Buckling of built-up columns.

Ghada Mohamed. Elmahdy

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BUCKLING OF BUILT-UP COLUMNS

by

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B.Sc., M.A.Sc.

A Dissertation
Submitted to the Faculty of Graduate Studies and Research through
the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy at the
University of Windsor

Windsor, Ontario, Canada
1997
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0-612-30272-5
ABSTRACT

The problem of built-up columns has been under investigation since the collapse of the Quebec Bridge in 1907, which occurred due to the failure of the lower chords. It was reported that these elements were not designed as latticed columns. It is widely accepted that built-up columns exhibit reduced shear stiffness resulting in an increase in lateral deflection. This reduction in shear stiffness leads to a reduction in the strength of the column, which may be accounted for by increasing the slenderness ratio of the built-up member to an equivalent slenderness ratio.

One of the questions addressed by this research was to determine the appropriate value of this equivalent slenderness ratio. A differentiation was made between the action of a battened and a buttoned column. The second problem tackled was the need to limit the slenderness ratio of the individual members between interconnectors to something less than the slenderness ratio of the integral member. It was found that increasing this slenderness ratio decreased the strength of the column, but this could be accounted for by using the equivalent slenderness
ratio. The second concern with increasing the slenderness ratio of the individual members between interconnectors was the occurrence of simultaneous local and global buckling. This interaction could also lead to a decrease in the strength of the column resulting from imperfection sensitivity.

An experimental program was designed to investigate these problems. Three groups of specimens were tested. The first two groups were composed of channel sections, interconnected by batten plates, placed toe-to-toe forming a box-like section. The ends were hinged about the axis passing through the web, the $X$ axis, and fixed about the other axis, the $Y$ axis. The specimens with one and two interconnectors buckled about the $Y$ axis, whereas, the specimens with more interconnectors buckled about the $X$ axis. The third group was composed of channel sections, placed back-to-back, interconnected by batten or button plates. These specimens were hinged about the $Y$ axis and fixed about the $X$ axis, and buckled about the $Y$ axis.

The specimens of the experimental program were modelled using the finite element method and results were compared.
To my dear Mother
for her encouragement throughout
the work for this degree
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor Dr. M.C. Temple for his supervision, advice, and support throughout this research. I would also like to acknowledge Dr. N. Zamani for his suggestions in the finite element modelling.

I would like to thank Mr. Richard Clark for preparing the test specimens and set-up and his assistance during the testing. Thanks are also due to Mr. Dieter Liebsch in the Technical Support Centre for his assistance in preparing the specimens. I am also grateful to Mr. Gregory Kuhun for his assistance with the electronic equipment during the testing of the specimens.

I would like to thank Mr. Robert Mavrinac for his help with computing matters in the Faculty of Engineering, and Ms. Jaya Sreedharan for her help with network problems. I am also grateful to Miss Anne-Marie Bartlett for typing the references, and the Interlibrary Loan Services at Leddy Library for providing unavailable references.

I would like to thank NSERC for the funding of this research and the Faculty of Graduate Studies and Research for their financial help through scholarships and graduate assistantships.

Finally I would like to express my appreciation to Dr. A.F. Asfour, Dr. G. Abdel-Sayed, Dr. H. ElMaraghy, and Dr. W. ElMaraghy for their kindness during this degree.
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NOMENCLATURE

\(a\) centre-to-centre distance between interconnectors; length of local column
\(a, b, p\) finite element nodes
\(A\) integral cross-sectional area
\(A_0\) original cross-sectional area of tension coupon
\(A_1, B_1\) constants
\(A_b\) sum of cross-sectional areas of battens on one level
\(A_f\) area of flanges in one section
\(A_i\) cross-sectional area of one component (i.e. a single channel)
\(b\) distance between centroids of individual components (main members)
\(b'\) thickness of button plate or width of batten plate
\(b_1\) coefficient
\(B\) arbitrary point on the cross section
\(c\) half of distance between flanges
\(C\) spring stiffness
\(C_1, C_2\) spring stiffness
\[ C_i \] spring stiffness
\[ C_r \] compressive resistance
\[ d_1 \] clear distance between batten plates
\[ d_2 \] depth of batten plate
\[ D_i, E_i \] coefficients
\[ D^{ei} \] elasticity matrix
\[ E \] Young's modulus of elasticity
\[ E A^*_i \] reduced axial stiffness of flanges
\[ F_1, G_i \] coefficients
\[ F_y \] yield stress
\[ G \] shear modulus
\[ G_A, G_B \] factors for Julian and Lawrence nomograph
\[ G_R \] ratio of column stiffness to beam stiffness
\[ i \] iteration number
\[ I \] effective second moment of area of integral cross section including plasticity effects
\[ I_b \] sum of second moments of area of battens on one level
\[ I_f \] second moment of area of a main component about its centroidal axis parallel to the axis of buckling
\[ I_o \] second moment of area of the integral section about the \( Y \) axis neglecting the second moment of area of individual components about their own centroidal axis; second moment of area of column as a truss
\[ I_{ps} \] polar second moment of area of a single channel about its shear centre
\( I_x \)  
second moment of area of a single channel about the \( x \) axis

\( I_y \)  
second moment of area of one main member (\( i.e. \) a single channel) about the \( y \) axis (the axis normal to the plane of the battens)

\( I_{ps} \)  
polar second moment of area of integral cross section about the shear centre

\( I_r \)  
second moment of area of complete cross section such that \( I_r = 2I_y + I_o \)

\( I_{x}, I_{y} \)  
second moment of area of integral cross section about the \( X \) and \( Y \) axes, respectively

\( J \)  
torsion constant

\( J_i \)  
torsion constant of single channel

\( k_1 \)  
constant equal to \( b/L \)

\( k_b \)  
buckling stress coefficient

\( k_i \)  
local effective length factor in equivalent slenderness ratio formula

\( k_p, k_s \)  
differential equation coefficients

\( K \)  
effective length factor of column

\( K_i \)  
effective length factor of local column between interconnectors

\( K_{ix}, K_{iy} \)  
effective length factor for local flexural buckling about the local \( x \) and \( y \) axes, respectively

\( K_{i(\_)} \)  
effective length factor for local torsional buckling

\( K_{Tr+1}^i \)  
tangent stiffness for all iterations of the \( n+1^{st} \) increment in the modified Newton-Raphson method

\( K_{Tr+1}^i \)  
tangent stiffness of the \( i^{th} \) iteration of the \( n+1^{st} \) increment in the Newton-Raphson method

\( K_{X}, K_{Y} \)  
effective length factors for global buckling about the \( X \) and \( Y \) axes,
respectively

\( K_\phi \)  
**effective length factor for global torsional buckling**

\( l \)  
**current value of gauge length**

\( l' \)  
**distance from centre of end connector to end plate of column**

\( l_0 \)  
**original gauge length of tension coupon**

\( L \)  
**total length of column; length of rigid bar**

\( m \)  
**total length of middle line of cross section**

\( m_c \)  
**number of components or groups of components connected by batten plates**

\( m_i \)  
**length of each segment of cross section**

\( M \)  
**internal moment in column**

\( M_1, M_2 \)  
**end moments on local column with partial restraints**

\( M_b \)  
**balanced end moments on local column with partial restraints**

\( M_c \)  
**bending moment acting on connection between batten plate and main member**

\( M_s \)  
**magnified moment in column**

\( n \)  
**increment number**

\( \bar{n} \)  
**number of degrees of freedom**

\( n_b \)  
**modification factor**

\( n_p \)  
**number of panels**

\( n_s \)  
**shape factor**

\( N, N_E \)  
**component of internal force tangent to column axis**
\( N_H \) component of internal force normal to displaced and rotated cross section

\( o_1, o_2 \) midpoint modal amplitudes

\( O \) shear centre of integral cross section

\( p \) ratio of axial compressive force to Euler load

\( P \) axial compressive load on column; tension load on tension coupon

\( P' \) component of vertical load in direction of column axis

\( P_{cr} \) critical load of column

\( P_{cr}^0 \) critical load of perfect column

\( P_{cr1}, P_{cr2} \) uncoupled critical loads

\( P_d \) shear stiffness of column

\( P_f \) force in one chord

\( P_{fl} \) buckling load of flanges

\( P_{fl} \) local flexural-torsional buckling load of a single channel about the local \( x \) axis

\( P_l \) limit load for coupled mode

\( P_n \) total load at the start of the \( n+1^{th} \) increment

\( P_{test} \) experimental failure load

\( P_w \) buckling load of webs

\( P_x, P_y \) local flexural buckling load of a single channel about the local \( x \) and \( y \) axes, respectively

\( P_y \) yield load of column
$P_B$ overall buckling load

$P_E$ Euler buckling load

$P_G$ global buckling load

$P_L$ local buckling load

$P_{X}, P_{Y}$ global flexural buckling load about the $X$ and $Y$ axes, respectively

$P_{Y_{fl}}$ yield load of flanges

$P_{Y_w}$ yield load of webs

$P_{\phi}$ local torsional buckling load of a single channel about its shear centre

$P_{\phi}$ global torsional buckling load about the shear centre

$q$ inclination angle

$q_1, \ldots, q_{\bar{n}}$ generalized displacements

$\dot{q}_1, \ldots, \dot{q}_{\bar{n}}$ generalized velocities

$Q$ component of internal force in the direction normal to the column axis in the deformed state; shear force

$Q_0$ shearing component of vertical load at ends of column

$Q_1, Q_2$ internal shearing components in first and second panels

$Q_{E}, Q_{H}$ component of internal force in the direction that follows the material points of the displaced and rotated cross section

$r$ radius of gyration

$r_{eq}$ equivalent radius of gyration for local flexural-torsional buckling

$r_f$ radius of gyration of a main component about its centroidal axis parallel to the axis of buckling
\( r_i \)  
minimum radius of gyration of an individual component

\( r_n \)  
normal distance from the tangent at point \( B \) on cross section to shear centre for calculation of warping constant

\( r_o \)  
polar radius of gyration about the shear centre

\( r_x, r_y \)  
radius of gyration of a main member (i.e. single channel) about the \( x \) and \( y \) axes, respectively

\( r_X, r_Y \)  
radius of gyration of integral cross section about the \( X \) and \( Y \) axes, respectively

\( r_\phi \)  
equivalent radius of gyration for local torsional buckling

\( r_\Phi \)  
equivalent radius of gyration for global torsional buckling

\( R \)  
shape factor

\( R_l \)  
half the distance between chords of a laced column

\( R_n^{i+1} \)  
residual forces in the \( i^{th} \) iteration of the \( n+1^{th} \) increment

\( R_Y \)  
design yield strength

\( s \)  
distance along middle line of cross section from the origin to a point \( B \) for calculation of warping constant

\( S_v \)  
shear stiffness

\( t \)  
thickness of flange or plate; thickness along cross section

\( t_i \)  
average thickness of each segment of cross section

\( u, v \)  
displacement components in the \( x \) and \( y \) directions, respectively

\( u_1 \)  
local flange deflection

\( u_2 \)  
global flange deflection

\( u_a, u_b \)  
displacements of nodes \( a \) and \( b \)
$u_{n+1}$ total displacement at the end of the $i^{th}$ iteration of the $n+1^{th}$ increment

$u_o$ initial bow

$u_x, u_y, u_z$ displacements in the $X, Y, \text{and } Z$ directions, respectively

$u_x^a, u_y^a, u_z^a$ displacements in the $X, Y, \text{and } Z$ directions of nodes in node set HINGE, respectively

$u_x^t, u_y^t, u_z^t$ displacements in the $X, Y, \text{and } Z$ directions of nodes in node set TOPEND, respectively

$U$ elastic strain energy

$w$ width of flange or plate; displacement in the $z$ direction

$w_f$ axial shortening per length $a$ due to bending

$W$ work done by the loads

$x$ variable

$x, y, z$ distance along the $x, y, \text{and } z$ axes (or $X, Y, \text{and } Z$ axes), respectively

$x_0$ local initial imperfection

$x_1$ ratio of global to local buckling load

$x_1, x_2$ deflection along local $x_1$ and $x_2$ axes, respectively

$x_i$ distance from centroidal axis of integral cross section to parallel centroidal axis of single channel section

$x_{i1}, y_{1a}$ initial out-of-straightness in $X$ and $Y$ directions, respectively

$x_{o1}, y_{1o}$ distance of shear centre from centroid in the $x$ and $y$ directions (or $X$ and $Y$ directions), respectively

$Z$ semi-rigid connection constant
\( \alpha \) \hspace{1cm} \text{imperfection parameter}

\( \bar{\alpha} \) \hspace{1cm} \text{initial inclination (i.e. imperfection)}

\( \alpha_1, \alpha_2, \alpha_3 \) \hspace{1cm} \text{terms related to interaction of buckling modes}

\( \alpha_i \) \hspace{1cm} \text{general expression for } \alpha_1, \alpha_2, \alpha_3

\( \alpha_s \) \hspace{1cm} \text{separation ratio}

\( \beta \) \hspace{1cm} \text{ratio of centroidal distance between main members to thickness of button plate}

\( \gamma \) \hspace{1cm} \text{shear deformation}

\( \gamma_b \) \hspace{1cm} \text{shear deformation in a panel due to bending of batten and main member}

\( \gamma_{max} \) \hspace{1cm} \text{maximum shear strain}

\( \gamma_r \) \hspace{1cm} \text{ratio between Euler load of column and shear stiffness, } P_d, \text{ of web}

\( \gamma_s \) \hspace{1cm} \text{shear deformation in a panel due to shear deformations of batten and main member}

\( \gamma_t \) \hspace{1cm} \text{additional slope due to shear deformation}

\( \gamma_{xy} \) \hspace{1cm} \text{unit shearing strain in the } x-y \text{ plane}

\( \Gamma \) \hspace{1cm} \text{warping constant}

\( \Gamma_i \) \hspace{1cm} \text{warping constant of single channel}

\( \delta \) \hspace{1cm} \text{maximum lateral deflection of column}

\( \delta_0 \) \hspace{1cm} \text{midpoint initial imperfection of column}

\( \delta_1 \) \hspace{1cm} \text{lateral deflection of panel due to bending of batten}

\( \delta_2 \) \hspace{1cm} \text{lateral deflection of panel due to bending of main member}

\( \delta_3 \) \hspace{1cm} \text{lateral deflection of panel due to shear deformation of batten and main member}
member

\( \delta_s \) lateral deflection of panel due to semi-rigidity of connection

\( \delta u_n^i \) increment in displacement in the \( i^{th} \) iteration of the \( n+1^{th} \) increment

\( \Delta \) lateral deflection due to shear

\( \Delta \Delta \) amplitude of sidesway

\( \Delta l \) change in gauge length

\( \Delta u_n^i \) total increment in displacement at the end of the \( i^{th} \) iteration of the \( n+1^{th} \) increment

\( \Delta P_n \) load increment in the \( n+1^{th} \) increment

\( \Delta U \) strain energy stored in a structure caused by deformations

\( \Delta W \) work done by external loads as they move from one state of equilibrium to an adjacent state of equilibrium

\( \Delta \Pi \) net change in energy of the structure-load system

\( \varepsilon \) conventional or engineering strain

\( \varepsilon_1, \varepsilon_2 \) maximum and minimum principal strains, respectively

\( \varepsilon_{\sigma}, \varepsilon_{\nu}, \varepsilon_c \) strain read by strain gauges \( a, b, \) and \( c \) (or \( 1, 2, \) and \( 3 \)) in a strain rosette, respectively

\( \varepsilon_{avg} \) average unit normal strain at maximum shear strain

\( \varepsilon^\prime \) log elastic strain in finite strain problems

\( \varepsilon_{ln} \) natural, logarithmic, or true strain

\( \varepsilon_x, \varepsilon_y \) unit elongation in the \( x \) and \( y \) directions, respectively

\( \varepsilon_\theta \) unit elongation or contraction in any direction \( \theta \)
θ angle defining a direction

θ₁, θ₂ rotations at ends of local column with partial restraints

θₐ, θₖ, θᵉ angle of rotation of strain gauges a, b, and c from a datum

θ₉ bending rotation at each end of batten plate

θ₀ rotation of ends of column

θₚ acute angle of rotation of first principal strain or stress

θₚ₁, θₚ₂ directions of principal strains or stresses from the direction of εₓ or σₓ, respectively

θᵅ relative rotation between batten plate and main member

θₓ, θᵧ, θₜ rotation of axes for model axis rotation

θₓ, θᵧ rotations at the intersections of beams and columns in subassemblage models for Julian and Lawrence nomographs

κ load control parameter

λ nondimensional slenderness parameter in column formula equal to Λ(Fₜ/π²E)₁/₂

λₜ ratio of buckling load to local buckling load

λₑq nondimensional equivalent slenderness parameter

λₘ ratio of the sum of second moments of area of individual components to second moment of area of column as a truss

λₓ nondimensional integral slenderness parameter about the X axis

Λ slenderness ratio

Λₑq equivalent slenderness ratio

Λᵣ slenderness ratio of a main component between interconnectors about the
axis parallel to the axis of buckling

$\Lambda_{fm}$ modified slenderness ratio of a main component between adjacent connections about the axis parallel to the axis of buckling

$\Lambda_i$ maximum local slenderness ratio of a main member between interconnectors

$\Lambda_{im}$ modified maximum local slenderness ratio of the individual members between interconnectors

$\Lambda_{max}$ maximum slenderness ratio of built-up column

$\Lambda_o$ slenderness ratio of integral member acting as a unit

$\Lambda_{tr}$ equivalent slenderness ratio of a perfect laced column

$\Lambda^*$ equivalent slenderness ratio of an imperfect laced column

$\Lambda_X, \Lambda_Y$ slenderness ratio of integral column about the $X$ and $Y$ axes, respectively

$\mu$ plasticity coefficient

$\mu_1, \mu_2$ displacements

$\nu$ Poisson's ratio

$\nu_1, \nu_2$ imperfections

$\Pi$ potential energy

$\rho$ distance from shear centre to a point $B$ on cross section

$\sigma$ true or Cauchy stress in finite strain problems

$\sigma_1, \sigma_2$ maximum and minimum principal stresses, respectively

$\sigma_{avg}$ average unit normal stress at maximum shear stress

$\sigma_{cr}$ critical stress for plate buckling

xxxiv
\[\sigma_n\] nominal stress

\[\sigma_{true}\] true stress

\[\sigma_x, \sigma_y\] unit normal stresses in the \(x\) and \(y\) directions, respectively

\[\tau_{max}\] maximum shear stress

\[\tau_{xy}\] unit shear stress on the plane perpendicular to the \(x\) axis and parallel to the \(y\) axis and is equal in magnitude to \(\tau_{yx}\)

\[\phi\] resistance factor equal to 0.9 for compression

\[\phi_a, \phi_b\] rotations of nodes \(a\) and \(b\)

\[\phi_{bu}\] buckling factor of perfect column

\[\phi_{bu}^*\] buckling factor of imperfect column

\[\phi_x, \phi_y, \phi_z\] rotations about the \(X, Y,\) and \(Z\) axes, respectively

\[\omega\] warping function

\[\bar{\omega}\] average value of \(\omega\) along the entire cross section
CHAPTER 1

INTRODUCTION

1.1 General Discussion

Built-up columns have long been used in structural engineering due to the feasibility of using two or more standard rolled sections together to form a cross section with a higher second moment of area. The risk in doing this arises from the reduced compressive resistance of such a column as a result of the weaker shear resistance. Modern ideas of sparsely connecting the main members have led to a considerable decrease in the actual strength and efficiency of these structural members. This must be taken into account in the design of built-up columns by appropriately reducing the compressive resistance, and hence preventing premature failure.

The absence of a continuous web in built-up columns makes it necessary for there to be a secondary system to resist the shearing forces. These shearing forces may arise from the bending of the column, the presence of eccentricity in the axial
load, or any lateral load. This secondary system connects the main members and can be either batten plates (frame action), button plates (shear connectors), lacing bars (truss action), or perforated cover plates (frame action). Figure 1.1 shows the configuration of these interconnectors. This dissertation is only concerned with batten plates (battens) and button plates (buttons).

The modes of buckling of built-up columns that this dissertation is concerned with are:

(a) local flexural buckling of the main members between interconnectors,

(b) global flexural buckling about the $Y$ axis, and

(c) global flexural buckling about the $X$ axis.

These axes are shown in Fig. 1.2. Interaction between these modes of buckling can also occur.

In the battened cross section shown in Fig. 1.2, the $Y$ axis is the axis which passes through the batten plates and the $X$ axis is the axis which passes through the solid webs of the channels. Buckling about the $Y$ axis causes a relative deformation that produces shear forces in the interconnectors, as shown in Fig. 1.3. This mode of buckling involves a secondary frame action to carry the transverse shear forces in the main members. For buckling about the $X$ axis the interconnectors do not perform any specific task. They simply hold the two
members together and displace laterally during bending.

For buckling about the $Y$ axis the transverse shear, resisted by the secondary frame action, causes an increase in lateral deflection. This in turn causes a decrease in the compressive resistance of the column. A convenient way of expressing this decrease in the compressive resistance has been to increase the integral (overall) slenderness ratio of the member about this axis. This new slenderness ratio is often referred to as the equivalent slenderness ratio or the modified slenderness ratio. Naturally, this concept is not applicable to buckling about the $X$ axis.

In the buttoned cross section shown in Fig. 1.4, buckling about the $Y$ axis involves shear deformation of the button plates, leading to an increase in the integral slenderness ratio. The mechanism of this interconnector with negligible bending flexibility differs from that of a batten plate which has bending flexibility about the $Y$ axis.

The danger of optimizing the design of a built-up column is the simultaneous occurrence of local flexural buckling with global flexural buckling, which theoretically increases the imperfection sensitivity of the column. In general, Euler columns show neither stable nor unstable bifurcation, but neutral
bifurcation. Neutral bifurcation is only slightly imperfection sensitive. Only when
the column is in the intermediate range does imperfection sensitivity have a
significant effect on the compressive resistance. However, when two modes of
buckling coincide, such as local and global buckling, this interaction causes a more
severe case of imperfection sensitivity.

An experimental program was devised to investigate the effect of the
slenderness ratio of the main members between interconnectors on the compressive
resistance of the column. Specimens of double channel sections that buckled about
both the $X$ and $Y$ axes were tested. Both battened and buttoned columns were
tested. These specimens were designed to test the validity of the requirements of
the Canadian Standard CAN/CSA-S16.1-M89 "Limits states design of steel
structures" (1989a) with regards to the slenderness ratio of the main members
between interconnectors. The size and function of the interconnector was also
varied. Three groups of specimens were tested. Group I was composed of
battened specimens forming a box section, having an integral slenderness ratio of
120 about the $X$ axis and 60 about the $Y$ axis. These specimens were designated
as 120-63-3-1/4, for example, where the first number represents the integral
slenderness ratio about the $X$ axis, the second number represents the nominal depth
of the interconnector in mm, the third number represents the number of
interconnectors, and the final number represents the thickness of the
interconnectors in inches. If the final number is omitted it is automatically taken to mean a thickness of 1/8 inches. Group II was composed of battened specimens forming a box section, having an integral slenderness ratio of 70 about the X axis and 35 about the Y axis, and was designated in the same way as Group I. Group III was composed of battened and buttoned specimens with the channel sections placed back-to-back, having an integral slenderness ratio of 120 about the Y axis and 37.7 about the X axis. These specimens were designated as 120-A-3, for example, where 120 represents the integral slenderness ratio about the Y axis, A represents batten interconnectors and U represents button interconnectors, and the final number represents the number of interconnectors.

The experimental results were verified using a three-dimensional finite element model. The channel sections, batten plates, and end pieces were modelled as shell elements, and the button plates were modelled as solid elements.

1.2 Objectives

Canadian Standards for the limit states design of steel structures preceding the 1989 edition do not specify any equivalent slenderness ratio formula. For battened columns limits are given for the slenderness ratio of the main members between interconnectors and the member is assumed to have the capacity of that of the integral column. For buttoned columns the slenderness ratio of the main
members between interconnectors is restricted to that of the integral column and the member is assumed to have the capacity of that of the integral column. In 1989 the Canadian Standards Association published the next edition of the Canadian Standard CAN/CSA-S16.1-M89 (1989a). In this edition, an equivalent slenderness ratio formula is introduced in Clause 19.1.4 for buttoned columns that buckle in a mode that causes relative deformation which produces shear forces in the interconnectors. This equivalent slenderness ratio reduces the compressive resistance of sparsely connected buttoned columns. It is realized that the intent of Clause 19.1.4 is that the equivalent slenderness ratio was to be applied to buttoned columns only. Since some design engineers also applied this equivalent slenderness ratio to batted columns it was the intent of this research to indicate that using a factor of 0.65 is unconservative for batted columns. Theoretical and experimental results by Elmahdy (1992) have also shown that using a factor of 0.65 is unconservative for batted columns. This edition still specifies the same limits for the slenderness ratio of the main members between interconnectors for batted and buttoned columns.

In 1994, based on research by Temple and Elmahdy (1993, 1995) the latest edition of the Canadian Standard CAN/CSA-S16.1-94 (1994) eliminates the limits of the slenderness ratio of the main members between interconnectors for batted columns and only the condition that the slenderness ratio of the main members
between interconnectors must be less than that of the built-up member is specified. The equivalent slenderness ratio formula is modified so that a factor of 1.0 is specified for batten columns and buttoned columns with snug-tight bolts, and 0.65 for buttoned columns with welds or pretensioned bolts. This edition considers imperfection sensitivity due to interaction of buckling modes to be included in the specified compressive resistance.

The objective of this dissertation is to:

(a) study the effect of the local slenderness ratio on the strength and behaviour of the integral column,

(b) investigate theoretically the effect of the occurrence of simultaneous local flexural and global buckling;

(c) distinguish between the behaviour of a battenailed and a buttoned column, and

(d) determine whether or not the limitations of Clause 19.1.16 of S16.1-M89 are required.
CHAPTER II

LITERATURE REVIEW

2.1 Introduction

An extensive amount of literature is available on the use of built-up compression members. This literature shows that the main problems of built-up compression members composed of two interconnected rolled shapes is their shear flexibility and, theoretically, the occurrence of simultaneous local buckling of the individual main members between interconnectors and global buckling of the integral column. This chapter outlines the main developments dealt with on this subject.

2.2 Effect of Shear in Columns

The problem of shear in columns is a classical one and can be traced back to Engesser (1889, 1891). In fact, there are two schools of thought as to the proper formulation of the buckling analysis of columns with finite shear stiffness, the Engesser school and the Haringx school (1948). The Engesser school is
typically followed by structural engineers for problems such as the analysis of batted and laced columns. The Haringx school is followed by mechanical engineers for the analysis of helical and elastomeric springs and bearings in which the effect of shear is unusually large. These two approaches differ in the form in which the internal forces are decomposed. Nänni (1971) showed by means of the three-dimensional theory of elasticity that Engesser's approach was superior for bars. Ziegler (1982) again confirmed this result by means of a more fundamental one-dimensional approach, and showed that shortening of the bar must also be taken into account. The Engesser approach, the Haringx approach, as well as the approach used by Timoshenko and Gere (1961) were reviewed by Djukic and Atanackovic (1993). The decomposition of internal forces in these three approaches is illustrated in Fig. 2.1(b), (c), and (d), respectively. Djukic and Atanackovic gave an extension to the method proposed by Timoshenko and Gere (1961) where the axial force is assumed to act in the direction tangent to the deformed axis of the member, and the shear force is assumed to act in the direction normal to the deformed axis. Gjelsvik (1991) investigated the problem of columns with finite shear stiffness using the methods of both Engesser and Haringx. He concluded that when the usual shear stiffness of the column is used, the Engesser method is the correct one for columns modelled as continuous Timoshenko shear beams (1921). The shear beam theory is based on the kinematic assumption that plane sections do remain plane, but do not remain normal to the displaced axis of
the column as shown by Wang (1995).

2.3 Effect of Shear in Built-Up Columns

The importance of the proper design of shear resisting elements in built-up columns, as pointed out by Galambos (1988), was tragically demonstrated in 1907 by the failure of the first Quebec Bridge during construction. This was reported by the Royal Commission for the Quebec Bridge Inquiry (1908), and discussed by Engesser (1907). This led to many extensive studies on the behaviour of built-up columns.

Engesser (1909) again contributed to the understanding of the problem of shear in built-up columns by publishing a refined analysis taking into account the secondary effects of the shearing forces. Many attempts have been made to approach the problem of a built-up column as a framework to reach an exact solution. Examples of these given by Bleich (1952) are Mann (1909), Ljungberg (1922), von Mises and Ratzersdorfer (1925, 1926), Wentzel (1929), and Chwalla (1927). The importance of these studies was not the addition of new knowledge concerning the performance of built-up columns, but the confirmation of Engesser's results. The exact solutions differ from the approximate solution in that the number of panels \( n_P \), in which the column is subdivided appears only in the exact expressions of critical load. In practice, however, \( n_P \) is usually greater than four.
and so it is sufficiently accurate to obtain results by the approximate formulae. Tall (1974) also confirmed that if the number of panels in a laced or battened column is greater than four, the column can be considered as an ordinary column, rather than a framework.

The problem of built-up columns was also analyzed by Müller-Breslau (1910), Petermann (1926, 1931), Young (1936), Holt (1940), and Pippard (1948, 1952). Pippard derived an expression for the critical load of a battened column by assuming a continuous web, which can apply flexural restraint to the flanges but cannot transmit stress along the axis of the column, instead of a finite number of batten plates.

Bleich (1952) derived an effective length factor, and hence the equivalent slenderness ratio, for laced and battened columns using an energy method. This states that the transition from stable to unstable equilibrium of any elastic system is characterized by the energy condition that the strain energy due to deflection is equal to the work done by the external forces due to the displacement of their points of application. Hence, this permits the determination of the critical load at which there is a transition from the straight and stable configuration to the displaced and unstable form of equilibrium. As a battened column is a highly redundant structure, its analysis can be simplified by placing hinges at the
midpoints of the panels and interconnectors, converting it into a statically
determinate framework. This assumption is made from the theory of rectangular
frameworks. Bleich also established rules for the proper design of the details of
the lateral system itself in order to prevent premature failure of these members
prior to the primary failure of the column.

Timoshenko and Gere (1961) derived expressions for the shear flexibility
of built-up columns with laced, battened, or perforated cover plates as
interconnectors. For columns with battens and perforated cover plates points of
inflection were again assumed at the midpoints of the main members of each panel.
Hence, the critical loads of these columns were derived from which an expression
for the equivalent slenderness ratio can be formulated. The derivation of the
expression of the equivalent slenderness ratio as given by Timoshenko and Gere
is presented in expanded form in Chapter III.

Other researchers to study the problem of built-up columns were Ng (1947,
1951), Jones (1952), Koenigsberger and Mohsin (1956), Mohsin (1958, 1965),
Tamayo and Ojalvo (1965), Williamson and Margolin (1966), Lin et al. (1970),
Johnston (1971), Kennedy and Madugula (1972), Ballio and Mazzolani (1983), and
Libove (1985) studied the elastic post-buckling behaviour of built-up columns with two and three interconnectors, as shown in Fig. 2.2. His analysis, which was based on equilibrium equations, showed that the post-buckling behaviour of these columns was unstable. The post-buckling instability, which is in marked contrast to the virtually neutral stability of the solid elastic column, has important implications regarding imperfection sensitivity. Similar conclusions were reached by Chang (1984), who also concluded that the addition of interconnectors lessens the post-buckling instability and any optimization of the spacing of interconnectors to maximize the buckling load appears to worsen the post-buckling instability. Chang (1991) extended Engesser-Shanley's (Shanley; 1947) tangent modulus theory of inelastic buckling to his model with three interconnectors. Chang and Chung (1991) used the Koiter-Budiansky's (Koiter; 1945, Budiansky; 1974) general theory of buckling of elastic structures using a functional approach and perturbation technique to find the analytical expressions for buckling load, post-buckling behaviour, and peak load versus imperfection for the same model. Chang and Sher (1992) studied the elasto-plastic post-buckling behaviour of perfect and imperfect built-up columns of the type shown in Fig. 2.2. The material behaviour of the columns was assumed to obey a bilinear stress-strain and kinematic hardening relationship.

The concept of equivalent slenderness ratio as derived by Timoshenko and
Gere (1961) and by Bleich (1952) has been adopted by various design codes. However, there exists many variations and corrections to this slenderness ratio between different design codes. The AISC-LRFD (1986) Specification for Structural Steel Buildings included new rules affecting the design of built-up compression members composed of rolled shapes. Zahn and Haaijer (1987, 1988) provided the background information of these new rules. These rules were based on the results of recent research conducted by Zandonini (1985) and were verified with studies by Astanah et al. (1985) and Libove (1985). Zandonini (1985) conducted two series of tests on back-to-back channels, one connected by welded filler plates and the other by snug-tight bolted filler plates, as shown in Fig. 2.3. His test results were compared with the SSRC2 curve (Galambos; 1988) as the specimens had an initial out-of-straightness of \( L/i000 \). Astanah et al. conducted cyclic tests on back-to-back double angle struts, the first cycle of which provides valuable data. The strength of these specimens was compared with the provisions of the AISC-LRFD Specification (1986). This specified that a modified slenderness ratio be used of:

a) for snug-tight bolted interconnectors:

\[
\Lambda_{eq} = \sqrt{\Lambda_o^2 + \Lambda_i^2}
\]  

(2.1)

b) for welded interconnectors and for fully tightened bolted interconnectors as required for slip-critical joints:
\[ \Lambda_{eq} = \sqrt{\Lambda_o^2 + (\Lambda_i - 50)^2} \]  
\[ \Lambda_{eq} = \Lambda_o \]  

where \( \Lambda_{eq} \) is the modified or equivalent column slenderness of the built-up member; \( \Lambda_o \) is the column slenderness of the built-up member acting as a unit; \( \Lambda_i = a/r_i \) is the maximum slenderness ratio of an individual component between interconnectors; \( a \) is the distance between interconnectors; and \( r_i \) is the minimum radius of gyration of an individual component. Equation (2.1) is that used in previous European specifications, whereas Eq. (2.2) was verified in the report by Zahn and Haaijer (1987). Duan and Chen (1988b) proposed a unified formula which contains an imaginary local effective length factor \( k_i \) such that

\[ \Lambda_{eq} = \sqrt{\Lambda_o^2 + (k_i \Lambda_i)^2} \]  

where the factor \( k_i \) is assigned a value of 1.0 for snug-tight bolted interconnectors, and 0.65 for welded interconnectors and fully tightened bolted interconnectors as required for slip-critical joints. This formula, as well as Eqs. (2.1) and (2.2), were again checked using the experimental results from the tests carried out by Zandonini (1985) and Astenah et al. (1985). This proposed modified slenderness formula was found to be concise, simple, and physically clear and was adopted by the Canadian Standard S16.1-M89 (1989a). This specified, for built-up compression members composed of two or more rolled shapes in contact or separated from one another by intermittent fillers, an equivalent slenderness ratio.
of $\Lambda_{eq} = (\Lambda_o^2 + (k_i \Lambda_o)^2)^{1/2}$; where $k_i$ is an effective length factor taken as 1.0 when the interconnectors are fastened with snug-tight bolts and 0.65 when fastened with welds or pretensioned bolts.

