) and \( q_i (i = 0, 1) \) denote product \( i \)'s retail price and quantity, respectively. The first term of the right-hand side of Eq. (1) denotes the utility function of the representative consumer, which is assumed quadratic and strictly concave. This form of utility function gives rise to linear demand structure as shown by Eq. (2) when the representative consumer maximizes her consumer surplus (Singh and Vives, 1984).

From Eq. (1), we derive the inverse demand as

\[
p_i \left( q_i, q_j \right) = \alpha - q_i - \beta q_j, \quad i, j = 0, 1; i \neq j
\]  

(2)

where \( \alpha \) is the price cap and \( \beta \in [0, 1] \) measures the degree of product substitutability, with \( \beta = 0 \) and 1 representing the cases of completely independent and homogenous products, respectively.

Inverting Eq. (2), we obtain the direct demand as
\[ q_i(p_i, p_j) = \frac{(1-\beta)\alpha - p_i + \beta p_j}{1-\beta^2}, \quad i, j = 0,1; i \neq j \] (3)

In this section, we adopt Eq. (3) to examine Bertrand competition, and Cournot competition will be analyzed in Section 3 by using Eq. (2). To focus on the strategic effect, we assume that both firms' marginal costs are normalized to zero. Social welfare equals to the sum of consumer surplus and both channels' profits. Hereafter, social welfare and the private firm's profit are respectively denoted by \( SW \) and \( \Pi_{F1} \), where the subscript 'FI' indexes the private firm.

Throughout the paper, we use the superscript 'MN' to indicate various channel structures, where

The first element \( M = I \) or \( D \) indicates the public firm's integration or distributing through a private retailer (retailer 0), respectively.

The second element \( N = I \) or \( D \) denote the private firm's integration or distributing through a private retailer (retailer 1), respectively.

3.1. Channel structure II

Consider the first scenario in which both the public firm (firm 0) and the private firm (firm 1) choose to integrate. Then firm 0’s social welfare maximization problem and firm 1’s profit maximization problem are

\[
\begin{align*}
\begin{aligned}
\text{Max } SW^{II}(p_0, p_1) & = p_0 q_0(p_0, p_1) + p_1 q_1(p_0, p_1) + CS \\
\text{Max } \Pi_{F1}^{II}(p_0, p_1) & = p_1 q_1(p_0, p_1)
\end{aligned}
\end{align*}
\] (4)

Substituting Eq. (3) into Eq. (4) and solving the first-order conditions (FOCs) yield
\[ \begin{align*}
    p_0^{**} &= \frac{\beta(1-\beta)\alpha}{2-\beta^2}, \\
    p_i^{**} &= \frac{(1-\beta)\alpha}{2-\beta^2},
\end{align*} \]

where the asterisk '*' indicates the optimal status. Simple comparison yields
\[ p_0^{**} \leq p_i^{**} (0 \leq \beta \leq 1), \]
which means that the public firm sets a lower price than the private firm due to the social welfare concern. Based on the retail prices, we obtain the social welfare and the private firm's profit, which are summarized in Table 3 in the Appendix.

### 3.2. Channel structure ID

In this subsection we consider that the public firm (firm 0) integrates while the private firm (firm 1) decentralizes by distributing through a private retailer (retailer 1). By backward induction, we first analyze the last stage. Knowing firm 1's wholesale price \( w_1 \), firm 0 and retailer 1 solve

\[
\begin{align*}
    \max_{p_0} SW^{ID}(p_0, p_1) &= p_0 q_0(p_0, p_1) + p_1 q_1(p_0, p_1) + CS \\
    \max_{p_1} \Pi_{R1}^{ID}(p_0, p_1) &= (p_1 - w_1) q_1(p_0, p_1),
\end{align*}
\]

where the subscript 'R1' denotes retailer 1.

Solving the FOCs from Eq. (5), we obtain
\[
\begin{align*}
    p_0^{ID^*}(w_i) &= \frac{\beta[(1-\beta)\alpha + w_i]}{2-\beta^2} \\
    p_i^{ID^*}(w_i) &= \frac{(1-\beta)\alpha + w_i}{2-\beta^2}.
\end{align*}
\]
For firm 1’s wholesale price decision, we consider two vertical contracts, wholesale-price and two-part tariff contracts. Following Piccolo and Reisinger (2011), we examine the two vertical contracts as follows:

i). With a wholesale-price contract, firm 1’s objective is

\[ \text{Max } \Pi^{(w)_{ID}}_{F1} (p_0, p_1) = w_i q_i (p_0, p_1), \] (7)

where the bracketed ‘W’ in the superscript indicates the wholesale-price contract case.

ii). If firm 1 adopts a two-part tariff contract, it can extract all the profit from retailer 1. Thus, firm 1’s objective with a two-part tariff contract is

\[ \text{Max } \Pi^{(T)_{ID}}_{F1} (p_0, p_1) = (p_1 - w_i) q_i (p_0, p_1) + w_i q_i (p_0, p_1) = p_i q_i (p_0, p_1), \] (8)

where the bracketed ‘T’ in the superscript indicates the two-part tariff contract case.

Substituting Eq. (6) into Eqs. (7) and (8), respectively, we solve the two contract scenarios and obtain the optimal wholesale prices as

\[ w^{(w)_{ID^*}}_i = \frac{\alpha}{2(1 + \beta)} \]

and

\[ w^{(T)_{ID^*}}_i = \frac{\beta^2 \alpha}{2(1 + \beta)} \]

for the wholesale-price and two-part tariff contracts, respectively. Note that \( w^{(T)_{ID^*}}_i \) is lower than \( w^{(w)_{ID^*}}_i \) because the two-part tariff contract can coordinate firm 1 and retailer 1’s pricing decisions. Based on the wholesale price decisions, the social welfare and firm 1’s profit under both vertical contracts can be obtained and are shown in Table 3 in the Appendix.

