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Visual arguments and meta-arguments

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ABSTRACT: Visual arguments—arguments that appeal to visual elements essentially—are legitimate arguments. To show this, I first consider what I call (perfect) fit arguments—arguments in which the recognition that items fit together suggests that they were once conjoined, perhaps originally. This form of argumentation is a type of abduction or inference to the best explanation (IBE). I then consider mathematical visual meta-arguments—arguments in which the validity or soundness of a mathematical argument is confirmed or refuted by appeal to diagrams.

KEYWORDS: visual arguments, meta-arguments

1. INTRODUCTION

As studies in visual rhetoric flourish, it is surprising that the logical analysis of putative visual arguments has relatively fewer proponents. One way to make the case for the logical, as opposed to the rhetorical, analysis of putative visual argumentation is to show not just the ubiquity of appeals to visual elements in reasoning, but that there are distinctive logical manoeuvres involving such visuals. To this end, in this paper I begin by considering an argument from the history of geology that requires such a manoeuver. Although it is possible to reconstruct the reasoning in purely verbal/symbolic fashion, I suggest that the reconstruction depends upon both the visual recognition of a property—that some items fit together—as well as the notion that such fit implies prior connection. This form of argument goes back at least to C.S. Pierce (1878). It is a form of what Pierce calls hypothesis and, later, abduction, though it is usually called Inference to the Best Explanation (IBE) now.

According to Peter Lipton (1991), one good way to understand IBE requires that there be competing or contrasting explanations. In the competition for explanation, the best explanation, where “best” is understood in terms of beauty or what-have-you, is inferred to be correct. For arguments from fit, I suggest that the contrasting explanation is chance or coincidence. The relative goodness of fit—where the fit is recognized visually—undermines chance or coincidence as the correct explanation of the phenomena. In this way, fit implies connection; and, the better the fit, the stronger the implication.
As a second type of visual argument, I consider the use of visuals as counterexamples. I take counterexamples to fit naturally within the concept of meta-argumentation, i.e., arguments about arguments (see Finocchiaro 2013). In particular, I consider an early attempted proof of what we now refer to as the four-color theorem. This theorem was proven with the aid of computers in the 1970s. But there was an earlier proof of the theorem that the mathematical community accepted for about a decade. It’s acceptance as a proof crumbled upon the discovery of a counterexample. Interestingly, the counterexample didn’t undermine the theorem but the method of proof. This counterexample, therefore, isn’t merely an assertion gainsaying the conclusion. Rather, the dialectical employment of the counterexample in this particular incident from the history of mathematics speaks to the invalidity of a method. That is, this counterexample fits nicely within a particular argumentative context. Moreover, in this case, the counterexample is wholly visual. Insofar as some visuals are counterexamples to some arguments, it follows that visuals can carry some parts of some arguments.

2. ARGUMENTS FROM (PERFECT) FIT

Putting together children’s toys can be an exercise in futility. In some cases I’ve ended up with leftover parts. In others, I’ve used every part but the darn thing still didn’t work properly because it seemed to need more nuts or bolts or random pieces of plastic or metal than were supplied. But of the many ways that assembling these toys can be frustrating, perhaps the most common is when the instructions are absent or unhelpful. In such cases, trial and error is my method. To know whether two parts ought to be joined you can either physically attempt to conjoin them, or you can inspect them to see whether they appear to fit together. When two pieces fit perfectly together, joy of joys, I happily move on to another piece or set of pieces; happy in the knowledge that the pieces I’ve just joined together were meant to be so. This contented knowledge is justified, I suggest, on the basis of a kind of inference to the best explanation. The pieces are unlikely to fit together unless they were intended to. Hence, when the pieces fit, one can infer the intent.