The new AISC-LRFD Specification for Structural Steel Buildings (1993) introduced a new formula for the modified slenderness ratio, for columns with welded or fully-tensioned bolted interconnectors, based on an investigation conducted by Aslani and Goel (1991b). Aslani and Goel presented a generalized version of the analytical equation proposed by Bleich (1952). Bleich’s equation is applicable to all built-up struts with general end conditions and his generalized modified slenderness is given as

$$\Lambda_{eq} = \sqrt{\Lambda_o^2 + \frac{\pi^2 I_o}{12 I_y} \Lambda_y^2}$$

(2.4)

where $I_o = A_i b^2 / 2$ is the second moment of area of the integral section about the buckling axis neglecting the second moment of area of the individual components about their own centroidal axis; $A_i$ is the cross-sectional area of one component; $b$ is the distance between centroids of individual components; $I_y$ is the total second moment of area of the integral section about the axis of buckling; and $\Lambda_y$ is the slenderness ratio of a main component between interconnectors about the axis parallel to the axis of buckling. In this way, the effect of separation of the main
components in the built-up strut can be taken into account. Aslani and Goel also replaced the term $\pi^2 I_c/12 I_y$ with $0.82 \alpha_r^2/(1+\alpha_r^2)$ where $\alpha_r = b/2r_f$ is a separation ratio, and $r_f$ is the radius of gyration of a main component about its centroidal axis parallel to the axis of buckling. This equation was verified by a parametric study and by test results. Hence the format adopted by the AISC-LRFD Specification (1993) is

$$\Lambda_{eq} = \sqrt{\Lambda_o^2 + 0.82 \frac{\alpha_r^2}{(1+\alpha_r^2)} \Lambda_f^2}$$ (2.5)

Other values for the factor in the second term of the radicand of Eq. (2.5) have been proposed. For example, Ballio and Mazzolani (1983) suggested that the term 0.82 be multiplied by 0.8 giving a factor of 0.66. The factor 0.8 accounts for the overall flexural behaviour of this form of built-up member (i.e. compound struts) where the contribution to the chord bending cannot be disregarded. From the results of tests conducted by Temple and Elmahdy on model struts composed of bars interconnected by welded battens (1993) and built-up columns composed of face-to-face channels interconnected by welded battens (1995), it was concluded that it was unconservative to use an effective length factor of 0.65 in the second term of the radicand of Eq. (2.3). The new version of the Canadian Standard CAN/CSA-S16.1-94 (1994) left the effective length factor for compact built-up struts unchanged. Whereas, for built-up members composed of two interconnected
rolled shapes separated by lacing or batten plates, the maximum slenderness ratio of component parts between fasteners or welds was based on an effective length factor of 1.0 for both snug-tight and pretensioned bolts and for welds.

Many studies on the behaviour and arrangement of double angle struts have been made. These include those by Temple et al. (1986, 1987, 1988), Astaneh and Goel (1984), Aslani and Goel (1991a), and Krige and Wolmarans (1991). The interconnection of starred angle compression members was studied by Temple et al. (1986) and Krige and Wolmarans (1991). The former suggested a minimum of two welded interconnectors at the third points to ensure buckling as an integral unit about the minor principal axis. The latter suggested a minimum of three bolted interconnectors one at the midpoint and one at each sixth or quarter points in a plane perpendicular to that of the interconnector at mid-height. The interconnection of boxed angle compression members was studied by Temple et al. (1987) who recommended that at least two interconnectors be used for boxed angle compression members so that they would buckle as an integral unit about the minor principal axis. The interconnection of widely spaced angles was studied by Temple and Tan (1988) who suggested that the use of only one interconnector at mid-height was sufficient. Studies on the seismic resistance of double angle bracing were made by Astaneh and Goel (1984), Astaneh et al. (1985), and Aslani and Goel (1991a).
Gjelsvik (1990, 1993) presented a method of analysis of built-up columns where the two column chords are treated as beams connected by a shear panel simulating a web. The governing differential equation then changes from a fourth to a sixth order differential equation with a corresponding increase in the number of boundary conditions. In this manner he included the effect of the stay plates (end connectors) on the buckling load. The purpose of the stay plates is to increase the shear strength of the column at the ends where the shear force is largest and to suppress the shear deformation at the column ends. It is found that the stay plates have a very significant effect on the buckling load, and that the buckling load depends not only on the bending and shear stiffness of the column but also on the relative value of the bending stiffness of the chords to the bending stiffness of the overall column. The solution of the general equation derived is shown in Fig. 2.4. The limiting cases of this solution is the Engesser solution where the second moment of area of the chords equals zero, the Johnston solution where the shear stiffness equals zero, and the Euler solution. Paul (1993, 1994a, 1994b) generalized Gjelsvik's solution by modelling the web as being continuously connected to the chords and being capable of sharing the axial and flexural stresses with the chords.

The problem of torsional-flexural buckling of built-up columns was investigated by Toossi (1972) and Kitipornchai and Lee (1986).
2.4 Coupled Instabilities

Researchers have often studied the problem of interaction between buckling modes, in particularly local and global flexural buckling modes, considering them as independent buckling modes. This is definitely a theoretical study and is not substantiated by experimental results. For the sake of optimization of design, it is logic to make the failure loads of all buckling modes equal. However, the dangers of this type of structural optimization as outlined by Thompson and Hunt (1973, 1974) is the tendency, at least theoretically, to generate even more severe instabilities in the continuing search for greater structural efficiency. This means that the idealized perfect structure exhibits an unstable and often compound branching point and would fail by an explosive instability while nominally perfect real structures containing inevitable initial imperfections fail at scattered loads which can be considerably lower than that of the idealization. As shown in the sketch in Fig. 2.5, an increasing degree of optimization implies an increasingly severe instability by demanding in turn an unstable bifurcation, a very unstable bifurcation, and finally a very unstable compound bifurcation. The last of these is especially dangerous as it may involve an unexpected nonlinear coupling action. Koiter and Skaloud (1962) predicted that the interaction between several independent buckling modes at (nearly) the same critical stress may result in a highly unstable post-buckling behaviour, even if the post-buckling behaviour in each of the separate modes was stable.
A common problem is the erosion of nominally optimum designs by the imperfection-sensitivity of coupled failure modes. This was studied by Thompson and Supple (1972), Thompson and Hunt (1973), and Bažant and Cedolin (1991), for a simple two-degree-of-freedom buckling model which exhibits this unexpected coupling action. It was shown that the compound imperfection-sensitivity can quite seriously modify, and perhaps destroy, the apparent optimum solution.

An important application to this phenomenon in columns is the interaction of local plate buckling and overall buckling of thin-walled compression members, which are subjected to column failure (bending), torsional-flexural buckling, and local buckling. This problem has been studied by van der Neut (1968), Graves-Smith (1969), Koiter and Kuiken (1971), Thompson and Lewis (1972), Svensson and Croll (1975), and Fan (1989). The interaction of local buckling and column failure of thin-walled compression members was first modelled by van der Neut (1968) using two load carrying flanges of width $w$, thickness $t$, separated at a distance of $2c$ by webs, which are rigid in shear and are rigid laterally but have no longitudinal stiffness. The webs offer simple support to the flanges. This model is shown in Fig. 2.6. He found that the reduced longitudinal stiffness of plates in the post-buckling state affects the column strength. Columns without imperfections fail explosively at the local buckling load $P_L$ over a range of $P_L/P_L$ above unity, where $P_L$ is the global Euler buckling load. Initial waviness of the composing thin-
walled elements of the column reduces the column strength $P_B$ to values below $P_L$ even when $P_E$ is much larger than $P_L$. For $P_E/P_L$ close to unity this reduction is explosive. The more realistic problem of a square tube with thin walls has been studied by Graves-Smith (1969). The effect of the interaction between overall buckling and local buckling is less pronounced in this case, due to the bending stiffness of the webs connecting the inner and outer flange in overall buckling. More recently, Koiter and Kuiken (1971) have confirmed van der Neut's findings through the use of Koiter's general nonlinear theory of elastic stability (1945), which yields almost identical numerical solutions as well as useful asymptotic formulae. Thompson and Lewis (1972) also used van der Neut's model to discuss the validity of simultaneous buckling as a criterion of optimization of design of thin-walled compression members. The optimum is seen to erode and shift quite significantly for small imperfections, and for larger ones it can be entirely eliminated. Svensson and Croll (1975) deduced that the strong optimum indicated that for coincident mode design the critical load is effectively diminished when this imperfection sensitivity as well as effects of plasticity are included for box columns. Fan (1989) discussed the same problem for stiffened panels with symmetrical cross sections. He states that if deflections in a primary local mode occur on both sides of the neutral axis, the analysis should be improved by taking into account a second local mode.
The degrading effect of small shape imperfections on the buckling strength of imperfection sensitive optimized structures was investigated by Palassopoulos (1989). A general method is presented, which reduces the imperfection effect to an eigenvalue problem, and is applied to the optimization of a column on a linear elastic foundation. The significant effect of the shape imperfections on the optimal design is demonstrated. The same author (1991) developed a general method for the reliability-based design of imperfection-sensitive structures. The approach results in simple analytical formulae for the design value of the imperfection magnitude at any desired reliability level. All the imperfection modes are taken into account, and no restrictions, besides the usual smoothness requirements, need be imposed on the imperfection patterns. This method is further extended and generalized to cover all possible sources of structural imperfections including the shape imperfections by Palassopoulos (1993). Ikeda and Murota (1990) introduced a method for determining the critical initial imperfection for discretized structures that decrease the load bearing capacity most rapidly.

Many researchers have investigated the interaction of local plate buckling and overall buckling of symmetrical and monosymmetrical thin-walled steel columns. Migita et al. (1992) derived an empirical formula based on compression tests to predict local buckling strength and interaction strength between local and overall buckling strengths of closed symmetric polygonal steel columns. For
monosymmetrical cross sections the possibility of flexural-torsional buckling must be included in the analysis. Gioncu et al. (1983) investigated the interaction between flexural-torsional buckling and local buckling of the flanges of thin-walled channel members. Their method considers the decrease of flange stiffness due to local buckling and geometrical imperfections. In a study made by Loughlan and Upadhya (1984), it is shown that channel columns are extremely sensitive to small local imperfections. Pignataro and Luongo (1985) analyzed the simultaneous buckling modes and imperfection sensitivity of simply supported stiffened and unstiffened channels under uniform compression by means of the finite strip method. It was found that the Euler/local buckling mode interaction is more dangerous than the flexural-torsional/local buckling mode interaction since the channel is more sensitive to initial imperfections. Rasmussen and Hancock (1992) presented a detailed comparison of nonlinear analyses of thin-walled channel section columns with tests. The columns are analyzed as beam-columns, and the nonlinear constitutive relationships are obtained firstly, by assuming elastic material behaviour and allowing local buckling of both flanges and web and secondly, by assuming elastic-plastic material behaviour and confining local buckling deformations to the web. They also investigate the applicability of combining an elastic nonlinear analysis with a spatial plastic mechanism analysis to describe the structural behaviour both before and after the ultimate load. The same authors (1993) pointed out that the strength of a fixed-ended channel section column
exceeds that of a pin-ended column of the same effective length owing to the fact that local buckling does not induce bending of fixed-ended channel section. Fixed-ended channel section columns are also less sensitive to imperfections. Gioncu et al. (1992) analyzed the interaction between flexural and flexural-torsional buckling in the case of monosymmetrical steel compression members, namely channel, angle, and tee sections. Their analysis has led to the conclusion that this interaction can be classified as weak or medium.

The interaction of local and overall buckling in thin-walled columns is again considered by Benito and Sridharan (1983) and Sridharan and Benito (1984). The purpose of their work was to outline a more comprehensive analysis which can deal with any arbitrary cross section (symmetrical or monosymmetrical) as well as nonuniform loading, to model a variety of interaction but in particular the interaction of local and flexural-torsional buckling, and to demonstrate that if the load were applied suddenly there can be a further reduction on the collapse load than would be caused by static interactive buckling.

Sridharan and Ali (1985) developed a new analytical model to study thin-walled beam-columns having doubly symmetrical cross sections. It incorporated the interaction of overall buckling/bending with two companion local modes. The second local mode was induced into the column due to the bending of the column in
overall buckling. The model also accounted for the phenomenon of amplitude modulation and can model any set of realistic end conditions. Ali and Sridharan (1987, 1988) extended this analytical model to include columns with sections having a single axis of symmetry. This model can treat the combined interaction of flexural-torsional and flexural modes of buckling with local buckling. In this case, overall bending occurs in two perpendicular directions and thus it calls for yet a third local mode, making one primary local mode and two secondary local modes. It was shown that channel section columns of commonly used proportions are highly imperfection sensitive in the context of combined interaction of enumerated modes of buckling.

A similar phenomenon to that of imperfection sensitivity of thin-walled columns such as was shown in van der Neut's model occurs in the buckling of a reticulated or triangulated column (Thompson and Hunt; 1973, 1974), such as that shown in Fig. 2.7. However, the modes of buckling in this case are local buckling of the main members between lacing bars and overall buckling of the column. A similar analysis as that carried out by Koiter and Kuiken (1971) on a thin-walled column gave identical imperfection indices for a reticulated column. That is, referring to Fig. 2.8, the power indices due to imperfections are 2/3 for \( x_1 > 1 \), 1/2 for \( x_1 = 1 \), and 2 for \( x_1 < 1 \), where \( x_1 = P_0 / P_L \), \( P_0 \) being the global buckling load. These results are identical to those of Koiter and Kuiken shown in Fig. 2.9. It will be noted that the imperfection sensitivity has the maximum power at the point of
interaction between local and overall buckling.

Other researchers to have tackled the problem of interactive buckling of latticed columns are Byskov (1979), Crawford and Hedgepeth (1975), Miller and Hedgepeth (1979), Crawford and Benton (1980), Svensson and Kragerup (1982, 1983), Bâlut et al. (1982), Gioncu (1989, 1990), and Tong and Chen (1989). Byskov (1979) studied the applicability of an asymptotic expansion for elastic buckling problems with mode interaction. Miller and Hedgepeth (1979) presented an analysis for determining the buckling load of triangular lattice columns with stochastic imperfections. For latticed columns a slight curvature in the overall centre line causes the compressive end load to induce increased loads in the local members resulting in premature crippling in those local members. On the other hand, local imperfections of the cross section produce a reduction of the effective column bending stiffness and thereby induce premature column buckling. Therefore, the proper design of such slender lattice columns must take into account the simultaneous effects of random local and deterministic overall imperfections. In a preliminary deterministic analysis, the combined effects of local and overall imperfections on the buckling load are calculated for a broad range of imperfection amplitudes. In the more general case of random local imperfections, estimators for the mean and standard deviation of the random buckling load are developed, which give an accurate basis for predicting the minimum buckling load. Svensson and
Kragerup (1982, 1983) investigated the imperfection sensitivity of laced columns, the imperfection consisting of both a local and an overall geometrical imperfection. They showed that a laced column made of linear elastic material, after having reached the collapse load, behaves almost like a shell and must, therefore, be expected to be sensitive to imperfections. Also, for a linear elastic-ideally plastic column the collapse load was approximated with the load where yielding initially occurs. For both cases realistic imperfections have been shown to reduce the capacity by approximately 50%. Bälat et al. (1982) suggested limiting the slenderness ratio of the chords of laced columns to counteract the reduction of bending stiffness of laced columns and consequently of their buckling loads. As shown in Fig. 2.10 the imperfection sensitivity is more important when $\Lambda_i$, the maximum local slenderness ratio of a chord and, $\Lambda_{ir}$, the equivalent slenderness ratio of the perfect laced column have nearly equal values. The case of $\Lambda_i > \Lambda_{ir}$ is not shown, being unusual. The reduction in load can reach up to 40-50% of the overall buckling load. Therefore, in order to reduce this reduction to about 3-4% the authors suggested the following limits for the local slenderness ratio:

for I-shaped chords

\[
\begin{align*}
\text{for } & \Lambda_{ir} \leq 40 & \Lambda_i & \leq 40 \\
\text{for } & \Lambda_{ir} > 40 & \Lambda_i & \leq 40 + 0.5 \frac{210}{R_f} (\Lambda_{ir} - 40) \\
\end{align*}
\]

(2.6)

for channel-shaped chords
\begin{align}
    \Lambda_i &\leq 40 - 10 \frac{210}{R_y} \\
    \Lambda_i &\leq 40 + 0.5 \frac{210}{R_y} (\Lambda_p - 60)
\end{align}

where $R_y$ represents the design yield strength in MPa. Gioncu (1989) emphasized that the coupling effect of two instability forms leads to the increase of imperfection sensitivity. In fact the coupling of two stable post-critical curves results in an unstable post-critical curve, and hence imperfection sensitivity. He used the examples of the coupling of overall flexural buckling with local buckling of thin walled compression members, the coupling of flexural buckling with torsional-flexural buckling of monosymmetrical cross-sections, the coupling of overall and individual local buckling of laced columns, and the coupling of general and local buckling of reticulated curve structures. Figure 2.11 shows the coupling action of an instability of neutral post-critical behaviour with: (a) an instability with stable post-critical behaviour; (b) an instability with neutral post-critical behaviour; and (c) an instability with unstable post-critical behaviour. Tong and Chen (1989) presented an interactive buckling theory for built-up beam-columns by introducing the concept of reduced axial stiffness of an imperfect bar, and extended it to apply to centrally compressed built-up members. In this way various adverse influences of imperfections were taken into account. Finally, Bažant and Cedolin (1991) presented an analysis to predict the imperfection sensitivity of laced columns. This is modified in Chapter III to apply to battened columns.
Earlier research, as reported by Koenigsberger and Mohsin (1956), has given optimum values of the local slenderness ratio $\Lambda$, as: (1) 28 according to an ASCE report (1931); (2) 50 according to Ng (1947) corresponding to $\Lambda/\Lambda_r \approx 0.6$ where $\Lambda_r$ is the overall slenderness ratio; (3) 40 according to Petermann (1926, 1931) who based his finding on tests conducted by Müller-Breslau (1910); and (4) 30 based on tests run by Petermann, corresponding to $\Lambda/\Lambda_r \approx 0.75$. The reason for these variations is that each researcher quoted only the value of $\Lambda$, suitable for the arrangement of built-up member studied.
CHAPTER III

THEORETICAL ANALYSIS

3.1 Introduction

There are two accepted design considerations for built-up columns. The first is the design concept that the shear produced in the column by bending must be resisted by a secondary system. This leads to a reduction of the compressive resistance of the column, and may be accounted for by increasing the slenderness ratio of the column to an equivalent slenderness ratio. The second concern is the interaction between local and global buckling, which leads to an increased imperfection sensitivity and, in the past, has been avoided by reducing the local slenderness ratio of the main members. It is interesting to note that European Codes (viz. Eurocode 3; 1992) are based on a third concept known as the second order design concept (Gioncu; 1990). This determines the strength of a built-up column on the basis of the axial force, due to axial loading and global bending, in the most stressed member.
The equivalent slenderness ratio concept is applicable when the buckling mode involves deformation of the main members, as shown in Fig. 3.1, in such a manner that shear forces are produced in the interconnectors. Although both battened and buttoned columns transfer shear forces through the interconnectors, the mechanism of shear transfer differs. Battens deform by bending and shear, whereas buttons deform, predominantly, by shear.

Even though using the equivalent slenderness ratio concept is sufficient to predict the compressive strength of a built-up column, an additional concern is the occurrence of local and global buckling simultaneously. In this dissertation local buckling is defined as the flexural or torsional-flexural buckling of a main member between interconnectors. The phenomenon of simultaneous local and global buckling, called the interaction of buckling modes, increases the columns sensitivity to imperfections, and thus reduces the ideal value of the buckling load. For the interaction analysis, it is assumed that local buckling of the main members between interconnectors is not sidesway prevented. The occurrence of the interaction of buckling modes is a product of the optimization of design, in which the local slenderness ratio of the main members between interconnectors is made equal to the global slenderness ratio of the column. This is, theoretically, an undesired practice.
The difference between a batten and a button interconnector is also reviewed, when the buckling mode involves deformations that produce shear forces in the interconnectors. As a button interconnector is not flexible the theory of shear transfer of batten columns is not applicable to buttoned columns.

3.2 Effect of Shear

Unlike solid columns, the effect of a shear force, \( Q \), on the deflections must be considered in built-up columns. This is true when they are analyzed as homogeneous columns and the buckling mode involves deformations in the main members that cause shear forces in the interconnectors (Timoshenko and Gere; 1961). The increased lateral deflection in a batten column due to shear is shown in Fig. 3.2. In this analysis the normal force is taken in the direction of the column centre line and the shear force is taken in the direction perpendicular to the column centre line. For the column shown in Fig. 3.3(a), the transverse shearing force is a component of the vertical load, \( P \), and is produced by the bending of the column. Other sources of transverse shear forces are the eccentricity of the axial load and the presence of lateral loads. Figure 3.3(b) shows the shearing force acting on an element of the column of length \( dz \). It can be seen from Figs. 3.3(b) and (c) that the magnitude of this shearing force is \( Q = P dx/dz \). For a built-up column this shearing force is not carried by a solid web, and so a secondary system is required. This secondary system is the frame action (Vierendeel beam
action) of the main members and batten plates for battened columns, and the shear connector action of the button plates for buttoned columns.

As a result of the shearing force in the main members, the lateral deflection of the column is increased. The change in slope of the deflection curve produced by the shearing force is \( Q/P_d \) for a column, where \( P_d \) is the shear stiffness of the column. The rate of change of slope of the deflection curve due to shearing force represents the additional curvature due to shear and is equal to

\[
\frac{1}{P_d} \frac{dQ}{dz} = \frac{P}{P_d} \frac{d^2x}{dz^2}
\]  

(3.1)

The total curvature of the deflection curve is the sum of the curvature due to shear and the curvature due to bending. For a pinned column the differential equation of the deflection curve is

\[
\frac{d^2x}{dz^2} = -\frac{P_d}{EI_y} \frac{P}{P_d} \frac{d^2x}{dz^2}
\]  

(3.2)

where \( EI_y \) is the bending rigidity of the column about the buckling axis. Rearranging this gives

\[\frac{d^2x}{dz^2} + k_s^2 x = 0\]

where

\[k_s = \sqrt{\frac{P}{EI_y \left(1 - \frac{P}{P_d}\right)}}\]

(3.3)
The solution for this equation is

\[
\frac{P_\sigma L^2}{EI_y \left(1 - \frac{P_\sigma}{P_d}\right)} = \pi^2
\]  

(3.4)

where \(P_\sigma\) is the critical load of the column; and \(L\) is the length of the column. Rearranging this and taking the Euler load of the column as \(P_e = \pi^2 EI_y/(KL)^2\), where \(K\) is the effective length factor, gives

\[
P_\sigma = \frac{P_E}{1 + \frac{P_E}{P_d}}
\]  

(3.5)

Thus Eq. (3.5) shows that the critical load is reduced by the ratio \(1/(1 + P_E/P_d)\). This is very close to unity for solid columns but is of practical importance in built-up columns.

### 3.3 The Equivalent Slenderness Ratio

The effect of the shear force on the critical load depends on the type and arrangement of the secondary system carrying the shear force. For built-up columns, in general, the factor \(1/P_d\) in Eq. (3.5) is the quantity by which the shear force, \(Q\), is multiplied in order to obtain the additional slope, \(\gamma_s\), of the deflection curve due to shear. The lateral displacements caused by the shear forces must be evaluated to determine the value of \(1/P_d\).
A battened column, as shown in Fig. 3.4(a), is actually a frame and it would be proper to analyze it as such. However, if there are a large number of panels it can be treated, approximately, as a homogeneous column (Timoshenko and Gere; 1961). To solve the problem of shear the following assumption can be made. Referring to the deformed axis in Fig. 3.2, points of inflection (i.e. hinges) in the deflection curve can be assumed at the midpoints of the panels and at the midpoints of the battens. Thus the statically determinate system used to determine the effect of the shear force in the column is as shown in Fig. 3.4(b).

Considering the effect of shear on one panel (m-n) of length \( a \), the shear force can be assumed to act at the hinges of the main members, half on each member as shown in Fig. 3.5(a). This would cause the bending moment shown in Fig. 3.5(b). The lateral deflection in a panel due to shear, as shown in Fig. 3.6, consists of five components which are the deflection due to:

(a) the bending of the batten, \( \delta_1 \),

(b) the bending of the main member, \( \delta_2 \),

(c) the shear deformation of the batten,

(d) the shear deformation of the main member, \( \delta_3 \) is composed of (c) and (d), and

(e) the semi-rigidity of the connection, \( \delta_4 \).

The moment, \( Qa/2 \), acting at the ends of the batten results in an angle of
rotation at each end equal to

\[ \theta_b = \frac{Qab}{12EI_b} \quad (3.6) \]

where \( b \) is the distance between centroids of the main members; and \( EI_b \) is the sum of the flexural rigidities of the battens on one level. This produces a lateral deflection, \( \delta_1 \), of

\[ \delta_1 = \frac{\theta_b a}{2} = \frac{Qa^2 b}{24EI_b} \quad (3.7) \]

The bending of the main member as a cantilever caused by the shearing force \( Q/2 \) gives the second component of deflection which is

\[ \delta_2 = \frac{Q}{2} \left( \frac{a}{2} \right)^3 \frac{1}{3EI_y} = \frac{Qa^3}{48EI_y} \quad (3.8) \]

where \( EI_y \) is the flexural rigidity of one of the main members about its centroidal axis parallel to the axis of bending. Thus the angular displacement, \( \gamma_b \), produced by bending under the effect of the shearing force \( Q \) is

\[ \gamma_b = \frac{\delta_1 + \delta_2}{a} = \frac{Qab}{12EI_b} + \frac{Qa^2}{24EI_y} \quad (3.9) \]

In addition to \( \delta_1 \) and \( \delta_2 \), the contribution of the shear deformation in the batten and the main member caused by the angular displacement must be taken
into consideration. Figure 3.7 shows that the shear force in the batten is $Qa/b$ and
the shear force in the main member is $Q/2$. Hence the corresponding shear strain,
$\gamma_s$, is

$$\gamma_s = \frac{n_s Qa}{bA_b G} + \frac{n_s Q}{2A_i G} \quad (3.10)$$

where $A_b$ is the sum of the cross-sectional areas of the battens on one level; $A_i$ is
the cross-sectional area of one main member; $G$ is the shear modulus; and $n_s$ is a
shape factor which equals 1.2 for rectangular cross-sections\(^1\). The shearing strain,
$\gamma_s$, gives a lateral deflection of \(\delta_s = \gamma_s a/2\).

Finally, the possibility of lateral deflection due to the semi-rigidity of the
connection between the batten plate and the main members can be considered (Lin
et al.; 1970). From Fig. 3.5(b) it can be seen that the bending moment acting on
the connection of a batten plate to a main member is $M_c = Qa/2$. Should the
connection not be fully rigid as in the case of snug-tight bolts, this moment would
cause a relative rotation between the batten plate and the main member. The
magnitude of this relative rotation depends on the rigidity of the connection, and
is directly proportional to the magnitude of the moment. Therefore, the relative
rotation can be expressed as $\theta_r = ZM_c$ where $Z$ is a semirigid connection constant

\(^1\) For I sections and channel sections bent about the minor axis the shape factor can be taken as approximately $1.2A_i/A_p$, where $A_i$ is the total area of one section; and $A_p$ is the area of the two flanges of that section.
(i.e. \(Z=0\) for a rigid connection and \(Z=\infty\) for a hinged connection). This would give a lateral deflection of \(\delta_z = \frac{1}{4}a^2ZQ\). Adding the effect of \(\delta_3\) and \(\delta_4\) to Eq. (3.9) and dividing by \(Q\) gives the expression for \(1/P_d\) as

\[
\frac{1}{P_d} = \frac{ab}{12EI} + \frac{a^2}{24EI} + \frac{n_s a}{bA_b G} + \frac{n_s}{2A_i G} + \frac{aZ}{2} \quad (3.11)
\]

It should be noted that the amplification of the lateral deflection due to the axial load in the main members has been neglected.

Substituting Eq. (3.11) into Eq. (3.5) and using \(P_e = \pi^2 EI_y/(KL)^2\) gives the following expression for the critical load

\[
P_{cr} = \frac{\pi^2 EI_y}{(KL)^2} \frac{1}{1 + \frac{\pi^2 EI_y}{(KL)^2} \left( \frac{ab}{12EI} + \frac{a^2}{24EI} + \frac{n_s a}{bA_b G} + \frac{n_s}{2A_i G} + \frac{aZ}{2} \right)} \quad (3.12)
\]

To express the critical load in the form \(P_{cr} = \pi^2 EA/\Lambda_{eq}^2\); where \(A = 2A_i\) is the integral cross-sectional area; and \(\Lambda_{eq}\) is the equivalent slenderness ratio of the integral cross section about the axis of buckling (i.e. the \(Y\) axis), Eq. (3.12) must be rearranged so that

\[
P_{cr} = \frac{\pi^2 EA}{\Lambda_{eq}^2} = \frac{\pi^2 EA}{\Lambda_y^2 + \frac{\pi^2 Aab}{12I_b} + \frac{\pi^2 a^2}{12r_y^2} + \frac{2\pi^2 An_s a(1+\nu)}{bA_b} + 2\pi^2 n_s (1+\nu) + \frac{\pi^2 EAaZ}{2}} \quad (3.13)
\]

where \(\nu\) is Poisson's ratio; and \(r_y\) is the radius of gyration of a main member about
its minor axis (or the axis parallel to the buckling axis). The expression in the
denominator is known as the equivalent slenderness ratio of a batten column.
It is necessary now to determine which of the terms in this expression have a
significant effect on $P_{cr}$. The first term is of course the slenderness ratio of the
integral column and is a fundamental requirement of the equivalent slenderness
ratio formula. The following five terms are the secondary effects of additional
lateral deflection of the column due to shear forces. Two of these terms are
characterised by the bending and shear deformations in the main members, two by
the bending and shear deformations in the batten plates, and the final one by semi-
rigid connections. Naturally, the role played by the batten plates depends on the
dimensions of the batten plates. As this work is concerned with welded
connections, which are assumed to be fully rigid, it is reasonable to neglect the last
term in the denominator of Eq. (3.13).

The numerical properties of the built-up members used in the experimental
program (i.e. columns of practical dimensions), were used to test the significance
of the second to fourth terms of the equivalent slenderness ratio. The third term
in the equivalent slenderness ratio formula, the deformation due to bending of the
main members, is without a doubt the most influential term after the slenderness
ratio of the integral column having a value of up to 6.3 times that of the integral
slenderness ratio. It consists of the square of the slenderness ratio of the main
members between interconnectors multiplied by a factor of $\pi^2/12$. From a comparison made between the remaining three secondary terms and the third term, it was found that the fifth term, the deformation due to shearing force in the main members, is negligible having a maximum value of less than 3.8% of that of the third term. The second and fourth terms are a result of the bending and shear deformation of the batten plates, respectively. For square batten plates the ratio of second term to the third term is less than 14%. However, the ratio of the fourth term to the third term can be as high as 70%. This term seems to make a significant contribution to the equivalent slenderness ratio of a built-up column, but can be neglected when battens of considerable shear stiffness are used, which is the usual design procedure. Hence, the major contribution to the equivalent slenderness ratio formula are made by the first and third terms giving the expression

$$\Lambda_{eq} = \sqrt{\Lambda_y^2 + \frac{\pi^2}{12} \left( \frac{a}{r} \right)^2}$$  \hspace{1cm} (3.14)$$

It seems reasonable to assume that approximating the factor $\pi^2/12$ in the third term to 1.0, that is, increasing the third term by about 22% would help in compensating for the effect of bending and shear deformations in the batten plates. The equivalent slenderness ratio formula is thus reduced to
\[ \Lambda_{eq} = \sqrt{\Lambda^2 + \left(\frac{a}{r_y}\right)^2} \]  \hspace{1cm} (3.15)

Hence, it can be said that the equivalent slenderness ratio of a battened column is equal to the square root of the sum of the squares of the integral slenderness ratio and the slenderness ratio of a main member between interconnectors.

Other variations of the equivalent slenderness ratio formula are mentioned in Chapter II. It is interesting to note that most international standards and specifications (Australia, China, Eastern Europe, Japan, and North America), with the exception of Western Europe, prescribe an equivalent or effective slenderness ratio as shown in Appendix A.

3.4 Interaction of Buckling Modes

The second concern with built-up columns is the interaction of local and global buckling, also known as the occurrence of coupled instabilities. To economize on the cost of fabrication of built-up columns it is logic to make the maximum slenderness ratio of the individual main members between interconnectors equal to the slenderness ratio of the overall column. However, theoretically the danger of this type of structural optimization (Thompson and Hunt; 1973, 1974) is that it tends to increase the imperfection sensitivity. Post-buckling modes can be classified as
neutral, stable, and unstable (Chajes; 1974, and Esslinger and Geier; 1975), as shown in Fig. 3.8. Stable post-buckling is characterised by no imperfection sensitivity. Unstable post-buckling is characterised by severe imperfection sensitivity. Whereas, neutral post-buckling is characterised by mild imperfection sensitivity. An example of symmetric stable and symmetric unstable post-buckling behaviour is given in Appendix B. Both Euler local flexural buckling and Euler global buckling have a neutral post-buckling mode when they occur individually, which is virtually insensitive to imperfections. However, nonlinear coupling of these two buckling modes can generate a very unstable compound bifurcation and associated imperfection sensitivity. Experimentally, a built-up column exhibits an unstable buckling, characterized by a drop in the load after reaching its maximum load. Also, thinking of a built-up column as an integral member, the built-up action of the column does in fact reduce its load-carrying capacity, which is accounted for by using the equivalent slenderness ratio.