Comparing channel structure II with channel structure ID, we have
Lemma 1. Under Bertrand competition, when the public firm integrates, the private firm's channel strategies are as follows.

(i). With a wholesale-price contract, the private firm prefers decentralization to integration if \( \beta \geq 0.816 \), and prefers integration to decentralization otherwise.

(ii). With a two-part tariff contract, the private firm always prefers decentralization to integration.

Proof. See Appendix.

The intuitions underlying Lemma 1 are as follows. With a wholesale-price contract, if the private firm decentralizes, the introduction of an independent retailer will cause the well-known “double marginalization” problem, which tends to reduce the private firm’s profit. On the other hand, under decentralization the independent retailer serves as a buffer that alleviates competition between channels and tends to increase the private firm’s profit, which is the so-called “retailer buffer” effect (McGuire and Staelin, 1983; Wang et al., 2011). In general, between these two conflicting effects of decentralization, the “retailer buffer” effect may outweigh the “double marginalization” effect when products are highly substitutable, i.e., when the competition is intense. Part (i) of Lemma 1 is in line with this classic result: the private firm prefers decentralization when products are highly substitutable ( \( \beta \geq 0.816 \) ) and prefers integration otherwise.

With the above interpretations, part (ii) of Lemma 1 follows easily: with a two-part tariff contract, decentralization stills gives the “retailer buffer” effect, while no longer has the “double marginalization” problem as the private firm can always extract all the profit of its downstream retailer, and thus the private firm always prefer decentralization.
While it is convenient and consistent with the literature to use the two classic terms of “double marginalization” and “retailer buffer” to explain the behavior of the private firm, these two terms do not directly apply to the public firm. The reason is simple: the “double marginalization” effect and the “retailer buffer” effect are both about profit, while the public firm is not profit-oriented. Because of this, we shall revisit these two classic terms from a basic optimization or game theoretic perspective, which will help explain the behavior of the public firm in later analysis.

From an optimization perspective, the “double marginalization” effect of decentralization is simply about the “price-setting power”. That is, if the private firm integrates, it can directly set the retail price for profit maximization, while under decentralization, the independent retailer will set the retail price for its own profit maximization. Thus, decentralization will compromise the price-setting power of the private firm and thereby tend to reduce its profit.

The “retailer buffer” effect of decentralization could be viewed as a “first-mover advantage” from a game theoretic perspective. That is, when the private firm distributes through an independent retailer, the public firm (which does not decentralize) has to directly compete against the retailer in the market, and the market competition result depends on the private firm’s wholesale price as given by Eq. (6). In other words, by decentralization the private firm becomes a leader or first-mover in the game, which can set its wholesale price to control the downstream market competition.

The above interpretations using “price-setting power” and “first-mover advantage” are based on basic optimization and game theoretic analyses, and thus apply to both the private and the public firms. Hereafter, when we explain the private firm’s choice between integration and decentralization, to be consistent with the literature, we shall use the two classic terms of “double
marginalization” and “retailer buffer”. When we explain the public firm’s choice, we shall refer to the “price-setting power” (or “quantity-setting power” under Cournot competition) and the “first-mover advantage”.

3.3. Channel structure DI

Here we consider another channel structure where the public firm (firm 0) distributes its product through a private retailer (retailer 0) while the private firm (firm 1) integrates. Given firm 0's wholesale price, retailer 0 and firm 1 solve

\[
\begin{align*}
\max_{p_0} & \quad \Pi_{RI}^{DI}(p_0, p_1) = (p_0 - w_0)q_0(p_0, p_1) \\
\max_{p_1} & \quad \Pi_{R1}^{DI}(p_0, p_1) = p_1q_1(p_0, p_1)
\end{align*}
\]

(9)

where \( w_0 \) is firm 0's wholesale price, and the subscript 'R0' denotes retailer 0.

Solving the FOCs from Eq. (9), we have

\[
\begin{align*}
p^{*}_{0 DI}(w_0) &= \frac{(2 - \beta - \beta^2)\alpha + 2w_0}{4 - \beta^2} \\
p^{*}_{1 DI}(w_0) &= \frac{(2 - \beta - \beta^2)\alpha + \beta w_0}{4 - \beta^2}
\end{align*}
\]

(10)

Since firm 0 always maximizes social welfare, a wholesale-price contract and a two-part tariff contract between firm 0 and retailer 0 make no difference. Anticipating Eq. (10), firm 0 maximizes social welfare by choosing the optimal wholesale price as

\[
\begin{align*}
w^{*}_{0 DI} &= \frac{(1 - \beta)^2 (2 + \beta)^2 \alpha}{4 - 3\beta^2}.
\end{align*}
\]
Note here firm 0, for the welfare-enhancing purpose, subsidizes retailer 0 \( w_{0}^{st*} < 0 \) to induce a lower retail price. Substituting \( w_{0}^{st*} \) into Eq. (10) and then Eq. (9), we can derive the social welfare and firm 1’s profit, which are summarized in Table 3 in the Appendix.

Comparing channel structure \( II \) with channel structure \( DI \), we have

**Lemma 2.** Under Bertrand competition, when the private firm integrates, the public firm always prefers decentralization to integration, regardless of the vertical contract form.

