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A method for computing the angular momentum of the human body.

Andrew Ward. Smith
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THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED
A METHOD FOR COMPUTING THE ANGULAR
MOMENTUM OF THE HUMAN BODY

by

Andrew Ward Smith

A Thesis
submitted to the Faculty of Graduate Studies
through the Faculty of
Human Kinetics in Partial Fulfillment
of the requirements for the Degree
of Master of Human Kinetics at
The University of Windsor

Windsor, Ontario; Canada
1982
ABSTRACT

The purposes of the study were to develop and validate a simple, practical method for quantifying angular momentum, moments of inertia, and angular velocity from filmed trials of any uniplanar activity, without placing unusual constraints on the performer. The method was validated three ways: verification of the mathematical model by use of a pendulum, comparison of the method to that of Miller (1970) eliminating the effects of individual differences, and comparison of data from this method of selected springboard dives and long jumps to published data of Miller (1970) and Ramey (1973,1974).

Four subjects, two divers and two long jumpers, were filmed performing their respective activities by use of a 16 mm Locam camera and the resulting raw data was smoothed using a fourth-order, low-pass digital filter and processed by a 48K Apple microcomputer in a program designed to determine segmental and whole body centres of gravity (c.g.), segmental moments of inertia and angular velocity, and local and transfer terms of angular momentum. The mean angular momentum values for the activities were: subjects 1 and 2, pike dive: 16.90 and 13.90 kg·m²·sec⁻¹; 1-1/2 somersault dive: 33:80 and 25.70 kg·m²·sec⁻¹. Subjects 3 and 4 performed the long jump and their results were 1.20 and 9.80 kg·m²·sec⁻¹ respectively.

It was concluded that the method is a precise technique for quantifying angular momentum, moments of inertia, and angular velocity, as demonstrated by the pendulum experiment. The Smith/Miller methods comparison strongly suggested that, in removing between subject variance, a significant difference exists between the method of this study and Miller's method. It was revealed that local angular momentum is
dependent on angular velocity, while variation of transfer angular momentum occurs as a result of alterations in transfer moments of inertia. Future direction of work in the area of angular momentum could be toward applying the method of this study to coplanar activities, as well as exploring the relationship between angular momentum and performance.
DEDICATION

To my wife, Peggy, whose love, concern and personal drive for excellence served as an comfort and inspiration to me during the past two years, as it always has, this work is sincerely dedicated.

And to our parents, Ronald and Regina Smith, and Jack and Peggy Heffernan: their love and their confidence in my success was, and always will be, greatly appreciated.
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Chapter I

Introduction

The angular momentum relationship of moment of inertia and angular velocity is described as follows:

\[ \mathbf{\dot{H}} = \mathbf{I} \times \mathbf{\dot{W}} \] (1)

where \( \mathbf{H} \) is the angular momentum, \( \mathbf{I} \) is the moment of inertia, and \( \mathbf{W} \) is the angular velocity. If \( \mathbf{H} \) is held constant, as occurs in a system where no external torques are acting, then any alteration in \( \mathbf{I} \) results in an inverse change in \( \mathbf{W} \). This concept is referred to as the conservation of angular momentum. As long as the various body segments remain essentially unchanged in position relative to each other, this law is easily demonstrated. Miller (1970) developed an efficient method, based on Eq 1, which was used to determine the angular momentum of the airborne phase of a series of four dives. The method involved locating periods of small relative positional change between body segments and the determination of the body's moment of inertia and angular velocity during these periods. However, the number of activities where this kind of 'quasi-rigid' body position is maintained throughout an airborne phase is quite limited. Thus, in the past decade researchers have attempted to adapt analysis techniques to investigate a much broader spectrum of human movements.

Ramey (1973, 1974) developed a method of accurately determining changes in angular momentum through coordination of cinematographic and force platform data. This method assesses magnitudes of both vertical and horizontal forces as well as the location of the subject's centre of gravity (c.g.) at the instant of take-off. However, the Ramey technique is only useful for determining changes in angular
momentum and not investigating the angular momentum itself. As well, the use of a force platform, which is a rather expensive piece of equipment, represents a second limitation in the usefulness of the Ramey method.

The main thrust of more recent investigation into the calculation of angular momentum has been that of a segmental analysis, i.e., treating each segment of the body as a separate entity, linked to other similar segments, and relating each segment to the whole body c.g. in terms of moment of inertia and angular velocity. Two studies have addressed the problem from this perspective. Hay, et al (1977) and Dapena (1978), in order to increase the accuracy of angular momentum measurements, have divided angular momentum into two types: local, i.e., pertaining to each segment with respect to its own c.g., and transfer, i.e., dealing with each segment in regards to the whole body c.g. Hay, et al (1977) used a modification of Eq 1 to produce angular momentum data for four different movements, namely a front somersault, a long horse vault, a high jump, and a pole vault. The analysis involved various Body Segment Parameter Data (BSPD), including segment masses, segmental c.g. locations and moments of inertia. As well, displacement data from the filmed trials were smoothed using the cubic spline technique. While the results appeared to be acceptable, the analysis technique is complex and a simpler method which could produce equally valid data would be advantageous.

The study by Dapena (1978) used, with some modifications, the Hay method to address the problem of coplanar movement and the calculation of angular momentum therein. Specifically, Dapena (1978) examined the Fosbury Flop high jump and resolved the angular momentum of the
activity into three planes, X, Y, and Z. As with the Hay data, the results are reasonable. Dapena's method is understandably more complex, owing to the additional perspective of a coplanar analysis. However, a study by Smith (1981) revealed a potential problem with the technique. Calculation of the transfer term of the angular momentum in a forward Pike Dive, using Dapena's method adapted for a uniplanar analysis, produced atypical data. Further analysis revealed that Dapena's method fails to adequately account for the angular velocity of each segment about the whole body c.g. Since Dapena's findings (1978) are reported in terms of total angular momentum and did not include a breakdown of the angular momentum into its component parts of moment of inertia and angular velocity, the cause of the atypical data was not easy to extract from the published data.

It would appear that a modification of the Hay and Dapena methods could produce an analysis technique that would be simple to use, accurate and, above all, not require expensive equipment or be restricted in the types of movements that could be analyzed. As well, such a method would be easily adaptable, due to its simplicity, to the various forms of hardware tools currently in use in biomechanics facilities.

