A MEASUREMENT OF THE SINGLE POSITIVE HELIUM ION(DOUBLET-2S(1/2)-DOUBLET-2P(1/2)) LAMB SHIFT.

JAGDISH S. PATEL
University of Windsor

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A MEASUREMENT OF THE $\text{He}^+ (2^2S_{1/2} - 2^2P_{1/2})$ LAMB SHIFT

by

Jagdish S. Patel

A Dissertation submitted to the Faculty of Graduate Studies and Research through the Department of Physics in partial fulfilment of the requirements for the Degree of Doctor of Philosophy at the University of Windsor

Windsor, Ontario, Canada

1986
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ABSTRACT

A beam of unpolarized metastable He$^+$ $2s_{1/2}$ ions, quenched by a static electric field, $E$, has an anisotropy in the emitted radiation that can be used to determine the He$^+$ $2s_{1/2}-2p_{1/2}$ Lamb shift. A high precision measurement of this anisotropy for a field strength of $|E| = (370.97 \pm 0.06)$ V/cm has been made. The value of the field-free anisotropy, $A^{(0)}$, adjusted by small corrections is $(0.1178082 \pm 0.0000086)$, where the uncertainty is the statistical one standard deviation. Using much improved apparatus over Drake, Goldman and van Wijngaarden, our values for both the anisotropy $A^{(0)}$ and the Lamb shift, $S = (14022.67 \pm 1.06)$ MHz lie lower by 17 standard deviations from theirs. The great discrepancy is not understood, and may be attributed to the inherent difficulties of measuring light intensities by photon counting techniques.
To my Parents
ACKNOWLEDGEMENTS

My thanks and appreciation go to Dr. A. van Wijngaarden for his supervision during this work, and to Dr. G. W. F. Drake for his guidance in the theoretical matters. In addition to the present research, I was fortunate to be a part of their team on several other projects. The respective expertise of Mr. Bernard Massa in the Electronics shop and Mr. Werner Grewe and staff in the Machine shop is also appreciated.

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CHAPTER I
INTRODUCTION

In the one-electron Dirac theory, energy levels are degenerate for those states having the same main quantum number $n$, and total angular momentum quantum number $j$, but different orbital angular momentum quantum number $l$. The existence of an energy displacement between the $2s_{1/2}$ and $2p_{1/2}$ levels (in atomic hydrogen) was first shown experimentally by Lamb and Retherford (1947). This energy separation, called the Lamb shift, is explained by the theory of quantum electrodynamics (QED). In a hydrogenic system, the Lamb shift is the energy difference between two states having the same $n$ and $j$, but different $l=|j+1/2|$. For the 1s ground state, it is the difference between the shifted and un-shifted (non-QED) energy.

The Lamb shift is a result of the interaction of the atomic electron with its own (virtual) radiation field. The two largest contributions to the Lamb shift are the electron self energy and vacuum polarization, illustrated by the corresponding Feynman diagrams in Fig. 1.1. The electron self energy arises from the emission and re-absorption of virtual photons, and can be pictured as an interaction of the electron with the zero point oscillations or vacuum fluctuations of the quantized electromagnetic field (Welton, 1948). Hence, any bound electron undergoes an additional random “oscillation in space” (zitterbewegung) due to recoil processes, not
FIG. 1.1. Feynman diagrams for the (a) self energy and (b) vacuum polarization in one-electron atoms. The double lines represent a bound electron in the Coulomb field of the nucleus.

predicted by Dirac theory. This interaction causes a smearing of the electron charge over its mean square orbital radius (Bjorken and Drell, 1964) given by

\[ \langle (\delta r)^2 \rangle = 2\alpha^3 a^2 / \pi \ln(Z\alpha)^{-1} \]

Here, \( a = \hbar^2 / me^2 \) is the Bohr radius, \( \alpha = e^2 / \hbar c \) is the fine structure constant, and \( Z \) is the nuclear charge. For \( Z = 1 \) the numerical value of the root mean square smearing is \( \delta r = 5.8 \times 10^{-12} \text{ cm} \), which is small compared to the Bohr radius of \( 5.29 \times 10^{-9} \text{ cm} \). The contribution arising from the charge smearing is an upward energy shift for s-states of Drake, 1982

\[ \Delta E_n = (2\alpha^4 a^3 / 3\pi n^3) \ln(Z\alpha)^{-1} R_\infty \]  

(1.1)

where \( R_\infty = e^2 / 2a \) is the Rydberg energy for infinite nuclear mass. Equation 1.1 for \( n=2 \) and \( Z=1 \) then gives a frequency shift of \( \approx 1000 \text{ MHz} \). An electron in the s state spends much of its time near the origin, where the Coulomb potential
diverges, and is therefore affected more significantly than a p state electron that stays mostly in potentials of much smaller gradients.

The energy correction from vacuum polarization arises from the polarization of the virtual electron-positron pairs in the electric field of the nuclear charge. This results in a small downward energy shift given by (Drake, 1982)

$$\Delta E_{\text{VP}} \propto -\left(\alpha^3 Z^4 / 15\pi\right) R_\mu$$

(1.2)

where $R_\mu$ is the Rydberg constant for a particle of reduced mass $\mu$. For $Z=1$, this correction is $\alpha \approx 27$ MHz. Calculations of the precise Lamb shifts from the QED formalism are complex, and involve the evaluation of higher order terms in $(Z\alpha)$, given as a double expansion in $\alpha$ and $Z\alpha$, each term corresponding to a group of Feynman diagrams (reviews are presented in Taylor et. al., 1969, Lautrup et al., 1972, Brodsky and Mohr, 1978, and Drake, 1982). We can write the radiative contributions for an electron with quantum numbers $n,l,j$ in the form (Drake, 1982)

$$\Delta E = \frac{8 \alpha Z^4}{3 \pi^3 \hbar c} \left[ A_{40} + A_{41} \ln(Z\alpha)^{-2} + A_{50}(Z\alpha) ight. \
+ (Z\alpha)^2 \left( A_{62} \ln(Z\alpha)^{-2} + A_{61} \ln(Z\alpha)^{-2} + G(Z\alpha) \right) \
+ (\alpha/\pi) \left( B_{40} + O(Z\alpha) \right) + O(\alpha^2/\pi^2) \bigg] + \Delta E_M + \Delta E_R$$

(1.3)

where

$$G(Z\alpha) = A_{60} + O(Z\alpha)$$

(1.4)

and where the constants $A_{mn}$ have individual parts arising from electron self energy, vacuum polarization and anomalous
magnetic moment:

\[ A_{an} = A_{an}^{SE} + A_{an}^{VP} + A_{an}^{MN} \]

The terms \( A_{an}^{SE} \) and \( A_{an}^{VP} \) are finite nuclear mass and nuclear size corrections. \( G(Zx) \) in equation (1.4) represents higher order binding energy corrections that are difficult to evaluate, and has been treated by two different approaches by Erickson (1971, 1977) and Mohr (1976), with different end results. Their values for the Lamb shift together form the usual theoretical comparisons to experimental values. A subsequent calculation of \( A_{60} \) in \( G(Zx) \) for the \( 1s_{1/2} \) state by Sapirstein (1981) is in agreement with Mohr. It has been suggested (Borie, 1981) that a further contribution from finite nuclear size effects be included in the low order level shift of \( ns_{1/2} \) states, but that is still in question (Lepage et al., 1981).

Erickson's values lie higher than Mohr's, and these along with all precision measurements to date in hydrogenic systems are listed in Table 1.1 and illustrated by Fig. 1.2. Since the difference between the Erickson and Mohr values varies approximately as \( Z^6 \), the vertical scale has been scaled by \( Z^{-6} \), and the horizontal axis represents the average of their values for each ion.

Experiments provide the essential tests for the theories and have also progressed from the early radio frequency (rf) resonance measurement by Lamb and Retherford in H. Combined with other techniques, more sophisticated and accurate results in ions up to \( Z = 18 \) have been obtained (see Table 1.1;
TABLE 1.1. Experimental and theoretical values of the Lamb shift in hydrogenic ions (adapted from Drake, 1982).

<table>
<thead>
<tr>
<th>Ion</th>
<th>Reference</th>
<th>Technique</th>
<th>Value</th>
<th>Theory</th>
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<tr>
<td></td>
<td></td>
<td></td>
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<td>37910(380)</td>
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</table>

bThese values are as quoted in Drake (1982), except for Ne^14, where it is given in reference (18).cThese reference numbers correspond with those of Fig. 1.2.dThis measurement is also shown in Fig. 1.2, but has not been mentioned in some recent literature.eThe two experimental values are based on one experimental measurement of the 2p_{1/2}-2p_{3/2} energy splitting and assuming the two calculated values of the fine structure splitting by Erickson (above) and Mohr (below).
FIG. 1.2. Comparison between scaled theoretical and experimental Lamb shifts in hydrogenic ions, expressed as deviations from the theoretical mean of Erickson (1977) and Mohr (1976) values (adapted from Drake, 1982). Erickson's and Mohr's values are the upper and lower lines respectively for each ion, while the dashed lines are the Borie (1981) corrections to both theories (see text). A number below every experimental value refers to the entries of Table 1.1.
A summary of experimental results is given in Drake, 1982, and Pellegrin et al., 1985). The present experimental accuracy for the Lamb shift in hydrogen by microwave resonance (Lundeen and Pipkin, 1981) is sufficient to distinguish between the two theoretical values, and as can be seen in Fig. 1.2, it disagrees with both. A large contributor to the theoretical uncertainty for the H Lamb shift is from the experimental value for the proton size in the finite nuclear size correction. In comparison, the radius of the helium nucleus is known much more accurately (Carboni et al., 1977, Borie et al., 1978), making He a more important candidate for testing theory. At higher Z, the measurement of Gould and Marrus (1983) in Ar \(^{17+}\), despite its lower precision than H, is equally significant for the following reason. Uncertainties in the theoretical values for the Lamb shift come from 1) the term G(Z\(\alpha\)) of higher order binding corrections, and 2) as stated above, from nuclear size corrections. The first varies as \(Z^6\), while the second as \(Z^4\), and since the Lamb shift is also roughly proportional to \(Z^4\), the experimental precision required for a test of the G(Z\(\alpha\)) term varies as roughly \(\pm 10Z^2\) ppm (Drake, 1982).

In He, the present accuracy for the n=2 Lamb shift is only about 100 ppm, and needs to be improved to be a test for QED. Two of the three previous experiments (see Table 1.1) have used the method of microwave resonance. A fixed rf field is used while the \(2s_{1/2}-2p_{1/2}\) transition is brought into resonance by Zeeman shifting. In rf resonance experiments,
since the resonance width of 99.724 MHz is about 1/10th of the Lamb shift being measured (Drake, 1982), a major source of error is in determining the peak centre with sufficient accuracy.

An alternative method is used here, as in the experiment of Drake, Goldman and van Wijngaarden (1979), which in itself provides an independent check on the rf resonance experiments. Our aim is to improve the accuracy of the previous 206 ppm measurement by a factor of ≈ 4. The Lamb shift is derived in terms of the measured anisotropy in the electric-field induced quenching radiation of metastable 2s_{1/2} He^+ ions, described in the next chapter. The Stark-induced Ly-α intensities I_∥ and I_⊥ emitted in directions parallel and perpendicular to the applied electric field direction have an anisotropy defined by A = (I_∥ - I_⊥)/(I_∥ + I_⊥). This anisotropy which is roughly proportional to the Lamb shift, can be calculated as a function of the Lamb shift and the accurately known fine structure splitting 2P_{1/2} - 2P_{3/2} separation (Drake, 1986 and references therein). The Lamb shift is then deduced from a measured value of the anisotropy A.
CHAPTER II

THEORY

II.A Theory of the quenching radiation asymmetries

The electric field quenching of the metastable $2s_{1/2}$ state of one-electron systems can be described by the phenomenological Bethe-Lamb quenching theory (Lamb and Retherford, 1950, 1952), and is derived from the Wigner-Weisskopf (1930) analysis for time-dependent perturbations. Following is a summary of Drake's theory (1986) pertaining to the Lamb shift.

He begins with the time-dependent Schrödinger equation for an atom in an external field

$$i\hbar \frac{d\hat{a}}{dt} = \hat{H}(t)\hat{a}$$

(2.1)

$$\hat{H}(t) = E + F(t)\hat{V}$$

(2.2)

where $\hat{a}$ represents a column vector of state amplitudes, usually belonging to a finite basis set of states. $E$ is the diagonal matrix of field-free eigenvalues, $\hat{V}$ is the interaction matrix with the external field, and $F(t)$ describes its time dependence. In the phenomenological quenching theory, the field-free eigenvalues $E_j$ are replaced by $E_j - i\tau_j/2$, where the $\tau_j$ are the field free level widths. Then, in the absence of perturbations, the state amplitudes decay independently of one another according to
\[ |a_j(t)|^2 = |a_j(0)|^2 e^{-\Gamma_j t/\hbar} \]  

as expected. From quantum electrodynamics, it has been shown by Kelsey and Hacek (1977) and Hillery and Mohr (1980) that the Bethe-Lamb phenomenological formalism has a rigorous foundation at least to lowest relative order in \( \alpha/\pi \), where \( \alpha = e^2/\hbar c \) is the fine structure constant. It is therefore consistent to use relativistic wave functions for the evaluation of the matrix elements, together with relativistic eigenvalues that include the Lamb shift and imaginary level widths. Before finding the solutions to 2.1, we first review the theory of spontaneous transitions.

For a radiative transition, the probability in unit time for a transition between initial and final states is given by Fermi's golden rule:

\[ w = 2\pi/\hbar \langle f|V_{\text{int}}|i\rangle^2 \rho_f \]  

where \( V_{\text{int}} \) is the interaction energy operator and \( \rho_f \) is the number of final states per unit energy interval. For the emission of a photon of frequency \( \omega \), polarization \( \hat{e} \) and propagation vector \( \hat{k} \) \((|\hat{k}| = \omega/c\)\), the terms in (2.4) are

\[ \rho_f = k^2 d\Omega/(2\pi)^3 \hbar c \]  

and

\[ V_{\text{int}} = e \hat{\alpha} \cdot \hat{A}^* \]  

where \( \rho_f \) is the number of photon states of polarization \( \hat{e} \) per unit energy and solid angle in the arbitrary normalisation volume \( V \). The wave functions in (2.4) are assumed to be 4-component Dirac spinors and \( \hat{\alpha} \) is the usual 4x4 Dirac matrix. The photon vector potential \( \hat{A} \), normalized
to a field energy of $E_\Phi$ per unit volume is given by

$$\Phi = \frac{1}{k} \left( \frac{2\pi \hbar}{\nu} \right)^{1/2} e^{-i k \cdot r}.$$  \hspace{1cm} (2.7)

In unit time, $\Phi$ becomes

$$\Phi = \frac{e^{2k}}{2\pi \hbar} |\langle f | \alpha \cdot e^{-i k \cdot r} | i \rangle|^2 d\Omega$$ \hspace{1cm} (2.8)

When (2.8) is applied to the $2s_{1/2}$ state of a hydrogen atom, one finds that the electric dipole (E1) transitions to the $1s_{1/2}$ ground state are strictly forbidden by the parity selection rule. As illustrated by Fig. 2.1, the spontaneous magnetic dipole (M1) transitions are allowed when relativistic and retardation corrections are taken into account, but the dominant decay mode in the absence of an external field is the two photon electric dipole (2E1) transition to the ground state (see Drake, 1986).

In the presence of a small electric field, however, field-induced mixing of the $2s_{1/2}$ state with states of opposite parity, primarily $2p_{1/2}$ and $2p_{3/2}$ takes place, thus allowing both E1 and M2 decay modes. Both field-induced and spontaneous decay processes can be shown by expanding the photon vector potential $\Phi$ into electric and magnetic multipoles (see, for example, Akhiezer and Berestetskii, 1965). Retaining only the E1, M1 and M2 contributions,

$$\Phi = (3/8\pi)^{1/2} \sum_M \left( e_M \tilde{a}^{(1)}_M + i \langle k \cdot e \rangle_M \tilde{a}^{(0)}_M \right)$$

$$+ i (10/3)^{1/2} \left[ k \cdot e \tilde{a}^{(0)}_2, M \tilde{a}^{(0)*}_2, M \right]$$ \hspace{1cm} (2.9)

where $e_M$ denote the irreducible tensor components of the polarization vector.
FIG. 2.1. Energy levels of a hydrogenic atom for \( n=1 \) and \( n=2 \), showing the spontaneous and field induced decay modes from the \( 2s_{1/2} \) state. The dashed lines indicate mixing with the \( 2p_{1/2} \) and \( 2p_{3/2} \) states by an external field, leading to field induced E1 (electric dipole), and M2 (magnetic dipole) decay modes for the \( 2s_{1/2} \) state. Cross terms among all four single-photon decay modes produce interference effects and quantum beats.
\[ e_{\pm 1} = \pm \frac{1}{\sqrt{2}} (e_x \pm e_y), \quad e_0 = e_z \]  
\[ (2.10) \]

The \( \hat{a}_{LM} \) are the standard operators for electric multipole \( (\lambda = 1) \) and magnetic multipole \( (\lambda = 0) \) transitions given in the Coulomb gauge by

\[ \hat{a}_{LM} = \left( \frac{L}{2L+1} \right)^{1/2} \frac{g_{L+1}(kr)}{\sqrt{2L+1}} \hat{Y}_{LL+1M} + \left( \frac{L+1}{2L+1} \right)^{1/2} \frac{g_{L-1}(kr)}{\sqrt{2L+1}} \hat{Y}_{LL-1M} \]  
\[ (2.11) \]

and

\[ \hat{a}_{LM} = g_{L}(kr) \hat{Y}_{LLM} \]  
\[ (2.12) \]

where \( \hat{Y}_{LM} \) is a vector spherical harmonic (Edmonds, 1960), and the radial function \( g_{L}(kr) \) is given in terms of a spherical Bessel function \( j_{L}(kr) \):

\[ g_{L}(kr) = 4\pi i^{L} j_{L}(kr) \]  
\[ (2.13) \]

\[ j_{L}(z) = \frac{z^{L}}{(2L+1)!} \left(1 - \frac{z^{2}/2}{11(2L+3)} + \frac{(z^{2}/2)^{2}}{21(2L+3)(2L+5)} + \ldots \right) \]  
\[ (2.14) \]

Since \( kr = \omega r/c \), only the leading one or two terms of (2.14) normally need to be retained for low \( Z \) atoms (the long wavelength approximation). The notation \([a,b]_{2M}\) in (2.9) denotes the vector coupled product

\[ [a,b]_{2M} = \sum_{m_{1}, m_{2}} \langle 1m_{1} 1m_{2} | 2M \rangle a_{m_{1}} b_{m_{2}} \]  
\[ (2.15) \]

The first and the last terms of (2.9) representing the E1 and M2 modes are forbidden by parity, and (in the case of M2), triangular selection rules. In the presence of an electric field, both these terms contribute by field induced mixing of the s and p states. For low \( Z \) atoms, such as \( \text{H} \) or \( \text{He}^+ \), in field strengths of up to several \( \text{kV/cm} \), significant mixing only occurs in the manifold of states \( 2s_{1/2}, 2p_{1/2} \) and \( 2p_{3/2} \).
The analysis is further complicated if an atom has hyperfine structure (see Drake, 1986). In our case, \( \text{He}^+ \), there is no hyperfine structure, and assuming also that the electric field \( \vec{E} \) is switched on adiabatically, we write the perturbed 2s\(_{1/2} \) initial state as

\[
\psi(2s_{1/2}, m) = a(\vec{E}) \psi_0(2s_{1/2}, m) \\
+ \sum_m (b^{(1/2)}_{m,s, m'} \psi_0(2p_{1/2}, m') + b^{(3/2)}_{m,s, m'} \psi_0(2p_{3/2}, m'))
\]

(2.16)

where the matrices \( b^{(j)} \) (\( j = 1/2, 3/2 \)) are given by

\[
b^{(1/2)} = b^{(1/2)}(\vec{E}) \delta_{s, \vec{E}}
\]

(2.17)

and

\[
b^{(3/2)} = b^{(3/2)}(\vec{E}) \begin{pmatrix}
-r^3 E_{-1} & r^2 E_0 & -E_1 & 0 \\
0 & -E_1 & r^2 E_0 & -r^3 E_1
\end{pmatrix}
\]

(2.18)

and the \( \hat{E}_q \) (\( q = 0, \pm 1 \)) are the irreducible tensor components of the unit vector \( \vec{E} \) in the electric field direction as defined by (2.10). Using the Wigner-Eckart theorem, the coefficients of \( \hat{E}_q \) in (2.17) and (2.18) are

\[
\langle 2p_j, m, 1 | \hat{E}_q r | 2s_{1/2}, m \rangle = (-1)^{j-m} \begin{pmatrix} j & 1 & 1/2 \\ -m' & -q & m \end{pmatrix} \hat{E}_q \langle 2p_j | r | 12s_{1/2} \rangle.
\]

(2.19)

The upper and lower rows of the matrix in (2.18) correspond to \( m = 1/2 \), and \(-1/2 \) respectively, while \( m' \) ranges from \( m' = 3/2 \) to \(-3/2 \) across the rows.

Under external fields, the energies of the 2s\(_{1/2}, \pm 1/2 \)
states remain degenerate, since the perturbation operator $eE \cdot \vec{r}$ is an operator of odd parity. The energies are also independent of the field orientation, so that the forms of equations (2.17) and (2.18) remain valid to all orders of perturbation theory. The overall multiplying factors $a(|\vec{E}|)$, $b_{1/2}(|\vec{E}|)$ and $b_{3/2}(|\vec{E}|)$ have an explicit dependence on the field strength, and to lowest order in field strength, they are given by

$$a = 1 + O(|\vec{E}|^2)$$  \hspace{1cm} (2.20)

$$b_{1/2} = \frac{e|\vec{E}|<2p_{1/2} \mid \vec{r} \mid 12s_{1/2}>}{\sqrt{6} \left( A_L + i \Gamma/2 \right)} + O(|\vec{E}|^3)$$ \hspace{1cm} (2.21)

$$b_{3/2} = \frac{e|\vec{E}|<2p_{3/2} \mid \vec{r} \mid 12s_{1/2}>}{\sqrt{12} \left( A_F + i \Gamma/2 \right)} + O(|\vec{E}|^3)$$ \hspace{1cm} (2.22)

where $A_L = E(2s_{1/2}) - E(2p_{1/2})$ is the Lamb shift and $A_F = E(2s_{1/2}) - E(2p_{3/2})$ is the Lamb shift minus the fine structure splitting. $\Gamma$ is the level width of the $2p$ state.

The properties of the quenching radiation are determined by the matrix elements between the unperturbed $1s_{1/2}$ final state and the perturbed $2s_{1/2}$ initial state (Eq. 2.16):

$$A_{m,m'} = <1s_{1/2},m \mid e^{i \vec{k} \cdot \vec{r}} e^{-i \vec{k} \cdot \vec{r}} \mid 12s_{1/2},m'>$$ \hspace{1cm} (2.23)

Using the expansion for $e^{i \vec{k} \cdot \vec{r}}$ (Eq. 2.9), and applying the Wigner-Eckart theorem to express the matrix elements of $\alpha\lambda a_{LM}$ in terms of reduced matrix elements, the $2 \times 2$ transition matrix $A$ with elements $A_{m,m'}$, becomes
\[ A = V_+ \hat{e} \cdot \hat{E} \pm c \cdot \left( 1V_- (\hat{e} \times \hat{E}) + M(\hat{k} \times \hat{e}) \right) \]  
(2.24)

where

\[ V_+ = V_{1/2} + 2V_{3/2} \]  
(2.25a)

\[ V_- = V_{1/2} - V_{3/2} + M_{3/2} \]  
(2.25b)

\[ M = M_{1/2} + 21(\hat{k} \cdot \hat{E})M_{3/2} \]  
(2.25c)

and

\[ V_{1/2} = \frac{-b_{1/2}}{4 \times 1/2} \langle 1s_{1/2} | \hat{a} \hat{d}_{1}^{+} | 1 \rangle | 2p_{1/2} \rangle \]  
(2.26a)

\[ V_{3/2} = \frac{-b_{3/2}}{4(2\pi)^{1/2}} \langle 1s_{1/2} | \hat{a} \hat{d}_{1}^{+} \rangle | 2p_{3/2} \rangle \]  
(2.26b)

\[ M_{1/2} = \frac{-ia}{4 \times 1/2} \langle 1s_{1/2} | \hat{a} \hat{d}_{2}^{+} \rangle | 2s_{1/2} \rangle \]  
(2.26c)

\[ M_{3/2} = \frac{-b_{3/2}}{4(2\pi)^{1/2}} \langle 1s_{1/2} | \hat{a} \hat{d}_{2}^{+} \rangle | 2p_{3/2} \rangle \]  
(2.26d)

The values for the reduced matrix elements in (2.21), (2.22) and (2.26), including the leading relativistic corrections are summarised in Table 2.1. The \( V_{1/2} \) and \( V_{3/2} \) terms represent the amplitudes for the electric field quenching (with the emission of an E1 photon) of the \( 2s_{1/2} \) state via the admixture of \( 2p_{1/2} \) and \( 2p_{3/2} \) intermediate states respectively. The \( M_{1/2} \) term is the amplitude for spontaneous M1 transitions, which to a first approximation is independent of field strength, while the \( M_{3/2} \) term is a small M2 correction. The combinations \( V_+ \) and the E1 part of \( V_- \) come from transitions with \( \Delta m = 0 \) and \( \Delta m = \pm 1 \) respectively.