This manner of justification isn’t unique to assembling children’s toys. Indeed, in the late 1800s, Charles Sanders Pierce gives an example reminiscent of my experience with toys.¹

A certain anonymous writing is upon a torn piece of paper. It is suspected that the author is a certain person. His desk, to which only he has had access, is searched, and in it is found a piece of paper, the torn edge of which exactly fits, in all its irregularities, that of the paper in question. It is a fair hypothetic inference that the suspected man was actually the author. The ground of this inference is evidently that the two torn pieces of paper are extremely unlikely to fit together by accident. Therefore, of a great number of inferences of this sort, but a very small proportion would be deceptive. (Pierce, p. 475)

There are many interesting elements to consider here. Start with the exact fit of the

¹ Both John Anders and Dave Beisecker separately suggested this example to me.
torn pieces of paper. There is a limit to visual acuity. It is, therefore, humanly impossible to recognize exact or perfect fit visually. However, if one takes “exact fit” as the limit of visual indiscernibility, then there is nothing mysterious about claiming to recognize such fit.

This leads to a second consideration. Since the exactness of fit (a) comes in degrees, and (b) even when it is at the limit of visual acuity may still fail to be perfect, any inference from fit, though perhaps reliable, will fall short of logical entailment. Hence Pierce’s claim that only a small number of such inferences will be deceptive. Still, the fit acts as a “ground of the inference.” That is, if it fits, you can infer that the fit is intended. Part of the reliability of the inference is that fit is unlikely in the absence of intent or prior connection. From this we can get a rough measure of the inferential, i.e., logical, strength of such arguments: the better the fit, the stronger the inference; though always with the possibility of failure.

This isn’t just an abstract exercise built from toy examples. An argument of this type was historically important in geology. Alfred Wegener proposed an early theory of continental drift. His reason for thinking that the continents had drifted from an earlier configuration was based on an argument from fit. While on an expedition, Wegener wrote to his then wife-to-be.

Please reexamine the map of the world: doesn’t the east coast of South America fit exactly against Africa’s west coast, as if they had once been joined? (Wegener 1960, p. 262 [my translation])

The idea of continental drift was in the air in the late 1800s and early 1900s. And Wegener wasn’t the first to have noticed the apparent fit of the continents. But what we can see from the quote is that Wegener is coming close to having an argument from fit. We can tell that he doesn’t yet have the argument from the wording in the quote. Wegener writes, “as if they had once been joined” rather than “because they had once been joined.” By 1911 he seems to have solidified his own thinking that there was indeed an argument to be had. But before turning to his actual argument, I want to consider a kind of objection that this particular argument will face.

With puzzles, children’s toys, and things manufactured, the argument from fit can be cashed out in terms of intent. The manufacturer intended the pieces to fit together. But does this inference carry over to natural objects? Charles Schuchert, an American geologist, thought there were reasons to doubt that fit implies prior connection. He argued that in the case of continents natural geological processes would wear the shorelines in such a way that any apparent fit would likely be the result of coincidence rather than prior connection. Moreover, given that sea level is historically inconstant and there is variance in the elevation of continental coastlines, the shapes of the coastlines of the continents will be a function of things that would tend to make coincidence more likely as an explanation (see Drake, p. 2

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2 The term “intended” isn’t quite apt for all cases of the inference type I have in mind here. There will be examples where the fact that pieces fit together exactly implies that these pieces had previously been conjoined even though there was no intention to put the pieces together in the original configuration. For example, in reassembling a broken vase, the fit shows the correctness of the reconstruction rather than any intent.
41). Put another way, given the harsh nature of geological processes, fit implies coincidence.

In some ways, this is a serious objection. And ideas like this might explain some of the recalcitrance of the American geological community to the idea of continental drift—Wegener first proposed his theory in print in 1915, but it wasn’t until the 1960s and early 1970s that the Americans generally acknowledged something like drift. Still, even in its nascent form, Wegener’s pre-argument could account for this kind of objection. The rest of the quote from Alfred to Else Wegener gives the solution.

