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Quantum theory of longitudinal momentum transfer in above-threshold ionization

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In the ionization process, longitudinal momentum along the direction of propagation is transferred to the photoelectrons due to the action of the magnetic component of Lorentz force. In a recent experiment by Smeenk et al. [Phys. Rev. Lett. 106, 193002 (2011)], such a transfer is observed in the ionization of argon and neon atoms by circularly polarized light at 800 and 1400 nm in the intensity range of $10^{14}$–$10^{15}$ W/cm$^2$. They accounted for the results by a purely classical model. We present a fully quantum-mechanical calculation of the transfer of longitudinal momentum to the photoelectrons. The results are in agreement with the observations of Smeenk et al. at high intensities, but clear evidence for additional Coulomb interactions emerges at low intensities.

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In the multiphoton ionization of atoms by a strong light field, several photons are absorbed, resulting in the transfer of the photon energy, linear and angular momenta to the electron-ion system. Due to the smallness of the linear momentum of a single visible photon, it is usually neglected. Recently, however, the transfer of the photon linear momentum was clearly observed in the experiment of Smeenk et al. [1]. It is also known that a large number of photons can produce the macroscopic effect of radiation pressure [2,3]. Recently, the photon linear momentum transfer to the photoelectron, i.e., radiation pressure, has been invoked to account for the generation of terahertz radiation emitted by laser-generated filaments [4–6]. Motivated by the observations of Smeenk et al. [1], we present within the single active electron approximation, a theoretical formulation for the simultaneous transfer of both the photon linear and the angular momentum in the ionization process. We then restrict attention to the transfer of photon linear momentum.

In the dipole approximation, the expression $e^{ik\cdot r} = 1 + \frac{ik\cdot r}{i} + \cdots$ is replaced by unity when the radiation wavelength is large compared with a length characteristic of the system. Here, $k$ is the radiation-field propagation vector. This makes the vector potential $A$ space independent. To obtain longitudinal momentum transfer effects, a nondipole formulation is required. We start with the exact expression for the time-reversed transition amplitude from a ground state $\phi_i$ to a final continuum state $\Psi_f^-$ [7] (atomic units are used throughout).

\[
(S - 1)_h = -i \int_0^{\infty} dt \langle \Psi_f^- | V_L \phi_i \rangle.
\]  

The final state $\Psi_f^-$ is the solution of the time-dependent Schrödinger equation for an atomic electron interacting with a laser field,

\[
i\hbar\frac{d}{dt} - H_0 - V_L - V_A)\Psi_f^-(r,t) = 0.
\]

Here, $H_0$ is the free Hamiltonian, $V_A$ is the atomic Coulomb potential, and $V_L = \frac{1}{2}A(\varphi) \cdot \mathbf{P} + \frac{A(\varphi)}{2c^2}^2$ is the atom-laser interaction Hamiltonian. The operator $\mathbf{P} = -i \nabla$ is the momentum operator and $\mathbf{A} = A(\varphi) \mathbf{r} - \omega t$ is the vector potential of the laser field.

To solve Eq. (2), we carry out the Henneberger unitary transformation to the oscillating frame [8]. Let

\[
\Psi_f^- = e^{-i\int^t dt V_L(t)} \Phi = e^{-i(\mathbf{P} + U)\Phi},
\]

where

\[
\Phi = \frac{1}{c} \int^t dt A(t) - \frac{\beta}{\omega} \int^t d\varphi A(\varphi) d\varphi,
\]

\[
U = \int^t d\tau A^2(\tau) \frac{1}{2c^2} = \frac{1}{\omega} \int^\varphi d\varphi A^2(\varphi) \frac{1}{c^2}
\]

\[
\approx -\frac{1}{\omega}[U_1(\varphi) + \varphi U_p],
\]

where $U_p = I/(4a_0^3)$ is the ponderomotive energy, $I$ is the intensity, and $U_1(\varphi)$ is a periodic function in $\varphi$.

