Belief Sets and Commitment Stores

Roderic A. Girle

University of Auckland

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BELIEF SETS AND COMMITMENT STORES

Roderic A. Girle
Philosophy Department
University of Auckland
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Abstract:
In this paper we compare central elements of Dialogue Logic and Belief Revision theory. Dialogue Logic of the Hamblin/Mackenzie style, or Formal Dialectic, contains three main features. First, there is a rule governed interaction between dialogue participants—the minimal case being two participants. Second, each participant has a commitment store which changes as the dialogue progresses. Third, the changes in the commitment store are governed by rules for additions and withdrawals of material. Withdrawal of material is one major source of difficulty in proposing rules for commitment store change.

The classic Belief Revision theory is the AGM (Alchourrón, Gärdenfors and Makinson) theory. AGM theory is a theory about ideal rational believers who change their sets of beliefs by either expansion or contraction. Contraction is a major source of difficulty in belief revision theory. We claim that the commitment stores of dialogue logic include, in a sense, the belief sets of belief revision theory. Further, withdrawal and contraction are essentially the same process. We consider various kinds of withdrawal and contraction, and show how approaches to these processes illuminate certain of the formal fallacies.

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Introduction:
There are two approaches to belief change which will occupy us in this paper. They are Belief Revision Theory and Dialogue Logic. These approaches give different perspectives on belief change. The former is abstract, ideal and rational. The latter is dynamic, interactional and far more realistic. We begin with a brief account of the two approaches. We then focus on the problems of giving up belief. This leads to a discussion of how the giving up of beliefs can have more or less severe consequences. Finally we consider the denying the antecedent argument form.

Dialogue:
Dialogicians see dialogue as an activity much like argumentation. One would not expect a theory of argument to be no more than an empirical account of how people argue, or how they present arguments for points they wish to make. Nor should a theory of dialogue consist in nothing more than an empirical account of dialogues as they occur.
in everyday life. The dialogue-logic approach involves a logical account of interaction in terms of rules for particular kinds of responses and interactions. The resulting systems can be viewed as reactive systems (Girle [1992]) which can cope with any dialogue in which assertions, questions, reason giving, and the retraction of assertions occur.

An important feature of dialogue logic, given a dialogue between two or more participants, is that each participant has a growing and changing commitment store. Each time a participant asserts something, it is added to the participant's commitment store. Commitment stores are like sets of beliefs, but not exactly the same as them.

If a participant retracts an assertion, it is withdrawn from the participant's commitment store. In dialogue logic, commitment stores contain more than just assertions. In what follows we will set this out in more detail and show how these additional features of commitment stores meet some of the more intractable problems of belief revision theory.

**Formal Dialogue Systems**

We now turn to the work of the dialogicians. (We note in particular: Barth and Martens [1984], Hamblin [1970], Mackenzie [1979, 1984], Walton [1984], and Walton and Krabbe [1995]). These dialogicians have created several dialogue systems at a formal level. Despite some differences between the dialogue systems, the systems have several things in common.

There are two main elements in a dialogue-logic. There is a sequence of locution events, and there is a commitment store for each participant in the sequence. There is a set of rules to govern each of these elements. The dialogue-logic also has syntactic stipulations concerning the types of locutions with which the logic will deal. The syntactic list contains the sorts of locutions which are to be found in real discourse: statements, responses of various sorts, questions of various kinds, and withdrawals.

There are Interaction Rules which set out the proper sequence in which the locutions should be used in order to sustain the dialogue. For example, a question of the form "Why do you believe that P?" must be followed by the reasons from which one is to draw the conclusion that P. These rules immediately make the dialogue into a joint activity. Breach of the rules indicates a failure in the joint activity.

As well as the Interaction Rules there is a set of Commitment Store Rules for each participant in the dialogue. The commitment store is added to and subtracted from according to what statements, questions, commands, answers and withdrawals are used by participants during the dialogue. The content of the commitment store is subject to rules. A participant's commitment store does not have to be logically consistent. Its logical consistency becomes an issue only if the other participants in the dialogue detect prima facie logical inconsistency and demand that the inconsistency be resolved.

