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Behaviour of curved reinforced and prestressed concrete waffle slab bridges.

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University of Windsor

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BEHAVIOUR OF CURVED
REINFORCED AND PRESTRESSED CONCRETE
WAFFLE SLAB BRIDGES

by

Alaa Aly El-Sayed M. Aly

A Dissertation
submitted to the Faculty of Graduate Studies and Research
through the Department of Civil and Environmental Engineering
in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
at the University of Windsor

Windsor, Ontario, Canada
1993
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ABSTRACT

The use of horizontally curved bridges has increased considerably in recent years for highway bridges and interchanges in large urban areas. Due to the increasing demands for reduced dead weight and for efficient cross-section in distributing the wheel loads, waffle slab construction is introduced. In the past, bridge design engineers have shied away from using waffle slab construction since they considered it incompatible to the predominately one-way supporting system of a bridge. However, a horizontally-curved bridge can be subjected to large twisting moments. Therefore, the orthogonal rib system in a waffle slab construction provides an efficient resistance for such twisting moments. In this research, theoretical and experimental studies were carried out to determine the various rigidities of curved concrete waffle slab bridges and to investigate their behaviour under working and ultimate loads. In the analyses, the finite difference technique, the finite element method and the yield-line theory were used. Simplified yield-line expressions were developed to predict the collapse load of curved reinforced and prestressed concrete waffle slab bridges. These expressions were based on both parametric study as well as on laboratory test results on curved waffle slab bridges. Also,
the structural responses of curved prestressed concrete waffle slab and girder-slab bridges are examined for the effect of prestressing, working load and ultimate load. An analysis for the effect of the high radial forces, generated by the horizontal curvature of the prestressing tendons is proposed. Furthermore, a parametric study was conducted to examine the influence of bridge horizontal radius to width ratio, bridge aspect ratio, loading position and the presence of the transverse ribs on the elastic and post-elastic behaviour of such bridges. The experimental program is divided into two groups: the first group deals with the rigidities of curved waffle slab structures and consists of nine tests on horizontally curved concrete waffle slab. The second group deals with the behaviour of curved concrete waffle slab bridges. It consists of testing three curved concrete slab bridges: the first model was a curved reinforced concrete waffle slab bridge; the second model was a curved prestressed concrete waffle slab bridge, while the third model was a curved prestressed concrete girder-slab bridge to study the effect of the transverse ribs on the behaviour of curved bridges. Good agreement between the theoretical and experimental results is found. Both the analytical and experimental results demonstrated that both the elastic and post-elastic response of curved waffle slab bridges are superior to those of girder-slab bridges. Results from the parametric study revealed that the bridge horizontal radius to width ratio, aspect ratio, loading position and the presence of the transverse ribs significantly influence the elastic behaviour as well as the ultimate load capacity of such structures.
TO MY FAMILY
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NOMENCLATURE

$A_0$  cross sectional area of steel diaphragm

$A_o$  cross sectional area of steel girder

$A_t$ ($A_t'$)  area of tension steel in the longitudinal (transverse) rib

$b_o$ ($b_o'$)  width of longitudinal (transverse) rib

$b_e$ ($b_e'$)  effective width of the concrete deck slab of composite bridge in the longitudinal (transverse) direction

$D$  flexural rigidity of the concrete deck slab with respect to its middle plane

$D_o$  tangential flexural rigidity per unit width

$D_r$  radial flexural rigidity per unit width

$D_{o\theta}$  torsional rigidity per unit width

$d_o$ ($d_o'$)  depth of the longitudinal (transverse) rib

$E_e$  modulus of elasticity of the concrete

$e_o$ ($e_o'$)  depth of the neutral plane from the top fibre for bending in the $\theta$ ($r$) direction

$f_c'$  28-day concrete cylinder strength in psi

$G$  shear modulus

$2H$  effective torsional rigidity

$h$  thickness of the concrete deck slab
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{cs}$ ($I_n$)</td>
<td>moment of inertia of concrete in compression about the neutral axis in the $\theta$ ($r$) direction</td>
</tr>
<tr>
<td>$I_{sr}$ ($I_{nr}$)</td>
<td>moment of inertia of the transformed reinforcing steel about the neutral axis in the $\theta$ ($r$) direction</td>
</tr>
<tr>
<td>$J$</td>
<td>torsional constant of the cross-section</td>
</tr>
<tr>
<td>$2L$</td>
<td>bridge span at the longitudinal center line</td>
</tr>
<tr>
<td>$M_0$ ($M_s$)</td>
<td>moment vector in the $\theta$ ($r$) direction</td>
</tr>
<tr>
<td>$M_{tr}$</td>
<td>twisting moment vector in the $\theta$-$r$ direction</td>
</tr>
<tr>
<td>$m_1$</td>
<td>ultimate positive moment of resistance per unit width about radial axis ($r$)</td>
</tr>
<tr>
<td>$m_2$</td>
<td>ultimate negative moment of resistance per unit width about radial axis ($r$)</td>
</tr>
<tr>
<td>$\mu m_2$</td>
<td>ultimate negative moment of resistance per unit width about $\theta$ - axis</td>
</tr>
<tr>
<td>$m_t$</td>
<td>ultimate torsional moment of resistance of the cross section per unit width</td>
</tr>
<tr>
<td>$n$</td>
<td>modular ratio</td>
</tr>
<tr>
<td>$P_u$</td>
<td>ultimate collapse load of the bridge</td>
</tr>
<tr>
<td>$q_0$</td>
<td>dead weight of the bridge per unit area</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of Curvature of the Inner Edge</td>
</tr>
<tr>
<td>$R_0$ ($R_1$)</td>
<td>reaction acting on the face normal to the tangent (radial)</td>
</tr>
<tr>
<td>$S_0$ ($S_1$)</td>
<td>spacing of the longitudinal (transverse) ribs</td>
</tr>
</tbody>
</table>
\( V_0 \) (\( V_r \)) shear acting on the face normal to the tangent (radial)

\( W \) lateral deflection

\( W_s \) width of the slab

\( W_{00} \) (\( W_r \)) curvatures associated with moment \( M_0 \) (\( M_r \))

\( W_{tr} \) twisting curvature associated with the twisting moment \( M_{tr} \)

\( \alpha \) \( D_0/D_r \)

\( \beta \) \( D_0/D_r \)

\( \lambda \) radial spacing between the mesh points

\( \mu \) Poisson's ratio of concrete

\( \kappa \) torsional parameter

\( \theta_0 \) angular spacing between the mesh points (radian)

\( \theta_T \) total connecting angle (radian)
CHAPTER I

INTRODUCTION

1.1 General

The increasing use of horizontally curved bridges can be attributed mainly to their aesthetics and to the need of such geometries encountered frequently in modern highway networks. The principal requirement for an efficient bridge cross-section is the ability to transfer and distribute wheel loads over a wide area so that stresses are kept within allowable limits. Solid concrete slabs are highly efficient in distributing concentrated wheel loads. However, for spans exceeding 15 m (50 ft), the dead weight stress can be high and may exceed the allowable stress rendering the solid slab section uneconomical. To enhance structural efficiency and to reduce the dead weight without weakening the section, voided slab bridges were introduced. Observations of the performance of voided-slab bridges have revealed that this structure suffers from excessive longitudinal cracks (Csagoly and Holowka 1975). A more attractive alternative is the waffle slab construction which is often used in large auditoriums, parking garages and marine structures.

Although waffle slab construction has been used regularly in buildings, its usage
in bridges has so far been limited. The only example is the Hegenberger Overpass which is a curved prestressed concrete waffle slab bridge, designed by Lin et al. (1969), with eight lanes of highway traffic, as shown in Fig. 1.1. The reason for this neglect is that many bridge design engineers regard the two-way structural system of a waffle slab to be incompatible with the predominately one-way supporting system of a bridge. However, since a horizontally-curved bridge is subjected to large twisting moments, the orthogonal rib system in a waffle slab construction can provide an efficient resistance for such twisting moments.

In the past, much research was carried out on curved reinforced and prestressed concrete box-girder bridges. Comparatively little attention was directed toward the behaviour of curved reinforced and prestressed concrete waffle slab bridges. Moreover, no research effort was directed to predict the collapse load of curved reinforced and prestressed concrete waffle slab bridges.

In this dissertation, theoretical expressions are derived for predicting the flexural and torsional rigidities of curved waffle slab structures. Also, the elastic and post-elastic structural responses of curved reinforced and prestressed concrete waffle slab bridges are examined. Furthermore, the effect of bridge radius to width ratio, bridge aspect ratio, loading position and the presence of the transverse ribs on the elastic and post-elastic behaviour of such bridges are also studied. The theoretical analyses are verified and substantiated by results from tests on curved concrete waffle slab as well as girder-slab bridges.

In this work, curved reinforced and prestressed concrete waffle slab bridges will
be referred to as RWS and PWS, respectively. On the other hand, curved reinforced and prestressed concrete girder-slab bridges will be referred to as RGS and PGS, respectively.

1.2 Objectives and Scope

This study is an attempt to examine the behaviour of curved reinforced and prestressed concrete waffle slab bridges in the elastic and post-elastic loading ranges. In this respect, experimental and analytical investigations were carried out to examine the behaviour of such bridges. The following theoretical approaches were used in the analyses:

1- Finite Difference Technique.

2- Finite Element Method.

3- Yield Line Theory.

The primary objectives of this study may be summarized as follows (see Figs. 1.2 to 1.6):

1- To develop theoretical expressions for predicting the flexural and torsional rigidities of curved waffle slab and composite structures before and after cracking. In this respect, the experimental results from tests on curved reinforced concrete waffle slab models are used to verify the predicted rigidities.

2- To examine the feasibility of using an equivalent orthotropic plate system to predict the behaviour of curved concrete waffle slab structures. In this respect, the finite difference technique with the predicted rigidities from (1) are used in the analyses.
3- To study, theoretically and experimentally, the elastic and post-elastic behaviour of curved reinforced concrete waffle slab bridges.

4- To examine the elastic response of curved reinforced concrete girder-slab as well as solid slab bridges and compare their behaviour with the waffle slab bridge.

5- To study the responses of a curved prestressed concrete waffle slab as well as girder-slab bridges under the effect of prestressing. In this respect, an analysis for the effect of the high radial forces, generated by the horizontal curvature of the prestressing tendons, is proposed.

6- To examine theoretically and experimentally the elastic and post-elastic behaviour of curved prestressed waffle slab and girder-slab bridges.

7- To develop, using the yield line theory, simple expressions to predict the collapse load of curved reinforced and prestressed waffle slab bridges. In this respect, the experimental and finite element analytical results are used to provide the relevant information required to develop these expressions.

8- To examine the influence of the aforementioned factors on the elastic and post-elastic behaviour of curved waffle slab as well as girder-slab bridges.

The contributions of this candidate to the advancement of knowledge in this field of structural engineering are that all the above objectives were achieved.

The experimental program was divided into two groups: the first group dealt with the rigidities of curved waffle structures. Nine tests were carried out on horizontally curved concrete waffle slab, three being square in plan for the pure twisting test, while the other six were rectangular in plan for the pure bending tests. The second group dealt
with the behaviour of curved concrete waffle slab bridges. It consisted of testing three
curved concrete slab bridges: the first model was a curved RWS bridge, the second
model was a curved PWS bridge, while the third model was a curved PGS bridge to study
the effect of the transverse ribs on the behaviour of curved bridges. All three models
were identical in plan and cross sectional dimensions.

The contents of this dissertation are as follows:

Chapter II  Contains a review of the different methods used to determine the rigidities
of stiffened plate. Also, this chapter reviews the historical literature on the waffle slab
system, and on the elastic and post-elastic behaviour of curved bridges.

Chapter III  Deals with developing theoretical expressions for predicting the different
rigidities of curved concrete waffle slab as well as composite slab bridges before and after

Chapter IV  Presents the finite difference technique and its application to curved waffle
slab bridges using the equivalent orthotropic plate theory.

Chapter V  Focuses on the finite element formulation and gives a brief description for
the parts of the 'ABAQUS' computer code (Hibbitt et al. 1989) used in the analysis.

Chapter VI  Deals with developing simple expressions, using the yield line theory, to
predict the collapse load of curved reinforced and prestressed concrete waffle slab bridges under different loading conditions.

Chapter VII Deals with the experimental program with a full description of the tested models, the prestressing procedure, the equipment and material used to accomplish the experimental program.

Chapter VIII Presents the discussion of results from the experimental and theoretical analyses and the effect of different parameters on the elastic and post-elastic behaviour of curved concrete waffle slab bridges.

Chapter IX Contains the summary and conclusions of the study as well as recommendations for future research.
CHAPTER II

LITERATURE REVIEW

2.1 General

In recent years, research interest in horizontally curved bridges has intensified considerably. This reflects the increase in usage of such structures due to their economic and aesthetic advantages. Reasonable research effort was carried out to investigate the elastic and post-elastic response of horizontally curved voided-slab bridges. However, no research effort was carried out to study the elastic and post-elastic behaviour of curved reinforced and prestressed concrete waffle slab bridges.

This chapter presents the background for the present investigation in four parts. The first part is concerned with the different methods which determine the rigidities of stiffened plates. The second part deals with the analysis of the waffle slab system and its usage in buildings and bridges. The third part presents the different methods; classical, numerical, and approximate methods used to predict the elastic response of curved bridges. The last part of this review is concerned with the post-elastic behaviour of curved prestressed concrete bridges.
2.2 Rigidity of Stiffened Plates

The wide use of stiffened structural form in engineering began in the nineteenth century, mainly with the use of steel plates in ships and with the development of steel bridges and aircraft structures. The analysis of stiffened plates may be simplified by replacing the plate stiffener combination by an equivalent homogenous orthotropic plate of constant thickness. This procedure requires the determination of the elastic rigidity constant. For such an approach to be valid, it is necessary that the stiffeners be spaced close enough to justify the assumption of homogeneity.

The study of the elastic constants of plates by bending and twisting tests began when Bergstrasser (1927) used a procedure suggested early by Nadai for experimentally applying pure bending and twisting moments on a plate of constant thickness. For applying pure bending moments, he used a rectangular plate supporting it at three points and loading it at three other points. A square plate was used for applying a pure twisting moment in the plate supported on two diagonally opposite corners and loaded on the other two corners. Bergstrasser assumed a relation between the applied bending moment and the resulting deflection to calculate the elastic constants.

Hearmon and Adams (1952) used the same procedure adopted by Bergstrasser. The principal difference between the two investigators is in the manner in which they measured the displacements, and in the equations used to determine the elastic constants.

Witt, Hoppman and Buxbaum (1953) gave the theoretical basis and an experimental method for determining the anisotropic elastic constants of a material by measuring deflections of the thin plate of the material subjected to couples on its
boundary.

The earliest treatment of stiffened slabs as orthotropic plates is due to Huber (1923). He investigated reinforced concrete slabs, with and without integral ribs. Huber's calculation of the twisting rigidity involved the two flexural rigidities.

Giencke (1955) presented an analysis of ribbed plates and considered the effect of Poisson's ratio throughout the system. However, the stress equations were formulated as if the transverse ribs were uniformly in contact with the entire length of the longitudinal ribs.

Huffington (1956) investigated theoretically and experimentally the method for the determination of rigidities for metallic rib-reinforced deck structures. It was applied to the case of equally spaced stiffeners, of rectangular cross-section, and symmetrically placed with respect to its middle plane.

Hoppman and Huffington (1956) applied the vibration technique to the determination of the elastic constants of various circular and rectangular plates.

A different method, the so-called curvature method, was presented by Beckett et al. (1963), for the experimental determination of the elastic constants of an orthogonal plate. This method utilizes the fact that pure bending and twisting moments can be expressed in terms of the curvature and twist of a surface. The method assumes that a region of the plate can be loaded by a uniform moment or a uniform twist. The surface of the plate in this region is carefully measured to determine the principal radii of curvature.

Rowe (1962) suggested an equation to calculate the torsional rigidity of ribbed
plates. The method suffered from ignoring the stiffening effect of orthogonal ribs on the torsional rigidity of the flange plate.

Tsai (1965) proposed a simple experimental method for the determination of the elastic constants of orthotropic plates. The special test fixture for the pure twisting test is extremely simple. The loading scheme uses three fixed and one loading points.

Kinloch and Harvey (1967) used the experimental method suggested by Bergstrasser to evaluate the elastic constants for corrugated panel decking. These tests have shown excellent agreement between the theoretical and experimental results.

Heins and Hails (1969) proposed a method to estimate the torsional and flexural rigidities of stiffened curved plates. The proposed method for the torsional rigidity was based on the membrane analogy; however, the analysis did not account for the stiffening effect of orthogonal ribs on the torsional rigidity of the flange plate; furthermore the method is not applicable to reinforced concrete slabs in the post-cracking stage.

Cusens et al. (1972) presented an elastic analysis for the determination of rigidities of orthogonally-ribbed plates in flexure and torsion; however, the analysis suffers from the same deficiencies as that of Heins for the torsion analysis.

Cardenas et al. (1972) investigated the in-plane and flexural stiffnesses of isotropically and non-isotropically reinforced concrete plates.

Szilard (1974) presented an analysis for ribbed plates and considered the stiffening effect of orthogonal ribs on the torsional rigidity of the flange plate; the method did not account for the continuity of the flange plate nor for Poisson's ratio when estimating the torsional and flexural rigidities respectively. Furthermore, the method is not applicable
to reinforced concrete slabs in the post-cracking stage.

Bali (1980) carried out theoretical and experimental studies to predict the rigidities of orthogonally and non-orthogonally rib-stiffened concrete slabs in the pre-cracking and post-cracking stages.

Little information is available as to how these rigidities might be assessed for the cracked sections of a concrete structure. Desayi and Kulkarni (1977) suggested the empirical reduction factors for the flexural rigidities of reinforced concrete slabs up to the yield load.

Clark and White (1978) carried out tests to determine the torsional stiffness of flexurally cracked slab elements.

2.3 Waffle Slab System

In recent years reinforced and prestressed concrete waffle slab constructions have been employed successfully in buildings. Considerable attention has been given recently to study the elastic and post-elastic behaviour of such system. Kennedy and Ghobrial (1980) carried out a study on waffle slab construction. Their study showed that waffle slab bridges, supported by isolated interior columns, are more economical, in terms of the required amount of steel reinforcement, than either the one-way ribbed or solid slab bridges. It was also concluded that the waffle slab system possesses other advantages such as: reduction in dead load moment and deflection leading to minimizing the amount of secondary stresses; easy accessibility to parts of the structure for inspection and repair, and, shallow depth of its cross-section.
Kennedy and El-Sebakhy (1980) presented a series solution to predict the elastic behaviour of prestressed concrete waffle slab bridges, of rectangular and skew planforms. The solution is formulated by the superposition of results from a bending analysis to those from an in-plane stress analysis. Test results from an experimental program on models of prestressed concrete waffle slab bridges subjected to concentrated load substantiate and verify the analytical solution and its convergency. A method of analysis was suggested to deal with prestressed waffle slab bridges on isolated interior column supports.

Kennedy and El-Sebakhy (1982) extended their previous elastic study on waffle slab bridges to include the post-elastic behaviour of such structures. The ultimate collapse load was estimated by means of the yield line theory. An experimental program consisting of five bridge models was carried out to verify and substantiate the elastic and ultimate load analyses.

A feasibility study by Kennedy and Bakht (1983) and Kennedy (1987) was undertaken to investigate the structural efficiency of a waffle slab bridge system in comparison with conventional alternatives, namely the solid slab bridge and the slab-on-girder bridge. Tools for design and analysis of waffle slab bridges for both serviceability and ultimate limit states were presented.

Kennedy (1983) carried out an experimental program on three bridge models to study the effect of orientation of ribs on the response of waffle slab skew bridges. The three structures were of orthogonal, nonorthogonal-rib waffle slab constructions and solid-slab construction. The study revealed that the orthogonal rib waffle slab construction for a skew bridge provides a better load distribution and ultimate load-carrying capacity when
compared to the other two systems.

Grace (1986) studied the dynamic response of waffle slab bridges. The analytical study was verified and substantiated by experimental results from tests on prestressed and reinforced waffle slab bridge models. The study showed that by increasing the stiffness by means of prestressing, the natural frequencies of the bridge are significantly enhanced.

Lawson and Mark (1986) carried out large-scale fire tests on various forms of ribbed concrete floors. The tests showed good resistance of the waffle slabs against fire.

Recently, Meli et al. (1988) studied the seismic behaviour of waffle-flat plate buildings. This study was carried out after the collapse of many buildings with waffle-flat slabs by the 1985 earthquake in Mexico city. The performance of specific buildings was evaluated and compared with the proposed design procedure.

2.4 Classical, Numerical, and Approximate Methods

Due to the increasing use of horizontally curved bridges, considerable volume of research efforts has been directed to this type of bridges. The analysis of horizontally curved bridges stiffened by a system of stiffeners can be performed by considering it as a structurally equivalent orthotropic plate (EOP); or by considering it as a plate and stiffeners individually and then matching their solutions. This method is sometimes referred to as the plate-stiffener system (PSS). It should be noted that analyzing a stiffened plate as an equivalent orthotropic plate (EOP) simplifies the analysis considerably and the analytical techniques available for orthotropic plates can be employed to analyze this type of bridges. However, the plate-stiffener system method
(PSS) is recommended when the stiffeners have different cross sections or when the stiffener spacing is large enough as not to ensure approximate homogeneity.

The recent work by Harik and Haddad (1987) can be cited as an example of the PSS approach. In their research, they studied the static response of stiffened sector plates by considering individual solutions to plate and stiffeners and then matching these solutions to obtain the complete response.

Many attempts were made in the past to include classical or numerical solutions for curved plates. Ceradini (1965) presented an analysis of curved bridge decks using a closed form solution.

Coull and Das (1967) presented an exact solution for the analysis of isotropic curved bridge decks subjected to concentrated loads. The deflection form of the curved bridge deck was expressed as a Fourier series in the spanwise direction. The concentrated loads were incorporated in the solution as a discontinuity in shear force along the load line. Results from experimental tests on model structures verified the proposed method.

Heins and Bell (1972) developed a Fourier Series Slope-Deflection Technique to analyze curved girder bridge systems. The determination of slopes and deflections, along each curved girder, is obtained by a Fourier series summation. The technique incorporates both pure and warping torsional effects as well as bending effects.

Harik and Pashanasangi (1984) carried out an analytical study to evaluate the influence of the state of orthotropy on bending of annular sector plates stiffened by angular and radial stiffeners. Basically, the method employs the classical method of separation of variables in which the basic function in the angular direction satisfies the
boundary conditions of the radial edges. By a suitable transformation, the governing partial differential equation of orthotropic plates in polar coordinates is converted to an ordinary differential equation. From the resulting fourth order differential equation, the deflection equation is generated.

Recently, various numerical methods such as finite difference, finite strip, and finite element have been employed to obtain approximate solutions. The first notable analytical effort to study the elastic behaviour of stiffened curved plate structure, using the finite difference technique, was perhaps due to Heins and Hails (1969). In this study, a stiffened curved plate model, subjected to static loading, is analyzed as an equivalent orthotropic plate (EOP). This analysis incorporates the orthotropic plate equation in polar coordinates, and is solved by the finite difference technique. A curved stiffened plate model was experimentally tested to verify the proposed method of analysis. The results of this study show the feasibility of using an equivalent orthotropic plate system in predicting the behaviour of a structural model. The study recommended further research in determining actual bridge stiffnesses.

Dey and Samuel (1980) carried out a static analysis of orthotropic curved bridge decks using a combination of Fourier series and finite difference technique. Basically, the proposed method is a combination of classical method of separation of variable and modern numerical procedure. The governing fourth order partial differential equation of orthotropic plates in polar coordinates is first converted to an ordinary differential equation and subsequently solved by finite difference method. The study demonstrated that the proposed method requires a minimum input data and small computer storage.
Brave and Day (1983) proposed a method which incorporates the concepts of isoparametry in the finite difference energy method making it more powerful and versatile to tackle complex plate bending problems with curved boundaries.

Azad et al. (1989) presented a static analysis of horizontally curved, continuous multigirder slab type bridge decks using the finite difference method in conjunction with the method of consistent deformation. The deck is idealized as a curved thin plate supported by flexible supports having both vertical and rotational flexibility. The Levy-type solution permits the use of one-directional finite difference equations only along the central radial line. First, the continuous deck is transformed into a simply supported deck with the removal of the redundant pier supports. The analysis is then performed by using the equations of a simply supported curved girder-slab deck, developed early by Azad et al. (1986), in conjunction with the deformation compatibility of the actual deck.

Recently, Verma and Dey (1991) carried out a static analysis of bridge superstructures which are curved in plane, having radial as well as circumferential beams, and continuous over axially flexible columns. They used a variational based finite difference energy method. The total potential energy of the system composed of different components of the bridge deck is discretized in terms of pivot displacements by finite difference operators. The principle of minimum total potential energy is applied to obtain the force-displacement relationship which was solved for pivot displacements.

The finite strip method was developed by Cheung (1969) in the wake of necessity for a method which reduces the labour of solving many algebraic equation. The method has been utilized for the analysis of orthotropic curved slabs.
Dey (1980) carried out a bending analysis for orthotropic curved bridge decks using an analytical finite strip method which leads to an efficient solution using few elements. The stiffness matrix of an individual element was derived using a homogeneous differential equation of an orthotropic plate in polar coordinates. The convergence of the solution was achieved merely by increasing the number of harmonics rather than by increasing the number of elements.

Cheung and Chan (1981) developed two and three dimensional finite strips for the static and dynamic analysis of thin and thick sectorial plates. The method can incorporate plates with isotropic or orthotropic properties, of constant or variable thickness, and can have different combinations of boundary conditions.

Dey and Balasubramanian (1984) carried out a study on the dynamic response of orthotropic curved bridge decks due to moving loads using the finite strips.

Cheung (1984) presented a numerical technique based on the finite strip and the force method for the analysis of continuous curved box-girder bridge. Curved two node low order and three node high order finite strips were used in the formulation.

Harik and Pashanasangi (1985) introduced an exact solution for the analysis of orthotropic curved bridge decks subjected to patch, uniform, line, and concentrated loads. The bridge deck is idealized as an assemblage of radially curved plate strips. The deflection of each plate strip is expressed as a Levy type Fourier series and the loads are represented in a series form.

Recently, Cheung et al. (1986) applied the spline-finite strip method to analyze curved slab bridge. This method possesses the advantages of both the finite element and
finite strip methods.

Finite element method provides an elegant and powerful tool in dealing with all types of complicated plate problems. The method includes not only linear elastic problems, but also those with geometric and material nonlinearities as well as plates with irregular planforms. A large number of publications, most of which have appeared since 1960, discuss the finite element method, particularly its application to curved plates.

Cheung et al. (1968) carried out an extensive study to analyze slab bridges with arbitrary shape and support conditions using the finite element method. The study included rectangular, skew, and curved bridges.

Olson and Lindberg (1970) have derived a plate bending elements in polar coordinates. They deduced the stiffness matrix of circular and annular sector elements to provide close fit to the boundaries.

High order sector element was developed by Sawko and Merriman (1971), as well as by Singh and Ramaswamy (1972).

Kohnke and Schnobrich (1972) presented a finite element analysis of circular cylinders reinforced with eccentric stiffeners using curved elements.

Cretu (1988) carried out a theoretical study on the analysis of curved bridges with large and medium span using the finite element method. He used two isoparametric thin-walled finite plate elements, formulated in terms of displacements.

Recently, Bhimaraddi et al. (1989) presented a finite element analysis of orthogonally stiffened annular sector plates by combining the annular sector plates element and the curved beam element, developed by Bhimaraddi et al. (1989). They
pointed out that the plate stiffener system may not always give correct results in spite of its rigorous mathematical justifications.

In addition to the aforementioned analytical and numerical methods for the analysis of stiffened curved plates, approximate methods were presented to analyze curved girder bridges. These structures were analyzed either as a curved beam or an assemblage of curved beams. This idealization leads to inaccurate results when the bridge deck is wide. Research work done on this topic was presented by MacManus et al. (1969).

Another substitute method for the analysis of such structures is to replace it by an equivalent open grillage. Sawko (1969) proposed a grid frame work model composed of interconnected circular and straight beams for decks horizontally curved in plane. This approach is incapable of representing the plate action fully since it is difficult to equate the overall twisting moment of the plate to the torsional properties of the beams.

2.5 Post-elastic Behaviour of Curved Prestressed Concrete Bridges

Due to the rapid increase in the axle loads of trucks in the last few decades, significant research efforts were directed towards the study of the behaviour of overloaded curved prestressed concrete bridges.

The first notable experimental effort to study the post-elastic behaviour of curved prestressed concrete cellular bridge, was due to Cheung and Gardner (1968). The work presented experimental results of a 1/6 scale model for the Tsing Fung Flyover in Hong Kong. The model was tested to destruction and the results indicated that the multi-box section possesses a very large torsional rigidity. The study recommended that the top
flange should be adequately reinforced in the lateral direction to safeguard against local distortion.

In the last thirty years, a considerable volume of research effort was directed to study the elastic and post-elastic behaviour of curved prestressed concrete box girder bridges. However, little attempts were made to examine the elastic and post-elastic response of horizontally curved prestressed concrete slab bridges (waffle slab and voided slab).

Possibly the largest stride in the design of curved prestressed slabs was taken by Lin et al. (1969) who were the first to use post-tensioned waffle construction in a bridge; they used large precast concrete pans to form a waffle layout for the bridge deck; the dead load on the bridge was partially balanced by prestressing to achieve a minimum amount of bending under dead load.

Recently, Hodge (1988) carried out a theoretical study to investigate the ultimate strength of longitudinally and transversally prestressed, continuous, curved, voided slab bridges. A simplified analysis was developed for incorporating the effect of transverse prestress on the ultimate moment capacity and modifications are proposed for calculating the ultimate capacity.

Earlier voided slab bridges prestressed only in the longitudinal direction, developed extensive cracking after short time in service. Czagoly and Holowka (1975) reported in their study on the cracking of voided posttensioned concrete bridge decks that many bridges in Ontario are suffering from extensive longitudinal cracking. To avoid these longitudinal cracks, both AASHTO (1983) and OHBDC (Ministry of Transportation and
Communications 1983) recommended that the bridge deck be additionally prestressed in the transverse direction.

Podolny (1985) carried out an extensive study to investigate the cause of cracking in post-tensioned concrete box girder bridges.

An appreciable amount of research effort has been done on the ultimate load analysis of rectangular and skew waffle slab bridges (Kennedy and El-Sebakhy 1982) and on composite bridges (Kennedy and Soliman 1992). However, no research effort has been carried out to determine the collapse load of curved reinforced and prestressed waffle slab bridges using the yield line theory. While this approach theoretically would lead to an upper bound solution to the collapse load (Jones and Wood 1967), it is shown later that the results obtained are quite close to the test results reported herein.
CHAPTER III

RIGIDITIES OF CURVED BRIDGES

3.1 General

Horizontally curved bridges are generally built in reinforced or prestressed concrete construction, or in composite construction with a concrete slab-on-rigid steel gridwork. To accurately predict the response of such curved structures to static loads, using the finite difference technique or the finite strip method, it is essential to use accurate estimates of the structure's various rigidities. The provision of an adequate number of closely-spaced steel diaphragms in composite bridges, or transverse ribs in waffle slab bridges, is necessary in order for the curved structure to resist the large torsional moments.

In this chapter, expressions are derived for predicting the flexural and torsional rigidities of horizontally-curved waffle slab and composite structures before and after cracking. Some existing methods of calculating the rigidities are also presented and compared to the proposed formulae.
3.2 Assumptions

The use of an adequate number of radial ribs or diaphragms rigidly connected to the longitudinal girders of horizontally-curved wide flat structures is necessary in order for the structure to resist the large torsional moments. In order to assess the various orthotropic rigidities of such structures, the following assumptions are made:

(i) The number of ribs or diaphragms is large enough for the real structure to be replaced by an idealized one with continuous properties.

(ii) The neutral plane in each of the two orthogonal directions coincides with the center of gravity of the total section in the corresponding direction.

(iii) The area of the concrete deck slab is magnified by the factor $1/(1-\mu^2)$ to allow for the influence of Poisson's ratio, $\mu$.

3.3 Rigidities of Uncracked Sections of Curved Waffle Slab

3.3.1 Flexural Rigidities

Figure 3.1 shows a typical section of a curved waffle slab. In order to calculate the rigidities, it is necessary to consider the contributions of the concrete deck slab as well as the orthogonal ribs. Based on the aforementioned assumptions, the tangential and radial flexural rigidities $D_\theta$, $D_\tau$ as well as the coupling rigidities $D_1$, $D_2$ due to Poisson's effect of an uncracked section can be expressed as follows:

$$D_\theta = D + \frac{E_c h (e_y-h/2)^2}{1-\mu^2} + \frac{E_c J_1'}{S_0}$$
\( D_r = D + \left[ \frac{E_h (e_r - h/2)^2}{1 - \mu^2} \right] + \frac{E J'_r}{S_r} \)  

\( D_1 = \mu D'_0 \)

\( D_2 = \mu D'_r \)

in which \( D = \) flexural rigidity of the concrete deck slab with respect to its middle plane,

and \( = E_s h^3/12(1 - \mu^2); \)

\( E_s = \) modulus of elasticity of the concrete;

\( f'_s = \) 28-day concrete cylinder strength;

\( h = \) thickness of the concrete deck slab;

\( \mu = \) Poisson's ratio of concrete;

\( S_{0,0}, S_{0,2} = \) spacing of the longitudinal (transverse) ribs;

\( e_0, e_r = \) depth of the neutral plane from the top fiber for bending in the \( \theta(r) \) direction, i.e.

\[
\begin{align*}
e_0 &= \frac{\left[ b_0 d_0 \left( h + \frac{1}{2} d_0 \right) + (n-1) A_s \left( h + d_0 - d' \right) + \frac{1}{2} \frac{S_0 h^2}{(1 - \mu^2)} \right]}{\left[ b_0 d_0 + (n-1) A_s + \frac{S_0 h}{(1 - \mu^2)} \right]} \\
e_r &= \frac{\left[ b_r d_r \left( h + \frac{1}{2} d'_r \right) + (n-1) A'_s \left( h + d_r - d''_r \right) + \frac{1}{2} \frac{S_r h^2}{(1 - \mu^2)} \right]}{\left[ b_r d_r + (n-1) A'_s + \frac{S_r h}{(1 - \mu^2)} \right]} 
\end{align*}
\]

(3.2)
\[ I_0' = b_0d_0 \left[ \left( \frac{h}{2} + \frac{1}{2}d_0 \right) - e_0 \right]^2 + (n-1) A_s \left[ \left( h + d_0 - d' \right) - c_e \right]^2 + \frac{1}{12} b_0 d_0^3 \]

\[ I_r' = b_r d_r \left[ \left( \frac{h}{2} + \frac{1}{2}d_r \right) - e_r \right]^2 + (n-1) A_s' \left[ \left( h + d_r - d'' \right) - c_r \right]^2 + \frac{1}{12} b_r d_r^3 \]  

(3.3)

in which \( n = \) modular ratio = \( E/E_c \);

\( b_0 \) (or \( b_r \)) = width of longitudinal (transverse) rib;

\( d_0 \) (or \( d_r \)) = depth of the longitudinal (transverse) rib;

\( d' \) (or \( d'' \)) = concrete cover to the center of the longitudinal (transverse) reinforcements.