In the oscillating frame, $\Phi_f^-$ is the wave function for an electron in an oscillating Coulomb center. For low-frequency laser fields, as is the case in the experiment of Smeenk et al. [1], the oscillating Coulomb center can be considered quasistatic, and therefore, $\Phi_f^-$ is given approximately by a Coulomb scattering state wave function $\Sigma^-_{A,P}:

\Phi_f^- \approx \Sigma^-_{A,P} \approx \frac{e^{-\alpha/2}G(1 + i\alpha)}{(2\pi)^{1/2}} F_1(-i\alpha,1,-i(p \cdot r + pr)) \times e^{ip\cdot r - iE_pt},

where $E_p = p^2/2$, $a = Z/p$, and $Z$ is the effective atomic core charge. Substituting Eq. (6) into Eq. (3), then the final-state wave function $\Psi_f^-$ is

\[
\Psi_f^- \approx e^{-i\int^t dt V_L(t)} \Sigma^-_{A,P} \approx e^{-(\mathbf{P} + U)\Sigma^-_{A,P}}.
\]

As we see, embedded in the final-state wave function as given by Eq. (7) is a dependence on $p_z$. Moreover, it treats both the laser field and the Coulomb potential on equal footing and, therefore, provides the most accurate account of Coulomb effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state. The recently observed low-energy effects in the final state.
the transition amplitude given by Eq. (1) becomes more difficult. This is because the Coulomb scattering state wave function $\Psi_{A,p}^-$ is not an eigenvector of the operator $e^{-i(\mathbf{p} + U)}$. We begin by writing $\Psi_{A,p}^-(\mathbf{r}) = (2\pi)^{-3/2} \int d\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}} \tilde{\Psi}_{A,p}^-(\mathbf{q})$ and $\Phi_0(\mathbf{r}) = (2\pi)^{-3/2} \int d\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}} \hat{\phi}(\mathbf{q})$. Due to the transversality condition ($\mathbf{k}$ is perpendicular to both $\mathbf{A}$ and $\mathbf{a}$), the plane wave $e^{i\mathbf{q}\cdot\mathbf{r}}$ is an eigenvector of $V_1$ and $e^{-i\int V_1(\mathbf{r}) \, d\mathbf{r}}$. Using Eqs. (4) and (5) and the Fourier components introduced above, Eq. (1) reads

$$(S - 1)_{\mathbf{h}} = -i \int_{-\infty}^{\infty} dt \int d\mathbf{q} \int d\mathbf{q} \int d\mathbf{r} \frac{e^{iE_{\mathbf{r}}t}}{(2\pi)^3} \tilde{\Psi}_{A,p}^+(\mathbf{q}) \hat{\phi}(\mathbf{q}^\prime) \times \exp \left( -i \frac{U_{\mathbf{A}}(\mathbf{k} \cdot \mathbf{r} - \omega t) \right) \exp \left( -i \frac{\mathbf{k} \cdot \mathbf{q} + U_1(\mathbf{q}) \right) \times V_1(\mathbf{q}^\prime, \mathbf{q}) e^{i\mathbf{q}^\prime \cdot \mathbf{r}},$$

where $E_T = E_p + E_B$ and $E_B$ is the ground-state binding energy. Now, $\exp \left( -\frac{i}{\omega} [\mathbf{q} \cdot \mathbf{q} + U_1(\mathbf{q})] \right) V_1(\mathbf{q}^\prime, \mathbf{q})$ is periodic in $\mathbf{q}$ with a period $2\mathbf{p}$. Thus, we write $\exp \left( -\frac{i}{\omega} [\mathbf{q} \cdot \mathbf{q} + U_1(\mathbf{q})] \right) V_1(\mathbf{q}^\prime, \mathbf{q}) = \sum a_n(\mathbf{q}, \mathbf{q}^\prime) e^{i\mathbf{q} \cdot \mathbf{n}}$ with the Fourier components $a_n(\mathbf{q}, \mathbf{q}^\prime) = \frac{1}{2\pi} \int_0^{2\pi} \exp \left( -\frac{i}{\omega} [\mathbf{q} \cdot \mathbf{q} + U_1(\mathbf{q})] \right) - i \phi_n(\mathbf{q}^\prime) \, d\mathbf{q}$. With the definitions $\Delta^{(n)} = n\omega - U_0$ and $\delta^{(n)} = \frac{\mathcal{A}^{(n)}}{\omega} \mathbf{k}$, the expression for the transition probability then reads