**The Dialogue Logic: DL3**

There are many formal dialogue systems. Despite differences between the systems, they have several things in common.

There are four main elements in most dialogue-logics. The first element is a taxonomy of locutions. The locutions include: statements, responses of various sorts, questions of various kinds, and withdrawals. Locutions are used by
the participants in a dialogue to form a sequence of locution events. A dialogue is a sequence of locution events. In setting out a dialogue we number locutions to indicate their order in the dialogue. These numbers are somewhat like the numberings of formulas in a proof.

The second element is a set of commitment stores, one for each participant in the sequence. Commitment stores are neither deductively closed nor necessarily logically consistent.

The third element is a set of Commitment Store Rules. Each participant's commitment store is added to and subtracted from according to what statements, questions, answers and withdrawals are used by participants in the dialogue. For example, if a participant asserts that \( P \), then \( P \) is added to everyone's commitment store. If anyone disagrees, then they must explicitly deny \( P \). Such a condition gives expression to the notion that we mostly believe what people say.

The fourth element is a set of Interaction Rules to stipulate the legal sequence of locution events. For example, a question of the form "Why do you believe that \( P \)?" must be followed either by the reasons from which one is to draw the conclusion that \( P \) or a denial that one believes that \( P \).

We set out the rules for a dialogue-logic, \( \text{DL3} \) (Girle [1996]), which is based on the systems \( \text{DL} \) (Girle [1993]), \( \text{DL2} \) (Girle [1994]), and \( \text{BQD} \) (Mackenzie [1979, 1984]). For \( \text{DL3} \) there are just two participants, \( X \) and \( Y \). In setting out rules below we will use \( S \) for the speaker and \( H \) for the hearer. There are nine sorts of locutions allowed: statements of three kinds, declarations, withdrawals, tf-questions, wh-questions, challenges, and resolution demands.

- The categorical statements are statements such as \( P \), not \( P \), \( P \) and \( Q \), \( P \) or \( Q \), If \( P \) then \( Q \) and statements of ignorance (I do not know whether or not \( P \)). The last is abbreviated to \( \top \) \( P \).
- The reactive statements are grounds (Because \( P \)), abbreviated to \( :P \).
- The logical statements are immediate consequence conditionals such as: If \( P \) and \( P \) implies \( Q \), then \( Q \).
- A term declaration is the utterance of some term, say \( t \).
- The withdrawal of \( P \) is of the form I withdraw \( P \), I do not accept \( P \), not \( P \), or I no longer know whether \( P \). The first and second are abbreviated as \(-P\).
- The tf-questions are of the form Is it the case that \( P \) ?, abbreviated to \( P \) ?.
- The wh-questions are of the form What (when, where, who, what, which) is an (the) \( F \) ? The strict logical form is \( (Qx)Fx \), where \( Q \) is the interrogative quantifier, and for each such formula there will be an associated statement \( (\exists x)Fx \). (Mackenzie [1987])
- A challenge is of the form Why is it supposed to be that \( P \) ?, abbreviated to Why \( P \) ?.
- The resolution demands are of the form Resolve \( P \).

Each locution event is represented in the formal representation of a dialogue in an ordered triple of a number, an agent and the agent's locution. The number is the number of an event in the dialogue sequence. For example, the statement \( P \) uttered at the \( n \)th step in the dialogue by \( X \) is represented as \( <n, X, P> \). We also allow for
justification sequences. They are four-tuples consisting of the antecedent of a conditional, the conditional, its consequent, and a challenge of the consequent. For example:

\[ P, \text{If } P \text{ then } Q, Q, \text{Why } Q? \]

We now set out the rules of DL3 with comments on their significance and operation.

There are seven **Commitment Store Rules**:

1. **Statements**: After an event \(<n, S, P>\), where \(P\) is a statement, unless the preceding event was a challenge, \(P\) goes into the commitment stores of both participants.

   (It is assumed that everyone agrees with statements unless and until they deny them or withdraw them. The inclusion of the full ordered pair is so that there is a record in the commitment store of the historical order of the locutions included.)