\( A_s \) (or \( A_s' \)) = area of tension steel in the longitudinal (transverse) ribs.

\( D_0' \) (or \( D_r' \)) = flexural rigidity of the concrete deck slab with respect to the neutral plane of the gross cross section associated with bending in the \( \theta \) (or \( r \)) direction; i.e.

\[ D_0' = D + \frac{E_c h \left( e_0 - \frac{h}{2} \right)^2}{1 - \mu^2} \]

\[ D_r' = D + \frac{E_c h \left( e_r - \frac{h}{2} \right)^2}{1 - \mu^2} \]  

(3.4)

It should be noted from equation 3.1 that the flexural rigidity of the concrete deck slab was calculated first about the slab's neutral axis then transferred to the neutral plane.
of the total cross section. The same procedure was followed in calculating the contributions of the ribs to the flexural rigidities, as shown in equation 3.3. Furthermore, the effect of Poisson's ratio was applied only to the concrete deck slab. This is unlike the method of Heins and Hails (1969) which applies this effect to the total cross section, leading to gross overestimation of the flexural rigidities. It should be remarked that this effect was completely neglected by Szilard (1974) and Giencke (1955).

3.3.2 Torsional Rigidity

The torsional rigidity of an uncracked section of a curved concrete waffle slab is estimated by the membrane analogy method (Timoshenko and Goodier 1970). The analysis herein accounts for the stiffening effect of orthogonal ribs on the torsional rigidity of the concrete deck slab. A typical tee-section of a waffle slab is shown in Fig. 3.2; the torsional constants for the rectangular sections 1, 2 and 3 are calculated and added to yield the torsional constant $J$ of the section.

Thus for a section normal to the $\theta$-axis,

$$J_\theta = J_1 + J_2 + J_3$$  \hspace{1cm} (3.5)

in which

$$J_1 = \frac{1}{2} K_1 S_\theta h^3$$

$$J_2 = K_1 d_\theta b_\theta^3$$
\[ J_3 = \frac{4}{\pi} K_1 (n-1) A^{'2} \]  
(3.6)

where, \( K_1 \) is the constant for a rectangular section in torsion (Timoshenko and Goodier 1970). The contribution of the reinforcing or prestressing steel to the torsional rigidity at the precracking stage is quite small, and hence \( J_3 \) can be ignored. It should be noted that the continuity of the concrete deck slab has been taken into consideration through the reduction factor of 1/2 for \( J_1 \) (Bali 1980).

To take into account the presence of the transverse rib, the torsional constant \( J_1 \) of the concrete deck slab in one direction should be magnified and modified as follows:

\[ (J_1) \text{ modified} = (J_1) \frac{(J_{S}+J_{W})}{J_{S}} \]  
(3.7)

in which \( J_{S} = \) the torsional constant of the concrete deck slab; and \( J_{W} = \) the torsional constant of the transverse rib.

Thus Eq.3.5 becomes

\[ J_\theta = (J_1) \text{ modified} + J_2 + J_3 \]  
(3.8)

The torsional rigidity \( D_{\theta r} \) is now given by:

\[ D_{\theta r} = \frac{G J_\theta}{S_\theta} \]  
(3.9)

Similarly, the torsional rigidity \( D_{\theta} \) of a section normal to the \( r \) axis is
\[ D_{r_0} = \frac{G J_r}{S_r} \]  

where \( G = \text{shear modulus} = \frac{E}{2(1+\mu)} \)

It should be noted that the present method accounts for the stiffening effect of the transverse ribs on the torsional rigidity by using a magnification factor shown in Eq. 3.7 while the method presented by Heins and Hails (1969) does not and consequently underestimates the torsional rigidity. The torsional rigidity proposed by Szilard (1974) takes into consideration the effect of the transverse ribs by averaging the individual torsional rigidities of the two orthogonal ribs and adding them to the torsional rigidity of the concrete deck slab. This overestimates the torsional rigidity. The method proposed by Rowe (1962) has the same deficiency as that of Heins and Hails (1969) in neglecting the stiffening effect of the orthogonal ribs.

### 3.4 Rigidities of Cracked Sections of Curved Waffle Slab

After cracking of the concrete section, the structure continues to behave elastically, provided that the stress in the steel is below the yield point and the compressive strength in the concrete does not exceed 0.5 \( f'_c \). These conditions prevail in practice and therefore used generally in the design of concrete structures. In addition to the assumptions made earlier and for simplicity, it is assumed that the tension cracks have propagated to the neutral axis (assumed to be in the concrete deck slab, which is generally the case). Furthermore, the concrete in the tension zone of the section is neglected in the flexural
analysis and design computation, and the reinforcing steel is assumed to resist the total tensile force. This leads to conservative results.

3.4.1 Flexural Rigidity

The rigidities are calculated based on the transformed section, consisting of the concrete and n times the area of the tension steel. Thus, the rigidities of the cracked section can be expressed as:

\[
D_0 = \frac{E_c}{S_0} [U_{S0} + \frac{I_{cb}}{(1-\mu^2)}]
\]

\[
D_r = \frac{E_c}{S_r} [U_{Sr} + \frac{I_{cr}}{(1-\mu^2)}]
\]

\[
D_1 = \mu \ D_0'
\]

\[
D_2 = \mu \ D_r'
\]

where \(I_{cb}\) and \(I_{cr}\) = moments of inertia of the concrete in compression about the neutral axis for bending in the \(\theta\) and \(r\) directions, respectively; or,

\[
I_{cb} = \frac{1}{3} \ S_0 \ (kd_0)^3
\]

\[
I_{cr} = \frac{1}{3} \ S_r \ (kd_r)^3
\]
\( I_{s\theta} \) and \( I_{sr} \) = moments of inertia of the transformed reinforcing steel about the neutral axis for bending in the \( \theta \) and \( r \) directions, respectively; or,

\[
I_{s\theta} = nA_s \left[ (h + d_\theta - d') - kd_\theta \right]^2
\]

\[
I_{sr} = nA_s' \left[ (h + d_r - d'') - kd_r \right]^2
\]

(3.13)

\( D'_\theta \) and \( D'_r \) = flexural rigidities of the concrete deck slab with respect to the neutral plane of the total cracked section associated with bending in the \( \theta \) and \( r \) directions, respectively.

The location of the neutral axis \( kd_\theta \) (or \( kd_r \)) is determined by equating the tension force to the compression force on the section. Assuming the neutral axis lies in the concrete deck slab, \( kd_\theta \) and \( kd_r \) are given by,

\[
nA_s \left[ (h + d_\theta - d') - kd_\theta \right] - \frac{S_\theta \left( kd_\theta \right)^2}{2 \left( 1 - \mu^2 \right)} = 0
\]

(3.14)

\[
nA_s' \left[ (h + d_r - d'') - kd_r \right] - \frac{S_r \left( kd_r \right)^2}{2 \left( 1 - \mu^2 \right)} = 0
\]

(3.15)

3.4.2 **Torsional Rigidity**

For a rib-stiffened slab construction, it can be shown (Rowe 1962) that,

\[
(D_{br} + D_{r0} + D_1 + D_2) = 2\kappa \left( D_0 D_r \right)^{1/2}
\]

(3.16)

or
\[
\kappa = \frac{1}{2} \frac{(D_{br} + D_{r\theta} + D_1 + D_2)}{(D_0 D_\rho)^{1/2}}
\]  

(3.17)

The coefficient \( \kappa \) is a torsional parameter with a lower limit of zero and an upper limit of one. Experimental studies conducted have shown that for this type of slab

\[
D_{br} = D_{r\theta}
\]  

(3.18)

Thus Eq. (3.17) reduces to,

\[
\kappa = \frac{1}{2} \frac{(2D_{br} + D_1 + D_2)}{(D_0 D_\rho)^{1/2}}
\]  

(3.19)

It is reasonable to assume that the coefficient \( \kappa \) remains the same before and after cracking of the concrete. Thus \( \kappa \) can be calculated from Eq. 3.19 using the pre-cracking flexural and torsional rigidities from Eqs. 3.1 and 3.10, respectively; hence,

\[
D_{br} = \kappa \frac{(D_0 D_\rho)^{1/2}}{} - \frac{1}{2} \left( \frac{D_1 + D_2}{2} \right)
\]  

(3.20)

Therefore, the post-cracking torsional rigidity \( D_{br} \) can be determined from Eq. 3.20 in which \( D_0, D_r, D_1 \) and \( D_2 \) are estimated from Eq. 3.11 for the post-cracking condition.
3.5 Rigidities of Horizontally-Curved Uncracked Composite Structures:

3.5.1 Flexural Rigidities

Although no experimental program was carried out to investigate the orthotropic rigidities of horizontally curved composite structures, the proposed equations can predict accurately the various rigidities of such structures, since the proposed equations are based on the same assumptions as those for waffle slabs. Figure 3.3 shows the plan and cross-section of a horizontally-curved composite structure with rigidly-connected diaphragms. This type of connection is capable of transferring both moment and shear between the main girders and the diaphragms. It has to be noted that the structure's curvature affects a change in the spacing of the radial diaphragms which has to be accounted for in calculating the rigidities. Based on the aforementioned assumptions the orthotropic rigidities of a curved composite structure with rigidly connected diaphragms and an uncracked concrete deck slab can be expressed as:

\[
D_0 = \frac{b_{c0}D + \frac{b_{c0}h}{(1 - \mu^2)} E_c \left( e_0 - \frac{h_0^2}{2} \right) + nE_c f_0'}{S_0} \tag{3.21}
\]

\[
D_1 = \mu D_0' = \mu \left( D_0 - \frac{nE_c f_0'}{S_0} \right) \tag{3.22}
\]

\[
D_r = \frac{b_{c0}D + \frac{b_{c0}h}{(1 - \mu^2)} E_c \left( e_r - \frac{h_r^2}{2} \right) + nE_c f_r'}{S_r} \tag{3.23}
\]
\[ D_2 = \mu D'_r = \mu \left( D_r - \frac{nE_c I'_r}{S_r} \right) \] (3.24)

in which

\[ e_\theta = \frac{nA_G \left( \frac{1}{2} d_\theta + h \right) + \frac{h b_{\theta \theta}}{(1 - \mu^2)} \frac{h}{2}}{nA_G + \frac{h b_{\theta \theta}}{(1 - \mu^2)}} \] (3.25)

\[ e_r = \frac{nA_D \left( \frac{1}{2} d_r + h \right) + \frac{h b_{rr}}{(1 - \mu^2)} \frac{h}{2}}{nA_D + \frac{h b_{rr}}{(1 - \mu^2)}} \] (3.26)

\[ I'_\theta = I_G + A_G \left( \frac{1}{2} d_\theta + h - e_\theta \right)^2 \] (3.27)

\[ I'_r = I_D + A_D \left( \frac{1}{2} d_r + h - e_r \right)^2 \] (3.28)

in which \( I_0 \) and \( I_D \) = moments of inertia of steel girder and diaphragm respectively, with respect to their individual neutral axes;

\( A_G, A_D = \) cross section area of steel girder and diaphragm, respectively.;

\( S_0, S_r = \) spacing of steel girders and diaphragms, respectively;

\( b_{\theta \theta} = \) effective width of the concrete deck slab in the longitudinal direction; and,

\( b_{rr} = \) effective width of the concrete deck slab in the transverse direction.
Here the following two cases should be considered: if $S_t$ is less than $S_0$ then $b_{oe} = \alpha S_t$, as required by the AASHTO (1983); if $S_t$ is equal or greater than $S_0$, then $b_{oe} = b_{oe}$. It should be noted that the steel diaphragms are also attached to the concrete deck slab with shear connection providing full interaction and thus better load distribution.

3.5.2 Torsional Rigidity

Based on the membrane analogy method proposed by Timoshenko and Goodier (1970), the torsional rigidity of a horizontally-curved composite structure is estimated assuming full interaction between the steel grid work and the concrete deck slab. Referring to Fig. 3.4, the torsional constant $J_\theta$ for a section normal to the $\theta$-axis can be expressed as:

$$J_\theta = J_1 + J_2$$  \hspace{1cm} (3.29)

in which,

$$J_1 = \frac{1}{6} \frac{S_0}{n} h^3$$

$$J_2 = K_1 (2 b_\theta t_{e_1}^3 + h_\theta t_{e_2}^3)$$  \hspace{1cm} (3.30)

Because of the transverse diaphragms, $J_1$ is modified as:

$$(J_1) \text{ modified} = (J_1) \left( \frac{J_S + J_P}{J_S} \right)$$  \hspace{1cm} (3.31)
in which $J_5$ = the torsional constant of the deck slab, and $J_D$ = the torsional constant of the transverse diaphragm. Thus,

$$J_0 = (J_1) \text{ modified} + J_2$$

(3.32)

Hence,

$$D_{0r} = \frac{G J_0}{S_0}$$

(3.33)

in which,

$$G = \frac{n E_c}{2 (1 + \mu)}$$

(3.34)

and $S_0$ = spacing between the longitudinal girders. Similarly,

$$D_{r0} = \frac{G J_r}{S_r}$$

(3.35)

3.6 Rigidities of Horizontally-Curved Cracked Composite Structures

For a continuous composite structure only transverse cracks are likely to develop at an interior support. Thus for a section at an interior support the rigidities $D_r$, $D_2$ and $D_{r0}$ can still be calculated from Eqs. 3.23, 3.24 and 3.35, respectively. The cracked rigidities ($D_0)_e$, ($D_1)_e$ and ($D_{0e})_e$ corresponding to the longitudinal axis of the bridge can be calculated as:
\[(D_9)_c = \frac{n E_c I'_{bc}}{S_9}\]  
\(\text{(3.36)}\)

\[(D_1)_c = 0.0\]  
\(\text{(3.37)}\)

\[(D_{0v})_c = \frac{G J_6}{S_9}\]  
\(\text{(3.38)}\)

with

\[J_6 = J_2\]  
\(\text{(3.39)}\)

in which

\[I'_{bc} = I_G + A_G \left(\frac{1}{2} d_8 + h - e_0\right)^2 + A_s (e_0 - d')^2 + A_s' (e_0 - h + d'')^2\]  
\(\text{(3.40)}\)

\[e_0 = \frac{n A_G \left(\frac{1}{2} d_8 + h\right) + n A_s d' + n A_s' (h - d'')}{n (A_G + A_s + A_s')}\]  
\(\text{(3.41)}\)

\(d'\) (\(d''\)) = distance from top (bottom) reinforcing steel of the concrete deck slab to the top (bottom) of the concrete deck slab; and, \(A_s (A_s') = \) area of longitudinal reinforcing steel at top (bottom) of the concrete deck slab. Experience has shown that the effect of the reinforcing steel in the concrete deck slab on the torsional rigidity \(D_9\) is relatively small and therefore can be ignored. In the above expression, it is assumed that the concrete in the deck slab has cracked and that the tension force is entirely carried by the reinforcing steel.
steel.

3.7 **Mathematical Formulation for the Experimental Study**

The relation between moments, curvatures and twist for a curved waffle slab can be approximated as (Lekhnitskii 1968):

\[
M_\theta = -(D_\theta W_{\theta \theta} + \mu D_{\theta} W_{rr})
\]

\[
M_r = -(D_r W_{rr} + \mu D_r W_{\theta \theta})
\]

\[
M_{\theta r} = D_{\theta r} W_{\theta r}
\]

(3.42)

where

\[
W_{\theta \theta} = \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2}
\]

\[
W_{rr} = \frac{\partial^2 W}{\partial r^2}
\]

\[
W_{\theta r} = \frac{1}{r} \frac{\partial^2 W}{\partial \theta \partial r} - \frac{1}{r^2} \frac{\partial W}{\partial \theta}
\]

(3.43)

in which \(M_\theta\) and \(M_r\) = moment vector in the \(\theta\) and \(r\) directions, respectively; \(M_{\theta r}\) = twisting moment vector in the \(\theta\)-\(r\) direction; \(W_{\theta \theta}\) and \(W_{rr}\) = curvatures associated with moments \(M_\theta\) and \(M_r\), respectively; \(W_{\theta r}\) = twisting curvature associated with the twisting moment \(M_{\theta r}\); and, \(W\) = lateral deflection in the \(z\)-direction. Thus, by applying a known
moment $M_0$ to a rectangular specimen, while $M_r = 0$, and measuring the resulting curvatures will yield two equations in four unknown rigidities. Subjecting another identical waffle specimen to a known moment $M_r$ with $M_0 = 0$, and measuring the resulting curvatures, will result in another two equations. Solving the four equations will yield the four flexural rigidities $D_0$, $D_r$, $D'_0$, and $D'_r$. To deduce the torsional rigidity $D_\theta$, a third specimen is subjected to a known twisting moment $M_\theta$ and the resulting curvature $W_\theta$ is measured. Then using Eq. 3.42 the torsional rigidity can be calculated readily.
CHAPTER IV

FINITE DIFFERENCE TECHNIQUE

4.1 General

The solution of plate problems by means of the classical methods is limited to relatively simple plate geometry, load and boundary conditions. If these conditions are more complex, as in the case of curved waffle slab bridges, the analysis becomes increasingly tedious and even impossible. In such cases numerical and approximate methods are the only approach that can be employed. Fortunately, numerical treatment of differential equations can yield approximate results, acceptable for most practical purposes. Among the numerical techniques presently available, the finite difference method is one of the most general. In applying this method, the derivatives in the general differential equation of the orthotropic plate are replaced by difference quantities at some selected points.

4.2 Equivalent Orthotropic Plate Theory

Orthotropic elements can be classified into two types. The first type belongs to
the elements which possess different physical properties in two perpendicular directions. Natural timber is a typical example of such a material. The second type belongs to those which have different rigidities in two perpendicular directions. The waffle slab system can be cited as an example of that case.

In dealing with the waffle slab system, the analysis may be simplified by using the equivalent orthotropic plate approach. In this approach, the waffle slab system is reduced to a statically equivalent system with uniformly distributed stiffnesses in both directions. Such an idealized orthotropic plate reflects the characteristic properties of the actual system.

4.3 Assumptions

It should be understood that the actual waffle slab system cannot be equivalent to the orthotropic plate in every aspect. Theoretical investigations and experimental data indicate that the equivalent orthotropic plate theory is applicable only under the following assumptions:

1- The ratios of stiffener spacing to plate boundary dimensions are small enough to insure approximate homogeneity of stiffness.

2- It is assumed that the rigidities are uniformly distributed in both directions.

3- Flexural and twisting rigidities do not depend on the boundary conditions of the plate or on the distribution of the vertical load.

4- Perfect bond exists between plate and stiffeners.

In dealing with curved concrete waffle slab bridges, it has to be noted that the
rigidity in the radial direction changes from one location to another, as mentioned in chapter III. In spite of this change in the orthotropic rigidities, the equivalent orthotropic plate theory can still be applied at these different locations. This approximate method will yield acceptable results for most practical purposes, as will be discussed later.

4.4 General Differential Equation

The analytical technique to be used herein is an extension to the work developed early by Heins and Hails (1969). The governing differential equation of equilibrium for an orthotropic plate in polar coordinates can be shown to be (Lekhnitskii 1968):

\[
D_r \left[ \frac{\partial^4 W}{\partial r^4} + \frac{2}{r} \frac{\partial^3 W}{\partial r^3} \right] + 2H \left[ \frac{1}{r^2} \frac{\partial^2 W}{\partial r^2 \partial \theta^2} - \frac{1}{r^3} \frac{\partial^2 W}{\partial r \partial \theta^2} + \frac{1}{r^4} \frac{\partial^2 W}{\partial \theta^2} \right] \\
+ \frac{1}{r^4} \frac{\partial^4 W}{\partial \theta^4} + \frac{1}{r^2} \frac{\partial^2 W}{\partial r^2} + \frac{2}{r^4} \frac{\partial^2 W}{\partial \theta^2} + \frac{1}{r^3} \frac{\partial W}{\partial r} = q(r, \theta) \tag{4.1}
\]

where

\( W \) = transverse displacement at point \((r, \theta)\)

\( 2H = D_1 + D_2 + D_\theta + D_\phi \) = effective torsional rigidity;

\( D_\theta, D_r \) = flexural rigidities in the tangential and radial directions, respectively;

\( D_1, D_2 \) = coupling rigidities due to Poisson's ratio;

\( D_\theta, D_\phi \) = torsional rigidities; and,

\( q(r, \theta) \) = intensity of load.

Equation 4.1 describes the load-deformation relationship of an angular plate element, subjected to transverse load, resisted by generalized forces. These forces, shown
in Fig. 4.1, are related to the displacement $W$ by the following equations (Lekhnitskii 1968):

\[ M_\theta = -[D_r \left( \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2} \right) + D_1 \frac{\partial^2 W}{\partial r^2}] \quad (4.2) \]

\[ M_r = -[D_r \frac{\partial^2 W}{\partial r^2} + D_2 \left( \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta^2} \right)] \quad (4.3) \]

\[ M_{r\theta} = D_{r\theta} \left( \frac{1}{r} \frac{\partial^2 W}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial W}{\partial \theta} \right) \quad (4.4) \]

\[ V_r = -[D_r \left( \frac{\partial^3 W}{\partial r^3} + \frac{1}{r} \frac{\partial^3 W}{\partial r^2} \right) + H \left( \frac{1}{r^2} \frac{\partial^3 W}{\partial r \partial \theta^2} - \frac{1}{r^3} \frac{\partial^2 W}{\partial \theta^2} \right) \]
\[ - D_\theta \left( \frac{1}{r^2} \frac{\partial W}{\partial r} + \frac{1}{r^3} \frac{\partial^3 W}{\partial \theta^3} \right) ] \quad (4.5) \]

\[ V_\theta = -[H \left( \frac{1}{r} \frac{\partial^3 W}{\partial r^2 \partial \theta} \right) + D_\theta \left( \frac{1}{r^2} \frac{\partial^3 W}{\partial \theta \partial r} + \frac{1}{r^3} \frac{\partial^3 W}{\partial \theta^3} \right) ] \quad (4.6) \]

and the reactions are as follows:

\[ R_r = -[D_r \left( \frac{\partial^3 W}{\partial r^3} + \frac{1}{r} \frac{\partial^3 W}{\partial r^2} \right) + (H + D_\theta) \left( \frac{1}{r^2} \frac{\partial^3 W}{\partial r \partial \theta^2} - \frac{1}{r^3} \frac{\partial^2 W}{\partial \theta^2} \right) \]
\[ - D_\theta \left( \frac{1}{r^2} \frac{\partial W}{\partial r} + \frac{1}{r^3} \frac{\partial^3 W}{\partial \theta^3} \right) ] \quad (4.7) \]
\[
R_\theta = - [D_\theta \left( \frac{1}{r^2} \frac{\partial^2 W}{\partial \theta \partial r} + \frac{1}{r^3} \frac{\partial^3 W}{\partial \theta^3} \right) + (H + D_{r\theta}) \left( \frac{1}{r} \frac{\partial^3 W}{\partial r^2 \partial \theta} \right) \\
+ 2D_{r\theta} \left( - \frac{1}{r^2} \frac{\partial^2 W}{\partial r \partial \theta} + \frac{1}{r^3} \frac{\partial^3 W}{\partial \theta^3} \right) ]
\]

(4.8)

where

\[r = \text{Radius to the differential element}\]
\[\theta = \text{Angle to the differential element}\]
\[M_r = \text{Bending moment in the radial direction}\]
\[M_\theta = \text{Bending moment in the tangential direction}\]
\[M_{r\theta} = \text{Torsional moment}\]
\[V_r = \text{Shear acting on the face normal to the radius}\]
\[V_\theta = \text{Shear acting on the face normal to the tangent}\]
\[R_r = \text{Reaction acting on the face normal to the radius}\]
\[R_\theta = \text{Reaction acting on the face normal to the tangent}\]

4.5 Finite Difference Solution

To solve the orthotropic plate equation, Eq. 4.1, the actual deflection surface \(W(r, \theta)\) of the plate is assumed to have the following form:

\[
\bar{W} = Ar^2 + Br + C + Dr\theta + Er\theta^2
\]

(4.9)

such that at each location \((r, \theta)\), \(\bar{W}\) is identical to the actual deflection \(W\). Imposing these conditions at the grid points o, r, l, a and b shown in Fig. 4.2, and solving for the
arbitrary constants A, B, C, D, E in equation 4.9, yield the following equation:

\[
\bar{W} = \frac{(W_l - 2W_o + W_r)}{2\lambda^2} r^2 + \frac{[- W_l (2r + \lambda) + 4rW_o - W_r (2r - \lambda)]}{2\lambda^2} r
\]

\[
+ \frac{[W_l (\lambda + r) r + 2W_o (\lambda^2 - r^2) + W_r (-\lambda + r) r]}{2\lambda^2} + \frac{(W_a - W_b)}{2\theta_o} \theta
\]

\[
+ \frac{(W_a - 2W_o + W_r)}{2\theta_o^2} \theta^2
\]

(4.10)

in which

\[\lambda = \text{Radial spacing between the mesh points}\]

\[\theta_o = \text{Angular spacing between the mesh points (radians)}\]

Eq. 4.10 now represents the surface of the plate. In order to use this equation in solving the differential Eq. 4.1, partial derivatives of Eq. 4.10 must be evaluated and substituted into Eq. 4.1. This will yield the following general orthotropic plate finite difference equation:

\[
W_o \left[6r^4\theta_o^4 + 12\beta r^2\lambda^2\theta_o^2 + 6\alpha \lambda^4 + 2\alpha r^2\lambda^2\theta_o^4 - 4(\alpha + \beta)\theta_o^2\lambda^4\right]
\]

\[+(W_a + W_o) \left[-4\beta r^2\lambda^2\theta_o^2 - 4\alpha \lambda^4 + 2(\alpha + \beta)\theta_o^2\lambda^4\right]
\]

\[+ W_r (-4r^4\theta_o^4 - 4\beta r^2\lambda^2\theta_o^2 - 2\lambda r^3\theta_o^4 + 2\beta \lambda^3\theta_o^2 r - \alpha r^2\lambda^2\theta_o^4
\]

\[+ \frac{\alpha}{2}\theta_o^4 \lambda^3) + W_l (-4r^4\theta_o^4 - 4\beta r^2\lambda^2\theta_o^2 + 2\lambda r^3\theta_o^4 - 2\beta \lambda^3\theta_o^2 r
\]

\[-\alpha r^2\lambda^2\theta_o^4 - \frac{\alpha}{2}\theta_o^4 \lambda^3) + W_{rr} (r^4\theta_o^4 + \lambda r^3\theta_o^4)
\]
\[ + W_a (r^4 \theta_o^4 - \lambda r^3 \theta_o^3) + W_a (2 \beta r^2 \lambda^2 \theta_o^2 + \beta \lambda^3 \theta_o^2 r) \\
+ W_{ar} (2 \beta r^2 \lambda^2 \theta_o^2 - \beta \lambda^3 \theta_o^2 r) + W_{br} (2 \beta r^2 \lambda^2 \theta_o^2 + \beta \lambda^3 \theta_o^2 r) \\
+ W_{br} (2 \beta r^2 \lambda^2 \theta_o^2 - \beta \lambda^3 \theta_o^2 r) + (W_{aa} + W_{bb}) (\alpha \lambda^4) = q \frac{\lambda^4 r^4 \theta_o^4}{D_r} \]

(4.11)

in which

\[ \alpha = \frac{D_o}{D_r} \]

\[ \beta = \frac{D_o}{D_r} \]

Equation 4.11 is the final solution of the differential plate equation, Eq. 4.1, in finite difference form. The moments Eqs. 4.2 and 4.3, and reaction Eq. 4.7, at an interior point \( o \), may also be expressed in finite difference form, assuming Poisson's ratio equal zero for simplicity, as follows:

\[ (M_o) = -\frac{D_r}{\lambda^2} (W_i - 2W_o + W_o) \]

(4.12)

\[ (M_o) = -\frac{D_o}{r^2 \theta_o^2} [(W_o - 2W_o + W_o) + (W_o - W_o) \frac{r \theta_o^2}{2 \lambda}] \]

(4.13)
\[ \begin{align*}
(R_{ro})_o = & -D_r \left[ (W_{rr} - 2W_r + 2W_l - W_d) \frac{1}{2\lambda^2} + (W_l - 2W_o + W_r) \frac{1}{\lambda^2r} \\
& + 2\beta \left( (W_{al} - W_{dl} - 2W_r + 2W_i + W_{br} - W_{bl}) \frac{1}{2r^2\theta_o^2\lambda} \\
& - (W_a - 2W_o + W_b) \frac{1}{r^2\theta_o^2} \right) - \alpha \left( (W_r - W_i) \frac{1}{2r^2\lambda} \\
& + (W_a - 2W_o + W_b) \frac{1}{r^2\theta_o^2} \right) \right] 
\end{align*} \]

(4.14)

4.6 Mathematical Model

To apply the previously derived finite difference equations, a specific type of plate and specified boundary conditions must be defined. A simply supported waffle slab, with free supports at the inner and outer edges, as shown in Fig. 4.3 is to be analyzed herein. With the model specified, the general finite difference equation, Eq. 4.11, can be modified to incorporate the specified boundary conditions. These boundary conditions are as follows:

\[ \begin{align*}
M_r = R_r = 0 & \quad \text{at } r = R , \ r = R + W_s \\
W = M_o = 0 & \quad \text{at } \theta = 0 , \ \theta = \theta_r 
\end{align*} \]

(4.15)

where \( R = \) minimum radius, \( W_s = \) width of the slab

Applying these conditions results in fifteen new plate equations. Figure 4.4 describes the locations of these various finite difference expressions. The fifteen conditions which are necessary for a complete solution are given in detail by Heins and Looney (1967).
4.7 Computer Program

Utilizing the finite difference equations, the mathematical model can be constructed such that it will represent a physical bridge structure which is to be analyzed. The bridge deck is divided into a polar grid mesh pattern consisting of a minimum and maximum radius, a total enclosed angle, and subdivisions of these radii and angles which are listed as \( \lambda \) and \( \theta_0 \). At each intersection of the grid, there are three rigidities as well as any applied load. Thus, if the grid system is fine enough, it can be visualized that any loading can be described and any change in the rigidities can be incorporated into the analysis. The refinement of the grid system is necessary to study the convergence of the solution. However, for each additional point on the grid a simultaneous equation must be written. Therefore, the accuracy of the results as well as the execution time will be the limiting factor for this refinement.

The generation and the solution of the finite difference equations are accomplished by a computer program. The computer program was first developed by Heins and Looney (1967), while the current program has the additional capability of taking into consideration the change in the flexural and torsional rigidities of the curved waffle slab structure.

The geometries and rigidities are the input parameters of the program, along with the desired mesh grid and the particular cases of loading. The program then generates a set of simultaneous equations, one for every point on the grid, incorporating the effects of the boundary conditions. These equations are set up in a matrix form and solved internally using the Gaussian reduction method. The solution is a set of deflections prescribing the deformed shape of the bridge model. The computer program list as well
as the input parameters are given in appendix A-1.

4.8 Parametric Study

In order to study the effect of changing the plate rigidities on the behaviour of curved concrete waffle slab bridges, four types of slabs were studied under the effect of a concentrated load applied at mid-span. The four types of slabs studied were:

I  curved concrete waffle slab.

II curved concrete solid slab having the same volume of concrete as in the waffle slab.

III curved concrete solid slab having the same torsional rigidity as that of the waffle slab.

IV curved concrete solid slab having the same flexural rigidities as those of the waffle slab.
5.1 General

The finite element method was developed simultaneously with the increasing use of high speed digital computers. The method has become a reliable tool in determining the response of a structure during the linear and nonlinear stages of loading. Also, it has the capability of providing a theoretical simulation of both the reinforced and prestressed concrete structural components.

In this chapter, a general description of the finite element technique used in the analysis is presented. The theoretical modelling of the concrete, reinforcement, and prestressing is explained. A general description of the finite element program 'ABAQUS' (1989) used throughout this analysis is presented. The theoretical modelling of curved reinforced and prestressed concrete waffle slab as well as girder-slab bridge models is established.
5.2 Finite Element Analysis

The basis of the finite element method is the discretization of the structure into an equivalent system of finite elements. Each finite element is interconnected with the adjacent elements by nodal points. Simple functions are chosen to approximate the distributions of the actual displacements over each finite element. Hence, the basic approximation of the finite element method is introduced at this stage. The stiffness matrix consists of the coefficients of the equilibrium equations derived from the material and geometric properties of an element and obtained by use of the principle of minimum potential energy. The equilibrium relation between the stiffness matrix, nodal force vector, and nodal displacement vector is expressed in a set of simultaneous equations. Applying the loading and boundary conditions for the structural problem, the assembled set of equations can be solved for the unknowns. These values are substituted back to each element formulation to provide the distributions of displacement and stress everywhere within each element.

The basic formulation of the finite element method relating the global displacements and loads, in a matrix form, can be expressed as:

\[ [K][U] = [P] \] (5.1)

Where

\([K]\) = structure stiffness matrix;

\([U]\) = displacement vector at the nodes; and

\([P]\) = applied loads vector at the nodes. The structure stiffness matrix \([K]\) is the assemblage of the element stiffness matrices \([K]\), given by:
\[ [K] = \Sigma[K]_i \] (5.2)

Where

\[ [K]_i = \text{element i stiffness matrix} \]

and

\[ [K]_i = \int [B]^T[E][B]d\nu \] (5.3)

in which the integration in Eq. 5.3 is performed over the volume \( \nu \) and;

\[ [B] = \text{strain displacement matrix}; \text{ and} \]

\[ [E] = \text{elasticity matrix}. \]

In a linear problem, loads are applied to a structure and the response can be obtained directly. Non-linear finite element problems are usually solved by taking several linear steps. Many solution procedures have been proposed to solve non-linear problems. ABAQUS (1989) uses the well known Newton's method as a numerical technique for solving the non-linear equilibrium equations.