Proof. See Appendix.

The underlying intuitions of Lemma 2 are as follows. Given the private firm’s choice of integration, if the public firm integrates, it can directly set its retail price to ensure welfare maximization, i.e., it has direct “price-setting power”. On the other hand, if the public firm decentralizes, it can enjoy the “first-mover advantage” (i.e., setting \( w_0 \) before product market competition) without losing its “price-setting power” as it can always use its wholesale price to adjust downstream competition. Specifically, Eq. (10) shows that the “first-mover advantage” under decentralization does not contradict the “price-setting power” of firm 0: reducing \( w_0 \) will reduce \( p_0 \) (price-setting power that lowers the retail price of its own channel for welfare enhancement) and will also reduce \( p_1 \) (first-mover advantage that lowers its competitor’s retail price for welfare enhancement). Simply speaking, when the private firm integrates, the public firm always prefers decentralization because decentralization offers first-mover advantage without compromising its price-setting power.
3.4. Channel structure DD

Now we move on to consider a fully decentralized channel structure, where both the public and the private firm distribute their products exclusively through independent private retailers, retailer 0 and retailer 1, respectively. Knowing upstream wholesale prices \( w_0 \) and \( w_1 \), retailer 0 and retailer 1 maximize their profits given by

\[
\begin{align*}
\max_{p_0} \Pi_{R0}^{DD} (p_0, p_1) &= (p_0 - w_0) q_0 (p_0, p_1) \\
\max_{p_1} \Pi_{R1}^{DD} (p_0, p_1) &= (p_1 - w_1) q_1 (p_0, p_1)
\end{align*}
\]  

(11)

Solving the FOCs from Eq. (11) yields

\[
\begin{align*}
p_0^{DD*} (w_0, w_1) &= \frac{(2 - \beta - \beta^2) \alpha + 2w_0 + \beta w_1}{4 - \beta^2} \\
p_1^{DD*} (w_0, w_1) &= \frac{(2 - \beta - \beta^2) \alpha + 2w_1 + \beta w_0}{4 - \beta^2}
\end{align*}
\]  

(12)

Recall that the two vertical contracts make no difference to firm 0's decision rule, while firm 1 has different objectives under different contracts. Therefore, observing Eq. (12), firm 0 and firm 1 optimize their objectives

\[
\begin{align*}
\max_{w_0} SW^{(w)DD} &= p_0 q_0 (p_0, p_1) + p_1 q_1 (p_0, p_1) + CS \\
\max_{w_1} \Gamma^{(w)DD}_{F1} &= w_1 q_1 (p_0, p_1)
\end{align*}
\]  

(13)

under the wholesale-price contract scenario, and solve

\[
\begin{align*}
\max_{w_0} SW^{(T)DD} &= p_0 q_0 (p_0, p_1) + p_1 q_1 (p_0, p_1) + CS \\
\max_{w_1} \Gamma^{(T)DD}_{F1} &= p_1 q_1 (p_0, p_1)
\end{align*}
\]  

(14)

under the two-part tariff contract scenario.

Substituting Eq. (12) into Eqs. (13) and (14), we can solve the equilibrium wholesale prices under the two contract scenarios, and then obtain the respective equilibrium social welfare and
firm 1’s profit under the two contract scenarios, which are all summarized in Table 3 in the Appendix.

Comparing channel structure ID with channel structure DD, we have

**Lemma 3.** Under Bertrand competition, if the private firm decentralizes, the public firm always prefers decentralization to integration, regardless of the vertical contract form.

Proof. See Appendix.

The intuition behind Lemma 3 is similar to that of Lemma 2. Given that the private firm decentralizes, if the public firm integrates, it has direct “price-setting power” of its own channel but this “price-setting power” is dominated by the private firm's overall “first-mover advantage” as shown by Eq. (6) (\( w_i \) determines both \( p^{ID*}_0 \) and \( p^{ID*}_1 \)). On the other hand, from Eq. (12), if the public firm decentralizes, it can not only influence the downstream retail prices, but also counteract the private firm’s profit-oriented behaviour prior to market competition (\( p^{DD*}_0 \) and \( p^{DD*}_1 \) depend on both \( w_0 \) and \( w_1 \), not \( w_i \) alone). More specifically, from Eq. (12), the public firm can reduce its wholesale price \( w_0 \) to lower \( p^{DD*}_0 \) (price-setting power that lowers the retail price of its own channel for welfare enhancement) and also lower \( p^{DD*}_1 \) (counteracting its competitor’s first-mover advantage to lower its competitor’s retail price for welfare enhancement). Simply speaking, when the private firm decentralizes, the public firm always prefers decentralization because decentralization counteracts the private firm’s first-mover advantage and retains the public firm’s price-setting power.
Based on Lemma 2 and 3, we see that, under Bertrand competition, the public firm always prefers decentralization to integration, regardless of the private firm's channel strategy. As will be seen in next section, this result does not hold under Cournot competition.

Comparing channel structure DI with channel structure DD, we have

**Lemma 4.** Under Bertrand competition, when the public firm decentralizes, the private firm's channel strategies are as follows.

(i). With a wholesale-price contract, the private firm prefers decentralization to integration if \( \beta \geq 0.903 \), and prefers integration to decentralization otherwise.

(ii). With a two-part tariff contract, the private firm prefers decentralization to integration.

Proof. See Appendix.