In addition to the \( \hat{e} \) and \( \hat{k} \) vectors describing the photon polarization and direction of the emitted photons, the quench-
TABLE 2.1. Values of matrix elements for fine structure transitions in hydrogenic ions* (from Drake, 1986).

<table>
<thead>
<tr>
<th>Matrix Element</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt;2p_{1/2}</td>
<td></td>
</tr>
<tr>
<td>( &lt;2p_{3/2}</td>
<td></td>
</tr>
<tr>
<td>( &lt;1s_{1/2}</td>
<td></td>
</tr>
<tr>
<td>( &lt;1s_{1/2}</td>
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</tr>
<tr>
<td>( &lt;2s_{1/2}</td>
<td>z</td>
</tr>
<tr>
<td>( &lt;2s_{1/2}</td>
<td>z</td>
</tr>
<tr>
<td>( &lt;1s</td>
<td>z</td>
</tr>
<tr>
<td>( &lt;2p</td>
<td>z</td>
</tr>
</tbody>
</table>

* The \( \hat{a}_l^{(1)}(\omega r / a) \) are evaluated at \( \hbar \omega = E(2s_{1/2}) - E(1s_{1/2}) \), and the reduced matrix elements are defined in terms of 3-j symbols by

\[
\frac{n'1'j'm'}{\sqrt{\mathcal{L}_m}} \left| \hat{a}_l^{(\lambda)} \right| n_{ijm} = (-1)^{j'-m'} \left( \begin{array}{ccc} j' & L & j \\ m' & M & m \end{array} \right) \begin{array}{c} n'1'j'm' \end{array}
\]
ing radiation also depends on the electron-spin polarization of the initial $2s_{1/2}$ state. We specify this by the density matrix

$$\rho = \frac{1}{2} (1 + \sigma \cdot \vec{P})$$  \hspace{1cm} (2.27)

where $\vec{P}$ is the polarization vector, whose magnitude is 0 for an incoherent source and 1 for maximum polarization. The decay rate per solid angle, summed over final atomic states and averaged over initial states is then

$$w d\Omega = \frac{e^{2k}}{2\pi \hbar} \text{Tr} [\rho A^+ A] d\Omega$$  \hspace{1cm} (2.28)

The terms in (2.28) are multiplied out by making repeated use of the identity

$$(\sigma \cdot a)(\sigma \cdot b) = a \cdot b + i \sigma \cdot a \times b$$  \hspace{1cm} (2.29a)

together with

$$\text{Tr}(\sigma) = 0.$$  \hspace{1cm} (2.29b)

Before obtaining the final results, we choose two orthogonal photon polarization vectors, $\hat{e}_1$ and $\hat{e}_2$, both perpendicular to $\hat{k}$ so that in general, elliptical polarization is given by

$$\hat{e} = \cos \beta \hat{e}_1 + i \sin \beta \hat{e}_2.$$  \hspace{1cm} (2.30)

Hence, $\beta = 0, \pi/2, \ldots$ corresponds to linearly polarized light, and $\beta = \pi/4, 3\pi/4, \ldots$ corresponds to circularly polarized light. Then, Eq. (2.28) becomes

$$w(\hat{e}, \hat{k}, \vec{P}) d\Omega = \frac{e^{2k}}{2\pi \hbar} \left( I_0 + \vec{P} \cdot \vec{J}_0 + \vec{P} \cdot \vec{J}_1 \sin 2\beta ight.$$

$$+ \hat{E}(\hat{e}_1 \cdot \hat{e}_1 - \hat{e}_2 \cdot \hat{e}_2) \vec{J}_2 \cos 2\beta \left.) \right) d\Omega$$  \hspace{1cm} (2.31)

where
\[ I_0 = \frac{\mathbf{k} \times \mathbf{E}}{4} \left[ 1 - (\mathbf{E} \cdot \mathbf{k})^2 \right] + \frac{\mathbf{k} \times \mathbf{E}}{4} \left[ 1 + (\mathbf{E} \cdot \mathbf{k})^2 \right] \]

\[ + 2 \text{Im}(\mathbf{V}_+ \mathbf{V}_-)(\mathbf{k} \times \mathbf{E}) + \mathbf{|E|^2} \]  

(2.32)

\[ \mathbf{J}_0 = (\mathbf{k} \times \mathbf{E}) \left( \text{Re}(\mathbf{V}_+^* + \mathbf{V}_-) - \text{Im}(\mathbf{V}_+^* \mathbf{V}_-)(\mathbf{k} \times \mathbf{E}) \right) \]  

(2.33)

\[ \mathbf{J}_1 = |\mathbf{V}_+^*|^2 (\mathbf{k} \times \mathbf{E}) \mathbf{k} \times \mathbf{E} + \text{Re}(\mathbf{V}_+^* \mathbf{V}_-) - \text{Im}(\mathbf{V}_+^* (\mathbf{V}_+ + \mathbf{V}_-)) \mathbf{k} \times \mathbf{E} \]

- \text{Im}(\mathbf{V}_+^* (\mathbf{V}_+ - \mathbf{V}_-)) (\mathbf{k} \times \mathbf{E}) \mathbf{k} - \mathbf{|E|^2} \mathbf{k} \]  

(2.34)

\[ \mathbf{J}_2 = \frac{\mathbf{V}_+^2 - |\mathbf{V}_-|^2}{4} \mathbf{k} \times \mathbf{E} + \text{Re}(\mathbf{V}_+^* (\mathbf{V}_+ - \mathbf{V}_-)) \mathbf{P} \times \mathbf{k} \]

+ \text{Im}(\mathbf{V}_+^* \mathbf{V}_-) \mathbf{E} \mathbf{k} \]  

(2.35)

The dyadic notation \( \mathbf{d} \cdot \mathbf{e} \) in (2.31) is defined by

\[ \mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c}) \mathbf{d} = (\mathbf{a} \cdot \mathbf{b}) \mathbf{c} \cdot \mathbf{d} \]  

Since \( \mathbf{H} \) is smaller than \( \mathbf{V}_+ \) by a factor of \( O(\alpha^2 Z^2) \), the dominant terms are those containing \( \mathbf{V}_+ \) and \( \mathbf{V}_- \). The term \( I_0 \) which is dealt with in this work is independent of the electron polarization. The angular distribution of radiation from \( I_0 \) is illustrated by the outer elliptical curve in Fig. 2.2. The term \( \text{Im}(\mathbf{V}_+^* \mathbf{V}_-) \mathbf{k} \times \mathbf{E} \) in \( \mathbf{J}_0 \) is sensitive to the imaginary level widths in the denominators of (2.21) and (2.22), producing the clover leaf distribution in Fig. 2.2 (Drake, Patel and van Wijngaarden, 1983). The remaining part of \( \mathbf{J}_0 \), \( \text{Re}(\mathbf{V}_+^* (\mathbf{V}_+ + \mathbf{V}_-)) \mathbf{P} \times \mathbf{E} \) produces the innermost pattern in Fig. 2.2. By measuring this term, the 2s - 1p magnetic dipole matrix element can be determined (van Wijngaarden, Patel and Drake, 1986).

In the electric dipole approximation, where only the \( \mathbf{V}_+ \) terms are retained, a general relationship exists between the
FIG. 2.2. Polar diagram (not to scale) of the contributions to the quenching radiation asymmetries produced by an electric field $\hat{E}$ acting on the $2s_{1/2}$ state. The outer elliptical shape is the main Lamb shift anisotropy for unpolarized atoms. The clover leaf pattern is the E1-E1 damping asymmetry for atoms polarized so that $\vec{B}$ points into the page, and the inner dashed curve corresponds to the E1-M1 interference asymmetry. The (+) and (-) signs indicate positive and negative additions to the total intensity.
polarization of the emitted radiation and the angular anisotropy. We define the z axis to be an axis of rotational symmetry by an electric field as shown in Fig. 2.3. The observed radiation intensities depend on the orientation of \( \hat{e} \) relative to the z-axis. Then the polarization in the x-direction is

\[
P = \frac{(I_y - I_z)/(I_y + I_z)}\]

The rotational anisotropy defined in terms of the total intensities observed in directions parallel (\( I_\parallel = I_x + I_y \)) and perpendicular (\( I_\perp = I_y + I_z \)) to the symmetry axis is

\[
A = \frac{I_\parallel - I_\perp}{I_\parallel + I_\perp} = \frac{I_x - I_z}{I_x + 2I_y + I_z} \quad (2.37)
\]

Since the z-axis is defined the symmetry axis, \( I_x = I_y \), hence

\[
P = 2A/(1 - A) \quad (2.38)
\]

II.B The Lamb shift anisotropy

The Lamb shift anisotropy is the radiation asymmetry predicted by Eq. (2.31), for an unpolarized beam \( |\vec{P}| = 0 \), and summed over photon polarization directions (in practice, with polarization insensitive detectors). An unpolarized beam may also be used provided the intensity emitted perpendicular to the electric field is taken as the average of the two intensities in opposite directions as shown in Fig. 2.2. This effectively eliminates the El-M1 interference term. Under these conditions, (2.31) reduces to

\[
w(\hat{k})d\alpha = \frac{e^2}{\kappa \hbar} I_0(\hat{k})d\alpha \quad (2.39)
\]

The \( \text{Im}_{1/2} I^2 \) term in \( I_0 \) (Eq. 2.32) is completely negligible for
FIG. 2.3. Illustration of the relationship between the polarization of quenching radiation in the x-direction and the anisotropy in the total intensity (summed over polarizations) in the x- and z-directions. The arrows signify photon polarization vectors. In the experiment, the observation direction, $\hat{k}$ is restricted to the x-z plane.
realistic quenching fields. When the measured intensities are averaged over the directions \( \hat{k} \) and \(-\hat{k}\), the \( \text{Im}(H^{*}_{1/2}V) \) term drops out. If the contributions from the \( H^{3/2} \) term due to the \( 2p_{3/2} \rightarrow ls_{1/2} \) magnetic quadrupole transition are neglected for now, and its effects calculated as a small correction later, then \( I_0 \) becomes

\[
I_0(k) \propto |V_{1/2}|^2 + \text{Re}(V^{*}_{1/2}V_{3/2})(1 - 3\cos^2\theta) \\
+ 1/2|V_{3/2}|^2(5 - 3\cos^2\theta)
\]

(2.40)

where \( \hat{k} \cdot \hat{E} = \cos\theta \). Introducing a dimensionless parameter \( \rho \),

\[
\rho = \frac{V_{3/2}}{V_{1/2}}
\]

(2.41)

and using the first order perturbation expressions (2.21) and (2.22) in the limit of weak electric fields, together with the nonrelativistic parts of the matrix elements in Table 2.1, we obtain

\[
\rho \propto \frac{\Delta L + 1\pi/2}{\Delta T + 1\pi/2}
\]

(2.42)

and

\[
I_0(k) = |V_{1/2}|^2(1 + \text{Re}(\rho)(1 - 3\cos^2\theta) + 1/2|\rho|^2(5 - 3\cos^2\theta))
\]

(2.43)

An approximate value for the anisotropy is then \( A \).

\[
A \approx [I_{||} - I_{\perp}]/[I_{||} + I_{\perp}] \propto A^{(0)}
\]

(2.44)

where

\[
A^{(0)} = \frac{-3\text{Re}(\rho) + 3/2|\rho|^2}{2 - \text{Re}(\rho) + 7/2|\rho|^2}
\]

(2.45)

We note that \( A^{(0)} \) is determined primarily by the ratio of
the Lamb shift to the \(2s_{1/2} - 2p_{3/2}\) energy difference. A measurement of \(A\), along with the accurate value for the fine structure splitting, which is to lowest order a known non-QED effect, allows us to measure the Lamb shift. A correction due to the effects of a finite electric field can be made by calculating higher order perturbation corrections for the mixing coefficients \(a, b_{1/2}\) and \(b_{3/2}\) (Eqs. 2.20 to 2.22). The result is a perturbation expansion for \(A\) of the form (Drake, Goldman and van Wijngaarden, 1979)

\[
A_0 = A^{(0)} + A^{(2)} \epsilon^2 + A^{(4)} \epsilon^4 + \ldots
\]

(2.46)

where \(\epsilon\) is the DC field strength in kV/cm. For ions having non-zero nuclear spin, although \(A^{(0)}\) remains well defined, the Stark shifts to the energy levels may not be small compared with the hyperfine structure splittings (at practical quenching field strengths), and the expansion (2.46) does not apply. For \(\text{He}^+\), Eq. (2.46) is accurate to \(\approx 1\) ppm for \(\epsilon < 1\) kV/cm, and the \(A^{(n)}\) coefficients are shown in Table 2.2.

In the following section, we describe small corrections to the asymmetry by:

1) Contributions from intermediate \(p\) states with \(n > 2\), and contributions from final-state perturbations,

2) Relativistic corrections to the matrix elements

3) Correction due to the \(2p_{3/2} - 1s_{1/2}\) magnetic quadrupole term, \(M_{3/2}\).

II.C Small corrections

The contribution from intermediate \(p\) states with \(n > 2\)
and perturbation corrections to the $1s_{1/2}$ state is described in terms of second-order electric dipole perturbation expressions analogous to $V_{1/2}$ and $V_{3/2}$ of the form (Drake and Grimley, 1973)

\[
B = \sum_{n=2}^{\infty} \frac{\langle 2s|z|np\rangle \langle np|z|1s\rangle}{E(2s) - E(np)}
\]

(2.47)

\[
C = \sum_{n=2}^{\infty} \frac{\langle 2s|z|np\rangle \langle np|z|1s\rangle}{E(1s) - E(np)}
\]

(2.48)

Here, $B$ represents the contributions from virtual transitions through high np states, and $C$, the contribution from final state perturbations. From the analysis of Drake and Grimley (1973), the fractional correction to the asymmetry $A_0$, neglecting level widths, and in the limit of weak fields is

\[
\left( \frac{\delta A}{A_0} \right)_{\text{B+C}} = 2(B+C) \frac{\Delta E_F}{N} \frac{(1 + 2\rho)}{(2 + \rho)} (1 + A_0)
\]

(2.49)

where

\[
\Delta E_F = E(2s_{1/2}) - E(2p_{3/2}) \quad \text{and}
\]

\[
N = \langle 1s|z|2p\rangle \langle 2p|z|1s\rangle.
\]

Using the values $B = 12.416 \ Z^{-4}$ a.u., $C = -3.476 \ Z^{-4}$ a.u., and $N = (-2^7 2^{16}/81)Z^{-2}$ a.u., and expressing $\Delta E_F$ in a.u., then

\[
\left( \frac{\delta A}{A_0} \right)_{\text{B+C}} = 8 \ \frac{\Delta E_F}{Z^2} \frac{(1 + 2\rho)}{(2 + \rho)} (1 + A_0).
\]

(2.50)

The numerical values are approximately $-6 \times 10^{-6} Z^2$ for $Z<20$.

An error of less than 1 ppm is introduced by neglecting the level widths and field dependent corrections to Eq. (2.50).

Relativistic corrections to the matrix elements enter by modifying the value of the dimensionless parameter $\rho$ defined
by (2.41). The correction to the approximate expression in
Eq. (2.42) is (Drake, 1986)

\[
\delta \rho / \rho = \mu_{3/2} + \mu_{3/2}' - \mu_{1/2} - \mu_{1/2}'
\]

where the \( \mu_j \) and \( \mu_j' \) are the fractional corrections of \( O(\alpha^2 Z^2) \)
to the matrix elements

\[
\langle 2p_j \mid r \mid 1s_{1/2} \rangle \; \text{and} \; \langle 1s_{1/2} \mid \alpha \dagger (1) \mid 1s_{1/2} \rangle
\]
shown in Table 2.1. The values are

\[
\mu_{3/2} = -1/6 \, (\alpha Z)^2 ,
\]

\[
\mu_{1/2} = -5/12 \, (\alpha Z)^2 ,
\]

\[
\mu_{3/2}' = (-11/48 - 5/4 \ln 2 + 3/4 \ln 3) \, (\alpha Z)^2 , \; \text{and}
\]

\[
\mu_{1/2}' = (-11/96 - 3/2 \ln 2 + \ln 3) \, (\alpha Z)^2
\]

The correction to the anisotropy, neglecting again the level
widths, is

\[
(\delta A/A_0)_{\text{rel}} = 2(\mu_{3/2} + \mu_{3/2}' - \mu_{1/2} - \mu_{1/2}') \left( \frac{1+2\rho}{2+\rho} \right) \left( \frac{1+A_0}{1-\rho} \right)
\]

\]

\[
(2.52)
\]

The numerical value is roughly \( 1.7 \times 10^{-6} Z^2 \) for \( Z < 20 \).

Next, we consider a correction arising from the \( 2s_{1/2} - 2p_{3/2} - 1s_{1/2} \) magnetic quadrupole decay term \( M_{3/2} \), first
pointed out by Hillery and Mohr (1980). The \( M_{3/2} \) terms make an
additional contribution to \( I_0 \) (Eq. 2.32) of

\[
A_{10} = \text{Re} \left( M_{3/2}^* \left( V_{1/2} - V_{3/2} \right) \right) \left( 1 - 3\cos^2 \theta \right) + \text{Re} M_{3/2}^2 \left( 1 + \cos^2 \theta \right).
\]

\]

\[
(2.53)
\]

Keeping terms linear in \( M_{3/2} \) and neglecting level widths, the
correction to the anisotropy is
\[
\frac{\delta A}{A_0 M_2} = \frac{-9e^2 Z^2}{32} \frac{(1 - \rho)(1 - A_0/3)}{(1 + \rho/2)} \tag{2.54}
\]

This correction varies as approximately \(-16 \times 10^{-6} Z^2\) for \(Z < 20\).

The final expression for the theoretical asymmetry, incorporating the corrections in Eqs. 2.50, 2.52 and 2.54 is

\[
A_T = A^{(0)} \left[ 1 + \left( \frac{\delta A}{A_0} \right)_{\text{B+C}} + \left( \frac{\delta A}{A_0} \right)_{\text{rel}} + \left( \frac{\delta A}{A_0} \right)_{M_2} \right] \\
+ A^{(2)} \varepsilon^2 + A^{(4)} \varepsilon^4. \tag{2.55}
\]

The \(\text{He}^+\) values for the corrections \(\delta A/A_0\) are listed in Table 2.2.

**TABLE 2.2. Anisotropy values \(A^{(n)}\) and small correction factors for \(\text{He}^+\).**

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E(2s_{1/2} - 2p_{1/2}))</td>
<td>14042.05 MHz*</td>
</tr>
<tr>
<td>(E(2p_{3/2} - 2p_{1/2}))</td>
<td>175594.0 MHz</td>
</tr>
<tr>
<td>(r(2p))</td>
<td>(1.0028 \times 10^{10} \text{ s}^{-1})</td>
</tr>
<tr>
<td>(A^{(0)})</td>
<td>0.1179656</td>
</tr>
<tr>
<td>(A^{(2)})</td>
<td>0.0005822 ((\text{kV/cm})^{-2})</td>
</tr>
<tr>
<td>(A^{(4)})</td>
<td>-0.0000037 ((\text{kV/cm})^{-4})</td>
</tr>
<tr>
<td>(\langle \delta A/A_0 \rangle_{\text{B+C}})</td>
<td>(-2.37 \times 10^{-5})</td>
</tr>
<tr>
<td>(\langle \delta A/A_0 \rangle_{\text{rel}})</td>
<td>(6.54 \times 10^{-5})</td>
</tr>
<tr>
<td>(\langle \delta A/A_0 \rangle_{M_2})</td>
<td>(-6.54 \times 10^{-5})</td>
</tr>
</tbody>
</table>

*The Lamb shift value is assumed for the purposes of finding \(A^{(n)}\).
CHAPTER III
EXPERIMENTAL APPARATUS

III.A Overall plan

A schematic diagram of the apparatus is shown in Fig. 3.1. A monoenergetic beam of ground state He\(^+\)(1s) ion beam of about 115 keV passes through a helium gas cell. The emerging beam consisting of a small fraction of He\(^+(2s_{1/2})\) metastables passes successively through a pre-quencher, a collimator, the quenching cell proper and is finally monitored with a Faraday cup. The quenching cell consists of four metal rods mounted in a quadrupole arrangement to which DC potentials are applied. The resulting electric field induces Lyman-\(\alpha\) photon emission, whose intensity is measured simultaneously by four UV detectors A, B, C and D that view the radiation in mutually orthogonal directions, perpendicular to the ion beam. All data is obtained with a single field strength of 370.97 V/cm. The electric field direction can assume an angle of 0, \(\pi/2\), \(\pi\) or 3\(\pi/2\) radians with respect to any one particular observation axis.

A pre-quencher potential of 3500 volts is only switched on for noise determinations. The quenching electric field thus created is sufficiently strong to destroy nearly all the metastable He\(^+(2s_{1/2})\) ions in the beam. A microcomputer switches the potentials to the quadrupole rods and to the pre-quencher. The microcomputer also collects the counter

20
Fig. 3.1. Schematic diagram of the apparatus (not to scale).
data and analyses it briefly (see Chapter V).

III.B Beam preparation

The He\(^+(2s)\) ions are obtained from a radio frequency gas discharge source and after acceleration through an accelerator tube are momentum analysed by a magnetic sector field. For each run, the beam energy is nominally kept at 115 keV. This is measured to a precision of \(\approx 0.5\%\) by recording the accelerating potential. The main requirement on the ion source is to produce a stable high beam current, typically 100 mA. After collimation the maximum available beam current passing through the observation region is about 2 mA.

III.C Metastable production

III.C.1 Helium gas cell

The He\(^+(1s)\) beam entering the gas cell is collisionally excited by helium gas. The gas cell consists of two concentric chambers (see Fig. 3.2). Helium gas is leaked into the inner chamber while the outer chamber is differentially pumped by a diffusion pump (Edwards model E04) with a liquid nitrogen trap, for a combined pumping speed, \(S\) for helium of nearly 500 l/sec. The entire gas cell is pumped from both ends by two more diffusion pumps and associated liquid nitrogen traps. The beam traverses through four rectangular slits \(S_a, S_b, S_c, S_d\) mounted at the ends of the inner and outer chambers. All slit dimensions are \((0.15 \times 0.51)\) cm, and each is movable transversely to the beam for alignment purposes.
Fig. 3.2. Gas cell for metastable production (not to scale). The cross section of all the rectangular slits is (0.15 cm x 0.51 cm)
The ratio of metastables to the ground state helium atoms produced by such a gas cell has previously been studied qualitatively (Drake, Goldman and van Wijngaarden, 1979). This fraction varies with the gas used, the cell pressure and the energy of the beam. Of the three gases used, He, N\(_2\) and Ar, helium is the most efficient in producing metastables, being four times as effective as argon. As the gas pressure in the cell is increased, the fraction first increases, reaches a maximum and then decreases.