**Statement of the Problem**

The purposes of the study were to develop and validate a simple, practical method of quantifying angular momentum, moment of inertia, and angular velocity from filmed trials of any uniplanar movement, without placing unusual constraints on the performer.

**Limitations**

The limitations of this study were threefold: first, there was a
certain amount of error inherent in the use of high speed cinemato-
ography and its subsequent analysis. Some examples of this error in-
cluded lens characteristics, small movements of the film during expos-
ure and projection, errors in joint centre location, movement of the
subject out of the plane of filming, and so on. Estimates of the
amount of this kind of error can range as high as 10-15%, and this
necessitated the use of an appropriate data smoothing technique in the
analysis.

Second, the size of the sample in the validation of the analysis
method was a limitation. Any limitation involved in using a group of
two long jumpers and two divers was reduced to a minimum by selecting
relatively highly skilled performers. Also, since this study in-
vestigated a measurement technique and was not concerned with making
broad generalizations about either long jumpers or divers, the size of
the sample was not a major limitation.

Third, the analysis technique utilized BSPD from both Dempster
(1955) and Clauser (1969) to determine segmental masses, lengths and
c.g. locations. Since BSPD are generally based on cadaver studies of
older, non-athletic male subjects, certain small discrepancies may
exist. However, the use of BSPD is widely accepted as the best
approach to the determination of inertial properties in spite of the
inherent error.

**Definition of Terms**

The major terms used in this study are defined as follows:

**Centre of Gravity (c.g.)** is the theoretical point where all of the mass
of a body is seen to exist.

**Moment of Inertia (I)** is the resistance to rotation possessed by a body
as determined by its mass and the manner in which the mass is distributed relative to the axis of rotation.

Angular Displacement ($\theta$) is the change in the positions of a line connecting two points in space, relative to the proximal point, and measured in radians from a right horizontal in consecutive frames.

Angular Momentum ($H$) is a property possessed by a rotating body and is equal to the product of its moment of inertia and the change in angular displacement with respect to time.

Take-off (t.o.) is the instant in time where the performer loses contact with the ground or diving board.

Airborne Phase is that portion of the activity following take-off where the only forces acting on the performer are those of gravity and air resistance, and ends with contact with the ground or the surface of the water.

Transverse Axis is the rotation axis extending through the c.g., perpendicular to the plane of motion.
Chapter II

Review of Literature

The purposes of the study were to develop and validate a simple, practical analysis technique for determining angular momentum, moment of inertia, and angular velocity from filmed trials of any uniplanar movement without placing unusual constraints on the performer. The review of literature was divided into two sections: angular momentum and methodology studies.

Angular Momentum

The law of the conservation of angular momentum is directly related to the rotational analogues of Newton's First and Third Laws, and states, "In the absence of an external, unbalanced moment of force, a rotating body has an angular momentum which is constant in both magnitude and direction" (Simonian, 1981, p. 111). Since, in a situation where the human body is rotating around its own C.G., the angular momentum of the person must be conserved, changes in the two components of angular momentum are possible when the relationship between moment of inertia and angular velocity (EQ 1) is taken into consideration. By bringing body segments closer to the axis of rotation, as the diver does in assuming a tuck position, the body's moment of inertia is decreased. In order to keep the angular momentum constant, the angular velocity of the body will correspondingly increase. Because the diver is not in contact with the ground, it is not possible to affect the angular velocity directly without first altering the moment of inertia (Simonian, p.112). As the diver approaches the water, the angular velocity of the body must be quickly reduced to near zero in order to enter the water properly. This is achieved by extending the body
segments and thus increasing the moment of inertia.

Methodology Studies

A typical problem with which a movement analyst must contend before any study of motion is undertaken is that of modifying deterministic models to suit real-life situations. Specifically, most kinetic equations used in biomechanics are based on principles dealing with particles and centres of mass. The human body is a complex structure, composed of various segments, each of which has a different mass, volume, and density, linked together by several kinds of joints. Thus, it is clear that to apply equations normally used to analyze a particle to a non-particle structure, three options are available: (a) modify the equations, (b) subdivide the structure into its particle components, or (c) a combination of both (a) and (b). The majority of studies in biomechanics attempt to utilize the third option.

The general pattern of analysis used by researchers studying angular momentum is, first, devising a human body model which both satisfies the parameters of the necessary equations and reasonably fits the real-life limitations and capabilities of the body. Second, the basic equations which normally handle less complex structures must be modified in a logical and consistent manner in order to fit the human body model and still measure what they are designed to measure. Depending upon the complexity of the original human model, the sophistication of the different kinds of analyses is quite variant.

One of the first researchers into the problem of determining the angular momentum of the human body was Miller (1970). Examining springboard dives, Miller used a 'quasi-rigid' model of the body. Periods of the dive where there was no motion of the body segments
relative to one another were located and the model was defined as a line connecting two widely separately landmarks. The moment of inertia of the entire body around its transverse axis was then used to calculate the angular momentum. Thus, Miller (1970) undertook the analysis by utilizing a simple model requiring very little modification of Eq 1. Miller's technique produced values which ranged from 26.49 to 105.95 kg·m²·sec⁻¹ for the different dives studied. However, the accuracy of the method was dependent upon the strictness with which 'quasi-rigid' was defined and it was this inflexibility which did not allow Miller's method to be suitable for all activities.

Ramey (1973, 1974) utilized a 14 segment model of the body in order to, first, determine the location of the c.g. at take-off, and second, coordinate force platform data with cinematography in assessing the change in angular momentum in long jump take-offs. The equation used was:

$$\Delta H = \int (\mathbf{F} \times \mathbf{r}) \, dt$$  \hspace{2cm} (2)

where: $\Delta H$ is the change in angular momentum; $\mathbf{F}$ is the force produced at t.o.; $\mathbf{r}$ is a position vector between the c.g. and the point of force application; and $dt$ is the elapsed time. (Ramey, 1973)

The above equation (Eq 2) was based on the angular analogue to the momentum-impulse relationship and as such yielded the change in, and not the actual, angular momentum (Hay, et al, 1977). Thus, Ramey's method was only effective if the amount of angular momentum was known prior to contact with the force platform. Hay, et al (1977) felt that, in addition to the fact that a force platform seriously limited the use of Ramey's technique in competitive situations, the procedure was very sensitive to small errors in the determination of the X-coordinate of
the c.g., thereby placing a heavy demand on accuracy in the coordination of film and force platform data.