2. **Defences**: After the event \(<n, S, \vdash P>\), when:
   - Why \(Q\)? and \(Q\) are in the speaker's commitment store,
   - the justification sequence: \(<P, \text{If } P \text{ then } Q, Q, \text{Why } Q?>\), and
   - \(P\) and \(\text{If } P \text{ then } Q\) go into the commitment stores of both participants, and
   - the challenge: Why \(Q\)? is removed from the commitment stores of both participants.

(If someone gives reasons for a statement \(Q\), then the reason, its assumed conditional connection, and exactly what is justified go into the commitment stores of both participants. This allows us to keep track of why statements are in the commitment stores.)

3. **Withdrawals**: After any of the following three events:
   - \(<n, S, \neg P>\) or
   - \(<n, S, \lnot P>\) or
   - \(<n, S, \iota P>\), where \(P\) is in \(S\)'s commitment store, then
   (a) the statement \(P\) is removed from the speaker's commitment store, and
   (b) if the withdrawal was of form \(\lnot P\) or \(\iota P\), then the withdrawal goes into the commitment stores of both participants, and
   (c) if \(P\) is a statement associated with a wh-question, say \((Qx)Fx\), then \(P\) is removed from the hearer's commitment store and \((Qx)Fx\) is removed from the commitment stores of both participants, and
   (d) if the withdrawal was preceded by the event \(<n-I, H, \text{Why } P?>\), then Why \(P\)? is removed from the commitment stores of both participants, and
   (e) if \(<P, \text{If } P \text{ then } Q, Q, \text{Why } Q?>\) is in the speaker's commitment store, then it is removed; and if \(Q\) is in either participants' store, then it is removed.

(This is a fairly gentle, but not the gentlest, withdrawal rule. Given that commitment stores are neither deductively closed nor essentially consistent, only the obvious justification is withdrawn. A ruthless recursive withdrawal rule is found in Girle [1994]. A far more complex approach is in Walton and Krabbe [1995].)

4. **Challenges**: After the event \(<n, S, \text{Why } P?>\), the challenge, Why \(P\)?, goes into the commitment
stores of both participants.
If \( P \) is not in the hearer's commitment store then: \( P \) goes into the hearer's commitment store.
If \( P \) is in the speaker's commitment store, it is removed.
If the \( P \) is present in the speaker's commitment store as part of a justification sequence, the justification sequence is removed.

(Although it might seem strange to put \( P \) into the hearer's commitment store, the hearer can withdraw it or deny it (see (v)(a) below and C3 above). Also, if \( P \) is in the speaker's commitment store it is withdrawn because, if the speaker has no problem about the statement, the challenge should not have been issued. It should be noted that this is not an altogether unproblematic explanation. The speaker might want to discern whether or not the hearer has reasons for asserting \( P \) other than the speaker's. We will bypass this point for the moment.)

(C5) Information: After the event \( <n, S, (Qx)Fx> \), the associated statement, \( (\exists x)Fx \), goes into the commitment stores of all participants, and the wh-question, \( (Qx)Fx \), goes into the hearer's commitment store.

(If the speaker asks "What is the \( F \)?", then it is assumed that there is at least one \( F \).)

(C6) Reply: After the declaring of a term in an event, say \( <n, S, t> \), when the previous event was a wh-question, say \( (Qx)Fx \), then \( Ft \) goes into the commitment stores of both participants. \( Ft \) is known as the wh-answer to \( (Qx)Fx \).

(C7) True/false: After the event \( <n, S, P ?> \), if the statement \( P \) is in the speaker's commitment store it is removed.

There are eight Interaction Rules:

(i) Repstat: No statement may occur if it is in the commitment stores of both participants.

(This rule prevents vain repetition and helps stop begging the question. From an everyday rhetorical perspective it is unrealistic, but in the ideal dialogue it is appropriate.)

(ii) Imcon: A conditional whose consequent is an immediate consequence of its antecedent must not be withdrawn.

(iii) LogChall: An immediate consequence conditional must not be withdrawn.

(These rules, (ii) and (iii), prevent the withdrawal or challenge of logical principles.)

(iv) TF-Quest: After \( <n, S, Is it the case that P?> \) the next event must be \( <n+1, H, Q> \), where \( Q \) is either
   (a) a statement that \( P \), or
   (b) a statement that \( \neg P \), or
   (c) a withdrawal of \( P \), or
   (d) a statement of ignorance (I do not know whether or not \( P \)).