$$(S - 1)_{\mathbf{h}} = -i \sum_n \int_{-\infty}^{\infty} dt \int d\mathbf{q} \int d\mathbf{q} \int d\mathbf{r} \frac{e^{iE_{\mathbf{r}}(\Delta^{(n)} - \mathbf{A}^{(n)})}}{(2\pi)^3} \times \tilde{\Psi}_{A,p}^+(\mathbf{q}) \tilde{\Psi}_{A,p}^+(\mathbf{q}) \hat{\phi}(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{r}},$$

where we used $\alpha = -\beta / \omega$. Now, $\tilde{\Psi}_{A,p}^+(\mathbf{q})$ is the Fourier transform of the Coulomb scattering state, which, as a limiting process, can be evaluated as a Nordsieck integral [14], thus, yielding

$$\tilde{\Psi}_{A,p}^+(\mathbf{q}) = -\frac{\Gamma^2(1 - i\alpha)e^{e\alpha/2}}{2\pi^2} \lim_{\lambda \to 0} \frac{\partial}{\partial \lambda} \int_{-\infty}^{\infty} \left[ \frac{\mathbf{q}^\prime - (\mathbf{p} - i\lambda)^2}{(\mathbf{p} - i\lambda)^2 + \lambda^2} \right]^{i\alpha}$$

where $\alpha$ is the minimum number of photons required for threshold ionization. Since $dW/dp_i$ is not an even function of $\mathbf{p}_i$, then $\int_{-\infty}^{\infty} dp_i d\mathbf{p}_i \neq 0$, implying a net longitudinal momentum transfer (see Fig. 1).

For the case of ionization by a linearly polarized laser, the Bessel function in Eq. (13) is replaced by a generalized Bessel function. This implies that linearly and circularly polarized pulses will transfer very different longitudinal momenta to the photoelectron. (In circular polarization, photoelectrons are
more energetic, $E_p > U_p$, than those of linear polarization where $E_p$ is in the vicinity of the threshold.)

In the following, we confirm that the net longitudinal momentum transferred $p_z = E_p/c$. From Eq. (12), the ionization rate per unit energy to detect an electron, at constant intensity, is

$$
\frac{d\bar{W}(I)}{dE_p} = 32\pi^5 \sum_{n=0}^{\infty} \frac{\delta(E_T - \Delta(n))\sqrt{2E_p}}{\sqrt{Z}} \int_0^\pi d\theta \sin \theta \Delta(n)^2 \frac{J_n^2(2\sqrt{\bar{U}_p E_p} \sin \theta)}{\frac{I}{Z^2} + 2E_p + \frac{1}{c^2}\Delta(n)^2 - \frac{2}{c^2}\Delta(n)^2 \cos \theta},
$$

(14)

where $\theta$ is the angle between $p$ and $k$ and $U_p = \frac{I}{Z^2}$. For a Gaussian pulse envelope, as in the case of the experiment of Smeenk et al. [1], the intensity distribution within the focal volume is [15]

$$
\frac{I(\rho, z)}{I_0} = \left( \frac{\omega_0}{\omega(z)} \right)^2 \exp \left[ -4 \ln(2)(\varepsilon - c t)^2 / (c^2 \tau^2) \right] - 2\rho^2 / (\omega(z)^2),
$$

(15)

where $\omega(z) = \omega_0 [1 + (z/\zeta_0)^2]^{1/2}$, $\omega_0 = \sqrt{\kappa_0} \zeta_0 / \pi$ is the Rayleigh range, and $\tau$ is the temporal full width at half maximum in intensity. The focal volume consists of surfaces of constant intensity. The differential volume element is a shell between two surfaces of constant intensity $I$ and $I + dI$. Atoms lying within a given differential volume element are ionized at a constant intensity $I$. Then, within the focal volume, the total number of electrons generated per unit of energy is

$$
\frac{dN}{dE_p} = 2\pi \rho_d \zeta_0 \omega_0^2 \int_0^I \frac{dI}{I^{5/2}} \sqrt{I_0 - I} \frac{d\bar{W}(I)}{dE_p} \times \int_1^1 d\eta [I + (I_0 - I)\eta^2] \left[ \ln \frac{I_0}{I + (I_0 - I)\eta^2} \right]^{1/2},
$$