The non-linear solution of the problem is obtained iteratively by solving a series of linear problems. Let \([P_o]\) and \([U_o]\) be the initial load and displacement, and \([K_o]\) is the tangent stiffness at \([U_o]\), \([P_o]\). For the \(i^{th}\) cycle of the iteration process, the necessary load is determined by

\[ [P_i] = [P] - [P_{i-1}] \] (5.4)

where \([P]\) is the total load to be applied and \([P_{i+1}]\) is the load equilibrated after the previous step.
An increment to the displacement is computed during the $i^{th}$ step by the relation

$$[K_i][\Delta U_i] = [P_i]$$ \hspace{1cm} (5.5)

and, the total displacement after the $i^{th}$ iteration is computed from

$$[U] = [U_0] + \sum_{j=1}^{i}[\Delta U_j]$$ \hspace{1cm} (5.6)

the procedure is repeated until the increments of displacements or the unbalanced forces become null or sufficiently close to null according to some preselected criterion.

Two methods can be used to compute the stiffness matrix $[K_i]$. One method is the tangent stiffness matrix at the end of the previous iterative step, that is, the slope of the $[P]-[U]$ curve at the point $[P_{i+1}], [U_{i+1}]$

$$[K_i] = [K_{i-1}]$$ \hspace{1cm} (5.7)

Instead of computing a different stiffness for each iteration, a modified iterative technique has been employed which utilizes only the initial stiffness $[K_0]$. The basic and modified iterative procedures are shown in Figs. 5.1 and 5.2.

**5.3 Scope of Analysis**

In this work, the goal of using the finite element method is to study the elastic and post-elastic behaviour of curved reinforced and prestressed concrete waffle slab bridges. The influence of the transverse ribs on the behaviour of horizontally curved concrete bridges was investigated by analyzing curved reinforced and prestressed concrete girder-slab bridge models. The finite element method was also used to predict the most
probable yield-line patterns of failure, and consequently the collapse load of curved reinforced and prestressed concrete waffle slab bridges.

The overall objectives of the theoretical analysis using the finite element technique can be summarized as follow:

1- To study the elastic and post-elastic behaviour of curved reinforced concrete waffle slab bridges.

2- To examine the elastic and post-elastic behaviour of curved reinforced concrete girder-slab bridges.

3- To investigate the response of curved reinforced concrete solid slab bridges, having the same volume of concrete and amount of reinforcement as the waffle slab bridge.

4- To study the response of both curved prestressed concrete waffle slab and girder-slab bridges due to the effect of prestressing.

5- To examine the elastic and post-elastic behaviour of curved prestressed concrete waffle slab and girder-slab bridges.

6- To predict the most expected yield-line patterns of failure and hence, the maximum capacity of curved concrete waffle slab bridges.

In addition to the aforementioned objectives, the finite element analysis was used to carry out a parametric study on curved waffle slab and girder-slab bridges to examine the influence of the bridge horizontal radius to width ratio, aspect ratio and applied load locations on the behaviour of the two systems, as well as the collapse load of waffle slab bridges.
5.4 The Finite Element Program 'ABAQUS'

The commercial finite element program used in the analysis ABAQUS (Hibbit et al. 1989) is a world-wide program used to estimate both linear and non-linear responses of different types of structures. ABAQUS program is well organized and quality assurance tested and is continuously being enhanced with additional capabilities. ABAQUS is a batch program and therefore, a data deck which describes the problem has to be prepared and provided to the program. A data deck for ABAQUS contains model data and history data. Model data defines a finite element model: the elements, nodes, element properties, material definitions and nodal constraints, while history data defines what happens to the model, the sequence of loadings for which the model's response is sought. This history is divided into a sequence of steps. Each step is a period of response of a particular type such as static loading or dynamic response. The definition of a step includes the procedure type, the control parameters for the non-linear solution procedures, the loading and the output request.

ABAQUS also has many types of elements, from which the shell and beam elements were used in the analytical study presented herein. Two different nodal constraints were used:

1- Boundary constraints: in which a specific boundary condition is defined for the nodes, and

2- Multi-point constraints: which allows constraints to be imposed between different degrees of freedoms of the model.

All data definitions in ABAQUS are accomplished with option blocks-sets of data
describing a part of the problem definition. The user chooses those options that are relevant for a particular application.

5.5 Finite Element Model

The finite element mesh is usually chosen based on a convergence study as well as on the required execution time. In the following sections, the element types used in the analyses, material modelling and the non-linearity control in the program are presented.

5.5.1 Element Types

Among the many types of elements available in the ABAQUS finite element program directory, it was decided to use two elements after testing several element types in the pilot runs. These were: the four-node shell element (S4R) with six degree of freedom \( (U_1, U_2, U_3, \phi_1, \phi_2, \phi_3) \) assigned to each node, and the three dimensional two-node beam element (B31H) with six degrees of freedom \( (U_1, U_2, U_3, \phi_1, \phi_2, \phi_3) \) at each node. Figure 5.3 and 5.4 show the characteristics of the shell and beam finite elements used in the analysis, respectively.

5.5.2 Reinforced Concrete Modelling

The reinforced concrete model in 'ABAQUS' can be accomplished by combining the CONCRETE option, which is used to define the properties of plain concrete outside the elastic range, with the REBAR option, defining the properties and locations of
reinforcement bars in the concrete elements. Rebars are defined singly or embedded in oriented surfaces, that use a one dimensional strain theory, and which may be used to model the reinforcing itself. These elements are superposed on the mesh of plain concrete elements, and are used with standard metal plasticity models that describe the behaviour of the rebar material. Effects associated with the rebar/concrete interface, such as bond slip and dowel action, may be modelled approximately by introducing some "tension stiffening" to simulate load transfer across cracks through the rebar. The local energy release and the concrete-rebar interaction that occur as the concrete begins to crack are of major importance in determining the response of such a structure. ABAQUS offers an indirect technique for modelling these effect by means of the 'TENSION STIFFENING' option. This option models the energy release as cracking propagates across the material, and the associated concrete-rebar interaction, as a gradual loss of strength beyond the failure strain, shown in Fig. 5.5.

When the principal stress components are dominantly compressive, the response of the concrete is modelled by elastic-plastic theory, using a simple form of yield surface written in terms of the equivalent pressure stress and the Mises equivalent deviatoric stress. When the concrete is loaded in compression it initially exhibits elastic response. As the stress is increased some non-recoverable (inelastic) straining occurs, and the response of the material softens. An ultimate stress is reached after which the material loses strength until it can no longer carry any stress. If the load is removed at some point after inelastic straining has occurred, the unloading response is softer than the initial elastic response, but this effect is ignored in the model. When a uniaxial concrete
specimen is loaded in tension it responds elastically until, at a stress that is typically 7% to 10% of the ultimate compressive stress, cracks form so quickly that, it is very difficult to observe the actual behaviour. ABAQUS models the cracking as strain softening, the softening rate being dependent on the size of elements in the cracked region.

The cracking and compression responses of concrete that are incorporated in the model are illustrated by the uniaxial response of a specimen shown in Fig. 5.6. In multiaxial stress states these observations may be generalized through the concept of surfaces of failure. These surfaces are fitted to experimental data. The surfaces used are shown in Fig. 5.7.

5.5.3 Prestressing Modelling

ABAQUS provides a simulation for post-tensioning of the existing rebars in the finite element mesh through the INITIAL CONDITIONS and PRESTRESS HOLD options. INITIAL CONDITIONS option is used to prescribe the prestress in the rebars. Due to this prestressing the initial stress state may not be an equilibrium state for the finite element model. Therefore an initial STATIC step, with no external loads applied, is included in the analysis to allow the program to check equilibrium and, if necessary, iterate to achieve equilibrium. During this step the prestress in the rebar may be allowed to change according to the straining of the structure as the self-equilibrating stress state establishes itself. Alternatively, by using the PRESTRESS HOLD option, the stress in the rebar may be kept constant during this equilibrium solution, as though the rebar were sliding through the concrete.
5.5.4 Non-Linear Analysis in ABAQUS

Non-linear static analysis requires the solution of non-linear equilibrium equations, and as mentioned before, the program uses Newton's method for this purpose. The solution usually is obtained as a series of increments, with iteration within each increment to obtain equilibrium. Increments must sometimes be kept small, but most commonly the choice of increment size is a matter of computational efficiency. If the increments are too large, more iteration will be required. ABAQUS has the capability of providing an automatic incrementation scheme which selects the increment sizes based on the convergence of the iteration process of each increment, until the specified load tolerances are achieved. Very tight tolerances will cost more iterations in the non-linear solution scheme, while loose tolerances will give incorrect answers, because the structure will not be in equilibrium. If the number of iterations exceeds the maximum allowed, the increment size is reduced. If this resulted in a smaller increment than was specified as a minimum in the input, the run is terminated. The user may define a maximum value of the load or specify a certain amount of displacement beyond which the solution is not of interest. If neither of the finishing conditions described above is specified, the analysis will continue for the number of increments defined at the beginning of the step.

5.6 Analysis of Curved Reinforced Concrete Waffle Slab Bridge Model

This analysis was aimed to study the elastic as well as the post-elastic response of curved reinforced concrete waffle slab bridge. Two meshes for the tested RIVS model were tested in the pilot runs, with different number of elements and element types. In the
first mesh, the concrete deck slab as well as the longitudinal and transverse ribs were modelled using the four-node shell element (S4R) with six degrees of freedom at each node, as shown in Fig. 5.8, while, in the second mesh the four-node shell element (S4R) was used to model the concrete deck slab and the three dimensional two-node beam element (B31H), with six degrees of freedom assigned to each node, was used to model the longitudinal as well as the transverse ribs, as shown in Fig. 5.9.

Multi point constraints (MPC) option was used to allow constraints between the different degrees of freedom of the shell nodes and the beam nodes. In another word, MPC option ensures full interaction between the deck slab and the interacting ribs. The definition of the MPC option used herein in the analysis is shown in Fig. 5.10. Both meshes give almost the same results; however, it was decided to use the second mesh due to the reduction in the number of elements, and consequently the execution time. In addition to the aforementioned advantages, the beam element in the second mesh simplifies the modelling of the reinforcement and of the prestressing wires as well.

ABAQUS has the capability of providing generation of nodes on a curved path. Thus, knowing the first and the last node on the path as well as the center of curvature, the nodes in between can be generated easily which save time and effort as well. The TRANSFORM option in ABAQUS was used which provides the boundary input and the nodes output in the radial and tangential direction respectively. Also, the ORIENTATION option was used to transform the element outputs in the radial and tangential direction respectively. Since this study was restricted to simply supported curved bridges, only the vertical displacement, W, was restricted for all nodal points on the support lines.
The curved reinforced concrete girder-slab bridge was analyzed to study the influence of the absence of transverse ribs on the behaviour of curved bridges. The same mesh of RWS model was used in the analysis but discarding the transverse ribs, as shown in Fig. 5.11.

Finally, the curved reinforced concrete solid slab, having the same volume of concrete and the amount of reinforcement as the waffle slab, was analyzed and compared with the waffle slab system. A typical mesh for this model is shown in Fig. 5.12.

5.7 Analysis of Curved Prestressed Concrete Bridge Models

The curved prestressed concrete waffle slab as well as the girder-slab bridge models were modelled, as shown in Figs. 5.13 and 5.14, using the prescribed shell and beam elements. Since the prestressing forces are transferred to the concrete at the end zones, a finer mesh at the boundaries of the model was necessary to model the distribution of the stresses. The finite element mesh of the RWS model, but with refinement at the ends, was used in the analysis of the PWS model. The prestressing in the wires was modelled, as indicated before, by applying an initial stress to the rebars in the beam elements using the INITIAL CONDITIONS option as well as the PRESTRESS HOLD option. Again, the same mesh of PWS model was used in the analysis of PGS model but discarding the transverse ribs, as shown in Fig. 5.14.

5.8 Parametric Study

In addition to the analytical work carried out on the previously mentioned curved
slab models, the ABAQUS computer program was used to conduct a parametric study. This parametric study was carried out by varying a number of factors which could influence the responses of curved waffle slab and girder-slab bridges. Such factors were: (i) radius to width ratio, (ii) the bridge aspect ratio, and (iii) locations of the load. The goals of the parametric study were:

1- To investigate the influence of the aforementioned factors on the elastic and post-elastic behaviour of curved waffle slab as well as girder-slab bridges.

2- To use the collapse load and the failure pattern results obtained in (1) to develop collapse load expressions for curved waffle slab bridges under different loading conditions using the yield-line theory.

3- To investigate the influence of the transverse ribs on the behaviour of curved as well as straight right bridges, having the same span length as the curved bridge.

The overall layout of the parametric study is shown in Figs. 5.15 and 5.16.
CHAPTER VI

YIELD LINE APPROACH

6.1 General

The yield line theory (Johansen 1962) has been used successfully to predict the ultimate load capacity of concrete slab bridges. It can be used for complex slab geometries, as the case of curved concrete waffle slab bridges, with some mathematical effort. The ultimate load method of analysis is extremely useful in finding the true safety factor and the failure pattern of the structure under critical loading conditions. It is well known (Jones and Wood 1967) that an analysis based on the yield line approach would lead to an upper bound solution to the collapse load. However, it is shown later that the results obtained by this approach are very close to the test results. This can be attributed mainly to neglecting some enhancing factors, such as strain hardening of steel and tension stiffening of concrete, resulting in a collapse load which, in practice, is lower than the actual collapse load.

The ultimate load analysis of the curved concrete waffle slab bridges, studied herein, was based on assumed yield line patterns of failure. The justifications of these
patterns of failure were: laboratory test results, previous experience and the results from an extensive parametric study using the finite element method.

6.2 Assumptions

In the yield line analysis the following assumptions were made to assist in the prediction of the yield line pattern:

1- Yield lines terminate at the slab boundaries;
2- Yield lines are straight;
3- Yield lines pass through the intersection of the axes of rotation;
4- Axes of rotation generally lie along lines of supports; and
5- Shear, punching and bond failure are precluded.

6.3 Yield Line Analysis (Virtual Work Method)

The first step in the yield line analysis is to postulate a yield line pattern. Once the failure pattern has been postulated, either the virtual work or the equilibrium method can be used. In this study the virtual work method has been adopted and used to predict the collapse load of curved concrete waffle slab bridges.

When the yield line pattern is formed, only an infinitesimal increase in the load is required to cause the structure to collapse. By the principle of virtual work, the ultimate collapse load is derived by equating the external and internal virtual work, \( W_e \) and \( W_i \), respectively. The external virtual work is generated by the work done by the external load on the structure at collapse, \( P_u \), as well as by the uniformly-distributed dead
load, \( q_d \), through virtual displacement \( \delta \). The internal virtual work is done by the ultimate moments of resistance per unit width, \( m_i \), along a yield line of length \( l \). Thus, from the principle of virtual work:

\[
W_e = W_i
\]

or

\[
\sum \int \int (q_d \, dx \, dy) \, \delta = \sum m_i \, \theta_i \, l \quad (6.1)
\]

where

\( q_d = \) Load per unit area at point \((x, y)\) over the slab; and,

\( \theta_i = \) The rotation of the slab normal to the yield line.

**x.4 Collapse Load of Curved Waffle Slab Bridges**

**6.4.1 General**

In the formulation of the collapse load, the internal work done included the following contributions:

1- Positive moment contribution;

2- Negative moment contribution; and

3- Torsional moment contribution.

The above contributions were considered in the longitudinal and transverse directions, as applicable. The external work done included the following contributions:

1- The applied external load; and

2- The structure's own weight.
In this study, several cases of loading were considered such as two concentrated loads each at the center of each lane, a concentrated load at the center of the outer edge, a concentrated load at the center of the inner edge, as well as a uniformly-distributed load. The study also included the predictions of the most probable yield line pattern of failure, and consequently the collapse load, for both relatively wide and relatively long span curved waffle slab bridges. These predictions are based on a parametric study using the finite element method and the test results on curved concrete waffle slab bridge models.

6.4.2 Curved Waffle Slab Bridge Under Concentrated Load at Center of Each Lane

Fig. 6.1 shows a simply supported horizontally curved waffle slab bridge with relatively weak edge beams at the inner and outer edges. Observations of the crack pattern of the tested models, RWS and PWS, show a single line failure crack pattern (f-f'). At collapse, it is assumed that the failure line deflects downward a distance δ at the inner edge and a distance δ (R + W_s)/R at the outer edge, in which:

\[ R = \text{Radius of curvature of the inner edge, and } W_s = \text{Width of the slab.} \]

When \( R \to \infty \), \( \frac{(R + W_s)}{R} = 1.0 \), and consequently \( (\delta)_{inner} = (\delta)_{outer} \), which is the case for a rectangular slab under the same load. This supports the validity of the assumption adopted for any value of \( R \), provided that \( R \) is greater than zero.
i- **External Work**

1- **Due to Own-Weight**

If one assumes a finite element dr × rdθ at a distance r from the center of curvature as shown in Fig. 6.1, then the external work due to own-weight can be as follows:

\[
W_{e1} = 2q_D \int_0^{\theta_T} \int_R^{R+W_s} r \, dr \, d\theta \, \delta \, \frac{r}{R} \frac{\theta}{\theta_T} \frac{1}{2}
\]

hence,

\[
W_{e1} = \frac{q_D \theta_T}{6R} \left( W_s^3 + 3RW_s^2 + 3R^2W_s \right) \delta \tag{6.2}
\]

in which \( q_D \) = Own weight of the bridge per unit area, and \( \theta_T \) = Connecting angle (radians).

2 **Due to Concentrated Load at Center of Each Lane**

Referring to Fig. 6.1, it can be shown that

\[
W_{e2} = \frac{P_U}{2} \delta \frac{(R + W_s/4)}{R} + \frac{P_U}{2} \delta \frac{(R + 3W_s/4)}{R}
\]

which may be re-expressed as:

\[
W_{e2} = P_U \delta \frac{(R + W_s/2)}{R} \tag{6.3}
\]
The summation of Eq. 6.2 and 6.3 will yield the total external work as follows:

\[ W_e = W_{e1} + W_{e2} \]

\[ W_e = \left[ P_U \frac{(R + W_s/2)}{R} + \frac{a_D \theta_T}{6R} (W_s^3 + 3RW_s^2 + 3R^2W_s) \right] \delta \]  \hspace{1cm} (6.4)

ii Internal Work

1- Slab

Assuming a finite strip \( dr \) at a distance \( r \) from the center of curvature as shown in Fig. 6.1, then:

\[ W_{l1} = 2m_1 \int_0^R \frac{dr}{r} \frac{\delta}{R} \frac{1}{r \theta_T/2} \]

hence,

\[ W_{l1} = 4m_1 \frac{\delta}{\theta_T R} W_s \]  \hspace{1cm} (6.5)

in which \( m_1 = \) Ultimate positive moment of resistance per unite width about radial axis (r).

2- Edge Beams

From Fig. 6.1
\[ W_{t_2} = 2M_B \delta \frac{(R + W_s)}{R} \frac{1}{(R + W_s) \theta_r/2} + 2M_B \delta \frac{1}{R \theta_r/2} \]

or,

\[ W_{t_2} = \frac{8M_B}{\theta_r R} \delta \] (6.6)

The summation of Eqs. 6.5 and 6.6 will yield the total internal work as follow:

\[ W_I = W_{t_1} + W_{t_2} \]

or,

\[ W_I = \left( \frac{4m_1 W_t}{R \theta_r} + \frac{8M_B}{R \theta_r} \right) \delta \] (6.7)

in which \( M_B = \) Ultimate moment of resistance of the edge beam.

Equating the external work, Eq. 6.4, to the internal work, Eq. 6.7, and rearranging the terms will yield the following expression for the ultimate collapse load \( P_u \). Thus,

\[ W_I = W_e \]

or,
\[
[P_U \left( \frac{R + W_f/2}{R} \right) + \frac{q_D \theta_T}{6R} (W_s^3 + 3RW_s^2 + 3R^2W_s)] \delta \\
= \left( \frac{4m_1 W_s}{R \theta_T} + \frac{8M_B}{R \theta_T} \right) \delta
\]

(6.8)

hence, the failure load, \( P_U \), is

\[
P_U = \frac{1}{(R + W_f/2)} \left[ \frac{(4m_1 W_s + 8M_B)}{\theta_T} - \frac{q_D \theta_T}{6} (W_s^3 + 3R^2W_s + 3RW_s^2) \right]
\]

(6.9)

It should be noted that Eq. 6.9 is valid for any horizontally curved waffle slab bridge with a radius \( R \). Also, this equation is valid for relatively wide as well as relatively long curved waffle slab bridges.

When

\[
R \to \infty \quad \lim_{R \to \infty} \frac{(R + W_f/2)}{R} = 1.0
\]

(6.10)

\[
\theta_T R = 2L
\]

and,

Substituting Eq. 6.10 into Eq. 6.8 will yield the following relation:

\[
P_U + q_D W_s L = \frac{4m_1 W_s}{2L} + \frac{8M_B}{2L}
\]

from which,

\[
P_U = \frac{(2m_1 W_s + 4M_B - W_s L^2 q_D)}{L}
\]

(6.11)
Eq. 6.11 gives the ultimate collapse load for rectangular waffle slab with span 2L and width \( W_r \). This equation is exactly the same as the one developed by Kennedy and El-Sebakhy (1982), which supports the assumption used herein.

### 6.4.3 Curved Waffle Slab Bridge Under Uniformly Distributed Load

For the case of a simply supported horizontally curved waffle slab bridge under a uniformly distributed load, it is assumed that the failure pattern is shown in Fig. 6.1. Following the same assumptions used in the case of a concentrated load for the deflection, the external and internal virtual works \( W_e \) and \( W_i \) can be shown to be as follows:

**i- External Work**

Referring to Fig. 6.1, it can be shown that:

\[
W_e = 2q_U \int_0^{\frac{\theta_T}{2}} \int_R^r r \, dr \, d\theta \, \delta \, \frac{r}{R} \frac{\theta}{\theta_T/2}
\]

or,

\[
W_e = \frac{q_U \theta_T}{6R} (W_s^3 + 3RW_s^2 + 3R^2W_s) \delta \quad (6.12)
\]

**ii- Internal Work**

The total internal energy is the same as that given by Eq. 6.7

Equating \( W_e \) to \( W_i \) yields
\[ q_U = \frac{24 \frac{\theta_T}{\theta_T^2 W_s}}{(W_s^2 + 3RW_s + 3R^2)} \] (6.13)

It should be noted that Eq. 6.13 is valid for any value of \( R \), provided that \( R \) is greater than zero.

When \( R \to \infty \), \( R \theta_T = 2L \)

and hence, the total internal and external work will be as follow:

\[ \lim_{R \to -} W_e = \lim_{R \to -} \frac{q_U \theta_T}{6R} (W_s^3 + 3RW_s^2 + 3R^2W_s) \ \delta \]

\[ W_e = q_U \ W_s \ L \ \delta \] (6.14)

\[ \lim_{R \to -} W_i = \lim_{R \to -} \frac{4}{R \theta_T} (m_1 W_s + 2M_p) \ \delta \]

\[ W_i = \frac{2}{L} (m_1 W_s + 2M_p) \ \delta \] (6.15)

Equating \( W_e \) to \( W_i \) yields the following equation:

\[ q_U = \frac{2}{L^2} \left( m_1 \frac{2M_p}{W_s} \right) \] (6.16)

This equation gives the ultimate uniform load for a rectangular waffle slab with span 2L and width \( W_s \), and is the same as the one developed by Kennedy and El-Sebakhy (1982).
6.4.4 Curved Waffle Slab Bridge Under Concentrated Load at Center of Inner Edge

For this case of loading, it is assumed that the failure pattern is as shown in Fig. 6.2. This pattern is more likely to develop in the most practical cases of relatively long span curved bridges as well as relatively wide curved bridges. At collapse, it is assumed that the failure line deflects downward a distance $\delta$ at the inner edge and a distance $\beta'^{\prime}\delta$ at the outer edge. The external and internal virtual work will be as follows.

i- **External Work**

1- **Due to Own Weight**

Assuming a finite strip $dr$ at a distance $r$ from the center of curvature, as shown in Fig. 6.2, then:

$$W_{e1} = 2 \left[ q_D \int_{R}^{R+W_s} \frac{\theta_T r}{2} dr \left[ (1 - \beta'^{\prime}) \frac{(R + W_s - r)}{W_s} + \beta'^{\prime} \right] \right] \frac{\delta}{2}$$

or,

$$W_{e1} = \frac{\theta_T q_D}{4} \left[ RW_s \left( 1 + \beta'^{\prime} \right) + \frac{2W_s^2}{3} \left( 0.5 + \beta'^{\prime} \right) \right] \delta$$

(6.17)

2- **Due to Concentrated Load**

Referring to Fig. 6.2, it can be shown that:

$$W_{e2} = P_y \delta$$

(6.18)

Adding $W_{e1}$ to $W_{e2}$ yields the total external virtual work $W_e$ as follows:
\[ W_e = P_u \delta + \frac{\theta_T q_D}{4} [RW_s (1 + \beta''') + \frac{2W_s^2}{3} (0.5 + \beta''')] \delta \]  

(6.19)

ii- Internal Work

1- slab

It can be shown that:

\[ W_{t1} = 2 \int_{R}^{R + W_s} m_1 dr \left( 1 - \beta'' \right) \frac{(R + W_s - r)}{W_s} + \beta'' \frac{\delta}{r \theta_f/2} \]  

(6.20)

2- Edge Beams

Assuming the failure line deflects downward a distance \( \delta \) at the inner edge and a distance \( \beta'' \delta \) at the outer edge, then:

\[ W_{t2} = 2M_B \frac{\delta}{R \theta_f/2} + 2M_B \frac{\beta'' \delta}{(R + W_s) \theta_f/2} \]  

(6.21)

Adding \( W_{t1} \) to \( W_{t2} \) yields the total internal work as follow:

\[ W_i = \frac{4m_1 \delta}{\theta_T} \left[ \xi \log_e \left( \frac{R + W_s}{R} \right) + \eta \right] + \frac{4M_B}{\theta_i} \frac{(2R \psi + W_s)}{R (R + W_s)} \delta \]  

(6.22)

Equating \( W_e \) to \( W_i \) yields the collapse load \( P_u \).
\[ P_U = \frac{4}{\Theta_T} [m_1 (\xi \log_e \left( \frac{R + W_s}{R} \right) + \eta) + M_B \left( \frac{2R\psi + W_s}{R (R + W_s)} \right) - \frac{g_D \Theta_T^2}{16} (\phi \sqrt{W_s^2 + 2\psi R W_s})] \]  

(6.23)

in which the constants \( \xi, \eta, \psi \) and \( \phi \) are defined as:

\[
\xi = \left( \frac{R}{W_s} - \beta'' \frac{R}{W_s} + 1 \right)
\]

\[
\eta = (\beta'' - 1)
\]

\[
\psi = \frac{(\beta'' + 1)}{2}
\]

\[
\phi = \frac{(2\beta'' + 1)}{3}
\]

(6.24)

where \( \beta'' \) = Ratio of the outer deflection to the inner deflection.

It should be noted that the constant \( \beta'' \), depends on aspect ratio as well as the ratio of the radius of curvature to the slab width. Based on the test results as well as from an extensive parametric study using the finite element method, a design curve for the determination of the coefficient \( \beta'' \) was developed and is given in Fig. 6.3. This design curve was developed for different aspect ratios of curved waffle slab bridges. Knowing the aspect ratio of the bridge and the ratio of the radius of curvature to the bridge width, the coefficient \( \beta'' \) can be determined readily from the design curve, and consequently the constants \( \xi, \eta, \psi \) and \( \phi \) can be calculated.
Equation 6.24 is valid for any value of R for both relatively long span and relatively wide curved waffle slab bridges. However, for some practical cases of relatively wide curved waffle slab bridges, the failure pattern might be changed to the one shown in Fig. 6.4, provided that the value of the radius R is too large, compared with the bridge width, and the aspect ratio 2L/Ws is small. For this case, at collapse it is assumed that the concentrated load P, deflects downward a distance δ. Thus the external and internal virtual works can be calculated as follows:

\[
W_e = P_U \delta + \frac{\theta_T R}{6} \rho W_s q_D \delta
\]  

\[
W_i = 2 \left( m_1 \rho W_s \frac{\delta}{R \theta_T /2} \right) + 2 M_B \frac{\delta}{R \theta_T /2} + 2 \left( m_2 \rho W_s \frac{\delta}{R \theta_T /2} \right) + 2 \left( \mu m_2 \frac{\theta_T}{2} \frac{\delta}{\rho W_s} \right) + 2 \left( m_2 \rho W_s \frac{\delta}{\rho W_s} \right)
\]  

(6.26)

in which \( m_2 \) = Ultimate negative moment of resistance per unit width about r-axis; \( \mu m_2 \) = Ultimate negative moment of resistance per unit width about θ-axis' and \( m_1 \) = Ultimate torsional moment of resistance of the cross section per unit width.

Equating \( W_e \) to \( W_i \) yields the collapse load \( P_U \) in terms of the length of the positive yield line \( \rho W_s \) as follows:

\[
P_U = \frac{4\rho W_s}{R \theta_T} \left( m_1 + m_2 \right) + \frac{R \delta_T}{\rho W_s} \mu m_2 + \frac{4M_B}{R \theta_T} + 2m_2 - R \delta_T \rho W_s \frac{q_D}{6}
\]

(6.27)
For the least collapse load, \( P \) is minimized with respect to the variable \( \rho \). Or

\[
\frac{\partial P}{\partial \rho} = 0
\]

and hence

\[
\rho = \left[ \frac{R^2 \theta_T^2 \mu m_2}{4 \bar{W}^2 (m_1 + m_2)} - R^2 \theta_T^2 \bar{W}^2 \frac{q_D}{6} \right]^{\frac{1}{2}}
\]

(6.28)

Substituting Eq. 6.28 in Eq. 6.27 will yield the minimum collapse load as follow:

\[
P_U = 2 \left[ \frac{2M_B}{R \theta_T} + m_r + \sqrt{\mu m_2 \left[ 4 \left( m_1 + m_2 \right) - R^2 \theta_T^2 \frac{q_D}{6} \right]} \right]
\]

(6.29)

6.4.5 Curved Waffle Slab Bridge Under Concentrated Load at Center of Outer Edge

For this case of loading, one possible yield line pattern of flexural failure is the one shown in Fig. 6.5. This assumed failure pattern is expected to develop in relatively long span curved waffle slab bridges and some particular cases of relatively wide ones (when the radius is small compared to the width of the slab). At collapse, it is assumed that the failure line deflects downward a distance \( \delta \) at the inner edge and a distance \( \beta' \delta \) at the outer edge, in which \( \beta' \) is greater than 1.0. The external and internal virtual work in this case can be shown to be as follows:
i- External Work

1- Due to Own-Weight

Referring to Fig. 6.5, it can be shown that:

\[ W_{e1} = 2 \left[ q_D \int \frac{\theta_T}{2} r \, dr \left[ \delta + (\beta' \delta - \delta) \left( \frac{r - R}{W_s} \right)^2 \right] \right] \]

from which,

\[ W_{e1} = \frac{\theta_T}{4} q_D \left[ RW_s \left( 1 + \beta' \right) + \frac{2W_s^2}{3} \left( 0.5 + \beta' \right) \right] \delta \quad (6.30) \]

2- Due to Concentrated Load

From Fig. 6.5

\[ W_{e2} = P_U \beta' \delta \quad (6.31) \]

Adding \( W_{e1} \) to \( W_{e2} \) yields the total external virtual work \( W_e \):

\[ W_e = P_U \beta' \delta + \frac{\theta_T}{4} q_D \left[ RW_s \left( 1 + \beta' \right) + \frac{2W_s^2}{3} \left( 0.5 + \beta' \right) \right] \delta \quad (6.32) \]

ii- Internal Work

1- slab

From Fig. 6.5, it can be shown that
\[ W_{i_1} = 2 \left[ \int_R^{r - W_s} m_1 \, dr \left[ \delta + (\beta' \delta - \delta) \left( \frac{r - R}{W_s} \right) \right] \frac{1}{r \frac{\theta_T}{2}} \right] \] (6.33)

2- Edge Beams

Assuming the failure line deflects downward a distance \( \delta \) at the inner edge and a distance \( \beta' \delta \) at the outer edge, then

\[ W_{i_2} = 2M_B \frac{R}{\theta_T} \frac{\delta}{2} + 2M_B \frac{\beta' \delta}{(R + W_s) \theta_T} \] (6.34)

Adding \( W_{i_1} \) to \( W_{i_2} \) yields the total internal work:

\[ W_i = 4m_1 \frac{\delta}{\theta_T} \left[ \xi' \log_e \left( \frac{R + W_s}{R} \right) + \eta' \right] + \frac{4M_B}{\theta_T} \frac{(2R \psi' + W_s)}{R (R + W_s)} \delta \] (6.35)

Equating \( W_s \) to \( W_i \) yields the collapse load as follows:

\[ P_U = \frac{4}{\beta' \theta_T} \left[ m_1 \left( \xi' \log_e \left( \frac{R + W_s}{R} \right) + \eta' \right) + M_B \frac{(2R \psi' + W_s)}{R (R + W_s)} \right. \]

\[- \frac{q_d}{16} \frac{\theta_T^2}{R} \left( \phi' W_s^2 + 2 \psi' R W_s \right) \] (6.36)
in which the constants $\xi'$, $\eta'$, $\psi'$ and $\phi'$ defined as:

\[
\xi' = \left( \frac{R}{W_s} - \beta' \frac{R}{W_s} + 1 \right)
\]

\[
\eta' = (\beta' - 1)
\]

\[
\psi' = \frac{(\beta' + 1)}{2}
\]

\[
\phi' = \frac{(2\beta' + 1)}{3}
\]  \hspace{1cm} (6.37)

where $\beta' = \text{Ratio of the outer deflection to the inner deflection}.$

Extensive parametric study was carried out to determine the value of the coefficient $\beta'$ and suggested values will be discussed later. Another possible yield line pattern of flexural failure is more likely to develop in some special cases of relatively wide curved waffle slab bridges as shown in Fig. 6.6. This occurs when the radius, $R$, is large compared with the width of the bridge $W_s$. The failure load in this particular case be estimated from Eq. 6.29 by substituting $(R+W_s)$ for $R$, and the equation for $P_u$ becomes:

\[
P_u = 2 \left[ \frac{2M_B}{(R + W_s) \theta_s} + m_r + \sqrt{\mu m_2 \left[ 4 \left( m_1 + m_2 \right) - (R + W_s)^2 \theta_s \frac{q_d}{6} \right]} \right]
\]  \hspace{1cm} (6.38)
6.5 Treatment of Truck Loading

The equation derived previously for the collapse load of a curved concrete waffle slab bridge are for a single concentrated load. However, in practice, the design is based on truck loadings, such as, the OHBDC loading (Ministry of Transportation and Communications 1991), and AASHTO HS 20 loading (AASHTO 1983). Fig. 6.7 shows a longitudinal section of a simply supported bridge subjected to three loadings:

1- A concentrated load $P$ at the center of span through which a yield line passes.