Please reexamine the map of the world: doesn’t the east coast of South America fit exactly against Africa’s west coast, as if they had once been joined? It agrees even more when one looks at the deep-sea shelves of the continents as opposed to the current continental coasts. (Wegener 1960, p. 262 [my translation])

Since the argument won’t depend on the fit of the coastlines but on the edges of the continental shelves, Wegener blunts Schuchert’s criticism—the continental shelves aren’t liable to the same geological forces and hence are more likely to retain their shape over a geological timeframe. Moreover the shape of the continental shelves aren’t a function of sea level. It is curious to me that Wegener blunted this criticism a decade before Schuchert gave it. But that is outside the concerns of the present paper.

Now let’s consider Wegener’s official argument. On the one hand, we have already seen the jigsaw element of the fit of the continents. Anyone who has looked a globe can see the apparent fit of the continents. Figure #1 is Wegener’s own rendering of the geometrical fit of continents.

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3 Shuchert levied the criticism at a conference in 1926, the proceedings of which were published two years later (see Drade, p. 41).
North America, with a little rotation, fits with Greenland, and together they fit with Europe and North-Western Africa. South America slides right in to western Africa. South Asia and Australia fit together. And Antarctica fills in the bottom.

The fit is more than just the geometrical shape. For, once the continents are realigned to their supposed original configuration, geological, paleontological, and other congruencies appear. For example, one can align mountain ranges to span from South America across to Africa. Other ranges cross from North America to Greenland to Scotland to Europe. Distinctive fossils of land-borne animals are seen across the continents, and more. Wegener explained these congruencies with an analogy reminiscent of Pierce.

It is just as if we put together the pieces of a torn newspaper by their ragged edges, and then ascertained if the lines of print ran evenly across. If they do, obviously there is no course but to conclude that the pieces were once actually attached in this way. (Wegener p. 56, Quoted in Giere, p. 281).

The additional lines of fit diminish the worry that this example is deceptive, to borrow the terminology from Pierce. Another way of thinking about this is to reconsider how we actually solve jigsaw puzzles. It is possible to complete a jigsaw puzzle by considering only the shape of the pieces. For example, you could turn the pieces face down. But, it is easier to recognize fit when the picture is available. One discerns how the picture needs to be aligned.
The combination of all of the elements of fit—the geometrical fit, the paleontological fit, the geological fit, etc.—underwrites the Wegener’s argument. Given the idea that the better the fit, the stronger the inference, Wegener’s argument seems robust.

To counter the apparent strength of the argument, one would need to make coincidence or chance a better explanation for fit that prior connection. Chester Longwell (cf. Giere, 1996, p. 285) offered what he thought was such a counterexample. Wegener, recall, uses the unlikelihood of fit without prior connection as reason for thinking that actual fit implies prior connection. And Wegener didn’t think that Australia was previously connected to Africa, Saudi Arabia, New Guinea and India in the manner presented in the visual (Fig. 3). So, the fact that the continent can fit into this space undermines the claim that fit implies prior connection. What is missing from the counterargument, though, is the non-geometrical elements of fit that exist in Wegener’s own argument. What clinched the case, at least for Wegener, was that the geological, paleontological, and other evidences could be seen linked across the continental divides once the continents had been re-fit into their apparent prior positions. At best, Longwell’s argument—and it is a visual counterargument—lessens the strength of the jigsaw component of Wegener’s argument. It does not, however, undermine the overall argument. Furthermore, the secondary fit of the geological evidence (etc.) across the continental divides further supports the geometrical fit. I take that to be one of the curious things about the argument. If there weren’t geometrical fit, then the secondary evidence wouldn’t be valuable—it probably wouldn’t even be evidence. But, given that there is geometrical fit, the fact that some geometrical fit doesn’t imply prior connection can be countermanded by the secondary evidence. Think of the metaphor of the torn newspaper. Perhaps we could cut some newspaper pages so that many different pairings of left and right sides seem to work geometrically. It is likely that the secondary evidence of aligned text would verify the particular true pairing; and likewise for continents. That the continents can be fit together as a global puzzle doesn’t guarantee the particular geometrical alignment. Yet once the secondary evidence aligns, the particular alignment is much (much much much) more likely. And this argument makes essential use of visual elements.
How do these visuals work? On the one hand, the visuals are not the objects to which the arguments apply. The maps represent the physical configurations of continents. For visual reasoning to work as I’ve suggested, there must be a way to explain the visual content of the representations. Fortunately, there are many theories purporting to do just that. If we were dealing with photographs of the continents, perhaps photographs taken from space, then one appealing way to explain the visual content of the photo was given by Kendall Walton (1984). Walton suggests that, because there is a direct causal story to explain the visual configuration of items in the photograph as dependent upon the physical configuration of the thing photographed, the photo allows one to see-through to the object itself. The photo, then, is analogous to any other visual device—binoculars, glasses, telescopes, microscopes, or even eyes. The photograph, according to Walton, is transparent. What we see is the object itself.