Consider a pinned built-up column in which the main members are interconnected by an unspecified web (i.e. batten plates) to maintain the integrity of the column, as shown in Fig. 3.9(a) (Bažant and Cedolin; 1991). The column has an overall length of $L$ and carries a load of $P$. The main members, which may be considered to be flanges, are a distance $b$ apart and have an axial stiffness of $EA$, and a bending stiffness of $EI_y$. Two basic buckling modes can occur, viz. local flexural
buckling of the main members and global buckling of the column as shown in Figs. 3.9(b) and (c), respectively. The flexibility of the interconnectors is very small compared to that of the main members, as shown in Appendix C. So fixed-end conditions are assumed for the local flexural buckling mode. Two local buckling modes are possible, the first with sidesway prevented as shown in Fig. 3.10, and the second with sidesway permitted as shown previously in Fig. 3.9. The latter buckling mode is the weaker mode, and so it is assumed that the local column chooses this mode when local and global buckling occur simultaneously, as shown by the experimental and finite element results. Figure 3.9(d) shows the simultaneous occurrence of local and global buckling. The buckling modes can be associated with:

(a) a local flange deflection $u_1$, based on the assumption of fixed-end conditions and the occurrence of sidesway,

(b) a global sinusoidal flange deflection $u_2$, and

(c) a local initial imperfection $x_0$ taking the shape of the local buckling mode to initiate this mode,

such that

$$u_1 = o_1 \left(1 - \cos \frac{\pi z}{a}\right) \quad u_2 = o_2 \frac{\sin \frac{\pi z}{L}}{L} \quad x_0 = \alpha a \left(1 - \cos \frac{\pi z}{a}\right)$$

(3.16)

where $o_1$ and $o_2$ are midpoint modal amplitudes; and $\alpha$ is an imperfection parameter ($\alpha a$ represents the initial imperfection midpoint amplitude i.e. local initial imperfection is a function of $a$ and imperfection amplitude $\alpha$). No initial global
imperfection is included as it is assumed that the local initial imperfection will
initiate buckling, and the purpose of this analysis is to study the effect of local
flexural buckling on the overall behaviour of the column. However, sinusoidal
global flange deflection is still assumed.

The column can carry a buckling load of \( P_s \), causing an axial force of
approximately \( \frac{1}{2}P_s \) in each flange. In the local flexural buckling mode the flange
may be considered to buckle as a series of fixed-end unbraced columns of length
\( a \). Therefore, the elastic local flexural and global flexural buckling loads can be
expressed as

\[
P_L = \frac{2\pi^2 EI_y}{a^2} \quad P_G = \frac{\pi^2 EA_i b^2}{2L^2}
\]

and correspond to the buckling loads for the built-up columns shown in Figs.
3.9(b) and (c), respectively. It should be noted that for the sake of algebraic
simplicity the global second moment of area of the column has been taken as
\( I_o = A_i b^3/2 \), neglecting the second moment of area of the individual main members
about their own centroidal axes. For widely spaced built-up columns the effect of
this approximation is negligible.

### 3.4.1 Local Buckling Alone

Referring to Fig. 3.9(b), considering \( o_2 = 0 \) gives just local flexural buckling
in the form of successive unbraced fixed-end columns of length \(a\) in which the midpoint deflections are magnified by a magnification factor of \(1/(1-P_B/P_L)\) (Chen and Lui; 1987) giving

\[
\alpha a + o_1 = \frac{\alpha a}{P_L} \quad \Rightarrow \quad o_1 = \frac{\alpha a P_B}{P_L - P_B} \quad (3.18)
\]

The axial shortening per length \(a\) due to bending, \(w_f\), can be derived from the expression

\[
w_f = \frac{1}{2} \int_0^a \left[ (x_0' + u_1')^2 - x_0'^2 \right] dz \quad (3.19)
\]

where the prime designates the first derivative with respect to \(z\). Substituting the derivatives of Eq. (3.16) into Eq. (3.19) and simplifying gives

\[
w_f = \frac{1}{2} \left( \frac{2 \pi^2 a o_1}{a} + \frac{o_1^2 \pi^2}{a^2} \right) \int_0^a \sin^2 \frac{\pi z}{a} dz \quad (3.20)
\]

Substituting the value of the integral as \(a/2\) and substituting \(o_1\) from Eq. (3.18) gives

\[
w_f = \frac{\pi^2 a^2 a P_B}{4(P_L - P_B)^2} (2P_L - P_B) \quad (3.21)
\]

Approximately the same result can be derived for slightly flexible end conditions as shown in Appendix C.
This axial shortening due to local bending of the flanges reduces the axial stiffness $EA_i^*$ of the flanges, or in other words increases the axial compliance $1/EA_i^*$ such that

$$\frac{1}{EA_i^*} = \frac{1}{EA_i} + \frac{1}{a} \frac{\partial w_f}{\partial P_B}$$

(3.22)

where the second term can be determined by partially differentiating Eq. (3.21) with respect to $P_B$ giving

$$\frac{\partial w_f}{\partial P_B} = \frac{\pi^2 a^2 a p_L^2}{2(P_L - P_B)^3}$$

(3.23)

Substituting this expression and the second expression of Eq. (3.17) into Eq. (3.22) yields the following expression for $1/EA_i^*$

$$\frac{1}{EA_i^*} = \frac{\pi^2 b^2}{2L^2 P_G} + \frac{\pi^2 a^2 P_L^2}{2(P_L - P_B)^3}$$

(3.24)

This is the modified axial compliance per unit length.

3.4.2 Interaction of Local and Global Buckling

The general idea of this analysis is to emphasize, at least theoretically, the influence of local flexural buckling on global buckling. It is suggested that the effect of local buckling on global buckling is another way of representing the effect
of shear on the integral column. As the local column gets weaker (i.e. its length \( a \) increases), the equivalent slenderness ratio of the integral column increases reducing the load-carrying capacity of the integral column. Taking simultaneous local flexural and global buckling into account, the elastic buckling load of the strut \( P_b \), when the global sinusoidal deflection \( u_2 \) is nonzero, can be approximately taken as the Euler load of a column with a bending stiffness of \( b^2EA_i^*/2 \) such that

\[
P_b = \frac{\pi^2 b^2 EA_i^*}{2L^2}
\]  

(3.25)

Substituting the expression for \( EA_i^* \) from Eq. (3.24) into the above formula and simplifying gives

\[
\frac{1}{\lambda_b} \frac{P_L}{P_G} = \frac{\alpha^2 L^2}{b^2 (1-\lambda_b)^3}
\]

where

\[
\lambda_b = \frac{P_b}{P_L}
\]  

(3.26)

This is the general equation for the interaction of local flexural buckling and global buckling. The relationship between the buckling load for various ratios of global buckling load to local flexural buckling load for an ideal column is plotted in Fig. 3.11. In this example, the local slenderness ratio is considered to be a constant and the global slenderness ratio is gradually increased. It can be seen that the buckling load increases linearly with the global buckling load until the local flexural buckling load is equal to or less than the global buckling load, hence the
buckling load is constant and equal to $P_L$. The buckling load of the built-up column is now governed by either: (a) global buckling; (b) simultaneous local and global buckling; or (c) local buckling. In the following sections these three buckling modes will be examined to determine the sensitivity of the buckling load to initial imperfections.

**Case I: Global buckling governs**

This case refers to failure of the column due to global buckling before the occurrence of local flexural buckling, but close to it. This situation is illustrated in Fig. 3.11. In other words $P_G < P_L$ (but close to $P_L$) such that $P_b \approx P_G$ and $\lambda_b \approx P_G/P_L$. Also, as the local buckling load is close to the global buckling load ($P_L \approx P_G$), it can be deduced from Eq. (3.17) that

$$\frac{2r_y^2}{a^2} \approx \frac{b^2}{2L^2} \Rightarrow \frac{r_y}{a} \approx \frac{b}{2L}$$

(3.27)

which implies that the slenderness ratio of the main member between points of interconnection and the slenderness ratio of the built-up column are nearly equal. Hence, introducing the constant $k_1 = b/L$, then substituting this into Eq. (3.26) and simplifying gives
\[
\frac{P_B}{P_G} = \frac{1}{1 + \left(\frac{\alpha}{\alpha_1}\right)^2}
\] where \[
\alpha_1 = k_1 \left[ \frac{(P_L - P_G)^{3/2}}{P_L P_G^{1/2}} \right]
\] (3.28)

Assuming that \(\alpha/\alpha_1\) is a small quantity taken as \(x\) the right hand side of this expression can be expressed in a Maclaurian Series as approximately equal to \(1 - x^2\).

Hence the above expression reduces to

\[
\frac{P_B}{P_G} = 1 - \left(\frac{\alpha}{\alpha_1}\right)^2
\] where \[
\alpha_1 = k_1 \left[ \frac{(P_L - P_G)^{3/2}}{P_L P_G^{1/2}} \right]
\] (3.29)

A built-up column that fails by global buckling has a mild sensitivity to an initial imperfection as indicated by the exponent 2.

**Case II: Simultaneous local and global buckling**

This case refers to failure of the column due to simultaneous local flexural and global buckling. Referring to Fig. 3.11, this would be at the point of intersection of the inclined and horizontal lines. The condition for this to occur is \(P_G = P_L\). As a simplifying assumption \(P_B\) is taken to be close to \(P_L\) that is \(\lambda_b \approx 1\).

Substituting this into Eq. (3.26) and using the approximation \(\lambda_b^{-1} - 1 \approx 1 - \lambda_b\) gives

\[
\frac{P_B}{P_L} = 1 - \left(\frac{\alpha}{\alpha_2}\right)^{1/2}
\] where \[
\alpha_2 = k_1
\] (3.30)
Thus a built-up column that buckles simultaneously in local flexural buckling and global buckling is severely sensitive to imperfections, which is characterized by the exponent of $1/2$. This case represents the optimum design based on the economic aspect of reducing the number of interconnectors.

**Case III: Local buckling governs**

This case refers to failure due to local flexural buckling before the global buckling load capacity has been reached. It specifically refers to situations where the local buckling load is less than the global buckling load but close to it as shown in Fig. 3.11. In other words $P_L < P_G$ (but close to $P_G$) which implies that $P_s \approx P_L$ giving $\lambda_s \approx 1$. Substituting this into Eq. (3.26) and simplifying gives

$$
\frac{P_s}{P_L} = 1 - \left( \frac{\alpha}{\alpha_3} \right)^{2/3} \text{ where } \alpha_3 = k_1 \left( 1 - \frac{P_L}{P_G} \right)^{1/2} \quad (3.31)
$$

The built-up column is said to be strongly sensitive to imperfections which is characterized by the exponent $2/3$. Hypothetically, as the ratio $P_G/P_L$ increases indefinitely the reciprocal of this ratio tends to zero. This also implies that the length of the column tends to zero and hence $k_1$ tends to infinity. If this condition was applied to Eq. (3.31), $\alpha_3$ would tend to infinity and $P_s$ would asymptotically approach $P_L$ at large ratios of $P_G/P_L$; (In other words, as $P_G/P_L \gg 1$, $P_L/P_G \to 0$, $\therefore L \to 0$
hence \( k_1 \rightarrow \infty \), which leads to \( \alpha_3 \rightarrow \infty \) hence \( P_b \approx P_L \). In short, as \( P_c / P_L \) increases \( P_b \) asymptotically approaches \( P_L \) regardless of the magnitude of the initial imperfection of the column.

In all three cases the imperfection sensitivity of the column is directly proportional to the term \( \alpha / \alpha_i \), where \( i = 1, 2, 3 \). In turn, the imperfection sensitivity is directly proportional to the local initial imperfection \( \alpha \). Whereas, the imperfection sensitivity is inversely proportional to the factor \( \alpha_i \), which in all three cases depends on \( k_1 = b / L \approx 4r_y / a \). Therefore using a column with \( L \gg b \) or \( a \gg 4r_y \) would increase the imperfection sensitivity. Figure 3.12 illustrates a plot of Eqs. (3.29), (3.30), and (3.31) for imperfections of \( \alpha = 0.0005, 0.0007, \) and \( 0.001 \) and a value of \( k_1 = 0.0157 \). This corresponds to a value of \( b = 55.8 \) mm and \( L = 3550 \) mm. These imperfections are assumed values and the value of \( k_1 \) was calculated using the dimensions of the specimens of Group I. It can be observed that the imperfect curves do not asymptotically reach the ideal column curve at values of \( P_c / P_L \gg 1 \) as the same column properties are used throughout, and so the length \( L \) does not approach zero. It is also interesting to note that as the analysis involved finding the effect of local flexural buckling on global buckling and not the reverse, it is understandable that the curve is not asymptotic to the ideal local buckling load.
This analysis was based on certain approximations and assumptions. These are:

(i) the use of a sinusoidal deflection curve for $u_2$ in Eq. (3.16), and the omission of an initial global imperfection,

(ii) neglecting the effect of shear on the global buckling load,

(iii) omitting the second moment of area of the main members for the global buckling load and basing it on $EA^*$,

(iv) using fully fixed-end conditions and the magnification factor in Eq. (3.18) for the unbraced local column, and

(v) the use of elastic buckling loads for local and global buckling.

3.4.3 Design Codes

Most international standards and specifications for the design of steel structures specify limits for the individual slenderness ratio of the main members between interconnectors, $\Lambda_i = a/r_i$. It is generally accepted that the slenderness ratio of the main members between interconnectors, $\Lambda_i$, must be less than the governing slenderness ratio of the overall column. This is to prevent premature local buckling. Many codes, however, go on to further reduce $\Lambda_i$ in order to prevent interaction of local and global buckling leading to the erosion of the buckling load caused by geometric imperfections.
All the editions of the Canadian Standard S16.1 prior to the 1994 edition specified stringent limits for the slenderness ratio of the main members between interconnectors for battened columns. These limits are specified in Clause 19.1.16 of S16.1-M89 (1989a) as

\[
\begin{align*}
(a) & \text{ If } \Lambda_Y \leq 0.8 \Lambda_X \\
& \frac{d_i}{r_i} \leq 50 \tag{3.32a} \\
& \frac{d_i}{r_i} \leq 0.7 \Lambda_X \\
(b) & \text{ If } \Lambda_Y > 0.8 \Lambda_X \\
& \frac{d_i}{r_i} \leq 40 \tag{3.32b} \\
& \frac{d_i}{r_i} \leq 0.6 \Lambda_Y
\end{align*}
\]

where \( \Lambda_X \) and \( \Lambda_Y \) are the slenderness ratios of the integral member about the \( X \) and \( Y \) axes, respectively; and \( d_i \) is the clear distance between batten plates, as shown in Fig. 3.13.

The first of these conditions is taken to mean that the overall slenderness ratio about the \( X \) axis is greater than the effective (equivalent) slenderness ratio about the \( Y \) axis so that \( X \) axis buckling governs. This clause assumes that \( \Lambda_Y/0.8 \) is enough to justify an equivalent slenderness ratio from which the mode of buckling can be determined. For \( X \) axis buckling increasing the number of interconnectors above that required to prevent local buckling does not increase the
strength of the column. The column behaves as two simple struts working together, and the interconnectors are simply compressed as shown in Fig. 3.14. Hence, increasing the number of interconnectors would not increase the buckling load. Furthermore, if the local slenderness ratio is equal to the slenderness ratio about the $X$ axis, then the equivalent slenderness ratio about the $Y$ axis would govern.

The second of these conditions refers to $Y$ axis buckling. In this case interaction is represented by the use of an equivalent slenderness ratio. However, for this mode of buckling increasing the number of interconnectors increases the strength of the column.

Whether this clause is important or not depends greatly on the actual degree of load reduction caused by geometrical imperfections. It was removed from the current Canadian Standard S16.1-94 in lieu of using a more conservative equivalent slenderness ratio formula.

The Canadian Standard S16.1-M89 also specifies for compression members composed of two or more rolled shapes in contact or separated from one another by intermittent fillers (buttons), or interconnected in general (S16.1-94) that
\[ \Lambda_i \leq \frac{\Lambda_x}{\Lambda_y} \] (3.33)

The requirements of other international codes regarding the maximum slenderness ratio of individual main members between interconnectors are listed in Appendix A.

3.5 Batten Versus Button Interconnector

When the buckling mode involves deformations that produce shear forces in the interconnectors (i.e. buckling about the Y axis), the difference in mechanism of shear transfer in a batten and a button interconnector is noteworthy. Using the theory of battened columns to solve the problem of shear, the column can be represented as a frame having hinges at the midpoints of the main members between battens and at the midpoints of the battens, as shown in Fig. 3.15. In this way, the shear forces in a battened column, which vary along the length of the member, are transferred in the manner shown in Fig. 3.16. Studying the deformation of the upper intermediate batten as shown in Fig. 3.17, it can be seen that the deformation of the batten is compatible with the internal forces acting on the ends of the batten. Thus the theory of battened columns is verified for batten interconnectors. However, applying the same theory to a buttoned column without using hinges at the midpoint of the buttons gives the system of internal forces
shown in Fig. 3.18. Again, the deformation of the upper intermediate button under the action of the internal forces acting on the button is shown in Fig. 3.19. It can be seen that the shear deformation of the flexurally rigid button interconnector is not compatible with deformation of the main members, unless a rigid body rotation occurs. This leads to the question as to whether or not the theory of batten columns applies to buttoned columns.

Experimental tests on built-up buttoned and battened columns have indicated that buttoned columns have a slightly higher compressive resistance to identical battened columns. However, experimental results of buttoned columns (Aslani and Goel; 1991b) have shown that Eqs. (2.2) and (2.4) satisfactorily predict the compressive strength of buttoned columns as described in Chapter II.
CHAPTER IV

FINITE ELEMENT ANALYSIS

4.1 Introduction

A model of each experimental specimen was analyzed using finite elements. The purpose of this analysis was to model the behaviour of a built-up column under axial load, hence, verify the analysis by the experimental results. The channel sections were modelled as shell elements, each section being divided into four elements, one for each flange and two for the web. The batten plate interconnectors were also modelled as shell elements. The button plate interconnectors were modelled as solid elements due to their thickness and to eliminate bending. As a first attempt to model the end plate and knife edge, solid elements were used, however, as these solid elements had no rotational degrees of freedom at the nodes, they did not allow for hinge action. To overcome this problem, the end plate was modelled as shell elements.

A commercial finite element package, ABAQUS Version 5.5 (1995), was
used to execute the finite element analysis. An example of the data deck of a

4.2 Finite Element Model

typical program is given in Appendix D.

The objective of the first part of the data deck was to define the geometry,

4.2.1 Nodes

kinematic constraints, and material properties of the finite element model. This

consisted of a definition of the nodes, elements, constraints, and material

properties.

The nodes were defined using the *NODE option assigning them to node

sets using the NSET parameter. The channel sections were modelled using the

parabolic out-of-straightness parameter, LINE=P to generate the nodes in the

*NGEN option. The nodes were also defined using the *NCOPY option. The

nodes in each line were numbered consecutively with an interval of one. Whereas

each line of nodes had an interval of 200. Figures 4.1(a) and (b) show the node

numbering of the lower cross section of a column where the channel sections were

arranged in the toe-to-toe and back-to-back configurations, respectively. The node

numbering of the cross section was selected to facilitate the generation of the

elements. The number of nodes along the length of the channel sections varied

from specimen to specimen. In general, the aim was to make the length of the
channel elements approximately equal to the length of the interconnectors, and at the same time to divide the length of the channel sections so that a whole element coincided with an interconnector. This facilitated the constraining of the interconnectors and end connectors to nearby corresponding nodes on the channel sections. Figure 4.2 shows a schematic representation of the node numbering along the channel sections of all the test specimens modelled. The nodes of each channel section were put in separate node sets, _viz._ MEM1 and MEM2. The channel sections were modelled such that the nodes were assigned along the centre line of the flanges and web. Nominal dimensions were used to assign coordinates, as specified in the Handbook of Steel Construction (CISC; 1991), not the actual dimensions measured for a sample of a channel section which varied from -1 to 2% from the nominal.

Additional nodes were defined for the end plates and knife edges. The geometry of the end plates and knife edges was simplified to model them as shell elements. Figure 4.3 illustrates the node numbering of the bottom end plate and knife edge for Groups I and II as well as Group III. As the specimens of Groups I and II were hinged about the _X_ axis and the specimens of Group III were hinged about the _Y_ axis, the orientation of the knife edge was changed for these two sets of specimens. The additional nodes of the bottom and top end pieces were put in node sets called BOTPL and TOPPL, respectively.
The final set of nodes defined were the interconnectors and end connectors. For simplicity these will be referred to collectively as the interconnectors. These elements were assigned independent node numbers, which were in turn constrained to the corresponding nodes on the main members. Each batten interconnector was divided into twenty-four elements, six along the width and four along the length of the interconnector. Each button interconnector was also divided into twenty-four elements, six along the width and four along the length of the interconnector. The nodes were numbered with an interval of one along the width and ten along the length. Figure 4.4(a) shows the node numbering of a typical pair of batten interconnectors. Figure 4.4(b) shows the node numbering of a typical button interconnector. The interconnectors were assigned to a node set called CONN.

4.2.2 Elements

The channel sections were modelled as shell elements. A typical shell element was defined by its nodes using the *ELEMENT option specifying the type of element using the TYPE parameter. Incremental generation of elements were made using the *ELGEN option by prescribing increments in the node and element numbers. The flanges were each modelled as one column of shell elements, and the web was modelled as two columns of shell elements. The element used for the channel sections was an 8-node shell element, S8R. This element is an 8-node, doubly curved shell with reduced integration and is suitable for thick shells or
when the transverse shear flexibility is important, which is the case for built-up columns. This uses a lower order integration to form the element stiffness matrix, which usually provides more accurate results (Zienkiewicz and Taylor; 1989). Figure 4.5(a) shows an example of the node numbering and integration points of an 8-node shell element. This element has 6 active degrees of freedom at each node, three displacements $u_x$, $u_y$, $u_z$, and three rotations $\phi_x$, $\phi_y$, $\phi_z$. The flange elements were assigned to an element set called FLANGE by the *ELSET option, and given a linearly varying thickness on the *SHELL SECTION option from 9.49 mm at the web to 3.98 mm at its free end. This was done by specifying the NODAL THICKNESS parameter on the *SHELL SECTION option which read the thickness specified on the *NODAL THICKNESS option. The web elements were assigned to an element set called WEB and given a uniform thickness of 4.3 mm.

The end plates were modelled using 8-node shell elements, S8R, as they were thick elements. The elements of the top and bottom end pieces were assigned to a node set called ENDS and given a uniform thickness of 38.1 mm.

The batten interconnectors were modelled as 4-node shell elements, S4R, being relatively thin shells. The thickness of these varied from specimen to specimen. Figure 4.5(b) shows an example of the node numbering and integration point of a four node shell element. This element also has 6 degrees of freedom at
each node. The button interconnectors were modelled as solid 8-node linear brick elements, C3D8, placed between the channel shell elements. This element has 3 active degrees of freedom at each node, \( u_x, u_y, u_z \). Figure 4.5(c) shows an example of the node numbering of an 8-node solid element. Both the batten and button interconnectors were assigned to an element set called CONN.

Figures 4.6-4.9 illustrate the finite element models of specimens 120-63-3-1/8, 70-63-1, 120-A-2, and 120-U-2, respectively.

4.2.3 Kinematic Constraints

Three types of kinematic constraints were used, multi-point constraints (*MPC), fixed boundary conditions (*BOUNDARY), and linear defined multi-point constraints (*EQUATION).

The first need to use constraints was to model the action of the welds which connect the interconnectors to the channel sections. This was done using multi-point constraints. It should be noted that the appropriate nodes of the interconnectors were constrained to the corresponding nodes of the flange elements leaving the degrees of freedom of the nodes of the latter unconstrained. For specimens of Groups I and II the batten plates were welded all around the interconnectors to the exterior side of the flanges of the channel sections. Hence,
the aim was to constrain all the exterior nodes of each interconnector to the closest node or nodes on the channel sections, with the exception of the central exterior nodes along the width of the interconnectors which were left free. Two types of MPCs were used to do this: TIE MPCs which make all active degrees of freedom equal at two nodes, $a$ and $b$, as shown in Fig. 4.10(a), and LINEAR MPCs which constrain all active degrees of freedom at node $p$ to be interpolated linearly from the corresponding degrees of freedom at nodes $a$ and $b$, as shown in Fig. 4.10(b). LINEAR MPCs are often used for mesh refinement of first-order elements. The corner nodes on the interconnectors and the exterior middle nodes along the length of the interconnectors were constrained to the line joining the corresponding nodes on the channel sections by TIEs. The rest of the exterior nodes on the interconnectors were constrained to the closest two nodes on the channel sections by LINEAR MPCs. Figure 4.11 gives a schematic representation of these constraints for a typical batten interconnector. Note that the middle nodes along the width of the interconnectors were left free. For the specimens of Group III the interconnectors, whether battens or buttons, were only welded to the channels along their length, with the exception of the end interconnectors which were slightly more constrained. To model the effect of friction between the button interconnectors and the webs of the channel, all the internal nodes of the button interconnectors were constrained.
There were two boundary conditions to be modelled, viz. the upper boundary condition and the lower boundary condition. The statical system of the finite element model was hinged about one axis at the bottom end and hinged about the same axis at the top end, but free to displace axially, as shown in Fig. 4.12. The row of nodes in the top end plate, along the axis of the hinge, was assigned to a node set called TOPEND (designated by a superscript $i$). This node set was prevented from displacing in the $X$ and $Y$ directions which prevented rotation about the $Z$ axis and the occurrence of sidesway. This was done using the *BOUNDARY option such that

$$u^i_x = u^i_y = 0 \tag{4.1}$$

Axial shortening in the $Z$ direction was permitted. For Groups I and II, as the node set TOPEND was along the $X$ axis, rotation about the $X$ axis was permitted. To prevent rotation about the $Y$ axis no relative displacement was permitted along this node set in the $Z$ direction. This was done by an *EQUATION constraining the degree of freedom $u^i_z$ of all the nodes in the node set TOPEND to be equal to each other. The row of nodes in the bottom end plate, along the knife edge, was assigned to a node set called HINGE (designated by a superscript $h$), which was prevented from displacing by the *BOUNDARY option type PINNED such that

$$u^h_x = u^h_y = u^h_z = 0 \tag{4.2}$$

In this way rotation was prevented about the $Y$ and $Z$ axes and permitted about the
line passing through the row of nodes in this node set, which was the $X$ axis for Groups I and II. For the specimens of Group III, where the knife edge was along the $Y$ axis, the above boundary conditions would prevent rotation about the $X$ and $Z$ axes and allow rotation about the $Y$ axis. This concludes the description of the constraints imposed on the finite element models of the built-up columns.

In addition to the kinematic constraints, a contact problem analysis was conducted to ensure that the channel sections and interconnectors did not penetrate each other during the run.

4.2.4 Material Definition

Each set of elements was assigned to a certain material model definition on the corresponding *SHELL SECTION option or *SOLID SECTION option. The definition of each material model was begun by the *MATERIAL option, which also assigned it a name by the NAME parameter. The material of the channel sections was named A1, the material of the interconnectors was named CONN, and the material of the end plate and knife edge was named ENDS.

All the material used to fabricate the built-up columns was structural steel that differed from one another in elastic and plastic properties. All steels are isotropic in nature, and so the engineering constants were uniform in all three
principal directions. These were obtained from tension tests carried out on coupons made from the appropriate materials. Details of these tests are given in Appendix E. As the material was isotropic a standard MISES yield surface model with associated plastic flow was used, in which the uniaxial yield stress was a function of the uniaxial equivalent plastic strain. Associated plastic flow means that, as the material is yielding, the inelastic deformation rate is in the direction of the normal to the yield surface. All the models were intended to describe the yield and inelastic flow of a metal at relatively low temperatures, where creep effects were not important. The materials were modelled using an incremental theory in which the mechanical strain was decomposed into an elastic part and a plastic (inelastic) part. This theory formulates the model in terms of a yield surface. This generalizes the concept of: (a) a yield load which can be used to determine if the material will respond purely elastically at a particular state of stress; (b) a flow rule which defines the inelastic deformation that must occur if the material is no longer responding purely elastically; (c) an evolution law that defines the work hardening which is the way in which the yield or flow definitions change as inelastic deformation occurs.

The material of the channel sections was a linear elastic steel with a definite yield point (mild strength steel), so it was modelled as a linear elastic-perfectly plastic material. Young's modulus of elasticity and the yield stress of the steel
varied from specimen to specimen according to the bar from which the coupons were taken. The exact values of these properties were used to model each specimen and can be found in Appendix E. Poisson's ratio was not measured for this material and so it was set at the default value of 0.3. The *ELASTIC option was used to specify the elastic properties, $E$ and $\nu$, hence the total stress, $\sigma$, (true, or Cauchy stress in finite strain problems) was derived from the total elastic strain, $\varepsilon^{el}$, (log strain in finite strain problems) as

$$\sigma = D^{el} \varepsilon^{el} \quad (4.5)$$

where $D^{el}$ is the elasticity matrix. The parameter TYPE was assigned the default value of ISOTROPIC. The *PLASTIC option was used to model the mechanical response of the material as it undergoes nonrecoverable deformation in a ductile fashion. The parameter HARDENING was assigned the default value of ISOTROPIC. For this model perfect plasticity was defined, meaning that the yield stress does not change with plastic strain. Only the value of the yield stress was necessary to be input, at zero plastic strain.

The material of the interconnectors was a high strength structural steel with no definite yield point. Each plate thickness had a specific stress-strain relationship, as shown in Appendix E. This type of plasticity is known as isotropic hardening, in which the yield surface changes size uniformly in all directions, so
that the yield stress increases in all stress directions as plastic straining occurs. The elastic constants were defined in the *ELASTIC option. The plastic hardening was defined in the *PLASTIC option by defining the yield stress as a function of plastic strain. The true plastic stress, $\sigma_{\text{true}}$, and log plastic strain, $\varepsilon_{\text{in}}$, were given as explained in Appendix E.

The material of the end plate was similar to that of the interconnectors and a similar procedure was used to define it.

### 4.3 Preliminary Output

At this stage it is suitable to draw the model. The model can be drawn using the *DRAW option specifying which part of the model using the *DETAIL option. The viewpoint of the model can also be defined as explained below.

### 4.4 Loading History

The loading history data is the sequence of loadings or events that define what happens to the model, and what response variables are required from the analysis. In ABAQUS, this history is divided into a sequence of steps. Each step is initiated with the *STEP option and ended with the *ENDSTEP option, and is a period of response of a certain type (i.e. static, dynamic, etc...). The definition of a step includes: the procedure type with control parameters for time integration
or for the nonlinear solution procedure; the loading; and output requests. The procedure or response type defines the type of analysis (such as linear or nonlinear, static stress analysis, dynamic transient, transient heat transfer analysis) and includes control parameters for time integration for transient cases. The loading defines the type of loading and the point and direction of application of the load. If the amplitude of the load is defined, the load is assumed to vary either linearly over the step or to be applied instantaneously depending on the response type chosen. For nonlinear analysis the load is applied as a series of increments, where the increment size is chosen either directly or using tolerance parameters. The loading may also be a function of time, such as a ground motion, or a pressure of temperature transient in a fluid surrounding a structure. Finally, the output can be requested in the form of printed output, plotting of results, results file output for subsequent post-analysis use as a data file, and restart file output for subsequent continuation of the analysis. For more complicated history several steps can be used to change the procedure, or to reset conveniently the output edit commands. The *RESTART option can be used to divide the analysis, or to request output for subsequent post-processing.

ABAQUS makes a clear distinction between linear analysis and nonlinear analysis procedures. A linear perturbation analysis step provides the linear response of the system about the base state, which is the state at the end of the last
nonlinear analysis. During a linear perturbation analysis step the response of the
model is defined by its linear elastic stiffness at the base state, while all inelastic
effects are ignored. An analysis step in which the effect of any nonlinearity
present in the model are included is called a general analysis. In a general
nonlinear analysis step the starting condition for each step is the end condition
from the last general step with the state of the model evolving throughout the
history of steps as it responds to the history of loading. There are three sources
of nonlinearity: (a) material nonlinearity, where the material's properties are history
dependent, such as perfect plasticity or isotropic hardening, (b) geometric
nonlinearity, which means that large displacement effects are included in the
analysis, which is imperative in such analyses as bifurcation buckling, and (c)
boundary nonlinearity, which occurs in contact and friction problems. Geometric
nonlinearity is specified by including the NLGEOM parameter in the *STEP
option. Also, the maximum number of increments can be specified by the INC
parameter.

For nonlinear analysis, nonlinear material properties and large displacement
analysis depend on the loading history, and need to be corrected as the load is
applied. For this reason the load is applied in increments, for which iteration is
required within each increment to obtain equilibrium. In general, ABAQUS uses
Newton's method to solve the nonlinear equilibrium equations. According to
Zienkiewicz and Taylor (1989) Newton's method is the same as the Newton-Raphson method as it was simultaneously derived by Raphson (Bićanić and Johnson; 1979). The Newton method or the Newton-Raphson method (Desai and Abel 1972; Dhatt and Touzot 1984) is a procedure for iteration which uses the tangent stiffness of the model for iterations, as shown in Fig. 4.13(a). This is probably the most rapidly convergent solution of nonlinear problems, but it requires the formation and inversion of the stiffness matrix for every iteration, which involves a considerable amount of computation. The Newton-Raphson method has a finite radius of convergence, which means that too large an increment can prevent any solution from being obtained because the initial state is too far away from the equilibrium state that is being sought (i.e. it is outside the radius of convergence). When the approximation at an iteration is within the radius of convergence, that is, the tangent provides an improvement to the solution, the Newton-Raphson method has a quadratic conversion rate. It is interesting to note (Zienkiewicz and Taylor; 1989) that in certain cases, it is more economical and convenient to use the same stiffness matrix for iteration throughout an increment. This is known as the modified Newton method or the modified Newton-Raphson method, as shown in Fig. 4.13(b). Obviously the procedure will converge at a slower linear rate. The combination of incrementation and the Newton-Raphson method of iteration is depicted in Fig. 4.14.
ABAQUS provides two methods of incrementation, direct user control of increment size and automatic control. The former is useful when the user has considerable experience with a particular problem and can therefore select a more economic approach. The latter may increase the cost of the analysis over the cost when direct user control is adopted, but may save enormously over repeated user controlled running of a problem to obtain a satisfactory incrementation scheme, and is preferred for unexperienced users. For automatic incrementation the user defines or leaves to the default the initial time increment (0.1), the total step time period (1.0), the minimum time increment (10^{-5}), and the maximum time increment (1.0) on the *STATIC option. The minimum time increment ensures that the job will terminate if a solution cannot be developed within that time limit, and hence avoids using excessive computer time if, for example, the structure collapses and the load cannot be achieved. ABAQUS allows a maximum of 16 equilibrium iterations for each time increment. Convergence of an increment is assumed to have occurred when the equilibrium equations are within certain tolerances or error measures that can be specified or left to the default. For force or moment the convergence tolerance criteria for nonlinear problems are 5 \times 10^{-3} for residual force or moment, and 10^{-2} for displacement or rotation correction.