The intuition of Lemma 4 is exactly the same as that of Lemma 1. That is, in the wholesale-price contract case, the "retailer buffer" effect of decentralization outweighs the "double marginalization" effect only when products are highly substitutable, and thus the private firm prefers decentralization when products are highly substitutable and prefers integration otherwise. Under the two-part tariff contract scenario, the private firm always prefers decentralization because decentralization offers the "retailer buffer" effect without the "double marginalization" problem.

We end this section by summarizing Lemma 1 to 4 into the following proposition.

**Proposition 1.** Under Bertrand competition,
(i). In the wholesale-price contract case, channel structure $DI$ arises in equilibrium if $\beta \in (0, 0.903)$; otherwise, channel structure $DD$ is the equilibrium;

(ii). In the two-part tariff contract case, channel structure $DD$ always occurs in equilibrium.

Proof. From Lemma 2 and 3, the public firm always chooses decentralization, and thus the equilibrium channel structure simply follows from Lemma 4.

4. Channel strategies under Cournot competition

In this section we proceed to analyze the four channel structures, $II$, $ID$, $DI$ and $DD$, under Cournot competition.

4.1. Channel structure $II$

We first consider both firm 0 and firm 1 choose to integrate, so they determine their respective quantities to solve

$$\begin{align*}
\begin{cases}
\text{Max SW}^{II} (q_0, q_1) = p_0(q_0, q_1)q_0 + p_1(q_0, q_1)q_1 + CS \\
\text{Max } \Pi^{II}_{F_1} (q_0, q_1) = p_1(q_0, q_1)q_1
\end{cases}
\end{align*}$$

(15)

Using the inverse demand Eq. (2) and solving the FOCs from Eq. (15) yield

$$\begin{align*}
\begin{cases}
q_0^{II*} = \frac{(2 - \beta)\alpha}{2 - \beta^2} \\
q_1^{II*} = \frac{(1 - \beta)\alpha}{2 - \beta^2}
\end{cases}
\end{align*}$$

(16)

from which we have optimal social welfare and the private firm’s profit, which are summarized in Table 4 in the Appendix.
Simple comparison gives \( q_0^{\mu*} > q_i^{\mu*} (0 \leq \beta \leq 1) \), which means that the public firm supplies more products than the private firm does due to the social welfare concern.

### 4.2. Channel structure ID

In this subsection we consider that firm 0 integrates while firm 1 decentralizes by distributing through retailer 1. By backward induction, we first analyze the last stage. Given firm 1’s wholesale price \( w_1 \), firm 0 and retailer 1’s objectives are

\[
\begin{align*}
\text{Max } & SW^{ID} (q_0, q_1) = p_0 (q_0, q_1) q_0 + p_i (q_0, q_1) q_i + CS \\
\text{Max } & \Pi_{R1}^{ID} (q_0, q_1) = [p_i (q_0, q_1) - w_1] q_i
\end{align*}
\]

(17)

Solving the FOCs from Eq. (17), we obtain

\[
\begin{align*}
q_0^{ID*} (w_1) &= \frac{(2 - \beta) \alpha + \beta w_1}{2 - \beta^2} \\
q_i^{ID*} (w_1) &= \frac{(1 - \beta) \alpha - w_1}{2 - \beta^2}
\end{align*}
\]

(18)

Similar to subsection 3.2, firm 1 solves \( \text{Max } \Pi_{F1}^{(W)ID} (q_0, q_1) = w_i q_i (w_1) \) under a wholesale-price contract, and solves \( \text{Max } \Pi_{F1}^{(T)ID} (q_0, q_1) = p_i q_i (w_1) \) under a two-part tariff contract. Thus, observing Eq. (18), firm 1 sets its optimal wholesale price at \( w_i^{(W)ID*} = \frac{(1 - \beta) \alpha}{2} \) under a wholesale-price contract, and \( w_i^{(T)ID*} = -\beta^2 \alpha / (1 + \beta) \) under a two-part tariff contract. Correspondingly, the social welfare and firm 1’s profit can be obtained. We summarize all the results in Table 4 in the Appendix.

Comparing channel structure II with under channel structure ID, we have
Lemma 5. Under Cournot competition, if the public firm integrates,

(i). In the wholesale-price contract case, the private firm always prefers integration to decentralization.

(ii). In the two-part tariff contract case, the private firm always prefers decentralization to integration.

Proof. See Appendix.

The intuition of Lemma 5 is similar to Lemma 1. Recall that the “retailer buffer” effect of decentralization may outweigh the “double marginalization” effect only when the competition is intense. Because Cournot competition is not intense enough (less intense than Bertrand competition), the “retailer buffer” effect of decentralization actually never outweighs the “double marginalization” effect under Cournot competition. As a result, when both of the two effects exist (in the wholesale-price contract case), the private firm prefers integration. When only the “retailer buffer” effect exists (in the two-part tariff contract case), the private firm prefers decentralization.

4.3. Channel structure DI

Here we consider the channel structure where firm 0 distributes its product through retailer 0 while firm 1 integrates. Knowing firm 0's wholesale price \(w_0\), retailer 0 and firm 1 solve

\[
\begin{align*}
\max_{q_0} \Pi^{D_0}_{R_0}(q_0, q_1) &= \left[ p_0(q_0, q_1) - w_0 \right] q_0 \\
\max_{q_1} \Pi^{D_1}_{R_1}(q_0, q_1) &= p_1(q_0, q_1) q_1
\end{align*}
\]

which gives equilibrium quantities
Again, firm 0's welfare objective function remains unchanged under the two vertical contracts. Substituting Eq. (20) into firm 0's objective function and then solving it yield

\[
q_{0}^{DI*}(w_0) = \frac{(2-\beta)\alpha - 2w_0}{4-\beta^2} \\
q_{1}^{DI*}(w_0) = \frac{(2-\beta)\alpha + \beta w_0}{4-\beta^2}.
\]  

(20)

from which the social welfare and firm 1's profit are derived and summarized in Table 4 in the Appendix.