This won’t work, however, for visuals in which the production of the visual doesn’t have the strong causal connection between object and representation. In these cases, we need a theory of depiction. Again, fortunately, there is no dearth of competing theories of depiction. Though there may yet be debate regarding which theory is correct, unless one is sceptical of depiction generally, we can appeal to depiction to solve the current worry. So, for example, Richard Wollheim (1987), takes visual depiction to require a resemblance between the object depicted and the depiction itself. He calls the recognition of this resemblance seeing-in (Wollheim, pp. 46-47). In the process of recognition, then, one sees the depicted object in the depiction. Hence, the content of the depiction is the object. For our purposes, this is the way that the map of the continents depicts the continents—we see the
continents in the maps. We are thus justified in taking our conclusions about the visual fit of the represented continents to carry over to the actual continents.

3. VISUAL COUNTER-MODELS AS (PARTS OF) META-ARGUMENTS

Maurice Finocchiaro has urged classifying arguments about arguments as “meta-arguments” (cf. Finocchiaro, 2013, p. 1). One way to *argue* about an argument is to offer an objection to an argument. One can already sense a kind of dialectical entanglement for the notion of meta-argument, i.e., meta-arguments can be the explicit formulation of one’s dialectical obligation in an argumentative circumstance. Finocchiaro distinguishes two kinds of objections—one he calls simply *objections* and the other he terms *counterarguments* (Finocchiaro, p. 19). Both are arguments in themselves, but the former has as its conclusion the negation of either some grounds or some conclusion of the target argument. Counterarguments have as their conclusions the negation of the conclusion of a target (or what Finocchiaro terms *ground-level*) argument. For the present purposes, I am interested in objections that might best be termed *counter-models*. A counter-model is an objection to an argument that shows that the reasoning fails without necessarily countering the conclusion of the target argument. Arguments employing counter-models can keep their conclusions tacit—the conclusion is going to be an assessment of an argument’s strength or an inference’s strength. When the target is deductive, the counter-model shows the invalidity of the reasoning whether or not the conclusion is true.

Most students seem to have no problem accepting the transitivity of universal claims, e.g., if (1) all cats are mammals and (2) all mammals viviparous, then it follows that (3) all cats are viviparous—even when they don’t know what “viviparous” means. Moreover, they seem to have no problem understanding that particular claims are not (normally) transitive, e.g., even if (I) some whales are friendly, and (II) some friendly animals are kept in apartments as pets, it doesn’t follow that (III) some whales are kept in apartments as pets. Students don’t usually seem to need to be given examples to underwrite these claims. However, it has been my experience that students are less good at understanding the transitivity (or intransitivity) of most-claims. Formally, if (a) most A are B and (b) most B are C, are (c) most A also C? You can easily trick the students into thinking that most is transitive by choosing examples where “most” is weaker than the facts. So, for example you can choose examples of populations of A, B and C in which most A are B, most B are C, and most A are C. The clever students try the reasoning with different interpretations of A, B and C. But here is where the visual comes in. If we start with just the query regarding the possible transitivity of most, we can get students to *see* the correct answer by providing a useful visual.
Fig. 4 refutes the general claim that most is transitive. The general claim has the form, if most A are B and most B are C, then most A are C. The diagram in Fig. 4 makes true that most A are B and that most B are C, but it also makes false that most A are C.\footnote{Indeed, it makes the stronger claim: No A are C.} Lest you think Fig. 4 merely illustrates the verbal point, consider this fact. We wouldn’t think of a verbal counter-model in this case as merely illustrating the intransitivity. Rather, the verbal counter-model shows the failure of transitivity. Yet, this is precisely what happens in the visual case. Fig. 4 offers a visual model in which most A are B, most B are C, but it’s not the case that most A are C.