(This rule must be read in conjunction with C1 and C3.)

(v) Chall: After \( <n, S, Why P?> \) the next event must be \( <n+1, H, Q> \), where \( Q \) is either
(a) a withdrawal or denial of \( P \), or
(b) the resolution demand of an immediate consequence conditional whose consequent is \( P \) and whose antecedent is a conjunction of the statements to which the challenger is committed, or
(c) a statement of grounds acceptable to the challenger.

We require, at this point, a definition of what an acceptable statement of grounds is:

A statement of grounds, Because \( P \), is acceptable to participant \( S \) if either \( P \) is not under challenge by \( S \), or if \( P \) is under challenge by \( S \) then there is a set of statements to each of which \( S \) is committed and to none of which is \( S \) committed to challenge, and \( P \) is an immediate modus ponens consequence of the set.

This definition is discussed at length in Mackenzie [1984].

(When the challenge is issued, see C4, the person challenged can either (a) deny any adherence to \( P \), or (b) throw the challenge back to the challenger by pointing out that the challenger is committed to \( P \), or (c) give a reason acceptable to the challenger.)

\( (vi) \) Resolve: The resolution demand in \( <n, S, \text{Resolve whether } P> \) can occur only if either
(a) \( P \) is a statement or conjunction of statements which is immediately inconsistent and to which its hearer is committed, or
(b) \( P \) is of the form If \( Q \) then \( R \) and \( Q \) is a conjunction of statements to all of which its hearer is committed, and \( R \) is an immediate consequence of \( Q \), and the previous event was either \( <n-1, H, I \text{ withdraw } P> \) or \( <n-1, H, \text{ Why } R?>. \)

(The rule above opens the way for keeping statements consistent.)

\( (vii) \) Resolution: After the event \( <n, S, \text{Resolve whether } P> \) the next event must be \( <n+1, H, Q> \), where \( Q \) is either
(a) the withdrawal of one of the conjuncts of \( P \), or
(b) the withdrawal of one of the conjuncts of the antecedent of \( P \), or
(c) a statement of the consequent of \( P \).

\( (viii) \) Enlightenment: After the event, \( <n, S, (Qx)Fx> \),
(a) the next event must be \( <n+1, H, Q> \), where \( Q \) is either
(i) the declaration of some term, say \( t \), or
(ii) the withdrawal or denying of the associated statement \( (\exists x)Fx \), or
(iii) a statement of ignorance (I do not know whether or not \( (\exists x)Fx \)),
and

(b) in the case of the hearer declaring some term, at the earliest subsequent event the asker of the wh-question must state the wh-answer to the wh-question, or its denial, where the earliest subsequent event is the event separated from the term declaration by nothing other than the asker asking further wh-questions and their wh-answers, where the wh-questions contain only predicates and terms from the wh-answers immediately preceding them.

(This rule allows us to introduce the notion that an event or set of events constitute a discovery block with respect
to the wh-question and the participant who asked it. A discovery block ends with a statement by the asker of the wh-question, of the wh-answer for the wh-question. In other words, the asker shows that they are satisfied.)

It is simpler if we set out the key points in a Rule Operation Table, **TABLE 1**. There are rows for each of the speaker's, \( S \), locutions. There are two commitment store columns for the resultant entries to the speaker's, \( S \), and hearer's, \( H \), commitment stores. We use plus and minus to indicate what is being added to or subtracted from the commitment stores of speaker and hearer. There is a column for any *required* next locution from the hearer.