2- An OHBDC truck causing a yield line to form with the middle axle acting along the transverse center line.

3- An AASHTO HS20 truck causing a yield line to form with the middle axle acting along the transverse center line.

6.5.1 OHBDC Truck Load

Since the internal work generated by the single concentrated load and the equivalent OHBDC Truck loading is equal, their external work is also equal. Thus

$$P = [ \gamma_1 (1 - \frac{2d_1}{L}) + \gamma_2 (1 - \frac{2d_2}{L}) + \gamma_3 (1 - \frac{2d_3}{L}) + \gamma_4 (1 - \frac{2d_4}{L}) ] Q$$

(6.39)

in which $d_1$, $d_2$, $d_3$, and $d_4$ are the distance of the axle loads $\gamma_1 Q$, $\gamma_2 Q$, $\gamma_3 Q$ and $\gamma_4 Q$, respectively, from the axle load $Q$. For an OHBDC truck, $\gamma_1 = 0.7$, $\gamma_3 = 0.3$ and $\gamma_4 = 0.8$. Therefore, Eq. 6.39 reduces to:
\[ P = \left[ 3.5 - \frac{1.4 (d_1 + d_2) + 0.6 d_1 + 1.6 d_2}{L} \right] Q \quad (6.40) \]

Applying the live load factor \( B_l \) to \( Q \) (OHBDC 1983), the factored equivalent collapse load \( P_r \) becomes, from Eq. 6.40

\[ P_r = (B_l) P \quad (6.41) \]

Thus, in design, given the load on the bridge, the factored collapse load \( P_r \) can be derived from Eqs. 6.40 and 6.41. The equations derived herein are for one-lane loading, while, for the case of two, three and four-lane loadings, the total factored load for design is reduced by 0.9, 0.8 and 0.7, respectively.

6.5.2 AASHTO Truck Load

The relation between the single concentrated load and the equivalent AASHTO HS20 truck loading (AASHTO 1983), referring to Fig. 6.7 is as follows:

\[ P = \left[ 1 + \gamma_1 \left( 1 - \frac{2d_1}{L} \right) + \gamma_2 \left( 1 - \frac{2d_2}{L} \right) \right] Q \quad (6.42) \]

in which \( d_1 \) and \( d_2 \) are the distance of the axle loads \( \gamma_1 Q \) and \( \gamma_2 Q \), respectively, from the axle load \( Q \). For an AASHTO HS20 truck (AASHTO 1983), \( \gamma_1 = 0.25 \) and \( \gamma_2 = 1.0 \). Therefore, Eq. 6.42 reduces to:
\[ P = \left[ 2.25 - \frac{d_1 + 4d_2}{2L} \right] Q \]  \hspace{1cm} (6.43)

Applying the live load factor \( \gamma B_L \) to \( Q \) (AASHTO 1983), the factored equivalent collapse load \( P_f \) becomes, from Eq. 6.43

\[ P_f = (\gamma B_L) P \]  \hspace{1cm} (6.44)

Thus, in design, given the loading on the bridge, the factored collapse load \( P_f \) can be derived from Eqs. 6.43 and 6.44. With an estimated factored value for \( q_{D0} \), estimates of the required ultimate moments of resistance can be calculated from the relevant equations derived here. In analysis, the reverse procedure is applied as follows: given the ultimate moments of resistance and the factored value for \( q_{D0} \), a value of \( P_U \) can be derived from the relevant derived equations. Equating \( P_U \) to \( P_f \), given be Eqs. 6.43 and 6.44, will yield the actual live load factor \( \gamma B_L \).

The equations derived herein are for one-lane and two-lane loadings, while, in the case of three and four-lane loadings, the total factored load for design (AASHTO 1983) is reduced by 0.9 and 0.75, respectively.
CHAPTER VII

EXPERIMENTAL INVESTIGATION

7.1 General

In this study the experimental program can be classified into two main groups as shown in Figs 1.5 and 1.6. The first group deals with the rigidities of curved concrete waffle slab bridges, while the second group deals with the structural response of curved concrete waffle slab bridges under working and ultimate loads.

7.2 Rigidities of Curved Concrete Waffle Slab Bridges

7.2.1 Scope of the Experimental Program

The experimental program consisted of tests on nine curved concrete waffle slab models. There were three series of tests, each series consisting of three types of specimens; two specimens were rectangular in plan for pure bending in the tangential and radial directions, and the third specimen was square in plane for pure torsion. These tests were intended to predict the flexural and torsional rigidities during the pre-cracking and post-cracking stages, and to verify the analytical expressions developed in Chapter III.

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The mathematical formulations developed in chapter III were used to calculate the flexural and torsional rigidities from the experimental results.

7.7.2 Materials

7.2.2.1 Concrete

High early strength portland cement Type 30 manufactured by Canada Cement Company was used in all the slab specimens. The maximum size of the aggregate was restricted to 0.25 in. (6.4 mm) due to the narrow dimensions between the sides of the formwork. The combined aggregate was prepared according to the ACI manual of concrete practice (ACI 1989). This gave a well graded combined aggregate with a fineness modulus equal to 2.56. A concrete mix design example is given in Appendix A-4. Natural tap water was used in the concrete mix. Water cement ratio of 0.4 was selected to achieve the required 28-concrete strength of 6000 psi (44 MPa). Mixing of the concrete was performed in an Eerich Counter Current Mixer, Model EA2 (2W) with a capacity of five cubic feet (0.14 m$^3$).

7.2.2.2 Reinforcement

The steel reinforcement used in the rigidity models consisted of 0.25 in. (6.4 mm) diameter mild steel bars with a modulus of elasticity of 30.8x10$^6$ psi (213x10$^3$ MPa).

7.2.2.3 Formwork

Three forms were made from plywood, 0.75 in. (19 mm) thick, with wood joist
2x6 in. in cross section used as stiffeners. The first form was square in plan and used for
casting the slab model T for the pure twisting test as shown in Fig. 7.1. The second and
the third specimens were rectangular in plan and used for casting the slab models F₀ and
Fᵣ for the pure bending tests in the tangential and radial directions, respectively. Figure
7.2 shows the form for the bending specimen in the radial direction. Styrofoam blocks
with different dimensions were used for producing the waffle shapes and were fixed on
the wood surface by a water resistance glue.

7.2.3 Description of the Specimens

A total of nine specimens were tested for the bending and twisting tests. All
specimens had a continuous slab with a thickness of 1 in. (25.4 mm) and the depth of the
ribs was 3 in. (76 mm). In each specimen, two steel reinforcing layers, perpendicular to
each other, were placed at 0.5 in. (12.5 mm) clear cover from the bottom surface.
Geometric description of these specimens is given in Table 7.1. The nine specimens were
classified into three series, each series with three specimens.

The dimensions of the bending specimen in the tangential direction, for all the
three specimens, were 84x45.5 in. (2.13x1.15 m) in plan, and the spacing of the ribs in
the tangential direction was kept constant at 5.5 in. (140 mm). The spacings in the radial
direction, along the longitudinal center line of the slab, were 5.5 in. (140 mm), 7.0 in.
(178 mm) and 9.0 in. (299 mm) for the three specimens, respectively.

The dimensions of the bending specimen in the radial direction for all the three
specimens were 114.5x45.5 in. (2.9x1.15 m) in plan, and the spacing between the ribs,
in both directions, were the same as the bending specimens in the tangential direction.

The three twisting specimens were 45.5x45.5 in. (1.15x1.15 m) in plan, and the spacing of the ribs in the tangential direction was kept constant at 5.5 in. (140 mm). The spacings between the ribs in the radial direction for the three specimens, along the longitudinal center line of the slab, were 5.5 in. (140 mm), 7.0 in. (178 mm) and 9.0 in. (229 mm), respectively.

7.2.4 Construction of the Specimens

The required wood forms were prepared as shown in Figs. 7.1 and 7.2. The reinforcement was placed in the center of each rib for both directions following a marked pattern on the wood. Due to the curvature of the ribs in the tangential direction, the reinforcing steel bars were cold bent horizontally with different curvatures using a rolling machine.

For the tangential bending specimens, the bent bars were paced first on 5/8 in. (16 mm) staple, connected to the bottom of the form, to keep it 0.5 in. (12.7 mm) from the bottom surface of the form. These steel bars were then tied down to the staple using stove thin wires. The steel bars in the radial direction were placed on the top of the first layer, and tied together for stability during casting of the concrete.

For the radial bending specimens, the reinforcing steel bars in the radial direction were placed first on the staple followed by the ones in the tangential direction. Again, both layers of reinforcement were tied together with thin wires.

One hour before mixing the concrete, the top surface and the sides of the form
were soaked with heavy industrial oil. The concrete was then cast, vibrated and trowelled to a smooth finish. All the specimens were moist cured for 7 days. To determine the compressive strength of the concrete, three 6x12 in. (152x305 mm) concrete cylinders were cast with each specimen. They were cured in the same way as the slab models, and were subjected to a compression test at the time of testing the specimens.

7.2.5 Instrumentation

The deflection of the concrete specimens for bending and twisting tests were measured by means of mechanical dial gauges with a sensitivity of 0.001 in. (0.025 mm). All the dial gauges were placed at the top surface of the slabs, and supported on a steel frame.

Two T-shaped loading frames were made for the bending tests in the tangential and radial directions, while a straight loading frame was fabricated for the twisting test. The T-shaped loading frame for the tangential bending test was made of 3x3x1/8 in. (76x76x3.2 mm) steel hollow section, while the T-shaped loading frame for the bending test in the radial direction was fabricated using S 130x15 for the beam and 3x3x1/8 in. (76x76x9.6 mm) for the frame legs. The straight loading frame for the torsion test was made of 3x3x3/8 in. (76x76x9.6 mm) steel hollow section, stiffened on the top and bottom surfaces by steel plate of 2x1/2 in. (51x12.7 mm). A bearing plate was welded to the legs of T-shaped frame to transmit load to the specimen. Two cylindrical load cells with a maximum capacity of 10 kips (44.8 kN) were used to transmit the load to the two corners of the torsion specimens as shown in Fig. 7.6. The concentrated downward load
was applied to the specimen through a 50-kip (222.5 kN) hydraulic jack through a calibrated cylindrical load cell.

7.2.6 Experimental Set-up and Test Procedure

The rectangular specimens were subjected to load producing a uniform bending condition in the middle region of the specimens, using the three-point loading and support system suggested by Bergstrasser (1927), as shown in Figs. 7.4 and 7.5. The square specimens were subjected to pure twisting moment by supporting the diagonally opposite corners and loading the other two corners, as shown in Fig. 7.3.

Deflections were measured by dial gauges, mounted rigidly at seventeen grid locations for the flexural test, and at fourteen grid locations for the torsion test. The test set-up is shown in Figs. 7.6, 7.7 and 7.8 for the torsion, tangential bending and radial bending specimens, respectively. The dial gauges were mounted some distance away from the loading points and the readings for loading and unloading were averaged for the linear stage of loading. Loading was then continued at reduced load increments until failure was evident by wide cracks and large deflections.

7.3 Curved Concrete Waffle Slab and Girder-Slab Bridges

In order to verify the results from analytical formulation, finite difference technique, finite element approach, and the yield line method, a series of experimental tests were carried out on curved concrete slab bridge models.

The experimental program consisted of testing three curved concrete slab bridge
models. The first model was a curved reinforced concrete waffle slab bridge RWS, the second model was a curved prestressed concrete waffle slab bridge PWS, while the third model was a curved prestressed concrete girder-slab bridge PGS. All the three models were simply supported along the two radial edges and free along the sides parallel to the traffic flow. The prestressing wires in the slab models PWS and PGS were horizontally curved in the tangential direction and straight in the transverse direction.

The testing program steps can be summarized as follows:

1- Elastic (service load) testing of the curved concrete waffle slab and girder-slab bridge models, and

2- Ultimate (collapse load) testing.

Throughout the testing program, necessary measurements were taken to examine the overall structural response of the curved reinforced and prestressed concrete waffle slab bridges under several loading conditions. These measurements were also used to examine the influence of the slab type, either waffle slab or girder slab, on the structural response of curved bridges subjected to service and ultimate load and to provide a reliable theoretical model.

7.3.2 Materials

7.3.2.1 Concrete

Complete descriptions of the concrete material and the mix design example are given in Sec. 7.2.2.1 and Appendix A-4, respectively.
7.3.2.2 Reinforcement

Mild steel bars of 0.25 in. (6.4 mm) diameter were used for the reinforcement of slab model RWS, while 3/16 in. (4.7 mm) diameter were used as a non-prestressed steel reinforcement for slab models PWS and PGS. In all the slab models, the solid slab part was reinforced in two perpendicular directions by 1/8 in. (3 mm) bars and spaced 3 in. (76 mm). High tensile steel wires of 0.276 in. (7 mm) diameter were used for prestressing the two slab models PWS and PGS. The stress-strain relationship for the high tensile steel wires is shown in Appendix A-3.

7.3.2.3 Formwork

One form was made to accommodate the three slab models. This form was fabricated from plywood, 0.75 in. (19 mm) thick, with wood joist, 2x6 in. (51x152) in cross section used as stiffeners. The two radial sides of the form were made from plywood, while the two tangential sides were fabricated using aluminum sheet to facilitate the bending of such sides. Styrofoam blocks were used for producing the waffle shapes for slab models RWS and PWS and the space between the girders for the slab model PGS. The styrofoam blocks were fixed on the wood surface, by a water-resistance glue, following the marked patterns.

7.3.3 Models Description

Bridge models RWS and PWS were identical in all aspects except for the type of reinforcement: mild steel in the RWS model, and prestressed high tensile steel in the PWS.
model. The overall dimensions of these slab models were 34 in. (0.86 m) in width and 70 in. (1.78 m) in span along the longitudinal center line of the bridge model. The total connecting angle between the left and right supports was 71.9 degrees and the radius of curvature at the longitudinal center line was 55.75 in. (1.41 m). The spacing between the ribs in the tangential direction was 7.0 in. (178 mm), while the spacing in the radial direction was 7.0 in. (178 mm) at axis 3-3. The plan layouts of RWS and PWS models are shown in Fig. 7.9. The slab thickness of the waffle section was 1.5 in. (38 mm) and the rib depth was 4.0 in. (101 mm). The cross-sectional dimensions are shown in Fig. 7.10.

Bridge model PGS was identical to bridge models RWS and PWS except for the transverse ribs. The plan layout and the cross-sectional dimensions are shown in Figs. 7.11 and 7.12. Figures 7.13 and 7.14 show the curved waffle slab and girder slab bridge models, respectively.

The bridge model PWS was prestressed by fourteen high tensile steel wires in both directions, five horizontally curved wires in the longitudinal direction and nine straight wires in the transverse direction, while bridge model PGS was prestressed only in the longitudinal direction with five wires.

It should be noted that all bridge model dimensions were judged to be the most suitable in accordance with the available space in the laboratory.
7.3.4 Experimental Equipment

7.3.4.1 Prestressing Equipment

A prestressing hydraulic jack of 20 kips (89 kN) capacity, manufactured by Cable Covers Ltd., England, was used for post-tensioning as shown in Fig. 7.15. The mechanical gripping devices with open grip type were used. Black wax lubricant was applied to the wedges to make it easier to release the grips after completing the prestressing operation. Bearing plates, 0.25 in. (6.4 mm) thick, with holes of 0.276 in. (7 mm) diameter and of 1.5x3.0 in. (38x76 mm) dimensions, were used at both ends of the wires to transfer and distribute the prestressing force from the steel to the concrete.

7.3.4.2 Automatic Strain Indicator

An automatic strain indicator, manufactured by Visha Intertechnology Inc., Malvern, U.S.A, was used to automatically record the strain readings during the experiment as shown in Fig. 7.16. The strain indicator includes mainly four devices: The V1E-21 Switch Balance, the V1E-20 Digital Strain Indicator, Scan Controller V1E-25, and the Automatic Printer V1E-22. The V1E Strain Indicator provides a method of sequentially reading the outputs of up to ten channels of strain gauge information. Ten V1E-21's were connected to the same indicator which gave a capability of recording readings up to 100 strain gauges.

7.3.4.3 Vertical Loading Equipment

A hydraulic jack with a maximum capacity of 50 kip (222.5 kN) was used for
applying concentrated load to the slab models at different point locations. The length of
the jack was 20 in. (510 mm) with a 17 in. (430 mm) travel piston. The jack was
attached to the loading frame by means of heavy C-clamps. The jack with this system
can move only in one direction. This loading system is shown in Fig. 7.29.
Steel blocks of 50 lb. (222 N) each, were used to apply uniform load on the bridge
models. Styrofoam strips were used to prevent the weights from contact with the strain
gauges located at loading positions. The uniformly distributed load system is shown in
Fig. 7.35.

7.3.5 Instrumentation

7.3.5.1 Electrical Strain Gauges

Concrete strains were measured on the top and bottom surfaces of the bridge
models, using electric foil gauges, each 30 mm in length, type N11-FA-30-120-11 with
an average resistance of 119.8 Ohms and a gauge factor of 2.16. The concrete surface
at the gauge locations was prefinished to mount the gauges in accordance with the
requirements for concrete gauge application. After soldering the wires to the gauges, the
latter were moisture-proofed. The gauges were then connected to the strain indicator,
balanced to zero and made ready for testing. The locations of the strain gauges on the
top and bottom surfaces for waffle slab models, RWS and PWS, and girder-slab model,
PGS, are shown in Figs. 7.17, 7.18, and Fig.7.19, respectively.
7.3.5.2 Mechanical Dial Gauges

The deflection resulting from both the prestressing and transverse loading were measured using mechanical dial gauges with a sensitivity of 0.001 in (0.025 mm). All the dial gauges were placed at the bottom surface of the slabs, and supported on a light steel frame as shown in Fig. 7.20. The location of the dial gauges for the curved waffle slab and girder-slab bridge models are shown in Figs. 7.21 and 7.22, respectively.

7.3.5.3 Load Cells

A universal flat load cell with a capacity of 50 kips (222.5 kN), was used in the three slab models to measure the applied concentrated load to the slab through a hydraulic jack. The calibration of this load cell is given in Appendix A-3. Fourteen cylindrical load cells were used to measure the prestressing forces in the longitudinal and transverse directions. Another four cylindrical load cells were used to measure the negative vertical and horizontal reactions at the inner edge, as shown in Fig. 7.23. The calibration of these load cells is given also in Appendix A-3.

7.3.6 Construction of the Slab Models

The dimensions of the three bridge models are shown in Figs. 7.9 and 7.11, while the cross-sectional dimensions are shown in Figs. 7.10 and 7.12. The required wood forms for bridge models PWS and PGS are shown in Figs. 7.26 and 7.27, respectively. For RWS bridge model, the longitudinal steel bars were cold bent horizontally with different curvature and placed on 5/8 in. (16 mm) staple, which provided a clear cover
of 0.5 in. (12.7 mm) from the bottom surface. The steel bars in the radial direction were then placed on the top of the first layer and were tied together to ensure stability during the casting of concrete. Styrofoam blocks were then glued to the bottom of the form following a marked pattern on the wood. The steel mesh was placed in position using small wood blocks. The steel mesh was then tied down to the reinforcing steel bars to support it against floating during casting of concrete. The reinforcing of bridge model RWS is shown in Fig. 7.24.

For PWS bridge model, the non-prestressed steel wires were placed first, following the same procedure as that for RWS bridge model. Rubber hoses having a diameter of 5/16 in. (7.9 mm) were used to cover the bent bars during the casting of concrete, and hence prevent bonding between the steel and the concrete. The longitudinal steel bars, with different curvature, were inserted in the rubber hoses and supported in their position on the two radial sides of the form. To avoid sagging of the longitudinal steel bars, S shape hooks were used to hang the longitudinal steel bars on the transverse ones. Three inch (75 mm) nails were placed, staggered to keep the longitudinal steel bars in their horizontal position during casting of concrete. Fig. 7.25 shows the prestressing wire locations in the cross-section.

One hour before mixing the concrete, the inside of the forms was painted with grease material, for easy form release. The form was placed on a special steel bed, having an attached vibrator as shown in Fig. 7.27. Care was taken during casting and vibration to ensure that the longitudinal wires and the top steel mesh did not move. The top surface of the slab was levelled and smoothed after casting and then covered with a
plastic sheet. After six hours, water-curing of the slab was started and continued for 7 days. After hardening of the concrete, the steel bars as well as the rubber hoses were pulled out and the prestressing wires in the longitudinal and transverse directions were inserted in the holes. To determine the compressive strength of the concrete, three 6x12 in. (152x305 mm) cylinders were cast with each model, following the same procedure as before.

7.3.7 Experimental Setup and Testing Procedures

Adequate precautions and extreme care were taken during taking the forms off the models and transferring them from the casting bed to their final position for testing. An overhead crane was used to transfer the slab models. One system of supports was used for all the bridge models since they have the same plan geometry. The models were supported at the two radial edges on 1.5 in. (38 mm) diameter steel rollers. Steel and rubber shims were used between the supports and the concrete surface to insure full contact between both surfaces. To prevent uplift of the bridge models, which may occur under some loading conditions, a holding-down system shown in Fig. 7.28 was used. This system with cylindrical load cells attached, can measure the negative reactions as shown in Fig. 7.23. To measure these negative reactions, a pre-compression force was applied first to the vertical load cells then any subsequent change in this force during loading of the bridge model will measure the negative reactions. The testing sequence for the three bridge models was divided in two phases: the elastic (service load) phase and the failure (ultimate load) phase. The first phase consisted of concentrated and
distributed load applied at different positions of the bridge models. In the second phase, each of the three bridge models was loaded to failure. In this latter phase, the bridge models were loaded by two concentrated loads at mid-span, each load at the center of each lane, until complete collapse was achieved. Figure 7.28 shows the test set-up and the supporting system.

7.3.7.1 **Curved Reinforced Concrete Waffle Slab Bridge Model-RWS**

The dimensions of the RWS bridge model are shown in Fig. 7.9, and its loading system is presented in Fig. 7.29. The slab was mounted with forty-two strain gauges to measure the strain on the top and bottom surfaces of the concrete, as shown in Figs. 7.17a and 7.17b. On the other hand, twenty-one mechanical dial gauges were used to measure the lateral deflection, as shown in Fig. 7.21. Three cases of loading were considered in the elastic phase, namely: a concentrated load at the center of the bridge model, as shown in Fig. 7.29; and a uniformly-distributed load applied on the outer and inner lanes, respectively. Finally, the model was loaded until collapse by two concentrated loads, each applied at the center of each lane, as shown in Fig. 7.30.

7.3.7.2 **Curved Prestressed Concrete Waffle Slab Bridge Model-PWS**

The dimensions of the PWS bridge model are shown in Fig. 7.9 and the reinforcing steel as well as prestressing steel locations are shown in Fig. 7.25. Fifty-six strain gauges were used to monitor the strain on the top and bottom surfaces of the concrete as shown in Figs. 7.18a and 7.18b, while twenty-one mechanical dial gauges
were used to measure the lateral deflection produced by the prestressing as well as the transverse load, as shown in Fig. 7.21. The bridge model was prestressed by fourteen high tensile steel wires, one in each rib, connected to a cylindrical load cell to measure the prestressing force. Great care and adequate precautions were taken during the prestressing process, especially for prestressing in the curved ribs. To avoid any distortion or local failure, the wires in the radial direction were tensioned first followed by the ones in the tangential direction. In the radial direction, the wires were fully prestressed, with the design force, starting with the middle rib E-E followed by the adjacent ones to the right and to the left alternatively. On the other hand, the wires in the longitudinal direction were partially prestressed with half the design force following the same sequence as those in the radial direction. Finally, the ribs in the tangential direction were fully prestressed with the remainder of the design force in the same sequence. Figures 7.31 and 7.33 show the prestressing operation in the radial and tangential directions, respectively. For practical purposes, the prestressing forces in the tangential direction were designed to vary between the inner and outer ribs to accommodate the radial changes in the span length. Figure 7.33 shows the final amount of prestressing forces.

The bridge model was tested by a single concentrated load applied at eight various positions using a loading beam to transfer the load from the hydraulic jack to the loading position, as shown in Fig. 7.34. Also, the model was loaded by uniformly-distributed loads on the inner and outer lane, as shown in Fig. 7.35. Finally, the bridge model was loaded by two concentrated loads at mid-span, each load at the center of each lane, up to
failure. Figure 7.36 shows the loading system for the ultimate load case.

7.3.7.3 Curved Prestressed Concrete Girder Slab Bridge Model-PGS

Figure 7.11 shows the dimensions of bridge model PGS. Forty-six strain gauges were used to record the strain on the top and bottom surfaces of the concrete and twenty-one mechanical dial gauges were mounted to measure the deflections. Figures 7.19 and 7.22 show the location of these gauges. The bridge model was prestressed by five high tensile wires in the tangential direction, following the same sequence mentioned in section 7.3.7.2. Figure 7.37 shows the final amount of prestressing forces for bridge model PGS. The loading of the bridge model, during both the elastic and ultimate phases, was exactly the same as the one applied to the bridge model PWS. Figure 7.38 shows the bridge model under external load.
CHAPTER VIII

RESULTS AND DISCUSSIONS

8.1 General

An experimental program consisting of nine tests on curved concrete waffle slabs was carried out to confirm the validity of the analytical expressions, developed in this study, for predicting the flexural and torsional rigidities of curved concrete waffle slab bridges during the pre-cracking and post-cracking stages. Results derived from other existing methods are also presented and compared with both the experimental and theoretical results achieved.

The experimental study on the elastic and post-elastic behaviour of curved prestressed and reinforced concrete waffle slab bridges was carried out by testing two laboratory bridge models. The finite element method, the finite difference technique and the yield line theory were used in the analyses. The influence of the transverse ribs on the response of horizontally curved bridges subjected to elastic and post-elastic loadings is investigated by also testing a curved prestressed concrete girder-slab bridge model. The experimental and theoretical results achieved for the latter are presented and compared.
The analytical effort was extended to cover other curved bridge systems and their responses were compared. The influence of the bridge horizontal radius to width ratio, aspect ratio and applied load locations on the structural response of curved waffle slab bridges were also examined.

A- RIGIDITIES OF CURVED SLAB BRIDGES

8.2 General

To demonstrate the validity of the analytical expressions, derived in Chapter III for predicting the flexural and torsional rigidities of curved concrete bridges, an experimental program consisting of nine tests on curved concrete waffle slab models was carried out. The theoretical and experimental results achieved for the flexural and torsional rigidities of curved concrete waffle slabs during the pre-cracking and post-cracking stages are presented and compared. Results of both the flexural and torsional rigidities, during the pre-cracking stages, from other existing methods are also presented and compared. Deficiencies in these different methods for calculating the rigidities are discussed. A method to account for the effect of concrete creep on the rigidities is also presented. The influence of the type of cross section on the values of the rigidities is discussed.

8.3 Rigidities of Curved Concrete Waffle Slab

The load-deflection curve for each specimen at every dial gauge was plotted. The
typical load-deflection behaviour of the uncracked specimen appears to be elastic and linear as shown in Fig. 8.1. Furthermore, the rate of change of the load-deflection behaviour of the cracked specimen continued to be almost linear over a considerable range of loading. Due to this observation and for simplicity, two linear straight lines were fitted to the deflection results at every dial gauge location. Experimental results close to the collapse load of the cracked specimen were omitted since they did not fit the assumed bi-linear approximation.

8.3.1 Tangential Flexural Rigidity

The predicted theoretical flexural rigidities in the tangential direction $D_\theta$, calculated from Eq. 3.1, are compared with the experimental results at five stations I, II, III, IV and V as shown in Fig. 7.4, for the three specimens $F_{61}$, $F_{62}$, and $F_{63}$. Tables 8.1, 8.2 and 8.3 show the comparison of tangential rigidity values at the pre-cracking stage for specimens $F_{61}$, $F_{62}$, and $F_{63}$, respectively. Close agreement can be noted between the experiment and the present theory. The percent difference at any station does not exceed 5% for specimen $F_{61}$, 10% for specimen $F_{62}$ and 6% for specimen $F_{63}$. It should be noted that the tangential flexural rigidity, for each specimen, at the five stations across the width of the slab are equal as shown in tables 8.1, 8.2 and 8.3. This is because the ribs in the tangential direction are equally spaced.

Results for the tangential flexural rigidity by other investigators are also presented as shown in Tables 8.1, 8.2 and 8.3. Comparison of these results with the results from the present theory shows that the rigidities calculated using Hein's method (1969) are
overestimated since in this method the effect of Poisson's ratio is applied to the total section and not to the slab only. The methods of Szilard (1974) and Giencke (1955) underestimate the flexural rigidities because these methods ignore the effect of Poisson's ratio. Rigidities calculated by means of the method of Cusens (1972) show good agreement with the present method since Cusens accounted for the effect of Poisson's ratio.

For comparison, results for the tangential flexural rigidities were obtained for two types of structures: the curved waffle slab, investigated herein, and an orthotropic slab of uniform thickness, both having the same volume of concrete and reinforcing steel. Table 8.1, 8.2 and 8.3 show these comparisons. It can be noted that these rigidities are approximately one-half of the curved waffle slab investigated herein. Due to the lower value of the flexural rigidity for the solid slab, as compared to that for the waffle slab, the former will exhibit much larger deflections and higher stresses for the same load. It should be noted that one advantage of the waffle slab structure over slabs of uniform thickness is in prestressed construction, where the presence of ribs contributes to the increased eccentricities for prestressing in waffle slabs. This results in higher efficiency of a prestressed waffle slab in carrying loads as compared with that of solid slabs.

Tables 8.4, 8.5 and 8.6 show the comparison of tangential rigidity values at the post-cracking stage for specimens F_{61}, F_{62}, and F_{63}, respectively. Close agreement can be noted between the experiment and the present method. The percent difference at any station does not exceeding 9% for specimen F_{61}, 5% for specimen F_{62}, and 4% for specimen F_{63}. 

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8.3.2 Radial Flexural Rigidity

The theoretical radial flexural rigidity $D_R$, calculated using the proposed equations, is compared to the experimental results at five stations I, II, III, IV and V, as shown in Fig. 7.5, for the three specimens. Tables 8.7, 8.8 and 8.9 present the comparison between the theoretical and experimental results of the radial rigidity values at the pre-cracking stage for $F_{R1}$, $F_{R2}$, and $F_{R3}$, respectively. These comparisons reveal good agreement between the experiment and the present method. The percent difference at any station does not exceed 5% for any of the three specimens.

It should be noted that these rigidities change from one station to another across the width of the slab as shown in Tables 8.7, 8.8 and 8.9. This is because the spacing of ribs in the radial direction becomes smaller toward the center of curvature. The percent difference between the rigidities at the first station and the fifth station, taking the fifth one as a reference, is 29%, 31% and 33% for specimens $F_{R1}$, $F_{R2}$, and $F_{R3}$, respectively. Based on this observation, it can be concluded that idealization of a curved waffle slab bridge as an orthotropic slab with constant tangential and average radial flexural rigidities, through the width of the slab, will lead to inaccurate results. Results for the radial rigidity by other investigators are also presented in Tables 8.7, 8.8 and 8.9, respectively. The efficiency and deficiency of each method are the same as explained above when discussing the tangential flexural rigidity. Tables 8.7, 8.8 and 8.9 also show the radial rigidity of an orthotropic slab with uniform thickness, having the same volume of concrete and amount of steel as the waffle one. It can be noted that these rigidities are constant through the width of the slab and are approximately one-half of those of the
curved waffle slab.

Tables 8.10, 8.11 and 8.12 show the comparison between the theoretical and experimental results of the radial rigidity values at the post-cracking stage for specimens $F_{R1}$, $F_{R2}$, and $F_{R3}$, respectively. Close agreement can be noted between the results from the tests and the present theory. The percent difference at any station for any of the three specimens does not exceed 7%.

8.3.3 Torsional Rigidity

Tables 8.13, 8.14 and 8.15 present the comparison between the theoretical and experimental results of the torsional rigidity $D_{DR}$ at five stations I, II, III, IV and V, as shown in Fig. 7.3, for specimens $T_1$, $T_2$ and $T_3$ at the pre-cracking stage. These comparisons reveal good agreement between the experiment and present method. The percent difference at any station does not exceed 3%, 4% and 3% for specimens $T_1$, $T_3$ and $T_3$, respectively. It should be noted that the present method takes into account the stiffening effect of the orthogonal ribs on the torsional rigidity. Thus the torsional rigidity changes from one station to another through the width of the slab, as shown in Tables 8.13, 8.14 and 8.15. The percent differences between the rigidities at the first station and the fifth station, taking the fifth station as a reference, are 21%, 18% and 14% for specimens $T_1$, $T_2$ and $T_3$, respectively. It can be concluded that, analyzing a curved waffle slab bridge as a wide beam with constant torsional rigidity is incorrect and leads to unreliable results. Results for the torsional rigidity by other methods are also presented in Tables 8.13, 8.14 and 8.15. Comparison of these results with the present theory shows
that the values of the torsional rigidity based on Huber's method (1923) and calculated from

$$D_{\phi} = (1 - \mu) (D_x D_\theta)^{1/2}$$  \hspace{1cm} (8.1)

are significantly different and are unreliable. The torsional rigidities calculated using Hein's method (1969) underestimate the true value. The reason for this discrepancy is that it ignores the stiffening effect of the ribs in the orthogonal direction. Ignoring this effect will lead to a constant rigidity value through the width of the slab as shown in Tables 8.13, 8.14 and 8.15. Hein's method was modified by the author to include the effect of the ribs in the orthogonal directions. The results based on this modification are shown in Tables 8.13, 8.14 and 8.15. It can be observed that the modified values of the tangential rigidity are now closer to the experimental results. The torsional rigidity calculated by means of Szilard's method (1974) overestimates the actual rigidity because the continuity of the deck slab is not accounted for; whereas Rowe's method (1962) underestimates it since it neglects the stiffening effect of the ribs.

Tables 8.13, 8.14 and 8.15 show the values of the torsional rigidity of an orthotropic slab of uniform thickness having the same volume of concrete and amount of reinforcement; it can be noted that these rigidities are approximately 2 1/2 times those of the curved waffle slab investigated herein. These tables also shown a comparison between the torsional rigidity of a waffle slab and the sum of the individual torsional rigidities of the deck slab and the grid, taken as separate units. The results indicate that the waffle slab has about 40 % greater torsional rigidity in this case.

Tables 8.16, 8.17 and 8.18 show the comparison of the torsional rigidity values
at the post-cracking stage for specimens $T_1$, $T_2$ and $T_3$, respectively. Good agreement can be noted between the experiment and present method. The percent difference at any station for any of the three specimens does not exceed 7%. Figure 8.2 shows a torsion specimen after collapse.