A similar technique to Ramey's was used by Bedi and Cooper (1977) to determine the change in angular momentum of long jumps. The authors made the assumption that the change in angular momentum equalled the change in angular impulse, which was identical to Ramey's premise. As such, the Bedi/Cooper method was subject to the same limitations as Ramey's and nowhere was this more evident than in their respective results. Recording data from virtually identical experimental designs, Ramey (1974) reported a $\Delta H$ of 20.31 kg m$^2$ sec$^{-1}$ (15 ft lb sec) while Bedi and Cooper (1977) found a mean value of 99 kg m$^2$ sec$^{-1}$ (12.2 ft lb sec). While, in some specific applications the force platform method may be useful, it did not provide the kind of general, all-purpose analysis technique that this study proposed to determine.

In order to maximize the similarity between a human model and a system of particles, the segmental approach has been universally accepted in a wide range of biomechanics analyses, including angular momentum studies. The approach treats the human body as a system of linked segments, each having a mass and a c.g. location expressed as a proportion of the total body mass and segment length respectively.

Three studies approached angular momentum from a segmental viewpoint: Hay, et al (1977), Dapena (1978), and Smith (1981). All three studies used similar models and each subdivided angular momentum into two types, local and transfer. Local angular momentum referred to the segmental angular momentum about its own transverse axis, while transfer was the angular momentum of the segment with respect to the body's principal transverse axis. By summing each type for all 14 segments
and adding the totals an accurate measure of the body's angular momentum was made. The studies by Hay, et al (1977) and Smith (1981) dealt with uniplanar movement, while Dapena's study (1978) involved a three dimensional analysis.

Dapena's study (1978) was the most complex due mostly to the fact that segments and c.g.'s were located in terms of X, Y, and Z coordinates. Analyzing the Fosbury Flop high jump, Dapena (1978) reported values of approximately -26, 3, and -12 kg·m²·sec⁻¹ about the X, Y, and Z axes respectively. The study by Smith (1981) adapted the Dapena method for a uniplanar analysis and, in doing so, it was discovered that the method failed to account for the angular velocity component of the transfer term of the performer's angular momentum. Therefore, results by Smith (1981) of a two dimensional analysis of a forward pike dive were atypical.

Summary

A review of the research pertaining to angular momentum has revealed that there are three different types of methods for calculating the angular momentum of the human body, namely the Miller (1970), the Ramey (1973, 1974), and the Hay; et al (1977) methods, the latter of which is also the basis of both the Dapena (1978) and Smith (1981) methods. The papers reviewed can be summarized as follows: The Miller technique provided accurate data in very specific situations, as did all of the reviewed studies. However, it was not flexible enough to cover all uniplanar activities. The coordination of force platform and film data, as used in the Ramey and Bedi/Coooper methods was also not the most practical due to the expense and inconvenience of the equipment, and the oversensitivity of the analysis technique in certain
situations. While the Hay and Dapena methods are the most versatile, both, perhaps unnecessarily, sacrificed simplicity for versatility. As well, possible flaws in their respective equations may lead to inaccurate data.
Chapter III

Methodology

The purposes of the study were to develop and validate a practical analysis technique to quantify angular momentum, moment of inertia, and angular velocity in any uniplanar activity without placing unusual constraints on the performer. As well, the study used data from filmed trials of long jumps and selected springboard dives to verify the technique through comparison of the resulting data to data from similar studies. Specifically, the study dealt with the following parameters:

1) Locations of both segmental and whole body c.g.

2) Determinations of both local and transfer aspects of moment of inertia, angular velocity, and angular momentum.

3) Summation of both local and transfer angular momenta to determine a total amount of angular momentum for the entire body at any given point in time.

Development of the Model

The analysis involved the determination of 15 segmental endpoints, numbered 0 to 14 (FIG. 1), by use of a Numerics Corp. graphics calculator. These data were smoothed using a fourth order low-pass digital filter (Butterworth type) (Winter, 1979), and processed by an Apple II, 48K microcomputer in a program which determined segmental and whole body c.g., moments of inertia, angular displacements, and angular momenta in a segmental method. Specifically, the method utilized a model where the human body is a system of 14 linked segments, numbered 0 to 13 (FIG. 2). The equations, along with the model, used in the analysis technique were adapted largely from three previous analyses,
FIGURE 1: The basic schema for the numbering of the body segmental endpoints.

FIGURE 2: The basic schema for the numbering of the body segments.

This analysis method made the following assumptions: if a transverse axis, "A", was defined as passing through the c.g. of the body, the angular momentum of the whole body about "A" was seen to equal the sum of all the angular momenta of the segments about "A". Further, angular momentum was seen to be of two distinct types, local and transfer. Local angular momentum was the angular momentum of a segment about an axis "A'", parallel to "A" and passing through its c.g., while transfer angular momentum was that of a segment about "A". Thus, the total angular momentum at any given point in time was the sum of both the local and transfer terms (Smith, 1981).

Whereas previous methods have required rigid body positions, sensitive and expensive equipment, or more complex and possibly erroneous equations, this method attempted to simplify the necessary calculations while achieving a high level of accuracy. Using BSPD, the mass and height of the subject, and the temporal and linear factors, the analysis technique determined the required parameters by use of the following equations:

A) Whole Body C.G.:

(a) \( C_1 = (P_1 \times (1-q_1)) + (D_1 \times q_1) \) \( (3) \)

where: \( C_1 \) is the coordinate of the c.g. of the ith segment; \( P_1 \) and \( D_1 \) are the proximal and distal coordinates of the ith segment; and \( q_1 \) is the fractional distance from the c.g. of the ith segment to the proximal endpoint.

(b) \( C^* = \sum_{i=0}^{13} [(C_i \times m_i \times M)]/M \) \( (4) \)

where: \( C^* \) is the coordinate of the whole body c.g.; \( m_i \) is the fractional proportion of the mass of the ith segment; and \( M \) is the total mass.
B) Segmental Moment of Inertia:

\[ I_i = (m_i \times \bar{M}) \times (SL_i \times k_i)^2 \times \kappa \]  

(5)

where: \( I_i \) is the moment of inertia of the \( i \)th segment; \( SL_i \) is the computed length of the \( i \)th segment; \( k_i \) is the fractional radius of gyration of the \( i \)th segment; and \( \kappa \) is the correction constant for out of plane movement determined as follows: if \( SL_i \) is equal to the actual length then \( \kappa = 1 \); if \( SL_i \) is not equal to the length then \( \kappa = (\text{actual}/SL_i)^2 \).