**TABLE 1**

<table>
<thead>
<tr>
<th><strong>S LOCUTION</strong></th>
<th><strong>S STORE</strong></th>
<th><strong>H STORE</strong></th>
<th><strong>H RESPONSE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>at Step ( n )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>categorical statement: ( P )</td>
<td>+( P )</td>
<td>+( P )</td>
<td></td>
</tr>
<tr>
<td>reactive statement: ( :R )</td>
<td>+( R )</td>
<td>+( R )</td>
<td></td>
</tr>
<tr>
<td>( \text{If } R \text{ then } S )</td>
<td>+( \text{If } R \text{ then } S )</td>
<td>+( \text{If } R \text{ then } S )</td>
<td></td>
</tr>
<tr>
<td>( \text{\textless } \ldots \text{ Why } S? \rangle \text{?} )</td>
<td>+( \text{\textless } \ldots \text{ Why } S? \rangle \text{?} )</td>
<td>+( \text{\textless } \ldots \text{ Why } S? \rangle \text{?} )</td>
<td></td>
</tr>
<tr>
<td>(-\text{Why } S? )</td>
<td>+(-\text{Why } S? )</td>
<td>+(-\text{Why } S? )</td>
<td></td>
</tr>
<tr>
<td>term declaration: ( t )</td>
<td>+( F_t )</td>
<td>+( F_t )</td>
<td>at some later point ( F_t \text{ or } \neg F_t )</td>
</tr>
<tr>
<td>withdrawal</td>
<td>- ( P )</td>
<td>- ( P )</td>
<td></td>
</tr>
<tr>
<td>(- P )</td>
<td>- ( P )</td>
<td>- ( P )</td>
<td></td>
</tr>
<tr>
<td>(- (\exists x)F_x )</td>
<td>-( (\exists x)F_x )</td>
<td>-( (\exists x)F_x )</td>
<td></td>
</tr>
<tr>
<td>(- (\forall x)F_x )</td>
<td>-( (\forall x)F_x )</td>
<td>-( (\forall x)F_x )</td>
<td></td>
</tr>
<tr>
<td>tf-question ( P )</td>
<td>- ( P )</td>
<td>- ( P )</td>
<td>one of ( P, \neg P, \top P ) or (-P )</td>
</tr>
<tr>
<td>wh-question ( (\forall x)F_x )</td>
<td>+( (\forall x)F_x )</td>
<td>+( (\forall x)F_x )</td>
<td>one of ( t, \neg(\exists x)F_x )</td>
</tr>
<tr>
<td>( (\exists x)F_x )</td>
<td>+( (\exists x)F_x )</td>
<td>+( (\exists x)F_x )</td>
<td>( (\exists x)F_x \text{ or } \neg(\exists x)F_x )</td>
</tr>
<tr>
<td>challenge ( \text{Why } S? )</td>
<td>-( S )</td>
<td>+( S )</td>
<td>one of acceptable ( :R )</td>
</tr>
<tr>
<td>( \text{Why } S? )</td>
<td>+( \text{Why } S? )</td>
<td>+( \text{Why } S? )</td>
<td>( \text{or } \neg S \text{ or } - S \text{ or a resolution demand as in (v) } )</td>
</tr>
<tr>
<td>resolution</td>
<td>+( P )</td>
<td>+( P )</td>
<td>withdraw part ( P ) or</td>
</tr>
</tbody>
</table>
There are three points to note. First, commitment stores contain much more than just categorical statements. They contain relevant portions of the dialogue content. Questions and challenges are important parts of that content.

Second, a participant's commitment store does not have to be logically consistent. Its logical consistency becomes an issue only if the other participants in the dialogue detect *prima facie* logical inconsistency and demand that the inconsistency be resolved.

Third, some of the allowed responses are more complex than can be fitted into the box in the table. This is particularly so with respect to the responses to challenges, wh-questions and resolution demands. The response to a challenge must satisfy conditions of "acceptability", as we have seen in *(vi)* *Chall:* above. The response to a wh-question must be followed by a questioner's acknowledgement or rejection of the answer. The resolution demands require both a set of pre-conditions and a response which can be one of several kinds.

The operations table shows what constraints the logic imposes on a dialogue. These rules have been used in formal debates in classrooms. They impose a discipline, but can allow utterly inconsequential debates (see Stewart-Zerba and Girle [1993]).

Here is an example sequence in which there are two participants, X and Y. We shall use X+P for P is added to X's commitment store if not already there, and X-P for P is taken out of X's commitment store (if it is there).

**Example 1.**

<table>
<thead>
<tr>
<th>Locution</th>
<th>Store</th>
<th>Event</th>
<th>Changes</th>
<th>Commitment Stores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt;1, X, P&gt;</td>
<td>X+P</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Y+P</td>
<td></td>
</tr>
</tbody>
</table>

(The statement goes into both stores (C1).)

