May 15th, 9:00 AM - May 17th, 5:00 PM

Commentary on Boger

John Woods

Author: John Woods
In Response To: George Boger's *Aristotle: an ancient mathematical logician*

Not only was Aristotle a mathematical logician but, as George Boger sees it, a mathematical logician whose work has a particular resonance for the modern logician. Boger sees this modern resonance in five features of Aristotle's work on logic: (1) Aristotle's "logic", says Boger "is taken as part of epistemology; (2) syllogistic deduction is treated methodologically, (3) rules of natural deduction are explicitly formulated, (4) the syllogistic system is modeled to demonstrate logical relationships among its rules, and (5) logical syntax is distinguished from semantics".[1]

These are important claims. If true, they help make the case for Aristotle's "genius as a logician of considerable originality and insight". I find myself wholly in agreement with this assessment, notwithstanding reservations about two of Boger's five claims. I think that Boger is entirely right about propositions (2), (3) and (4). I have doubts about (1) and (5), as well as about the paper's subtitle. Let me begin there. What is it to be a mathematical logician? Here is Barwise on the point: "Mathematical logic is traditionally divided into four parts: model theory, set theory, recursion theory and proof theory."[2] By Kleene's lights, "the study of mathematical logic begins properly only" in the theory of computibility and decidability.[3] Shoenfield sees things in much the same way as Barwise. "The central topics of mathematical logic [...are...] proof theory, model theory, recursion theory, axiomatic number theory, and set theory".[4] Even Church, whom Boger cites, has the same view of the subject, as witness the chapter headings of volume one of Introduction to Mathematical Logic and the tentative table of contents of volume two.[5] Of these four or five "central topics" one finds no trace in the Prior Analytics, except for the sort of proof theory that a theory of natural deduction often is. But is this proof theory in the modern sense of an exercise in logical syntax?

Boger finds that "Aristotle and modern logicians share the same notion that establishing knowledge of logical consequence is central to the study of logic" (2). Further, "the study of such an instrument is focused precisely on the formal conditions of logical consequence" (2). Concerning syllogisms, Aristotle "demonstrated which of these patterns have only valid argument instance" (3). And "patterns with only valid instances [...are...] sullogismoi." (3).

Logical consequence enters Aristotle's account in three ways. It occurs as the converse of the primitive notion of necessitation or entailment. It is the converse of the defined relation of syllogistic implication. And it drives the non-syllogistic perfection proofs of the metalogic in the Prior Analytics. In no case is it discernible that consequence is an artefact of logical syntax. As the converse of necessitation, it imbibes the overtly semantic character of that relation. As the converse of syllogistic implication, it is a restriction of a core semantic relation, even though some of the restrictions might be seen as purely syntactic constraints. As the relation that drives the perfection proofs of the Prior Analytics, it makes indispensable use of deduction rules which embed the notion of syllogistic implication. I conclude, then, that in its treatment of logical consequence there is no case to be made for a modern syntax–semantics distinction in Aristotle's logic.

In this regard it is useful to bear in mind that in no sense is Aristotle's a logic of validity. His principal target is the syllogism. Syllogisms are restrictions of valid arguments. They are valid arguments satisfying further conditions. One is that there be no idle premisses. Another is that conclusions not repeat a premiss. A third is that syllogisms have no multiple conclusions (in the manner of Gentzen, for example). These are weighty constraints. Jointly they provide that the core theory of syllogisms is an intuitionistic, relevant, nonmonotonic logic.6

Bearing on the question of how much of a syntaxicist Aristotle was in his natural deduction theory is the extent to which he makes serious theoretical use of the notion of logical form. Boger thinks it "worth noting...that Aristotle employed, at least implicitly, the semantic principle of form". "Indeed", says Boger, "we see that Aristotle developed an artificial language, although not strictly a formal language, to help model his logic better to reveal its properties. Although", Boger points out, "it is stretching the point to say that Aristotle approximated inventing an uninterpreted language, we might, nevertheless, recognize a move in this direction".(5)

I myself doubt it. It is true that Aristotle's theory of syllogisms is tied to a canonical language, but it is strictly a sublanguage of Greek, viz., the set of categorical propositions. "Proposition" is a technical term for Aristotle. We ourselves use the equivalent expression "categorical proposition". No proposition contains any expression which is not either a general term of Greek, or a symbol for predicate negation (or term complementation) or a quantifier expression, "All...are—" , "Some...are—" and "No...are—" (in which the apparent copula is an undetachable part). What makes categorical Greek Aristotle's choice of canonical notation for logic is the thesis of propositional simplification7 ,which asserts that anything statable in Greek is statable without relevant loss in the language of [categorical] propositions. Thus the language of the theory of syllogism is wholly without artifice. It is true that when he makes general remarks about propositions and syllogisms that are always triples of them, Aristotle often uses the device of what Quine calls dummy or schematic letters. But, as Bolger points out, these aren't variables. "We believe", he says, "that Aristotle took each letter to be a schematic letter, and not a logical variable, in ways similar to W.O. [sic] Quine's meaning of 'a dummy to mark a position'." (Ft.nt. 21). So then not even in the language of metalogic do we find variables.