8.4 Rigidities of Horizontally-Curved Composite Structures

Although no experimental program was undertaken to calculate the orthotropic rigidities of horizontally-curved composite structures, the proposed equations, given in Chapter III, can predict accurately the various rigidities of such structures during the pre-cracking and post-cracking stages. This is simply because the proposed equations for calculating the orthotropic rigidities of horizontally-curved composite structures were based on the same assumptions as those applied to the waffle slab structure, and followed the same procedures. It should be noted that the deck slab of a curved composite structure contributes about 90% of the total torsional rigidity of the cross section, whereas in the case of a curved concrete waffle slab structure the contribution is about 60% of the total torsional rigidity of the cross section, depending on the spacing of the ribs.

It should be mentioned that the rigidities are affected by creep in the concrete. This influence can be accounted for by adjusting the modulus of elasticity of the concrete, $E_c$, to a time-dependent modulus of deformation, $E_e$ (Saeed and Kennedy 1970). Finally, the discrepancies between the experimental and theoretical results can be attributed to several sources of error, such as: the assumptions made in the theory; geometry and
loading of the specimens; the deflection measurements and the calculation of the respective curvatures. However, notwithstanding the above sources of error, the proposed formulae for the calculation of the rigidities appear to predict well the actual rigidities of horizontally-curved structures before as well as after cracking of the concrete.

**B- FINITE DIFFERENCE TECHNIQUE**

8.5 General

To demonstrate the validity of the finite difference technique in predicting the elastic behaviour of curved reinforced concrete waffle slab bridge, experimental elastic tests were carried out on a curved bridge model.

The analytical solution used herein is based on the equivalent orthotropic plate theory in which the various rigidities required for the solution are estimated using the formulas developed in Chapter III. The generation and solution of the finite difference equations are accomplished by the computer program given in Appendix A-1. The use of double precision improved the results in cases involving large matrices.

The experimental deflections were compared to the analytical solution from both the finite difference technique and the finite element method. The effect of changing the orthotropic plate rigidities on the behaviour of curved concrete slabs is discussed.
8.6 Curved Concrete Waffle Slab Bridge Model-RWS

It should be noted that for curved waffle slab structures, the radial flexural rigidity as well as the torsional rigidity change from one station to another as explained earlier. Table 8.19 presents the different values of the rigidities at each tangential axis. Referring to Fig. 7.9, if axis 1-1 is taken as a base, then the percent difference in the radial rigidity at axis 5-5 is 27 %, while the percent difference in the torsional rigidity is 24 %.

Three different mesh configurations were considered to study the convergence of the finite difference solution. The first mesh pattern, as shown in Fig. 8.3, consisted of 45 mesh points with \( \lambda = 7.0 \text{ in.} \) (178 mm) and \( \theta_c = 0.1255 \text{ radian} \), resulting in 45 simultaneous equations. The second mesh pattern was refined by decreasing \( \lambda \) to 3.5 in. (89 mm) creating 81 points for the mesh pattern, as shown in Fig. 8.4. Finally, the third mesh was refined further by decreasing \( \theta_c = 0.0628 \) resulting in 171 mesh points, as shown in Fig. 8.5. Figure 8.6 describes the transverse deflected shape for axis E-E due to load at mid-span for the three meshes. As shown in Fig. 8.6, the differences in the results from the three meshes are quite small. The curved concrete waffle slab bridge model RWS was tested elastically under the effect of three cases of loading, as shown in Fig. 8.19. The first case was a concentrated load of 1.3 kip. (5.8 kN) at the center of the bridge model, point E-3. The second case was a uniformly-distributed load of 1.0 psi (6.9 kPa) on the outer lane, while the third case was a uniformly-distributed load of 0.87 psi (5.9 kPa) on the inner lane. The deflection results were compared to the analytical solution from both the finite difference technique and the finite element method. The finite difference results were based on the 45-mesh points. Figures 8.7, 8.8 and 8.9 show
the deflection results for axis 1-1, 2-2 and 3-3, respectively. Comparison of these results shows that there is a good agreement between the theoretical and experimental results. The analytical results from the finite element analysis appear to underestimate the test results, while the results from the finite difference overestimate them. The differences are larger for deflections along the outer edge than for the ones along the inner edge.

Figures 8.10 through Fig. 8.12 and Figures 8.13 through Fig. 8.15 show the deflection results for axis 1-1, axis 3-3 and axis 5-5 due to uniform load on the outer lane and the inner lane, respectively. Comparison of these results reveals good agreement between the theoretical and experimental results. As expected, the difference in the maximum deflection between the experimental results and the finite difference results is less than that for the case of the concentrated load at center. Based on this observation, it can be concluded that the convergence of the finite difference solution in the case of uniformly-distributed load was better than the ones in the case of concentrated load, as expected.

To study the effect of changing the rigidities of curved slab structures on its behaviour, the various slab types I, II, III and IV, presented in Chapter IV, were analyzed. Comparing the curved waffle slab, type I, with the solid slab, type II, reveals that the flexural rigidities of the solid slab are lower by about 42 %, while its torsional rigidity is higher by about two times and half. From simple beam theory one would expect also a 42 % increase in the deflection. However, as shown in Fig. 8.16 the maximum deflection increases by only 25 %. This is mainly due to the substantial increase in the torsional rigidity of the solid slab. Furthermore, the differences in deflections along the
inner and outer edges of the curved solid slab are much less than the differences in the curved waffle slab. This can be attributed mainly due to the high torsional rigidity of the former.

Figure 8.17 shows the deflection results for the curved waffle slab I and the curved solid slab III with the same torsional rigidity. Again, it is noted that while the flexural rigidities of the latter are reduced to 1/4 those of the former, the maximum deflections of the latter are only 2 3/4 those of the former. This again demonstrates the importance of the torsional rigidity in controlling deflection of curved slab structures.

The deflection of the curved waffle slab I and those of the curved solid slab IV with the same flexural rigidities are compared and shown in Fig. 8.18. It can be observed that deflections of the curved solid slab IV are lower by 32 % when compared to those of the curved waffle slab I; this is mainly due to the substantial increase in its torsional rigidity. One should also note that the concrete volume in the curved solid slab IV is 20 % greater than that in the curved waffle slab I. Table 8.20 presents a comparison between the four types of curved slab structures.
C- FINITE ELEMENT APPROACH

8.7 Elastic Response

8.7.1 Curved Reinforced Concrete Waffle Slab Bridge-RWS

The elastic response of the curved reinforced concrete waffle slab bridge model RWS was examined by applying both a uniformly-distributed load and a single concentrated load over the bridge model. The locations of the applied load are shown in Figs. 8.19 to 8.22.

Figures 8.23, 8.24 and 8.25 show the theoretical and experimental results for the deflections due to a single concentrated load at center, E-3, a uniformly-distributed load on the outer lane and a uniformly-distributed load on the inner lane, respectively. The results show good correspondence between the theoretical and experimental results. A study of these figures reveal that in the case of concentrated load, the deflection at the center of the outer edge was almost three times that of the inner edge. This can be attributed mainly to the greater flexibility of the longer outer edge when compared to that of the inner edge. Also, it can be observed that the maximum deflection due to uniformly-distributed load on the outer lane was almost 2 1/2 times the resulting deflection for the same load acting on the inner lane.

Figures 8.26, 8.27 and 8.28 show the compressive strain distribution at the top surface of the slab due to a single concentrated load and to a uniformly-distributed load, respectively. Close agreement can be observed between the theoretical and experimental results. The good correspondence between the theoretical and experimental results
confirms that if a suitable and more refined mesh is chosen for the finite element method, the designer can rely on this method for analyzing slab bridges. Also, from Fig. 8.26 it can be noticed that the maximum strains are under the concentrated load and then decrease rapidly moving away from the loaded point. Comparing the strain distributions for the cases of uniformly-distributed load on the outer and inner lanes, respectively, one can observe that the maximum strains due to the uniformly-distributed load on the outer lane are at the center of the outer edge and are about twice the strains at the inner edge. However, for the case of a uniformly-distributed load on the inner lane, the strains are almost uniform across the mid-span. This difference between the two cases of loading can be attributed to the difference in the load eccentricity from one case to another, and consequently the resulting torsional moment.

Although the curved bridge model RWS was tested three times at different loading positions, the results show that the response of the structure continued to be linear elastic. To ensure such response, the bridge model was tested at half the cracking load to guarantee that there was no microcracking in the model. The transverse strain distributions at the top surface across mid-span, axis E-E, and across axis G-G are given in Figs. 8.29 to 8.31. Close agreement can be noted between the theoretical and experimental results. A study of these figures reveals that the transverse strains are relatively small in comparison to those in the longitudinal direction. This can be attributed to the higher radial flexural rigidity of the waffle slab system. All the transverse strains at top surface, due to the uniformly-distributed load and to the single concentrated load, were tensile strains. Compressive strains were observed under the
concentrated load, as shown in Fig. 8.29.

Figure 8.32 shows the transverse strain distribution across the mid-span for a single concentrated load at center and a uniformly-distributed load, having the same value as the concentrated load but distributed over the whole bridge. It can be observed that unlike the case of the uniformly-distributed load, the resulting strains in the transverse direction due to a single concentrated load were relatively significant. This shows that when a waffle slab bridge is subjected to a concentrated wheel load, it will exhibit excellent transverse load distribution characteristics.

The load cells at the tying down system did not show any indication of uplift at the supports for the case of a single concentrated load at center. However, the finite element model did exhibit insignificant values for the negative reactions at the supports.

The analytical study, using the finite element model RWS, was extended to accommodate other cases of loading across the mid-span. Figure 8.33 presents a comparison between the deflections, across mid-span, due to a single concentrated load at the inner edge, center and outer edge respectively. It is observed that for the case of a single concentrated load at the outer edge, the maximum deflection at the outer edge was almost five times that of the inner edge. This value drops to three for the case of a single concentrated load at the center. Furthermore, the deflection distribution pattern becomes uniform across the mid-span when the load acts at the inner edge. Figure 8.34 presents a comparison between the longitudinal compressive strains at the top surface across the mid-span under the same loads. It can be noted that the maximum compressive strains always occur under the concentrated load as expected.
8.7.1.1 **Comparison Between Curved Waffle Slab Bridge and Girder-Slab Bridge**

An analytical study using the finite element model was carried out to compare the behaviour of two types of curved bridges under the effect of a uniformly-distributed load, single concentrated load and two equal concentrated loads. The first type is a curved waffle slab bridge RWS, and the second one is a curved girder-slab bridge RGS, both having the same plan geometry and cross-sectional dimensions. This study was undertaken to investigate the structural efficiency of the curved waffle slab bridge over the girder slab bridge.

According to the proposed formulae for the flexural and torsional rigidities, presented in Chapter III, the values of the tangential flexural rigidity of both types are the same. However, the radial flexural rigidity of the girder-slab bridge is only 5% that of the waffle slab bridge. Moreover, the torsional rigidity of the former is almost half that of the latter. Figures 8.35 and 8.36 show the comparison for deflections between the waffle slab bridge and the girder-slab bridge due to a single concentrated load at the center of mid-span and two equal concentrated loads at the center of each lane. It can be observed from these figures that the waffle slab bridge exhibits much smaller deflection than that of girder-slab bridge. For example, the curved concrete waffle slab bridge when subjected to a single concentrated load at the center, E-3, has a maximum deflection, at the center of the outer edge, equal to 70% that of the girder-slab bridge. Since both slabs have the same tangential flexural rigidity, one would expect the deflection of both systems to be the same. However, the substantial increase in the torsional rigidity of the waffle slab affects a decrease in the deflection by about 30%.
It is interesting to note that the deflection of the waffle slab bridge at the center of the outer edge is three times that of the inner edge. However, this ratio increases to 5.0 for the girder-slab bridge. This can be attributed mainly to the lower torsional rigidity of the girder-slab bridge. Thus, the former exhibits higher twist, at mid-span, than that of the latter.

It should be noted that, for the curved girder slab bridge, the maximum deflection due to a single load system was 85% of that due to the two-equal concentrated load system. However, for the curved waffle slab bridge the resulting deflection for both cases is almost the same.

Figures 8.37 and 8.38 show the comparison for strains, at the top surface, between the waffle slab bridge and the girder-slab bridge due to a single concentrated load at center, E-3, and the two-equal concentrated loads at the center of each lane. It is observed that the tangential compressive strain, at the top surface, for the girder-slab bridge is always higher than that of the waffle slab bridge. Furthermore, the strain in the girder-slab bridge was much more localized than that of the waffle slab bridge. It is interesting to note in Figs. 8.37 and 8.38 that the longitudinal strains in the outer edge of the girder-slab bridge is four times that of the inner edge. However, this ratio is only 1 1/2 times in the waffle slab bridge. This indicates that the waffle slab bridge has excellent transverse load distribution characteristics, when compared to the girder slab bridge, due mainly to its superior radial flexural and torsional rigidities.

Figures 8.39 and 8.40 show the transverse strain patterns across the mid-span for the waffle slab and girder-slab bridges. It can be noted that the girder-slab bridge exhibits
higher strains than those in the waffle slab bridge. Furthermore, the transverse strains in the curved-girder slab bridge are relatively large when compared to those in the longitudinal direction. This can be attributed mainly to the lower radial flexural rigidity of the curved-girder slab.

Figure 8.41 through Fig. 8.44 present comparisons between the deflections and the strains, across the mid-span, for curved waffle slab and girder-slab bridges due to a single concentrated load and a uniformly-distributed load at different positions. It can be observed that the deflection patterns for waffle slab and girder-slab bridges are identical when the concentrated load is at the inner edge. On the other hand, the maximum deflection of the girder-slab bridge is about twice that of the waffle slab bridge when the load is at the outer edge. The same can be observed for the case of the uniformly-distributed load on the inner and outer lanes, respectively.

It can be concluded that an eccentric load on the outer edge of a curved slab bridge would give rise to a twisting moment that is much greater in magnitude than that caused by the same load applied at the center. Thus, the curved waffle slab bridge, with its significant torsional rigidity, is able to distribute the eccentric load transversely more effectively than a curved girder slab bridge. Furthermore, it is interesting to note that the transverse ribs in straight right bridges act only as secondary members in maintaining structural integrity. However, due to the inherent interaction of bending and torsion in horizontally curved bridges, these ribs become major load-carrying elements in the curved waffle slab bridge.
8.7.1.2 Comparison Between Curved Waffle Slab Bridge and Solid Slab Bridge

The analytical finite element model was used to compare the behaviour of the curved waffle slab with a solid slab having the same volume of concrete and reinforcing steel. Figures 8.45 to 8.49 present comparisons between the waffle slab bridge and the solid slab bridge for deflection and strain patterns due to a single concentrated load as well as two-equal concentrated loads. It can be observed that the waffle slab bridge exhibits a much smaller deflection than the solid slab bridge. Furthermore, the deflection of the waffle slab bridge at the center of the outer edge is three times that of the inner edge. However, this ratio drops to 2 1/2 times in the case of a solid slab bridge. This can be attributed to the higher flexural rigidities of the waffle slab when compared to the solid slab, and the high torsional rigidity of the solid slab, respectively. It is interesting to note that the resulting deflection patterns due to a single concentrated load and two-equal concentrated loads are identical. A study of Figures 8.46 to 8.49 reveal that both the waffle slab bridge and the solid slab bridge have the same behaviour under the effect of transverse load. However, the solid slab bridge exhibits higher strains in the longitudinal and transverse directions when compared to the waffle slab bridge. Figures 8.50 to 8.53 present a comparison between the deflections and strains for the curved waffle slab and solid slab bridges under the effect of a single concentrated load as well as a uniformly-distributed load applied at different positions across the mid-span. It is observed that the solid slab bridge exhibits larger deflections, for all cases, when compared to the waffle slab bridge. The maximum difference in deflection between both systems was 20%. The same observation was noticed for the case of a uniformly-
distributed load.

A study of Figs. 8.52 and 8.53 reveals that the solid slab bridge has higher strains for all cases of loading. This is again due to the higher flexural rigidity of the waffle slab bridge as compared to the solid slab bridge. For a single concentrated load acting at the center of the outer edge, the resulting strain at the outer edge for the solid slab bridge is 1 1/2 times that at the inner edge. However, this ratio increases about four for a waffle slab bridge. This again is due to the high torsional rigidity of the solid slab structure.

From this study, it can be concluded that in a relatively wide curved bridge, a waffle slab system has the potential of being a more functional and economical alternative to solid slab and girder-slab systems. This is due to its significant flexural and torsional rigidities when compared to the other two alternatives.

8.7.2 Curved Prestressed Concrete Waffle Slab Bridge-PWS

8.7.2.1 At Transfer

The curved prestressed concrete waffle slab bridge model PWS was prestressed in both the tangential and radial directions. At transfer, the design called for no tension in the concrete due to the prestressing in the tangential direction. For practical reasons and to accommodate the changes in the span length in the tangential direction, the amount of prestressing was varied between the inner and the outer ribs as shown in Fig. 7.33. The outer rib had a higher prestress value than that of the inner rib to counteract the greater flexibility of the former. The proper sequence of posttensioning the longitudinal cables, as explained in Chapter VII, is also important. Without the proper sequence
distortion of cross section and/or initiation of cracks will result.

It should be kept in mind that earlier voided slab bridges were prestressed only in the longitudinal direction and this led to extensive cracking at transfer and at working loads due to the tensile stresses generated in the transverse direction. To counteract the tensile stresses in the concrete both OHBDC (Ministry of Transportation and Communications 1991) and AASHTO (1983) specifications recommend that the bridge deck be additionally prestressed in the transverse direction. To avoid splitting tensile stresses in the transverse direction due to posttensioning the longitudinal tendons, transverse prestressing should precede the longitudinal one. In a horizontally curved bridge with a small radius of curvature, serious attention should be given to the very high radial forces generated by the horizontal curvature of the prestressing tendons. The radial force, $q_r$, resulting from the tendon force $T_p$ is given as:

$$q_r = \frac{T_p}{R} \quad (8.2)$$

where $R$ is the horizontal radius of curvature of the tendon. In the PWS model, the maximum value of this force was 116 lb/in. (20.3 N/mm) in the inner rib. The response of a horizontally curved waffle slab bridge under the effect of such radial forces can be divided into the following three separate actions, as shown in Fig. 8.54:

1- Local slab action of the concrete cover over the tendons; 2- Plate action of the part of the longitudinal ribs which lies between two adjacent transverse ribs; and 3- Global or overall bridge action due to the radial forces.

The local slab action is probably the most important one since it may cause
failure of bridge structure (Podolny 1985). For analysis of the prestressed model PWS the cover C between the tendon and the inner face of the rib was considered as a plain concrete slab of thickness C. The punching shear failure stresses between the tendon and the slab of thickness C were calculated. These stresses were not to exceed the allowable shear strength of the concrete to avoid tendon splitting out of concrete. Following the ACI 318-89 Code (1989), the maximum nominal resisting shear strength of plain concrete \( V_c \) is not to exceed \( 2 \left( f_c' \right)^{1/2} \) for beams and one-way action slab. For PWS model, \( V_c = 158 \text{ psi} \) (1.0 MPa), and the maximum shearing stress in any longitudinal rib did not exceed 53 psi (0.37 MPa). It should be noted that any horizontal movement of the tendon during the construction of this structure will significantly affect the resisting shear strength. Thus extreme care should be taken during the erection and prestressing of such structures.

The plate action: consider the part of the longitudinal ribs lying between two adjacent transverse ribs as an individual plate fixed on three edges: the top of the slab and the two transverse ribs, and free on the bottom edge. This plate is acted upon by the radial force from the tendon and the partially counteracting force due to the arching effect of the longitudinal compression due to the tendon prestress. In other words, the tendency to push radially inward by the curved tendons is counteracted to some degree by the tendency to push radially outward by the concrete as shown in Fig. 8.54. These forces are resisted by the plate action without any overstress due to its higher rigidity.

The global or the overall bridge action due to the radial forces can be treated by considering the curved bridge as a curved deep beam in resisting these forces in its plane.
These forces create tensile stresses in the longitudinal direction at the inner edge and compressive stresses at the outer edge. This can be observed in Fig. 8.56 which shows the distribution of the longitudinal strains at the top surface of model PWS due to prestressing only.

Figures 8.55 to 8.57 show the theoretical and experimental results for deflection, strains at top and bottom surfaces due to prestressing bridge model PWS. Figure 8.55 shows the distribution of the upward deflection (camber) due to prestressing, with good correspondence between the theory and experiment. It should be noted that the deflection at the center of the outer edge is about two times that of the inner edge. This is again due to the greater flexibility of the outer edge. Figures 8.56 and 8.57 show the resulting strains at the top and bottom surfaces of the slab, due to prestressing. Close agreement is evident between the theoretical and the experimental strains. All the stresses at the top surface are compressive except at the center of the inner edge. There small tensile stresses were recorded and these are mainly due to the global action of the slab under the effect of the horizontally radial forces. The maximum compressive stress at mid-span was 420 psi (2.9 MPa). Figure 8.58 presents the transverse strain distribution at the top surface due to prestressing in both directions. Close agreement can be noticed between the theoretical and the experimental results. It is interesting to note that the transverse compressive strain at the inner edge is higher than that at the outer edge. This is because the transverse ribs near the inner edge are spaced closer than at the outer edge.
8.7.2.2 At Working Stage

The elastic response of the curved prestressed concrete waffle slab bridge under working load condition was examined by applying both a uniformly-distributed load and a single concentrated load over the bridge model. The uniformly-distributed load was applied on the outer lane as well as on the inner lane, while the single concentrated load was applied at different locations across axis E-E and G-G in the manner shown in Figs. 8.59 to 8.61. Figures 8.62 to 8.69 show the deflection profiles due to a single concentrated load and a uniformly-distributed load. The results show good agreement between the theoretical and experimental results. A study of these figures reveals that the curved prestressed concrete waffle slab bridge exhibits the same behaviour as the reinforced concrete one and the same conclusions can be reached under the different load positions. It can be observed that the deflection at the center of the outer edge due to a single concentrated load at point E-3 is almost 1 1/2 times the resulting deflection for the same load acting at point G-3.

Figures 8.70 to 8.87 present the strain distribution at the top and bottom surfaces of the bridge model due to a single concentrated load and uniformly-distributed load. The results show good agreement between the theoretical and experimental results. It can be noted that the maximum strains occur under the single concentrated load and decrease rapidly moving away from the loading point in both directions. On the other hand, the distribution of the strain under the effect of uniformly-distributed load appears to be uniform and no tensile stresses were recorded at the bottom surface.

The transverse strain distribution at the top surface across the mid-span, axis E-E
and axis G-G during the transverse prestressing, full prestressing and working load conditions are shown in Figs. 8.88 to 8.95. A study of these figures reveals that all the strains at the top surface due to transverse prestressing was compression with a maximum value at the inner edge. After applying the longitudinal prestressing, these strains reduced by about 30%. This can be attributed mainly to the tensile stresses generated in the transverse direction by the longitudinal prestressing. It should be noted, that almost all the transverse strains at the top surface under the effect of uniformly-distributed load are compressive strains. However, some tensile strains did develop under the effect of a single concentrated load, as shown in Figs. 8.92 and 8.95. It can be concluded that transverse prestressing is very important in eliminating all the tensile stresses and thus controlling the longitudinal cracks which may be developed in non-transversely prestressed bridges.

Negative reactions were recorded in the case of a single concentrated load at the outer edge and a uniformly-distributed load on the outer lane. This indicates that the model was subjected to uplift under the eccentric load conditions and that the tying-down system was essential in order to model the system properly. This is generally the case in horizontally curved bridges with small radius of curvature. However, in prototype bridges, the self-weight of the bridge may be sufficient to counterbalance the uplift.

It is interesting to note that the elastic response of a prestressed concrete waffle slab bridge can be predicted more accurately than a reinforced concrete one, due to the absence of local cracking and microcracking.
8.7.3 Curved Prestressed Concrete Girder-Slab Bridge-PGS

8.7.3.1 At Transfer

The curved prestressed concrete girder-slab bridge model was prestressed in the tangential direction with no tension allowed in the concrete at the transfer stage. To resist the high flexibility of the outer edge, the outer rib was subjected to a higher prestress than that of the inner rib. The proper sequencing, explained in Chapter VII, was applied. The analysis for the effect of horizontally radial forces was similar to the case of PWS bridge. The first two actions, the local slab action and the overall action, are identical to those discussed in curved prestressed concrete waffle slab bridge. On the other hand, for the plate action each longitudinal rib is considered as a one way plate fixed on the top slab from one side and free on the other side. The plate is acted upon by the radial forces from the tendons and the arching effect from the compressive stresses generated from prestressing. The stresses, due to this plate action, at the junction between the deck slab and the longitudinal ribs should not exceed the rupture stress of the concrete. In the studied model PGS, these stresses did not exceed 10 % of the allowable rupture stresses stipulated by the Code (ACI 1989). Such low stresses are attributed to the counteracting of the different radial forces affecting the longitudinal ribs.

Figure 8.96 shows good correspondence between the theory and experiment for the upward deflection distribution (camber) due to prestressing. Figures 8.97 and 8.98 show the strain distribution at the top and bottom surfaces due to prestressing, with good agreement between the theory and experiment.
8.7.3.2 At Working Stage

To study the behaviour of curved prestressed girder-slab bridge in the elastic domain, both a uniformly-distributed load and a single concentrated load were applied over the bridge model in the manner shown in Fig. 8.59. The deflection profiles due to a single concentrated load as well as a uniformly distributed load at different locations were presented in Figs. 8.99 to 8.103. The results show good agreement between the theoretical and the experimental results. It can be noticed from these figures that the curved prestressed concrete girder-slab bridge exhibits greater deflection, under the same loading conditions, when compared to those obtained from the curved prestressed concrete waffle slab bridge. This is attributed to the lower torsional rigidity of the former bridge system. Also, it can be observed that the deflection at the center of the outer edge due to a single concentrated load at the same position is about three times the deflection for the same load acting at the center of the bridge model. The deflection at the center of the outer edge due to a uniformly-distributed load on the outer lane is about four times the resulting deflection for the same load acting on the inner lane.

Figures 8.104 to 8.113 show the strain distributions at the top and bottom surfaces of the bridge model due to a single concentrated load and uniformly-distributed load as well. Close agreement can be noted between the theoretical and experimental results. It can be observed that the girder-slab bridge model exhibits higher strains, under the same loading conditions, when compared to the waffle slab bridge model. In the case of a single concentrated load, high strains occur under the load and decrease rapidly moving away from the position of the load. However, for the case of a uniformly-distributed
load, unlike the case of the single concentrated load, the strain distributions are almost uniform. Furthermore, the strains in the girder-slab bridge were much more localized than those in the waffle slab bridge. This is due to absence of the transverse ribs which assist in distributing the load transversely in an orthotropic manner. The transverse strain distributions at the top surface across the mid-span, axis E-E, and across axis G-G due to prestressing as well as working load conditions are shown in Figs. 8.114 to 8.119. It can be observed that tensile strains were developed in the transverse direction due to the longitudinal prestressing. These strains increase by applying the transverse loads, uniformly-distributed load or single concentrated loads, as shown in Figs. 8.114 to 8.119, respectively. Furthermore, compressive strains in the transverse direction were observed under the single concentrated load and these strains changes rapidly to tensile strains, when moving transversely away from the position of the load as shown in Fig. 8.114.

For the purpose of comparison, two identical models of prestressed waffle slab bridge and girder slab bridge, having the same prestressing forces and the same material properties, were analyzed using the finite element approach. This study was carried out to show the structural efficiency and deficiency of each system under various loading conditions. Figures 8.120 to 8.124 show the comparison between the deflection distributions across the mid-span for the two systems under the effect of a single concentrated load, at different positions, and uniformly-distributed load on the outer and inner lanes, respectively. It can be observed that the girder-slab bridge exhibits larger deflections than that of the waffle slab bridge system, especially when the load is at the outer edge. This is due to the lower torsional rigidity of the former system, as indicated
Figures 8.125 to 8.129 present comparisons between the strain distributions at top surface, across mid-span, for both systems. It can be noticed that the girder slab bridge has higher strains than that of the waffle slab bridge. Furthermore, the strain in the former system was more localized than that of the latter, due to relatively lower radial flexural rigidities of the girder slab system.

Figures 8.130 to 8.132 show the transverse strain patterns at axis E-E for the two systems under the effect of prestressing, single concentrated load and uniformly-distributed load. It is interesting to note that, unlike the case of girder-slab bridge, the transverse prestressing in the waffle slab bridge eliminates all the tensile stresses at the top surface. In addition, the girder-slab bridge exhibits higher strain, in the transverse direction, than that in the waffle slab due to the lower radial flexural rigidity of the former system.

8.8 Post-Elastic Response

8.8.1 Curved Reinforced Concrete Waffle Slab Bridge-RWS

After completing the elastic loading (service load) tests of the bridge model RWS, the model was loaded beyond its elastic load to failure by a two-equal concentrated loads at mid-span, each at the center of each lane. Figures 8.133 and 8.134 show the experimental and analytical load-deflection and load-strain results at the center of the bridge model, point E-3. It is observed that there is an excellent agreement between theory and experiment in the elastic range and reasonable agreement in the post-elastic...
range. Figures 8.135 and 8.136 demonstrate the difference in response between the center of the inner edge, center of the model and center of the outer edge. It is interesting to note that at failure there was almost no difference in the longitudinal strain across mid-span. This indicates that the transverse ribs in the waffle slab system functioned satisfactorily, at all intensities of load, in distributing the load transversely across the width of the slab. On the other hand, the deflection across the mid-span varied between the outer and inner edges. This, as indicated before for the elastic range, is due to the greater flexibility of the outer edge.

Figures 8.137 and 8.138 show the crack propagation of the bridge model RWS due to two-equal concentrated loads applied at mid-span at the center of each lane, by means of the finite element analysis. The first two cracks appeared directly under the load at the bottom surface of the longitudinal ribs 2-2 and 4-4, at a load of 2.3 kip (10.2 kN) as shown in Fig. 8.137. Upon increasing the load, other elements at the bottom surface of the longitudinal ribs were severely cracked across the center line of the bridge model. The cracks propagated both transversely and vertically through the depth and the analysis was terminated when the cracks reached the bottom surface of the deck slab, at a load of 4.0 kip (17.8 kN) as shown in Fig. 8.138. Figures 8.139 to 8.141 show the crack propagation and the single-line crack pattern of the tested bridge model RWS which confirm the theoretical results from the finite element model.

8.8.2 Curved Prestressed Concrete Waffle Slab Bridge-PWS

The bridge model PWS was loaded to failure by two-equal concentrated loads at
mid-span and at the center of each lane. The collapse load as well as the orientation of the cracks were observed and compared to the theoretical predictions. Load versus deflection as well as load versus strains at different locations of the slab model were recorded.

Figures 8.142 and 8.143 show the experimental and analytical load-deflection and load-strain results at the center of the bridge model, point E-3. Close agreement can be observed between theory and experiment in the elastic range and reasonable agreement in the post-elastic range. Figures 8.144 and 8.145 demonstrate the difference in response between the center of the inner edge, center and the center of the outer edge of the tested bridge models. It can be observed that the longitudinal strains across the mid-span, axis E-E, are almost uniform near the collapse load. This indicates excellent transverse-load distribution capability of the waffle slab system and that the model exhibits single-line crack pattern as shown in Fig. 8.150. However, the deflections varied across the mid-span due to the greater flexibility of the outer edge.

Figures 8.146 and 8.147 show the propagation of cracks in the bridge model PWS at all intensities of load up to failure load. The first cracks appeared directly under the load of 4.5 kip (20 kN) as shown in Fig. 8.146, and they extended along the center line of the model, axis E-E, intersecting all the longitudinal ribs. The cracks propagated through the depth of the bridge mode! and a crushing failure of the concrete deck slab occurred at a load of 7.3 kip (32.5 kN) as shown in Fig. 8.147. The experimental crack propagation and the model after collapse are shown in Figs. 8.148 to 8.151.
8.8.3 Curved Prestressed Concrete Girder-Slab Bridge-PGS

The curved prestressed girder-slab bridge model PGS, like the case of waffle slab bridge models, was tested to failure by two-equal concentrated loads at mid span, and at the center of each lane. The cracking load, crack propagation, failure mode and collapse load were recorded and compared to the theoretical predictions. Figures 8.152 and 8.154 present the experimental and analytical load-deflection and load-strain results at the center of the bridge model. Close agreement can be noted between theory and experiment in the elastic range and reasonable agreement in the post-elastic range. This is mainly due to the assumptions adopted in the analysis such as ignoring the strain hardening of the steel material as well as the tension stiffening of the concrete. Figures 8.135 and 8.155 show the difference in response between the center of the inner edge, center of the model and center of the outer edge. It can be observed that the deflection varies considerably across the mid-span, axis E-E, in both the elastic and post-elastic ranges. This is again due to the lower torsional rigidity of the PGS model. Unlike the case of the waffle slab bridge, considerable difference can be noted between the longitudinal strains, across the mid span, close to the collapse load. This can be attributed mainly to the absence of the transverse ribs which help in transmitting the loads transversely across the width of the bridge. The first crack was observed at the bottom surface of the rib on axis 2-2 under the applied load at 4.0 kip (17.8 kN). This flexural crack was formed when the tensile stress in the bottom surface of the concrete, due to the pure bending moment, reached its modulus of rupture. Upon increasing the loading further, the cracks propagated across the width of the bridge model but it did not reach the inner rib. The flexural crack at
mid-span started to propagate vertically through the depth of the bridge model at 4.5 kip (20 kN) and reached almost half the depth at 5.5 kip (24 kN). This crack stopped propagating vertically, while another two inclined cracks developed suddenly. By increasing the load further, large cracks, caused by severe twisting, appeared on all the longitudinal ribs except the inner one as shown in Figs. 8.156 to 8.158. The model failed at 6.5 kip (28.9 kN) when the ultimate capacity of the longitudinal ribs, without the contribution of the deck slab, was reached due to the combined action of torsion and bending. It should be noted that the absence of vertical steel ties between the longitudinal ribs and the deck caused complete separation between the longitudinal ribs and the deck slab at failure, as shown in Fig. 8.157. The failure can be attributed mainly due to the shear failure of the longitudinal ribs without any sign of failure in the concrete deck slab. No yielding of reinforced steel or prestressed wires was observed.

Figures 8.159 and 8.160 show the crack propagation from the finite element analysis. Good correspondence can be noted between the experimental and analytical results. The theoretical cracking load was 3.85 kip (17.1 kN) with good agreement with the experimental results. However the analysis was terminated at a maximum load of 5.7 kip (25.4 kN) due to cracking of a considerable number of elements and the finite element model could not sustain any further load.