Comparing channel structure II with channel structure DI, we have

Lemma 6. Under Cournot competition, if the private firm integrates, the public firm always prefers decentralization to integration, regardless of the vertical contract form.

Proof. See Appendix.

At the first glance, Lemma 6 here seems just the counterpart of Lemma 2 under Bertrand competition, while the underlying intuitions and explanations are actually quite different. We will see the difference more clearly later together with Lemma 8, and because of this, we will leave the discussions of Lemma 6 later with Lemma 8.

4.4. Channel structure DD

Now we move on to consider fully decentralized channel structure DD, where firm 0 and firm 1 distribute their products exclusively through independent private retailers, retailer 0 and
retailer 1, respectively. Knowing upstream wholesale prices $w_0$ and $w_1$, retailer 0 and retailer 1 solve
\[
\begin{align*}
\text{Max } & \Pi^{DD}_{R0} (p_0(q_0,q_1), p_1(q_0,q_1)) = [p_0(q_0,q_1) - w_0]q_0 \\
\text{Max } & \Pi^{DD}_{R1} (p_0(q_0,q_1), p_1(q_0,q_1)) = [p_1(q_0,q_1) - w_1]q_1.
\end{align*}
\] (21)

Solving the FOCs from Eq. (21), we obtain
\[
\begin{align*}
q_0^{DD^*} (w_0, w_1) = \frac{(2 - \beta)\alpha - 2w_0 + \beta w_1}{4 - \beta^2} \\
q_1^{DD^*} (w_0, w_1) = \frac{(2 - \beta)\alpha - 2w_1 + \beta w_0}{4 - \beta^2}.
\end{align*}
\] (22)

Similar to Eqs. (13) and (14) in subsection 3.4, here firm 0 and firm 1’s objectives are
\[
\begin{align*}
\text{Max } & SW^{(W)DD}_{w_0} = p_0 (q_0, q_1) q_0 + p_1 (q_0, q_1) q_1 + CS \\
\text{Max } & \Pi^{(W)DD}_{w_1} = w_1 q_1
\end{align*}
\] (23)

under the wholesale-price contract scenario, and
\[
\begin{align*}
\text{Max } & SW^{(T)DD}_{w_0} = p_0 (q_0, q_1) q_0 + p_1 (q_0, q_1) q_1 + CS \\
\text{Max } & \Pi^{(T)DD}_{w_1} = p_1 q_1
\end{align*}
\] (24)

under the two-part tariff contract scenario.

Substituting Eq. (22) into Eqs. (23) and (24), we can solve the equilibrium wholesale prices under the two contract scenarios, and then obtain the respective equilibrium social welfare and firm 1’s profit under the two contract scenarios, which are all summarized in Table 4 in the Appendix.

Comparing channel structure $DI$ with channel structure $DD$, we have

**Lemma 7.** Under Cournot competition, if the public firm decentralizes,
(i). In the wholesale-price contract case, the private firm always prefers integration to decentralization.

(ii). In the two-part tariff contract case, the private firm always prefers decentralization to integration.

Proof. See Appendix.

The interpretation of Lemma 7 is obviously the same as that of Lemma 5. Combining Lemma 5 and 7, we can see that, under Cournot competition, the private firm always prefers integration under a wholesale-price contract, and always prefers decentralization under a two-part tariff contract, regardless of the public firm’s strategy.

Comparing channel structure $ID$ with channel structure $DD$, we have

**Lemma 8.** Under Cournot competition, if the private firm decentralizes, the public firm prefers integration to decentralization, regardless of the vertical contract form.

Proof. See Appendix.

Combining Lemma 6 and 8 here under Cournot competition, and comparing them with Lemma 2 and 3 under Bertrand competition, we can see a key difference: under Bertrand competition, the public firm always prefers decentralization; while under Cournot competition, the public firm’s preference depends on the private firm’s choice. This difference arises from how the “first-mover advantage” of decentralization will affect the “price or quantity setting power” of the public firm under the two different competition modes.
Under Bertrand competition, as mentioned before, from Eqs. (10) and (12) we can see that the “first-mover advantage” of decentralization does not contradict the “price-setting power” of the public firm: reducing $w_0$ will reduce $p_0$ (price-setting power that lowers the retail price of its own channel for welfare enhancement) and will also reduce $p_1$ (first-mover advantage that lowers its competitor’s retail price for welfare enhancement). This means if the public firm chooses decentralization, it will enjoy the “first-mover advantage” without losing its “price-setting power”, and thus the public firm will always choose decentralization under Bertrand competition.

However, under Cournot competition, from Eqs. (20) and (22) we can see that the “first-mover advantage” of decentralization conflicts with the “quantity-setting power” of the public firm: for “quantity-setting power” of its own channel, $w_0$ needs to be reduced to increase $q_0$ for welfare improvement; while for “first-mover advantage” to affect its competitor, $w_0$ needs to be increased to increase $q_1$ for welfare improvement. This conflict means that, if the public firm chooses decentralization, it will enjoy “first-mover advantage” at the expense of its “quantity-setting power”. Note that the “quantity-setting power” is very important under Cournot competition because the competition is not intense and Retailer 0 tends to set $q_0$ very small for its own profit purpose. This can be seen by simply comparing $q_0''$ given by Eq. (16) with $q_0^{pr*}$ given by Eq. (20): without subsidy ($w_0 = 0$), $q_0^{pr*}$ will be less than half of $q_0''$ due to Retailer 0’s self-optimization. Therefore, only when the “first-mover advantage” is significant enough will the public firm choose decentralization. It is intuitive that, when the private firm chooses integration, the “first-mover advantage” of the public firm under decentralization will be big because the public firm will be the only first-mover; when the private firm chooses
decentralization, the “first-mover advantage” of the public firm under decentralization will not be that big because the public firm is not the only first-mover. As a result, the public firm will choose decentralization in the former case (Lemma 6), and will choose integration in the latter case (Lemma 8).