<table>
<thead>
<tr>
<th>X's Store</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y's Store</td>
<td>P</td>
</tr>
</tbody>
</table>

<2, Y, Why P?>          Y+Why P?

Y-P
X+Why P?

(This is a challenge to P. See rule (C4).)

<table>
<thead>
<tr>
<th>X's Store</th>
<th>P Why P?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y's Store</td>
<td>Why P?</td>
</tr>
</tbody>
</table>

<3, X, :Q>  

X+<Q, If Q then P, P, Why P?>

X+Q

X+If Q then P

X-Why P?

Y+<Q, If Q then P, P, Why P?>

Y+Q

Y+<If Q then P>

Y+P

Y-Why P?

(Grounds are given and added to both stores together with the conditional which links the grounds to what they support (v). At this stage Q is not yet under challenge. Since the challenge to P has been met, it is removed from both stores.)

<table>
<thead>
<tr>
<th>X's Store</th>
<th>P If Q then P &lt;Q, If Q then P, P, Why P?&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y's Store</td>
<td>Why P? &lt;Q, If Q then P, P, Why P?&gt;</td>
</tr>
</tbody>
</table>

<4, Y, Why Q?>  

Y+Why Q?

Y-Q

Y- <Q, If Q then P, P, Why P?>
Why Q?

(This is the challenge to Q. Q is removed from Y's store, and the justification sequence in which Q occurs is also removed. The removal of anything else awaits further challenges or withdrawals.)

<table>
<thead>
<tr>
<th>X's Store</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q If Q then P</td>
<td></td>
</tr>
<tr>
<td>&lt;Q, If Q then P, P, Why P?&gt;</td>
<td></td>
</tr>
<tr>
<td>Why Q?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Y's Store</th>
<th>Why P?</th>
</tr>
</thead>
<tbody>
<tr>
<td>If Q then P</td>
<td></td>
</tr>
<tr>
<td>Why Q?</td>
<td></td>
</tr>
</tbody>
</table>

<5, X, - Q>    

X-Q

X-<Q, If Q then P, P, Why P?>

X-Why Q?

X-P

Y-Why Q?

Y-P

(Armed almost everything in the stores now has to be removed. If Q then P does not have to be withdrawn under (C3). But, P has to go under (C3), because its proposed justification has been taken away. P will have to be restated if X wants to continue to hold to it.)

<table>
<thead>
<tr>
<th>X's Store</th>
<th>If Q then P</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Y's Store</th>
<th>If Q then P</th>
</tr>
</thead>
</table>

The commitment stores have not much left in them. Principles of minimal change might have left P in the commitment stores of both X and Y at the last step in the dialogue. So, (C3) is not a minimal change rule. There are a range of possibilities for withdrawal rules. We have taken the "middle way".

Belief Revision

Classic Belief Revision Theory is given its best expression in the AGM theory (Alchourrón, et.al. [1985]). The
theory gives an account of changes in sets of beliefs, and is a theory based on an ideal epistemic agent who has a belief set, that is, a set of beliefs.

The belief sets considered in AGM theory have three main organizational features. The sets are deductively closed, classical consistent, and epistemically ordered. It follows from their being deductively closed that such belief sets are infinite.

The believers are ideal agents who believe all the deductive consequences of what they believe. Given that $K$ is a belief set we have:

(Def BS) A set $K$ of sentences is a (non-absurd) belief set iff

(i) $\perp$ is not a logical consequence of the sentences in $K$, and

(ii) if $K \models B$, then $B \in K$

A belief set is a logical theory.

The Definition of a Consequence Set, where "Cn($K$)" is read as "the set of logical consequences of $K"$, is:

$B \in \text{Cn}\{A\} \iff B \in \{P: A \models P\}$

There is the general principle that Belief Sets, such as $K$, are Equilibrium States:

(Cn) $K = \text{Cn}(K)$

The deductive closure of belief sets reinforces the picture of ideal belief agents' being deductively omniscient. This picture can be made more realistic if belief sets are not interpreted as the beliefs of an agent but as the propositions or beliefs to which the agent's actual beliefs commit them. This interpretation is suggested by Fuhrmann [1988]. This interpretation is more or less the same as the idea that belief sets give the implicit beliefs of the agent, beliefs consequential on the explicit beliefs of the agent.