C) Local Angular Momentum:

\[ H(\text{local}) = \sum_{i=0}^{13} [I_i \times (\sin \theta_i/\text{dt})] \]  

(6)

where: \( H(\text{local}) \) is the local term of the angular momentum about "A"; \( \theta_i \) is the angular displacement of the \( i \)th segment; and \( \text{dt} \) is the elapsed time.

D) Transfer Angular Momentum:

\[ H(\text{trans}) = \sum_{i=0}^{13} [m_i \times \bar{M} \times (v_1)_i \times (v_2)_i] \times [\sin \phi_i/\text{dt}] \]  

(7)

where: \( H(\text{trans}) \) is the transfer term of the angular momentum about "A"; \( v_1 \) and \( v_2 \) are location vectors between "A" and "A'" in two consecutive frames for the \( i \)th segment; and \( \phi_i \) is the angular displacement of a line connecting "A" and "A'" for the \( i \)th segment.

E) Total Angular Momentum:

\[ H(\text{total}) = H(\text{local}) + H(\text{trans}) \]  

(8)

where: \( H(\text{total}) \) is the total angular momentum about "A".

Experimental Procedures

A pendulum system was used to further validate this method.

A small weight was suspended in the system, set into motion and filmed at 30 frames per second. Three different lengths of the pendulum arm were used in this additional investigation and the angular momentum of the system was then measured from the film using an adaptation of the
method of this study. These measurements were compared to *ap priori* calculations of the angular momentum of the system using the period of the pendulum as the temporal factor.

As well, a comparison between the Miller (1970) method and the method of this study was conducted in an experiment which eliminated individual differences. Raw segmental endpoint data were used in a replication of the method used by Miller (1970), as well as in the method of this study.

**Subjects**

Four subjects participated in the study. Two subjects performed the Long Jump. One subject was from a Detroit Track Club and the other was a member of the University of Windsor Track Team. The remaining subjects were members of the University of Windsor Diving Team.

**Task: Long Jump**

Each jumper warmed-up as for competition and, following several practice jumps, performed three filmed trials each of a Long Jump. Subjects were encouraged to simulate the competitive situation.

**Task: Diving**

Following a sufficient warm-up, each diver was filmed performing two trials each of a forward Pike and a forward 1-1/2 Somersault Dive. A low-splash entry into the water was stressed to encourage good performances by the subjects.

**Filming Areas and Protocol**

The Long Jumps took place at the Long Jump pit at the University of Windsor's indoor track facilities, while the dives were performed from the one-meter springboard at the University of Windsor pool. The
filming was done in three separate sessions, two for the long jumps and one for the dives.

The trials of each performer were recorded on film by use of a 16 mm Locam camera, placed normal to the plane of motion and approximately 30 meters away for both activities. For the long jumps, the action filmed included the last stride of the approach run to the landing while the diving included the hurdle to the entry of the hands into the water.

The film speed was set at 100 frames per second and this was verified by use of an internal light emitting diode (L.E.D.) operating at 100 Hz. Linear data was standardized for each roll of film by filming a meterstick in the plane of motion. In addition, prior to filming, data pertaining to each subject's mass and height was recorded for use in the analysis.

Each subject's filmed trials were viewed and the best performance for each was selected for analysis. The main criteria for the trial selections was the longest trial for the Long Jump and the least amount of water displaced at entry for the Dives.
Chapter IV

Results and Discussion

The purposes of the study were to develop and validate a simple, practical method of quantifying angular momentum, moment of inertia, and angular velocity from filmed trials of any uniplanar movement, without placing unusual constraints on the performer. The discussion which follows will focus first, on the development of the angular momentum model as presented in the previous chapter, and second, on the testing of that model.

Development of the Model

In order to develop a better method of calculating the angular momentum of the human body, other available methods were objectively assessed in terms of their strengths and weaknesses. The Miller (1970) method provides a relatively simple, four-segment model of the human body. Further, the angular momentum is measured in the form of both local and transfer terms, the use of which improves the accuracy of analysis. However, Miller (1970) depends on the maintenance of a 'quasi-rigid' body position throughout the measurement period, and this requirement accounts for the inflexibility of the method. The method used in this study took from the Miller method the concept of the two terms of the angular momentum as well as the basic goal of simplicity. However, it was decided that any improvement on the Miller method should be in the area of increasing the flexibility of the analysis technique. This improvement should be achieved by ensuring that there be no requirement of a 'quasi-rigid' period during the movement.

The major strength of the Ramey (1973, 1974) method is the use of a fourteen-segment model of the human body. A deterrent to the
widespread use of the Ramey method is the requirement of the force platform which is not only an expensive piece of equipment, but one which limits the practicality of the method in terms of its use in all situations. Drawing on the Ramey method, the method of this study defined the human body as being fourteen rigid segments, connected at various hinged joints.

The methods of Hay, et al (1977) and Vapena (1978) are the most versatile of the methods used to date and as such, provide a number of important elements which were incorporated into the method of this study. Specifically, the Hay method made important contributions in terms of providing an efficient and logical algorithm for calculating the angular momentum of the human body. Also, the Hay method contributes a technique for determining angular displacement which forms the basis of the definition of 'angular displacement' used in this study. The Hay method is based on the following equation:

\[ H_Z = \sum_{i=0}^{13} \left[ I_{Zi} (W_{zi}) + m_i (r_{SZ_i}) (w_{SZ_i}) \right] \]  \hspace{1cm} (9)

where: \( H_Z \) is the angular momentum about a transverse axis \( Z^* \); \( I_{Zi} \) is the moment of inertia about \( Z^* \); \( W_{zi} \) is the angular velocity relative to \( Z^* \); \( m_i \) is the segment's mass; \( r_{SZ_i} \) is the distance of the segment's c.g. (S) to \( Z^* \); and \( w_{SZ_i} \) is the angular velocity of S about \( Z^* \). (Hay, et al, 1977)

The Hay method requires the usual BSPD, i.e., segmental masses and c.g. locations, along with segmental moments of inertia as calculated in cross-sectional studies (Hay, et al, 1977). This tends to add more potential for BSPD error than would occur without the use of moment of inertia data. Out of plane motion by the segments is corrected by the following equation:

\[ I_{Z^*} = I(S) \times (lp / la)^2 \]  \hspace{1cm} (10)
where: \( I(S) \) is the calculated moment of inertia from film data; and \( l_p \) and \( l_a \) are the projected and actual lengths of the segment (Hay, et al, 1977).