For a weightier example, consider the famous four-color theorem—that for any planar map, four colors will suffice so that no countries sharing a border will need to have the same color. I take it that no one will doubt that there are maps that require more than one color.\footnote{For a proof, consider just regions C and D in Fig. 5. Such a map requires two colors.}
Likewise, the following is a proof that three colors are insufficient for the task at hand.

Figure 6: Three Colors are Insufficient

In this case, there is no way to choose a color from regions B, C, or D to color region A such that the coloring would suffice. Hence, a fourth color is required.

It is, perhaps, tempting to think that the pattern of example offered in Fig. 5 and Fig. 6 can be extended to construct a counterexample 4CT. You may have noticed a pattern. The following diagram (Fig. 7) continues the pattern in an effort to increase the minimum number of colors needed.

Figure 7: Do We Need Five?

This map can be colored using just the four already-present colors: color region E using the color from region B. Since E and B do not share a border, B’s color is available to E. It isn’t, or at least it wasn’t prior to Appel and Haken’s proof of 4CT, irrational to attempt to construct counterexamples to the theorem. Indeed, even after the theorem was a theorem, many eminent mathematicians expressed some skepticism regarding the result. Still, before Appel and Haken, there was a putative
proof that convinced the mathematical community of 4CT for more than a decade before someone discovered a counterexample.

Kempe’s proof set out to give a general method for constructing any map such that: if it was four-colorable at a given stage in its development, then it was four-colorable at the next stage. A map with four or fewer regions is four-colorable. When a map contains five or more regions, the question whether it is four-colorable arises only when the new region has at least four contiguous neighbors. If a region/country has fewer than three neighbors, simply choose any of the colors different from its neighbors (see Fig. 8 and Fig. 9 for examples).

![Figure 8: Color it Orange!](image1)

![Figure 9: Color it Orange!](image2)

When the region needing color has at least four neighbors, then it becomes more interesting. For example, when there are exactly five regions and the one to be colored is neighbor to all the other four regions, then some re-coloration is required. Perhaps it is easier to see with a visual example (see Fig. 10a-c). In this example, the central region needs color. But four colors are already used. So, one cannot simply choose a new color. Instead, one must reuse a color. But, this will make the map, at least temporarily, fail to be four-colorable. To get it back to being four-colorable, just change the region that lent its color to the central region to any of its non-neighbor’s colors.

![Figures 10(a-c): Preserving Four-Color-Ability Through Swapping](image3)

Choose blue (though we could have chosen any of the four available colors). Then, the map has a blue region that borders another blue region. To get the map back to four-colorable, change the original blue to green, since none of its neighbors are green.