We now summarize Lemma 5 to 8 into the following proposition.

**Proposition 2.** Under Cournot competition,

(i). Under a wholesale-price contract, channel structure $DI$ arises in equilibrium;

(ii). Under a two-part tariff contract, channel structure $ID$ arises in equilibrium.

Proof. Lemma 5 and 7 state that the private firm always prefers integration under a whole-price contract, and always prefers decentralization under a two-part tariff contract. Then the equilibrium channel structures simply follow from Lemma 6 and 8.

5. Concluding remarks

This paper examines distribution channel strategies in a mixed market under different contract and competition scenarios. Our results suggest that the equilibrium channel structures depend on the market competition mode, the vertical contract form, and the level of product substitutability. Specifically, the channel strategy of the private firm depends mainly on the vertical contract form: in the two-part tariff contract case, the private firm always chooses decentralization; in the wholesale-price contract case, the private firm chooses integration for most scenarios except for highly substitutable products under Bertrand competition (i.e., under very intense competition). The channel strategy of the public firm depends mainly on the competition mode: under Bertrand competition, the public firm always chooses decentralization; under Cournot competition, the
public firm always chooses the opposite of the private firm’s strategy. Note that, due to scope limitation and to keep the paper concise, we focus on endogenous channel strategies and assume that other settings (i.e., vertical contracts, competition modes) are exogenously given. We believe it is worthwhile to examine other non-linear contracts such as revenue sharing and return policy, which are left for future research.

This research sheds some light on policy considerations regarding privatization. In particular, there are some differences in channel strategies between private duopoly and mixed duopoly. For instance, channel structure II (both firms adopting an integrated channel strategy) is always one equilibrium in a private duopoly, while it is never an equilibrium in our mixed duopoly setting, i.e., at least one firm will choose to decentralize in a mixed duopoly. Therefore, privatization of a public firm may lead to changes of channel structures. This indicates that a policy maker should carefully design its policy to induce the market players to arrive at a desired equilibrium.

Appendix.

Table 3. Equilibrium solutions under Bertrand competition

<table>
<thead>
<tr>
<th>Channel Structure</th>
<th>II</th>
<th>ID</th>
<th>DI</th>
<th>DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price</td>
<td>N/A</td>
<td>$w_1^w = \frac{\alpha}{2(1+\beta)}$</td>
<td>$w_0^o = \frac{(1-\beta)^2 (2+\beta)^2 \alpha}{4-3\beta^2}$</td>
<td>$w_0^w = \frac{(1-\beta)(2+\beta)C\alpha}{5\beta^2 - 20\beta^2 + 16}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$w_1^t = \frac{\beta^2 \alpha}{2(1+\beta)}$</td>
<td>$w_1^t = \frac{(1-\beta)(2-\beta^2)(4-\beta^2)\alpha}{5\beta^2 - 20\beta^2 + 16}$</td>
<td>$w_0^o = -\frac{(1-\beta)D\alpha}{\beta^2 - 8\beta^2 + 8}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$w_1^t = \frac{\beta^2 (1-\beta)(2-\beta^2)\alpha}{\beta^2 - 8\beta^2 + 8}$</td>
<td></td>
</tr>
<tr>
<td>Retail price</td>
<td>$p_s = \frac{\beta(1-\beta)\alpha}{2-\beta^2}$</td>
<td>$p_i = \frac{\beta(3-2\beta^2)\alpha}{2(1+\beta)(2-\beta^2)}$</td>
<td>$p_i = \frac{(3-2\beta^2)\alpha}{2(1+\beta)(2-\beta^2)}$</td>
<td>$p_i = \frac{\beta(1-\beta)\alpha}{4-3\beta^2}$</td>
</tr>
<tr>
<td>Sales quantity</td>
<td>$q_i = \frac{\alpha}{1+\beta}$</td>
<td>$q_i = \frac{\alpha}{(1+\beta)(2-\beta^3)}$</td>
<td>$q_i = \frac{\alpha}{(1+\beta)(2-\beta^3)}$</td>
<td>$q_i = \frac{(4+\beta-3\beta^2-\beta^3)\alpha}{(1+\beta)(4-3\beta^2)}$</td>
</tr>
<tr>
<td>Retailer profit</td>
<td>N/A</td>
<td>$\Pi_{ri} = \frac{(1-\beta)\alpha^2}{4(1+\beta)(2-\beta^2)^2}$</td>
<td>$\Pi_{ri} = \frac{(1-\beta)(4+\beta-3\beta^2-\beta^3)\alpha^2}{(1+\beta)(4-3\beta^2)^2}$</td>
<td>$\Pi_{ri} = \frac{(1-\beta)(\beta^4-\beta^3-7\beta^2+\beta+7)}{(1+\beta)((5\beta^2-20\beta^2+16)^2)}$</td>
</tr>
<tr>
<td>Private firm profit</td>
<td>$\frac{(1-\beta)\alpha^2}{(1+\beta)(2-\beta^2)^2}$</td>
<td>$\Pi_{ri} = \frac{(1-\beta)\alpha^2}{4(1+\beta)^2(2-\beta^2)^2}$</td>
<td>$\Pi_{ri} = \frac{(1-\beta)(2-\beta^2)^2\alpha^2}{(1+\beta)(4-3\beta^2)^2}$</td>
<td>$\Pi_{ri} = \frac{(1-\beta)(\beta^4-\beta^3-7\beta^2+\beta+7)}{(1+\beta)((5\beta^2-20\beta^2+16)^2)}$</td>
</tr>
<tr>
<td>Social welfare</td>
<td>$\frac{A\alpha^2}{2(1+\beta)(2-\beta^2)^2}$</td>
<td>$SW^w = \frac{B\alpha^2}{8(1+\beta)^2(2-\beta^2)^2}$</td>
<td>$SW^r = \frac{(7+8\beta^2-\beta^3)\alpha^2}{2(1+\beta)(4-3\beta^2)^2}$</td>
<td>$SW^w = \frac{G\alpha^2}{(1+\beta)((5\beta^2-20\beta^2+16)^2)}$</td>
</tr>
</tbody>
</table>