A summary of results for the cracking loads, collapse loads and the failure modes for the three bridge models RWS, PWS and PGS are presented in Table 8.21. It can be observed that there is a close agreement between theory and experiment. Moreover, the prestressed concrete bridge model was much stiffer and stronger than the reinforced
concrete model. Also, it can be noted that the presence of the transverse ribs in curved bridges affect significantly the structural response of such bridges and their failure mode.

8.9 Effect of Bridge Horizontal Radius to Bridge Width Ratio on the Behaviour of Curved Bridges

A parametric study was carried out on RWS and RGS curved bridges, using the finite element approach, to examine the influence of the bridge horizontal radius to width ratio (R/W) on the elastic behaviour of such structures. Figures 8.161 to 8.164 show the deflection patterns across the mid-span, axis E-E, for bridge models RWS and RGS, with aspect ratio equals two, due to a single concentrated load as well as a uniformly-distributed load. It should be noted that the radius to width ratio (R/W) plays a significant part in controlling the distribution of the deflections across the width of the slab in both systems. For example, the larger the R/W ratio the smaller the differences in deflection between the outer and the inner edge. It can be observed from these figures that RWS bridge, in general, has smaller deflections than those in the RGS bridge due to its higher torsional rigidity. This difference in deflection between the two systems reduces with increase in the radius to width ratio. Finally, it is interesting to note that the behaviour of a curved waffle slab bridge as well as a girder-slab bridge with radius to width ratios equal to 4.0, 6.0 and 8.0 are almost identical. As a result, both systems with radius to width ratio greater than 8.0 can be simplified and treated as straight bridges and the effect of curvature can be ignored.

Figures 8.165 to 8.168 compare the deflection distributions of curved RWS and

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RGS bridges, having a small radius to width ratio equal to 1.15, to straight RWS and RGS bridges, having a radius to width ratio approaching infinity. This comparison was carried out under the effect of uniformly-distributed load as well as for an equivalent single concentrated load at different positions. A study of these figures reveals that a curved RWS bridge has a smaller deflection than that of a curved RGS bridge, as discussed before. However, in straight bridges both systems have almost the same deflection, when subjected to single concentrated load or uniformly-distributed load, since they have the same flexural rigidity in the longitudinal direction. It can be concluded that unlike the case of straight bridges, torsional rigidity, as well as flexural rigidity, plays an important role in controlling the behaviour of horizontally curved bridges with a small radius of curvature.

Figures 8.169 to 8.172 present the longitudinal strain distribution at the top surface for RWS as well as RGS models under the effect of different loading conditions. The comparison was carried out on bridges with different horizontal radius to width ratio to investigate the influence of the bridge curvature on the strain distribution. It can be observed that the larger the radius to width ratio the better is the distribution of the longitudinal strains across the width of the slab. Again, it can be noted that the strains are almost identical for slabs with a radius to width ratio greater than 4.0. As indicated before, the RGS bridge exhibits higher strains than RWS bridge for all radius to width ratios.

Figures 8.173 to 8.176 present a comparison of the longitudinal strains in curved and straight bridges. It can be observed that a curved RGS bridge has higher strains than
a RWS bridge under the effect of concentrated loads or uniformly-distributed loads. However, the strain distributions in straight RWS and RGS bridges are identical for the case of uniformly-distributed load and more localized for the RGS bridge than for RWS bridge for the case of single concentrated load. Figures 8.177 to 8.180 show the transverse strain distributions for the two systems with higher strain values for the RGS bridge than for the RWS bridge.

D- YIELD LINE APPROACH

8.10 Collapse Load of the Tested Models Using Yield-Line Formulae

The yield line theory was used to predict the collapse load of curved reinforced and prestressed concrete waffle slab bridges. Tables 8.22 and 8.23 present a comparison of the theoretical and experimental collapse loads of the curved RWS and PWS bridge models. The theoretical analyses were carried out using the developed yield line formulas, based on the assumed crack pattern, and the finite element technique.

The reinforced concrete bridge model, RWS, was tested to collapse by two-equal concentrated loads at mid-span, axis E-E. The first crack was observed at 2.5 kip (11.1 kN) on the bottom surface of the longitudinal rib 3-3. The cracks propagated through the width and the depth of the bridge model, forming a single-line crack pattern as shown in Fig. 8.141. Finally, the model collapsed at 5.0 kip (22.3 kN) due to extensive yielding of the reinforcing steel associated by large deflection at mid-span, as shown in Fig. 8.140. The model failed in flexure at twice the cracking load and with no sign of distress due
to shear or torsion. The theoretical collapse load was 4.6 kip (20.5 kN) using the
developed yield line formulae, (see Appendix A-6). It can be observed that the theoretical
collapse load, using the yield line approach, is in good agreement with the experimental
collapse load, the former being conservative and lower than the latter, due to the reasons
discussed before.

The curved prestressed concrete bridge model, PWS, was tested also to collapse
under the same type of load used for the RWS bridge model. The cracking load, collapse
load and the crack pattern of failure were recorded and compared with those from the
analytical studies, finite element and yield line formulas. At 4.5 kip (20 kN) the first
crack was observed at the bottom surface of the longitudinal rib 3-3 and adjacent to the
transverse rib E-E. The cracks propagated toward the outside edge and reached it at a
load of 5.0 kip (22.3 kN). Upon increasing the load further, the cracks propagated slowly
through the depth of the model and widened considerably. The development of a single,
or a few, wide cracks is characteristic of a prestressed concrete member with unbonded
tendons. The model failed in flexure at almost twice the cracking load at a value of 8.5
kip (37.8 kN). The large extension in the prestressing wires caused the crack at mid-span
to widen to 3/8 in. (9.5 mm), pushing up the neutral axis, and resulting in rushing of the
concrete immediately above the crack. Figures 8.148 and 8.149 show the failure of the
PWS bridge model. There was no indication of shear or torsion cracks due to the high
torsional rigidity of the waffle slab system. After releasing the load, the remaining
prestressing forces in the tendons caused the buckling of the non-prestressed steel, as
shown in Fig. 8.150.
It should be noted that, at failure, the maximum deflection of the prestressed concrete bridge model is much smaller than that of the reinforced concrete bridge model. This can be attributed mainly to the different modes of failure for both systems, yielding of the reinforcing steel in the latter and the crushing failure of the concrete deck slab of the former.

The theoretical collapse load in prestressed bridges with unbonded tendons may be difficult to assess since the stress in the tendon cannot be accurately predicted at collapse. However, there were three available methods to compute the stress in the tendons, based on the experimental results, as shown in Appendix A-2. The theoretical collapse loads of the PWS model based on the developed yield line formulas, and using the different methods for calculating the stress in the tendons are given in Table 8.23, while the calculations are presented in Appendix A-6. It should be noted that using the ACI Code (1989) and CAN Code (Canadian Standards Association 1984) the estimated theoretical collapse load is in good agreement with the experimental collapse load. However, the method suggested by Kennedy and El-Sebakhy (1982) gives a conservative value when compared to the other two methods. Of course, it should be remembered that whether it is a design or an analysis problem, proper load and performance factors should be incorporated with the derived formulae for the yield-line method.

The theoretical analysis, using the yield line formulae and the finite element method, were extended to predict the collapse load of RWS and PWS due to a single concentrated load at the inner and outer edges respectively. It should be noted that these cases of loading were not covered in the experimental program. However, the good
correspondence between the finite element and the yield-line results, as presented in Tables 8.22 and 8.23, confirms the assumed crack pattern at failure and consequently, the developed formulae for estimating the collapse load. Finally, it is interesting to note that the collapse load for both RWS and PWS bridge models due to a single concentrated load at the inner edge is about 1 1/2 times that of the load at the outer edge. This can be attributed to the lower flexibility of the inner edge.

8.11 Parametric Study

A parametric study was carried out on RWS curved bridges, using the yield-line theory and the finite element approach, to investigate the influence of the aspect ratio \(2L/W\), bridge horizontal radius to width ratio \(R/W\) and the applied load locations on the collapse load. Table 8.24 shows good correspondence between the calculated theoretical collapse load, from the finite element method and the developed yield-line formulae, with good correspondence between the two methods. It should be noted that the aspect ratio, horizontal radius to width ratio and applied load location play a significant role in determining the magnitude of the collapse load. For example, the larger the radius to width ratio the smaller the collapse load when acting at the inner edge and the larger for those with load at the outer edge. This is due to the increasing length of the of the inner edge and decreasing length of the outer edge. It is interesting to note that the collapse loads of RWS bridge due to load at the center of the outer edge and at the center of the inner edge are almost identical for radius to width ratio equals 8.0. This indicates that the RWS model can be treated as a straight bridge, in determining the
collapse load, when the radius to width ratio exceed a certain value, which also depends on the width of the slab and the aspect ratio as well. Furthermore, the developed yield-line formulae in this study were found to be the same as the ones developed by Kennedy and El-Sebakhy (1982), for rectangular waffle slab bridge, when the radius R approaches infinity. This observation supports the reliability of using the proposed yield line equations presented herein to predict the collapse load of both curved and straight waffle slab bridges.

Finally, it should be mentioned that the collapse loads for the different bridge models, given in Table 8.24, were based on the first mode of failure, given in Figs. 6.2 and 6.5. However, the second mode of failure, Figs. 6.4 and 6.6, might be developed for some particular cases of small aspect ratio and large radius to width ratio. Thus, in such cases the collapse load should be determined from both failure patterns and the lowest collapse load should be used for conservative design, as shown in Table 8.24.

8.12 Determination of the Constants \( \beta'' \) and \( \beta' \)

From the derived yield-line equations for the cases of load at the inner and outer edges, it is observed that the predicted collapse load \( P_u \) for a mode of failure shown in Figs. 6.4 and 6.6, is a function of the constants \( \beta'' \) and \( \beta' \), see Eqs. 6.24 and 6.37. The constants, \( \beta'' \) and \( \beta' \) are the ratio of the outer edge deflection to the inner edge deflection for the cases of load applied at the inner and outer edges, respectively.

A design curve was developed for determining the coefficient \( \beta'' \) and given in Fig. 6.3. This design curve was constructed for different aspect ratios and radius to width
ratios as well. On the other hand, suggested values for the coefficient $\beta'$ were derived from the extensive results obtained from the parametric study. For example, a curved waffle slab bridge with aspect ratio equals to 2.0 and the radius to width ratio falling between 1.0 and 8.0, the coefficient $\beta'$ can be taken as 4.0, while for an aspect ratio equal to 1.25 and the radius to width ratio falling between 1.0 and 8.0, the coefficient $\beta'$ can be taken as 8.0.
CHAPTER IX

SUMMARY, CONCLUSIONS AND FUTURE RESEARCH

9.1 Summary

Extensive experimental and theoretical studies were carried out to determine the various rigidities of curved concrete waffle slab bridges and to investigate its behaviour under service and ultimate loads. A detailed literature review was conducted in order to establish the foundation for this study. It was observed that little research effort has been directed toward the behaviour of such structures. From the theoretical and experimental study, simplified expressions were developed to estimate the various rigidities of curved concrete waffle slab structures. Moreover, it was found that the curved waffle slab bridge has the potential of being a more functional and economical alternative to the solid slab and girder-slab system due to its significant flexural and torsional rigidities. The analytical studies were based on the finite difference technique, finite element method and yield-line theory. Simplified yield-line expressions were developed to predict the collapse load of curved reinforced and prestressed concrete waffle slab bridges. Good agreement
between the analytical and experimental results were noted.

A parametric study was conducted to examine the influence of a number of variables on the behaviour of curved concrete waffle slab bridges. The results revealed that the bridge horizontal radius to width ratio, aspect ratio, loading position and the presence of the transverse ribs significantly influence the elastic behaviour as well as the ultimate load capacity of such structures.

9.2 Conclusions

Based on the limited results from the experimental and parametric studies the following conclusions are drawn:

1- The good agreement between the theoretical results, using the proposed equations, and the experimental results supports the reliability of the proposed formulae for estimating the various rigidities of curved concrete waffle slab structures.

2- The expressions developed for estimating the various rigidities are applicable to prestressed and reinforced concrete waffle slab structures as well as to composite structures for both the pre-cracking and post-cracking stages of concrete.

3- The idealization of a curved waffle slab bridge as a wide beam with constant rigidities is incorrect and unreliable. This is because the torsional and transverse flexural rigidities of curved waffle slab structure change considerably across the width of the bridge.

4- The good correspondence between the theoretical results from the finite difference technique, finite element analysis and those from tests confirm the validity of
the proposed formulae for predicting the rigidities of curved concrete waffle slab structures.

5- The results from the finite difference technique show the feasibility of using an equivalent orthotropic plate system to predict the behaviour of curved concrete waffle slab structures.

6- For curved slab structures the flexural rigidities as well as the torsional rigidity play an important role in controlling the transverse deflection.

7- A curved waffle slab bridge helps in reducing local deformation due to heavy concentrated loads due mainly to its significant transverse flexural and torsional rigidities. It also able to distribute eccentric loads transversely quite effectively in comparison to a curved girder-slab bridge.

8- The longitudinal and transverse flexural rigidities of curved concrete waffle slab structures are much higher than those of an identical curved solid slab with the same geometry and volume as that of the curved waffle slab. As a result, the former are structurally more efficient than the latter.

9- The transverse ribs in straight right-angled bridges act predominantly as secondary members in maintaining structural integrity. However, due to the inherent interaction of bending and torsion in horizontally-curved bridges, the transverse ribs become major load-carrying elements.

10- Curved prestressed concrete waffle slab construction is much stiffer and stronger than a curved reinforced concrete waffle slab construction.

11- Proper sequencing of tendons posttensioning in both directions in curved
prestressed concrete waffle slab structures is important. Otherwise distortion of the cross section and/or initiation of cracks may result.

12- In a horizontally curved prestressed concrete waffle slab and girder-slab bridges, serious attention should be given to the very high radial forces generated by the horizontal curvature of the prestressing tendons. In this respect, sufficient thickness of concrete should be provided between the tendons and the inner face of the ribs to ensure against the "popping out" of the tendons.

13- In curved prestressed concrete waffle slab bridges, transverse prestressing can eliminate all tensile stresses in the top surface of the deck, and consequently control the longitudinal cracks which may develop in bridges with no transverse prestressing.

14- Horizontally-curved wide bridges may be subjected to uplift at the supports under partial loads, depending on the width, radius of curvature of the bridge, and self weight of the bridge. In such cases, a tying down system should be provided at the supports.

15- The transverse ribs in horizontally curved concrete bridges play an important role in determining the failure mode of such structures, and consequently their ultimate load capacity.

16- The proposed yield-line formulae can effectively predict the collapse load of curved reinforced and prestressed concrete waffle slab bridge provided that the moments of resistance of the bridge are assessed correctly.

17- The bridge aspect ratio as well as the bridge horizontal radius to width ratio significantly affect the elastic behaviour and ultimate load capacity of curved concrete
waffle slab bridges.

9.3 Recommendations for Future Research

It is recommended that further research efforts be directed towards the following:

1- The study of the behaviour of continuous curved prestressed and reinforced concrete waffle slab bridges over interior isolated column or continuous supports.

2- The study of the dynamic response of curved concrete waffle slab bridges.

3- The study of the static and dynamic responses of horizontally curved composite steel-concrete bridges with simple- and rigid-connected diaphragms.
Table 7.1: Geometries of Test Specimens to Determine Rigidities

<table>
<thead>
<tr>
<th>Specimen Identification</th>
<th>Plan Dimensions (in. x in.)</th>
<th>Sectional Details (in.)</th>
<th>Spacing of Ribs</th>
<th>Slab Details (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$b_0=b_r$, $d_0=c_r$, $h$</td>
<td>$S_0$(in.)</td>
<td>$S_1$(in.)</td>
</tr>
<tr>
<td>$F_{01}$</td>
<td>84.0 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$F_{R1}$</td>
<td>114.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$T_1$</td>
<td>45.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>5.5</td>
</tr>
<tr>
<td>$F_{02}$</td>
<td>84.0 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>7.0</td>
</tr>
<tr>
<td>$F_{R2}$</td>
<td>114.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>7.0</td>
</tr>
<tr>
<td>$T_2$</td>
<td>45.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>7.0</td>
</tr>
<tr>
<td>$F_{03}$</td>
<td>84.0 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$F_{R3}$</td>
<td>114.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$T_3$</td>
<td>45.5 x 45.5</td>
<td>1.5 3.0 1.0</td>
<td>5.5</td>
<td>9.0</td>
</tr>
</tbody>
</table>

* spacing between longitudinal ribs at the longitudinal center line of the slab.
** radius from the center of curvature to the longitudinal center line of the slab.
*** spacing between radial ribs in degrees.

Note: 1 in. = 25.4 mm
Table 8.1: Comparison of Tangential Rigidity Values at Precracking Stage (Specimen $F_{91}$)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tangential Rigidity (lb.in$^2$/in)$\times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location</td>
</tr>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Method</td>
<td></td>
</tr>
<tr>
<td>Present Theory</td>
<td>12.74</td>
</tr>
<tr>
<td>Experiment</td>
<td>12.27</td>
</tr>
<tr>
<td>Heins</td>
<td>13.28</td>
</tr>
<tr>
<td>Szilard</td>
<td>12.45</td>
</tr>
<tr>
<td>Giencke</td>
<td>12.45</td>
</tr>
<tr>
<td>Cusens</td>
<td>12.54</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>6.93</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. $\times 10^6$ = 113 GN.mm$^2$/mm
Table 8.2: Comparison of Tangential Rigidity Values at Pecracking Stage (Specimen $F_{02}$)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tangential Rigidity (lb.in$^2$/in)$\times10^6$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Theory</td>
<td>12.74</td>
<td>12.74</td>
</tr>
<tr>
<td>Experiment</td>
<td>13.11</td>
<td>12.38</td>
</tr>
<tr>
<td>Heins</td>
<td>13.28</td>
<td>13.28</td>
</tr>
<tr>
<td>Szilard</td>
<td>12.45</td>
<td>12.45</td>
</tr>
<tr>
<td>Giencke</td>
<td>12.45</td>
<td>12.45</td>
</tr>
<tr>
<td>Cusens</td>
<td>12.54</td>
<td>12.54</td>
</tr>
<tr>
<td>Solid Slab</td>
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<td>6.01</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. $\times10^6 = 113$ GN.mm$^2$/mm
Table 8.3: Comparison of Tangential Rigidity Values at Precracking Stage (Specimen F_{03})

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tangential Rigidity (lb.in^2/in)*10^5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_{03})</td>
<td>Location</td>
</tr>
<tr>
<td>Method</td>
<td>I</td>
</tr>
<tr>
<td>Present Theory</td>
<td>12.28</td>
</tr>
<tr>
<td>Experiment</td>
<td>11.79</td>
</tr>
<tr>
<td>Heins</td>
<td>12.77</td>
</tr>
<tr>
<td>Szilard</td>
<td>12.03</td>
</tr>
<tr>
<td>Giencke</td>
<td>12.03</td>
</tr>
<tr>
<td>Cusens</td>
<td>12.10</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>5.12</td>
</tr>
</tbody>
</table>

Note: 1 lb.in^2/in. x 10^5 = 113 GN.mm^2/mm
Table 8.4: Comparison of Tangential Rigidity Values at Postcracking Stage (Specimen F_{61})

<table>
<thead>
<tr>
<th>Specimen (F_{61})</th>
<th>Tangential Rigidity (lb.in^2/in)*10^6</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.47</td>
<td>2.47</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.54</td>
<td>2.35</td>
</tr>
</tbody>
</table>

Table 8.5: Comparison of Tangential Rigidity Values at Postcracking Stage (Specimen F_{62})

<table>
<thead>
<tr>
<th>Specimen (F_{62})</th>
<th>Tangential Rigidity (lb.in^2/in)*10^6</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.47</td>
<td>2.47</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.58</td>
<td>2.55</td>
</tr>
</tbody>
</table>

Note: 1 lb.in^2/in.*10^6 = 113 GN.mm^2/mm
Table 8.6: Comparison of Tangential Rigidity Values at Postcracking Stage (Specimen F$_{83}$)

<table>
<thead>
<tr>
<th>Specimen (F$_{83}$)</th>
<th>Tangential Rigidity (lb.in$^2$/in)$\times10^6$ Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Method</td>
<td></td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.39</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.31</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. $\times10^6 = 113$ GN.mm$^2$/mm
Table 8.7: Comparison of Radial Rigidity Values at Precracking Stage (Specimen FR1)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Radial Rigidity (lb.in²/in) x 10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1</td>
<td></td>
<td>Location</td>
</tr>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>13.38</td>
<td>12.53</td>
</tr>
<tr>
<td>Experiment</td>
<td>13.94</td>
<td>13.09</td>
</tr>
<tr>
<td>Heins</td>
<td>13.84</td>
<td>12.96</td>
</tr>
<tr>
<td>Szilard</td>
<td>13.11</td>
<td>12.28</td>
</tr>
<tr>
<td>Giencke</td>
<td>13.11</td>
<td>12.28</td>
</tr>
<tr>
<td>Cusens</td>
<td>13.52</td>
<td>12.60</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>6.32</td>
<td>6.32</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. x 10⁶ = 113 GN.mm²/mm
Table 8.8: Comparison of Radial Rigidity Values at Precracking Stage (Specimen \( F_{R2} \))

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Radial Rigidity (lb.in(^2)/in) ( \times 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location</td>
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<tr>
<td></td>
<td>I</td>
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<tr>
<td>Present Theory</td>
<td>12.01</td>
</tr>
<tr>
<td>Experiment</td>
<td>11.57</td>
</tr>
<tr>
<td>Heins</td>
<td>12.49</td>
</tr>
<tr>
<td>Cusens</td>
<td>12.04</td>
</tr>
<tr>
<td>Solid Solid</td>
<td>5.77</td>
</tr>
</tbody>
</table>

Note: 1 lb.in\(^2\)/in. \( \times 10^5 \) = 113 GN.mm\(^2\)/mm
Table 8.9: Comparison of Radial Rigidity Values at Precracking Stage (Specimen $F_{R3}$)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Radial Rigidity (lb.in$^2$/in)$\times 10^6$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(F_{R3})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>10.16</td>
<td>9.38</td>
</tr>
<tr>
<td>Experiment</td>
<td>9.83</td>
<td>9.77</td>
</tr>
<tr>
<td>Szilard</td>
<td>9.98</td>
<td>9.21</td>
</tr>
<tr>
<td>Giencke</td>
<td>9.98</td>
<td>9.21</td>
</tr>
<tr>
<td>Cusens</td>
<td>10.18</td>
<td>9.39</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>5.08</td>
<td>5.08</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. $\times 10^6 = 113$ GN.mm$^2$/mm
Table 8.10: Comparison of Radial Rigidity Values at Postcracking Stage (Specimen $F_{R1}$)

<table>
<thead>
<tr>
<th>Specimen ($F_{R1}$)</th>
<th>Radial Rigidity (lb.in$^2$/in)$\times10^6$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.52</td>
<td>2.27</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.44</td>
<td>2.16</td>
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</table>

Table 8.11: Comparison of Radial Rigidity Values at Postcracking Stage (Specimen $F_{R2}$)

<table>
<thead>
<tr>
<th>Specimen ($F_{R2}$)</th>
<th>Radial Rigidity (lb.in$^2$/in)$\times10^6$</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.05</td>
<td>1.85</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.95</td>
<td>1.77</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. $\times10^6$ = 113 GN.mm$^2$/mm
Table 8.12: Comparison of Radial Rigidity Values at Postcracking Stage (Specimen FR3)

<table>
<thead>
<tr>
<th>Specimen (FR3)</th>
<th>Radial Rigidity (lb.in²/in)*10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Method</td>
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<tr>
<td>Present Theory</td>
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<td>1.47</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.55</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. x 10⁶ = 113 GN.mm²/mm
Table 8.13: Comparison of Torsional Rigidity Values at Precracking Stage (Specimen $T_1$)

<table>
<thead>
<tr>
<th>Specimen (T$_1$)</th>
<th>Torsional Rigidity (lb.in$^2$/in)*10$^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Location</td>
</tr>
<tr>
<td>Method</td>
<td>I</td>
</tr>
<tr>
<td>Present Theory</td>
<td>2.11</td>
</tr>
<tr>
<td>Experiment</td>
<td>2.05</td>
</tr>
<tr>
<td>Huber</td>
<td>9.79</td>
</tr>
<tr>
<td>Heins</td>
<td>1.81</td>
</tr>
<tr>
<td>Heins (mod.)</td>
<td>1.95</td>
</tr>
<tr>
<td>Szilard</td>
<td>2.44</td>
</tr>
<tr>
<td>Rowe</td>
<td>1.36</td>
</tr>
<tr>
<td>Grid + Solid Slab</td>
<td>1.22</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>4.95</td>
</tr>
</tbody>
</table>

Note: 1 lb.in$^2$/in. x 10$^6$ = 113 GN.mm$^2$/mm
**Table 8.14:** Comparison of Torsional Rigidity Values at Precracking Stage (Specimen T₂)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Torsional Rigidity (lb.in²/in) × 10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Theory</td>
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<td>1.69</td>
</tr>
<tr>
<td>Experiment</td>
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<td>1.72</td>
</tr>
<tr>
<td>Huber</td>
<td></td>
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<tr>
<td>Heins</td>
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<td>1.41</td>
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<tr>
<td>Heins (mod.)</td>
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<td>1.51</td>
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<td>2.01</td>
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<td>Rowe</td>
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<td>0.92</td>
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<tr>
<td>Solid Slab</td>
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<td>3.93</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. × 10⁶ = 113 GN.mm²/mm
Table 8.15: Comparison of Torsional Rigidity Values at Precracking Stage (Specimen T3)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Torsional Rigidity (lb.in²/in)*10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T3)</td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present Theory</td>
<td>1.59</td>
<td>1.52</td>
</tr>
<tr>
<td>Experiment</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>Huber</td>
<td>8.03</td>
<td>7.72</td>
</tr>
<tr>
<td>Heins</td>
<td>1.28</td>
<td>1.28</td>
</tr>
<tr>
<td>Heins (mod.)</td>
<td>1.37</td>
<td>1.32</td>
</tr>
<tr>
<td>Szilard</td>
<td>1.95</td>
<td>1.88</td>
</tr>
<tr>
<td>Rowe</td>
<td>1.29</td>
<td>1.29</td>
</tr>
<tr>
<td>Grid + Solid Slab</td>
<td>0.82</td>
<td>0.79</td>
</tr>
<tr>
<td>Solid Slab</td>
<td>3.63</td>
<td>3.63</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. x 10⁶ = 113 GN.mm²/mm
Table 8.16: Comparison of Torsional Rigidity Values at Postcracking Stage (Specimen T₁)

<table>
<thead>
<tr>
<th>Specimen (T₁)</th>
<th>Torsional Rigidity (lb.in²/in)×10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.53</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 8.17: Comparison of Torsional Rigidity Values at Postcracking Stage (Specimen T₂)

<table>
<thead>
<tr>
<th>Specimen (T₂)</th>
<th>Torsional Rigidity (lb.in²/in)×10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>0.40</td>
<td>0.37</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.39</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. × 10⁶ = 113 GN.mm²/mm
Table 8.18: Comparison of Torsional Rigidity Values at Postcracking Stage (Specimen T₃)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Torsional Rigidity (lb.in²/in) x 10⁶</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T₃)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Present Theory</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.38</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/in. x 10⁶ = 113 GN.mm²/mm
Table 8.19: Waffle Slab Rigidities

<table>
<thead>
<tr>
<th>Axis</th>
<th>$D_0 \times 10^6$ (lb.in²/fin)</th>
<th>$D_c \times 10^6$ (lb.in²/fin)</th>
<th>$D_{as} \times 10^6$ (lb.in²/fin)</th>
<th>$H$ (lb.in²/fin)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-5</td>
<td>43.37</td>
<td>50.71</td>
<td>9.70</td>
<td>14.49</td>
<td>0.8553</td>
<td>0.2857</td>
</tr>
<tr>
<td>4-4</td>
<td>43.37</td>
<td>46.31</td>
<td>8.83</td>
<td>13.28</td>
<td>0.9365</td>
<td>0.2868</td>
</tr>
<tr>
<td>3-3</td>
<td>43.37</td>
<td>43.37</td>
<td>8.20</td>
<td>12.43</td>
<td>1.0</td>
<td>0.2866</td>
</tr>
<tr>
<td>2-2</td>
<td>43.37</td>
<td>39.70</td>
<td>7.75</td>
<td>11.70</td>
<td>1.0922</td>
<td>0.2947</td>
</tr>
<tr>
<td>1-1</td>
<td>43.37</td>
<td>37.16</td>
<td>7.34</td>
<td>11.06</td>
<td>1.1671</td>
<td>0.2976</td>
</tr>
</tbody>
</table>

Note: 1 lb.in²/fin. x $10^6 = 113$ GN.mm²/mm
Table 8.20: Effect of Rigidities on Deflection

<table>
<thead>
<tr>
<th>Slab Type</th>
<th>Slab Thickness in.</th>
<th>$D_0 \times 10^6$ lb.in$^2$/in</th>
<th>$D_f \times 10^6$ lb.in$^2$/in</th>
<th>$D_{0f} \times 10^6$ lb.in$^2$/in</th>
<th>$H$ lb.in$^2$/in</th>
<th>% Difference in Volume Compared with (I)</th>
<th>% Difference in Maximum Deflection Compared with (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.5</td>
<td>43.37</td>
<td>43.37</td>
<td>8.21</td>
<td>14.49</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>II</td>
<td>3.85</td>
<td>25.36</td>
<td>25.36</td>
<td>19.02</td>
<td>25.36</td>
<td>N/A</td>
<td>+25%</td>
</tr>
<tr>
<td>III</td>
<td>2.9</td>
<td>10.84</td>
<td>10.84</td>
<td>8.21</td>
<td>10.84</td>
<td>-25%</td>
<td>+173%</td>
</tr>
<tr>
<td>IV</td>
<td>4.6</td>
<td>43.37</td>
<td>43.37</td>
<td>32.45</td>
<td>43.37</td>
<td>+20%</td>
<td>-32%</td>
</tr>
</tbody>
</table>

Note: $1 \text{ lb.in}^2/\text{in. } \times 10^6 = 113 \text{ GN.mm}^2/\text{mm}$, 1 in. = 25.4 mm
Table 8.21: Comparison of Experimental and Theoretical Cracking and Ultimate loads for the Tested Bridge Models

<table>
<thead>
<tr>
<th>Bridge Model</th>
<th>Cracking Load (kip)</th>
<th>Ultimate Load (kip)</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F.E</td>
<td>Exp.</td>
<td>F.E</td>
</tr>
<tr>
<td>RWS</td>
<td>2.3</td>
<td>2.5</td>
<td>4.1</td>
</tr>
<tr>
<td>PWS</td>
<td>4.5</td>
<td>4.5</td>
<td>7.2</td>
</tr>
<tr>
<td>PGS</td>
<td>3.85</td>
<td>4.0</td>
<td>5.70</td>
</tr>
</tbody>
</table>

Note: 1 in.=25.4mm, 1 kip=4.45 kN
Table 8.22: Comparison of Experimental and Theoretical Collapse Load for RWS Model

<table>
<thead>
<tr>
<th>Load Position</th>
<th>Experimental (kip)</th>
<th>Theory (kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Finite Element</td>
</tr>
<tr>
<td>Load at center of Each Lane</td>
<td>5.0</td>
<td>4.1</td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td>—</td>
<td>4.8</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td>—</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Note: 1 in.=25.4mm, 1 kip=4.45 kN
Table 8.23: Comparison of Experimental and Theoretical Collapse Load for PWS Model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Load at center of Each Lane</td>
<td>8.50</td>
<td>7.2</td>
<td>8.6</td>
<td>8.3</td>
<td>7.7</td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td>8.5</td>
<td></td>
<td>9.1</td>
<td>8.7</td>
<td>8.0</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td>4.8</td>
<td></td>
<td>5.3</td>
<td>5.0</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Table 8.24: Comparison Between Theoretical Collapse Loads Using F.E and the Developed Yield Line Formula.