In practice, the leak valve is adjusted for an optimum metastable content in the beam. The measured gas pressure in the outer chamber is then \(\approx 1 \times 10^{-5}\) Torr. For a single slit, the conductance, \(C\) for He gas from the inner to the outer chamber is \(C = \langle v \rangle A / 4\). Here \(\langle v \rangle = \sqrt{(8kT/\pi m)} = 1.3 \times 10^5\) cm s\(^{-1}\) is the mean thermal velocity of the atoms, and \(A = 7.7 \times 10^{-2}\) cm\(^2\) is the cross-sectional area of the slit. The throughput from two slits is then \(Q = 2C\Delta p\), where \(\Delta p\) is the pressure difference between the inner and outer chambers. The actual value for the throughput is known in terms of the pumping speed, \(S = 500\) l/sec and the pressure in the outer chamber, \(p_o = 1 \times 10^{-5}\) Torr by: \(Q = p_o S\). Hence the pressure in the inner chamber is \(1 \times 10^{-3}\) Torr. Over a length of 15 cm for the inner chamber, this is equivalent to an effective gas "thickness" of a few tenths of a mono-layer.

As the gas pressure in the cell is increased, the ion current falls, while the photon count from metastables increases. The photon count rate reaches a maximum when the
beam current has fallen by 30%. The fall in the beam current has its origin in charge neutralization and beam divergence in the gas cell. That the maximum occurs at such a low pressure arises from the competition between creation and subsequent destruction of the metastables. On the basis of this reasoning, one would expect the optimum pressure for production of metastables to be less than that corresponding to a mono-layer, consistent with observation.

On emerging from the gas cell, the beam has in addition to the \( \text{He}^+ (2s_{1/2}) \) metastable ions, other excited but very short-lived helium ions and neutrals. To improve the signal to noise ratio, these neutrals are removed by slightly deflecting the main beam by a weak magnetic field of a permanent magnet. The flight path from the gas cell is about 170 cm from the observation region. The beam traverses this distance in \( 7.2 \times 10^{-7} \) seconds, a time sufficiently long for all excited states to decay except the \( 2s_{1/2} \) metastable state (see Sec. III.D.2).

III.C.2 Metastable content of ion beam

The metastable content of the ion beam is obtained by comparing the calculated \( \text{He}^+ (2s_{1/2}) \) current, \( I_m \), with the total beam current, \( I \). The value for the measured count rate is known, and is given by

\[
C = \left( \frac{d^2}{16b^2} \right) \eta L \tag{3.1}
\]

where a point source is assumed, a distance \( b = 21.895 \) cm from the exit slit \( S_2 \) of diameter \( d = 1.016 \) cm (see Fig. 3.4), and \( \eta \)
is the channeltron detection efficiency of about 10% (Mack et al. 1976). \( I \) is the number of photons per second liberated inside the beam:

\[
L = \left( \frac{I \cdot l_{\text{eff}}}{e \cdot v_y} \right) R_s. \tag{3.2}
\]

The term inside the brackets in (3.2) is the number of He\(^+\) ions in the 2s\(_{1/2}\) state. The portion of the beam seen by the detector through slits \( S_1 \) and \( S_2 \) has an effective length \( l_{\text{eff}} = 0.780 \text{ cm} \), determined by the geometry of the photon collimator (see Sec. III.F.3). The velocity of the particles in the beam is \( v_y = 2.36 \times 10^8 \text{ cm/s} \). \( R_s \) is the electric field induced quench rate of the 2s\(_{1/2}\) state (see for example Gould and Marrus, 1983):

\[
R_s \propto \frac{[V_{ps}]^2 r_p}{\hbar^2 \left( \frac{2}{r_s^2} + 4r_p^2 \right)} \propto (V_{ps})^2 \frac{r_p}{\hbar^2} \tag{3.3}
\]

where the dipole matrix element \( V_{ps} \) is

\[
V_{ps} \propto -e \langle \text{e.s.u.} \rangle E(V/\text{cm}) \frac{3}{300} \frac{a}{Z}. \tag{3.4}
\]

\( a \) is the Bohr radius, \( r_p = 1.0028 \times 10^{10} \text{ s}^{-1} \) is the natural decay rate of the 2p\(_{1/2}\) state in He, and \( \omega / 2 \pi = 14.042 \text{ GHz} \) is the Lamb shift frequency for \( n=2 \) in He (see Table 2.2). Substituting the stated values into (3.3) and (3.4) yields

\[
R_s = 62.4 \left( E(V/\text{cm}) \right)^2. \tag{3.5}
\]

At our electric field of 371 V/cm, and a He beam current of \( I = 2 \mu A \), the observed photon count rate \( C \) is 10 kHz. Evaluating (3.1) for the metastable current yields

\[
I_m \propto 4.2 \times 10^{-9} \text{ A}.
\]
This indicates that the 2s metastable content is about 0.2% of the total beam current.

III.D Pre-quencher for noise determination

We define the noise background as the photon signal still observed with the metastable content reduced to zero by pre-quenching, rather than as the signal still observed for an unquenched beam in the absence of a quenching field in the quenching cell. Our noise level from pre-quenching increases slightly with the electric field in the quenching cell. This increase in noise level is a consequence of the formation of He\textsuperscript{+} (2s) metastables by the interaction of the ground state He\textsuperscript{+} ions with the remaining gas in the observation region (5x10\textsuperscript{-8} Torr). Therefore, in addition to pre-quenching, the noise level is measured with the same field value as the "raw" signal, and as a result, the noise level depends weakly on the field orientation. The resulting ratio of the intensities parallel and perpendicular to the field, \( r = I_\parallel / I_\perp \) for noise is similar to that for signal. As will be seen in Chapter IV, with our definition of the noise, which is subtracted from the raw signal makes the asymmetry in the noise inconsequential.
III.D.1 Pre-quencher

The pre-quencher is placed behind the entrance slit \( S_e \) of the collimator as shown in Fig. 3.3. It has cylindrical symmetry and consists of several metal apertures maintained alternatively at 3500 volts and ground potential. During a noise measurement, the high electric field \((\approx 3kV/cm)\) between the apertures de-excites virtually all the metastables.

III.D.2 The effects of cascading on noise

Next, we investigate the effect of long-lived excited states in the metastable beam and demonstrate that to a first approximation, cascading cannot affect our final results since the noise signal is subtracted from the raw signal, and the signal arising from cascading, if present, in nearly the same for both. Because the raw is measured under different conditions from the noise (the raw is obtained without pre-quenching), it would include photons from long-lived highly excited states that are absent for the noise. A consequence of this beam contamination is that it would lower the observed Lamb shift ratio \( r = I^4 / I^1 \). We now present two reasons why this effect is believed to be negligibly small.

Firstly, when the raw signal is measured, the beam passes through a distance of nearly half the quenching cell-length, corresponding to a time interval of \( \approx 30 \text{ ns} \), in a
field approaching 370 V/cm (see Sec. III.F). This pre-
quenching field effectively reduces the lifetime of long-
lived highly excited s states* before they enter the
observation region.

Secondly, the cross-section for excitation of the He+
ground state ions in passing through the cell varies
approximately as $n^{-3}$ for $n \geq 3$ (Inokuti et al., 1969, de
Heer, 1966). This certainly minimises the presence of
excited states of $n > 6$ by a factor of 40. The dominant beam
contamination of ions will then be in excited states of
$3s \leq n \leq 6$. For $n = 6$, the maximum lifetime for cascading to the
ground state is 38.1 ns (from lifetimes in Wiese et al.,
1966). This time interval is short compared to the flight
time of 720 ns from the gas cell to the observation
region. During this time, the $n = 6$ contamination is reduced
relative to the $2s$ state by $e^{-(720/38.1)} = 6 \times 10^{-9}$.

III.F Beam collimator

The collimator to the quenching cell defines the beam
direction precisely. The placement and sizes of the slits $S_e$,
$S_f$, $S_g$, $S_h$ and $S_i$ are chosen to minimise any slit scattered
particles entering the quenching cell. As a result, the beam

*This is because the $ns^{1/2} - np^{1/2}$ Lamb shift separation
varies as $n^{-3}$. 
Fig. 3.3. Pre-quencher and collimator assembly. All slit diameters and slit separations are in cm.
divergence and beam diameter are limited to (0.433±0.005) degrees and 2P=(0.165±0.030) cm respectively. To test for precise alignment of the collimator with the central path of the beam, deflection plates are mounted just before the collimator, and inside the collimator behind the pre-quencher (not shown in Fig. 3.3). Application of DC potentials allowed us to slightly deflect the beam in two mutually orthogonal directions, perpendicular to the beam axis. The beam is moved with respect to the collimator until a maximum beam current is observed with all the deflection plates at 0 volts. This criteria for beam alignment can only be met by cancelling the earth's magnetic field over the observation region with Helmholtz coils. As shown in Fig. 3.3, the collimator and quenching cell are mounted on a single rigid structure with a high value of area-moment of inertia to ensure precise alignment of the central axis of the collimator with that of the quenching cell.

III.F Quenching cell

III.F.1 Quadrupole and electric field calculation

The quadrupole seen in Fig. 3.4 has a set of four stainless steel rods mounted on insulators in an aluminium box of square cross section. Each of the rods can be applied with a potential separately. The field at the beam axis is produced by connecting together the rods in two nearest-neighbour pairs, with two pairs held at opposite potentials, +V and -V. The box, together with the end plates holding the entrance
Fig. 3.4. Cross-section of the quenching cell with one of the photon collimators. The four rods are of diameter $2b=1.270$ cm and rod separation $2a=4.064$ cm. The inner dimension of the square box is $2W=14.224$ cm. Slits $S_1$ and $S_2$ define the solid angle for the detector. Important dimensions are given in the text.
and exit slits are kept at ground potential. This provides a well-defined boundary condition for the field inside.

The field strength at the beam axis must be known accurately in order to calculate the anisotropy. For this reason, the quadrupole system is precision machined in all critical dimensions. The rods have a diameter of $2b=1.2700\pm0.0005$ cm, and adjacent rods are separated by a distance of $2a=4.0640\pm0.0006$ cm. The inside dimension of the box of square cross section is $2W=14.224\pm0.003$ cm. The ratios $b/a=5/16$ and $W/a=56/16$ are chosen to simplify numerical solution to the Laplace equation for the field calculation in the region of the ion beam (van Wijngaarden and Drake, 1978). These simple ratios allow a convenient subdivision of the quenching cell into a square grid. Laplace's equation $\nabla^2\phi=0$ with boundary conditions $\phi_i=\pm V$ on the rods and $\phi=0$ on the walls can now be solved by demanding that the potential at each grid point, $i$, be the average of that of the neighbours. For example, in 1 dimension,

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right) = \frac{(\phi_{i+1}-\phi_i)}{(x_{i+1}-x_i)} = 0$$

from which it follows that

$$\frac{\phi_{i+1}-\phi_i}{x_{i+1}-x_i} = \frac{\phi_i-\phi_{i-1}}{x_i-x_{i-1}}$$

Thus, for constant grid size,

$$\phi_i = \frac{1}{2} (\phi_{i+1}+\phi_{i-1})$$

The variation in field along the beam axis is shown in Fig. 3.5, where $y_0$ is the distance from the entrance slit of the
Fig. 3.5. The field variation along the beam axis inside the quenching cell. The beam enters the cell at \( y = 0 \). The photon collimator axis intercepts the beam axis at \( y \).
quenching cell to the observation region where the field reaches a maximum \( E = E_0 \). From \( y = y_0 \) to \( 2y_0 \), the field is a mirror image of that in Fig. 3.5. The length \( L=2y_0 \), with \( L = \left( 15.240 \pm 0.003 \right) \text{ cm} \) is a compromise between having a sufficiently small field gradient \( \partial E / \partial y \) in the observation region along the beam axis, and not having too large a loss in meta-stable content by quenching prior to observation. The field \( E_0 \) at \( y_0 \) is related to the applied potential \( zV \) by

\[
E_0 = \left( 0.8865 \pm 0.0001 \right) \text{ V/cm}
\]  
(3.6)

For our potential, \( V=\pm \left( 850.32 \pm 0.02 \right) \), the field is

\[
E_0 = \left( 370.97 \pm 0.06 \right) \text{ V/cm}
\]  
(3.7)

### III.F.2 Beam deflection

In passing from the entrance slit to the observation region, the ion beam obtains a deflection \( \Delta x \) due to the electric field. For the UV detectors viewing radiation perpendicular to the field, it is important that \( \Delta x \) be small so that the distance from those detectors to the source remains unchanged. The deflection \( \Delta x \) is found in the following manner. The transverse velocity of the beam, \( v_x \), after traveling a distance \( y \) along the beam axis is given by

\[
v_x \left( \frac{y}{y_0} \right) = \frac{qE_0y_0}{mv_y} \int_0^{y/y_0} \frac{E}{E_0} \frac{d\left( \frac{y'}{y_0} \right)}{d\left( \frac{y}{y_0} \right)}
\]  
(3.8)

where \( mv_y \) is the primary momentum of the beam.

\[
\Delta x = \frac{E_0y_0^2}{2v_a} \int_0^1 \left[ \int_0^{y/y_0} \frac{E}{E_0} \frac{d\left( \frac{y'}{y_0} \right)}{d\left( \frac{y}{y_0} \right)} \right] d\left( \frac{y}{y_0} \right)
\]  
(3.9)

Numerical solution using the field variation in Fig. 3.5, and
the accelerating potential $V_a = 115$ kV yields
\[ \Delta x = 0.035 \text{ cm.} \] (3.10)
The decrease in intensity resulting from this deflection for
the UV detectors viewing the radiation perpendicular to the
field is negligible.

III.F.3 Photon collimator

The photon collimators for the four UV detectors are
identical, and one of these is shown in Fig. 3.4. To achieve
accurate location, the slit system is mounted directly on the
quenching cell. The machining tolerance used in constructing
the quenching cell and photon collimator lead to an uncertain-
tainty in the location of the viewing axis of only $\Delta \theta = 0.02^\circ$. A rectangular entrance slit $S_1$ has an opening of $\alpha = (0.635 \pm 0.001)$ cm. Slits $S_3$ and $S_4$ are skimmer slits to reduce
scattered particles from entering the photon detector. An
exit slit $S_2$ of diameter $(1.016 \pm 0.002)$ cm, together with the
entrance slit $S_1$, define the effective solid angle for
observation.

III.F.4 Thin film noise filter

To stop low energy particles produced in the interaction
of the fast ion beam with the remaining gas, slits $S_2$ and $S_5$
are covered by thin, self-supporting aluminium films. The
films are of 500-700 Å thickness, and are transparent to the
Lyman alpha radiation. Since the films unavoidably have a few
pin holes, two films behind one another insure that the
detector entrance is completely covered. The presence of the films results in a reduction in the noise counting rate by a factor of 50. With the exclusion of low energy particles, the noise counts arise only from UV photons, the detectors being insensitive to longer wavelengths. The noise is proportional to the residual gas (see Sec. III.D), and at an operating pressure of \(5 \times 10^{-8}\) Torr, the average signal to noise ratio is 168:1. The signal to noise ratio is roughly proportional to the square of the electric field, because most of the noise arises from the relatively strong component of ground state \(\text{He}^+\) ions in the beam.

III.E.5 Wall scattering of UV radiation

Light reflection from the cell walls can influence the measured ratio in intensities for two adjacent detectors. With reference to Fig. 3.4, it can be seen that a photon detector views the direct radiation from the beam as well as the radiation that is reflected from part of the wall opposite the detector. The reflected intensity arises from a long beam section in the cell and the observed part originates from a large range of angles centred about the viewing axis (\(\theta = 0^\circ\)). This lowers the observed intensity parallel to the field direction relative to that in the perpendicular direction, and thus the observed ratio \(r = I_\parallel / I_\perp\) will also be lowered. In fact we calculate that for a 1% wall reflection, \(r\) reduces by 10 parts per million, corresponding to a reduction of 38 parts per million in the asymmetry (see Ch. IV).
To reduce these harmful reflections, all surfaces visible to the UV detectors are coated with black carbon soot. To estimate its reflectivity, we measured in a separate experiment the ratio between the reflectivity for a sooted surface and a gold coated mirror, since no data is available in literature. The results of the experiment are shown in Table 3.1.

<table>
<thead>
<tr>
<th>UV Radiation</th>
<th>Reflection of soot relative to gold for 60° incidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>584 A (He)</td>
<td>(7.5 ± 0.4)x10^-3</td>
</tr>
<tr>
<td>950 A (N₂)</td>
<td>(7.9 ± 0.5)x10^-3</td>
</tr>
</tbody>
</table>

Although it is apparent that for the wavelengths shown, the reflectivity from soot is small, we need to know it for our Ly-α (304 A) radiation. Unfortunately, a 304 A source was not readily available but from literature (Samson, 1980 and Canfield et. al., 1964) it is known that the reflectivity decreases as the wavelength decreases. The reflectivity of 400 A radiation from gold at normal incidence is 4% or less. Assuming this value for 304 A (as an upper limit), we infer the reflectivity of soot for 304 A to be less than 0.03%. Therefore, the reflectivity can reduce our asymmetry value by at most 1.3 parts per million.
III.G Photon detection

III.G.1 Electron multipliers - an earlier investigation

A previous measurement of the Lamb shift in deuterium (van Wijngaarden and Drake, 1978) and in helium (Drake, Goldman and van Wijngaarden, 1979) by the quenching-radiation method employed channeltrons for photon detection. The limited lifetime of those channeltrons (Galileo Optics model CEM 4028, bias current of 2.8μA) was a problem; after counting a few times $10^8$ counts, the channeltrons became non-linear and they had to be replaced frequently. The quantum efficiency of the channeltrons was count rate dependent at high counting rates. It is not certain whether or not other problems in the electronics that are investigated in this work also affected the photon detection efficiency in previous experiments.

To overcome the difficulties with the model CEM 4028 channeltrons, the detectors were replaced by electron multipliers (Schlumberger, model 510-00-16-M2), with Be-Cu dynodes. A disadvantage of the electron multipliers is that the conversion dynode has no cylindrical symmetry and consequently its detection efficiency is highly polarization sensitive (McConkey et. al., 1982, and references therein). Our experiment however requires photon polarization insensitive detectors. To fulfill this requirement, we used as conversion dynode a cone with unit gain as shown in Fig. 3.6a. The cones are custom built by Galileo optics, with a high degree of cylindrical symmetry, and are therefore
Fig. 3.6  a) Electron multiplier with conversion dynode (channeltron cone).  
b) The voltage dependence of current gain.  
c) Pulse height distribution.
insensitive to photon polarization. This detection system proved to be successful in a measurement of the asymmetry due to the E1-E1 damping interference in He\textsuperscript{+} at the 0.026% precision level (Drake, Patel and van Wijngaarden, 1983). Based on this finding, we believed that the detection system would be sufficiently capable for a measurement of the Lamb shift asymmetry to an accuracy of 0.002%.

The electron multipliers have 16-stage secondary emission dynodes, and the manufacturer's specified gain increases rapidly with applied voltage; from a gain of 10\textsuperscript{7} at 2300 volts to 10\textsuperscript{8} at 2800 volts (see Fig. 3.6b). The typical pulse height distribution (Fig. 3.6c) shows that a large number of small pulses is obtained. Ideally, a discriminator is needed whose acceptance level is set at the dip in the pulse height distribution, so as to accept only higher pulses. By doing this, small instabilities in the discriminator level, or in the form of the pulse height distribution have negligibly small effects in the observed counts.

In practice, we fed the electron multiplier signal to an amplifier (ORTEC, model 474), and the amplified output was sent via a discriminator (ORTEC, model T105/N) to a counter. The discriminator was set to operate with a dead time defined by a delay cable of 55 ns, while an acceptance threshold was chosen so as to inhibit noise from our detection electronics. The electron multiplier operating voltage was set at the voltage plateau, the region for which the count rate is independent of the voltage.
A series of runs was taken to measure the Lamb shift ratio \( r \). The method used to determine the ratio is the same as the final experiment, and will be described in detail in Ch. IV. The results of the first 37 runs are shown in chronological order in Fig. 3.7 and a corresponding runs test (Sec. IV.E), performed on the run-means is summarized in Table 3.2.

Table 3.2. Runs test results on the run-means of the first 37 runs.

<table>
<thead>
<tr>
<th>Run length</th>
<th>Low runs</th>
<th>High runs</th>
<th>Expected number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
<td>( 4.6 \pm 1.9 )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>( 2.3 \pm 1.4 )</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>( 1.2 \pm 1.0 )</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>( 0.58 \pm 0.69 )</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>( 0.29 \pm 0.50 )</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>( 0.15 \pm 0.36 )</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>( 0.07 \pm 0.26 )</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>7</td>
<td>( 9.3 \pm 3.0 )</td>
</tr>
</tbody>
</table>

From the table, it is evident that two high runs (of lengths 6 and 7) are outside statistical prediction. This is also intuitively obvious from the time dependent drift of the average about the overall mean (shown by a dashed line in Fig. 3.7). This mean, corrected only for noise, has the value

\[ R = 1.267400 \pm 0.000018. \]  \( \text{(3.11)} \)

Upon a careful re-evaluation of the experiment, it was concluded that the slow time-dependent drifts in the means had its origin in the pulse counting of the photon detection
Fig. 3.7. The Lamb shift ratio for 37 runs in chronological order measured with the detection system of Fig. 3.6a. Each measurement is the average of several individual measurements whose number appears on top of the figure. The dashed line is the overall average $R=1.267400 \pm 0.000018$. The dash-dot line is the average of the first 9 runs.
system. After the passage of a pulse through our 474 amplifier, the base line is not restored to zero level immediately, and a subsequent pulse rides on this offset. If the subsequent pulse was just too small to cross the discriminator threshold without the offset, then with the added base line shift, it will cross the threshold and be counted. As the pulse rate in the two observation directions, parallel and perpendicular to the field differ by 26.7%, the average shift in the base line for the counter viewing the radiation parallel to the field will be higher than that for the other counter. As a result, the first counter counts a larger fraction of the smaller pulses than the second, and the measured ratio $I_\parallel/I_\perp$ will be too large.

To eliminate the base line shift, the 474 amplifier was replaced by a fast filter amplifier (Ortec, model 579). As well as a faster response, the amplifiers have additional circuitry to provide base line restoration (BLR). To investigate the new detection system, the electron multipliers were set on their voltage plateaux, and the gains of the new amplifiers were varied. A criteria for proper operation is that the ratio $r = I_\parallel/I_\perp$ must be independent of the amplifier gain, at least over a small range. Fig. 3.8 is a plot of the counting rate versus the amplifier gain for one of the counters. The results from three sets of runs at gain settings of X125 (just above the knee), X190 (intermediate) and X250 (on the gain plateau) are shown in Fig. 3.9. Shown at the extreme right of each set of runs is the mean and standard deviation
FIG. 3.8. The amplifier gain dependence of the observed counting rate for a fixed number of incident photons.

of that set. It is clear that a definite tendency exists for the ratio to be higher with a lower gain setting. This probably arises from a combination of the awkward pulse height distribution of these counters, and a possible count rate dependent "cleaning" effect on which we now speculate.

The electron multiplier is not inside a perfect vacuum, and the Be-Cu dynodes are exposed to the residual gas which they adsorb. We therefore expect that for higher counting rates, the surface contamination decreases by the removal of adsorbed gas and the average secondary emission coefficient increases. The result of this is that the pulse height
Fig. 3.9. The observed Lamb shift ratios for the three amplifier-gain settings shown in the insets above. Each set consists of several runs whose average is indicated by a solid circle.
distribution shifts towards the larger pulse heights, and makes the counter "super-linear" in the sense that the detection efficiency increases with count rate. On the basis of this argument, one would expect a higher ratio for a lower amplifier gain setting, as observed.

A shift in the pulse height distribution for channeltrons (see Fig. 3.10 for a typical distribution) will not make them super-linear since the discriminator level for them can be set at a sufficiently low value such that virtually all pulses are counted despite any shift. It was realised that the only choice for pulse counting was to use channeltron detectors, but the disadvantage of conventional channeltrons is that they are linear only at low counting rates. At high counting rates, which are required in the experiment, "charge depletion" occurs, resulting in non-linear behaviour. This phenomenon has its origin in the high input-to-output resistance of the channeltron (≈1.0×10^9 Ω). Thus when two pulses are separated by a small time interval, the power supply for the channeltron does not adequately replenish the charge to the channel wall for the second pulse, whose pulse height is then decreased. This results in an overall gain depression for count rates above f=10 kHz (Kurz, 1979). The time averaged current drawn by the output signal is

\[ i_o = f \cdot G \cdot e \]

where \( G \approx 10^8 \) is the gain. At our count rate, \( i_o \) is 0.16 μA. Thus gain depression becomes noticeable when the output current exceeds 6% of the bias current, \( i_b = 2.8 \mu A \).
Fig. 3.10. Typical pulse height distribution for a channeltron electron multiplier for various supply voltages.
III.G.2 High bias-current channeltrons

To overcome gain depression, we installed new low resistance ($1.9 \times 10^{-7}$) channeltrons, with a bias current of 15μA, specially constructed for this experiment by Galileo Optics (model SPCL CEM 4503). Gain depression for these will only become noticeable at count rates greater than 1 MHz, which is far below our counting rate of 10 kHz.