The load was specified using the *CLOAD option. The load was applied to the top end plate of the column, on the node set called TOPEND, in the
direction of the axis of the column.

The final part of the step definition is the specification of the output. ABAQUS offers three output options, PRINT, FILE, and PLOT. Printed output can be requested for both element and nodal variables. File output allows results to be stored in a results file for post-processing. Time history plots and variable-variable plots can be generated by post-processing the results file. Plot output can be requested in the form of contour, displaced configuration, and vector plots. The element variable output was specified using the *EL PRINT option which gives the output in the form of a table. The element set for which the output was required was specified using the ELSET parameter, and the position at which the output was required (i.e. at integration points, averaged at nodes, etc...) was specified using the POSITION parameter. The element variables need not be printed for every increment and this can be specified using the FREQUENCY parameter. A summary of the maximum and minimum values in each column of the table is given by default and must be suppressed by the SUMMARY parameter. Also a total of each column in the table can be requested by using the TOTALS parameter. The nodal variable output was specified using the *NODE PRINT option defining the node set through the NSET parameter. The FREQUENCY, SUMMARY, and TOTALS parameters can also be used as above. No result file was requested.
The basic layout of the plots was defined by the *PLOT option, and the frequency of the plots was specified by the FREQUENCY parameter. Several displaced configuration plots were requested at different orientations. Each orientation was defined using the *VIEWPOINT option. On the screen or paper the X axis is taken as the horizontal axis, the Y axis as the vertical axis, and the Z axis as the axis out of the screen or sheet of paper. The method used to define the viewpoint consisted of assuming that the model and screen coordinate axis systems coincide initially, then the model is rotated with respect to the screen coordinate axis system. This rotation is defined as a series of rotations first about the X, then the Y, then the Z axis of the model or screen coordinate system. This method is specified using the DEFINITION parameter which has two options. By setting the DEFINITION parameter to MODEL AXIS ROTATION the orientation is defined by rotating the model with respect to the screen, and defining the rotations first about the X axis, then about the rotated Y axis of the model, then about the rotated Z axis of the model. This method of rotation is illustrated in Fig. 4.15. The displaced configuration plots were requested using the *DISPLACED option with the nodal displacements and rotations as variables. The original geometry is drawn in dotted lines, and the deformed geometry is drawn in solid lines. Hidden lines or surfaces can be removed if the HIDE parameter is included. Figs. 4.16-4.21 show the displaced configurations of the models of specimens 120-63-1, 120-63-2, 120-63-4, 70-63-3, 120-A-2, and 120-U-2, respectively. It is
interesting to note how the failure mode changed from buckling about the $Y$ axis in Fig. 4.16 to a combination of buckling about the $X$ and $Y$ axes in Fig. 4.17, and finally to buckling about the $X$ axis in Fig. 4.18.

The final command required to end a step is the *END STEP option. This ends the loading history and in fact the whole finite element program.

4.5 Approximations in the Finite Element Analysis

There were three sources of approximations in the finite element analysis; geometric, material, and loading approximations.

4.5.1 Geometric Approximations

The geometric approximations are the following:

(a) End plates were modelled the same size as the channel cross sections, and given a uniform thickness.

(b) Fillets in the channel sections were ignored.

(c) To model the welds, the outer nodes of the batten interconnectors were constrained to the nearest nodes on the channel sections and not directly to the face of the channel sections.

(d) The out-of-straightness was approximated to a parabolic curve with nominal maximum amplitude at the mid-section of $L/1000$. 
4.5.2 Material Approximation

The material approximations are:

(a) The material properties of the channel sections were approximated by linear elastic-perfectly plastic with no strain hardening.

(b) The nonlinear material properties of the interconnectors and end plates were approximated to a group of broken lines.

(c) The material of the welds was not modelled.

4.5.3 Loading Approximations

There were two loading approximations which are:

(a) The self-weight of the column was neglected. As the columns were tested horizontally, the initial bending stresses were also neglected.

(b) The heat analysis due to heating and cooling during fabrication and welding was neglected and hence the initial residual stresses due to hot rolling or welding were not included in the analysis.
CHAPTER V

EXPERIMENTAL PROGRAM

5.1 Purpose

The objective of this experimental program was to design and test a group of built-up columns that demonstrate the different requirements of Clause 19 of CAN/CSA-S16.1-M89 (1989a) with regards to battened and buttoned columns. At the beginning of this experimental program, which was in 1993, the current edition of the Canadian Standard CAN/CSA-S16.1-94 (1994) had not yet been issued. These requirements include the need to limit the slenderness ratio of the main members between interconnectors, $\Lambda_r$, the need for an equivalent slenderness ratio, $\Lambda_{eq}$, and the difference between a battened and a buttoned column.

The first question that arose was, is it necessary to use an equivalent slenderness ratio formula, and if so, when should it be used and what factor should be used as an imaginary effective length factor? The equivalent slenderness ratio formula stated in Eq. (2.3) consists of the square root of the sum of the squares of
the integral slenderness ratio and the slenderness ratio of the main members between points of interconnection, \( \Lambda_i \), multiplied by an imaginary effective length factor, \( k_i \). In the theoretical derivation given in Chapter III, for batteneted columns, it was shown that the equivalent slenderness ratio formula need only be used if the buckling mode involves deformations that produce shear forces in the interconnectors (i.e. buckling about the \( Y \) axis). Furthermore, the square of this imaginary effective length factor, \( k_i^2 \), is \( \pi^2/12 \) or conservatively, to account for approximations used in the derivation, can be taken as 1.0. Chapter III also outlines that the mechanism for load transfer of a button interconnector must differ from that of a batten interconnector as a button interconnector is not flexible. This fact along with past experimental results conducted on buttoned columns (Zandonini 1985; Astaneh et al. 1985) have led to the use of an imaginary effective length factor of \( k_i = 0.65 \) (or \( k_i^2 = 0.42 \)). For both batteneted and buttoned columns, buckling about the \( X \) axis does not involve any shear forces in the interconnectors and the column is expected to carry a load equal to the load of the integral column.

The second question that arose was, is it necessary to limit the slenderness ratio of the main members between interconnectors for batteneted columns, and does this also apply for buttoned columns? In editions of the Canadian Standard prior to the 1989 edition, no equivalent slenderness ratio formula was specified and both batteneted and buttoned columns were expected to have the capacity of the integral
column. In order to achieve this capacity the slenderness ratio of the main members between interconnectors was limited. For buttoned columns the Standard specified a local slenderness ratio of not more than the global slenderness ratio about any axis as given in Eq. (3.33). This was considered to suffice for integral action. For batten columns the Standard set limits for the local slenderness ratio for both \( X \) axis and \( Y \) axis buckling as given in Eqs. (3.32a) and (3.32b). These limits were intended to prevent a decrease in the capacity of the column due to the interaction between local and global buckling or the effect of shear and hence ensure that the capacity of batten columns reached that of the integral column. In the 1989 edition of the Canadian Standard, an equivalent slenderness ratio formula was introduced for buttoned columns. The introduction of an equivalent slenderness ratio formula for batten columns in the 1994 edition of the Standard eliminated the need to limit the local slenderness ratio further than to prevent local buckling.

Finally, the third question that arose was, is the difference in the mechanical action of a batten and a button interconnector relevant enough to differentiate between the imaginary effective length factor in the equivalent slenderness ratio formula? The equivalent slenderness ratio formula was derived for a batten column, that is a column in which the interconnectors have bending flexibility and are rigidly fixed to the main members. Since, in a buttoned column, the
interconnectors cannot bend and can only transfer forces through shear, it is logical to presume that the derivation of the equivalent slenderness ratio formula does not necessarily apply for buttoned columns.

5.2 Design

Three groups of specimens were designed to investigate the requirements of the Canadian Standard S16.1 for batted and buttoned columns. The first and second groups were designed with the purpose of testing the requirements of the slenderness ratio of the main members between interconnectors for X axis buckling of batted columns (i.e. Clause 19.1.16(a) of S16.1-M89), as well as to determine the value of the equivalent slenderness ratio.

Appendix F shows calculations of the cross-sectional properties of these three groups as well as illustrations of their configuration.

5.2.1 Group I

The first group of specimens was composed of two C 75 x 6 sections placed toe-to-toe so that they formed a box-like section as shown in Fig. F.2. These channels were separated from each other by a distance of 7.56 mm and interconnected by batten plates. This gave equal second moments of area about both the X and Y axes of \( I_x = I_y = 1.34 \times 10^6 \text{ mm}^4 \).
The purpose of testing this group was to study the elastic behaviour of the column when it buckled in a mode that did not involve deformations that caused shear forces in the interconnectors \((i.e.\) buckling about the \(X\) axis). They had pinned end conditions about the \(X\) axis giving an integral slenderness ratio of \(\Lambda_x=120\), and fixed end conditions about the \(Y\) axis giving an integral slenderness ratio of \(\Lambda_y=60\). Naturally, the local slenderness ratio of the main members affects the equivalent slenderness ratio of the column about the \(Y\) axis, but has no effect on the integral slenderness ratio of the column about the \(X\) axis. The local slenderness ratio was varied by increasing the number of interconnectors \((i.e.\) decreasing the distance between interconnectors).

The interconnectors used were square plates having dimensions of 63.5x63.5 mm \((2\frac{1}{2}\times2\frac{1}{2} \text{ in.})\), with the exception of the first specimen which had one interconnector with a depth of 40 mm. The general thickness of the interconnectors was 3.18 mm \((1/8 \text{ in.})\), with the exception of the specimens with three interconnectors for which two additional specimens were added in which the thickness of the interconnectors were 4.76 and 6.35 mm \((3/16 \text{ and } 1/4 \text{ in.})\). These two specimens were added to determine the effect of the thickness of the interconnectors on \(X\) axis buckling. The different dimensions of these interconnectors are shown in Fig. F.2. In addition to the interconnectors, end connectors were added to these specimens.
The aim of testing this group was to examine the need for limiting the slenderness ratio of the individual main members of baffled columns for the case of \( \lambda_y \leq 0.8 \lambda_x \), meaning \( X \) axis buckling. This was achieved by increasing the number of interconnectors from one to the number required to reduce the slenderness ratio of the individual main members to 50 or 0.7\( \lambda_x \), which was five. Altogether eight specimens were fabricated and tested in this group. Two with one interconnector, three with three interconnectors, and three with two, four, or five interconnectors.

For the specimens with one and two interconnectors, the equivalent slenderness ratio about the \( Y \) axis was greater than the integral slenderness ratio about the \( X \) axis, which made \( Y \) axis buckling predominant.

5.2.2 Group II

The second group of specimens had identical cross sections and end conditions to the first group but differed in integral slenderness ratio. This group was designed to buckle inelastically and so had an integral slenderness ratio about the \( X \) axis of \( \lambda_x = 70 \), and about the \( Y \) axis of \( \lambda_y = 35 \).

All the specimens had interconnectors with dimensions 63.5x63.5x3.18 mm. This group was composed of three specimens with one, two, and three
interconnectors, respectively. Again, the specimens with one and two interconnectors gave an equivalent slenderness ratio about the Y axis greater than the integral slenderness ratio of the column about the X axis making Y axis buckling predominant.

5.2.3 Group III

The third group of specimens consisted of two C 75 x 6 sections placed back-to-back so that they formed the sections shown in Figs. F.3(a) and (b). The two channels were separated from each other by a distance of 9.53 mm (3/8 in.) and interconnected by either batten plates or button plates. The cross section had a second moment of area about the Y axis of \( I_y = 0.528 \times 10^6 \) mm\(^4\) and about the X axis of \( I_x = 1.34 \times 10^6 \) mm\(^4\).

The purpose of testing this group of specimens was to compare the behaviour of a buttoned column to that of a similar batten column, as well as to determine the correct imaginary effective length factor, \( k_n \), for each case. Eight columns were designed, four batten columns and four analogous buttoned columns. The button interconnectors had dimensions of 86.0x69.9x9.53 mm so they just fitted in between the main members. The batten interconnectors were square plates with dimensions of 69.9x69.9x4.76 mm and were placed on either side of the channel section. To ensure buckling in a mode that causes shear forces
in the interconnectors, the specimens were pinned about the $Y$ axis giving an integral slenderness ratio of $\Lambda_y = 120$ and fixed about the $X$ axis giving and integral slenderness ratio of $\Lambda_x = 37.7$.

This group of columns fell into the range $\Lambda_y > 0.8 \Lambda_x$, and therefore to satisfy the requirements for the slenderness ratio of the main members between interconnectors for battened columns the number of interconnectors was increased from one to the number required to reduce this slenderness ratio to 40 or $0.6 \Lambda_y$, which was four. Although for buttoned columns it is only necessary to limit the slenderness ratio of the main members between interconnectors to that of the integral column, the same set of specimens were designed for comparison.

5.3 Apparatus

5.3.1 Load Cell

Two load cells were used during the experimental program, a 50 kip (224 kN) load cell and an 100 kip (448 kN) load cell. The former was used to measure the load applied to the specimens in Group I and the latter was used for Groups II and III. The load cells were calibrated twice, once using a strain indicator to obtain a calibration curve, and once electronically using the same MEGADAC which was used in the compression tests. The calibration curves are shown in Figs. 5.1(a) and (b).
5.3.2 Hydraulic Pump System

The compressive load was applied to the column specimens with a hydraulic jack controlled by a hydraulic pump. Both the ENERPAC jack and the OTC pump had a capacity of 10 ksi (68.9 MPa). Figure 5.2 shows a photograph of these items.

5.3.3 Displacement Measuring Instruments

When the experimental program was first designed it was planned to use mechanical devices to measure the deflection (i.e. dial gauges). However, the availability of electronic equipment to measure load, strain, and deflection led to the use of LVDT's and hence potentiometers. An LVDT is a linear variable-differential transformer, which is composed of a metal barrel out of which a metal piston protrudes, as shown in Fig. 5.3. When the piston moves it causes a change in potential which in turn is a measure of displacement. The deficiency of this instrument was that it only offered a stroke of ±38.1 mm (±1½ in.) which led to the interruption of loading of the specimen during a test to reset it. There were only two available LVDTs that had a stroke of ±76.2 mm (±3 in.). Furthermore, because it did not have a recoiling system it was necessary for the horizontal LVDTs to be attached to the column during testing, as shown in Fig. 5.4, which resulted in the bending of the piston with the bending of the specimen in two orthogonal directions. These discrepancies led to the use of potentiometers.
The potentiometer had a stroke of ±76.2 mm (±3 in.) in either direction. It was not necessary to attach them to the specimen as the piston had a recoiling system. The potentiometer was a much more sophisticated piece of displacement measuring apparatus and gave a more satisfactory performance than the LVDT. The potentiometers were held into position by dial gauge stands adapted to support them. Figure 5.5 illustrates the potentiometers during one of the tests. Prior to testing, the potentiometers were calibrated against a scale.

5.3.4 Strain Gauges

Electric resistance strain gauges were used to measure strains for the specimens of Groups I and II. The strain gauges were chosen with a resistance of 350 Ω in order to feed the data to the MEGADAC simultaneously with the load cell and potentiometers. The strain gauges had a gauge length of 5 mm. Two types of gauges were used, uniaxial gauges to measure the strain in the longitudinal direction and strain rosettes to measure the strain in three directions and hence determine the principal strains and stresses from the readings. A diagram of each of these gauges is shown in Figs. 5.6(a) and (b), respectively.

5.3.5 MEGADAC 3008

The MEGADAC 3008 is a low-cost data acquisition system which is ideal when accurate, high-speed data acquisition is essential. It was used to acquire data
from the load cell, potentiometers, and strain gauges during the tests.

5.3.6 Computer

An IBM Compatible 486 DX40 computer was used with an IEEE 488 interface card to receive data from the MEGADAC and convert it into a database using TCS (Test Control Software). This database was then converted into ASCII from which it could be imported to a spreadsheet program.

5.3.7 Testing System

The load was applied to the specimen by the piston of the hydraulic jack, which was controlled by the hydraulic pump. The load cell, through the MEGADAC, measured the applied load. The deflections and strains were measured by the potentiometers and strain gauges, respectively, and were compiled by the MEGADAC. All the data was sent to the computer and could be read off the screen during the test. After the test, the data was converted to ASCII. The diagrammatic representation of this system is shown in Fig. 5.7.

5.4 Set-Up

The specimens of Group I had a length greater than any of the testing machines in the Structural Laboratory at the University of Windsor, and so the tests were carried out on a horizontal testing frame. The set-up consisted of two end
brackets, manufactured in the Technical Support Centre at the University of Windsor, fixed to the table by four 25.4 mm (1 in.) bolts and also by welds. The end brackets were composed of a 406x406x38.1 mm base plate to which was welded a 406x305x50.8 mm vertical plate stiffened by two welded triangular stiffeners of thickness 25.4 mm. Figure 5.8 shows a 3-dimensional drawing of this end bracket.

The hydraulic jack was attached to the end bracket at one end of the set-up by four 19.1 mm (3/4 in.) bolts. The load cell was then connected to the jack through a screw-in cylinder. The end pieces were then attached to the load cell at one end and to the second end bracket at the other end, as shown in Figs. 5.9 and 5.10, respectively. These end pieces were 203x203x38.1 mm plates, which had a central 135° groove in one direction. Figure 5.11 shows this piece and Fig. 5.12 shows the whole set-up.

All these pieces were carefully aligned during assembly. The position of the second end bracket was adjusted twice to accommodate the different lengths of the specimens of Groups II and III.

5.5 Specimens

5.5.1 Fabrication

The specimens were fabricated in the Structural Laboratory and the
Technical Support Centre. The total lengths of the specimens from knife edge to knife edge was 3552, 2072, and 2233 mm for Groups I, II, and III, respectively. However, the channel sections themselves were cut 113 mm shorter than this and then milled an extra 10 mm to the correct length to ensure the ends were square. The remainder of the length of the specimens was made up by the end plates and knife edges.

For each specimen the two channel sections were clamped together so that the spacing between them was maintained by a spacer. At each end of the specimen a 127x127x12.7 mm end plate was adjusted and clamped so that the cross section was centred with the end plate as shown in Fig. 5.13. The end plate was then welded onto the channel sections by 5 mm welds applied all around the external perimeter of the cross section. For reasons of economy the end plate was split into two parts, the permanent welded part that has already been described and a second part that had a thickness of 25.4 mm (1 in.) and was fixed to the first by four 4.76 mm (3/16 in.) hex-socket cap screws. This enabled the latter to be disconnected and used for every specimen. The second part of the end plate was tapered near the edges of the outer side to prevent obstruction of rotation, and also had a 90° groove running centrically in one direction. In this groove a 25.4x25.4x127 mm square was placed for a knife edge. Details of this end plate and knife edge are shown in Fig. 5.14.
The interconnectors were then clamped to the specimens in preparation for welding. For Groups I and II, the interconnectors were welded all around, with a fillet welds, to the specimens as shown in Fig. 5.15. For Group III the interconnectors were welded along the longitudinal edges only as shown in Fig. 5.16. The finished specimens are shown in Figs. 5.17-5.19.

5.5.2 Measurement of Out-of-Straightness

The initial out-of-straightness was measured in at least two orthogonal directions. This was done by measuring the distance between a plastic wire, held tautly over the specimens by means of weights tied to each end of it, and to the specimen itself. This distance was measured by a vernier calliper with an accuracy of 0.02 mm. Figure 5.20 illustrates this procedure. An example of the initial out-of-straightness measured, for specimen 120-63-3-1/4, is shown in Fig. 5.21. The measured initial out-of-straightness varied from approximately $L/230\ 000$ to $L/900$.

5.5.3 Strain Gauges

Electric resistance strain gauges were used for Groups I and II to measure the strains in the midpoint interconnectors as well as in the main members at the midpoint. Uniaxial strain gauges were used to measure the axial strains in the main members, and strain rosettes were used to measure the principal strains in the interconnectors. A polished surface for each gauge was prepared and cleaned prior
to fixing the strain gauges to the specimens. The experimental scheme of strain
gauges for Groups I and II is shown in Fig. 5.22.

5.6 Tension Tests

Details and results of tension tests performed on coupons taken from the
channel sections, interconnectors, and end plates are shown in Appendix E. It is
interesting to note, however, that the material of the channel sections had a definite
yield point, whereas the material of the interconnectors and end plates did not.

5.7 Compression Tests

The compression tests were conducted for all three groups of specimens in
the Structural Laboratory on a horizontal test frame prepared for them. Before
each test the specimen was positioned in the set-up so that the expected mode of
buckling was in a horizontal plane. As some of the specimens buckled about the
$Y$ axis and others about the $X$ axis, the end pieces had to be rotated by 90° to
accommodate this. The specimen was aligned so that the knife edges corresponded
to the end pieces, and was supported by two stands at its ends.

For the specimens with one interconnector in Group I, five LVDTs were
used to measure the deflection of half the specimen in the plane of buckling and
one to measure the midpoint deflection in the orthogonal direction. The objective
of using five potentiometers was to detect any local buckling. This is shown in Fig. 5.23. For the rest of the specimens in Group I only two LVDTs or potentiometers were used, one to measure the midpoint deflection in either orthogonal direction, as shown in Fig. 5.24. For the specimen with one interconnector in Group II, which had a shorter length, five potentiometers were used to measure the deflection along the whole length of the specimen in the plane of buckling, and one potentiometer was used to measure the midpoint deflection in the other orthogonal direction, as shown in Fig. 5.25. For the rest of the specimens in Group II two potentiometers were used to measure the midpoint deflection in either orthogonal direction, as shown in Fig. 5.26. For all the specimens of Group III, five potentiometers were used to measure the deflections along the length of the member in the plane of buckling and one to measure the midpoint deflection in the orthogonal direction, as shown in Fig. 5.27.

After the potentiometers were set in their stands and adjusted, they were connected along with the strain gauges, where applicable, and the load cell to the MEGADAC. All of this equipment was balanced and zeroed before starting the test.

A small load was applied to begin with to hold the specimen firmly in place, then the two stands were removed. The load was then applied slowly in intervals
of approximately 10 kN, and then in much smaller intervals as the load approached the buckling load. If the deflection exceeded the capacity of the potentiometer or if the potentiometer needed repositioning, the loading was stopped and the potentiometer adjusted, then the loading recommenced. This adjustment in reading of deflection was corrected after the test. When the member had yielded or the load started to drop, the load was slowly removed, and the stands replaced under the specimens. Residual strains and deflections were noted.
CHAPTER VI

DISCUSSION OF RESULTS

6.1 General Discussion

The main objective of this research was to examine the possibility of eliminating the use of an excessive number of interconnectors for built-up columns. This required a study of the relevance of Clause 19.1.16 of the Canadian Standard CAN/CSA-S16.1-M89 (1989a), which limits the slenderness ratio of the main members between interconnectors. The evaluation of the local effective length factor, $k_n$, in the equivalent slenderness ratio formula was also required for battened as well as buttoned welded built-up columns. The experimental results were compared with the column design curve specified in CAN/CSA-S16.1-M89 (which is based on SSRC Column Curve 2). The design curves specified in CAN/CSA-S16.1-M89 and CAN/CSA-S16.1-94 (1994) are compared in Fig. 6.1(a). For CAN/CSA-G40.21-M92 (1992) 300W, the standard weldable steel used for most steel construction, the slenderness parameter $\lambda$ is limited to 2.46 since the standard limits the slenderness ratio to 200. The new compressive resistance equation is a
double exponential equation that replaces the five part polynomial equations in CAN/CSA-S16.1-M89. Figure 6.1(b) shows that the percent difference between these two curves does not exceed ± 2.6%.

6.2 Buckling Mode and Buckling Load of Specimens

6.2.1 Experimental Results

The buckling mode of the specimens were, in general, either X axis flexural buckling or Y axis flexural buckling. The experimental results of the test specimens will be discussed by group for convenience.

Group I:

The purpose of the research on the specimens in this group was to establish the interconnection requirements for X axis buckling of specimens that buckled elastically. For this reason the knife-edges were placed along the X axis giving an integral slenderness ratio of 120 about this axis and 60 about the Y axis. It seems that buckling should occur about the X axis. For the specimens with one or two interconnectors, however, the equivalent slenderness ratio about the Y axis, calculated using a local effective length factor of 1.0, exceeded the integral slenderness ratio about the X axis. This indicated that buckling should occur about the Y axis, which was in fact what was observed. The experimental buckling mode, buckling load, and slenderness ratios of these specimens are shown in Table
6.1. The number of interconnectors was increased from one to five, decreasing the slenderness ratio of the main members from centre-to-centre of the interconnectors from 165.8 to 54.9 (or for the clear distance between interconnectors from 161.5 to 48.6). The last of these specimens, which had five interconnectors (120-63-5), satisfies the requirements of Clause 19.1.16(a) which requires the local slenderness ratio between battens to be less than $0.7\Lambda_{X}$ (84) or 50 for $X$ axis buckling. The specimens with three, four, or five interconnectors, with the exception of specimens 120-63-3-3/16 and 120-63-3-1/4, buckled about the $X$ axis at approximately the same buckling load ($\approx 215.2$ kN). The two specimens mentioned above had slightly higher failure loads of 230.4 and 227.8 kN, respectively. This was attributed to the greater second moment of area of the specimen at the mid-section due to the increase in thickness of the interconnectors. However, the fact that all the specimens with 1/8 in. thick interconnectors failed at approximately the same load, which corresponds well with the Euler load for $X$ axis buckling as shown in Table 6.2, indicates that the restrictions in Clause 19.1.16(b) are unnecessary.

**Group II:**

The second group of specimens served the same purpose as the first but the buckling was inelastic. This group had the same knife-edge conditions as for Group I specimens. The integral slenderness ratio about the $X$ axis was 70 and about the $Y$ axis was 35. Again, it appeared that buckling should occur about the
$X$ axis. However, for the specimens with one and two interconnectors the equivalent slenderness ratio about the $Y$ axis, calculated using a local effective length factor of 1.0, was greater than the integral slenderness ratio about the $X$ axis. Hence, buckling about the $Y$ axis, as shown by the slenderness ratios in Table 6.1, was predominant. The number of interconnectors was increased from one to three, decreasing the slenderness ratio of the main members from centre-to-centre of the interconnectors from 91.4 to 45.7 (or for the clear distance between interconnectors from 85.1 to 39.4). The last of these specimens, which had three interconnectors (70-63-3), satisfies Clause 19.1.16(a). This clause requires that the local slenderness ratio between battens be less than $0.7\Lambda_x$ (49) or 50 for $X$ axis buckling. This specimen deflected mostly about the $Y$ axis at the beginning of the test, but failed about the $X$ axis. The compressive resistance was greater than predicted by S16.1-M89 having a ratio of test to predicted compressive resistance of 1.184 (1.212 for S16.1-94), as shown in Table 6.2. This result also indicates that the restrictions in Clause 19.1.16(b) are unnecessary. Figure 6.2 compares the experimental results of Groups I and II with the compressive resistance calculated in accordance with the requirements of S16.1 and the Euler load. In this figure $\Lambda_{eq}$ is calculated using a local effective length factor of $k_f=1.0$.

**Group III**

The purpose of the third group of specimens was to examine the difference
between battened columns and buttoned columns, and in particular to determine the local effective length factor, \( k_p \), in the equivalent slenderness ratio formula. For this reason, the knife-edges were positioned along the \( Y \) axis giving the specimens an integral slenderness ratio of 120 about this axis and 37.7 about the \( X \) axis. Hence, it is evident that failure should be due to flexural buckling about the \( Y \) axis which, as shown in Table 6.1, is what was observed. Note that \( A \) designates batten interconnectors and \( U \) designates button interconnectors. The number of interconnectors was increased from one to four, decreasing the slenderness ratio of the main members from centre-to-centre of the interconnectors from 99.0 to 39.6 (or for the clear distance between interconnectors from 92.1 to 32.7). The final pair of these specimens, with four interconnectors, satisfies the requirements of Clause 19.1.16(b), which requires the local slenderness ratio between battens to be less than \( 0.6A_y \) (72) or 40 for \( Y \) axis buckling.

From the experimental failure loads of these specimens, shown in Table 6.2, it can be seen that the battened and buttoned specimens with one interconnector had similar compressive resistance. The specimens with two and three interconnectors, however, showed a definite increase in strength in buttoned columns over that of battened columns. For the specimens with four interconnectors, there was an unexpected drop in failure load for both the battened and buttoned columns. The reason for this drop might be due to an increase in
residual welding stresses. For the sake of consistency, these results are not used further in the analysis.

6.2.2 Finite Element Results

The finite element results are presented in Table 6.3. The finite element model had a nominal initial out-of-straightness of \( L/1000 \) to enable the comparison of results. The measured out-of-straightness varied from approximately \( L/230 \ 000 \) to \( L/900 \) with a mean of \( L/4 \ 100 \). Residual stresses were not included in the finite element analysis because no attempt was made to measure them.

The finite element models buckled in identical modes to the test specimens, with the exception of the specimens with two interconnectors in Groups I and II. For specimen 120-63-2 in Group I using the nominal initial imperfection of 3.55 mm in both the \( X \) and \( Y \) directions caused buckling to occur about the \( X \) axis at a load of 184.3 kN. Whereas using an imperfection of 2.0 mm in the \( X \) direction and 0.2 mm in the \( Y \) direction did cause buckling about the \( Y \) axis at a load of 206.1 kN which compares well with the experimental failure load of 208.1 kN, and using an imperfection of 3.55 mm in the \( X \) direction and 0.2 mm in the \( Y \) direction also caused buckling about the \( Y \) axis at a load of 184.3 kN. This shows that the buckling mode of this specimen is highly sensitive to initial imperfections. Referring to Table 6.1, the slenderness ratio of this column is approximately equal about the \( X \) and \( Y \)
axes when using a factor of \( k_i = 1.0 \) in the equivalent slenderness ratio formula. It is interesting to note that using an initial imperfection of 0.2 mm in both the \( X \) and \( Y \) directions caused buckling about the \( X \) axis at a load of 218.7 kN. Again, for specimen 70-63-2 of Group II using an initial nominal imperfection of 2.07 mm in both the \( X \) and \( Y \) directions caused bending about both the \( X \) and \( Y \) axes, but failure was due to buckling about the \( X \) axis at a load of 391.4 kN, whereas using an initial imperfection of 0.2 mm in the \( X \) direction and 2.07 mm in the \( Y \) direction gave buckling about the \( Y \) axis at a failure load of 398.8 kN. Referring again to Table 6.1, the slenderness ratio of this column is approximately equal about the \( X \) and \( Y \) axes when using a factor of \( k_i = 1.0 \) in the equivalent slenderness ratio formula.

For Group I the finite element analysis predicted a mean ratio of test failure load to finite element failure load of 107.0±2.8%. For buckling about the \( Y \) axis the mean ratio of failure loads was 105.2±0.0%, whereas for buckling about the \( X \) axis, excluding specimen 120-63-2 which was imperfection sensitive, it was 106.5±1.8%. Figure 6.3 shows the finite element failure loads for the specimens in Group I. For the specimens which buckled about the \( X \) axis the increase in number of interconnectors had little effect on the load. For Group II the compressive resistance had a mean ratio of test failure load to finite element failure load of 109.1±9.95%. For the battened columns of Group III simulating the welds along the depth of the battens gave results that were too low. As these welds often
went around the corner of the batten, an attempt was made to account for this by constraining the outermost intermediate node on the perimeter of each interconnector. This gave results that were too high. As a final attempt to model the action of the welds, the outermost intermediate node on the perimeter of only the end connectors was constrained. The results of this model correlated best with the experimental results. For the buttoned columns of Group III, again modelling the welds which were only along the depth of the buttons gave results that were too low. Therefore, to simulate the action of friction between the button interconnector and the web of the channel sections, all the nodes of the interconnectors were constrained to the appropriate nodes of the channel sections. This improved the results and gave buckling loads close to the experimental failure loads. Using constraints only along the depth of the interconnectors showed that the buttoned columns were in fact stronger than the corresponding batten columns, however, using the constraints described above made the batten columns stronger than the buttoned columns as shown in Table 6.3. For Group III the batten columns had a mean ratio of test failure load to finite element failure load of 99.0±5.9%, whereas it was 113.3±13.4% for buttoned columns.

6.3 Load-Deflection Curves

6.3.1 Experimental Results

As mentioned in Chapter V, the first attempt to measure displacements was
made by the use of manual dial gauges. As the MEGADAC, data acquisition unit, became available an electronic method for measuring displacements became possible. A second attempt was made to measure the deflections using linear variable-differential transformers (LVDTs). LVDTs were used to measure the deflection of specimens 120-40-1, 120-63-1, 120-63-2, and 120-63-3-1/8. In general, the LVDTs gave satisfactory results with the exception of the midpoint LVDT in the direction of buckling. As the LVDTs did not have a recoil system, it was necessary to attach them to the column by screwing the end of the piston into a small plate which was bonded to the specimen. The problem with this method is that at the mid-section where the column deflects the most in both principal directions, the LVDT tends to bend permanently and to stick. This problem can be seen in the load-deflection curves of these four specimens.

For specimen 120-40-1 LVDTs were used to measure the deflection in the $X$ direction of half the column and the midpoint deflection in the $Y$ direction, as shown in Fig. 6.4. The results of these readings, plotted against the load are shown in Fig. 6.5(a). An initial out-of-straightness is included in the deflection readings. The deflections at the midpoint for buckling about the $X$ and $Y$ axes shows that $Y$ axis buckling governs as shown in Fig. 6.5(b). The sticking of the midpoint LVDT is apparent in these figures. Figure 6.6 shows the deflection of half of this specimen at loads 15.6, 80.1, and 101.5 kN. Local buckling was not
detected.

The same configuration of LVDTs was used for specimen 120-63-1 and the results are shown in Fig. 6.7(a). The deflections at the midpoint for buckling about the $X$ and $Y$ axes are shown in Fig. 6.7(b). Again the sticking of the LVDT is apparent. Figure 6.8 shows the deflection of half a specimen at loads 13.1, 54.9, and 109.5 kN.