<table>
<thead>
<tr>
<th>Bridge Model Description (RWS)</th>
<th>Collapse Load (kip)</th>
<th>Yield Line Theory</th>
<th>Finite Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aspect Ratio $2L/W_s = 2.0$</td>
<td>4.9</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 1.14$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio $2L/W_s = 2.0$</td>
<td>3.4</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 4.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio $2L/W_s = 2.0$</td>
<td>3.1</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 8.0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio $2L/W_s = 1.25$</td>
<td>6.3</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 1.14$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio $2L/W_s = 1.25$</td>
<td>5.1</td>
<td>6.3</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 4.0$</td>
<td>6.8$^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aspect Ratio $2L/W_s = 1.25$</td>
<td>4.8</td>
<td>5.9</td>
<td></td>
</tr>
<tr>
<td>Radius / Span $R/W_s = 8.0$</td>
<td>6.7$^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load at Inner Edge</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8.24: Continued

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 2.0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 1.14</td>
<td>2.7</td>
<td>2.3</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 2.0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 4.0</td>
<td>2.8</td>
<td>2.6</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 2.0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 8.0</td>
<td>2.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 1.25</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 1.14</td>
<td>4.1</td>
<td>4.6</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 1.25</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 4.0</td>
<td>4.3</td>
<td>5.4</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aspect Ratio (2L/Ws) = 1.25</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius / Width (R/Ws) = 8.0</td>
<td>4.5</td>
<td>5.6</td>
</tr>
<tr>
<td>Load at Outer Edge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>* Calculated based on the second mode of failure, figs. 6.4 and 6.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: 1 in=25.4 mm, 1 kip=4.45 kN
Hegenberger Overpass, Oakland, California
Fig. 1.2 Theoretical Study Layout (Section I: Rigidities of Curved Flat Structures)
Fig. 1.3 Theoretical Study Layout (Section II: FDM and FEM)
Note:

FDM = Finite Difference Method
FEM = Finite Element Method

Fig. 1.4 Theoretical Study Layout (Section III: Yield Line Method)
Fig. 1.5 Experimental Study Layout
(Part I: Rigidities of Curved Concrete Waffle Slab Bridges)
Fig. 1.6 Experimental Study Layout

(Part II: Behaviour of Curved Concrete Slab Bridge)
Fig. 3.1 Geometry of Curved Waffle Slab With Reference Axes

(a) Plan

(b) Cross-Section

Tangential ribs
Radial ribs

$S_r$, $S_{r'}$, $e_0$, $e_1$, $A_s$, $A_s'$, $b_0$, $b_1$, $d_0$, $d_1$, $d_{r'}$, $d_{r''}$, $N, A$. $S_0$, $S_r$.
Fig. 3.2 Partitioning of Waffle Cross-Section for Torsional Calculations
Fig. 3.3  Geometry of Curved Composite Flat Deck Structure
Fig. 3.4 Partitioning of Composite Cross-Section for Torsional Calculations
Fig. 4.1 Geometry and Forces on Wedge Plate
Fig. 4.3 Circular Plate Segment Boundaries
Fig. 5.1 Tangent Stiffness Procedure
Fig. 5.2 Modified Procedure
Fig. 5.3 Shell Element S4R

- Four Nodes Element
- Degrees of Freedom: $U_1, U_2, U_3, \psi_1, \psi_2, \phi_3$
- Output Forces: $S_{F1}, S_{F2}, S_{F3}, S_{F4}, S_{F5}$
- Output Moments: $S_{M1}, S_{M2}, S_{M3}$
- Stress Components: $S_{I1}, S_{I2}$, Available in Two Directions

$SM3, SF4, SF5$
Fig. 5.6  Uniaxial Behaviour of Plain Concrete
Fig. 5.7 Yield and Failure Surfaces of Concrete in Plane Stress
Fig. 5.10 Multi Point Constraints Definition
Fig. 5.12 Finite Element Mesh for Bridge Model RSS
Fig. 5.14 Finite Element Mesh for Bridge Model PCS
Fig. 5.15 Elastic Parametric Study Layout
Fig. 5.16 Post-Elastic Parametric Study Layout (Collapse Load)
Fig. 6.1  Yield Line Failure Pattern for Curved Waffle Slab Under Two Concentrated Loads Each at the Center of Each Lane
Fig. 6.2 Yield Line Failure Pattern for Curved Waffle Slab Under Concentrated Load at Center of Inner Edge – Mode (1)
Fig. 6.3 Design Curves for Determination of the Coefficient $\beta''$
Fig. 6.4 Yield Line Failure Pattern for Curved Waffle Slab Under Concentrated Load at Center of Inner Edge – Mode (2)
Fig. 6.5 Yield Line Failure Pattern for Curved Waffle Slab Under Concentrated Load at Center of Outer Edge – Mode (1)
Fig. 6.6 Yield Line Failure Pattern for Curved Waffle Slab Under Concentrated Load at Center of Outer Edge – Mode (2)
(a) OHBDC Truck Load

(b) AASHTO Truck Load

(c) Equivalent Concentrated Load

Fig. 6.7 Longitudinal Section of a Curved Waffle Slab Bridge under OHBDC and AASHTO Truck Loads
Fig. 7.1 Square Form for the Pure Twisting Test
Fig. 7.2 Rectangular Form for the Pure Radial Bending Test
Fig. 7.3 Loading Scheme and Support System for Test Specimen in Torsion

KEY

- Support point
- Load point
- Dial gage location
KEY

▲ Support point
● Load point
• Dial gage location

Fig. 7.4 Loading Scheme and Support System for Test Specimen in Flexure (Tangential Direction)
Fig. 7.5 Loading Scheme and Support System for Test Specimen in Flexure (Radial Direction)
Fig. 7.6 Test Set-up for the Torsion Specimens
Fig. 7.7  Test Set-up for the Tangential Bending Specimens
Fig. 7.8 Test Set-up for the Radial Bending Specimens
Fig. 7.9 Plan Layout of Reinforced and Prestressed Concrete Waffle Slab Models
Fig. 7.10 Cross - Sectional Dimensions for RWS and PWS Models

1 in = 25.4 mm

4.5''
4.0''
3.0''
4 x 7''
Fig. 7.11 Plan Layout of Prestressed Concrete

Girder Slab Model
Curved Concrete Girder-Slab Bridge Model
Fig. 7.15  Hydraulic 20 kip Prestressing Jack
Fig. 7.16 Automatic Strain Indicator
Fig. 7.17a Strain Gauges Locations on Top Fibre
RWS Model
Fig. 7.17b Strain Gauges Locations on Bottom Fibre
RWS Model
Fig. 7.18a Strain Gauges Locations on Top Fibre
PWS Model
Fig. 7.18b Strain Gauges Locations on Bottom Fibre PWS Model
FIG. 7.21 Locations of Dial Gauges for RWS and PWS Models
Fig. 7.22 Locations of Dial Gauges for PGS Model
Fig. 7.25 Reinforcement Details for Bridge Model PWS
Fig. 7.28 Testing Set-Up for Curved Waffle Slab Models
Fig. 7.30 Bridge Model RWS under Two Concentrated Loads - Ultimate Case
Fig. 7.31  Prestressing the Radial Ribs of Bridge Model PWS
Fig. 7.32 Prestressing the Tangential Ribs of Bridge Model PWS
Fig. 7.34 Bridge Model PWS under Single Concentrated Load at the Outer Edge
Fig. 7.36 Bridge Model PWS Tested to Failure
Fig. 7.37  Prestressing Forces for Bridge Model PGS
Fig. 7.38 Bridge Model PGS Tested to Failure
Fig. 8.1 Typical Load–Deflection Relationship

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.4.81 Mesh Points — Bridge Model (RWS)
Fig. 8.6 Deflected Shape for RWS Model Along Axis E–E With Different Mesh Configurations

\( (P = 1.3 \text{ kip at Center}) \)

Note: 1 in. = 25.4 mm, 1 kip = 4448 kN
Fig. 8.7 Deflected Shape for RWS Model Along Axis 1–1
(P = 1.3 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.9 Deflected Shape for RWS Model Along Axis 5–5
(P = 1.3 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.10 Deflected Shape for RWS Model Along Axis 1–1
( U.D.L = 1.0 psi on Outer Lane )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.11 Deflected Shape for RWS Model Along Axis 3–3
(U.D.L. = 1.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.12 Deflected Shape for RWS Model Along Axis 5–5
(U.D.L. = 1.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 6.13 Deflected Shape for RWS Model Along Axis 1–1
(U.D.L = 0.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.14 Deflected Shape for RWS Model Along Axis 3–3
(U.D.L = 0.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.15 Deflected Shape for RWS Model Along Axis 5-5
(U.D.L. = 0.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.16 Comparison of Deflections of Waffle Slab and Solid Slab Type II
\( (P = 1.3 \text{ kip at E-3}) \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.17 Comparison of Deflections of Waffle Slab and Solid Slab Type III
( \( P = 1.3 \) kip at \( E = 3 \) )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.18 Comparison of Deflections of Waffle Slab and Solid Slab Type IV
( P = 1.3 kip at E-3 )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.19 Applied Load Locations for the Tested Bridge Model

RWS
Fig. 8.21  Bridge Model WWS under Uniformly-Distributed Load on the Outer Lane
Fig. 8.22  Bridge Model RWS under Uniformly-Distributed Load on the Inner Lane
Fig. 8.23 Deflection Pattern for RWS Model - ( P = 1.3 kip at E-3 )

- Upward
+ Downward,

Deflection X 10^3 (in.)
1 in.=25.4 mm, 1 kip=4.45 kN
Deflection $\times 10^3$ (in.)
1 in. = 25.4 mm, 1 kip = 4.45 kN

Fig. 8.24 Deflection Pattern for RWS Model - (U.D.L = 1.0 psi on Outer Lane)
Fig. 8.25 Deflection Pattern for RWS Model - (U.D.L. = 0.87 psi on Inner Lane)

- Upward
+ Downward

Deflection $\times 10^3$ (in.)

1 in.$=25.4$ mm, 1 kip = 4.45 kN

Theoretical
Experimental
Fig. 8.26 Strain Distribution at Top Surface for RWS Model - ( P = 1.3 kip at E-3 )
Fig. 8.27 Strain Distribution at Top Surface for RWS Model - (U.D.L. = 1.0 psi on Outer Lane)
Fig. 8.28 Strain Distribution at Top Surface for RWS Model - (U.D.L = 0.87 psi on Inner Lane)
Fig. 8.29 Transverse Strain at Top Surface for RWS Model

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.30 Transverse Strain at Top Surface for RWS Model
(U.D.L = 1.0 psi on Outer Lane)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.31 Transverse Strain at Top Surface for RWS Model
( U.D.L = 0.87 psi on Inner Lane )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.32 Transverse Strain at Top Surface for RWS Model Along Axis E–E
Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.33 Deflected Shape for RWS Model Along Axis E–E

(P = 1.3 kip)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.34 Longitudinal Strain at Top Surface for RWS Model Along Axis E-E
( P = 1.3 kip )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.35 Comparison of Deflections of RWS and RGS Models
( P = 1.3 kip at E-3 )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.36  Comparison of Deflections of RWS and RGS Models
(P = 0.65 kip at E–2 and E–4)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.37  Comparison of Strains at Top Surface of RWS and RGS Models
( P = 1.3 kip at E−3 )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.38 Comparison of Strains at Top Surface of RWS and RGS Models
(P = 0.65 kip at E-2 and E-4)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.39 Comparison of Transverse Strains at Top Surface for RWS and RGS Models
(P = 1.3 kip at E-3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.40 Comparison of Transverse Strains at Top Surface for RWS and RGS Models
(P = 0.65 kip at E-2 and E-4)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.41 Comparison of Deflected Shapes of RWS and RGS Models Along Axis E-E
(P = 1.3 kip)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.42 Comparison of Deflected Shapes of RWS and RGS Models Along Axis E-E

(\text{U.D.L.} = 1.0 \text{ psi})

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.43 Comparison of Longitudinal Strains at Top Surface for RWS and RGS Models Along Axis E-E

\( P = 1.3 \text{ kip} \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.44 Comparison of Longitudinal Strains at Top Surface for RWS and RGS Models Along Axis E–E
(U.D.L = 1.0 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.45 Comparison of Deflections of RWS and Solid Slab Models
(P = 1.3 kip at E-3)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.46 Comparison of Strains at Top Surface of RWS and Solid Slab Models (P = 1.3 kip at E-3)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.47 Comparison of Strains at Top Surface of RWS and Solid Slab Models

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN

(P = 0.65 kip at E-2 and E-4)
Fig. 8.48 Comparison of Transverse Strains at Top Surface for RWS and Solid Slab Models

\( P = 1.3 \text{ kip at E-3} \)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.49: Comparison of Transverse Strains at Top Surface for RWS and Solid Slab Models

(\( P = 0.65 \text{ kip at E-2 and E-4} \) )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.50 Comparison of Deflected Shapes of RWS and Solid Slab Models Along Axis E-E

( P = 1.3 kip )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.51 Comparison of Deflected Shapes of RWS and Solis Slab Models Along Axis E-E
(U.D.L = 1.0 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.52 Comparison of Longitudinal Strains at Top Surface for RWS and Solid Slab Models Along Axis E–E
(P = 1.3 kip)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.53 Comparison of Longitudinal Strains at Top Surface for RWS and Solid Slab Models Along Axis E–E (U.D.L = 1.0 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.54 Waffle Slab Actions due to Horizontally Curved Tendons
Fig. 8.55  Deflection Pattern for PWS Model - (Due to Prestressing)
Fig. 8.56 Strain Distribution at Top Surface for PWS Model - (Due to Prestressing)
Fig. 8.57 Strain Distribution at Bottom Surface for PWS Model - (Due to Prestressing)
Fig. 8.58 Transverse Strain Distribution at Top Surface for PWS Model - (Due to Prestressing)
Fig. 8.59 Applied Load Locations for the Tested Bridge Models
PWS and PGS
Fig. 8.60  Bridge Model PWS under Single Concentrated Load at the Inner Edge
Fig. 8.61 Bridge Model PWS under Uniformly-Distributed Load on the Inner Lane
Fig. 8.62 Deflection Pattern for PWS Model - (P = 2.0 kip at E-1)

Deflection $\times 10^3$ (in.)

1 in. = 25.4 mm, 1 kip = 4.45 kN

- Upward
+ Downward
Fig. 8.63 Deflection Pattern for PWS Model - (P = 2.5 kip at E-3)

- Theoretical
- Experimental

Deflection $\times 10^3$ (in.)

1 in.$=25.4$ mm, 1 kip.$=4.45$ kN

- Upward
- Downward

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Fig. 8.6 Deflection Pattern for PWS Model - (P = 0.2 kip at G-1)

Deflection $\times 10^2$ (in.)

1 in. = 25.4 mm, 1 kip = 4.45 kN

- Upward
+ Downward

Theoretical
Experimental
Fig. 8.67 Deflection Pattern for PWS Model - (P = 2.5 kip at G-5)
Fig. 8.68 Deflection Pattern for PWS Model - (U.D.L = 2.0 psi on Outer Lane)

Deflection $\times 10^3$ (in.)
1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.69 Deflection Pattern for PWS Model - (U.D.L. = 1.87 psi on Inner Lane)
Fig. 8.70 Strain Distribution at Top Surface for PWS Model - (P = 2.0 kip at E-1)
Fig. 8.72 Strain Distribution at Top Surface for PWS Model - (P = 2.5 kip at E-3)
Fig. 8.73 Strain Distribution at Top Surface for PWS Model - (P = 2.5 kip at E-4)

Strain X 10^6 (in./in.)
1 in. = 25.4 mm, 1 kip = 4.45 kN

- Compression
+ Tension

Theoretical
Experimental
Fig. 8.74 Strain Distribution at Top Surface for PWS Model - (P = 2.5 kip at E-5)
Fig. 8.75 Strain Distribution at Top Surface for PWS Model - ( P = 2.0 kip at G-1 )
Fig. 8.77 Strain Distribution at Top Surface for PWS Model - (P = 2.5 kip at G-5)

Strain $X \times 10^6$ (in./in.)

1 in. = 25.4 mm, 1 kip = 4.45 kN

- Compression
- Tension

Theoretical
Experimental
Fig. 8.78 Strain Distribution at Top Surface for PWS Model - (U.D.L = 2.0 psi on Outer Lane)
Fig. 8.79 Strain Distribution at Top Surface for PWS Model - (U.D.L. = 1.87 psi on Inner Lane)
Fig. 8.80 Strain Distribution at Bottom Surface for PWS Model - ( P = 2.0 kip at E-1 )
Fig. 8.82 Strain Distribution at Bottom Surface for PWS Model - (P = 2.5 kip at E-5)
Fig. 8.84 Strain Distribution at Bottom Surface for PWS Model - (P = 2.5 kip at G-3)
Fig. 8.85 Strain Distribution at Bottom Surface for PWS Model - (P = 2.5 kip at G-5)
Fig. 8.86 Strain Distribution at Bottom Surface for PWS Model - (U.D.L = 2.0 psi on Outer Lane)
Fig. 8.87 Strain Distribution at Bottom Surface for PWS Model - ( U.D.L. = 1.87 psi on Inner Lane )
Fig. 8.88 Transverse Strain at Top Surface for PWS Model Along Axis E–E

(U.D.L. = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.89 Transverse Strain at Top Surface for PWS Model Along Axis G–G
(U.D.L = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.90 Transverse Strain at Top Surface for PWS Model Along Axis E–E
(U.D.L = 1.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.91 Transverse Strain at Top Surface for PWS Model Along Axis G-G

Note: 1 in = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.92 Transverse Strain at Top Surface for PWS Model Along Axis E–E
(P = 2.5 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.93 Transverse Strain at Top Surface for PWS Model Along Axis G–G
(\( P = 2.5 \) kip at \( E-3 \))

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.94  Transverse Strain at Top Surface for PWS Model Along Axis E–E  
(  P = 2.5 kip at G–3 )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.96 Deflection Pattern for PGS Model - (Due to Prestressing)
Fig. 8.97 Strain Distribution at Top Surface for PGS Model - (Due to Prestressing)
Strain \( X \times 10^6 \) (in./in.)
1 in. = 25.4 mm, 1 kip = 4.45 kN

Fig. 8.98 Strain Distribution at Bottom Surface for PGS Model - (Due to Prestressing)
Fig. 8.99 Deflection Pattern for PGS Model - (P = 1.5 kip at E-1)

Deflection \times 10^3 \text{ (in.)}
1 \text{ in.}=25.4 \text{ mm}, 1 \text{ kip}=4.45 \text{ kN}

- Upward
+ Downward

Theoretical
Experimental
Fig. 8.100 Deflection Pattern for PGS Model - (P = 2.5 kip at E-3)

Deflection $\times 10^3$ (in.)
1 in. = 25.4 mm, 1 kip = 4.45 kN

- Upward
+ Downward

Theoretical
Experimental
Fig. 8.101 Deflection Pattern for PGS Model - (P = 2.5 kip at E-5)
Fig. 8.102 Deflection Pattern for PGS Model - (U.D.L = 2.0 psi on Outer Lane)
Fig. 8.103 Deflection Pattern for PGS Model - (U.D.L. = 1.87 psi on Inner Lane)
Fig. 8.105 Strain Distribution at Top Surface for PGS Model - ( P = 2.5 kip at E-3 )
Fig. 8.106 Strain Distribution at Top Surface for PGS Model - (P = 2.5 kip at E-5)
Fig. 8.107 Strain Distribution at Top Surface for PGS Model - ( U.D.L = 2.0 psi on Outer Lane )
Fig. 8.108 Strain Distribution at Top Surface for PGS Model - (U.D.L. = 1.87 psi on Inner Lane)
Fig. 8.109  Strain Distribution at Bottom Surface for PGS Model - ( P = 1.5 kip at E-1 )
Fig. 8.110 Strain Distribution at Bottom Surface for PGS Model - (P = 2.5 kip at ε-3)
Fig. 8.111 Strain Distribution at Bottom Surface for PGS Model - (P = 2.5 kip at E-5)
Fig. 8.112 Strain Distribution at Bottom Surface for PGS Model - (U.D.L = 2.0 psi on Outer Lane)
Fig. 8.113 Strain Distribution at Bottom Surface for PGS Model - ( U.D.L. = 1.87 psi on Inner Lane )
Fig. 8.114 Transverse Strain at Top Surface for PGS Model Along Axis E–E
( P = 2.5 kip at E–3 )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.115 Transverse Strain at Top Surface for PGS Model Along Axis G–G
( P = 2.5 kip at E–3 )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.116 Transverse Strain at Top Surface for PGS Model Along Axis E–E
( U.D.L = 2.0 psi on Outer Lane )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.117 Transverse Strain at Top Surface for PGS Model Along Axis G–G
(U.D.L = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.118 Transverse Strain at Top Surface for PGS Model Along Axis E–E
(U.D.L. = 1.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.119 Transverse Strain at Top Surface for PGS Model Along Axis G–G
(U.D.L = 1.87 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.120 Comparison of Deflected Shapes of PWS and PGS Models
Along Axis E–E
( P = 2.0 kip at E–5 )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.121 Comparison of Deflected Shapes of PWS and PGS Models 
Along Axis E–E 
( P = 2.0 kip at E–3 )

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.122 Comparison of Deflected Shapes of PWS and PGS Models  
Along Axis E–E  
( P = 2.0 kip at E–1 )  

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.123 Comparison of Deflected Shapes of PWS and PGS Models Along Axis E–E
(U.D.L = 2.0 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.124 Comparison of Deflected Shapes of PWS and PGS Models Along Axis E–E
(U.D.L = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 6.125 Comparison of Longitudinal Strains at Top Surface for PWS and PGS Models Along Axis E–E
(P = 2.0 kip at E–5)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.126 Comparison of Longitudinal Strains at Top Surface for PWS and PGS Models Along Axis E–E

(P = 2.0 kip at E–3)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.127 Comparison of Longitudinal Strains at Top Surface for PWS and PGS Models Along Axis E-E
(P = 2.0 kip at E-1)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.128 Comparison of Longitudinal Strains at Top Surface for PWS and PGS Models Along Axis E-E (U.D.L. = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4,45 kN
Fig. 8.129 Comparison of Longitudinal Strains at Top Surface for PWS and PGS Models Along Axis E–E
(U.D.L = 2.0 psi on Inner Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
**Fig. 8.130** Comparison of Transverse Strains at Top Surface for PWS and PGS Models Along Axis E–E

(\( P = 2.0 \text{ kip at E–3} \))

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.131 Comparison of Transverse Strains at Top Surface for PWS and PGS Models Along Axis E–E
(U.D.L = 2.0 psi on Outer Lane)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.132 Comparison of Transverse Strains at Top Surface for PWS and PGS Models Along Axis E–E
(U.D.L = 2.0 psi on Inner Lane)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.133 Load-Deflection Relationship for RWS Model

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.134 Load–Strain Relationship for RWS Model

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.135 Load–Deflection Relationship for RWS Model Along Axis E–E

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.136 Load–Strain Relationship for RWS Model Along Axis E–E

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.137 Crack Pattern of RWS at Cracking Load
Fig. 8.138 Crack Pattern of RWS at Failure Load
Fig. 8.142 Load–Deflection Relationship for PWS Model

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.143 Load-Strain Relationship for PWS Model

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.144 Load–Deflection Relationship for PWS Model Along Axis E–E

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.145 Load–Strain Relationship for PWS Model Along Axis E–E

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.147 Crack Pattern of PWS at Failure Load
Fig. 8.150  Single-Line Crack Pattern of Bridge Model PWS
Fig. 8.152 Load–Deflection Relationship for PGS Model

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.153  Load–Strain Relationship for PGS Model

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.154 Load–Deflection Relationship for PGS Model Along Axis E–E

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.155 Load-Strain Relationship for PGS Model Along Axis E-E

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.157  Separation between Longitudinal Ribs and Deck Slab after Failure of Bridge Model PGS
Fig. 8.158  Combined Flexural and Torsional Cracks in the Longitudinal Ribs of Bridge Model PGS
Fig. B.159 Crack Pattern of PGS at Cracking Load

Beam Cracks
Fig. 8.160 Crack Pattern of PGS at Failure Load
FIG. 8.161 Deflected Shape for RWS Model Along Axis E–E With Different Radius to Width Ratios
(P = 1.0 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
FIG. 8.162 Deflected Shape for RGS Model Along Axis E–E With Different Radius to Width Ratios

$\frac{R}{W_s} = 0.0$  
$\frac{R}{W_s} = 1.15$  
$\frac{R}{W_s} = 2.0$  
$\frac{R}{W_s} = 4.0$  
$\frac{R}{W_s} = 6.0$

(\( P = 1.0 \text{ kip at E-3} \))

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN

Deflection $\times 10^3$ (in.)
Fig. 8.163 Deflected Shape for RWS Model Along Axis E–E With Different Radius to Width Ratios

(U.D.L = 0.5 psi)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.164 Deflected Shape for RGS Model Along Axis E–E With Different Radius to Width Ratios
(U.D.L = 0.5 psi)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.165  Comparison of Deflections of Curved and Straight
RWS and RGS Models Along Axis E–E

(P = 1.0 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.166 Comparison of Deflections of Curved and Straight RWS and RGS Models Along Axis E-E (U.D.L. = 0.5 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.167 Comparison of Deflections of Curved and Straight
RWS and RGS Models Along Axis E–E

\[ P = 1.0 \text{ kip at } E-1 \]

*Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN*
Fig. 8.168 Comparison of Deflections of Curved and Straight RWS and RGS Models Along Axis E–E

( P = 1.0 kip at E–5 )

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
FIG. 8.169 Longitudinal Strains at Top Surface for RWS Model Along Axis E–E With Different Radius to Width Ratios
(P = 1.0 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
FIG. 8.170 Longitudinal Strains at Top Surface for RGS Model Along Axis E–E With Different Radius to Width Ratios

\( P = 1.0 \text{ kip at E–3} \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
FIG. 8.171 Longitudinal Strains at Top Surface for RWS Model Along Axis E-E With Different Radius to Width Ratios (U.D.L. = 0.5 psi)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN

Center

Outer Edge

Inner Edge

\[ \text{Strain } \times 10^6 (\text{in.}/\text{in.}) \]
FIG. 8.172 Longitudinal Strains at Top Surface for RGS Model Along Axis E–E With Different Radius to Width Ratios
(U.D.L = 0.5 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.173 Comparison of Longitudinal Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E
(P = 1.0 kip at E–3)

Note: 1 in.=25.4 mm, 1 kip=4.45 kN
Fig. 8.174 Comparison of Longitudinal Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E
(U.D.L. = 0.5 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.175 Comparison of Longitudinal Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E

\( P = 1.0 \text{ kip at } E-1 \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.176 Comparison of Longitudinal Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E

\( P = 1.0 \text{ kip at E–5} \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.177 Transverse Strains at Top Surface for RWS Model Along Axis E–E With Different Radius to Width Ratios
\( P = 1.0 \text{ kip at } E-3 \)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.178 Transverse Strains at Top Surface for RGS Model Along Axis E-E with Different Radius to Width Ratios (P = 1.0 kip at E-3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.179 Comparison of Transverse Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E
(P = 1.0 kip at E–3)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
Fig. 8.180 Comparison of Transverse Strains at Top Surface of Curved and Straight RWS and RGS Models Along Axis E–E (U.D.L. = 0.5 psi)

Note: 1 in. = 25.4 mm, 1 kip = 4.45 kN
APPENDIX A-1

FINITE DIFFERENCE COMPUTER PROGRAM AND

INPUT PARAMETERS
FINITE DIFFERENCE COMPUTER PROGRAM

C *****************************************************
C ANALYSIS OF CURVED WAFFLE SLAB BY THE FINITE DIFFERENCE
TECHNIQUE
C *****************************************************

IMPLICIT REAL*8(A-H,O-Z)
COMMON AA
DIMENSION AA (400,400),BB(400),C(400),ALP(20),BET(20)
REAL MINRAD
REAL LAM, LAM2, LAM3, LAM4
INTEGER ROW, COL

C ***********************************************
C INPUT DATA
C ***********************************************

READ(5,100)N,NN
100 FORMAT (2I5)
   READ(5,182)MINRAD,LAM,OTHETA
182 FORMAT (3F10.0)
   READ(5,*) ((ALP(I)),I=1,NN)
   READ(5,*) ((BET(I)),I=1,NN)

C ***********************************************
C GENERATION OF THE FINITE DIFFERENCE EQUATIONS
C ***********************************************

NUMROW= N/NN
C WRITE (6,*) ((ALP(I)),I=1,NN)
C WRITE (6,*) ((DR(I)),I=1,NN)
C WRITE (7,*)N,NN,MINRAD,LAM,OTHETA
C
DO 111 I=1,N
  DO 111 J=1,N
111 AA(I,J)=0.0
  TEMP=NUMROW+1
  THETA=OTHETA * TEMP
  LAM2 = LAM ** 2
  LAM3 = LAM ** 3
  LAM4 = LAM ** 4
  OTHET2 = OTHETA ** 2
  OTHET4 = OTHETA ** 4
  COL = 0
  RAD = MINRAD
NTEST = (N/2)*2
IF(NTEST.NE.N) NCENT= (NUMROW/2)+1
NC=0
DO 21 K =1,N,NN
M = K + NN - 1
ROW = ROW + 1
TEMP = NUMROW + 1 - ROW
XTHETA = TEMP * OTHETA
DO 20 I = K , M
IF (.GT.NN) GO TO 222
JK=I
GO TO 777
222 JK=I-NC*NN
777 RAD3 = (RAD*OTHETA)**3
   C1 = BET(JK) * RAD * OTHET2 * LAM2
   C2 = 2.0 * RAD ** 2
   C3 = 2.0 * BET(JK) * LAM2 * OTHET2
   C4 = 2.0 * ALP(JK) * LAM3
   C5 = LAM * OTHET2
   C6 = RAD * OTHET2
   C7 = RAD * LAM
   C8 = RAD ** 4 * OTHET4
   C9 = LAM * RAD ** 3 * OTHET4
   C10 = BET(JK) * RAD * LAM3 * OTHET2
   C11 = BET(JK) * RAD ** 2 * LAM2 * OTHET2
   C12 = ALP(JK) * RAD * OTHET4 * LAM3
   C13 = ALP(JK) * RAD ** 2 * OTHET4 *LAM2
   C14 = RAD ** 4 * OTHET4
   C15 = LAM * RAD **3 * OTHET4
   C16 = 4.0 * LAM2 * OTHET2 * BET(JK)
   C17 = 2.0 * RAD ** 2
   C18 = LAM2
   C19 = RAD * LAM
   C20 = ALP(JK) * LAM2
   C21 = RAD ** 2 * OTHET4
   C22 = LAM2 * OTHET2
   C23 = LAM * RAD * OTHET2
   C24 = LAM * RAD * OTHET4
COL = COL + 1
IF (ROW .EQ. 1) GO TO 35
IF (ROW .EQ. 2) GO TO 32
J1 = I - 2 * NN
AA(I,J1)=ALP(JK)*LAM4
32 IF (COL .EQ. 1) GO TO 33
   J2 = I - NN - 1
IF(COL.EQ.NN) AA(I,J2) = 2.0*C1*(2.0*RAD+3.0*LAM)
IF(COL.NE.NN) AA(I,J2) = C1*(2.0*RAD+LAM)
33 J3 = I - NN
IF(COL.EQ.NN) AA(I,J3)=C3*(3.0*LAM2-C2-C7)+C4*(2.0*C5-2.0*LAM+C6)
IF(COL.EQ.1) AA(I,J3)=C3*(3.0*LAM2-C2+C7)+C4*(2.0*C5-2.0*LAM-C6)
429
IF(COL.NE.1 .AND. COL.NE.NN) AA(I,J3)=C3*(LAM2*C2)+ C4*(C5-2.0*LAM)
IF (COL.EQ. NN) GO TO 35
Jd = 1 - NN + 1
IF (COL.EQ.1) AA(I,J4)=2.0*C1*(2.0*RAD-3.0*LAM)
IF (COL.NE.1) AA(I,J4)=C1*(2.0*RAD-LAM)
35 IF (COL.LE.2) GO TO 36
IF (COL.EQ.NN) AA(I,J-2)=2.0*C8
IF (COL.NE.NN) AA(I,J-2)=C8+C9
36 IF(COL.EQ.1) GO TO 37
IF (COL.EQ.NN) AA(I,J-1)=4.0*C8-12.0*C10-8.0*C11-3.0*C12-2.0*C13
IF (COL.EQ.2) AA(I,J-1)=-2.0*C8+2.0*C10-4.0*C11-0.5*C12-C13
IF(COL.NE.NN .AND. COL.NE.2) AA(I,J-1)=-4.0*C8+2.0*C9-2.0*C10
1-4.0*C11-0.5*C12-C13
37 IF (ROW.EQ.1 .OR. ROW.EQ.NUMROW) GO TO 38
IF(COL.EQ.2) AA(I,J)=5.0*C14+C15+C16*(C17-C18)+2.0*C20*(3.0*C18
1+C21-2.0*C22)
IF(COL.EQ.NN-1) AA(I,J)=5.0*C14-C15+C16*(C17-C18)+2.0*C20*(3.0*C18
1+C21-2.0*C22)
IF (COL.EQ.1) AA(I,J)=2.0*C14+C16*(C17-3.0*C18-C19)+C20*(6.0*C18
1+2.0*C21-8.0*C22+4.0*C23-3.0*C24)
IF (COL.EQ.NN) AA(I,J)=2.0*C14+C16*(C17-3.0*C18+C19)+C20*(6.0*C18
1+2.0*C21-8.0*C22-4.0*C23+3.0*C24)
IF(COL.GT.2 .AND. COL.LT.NN-1) AA(I,J)=6.0*C14+C16*(C17-C18)+2.0*C20
1*(3.0*C18+C21-2.0*C22)
GO TO 39
38 CK=+1.0
IF (ROW.EQ.NUMROW) CK = 1.0
IF(COL.EQ.2) AA(I,J)=5.0*C14+C15+C16*(C17-C18)+C20*(5.0*C18+2.*C21
1-4.0*C22)
IF(COL.EQ.NN-1) AA(I,J)=5.0*C14-C15+C16*(C17-C18)+C20*(5.0*C18+2.*
1*C21-4.0*C22)
IF(COL.EQ.1) AA(I,J)=2.0*C14+C16*(C17-3.0*C18-C19)+C20*(5.0*C18
1+2.0*C21-8.0*C22+4.0*C23-3.0*C24)
IF (COL.EQ.NN) AA(I,J)=2.0*C14+C16*(C17-3.0*C18+C19)+C20*(5.0*C18
1+2.0*C21-8.0*C22-4.0*C23+3.0*C24)
IF(COL.GT.2 .AND. COL.LT.NN-1) AA(I,J)=6.0*C14+C16*(C17-C18)+2.0*C20
1*(2.5*C18+C21-2.0*C22)
39 IF (COL.EQ.NN) GO TO 40
IF (COL.EQ.1) AA(I,J+1)=-4.0*C8+12.0*C10-8.0*C11+3.0*C12-2.0*C13
IF(COL.EQ.NN-1) AA(I,J+1)=-2.0*C8+2.0*C10-4.0*C11+0.5*C12-C13
IF(COL.NE.1 .AND. COL.NE.NN-1) AA(I,J+1)=-4.0*C8-2.0*C9+2.0*C10
1-4.0*C11+0.5*C12-C13
40 IF(COL.GE.NN-1) GO TO 41
IF (COL.EQ.1) AA(I,J+2)=2.0*C8
IF (COL.NE.1) AA(I,J+2)=C8+C9
41 IF (ROW.EQ.NUMROW) GO TO 45
IF (COL.EQ.1) GO TO 444
J5=I+NN-1.0
IF(COL.EQ.NN) AA(I,J5)=2.0*C1*(2.0*RAD+3.0*LAM)
IF (COL.NE.NN) AA(I,J5)=C1*(2.0*RAD+LAM)
430
444 J6=I+NN
IF(COL.EQ.1) AA(I,J6)=C3*(3.0*LAM2-C2+C7)+C4*(2.0*C5-2.0*LAM-C6)
IF(COL.EQ.NN) AA(I,J6)=C3*(3.0*LAM2-C2-C7)+C4*(2.0*C5-2.0*LAM+C6)
IF(COL.NE.1.AND.COL.NE.NN) AA(I,J6)=C3*(LAM2-C2)+C4*(C5-2.0*LAM)
IF (COL.EQ.NN) GO TO 43
J7 = I+NN+1
IF(COL.EQ.1) AA(I,J7)=2.0*C1*(2.0*RAD-3.0*LAM)
IF(COL.NE.1) AA(I,J7)=C1*(2.0*RAD-LAM)
43 IF(ROW.EQ.NUMROW-1) GO TO 45
J8=I+2*NN
AA(I,J8)=ALP(JK)*LAM4
45 RAD=RAD+LAM
IF(COL.EQ.NN) RAD=MNRAD
20 IF(COL.EQ.NN) COL=0
NC=NC+1
21 CONTINUE
C WRITE (6,9) ((AA(I,J),I=1,N),J=1,N)
C 9 FORMAT (3E18.8)
READ (5,*) LL
DO 556 I=1,N
556 BB(I)=0.0
DO 1156 L=1,LL
READ (5,*) I,BB(I),R,DR1
X=LAM**3*R**3*OTHETA**3
CX=X/DR1
BB(I)=BB(I)*CX
1156 CONTINUE
C WRITE (6,*) (BB(I),I=1,N)
C FORMAT (3E18.8)
CALL SOLVE (AA,BB,N)
STOP
END

C SUBROUTINE SOLVE TO SOLVE THE SIMULTANEOUS EQUATIONS

SUBROUTINE SOLVE (AA,BB,N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AA(400,400),BB(400)
K=0
508 K=K+1
C A=AA(K,K)
DO 510 J=1,N
AA(K,J)=AA(K,J)/CA
510 CONTINUE
BB(K)=BB(K)/CA
IF (K.EQ.N) GO TO 540
KK=K+1
DO 530 I=KK,N

431
AAA=AA(I,K)
DO 520 J=1,N
   AA(I,J)=AA(I,J)-AA(K,J)*AAA
520 CONTINUE
   BB(I)=BB(I)-BB(K)*AAA
530 CONTINUE
   GO TO 508
540 K=N
545 KK=K-1
   DO 570 I=1, KK
      AAAA=AA(I,K)
   DO 550 J=1,N
      AA(I,J)=AA(I,J)-AA(K,J)*AAAA
550 CONTINUE
   BB(I)=BB(I)-BB(K)*AAAA
570 CONTINUE
   K=K-1
   IF (K-1) 580,580,545
580 CONTINUE
WRITE (6,600)
600 FORMAT(/2X,'TABLE 1. NODE DEFLECTIONS'
      .   ////2X,'---------------------'
      .   /2X,' NODE   DEFLECTION '
      .   /2X,'NUMBER   (ft.) '
      .   /2X,'---------------------')
WRITE (6,601) (I,BB(I),I=1,N)
601 FORMAT (3X,I3,9X,D10.3)
WRITE (6,602)
602 FORMAT(2X,'---------------------')
RETURN
END
INPUT PARAMETERS

The following describes the necessary input data for the finite difference computer program:

Card 1 - N, NN

where

N = Total number of mesh points
NN = Number of mesh points per row

Card 2 - MINRAD, LAM, OTHETA

where

MINRAD = Minimum radius (r)
LAM = Radial spacing (λ)
OTHETA = Angular spacing (θ₀) in radians

Card 3 - ALP(I)

where

ALP(I) = D₀/Dᵣ and I = 1, NN

Card 4 - BET(I)

where

BET(I) = D₀/Dᵣ and I = 1, NN

Card 5 - LL

where

LL = Number of loaded points on the bridge
Card 6 - I, BB(I), R, DR

where

I = Location of load, mesh point number
BB(I) = Value of load at mesh point I
R = Radius to the location of the load
DR = Radial rigidity at the load location

Enter as many cards as needed to represent the number of loaded points.