The channeltron, illustrated by Fig. 3.11a, consists of two sections joined together by a metal sleeve: a cone (the conversion dynode) and a spiral amplifier (electron multiplier). The cone input aperture is 1.74 cm in diameter and it is secured to the detector assembly by spring clips. The channeltron input and the thin aluminium film at $S_5$ (see Fig. 3.4) are at negative high voltage while the output is at ground. An advantage of this mode of operation is to prevent the detection of electrons. Each of the four counters is connected to a separate power supply and potential divider. The potentials across the cone and spiral amplifier are 430 V and 2350 V respectively. To ensure complete electron collection, the output of the spiral is at -150 V with respect to the collector. Prior to every run, the voltage dependence of the counting rate for all channeltrons is obtained. This allowed us to determine the proper operating points. A typical plot is shown in Fig. 3.11b.

III.G.3 Signal interfacing and dead time measurement

The detector signal is brought to the amplifier (Ortec
Fig. 3.11 a) Electrical connections to the channeltron
b) The voltage dependence of the observed number of counts
for the detection system of Fig. 3.11a. The test point is
described in the text in Sec. VI.A.
model 579) by a short coaxial lead as shown by Fig. 3.11a. As described in Sec. III.G.1, the amplifier incorporates baseline restoration. An internal filter integrates the signal with a 10 ns time constant to remove high frequency noise. The typical pulse duration is 15 ns, observed at the amplifier output by an oscilloscope, and while the pulse height distribution of the CEMs was not measured, it is of the form shown in Fig. 3.10. The amplifier output is fed to a discriminator (Ortec model T 105/N), placed at 60–65 mV, to reject noise pulses. The discriminator level with the present pulse height distribution is not critical.

The dead time of the counting system is an important correction in the measurement of the asymmetry, and it is conveniently set at the discriminator module by a length of delay cable. It is accurately measured with the setup shown in Fig. 3.12a. A pulse generator sends pulses to a buffer amplifier at a rate of 5 kHz. The output pulses from the buffer amplifier are shortened to about 5 ns. A 2-input OR function gate receives one input directly from the buffer amplifier and the other input delayed by a pre-calibrated delay box. From the OR gate output, pulse pairs at 5 kHz, separated by a delay $\delta t$, (set by the delay box) enter the counting system. As $\delta t$ is gradually decreased from a high value to a lower one where it equals to the dead time $t$, the count rate drops very sharply from 10 kHz to 5 kHz, illustrated by Fig. 3.13. The value of $\delta t$ as shown in the figure is not precisely given by the reading on the delay box since
Fig. 3.12. Block diagrams for dead time determination.
a) set up for measuring $\Delta t$. The pulse trains are described in the text.
b) Set up for calibrating delay box and associated electronics.
FIG. 3.13. Variation in the count rate with delay $\delta t$.

It includes the delay in the connecting cables and the OR gate circuitry.

Measurement of the delay box and cabling delay is performed by the circuitry of Fig. 3.12b. The set up is similar to the previous one except that a time to amplitude converter (TAC) and a pulse height analyser (PHA) replace the counting system. The pulse frequency is monitored at one of the buffer amplifier outputs, "scoop". A second output of the buffer amplifier triggers the start of the TAC. The time delayed pulse of the pulse pair at the output of the OR gate gives the TAC its stop signal. The output of the TAC is translated by the PHA to show the separation in time between the two pulses of a pulse pair in terms of the known separation between pulse pairs. The final results of the dead times for
the four counting systems are

A: (56.1 ± 0.1) ns  B: (54.1 ± 0.1) ns
C: (55.8 ± 0.1) ns  D: (56.3 ± 0.1) ns.

III.H Normalization to beam current

The duration of the counting interval for the four counters is set for a pre-determined particle flux. This is achieved as follows. An electrometer monitors the ion beam current collected in a Faraday cup, (see Fig. 3.1), placed 26 cm from the centre of the quadrupoles. A proportional anologue voltage signal from the electrometer is supplied to a voltage-to-frequency converter. The pulses from the converter are counted by a timer-counter (Ortec model 773), which stops all four counters upon reaching a preset number of counts.

It is important to measure the incident current accurately because a measurement of the noise must be carried out for precisely the same time interval as that for the signal (see Ch. IV). To ensure this, a repeller plate placed in front of the Faraday cup is kept at -90 V to prevent secondary electrons from leaving the cup. To minimise ion back-scattering, the end of the Faraday cup has a beryllium beam dump. This low Z material has a very low Rutherford back-scattering cross section ($\propto Z^2$). The opening to the Faraday cup and the repeller plate aperture are 3 times the deflection distance of the ion beam by the electric field. This ensures that the ion beam does not strike the cylindrical wall of the Faraday cup but only the beam dump.
CHAPTER IV
DATA COLLECTION AND ANALYSIS

Data is obtained from the counters with the electric field attaining one of four directions shown in Fig. 4.1 (a cross section of Fig. 3.1 in the observation plane). The data is combined with a view to eliminate instrumental effects resulting from:

1) Different photon counting efficiencies for each UV detector,

2) Differing counting periods for each field setting,

3) Beam deflection by the electric field,

4) Small stray fields in the observation region.

It will be shown that a measurement of the ratio r of the intensities parallel and perpendicular to the electric field rather than a direct measurement of the asymmetry itself allows us to combine the data in such a manner as to eliminate all instrumental effects to first order. The first section derives the relation between the error in the asymmetry and that in the ratio. The method of combining the data is then described, deriving as well, an expression for the theoretical standard deviation in the ratio for a given number of photon counts. The procedure for eliminating any remaining polarization sensitivity in the detectors is shown next, followed by the derivation of the dead time and solid angle corrections. Finally, a description of the runs test is provided.
FIG. 4.1. The four electric field directions used in the intensity ratio measurements. The ion beam passes through the centre of each diagram, and A, B, C and D are the detectors.

IV.A. Relationship between the uncertainties in the asymmetry and in the ratio

The Lamb shift asymmetry $A$ is $(I_\parallel - I_\perp)/(I_\parallel + I_\perp)$, where $I_\parallel$ and $I_\perp$ refer to the intensities observed parallel and perpendicular to the field directions. In practice, a measurement is made of the ratio

$$r = I_\parallel / I_\perp,$$  \hspace{1cm} (4.1)

which is related to the asymmetry $A$ by

$$A = (r-1)/(r+1).$$ \hspace{1cm} (4.2)

The error $\delta r$ in the ratio $r$ in terms of the error $\delta A$ in the asymmetry is given by

$$\frac{\delta A}{A} = \sqrt{\frac{\delta r^2}{(r+1)^2} + \frac{\delta r^2}{(r-1)^2}}.$$ \hspace{1cm} (4.3)
\[ \frac{\delta r}{r^2-1} \sqrt{\frac{(r-1)^2 + (r+1)^2}{2(r^2 + 1)}} \]  

Using the expected value for \( r = 1.2677 \), for our field value of 370V/cm, \( \delta A/A = 3.76\delta r \). Hence a 120 parts per million (ppm) measurement for the ratio is equivalent to a 450 ppm value for the asymmetry, typically achievable in a single day's run.

IV.B Method of data analysis

Measurements of the intensities are taken with the electric field in one of the four settings shown in Fig. 4.1. The counts obtained are the "raw" data, to be further corrected for background. The background or noise is the counts obtained when the pre-quencher is switched on. Because it constitutes only 0.6% of the raw, the noise is measured once after a number \( q \) of consecutive raw measurements, to still give adequate precision as will be seen in Sec. IV.D. The "signal" is defined as the raw-minus-noise value, and for simplicity, the following expressions refer to signal rather than (raw - noise).

For the field direction \( \vec{E}_0 \) shown in Fig. 4.1, we now focus attention to the photon intensity ratio for counter pair A and B. The observed number of counts in a given counting period \( \tau_0 \) for the detectors A and B are given by
\[ N_{\text{AB}} = I_A^\alpha \tau_0 \] and
\[ N_{\text{B} \perp} = I_B^\beta \tau_0 \]
where \( I \) refers to the number of photons per second entering the detector and \( \alpha \) and \( \beta \) are detector efficiencies. The resulting ratio
\[ \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_0 = \frac{N_{\text{AB}}}{N_{\text{B} \perp}} = \frac{(I_A^\alpha)}{(I_B^\beta)} \]
still depends on the relative efficiencies but the \( \tau \)'s cancel. Next we rotate the field to \( \vec{E} \pi/2 \) thereby reversing the roles of detectors A and B. The ratio now is
\[ \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_{\pi/2} = \frac{N_{\text{AB}}}{N_{\text{A} \perp}} = \frac{(I_B^\beta)}{(I_A^\alpha)} \]
This ratio also depends on the relative efficiencies. The product of the ratios however is independent of the efficiencies:
\[ \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_0 \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_{\pi/2} = \left( \frac{I_A^\alpha}{I_B^\beta} \right)_x \left( \frac{I_B^\beta}{I_A^\alpha} \right) \]
Moreover, since \( \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_0 \) and \( \left( \frac{r_{\text{AB}}}{r_{\text{B} \perp}} \right)_{\pi/2} \) have the same physical interpretation, we write
\[ r_{\text{AB}} = \sqrt{\frac{I_{\text{AB}}}{I_{\text{B} \perp}} \frac{I_B^\beta}{I_A^\alpha}} \] \hspace{1cm} (4.5)
Writing for compactness \( I_{\text{AB}}/I_{\text{B} \perp} \) as \( (A/B)_\theta \), where
\( \theta = 0, \pi/2, \pi \) or \( 3\pi/2 \) is the angle of the field direction relative to the observation direction of counter A, the last equation is
\[ r_{\text{AB}} = \frac{(A/B)_0}{(B/A)_{\pi/2}} \]
As the field is switched through the four directions, four values of \( r_{\text{AB}}^2 \) are obtained, namely
\[
\begin{pmatrix}
A \\
B
\end{pmatrix}
_0 
= 
\begin{pmatrix}
B \\
A
\end{pmatrix}
_\pi/2,
\begin{pmatrix}
A \\
B
\end{pmatrix}
_0 
= 
\begin{pmatrix}
B \\
A
\end{pmatrix}
_3\pi/2,
\begin{pmatrix}
A \\
B
\end{pmatrix}
_\pi/2 
= 
\begin{pmatrix}
B \\
A
\end{pmatrix}
_\pi/2,
\begin{pmatrix}
A \\
B
\end{pmatrix}
_\pi 
= 
\begin{pmatrix}
B \\
A
\end{pmatrix}
_3\pi/2.
\]

(4.6)

The order of applied field directions is
\[F_0 \rightarrow F_{3\pi/2} \rightarrow F_\pi \rightarrow F_{\pi/2} \]. Hence each detector measures
I\parallel and I\perp alternatively, thereby randomising the effect of slow beam drifts and minimising possible systematic errors.

To this purpose, a complete measurement is split into two "rounds", having clockwise followed by anti-clockwise applications of the electric field (8 constitutional measurements). The number of photon counts for the same field directions are added up before evaluating ratios.

An arithmetic mean of the terms in Eq. 4.6 is formed:
\[
r_{AB}^2 = \frac{1}{4} \left[ \left( \begin{pmatrix}
A \\
B
\end{pmatrix}
_0 + \begin{pmatrix}
A \\
B
\end{pmatrix}
_\pi/2 + \begin{pmatrix}
A \\
B
\end{pmatrix}
_3\pi/2 \right) \right].
\]

(4.7)

It is clear, for counter A as an example, that small beam deflections, (see Sec. III.F.2), in the direction of the electric fields \(F_0\) and \(F_\pi\) produce opposite intensity changes \(\delta A\parallel\), and thus cancel on average. Furthermore, Eq. 4.7 cancels the effect of small stray fields to be discussed below.

It is possible to combine the terms in Eq. 4.6 in other ways, for example as a geometric mean:

\[
r_{AB,\text{geom}} = \left[ \begin{pmatrix}
A \\
B
\end{pmatrix}
_0 \begin{pmatrix}
B \\
A
\end{pmatrix}
_\pi/2 \begin{pmatrix}
A \\
B
\end{pmatrix}
_0 \begin{pmatrix}
B \\
A
\end{pmatrix}
_3\pi/2 \right]^{1/8}.
\]

It can be shown that this mean only gives but partial cancellation of the beam deflection and residual stray field.
effects.

The effect of a stray field, for instance the vertical component of the residual earth's magnetic field (which is cancelled to 0.1 Gauss - see Sec. III.E) is to cause a motional electric field

$$\mathbf{E}_{\text{motional}} = \frac{\mathbf{v}}{c} \times \mathbf{B} \text{ statvolts/cm } \approx 0.2 \text{ V/cm.}$$

This field adds to and subtracts from the applied electric fields $E_{x/2}$ and $E_{3x/2}$ respectively so that its effect cancels to first order in the arithmetic mean of Eq. 4.7.

Ratios $r_{BC}$, $r_{CD}$ and $r_{DA}$ are formed similar to Eq. 4.7. The average ratio for a complete measurement is then

$$r = \frac{(r_{AB} + r_{BC} + r_{CD} + r_{DA})}{4}.$$  \hspace{1cm} (4.8)

To emphasise that the symbols in Eq. 4.7 refer to signal counts, consisting of raw (N) and noise (n) counts, we re-write it as

$$r_{AB} = \frac{1}{2} \sqrt{\left[ \left( \frac{N_A - n_A}{N_B - n_B} \right)_{0} + \left( \frac{N_A - n_A}{N_B - n_B} \right)_{\pi} \right]}
\times \left[ \left( \frac{N_B - n_B}{N_A - n_A} \right)_{x/2} + \left( \frac{N_B - n_B}{N_A - n_A} \right)_{3x/2} \right]}.$$  \hspace{1cm} (4.7b)

**IV.C. Weighted mean and standard deviation**

The ratio measurements are combined as follows. Assume several statistical measurements and their standard deviations:

$$a_i \pm \sigma_i, \quad i = 1, 2, \ldots s$$  \hspace{1cm} (4.9)

where each mean $a_i$ is found from $N_i$ identical trials. Then
\[ \sigma_i = \frac{1}{N_i} \quad \text{or} \quad \sigma_i = \frac{c}{N_i^{\frac{1}{2}}} \]

where \( c \) is a constant dependent on the nature of the experiment. The overall mean \( \bar{a} \) of these measurements is

\[ \bar{a} = \frac{N_1 a_1 + N_2 a_2 + \ldots + N_s a_s}{N_1 + N_2 + \ldots + N_s} \quad (4.10) \]

\[ = \frac{a_1 \frac{c^2}{\sigma_1^2} + a_2 \frac{c^2}{\sigma_2^2} + \ldots + a_s \frac{c^2}{\sigma_s^2}}{\frac{c^2}{\sigma_1^2} + \frac{c^2}{\sigma_2^2} + \ldots + \frac{c^2}{\sigma_s^2}} \]

Hence,

\[ \bar{a} = \frac{\sum_{i=1}^{s} a_i / \sigma_i^2}{\sum_{i=1}^{s} 1 / \sigma_i^2} \quad (4.11) \]

The overall standard deviation is found by re-examining Eq. 4.10:

\[ \bar{a} = \frac{N_1 a_1 + N_2 a_2 + \ldots + N_s a_s}{\sum N_i} \]

Hence,

\[ \sigma = \sqrt{\frac{\sum N_i \sigma_i^2}{N_1} + \frac{\sum N_i \sigma_i^2}{N_2} + \ldots + \frac{\sum N_i \sigma_i^2}{N_s}} \]

and the standard deviation per measurement is

\[ \text{Error} = \frac{\sigma}{N_i^{\frac{1}{2}}} = \frac{\sum_{i=1}^{s} N_i \sigma_i^2}{\sum_{i=1}^{s} N_i} \quad (4.12) \]
LV.D  Theoretical standard deviation

The expected standard deviation, or the "theoretical" standard deviation, based only on the grand number of photon counts measured by each counter is now evaluated. This value is used to compare with the "experimental" standard deviation (Eq. 4.12), to detect the presence of errors other than that from counting statistics, since any additional errors always increase the experimental standard deviation.

To find the theoretical standard deviation, we note that the ratio $r$ is a function of the grand total of observed raw counts $N_i$ and the corresponding noise counts $n_i$:


We first assume that the noise is measured after every measurement of the raw, and later make a modification for taking $q$ consecutive raw measurements between every noise measurement. For purposes of simplifying the calculation of $\sigma_r$, we note that for the field orientation of $E_0$ or $E_\pi$, the field reversed cases shown in Fig. 4.1, the form of the ratio is

$$r_{0,\pi} = \frac{1}{2} \left( \frac{N_A - n_A}{N_B - n_B} + \frac{N_C - n_C}{N_D - n_D} \right) \quad (4.13)$$

where each of the statistical quantities $N_i$ and $n_i$ are the sums $(N_i)_0 + (N_i)_\pi$ and $(n_i)_0 + (n_i)_\pi$. Writing (4.13) as

$$r' = \frac{1}{2} (r_1 + r_2),$$

with $\partial r / \partial r_1 = \partial r / \partial r_2 = \frac{1}{2}$, the standard deviation $\sigma_r$ is
\[ \sigma_r^2 = \frac{1}{4} (\sigma_{r_1}^2 + \sigma_{r_2}^2) \]  

(4.14)

and for \( r_i = (N_A - n_A)/(N_B - n_B) \),

\[ \sigma_{r_1} = r_1 \sqrt{\frac{\sigma_{N_A-n_A}^2}{N_A-n_A} + \frac{\sigma_{N_B-n_B}^2}{N_B-n_B}} \]  

(4.15)

where

\[ \sigma_{N_A-n_A} = \sqrt{\sigma_{N_A}^2 + \sigma_{n_A}^2} = \sqrt{N_A + n_A}, \text{ and} \]  

(4.16a)

\[ \sigma_{N_B-n_B} = \sqrt{N_B + n_B}. \]  

(4.16b)

On substitution into (4.15), we get

\[ \sigma_{r_1} = r_1 \sqrt{\frac{N_A+n_A}{(N_A-n_A)^2} + \frac{N_B+n_B}{(N_B-n_B)^2}} \]

Substituting \( \sigma_{r_1} \) together with a similar expression for \( \sigma_{r_2} \) into (4.14) gives

\[ \sigma_r = \frac{r}{2} \sqrt{\frac{N_A+n_A}{(N_A-n_A)^2} + \frac{N_B+n_B}{(N_B-n_B)^2} + \frac{N_C+n_C}{(N_C-n_C)^2} + \frac{N_D+n_D}{(N_D-n_D)^2}} \]  

(4.17)

Here, \( r_1 = r_2 = r \) is assumed. We now make allowance for measuring noise once every \( q \) measurements. The grand total of noise \( n_i \) is therefore an inferred quantity, and the standard deviation in \( n_i \) is \( \sqrt{q} \) greater than that stated in Eq. 4.16a, which modifies to

\[ \sigma_{N_A-n_A} = \sqrt{\sigma_{N_A}^2 + (\sqrt{q} n_A)^2} = \sqrt{N_A + qn_A} \]  

(4.18)

Hence the expression for \( \sigma_r \) is now
\[
\sigma_r = \frac{r}{2} \sqrt{\frac{N_A + qn_A}{(n_A - n_A)^2} + \frac{N_B + qn_B}{(n_B - n_B)^2} + \frac{N_C + qn_C}{(n_C - n_C)^2} + \frac{N_D + qn_D}{(n_D - n_D)^2}}
\]

(4.19)

The same expression as 4.19 holds for a ratio measured with the field of \( E_{\pi/2} \) or \( E_{3\pi/2} \), where instead of the ratio in Eq. 4.13, we have

\[
r_{\pi/2, 3\pi/2} = \frac{1}{2} \left( \frac{N_B - n_B}{n_A} + \frac{N_D - n_D}{n_C} \right)
\]

(4.13a)

Since the average ratio is \( \bar{r} = \frac{1}{2}(r_{0, \pi} + r_{\pi/2, 3\pi/2}) \),

the corresponding expected standard deviation is

\[
\sigma_r^2 = \frac{1}{4} \left( \sigma_{r_{0, \pi}}^2 + \sigma_{r_{\pi/2, 3\pi/2}}^2 \right)
\]

(4.20)

For the purpose of choosing a value for \( q \), the number of consecutive raw measurements, we examine the error in \( N_A - n_A \) given by Eq. 4.18. We note that the value for \( q \) should be such that the error in \( N_A \) and in \( n_A \) is comparable:

\[
\sigma_{N_A}^2 = (r q n_A)^2
\]

Hence, \( q \) should equal the raw to noise ratio \( N_A/n_A \), which from experiment is \( \approx 170 \). However, because the noise slowly decreases in time due to the corresponding fall in the background pressure, it needs to be measured much more frequently. We choose \( q=5 \), since in such an interval, the pressure variations both in the observation chamber and in the gas cell are small.

The theoretical standard deviation as given by 4.20 has a statistical error \( \delta \sigma_r \). This error is related to the number
of measurements, \( N_{\text{meas}} \) in which \( \sigma_F \) has been obtained and is given by (Miller and Freund, 1985).

\[
\delta \sigma_F = \frac{\sigma_F}{\sqrt{2N_{\text{meas}}}}.
\]

IV.E Runs test

The runs test provides an assurance about an observed sample of measurements as being random. In this test, a "high" (or "low") run contains a sequence of values all of magnitude larger (or smaller) than the average value. The total number of runs for \( n \) trials gives a statistic which tests whether or not the arrangement of the high and low ratios is random. For instance in a sample of 20 measurements, if there were 10 high values followed by 10 low values (2 runs), then we might suspect the probability of obtaining high (or low) runs as not remaining constant. On the other hand if we only had alternating high and low values for the entire sample of 20 trials (20 runs) then we might suspect that the trials were not independent.

Runs may also be categorised in run lengths; the total number of consecutive high (or low) measurements is called the run length, \( i \). From statistics, the number of runs of a given run length \( i \) is asymptotically normally distributed. For \( n \) trials, drawn from a random population with probability \( e_1 \) of falling above the mean and \( e_2 \) below, \((e_1 + e_2 = 1)\), the expected number of high runs of length \( i \) is asymptotically (Mood, 1940):
\[ s_{11} = n e_1^2 e_2 \]  
\[ \text{(4.21a)} \]

with variance
\[ \sigma_{11}^2 = n (e_1^{21-1} e_2^3 (e_1^{21} e_2 - 1) e_1^2 e_2^2 + e_1 e_2) \]  
\[ \text{(4.21b)} \]

For our purposes, the distribution of observed values is symmetric, so that \( e_1 = e_2 = \% \).

IV.F Elimination of detector polarization effects from the measured Lamb shift ratio

The radiation to be detected is polarized in the electric field direction. Although the detectors have no expected sensitivity owing to the cylindrical symmetry of their conversion dynodes (see Sec. III.G.2), small sensitivities can affect our results as we will see below. To remove the resulting systematic error, the following procedure is used.

After performing one half the total number of measurements with the conversion dynodes at a given fixed position, \( \theta_c = 0^\circ \) (see illustration in Fig. 4.2), the second half of the experiments are performed with all four conversion dynodes rotated about their detection axes through 90 degrees of their original positions, \( \theta_c = 90^\circ \). In this way, each detector's predominant polarization axis (if any) goes through a 90 degree rotation while the polarization of the incident radiation remains fixed.

For the purposes of this analysis, we consider two counters, A and B along the \( \hat{z} \) and \( \hat{x} \) directions respectively. We assume that detector A is insensitive to polarization, but
FIG. 4.2. Diagram showing positions of detectors A and B viewing the radiation parallel and perpendicular to \( \hat{E} \) respectively. Two experiments are performed: one with conversion dynodes at \( \theta_c = 0^\circ \) and the other with the conversion dynodes rotated by \( 90^\circ \) about the viewing axes. The \( \theta_c = 0^\circ \) position is marked on each detector prior to installation. The intensities along the three axes are described in the text.