For specimen 120-63-2 only midpoint deflections for buckling about the $X$ and $Y$ axes were measured and the results are shown in Fig. 6.9. In this case the faulty LVDT was placed so it measured the deflection in the $Y$ direction for buckling about the $X$ axis. Both bending about the $X$ and $Y$ axes occurred at first, but, as the equivalent slenderness ratio about the $Y$ axis was greater than the slenderness ratio about the $X$ axis ($\Lambda_{xy} > \Lambda_x$) buckling about the $Y$ axis was predominant giving a deflection of 17.3 mm as opposed to the 7.6 mm of $X$ axis buckling at the maximum load.

Figure 6.10 shows the results of the midpoint deflections for buckling about the $X$ and $Y$ axes for specimen 120-63-3-1/8. At the maximum load, it can be seen that the column failed by buckling about the $X$ axis at a deflection of 35.3 mm whereas the deflection for buckling about the $Y$ axis was only 3.0 mm.
The deflection of the remainder of the specimens in Group I and the specimens in Groups II and III were measured using potentiometers. The potentiometers were superior to the LVDTs. They had longer barrels (capable of measuring larger deflections) and a recoil system so that there was no need to attach them to the member.

The first specimen to be measured using the potentiometers was specimen 120-63-3-3/16. The results of the midpoint deflections are shown in Fig. 6.11, showing that buckling about the $X$ axis governs. The load-deflection curves for specimens 120-63-3-1/4, 120-63-4, and 120-63-5, shown in Figs. 6.12, 6.13, and 6.14, respectively, also show that $X$ axis buckling occurred.

Deflections were measured in the $X$ direction all along specimen 70-63-1 and at the midpoint in the $Y$ direction as illustrated in Fig. 6.15. The results of these measurements are plotted against the load in Fig. 6.16(a). Figure 6.16(b) shows the midpoint deflection for buckling about the $X$ and $Y$ axes indicating that buckling about the $Y$ axis occurred.

Figure 6.17 shows the midpoint deflections of specimens 70-63-2 indicating a definite buckling about the $Y$ axis with substantial yielding occurring. This verifies that the equivalent slenderness ratio about the $Y$ axis is greater than the
slenderness ratio about the $X$ axis ($\Lambda_{eq} > \Lambda_x$). Failure finally occurred due to the forming of a plastic hinge. The curves shown in Fig. 6.17 are of a repeated test made of this specimen after an adjustment was made to the set-up. The specimen was first loaded to a load of 375.7 kN at a deflection of 10 mm. Due to a failure in the set-up, the specimen was then unloaded and reloaded. The out-of-straightness after the first test was not measured.

Figure 6.18 shows the results of specimen 70-63-3. This is an interesting specimen as it appeared that buckling about the $Y$ axis would be the cause of failure. At the maximum load the deflection was 5.2 mm for buckling about the $X$ axis and 13.9 mm for buckling about the $Y$ axis. However, when yielding occurred, buckling about the $X$ axis was the critical buckling mode collapsing at a deflection of 35.5 mm as opposed to 18.4 mm for $Y$ axis buckling. This verifies that $\Lambda_{eq} < \Lambda_x$.

The specimens of Group III were tested using potentiometers to measure the deflections. The deflections were measured at the ends of the column, at the quarter points of the column and at the midpoint of the column for buckling about the $Y$ axis as shown in Fig. 6.19. It should be noted that all the specimens of Group III buckled elastically about the $Y$ axis. The midpoint deflection was also measured for buckling about the $X$ axis. Figure 6.20(a) shows the load-deflection
curves of specimen 120-A-1. It can be seen that the column appears to have moved into position at the left end at the beginning of loading. Figure 6.20(b) shows just the midpoint deflections. Figures 6.21(a) and (b) to 6.27(a) and (b) show similar results for the rest of the specimens of Group III. The load-deflection curves for the batten and buttoned columns with 1, 2, 3, and 4 interconnectors are shown in Figs. 6.28 and 6.29, respectively. With the exception of the specimens with four interconnectors, the compressive resistance increased with the increase of interconnectors.

6.3.2 Finite Element Results

To compare finite element results only load-deflection curves for models with nominal out-of-straightness were used. The load-deflection curves of the specimens in Group I with one and two interconnectors are shown in Fig. 6.30(a). It can be seen how the load increases significantly with the increase in number of interconnectors, whereas the maximum deflection decreases (i.e. the column becomes less flexible). Figure 6.30(b) shows the load-deflection curves for the specimens in Group I which buckled about the X axis. These specimens gave similar curves. Figure 6.31 shows the load-deflection curves for buckling about the X and Y axes for specimens 120-63-3-1/8, 120-63-4, and 120-63-5 showing the finite element results when only the number of interconnectors is increased without varying the thickness for buckling about the X axis. Figure 6.32 shows the load-
deflection curves for the specimens of Group II for buckling about both the $X$ and $Y$ axes. These short specimens gave little deflection but there was a significant increase in load as buckling switched from $Y$ axis to $X$ axis buckling. Figures 6.33(a) and (b) show the load-deflection curves of the battened and buttoned columns in Group III, respectively. Increasing the number of interconnectors to three increases the compressive resistance of the column but decreases its flexibility.

6.4 Nature of Strains in Midpoint Interconnectors and Main Members

6.4.1 Buckling About the $Y$ Axis

6.4.1.1 Strain Gauge Readings

All the specimens of Groups I and II with one and two interconnectors buckled about the $Y$ axis. Only the strains in the midpoint interconnector were measured. This means that only the strains were measured in the midpoint interconnector of the specimens with one interconnector. An electric resistance strain rosette was centrally attached to the midpoint interconnector connected to the member on the concave side (i.e. the compression side) of specimens 120-40-1, 120-63-1, and 70-63-1. The results of these strain readings, for specimens 120-63-1 and 70-63-1, are shown in Figs. 6.34(a) and (b), respectively. Theoretically there are no shear forces or bending moments in the mid-interconnector for $Y$ axis buckling. However, due to the shortening of the column under compressive
stresses there is some strain in this interconnector. It can be seen that strain gauge 1, which is parallel to the axis of the column, measures mainly compressive strains and strain gauge 3, which is perpendicular to the axis of the column, measures tension strains due to the effect of Poisson's ratio. Strain gauge 2, which is positioned at an angle of 45° to the axis of the column, measures a component of the compressive strain.

Due to local flexural buckling, as noted in the finite element analysis, the stresses in the flanges of the main member on the concave side of the column become tensile as the deflection increases. For specimen 120-63-1, the strain in the member on the concave side of the column becomes tensile at approximately the maximum load. This is illustrated in Fig. 6.35(a). Strain gauges 4 and 5 are positioned centrally, one on each main member, as close as possible to the midpoint interconnector as shown in the icon in this figure. As the member on the concave side goes into tension (gauge 4), the compression in the member on the convex side significantly increases (gauge 5) at approximately the maximum load. The reversal in direction of the axial strain in the member on the concave side of the column implies the occurrence of elongation (tension stresses) on one side of the interconnector. The longitudinal strains, E22, at the maximum load are shown in the contour plot of this interconnector in Fig. 6.36(a). This explains the reversal of strains in gauge 1 from compression to tension at approximately the maximum
load, shown in Fig. 6.35(a). Gauge 1 is more to the tension side of the interconnector, therefore, gauge 3 is on the compression side of the interconnector and so Poisson's effect is increased, increasing the tension strains notably at about the maximum load. These transverse strains in the perpendicular direction, E11, are shown in the contour plot in Fig. 3.36(b).

Figure 6.35(b) shows that the strains in gauges 6 and 7 of this specimen, which are placed on either side of the back of the channel on the concave side of the column at the midpoint, are approximately equal.

Specimen 70-63-1 did not show any significant reversal in strain in the member on the concave side of the column as shown by gauge 4 in Fig. 6.37(a). The reason for this is that the overall midpoint deflection of this member is about 8.6 mm, which is trivial compared to the 41.9 mm of specimen 120-63-1 at maximum load, shown in the load-deflection curves. Hence, there is little moment in the overall column due to the P-δ effect. On the other hand, the compression load in this specimen reached 344.4 kN, which is quite high compared to that of specimen 120-63-1 which reached 109.4 kN, so tensile stresses due to bending are much less significant. This result is also reflected in the readings of the strain rosette, shown in Fig. 6.35(b). Gauge 1 stays in compression, and gauge 3 shows no rapid increase in tensile strains. Again, Fig. 6.37(b) shows that the strains in
gauges 6 and 7, attached to the back of the channel on the concave side of the column, are approximately equal.

Figure 6.38(a) shows the results of the strain gauges positioned at the midpoint of the concave side of the column, one on each member, for specimen 120-63-2. It can be seen that the strain in the two members are approximately equal, but a little strain reversal occurs. The nature of these strains is due to the occurrence of simultaneous buckling about both the $X$ and $Y$ axes as shown in Fig. 6.9.

Specimen 70-63-2 also had a strain gauge fixed at the centre of the flange of each member at the midpoint of the concave side of the column, as shown in Fig. 6.38(b). These gauges clearly indicate the difference in strain in each member due to bending.

6.4.1.2 Principal Strains and Stresses

The principal strains and stresses were calculated using the formulation given in Appendix H for strain rosettes. Figures 6.39(a) and (b) show the principal strains and stresses in the midpoint interconnector of specimen 120-63-1, respectively. The minimum principal strain, $e_2$, reverses direction at about the same load as the flange of the member on the concave side of the column goes
into tension. The minimum principal stress, \( \sigma_2 \), reverses direction slightly earlier, but still at approximately the maximum load. The acute angle, \( \theta_p \), from the axis of the column (direction of gauge 1) to the direction of the first principal strain (or stress) of this specimen in the direction from gauge 1 to gauge 2 is shown in Fig. 6.39(c). It can be seen that after an initial perturbation the angle falls from an angle of 44.1° to an angle of -14.1°. Specimen 70-63-1, has values of principal strains and stresses as shown in Figs. 6.40(a) and (b), respectively. Figure 6.40(c) shows that the angle \( \theta_p \) is approximately -6.13\( \pm 0.63^\circ \) all the way through after an initial perturbation and end rise.

6.4.2 Buckling About the \( X \) Axis

6.4.2.1 Strain Gauge Readings

The results of the rosette readings for specimens 120-63-3-1/8, 120-63-3-3/16, 120-63-5, and 70-63-3, which had midpoint interconnectors and buckled about the \( X \) axis, are shown in Figs. 6.41(a)-(d). Specimen 120-63-3-1/4 had a faulty terminal and is not shown. For \( X \) axis buckling, strain due to the shortening of the column and also the global bending of the column is induced into the interconnector. The rosettes were attached to the concave side of the column. Therefore, strain gauge 1 measured compressive strains and strain gauge 3 measured tension strains due to Poisson's effect. Figure 6.42 shows the magnitude of strains in gauge 1 for specimens 120-63-3-1/8, 120-63-3-3/16, and 120-63-3-1/4.
The magnitude of the strain is not affected by the increase in thickness of the interconnector.

For the slender specimens with three interconnectors, the readings of gauges 4 and 5 (next to midpoint interconnector at the centre of each flange on the concave side of the column) shown in Figs. 6.43(a), 6.44(a), and 6.45(a), vary with difference in deflection at the midpoint for buckling about both the $X$ and $Y$ axes, shown in Figs. 6.10 to 6.12. For specimen 120-63-3-1/8, Fig. 6.43(a) shows that gauges 4 and 5 are approximately equal with no strain reversal due to the small deflection (4.8 mm) of buckling about the $Y$ axis in the $X$ direction. Gauges 6 and 7 (on the back of the channel on the concave side of the column) show a definite process of strain reversal for this specimen in Fig. 6.43(b). For specimen 120-63-3-3/16 due to the increased deflection in the $X$ direction (7.6 mm) with respect to that in the $Y$ direction (34.0 mm) at maximum load, as shown in Fig. 6.11, gauges 4 and 5 were not quite equal, as shown in Fig. 6.44(a). Strain reversal occurs and gauge 4 indicates that one member even went into tension. Figure 6.44(b) shows the effect of bending about the $X$ axis through gauges 6 and 7. The readings of gauges 4 and 5 of specimen 120-63-3-1/4 are approximately equal till the member on the concave side yields from bending about the $Y$ axis, as shown in Fig. 6.45(a). It can be seen in this figure that the strain gauges used can only measure up to 12 000 microstrains (1.2%). Figure 6.45(b) shows the strain reversal of
gauges 6 and 7 of this specimen due to bending about the X axis.

Specimens 120-63-4 and 120-63-5 also show signs of yielding of the member on the concave side in Figs. 6.46 and 6.47(a), respectively. Figure 6.47(b) shows strain reversal in gauges 6 and 7 of specimen 120-63-5.

Due to the high failure load of specimen 70-63-3, the strain gauges show no signs of strain reversal as shown in Figs. 6.48(a) and (b).

6.4.2.2 Principal Strains and Stresses

A typical example of the principal strains and stresses in the midpoint interconnector, on the concave side of the column, for the specimens that buckled about the X axis are shown in Figs. 6.49(a) and (b) for specimen 120-63-3-3/16. No strain or stress reversal occurred. The angle $\theta_p$, shown in Fig. 6.49(c), after an initial drop is constant at a value of $-9.91\pm0.35^\circ$ till approximately the maximum load where it drops to $-29.7^\circ$.

6.5 Valid Effective Length Factor for the Slenderness Ratio of Individual Members

6.5.1 Battened Columns

For Group I, the experimental failure loads correlate well with an equivalent
slenderness ratio calculated using an effective length factor, $k_i$, of 1.0 multiplied by the local slenderness ratio. For the specimens with one and two interconnectors, Table 6.2 shows that using a factor of 1.0 gives an average ratio of test to predicted failure load of 1.041 using the Euler buckling load and an average ratio of 1.169 using the S16.1-M89 buckling load (1.161 using the S16.1-94 buckling load). These are much better results than those obtained using a factor of $k_i=0.65$ (i.e. $k_i^2=0.42$) in the equivalent slenderness ratio formula as often mistook to be specified for batten columns by S16.1-M89, which gave average ratios of 0.533 and 0.689 (0.669), respectively. Table 6.4 shows that the exact effective length factor required is 0.976 for the Euler load and 0.889 for the S16.1-M89 load. Thus, both the experimental load and the failure mode confirm the use of a factor of 1.0 in the equivalent slenderness ratio. The results of Group I are shown in Fig. 6.50. Increasing the factor $k_i$ in the equivalent slenderness ratio formula from 0.65 to 1.0 helps bring the compressive resistance of the slender columns closer to the Euler load. It also ensures that the compressive resistance of the column is greater than that required by S16.1.

For Group II, the experimental failure load of the specimens with one and two interconnectors correlate well with the compressive resistance determined using an equivalent slenderness ratio calculated using an effective length factor, $k_i$, of 1.0 multiplied to the local slenderness ratio, being on the safe side. Table 6.2 shows
that specimen 70-63-1 has a ratio of test to predicted failure load of 1.037 using the Euler load, and specimen 70-63-2, which is in the intermediate range, has a ratio of 1.139 using the S16.1-M89 load (1.167 using the S16.1-94 load). Table 6.4 shows that the exact factor required is 0.980 for specimen 70-63-1 using the Euler load, and 0.754 for specimen 70-63-2 using the S16.1-M89 buckling load. It should be noted from Table 6.2 that the experimental failure loads of these two specimens correlate, but 
unconservatively, with the failure loads calculated using S16.1-M89 and a factor of 0.65 in the equivalent slenderness ratio formula; however, the failure mode predicted using this factor does not verify the experimental failure mode. Thus, both the experimental failure load and the failure mode confirm the use of a factor of 1.0 in the equivalent slenderness ratio formula. Fig. 6.50 again shows that using a factor of 1.0 ensures that the compressive resistance of the column meets the requirements of S16.1.

For Group III, the experimental failure loads of the specimens with one, two, and three interconnectors are compared with the theoretical failure loads calculated using an effective length factor, \( k_e \), of 1.0 and 0.65 in the equivalent slenderness ratio formula. The results of the battened columns indicate that the experimental failure loads correlate well with the theoretical Euler loads calculated using a factor of 1.0 in the equivalent slenderness ratio formula. Column (11) of Table 6.2 shows that the average ratio of test failure load to predicted Euler load
is 1.145 including specimen 120-A-1, and 1.065 excluding specimen 120-A-1, which is too conservative. Figures 6.51(a) and (b) also confirm these results. As the results were not clear in Fig. 6.51(a) a small portion of the column curve was plotted in Fig. 6.51(b). Referring to this figure, it can be seen that using a factor of 0.65 satisfies the requirements of S16.1-M89, but not the Euler load. Whereas using a factor of 1.0 brings the results very close to and well above the Euler load.

In Table 6.4 the factor required, in the equivalent slenderness ratio formula, for the battened columns is given calculated using both the compressive resistance calculated in accordance with the requirements of S16.1-M89 and the Euler load. For most of the specimens, the load predicted by S16.1-M89 is so conservative that an equivalent slenderness ratio formula is not applicable with the exception of the specimens with one interconnector, which required a factor of only 0.286. Using the Euler load the results obtained are more realistic, requiring an average factor for battened columns of 0.770 including specimen 120-A-1 and 0.830 excluding specimen 120-A-1. Of course if the results of the specimens with four interconnectors were included these values would change.

6.5.2 Buttoned Columns

In Table 6.2 the results of the buttoned columns indicate that the experimental failure loads best correlate with the predicted Euler loads calculated
using a factor of 0.65 in the equivalent slenderness ratio formula. Column (13) of Table 6.2 shows that the average ratio of test failure load to predicted Euler load is very accurate having a value of 1.015 when specimen 120-U-4 is excluded.

Again, in Table 6.4 the factor required in the equivalent slenderness ratio formula for the buttoned columns is given calculated using both the failure load of S16.1-M89 and the Euler load. Only for the specimen with one interconnector is a factor of 0.284 required when using the load predicted by S16.1. However, using the Euler load again gives results that are more realistic, requiring an average factor for buttoned columns of 0.587. Including the results of the specimen with four interconnectors changes this value. Figure 6.52 plots these results. Viewing the clearer plot in Fig. 6.52(b) shows that using a factor of 0.65 for buttoned columns satisfies the Euler column load (with the exception of specimen 120-U-4), and hence there is no need to increase this factor to 1.0.

A very good usage of the finite element analysis is to extend these results beyond that obtained in the experimental program.

6.6 Effect of Variations in Interconnectors

6.6.1 Effect of Number of Interconnectors

The number of interconnectors effects the compressive strength of a built-up
column only when buckling about the Y axis occurs. Theoretically this is taken into account by using the equivalent slenderness ratio formula, \( \Lambda_{eq} = \left( \Lambda_a^2 + (k/L_c)^2 \right)^{1/2} \), which contains the term \( a \), the centre-to-centre distance between interconnectors. The number of interconnectors does not affect the compressive strength of the column when buckling about the X axis occurs. This can be seen in Fig. 6.53 for Group I. The compressive strength of the specimens with one and two interconnectors, which buckled about the Y axis, significantly increased when the number of interconnectors was increased from one to two. There was a corresponding decrease of \( a \) from 1663 mm to 1109 mm in the equivalent slenderness ratio formula. Whereas the specimens with three to five interconnectors, which all buckled about the X axis, showed little variation in compressive resistance, all failing at about 220.8±7.1 kN. Group II also showed a significant increase in compressive resistance when the number of interconnectors was increased from one to two as shown in Fig. 6.54. Group III also indicated an increase in compressive resistance with an increase in the number of interconnectors for the same type of connector as shown in Fig. 6.55. However, it is to be noted that the compressive resistance dropped for the specimens with four interconnectors. This is probably caused by the residual stresses due to the heating and cooling process of welding a greater number of interconnectors onto the column.
6.6.2 Effect of Size of Interconnectors

The effect of the size of the interconnector can be studied in Group I for the specimens with one interconnector and three interconnectors.

The effect of increasing the depth of the interconnectors, for buckling about the $Y$ axis, is shown in Fig. 6.56 for specimens 120-40-1 and 120-63-1. The experimental failure load is seen to increase with the increase in the depth of the interconnector from 40 mm to 63.5 mm. The theoretical Euler failure load, calculated using Eq. (3.13) is also seen to increase with the increase of the depth of battens. It should be noted that the dimensional properties of the ends of these specimens are constant, the exterior end of the end connector being fixed at a distance of 20 mm from the end plate. Hence, decreasing the depth of the interconnectors (including end connectors) serves to: (1) increase the length of $a$, the centre-to-centre distance between interconnectors; and (2) weaken the resistance of the interconnector in flexure and shear. Therefore, if the equivalent slenderness ratio of the column is calculated according to the denominator of Eq. (3.13), assuming a perfectly rigid connection between interconnector and main member, the compressive resistance would increase with an increase in the depth of the batten. This is illustrated in Fig. 6.56. In this figure the experimental failure loads for specimens 120-40-1 and 120-63-1 are slightly greater than the Euler load but correlate quite well with it. It is reasonable to assume that increasing the thickness
of the interconnectors, for buckling about the \( Y \) axis, would have the same effect of increasing the compressive resistance of the column due to the increased rigidity and shear strength of the interconnectors.

The effect of increasing the thickness of the interconnectors, for buckling about the \( X \) axis, is illustrated in Fig. 6.57. The theoretical compressive resistance of the column is calculated by assuming the built-up column acts as a solid column when buckling occurs about the \( X \) axis. To account for the increase in thickness of interconnector the cross-sectional area and second moment of area are calculated using the area of the interconnector as a percent of the length of the column. In other words the cross-sectional area of the interconnector is multiplied by the ratio of the total length of the interconnectors along the column to the total length of the column. In Fig. 6.57 the experimental failure loads of specimens 120-63-3-1/8, 120-63-3-3/16, and 120-63-3-1/4 can be seen to vary around the Euler load verifying this assumption.

6.6.3 Effect of Type of Interconnectors

The effect of the type of interconnector can be determined from the results of Group III. Batten and button interconnectors were used. An increase in strength was noticed in the buttoned columns as shown in Fig. 6.58. This goes back to the greater flexural rigidity in buttoned interconnectors.
6.7 Interactive Buckling

As described in Chapter III, the effect of interactive buckling between local flexural buckling and global buckling is a decrease in the failure load due to imperfection sensitivity. When global buckling occurs, local buckling is taken as a column between consecutive interconnectors where sidesway is permitted to occur. In this way the local column has the choice of either buckling in a mode where sidesway is allowed or buckling in a mode where sidesway does not occur. Naturally, the mode where sidesway is permitted is the weaker mode, and referring to the deflection line obtained from the finite element results, this is the mode the local column chooses for the first panel at least. The effective length factor for local buckling of each specimen, $K_n$, can be calculated from the Julian and Lawrence nomographs or their characteristic equations (Chen and Lui; 1991), as shown in Fig. 6.59, for the cases of sidesway prevented and sidesway permitted. To account for the antisymmetric nature of the beam (interconnector) deflection, a modification factor of 3 is applied to the beam stiffness, for the case of sidesway prevented, to calculate the factors $G_A$ and $G_B$ needed to calculate the effective length factor similar to those derived by Chen and Lui (1991). Modifications for the end conditions of surrounding columns were made according to the recommendations made by Duan and Chen (1988a). The local effective length factors for the test specimens for the cases of sidesway prevented and sidesway permitted are listed in Tables 6.5(a) and (b), respectively. Most of the local
columns have an effective length factor close to 0.5 for sidesway prevented and close to 1.0 for sidesway permitted, implying that the interconnectors cause fixed-end conditions.

Figures 6.60(a)-(d) show the perfect and imperfect failure paths for the battened specimens of Group III, assuming a local column with sidesway prevented, with the experimental failure load also plotted on the same curve. Figures 6.61(a)-(d) show the perfect and imperfect failure paths for the battened specimens of Group III, assuming a local column with sidesway permitted, also with the experimental failure load plotted on the same curve. Both these figures show a good correlation between the experimental failure loads and the theoretical curves.

A curve similar to that derived by Thompson and Hunt (1973, 1974) was generated using Eqs. (3.29), (3.30), and (3.31), as shown in Fig. 6.62. It should be noted that the curves for the perfect and imperfect column are not dependent on the value of the local buckling load. Therefore, an arbitrary local buckling load was used. However, plotting the experimental results on this curve was definitely affected by the local buckling load. Hence, the exact local effective length factor for each specimen was used from Tables 6.5(a) and (b). The discrepancy of this curve in the region $P_L/P_G<1.0$ (or $2KLr/Kba<1.0$) is that the imperfect column
curve does not asymptotically reach the perfect column curve. Two reasons for this are proposed. The first is that for the ratio $P_l/P_G$ to be very small the column would have to be infinitely short. The second is that Eqs. (3.29), (3.30), and (3.31) were derived on the basis of the effect of local buckling on global buckling. So for the region $2KLr/K, ba<1.0$ where local buckling governs, it may be necessary to derive the effect of global buckling on local buckling. The experimental results of Groups I and II are not shown on these curves due to the irregular end conditions and buckling modes of these specimens. However, the results of the battened and buttoned specimens of Group III are shown in Figs. 6.62(a) and (b), for the cases of local sidesway is prevented and local sidesway is permitted, respectively. Both these cases show good correlation with the theoretical curves.
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

Based on the experimental and theoretical results it can be concluded that:

1- As buttoned columns have been found to be stronger than batten columns for buckling about the $Y$ axis, it is suggested that a factor of $k_r=0.65$ for welded button interconnectors and of $k_r=1.0$ for welded batten interconnectors be used in the equivalent slenderness ratio formula.

2- For batten columns, it is not necessary to use a combination of an equivalent slenderness ratio and limitations to the slenderness ratio of the main members between interconnectors. Either the concept of a reduced compressive strength is used taking precautions not to allow any premature failures in another mode, or the column is well interconnected so that it has the strength of an integral member.

3- A similar conclusion as that stated in 2 applies to buttoned columns.

4- When buckling about the $X$ axis governs, the slenderness ratio of the main
members between interconnectors does not affect the strength of the column.

5-Experimental and theoretical buckling loads of built-up columns made of rolled shapes show that the effects of coupled buckling can be accounted for by using the equivalent slenderness ratio concept.

7.2 Recommendations

It is recommended that the correct procedure for the design of built-up columns is to use enough interconnectors to reach the required compressive strength. As an additional precaution, the slenderness ratio of the individual main members between interconnectors should not be made equal to the governing integral slenderness ratio to avoid any possibility of coupled buckling. An appropriate algorithm to do this would be to:

1- Determine the compressive strength of the individual main members between interconnectors taking into consideration the possibility of flexural, torsional-flexural, and torsional buckling of the individual main members, allowing for the occurrence of sidesway between interconnectors.

2- Determine the compressive strength of the integral member about the $Y$ axis (the axis that involves deformations causing shear forces in the interconnectors) using the equivalent slenderness ratio formula for flexural buckling and torsional-flexural buckling of the integral member.

3- Determine the compressive strength of the integral member about the $X$ axis.
4- Care should be taken to make sure that buckling modes do not coincide simultaneously. However, erosion of optimum values of buckling loads is probably taken into account when the S16.1 curve is used to calculate the compressive strength, as well as when an equivalent slenderness ratio is used.

7.3 Future Work

1- To study the effect of the semi-rigidity of the connection between the interconnectors and the main members.

2- To study the shear stiffness of a built-up column.

3- To conduct more tests to establish a resistance factor using the procedure outlined by Kennedy and Gad Aly (1980).

4- Use the finite element analysis to include built-up members not included in the experimental program.
REFERENCES


ASCE. 1931. Steel column research. Second progress report of the special committee. Transactions, American Society of Civil Engineers, 95: 1152-1254.


Boca Raton, FL.


DDR-Standard. 1982. Stahlbau, Stabilität von Stahltragwerken, Grundlagen (Steel
structures, stability of steel supporting structures, fundamentals). TGL 13503/01 82, Amt für standardisierung, Meßwesen and Warenprüfung, Berlin.


Engesser, F. 1891. Die Knickfestigkeit gerader Stäbe (The buckling resistance of

Engesser, F. 1907. Zum Einsturz der Brücke über den St. Lorenzstrom bei Quebec (About the collapse of the bridge over the St. Lawrence River in Quebec).

Zentralblatt der Bauverwaltung, 27: 609.


Gioncu, V. 1990. Buckling of built up members. Proceedings of the International Colloquium on Stability of Steel Structures, Budapest, Hungary, Final Report,
pp. 41-60.


Haringx, J.A. 1948. On highly compressible helical springs and rubber rods, and their
application for vibration-free mountings, I. Philips Research Reports, 3(6): 401-449.


Koiter, W.T. 1945. Over de stabiliteit van het elastisch evenwicht (On the stability


Mohsin, M.E. 1965. The critical load of unsymmetrical welded battened struts. The


Ng, W.H. 1951. The behaviour and design of battened structural members. Welding Research, 5(3): 175r-188r.


Petermann. 1931. Knickversuche mit Rahmenstäben aus St. 48 (Buckling tests of built-up struts made of special steel St. 48). Der Bauingenieur, 12(28): 509-515.


PN. (1987). Konstrukcje Stalowe, Obliczenia Statyczne i Projektowaine (Steel structures, design rules). PN 80/B-03200 87, Polska norma, Warsaw, Poland.


SAA. 1981. SAA steel structures code. AS 1250 81, Standards Association of Australia, Australia.


Supple, W.J. 1967. Coupled branching configurations in the elastic buckling of symmetric structural systems. International Journal of Mechanical Sciences,


Temple, M.C., and Elmahdy, G. 1995. Local effective length factor in the equivalent


U.S.S.R. Standard. 1982. Stalnve konstrukcii (Steel Structures). SNiP II-23-81 82,


compressive strength. Materials and Member Behavior. Proceedings of the Sessions at Structures Congress '87, Structural Division of the American Society of Civil Engineers, pp. 199-212.


FIG. 1.1: Types of interconnectors
FIG. 1.2: Cross-sectional axes
FIG. 1.3: Buckling about the Y axis
FIG. 1.4: Buttoned cross section
FIG. 2.1: Decomposition of internal forces (Djukic and Atanackovic)
FIG. 2.2: Unbuckled and buckled configurations of Libove's model
Series A

Series B

Note: All dimensions are in mm

FIG. 2.3: Connection details of Zandonini's specimens
\[ \lambda_m = \frac{2I_y}{I_o} \]

\[ \gamma_r = \frac{\pi^2 EI_T}{PL^2} \]

\[ p = \frac{PL^2}{\pi^2 EI_T} \]

FIG. 2.4: Gjelvik’s form of solution to general equations
FIG. 2.5: Optimization as a generator of structural instability 
(Thompson and Hunt’s model)
FIG. 2.6: van der Neut's model
FIG. 2.7: Thompson and Hunt's reticulated column
FIG. 2.8: Sensitivity indices for the three failure domains of a reticulated column
FIG. 2.9: Sensitivity indices for the three failure domains of a thin-walled column
FIG. 2.10: Imperfection sensitivity of laced columns (Balut et al.)
FIG. 2.11: Coupling action of two instabilities
FIG. 3.1: Deformation that causes shear forces in the interconnectors
FIG. 3.2: Increased lateral deflection due to shear in battened columns
FIG. 3.3: Effect of shear
FIG. 3.4: Stetical system of a batten column in shear
(a) effect of shear on one panel ($m-n$)

(b) bending moment in one panel due to shear

FIG. 3.5: Statical system of one panel in shear
FIG. 3.6: Total lateral deflection of a panel due to shear
FIG. 3.7: Shear force in a batten
FIG. 3.8: Modes of post-buckling
FIG. 3.9: Interaction of buckling modes with local sidesway permitted
FIG. 3.10: Interaction of buckling modes with local sidesway prevented
FIG. 3.11: Buckling load of an ideal column

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FIG. 3.12: Interaction of buckling modes
FIG. 3.13: Details of Clause 19.1.16 of CAN/CSA-S16.1-M89
FIG. 3.14: Buckling of a built-up column about the X axis
FIG. 3.15: Theory of battened columns
FIG. 3.16: Schematic representation of shear forces in a battened column
The relative displacements between the ends of the batten interconnector are not shown.

\[
\frac{(Q_1 + Q_2)\alpha}{4} \quad \Rightarrow \quad \frac{(Q_1 + Q_2)\alpha}{2b} \quad \Rightarrow \quad \frac{(Q_1 + Q_2)\alpha}{2b}
\]

\[b\]

FIG. 3.17: Deformation of a batten interconnector
FIG. 3.18: Schematic representation of shear forces in a buttoned column
The rigid body rotation of the button interconnector under the effect of the moments is not shown.

FIG. 3.19: Deformation of a button interconnector
FIG. 4.2: Schematic representation of node numbering along length of channels
FIG. 4.3: Node numbering of bottom end plate and knife edge
FIG. 4.4: Node numbering of typical interconnectors
(a) 8-node reduced integration shell element

(b) Integration point

(c) S4R 4-node reduced integration shell element

FIG. 4.5: Node numbering of typical finite elements
FIG. 4.6: Finite element model of specimen 120-63-3-1/8
FIG. 4.7: Finite element model of specimen 70-63-1
FIG. 4.8: Finite element model of specimen 120-A-2
FIG. 4.9: Finite element model of specimen 120-U-2
Each degree of freedom at node $p$ is interpolated linearly from the corresponding degrees of freedom at nodes $a$ and $b$.

FIG. 4.10: Example of a TIE and LINEAR type MPC
FIG. 4.11: MPCs used to connect a typical interconnector to the main members in Groups I and II
Upper boundary conditions
\[ u_x^t = u_y^t = 0 \]
\[ u_z^t \neq 0 \]

Lower boundary conditions
\[ u_x^b = u_y^b = u_z^b = 0 \]

FIG. 4.12: Statical system of finite element model for Groups I and II
(a) The Newton–Raphson method

(b) The modified Newton–Raphson method

FIG. 4.13: Methods of iteration

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FIG. 4.14: Incremental loading using the Newton-Raphson method of iteration
(i) Rotation about the $X$ axis, $\theta_x$

(ii) Rotation about the $Y$ axis ($Y^i$ axis), $\theta_y$

(iii) Rotation about the $Z$ axis ($Z^i$ axis), $\theta_z$

FIG. 4.15: Model axis rotation
FIG. 4.19: Displaced configuration of specimen 70-63-3
FIG. 4.20: Displaced configuration of specimen 120-A-2
FIG. 4.21: Displaced configuration of specimen 120-U-2
FIG. 5.1: Calibration curves
FIG. 5.3: An example of a ±38.1 mm LVDT

FIG. 5.4: Horizontal LVDT attached to column
(a) Uniaxial strain gauge

(b) Strain rosette

FIG. 5.6: Strain gauges
FIG. 5.7: Diagrammatic representation of testing system
Dimensions are in mm.