The output data are the node number and the corresponding deflection.
APPENDIX A-2

MOMENT-CARRYING CAPACITY OF WAFFLE SLAB BRIDGES
MOMENT-CARRYING CAPACITY OF
WAFFLE SLAB BRIDGES

The calculations of the ultimate moment of resistance of waffle slab bridges are based on
the following assumptions:

i- Strain distribution is assumed to be linear;

ii- Concrete in the tension zone of the section is neglected;

iii- At the ultimate stage all reinforcing steels in the tension zone reach their yield stress;

and

iv- All calculations for determining the ultimate moment of resistance are based on
smeared values.

1- Reinforced Concrete Waffle Slab Bridges

According to the ACI Code (1989) the stress distribution is taken as shown in Fig. A-2.1,
assuming the neutral axis lies in the slab. Equating the compressive force \( C \) in the concrete to
the tensile force \( T \) in the steel:

\[ T = C \]

yields

\[ A_s f_y = \beta_1 f'_c S a \]

which leads to

\[ M_u = \frac{A_s f_y (d - a/2)}{S} \]
where

\[ \beta_i = 0.85 \]

\[ A_s = \text{area of steel in in}^2; \]

\[ f_y = \text{yield stress of the steel}; \]

\[ f_c' = \text{ultimate strength of the concrete}; \]

\[ S = \text{spacing between ribs}; \text{ and,} \]

\[ a = \text{depth of the compression stress block}. \]

2. Prestressed Concrete Waffle Slab Bridges

An accurate estimation for the ultimate moment of resistance of unbonded section is more difficult than that of bonded ones. This is because the stress in the steel at rupture of the section cannot be closely computed. The stress distribution of a prestressed concrete waffle section with prestressed and non-prestressed steel can be taken as shown in Fig. A-2.2. Equating the compressive force \( C \) in the concrete to the tensile force \( T \) in the steel:

\[ T = C \]

or,

\[ T = T_p + T_s = 0.85 \ v^' \ a \ S \]

thus,

\[ f_{ps} A_{ps} + f_y A_s = 0.85 \ v^' \ a \ S \]

from which
\[ a = \frac{f_{ps} A_{ps} + f_y A_s}{0.85 f_c' S} \]

\[ M_s = \frac{(f_{ps} A_{ps} + f_y A_s) (d - a/2)}{S} \]

in which

- \( A_{ps} \) = Area of prestressed wires;
- \( A_s \) = Area of non-prestressed steel;
- \( f_y \) = Yield stress of the steel;
- \( f_{ps} \) = Ultimate stress of the prestressed wires at failure; and,
- \( d \) = Distance between top fiber of the section to the resultant tension force \( T \).

The ACI Code (1989) recommends the following equation for unbonded members:

\[ f_{ps} = f_{se} + 10,000 + \frac{f_c'}{100 \rho_p} \] (psi)

in which

- \( f_{se} \) = The effective prestress in the wires \( \text{ psi} \)
- \( f_{py} \) = The yield stress of the prestress wires \( \text{ psi} \)
- \( \rho_p = A_{ps} / bd \)

It should be noted that the value of \( f_{se} \) shall not taken greater than \( f_{py} \) nor \( (f_{se} + 60,000) \).

The CAN (1984) suggested another equation as follows:
\[ f_{ps} = f_{se} + \frac{5000}{l_o} (d_p - c_y) < f_{py} \]

in which

\( c_y \) = Distance from extreme compression fibre to neutral axis calculated using factored material strengths and assuming a tendon force of \( \phi_p A_p f_{py} \) (mm);

\( d_p \) = Distance from the extreme compression fiber to the centroid of the prestressing steel, in (mm);

\( l_o \) = Length of tendon between anchors divided by the number of plastic hinges required to develop a failure mechanism in the span under consideration, in (mm);

\( \phi_p \) = Resistance factor for prestressing steel = 0.9; and,

\( \phi_c \) = Resistance factor for concrete = 0.60.
Fig. A-2.1 Ultimate Moment Capacity of Reinforced Concrete Waffle Slab Section
Fig. A-2.2 Ultimate Moment Capacity of Prestressed Concrete Waffle Slab Section
APPENDIX A-3

STRESS-STRAIN CURVE FOR PRESTRESSING STEEL AND CALIBRATION OF LOAD CELLS
Fig. A–3.1 Typical Stress–Strain Curve for the Prestressing Steel

Note: 1 ksi=6.89 MPa
Fig. A–3.2 Typical Calibration Chart of Cylindrical Load Cell
Fig. A-3.3 Calibration Chart of 50 kip - Load Cell
CONCRETE MIX DESIGN

A.4.1 General

The concrete mix design for the laboratory bridge models was based on the

A.4.2 Design Parameters

- High Early Strength Portland Cement (Type 30)
- Fineness modulus for fine aggregate = 2.56
- Compressive strength $f'_c = 6000$ psi (41.3 MPa)
- Slump = 3 in. (80 mm)
- Air Content = 0.0 %

A.4.3 Design Steps

Step (1) - Max. Size of Aggregate:

- Max. size $\leq b/5 \leq 2.5/5 \leq 0.5$ in. (13 mm)

Take max. size = 3/8 in. (10 mm)

Step (2) - Estimation of Mixing Water Content:

According to Table 5.3.3, for slump = 3 - 4 in., nominal max. size of aggregate

= 3/8 in;

$W = 385 \text{ lb/}yd^3 = 228 \text{ kg/m}^3$
Step (3) - Selection of Water-Cement Ratio:

According to Table 5.3.4(a), W/C = 0.4

Step (4) - Calculation of Cement Content:

Cement = 228/0.4 = 570 kg/m³ (962.5 lb/yd³)

Step (5) - Estimation of Coarse Aggregate Content:

From Table 5.3.6, for fineness modulus of 2.56 and max. size of 3/8 in. (10 mm), the coarse aggregate content = 0.484. As the rodded-weight of coarse aggregate = 1600 kg/m³ (2700 lb/yd³), the dry weight of coarse aggregate = 0.484 x 1600 = 775 kg/m³ (1308 lb/yd³).

Step (6) - Estimation of Fine Aggregate Content

From Table 5.37.1, the estimated weight of fresh concrete = 3840 lb/yd³ (2278 kg/m³), hence:

fine aggregate = 2278 - (228 + 570 + 775) = 705 kg/m³

Step (7) - The Final Estimated Batch:

a) Water = 228 kg/m³ (14.30 lb/ft³)
b) Cement = 570 kg/m³ (35.6 lb/ft³)
c) Coarse aggregate = 775 kg/m³ (48.4 lb/ft³)
d) Fine aggregate = 705 kg/m³ (44.0 lb/ft³)

This lead to the following mix proportion:

(W : C : CA : FA) = (1 : 2.5 : 3.4 : 3.1)
APPENDIX A-5

RIGIDITIES OF CURVED WAFFLE SLAB STRUCTURES
RIGIDITIES OF CURVED CONCRETE
WAFFLE SLAB STRUCTURES

A-5.1 Uncracked Rigidities of Waffle Slab

A-5.1.1 Tangential Flexural Rigidity (Specimen \( F_{et} \))

\[ f_{c2} = 7665 \text{ psi (53.0 MPa)} \]
\[ E_c = 5700 \times (7665)^{0.5} = 4.99 \times 10^6 \text{ psi (34.4 GPa)} \]
\[ \mu = (f_e)^{0.5}/350 = 0.25 \]
\[ n = E_c/E_e = 30 \times 10^6/4.99 \times 10^6 = 6.0 \]

\[ e_0 = \frac{(1.5)(3.0)(2.5) + \left(\frac{(5.5)(1.0)}{2(1-0.25^2)}\right)}{1.5 \times 3.0 + \left[\frac{5.5 \times 1.0}{(1-0.25^2)}\right]} = 1.36 \text{ in. (33.4 mm)} \]

\[ I_0 = \frac{1.5(3.0)^3}{12} + 1.5 \times 3.0(2.5 - 1.367)^2 = 9.15 \text{ in}^4 \approx 3.81 \times 10^6 \text{ mm}^4 \]

\[ D_0 = \frac{4.99 \times 10^6(1.0)^3}{12(1-0.25^2)} + \left[\frac{4.99 \times 10^6 \times (1.367 - 0.5)^2}{(1-0.25^2)}\right] + \frac{4.99 \times 10^6 \times 9.1!}{5.5} \]

\[ = 0.44 \times 10^6 + 4.0 \times 10^6 + 8.30 \times 10^6 = 12.74 \times 10^6 \text{ lb.in}^2/\text{in.} \]

(1440 kN.m²/m)
A-5.1.2 Radial Flexural Rigidity (Specimen FR1)

\[ f_{cs} = 6500 \text{ psi (44.8 MPa)} \]

\[ E_c = 4.6 \times 10^6 \text{ psi (31.74 GPa)} \]

\[ \mu = 0.23 \]

\[ n = 6.50 \]

**Station I**

\[ S_r = 6.58 \text{ in. (167 mm)} \]

\[ e_r = 1.28 \text{ in. (32.5 mm)} \]

\[ R'_r = \frac{1.5(3.0)^3}{12} + 4.5(2.5-1.28)^2 = 10.07 \text{ in}^4 (4.19 \times 10^6 \text{ mm}^4) \]

\[ D_x = 0.405 \times 10^6 + 4.6 \times 10^6 \frac{(1.28-0.5)^2}{(1-0.23^2)} + 4.6 \times 10^6 \times 10^7 \]

\[ = 0.405 \times 10^6 + 2.955 \times 10^6 + 7.039 \times 10^6 = 10.39 \text{ lb.in}^2 / \text{in. (1170 kN.m}^2/\text{m}) \]

**Station II**

\[ S_r = 6.0 \text{ in. (125 mm)} \]

\[ e_r = 1.32 \text{ in. (33.5 mm)} \]

\[ L_r = 9.64 \text{ in}^4 (4.01 \times 10^6 \text{ mm}^4) \]

\[ D_x = 0.4047 \times 10^6 + 3.2658 \times 10^6 + 7.39 \times 10^6 \]

\[ = 11.06 \times 10^6 \text{ lb.in}^2 / \text{in. (1250 kN.m}^2/\text{m}) \]

**Station III**

\[ S_r = 5.5 \text{ in. (140 mm)} \]
$e_r = 1.36$ in. (34.5 mm)

$I_{r}' = 9.22$ in.$^4$ (3.84 X $10^6$ mm$^4$)

$D_r = 0.4047 \times 10^6 + 3.592 \times 10^6 + 7.71 \times 10^6$

$= 11.71 \times 10^6$ lb.in.$^2$/in (1320 kN.m$^2$/m)

**Station IV**

$S_r = 4.95$ in. (126 mm)

$e_r = 1.42$ in. (36 mm)

$I_{r}' = 8.62$ in.$^4$ (3.59 X $10^6$ mm$^4$)

$D_r = 0.4047 \times 10^6 + 4.11 \times 10^6 + 8.01 \times 10^6$

$= 12.53 \times 10^6$ lb.in.$^2$/in (1420 kN.m$^2$/m)

**Station V**

$S_r = 4.41$ in. (112 mm)

$e_r = 1.48$ in. (37.5 mm)

$I_{r}' = 7.97$ in.$^4$ (3.32 X $10^6$ mm$^4$)

$D_r = 0.4047 \times 10^6 + 4.66 \times 10^6 + 8.31 \times 10^6$

$= 13.38 \times 10^6$ lb.in.$^2$/in (1510 kN.m$^2$/m)

**A-5.1.3 Torsional Rigidities (Specimen T1)**

$f_{cis'} = 6800$ psi (47.0 MPa)

$E_c = 4.7 \times 10^6$ psi (32.4 GPa)

$\mu = 0.20$
-Station I

\( S_r = 6.58 \text{ in. (167 mm)} \)

\( J_1 = \frac{1}{2} \times 0.291 \times 5.5 \times (1.0)^3 = 0.8 \text{ in}^4. \ (0.33 \times 10^6 \text{ mm}^4) \)

\( J_2 = 0.229 \times 3.0 \times (1.5)^3 = 2.318 \text{ in}^4. \ (0.96 \times 10^6 \text{ mm}^4) \)

Neglect the torsional constant of the reinforcing steel \( J_0 \)

\( J_1 \text{ (modified)} = 0.8 \times \left[ (0.979 + 2.318)/0.979 \right] = 2.695 \text{ in}^4. \ (1.11 \times 10^6 \text{ mm}^4) \)

\( J_0 = J_1 \text{ (modified)} + J_2 = 2.695 + 2.318 = 5.01 \text{ in}^4. \ (2.09 \times 10^6 \text{ mm}^4) \)

\( G = 4.7 \times 10^6 /[2(1 + 0.20)] = 1.91 \times 10^6 \text{ psi} \ (13.20 \text{ GPa}) \)

\( D_{\theta r} = D_{\theta e} = 1.91 \times 10^6 \times 5.01/5.5 = 1.74 \times 10^6 \text{ lb.in}^2/\text{in.} \)

\( (197 \text{ kN.m}^2/\text{m}) \)

-Station II

\( S_r = 6.0 \text{ in. (152 mm)} \)

\( J_1 = 0.8 \text{ in}^4. \ (0.33 \times 10^6 \text{ mm}^4) \)

\( J_2 = 2.32 \text{ in}^4. \ (0.96 \times 10^6 \text{ mm}^4) \)

\( J_1 \text{ (modified)} = 2.91 \text{ in}^4. \ (1.20 \times 10^6 \text{ mm}^4) \)

\( J_0 = 5.23 \text{ in}^4. \ (2.18 \times 10^6 \text{ mm}^4) \)

\( D_{\theta r} = D_{\theta e} = 1.82 \times 10^6 \text{ lb.in}^2/\text{in.} \ (206 \text{ kN.m}^2/\text{m}) \)

-Station III

\( S_r = 5.5 \text{ in. (140 mm)} \)

\( J_1 = 0.8 \text{ in}^4. \ (0.33 \times 10^6 \text{ mm}^4) \)

\( J_2 = 2.32 \text{ in}^4. \ (0.96 \times 10^6 \text{ mm}^4) \)

\( J_1 \text{ (modified)} = 3.12 \text{ in}^4. \ (1.30 \times 10^6 \text{ mm}^4) \)

\( J_0 = 5.44 \text{ in}^4. \ (2.26 \times 10^6 \text{ mm}^4) \)
\[ D_{or} = D_{r0} = 1.89 \times 10^6 \text{ lb.in}^2/\text{in.} \quad (214 \text{ kN.m}^2/\text{m}) \]

-Station IV

\[ S_r = 4.95 \text{ in.} \quad (126 \text{ mm}) \]

\[ J_1 = 0.8 \text{ in}^4. \quad (0.33 \times 10^6 \text{ mm}^4) \]

\[ J_2 = 2.32 \text{ in}^4. \quad (0.96 \times 10^6 \text{ mm}^4) \]

\[ J_1 \text{ (modified)} = 3.37 \text{ in}^4. \quad (1.40 \times 10^6 \text{ mm}^4) \]

\[ J_0 = 5.69 \text{ in}^4. \quad (2.37 \times 10^6 \text{ mm}^4) \]

\[ D_{or} = D_{r0} = 1.97 \times 10^6 \text{ lb.in}^2/\text{in.} \quad (223 \text{ kN.m}^2/\text{m}) \]

-Station V

\[ S_r = 4.41 \text{ in.} \quad (112 \text{ mm}) \]

\[ J_1 = 0.8 \text{ in}^4. \quad (0.33 \times 10^6 \text{ mm}^4) \]

\[ J_2 = 2.32 \text{ in}^4. \quad (0.96 \times 10^6 \text{ mm}^4) \]

\[ J_1 \text{ (modified)} = 3.75 \text{ in}^4. \quad (1.55 \times 10^6 \text{ mm}^4) \]

\[ J_0 = 6.07 \text{ in}^4. \quad (2.53 \times 10^6 \text{ mm}^4) \]

\[ D_{or} = D_{r0} = 2.11 \times 10^6 \text{ lb.in}^2/\text{in.} \quad (238 \text{ kN.m}^2/\text{m}) \]

A-5.2 Cracked Rigidities of Waffle Slab

A-5.2.1 Tangential Flexural Rigidity (Specimen F61)

The location of the neutral axis is determined from Eq. 3.15 as follows:

\[ 6.0 \times 0.05 \left[ (4.0 - 0.625) - Kd_0 \right] - [5.5 \left( Kd_0 \right)^2 / 2 (1 - 0.25^2)] = 0.0 \]

\[ Kd_0^2 + 0.103 Kd_0 - 0.346 = 0.0 \]

\[ Kd_0 = 0.539 \text{ in.} \quad (14 \text{ mm}) \]

\[ I_{d0} = 5.5(0.539)^3 / 3 = 0.287 \text{ in}^4. \quad (0.12 \times 10^6 \text{ mm}^4) \]
\[ I_{x0} = 6.0 \times 0.05 \left[ 3.375 - 0.539 \right]^2 = 2.42 \text{ in}^4 \times (1.01 \times 10^6 \text{ mm}^4) \]

\[ D_\theta = 4.99 \times 10^6 \left[ 2.42 + 0.287/(1 - 0.25^2) \right]/5.5 \]

\[ = 2.47 \times 10^6 \text{ lb.in}^2/\text{in.} \times (279 \text{ kN.m}^2/\text{m}) \]

**A-5.2.2 Radial Flexural Rigidities (Specimen FR1)**

**Station I**

\[ S_r = 6.58 \text{ in.} \times (167 \text{ mm}) \]

\[ Kd_r^2 + 0.0939 \: Kd_r - 0.2936 = 0.0 \]

\[ Kd_r = 0.497 \text{ in.} \times (13 \text{ mm}) \]

\[ I_{xr} = 0.269 \text{ in}^4 \times (0.11 \times 10^6 \text{ mm}^4) \]

\[ I_r = 2.25 \text{ in}^4 \times (0.94 \times 10^6 \text{ mm}^4) \]

\[ D_r = 4.60 \times 10^6 \left[ 2.25 + 0.269/(1 - 0.23^2) \right]/6.58 \]

\[ = 1.77 \times 10^6 \text{ lb.in}^2/\text{in.} \times (200 \text{ kN.m}^2/\text{m}) \]

**Station II**

\[ S_r = 6.0 \text{ in.} \times (152 \text{ mm}) \]

\[ Kd_r^2 + 0.103 \: Kd_r - 0.322 = 0.0 \]

\[ Kd_r = 0.519 \text{ in.} \times (13 \text{ mm}) \]

\[ I_{xr} = 0.279 \text{ in}^4 \times (0.12 \times 10^6 \text{ mm}^4) \]

\[ I_r = 2.214 \text{ in}^4 \times (0.92 \times 10^6 \text{ mm}^4) \]

\[ D_r = 4.60 \times 10^6 \left[ 2.214 + 0.279/(1 - 0.23^2) \right]/6.0 \]

\[ = 1.92 \times 10^6 \text{ lb.in}^2/\text{in.} \times (217 \text{ kN.m}^2/\text{m}) \]

**Station III**

\[ S_r = 5.5 \text{ in.} \times (140 \text{ mm}) \]

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\[ K_d^2 + 0.112K_d - 0.351 = 0.0 \]

\[ K_d = 0.539 \text{ in. (14 mm)} \]

\[ I_w = 0.287 \text{ in.}^4 (0.12 \times 10^6 \text{ mm}^4) \]

\[ I_w = 2.18 \text{ in.}^4 (0.91 \times 10^6 \text{ mm}^4) \]

\[ D_r = 4.60 \times 10^6 \left[ 2.18 + \frac{0.287}{(1 - 0.23^2)} \right]/5.5 \]

\[ = 2.07 \times 10^6 \text{ lb.in}^2/\text{in.} (234 \text{ kN.m}^2/\text{m}) \]

**Station IV**

\[ S_r = 4.95 \text{ in. (126 mm)} \]

\[ K_d^2 + 0.1248K_d - 0.39 = 0.0 \]

\[ K_d = 0.565 \text{ in. (14 mm)} \]

\[ I_w = 0.297 \text{ in.}^4 (0.12 \times 10^6 \text{ mm}^4) \]

\[ I_w = 2.136 \text{ in.}^4 (0.89 \times 10^6 \text{ mm}^4) \]

\[ D_r = 4.60 \times 10^6 \left[ 2.136 + \frac{0.297}{(1 - 0.23^2)} \right]/4.95 \]

\[ = 2.27 \times 10^6 \text{ lb.in}^2/\text{in.} (256 \text{ kN.m}^2/\text{m}) \]

**Station V**

\[ S_r = 4.41 \text{ in. (112 mm)} \]

\[ K_d^2 + 0.1402K_d - 0.438 = 0.0 \]

\[ K_d = 0.595 \text{ in. (15 mm)} \]

\[ I_w = 0.31 \text{ in.}^4 (0.13 \times 10^6 \text{ mm}^4) \]

\[ I_w = 2.09 \text{ in.}^4 (0.87 \times 10^6 \text{ mm}^4) \]

\[ D_r = 4.60 \times 10^6 \left[ 2.09 + \frac{0.31}{(1 - 0.23^2)} \right]/4.41 \]

\[ = 2.517 \times 10^6 \text{ lb.in}^2/\text{in.} (284 \text{ kN.m}^2/\text{m}) \]
A-5.2.3 Torsional Rigidities (Specimen T₁)

Station I

\[ S_r = 6.58 \text{ in. (167 mm)} \]

\[ D_{θ} = 1.74 \times 10^6 \text{ lb.in}^2./\text{in. (197 kN.m}^2/\text{m)} \]

\[ D_θ = 12.0 \times 10^6 \text{ lb.in}^2./\text{in. (1360 kN.m}^2/\text{m)} \]

\[ D_r = 10.65 \times 10^6 \text{ lb.in}^2./\text{in. (1200 kN.m}^2/\text{m)} \]

\[ D_1 = 0.967 \times 10^6 \text{ lb.in}^2./\text{in. (109 kN.m}^2/\text{m)} \]

\[ D_2 = 0.814 \times 10^6 \text{ lb.in}^2./\text{in. (92 kN.m}^2/\text{m)} \]

From Eq. 3.20, the torsional parameter \( k = 0.233 \).

The post-cracking flexural rigidities are as follows:

\[ D_θ(\text{cracking}) = 2.326 \times 10^6 \text{ lb.in}^2./\text{in. (236 kN.m}^2/\text{m)} \]

\[ D_1(\text{cracking}) = 1.81 \times 10^6 \text{ lb.in}^2./\text{in. (205 kN.m}^2/\text{m)} \]

\[ D_1(\text{cracking}) = 0.614 \times 10^5 \text{ lb.in}^2./\text{in. (7.0 kN.m}^2/\text{m)} \]

\[ D_2 = 0.475 \times 10^5 \text{ lb.in}^2./\text{in. (5.0 kN.m}^2/\text{m)} \]

Substituting the post-cracking flexural rigidities and the torsional parameters \( k \) in Eq. 3.21 the post-cracking torsional rigidities will be as follows:

\[ D_{θ} (\text{cracking}) = 0.233 \times 2.326 \times 1.81^{1/2} \times 10^6 - [(0.614 + 0.475)/2] \times 10^5 \]

\[ = 4.23 \times 10^5 \text{ lb.in}^2./\text{in. (48 kN.m}^2/\text{m)} \]

Station II

\[ S_r = 6.0 \text{ in. (152 mm)} \]

\[ k = 0.235 \]

\[ D_{θ} (\text{cracking}) = 0.235 \times 1.96 \times 2.326^{1/2} \times 10^6 - [(0.614 + 0.542)/2] \times 10^5 \]

\[ = 4.44 \times 10^5 \text{ lb.in}^2./\text{in. (50 kN.m}^2/\text{m)} \]
Station III

\[ S_r = 5.5 \text{ in. (140 mm)} \]

\[ k = 0.237 \]

\[ D_{re} \text{ (cracking)} = 0.237 (2.12 \times 2.33)^{1/2} \times 10^6 \times \left[ (0.614 + 0.614)/2 \right] \times 10^5 \]

\[ = 4.64 \times 10^5 \text{ lb.in}^2/\text{in. (52 kN.m}^2/\text{m)} \]

Station IV

\[ S_r = 4.95 \text{ in. (126 mm)} \]

\[ k = 0.242 \]

\[ D_{re} \text{ (cracking)} = 0.242 (2.32 \times 2.33)^{1/2} \times 10^6 \times \left[ (0.614 + 0.699)/2 \right] \times 10^5 \]

\[ = 4.96 \times 10^5 \text{ lb.in}^2/\text{in. (56 kN.m}^2/\text{m)} \]

Station V

\[ S_r = 4.4 \text{ in. (112 mm)} \]

\[ k = 0.249 \]

\[ D_{re} \text{ (cracking)} = 0.249 (2.57 \times 2.33)^{1/2} \times 10^6 \times \left[ (0.614 + 0.82)/2 \right] \times 10^5 \]

\[ = 5.37 \times 10^5 \text{ lb.in}^2/\text{in. (60.7 kN.m}^2/\text{m)} \]
APPENDIX A-6

CALCULATION OF THEORETICAL COLLAPSE LOAD
CALCULATION OF THEORETICAL COLLAPSE LOAD

All the calculations are performed with reference to Figs. A-2.1 and A-2.2.

The relevant data are as follows:
span length (along center line) = 2L = 70 in. (1778 mm); bridge width = W = 34 in. (864 mm); radius of curvature (at inner edge) = R = 38.75 in. (984 mm); the connecting angle \( \theta_r = 71.94 \) degrees (1.255 radian); the cross sectional dimensions are shown in Fig. 7. 10, and unfactored dead weight \( q_d = 0.33 \) psi (2.27 kPa).

A-6.1 Bridge Model RWS Subjected to Concentrated Load at Center of Each Lane

\[ A_s = 0.0491 \text{ in}^2 \text{ (32 mm}^2\text{)}; \ t_f = 60 \text{ ksi (414 MPa)} \text{ and } f_c' = 7.8 \text{ ksi (54 MPa)}. \]

From \( C = T \), or \( 0.85 f_y' a S = A_s f_y' \), from which \( a = 0.063 \) in. (1.6 mm). Therefore, \( m_t = A_s f_y \) (lever arm)/ \( S = (0.0491)(60,000)(5.5-0.625-0.032)/7.0 = 2038 \) lb.in./in. (9.0 kN.m/m). From Eq. 6.9 with \( q_d = 0.33 \) psi (2.27 kPa) and the moment of resistance of the edge beam \( M_0 = 9.47 \) kip.in. (41.7 kN.m), \( P_u = 4.6 \) kip. (20.6 kN).

A-6.2 Bridge Model RWS Subjected to Concentrated Load at the Center of the Inner Edge

Take \( f'_c = 7.8 \) ksi (54 MPa) as before, \( m_t = 2038 \) lb.in./in. (9.0 kN.m/m). Take the coefficient \( \beta'' = 1.0 \), and hence from Eq. 6.24, \( \zeta = 1.0; \eta = 0.0; \psi = 1.0 \) and \( \phi = 1.0 \). Substitute these values in Eq. 6.23, the ultimate collapse load will be \( P_u = 4.90 \) kip (21.9 kN).
A-6.3 Bridge Model RWS Subjected to Concentrated Load at the Center of the Outer Edge

Take $f'_c = 7.8$ ksi (54 MPa) as before, $m_1 = 2038$ lb.in./in. (9.0 kN.m/m). Take the coefficient $\beta' = 4.0$, and substitute it in Eq. 6.37, and from which $\xi = -2.42$; $\eta = 3.0$; $\psi = 2.5$ and $\phi = 3.0$. Substitute these values in Eq. 6.36, the ultimate collapse load will be $P_u = 2.70$ kip (12.1 kN).

A-6.4 Bridge Model PWS Subjected to Concentrated Load at Center of Each Lane

$f'_c = 6.25$ ksi (43.1 MPa); $A_{ps} = 0.0598$ in.$^2$ (38.5 mm$^2$); $A_s = 0.0276$ in.$^2$ (18.0 mm$^2$); average prestressing force in each rib = 5.3 kip (23.7 kN); $f_{ps} = 5300/0.0598 = 88.6$ ksi (610 Mpa). Follow the ACI-ASCE Code to determine $f_{ps}$ (Appendix A-2), $f_{ps} = 122.4$ ksi (844 MPa). Therefore, $T_p = f_{ps} A_{ps}$ or $T_p = (122.4)(0.0598) = 7.3$ kip (32.7 kN) and $T_s = f_s A_s$ or $T_s = (60)(0.0276) = 1.65$ kip (7.4 kN), the total tension force $T = 7.3 + 1.65 = 8.95$ kip (40 kN). From $C = T$ or $8.95 = (0.85)(6.25)(7.0)(a)$, from which $a = 0.24$ in. (6.0 mm) and hence $Z = (5.5)-(1.95)-(0.12) = 3.43$ in. (87 mm). The ultimate moment $m_1 = T Z / S = (8.95)(3.43)/(7.0) = 4380$ lb.in./in. (19.5 kN.m/m). From Eq. 6.9 with $q_D = 0.33$ psi (2.27 kPa) and $M_b = 5.4$ kip.in. (23.8 kN.m), the ultimate collapse load will be $P_u = 8.6$ kip (38.5 kN).

Follow the Canadian Code CAN3-A23.3-M84 to determine $f_{ps}$ (Appendix A-2), $f_{ps} = 114.4$ ksi (789 MPa). Therefore, $T_p = f_{ps} A_{ps}$ or $T_p = (114.4)(0.0598) = 6.8$ kip (30.5 kN) and $T_s = 1.65$ kip (7.4 kN), the total tension force $T = 6.8 + 1.65 = 8.5$ kip (38.0 kN). From $C = T$, $a = 0.23$ in. (6 mm) and hence, $Z = 3.46$ in. (88 mm), the ultimate moment of capacity $m_1 = (8.5)(3.46)/(7.0) = 4.2$ kip.in/in. From Eq. 6.9 with $q_D = 0.33$ psi (2.27...
kPa) and $M_B = 5.4$ kip.in., the ultimate collapse load will be $P_u = 8.3$ kip (37.0 kN).

Following the method suggested by Kennedy and El-Sebakhy (1982), the stress in the prestressing steel at failure, $f_{ps} = 1.15 f_{se}$ or $f_{ps} = (1.15)(88.6) = 101.9$ ksi (703 MPa). The ultimate collapse load, following the same procedures as mentioned before, will be $P_u = 7.7$ kip (34.5 kN).

**A-6.5 Bridge Model PWS Subjected to Concentrated Load at the Center of the Inner Edge**

$f'_{c} = 6.25$ ksi (43.1 MPa); $A_{ps} = 0.0598 \text{ in}^2. (38.5 \text{ mm}^2)$; $A_s = 0.0276 \text{ in}^2. (18.0 \text{ mm}^2$); $f_{se} = 88.6$ ksi (610 MPa); $f_{ps} = 122.4$ ksi (844 MPa). Follow ACI Code (1989); $m_1 = 4.38$ kip.in./in. (19.5 kN.m/m); $M_B = 5.4$ kip.in (23.8 kN.m). Take the coefficient $\beta'' = 1.0$, and hence, from Eq.6.24, $\xi = \psi = \phi = 1.0$ and $\eta = 0.0$. Substitute these values in Eq. 6.23, the ultimate collapse load will be, $P_u = 9.10$ kip (40.8 kN).

**A-6.6 Bridge Model PWS Subjected to Concentrated Load at the Center of the outer Edge**

$f'_{c} = 6.25$ ksi (43.1 MPa); $A_{ps} = 0.0598 \text{ in}^2. (38.5 \text{ mm}^2)$; $A_s = 0.0276 \text{ in}^2. (18.0 \text{ mm}^2$); $f_{se} = 88.6$ ksi (610 MPa); $f_{ps} = 122.4$ ksi (844 MPa). Follow ACI Code (1989); $m_1 = 4.38$ kip.in./in. (19.5 kN.m/m); $M_B = 5.4$ kip.in (23.8 kN.m). Take the coefficient $\beta' = 4.0$, and hence, from Eq.6.37, $\xi = -2.42$; $\psi = 3.0$; $\phi = 3.0$ and $\eta = 2.5$. Substitute these values in Eq. 6.36, the ultimate collapse load will be, $P_u = 5.30$ kip (40.8 kN).
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