where

\[
A = 2\beta^4 - \beta^3 - 7\beta^2 + \beta + 7,
\]
\[
B = 8\beta^5 + 4\beta^4 - 32\beta^3 - 20\beta^2 + 32\beta + 23,
\]
\[
C = 2\beta^4 + \beta^3 - 8\beta^2 - 4\beta + 8,
\]

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\[ D = \beta^4 - 2\beta^3 - 8\beta^2 + 8, \]
\[ E = 2\beta^5 + 5\beta^4 - 8\beta^3 - 20\beta^2 + 6\beta + 16, \]
\[ F = (\beta^4 - 2\beta^3 - 8\beta^2 + 2\beta + 8)^2, \]
\[ G = 6\beta^9 + 19\beta^8 - 50\beta^7 + 150\beta^6 + 416\beta^5 - 414\beta^4 - 174\beta^3 - 466\beta^2 + 72\beta + 184, \]
\[ H = \beta^8 - 16\beta^6 + 6\beta^5 + 74\beta^4 - 14\beta^3 - 114\beta^2 + 8\beta + 56. \]

Table 4. Equilibrium solutions under Cournot competition

<table>
<thead>
<tr>
<th>Channel Structure</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>II</td>
<td>ID</td>
<td>DI</td>
</tr>
<tr>
<td>Wholesale price</td>
<td>N/A</td>
<td>( w_i^w = \frac{(1 - \beta)\alpha}{2} )</td>
<td>( w_o^w = -\frac{(2 - \beta)^2\alpha}{4 - 3\beta^2} )</td>
</tr>
<tr>
<td>Retail price</td>
<td>( p_o = 0 )</td>
<td>( p_i = \frac{(1 - \beta)\alpha}{2 - \beta^2} )</td>
<td>( p_i = \frac{2(1 - \beta)\alpha}{4 - 3\beta^2} )</td>
</tr>
<tr>
<td>Sales quantity</td>
<td>( q_i = \frac{(1 - \beta)\alpha}{2 - \beta^2} )</td>
<td>( q_i^w = \frac{(2 + \beta)\alpha}{2(1 + \beta)} )</td>
<td>( q_i = \frac{2(1 - \beta)\alpha}{4 - 3\beta^2} )</td>
</tr>
<tr>
<td>Retailer profit</td>
<td>N/A</td>
<td>$\Pi_{yi} = \frac{(1-\beta)^2 \alpha^2}{4(2-\beta^2)^2}$</td>
<td>$\Pi_{xi} = 0$</td>
</tr>
<tr>
<td>----------------</td>
<td>-----</td>
<td>------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>Private firm profit</td>
<td>$\frac{(1-\beta)^2 \alpha^2}{2(2-\beta^2)^2}$</td>
<td>$\Pi_{yi} = \frac{(1-\beta)^2 \alpha^2}{4(2-\beta^2)^2}$</td>
<td>$\Pi_{xi} = \frac{(1-\beta) \alpha^2}{4(1+\beta)}$</td>
</tr>
<tr>
<td>Social welfare</td>
<td>$\frac{2\beta^2-2\beta^3-6\beta+7}{2(2-\beta^2)} \alpha^2$</td>
<td>$SW^w = \frac{1\alpha^2}{8(2-\beta^2)^2}$</td>
<td>$SW^r = \frac{(7-6\beta)\alpha^2}{2(4-3\beta^2)}$</td>
</tr>
</tbody>
</table>

where

$$I = \beta^4 + 6\beta^3 - 12\beta^2 - 14\beta + 23,$$

$$J = 2\beta^7 - 5\beta^6 - 40\beta^5 + 232\beta^4 - 388\beta^3 - 224\beta^2 - 224\beta + 368,$$

$$K = 2\beta^7 - 5\beta^6 - 24\beta^5 + 40\beta^4 + 104\beta^3 - 132\beta^2 - 96\beta + 112.$$

**Proof of Lemma 1.**

Under Bertrand competition, comparing channel structure $II$ with channel structure $ID$ in Table 3, we have

i). Under the wholesale-price contract scenario:

$$\Pi^H_{F1} - \Pi^{(w)ID}_{F1} = \frac{2-3\beta^2}{4(1+\beta)^2(2-\beta^2)} \alpha^2 \begin{cases} 
\geq 0, & \text{if } \beta \leq 0.816 \\
< 0, & \text{otherwise}
\end{cases}$$

ii). Under the two-part tariff contract scenario:

$$\Pi^H_{F1} - \Pi^{(T)ID}_{F1} = -\frac{\beta^4 \alpha^2}{4(1+\beta)^2(2-\beta^2)} < 0.$$