The use of EQ 10 tends to underestimate the true magnitude of \( I_{Z^*} \) as the following example demonstrates: Given that most out of plane movement causes the filmed segment to appear shorter than it actually appears in real-life. Therefore:

\[
\begin{align*}
\text{IF } m &= 1.90 \text{ kg} \\
k &= 0.28 \\
l_a &= 0.28 \text{ m} \\
l_p &= 0.25 \text{ m} \\
\text{AND } I &= m \times (l \times k)^2 \\
\text{THEN } I_{Z^*} &= 0.012 \text{ kg m}^2 \text{ (calculated)} \\
\text{AND } I(S) &= 0.009 \text{ kg m}^2 \\
\text{--using Hay's correction technique, } I_{Z^*} \text{ becomes } 0.007 \text{ not } 0.012 \text{ kg m}^2.
\end{align*}
\]

Thus, the greater the difference between \( l_a \) and \( l_p \), the greater the underestimation of \( I_{Z^*} \). The correction factor for out of plane movement used in this study is:

\[
I_{Z^*} = I(S) \times (l_a / l_p)^2
\]

This results in the proper adjustment in the calculated lengths of segments, regardless of the difference between \( l_a \) and \( l_p \). Lastly, the equation used by Hay, et al to calculate the local term of the angular momentum is more complex than necessary due to the error in the above correction factor for out of plane motion (EQ 10). The particular equation is:

\[
H(\text{local}) = \sum_{i=0}^{13} \sqrt{(I_{Z^*1})^2 \times (I_{Z^*2})} \times \sin \theta_i \times dt \tag{12}
\]

where: \( H(\text{local}) \) is the local term of the angular momentum, \((I_{Z^*})_1\) and \((I_{Z^*})_2\) are the moments of inertia of segment i about an axis passing through the segmental c.g and parallel to an axis \( Z^* \) passing through the whole body c.g., \( \theta_i \) is the angular displacement, and \( dt \) is the elapsed time. Hay, et al (1977)
A minimum of two frames where $I_{Z*}$ is calculated is required for
EQ 12, and Hay et al indicated that $I_{Z*}$ is determined for each frame
studied. By utilizing the correction factor in EQ 11, $I_{Z*}$ need only
be determined once and the values obtained for each segment then used
throughout the subsequent analysis. Thus, the calculations for the
local term of the angular momentum are made more precise and simpler
to calculate at the same time.

While the Hay method has many strengths, certain improvements
are possible. Therefore, the method of this study drew on these strengths
by including a similar algorithm for determining angular momentum, as
well as using the same technique for calculating angular displacement.
The method used in this study also used the out of plane correction
tactor of the Hay method as the basis for its own correction constant.

The contribution of Dapena’s method to the method of this study,
while being largely an intangible one, is nevertheless a very important
one. The discovery of the failure of the Dapena method to adequately
account for the angular velocity of the transfer term of the angular
momentum (Smith, 1981), served as the impetus for this particular paper.
In addition, Dapena’s method offers one of the simplest techniques for
determining both segmental and whole body c.g. locations.

In summary, the development of the method of this study to deter-
mine the angular momentum of the human body included the utilization
of certain features found in prior methods. Specifically, the
Miller (1970) method provided both the concept of a 'simple' analysis
as well as the division of angular momentum into local and transfer
terms in order to increase the precision of the technique. The use
of a fourteen-segment model was taken from Ramey (1973,1974) in an attempt to improve the mathematical concept of the human body. The closely-related methods of Hay, et al (1977) and Dapena (1978) contributed both the impetus for this study, in the case of Dapena, as well as several important mathematical factors. These included a technique for measuring angular displacement, a modified way of correcting out of plane motion, a simple way of locating c.g., and a logical algorithm for solving the problem of quantifying angular momentum. In terms of the direction of future work in the area of the determination of angular momentum of man in motion, the primary alteration to be made in the method of this study would be to adapt it to coplanar movement. In this manner, activities such as high jumping, gymnastic tumbling, twisting dives, as well as non-sport movement patterns could be analyzed thus, increasing the body of knowledge in the area of angular momentum.

**Testing of the Model**

Having determined a method designed to quantify angular momentum, and drawing on the strengths of other methods, it was necessary to test the method. This was achieved three ways: verification of the mathematical soundness of the method, evaluation of the method in a direct comparison with the Miller method to elimate individual differences, and a comparison of angular momentum values obtained with the method of this study for selected springboard dives and long jumps to published data from Ramey (1973,1974) and Miller (1970).

**Pendulum Experiment**

In drawing conclusions regarding the relative merit of any one method of calculating angular momentum over another, there is a potential source of error as yet not discussed. If all methods have a
form of inherent error which has gone undetected, then any comparison would be more a reflection of the various degrees of error present rather than an assessment of the absolute accuracy of the methods. Thus, some kind of baseline is necessary to determine both the absolute accuracy of the method of this study, and the limits of possible error present.

In addition to the procedure stated in Chapter III, the method of this study was validated by comparing priori calculations of the angular momentum of a pendulum system to that calculated from film data of the same pendulum using three different lengths of the pendulum arm and a single mass (FIG. 3). A small metal weight was suspended by thread to form a pendulum. The lengths used were 0.60, 0.40, and 0.25 m for the three trials. The mass of the weight was determined by a Sartorius gram weight scale as being 266.73 gm. For each trial, the mass was displaced 0.174 radians from the equilibrium point and then released. Three trials for each length were filmed at 30 frames per second, by use of a 16 mm Locam camera placed normal to the plane of motion of the pendulum arm and approximately 3 meters away. The camera speed was verified by an quartz crystal L.E.D. operating at 100 Hz.