(16)

where $\rho_d$ is the atomic density. If $N_t = \int_0^{\infty} dE_p \frac{dN}{dE_p}$ denotes the total number of electrons with energy $E_p \geq 0$, then the average kinetic energy of electrons is

$$
\bar{E}_p = \frac{1}{N_t} \int_0^{\infty} dE_p E_p \frac{dN}{dE_p} = \frac{\int_0^{\infty} dE_p E_p \frac{dN}{dE_p}}{\int_0^{\infty} dE_p \frac{dN}{dE_p}},
$$

(17)

where $\frac{dN}{dE_p}$ is given by Eqs. (16) and (14).

In Fig. 2, the results of the theoretical calculations for $p_z$ versus the average intensity at ionization $I_{av}$ are superimposed on the experimental data of Smeenk et al. [1]. The values used for the binding energy are 21.5645 eV for Ne and 15.7596 eV for Ar. The dashed curves are the results of the calculations without focal averaging. In this case, it is clear that the dashed curves are well above the experimental data. This implies that the Lorentz force due to a constant pulse envelope alone does not account for the experimental data. The solid curves are the results of the calculations, including the focal volume averaging.

The agreement between theory and experiment is satisfactory in the case of Ar at 1400 nm (upper curve) and Ne at 800 nm (middle curve), but the experimental points fall progressively below theory for Ar at 800 nm (lower curve), especially in the intensity range of $1 \times 10^{14}$ to $1.5 \times 10^{14}$ W/cm$^2$ where the experimental data exhibit a negative $p_z$. Thus, we conclude from the first two comparisons that the focal volume averaging accounts for the Lorentz force due to the gradient of the pulse envelope, producing a negative force counteracting the force due to a constant envelope. We attribute the poor agreement in the case of Ar(800) to Coulomb effects, which are neglected in the numerical calculations. As discussed by Reiss [7], Coulomb effects can be neglected provided that the intensity parameter $z_1 = 2U_p/E_0$ satisfies the condition $z_1 > 1$. The actual values at the lowest intensity $I = 1 \times 10^{14}$ W/cm$^2$ in Fig. 2 are $z_1 = 2.322$ for Ar(1400) where agreement is good but $z_1 = 0.758$ for Ar(800) where agreement is poor. For Ne(800), $z_1 = 1.66$ at the lowest intensity.
measured intensity point of $3 \times 10^{14}$ W/cm$^2$ in harmony with satisfactory agreement at higher intensities. However, $z_1$ would decrease linearly with intensity to only $z_1 = 0.553$ at $I = 1 \times 10^{14}$ W/cm$^2$. One would, therefore, expect to see similarly significant deviations for Ne(800) at low intensities. There may be some evidence for this at the low-intensity end in Fig. 2. Further measurements for Ne(800) in the low-intensity range would be very useful.

The results for Ar(800), thus, demonstrate the influence of Coulomb effects at low intensities and especially, the negative values of $p_z$. The inclusion of Coulomb effects in the numerical calculations would account for the attractive Coulomb force along the $z$ direction and would explain negative values of $p_z$. Similar to the low-energy structures recently observed in ionization by linearly polarized light [9], we believe that the experimental data for Ar(800) at low intensities provide evidence of Coulomb effects in ionization by circularly polarized light. This will be investigated further in a future paper.

In conclusion, we have presented a quantum formulation for the transfer of longitudinal momentum in the ionization process. Nondipole effects account for the Lorentz magnetic force due to a constant envelope, thus, giving rise to the radiation pressure, which is the essential mechanism for the transfer of longitudinal momentum. However, this by itself is not sufficient to account for the experimental data. At high intensities, the combined effect of both the nondipole corrections and the pulse envelope accurately account for the experimental data. In this limit, we have demonstrated the validity of the assertion that $p_z = E_p/c$. At low intensities, where Coulomb effects cannot be ignored, $p_z$ is evidently reduced by the work performed against the attractive Coulomb force along the $z$ axis and may even become negative.

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