FIG. 5.8: End bracket
FIG. 5.9: End piece attached to load cell

FIG. 5.10: End piece attached to bracket

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FIG. 5.11: End piece of knife edge

Dimensions are in mm.
FIG. 5.12: Set-up
Dimensions are in mm.

FIG. 5.13: Orientation of cross section with end plate
Dimensions are in mm.

FIG. 5.14: End plate and knife edge
Dimensions are in mm.

FIG. 5.15: Details of interconnector of Groups I and II

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FIG. 5.16: Details of interconnectors of Group III

Dimensions are in mm.

(a) batten interconnector
(b) button interconnector
120-40-1  120-63-1  120-63-2  120-63-3-\frac{1}{8}

Dimensions are in mm.

FIG. 5.17: Test specimens of Group I
120-63-3-$\frac{3}{16}$  120-63-3-$\frac{1}{2}$  120-63-4  120-63-5

Dimensions are in mm.

FIG. 5.17(cont.): Test specimens of Group I

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Dimensions are in mm.

FIG. 5.18: Test specimens of Group II

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Dimensions are in mm.

FIG. 5.19: Test specimens of Group III

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Dimensions are in mm.

FIG. 5.19(cont.): Test specimens of Group III
FIG. 5.20: Measurement of out-of-straightness
(a) Measured initial out-of-straightness

(b) Initial out-of-straightness after correction for deflection under its own weight

Fig. 5.21: Initial out-of-straightness of specimen 120-63-3-\(\frac{1}{2}\)
FIG. 5.22: Experimental scheme of strain gauges
FIG. 5.23: Position of LVDTs for specimen 120-63-1

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FIG. 5.24: Position of potentiometers for specimen 120-63-3-1/8
FIG. 5.25: Position of potentiometers for specimen 70–63–1

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FIG. 5.26: Position of potentiometers for specimen 70-63-2

FIG. 5.27: Position of potentiometers for specimen 120-A-3

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FIG. 6.1: Comparison and difference in new design curve
FIG. 6.2: Column curves of Groups I & II for buckling about the X & Y axes
FIG. 6.3: Finite element results of Group I using nominal imperfections

out-of-straightness = \( L/1000 \)

- Y axis buckling
- X axis buckling

\( P \) (kN)

<table>
<thead>
<tr>
<th>no. of interconnectors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>4</th>
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FIG. 6.4: Position of LVDTs for specimen 120-40-1
FIG. 6.5: Experimental load-deflection curves: specimen 120-40-1
FIG. 6.7: Experimental load-deflection curves: specimen 120-63-1
FIG. 6.10: Experimental load-deflection curves: specimen 120-63-3-1/8
FIG. 6.12: Experimental load-deflection curves: specimen 120-63-3-1/4
FIG. 6.14: Experimental load-deflection curves: specimen 120-63-5
FIG. 6.16: Experimental load-deflection curves: specimen 70-63-1
FIG. 6.18: Experimental load-deflection curves: specimen 70-63-3
FIG. 6.20: Experimental load-deflection curves: specimen 120-A-1
FIG. 6.21: Experimental load-deflection curves: specimen 120-U-1
FIG. 6.23: Experimental load-deflection curves: specimen 120-U-2
FIG. 6.24: Experimental load-deflection curves: specimen 120-A-3
FIG. 6.25: Experimental load-deflection curves: specimen 120-U-3
FIG. 6.26: Experimental load-deflection curves: specimen 120-A-4
FIG. 6.27: Experimental load-deflection curves: specimen 120-U-4
FIG. 6.28: Load versus midpoint deflection - batten columns of Group III
FIG. 6.29: Load versus midpoint deflection - buttoned columns of Group III
FIG. 6.30: Load-deflection curves - finite element results of Group I
Specimens with 1/8" thick interconn.

FIG. 6.31: Load-deflection curves - finite element results of specimens of Group I with 3, 4, and 5 interconnectors
FIG. 6.32: Load-deflection curves - finite element results of Group II
(a) Battened specimens

(b) Buttoned specimens

FIG. 6.33: Load-deflection curves - finite element results of Group III
FIG. 6.34: Load-strain curves of strain rosettes for buckling about the Y axis
FIG. 6.35: Load-strain curves of experimental results of specimen 120-63-1
CONTOUR PLOT

(a) longitudinal strain, E22

(b) transverse strain, E11

FIG. 6.36: Contour plots of the strains around the midpoint interconnector of specimen 120-63-1
FIG. 6.37: Load-strain curves of experimental results of specimen 70-63-1
FIG. 6.38: Load-strain curves for buckling about the Y axis
FIG. 6.39: Load versus principal strain, principal stress, and $\theta_p$ for specimen 120-63-1
FIG. 6.40: Load versus principal strain, principal stress, and $\theta_p$
for specimen 70-63-1

267
FIG. 6.41: Load-strain curves of strain rosettes for buckling about the X axis
FIG. 6.43: Load-strain curves of experimental results of specimen 120-63-3-1/8

270
FIG. 6.44: Load-strain curves of experimental results of specimen 120-63-3-3/16
FIG. 6.45: Load-strain curves of experimental results of specimen 120-63-3-1/4

272
FIG. 6.46: Load-strain curves of experimental results of specimen 120-63-4
FIG. 6.47: Load-strain curves of experimental results of specimen 120-63-5
FIG. 6.48: Load-strain curves of experimental results of specimen 70-63-3
FIG. 6.49: Load versus principal strain, principal stress, and $\theta_p$
for specimen 120-63-3-3/16
FIG. 6.50: Column curves of Groups I & II for buckling about the Y axis
FIG. 6.51: Column curves of battened columns of Group III
FIG. 6.52: Column curves of buttoned columns of Group III
FIG. 6.53: Effect of number of interconnectors for Group I
FIG. 6.54: Effect of number of interconnectors for Group II
FIG. 6.55: Effect of number of interconnectors for Group III
FIG. 6.56: Effect of size of interconnectors for specimens with one interconnector
FIG. 6.57: Effect of thickness of interconnectors for specimens with three interconnectors
FIG. 6.58: Effect of type of interconnectors for Group III

285
(a) sidesway prevented

(b) sidesway permitted

FIG. 6.59: Subassemblage models for Julian and Lawrence nomographs
FIG. 6.60: Interactive buckling for the batted specimens of Group III with local sideways not permitted
FIG. 6.61: Interactive buckling for the battened specimens of Group III with local sideway permitted
(a) local effective length factors for the case of no sidesway were used to plot the experimental results

(b) local effective length factors for the case of sidesway permitted were used to plot the experimental results

FIG. 6.62: Interactive buckling - results of Group III
Table 6.1: Experimental buckling load, buckling mode, and slenderness ratios of the specimens of Groups I, II, and III.

<table>
<thead>
<tr>
<th>Specimen Number (1)</th>
<th>Buckling Mode</th>
<th>$P_{cr}$ (kN) (3)</th>
<th>$E$ (10$^3$ MPa) (4)</th>
<th>$F_y$ (MPa) (5)</th>
<th>$L$ (mm) (6)</th>
<th>$d$ (mm) (7)</th>
<th>Slenderness Ratios</th>
<th>Slenderness Ratios</th>
<th>Slenderness Ratios</th>
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<td>$Y$ axis</td>
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<td>2233</td>
<td>400</td>
<td>37.7</td>
<td>120</td>
<td>39.6</td>
</tr>
</tbody>
</table>

NOTES:

1. $a$ is taken as the centre-to-centre distance between interconnectors.
2. The specimen number is designated by the integral slenderness ratio about the $X$ axis, the depth of the interconnector (mm), and the number of interconnectors. If no extension is added to the specimen number the thickness of the interconnector is 1/8 inches. Otherwise the thickness of the interconnector is as given in inches. The batten interconnector has a width of 63.5 mm (2.5 in.).
3. The specimen number is designated by the integral slenderness ratio about the $Y$ axis, A for batten column and U for buttoned columns, and the number of interconnectors.
4. The batten interconnector has a dimension of 69.9 x 69.9 x 4.76 mm (2.75 x 2.75 x 1/16 in.).
5. The button interconnector has a dimension of 69.9 x 86.0 x 9.53 mm (2.75 x 3.38 x 3/8 in.).
Table 6.2: Experimental and predicted failure loads of the specimens of Groups I, II, and III.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>GROUP I</th>
<th>GROUP II</th>
<th>GROUP III</th>
</tr>
</thead>
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<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<td>(p_{max} = 101.5)</td>
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<td>(p_{max} = 101.3)</td>
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<td>(p_{max} = 208.1)</td>
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<td>S16.1</td>
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<td>S16.1</td>
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<td>S16.1</td>
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<td>(p_{max} = 208.1)</td>
<td>S16.1</td>
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</tbody>
</table>

Notes:
- \(p_{max}\) represents the predicted failure load.
- S16.1 indicates the section properties used in the analysis.
- Euler indicates the Euler buckling load.
- The table entries are rounded to two decimal places.
Table 6.3: Experimental and finite element results.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>$P_{	ext{ext}}$ (kN)</th>
<th>Experimental Midpoint Deflections (mm)</th>
<th>F.E. Load (kN)</th>
<th>Initial F.E. Midpoint Out-of-Straightness (mm)</th>
<th>Final F.E. Midpoint Deflections (mm)</th>
<th>Ratio of Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
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<td>$Y$ axis buckling</td>
<td>$X$ axis buckling</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
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<td>96.6</td>
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Nominal out-of-straightness ($L/1000$)
Table 6.4: Local effective length factor in equivalent slenderness ratio formula.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Buckling Mode</th>
<th>$P_{cr}$ (kN)</th>
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<th>$k_i$ Euler</th>
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Table 6.5(a): Local effective length factors according to Julian and Lawrence nomograph for the case of sidesway prevented.

<table>
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<th>Specimen Number</th>
<th>$l'$ (mm)</th>
<th>$a$ (mm)</th>
<th>$b$ (mm)</th>
<th>$l_e$ (mm)</th>
<th>$I_0$ (mm$^4$)</th>
<th>$G_a$</th>
<th>$G_b$</th>
<th>$G_a$</th>
<th>$G_b$</th>
<th>$K_f$</th>
<th>$K_i$</th>
</tr>
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<tbody>
<tr>
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<td>55.8</td>
<td>77</td>
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<td>N/A</td>
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<td>N/A</td>
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<td>N/A</td>
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<td>0.510</td>
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</tr>
<tr>
<td>120-60-3-1/8</td>
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<td>77</td>
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<td>0.017</td>
<td>0.537</td>
<td>0.509</td>
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<td>0.013</td>
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<tr>
<td>70-63-1</td>
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<td>55.8</td>
<td>77</td>
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<td>0.023</td>
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<td>N/A</td>
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<td>70-63-3</td>
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<td>31.3</td>
<td>77</td>
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<td>N/A</td>
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<td>120-U-1</td>
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<tr>
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<tr>
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Table 6.5(b): Local effective length factors according to Julian and Lawrence nomograph for the case of sidesway permitted.

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<tr>
<th>Specimen Number</th>
<th>$l^*$ (mm)</th>
<th>$a$ (mm)</th>
<th>$b$ (mm)</th>
<th>$I_T$ (mm$^4$)</th>
<th>$I_b$ (mm$^4$)</th>
<th>$G_A$</th>
<th>$G_B$</th>
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<th>$G_B$</th>
<th>$K_I$</th>
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<tbody>
<tr>
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<td>0.002</td>
<td>0.002</td>
<td>1.001</td>
<td>1.001</td>
</tr>
</tbody>
</table>

Diagram:

```
\begin{figure}
\centering
\begin{tikzpicture}[scale=0.5]
  \draw[->,thick] (0,0) -- (0,5) node[right]{$l_e$};
  \draw[->,thick] (0,0) -- (5,0) node[above]{$b$};
  \draw[->,thick] (0,0) -- (0,10) node[below]{$l_b$};
  \draw[->,thick] (0,0) -- (10,0) node[right]{$l_T$};
  \draw[->,thick] (0,0) -- (0,20) node[above]{$l_{end\ panel}$};
  \draw[->,thick] (0,0) -- (20,0) node[below]{$l_{second\ of\ intermediate\ panel}$};
\end{tikzpicture}
\caption{Diagram of the specimen configuration.}
\end{figure}
```

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APPENDIX A

INTERNATIONAL CODES

This appendix presents a comparison of the requirements of International Standards, Codes, and Specifications regarding the design of built-up battened columns, as outlined by Beedle (1991), with additions from later editions of these codes where available. The nomenclature has been standardized wherever possible to reduce unnecessary definitions.

A.1 Column Strength of Battened Members

<table>
<thead>
<tr>
<th>COUNTRY AND CODE NO.</th>
<th>SPECIFICATION</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AUSTRALIA:</strong> AS 1250 81</td>
<td>Battened members or connections, provided to columns composed of two or more main components which are parallel to one another, are designed to act as a single integral member with an effective or equivalent slenderness ratio: [ \Lambda_{eq} = (\Lambda_x^2 + \Lambda_y^2)^{1/2} ]</td>
<td>where: [ \Lambda_x = \text{slenderness ratio of the integral member}; ] [ \Lambda_y = \text{slenderness ratio of a main component between adjacent connections} ] ( (a/h) ). See Fig. A.1(a).</td>
</tr>
<tr>
<td><strong>CHINA:</strong> GBJ 17 88</td>
<td>(a) The slenderness ratio about the solid-web axis of built-up members shall be computed on the same basis as prescribed for solid compression members ( (X \text{ axis in Fig. A.1(b)}) ). (b) The slenderness ratio about the open-web axis of built-up members shall be computed by using an equivalent slenderness ratio as follows: For members comprised of two longitudinal components as shown in Fig. A.1(b): [ \Lambda_{eq} = (\Lambda_x^2 + \Lambda_y^2)^{1/2} ]</td>
<td>The smallest radius of gyration of the individual components should be used to calculate ( \Lambda_x ). The formula for the equivalent slenderness ratio is used to evaluate the reduction in column capacity due to shear deformation in built-up members.</td>
</tr>
</tbody>
</table>
| **EAST EUROPE:** | The equivalent slenderness ratio shall be taken as:
| **ČSN 73 1401 84** | \( \Lambda_{eq} = (1^3 + \Lambda_{pm}^3)^{1/3} \)
| (Czechoslovakia) | For columns with batten plates the slenderness ratio \( \Lambda_{pm} \) is:
| | for \( n_p = f_p/(f_p,0) > 0.2 \) \( \Lambda_{pm} = a/r_p (0.82(1+n_p))^{1/2} \)
| | for \( n_p \leq a/r_p \) \( \Lambda_{pm} = a/r_p \)
| **PN 80/B-03200 87** | The formulas for the calculation of the equivalent slenderness can be adapted to the general form:
| (Poland) | \( \Lambda_{eq} = (\Lambda_x^3 + \Lambda_{pm}^3)^{1/3} \)
| | The corresponding slenderness \( \Lambda_{eq} \) shall be determined from:
| | \( \Lambda_{eq} = a/r_p \leq 0.5 \Lambda_x \) or \( \Lambda_{eq} \leq 50 \)
| | when the limitation \( a/r_p \leq 50 \) is used.
| **SNI P II-23-81 82** | The following rule is given for built-up members in compression made up of two, three, or four parallel chords interconnected by batten plates placed into two, three (triangle section), or four (box section) planes as shown in Fig. A.1(c):
| (U.S.S.R) | (a) Slenderness ratios about the solid-web axis of the built-up member shall be determined on the same basis as prescribed for solid compression members.
| | (b) The general format of the design criterion for the buckling of built-up compression members in their open-web planes is the same as for solid members, except for the determination of buckling coefficient where the equivalent slenderness ratio \( \Lambda_{eq} \) shall be used. The same rules as given in ČSN 73 1401 84 apply.
| **TGL 13503/01 82** | The equivalent slenderness ratio of built-up members regarding buckling in the open-web planes may be approximated by the formula:
| (East Germany) | \( \Lambda_{eq} = (\Lambda_x^3 + (m_s/2)\Lambda_{pm}^3)^{1/3} \)
| | where for batten plates or separators:
| | \( \Lambda_{pm} = a/r_p \leq 0.5 \Lambda_x \) or \( \Lambda_{pm} \leq 50 \)
| | \( r_p \) the smallest radius of gyration of components.
| | \( \Lambda_x \) slenderness ratio of the built-up member about the solid-web axis.
| | \( \Lambda_{eq} \) the sum of the second moments of area of the batten plates on a certain level about their strong centroidal axis. See Fig. A.1(b) for rest of nomenclature.
| | For built-up members composed of single angle components more detailed design provision are available in the complete specification.

where:
- \( m_s \) number of components or groups of components connected by batten plates (i.e. \( m_s = 2 \) for the arrangement in Fig. A.1(a)).
- \( r_p \) the smallest radius of gyration of components.
- \( \Lambda_x \) slenderness ratio of the built-up member about the solid-web axis.
| JAPAN:   | (a) Slenderness ratios about the solid-web axis of built-up members shall be calculated on the same basis as prescribed for compression members composed of a single element.  
(b) Calculations with regard to buckling of built-up compression members in their open-web planes shall be computed by using an equivalent slenderness ratio as follows:  
\[ \Lambda_w = (\Lambda_0^2 + (m/2)\Lambda_o^2)^{1/2} \] 
or, if \( \Lambda_o \leq 20 \) then:  
\[ \Lambda_{eq} = \Lambda_o \]  
The value of \( \Lambda_i \) shall be calculated for built-up members connected with tie plate separators using the formula  
\[ \Lambda_i = a/r, \]  
where:  
\( r \) = the smallest radius or gyration of components. |
| NORTH AMERICA: | Allowable stresses are used for built-up members. No reduction in allowable axial stress is specified to allow for reduced column strength by shear deformation in members with lacing and perforated cover plates.  
AISC-ASD 89 | Allowable stresses are used for built-up members. No reduction in allowable axial stress is specified to allow for reduced column strength by shear deformation in built-up members.  
For compression members composed of two or more rolled shapes separated by intermittent fillers, at least two intermediate connectors shall be used along the length of the built-up member.  
The design strength of built-up members composed of two or more shapes is determined in accordance with the AISC-LRFD 86 formulas for columns, including the flexural-torsional effects if present, and subject to the following modification. If the buckling mode involves a relative deformation that produces shear deformation forces in the connectors between individual shapes, the usual slenderness ratio, \( \Lambda_o \), for compression members must be replaced by a modified slenderness ratio \( \Lambda_{eq} \) in determining the compressive strength of built-up members as follows:  
for snug tight bolted connections:  
\[ \Lambda_w = (\Lambda_0^2 + \Lambda_o^2)^{1/2} \]  
for welded connectors and for fully tightened bolted connectors as required for slip-critical joints:  
with \( \Lambda_o > 50 \)  
\[ \Lambda_w = (\Lambda_0^2 + (\Lambda_o - 50)^2)^{1/2} \]  
with \( \Lambda_o \leq 50 \)  
\[ \Lambda_w = \Lambda_o \]  
Permitted types of steel have yield points ranging from 248 to 690 MPa for AASHTO and AREA bridge specifications and AISC-ASD.  
N.B. Built-up columns connected by batten plates are not specified by the AASHTO and AREA bridge specifications as well as the AISC-ASD and AISC-LRFD building specifications.  
This formula is the same as that used in the Italian code as well as in other European specifications.  
where:  
\( \Lambda_i = a/r \) is calculated using the minimum radius of gyration of individual component. |
| AISC-LRFD 93 | The design strength of built-up members composed of two or more shapes is determined in accordance with the AISC-LRFD 93 formulas for columns, including the flexural-torsional effects if present, and subject to the following modification. If the buckling mode involves a relative deformation that produces shear deformation forces in the connectors between individual shapes, the usual slenderness ratio, \( \Lambda \), for compression members must be replaced by a modified slenderness ratio \( \Lambda_{eq} \) in determining the compressive strength of built-up members as follows:

(a) for intermediate connectors that are snug-tight bolted:
\[
\Lambda_{eq} = (\Lambda_1^2 + \Lambda_2^2)^{1/2}
\]
(b) for intermediate connectors that are welded or fully-tensioned bolted:
\[
\Lambda_{eq} = (\Lambda_1^2 + 0.82(\alpha_2^2/(1+\alpha_2^2))\Lambda_2^2)^{1/2}
\]

where:
\( \Lambda \) = largest column slenderness of individual components.
\( \Lambda_{eq} \) = modified slenderness ratio.
\( \alpha_2 \) = separation ratio = \( b/2r \) for \( b \) see Fig. A.1(a).

| AREA 87 | Allowable compressive stresses are used. No reduction in allowable axial stress is specified to allow for reduced column strength by shear deformation in built-up members.

| CSA-S6-M89 | Members with battens are permitted. Battened members are allowed only when the member does not carry calculated bending moments in the plane of the batten.

| CSA-S16.1-M89 | For compression members composed of two or more rolled shapes in contact or separated from one another by intermittent fillers, the equivalent slenderness ratio shall be taken as:
\[
\Lambda_{eq} = (\Lambda_1^2 + (k_n^2 \Lambda_2^2))^{1/2}
\]
where \( k \Lambda_1 \) is the maximum slenderness ratio of component part of a built-up member between fasteners based on an effective length factor, \( k_n \), of 1.0 when the interconnectors are fastened with snug-tight bolts and 0.65 when fastened with welds or pretensioned bolts.

| CSA-S16.1-94 | The compressive resistance of the built-up member shall be based on:
(a) The slenderness ratio of the built-up member with respect to the appropriate axis when the buckling mode does not involve relative deformation that produces shear
in the interconnectors.

(b) An equivalent slenderness ratio, with respect to the axis orthogonal to that in (a), when the buckling mode involves relative deformation that produces shear forces in the interconnectors, taken as:
\[ \Lambda_z = (\Lambda_z^2 + (k_i \Lambda_i)^2)^{1/2} \]
where \( k_i \Lambda_i \) is the maximum slenderness ratio of component part of a built-up member between interconnectors.

(c) For built-up members composed of two interconnected rolled shapes, in contact or separated only by filler plates, such as back-to-back angles or channels, the maximum slenderness ratio of component parts between fasteners or welds shall be based on an effective length factor, \( k_i \), of 1.0 when the fasteners are snug-tight bolts and 0.65 when welds or pretensioned bolts are used.

(d) For built-up members composed of two interconnected rolled shapes separated by lacing or batten plates, the maximum slenderness ratio of component parts between fasteners or welds shall be based on an effective length factor of 1.0 for both snug-tight and pretensioned bolts and for welds.

**WEST EUROPE:**

**EC 3 88**

Built-up compression members consisting of two or more main components connected together at intervals to form a single integral member are designed incorporating an equivalent geometric imperfection comprising an initial bow \( (u_a) \) of not less than \( L/500 \), where \( L \) is the length of the member.

The deformation of the built-up member is taken into account in determining the internal forces and moments in the main components, internal connections, and any subsidiary components such as battens. The design of the main and subsidiary components shall be checked using the methods given for struts.

In addition to the axial forces, allowance must also be made for any other forces or moments applied to the members such as the effects of its self-weight or wind loading acting on the member.

Only one West European code (namely EC 3 88) is mentioned because, from 1992 on, codes of the twelve member countries will be, in practice, replaced by EC 3 88.
**Battened Members:**
The design procedure is for a compressive force \( P \) applied to a built-up member consisting of two similar parallel chords of uniform cross section, spaced apart and interconnected by means of battens, which are rigidly connected to the chords and uniformly spaced throughout the length of the member. Battens shall be provided at each end of the member. Battens shall also be provided at intermediate points where load or lateral restraint is applied. Intermediate battens should divide the length of the member into at least 3 panels and there should be at least 3 panels between points which are laterally restrained in the plane of the battens. In general, intermediate battens should be spaced and proportioned uniformly throughout the length of the member. Where parallel planes of battens are supplied, the battens in each plane should be arranged opposite one another. When \( S_n \), the shear stiffness, is evaluated by disregarding the flexibility of the batten plates, the depth of an end batten along the length of the member should not be less than \( b \), the distance between the centroids of the chords. The depth of an intermediate batten should not be less than \( 0.5b \).

Unless the flexibility of the batten plates is explicitly taken into account in the evaluation of \( S_n \), the battens should also satisfy:

\[
\frac{I}{b} \geq 10I/a
\]

**A) Second Moment of Area:**
The effective second moment of area \( I \) of a battened compression member with two main components should be taken as:

\[
I = 0.5b^2A_r + 2\mu I_r
\]

with \( \mu \) obtained from the following:

- \( \Lambda_r \leq 75 : \quad \mu = 1 \)
- \( 75 < \Lambda_r < 150 : \quad \mu = 2 - \Lambda_r/75 \)
- \( \Lambda_r \geq 150 : \quad \mu = 0 \)

where:

\[
\Lambda_r = \frac{L}{r_y}
\]

\[
r_y = (0.5I_r/A)\frac{1}{2}
\]

\[
I_r = \text{value of } I \text{ with } \mu = 1.
\]

The chords may be solid members or may themselves be laced or battened in the perpendicular plane.

Where:

\( I_b = \) sum of second moments of area of the battens on one level.

\( I_r = \) second moment of area of one chord.

\( \mu \) is a plasticity coefficient

(Uhlmann and Ramm; 1982). For solid members where the flanges are continuously interconnected by the web, higher load levels can be reached in the elastic-plastic range. Whereas, for built-up members it is not important for short members because the full member does not display any significant plastic deformation before exceeding the critical design load. Slender built-up struts exhibit a complex...
(B) Chord Forces at Mid-Length should be determined from:

\[ P = 0.5(P + M_d b A / l) \]

where:

\[ M_d = P u_y / (1 - P / P_c - P / S_y) \]

\[ P_c = \pi^2 E I / L^2 \]

\[ u_y = l / r_y \]

The design shear stiffness is given by:

\[ S_y = 2 \pi^2 E I / a^2 \]

provided that \( I / b \geq 10 l / a \)

When this criterion is not satisfied, the flexibility of the batten plates should be taken into account by obtaining \( S_y \) from:

\[ S_y = 24 E I / (a^2(1 + 2 l / l_0 / a)) \text{ but } \geq 2 \pi^2 E I / a^2 \]

(C) Buckling Resistance of Chords

The buckling length of a chord in the plane of battens should be taken as the system length between centre lines of battens.

interaction between the influences of the full member and its single parts on the overall behaviour. Both components are of similar dimensions in this case so that plastic deformations can occur at comparatively low load levels.

A.2 Limit Values for Local Slenderness of Battened Columns

<table>
<thead>
<tr>
<th>COUNTRY AND CODE NO.</th>
<th>SPECIFICATION</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSTRALIA: AS 1250 81</td>
<td>( \lambda_i \leq 0.6 \lambda_{em} ) or ( \lambda_i \leq 50 )</td>
<td>( \lambda_{em} ) maximum slenderness of main component between adjacent connections.</td>
</tr>
<tr>
<td>CHINA: GBJ 17 88</td>
<td>( \lambda_i \leq 0.5 \lambda_{max} ) where ( \lambda_{max} ) is the maximum slenderness ratio of the built-up member.</td>
<td>( \lambda_{max} ) The slenderness ratio limitations of components account for the effects of the geometrical imperfections.</td>
</tr>
<tr>
<td>EAST EUROPE: SNIIP II-23-81 82</td>
<td>( \lambda_i \leq \lambda_{em} ) or ( \lambda_i \leq 40 )</td>
<td>The slenderness ratio limitations of individual sub-members are to account for the influence of geometrical imperfections upon the value of equivalent slenderness of a built-up member which are not otherwise considered. Generally the limit states design approach is used with factored loads and resistance. ( \lambda_{em} ) slenderness ratio of the built-up member about the solid-web axis.</td>
</tr>
<tr>
<td>TGL 13503/01 82</td>
<td>( \lambda_i \leq \lambda_x ) or ( \lambda_i \leq 50 )</td>
<td></td>
</tr>
<tr>
<td>JAPAN: AIJ 70</td>
<td>( \lambda_i \leq 50 )</td>
<td></td>
</tr>
<tr>
<td><strong>NORTH AMERICA:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>---------------------------</td>
<td></td>
</tr>
<tr>
<td>AISC-ASD 89</td>
<td>( \Lambda_i \leq \frac{4}{5} \Lambda_{max} )</td>
<td></td>
</tr>
<tr>
<td>AISC-LRFD 86</td>
<td>( \Lambda_i \leq \frac{4}{5} \Lambda_{y} )</td>
<td></td>
</tr>
<tr>
<td>AISC-LRFD 89</td>
<td>( \Lambda_i \leq \frac{4}{5} \Lambda_{y} )</td>
<td></td>
</tr>
<tr>
<td>CSA-S16.1-M89 When ( \Lambda_Y \leq 0.8 \Lambda_X )</td>
<td>( \Lambda_i \leq 50 \quad \text{or} \quad \Lambda_i \leq 0.7 \Lambda_X )</td>
<td></td>
</tr>
<tr>
<td>When ( \Lambda_Y &gt; 0.8 \Lambda_X )</td>
<td>( \Lambda_i \leq 40 \quad \text{or} \quad \Lambda_i \leq 0.6 \Lambda_Y )</td>
<td></td>
</tr>
<tr>
<td>CSA-S16.1-94</td>
<td>( \Lambda_i \leq \Lambda_X \quad \text{or} \quad \Lambda_i \leq \Lambda_Y )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>WEST EUROPE:</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>EC 3 88</td>
<td>Not Applicable.</td>
</tr>
</tbody>
</table>

This applies to members composed of two or more rolled shapes separated by intermittent fillers, or members interconnected by lacing having tie plates at each end and at intermediate points. **Batten plates are not specified.**

It is to be noted that the use of batten plates is not specified in the AISC-LRFD Specification.

where:
- \( \Lambda_Y \) = the integral slenderness ratio about the axis perpendicular to the plane of the battens.
- \( \Lambda_X \) = the integral slenderness ratio about the axis parallel to the plane of the battens.
- \( \Lambda_Y \) = the maximum slenderness ratio of individual components between ends of adjacent battens.

### A.3 Why Are There Differences?

Different assessment schemes are applied in different specifications around the world. Namely, the bifurcation theory is now the basis for some codes whereas the second order theory considering initial geometrical and mechanical imperfections has been or is being adopted in Europe.

Further differences arise from either taking advantage of, or ignoring the scatter between the first yielding and the collapse of the structure. In Europe more attention has been given recently to a more realistic elastic plastic collapse mechanism of compressed members.
The limit values given for local slenderness are more related to an effort to simplify the design formulas than to a real need.

Another source of differences is the evaluation of initial imperfections and their application in the design formulas. It seems that the differences are also related to the major or minor need of simplification and unification of structural detailing standards.
FIG. A.1: Nomenclature of international standards, codes, and specifications

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APPENDIX B

TYPES OF BIFURCATION

B.1 Introduction

This appendix describes the different types of post-buckling behaviour using the potential energy theory. Types of bifurcation and imperfection sensitivity of simple struts were studied by Supple (1973), Thompson (1973), Thompson and Hunt (1973), Huseyin (1975), and Bažant and Cedolin (1991). The latter used the energy approach to analyze the stability of discretized elastic systems using the Lagrange-Dirichlet theorem. This theorem states that "Assuming the total energy to be continuous, the equilibrium of a system containing only conservative forces (i.e. the work done by the force is independent of the path taken and depends only on the initial and final positions) and dissipative forces (i.e. nonconservative forces that do work as the structure moves and dissipate energy) is stable if the potential energy of the system has a strict minimum". The diverse types of post-critical behaviour leads to an important classification of conservative stability problems into stable, unstable, and neutral stability.
B.2 Potential Energy for Discrete Elastic Systems

A discrete system is a system with a finite number, \( \bar{n} \), of degrees of freedom, characterized by the generalized displacements \( q_1, \ldots, q_{\bar{n}} \) (kinematic variables) that may represent actual displacements or rotations, or parameters of some deformation modes. According to the Lagrange-Dirichlet theorem the solution of the stability analysis is reduced to finding a strict minimum for the energy function of the structure-load system, called the potential energy \( \Pi \). The dissipative forces (permitted by the Lagrange-Dirichlet theorem) must be such that \( \Pi \) is a function of the generalized displacements \( q_1, \ldots, q_{\bar{n}} \) in the vicinity of the equilibrium state under consideration, such that the variation of \( \Pi \) with \( q_1, \ldots, q_{\bar{n}} \) be path independent, that is, reversible. Consequently, the loads must have a potential (\( i.e. \) must be conservative), at least in the vicinity of the equilibrium state, and dissipative forces, if present, may depend only on velocities \( \dot{q}_1, \ldots, \dot{q}_{\bar{n}} \), which is the case for damping; they vanish when static deformations are analyzed.

The potential energy \( \Pi \) consists of the elastic strain energy \( U \) and the work of the loads \( W \). According to the principle of conservation of energy, the work done by the external loads as they move from one state of equilibrium to an adjacent state of equilibrium must be equal to the strain energy stored in the structure caused by deformations. That is to say \( \Delta U - \Delta W = 0 \), where \( \Delta U - \Delta W \) is the net change of energy of the structure-load system. Therefore, the potential energy
is defined as

\[ \Pi = U - W \]  

(B.1)

and the condition of stability is that \( \Delta \Pi \) has a strict minimum.