Two equations were used in the analysis, one for the real-life calculations and the other for the calculations of the data from the film. Both equations were based on EQ. 1, with the film data equation being a direct adaptation of the equations used in this method. They were as follows:

Real-life:

\[ H_r = m \cdot (1)^2 \cdot \left[ 0.698 / (2 \pi \sqrt{L/g}) \right] \]  
(13)
FIGURE 3: Apparatus used in pendulum experiment designed to compare predicted angular momentum to that calculated from cinematographic record.

where: \( H_t \) is the real-life angular momentum; \( m \) is the mass of the weight (the mass of the thread was assumed to be zero); \( l \) is the length of the thread; \( \pi \) is equal to 3.141529; and \( g \) is the acceleration due to gravity (\( g = 9.80 \text{ m/sec/sec} \)).

The expression, \( m \cdot (1/2) \), represented \( I \) and \( W \) was evaluated by the expression, \( 0.698 / (2 \pi \sqrt{l/g}) \), where the numerator was the displacement in radians and the denominator was the time as expressed by the period of the pendulum.

Film data:

\[
Nf = \sum_{i=0}^{n} \left[ m \cdot (P_i - CC_i)^2 \cdot \sin \sigma / dt \right] \quad (14)
\]

where: \( Nf \) is the angular momentum determined from the film data; \( P_i \) is the coordinate of the pivot of the pendulum in the \( i \)th frame; \( CC_i \) is the c.g. coordinate of the mass in the \( i \)th frame; \( \sigma \) is the angular displacement of the thread/mass unit in the \( i \)th frame, and \( dt \) is the elapsed time in seconds.

The results of the pendulum experiment are presented in Table I. As is evident, EQ 14 produces values for the angular momentum of a pendulum within 9% of the real-life values. The decision to use a pendulum to validate the method of this study was based on the
TABLE I

Comparison of Predicted and Calculated Angular Momenta of a Pendulum System

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>Mass (g)</th>
<th>Angular Momentum (kg · m² · sec⁻¹)</th>
<th>Percent Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>266.73</td>
<td>Predicted 0.043</td>
<td>Calculated 0.042</td>
</tr>
<tr>
<td>0.40</td>
<td>266.73</td>
<td>Predicted 0.024</td>
<td>Calculated 0.026</td>
</tr>
<tr>
<td>0.25</td>
<td>266.73</td>
<td>Predicted 0.012</td>
<td>Calculated 0.012</td>
</tr>
</tbody>
</table>

Similarity between a pendulum and individual body segments. In calculating the local and transfer terms of angular momentum, each of the fourteen body segments is considered to be a centre of mass rotating about a fixed point in space just as the mass of the pendulum rotates about the fixed pivot in that system. Thus, the total angular momentum of the human body at any given point in time is the sum of 28 separate calculations of the angular momentum of a mass centre about either its own transverse axis of rotation or that of the entire fourteen-segment structure. If the estimation of the angular momentum of the pendulum is within 9% of the real-life value, then the 28 similar calculations will be within the same error estimate. Furthermore, the actual methods of this study, when applied to the filmed data collected over a longer period of time, included a mathematical error reduction process (digital filtering) designed to eliminate data which did not fall within predetermined limits. Therefore, from a mathematical viewpoint, the method of this study is a precise way of determining the
angular momentum of the human body.

Miller (1970) Method Comparison Experiment

An attempt was also made to eliminate the effect of individual differences in the comparison between the method of this study and that of Miller (1970). In order to isolate variations in the two methods not attributable to the subjects, data from subjects 1 and 2, for both the Pike and 1-1/2 Somersault Dives, were used in an analysis which replicated Miller's technique. The choice of a diving comparison was made for two reasons. First, the divers' skill in this study was probably closer to that of the subjects which Miller used, and second, a replication of Ramey's (1973, 1974) method necessitated the use of a force platform which was not available.

Six consecutive frames of a single trial were selected in each of the two divers' different dives. The selection criterion was based on the definition of 'quasi-rigid' offered by Wilson and Hay (1976) and was as follows: the body was considered to be in a 'quasi-rigid' position if its joint angles changed by less than 0.09 radians over the six frames (Wilson and Hay, 1976).

The following equation was used to evaluate angular momentum as calculated by Miller (1970):

\[ H(\text{Miller}) = \sum_{i=0}^{4} \left( (I_i \times W_i) + (R_i \times M_i \times dR_i/dt) \right) \]  

where: \( H(\text{Miller}) \) is the angular momentum of the subject as calculated by Miller; \( I_i \) is the moment of inertia of the \( i \)th segment; \( W_i \) is the angular velocity of the \( i \)th segment; \( R_i \) is a position vector joining the \( i \)th segment's c.g. to the whole body c.g.; \( M_i \) is the mass of the \( i \)th segment; and \( dt \) is the elapsed time.

Dependent t-tests were performed on the data taken from both methods, and the t-ratios are found in Table II. It is evident that
TABLE II

Summary Table Comparing the Miller and the Smith Methods of Computing Total Angular Momentum (See Appendix "A", Table V)

<table>
<thead>
<tr>
<th>Activity</th>
<th>T-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Pike Dive (Subject 1)</td>
<td>0.58</td>
</tr>
<tr>
<td>Forward Pike Dives (Subject 2)</td>
<td>8.97 *</td>
</tr>
<tr>
<td>Forward 1-1/2 Somersault Dive (Subject 1)</td>
<td>4.20 *</td>
</tr>
<tr>
<td>Forward 1-1/2 Somersault Dive (Subject 2)</td>
<td>13.27 *</td>
</tr>
</tbody>
</table>

degrees of freedom = 4, * p< 0.01

there is a significant difference between the the results of the method of this study and that of Miller (1970) in three of the four dives. This indicates that there is a difference in methods not attributable to the variance between subjects. There are three possible explanations which might account for these results. First, the selection of the six frames of raw data may not have, in fact, satisfied the requirement for a 'quasi-rigid' body position. Second, the raw data was smoothed for the method of this study, but was not for the Miller comparison due to the fact that the data list was too short. Third, the differences found were probably a result of the fact that, in progressing from a four-segment model to a fourteen-segment model, the diver's body was more clearly-defined, in a mathematical sense, and, in coming closer to a real-life representation of the human body, the method of this study was able to produce more consistent and realistic data.
TABLE III

Means, Standard Deviations, and Coefficients of Variance of Angular Momenta for Pike Dive, 1-1/2 Somersault Dive, and Long Jumps