The example in Figure 10 was simple insofar as the regions needing color and re-coloration had both simple connections and few neighbors. Things get more complex when the number of regions and neighbors increase. One kind of problem that may arise concerns chains of neighbors that contravene a simple swapping of
colors. Consider Fig. 11. There is a continuous chain of red/green pairs connected to \( n \). Unlike in Fig. 10 where the swap of colors facilitated four-colorization, in Fig. 11, making \( n \) either red or green and swapping red for green throughout the chain will not work. Call such a chain a complete Kempe Chain. To facilitate colorization, then, we simply choose a neighbor of \( n \) that isn’t a complete Kempe Chain and swap the colors as before. See Fig. 13a-b for two different re-colorizations. I don’t know how many maps Kempe made to test his method, but he seems to have simply assumed that the method could be extended from cases where \( n \) has four neighbors to cases where \( n \) has five neighbors. Regardless, Kempe’s proof was sufficiently convincing to mathematicians that the proposition was called the four-color theorem. The proof stood for more than a decade.

Percy Heawood discovered a counterexample to Kempe’s method. It is very important to note that Heawood’s example is four-colorable—Heawood does not demonstrate the falsity of the theorem. Instead, his example undermines the re-coloration method. The clever configuration is that there are separate but incomplete Kempe chains that, in combination, interfere with the re-coloration technique.

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\(^6\) This terminology is not standard. The notion of completeness is that the chain connects completely to the original region to be colored. In Fig. 12 the chain goes: \( A \rightarrow E \rightarrow F \rightarrow G \rightarrow I \rightarrow C \rightarrow n \rightarrow A \).
In Fig. 13, \( n \) marks the region needing color. It has five neighbors. There is a blue/purple complete Kempe chain as well as a green/blue complete Kempe chain. Hence, we must choose one of the other neighbors to do the re-colorization. Moreover, purple and green are neighbors of \( n \) and of each other, as are both red and blue on the one hand and red and green on the other. Kempe’s method would have us color \( n \) red and swap out red for green from one of \( n \)’s red neighbors, and red for purple from the other. Unfortunately, there is a red/green Kempe chain, though not a complete Kempe chain, that ends as a neighbor to a purple region that is the endpoint of a red/purple Kempe chain, again though not a complete Kempe chain, from the other. This means that the required swap won’t work. The swap would make red neighbor red. Neither Kempe nor Heawood could fix the problem (at least not systematically).

The counterexample doesn’t merely illustrate the claim, nor is it merely a heuristic device by which one comes to understand the reasoning presented in another way. Instead, the picture is the counterexample. Without the picture, there is no counterexample at all.

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7 Heawood didn’t actually color his example, but merely labeled the colors for the regions.
Now, whereas in the cases of maps and photographs we needed some theory to explain the content of the visuals, we need no such theory for the present examples. The reason is that these visual examples don’t represent other objects. Instead, they are themselves the objects that constitute the counter-models. It is perhaps easier to see this in the simple case of the counter-model to the claim that most is transitive. The diagram is the counter-model. Most of the A-region of the diagram is inside of the B-region of the diagram. And most of the B-region of the diagram is inside the C-region of the diagram. But it is not the case that most of the A-region of the diagram is inside the C-region. Hence, “most” is not transitive. Such cases require visual inspection to determine the content. But like the cases of depiction considered in the previous section, unless one is generally skeptical regarding visual inspection, there isn’t a good reason to doubt that humans can recognize the visual elements under consideration here. Hence, some visuals can be parts of some meta-arguments.

4. CONCLUSION

There is a tendency on the part of visual argument skeptics to require more machinery to explain the possibility of visual arguments than they require of verbal arguments. Even if we grant that verbal arguments are the exemplars for argumentation, it isn’t as if people rarely appeal to visuals in reasoning. They do. In an effort to avoid a technical foray into semiotics on the one hand or theories of depiction or representation on the other, I’ve termed the method of appealing to visuals extraction. We extract information from a visual element as part of a process of reasoning. There is nothing mysterious about this process. If there is a difference of opinion regarding the extraction, this can be remedied. Compare this with the case of ambiguity in verbal arguments. If there is a preferred meaning, the arguer can clarify his or her argument—and likewise for visual elements. Moreover, these clarifications can be verbal. It is more important to analyse and evaluate examples of visual reasoning than it is to continue to argue about their existence.

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