The loading may be considered to change as a function of some control parameter \( \kappa \), which may represent the load \( P \), or the parameter of a system of loads, or the prescribed displacement, or the parameter of a system of prescribed displacements. Let \( \delta q_1, \ldots, \delta q_n \) be small variations of the generalized displacements from the equilibrium state assumed to occur at constant \( \kappa \). Assuming \( \Pi \) to be a smooth function, it may be expanded into a Taylor series about the equilibrium state giving

\[
\Delta \Pi = \Pi(q_1 + \delta q_1, \ldots, q_n + \delta q_n; \kappa) - \Pi(q_1, \ldots, q_n; \kappa) = \delta \Pi + \delta^2 \Pi + \delta^3 \Pi + \ldots
\]  

(B.2)

in which

\[
\delta \Pi = \frac{1}{1!} \sum_{i=1}^{n} \frac{\partial \Pi(q_1, \ldots, q_n; \kappa)}{\partial q_i} \delta q_i
\]

\[
\delta^2 \Pi = \frac{1}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 \Pi(q_1, \ldots, q_n; \kappa)}{\partial q_i \partial q_j} \delta q_i \delta q_j
\]  

(B.3)

\[
\delta^3 \Pi = \frac{1}{3!} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{\partial^3 \Pi(q_1, \ldots, q_n; \kappa)}{\partial q_i \partial q_j \partial q_k} \delta q_i \delta q_j \delta q_k
\]

\( \delta \Pi, \delta^2 \Pi, \delta^3 \Pi, \ldots \) are called the first, second, third, etc., variations of the potential
energy. The conditions of equilibrium are

\[ \delta \Pi = 0 \quad \text{for any } \delta q_i \quad \text{giving } \quad \partial \Pi / \partial q_i = 0 \quad \text{for each } i \quad (B.4) \]

This is also the critical state condition. The equilibrium state is stable for those values of the control parameter \( \kappa \) for which

\[ \delta^2 \Pi > 0 \quad \text{for any } \delta q_p, \delta q_j \quad (B.5) \]

When, for some value of \( \kappa \), \( \delta^2 \Pi = 0 \) for some \( \delta q_p, \delta q_j \) and \( \delta^2 \Pi > 0 \) for some other \( \delta q_p, \delta q_j \), the system may be, but need not be, unstable for that \( \kappa \) value, depending on the higher-order variations of \( \Pi \). When \( \delta^2 \Pi = 0 \) identically for all \( \delta q_p, \delta q_j \) values the system may or may not be stable, but it will be stable if, in addition, \( \delta^4 \Pi = 0 \) and \( \delta^4 \Pi > 0 \) for all \( \delta q_p, \delta q_j \). When, for some \( \kappa \) value, \( \delta^2 \Pi < 0 \) for some \( \delta q_p, \delta q_j \), the system is unstable for that \( \kappa \) value.

These relationships are shown in Figs. B.1(a)-(f) for the case of a ball in a gravity field with a single degree of freedom. The potential energy of a ball in a gravity field is proportional to its vertical coordinate. In Fig. B.1(g) the various surfaces \( \delta \Pi, \delta^2 \Pi, \delta^3 \Pi, \delta^4 \Pi \) are shown for the case of Fig. B.1(e).

### B.3 Post-Critical Behaviour and Types of Bifurcation

To demonstrate the difference between stable and unstable bifurcation an example of each type is given using a simple single degree of freedom system with
large-deflection.

B.3.1 Symmetric Stable Bifurcation

The rigid bar shown in Fig. B.2(a) has a length of $L$ and is held upright by a rotational spring of stiffness $C$. The load $P$ remains constant and vertical and the column has an initial inclination angle $\bar{\alpha}$, and $q$ is the inclination angle of the bar.

$$\Pi=U-W=\frac{1}{2}C(q-\bar{\alpha})^2-PL(\cos\bar{\alpha}-\cos q)$$ \hspace{1cm} (B.6)

The equilibrium condition is $\partial\Pi/\partial q=0$ giving

$$P=\frac{C}{L}\left(\frac{q-\bar{\alpha}}{\sin q}\right)$$ \hspace{1cm} (B.7)

Figure B.2(b) shows the equilibrium path of $P(q)$ for various values of initial imperfections, $\bar{\alpha}$. Setting $\partial^2\Pi/\partial q^2=0$, the critical states can be determined. These lie on the curve (dash-dot curve in Fig. B.2(b)), and are

$$P_{cr}=\frac{C}{L\cos q}$$ \hspace{1cm} (B.8)

The critical load of the perfect column ($\bar{\alpha}=0$) is $P_{cr}^0=C/L=\lim P_{cr}$ for $q\to0$. The perfect column has a point of bifurcation at $P=P_{cr}^0$, and for $P>P_{cr}^0$ it follows a rising post-buckling path. Therefore, this type of bifurcation is called stable. It is also symmetric because the post-buckling path is symmetric with $q$. Figure B.2(c) shows that the critical load increases with increasing imperfection.
The shape of the potential energy function near the equilibrium state at various stages of loading is sketched for the case $\bar{\alpha}=0$ in Fig. B.2(d). Positive curvature indicates stability.

**B.3.2 Symmetric Unstable Bifurcation**

Consider a slightly different column, as shown in Fig. B.2(e), in which the rotational spring at the base is replaced by a horizontal, vertically sliding spring at the top of the column of stiffness $C$. Unlike the previous example, the resisting force about the hinge at the base is not proportional to the rotation at the base, because the lever arm of the spring decreases as the column deflects. The potential energy is

$$\Pi = \frac{1}{2}CL^2(\sin \alpha - \sin \bar{\alpha})^2 - PL(\cos \bar{\alpha} - \cos q)$$

(B.9)

The equilibrium path can be found from $\partial \Pi / \partial q = 0$ as

$$P = CL \left(1 - \frac{\sin \alpha}{\sin q}\right) \cos q$$

(B.10)

The equilibrium diagrams $P(q)$ for various values of the initial imperfection $\bar{\alpha}$ are plotted in Fig. B.2(f). Setting $\partial^2 \Pi / \partial q^2 = 0$, the critical states, which lie on the dash-dot curve in Fig. B.2(f), are determined to be

$$P_\sigma = CL \frac{\cos 2q + \sin \bar{\alpha} \sin q}{\cos q}$$

(B.11)
The critical load of a perfect column ($\alpha=0$) is $P_{cr}^0 = CL = \lim_{q \rightarrow 0} P_{cr}$ for $q \rightarrow 0$. The rising portion of the equilibrium curves in Fig. B.2(f) are stable, the declining portions are unstable, and the critical states occur at the limit points, that is the points with a horizontal tangent.

It should be noticed that the critical states of the imperfect system occur at loads less than the critical load of the perfect system. For such systems, it is important to quantify the dependence of the critical load $P_{cr}$ of the imperfect system on the magnitude $\alpha$ of the imperfection. Structures for which the critical load (i.e. maximum load) decreases at increasing imperfection are called *imperfection sensitive*, as shown in Fig. B.2(g), while those with the opposite behaviour are called *imperfection insensitive*. If the drop of $P_{cr}$ is large the typical imperfect structure is said to be *strongly imperfection sensitive*. For columns and frames, the drop of $P_{cr}$ due to actual imperfections is usually quite small. The equilibrium curve emanating from the bifurcation decreases with displacement symmetrically and for this reason the bifurcation is termed *unstable symmetric*.

The shape of the potential energy function near the equilibrium state at various stages of loading is sketched for the case of $\alpha=0$ in Fig. B.2(h). Note that neutral equilibrium occurs at the critical state.
Finally, although there is no simple model of neutral equilibrium it is to be noted that a simple column buckles in a state of neutral equilibrium (Chajes; 1974, and Esslinger and Geier; 1975) shown in Fig. 3.8. Neutral equilibrium is only slightly imperfection sensitive.
FIG. B.1: Potential energy of a ball in a gravity field with a single degree of freedom
Symmetric Stable Bifurcation of a Rigid-Bar Column

Symmetric Unstable Bifurcation of a Rigid-Bar Column

FIG. B.2: Types of bifurcation

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APPENDIX C

PARTIAL END RESTRAINT OF COLUMNS

C.1 Introduction

Individual members between interconnectors behave as local columns. These columns are restrained at their ends by the interconnectors causing partially fixed-end conditions. The axis of the member, which is deformed due to global buckling, is considered to be the local column axis. As the global deformation causes a relative displacement between the ends of the local column, the local buckling mode is considered to permit sidesway. Therefore, the two possible modes of local buckling are that of a virtually fixed-fixed column with no sidesway permitted or that of virtually fixed-fixed column where sidesway is permitted. The latter buckling mode is the weaker buckling mode and hence is assumed to be the governing local buckling mode. The shape of this buckling mode also correlates well with finite element results as well as with the effect of shear on a built-up column.
In the analysis of the interaction between local and global buckling, it was assumed that the interconnectors were fully rigid giving the local column an effective length factor of 1.0. In actual fact, the interconnectors act as partial restraints of stiffness $C_i$. Attempts have been made to study the effect of end restraint on column strength, braced columns in particular, by Chen (1980), Razzaq and Chang (1981), and Sugimoto and Chen (1982), Shen and Lu (1983), Bjorhovde (1984). It is the purpose of this appendix to study the effect of partial end restraint of unbraced columns.

C.2 Deflection of Partially Restrained Columns

Consider the column shown in Fig. C.1(a). The ends are restrained by springs of stiffness $C_1$ and $C_2$. The column has a length of $a$ and a bending rigidity of $EI_x$, and its upper end is permitted to displace laterally. If a load $P$ is applied to the ends of the column so that it deflects and displaces laterally, the springs would cause end moments of magnitude $M_1=C_1\theta_1$ and $M_2=C_2\theta_2$ to act at the ends of the column. If $C_1$ and $C_2$ are large, these end moments cause the curvature of the column to approach a fixed-end column. These moments are balanced by a couple, $P\Delta$, formed by the compressive forces acting at the ends of the column, as shown in Fig. C.1(a). Studying the equilibrium of the free body diagram shown in Fig. C.1(b) the internal moment at a point is
\[
M = Px - M_2
\]  
(C.1)

Substituting this into the differential equation of equilibrium \(x'' = -M/El\) gives

\[
x'' + k_p^2 x = \frac{M_2}{EI_y}
\]  
(C.2)

where \(k_p^2 = P/El\). Assuming the special case of \(C_1 = C_2 = C\), then the end moments can be taken as \(M_1 = M_2 = M_b\). Hence, the solution to this differential equation is

\[
x = A_1 \sin k_p z + B_1 \cos k_p z + \frac{M_b}{P}
\]  
(C.3)

where \(A_1\) and \(B_1\) are constants. The boundary conditions of this column are

\[
x |_{z=0} = 0 \quad \text{ (a)} \]
\[
x' |_{z=0} = \frac{M_b}{C} \quad \text{ (b)}
\]  
(C.4)

Substituting these boundary conditions into Eq. (C.3) gives

\[
B_1 = -\frac{M_b}{P} \quad \quad A_1 = -\frac{M_b}{Ck_p}
\]  
(C.5)

Therefore the general solution is

\[
x = \frac{M_b}{P} \left( k_p^2 EI_y \sin k_p z - \cos k_p z + 1 \right)
\]  
(C.6)

Using the boundary condition \(x\big|_{z=\infty} = \Delta\) and also \(\Delta = 2M_b/P\) gives
\[ 1 + \cos k_p a - k_p a G_R \sin k_p a = 0 \]  \hspace{1cm} (C.7)

where \( G_R = (EI_y/a)/C \).

The problem now is to determine the value of \( G_R \) from the stiffness of the interconnectors, and hence, evaluate the variable \( k_p \) in Eq. (C.7). Referring to Fig. C.2(a), the spring constant can be derived from the slope-deflection relationship of the battens as \( C = 6EI_y/b \), where \( EI_y \) is the bending stiffness of the beam or battens and \( b \) is the distance between the centroidal axes of the main members. For practical reasons the width of the batten plates or the thickness of the button plates, \( b' \), is used to calculate \( C \). As the local column is assumed to be continuous, as shown in Fig. C.2(b), half the stiffness of the interconnectors is assumed to act on the column to the right and half on the column to the left of the spring. This gives \( C \) a value of \( 3EI_y/b \) from which \( G_R \) may be determined. The value of \( k_p a \) can be found by solving Eq. (C.7). From this, \( k_p \) can be evaluated and, hence, \( P_{cr} = k_p^2 EI_y \) can be determined. It is then possible to determine the value of \( K_i \) from the expression \( K_i = (P_i/P_c)^{1/2} \). Table C.1 gives the values of \( G_R \) and \( k_p a \) in columns (3) and (4) for the specimens of the experimental program.

It can be seen from Table C.1 that \( K_i \) is close to 1.0 for all cases. Hence, the assumption of fixed-end conditions is acceptable.
An alternative procedure to determine the value of $K_i$ is to use the Julian and Lawrence nomograph as explained in Chapter VI. This gives values of $K_i$ shown in Table 6.5(b).

**C.3 Effect of End Restraints on the Shortening of a Column**

The shortening of a column was examined in Chapter III for a fixed-end column (i.e. $K_i$=1.0). In this section the effect of end restraint will be discussed. Considering the special case of $C_1$=$C_2$=$C$, Eq. (C.6) can be rewritten as

$$x = \frac{o_1}{E_1} \left(1 + D_1 \sin \frac{b_1 \pi z}{a} - \cos \frac{b_1 \pi z}{a} \right)$$

(C.8)

where $o_1$ is the midpoint deflection and

$$b_1 = \frac{k_p a}{\pi} \quad D_1 = \frac{k_p E I}{C} \quad E_1 = 1 + D_1 \sin \frac{b_1 \pi}{2} - \cos \frac{b_1 \pi}{2}$$

(C.9)

Therefore, assuming an initial local imperfection in the form of a slightly swaying fixed-end column, the initial local imperfection, $x_0$, and the deflection of the column, $u_1$, can be expressed as

$$x_0 = a a \left(1 - \cos \frac{\pi z}{a} \right) \quad u_1 = \frac{o_1}{E_1} \left(1 + D_1 \sin \frac{b_1 \pi z}{a} - \cos \frac{b_1 \pi z}{a} \right)$$

(C.10)

The formula for the shortening of a column, $w_p$, is
\[
\omega_f = \frac{1}{2} \int_0^a \left( (u'_1 + x_0')^2 - x_0'^2 \right) dz
\]  

(C.11)

where \( u'_1 \) and \( x_0' \) are the first derivatives of \( u_1 \) and \( x_0 \) with respect to \( z \). This gives

\[
w_f = \frac{1}{2} \int_0^a \left\{ \left( \frac{b_1 D_1 o_1 \pi}{E_1 a} \right)^2 \cos^2 \frac{b_1 \pi z}{a} + \left( \frac{b_1 o_1 \pi}{E_1 a} \right)^2 \sin^2 \frac{b_1 \pi z}{a} \right\} dz
\]

\[
+ \frac{2b_1 D_1 \pi^2 o_1}{E_1 a} \sin \frac{\pi z}{a} \cos \frac{b_1 \pi z}{a} + \frac{2b_1 \pi^2 o_1}{E_1 a} \sin \frac{\pi z}{a} \sin \frac{b_1 \pi z}{a}
\]

(C.12)

By solving (C.12) and substituting the values of \( b_1, D_1, \) and \( E_1 \) for the specimens of the experimental program this equation can be expressed as

\[
w_f = \frac{1}{2} \left\{ F_1 \left( 2\pi^2 o_1 \right) + G_1 \left( \frac{\pi^2 o_1^2}{a} \right) \right\}
\]  

(C.13)

The values of \( F_1 \) and \( G_1 \) were found to equal 0.5 in Chapter III for fixed-end columns braced and unbraced columns as well as for unbraced pinned-end columns. The values of \( F_1 \) and \( G_1 \) are shown in columns (9) and (10) of Table C.1 for the specimens of the experimental program using partial end restraints as described above. It can be seen that these values are close to 0.5, therefore, assuming fixed-end conditions does not affect the accuracy of the solution to any appreciable amount.
FIG. C.1: Partially restrained unbraced column

$M_1 = C_1 \theta_1$

$M_2 = C_2 \theta_2$

$EI_y$

$P$

$\Delta_s$

$\theta_1$

$\theta_2$

$x$

$z$

$a$

$n$

$P$

$M = -EI_y x''$

$322$
FIG. C.2: Evaluation of spring stiffness
Table C.1: Values of local effective length factors and coefficients for shortening of a partially restrained unbraced column.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>C (10^3 N·mm)</th>
<th>G_R (10^3)</th>
<th>k_f α</th>
<th>K_i</th>
<th>b_1 (10^3)</th>
<th>D_i</th>
<th>E_1</th>
<th>F_1</th>
<th>G_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>120-40-1</td>
<td>0.320</td>
<td>28.7</td>
<td>2.97</td>
<td>1.06</td>
<td>0.946</td>
<td>85.4</td>
<td>1.00</td>
<td>0.487</td>
<td>0.476</td>
</tr>
<tr>
<td>120-63-1</td>
<td>1.28</td>
<td>7.23</td>
<td>3.10</td>
<td>1.02</td>
<td>0.986</td>
<td>22.4</td>
<td>1.00</td>
<td>0.497</td>
<td>0.493</td>
</tr>
<tr>
<td>120-63-2</td>
<td>1.28</td>
<td>10.9</td>
<td>3.08</td>
<td>1.02</td>
<td>0.979</td>
<td>33.4</td>
<td>1.00</td>
<td>0.495</td>
<td>0.490</td>
</tr>
<tr>
<td>120-63-3-1/8</td>
<td>1.28</td>
<td>14.5</td>
<td>3.05</td>
<td>1.03</td>
<td>0.972</td>
<td>44.2</td>
<td>1.00</td>
<td>0.493</td>
<td>0.487</td>
</tr>
<tr>
<td>120-63-3-3/16</td>
<td>1.92</td>
<td>9.646</td>
<td>3.08</td>
<td>1.02</td>
<td>0.981</td>
<td>29.7</td>
<td>1.00</td>
<td>0.495</td>
<td>0.491</td>
</tr>
<tr>
<td>120-63-3-1/4</td>
<td>2.56</td>
<td>7.23</td>
<td>3.10</td>
<td>1.05</td>
<td>0.986</td>
<td>22.4</td>
<td>1.00</td>
<td>0.497</td>
<td>0.493</td>
</tr>
<tr>
<td>120-63-4</td>
<td>1.28</td>
<td>18.1</td>
<td>3.03</td>
<td>1.04</td>
<td>0.965</td>
<td>54.8</td>
<td>1.00</td>
<td>0.492</td>
<td>0.484</td>
</tr>
<tr>
<td>120-63-5</td>
<td>1.28</td>
<td>21.7</td>
<td>3.01</td>
<td>1.04</td>
<td>0.959</td>
<td>65.3</td>
<td>1.00</td>
<td>0.490</td>
<td>0.481</td>
</tr>
<tr>
<td>70-63-1</td>
<td>1.28</td>
<td>13.0</td>
<td>3.06</td>
<td>1.03</td>
<td>0.975</td>
<td>39.9</td>
<td>1.00</td>
<td>0.494</td>
<td>0.488</td>
</tr>
<tr>
<td>70-63-2</td>
<td>1.28</td>
<td>19.5</td>
<td>3.02</td>
<td>1.04</td>
<td>0.962</td>
<td>59.1</td>
<td>1.00</td>
<td>0.491</td>
<td>0.483</td>
</tr>
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<td>70-63-3</td>
<td>1.28</td>
<td>26.1</td>
<td>2.99</td>
<td>1.05</td>
<td>0.951</td>
<td>77.8</td>
<td>1.00</td>
<td>0.488</td>
<td>0.478</td>
</tr>
<tr>
<td>120-A-1</td>
<td>2.33</td>
<td>6.62</td>
<td>3.10</td>
<td>1.01</td>
<td>0.987</td>
<td>20.5</td>
<td>1.00</td>
<td>0.497</td>
<td>0.494</td>
</tr>
<tr>
<td>120-U-1</td>
<td>308</td>
<td>0.0499</td>
<td>3.14</td>
<td>1.00</td>
<td>1.00</td>
<td>0.157</td>
<td>1.00</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>120-A-2</td>
<td>2.33</td>
<td>9.92</td>
<td>3.08</td>
<td>1.02</td>
<td>0.981</td>
<td>30.6</td>
<td>1.00</td>
<td>0.495</td>
<td>0.491</td>
</tr>
<tr>
<td>120-U-2</td>
<td>308</td>
<td>0.0749</td>
<td>3.14</td>
<td>1.00</td>
<td>1.00</td>
<td>0.235</td>
<td>1.00</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>120-A-3</td>
<td>2.33</td>
<td>13.2</td>
<td>3.06</td>
<td>1.03</td>
<td>0.974</td>
<td>40.5</td>
<td>1.00</td>
<td>0.494</td>
<td>0.488</td>
</tr>
<tr>
<td>120-U-3</td>
<td>308</td>
<td>0.0999</td>
<td>3.14</td>
<td>1.00</td>
<td>1.00</td>
<td>0.314</td>
<td>1.00</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>120-A-4</td>
<td>2.33</td>
<td>15.5</td>
<td>3.04</td>
<td>1.03</td>
<td>0.968</td>
<td>50.3</td>
<td>1.00</td>
<td>0.492</td>
<td>0.485</td>
</tr>
<tr>
<td>120-U-4</td>
<td>308</td>
<td>0.125</td>
<td>3.14</td>
<td>1.00</td>
<td>1.00</td>
<td>0.392</td>
<td>1.00</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

\[
C = \frac{3EI_b}{b'} \\
\frac{G_R}{C} = \frac{(EI_y/a)}{C} \\
P_{cr} = k_f^2EI_y \\
K_i = \sqrt{\frac{P_E}{P_{cr}}} \\
\]

\[
b_1 = \frac{k_f a}{\pi} \\
D_i = \frac{k_f EI_y}{C} \\
E_1 = 1 + D_i \sin \frac{b_1 \pi}{2} - \cos \frac{b_1 \pi}{2}
\]

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APPENDIX D

FINITE ELEMENT PROGRAM

An example of the data deck of a typical finite element program for ABAQUS Version 5.5 is presented. This example is for specimen 120-63-3-1/8.

and is as follows:

**************TOP OF FILE**************
**
**DATACHECK
*HEADING
3D model of a built-up column with three interconnectors with nominal cross-sectional dimensions consisting of two C 75 x 6 channel sections placed toe-to-toe forming a box-like section. The model had a nominal initial out-of-straightness of \( x_{\text{m}} = 3.55 \) mm and \( y_{\text{m}} = 3.55 \) mm. The interconnectors had a thickness of 3.175 mm. The model was loaded axially in increments.
*PREPRINT,ECHO=YES,HISTORY=NO,MODEL=YES
**
**************DEFINITION OF NODES**************
**
**************CHANNEL SECTIONS**************
**
*NODE,NSET=DATA
1,-36.63E-3,33.258E-3,0.0
2,-36.63E-3,33.258E-3,0.041
3,-36.63E-3,33.258E-3,0.081
56,-33.08E-3,36.808E-3,1.776
109,-36.63E-3,33.258E-3,3.471
110,-36.63E-3,33.258E-3,3.512
111,-36.63E-3,33.258E-3,3.552
*NGEN,NSET=DATA,LINE=P
  3.109156,-33.08E-3,36.808E-3,1.776
*NCOPY,CHANGE NUMBER=400,OLD SET=DATA,SHIFT,NEW SET=F1
  16.425E-3,1.414E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=400,OLD SET=F1,SHIFT,NEW SET=F2
  16.425E-3,1.414E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NSET,NSET=FL1
  DATA,F1,F2
*NCOPY,CHANGE NUMBER=200,OLD SET=DATA,SHIFT,NEW SET=F3
  0.0,-66.516E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=400,OLD SET=F3,SHIFT,NEW SET=F4
  16.425E-3,-1.414E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=400,OLD SET=F4,SHIFT,NEW SET=F5
  16.425E-3,-1.414E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NSET,NSET=FL2
  F3,F4,F5
*NSET,NSET=FLANGE1
  FL1,FL2
*NCOPY,CHANGE NUMBER=2400,OLD SET=DATA,SHIFT,NEW SET=W1
  0.0,-16.629E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=200,OLD SET=W1,SHIFT,NEW SET=W2
  0.0,-16.629E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=200,OLD SET=W2,SHIFT,NEW SET=W3
  0.0,-16.629E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NSET,NSET=WEB1
  W1,W2,W3
*NSET,NSET=MEM1
  FLANGE1,WEB1
*NCOPY,CHANGE NUMBER=1000,OLD SET=FL2,SHIFT,NEW SET=FL3
  40.41E-3,69.344E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=1400,OLD SET=FL1,SHIFT,NEW SET=FL4
  40.41E-3,-69.344E-3,0.0
  0.0,0.0,0.0,0.0,0.0,1.0,0.0
*NCOPY,CHANGE NUMBER=2000,OLD SET=WEB1,SHIFT,NEW SET=WEB2
73.26E-3,0,0,0,0
0,0,0,0,0,0,0,0,1,0,0
*NSET,NSET=MEM2
FL3,FL4,WEB2
**
*************END PLATES*************
**
*NSET,NSET=BOT1
2401,2601,2801
*NCOPY,CHANGE NUMBER=120,OLD SET=BOT1,SHIFT,NEW SET=BOT2
36.63E-3,0,0,0,0,0,0,0,1,0,0
*NODE,NSET=BOT3
121,0,0,35.761E-3,0,0
321,0,0,-35.761E-3,0,0
521,-3.78E-3,0,0,0,0
721,3.78E-3,0,0,0,0
*NSET,NSET=BOTPL
121,321,521,721,2521,2721,2921
*NCOPY,CHANGE NUMBER=1,OLD SET=BOTPL,SHIFT,NEW SET=TOPPL
0,0,0,0,3.552
0,0,0,0,0,0,0,0,1,0,0,0
**
*************INTERCONNECTORS*************
**
*NODE,NSET=CON1L
6000,-31.581E-3,39.758E-3,0.081
6010,-31.581E-3,39.758E-3,0.097
6020,-31.581E-3,39.758E-3,0.113
6030,-31.581E-3,39.758E-3,0.129
6040,-31.581E-3,39.758E-3,0.145
*NCOPY,CHANGE NUMBER=6,OLD SET=CON1L,SHIFT,NEW SET=CON1R
63.5E-3,0,0,0,0
0,0,0,0,0,0,0,0,0,1,0,0
*NFILL,NSET=CON1
CON1L,CON1R,6,1
*NCOPY,CHANGE NUMBER=400,OLD SET=CON1,SHIFT,NEW SET=CON2
0,0,-79.175E-3,0,0
0,0,0,0,0,0,0,0,0,1,0,0
*NSET,NSET=CONN1
CON1,CON2
*NCOPY,CHANGE NUMBER=75,OLD SET=CONN1,SHIFT,NEW SET=CONN2
2.400E-3,2.400E-3,0.831625
*NCOPY,CHANGE NUMBER=150,OLD SET=CONN1,SHIFT,NEW SET=CONN4
3.385E-3,3.385E-3,1.66325
0.0,0.0,0.0,0.0,0.0,1.0,0
*NCOPY,CHANGE NUMBER=225,OLD SET=CONN1,SHIFT,NEW SET=CONN6
2.400E-3,2.400E-3,2.494875
0.0,0.0,0.0,0.0,0.0,1.0,0
*NCOPY,CHANGE NUMBER=300,OLD SET=CONN1,SHIFT,NEW SET=CONN7
0.0,0.0,3.3265
0.0,0.0,0.0,0.0,0.0,1.0,0
*NSET,NSET=CONN
CONN1,CONN2,CONN4,CONN6,CONN7
*NSET,NSET=ALL
MEM1,MEM2,BOT1,BOT2,BOT3,HINGE,TOPPL,CONN
**
**************DEFINITION OF ELEMENTS**************
**
*ELEMENT,TYPE=S8R,ELSET=FLANGE
1,1,3,803,801,2,403,802,401
*ELGEN,ELSET=FLANGE
1,2,200,100,2,1200,200,55,2,1
*ELEMENT,TYPE=S8R,ELSET=WEB
401,1,3,2603,2601,2,2403,2602,2401
601,2601,2603,203,201,2602,2803,202,2801
*ELGEN,ELSET=WEB
401,2,2000,100,55,2,1
601,2,2000,100,55,2,1
*ELEMENT,TYPE=S4R,ELSET=CONN
800,600,6010,6011,6001
*ELGEN,ELSET=CONN
800,5,75,60
*ELGEN,ELSET=CONN
800,6,1,1,4,10,10,2,400,400
860,6,1,1,4,10,10,2,400,400
920,6,1,1,4,10,10,2,400,400
980,6,1,1,4,10,10,2,400,400
1040,6,1,1,4,10,10,2,400,400
*ELEMENT,TYPE=S8R,ELSET=ENDS
1620,1,2601,2721,121,2401,521,2521,401
1621,2601,201,321,2721,2801,601,2921,521
1622,121,2721,4601,2001,2521,721,4401,1601
1623,2721,321,2201,4601,2921,1801,4801,721
1630,111,2711,2722,122,2511,522,2522,511
1631,2711,311,322,2722,2911,711,2922,522
1632,122,2722,4711,2111,2522,722,4511,1711
1633,2722,322,2311,4711,2922,1911,4911,722
*ELSET,ELSET=ALL
FLANGE,WEB,CONN,ENDS
**

**************DEFINITION OF KINEMATIC CONDITIONS**************
**

**************INTERCONNECTOR CONSTRAINTS**************
**

*NSSET,NSSET=M1,GENERATE
  3,107,26
  4,108,26
  5,109,26
  203,307,26
  204,308,26
  205,309,26
*NSSET,NSSET=M2,GENERATE
  2003,2107,26
  2004,2108,26
  2005,2109,26
  2203,2307,26
  2204,2308,26
  2205,2309,26
*NSSET,NSSET=M3,GENERATE
  3,107,26
  4,108,26
  203,307,26
  204,308,26
*NSSET,NSSET=M4,GENERATE
  4,108,26
  5,109,26
  204,308,26
  205,309,26
*NSSET,NSSET=M5,GENERATE
  2003,2107,26
  2004,2108,26
  2203,2307,26
  2204,2308,26
*NSSET,NSSET=M6,GENERATE
  2004,2108,26
  2005,2109,26
  2204,2308,26
2205,2309,26
*NSET,NSET=M7,G2ENERATE
  3,107,26
  5,109,26
  203,307,26
  205,309,26

*NSET,NSET=M8,G2ENERATE
  403,507,26
  405,509,26
  603,707,26
  605,709,26

*NSET,NSET=M9,G2ENERATE
  803,907,26
  805,909,26
  1003,1107,26
  1005,1109,26

*NSET,NSET=M10,G2ENERATE
  1203,1307,26
  1205,1309,26
  1403,1507,26
  1405,1509,26

*NSET,NSET=M11,G2ENERATE
  1603,1707,26
  1605,1709,26
  1803,1907,26
  1805,1909,26

*NSET,NSET=M12,G2ENERATE
  2003,2107,26
  2005,2109,26
  2203,2307,26
  2205,2309,26

*NSET,NSET=C1,G2ENERATE
  6000,6300,75
  6020,6320,75
  6040,6340,75
  6400,6700,75
  6420,6720,75
  6440,6740,75

*NSET,NSET=C2,G2ENERATE
  6006,6306,75
  6026,6326,75
  6046,6346,75
  6406,6706,75
6426,6726,75
6446,6746,75
*NSET,NSET=C3,G3NENARATE
6010,6310,75
6030,6330,75
6410,6710,75
6430,6730,75
*NSET,NSET=C4,G3NENARATE
6016,6316,75
6036,6336,75
6416,6716,75
6436,6736,75
*NSET,NSET=C5,G3NENARATE
6001,6301,75
6041,6341,75
6401,6701,75
6441,6741,75
*NSET,NSET=C6,G3NENARATE
6002,6302,75
6042,6342,75
6402,6702,75
6442,6742,75
*NSET,NSET=C7,G3NENARATE
6004,6304,75
6044,6344,75
6404,6704,75
6444,6744,75
*NSET,NSET=C8,G3NENARATE
6005,6305,75
6045,6345,75
6405,6705,75
6445,6745,75
*MPC
TIE,C1,M1
TIE,C2,M2
LINEAR,C3,M3,M4
LINEAR,C4,M5,M6
LINEAR,C5,M7,M8
LINEAR,C6,M8,M9
LINEAR,C7,M10,M11
LINEAR,C8,M11,M12
**

***************MODELLING OF KNIFE EDGE***************
**
*NSET,NSET=HINGE
 2601,521,2721,721,4601
*NSET,NSET=TOPEND
 2711,522,2722,722,4711
*NSET,NSET=TOPSEC
 522,2722,722,4711
*EQUATION
 2
  TOPSEC,3,1.0,2711,3,-1.0
*BOUNDARY
  HINGE,PINNED
  TOPEND,1,2
**

***************SECTION DEFINITION***************
**
*NSET,NSET=CORN3,GENERATE
  1,111,1
  201,311,1
*NSET,NSET=CORN5,GENERATE
  2001,2111,1
  2201,2311,1
*NSET,NSET=CORN4,GENERATE
  801,911,1
  1001,1111,1
*NSET,NSET=CORN6,GENERATE
  1201,1311,1
  1401,1511,1
*NODAL THICKNESS
  CORN3,9.485E-3
  CORN4,3.983E-3
  CORN5,9.485E-3
  CORN6,3.983E-3
*NODAL THICKNESS,GENERATE
  CORN3,CORN4,2,400
*NODAL THICKNESS,GENERATE
  CORN6,CORN5,2,400
*SHELL SECTION,ELSET=FLANGE,MATERIAL=A1,NODAL THICKNESS
*SHELL SECTION,ELSET=WEB,MATERIAL=A1
  4.3E-3,3
*SHELL SECTION,ELSET=CONN,MATERIAL=CONN
3.175E-3,3
*SHELL SECTION,ELSET=ENDS,MATERIAL=ENDS
38.1E-3,7
**
**********DEFINITION OF CONTACT PROBLEM**********
**
*ELSET,ELSET=CON1,GGENERATE
800,835,1
*ELSET,ELSET=CON2,GGENERATE
860,895,1
*ELSET,ELSET=CON4,GGENERATE
920,955,1
*ELSET,ELSET=CON6,GGENERATE
980,1015,1
*ELSET,ELSET=CON7,GGENERATE
1040,1075,1
*ELSET,ELSET=CON8,GGENERATE
1200,1235,1
*ELSET,ELSET=CON9,GGENERATE
1260,1295,1
*ELSET,ELSET=CON11,GGENERATE
1320,1355,1
*ELSET,ELSET=CON13,GGENERATE
1380,1415,1
*ELSET,ELSET=CON14,GGENERATE
1440,1475,1
*ELSET,ELSET=E1
2,202
*ELSET,ELSET=E2
15,215
*ELSET,ELSET=E4
28,228
*ELSET,ELSET=E6
41,241
*ELSET,ELSET=E7
54,254
*ELSET,ELSET=E8
102,302
*ELSET,ELSET=E9
115,315
*ELSET,ELSET=E11
128,328
*ELSET,ELSET=E13
141,341
*ELSET,ELSET=E14
154,354
*SURFACE DEFINITION,NAME=E1
  E1,SPOS
*SURFACE DEFINITION,NAME=CON1
  CON1,SNEG
*CONTACT PAIR,INTERACTION=GAP1,ADJUST=0.0,SMLLL SLIDING
  E1,CON1
*SURFACE INTERACTION,NAME=GAP1
  5.04E-3
*SURFACE DEFINITION,NAME=E2
  E2,SPOS
*SURFACE DEFINITION,NAME=CON2
  CON2,SNEG
*CONTACT PAIR,INTERACTION=GAP2,ADJUST=0.0,SMLLL SLIDING
  E2,CON2
*SURFACE INTERACTION,NAME=GAP2
  5.04E-3
*SURFACE DEFINITION,NAME=E4
  E4,SPOS
*SURFACE DEFINITION,NAME=CON4
  CON4,SNEG
*CONTACT PAIR,INTERACTION=GAP4,ADJUST=0.0,SMLLL SLIDING
  E4,CON4
*SURFACE INTERACTION,NAME=GAP4
  5.04E-3
*SURFACE DEFINITION,NAME=E6
  E6,SPOS
*SURFACE DEFINITION,NAME=CON6
  CON6,SNEG
*CONTACT PAIR,INTERACTION=GAP6,ADJUST=0.0,SMLLL SLIDING
  E6,CON6
*SURFACE INTERACTION,NAME=GAP6
  5.04E-3
*SURFACE DEFINITION,NAME=E7
  E7,SPOS
*SURFACE DEFINITION,NAME=CON7
  CON7,SNEG
*CONTACT PAIR,INTERACTION=GAP7,ADJUST=0.0,SMLLL SLIDING
  E7,CON7
*SURFACE INTERACTION,NAME=GAP7
  5.04E-3
*SURFACE DEFINITION,NAME=E8
  E8,SNEG
*SURFACE DEFINITION,NAME=CON8
  CON8,SPOS
*CONTACT PAIR,INTERACTION=GAP8,ADJUST=0.0,SMALL SLIDING
  E8,CON8
*SURFACE INTERACTION,NAME=GAP8
  5.04E-3
*SURFACE DEFINITION,NAME=E9
  E9,SNEG
*SURFACE DEFINITION,NAME=CON9
  CON9,SPOS
*CONTACT PAIR,INTERACTION=GAP9,ADJUST=0.0,SMALL SLIDING
  E9,CON9
*SURFACE INTERACTION,NAME=GAP9
  5.04E-3
*SURFACE DEFINITION,NAME=E11
  E11,SNEG
*SURFACE DEFINITION,NAME=CON11
  CON11,SPOS
*CONTACT PAIR,INTERACTION=GAP11,ADJUST=0.0,SMALL SLIDING
  E11,CON11
*SURFACE INTERACTION,NAME=GAP11
  5.04E-3
*SURFACE DEFINITION,NAME=E13
  E13,SNEG
*SURFACE DEFINITION,NAME=CON13
  CON13,SPOS
*CONTACT PAIR,INTERACTION=GAP13,ADJUST=0.0,SMALL SLIDING
  E13,CON13
*SURFACE INTERACTION,NAME=GAP13
  5.04E-3
*SURFACE DEFINITION,NAME=E14
  E14,SNEG
*SURFACE DEFINITION,NAME=CON14
  CON14,SPOS
*CONTACT PAIR,INTERACTION=GAP14,ADJUST=0.0,SMALL SLIDING
  E14,CON14
*SURFACE INTERACTION,NAME=GAP14
  5.04E-3
**