Fuhrmann's interpretation certainly defuses the common criticism of standard belief revision theory, that it advocates deductively omniscient believers. But there are two problems. First, we are left with no real model of the believer. Second, if the closed theory is to be seen as the consequences of the base set of beliefs, then how are we identify the particular set of explicit beliefs of the agent from which the set is consequential? For the moment we leave this question unanswered.

When belief agents want to revise their beliefs, they usually have to retract certain beliefs and add others. The retraction of beliefs from a belief set is known as contraction, and the addition of beliefs is expansion (Gärdenfors [1988]). The expansion of belief sets is not generally a problematic activity from a theoretical or practical point of view. The main concerns for expansion centre around the maintenance of consistent sets of beliefs, or, if needs be, the maintenance of a non-trivial inconsistent belief sets, called "para-consistent" belief sets (Girle [1990a]).

We will not look in any detail at the consistency feature of belief sets, but we point out in passing that inconsistency does not, of necessity, lead to triviality. Para-consistent logic makes it possible for us to deal with non-trivial inconsistent sets of beliefs.

We turn to the ordering of belief sets by considering the contraction of belief sets. The major problems with belief
revision arise from the contraction of belief sets. Say, for example, an agent gains beliefs as a consequence of the following dialogue:

\[ B: \text{Susan is taking holidays.} \]
\[ A: \text{No! She must be taking sick leave.} \]
\[ B: \text{Why do you say that?} \]
\[ A: \text{Because she went into the dentist's rooms this morning.} \]
\[ B: \text{So what?} \]
\[ A: \text{People who have a dental appointment take sick leave, not holidays.} \]

In terms of both classical Belief Revision Theory and standard Dialogue Logic, participant \( A \) will have at least the following in their belief set and their propositional commitment store (they are numbered for ease of reference only):

1. \text{Susan is not taking holidays.}
2. \text{Susan is taking sick leave.}
3. \text{Susan went into the dentist's rooms this morning.}
4. \text{People who go into the dentist's rooms have a dental appointment.}
5. \text{People who have a dental appointment take sick leave, not holidays.}

In standard Dialogue Logic there is something extra. The fact that a participant offered an argument in the course of a dialogue is embedded in the commitment store. In the store there would be some explicit "linking" of the five propositions in a way which indicated that 1 and 2 two above were conclusions drawn from 3 to 5. But, this link is not to be noted simply because there is a logical relationship, the link is noted because there was a "Why" question in the dialogue. In response to the why question, \( A \) gives the premises as reasons for believing 1 and 2. In dialogue logic there is, as it were, a meta-belief to be noted: The reasons for 1 and 2 are 3 to 5. Specific argument links are not registered at all in AGM theory. There is a logical relationship, but we don't have to keep track of the "whys" of dialogue to keep track of logical relationships.

\textbf{The Justification Problem}

In Gärdenfors [1988] there is comment, in the context of expounding Doyle's truth maintenance system, that many models of epistemic states ignore the justifications of beliefs.

The idea of including justifications for the beliefs held seems fruitful. This aspect of belief systems seems neglected in other models of epistemic states. (page 35)

The question of justification is problematic in classical belief revision theory because of the closure of belief sets under logical consequence. We cannot use the classical material conditional to represent justification. For example, given two beliefs, say \( P \) and \( Q \), if we say that \( (P \rightarrow Q) \) means that a (the) justification of \( Q \) is \( P \), then given any two beliefs, one will always be a justification of the other because \( (P \rightarrow Q) \lor (Q \rightarrow P) \) is a tautology.

This problem is swiftly dealt with in dialogue-logic by virtue of the way in which commitment stores are built up, and by virtue of the fact that commitment stores contain not only beliefs but also Challenges.

Commitment stores are not closed under logical consequence. As beliefs are added they take their place in terms of the dynamic of the dialogue. They are automatically entrenched, again in terms of the dynamic of the dialogue. The
additional information in dialogue commitment stores provides a solution to Gärdenfor's problem.