B has unit sensitivity for light polarized in the \( \hat{y} \) direction and a sensitivity of \((1+\delta)\) for light polarized in the \( \hat{z} \) direction. As an example, we have that if \( \delta = -1 \), this detector is only sensitive to light polarized in the \( \hat{y} \) direction. For an electric field along the \( z \) axis, the amplitudes of a radiating dipole at the origin are \( \hat{x}A' \), \( \hat{y}A' \) and \( \hat{z}A \), as shown in Fig. 4.2. The intensities entering the solid angles of detectors A and B are

\[
I_\parallel = A'^2 + A'^2, \quad \text{and}
\]
\[ I_\perp^0 = A^2 + A_\perp^2. \]

Introducing as intensities, the notation \( a = A^2 \) and \( b = A_\perp^2 \), the ratio \( r_0 = \frac{I_\parallel^0}{I_\perp^0} \) becomes

\[ r_0 = \frac{2b}{a+b}. \]

The outputs for the polarization sensitive detectors are

\[ I_{A\parallel} = 2b \quad \text{and} \quad I_{B\perp} = (1+\delta)a + b, \]

and the observed ratio is then

\[ r(E_0) = \frac{2b}{(1+\delta)a + b}. \quad (4.22) \]

Next, we rotate the electric field to \( \hat{E}_{\parallel}/2 \), and the observed ratio will be

\[ r(E_{\parallel}/2) = \frac{(1+\delta)b + b}{a + b}. \quad (4.23) \]

We eliminate the detector efficiencies by finding a geometric mean of 4.22 and 4.23, as described in Sec. IV.B:

\[ r(\theta = 0) = \sqrt{\left( \frac{2b}{(1+\delta)a + b} \cdot \frac{b(2+\delta)}{a + b} \right)} \]

\[ \cong r_0 \left( 1 + \frac{\delta(b-a)}{4(a+b)} + \frac{\delta^2}{32(a+b)^2} \left( 7a^2 - b^2 - 6ab \right) \right) \quad (4.24) \]

to second order in \( \delta \). As the detectors are rotated as specified to \( \theta = 90^\circ \), detector A is unaffected, but detector B now has sensitivities of 1 and \( (1+\delta) \) for light polarized in directions \( \hat{z} \) and \( \hat{y} \) respectively. The ratio obtained now will be
\[ r(\theta_c = 90^\circ) \approx r_0 \left(1 - \frac{3(\delta^2)}{4(a+b)} + \frac{\delta^2}{32(a+b)^2} (-a^2 + 7b^2 - 6ab) \right) \] (4.25)

Taking for \( r_0 \), the expected value of 1.2677 for our field, and evaluating for \( a \) and \( b \), equations 4.24 and 4.25 are:

\[ r(\theta_c = 0^\circ) = r_0 \left(1 + 0.0676 - 0.0276^2 \right) \] (4.24a)

\[ r(\theta_c = 90^\circ) = r_0 \left(1 - 0.0676 + 0.0406^2 \right) \] (4.25a)

On averaging the ratios as proposed, we eliminate the first order effect entirely. The experimental finding for the maximum difference of the individual-counter ratios from the mean is \( \overline{\delta} = \delta \leq 3 \times 10^{-4} \). This corresponds to \( \delta \leq 4 \times 10^{-3} \), and it indicates that the remaining quadratic term will produce an effect of only 0.1 parts per million in the ratio.

### IV.G Dead time correction

The measured photon counts are affected by the existence of a dead time \( T \) in the counting system. To correct for this, we first note that the magnitude of the dead time for any one of the four counting systems \( T \approx 56 \times 10^{-9} \) sec (values shown in Sec III.G.3) is much smaller than the average time between pulses, \( T/n \approx 1 \times 10^{-4} \) sec. This implies that the probability for missing any more than one pulse is extremely small. Hence the fractional loss of counts from dead time is \( \delta n/n = Tn/T \). The ratio in the absence of dead time is

\[ r = \left(\frac{n_1 n_2'}{n_2 n_1'}\right)^{1/2} \] (4.26)

for two counters 1 and 2, where \( n_1 \) and \( n_2 \) are measured with
field $E_0$ and $n'_1$ and $n'_2$ are measured with field $E_{3x/2}$. Hence, with dead time, the ratio is

$$r_{\text{obs}} = \left[ \frac{n_1 (1 - T n_1 / T) n'_2 (1 - T n'_2 / T)}{n_2 (1 - T n'_2 / T) n'_1 (1 - T n'_1 / T)} \right]^{1/2}$$

$$= \left( \frac{n_1 n'_2}{n_2 n'_1} \right)^{1/2} \left[ 1 - \frac{1}{2} (n_1 + n'_2 - n'_1 - n_2) T / T \right]$$

$$= r \left[ 1 - \frac{1}{2} (n_1 + n'_2 - n'_1 - n_2) T / T \right]$$

Substituting the above into the expression for the asymmetry $A_{\text{obs}} = (r_{\text{obs}} - 1) / (r_{\text{obs}} + 1)$, we get

$$A_{\text{obs}} = \frac{\sigma}{(r-1)} \left[ 1 - \frac{(r T)}{2 T (r-1)} (n_1 + n'_2 - n'_1 - n_2) \right]$$

$$= \frac{\sigma}{(r-1)} \left[ 1 - \frac{(r T)}{2 T (r-1)} (n_1 + n'_2 - n'_1 - n_2) \right]$$

$$= A \left[ 1 - \frac{r T}{2 T} (n_1 + n'_2 - n'_1 - n_2) \left( \frac{1}{r-1} - \frac{1}{r+1} \right) \right]$$

But $n_1 / n_2 = r$ and $n'_2 / n'_1 = r$, thus

$$(A - A_{\text{obs}}) / A = \frac{T (n_1 + n'_2)}{T (r+1)}$$

Making the substitutions

$$n_1 = (n_1 + n'_2) n_1 / (n_1 + n_2) = (n_1 + n_2) r / (r+1)$$

and

$$n'_2 = (n'_1 + n'_2) r / (r+1)$$

we get $\delta A / A = T / T (n_1 + n_2 + n'_1 + n'_2) r / (r+1)^2$.

Noting that

$$1 - A^2 = 4 r / (r+1)^2$$

we get the final expression for the dead time correction:

$$\delta A / A = \frac{1}{2} \frac{T}{T} \langle n \rangle (1 - A^2),$$

(4.29)
where \( \langle n \rangle = \frac{1}{2} (n_1 + n_2 + n'_1 + n'_2) \). \hspace{1cm} (4.30)

Note that \( \langle n \rangle / T \) is the sum, for two counters 1 and 2, of their average count rates and the correction is directly proportional to the count rate. Therefore, knowing only these average count rates, the dead time correction can be applied to all measurements of a run simultaneously. To maintain full accuracy, the correction is made separately on the four run-means:

\[ r_{AB}, r_{BC}, r_{CD}, \text{ and } r_{DA} \]

(between pairs of counters AB, BC, CD and DA), taking for dead time \( T \), the corresponding pair-averages of dead times:

\[ T_{AB} = (55.08 \pm 0.07) \text{ ns} \]
\[ T_{BC} = (54.93 \pm 0.07) \text{ ns} \]
\[ T_{CD} = (56.05 \pm 0.07) \text{ ns} \]
\[ T_{DA} = (56.20 \pm 0.07) \text{ ns} \] \hspace{1cm} (4.31)

IV.H Solid angle correction

The ratio \( I_{\parallel}/I_{\perp} \), observed with detectors of a finite solid angle, differs from the "ideal" one for infinitesimally small solid angle. An expression for this correction will now be derived with the aid of Fig. 4.3. Here, the beam travels along the \( y \) axis, and the direction of the applied electric field is along the \( z \) axis. A radiating dipole at the origin has amplitudes \( \hat{x}A', \hat{y}A' \) and \( \hat{z}A \). A radius vector \( \vec{r} \) is associated with any observation point \( P(x, y, z) \). Only the projection of the amplitudes perpendicular to \( \vec{r} \) contribute to the
FIG. 4.3. Diagram illustrating two observation regions, 1 and 2, on the x and z axes. The electric field is along the z axis and the amplitudes from a radiating dipole at the origin are $\hat{z}A'$, $\hat{y}A'$ and $\hat{z}A$.

Intensity $I(x,y,z)$ at P:

$$I(x,y,z) = \frac{1}{4\pi l^2} \left[ (\hat{z}A')^2 + (\hat{y}A')^2 + (\hat{z}A)^2 \right]$$

$$= \frac{1}{4\pi l^2} \left[ A^2(x^2 + y^2) + A'^2(x'^2 + y'^2 + 2z^2) \right]$$

If $x', y', z'$ is a point inside the source then

$$I(x,y,z) = \frac{1}{4\pi l^2} \left[ A^2(x-x')^2 + (y-y')^2 \right.$$

$$\left. + A'^2(x-x')^2 + (y-y')^2 + 2z(z-z')^2 \right]$$

$$= \frac{1}{4\pi l^2} \left[ A^2 + A'^2 + (A^2 - A'^2) \left( \frac{(z-z')^2}{l^2} \right) \right]$$

(4.32)
To second order approximation, at position 1:

\[
\frac{1}{l_{r-r'}^2} \propto \frac{1}{x^2} \left[ \frac{1 + \frac{2x'}{x} + \frac{3x^2}{x^2} - \frac{(y-y')^2}{x^2}}{x} \right] \left[ \frac{1 + \frac{2y'}{y} - \frac{(z-z')^2}{z^2}}{y} \right]
\]

\[
\frac{1}{l_{r-r'}^4} \propto \frac{1}{x^4} \left[ \frac{1 + \frac{4x'}{x} + \frac{10x^2}{x^2} - \frac{2(y-y')^2}{x^2}}{x^2} \right] \left[ \frac{1 + \frac{2y'}{y} - \frac{(z-z')^2}{z^2}}{y} \right]
\]

Therefore the intensity for the detector at 1 is

\[
I_1(x,y,z) = \frac{1}{x^2} \left[ (A^2 + A'^2) \left( \frac{1 + \frac{2x'}{x} + \frac{3x^2}{x^2} - \frac{(y-y')^2}{x^2} - \frac{(z-z')^2}{z^2}}{x} \right) \right. \]

\[
\left. + \left( A^2 - A'^2 \right) \frac{(z-z')^2}{x^2} \right]
\]  \hspace{1cm} (4.33)

At position 2, also to second order,

\[
\frac{1}{l_{r-r'}^2} \propto \frac{1}{z^2} \left[ \frac{1 + \frac{2z'}{z} + \frac{3z^2}{z^2} - \frac{(y-y')^2}{z^2} - \frac{(x-x')^2}{z^2}}{z} \right]
\]

\[
\frac{1}{l_{r-r'}^4} \propto \frac{1}{z^4} \left[ \frac{1 + \frac{4z'}{z} + \frac{10z^2}{z^2} - \frac{2(y-y')^2}{z^2}}{z^2} \right] \frac{2(x-x')^2}{z^2}
\]

\[
(z-z')^2 = z^2 \left( 1 - \frac{2z'}{z} + \frac{z'^2}{z^2} \right).
\]

The intensity for the detector at position 2 is

\[
I_2(x,y,z) = \frac{1}{z^2} \left[ (A^2 + A'^2) \left( \frac{1 + \frac{2z'}{z} + \frac{3z^2}{z^2} - \frac{(y-y')^2}{z^2} - \frac{(x-x')^2}{z^2}}{z} \right) \right. \]

\[
\left. + \left( A^2 - A'^2 \right) \left( \frac{1 + \frac{2z'}{z} + \frac{3z^2}{z^2} - \frac{(y-y')^2}{z^2} - \frac{(x-x')^2}{z^2}}{z} \right) \right]
\]  \hspace{1cm} (4.34)

The intensities at positions 1 and 2 must now be integrated as a function of displacement over the source, and then over the solid angle observed through the photon collimator (see Fig. 3.4). At position 1, the integral to solve is of the form
\[
f = \frac{1}{x^2} \left[ 1 + \frac{ax^2}{y'^2} + \frac{b(y-y')^2}{x^2} + \frac{c(z-z')^2}{x^2} \right] \tag{4.35}
\]

where \( a = 3(A_r^2 + A_l^2) \),
\( b = -(A_r^2 + A_l^2) \) and
\( c = -(A_r^2 + A_l^2) + (A_r^2 - A_l^2) \) \tag{4.36}

To integrate over the source, we make the substitutions
\( x' = r' \cos \theta' \), \( z = r' \sin \theta' \) and \( dx'dz' = r'dr'd\theta' \):

\[
\bar{f} = \int_0^p r'dr' \int_0^{2\pi} f d\theta',
\]

where \( p \) is the radius of the beam. The integral w.r.t. \( d\theta' \)

\[
\frac{1}{x^2} \int_0^{2\pi} \left[ 1 + \frac{ar' \cos^2 \theta'}{x^2} + \frac{b(y-y')^2}{x^2}
+ \frac{c(z^2-2zr' \cos \theta' + r'^2 \cos^2 \theta')}{x^2} \right] d\theta',
\]

is

\[
= \frac{\pi}{x^2} \left[ 2 \frac{ar^2}{x^2} + \frac{2b(y-y')^2}{x^2} + \frac{2cz^2}{x^2} + \frac{cr'^2}{x^2} \right] \tag{4.37}
\]

Now we integrate w.r.t. \( r'dr' \):

\[
\frac{\pi}{x^2} \left[ 2r' + \frac{ar'^2}{x^2} + \frac{2r'b(y-y')^2}{x^2} + \frac{2r'cz^2}{x^2} + \frac{cr'^3}{x^2} \right] dr',
\]

\[
= \frac{\pi}{x^2} \left[ \frac{p^2}{x^2} + \frac{p^2(a+c)}{4x^2} \right] + \frac{p^2(b(y-y')^2 + cz^2)}{x^2},
\]

\[
\bar{f} = \frac{\pi p^2}{x^2} \left[ 1 + \frac{b(y-y')^2 + cz^2}{x^2} \right] + \frac{p^2(a+c)}{4x^2} \tag{4.38}
\]

Next for a given \( \delta y \), we integrate over \( dy' \) for the width of
the rectangular entrance slit \( S_1 \). The limits of integration,
\( y' = (x-y)t+y \) to \( y' = -(x+y)t+y \)
are determined by geometry for any point displaced by \( y \) from the centre of the detector exit slit \( S_{2a} \) as shown in Fig. 4.4. Here \( t = (c+d)/d \). Putting \( \eta = y' - y \), \( \delta y dy' = \delta y d\eta \), we have

\[
\int_{-t(a+y)}^{t(a-y)} \eta^2 d\eta = \frac{2}{3} (at)t(a^3 + 3ay^2).
\]

Hence, \( \bar{f}(y,z) = \int_{-t(a+y)}^{t(a-y)} f d\eta \)

\[
= \frac{xp^2}{x^2} \int \left[ 1 + \frac{b\eta^2}{x^2} + \frac{cz^2}{x^2} + \frac{(a+c)p^2}{4x^2} \right] d\eta
\]

\[
= \frac{xp^2}{x^2} 2at \left[ 1 + \frac{bt^2}{3x^2} (a^2+3y^2) + \frac{cz^2}{x^2} + \frac{(a+c)p^2}{4x^2} \right] \quad (4.39)
\]

Finally we integrate \( \bar{f}(y,z) \) over \( dy \) and \( dz \) for the circular exit slit \( S_2 \) of radius \( F \). The limits of integration are illustrated in Fig. 4.5.

\[
\bar{f} = \int_0^F dz \int_0^{\sqrt{F^2-z^2}} dy \bar{f}(y,z)
\]

\[
= \int_0^F dz \frac{xp^2}{x^2(2at)} \left[ 1 + \frac{cz^2}{x^2} + \frac{(a+c)p^2}{4x^2} + \frac{bt^2x^2}{x^2} \right](F^2-z^2)^{1/2}
\]

\[
+ \frac{bt^2(F^2-z^2)^{3/2}}{3x^2} \quad (4.40)
\]

Here we need the integrals

\[
\int_0^F (F^2-z^2)^{1/2} dz = \frac{\pi p^2}{4}.
\]
Fig. 4.4. Geometry of the photon collimator, as shown in Fig. 3.4, and the integration limits for the calculation of the solid angle correction.

Fig. 4.5. The integration limits for the circular slit $S_2$ shown in Fig. 4.4.
\[ \int_0^F z^2 (F^2 - z^2)^{1/2} \, dz = \frac{xF^4}{16}, \text{ and} \]

\[ \int_0^F (F^2 - z^2)^{3/2} \, dz = \frac{3xF^4}{16}, \text{ giving} \]

\[ f = \frac{xP^2}{x^2 (2\alpha t)(\frac{F^2}{4})} \left[ 1 + \frac{(a+c)P^2}{4x^2} + \frac{bt^2z}{3x^2} + \frac{F^2(c+bt^2)}{4x^2} \right] \quad (4.41) \]

On substituting \( a, b \) and \( c \) from Eq. 4.35, the intensity \( I_1 \) integrated over the source and the solid angle is

\[ I_1 = (A' + A^2) \left[ 1 + \frac{P^2}{2x^2} - \frac{t^2z}{3x^2} - \frac{F^2(1+t^2)}{4x^2} \right] \]

\[ + (A' - A^2) \left[ \frac{P^2}{4x^2} + \frac{F^2}{4x^2} \right] \quad (4.42) \]

The integrated intensity at position 2, \( I_2(x,y,z) \) is found similarly, yielding

\[ I_2 = (A' + A^2) \left[ 1 + \frac{P^2}{2x^2} - \frac{t^2z}{3x^2} - \frac{F^2(1+t^2)}{4x^2} \right] \]

\[ + (A' - A^2) \left[ 1 + \frac{P^2}{4z^2} - \frac{2t^2z}{3z^2} - \frac{F^2(1+t^2)}{2z^2} \right] \quad (4.43) \]

The lowered asymmetry \( Q \), corrected for solid angle by using the integrated intensities of (4.42) and (4.43) is

\[ Q = \left( I_2 - I_1 \right) / \left( I_2 + I_1 \right) \]

\[ = \frac{(A' - A^2) \left[ 1 + \frac{2t^2z}{3r^2} - \frac{F^2}{4r^2} (3 + 2t^2) \right]}{2(A' + A^2) \left[ 1 + \frac{P^2}{2r^2} - \frac{t^2z}{3r^2} - \frac{F^2}{4r^2} (1 + 2t^2) \right] + (A' - A^2) \left[ 1 + \frac{P^2}{2r^2} - \frac{2t^2z}{3r^2} - \frac{F^2}{4r^2} (1 + 2t^2) \right]} \]
which after algebraic manipulations yields \( Q = \)

\[
\frac{(A'-2-A^2) \left[ 1 - \frac{2}{3} t \frac{2z}{r^2} \frac{F^2}{4r^2}(3+2t^2) \right]}{(3A'-2+A^2) \left[ 1 + \frac{4}{3r^2} \frac{t^2}{3r^2} \frac{F^2}{4r^2} (1+t^2) \right] + (A'-2-A^2) \left[ \frac{1}{3} \frac{2z}{r^2} \frac{F^2}{4r^2} t^2 \right]}
\]

\( (4.44) \)

Dividing the numerator and denominator by \((3A'-2+A^2)\) and defining the "ideal" asymmetry \( Q_0 = (A'-2-A^2) / (3A'-2+A^2) \), we get

\[
Q = Q_0 \left[ 1 - \frac{2}{3} t \frac{2z}{r^2} \frac{F^2}{4r^2} (3+2t^2) \right]
\]

\[
1 + \frac{1}{2} \frac{P^2}{r^2} - \frac{t^2 z}{3r^2} - \frac{F^2}{4r^2} (1+t^2) - Q_0 \left[ \frac{a^2 t^2}{3r^2} + \frac{F^2 t^2}{4r^2} \right]
\]

\( \sim Q_0 \left[ 1 - \frac{1}{2} \frac{P^2}{r^2} - \frac{(1-Q_0)}{3r^2} t ^2 \frac{a^2}{r^2} + \frac{F^2}{4r^2} (2+t^2) + \frac{Q_0}{4r^2} F^2 t^2 \right] \)

\( \sim Q_0 \left[ 1 - \frac{1}{2} \frac{P^2}{r^2} - \frac{(1-Q_0) t^2 a^2}{2r^2} + \frac{F^2}{3} - \frac{F^2}{2r^2} \right] \)

\( (4.45) \)

The last equation expresses the observed (lower) asymmetry \( Q \) relative to the ideal asymmetry \( Q_0 \). The input variables are summarised in Table 4.1.
TABLE 4.1. Input data for the solid angle correction

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>radius of beam</td>
<td>1/2(0.165±0.030)</td>
</tr>
<tr>
<td>r(-d+c)</td>
<td>distance from source to detector</td>
<td>(21.895±0.005)</td>
</tr>
<tr>
<td>c</td>
<td>distance from source to slit S₁</td>
<td>(7.112±0.002)</td>
</tr>
<tr>
<td>α</td>
<td>half slit width of S₁</td>
<td>1/2(0.635±0.001)</td>
</tr>
<tr>
<td>F</td>
<td>radius of exit slit S₂</td>
<td>1/2(1.016±0.002)</td>
</tr>
<tr>
<td>t</td>
<td>= (c+d)/d = r/(r-c)</td>
<td>see above</td>
</tr>
</tbody>
</table>
CHAPTER V

COMPUTER CONTROL FOR DATA COLLECTION

V.A Outline

A listing of the program used by the Lamb shift experiment is shown in Fig. 5.1. Briefly, the computer sets the appropriate electric field in the quadruple, resets the photon counters, and waits for the time-up when the photon counts are received. For a single measurement, this process is repeated for the four electric field orientations. Every one in five raw measurements also measures the background. At the end of a measurement, the acquired data is partially analysed, and displayed on the monitor. For the purposes of later re-analysis and statistical tests, the data is printed out on a printer, and also, saved to a "file" on a magnetic disk. The next measurement starts at the conclusion of this activity, unless interrupted by the user. An ongoing statistical analysis, updated after every measurement is displayed on the monitor.

The next section describes the electronic interfaces in detail and shows the programming required for these interfaces. Following that, a description of the entire program is given, emphasising the sequence of events in taking a measurement.
Fig. 5.1. Listing of the BASIC program for the Lamb shift measurement. Remarks have been added for readability, and Commodore 64 screen characters such as (right), (white), etc have been written within curly brackets.