<table>
<thead>
<tr>
<th>Activity</th>
<th>No. of Frames</th>
<th>Angular Momentum (kg·m²·sec⁻¹)</th>
<th>Coef. of Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long jump: S3</td>
<td>26</td>
<td>( \bar{X} = -1.2 ) s = ± 2.30</td>
<td>1.92</td>
</tr>
<tr>
<td>: S4</td>
<td>30</td>
<td>( \bar{X} = -9.8 ) s = ± 1.50</td>
<td>0.15</td>
</tr>
<tr>
<td>Pike dive: S1</td>
<td>31</td>
<td>( \bar{X} = -16.9 ) s = ± 1.57</td>
<td>0.09</td>
</tr>
<tr>
<td>: S2</td>
<td>25</td>
<td>( \bar{X} = -13.9 ) s = ± 1.65</td>
<td>0.12</td>
</tr>
<tr>
<td>1-1/2 dive: S1</td>
<td>26</td>
<td>( \bar{X} = -33.8 ) s = ± 2.93</td>
<td>0.09</td>
</tr>
<tr>
<td>: S2</td>
<td>20</td>
<td>( \bar{X} = -25.7 ) s = ± 2.39</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Angular Momentum Data Comparison Experiment

A summary of the means, standard deviations, and coefficients of variance of the total angular momentum is presented in Table III. Values which are negative represent angular momenta derived from primarily forward rotations. By convention, positive and negative rotation is oriented in terms of the direction of that rotation relative to the performer. Regardless of the direction of the performer across the plane of motion, forward angular displacement, such as occurs when the performer bends forward at the hips, is negative, while backward rotation is positive. Thus, while the sign of any angular momentum is important for the sake of accuracy, it is the actual magnitudes which are vital for comparative purposes.

The final test of the method of this study was the analysis of selected springboard dives and long jumps. The angular momentum data
for the two activities are shown in Appendix "A", Figures 4-9. Each figure displays the local term, transfer term, and total angular momentum, as well as the mean of the total, with respect to time. As is evident, the data conform logically to each movement pattern. For the springboard dives (FIG. 4-7), the diver begins to assume either the pike or tuck position almost immediately upon take-off. Magnitudes of local angular momenta increase due to the increase in angular velocity. Transfer moments of inertia decrease and this is reflected in the decrease in transfer angular momentum. During the 'quasi-rigid' portion of each dive, the two terms of angular momentum are relatively stable. The preparation for entry into the water causes a decrease in the local term, due to decreased rotation, and an increase in the transfer term as a result of an increase in the moment of inertia of the body. Thus, it can be seen that local angular momentum is dependent on angular velocity, since the local moments of inertia are fixed, while transfer angular momentum depends largely on the transfer moments of inertia.

The data from the long jump trials (FIG. 8-9) are more complex than those of the dives. Generally, the trunk, neck, and head of each jumper are relatively free of rotation, such that variations in each angular momentum term are due to rotations and alterations in the moments of inertia of the upper and lower extremities. A major difference between diving and long jumping evident in Figures 8 and 9 is the manner with which subjects deal with the forward rotation produced at take-off. The divers rotate with the forward rotation, producing negative magnitudes for the two terms of the angular momentum. However, the rotations of the extremities of the performers in long jumping attempt to counter the tendency to rotate forward. This is
done to maximize the length of the jump by allowing the trunk to remain basically vertical throughout the jump.

In spite of the different styles used by the two jumpers, some similar trends exist. Most of the data exhibit a series of troughs and peaks, such that three of each occur in a distorted version of a periodic wave. This is possibly due to the jumpers' attempts to counter the forward rotation imparted at take-off. The parts of the curves where the slope is sharply negative are periods where this forward rotation is not being countered. Acutely positive slopes which immediately follow are the periods where, having detected a tendency to over-rotate in a forward direction, the jumper attempts to compensate with backward rotation of the extremities. It is possible that small contributions of the trunk, and the head and neck, which contain 58% of the body's mass (Hay, 1978, p. 136), may also occur to counter the forward rotation.

A comparison between the means of the total angular momentum found in Figures 4-9 from this study and the published results of both Miller (1970) and Ramey (1973, 1974) is presented in Table IV. There are two points to be made with respect to these data. First, the data from this study conform to the general pattern of increasing magnitudes of angular momentum from the long jump (sail) to the more rotational 1-1/2 Somersault Dive. This pattern relates directly to the attempt by the performer to apply the necessary torque at take-off to enable completion of the required rotations before landing. For the long jump (sail), a much smaller torque is needed than for the 1-1/2 Somersault Dive, hence the magnitude of angular momenta are less.

Second, the means from Figures 4-9 are from 1.5 to 4.5 times
31

TABLE IV


<table>
<thead>
<tr>
<th>Activity</th>
<th>Method</th>
<th>* Angular momentum (kg · m² · sec⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long jump (sail)</td>
<td>Smith</td>
<td>1.20</td>
</tr>
<tr>
<td>Long jump (sail)</td>
<td>Ramey</td>
<td>5.40</td>
</tr>
<tr>
<td>Long jump (hang)</td>
<td>Smith</td>
<td>9.80</td>
</tr>
<tr>
<td>Long jump (hang)</td>
<td>Ramey</td>
<td>14.22</td>
</tr>
<tr>
<td>Long jump (hitchkick)</td>
<td>Ramey</td>
<td>20.31</td>
</tr>
<tr>
<td>Forward dive (Pike,S2)</td>
<td>Smith</td>
<td>13.90</td>
</tr>
<tr>
<td>Forward dive (Pike,S1)</td>
<td>Smith</td>
<td>16.90</td>
</tr>
<tr>
<td>Forward dive (Pike)</td>
<td>Miller</td>
<td>26.49</td>
</tr>
<tr>
<td>Forward dive (layout)</td>
<td>Miller</td>
<td>28.74</td>
</tr>
<tr>
<td>Forward 1-1/2 dive (S2)</td>
<td>Smith</td>
<td>25.70</td>
</tr>
<tr>
<td>Forward 1-1/2 dive (S1)</td>
<td>Smith</td>
<td>33.80</td>
</tr>
<tr>
<td>Forward' 1-1/2 dive</td>
<td>Miller</td>
<td>95.38</td>
</tr>
<tr>
<td>Forward 2-1/2 dive</td>
<td>Miller</td>
<td>105.95</td>
</tr>
</tbody>
</table>

* For comparative purposes, all rotations are assumed to be primarily backward.

smaller in magnitude than the published data. If all three methods were relatively equal in their precision, the cause of these differences would probably be entirely due to skill levels of the different subjects. In fact, some of the differences may actually be a result of subject skill differences. However, most of the differences shown are due to the methods used to obtain the angular momentum values. It has been demonstrated that the Miller method is significantly different from the method of this study. Ramey's method is considered to be very sensitive and subject to potential error (Hay, et al., 1977). Since the pendulum experiment proved the validity of the mathematical model of this study, it is reasonable to conclude that the
method of this study is the most precise of the three. All things being equal, the difference due to skill levels might be in the order of 0.5 to 1.5 times. However, it is not reasonable to accept more than a four-fold difference in data as being largely due to differences between subjects.