**Proof of Lemma 2.**
Since the public firm always maximizes social welfare, here the vertical contract choice makes no difference. Under Bertrand competition, comparing channel structure $II$ with channel structure $DI$ in Table 3, we have

$$SW^{II} - SW^{DI} = -\frac{\beta^2 (1-\beta)^2 (1-\beta^2) \alpha^2}{2(4-3\beta^2)(2-\beta^2)^2} < 0.$$ 

**Proof of Lemma 3.**

Under Bertrand competition, comparing channel structure $ID$ with channel structure $DD$ in Table 3, we have

Under the wholesale-price contract scenario:

$$SW^{(W)ID} - SW^{(W)DD} = -\frac{\beta^2 (1344 - 4944\beta^2 + 7400\beta^4 - 5759\beta^6 + 2452\beta^8 - 540\beta^{10} + 48\beta^{12}) \alpha^2}{8(1+\beta)^2 (2-\beta^2)^2 (16 - 20\beta^2 + 5\beta^4)^2} < 0.$$ 

Under the two-part tariff contract scenario:

$$SW^{(T)ID} - SW^{(T)DD} = -\frac{\beta^2 (48 - 80\beta^2 + 32\beta^4 + \beta^6) \alpha^2}{8(1+\beta)^2 (8 - 8\beta^2 + \beta^4)^2} < 0.$$ 

**Proof of Lemma 4.**

Under Bertrand competition, comparing channel structure $DI$ with channel structure $DD$ in Table 3, we have

Under the wholesale-price contract scenario:

$$\Pi^{DI}_{F1} - \Pi^{(W)DD}_{F1} = \frac{2(1-\beta)(2-\beta^2)^2 (64 - 176\beta^2 + 164\beta^4 - 61\beta^6 + 8\beta^8) \alpha^2}{(1+\beta)(4-3\beta^2)^2 (16 - 20\beta^2 + 5\beta^4)^2} \begin{cases} \geq 0, \text{if } \beta \leq 0.903 \\ < 0, \text{otherwise} \end{cases};$$

Under the two-part tariff contract scenario:
\[ \Pi_{F1}^{II} - \Pi_{F1}^{(T)DD} = \frac{\beta^4 (1 - \beta)(4 - 2\beta^2 - \beta^4)(2 - \beta^2)^2}{(1 + \beta)(4 - 3\beta^2)(8 - 8\beta^2 + \beta^4)} < 0. \]

**Proof of Lemma 5.**

Under Cournot competition, comparing channel structure \(II\) with channel structure \(ID\) in Table 4, we have

Under the wholesale-price contract scenario:

\[ \Pi_{F1}^{II} - \Pi_{F1}^{(W)ID} = \frac{(1 - \beta)^2 (2 + \beta^2)\alpha^2}{4(2 - \beta^2)^2} > 0; \]

Under the two-part tariff contract scenario:

\[ \Pi_{F1}^{II} - \Pi_{F1}^{(T)ID} = -\frac{\beta^4 (1 - \beta)\alpha^2}{4(1 + \beta)(2 - \beta^2)^2} < 0. \]

**Proof of Lemma 6.**

Since the public firm maximizes social welfare, the forms of vertical contracts make no difference to Lemma 6. Comparing channel structure \(DI\) with channel structure \(DD\) in Table 4, we have

\[ SW^{DI} - SW^{II} = \frac{\beta^2 (1 - \beta)^2 \alpha^2}{2(4 - 3\beta^2)(2 - \beta^2)^2} > 0. \]

**Proof of Lemma 7.**

Under Cournot competition, comparing channel structure \(DI\) with channel structure \(DD\) in Table 4, we have

Under the wholesale-price contract scenario:

\[ \Pi_{F1}^{DI} - \Pi_{F1}^{(W)DD} = \frac{4(1 - \beta)^2 (2 - \beta^2)(64 - 48\beta^2 + 4\beta^4 + \beta^6)\alpha^2}{(4 - 3\beta^2)^2 (4 - 2\beta - \beta^2)^2 (4 + 2\beta - \beta^2)^2} > 0; \]
Under the two-part tariff contract scenario:

$$\Pi_{F1}^{DF} - \Pi_{F1}^{(T)DD} = -\frac{4\beta^4(1-\beta)^2(4-2\beta^2-\beta^4)}{(4-3\beta^2)(8-8\beta^2+\beta^4)} < 0.$$  

**Proof of Lemma 8.**

Under Cournot competition, comparing channel structure $ID$ with channel structure $DD$ in Table 4, we have

Under the wholesale-price contract scenario:

$$SW^{(W)ID} - SW^{(W)DD} = \frac{\beta^2(1-\beta)^2(192-240\beta^2+104\beta^4-17\beta^6+\beta^8)}{8(2-\beta^2)^2(4-2\beta-\beta^2)^2(4+2\beta-\beta^2)^2} > 0;$$  

Under the two-part tariff contract scenario:

$$SW^{(T)ID} - SW^{(T)DD} = \frac{\beta^2(1-\beta)(4-2\beta^2+\beta^3)(4-2\beta^2-\beta^3)}{8(1+\beta)(8-8\beta^2+\beta^4)^2} > 0.$$  

**References**