10 GOTO 2000

================================== obtain counts from RS232 channel ==================================
50 FOR T=1 TO 600: NET T: REM pause for relays to settle
55 GOSUB 1000: REM reset counters and start clock
60 IF PEEK(PB)<$PV THEN POK PB,PV: GOTO 50: REM check for correct relay setting
65 GET Z3$: IF Z3$="*" THEN Z1$=Z3$: GOTO 99: REM look for keys pressed at keyboard, if space-bar then halt
70 IF Z3$="(1)" THEN Z1$=Z3$: PRINT "(right)X": REM save request to stop after measurement
75 IF Z3$="R" THEN GOSUB 5005: GOTO 55: REM reset counters if "R" pressed
80 IF (TI-TI)TL THEN GOSUB 5005: GOSUB 5005: GOTO 55: REM if counting for too long, limit TL=2400 jiffies
85 GET Z2,Z9$: IF Z9$="O" THEN GOSUB 65: REM look for a numeric character from the RS232 channel, if present, counts are up
90 T=PEEK(CH): INPUT Z2,Z9$: Z2=Z3+Z2$: IF LEN(Z2$)>24 THEN Z3$="": GOTO 55
95 REM stop hardware clock, obtain remaining data, check if data is all there
99 RETURN

================================== FOR RAM: inter time, display counts, save counts in array I ===========================
100 NO=PEEK(CH): SD=PEEK CS: T0=PEEK CT: MI=NO: SI=SO: T1=T0: REM Keep 80 information, set the clock next time with T1 info
105 STR$=STR$(A): PRINT "(white)";SPC(7-LEN(STR$));STR$(T1); REM format output and display on screen (right justify for neatness)
110 STR$=STR$(B): PRINT SPC(7-LEN(STR$));ST$(T2); REM format output and display on screen
115 STR$=STR$(C): PRINT SPC(7-LEN(STR$));ST$(T3); REM format output and display on screen
120 STR$=STR$(D): PRINT SPC(7-LEN(STR$));ST$(T4);"(black)"
125 IF L$ THEN X(L,1)=A: X(L,2)=B: X(L,3)=C: X(L,4)=D: RETURN: REM L is the current field position
130 L9=9-L: X(L9,1)=X(L9,1)+A: X(L9,2)=X(L9,2)+B: REM if L is past 4 then the field positions are being repeated for "round 2", add up the "round 2" values to "round 1"
135 X(L9,3)=X(L9,3)+C: X(L9,4)=X(L9,4)+D: RETURN

================================== FOR NOISE: display counts, save counts in array I ================================
140 STR$=STR$(A): PRINT "black)";SPC(8-LEN(STR$));ST$(T1); REM noise displayed in black
145 STR$=STR$(B): PRINT SPC(7-LEN(STR$));ST$(T2); REM this is similar to RAW above
150 STR$=STR$(C): PRINT SPC(7-LEN(STR$));ST$(T3); REM this is similar to RAW above
155 STR$=STR$(D): PRINT SPC(7-LEN(STR$));ST$(T4);"(black)"
160 IF L$ THEN X(L,2)=A: X(L,4)=B: X(L,6)=C: X(L,8)=D: RETURN
165 L9=9-L: X(L9,2)=X(L9,2)+A: X(L9,4)=X(L9,4)+B
170 X(L9,6)=X(L9,6)+C: X(L9,8)=X(L9,8)+D: RETURN

================================== reset counters, and start clock with previous time ================================
180 POKE PB,PV+4: POKE CH,0: POKE CS,MI: POKE CS,S1: POKE PB,PV: POKE CT,T1: E1=T1: RETURN
**Fig. 5.1 — continued**

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200 L=1: PRINT LEFT$(D$N$2)"(black)"*: GOSUB 601: GOSUB 100: REM (L is field #), display "1" on screen for this field, get counts from RS232 and save counts in array I
230 IF Z14="* THEN 3030: REM check for space-bar press (halt measurement)

--- raw field 4 ---

231 PV=144: POKE PB,PV: FR=D=1 TO 600: NEXT T: GOSUB 180: REM next field=4, pause and reset counters
232 IF (WA(2) AND (W$5(2)) THEN 239:
233 IF W$5=2 THEN 236:
234 FOR K=1 TO 3: FOR K=1 TO 7 STEP 2:
235 PRINT#4, OLD(4,K): NEXT K: PRINT#4: NEXT J: WA(1)=GOTO 239
236 FOR J=1 TO 3: FOR K=1 TO 6: REM print raw only
237 PRINT#4, OLD(4,K): NEXT K: PRINT#4: NEXT J: WA(1)=
239 L=4: PRINT LEFT$(D$N$4)"*": GOSUB 601: GOSUB 100:
240 IF Z14="* THEN 3030

--- raw field 2 ---

241 PV=60 : POKE PB,PV: FR=D=1 TO 600: NEXT T: GOSUB 180: REM next field=2, pause and reset counters
242 IF (WA(0) AND (W$5=0) THEN 249:
243 IF W$5=1 THEN 245:
244 FOR K=1 TO 7 STEP 2: PRINT#4, OLD(4,K): NEXT K: WA(1)=GOTO 246:
245 FOR K=1 TO 6: PRINT#4, OLD(4,K): NEXT K: WA(1)=
246 PRINT#4,R$:R$: R$:M,S$: VR\$VS\$VY3
247 GOSUB 4601: PRINT#4,"A,B,C,D: "*CA; CB; CC; CD*
249 L=2: PRINT LEFT$(D$N$4)"*": GOSUB 601: GOSUB 100
250 IF Z14="* THEN 3030

--- raw field 3 ---

251 PV=96 : POKE PB,PV: FR=D=1 TO 600: NEXT T: GOSUB 180: REM next field=3, pause and reset counters
252 IF W$3=0 THEN 259:
253 PRINT: PRINT *(crsOnlDISK BUSY": GOSUB 5005: CLOSE: D=1=D=1: REM yes..close old one, increment tag character-ascii code in DI
254 OPEN B,B,B,B,0: "MM+CHR(DI)"","S,M" : INPUTS,AD$,BL$,CL$,DD$:REM open new file and check disk status
255 IF AD$="00" THEN 6000 REM trouble if not "00" (then) go to disk error handling at 6000
256 PRINT *(up)": M$=FILE OPEN: "MM+CHR(DI)+STR$(EL)+SEC": REM no error..erase disk busy
257 W$=0 : REM reset W$ flag back to 0 (disk duties done)
259 L=3: PRINT LEFT$(D$N$6,5)"*": GOSUB 601: GOSUB 100 : REM get counts and save in array I
260 IF Z14="* THEN 3030

--- raw field 3 again, round 2 ---

261 GOSUB 180: GOSUB 750: PRINT LEFT$(D$N$10)"*": REM field 3 relays still on, just reset counters,
find (individual) ratios in CA, CB, CC and CD and display...
262 ST=ST+CA(CA); PRINT LEFT$(ST,7); ST=ST+CC(CD); PRINT LEFT$(ST,7);  
263 ST=ST+CC(CC); PRINT LEFT$(ST,7); ST=ST+CC(CD); PRINT LEFT$(ST,7);  
264 ST=ST+CC( CA$B$C$); PRINT LEFT$(ST,9); IF W$6=0 THEN 265 REM is statistics in Gt updated?
265 SM=SM+$; G$D$=D$+1; D$=D$+1; VR$=VR$+1; IF SM=1 THEN 63=SM=ABS((D$-(6$G$D$G$)/I(GM-1)))) REM no..update it
266 PRINT LEFT$(D$N$12) TAB(31)"*"; G$=G$D$; REM display the n, mean, std in Gt (since last stopping)
267 ST=ST+GS(G$GN$); PRINT LEFT$(D$N$12) TAB(29) LEFT$(ST,9);  
268 ST=ST+GS(G$); PRINT LEFT$(D$N$14) TAB(29) LEFT$(ST,5);RIGHT$(ST,3); W$6=0
269 L=6: PRINT LEFT$(D$N$13)"*": GOSUB 601: GOSUB 100
270 IF Z14="* THEN 3030
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Fig. 5.1 - continued

---------- raw field 2 again, round 2 ----------
271 PV=80 : POKE PB, PV; FOR T=1 TO 600: NEXT T: GOSUB 180
275 L=7: PRINT LEFT$(DM,15)**2*$; GOSUB 60: GOSUB 100
280 IF Z$=" " THEN 3030

---------- raw field 4 again, round 2 ----------
281 PV=144: POKE PB, PV; FOR T=1 TO 600: NEXT T: GOSUB 180
285 L=5: PRINT LEFT$(DM,11)**4*$; GOSUB 60: GOSUB 100
290 IF Z$=" " THEN 3030

---------- raw field 1 again, round 2 ----------
291 PV=160: POKE PB, PV; FOR T=1 TO 600: NEXT T: GOSUB 180
297 IF WZ=0 THEN 299 : REM is meas# (W) been printed?
299 INT(N/9)=N/9 THEN PRINT$4, R$(R$): REM skip over page perforation
304 PRINT$(CHR$(14)) ; N+1:CHR$(15):MS$: REM print N in double width, and message in MS
305 MS$=""; WZ=0 : REM set new message to null, and turn switch WZ off
309 L=8: PRINT LEFT$(DM,17)**1*$; GOSUB 60: GOSUB 100: REM get counts and save in array I
310 IF Z$=" " THEN 3030

---------- Raw measured, noise next? ----------
305 IF WZ=0 THEN 380: REM skip over noise section if not to be measured

---------- noise field 1 ----------
306 PV=160: POKE PB, PV; FOR T=1 TO 600: NEXT T: POKE PB,172; POKE PB,168: GOSUB 750 :REM turn HV on and
field l, reset counters, calculate (individual) ratios again and display..
307 PRINT LEFT$(DM,19)**(white)**;
308 ST$=STR$(ST$); ST$=STR$(ST$); PRINT LEFT$(ST$); ST$=STR$(ST$); PRINT LEFT$(ST$);
310 ST$=STR$(ST$); PRINT LEFT$(ST$); ST$=STR$(ST$); PRINT LEFT$(ST$);
312 ST$=STR$(ST$); PRINT LEFT$(ST$); ST$=STR$(ST$); PRINT LEFT$(ST$);
314 ST$=STR$(ST$); PRINT LEFT$(ST$); ST$=STR$(ST$); PRINT LEFT$(ST$);
316 L=1: PRINT LEFT$(DM,3)**; GOSUB 55: GOSUB 150 :REM get raw counts and save in array I
315 IF Z$=" " THEN 3030

---------- noise field 4 ----------
320 L=4: PRINT LEFT$(DM,9)**; PV=152: POKE PB, PV; GOSUB 50: GOSUB 150
325 IF Z$=" " THEN 3030

---------- noise field 2 ----------
330 L=2: PRINT LEFT$(DM,5)**; PV=88: POKE PB, PV; GOSUB 50: GOSUB 150
335 IF Z$=" " THEN 3030

---------- noise field 3 ----------
340 L=3: PRINT LEFT$(DM,7)**; PV=104: POKE PB, PV; GOSUB 50: GOSUB 150
345 IF Z$=" " THEN 3030

---------- noise field 3 again, round 2 ----------
355 IF Z$=" " THEN 3030

---------- noise field 2 again, round 2 ----------
360 L=7: PRINT LEFT$(DM,16)**; PV=88: POKE PB, PV; GOSUB 50: GOSUB 150
365 IF Z$=" " THEN 3030

---------- noise field 4 again, round 2 ----------
370 L=5: PRINT LEFT$(DM,12)**; PV=152: POKE PB, PV; GOSUB 50: GOSUB 150
375 IF Z$=" " THEN 3030

---------- noise field 1 again, round 2 ----------
377 L=8: PRINT LEFT$(DM,18)**; PV=168: POKE PB, PV; GOSUB 50: GOSUB 150: PV=160: POKE PB, PV; REM HV off
379 IF Z$=" " THEN 3030
Fig. 5.1 - continued

380 GOSUB 600: Rem calculate ratios between pairs (raw)
381 IF X>Y THEN CX=ABS(R-US/X): PRINT: PRINT "(rsvm)"*R*CX*(rsvs)*"": IF (CX)<SUS3 THEN 700: REM: find the absolute difference between mean and ratio, if difference is greater than SSd then confirm
382 IF ZX<>** AND QZ0 THEN GOSUB 1505: Reset counters for next measurement (field still at 1)
383 N=M1; W2=1; IF INT(W/11)+W/11 THEN W2=1: increment no. of meas, N, and set switches for future printing and file renewal if needed (every 11 meas.)
385 IF N>1 THEN U3=SSR((ABS((U2-(US+US/N))/((N-1))))
386 PRINT "Rn, Rx, Rx, Rx, Rx, Rx": IF QX0 THEN W4=2: GOTO 620: REM: save N and Q to disk, branch to 620

---------- QX=0, raw & measurement ----------
390 W5=2: E1=T1: FOR J=1 TO 4: FOR K=1 TO 7 STEP 2: REM: save data to disk drive, add X array counts to Y array (holds total for run) and update OLD array with present I array for later printing, and subtract noise counts from raw
400 PRINT "Rn, Rx, Rx, Rx, Rx, Rx": IF QY=0 THEN W4=2: GOTO 620: REM: save data to disk drive, add X array counts to Y array (holds total for run) and update OLD array with present I array for later printing, and subtract noise counts from raw
402 OLD(J,K)=I(J,K)+I(J,K)+1
405 Y(J,K)=Y(J,K)+1
406 NEXT I
407 IF ZX<>** THEN 410: REM: if user request is pending, don't continue with next measurement
408 IF (TI-E1)<360 THEN 408: REM: wait for 6 seconds from the time at line 390 for HV to bleed to 0 volts
409 GOSUB 1505: REM: reset counters now

---------- find corrected ratios, instead of raw, X array holds signal ----------
420 IF N01 THEN V3=SSR((ABS((V2-(VS+VS/N))/((N-1))))
440 QZ=QZ1: IF QZ=5 THEN QZ=0: REM: cycle to next Q (QZ=0 is raw & noise, otherwise raw only)
445 PRINT "(clear)(rsvs)"X(black) A B C D "QZ:"(rsvs): REM: display results of ratios and means
450 PRINT LEFT("(mn,19)"*(yellow)-R--------MEAN--------SD-ERROR----(black)
452 ST=*STR(9): PRINT SPC(11-LEN(ST)): ST;
453 ST=*STR(9): PRINT SPC(12-LEN(ST)): ST;
460 PRINT "(yellow)-R--------"-"V3=SSR(W,N)"(black)
462 ST=*STR(9): PRINT SPC(11-LEN(ST)): ST;
464 ST=*STR(9): PRINT SPC(12-LEN(ST)): ST;
470 PRINT "(yellow)
472 E2=6041.62358(MAND240)+1(0AND15)+.62585(0AND240)+(0AND15)+.1(T0AND15): REM: change NO, SO and To binary coded decimal (BCD) values to normal, and find time taken for the measurement
473 E1=EL+E2: REM: elapse time so far for measuring raw
475 INPUT01,5,6D,6D,CD,DD: IF AD<>"00" THEN 6000: REM: disk status ok?
480 PRINT LEFT("(mn,24)"(black)(rsvs)"*(yellow)(rsvs)*"E2"SEC FILE: "MNCR(01): REM: display time taken, current filename and disk status
555 IF ZX<>** THEN GOSUB 600:GOTO 3030: REM: if user request pending, print out meas info, then to menu
560 GOTO 200: REM: otherwise continue loop again

---------- calculate ratios between pairs ----------
600 AR=.51SQR((X1,11/X1,3)+1(2,1)/X1,21/X1,31/X1,43/X1,41)
Fig. 5.1 - continued

620 FOR J=1 TO 4: FOR K=1 TO 7 STEP 2: PRINT#8,X(J,K);R6$: OLD(J,K)=I(J,K): REM save data to disk drive, add I array counts to Y array (holds total for run) and update OLD array with present I array for later printing and subtract noise counts from raw
630 Y(J,K)=Y(J,K)+I(J,K): X(J,K)=X(J,K)-X(J,K+1): Y(J,K+1)=Y(J,K+1)+X(J,K+1)
640 NEXT K: NEXT J: GOTO 410: REM continue at 410

700 REN OUT OF BOUNDS OF +/- 3+SD
710 GOSUB 5000: PRINT LEFT$(ODM,25)*(rvsOn)(white)R残留"R* ACCEPT (Y/N)"
715 INPUT ANS$: IF ANS$="Y" THEN 382
720 IF ANS$="N" THEN 3005
725 GOTO 700: REM INVALID ANSWER

740 REM COUNTERS INDEPENDENTLY, THIS MEANS
750 CA=(I(1,1)+I(2,1))/(I(2,2)+I(3,1)+I(4,1))
755 CB=(I(1,3)+I(4,3))/(I(1,1)+I(2,3))
760 CC=(I(1,5)+I(2,5))/(I(1,3)+I(4,5))
765 CD=(I(1,7)+I(4,7))/(I(1,7)+I(2,7))
770 RETURN

800 POKE PB,0 : REM INTERRUPT OCCURRED
810 IF W4>0 THEN 830
815 IF W5>0 THEN 850
820 GOTO 899

830 T=(I-W4)*(I-W4): FOR J=1 TO 4: FOR K=1 TO 7 STEP 2
840 PRINT#4,OLD(J,K);: NEXT K: PRINT#4: NEXT J: W4=0: GOTO 880
850 T=(I-W5)*(I-W5): FOR J=1 TO 4: FOR K=1 TO 8
860 PRINT#4,OLD(J,K);: NEXT K: PRINT#4: NEXT J: W5=0
880 PRINT#4,*A,B,C,D, CA;CB;CC;CD
890 GOSUB 4501: PRINT#4,*A,B,C,D, CA;CB;CC;CD
990 RETURN

1000 CLEAR: null variable holding user requests
1010 PRINT "clear"(rvsOff)(black) A B C D ";W1;"(rvsOn)";N+1;"(rvsOff)";
1020 GOSUB 1500: REM set field 1 and reset
1030 GOTO 200: REM enter the loop

1500 POKE PB,PV: FOR T=1 TO 900: NEXT T
1501 REM START AT 150S ALWAYS
1505 M1=0: S1=0: T1=0: GOSUB 180: RETURN: REM the clock will be set to 0.0 sec.
Fig. 5.1 - continued

================================ PROGRAM STARTS HERE =================================

2000 REM do quicker parts at the head of the program
2005 OPEN 2,2,0,CHR$(40)+CHR$(160); OPEN 4,4: OPEN 15,9,15: PRINT#4,CHR$(15);
2007 REM #2=RS232, #4-printer, #15-disk command/error 3 channels are opened
2010 POKE 53280,0: POKE 53281,8: REM screen and border colours
2020 POKE 56835,153: REM set mode on B255..mode 0, port B OUTPUT, A and C INPUT
2030 PB=56833: REM address of port B
2033 POKE 54294,200: POKE 54295,15: POKE 54296,31: POKE 54277,0: POKE 54278,240
2035 REM sound chip: fill freq, all filt'd, vol & low pass filter, AD=0, SR=240
2040 DW=*(home)+25down)*: R4=CHR$(13): REM DMA is used to go vertically down the screen
2050 DIM X(4,6): DIM Y(4,8): DIM OLD(4,8)
2052 CH=56587: CM=56586: CS=56585: CT=56584: MO=0: M1=0: SJ=0: S1=0: T0=0: T1=0: REM initialize
2055 FOR J=1 TO 4: FOR K=1 TO 8: X(J,K)=0: Y(J,K)=0: OLD(J,K)=0: NEXT K, J: REM clear arrays and these:
2056 I3%...holds user requests (Line 65),
2057 Z2% and Z9% hold received counter data, while A, B, C and D will hold that counter data (Line 75-90)
2058 E1 will hold start time counter reset, and TL the time limit, both in jiffies (Line 70, 180)
2059 ST4 will hold strings for print formatting,
2060 L and L9 are indices for field position (L=1 to 4, first round, and 5-8 when fields are repeated
2060 Z3 %= ; Z9 %= : Z2 %= : E1 %= : PV %= : A %= : B %= : C %= : D %= : ST4 %= : L %= : L9 %= TL=2400
2500 DI=65: REM ascii character code tagged to file name
2502 PRINT "(clear)(black)(down)(down)INSERT PROPER DISK FOR DATA STORAGE"
2504 PRINT "(down)(down)ENTER (UNIQUE) NAME OF FILE LIKE: " ; PRINT " AUSIS(white):" ; INPUTMM
2506 PRINT"(5)," T4 %= : REM "INITIALIZING REM DISK ID"
2507 INPUT1, A4%, D4%, C4%, B4%: IF AD4%="00" THEN 2522
2508 OPEN B,6,B,6: " +MM+%CHR$(DI)+",,M" : PRINT "-": REM open file for storage and check disk status
2520 INPUT1, AD4%, BS4%, DS4%, BD4%: IF AD4%="00" THEN 2530
2522 PRINT "(rvsOn)(down)(down) " ; ED4
2525 GOSUB 5000: PRINT "(black)(down)(down)CHECK DISK ERROR SHOWN ABOVE"
2527 PRINT "(down){} THEN RUN AGAIN BY TYPING: PRINT "(down)"RUN AND RETURN": END
2530 MSG=FILE OPEN: *+MM+%CHR$(DI): W2=1: REM W2 is set (reset) when message is to be (has been) printed

================================ MENU ================================================

3000 Q1=0: PRINT LEF$(DN%, 24)* (black) START WITH NOISE*: GOSUB 5005
3005 POKE PB,0: REM all relays off
3100 GET #2, Z2%: IF Z2% " THEN PRINT Z2%: INPUT#2, Z2%: PRINT Z2%: REM get data from RS232 and print
3105 if any, this keeps buffer empty
3105 R1=NOT(2): POKE 646,ABS(R1): PRINT LEF$(DN%, 25)* " (rvsOn) READY (black) (rvsOff) (15right) ;
3106 GET Z1%: IF Z1% " THEN FOR T=1 TO 75: NET: REM get keyboard entry
3107 IF Z1%="(11) THEN 1000: REM F1
3108 IF Z1%="(12) THEN GOSUB 6500: REM F2
3109 IF Z1%="(13) THEN 3000: REM F3
3110 IF Z1%="(14) THEN GOSUB 9000: REM F4
3111 IF Z4%="(15) THEN 3000: REM F5
3112 IF Z4%="(16) THEN GOSUB 9000: REM F6
3113 IF Z4%="(17) THEN GOSUB 4000: REM F7
3114 IF Z4%="(18) THEN GOSUB 7000: REM F8
3115 IF Z4%=" THEN 3005: REM SPACE
3116 GOTO 3100: REM reset 6% variables for n, mean, std
Fig. 5.1 - continued

============= print totals, ratio means, etc for run ==============
4000 PRINT
down)PRINTING TOTALS*
4010 PRINT$8;R$8;R$
4020 FOR J=1 TO 4: PRINT$4,CHR$(48+J); :REM print field direction
4030 FOR K=1 TO 9: PRINT$4,Y(J,K); :REM print totals for run
4050 NEXT K: PRINT$4: NEXT J
4060 FOR J=1 TO 4: FOR K=1 TO 7 STEP 2: TS=TS+Y(J,K); TN=TN+Y(J,K+1); NEXT K,J
4070 PRINT$4;TOTALS: SIG=TS*SIG/TN:REM print total raw and noise
4090 PRINT$4; TIME FOR SIGNAL COUNTS=";EL"SEC"
4100 PRINT$4;CHR$(14);"INDIVIDUALLY ";CHR$(15): GOSUB 4600
4120 PRINT$4;A: "CA B: "CB C: "CD D: "DE :REM counters individually
4150 PRINT$4;MEAN: "(CA+CD+CC+CD)/4
4200 PRINT$4,CHR$(14);"WITHOUT NOISE ";CHR$(15) :REM counter pairs, uncorrected
4250 PRINT$4;AB*UA/N BC*UB/N CDC UC/N DA*UD/N
4270 PRINT$4;MEAN: "US/VN SD=" U3
4400 PRINT$4,CHR$(14);"WITH NOISE ";CHR$(15) :REM counter pairs, corrected for noise
4450 PRINT$4;AB*VA/N BC*VB/N CDC VC/N DA*VD/N
4500 PRINT$4; MEAN: "US/VN SD: "V3 "N: "N
4500 RETURN

------------- calculate ratios for counters individually from run totals -------------
4600 REM COUNTERS INDIVIDUALLY
4601 CA=(Y(1,1)+Y(2,1)-Y(1,2)-Y(2,2))/(Y(1,1)+Y(4,1)-Y(3,2)-Y(4,2))
4602 CB=(Y(3,3)+Y(4,3)-Y(3,4)-Y(4,4))/(Y(1,3)+Y(2,3)-Y(1,4)-Y(2,4))
4604 CC=(Y(1,5)+Y(2,5)-Y(1,6)+Y(2,6))/(Y(3,5)+Y(4,5)-Y(3,6)+Y(4,6))
4606 CD=(Y(3,7)+Y(4,7)-Y(3,8)-Y(4,8))/(Y(1,7)+Y(2,7)-Y(1,8)-Y(2,8))
4610 RETURN
5000 REM BUZZER SOUND
5002 POKE 54273,4; POKE 54273,48; POKE 54276,32; REM FREQ: H1,L WAVEFORM
5004 FOR T=1 TO 600: NEXT : POKE 54276,0: RETURN
5005 REM BEEP SOUND
5007 POKE 54273,67; POKE 54273,68
5008 FOR T=1 TO 30: NEXT : POKE 54276,0: RETURN

================= Disk drive trouble =================
6000 REM DISK ERROR HANDLING
6010 IF A$="72" THEN PRINT "(clear)(white)(rvsOn)(down)"BD8;:GOTO 2525
6020 GOSUB 5000: PRINT R$"(down)(down)(white)(rvsOn)DISK IS ALMOST FULL;:PRINT "(black)YOU MAY EITHER"
6030 PRINT "(down)TRANSFER TO A NEW DISK WITH F2 OR"R$"(down)SEND RUN WITH F8;:GOTO 3005
6500 PRINT "(rvsOn)(white)(down)(down)A ACTION(black)R$"(down)"CURRENT FILE WILL BE CLOSED";
6510 PRINT "(down)A NEW DISK CAN THEN BE USED"
6515 PRINT "(down)WANT THIS ACTION (Y/N)"
6520 INPUTA$: IF A$="M" THEN 3005
6530 IF A$="Y" THEN 6500
6540 DI=DI+1: CLOSEB: PRINT "(down)FILE CLOSED"R$"(down)INSERT NEW DISK"
6550 PRINT "(down)PRESS RETURN WHEN READY"R$: INPUTA
6560 PRINT$15;"*: PRINT "-": REM "INITIALISING NEW DISK ID"
6570 OPEN 8,8,8,0:"S=M+CHR$(1)+"S=N" : PRINT "-"
6580 INPUT$15,A,B$,B$,C$,D$: IF A$="00" THEN 2530 : REM CONT FROM THERE
6590 GOSUB 5000: PRINT "(down)(white)(rvsOn)8D8"
6600 PRINT "(black)(down)(down)CHECK DISK ERROR SHOWN ABOVE"R$"(down)(down)AND TRY AGAIN": GOTO 6515
Fig. 5.1 - continued