This gives us two points to try to make something of. One is that in modern terms a natural language argument's logical form is its reconstruction in a semi-interpreted artificial language. There is nothing in Aristotle answering to any such conception of logical form. Then, too, on a modern understanding, logical forms embed variables irreducibly. But, there are no variables in Aristotle's logic. Twice-over the suggestion fails that Aristotle has even an implicit understanding of logical form, to say nothing of whether it does some of the heavy lifting in his logic. To the extent that there is no notion of logical form in Aristotle, and no notion of an artificial or uninterpreted canonical language, it is difficult to make the case that his logic harbours anything like our distinction between logical syntax and semantics.

What then of Boger's claim that Aristotle's logic exudes a modern cachet in so far as it seems itself as part of epistemology? Here, too, there are two questions to reflect upon. One is whether it is true that Aristotle sees his logic as part of epistemology. The other is whether this is also how modern logic sees itself. Let us proceed with these in reverse order. Epistemic logic itself aside, how would we begin to take the measure of the claim that modern logic is part of epistemology? It is true that Frege's second order logic has an epistemological motivation. Frege wanted to demonstrate the analyticity of arithmetic. This is clearly a epistemological objective. He thought that he could fulfill it by showing that number theory is reproducible without relevant loss in a theory of pure sets embedded in a quantification theory of second-order. Both these theories he took to be analytic, but there is little in Frege that counts as a well-shaped case for this assumption. In like fashion, constructionist logics are driven by a certain conception of the conditions under which mathematical knowledge is possible for beings like us. But it is one thing to have an epistemological motivation; it is another thing to be part of epistemology. There is a way of approaching our question which holds out some promise. Imagine that we are designing a course in epistemology. It is easy to suppose that the syllabus might direct our students to writings characteristic of the main positions in the theory of knowledge, both historically and contemporaneously. We would expect to find referrals to correspondence and coherence theories, to pragmatic theories, to internalism and externalism, to naturalism and reliablism, to the problem of à priori knowledge, to the nature of evidence and justification. But would we expect so much as a sentence directing the student to a system of logic? I think not. By modern standards, logic is not part of epistemology.[8]

If we pay attention to the Topics and Sophistical Refutations, we see that Aristotle wanted logic to be the theoretical core of a wholly general theory of argument. With Prior Analytics, the objective had narrowed; Aristotle sought a theory of demonstration, although he deferred its development, such as it is, to the Posterior Analytics. As conceived of by Aristotle, demonstrations embed two epistemologically significant notions. One is the idea of first principles. The other is the idea of certainty. However, what one does not find in the Prior Analytics are theoretical accounts of these things.

Of first principles one learns little more than that they neither require nor admit of demonstration. And of certainty, we learn little more than that demonstrations preserve truth but not certainty. So I conclude that it is a substantial overstatement to claim that Aristotle's logic is part of epistemology.

So much for what divides Boger and me. As to the three remaining points, I have nothing but support (and admiration). And it seems to me that these are far and away the most important points, on which Aristotle's reputation as a logician of astonishing prescience securely rests.

ENDNOTES

1 References here are to pages of Boger's manuscript.

2 Jon Barwise (ed.), Mathematical Logic, Amsterdam: North-Holland 1977, vii.

3 Stephen C. Kleene, Mathematical Logic, New York: John Wiley & Sons 1967, vii, xii.

4 Joseph R. Shoenfield, Mathematical Logic, Rending, MA: Addison-Wesley 1967, iii.

5 Alonzo Church, Introduction to Mathematical Logic, volume one, Princeton: Princeton University Press, 1956. See also p. 56, note 125.

6 See John Woods and Hans V. Hansen, "Hintikka on Aristotle's Fallacies", Synthese, 113 (1997), 217-239; 236-237, 238 (n. 17).

7 Aristotle, On Interpretation, 17a13, 18a19ff, and 18a24.

8 The young Heidegger is an exception. Logic, he says, studies "the conditions of knowing in general", Gesamtausgabe 1, 42-43. Such a view bears some similarity to that of certain informal logicians, who see their enterprise as inherently epistemological. See, for example, Harvey Siegel and John Biro, "Epistemic Normativity, Argumentation and Fallacies", in Frans H. van Eemeren, et al (eds.), Analysis and Evaluation, vol. II, Amsterdam: Sic Sat 1995, 286-299; Mark Weinstein; "Entailment in Argumentation", in Frans H. van Eemeren, et al. (eds.), Proceedings of the Second International Conference on Argumentation, vol. 1A, Amsterdam: Sic Sat 1991, 226-235; and James Freeman, "An Unrecognized Part of the Informal Program", in van Eemeren, et. al. [1991], 338-347.