The direction of future work in the area of angular momentum might be in exploring the relationship between angular momentum and performance. It is possible that this relationship may resemble the inverted "U" pattern of activation and performance. In this example, an optimum arousal state exists for obtaining a superior performance. A subject who is under- or over-aroused will not perform as well as possible. Similarly, this inverted "U" pattern possibly exists in the relationship between angular momentum and performance. Detrimental performances by subjects may not be entirely due to generating too little angular momentum. Over-production of angular momentum may also cause poor performances.
Chapter V

Summary and Conclusions

The purposes of the study were to develop and validate a method for quantifying angular momentum, moment of inertia, and angular velocity from filmed trials of any uniplanar activity without placing unusual constraints on the performer. The validity of this method was achieved three ways: verification of the mathematical soundness of the method, evaluation of the method in a direct comparison with the Miller method to eliminate individual differences, and a comparison of angular momentum values obtained with the method of this study for selected springboard dives and long jumps to published data from Ramey (1973, 1974) and Miller (1970).

The method of this study was developed in part by drawing on the strengths of other methods. Specifically, the methods examined were Miller (1970), Ramey (1973, 1974), Hay (1977), and Dapena (1978). Once the method of this study was developed, the mathematical model of the method was tested by way of a pendulum experiment. The results of the experiment confirmed the precision of the method. Individual differences were eliminated in a comparison between the Miller method and the method of this study designed to further validate the procedures. It was concluded that there was a significant difference between methods.

Four subjects were used in this study. Each subject performed several trials of either springboard dives or long jumps and the trials were filmed using a 16 mm Locam camera, set to operate at 100 frames per second. The best single performance of each subject was selected such that two Pike Dives, two 1-1/2 Somersault Dives, and two long jumps were analyzed. Prior to filming, data pertaining to each

33
subject's height and mass were recorded for use in the subsequent analysis.

Angular momentum was defined as being the summation of the local term, i.e., the angular momentum of the 14 body segments about each of their own c.g., and the transfer term, i.e., the angular momentum of the 14 body segments about the whole body c.g. The patterns associated with the relationship between local, transfer, and total angular momentum were investigated. It was found that the data was logically consistent with the movement patterns of both the springboard dives and the long jumps. It was concluded that variations in the local term of the angular momentum were dependent on the segmental angular velocities, since the moments of inertia are fixed. Also, the fluctuations in the transfer angular momentum were a result of changes in the transfer moments of inertia.

The means of the total angular momenta for the activities followed the pattern of increasing magnitudes found in the published data. The magnitudes ranged from the smallest for the long jump (sail) to the, 1-1/2 Somersault Dive having the largest. The cause of this pattern was the amount of torque needed at take-off to complete the necessary rotations in the activity. Finally, specific differences between magnitudes of angular momentum from this study and that of published results were concluded to be largely due to the actual methods used, with the method of this study being considered to be the most precise method.

Based on the results obtained in this study, the following conclusions are warranted:

1. As demonstrated by the pendulum experiment, the method of this study is a precise analysis technique.

2. Analysis of the local and transfer terms of the angular momentum
from selected trials of springboard dives and long jumps, expressed as a function of time, revealed that variations in the local term are dependent upon changes in angular velocity, since the local moments of inertia are fixed. Similarly, fluctuations in the transfer term are due to alterations in transfer moments of inertia.

3. The magnitudes of the total angular momentum produced by this method follow the pattern established by prior research with respect to activities with relatively smaller torques at take-off having small magnitudes of angular momentum (long jump) and those activities with greater torques having much larger magnitudes of angular momentum (Pike and 1-1/2 Somersault Dives).

4. The method of this study is a simple, practical technique for quantifying angular momentum, moment of inertia, and angular velocity from filmed trials of any uniplanar activity without placing unusual constraints on the performer.
References


Smith, A. Angular momentum, moment of inertia, and angular velocity relationships in springboard dives. Unpublished manuscript, University of Windsor, 1981.


APPENDIX "A"

Raw Angular Momentum Data
### TABLE V

Angular Momentum Values from Miller (1970) and Smith (1982) Methods

<table>
<thead>
<tr>
<th>Activity</th>
<th>Frame No.*</th>
<th>Angular Momentum (kg · m² · sec⁻¹)</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Miller</td>
<td>Smith</td>
</tr>
<tr>
<td>Pike Dive (S1)</td>
<td>1.5</td>
<td>41.1</td>
<td>15.6</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>30.5</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>22.9</td>
<td>15.0</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>4.9</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>5.5</td>
<td>9.8</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pike Dive (S2)</td>
<td>1.5</td>
<td>27.9</td>
<td>12.3</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>24.0</td>
<td>13.1</td>
</tr>
<tr>
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Degrees of freedom = 4,  ** p < 0.01

* Due to the method employed in determining the angular velocity in both methods, n + 1 frames were required to obtain n frames of angular momentum data.
FIGURE 4: Subject 1's local, transfer, and total angular momentum values for the Pike Dive.

FIGURE 5: Subject 1's local, transfer, and total angular momentum values for the 1-1/2 Somersault Dive.
FIGURE 6: Subject 2's local, transfer, and total angular momentum values for the Pike Dive.

FIGURE 7: Subject 2's local, transfer, and total angular momentum values for the 1-1/2 somersault Dive.
FIGURE 9: Subject 4's local, transfer, and total angular momentum values for the Dead Long Jump.
VITA AUCTORIS

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G. W. Marino, A. Smith and M. McDonald. Presented at C.A.S.S. Annual