============== (FB) pressed - end run ===============

7000 REM end of run
7005 PRINT "(down)(down)(down) DO YOU WANT FILE ";CHAIN(6);CHR$(DI);" CLOSED (Y/N): INPUT1$;
7020 IF ZD="Y" THEN RETURN: REM DON'T CLOSE, RETURN
7030 CLOSE: GOSUB 9150: GOSUB 4000 :REM get and print disk status, print totals, ratios, etc
7100 RETURN

9000 REM **************BKS HERE**************
9010 PRINT: PRINT "D-DIRECTORY": PRINT ">-DISK COMMAND"
9020 PRINT "X-RETURN TO PROGRAM": PRINT ">-DISK STATUS"
9030 GET ZD: IF ZD="**" THEN 9030
9040 IF ZD="D" THEN GOSUB 9200 : REM print disk directory
9050 IF ZD="*" OR ZD="." THEN GOSUB 9100 : REM send a disk command
9060 IF ZD="I" THEN RETURN : REM return to MENU
9070 IF ZD="S" THEN GOSUB 9150 : REM print disk status
9080 GOTO 9030
9120 PRINT BD$: IF BD$="" THEN 9110
9120 PRINT BD$: IF BD$=CHR$(13) THEN PRINT$15,CD$: GOSUB 9150: RETURN
9130 CD$=CD$+BD$: GOTO 9110
9150 INPUT$15,AD$,BD$,CD$,DD$: PRINT AD$","BD$","CD$","DD$: RETURN: REM get and print disk status
9200 OPEN 9,8,0,"";
9210 GET 9,AD$,AD$,AD$,AD$,AD$,BD$
9240 PRINT "C:0: IF AD$<>" THEN C=ASC(AD$)
9230 IF BD$="**" THEN C=C+ASC(BD$)+5
9240 PRINT "(rvsOn)m"STR$(C),2);TAB(3);"(rvsOff)
9250 JD=19: GOSUB 9300: PRINT BD$;
9270 JD=2 : GOSUB 9300: PRINT BD$;
9280 JD=2 : GOSUB 9300: PRINT BD$;
9300 GET 9,AD$,AD$,AD$,BD$
9310 IF ST$="O" THEN CLOSES: GOSUB 9150: RETURN
9320 C:0: IF AD$<>" THEN C=ASC(AD$)
9330 IF BD$="**" THEN C=C+ASC(BD$)+255
9340 PRINT "(rvsOn)m"STR$(C),2);TAB(3);"(rvsOff)
9350 JD=27: GOSUB 9300: ND$=BD$: PRINT ND$
9370 GOTO 9300
9380 BD$="" : FOR JD=0 TO JD: GET 9,AD$
9390 IF AD$<CHR$(56) AND AD$<CHR$(160) THEN BD$=BD$+AD$
9400 NEXT : RETURN
V.B Hardware Interfaces

A microcomputer was enhanced with two interfaces, one for serial communications (reference: Commodore 64 Programmer's Reference Guide, hereafter C64 ref.), and another for switching relays. A schematic of the first interface, for serial communications between the microcomputer's user input-output (I/O) port and the ORTEC 779 interface/controller (hereafter the "779 module") is shown in Fig 5.2. The microcomputer provides RS 232 support, but some signals are inverted and all are at transistor-transistor-logic (TTL) levels of 0 or +5V. The signals are modified externally to the corresponding RS 232 standard levels of $\xi -5V$ and $\xi +5V$.

An RS 232 serial interface requires both a positive and a negative power supply, of which only the positive supply is readily available from the microcomputer itself, at the user I/O port's pin 2. A negative supply is obtained off the receive (RCV) line from the 779 module which stays at the -12V level when no data is being sent. Diode D1 charges a supply capacitor C1 whenever the receive line is negative. Transistor T1, biased by this negative supply and +5 volts inverts the transmit (XMIT) signal from the microcomputer at user I/O port's pin M. The output pulses at the collector of T1 then swing greater than the minimum voltage levels required by the RS 232 standard.

The receive (RCV) signal from the 779 module is fed to the base of transistor T2. It is inverted and trimmed to TTL level required by the microcomputer at pins B and C of the
Fig. 5.2. The RS232 serial communication interface for the microcomputer.
user I/O port. The rest of the circuit in Fig 5.2 shows the "handshaking" required to keep the microcomputer informed of the ready status of a slower peripheral. The data set ready (DSR) signal from the peripheral goes negative when it is not ready. Transistor T3 inverts and trims the DSR signal, while transistor T4 inverts it again for the microcomputer DSR input at pin L of the user I/O port.

The RS 232 interface of the 779 module is set for a baud rate of 1200 bits per second. The character width is 8 bits, one stop bit and no parity checking. Four ORTEC model 770 counters, one for each photon detector, and an ORTEC model 773 timer-counter, fed by a voltage-to-frequency converter are all interconnected in a printing loop to the 779 module, as shown in Fig. 5.3. This allows a serial readout of the four counters and the timer, in sequence of their electrical connections, upon time up on the timer.

To set the computer for serial communication, the following BASIC statement is executed:

```
2005 OPEN 2, 2, 0, CHR$(40) + CHR$(160)
```

The first parameter after the OPEN statement is the channel number, here 2, which is chosen to be the same as the second parameter for convenience. The 2 for the second parameter refers to the RS 232 device (C64 ref.). Following the third dummy parameter are two characters, whose ASCII values are specified by the required baud rate, number of bits per character, and parity.

Information from the 779 module is received by the
FIG. 5.3. Interconnections to the Ortec printing loop. The printing loop allows the transfer of counter data from the 770 modules and the 773 module to the "779". A reset from the microcomputer starts the four counters and the timer-counter. The 773 module is used as a counter and stops at a preset count set by two thumbwheel switches. The 770 counters are set to the "SLAVE" mode, receiving their stop signals from the 773 timer which is set to the "MASTER" mode.

computer using either a GET#2, Z3$ or an INPUT#2, Z2$ statement. The first type of command receives a single character into the string variable Z3$, and the second receives an entire line that ends with a carriage return. While the counters and timer are accumulating counts, no data is sent out from the 779 module. Therefore, a GET#2 statement is used to look for a valid character, which when received, signals the completion of the counting period. Following this, the remainder of the readout is obtained by an INPUT#2 statement. The four counters and the timer each have 6 digits and the 779 module sends one line of 34 characters per readout,
received in variable Z2#, as for example: (space shown as b)
119911b229922b339933b449944b559955. The first four counters
named A, B, C, D then have the counts 11911, 229922, 339933,
449944, respectively, when separated by the following BASIC
statements:

85 A=VAL(LEFT$(Z2#,6))$: B=VAL(MID$(Z2#,8,6))
90 C=VAL(LEFT$(Z2#,15,6))$: D=VAL(MID$9Z2#,22,6))

The second interface to the computer allows the swit-
ching of relays that in turn apply the appropriate potentials
to the quadrupole. Fig 5.4 is a schematic diagram showing how
four relays, labelled I, II, III, IV are connected. Relay
switch positions required for any of the 8 different electric
field orientations are also shown. A fifth relay, labelled
HV, switches the input power to a 3500V power supply, used in
pre-quenching, for background noise measurement. Each of the
relays is energised by 115V AC across their coils. Triacs are
used as switching elements to drive the relays, but since the
computer must be protected from the 115V AC, optoisolators
(Motorolla model MOC 3031) are interposed between the triacs
and the computer. Fig. 5.5 shows the connection diagram of
one of the five relays.

The optoisolator device consists of an infrared light
emitting diode (IRED) isolated electrically from a silicon
detector performing the function of a triac driver. An added
feature of the MOC3031 optoisolator is to switch only at a
zero level crossing of the alternating voltage, thus prevent-
ing sudden current surges in the inductive load. To turn on
Fig. 5.4. Schematic diagram of the four electric field relays I, II, III and IV. Two power supplies provide +V and -V potentials relative to ground potential, shown as G. All relays are DPDT types, shown in their off positions. The four field orientations for the Lamb shift experiment in the middle of the figure are labelled 1, 2, 3 and 4.
FIG. 5.5. Circuit diagram of one of the five relay interfaces. Terminals I through HV are connected to the 8255 peripheral chip, shown in Fig. 5.6.

The IRED, an open collector inverter (7416) is used, which can also accommodate a red LED in series. External 120Ω resistors limit the current to ≈16 mA, for the correct operation of the IRED. The LED provides the user with a visual indication of the switch status of the relay, and of any malfunction in the IRED, since the two are connected in series. Inputs to the open collector inverters are in turn fed from latched output ports on the computer.

Since an insufficient number of ports were available, an Intel 8255 peripheral chip was interfaced to the microcomputer to provide 24 single-bit ports. The schematic diagram of the interface is shown in Fig. 5.6. The computer provides a
FIG. 5.6. Microcomputer interface to the INTEL 8255 peripheral chip.

chip select line for the purposes of interfacing external input/output devices, and it is addressed at 56832-57087. The 8255 peripheral chip contains four registers at addresses 56832-56835, listed in Table 5.1, where the first three addresses refer to ports A, B and C having 8 bits each. The last address, 56835 is a control register which sets the mode of operation for each port.
TABLE 5.1. The computer addresses for the registers of the 8255 peripheral chip when in mode 0.

<table>
<thead>
<tr>
<th>HEX</th>
<th>DEC</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE00</td>
<td>56832</td>
<td>PORT A, read or write</td>
</tr>
<tr>
<td>DE01</td>
<td>56833</td>
<td>PORT B, read or write</td>
</tr>
<tr>
<td>DE02</td>
<td>56834</td>
<td>PORT C, read or write</td>
</tr>
<tr>
<td>DE03</td>
<td>56835</td>
<td>CONTROL REGISTER, write only</td>
</tr>
</tbody>
</table>

The 8255 device has three modes of operation (see Intel reference): a basic input/output mode, a strobed input/output mode, and a strobed bi-directional bus input/output mode. In this application, it is used in the first mode, "mode 0", providing 24 1-bit ports, set for input or output in accordance to the options given in Table 5.2

TABLE 5.2. The values given below set the 8255 peripheral chip to mode 0 operation (basic input/output), and assign the 8-bit ports A, B and C to input or output. See Intel reference for other settings. In BASIC, the following is executed to set the given mode: POKE 56835,X.

<table>
<thead>
<tr>
<th>X</th>
<th>PORT A</th>
<th>PORT B</th>
<th>PORT C</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>OUTPUT</td>
<td>OUTPUT</td>
<td>OUTPUT</td>
</tr>
<tr>
<td>130</td>
<td>OUTPUT</td>
<td>INPUT</td>
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<td>137</td>
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<td>153</td>
<td>INPUT</td>
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<tr>
<td>155</td>
<td>INPUT</td>
<td>INPUT</td>
<td>INPUT</td>
</tr>
</tbody>
</table>

Only one 8-bit output port (port B) is required, of which five 1-bit ports are used for the relays, and a sixth
1-bit port to provide reset pulses to the 779 module as shown in Fig. 5.6 and summarised in Table 5.3.

**TABLE 5.3.** This table summarises the bit positions of port B assigned to the relays and the reset toggle line (RST). Any bit may be turned on by executing POKE 56833,X where X in decimal is given below.

<table>
<thead>
<tr>
<th>Bit number</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td>HV</td>
<td>RST</td>
<td>X</td>
<td>X</td>
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<td>X</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Before using the ports, the mode of operation must be set on the 8255 peripheral chip, by executing the following BASIC statement:

2020 POKE 56835,153.

This causes port B to be a latched output port, while A and C become input ports. To turn any one or more of the 8 bits of port B on (to 5V) or off (to 0V), the register at location 56833 is appropriately set. For example, if relays I and III need to be turned on, as they do for an electric field vertically up, (see Fig. 5.4), then the following BASIC statement is executed:

1500 PB=56833 : POKE PB,(128+32).

To turn on the HV relay also, the statement then becomes POKE PB,(128+32+8). To begin counting, the reset on the 779 module is toggled. Only the bit controlling the reset is changed.
from low to high and back to low again:

180 PV=128+32+8 : POKE PB,PV : POKE PB,PV+4 : POKE PB,PV.

To turn off all potentials on the quadrupole (and set them to
ground potential) and to turn off the pre-quenching high
voltage field, all bits are set to zero:

3005 POKE PB,0.

V.C The Control/Analysis Program

Following is a description of the program, listed in
Fig. 5.1. It is supplemented by flow chart diagrams in Figs.
5.7-5.9). In addition, a list of all variables used and the
associated statement numbers is presented in Table 5.4. A
sample of printed output from the program is given in Fig.
5.10. The program commences at statement 2000, where initia-
lation takes up lines 2000-2901. A sub-program is then
chosen from a menu in lines 3000-3100. During a measurement,
the computer runs in the "main loop", occupying lines 50-379.
The analysis section in lines 380-655 follows the main loop.
Other major sections of the program and their locations are:
a printing routine for raw totals, ratios, etc., (4000-4500),
a routine to handle disk drive "errors", (6000-6600), and a
collection of disk drive utility subroutines (9000-9400).

Due to timing considerations, those parts of the program
needing more rapid action are placed at the head of the
program. For the same reason, a number of other features have
to be introduced, making the program seem un-necessarily
complex. For example, all printer and disk drive activity
<table>
<thead>
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<th>105</th>
<th>124</th>
<th>126</th>
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<td>2060</td>
<td>2060</td>
</tr>
<tr>
<td>Z4</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
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<td>2060</td>
</tr>
<tr>
<td>Z5</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
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<td>Z6</td>
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<td>2060</td>
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<tr>
<td>Z7</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
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<tr>
<td>Z8</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
</tr>
<tr>
<td>Z9</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
<td>2060</td>
</tr>
</tbody>
</table>
(being time-consuming) is spread out over the span of an ongoing measurement, while the electric fields have been set and the counters are accumulating. This not only allows data gathering at different electric field settings in rapid succession, a necessity for correcting small beam drifts, but also saves considerable running time.

Three channels are opened in line 2005, the RS 232 serial communications (channel #2, device #2), the printer (channel #4, device #4), and the disk drive's command channel, needed for error checking while saving data to a disk (channel #15, device #15). Lines 2020-2030 deal with the 8255 peripheral chip (see previous section). An audible signal is ushered during certain parts of the program to alert the user, programmed in line 2033. In line 2040, a string DN* is defined as a series of cursor down characters, used throughout the program for placing information on the screen. Three arrays are declared in line 2050: X, OLD, and Y holding respectively the number of photon counts for the current measurement, N, the previous measurement, N-1, and the accumulated totals for all "accepted" measurements. All three arrays are of the form shown in Table 5.5.

An accurate timing clock is available in the microcomputer (C64 ref., page 431) which is used for determining the counting periods to 0.1 s resolution. These time periods are recorded for use later in the dead time correction. Line 2052 declares variables to hold the binary coded decimal (BCD) register values of the clock chip. M1, S1 and T1 hold the
TABLE 5.5. The structure of arrays X, Y and OLD, shown for the X array. Counter A is uppermost, for which the four electric field directions designated as 1, 2, 3 and 4 are as shown in Fig. 5.4.

<table>
<thead>
<tr>
<th>Counter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>raw noise</td>
<td>raw noise</td>
<td>raw noise</td>
<td>raw noise</td>
</tr>
<tr>
<td>1</td>
<td>X(1,1) X(1,2) X(1,3) X(1,4) X(1,5) X(1,6) X(1,7) X(1,8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X(2,1) X(2,2) X(2,3) X(2,4) X(2,5) X(2,6) X(2,7) X(2,8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X(3,1) X(3,2) X(3,3) X(3,4) X(3,5) X(3,6) X(3,7) X(3,8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>X(4,1) X(4,2) X(4,3) X(4,4) X(4,5) X(4,6) X(4,7) X(4,8)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

minutes, seconds and \(1/10\)th of a second values for a measurement, while the corresponding temporary variables W0, S0, and T0 hold the lapsed time for a part of the measurement.

Lines 2500-2901 deal with the disk storage. The character whose ASCII code is in variable DI, set initially at 65 (for character "A") is tagged to the end of the user-supplied file name in line 2508. To alleviate the loss of too much data resulting from a power failure, a small number of measurements is saved under this filename, and the file closed after which the tagged character is incremented to "B", "C", ..., for new files. Setting W2 to 1 in line 2530 allows a new-file-open message in MG8 to be printed in line 293. The variable W2 is set to zero again in line 295, and is one of several similar flag registers to keep order of pending operations of the disk drive and the printer.

The menu of different options, listed in Table 5.6, occupies lines 3000-3100. An option is chosen either in line 3020, or during the measurement in line 65. Program flow
TABLE 5.6. Table of options available, and their program locations. Note that pressing the space bar in line 65 halts a measurement in progress.

<table>
<thead>
<tr>
<th>KEY</th>
<th>LINE(S)</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1</td>
<td>1000</td>
<td>start a measurement and repeat until interrupted</td>
</tr>
<tr>
<td>f6</td>
<td>3000</td>
<td>begin next measurement taking both raw and noise</td>
</tr>
<tr>
<td>f3</td>
<td>3005</td>
<td>stop after the completion of current measurement</td>
</tr>
<tr>
<td>f7</td>
<td>4000-4500</td>
<td>print accumulated totals and results summary</td>
</tr>
<tr>
<td>f2</td>
<td>6500-6600</td>
<td>change a full disk to an empty one</td>
</tr>
<tr>
<td>f8</td>
<td>7000-7100</td>
<td>end of run: close file and do f7</td>
</tr>
<tr>
<td>f4</td>
<td>9000-9400</td>
<td>subroutines for the disk drive (for checking)</td>
</tr>
</tbody>
</table>

is redirected accordingly in lines 3030-3060.

An integer variable, Q% in line 3000 holds the number of successive measurements of raw counts only, and Q% is zero during a measurement of both raw and noise. Line 3010 obtains information from the serial communication channel, if any is received from manual resets of the 779 module. This keeps the buffer for the serial channel empty. Using variables GN, GS, G2 and G3, the program calculates and displays (in lines 265-268), a set of mean and standard deviation values of the ratio since the last user-interruption via the keyboard.

These G% variables become reset in line 3100.

On pressing the function key f1, the program flow diverts to line 1000. After a call to the subroutine at 1500, the computer enters the main loop of instructions to carry out continuous measurements beginning at line 200 and ending at line 560. The subroutine at 1500-1505 makes preparations to enter this loop: it sets the relay to the first field orientation (shown in Fig. 5.4 as "1") and resets the counters.

In line 1505, the subroutine call to line 180 resets the
Fig. 5.7. Flow chart of the initialisation and menu. Line numbers after asterisks show the starting point of routines.
counters, and starts the timing clock chip, reset to 0.0 s.
The clock chip starts timing when the tenth-of-a-second register at address CT=56584 is set. It is stopped after every counting period (for a given field orientation) in line 80 by reading the hours register, CH=56587. Calls to the subroutine at 180 start the clock with previous time settings, until a measurement is completed, after which the clock is again reset to 0.0 s.

In line 200, the variable L, an array index, is set to 1, indicating the field orientation. The subroutine call to line 60 is one of the entry points to the subroutine occupying lines 50-99 and is now described. Line 50 is a pause for the relays to settle in their new switch positions (as in line 1500). In line 60, a check is made to confirm that the port controlling the relays was correctly set, which occasionally differs due to transient spikes. The intended value to be placed into the port register is always given in variable PV. A user interrupt can be detected by lines 65-68. Counting must be stopped whenever the beam current is interrupted for long periods of time. For this reason, a "software" clock (see jiffy clock in C64 ref.) is used. Delays lasting longer than 40 seconds automatically reset the counters again. The first character from the 779 module is received in line 75, and the timing clock is stopped in line 80. After reading the remaining characters of the photon counts, and obtaining the values in the four variables A, B, C and D, control returns from the subroutine.
The next instruction in line 200 is a call to the subroutine at 100. This subroutine obtains the time from the clock chip, displays the values obtained in A, B, C and D and saves this data into the array X. The value of L is between 1 and 4 for the first round of measurements (at the four field orientations), and in the second round, when the fields are repeated in reverse order of the first round, L is between 5 and 8. The counts from the first and second rounds are combined in lines 126-128.

In statement 230, and in similar statements in other parts of the loop, if a user had interrupted the program, then the computer exits the loop to enter the menu section. In line 231, the next field is set, and the counters are reset, allowing the data to accumulate while the data of the previous measurement is printed out from the OLD array. "Flags" W4 and W5 indicate which data is to be printed. After receiving the counts for field orientation 4, the computer sets the next field (orientation 2), resets the counters, and sends the remaining data out to the printer in lines 242-247. This print-out (see Fig. 5.10) includes the ratio, mean, standard deviation, and the individual ratios for each counter, calculated at the completion of the previous measurement.

The main loop continues in a similar way; after the field has been set and the counters are reset, time is available for other activities. In line 252, if flag W3 was set to 1, it indicates that the current disk file is to be closed and the next one opened. If any disk drive related error occurred,
Fig. 5.8. Flow chart of the main loop.
such as a disk getting full, then the current measurement is abandoned, and the program enters line 6000, where it is handled. A new disk may then be exchanged for one that is full (lines 6000-6600, being part of option f2).

At line 261, the first round of raw measurements at the four field directions is completed. The ratios (for each counter individually) are calculated in the subroutine at lines 750-770. Also, the mean and the standard deviation of the ratios for the series of measurements since the last user-interruption is calculated in line 265. This information is displayed on the screen (lines 262-268) to allow monitoring of the data. The measurement number and the message in MG# are printed in lines 292-295. The message consists of the name of a new disk file, if any, and the total time taken for raw counts, up to that moment.

Line 305 marks the end of the second round of raw measurements and if noise is to be measured now, the loop continues at line 306, otherwise control passes to the analysis section beginning at line 380. If noise is to be measured, then the individual ratios are calculated a second time from the acquired raw measurements, and shown on the screen in lines 307-310. The loop continues in a similar way to the raw-measuring section from line 311 to 379, except that the X array is now assigned its values by the subroutine at line 150. This subroutine also differs from the one at line 100 in that the timing clock chip is not used since only the count rate during the raw measurement needs to be known. At the
conclusion of the noise measurements, the program enters the analysis section.

Line 380 calls a subroutine at 600 where the intensity ratios AR, BR, CR and DR for counter pairs AB, BC, CD and DA respectively, and their mean, R are calculated. These ratios are uncorrected for the background, and are referred to as the primed quantities. The difference between R' and the established mean' is compared with five times the standard deviation, σ' in line 381. If the difference, CK is exceeded, then program flow is diverted to line 700, where the user may intervene and reject the measurement. Although this feature is rarely used, it prevents a measurement from spurious data from ruining a run.

In line 382, the program resets the counters for the next measurement, provided that the field is still set at position 1 from a previous raw measurement (Q%>0). The number of measurements in variable N is incremented in line 383. Flag registers are set appropriately, and then the means of the primed ratios are calculated in line 384, and assigned to variables named Ux. The corresponding standard deviation is found and assigned to U3. In line 387, the measurement-identifying numbers N and Q%, are written to the disk file.

The program flow continues for a raw-and-noise measurement (Q%=0) at line 390, but diverts to line 620 otherwise. In lines 390-405 the array X is saved to the disk file, copied to the OLD array for later printing, and added to the Y array. A last array operation subtracts the noise from the
Fig. 5.9. Flow chart of the analysis section.
corresponding raw counts. The counters are reset for the next measurement, after a lapse of at least 6 seconds to allow the pre-quenching high voltage power supply to discharge to zero.

A similar sequence of array manipulations as above takes place in lines 620-655 for a raw-only measurement. Note that the noise measured at a previous raw-and-noise measurement is used again, and is also accumulated to the Y array. In line 655, the program jumps back to line 410, where the program continues for both cases (with Q%=0 or Q%>0), with the X array now containing signal in place of the raw counts. This results from the statement \(X(J,K) = X(J,K) - X(J,K+1)\) in lines 405 and 650.