Two Situations

Consider now two situations. The first situation is where $A$ discovers from some other impeccable source that Susan was, indeed, taking a vacation, and that this vacation coincided, fortuitously, with a dental appointment made months ago. What happens now? Obviously, $A$ would have to forego the belief that Susan is not taking holidays but is taking sick leave. It would normally follow from this derogation of belief that other beliefs, or commitments, would have to be abandoned. In particular, some or all of the premises (3 to 5) of the argument would have to be abandoned. The question for the rational person is, "Which of the beliefs (premises 3 to 5) should be abandoned (by contraction or withdrawal)?"

Belief sets are ordered by *epistemic entrenchment* in the AGM theory. The beliefs are placed into an order of epistemic importance to the believer. This order is first built on the assumption that logical deducibility is crucially important to order.

For example, if $Q$ is deducible from $P$, and we wish to remove $Q$ from the belief set, then $P$ must also be removed. If $P$ were not removed, and the set is deductively closed, then $Q$ would immediately re-appear.

The implication of this for entrenchment is that $P$ has to be seen as less well entrenched than $Q$. So,

$$P \models Q$$

is a sufficient condition for $Q > P$ (where "$>"$ means "is more deeply entrenched than"). This generally is seen as indicating that while $P$ must be removed if $Q$ is, $Q$ need not be removed if $P$ is. But this does not always make sense. An agent might believe $Q$ just because it believes $P$. In this case, if $P$ is rejected then it would be reasonable to assume that $Q$ would be withdrawn for lack of support.

If, in our case above, $A$ had *seen* Susan going into the dentist's rooms, then 3 would be a more entrenched belief than 4 and 5. The less entrenched beliefs are the ones first considered for derogation. So, in AGM theory there is an assumption that there is an entrenchment *ordering* of beliefs in any belief set.

As well as the ordering derived from logical relationships, there will also be ordering in terms of the agent's ordering from everyday belief and knowledge. This provides additional "finer grained" ordering. For example, we can tell what is a sensible ordering in the case of the dental appointment because we have contextual knowledge about work, holidays and dentists. Such knowledge will assist us in deciding about the entrenchment ordering of 4 and 5 above. We can, of course decide that they are of equal ranking. In that case, and if we assume they are of lesser ranking than 3, then both will have to be abandoned in the face of the new information about Susan's holidays.

The second situation is more interesting. This is the case where $A$'s information that Susan went into the dentist's rooms was based on observation, but it turns out that Susan has an identical twin, Sarah, and it was Sarah who was seen by $A$.

In this situation, most of us would think that, since a vital premise was now known to be false, the conclusions which depended on that premise would be, at the very least, consigned to the set of propositions no longer believed. These are the propositions which, in AGM theory, are either disbelieved or indeterminate. They are in either the "believed false" set or in the "agnostic" set.
But this is not what must happen under the AGM theory of belief revision. AGM theory is explicitly in favour of a minimalist approach to belief change. One contracts a belief set by the minimum amount to keep it consistent. So one would remove only the falsified premise, and not necessarily remove the dependant conclusion.

By some accounts of dialogue logic, one would remove the dependant conclusions as well as withdrawing the falsified belief. But this, at first sight commendable commitment store change, can look very much like the fallacy of denying the antecedent.

The dynamics of dialogue make it sensible to see that, at some instant in a dialogue, a participant could have a store containing:

\[
\begin{align*}
P \\
(P \rightarrow Q) \\
\therefore Q
\end{align*}
\]

At the next instant there could be a change represented by:

\[
\begin{align*}
\sim P \\
(P \rightarrow Q) \\
\therefore \sim Q
\end{align*}
\]

Outside of the dialogue context it would be easy to see the change as a change from a valid modus ponens to an invalid antecedent denial. In this, as with so many argument forms categorised as fallacious, the context shows that things are not simple. An unthinking "spot the fallacy" approach might lead one to draw all the wrong conclusions about wild arguments in their natural habitat.

**Conclusion**

The structure of commitment stores is more conducive to tractable change than present belief revision systems. Further investigation is needed into recursive withdrawal and contraction.

**References**


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