A second call to the subroutine at 600 calculates the noise-corrected ratios. This time, the various means and the standard deviation \(\sigma\) are the unprimed quantities. The corresponding variables used in lines 410-420 for these noise-corrected quantities begin with "V". Next, the number of raw-only measurements in Q% is incremented, and reset to zero after four have been measured. Lines 445-470 place some of the calculated information on the screen, and in line 472, the total lapsed time for raw counts is converted from BCD to decimal. The disk drive status is checked, and if not normal, the program diverts to line 6000, where the error is handled.

In line 555, a user's previous interrupt causes first, a jump to a subroutine at line 800, where all pending data is printed, and then, the computer enters the menu section. If no interrupt was detected, however, the analysis section at line
560 sends control back to the start of the loop at line 200. At this stage, the counters are still accumulating data for field position 1 of the next measurement.

Lines 4000-4500 form a subroutine that produces a more detailed print-out of the analysis (see Fig. 5.10). This subroutine is called by choosing option f7 (for printing totals) or f8 (for ending a run, see options in Table 5.6). Here, the total accumulated counts for raw and for noise held in array Y, as well as the values of the grand total raw and noise are printed by lines 4002-4005. A call to a subroutine occupying lines 4600-4606 calculates the ratios for each counter individually using the accumulated totals in the Y array. These ratios are printed in lines 4012-4015. The primed quantities for the four pairs of counters are then printed, followed by the mean' and the standard deviation'. Finally, the corresponding values of the unprimed ratios are printed.

When finishing a run, the function key f8 is pressed, which calls the subroutine at 7000. The current disk file is closed and a detailed print-out is obtained by the subroutine at 4000. The statements in lines 9000-9400 allow the user to perform disk drive activities from within the program. A submenu in lines 9040-9070 offers the choice of: 1) viewing the disk's directory, 2) to send a command to the disk drive, such as to initialise the disk drive, 3) to check the disk drive status or 4) to return back to the main menu. This subroutine is often used to check for space on a disk drive or to erase unwanted files.
Fig. 5.10. Sample print-out produced by the program.
CHAPTER VI
RESULTS

VI.A The experimental data

In all, two sets of data comprising 4113 measurements were obtained. One set of 2131 measurements were made with the photon detectors mounted in position $\theta_c = 0^\circ$, and another set of 1982 measurements were made after rotation of the detectors by 900 about the symmetry axes. All the data were collected in 28 separate runs. Each measurement contains on average $4.20 \times 10^6$ photon counts, for a combined total of $1.729 \times 10^{10}$ counts. Figure 6.1 shows the run-means, corrected only for noise, in chronological order.

The point marked "m" represents a test run to investigate the effect on the measured Lamb shift ratio with a shift in the operating point of the detectors. To this purpose, the DC channeltron potential for all the counters was lowered from the normal plateau position towards the "knee" as indicated by a dashed arrow in Fig. 3.11a. A run consisting of 100 measurements yielded the value of $(1.26686 \pm 0.00013)$, which is in agreement with the average value of $(1.26694 \pm 0.00003)$, for $\theta_c = 0^\circ$. Thus, our drastic change in the operating point has no observable effect. An absence of long drifts in the run-means of Fig. 6.1 is evident in comparison with the electron multiplier results of Fig. 3.7. Although the values are still subject to correction for solid angle
FIG. 6.1. The observed Lamb shift ratios for 28 runs in chronological order. The solid circles are the means for the two sets of runs for (a) $\theta = 0^\circ$ and (b) $\theta = 90^\circ$. A test run, marked "T" is discussed in the text.
and dead time effects, the overall means \( \bar{r}(\theta_c = 0^\circ) \) and 
\( \bar{r}(\theta_c = 90^\circ) \) differ by less than 20 ppm from the combined mean
of
\[
\bar{r}_{\text{grand}} = 1.266\,921\,6
\] (6.1)

Table 6.1 lists all 28 runs, showing the number of measurements, \( N \), the run-means, \( \bar{r} \), and the standard deviation 
\( \sigma_{\text{run}} \) for the run. The standard deviation in \( \bar{r} \), given by
\[
\sigma = \sigma_{\text{run}} / \sqrt{N}
\] (6.2)
is not shown. The average count rates for each counter are 
also given, and it is to be noted that the run of JUL19 has 
half the usual count rate for testing purposes, discussed in 
the next section.

VI.B Corrections to the asymmetry

As indicated in Sec. IV.G, the dead time correction (Eq. 4.29) is applied to the run-means of the counter pairs AB, 
BC, CD and DA separately. The dead time corrections \( \Delta_{\text{DT}} \) to 
each of the 28 runs is shown in Table 6.2. The typical error 
in the dead time correction is found below for the run of 
JUL9.1 as an example.

The total raw counts for this run is \( 6.858 \times 10^8 \), measured 
in a time of \( (21276.1 \pm 1.2) \) sec, giving an average count rate 
per counter of \( (3.2233 \pm 0.0002) \times 10^4 \) sec\(^{-1} \). For the purpose of 
this calculation, the average dead time is \( (55.565 \pm 0.035) \) ns, 
whose error dominates over that of the count rate. The dead 
time correction using the mean asymmetry value for the run,
TABLE 6.1. List of 28 runs, showing the number of measurements, N, the run-means of the ratios, $\bar{r}$, the run-standard deviation, and the average count rates for the four counters.

<table>
<thead>
<tr>
<th>$\theta_c$</th>
<th>Run date</th>
<th>N</th>
<th>$\bar{r}$</th>
<th>$\sigma_{\text{run}}$</th>
<th>Count rate (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>JUN28.3</td>
<td>100</td>
<td>1.2670008184</td>
<td>0.001278</td>
<td>7.50 7.79 7.52 7.75</td>
</tr>
<tr>
<td>0</td>
<td>JUN29.1</td>
<td>120</td>
<td>1.2671611876</td>
<td>0.001211</td>
<td>7.58 7.83 7.66 8.04</td>
</tr>
<tr>
<td>0</td>
<td>JUL1.3</td>
<td>120</td>
<td>1.2668669125</td>
<td>0.001038</td>
<td>8.53 7.99 6.78 7.19</td>
</tr>
<tr>
<td>0</td>
<td>JUL2.2</td>
<td>120</td>
<td>1.2669044742</td>
<td>0.001139</td>
<td>8.18 7.58 6.55 6.76</td>
</tr>
<tr>
<td>0</td>
<td>JUL4.1</td>
<td>130</td>
<td>1.2668720307</td>
<td>0.001225</td>
<td>8.82 8.10 6.93 7.22</td>
</tr>
<tr>
<td>0</td>
<td>JUL5.1</td>
<td>130</td>
<td>1.2670673676</td>
<td>0.001432</td>
<td>9.22 8.57 7.25 7.56</td>
</tr>
<tr>
<td>0</td>
<td>JUL6.1</td>
<td>150</td>
<td>1.2670856044</td>
<td>0.001226</td>
<td>9.28 8.63 7.25 7.66</td>
</tr>
<tr>
<td>0</td>
<td>JUL7.1</td>
<td>161</td>
<td>1.2668316811</td>
<td>0.001209</td>
<td>9.64 8.98 7.50 8.01</td>
</tr>
<tr>
<td>0</td>
<td>JUL9.1</td>
<td>150</td>
<td>1.2667907700</td>
<td>0.001364</td>
<td>9.06 8.53 7.06 7.57</td>
</tr>
<tr>
<td>0</td>
<td>JUL9.2</td>
<td>150</td>
<td>1.2669178325</td>
<td>0.001144</td>
<td>9.38 8.75 7.38 7.88</td>
</tr>
<tr>
<td>0</td>
<td>JUL10.1</td>
<td>160</td>
<td>1.2669930173</td>
<td>0.001111</td>
<td>9.42 8.88 7.33 7.89</td>
</tr>
<tr>
<td>90</td>
<td>JUL14.1</td>
<td>115</td>
<td>1.2669349456</td>
<td>0.001257</td>
<td>9.24 8.92 7.34 7.77</td>
</tr>
<tr>
<td>90</td>
<td>JUL15.1</td>
<td>115</td>
<td>1.2670866892</td>
<td>0.001132</td>
<td>9.37 8.83 7.40 7.93</td>
</tr>
<tr>
<td>90</td>
<td>JUL16.1</td>
<td>175</td>
<td>1.2668817972</td>
<td>0.001151</td>
<td>9.33 8.89 7.39 7.91</td>
</tr>
<tr>
<td>90</td>
<td>JUL17.N</td>
<td>160</td>
<td>1.2669008534</td>
<td>0.001199</td>
<td>11.29 10.83 9.00 9.55</td>
</tr>
<tr>
<td>90</td>
<td>JUL18.N</td>
<td>185</td>
<td>1.2667489568</td>
<td>0.001151</td>
<td>10.86 10.37 8.66 9.17</td>
</tr>
<tr>
<td>90</td>
<td>JUL18.H1</td>
<td>10</td>
<td>1.2672045421</td>
<td>0.001052</td>
<td>6.84 6.52 5.42 5.81</td>
</tr>
<tr>
<td>90</td>
<td>JUL19.N</td>
<td>100</td>
<td>1.2668026397</td>
<td>0.001498</td>
<td>5.17 5.01 4.21 4.47</td>
</tr>
<tr>
<td>90</td>
<td>JUL20.C1</td>
<td>50</td>
<td>1.2667670130</td>
<td>0.001103</td>
<td>9.91 9.59 7.94 8.30</td>
</tr>
<tr>
<td>90</td>
<td>JUL23.N</td>
<td>140</td>
<td>1.267050136</td>
<td>0.001133</td>
<td>8.26 7.89 6.58 6.95</td>
</tr>
<tr>
<td>90</td>
<td>JUL24.N</td>
<td>180</td>
<td>1.2669266222</td>
<td>0.001306</td>
<td>8.16 7.93 6.55 7.01</td>
</tr>
<tr>
<td>90</td>
<td>JUL25.N</td>
<td>200</td>
<td>1.266800816</td>
<td>0.001246</td>
<td>8.66 8.27 6.85 7.30</td>
</tr>
<tr>
<td>90</td>
<td>JUL26.N</td>
<td>200</td>
<td>1.2667692981</td>
<td>0.001336</td>
<td>8.79 8.45 7.00 7.45</td>
</tr>
<tr>
<td>90</td>
<td>JUL27.N</td>
<td>192</td>
<td>1.2669038507</td>
<td>0.001272</td>
<td>8.63 8.35 6.95 7.30</td>
</tr>
<tr>
<td>90</td>
<td>JUL28.N</td>
<td>160</td>
<td>1.2670039623</td>
<td>0.001269</td>
<td>8.92 8.61 7.14 7.53</td>
</tr>
<tr>
<td>0</td>
<td>JUL31.N</td>
<td>210</td>
<td>1.267005160</td>
<td>0.001254</td>
<td>9.21 8.75 7.27 7.65</td>
</tr>
<tr>
<td>0</td>
<td>AUG1.N</td>
<td>220</td>
<td>1.2667732667</td>
<td>0.001395</td>
<td>8.75 8.38 6.93 7.35</td>
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<tr>
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<td>210</td>
<td>1.2670231371</td>
<td>0.001433</td>
<td>8.46 8.15 6.71 7.16</td>
</tr>
</tbody>
</table>
TABLE 6.2  Dead time and solid angle corrections to the asymmetry values corresponding to the run-means, for Table 6.1.

<table>
<thead>
<tr>
<th>$\theta_c$</th>
<th>N</th>
<th>A</th>
<th>$\Delta DT$</th>
<th>$\Delta SA$</th>
<th>A\text{corr}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>0.117771160</td>
<td>0.0000492878</td>
<td>0.0000796544</td>
<td>0.1179060982</td>
</tr>
<tr>
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<td>0.0000502100</td>
<td>0.0000797338</td>
<td>0.1179694645</td>
</tr>
<tr>
<td>0</td>
<td>120</td>
<td>0.1177250023</td>
<td>0.0000491504</td>
<td>0.0000796618</td>
<td>0.117853145</td>
</tr>
<tr>
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<td>0.0000468799</td>
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<td>0.1178661707</td>
</tr>
<tr>
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<td>0.1178567336</td>
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<tr>
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<td>0.1179353113</td>
</tr>
<tr>
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<td>0.0000529599</td>
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<td>0.1179427871</td>
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<tr>
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<tr>
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<tr>
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<td>0.0000796755</td>
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<tr>
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<td>0.1179128246</td>
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<td>0.0000491678</td>
<td>0.0000796997</td>
<td>0.1179146689</td>
</tr>
</tbody>
</table>
\[ A = 0.1177 \]

\[ A_{\text{DT}} = A \times \frac{1}{2} \left( 55.565 \pm 0.035 \right) \times 10^{-9} \times \]
\[ \frac{1}{2} \left( 3.2233 \pm 0.0002 \right) \times 10^{-4} \times \left( 1 - A^2 \right) \]
\[ = (5.197 \pm 0.003) \times 10^{-5}. \]

The relative error in \( A \), resulting from the error in the dead time correction is:

\[ \delta A_{\text{DT}} / A = 0.3 \times 10^{-6} \]  \hspace{1cm} (6.3)

The average of the dead time-corrected values for the four counter pairs is used in further corrections:

\[ \bar{A} = \left( A_{AB} + A_{BC} + A_{CD} + A_{DA} \right) / 4. \]  \hspace{1cm} (6.4)

Since the solid angle correction (Eq. 4.45) itself depends on the asymmetry value, it is applied to every run separately, and these corrections are shown in Table 6.2. The error introduced in the asymmetry by the solid angle correction arises from the dimensional uncertainties of the photon collimator (see Table 4.1). The solid angle correction reduces to

\[ A \propto A_{\text{obs}} \left[ 1 + (1 - A_{\text{obs}}) \times \left( 4.521 \pm 0.013 \right) \times 10^{-4} \right. \]
\[ + \left. \left( 2.776 \pm 0.026 \right) \times 10^{-4} \right], \]  \hspace{1cm} (6.5)

and using the asymmetry of 0.11775 from the run of JUL9.1 as an example,

\[ A - A_{\text{obs}} = \Delta_{\text{SA}} = (7.965 \pm 0.031) \times 10^{-5}. \]

Here the error in \( \Delta_{\text{SA}} \) relative to the asymmetry is

\[ \delta \Delta_{\text{SA}} / A = 2.6 \times 10^{-6} \]  \hspace{1cm} (6.6)

The run-means adjusted for dead time and solid angle corrections are shown in the last column of Table 6.2. When these
corrections are applied to the investigative run of JUL19.N (performed with roughly one half the count rate of the other runs), its run-mean shows no anomalies; the run-mean of \((0.117810 \pm 0.000067)\) is within the error of the grand mean for \(\theta_c = 90^\circ\): \((0.117870 \pm 0.000012)\), shown below. We thus conclude that a decrease in the count rate by a factor of two produces no observable shift on the measured asymmetry. It strongly indicates the absence of count rate-dependent systematic errors resulting from our photon detection system.

Next, each measurements within a given run is adjusted by the difference between the corrected and uncorrected run-means. The resulting ensemble of 2131 measurements and 1982 measurements for \(\theta_c = 0^\circ\) and \(\theta_c = 90^\circ\) respectively is then used for all further analysis. The weighted mean and standard deviation for \(N\) measurements are found using Eq. 4.11 and 4.12:

\[
\bar{A} = \frac{\sum_{i=1}^{N} A_i \cdot n_i}{n_T}
\]

(6.7)

and

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{N} \left( A_i - \bar{A} \right)^2 \cdot n_i}{n_T(N-1)}}
\]

(6.8)

where \(A_i\) and \(n_i\) is the asymmetry and the number of counts respectively in the ith measurement, and \(n_T\) is the total number of counts. The resulting asymmetry values are summarised in Table 6.3. The means for \(\theta_c = 0^\circ\) and \(\theta_c = 90^\circ\) are in agreement with each other, indicating that there are no observable effects arising from polarization sensitivities of
the detectors. The two sets have roughly equal number of measurements and have the same standard deviations. Their combination automatically corrects for polarization effects to first order (see Sec. IV.F), and gives a grand mean and standard deviation shown in the last row.

TABLE 6.3. Weighted mean and standard deviations of dead time and solid angle corrected asymmetries. The uncertainties are from counting statistics alone.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>$\bar{A}$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0 = 0^\circ$</td>
<td>2131</td>
<td>0.117 886 8</td>
<td>0.000 012 1</td>
</tr>
<tr>
<td>$\theta_0 = 90^\circ$</td>
<td>1982</td>
<td>0.117 869 6</td>
<td>0.000 012 3</td>
</tr>
<tr>
<td>combined</td>
<td>4113</td>
<td>0.117 878 6</td>
<td>0.000 008 6</td>
</tr>
</tbody>
</table>

Further statistics are now calculated, on all 4113 measurements as an ensemble, in the following sections.

VI.C Comparison between the experimental and predicted standard deviations

We first describe the manner in which the error on the experimental standard deviation for the combined total of 4113 measurements is found. In conventional statistical analysis of experimental data, a Gaussian distribution is assumed. In actuality, this may not be so. For example, small systematic errors may distort the symmetry of the bell-shaped distribution. When this happens, the error on the standard
deviation derived for a Gaussian distribution is not reliable. To obtain a more meaningful error on the standard deviation, a "bootstrap" method, proposed by Efron (1979), was used. Referring to our specific needs, the bootstrap method involves:

1) The creation of 1113 new data points drawn randomly from the (original) 4113 observed data points. The new data points are called a bootstrap sample and are a subset of the original sample, some of which are selected zero times, some once, some twice, etc.

2) The weighted mean \( \bar{r}_1 \) and the standard deviation \( \sigma_1 \) for the randomly drawn sample are found using Eqs. 6.7 and 6.8.

3) Steps 1) and 2) are repeated an arbitrarily large number of times, where in each trial, an independent set of random numbers are used, yielding a new mean, \( \bar{r}_1 \) and standard deviation \( \sigma_1 \).

All standard deviations obtained can now be used as samples to evaluate a mean \( \bar{\sigma} \), and more importantly, the desired error \( \Delta \sigma \) in the standard deviation. The bootstrap method was implemented by a computer program that uses a software random number generator. After \( i \approx 525 \) trials, the error in the standard deviation approached a limiting value. The result is

\[
\sigma_{\text{expt}} = (8.64 \pm 0.10) \times 10^{-6}
\]  \hspace{1cm} (6.9)

The theoretical standard deviation is found using Eq. 4.20:

\[
\sigma_{\text{th}} = (8.84 \pm 0.10) \times 10^{-6}
\]  \hspace{1cm} (6.10)

The experimental and theoretical standard deviations as found above (from counting statistics alone) are just within the
error bars of each other, with the theoretical standard deviation being higher. This indicates an absence of time dependent systematic errors, which if present, would always increase the experimental standard deviation. That the errors in the experimental and theoretical standard deviations are the same indicates that the experimental data distribution can be treated as Gaussian for the purposes of statistical analysis.

VI.D Histogram and chi squared test

Figure 6.2 is a histogram of the ratios \( r_i \), corrected for dead time and solid angle. The abscissa is the standard normal variable

\[
z = \frac{(r_i - \bar{r})}{\sigma_{th}}
\]

where \( \bar{r} \) is the un-weighted mean and \( \sigma_{th} \) is the theoretical standard deviation of Eq. 6.10. The points represent the expected frequency, and is the sum of 28 normal distributions using the run-standard deviation of each run as given in Table 6.1. The chi squared test of the fit with the mean and the standard deviation as the only adjustable parameters yields

\[
\chi^2 = 44.3
\]

(6.11)

for 36 degrees of freedom, corresponding to a confidence limit of 16%. Despite the high value for \( \chi^2 \), the distribution is fairly symmetric about the mean as seen in the next section.
FIG. 6.2. Histogram of the distribution of the experimental data about the mean in units of the theoretical standard deviation. The points marked "•" represent the expected bar heights, described in the text.
VI.E. Runs test

The result of the runs test described in Sec. IV.E, is shown in Table 6.4.

<table>
<thead>
<tr>
<th>Run length</th>
<th>Low runs</th>
<th>High runs</th>
<th>Expected number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>481</td>
<td>504</td>
<td>514 ± 20</td>
</tr>
<tr>
<td>2</td>
<td>285</td>
<td>264</td>
<td>257 ± 14</td>
</tr>
<tr>
<td>3</td>
<td>117</td>
<td>128</td>
<td>129 ± 10</td>
</tr>
<tr>
<td>4</td>
<td>74</td>
<td>61</td>
<td>64 ± 7</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
<td>29</td>
<td>32 ± 5</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>23</td>
<td>16 ± 4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>7</td>
<td>8.0 ± 2.7</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>5</td>
<td>4.0 ± 2.0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1</td>
<td>2.01± 1.39</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>1.00± 0.99</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>0.50± 0.70</td>
</tr>
<tr>
<td>Total</td>
<td>1022</td>
<td>1022</td>
<td>1028 ± 16</td>
</tr>
</tbody>
</table>

From a total of 4113 measurements, 49.6% fall above the mean and 50.4% below. This is within the expected fraction of (50.0±0.8)% on each side, thus justifying the assumption of a symmetric distribution. The total number of high and low runs is 1022 each, consistent with the expected total of (1029 ±16). The agreement between the expected and observed number of runs is marginal for short run lengths. However, there are no long runs (>10), outside those predicted by theory, and we conclude therefore that there are no systematic effects in
the apparatus to cause a gradual time-dependent shift in the measurement during the entire time interval of 28 consecutive runs.

VI.F The Lamb shift

The final experimental value for the asymmetry is

\[ A = 0.1178786 \pm 0.0000086, \]

with the contributing errors shown in Table 6.5. The field independent part of \( A \) is

\[ A^{(0)} = 0.1178082 \pm 0.0000086. \]

This yields a corresponding value for the Lamb shift of

\[ \nu(2s_{1/2} - 2p_{1/2}) = 14022.67 \pm 1.06 \, \text{MHz} \]

with the error arising from the uncertainty in the field strength being less than 0.02 ppm in the Lamb shift value.

### TABLE 6.5. Sources of error for A.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \delta A/A ) (ppm)</th>
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</thead>
<tbody>
<tr>
<td>Counting statistics</td>
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</tr>
<tr>
<td>Corrections:</td>
<td></td>
</tr>
<tr>
<td>Effect of detector polarization</td>
<td>&lt; 0.4</td>
</tr>
<tr>
<td>Solid angle</td>
<td>2.6</td>
</tr>
<tr>
<td>Dead time</td>
<td>0.3</td>
</tr>
<tr>
<td>Total ( \left( \sum \delta A_i^2 \right)^{1/2}/A )</td>
<td>73.3</td>
</tr>
</tbody>
</table>
CHAPTER VII
CONCLUSIONS

Our values for the field-free anisotropy,
\[ A^{(0)} = 0.1178082 \pm 0.0000086 \] and \[ \text{He}^+ \ 2s_{1/2} - 2p_{1/2} \] Lamb shift
\[ S = (14022.67 \pm 1.06) \text{ MHz} \] both lie lower than those of Drake,
Goldman and van Wijngaarden (1979),
\[ A^{(0)} = 0.117951 \pm 0.000024 \] and \[ S = (14040.2 \pm 2.9) \text{ MHz} \] by
17 standard deviations. Our Lamb shift value is lower than
the theoretical values of Mohr and Erickson by about 20
standard deviations. Since the earlier measurement of Drake
et al., a considerable improvement in the apparatus has been
made. In particular, the photon detection system uses
channeltron electron multipliers of a much higher bias
current, and we expect no gain depression to have occurred
at our counting rates. If the measured anisotropy value is
taken to be correct then the validity of the quenching
radiation theory from which the Lamb shift is interpreted
must be questioned.

On the other hand, our experiments with photon counting
have shown the CEMs to be stable only in comparison with the
BeCu electron multipliers. The difference between these
detectors is just in their gain stages (electron
multiplication), and yet they have produced vastly different
results. To verify the present value for the anisotropy, an
alternative detection system must be employed that circumvents the difficulties inherent with photon counting techniques. It is hoped that a direct measurement of the photoelectric current from the surface of the conversion dynode will provide this alternative in a future experiment.
REFERENCES


VITA AUCTORIS


Here, he completed his post-secondary school education at Kilburn Polytechnic, and then spent a year working after which, he attended King's College, London and obtained an honours B. Sc. degree in Physics.

He then came to Canada, to the University of Windsor where he received a Master of Science degree in Physics, and is enrolled towards a PhD degree in Physics.