***************MATERIAL DEFINITION***************
**
*MATERIAL,NAME=A1
*ELASTIC
2.13E11,0.3
*PLASTIC
3.58E8,0.0
*MATERIAL,NAME=CONN
*ELASTIC
1.94E11,0.28
*PLASTIC
2.897E8,0.0
4.219E8,6.38E-4
4.662E8,1.172E-3
4.931E8,1.989E-3
5.082E8,3.9795E-3
*MATERIAL,NAME=ENDS
*ELASTIC
2.12E11,0.289
*PLASTIC
6.155E8,0.0
6.641E8,4.238E-3
7.026E8,2.6242E-2
**
***************PLOT OF MODEL**************
**
*PLOT,COLORS=4,OUTPUT=BINARY
DETAILS OF MESH
10,11,6,9,0.5,1.5,2.5,1
*VIEWPOINT,DEFINITION=MODEL AXIS ROTATION
-45.0,0.0,0,-45.0
*DETAIL,ELSET=ALL
*DRAW,HIDE
**
***************LOADING AND OUTPUT DEFINITION***************
**
*RESTART,WRITE,OVERRIDE
*STEP,NLGEOM,INC=50
*STATIC
0.1,1.0
*CLOAD
TOPEND,3,-50000.0
*EL PRINT,ELSET=ALL,FREQUENCY=0
S
E
*NODE PRINT,NSET=DATA
  U
*NODE PRINT,NSET=TOPEND
  U
*NODE PRINT,NSET=TOPEND,TOTALS=YES,SUMMARY=NO
  CF
  RF
*NODE PRINT,NSET=HINGE
  U
*NODE PRINT,NSET=HINGE,TOTALS=YES,SUMMARY=NO
  CF
  RF
**
*************PLOT OF DEFORMED MODEL*************
**
*PLOT,FREQUENCY=17,OUTPUT=BINARY
  DISPLACED COLUMN
  10,11,6,9,0.5,1.5,2.5,1
*VIEWPOINT,DEFINITION=MODEL AXIS ROTATION
  -45.0,0.0,-45.0
*DISPLACED,HIDE
  U,5
*VIEWPOINT,DEFINITION=MODEL AXIS ROTATION
  -90.0,0.0,0.0
*DISPLACED,HIDE
  U,5
*VIEWPOINT,DEFINITION=MODEL AXIS ROTATION
  -90.0,0.0,-90.0
*DISPLACED,HIDE
  U,5
*END STEP
**
*************END OF FILE*************
APPENDIX E

TENSION TESTS

E.1 Introduction

Fourteen tension tests were carried out on specimens prepared from the channel sections, one for each length of channel used in the experimental program, to determine their material properties. The tension coupons were taken out of the web of each section. Tension coupons were not taken out of the flanges as they are of variable thickness. The nominal dimensions of the tension coupons used are shown in Fig. E.1. In addition to the coupons taken from the channel webs, coupons were taken from the material used for the interconnectors and the material of the end plate, with the exception of the 1/4 in. interconnector which had no remaining material. The results obtained from the tension test were used in the finite element analysis. After both surfaces of the central part of the coupon were cleaned of the mill scale and prepared so that they were completely smooth, the exact cross-sectional dimensions were measured at the midpoint of the central part. These results as well as the specimen to which each coupon corresponds are shown
in Table E.1. It should be noted that coupons 5 to 8 got mixed up during the experimental program so the results of these coupons were taken as a batch, and the average mechanical properties were used for the corresponding specimens. Electrical resistance strain gauges were used on either side of each coupon. These gauges were positioned centrally on the central part of the coupon so that the gauge length corresponded with the axis of the coupon to measure axial strains. On the coupons taken from the interconnectors and the end plate a third gauge was attached horizontally to measure transverse strains caused by Poisson's effect. The reason for this was to determine a value for Poisson's ratio, \( v \), for these materials as the steel of these materials was a high tensile steel and not a linear elastic-ideally plastic steel with a definite yield.

\[ \text{E.2 Test Procedure} \]

The tension tests were carried out in the Universal Testing Machine located in the Strength of Materials Laboratory at the University of Windsor. The specimens were placed between the upper movable head and the lower fixed head as shown in Fig. E.2. The strain gauge wires were connected to a switch and balancing unit which in turn was connected to a strain indicator from which the strains were read. The average reading of these two gauges was taken as the strain. In this way any bending strains were eliminated. The load was read on the testing machine. The stress was calculated using the cross-sectional dimensions
listed in Table E.1. The tension coupons taken from the channel sections showed a definite yield, whereas, those taken from the interconnectors and end plate materials did not.

A typical stress-strain curve for coupon 11 taken from the channel sections is shown in Fig. E.3. The strain gauges for this coupon remained functional until a load close to the ultimate load. The stress-strain curves for the coupons taken from the 1/8, 3/16, and 3/8 in. interconnectors are shown in Figs. E.4(a)-E.6(a). The results of the 3/16 in. coupon were used to model the 1/4 in. interconnectors. The stress-strain curve of the 1/2 in. coupon, taken from the end plate, is shown in Fig. E.7(a). The offset yield strength (proof strength) for these materials was taken at 0.2% strain and is shown in Figs. E.4(a)-E.7(a). Poisson's ratio for the interconnectors and end plate materials was calculated by taking the negative ratio of the transverse strain to its corresponding axial strain. The variation of Poisson's ratio with increasing stress for these specimens is shown in Figs. E.4(b)-E.7(b).

The mechanical properties, the yield stress and the modulus of elasticity, derived from these tests are listed in Columns (4) and (5) of Table E.1. To eliminate the mistake made in coupons 5 to 8 an average yield stress and modulus of elasticity were calculated for this batch and are shown in Columns (6) and (7). The average value, within the elastic limit, of Poisson's ratio for the materials of
the interconnectors and end plate is shown in Column (8).

E.3 True Stress-Strain Curve

The nominal stress \( \sigma_n = P/A_0 \) and the conventional or engineering strain \( \varepsilon = \Delta l/l_0 \), where \( \Delta l \) is the change in gauge length, are calculated using the original cross-sectional area, \( A_0 \), and gauge length, \( l_0 \). To model the nonlinear behaviour of the interconnector and end plate material in the finite element analysis the true stress and true strain were required. The true stress is that stress calculated using the actual cross-sectional area at the time of application of the load, \( P \), (Mendelson 1968). Assuming there are no volume changes, the true stress would equal

\[
\sigma_{\text{true}} = \frac{Pl}{A_0 l_0} \quad (E.1)
\]

where \( l \) is the current value of the gauge length. This can be rewritten as

\[
\sigma_{\text{true}} = \sigma_n (1 + \varepsilon) \quad (E.2)
\]

The true strain can be calculated by integrating the increment of strain for a given length \( (dl/l) \) such that

\[
\varepsilon_{\ln} = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0} \quad (E.3)
\]

where \( \varepsilon_{\ln} \) is known as the natural, logarithmic, or true strain. Its relation to the conventional strain is
\[ \varepsilon_m = \ln(1 + \varepsilon) \] (E.4)

The true stress-strain curve is essentially the same as the conventional stress-strain curve up to and slightly above the yield point. Beyond this point the two types of plots diverge. The true stress always increases up to the rupture point and does not have a maximum at the point where the load starts dropping.

Figs. E.8 and E.9 show the true stress-strain curves for the materials of the interconnectors and end plate. The percent strain is used so the value of strain obtained from Eq. (E.4) is multiplied by 100. The ideal curve used to model the materials behaviour in the finite element analysis is shown on these figures as a dotted line.
FIG. E.1: Tension test coupon
FIG. E.2: Tension test specimen in universal testing machine
FIG. E.4: Tension test of interconnector with thickness 1/8 in.
FIG. E.5: Tension test of interconnector with thickness 3/16 in.
FIG. E.6: Tension test of interconnector with thickness 3/8 in.
FIG. E.7: Tension test of end plate with thickness 1/2 in.
FIG. E.8: True tension tests of interconnectors with thicknesses of 1/8 and 3/16 in.
FIG. E.9: True tension tests of interconnector with thickness of 3/8 in. and end plate with thickness of 1/2 in.

(a) Interconnector with thickness 3/8 in.

(b) End plate with thickness 1/2 in.
Table E.1: Tension test coupon allocation, cross-sectional dimensions, and material properties.

<table>
<thead>
<tr>
<th>Coupon No.</th>
<th>Corresponding Specimen</th>
<th>Cross-Sectional Dimensions (mm)</th>
<th>F_y (MPa)</th>
<th>E (GPa)</th>
<th>Batch Average</th>
<th>Poisson's Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120-40-1</td>
<td>12.80 x 3.90</td>
<td>341</td>
<td>202</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>120-63-1</td>
<td>12.74 x 3.78</td>
<td>359</td>
<td>212</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>120-63-2</td>
<td>12.72 x 3.92</td>
<td>351</td>
<td>210</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>120-63-3-1/8</td>
<td>12.66 x 3.88</td>
<td>358</td>
<td>213</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Batch consisting of 120-63-3-3/16, 120-63-3-1/4, 120-63-4, 120-63-5</td>
<td>12.58 x 3.82</td>
<td>354</td>
<td>209</td>
<td>351</td>
<td>205</td>
</tr>
<tr>
<td>6</td>
<td>70-63-1, 70-63-2</td>
<td>12.58 x 3.86</td>
<td>350</td>
<td>211</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>70-63-3</td>
<td>12.46 x 3.78</td>
<td>346</td>
<td>208</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>351</td>
<td>208</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>120-A-1, 120-U-1</td>
<td>12.66 x 3.78</td>
<td>353</td>
<td>213</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>120-A-2, 120-U-2</td>
<td>12.64 x 3.70</td>
<td>372</td>
<td>215</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>120-A-3, 120-U-3</td>
<td>12.74 x 3.82</td>
<td>351</td>
<td>211</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>120-A-4, 120-U-4</td>
<td>12.68 x 4.34</td>
<td>363</td>
<td>212</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>360</td>
<td>213</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/8 in.</td>
<td>Interconnector</td>
<td>12.66 x 3.00</td>
<td>595</td>
<td>217</td>
<td>0.331</td>
<td></td>
</tr>
<tr>
<td>3/16 in.</td>
<td>Interconnector</td>
<td>12.62 x 4.52</td>
<td>491</td>
<td>194</td>
<td>0.280</td>
<td></td>
</tr>
<tr>
<td>3/8 in.</td>
<td>Interconnector</td>
<td>12.62 x 9.20</td>
<td>623</td>
<td>213</td>
<td>0.291</td>
<td></td>
</tr>
<tr>
<td>1/2 in.</td>
<td>End Plate</td>
<td>12.62 x 12.14</td>
<td>645</td>
<td>212</td>
<td>0.287</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX F

CROSS-SECTIONAL PROPERTIES

F.1 Introduction

Two different cross sections were used for the experimental program, one for Groups I and II and one for Group III. Both cross sections consisted of two C 75 x 6 sections interconnected by either batten plates welded to the flanges of the channels or by button plates placed between the two channel sections and welded to the outer corners of the flange and web. The nominal dimensions and cross-sectional properties of a C 75 x 6 section, as listed in the Handbook of Steel Construction (1991), are shown in Fig. F.1. In addition to these the polar second moment of area about the shear centre of the section, $I_{ps}$, is shown.

In the first configuration, used for Groups I and II, the channel sections were arranged toe-to-toe separated by a distance of 7.56 mm as shown in Fig. F.2. The batten plates used to interconnect the two channels varied in dimension from specimen to specimen. The dimensions of these batten plates are shown in Fig.
F.2. In the second of these configurations, used for Group III, the channel sections were arranged back-to-back separated by a distance of 9.53 mm as shown in Fig. F.3. Either batten or button plates were used to interconnect these channels, as shown in Figs. F.3(a) and (b), respectively.

The cross-sectional properties of these two cross sections, calculated as shown below, are listed in Table F.1. The area of the interconnectors, whether battens or buttons, is not included in the calculation. It can be shown that although the interconnectors increase the area of the built-up member locally this effect is negligible. To include the area of the interconnectors in the calculation of the area and second moments of area of the cross section, etc., the area of the interconnector should be multiplied by the ratio of the sum of the lengths of the interconnectors to the total length of the column. However, it is suggested that the designer would consider this a trivial and unnecessary procedure. Furthermore, it is the main members of the column that provide bending stiffness whereas the interconnectors provide shear stiffness (Uhlmann and Ramm; 1982). So, conservatively, the area of the interconnectors should not be included in the analysis.

F.2 Properties

The cross-sectional properties of the two configurations were calculated
using the following formulas:

\[ A = 2A_i \]  \hspace{1cm} (F.1)

\[ I_x = 2I_z \]  \hspace{1cm} (F.2)

\[ I_y = 2(I_y + A_x x_i^2) \]  \hspace{1cm} (F.3)

\[ r_x = r_z = \sqrt{\frac{I_x}{A}} \]  \hspace{1cm} (F.4)

\[ r_y = \sqrt{\frac{I_y}{A}} \]  \hspace{1cm} (F.5)

\[ I_{PS} = I_x + I_y + A(x_0^2 + y_0^2) \]  \hspace{1cm} (F.6)

\[ J = \frac{1}{3} \sum m_i t_i^3 \]  \hspace{1cm} (F.7)

\[ \Gamma = \int_0^m (\bar{\omega}_s - \omega_s)^2 t \, ds \]  \hspace{1cm} (F.8)

where \( \omega_s = \int_0^s r_a \, ds \) also \( \bar{\omega}_s = \frac{1}{m} \int_0^m \omega_s \, ds \)

where \( A \) is the cross-sectional area of the integral cross section; \( A_i \) is the cross-sectional area of a single channel; \( I_x \) and \( I_y \) are the second moments of area of the integral cross section about the \( X \) and \( Y \) axes, respectively; \( I_z \) and \( I_y \) are the second
moments of area of a single channel about the $x$ and $y$ axes, respectively; $x_i$ is the distance from the centroidal axis of the integral cross section to the parallel centroidal axis of a single channel section; $r_x$ and $r_y$ are the radii of gyration of the integral cross section about the $X$ and $Y$ axes, respectively; $r_z$ is the radius of gyration of a single channel about the $z$ axis; $I_{ps}$ is the polar second moment of area of the integral cross section about the shear centre; $x_o, y_o$ are the distances of the shear centre from the centroid in the $X$ and $Y$ directions, respectively (i.e. for the double channel sections the shear centre coincides with the centroid and $x_o = y_o = 0$); $J$ is the torsion constant; $m_i$ is the length of each segment of the cross section; $t_i$ is the average thickness of each segment of the cross section; $\Gamma$ is the warping constant and was calculated for one channel about the shear centre of the integral cross section, $O$, then multiplied by 2; $m$ is the total length of the middle line of a cross section; $t$ is the thickness of thin-walled element along the cross section; $s$ is the distance along the middle line of the cross section from the origin to a point $B$; $\omega_z$ is the warping function; $r_z$ is the normal distance from the tangent at $B$ to the shear centre, as shown in Fig. F.4, and is taken positive if a vector along the tangent and in the direction of increasing $s$ acts counterclockwise about the axis of rotation (shear centre), and $\bar{\omega}_z$ is the average value of $\omega_z$ over the entire cross section (Timoshenko and Gere; 1961).
C 75 x 6

$A_i = 763 \text{ mm}^2$

$I_x = 0.670 \times 10^6 \text{ mm}^4$

$r_x = 29.6 \text{ mm}$

$I_y = 0.077 \times 10^6 \text{ mm}^4$

$r_y = 10.1 \text{ mm}$

$x_o = 22.3 \text{ mm}$

$J_i = 11.0 \times 10^3 \text{ mm}^4$

$\Gamma_i = 0.077 \times 10^9 \text{ mm}^6$

$I_{ps} = 1.126 \times 10^6 \text{ mm}^4$

Note: All dimensions are in mm.

---

FIG. F.1: Nominal cross-sectional properties of C 75 x 6

357
FIG. F.2: Channel sections arranged toe-to-toe – Groups I and II
FIG. F.3: Channel sections arranged back-to-back – Group III
FIG. F.4: Calculation of warping constant

360
Table F.1: Cross-sectional properties of test specimens.

<table>
<thead>
<tr>
<th>Group #</th>
<th>$A$ $\times 10^3$ mm$^2$</th>
<th>$I_x$ $\times 10^4$ mm$^4$</th>
<th>$I_y$ $\times 10^4$ mm$^4$</th>
<th>$r_x$ mm</th>
<th>$r_y$ mm</th>
<th>$I_{ps}$ $\times 10^6$ mm$^4$</th>
<th>$J$ $\times 10^3$ mm$^4$</th>
<th>$\Gamma$ $\times 10^6$ mm$^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I &amp; II</td>
<td>1.53</td>
<td>1.34</td>
<td>1.34</td>
<td>29.6</td>
<td>29.6</td>
<td>2.68</td>
<td>22.0</td>
<td>0.312</td>
</tr>
<tr>
<td>III</td>
<td>1.53</td>
<td>1.34</td>
<td>0.528</td>
<td>29.6</td>
<td>18.6</td>
<td>1.87</td>
<td>22.0</td>
<td>0.180</td>
</tr>
</tbody>
</table>

\[
A = 2A_i \\
I_x = 2I_x \\
I_y = 2(I_y + A_i x_i^2) \\
r_x = \sqrt{\frac{I_x}{A}}
\]

\[
r_y = \sqrt{\frac{I_y}{A}} \\
I_{ps} = I_x + I_y + A(x_o^2 + y_o^2) \\
J = \frac{1}{3} \sum m_i t_i^3
\]

\[
\Gamma = \int_0^m (\omega_s - \omega_s) t \, ds \quad \text{where} \quad \omega_s = \int_0^s r_n \, ds \quad \text{also} \quad \overline{\omega_s} = \frac{1}{m} \int_0^m \omega_s \, ds
\]
APPENDIX G

MODES OF FAILURE

The built-up channel sections described in Appendix F are doubly symmetric about the major and minor principal axes. Consequently, there is no possibility of flexural-torsional buckling occurring to the integral column about these axes. However, the individual channel section, forming a local column between interconnectors, has only one axis of symmetry as shown in Fig. F.1. Hence, there is a possibility of local flexural buckling occurring about the weak y axis (in the plane of symmetry), and local flexural-torsional buckling occurring about the strong x axis and the shear centre. Referring to the nomenclature in Appendix F, the possible modes of failure of the built-up channel sections are:

1- Global flexural buckling about the principal axes given by

\[ P_x = \frac{\pi^2EA}{\Lambda_x^2} \quad P_y = \frac{\pi^2EA}{\Lambda_{eq}^2} \quad (G.1) \]

where the appropriate value for the effective length factors, \( K_x \) and \( K_y \), is used;

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0.5 for fixed-end conditions and 1.0 for pinned-end conditions.

2- Global torsional buckling about the shear centre, which corresponds to point O in Figs. F.2 and F.3, given by

\[ P_{\phi} = \frac{\pi^2EA}{(KL/r)^2_{\phi}} \quad \text{where} \quad r_{\phi} = \sqrt{\frac{(KL)_{\phi}^2}{\pi^2E} \frac{GJ}{I_{PS}} + \frac{\Gamma}{I_{PS}}} \]  \hspace{1cm} (G.2)

and \( r_{\phi} \) is known as the global equivalent radius of gyration for torsional buckling. For the columns in question the knife edge prevents rotation about the global Z axis and so \( K_{\phi} \) can be taken as 0.5.

3- Local flexural buckling load of an individual channel about the local y axis given by

\[ P_y = \frac{\pi^2EA_i}{(K_y r)^2_y} \]  \hspace{1cm} (G.3)

where, according to Appendix C and the results of the Julian and Lawrence nomograph, \( K_y \) given in Table 6.5(b) is approximately 1.0 taking into consideration the unbraced nature of the local column.

4- Local flexural-torsional buckling of an individual channel about the local x axis given by
\[ P_z = \frac{1}{2R} \left[ P_x + P_\phi - \sqrt{(P_x + P_\phi)^2 - 4RP_xP_\phi} \right] \]

where \[ R = \left( 1 - \frac{x_0^2}{r_0^2} \right) \]

such that \[ r_o = \sqrt{\frac{I_{yz}}{A_t}} \] (G.4)

and \( I_{yz} \) is the polar second moment of area of a single channel section about its shear centre. In Eq. (G.4) \( P_x \) and \( P_\phi \) can be taken as

\[ P_x = \frac{\pi^2EA_i}{(K_a/r)^2_z} \]
\[ P_\phi = \frac{\pi^2EA_i}{(K_a/r)^2_\phi} \] (G.5)

The expression for \( r_o \) in Eq. (G.2) can be used in the above expression for \( r_o \) to calculate \( P_\phi \) using the cross-sectional properties of a single channel. The problem now is to estimate values for \( K_{ir} \) and \( K_{ip} \). Although the batten interconnectors are flexible about their centroidal axis parallel to the \( X \) axis, the action of two planes of interconnectors can give virtually fixed-end conditions to the local column. This, in addition to the unbraced nature of the local column, give \( K_{ir} \) a value of about 1.0, whereas the value of \( K_{ip} \) can be taken as 0.5 for local torsional buckling which is not affected by sidesway. In the case of button interconnectors, which are rigid about the local \( x \) axis, it is again appropriate to take them as 1.0 and 0.5, respectively. To determine the critical mode of buckling for local buckling it is easier to compare the radii of gyration. Hence, Eq. (G.4) can be written in the form
\[ P_f = \frac{\pi^2E A_i}{(K_i d/r_{eq})^2} \quad \text{where} \quad r_{eq}^2 = \frac{1}{2R} \left[ r_x^2 + r_y^2 - \sqrt{(r_x^2 + r_y^2)^2 - 4R^2 r_x^2 r_y^2} \right] \] (G.6)

where \( r_{eq} \) is the equivalent radius of gyration for flexural-torsional buckling.

5- Local plate buckling of the flanges or the web of the channel sections, which is rarely the cause of failure for hot rolled sections. The expression for this buckling mode is

\[ \sigma_{cr} = \frac{k_b \pi^2E}{12(1-v^2)} \left( \frac{t}{w} \right)^2 \] (G.7)

where \( \sigma_{cr} \) is the critical stress; \( k_b \) is a buckling stress coefficient taken as 0.425 for the flanges (one side simply supported one side free) and 4.0 for the webs (both sides simply supported) (Chajes; 1974); \( t \) is the average thickness of the plate; \( w \) is the width of the plate; and \( v \) is Poisson's ratio taken as 0.3. The formula in (G.7) predicted compressive resistances greater than the yield capacity such that \( P_f = 2884 \text{ kN} \) (\( \approx P_{yf} = 338.1 \text{ kN} \)) for the flanges and \( P_w = 1513 \text{ kN} \) (\( \approx P_{yw} = 228.8 \text{ kN} \)) for the webs. Hence, it can be concluded that this mode of buckling is not critical.

The buckling loads of the modes of buckling listed in 1-4 above are shown for the experimental specimens in Table G.1. To overcome the problem of
inelastic buckling the design equation in S16.1 (1994) was used instead of the Euler buckling load. The local buckling loads have been multiplied by two to account for both members. Table G.1 shows that either global flexural buckling about the $Y$ axis or global flexural buckling about the $X$ axis governs.
Table G.1: Theoretical buckling loads for all buckling modes.

<table>
<thead>
<tr>
<th>Specimen No. (1)</th>
<th>$P_x$ (kN) (2)</th>
<th>$P_y$ (kN) (3)</th>
<th>$P_\phi$ (kN) (4)</th>
<th>$P_\gamma$ (kN) (5)</th>
<th>$P_f$ (kN) (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120-40-1</td>
<td>173.5</td>
<td>90.2</td>
<td>508.5</td>
<td>100.7</td>
<td>362.2</td>
</tr>
<tr>
<td>120-63-1</td>
<td>173.5</td>
<td>91.2</td>
<td>508.5</td>
<td>102.0</td>
<td>363.7</td>
</tr>
<tr>
<td>120-63-2</td>
<td>173.5</td>
<td>162.6</td>
<td>508.5</td>
<td>198.6</td>
<td>433.6</td>
</tr>
<tr>
<td>120-63-3</td>
<td>173.5</td>
<td>221.0</td>
<td>508.5</td>
<td>288.8</td>
<td>467.3</td>
</tr>
<tr>
<td>120-63-4</td>
<td>173.5</td>
<td>263.4</td>
<td>508.5</td>
<td>358.9</td>
<td>487.4</td>
</tr>
<tr>
<td>120-63-5</td>
<td>173.5</td>
<td>293.0</td>
<td>508.5</td>
<td>408.7</td>
<td>500.5</td>
</tr>
<tr>
<td>70-63-1</td>
<td>340.5</td>
<td>233.4</td>
<td>510.1</td>
<td>255.1</td>
<td>456.2</td>
</tr>
<tr>
<td>70-63-2</td>
<td>340.5</td>
<td>339.4</td>
<td>510.1</td>
<td>381.3</td>
<td>493.4</td>
</tr>
<tr>
<td>70-63-3</td>
<td>340.5</td>
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<td>510.1</td>
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<td>510.7</td>
</tr>
<tr>
<td>120-A-1</td>
<td>478.7</td>
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<td>458.5</td>
<td>229.6</td>
<td>446.8</td>
</tr>
<tr>
<td>120-U-1</td>
<td>478.7</td>
<td>141.5</td>
<td>458.5</td>
<td>229.6</td>
<td>446.8</td>
</tr>
<tr>
<td>120-A-2</td>
<td>478.7</td>
<td>140.1</td>
<td>458.5</td>
<td>358.2</td>
<td>487.2</td>
</tr>
<tr>
<td>120-U-2</td>
<td>478.7</td>
<td>157.7</td>
<td>458.5</td>
<td>358.2</td>
<td>487.2</td>
</tr>
<tr>
<td>120-A-3</td>
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<td>153.1</td>
<td>458.5</td>
<td>432.3</td>
<td>506.6</td>
</tr>
<tr>
<td>120-U-3</td>
<td>478.7</td>
<td>164.2</td>
<td>458.5</td>
<td>432.3</td>
<td>506.6</td>
</tr>
<tr>
<td>120-A-4</td>
<td>478.7</td>
<td>159.9</td>
<td>458.5</td>
<td>472.0</td>
<td>516.9</td>
</tr>
<tr>
<td>120-U-4</td>
<td>478.7</td>
<td>167.4</td>
<td>458.5</td>
<td>472.0</td>
<td>516.9</td>
</tr>
</tbody>
</table>

Note: Loads were calculated using the formula in S16.1-94.

\[
P_x = \frac{\pi^2EA}{\Lambda_x^2} \quad P_y = \frac{\pi^2EA}{\Lambda_{eq}^2} \quad P_\phi = \frac{\pi^2EA}{(KL/r)_{\phi}^2}
\]

\[
r_\phi = \sqrt{\frac{(KL)_{\phi}^2 GJ}{\pi^2E I_{PS}} + \frac{\Gamma}{I_{PS}}}
\]

\[
P_y = \frac{\pi^2EA_i}{(K\alpha/r)_{y}^2} \quad P_x = \frac{\pi^2EA_i}{(K\alpha/r)_{x}^2} \quad P_\phi = \frac{\pi^2EA_i}{(K\alpha/r)_{\phi}^2}
\]

\[
P_f = \frac{1}{2R} \left[ P_x + P_\phi - \sqrt{(P_x + P_\phi)^2 - 4RP_x P_\phi} \right] \quad R = \left( 1 - \frac{x_0^2}{r_o^2} \right) \quad r_o = \sqrt{\frac{I_{PS}}{A_i}}
\]
APPENDIX H

STRAIN ROSETTES

H.1 Introduction

The normal strains at a point on a free surface and parallel to the free surface, hence the principal strains, can be determined using a cluster of three electric-resistance strain gauges arranged in a specific pattern called a strain rosette, as shown in Fig. H.1. The readings on the three gauges give the state of strain at a point. The strains are measured only in the plane of the gauges. Since the body is stress-free in a direction perpendicular to the gauges, the gauges are subjected to plane stress but not plane strain. In this regard, the normal to the free surface is a principal axis of strain. The principal strain along this axis is not measured by the strain rosette, and its effect on the in-plane measurements of the gauges is not important as this is a plane stress problem. The gauge readings can then be converted into principal strains and hence principal stresses as described in the following sections.
H.2 Transformation of Strains

Assuming that the state of strain at a point is such that the unit components of normal strain in the \( x \) and \( y \) directions and the unit shearing strain in the \( x-y \) plane are \( \varepsilon_x, \varepsilon_y, \) and \( \gamma_{xy}, \) respectively, the unit elongation in any direction \( \theta \) can be found (Timoshenko and Goodier, 1970). Consider a line element \( PQ \) between points \((x,y)\) and \((x+dx,y+dy)\) as shown in Fig. H.2(a). This element is translated, elongated (or contracted), and rotated into the line element \( P'Q' \) when deformation occurs, such that the displacement components of \( P \) are \((u,v)\), and of \( Q \) are

\[
\begin{align*}
\frac{u + \partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \\
\frac{v + \partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy
\end{align*}
\]  

(H.1)

If \( P'Q' \) is now translated so that \( P' \) is brought back to \( P \), as shown in Fig. H.2(b), it is now in the position \( PQ'' \), and \( QR, RQ'' \) represent the components of the displacement of \( Q \) relative to \( P \). Thus

\[
\begin{align*}
QR &= \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \\
RQ'' &= \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy
\end{align*}
\]  

(H.2)

The component of this relative displacement \( SQ'' \) along \( PQ'' \), ignoring the small angle \( QPS \) in comparison with \( \theta \), is

\[
SQ'' = QR \cos \theta + RQ'' \sin \theta
\]  

(H.3)

Hence, \( SQ'' \) gives the elongation of \( PQ \), and the unit elongation of \( P'Q' \), denoted by \( \varepsilon_\theta \), is \( SQ''/PQ \). Substituting Eq. (H.2) into Eq. (H.3) and dividing by \( ds \) gives
\[ \varepsilon_\theta = \cos \theta \left( \frac{\partial u}{\partial x} \frac{dx}{ds} + \frac{\partial u}{\partial y} \frac{dy}{ds} \right) + \sin \theta \left( \frac{\partial v}{\partial x} \frac{dx}{ds} + \frac{\partial v}{\partial y} \frac{dy}{ds} \right) \]  

(H.4)

Taking advantage of the trigonometric relationships in Fig. H.2(a) and the definition of \( \varepsilon_x, \varepsilon_y, \) and \( \gamma_{xy} \) gives the expression of the normal strain at a point in any direction \( \theta \) as

\[ \varepsilon_\theta = \varepsilon_x \cos^2 \theta + \varepsilon_y \sin^2 \theta + \gamma_{xy} \sin \theta \cos \theta \]  

(H.5)

Returning to Fig. H.1 the strain read by each gauge a, b, and c can be expressed as

\[ \varepsilon_a = \varepsilon_x \cos^2 \theta_a + \varepsilon_y \sin^2 \theta_a + \gamma_{xy} \sin \theta_a \cos \theta_a \]
\[ \varepsilon_b = \varepsilon_x \cos^2 \theta_b + \varepsilon_y \sin^2 \theta_b + \gamma_{xy} \sin \theta_b \cos \theta_b \]
\[ \varepsilon_c = \varepsilon_x \cos^2 \theta_c + \varepsilon_y \sin^2 \theta_c + \gamma_{xy} \sin \theta_c \cos \theta_c \]  

(H.6)

Solving these three equations would give expressions for \( \varepsilon_x, \varepsilon_y, \) and \( \gamma_{xy} \) in terms of the gauge readings. Considering the special case of the 45° rosette we can take \( \theta_a = 0^\circ, \theta_b = 45^\circ, \) and \( \theta_c = 90^\circ \). This would give the strains at a point as

\[ \varepsilon_x = \varepsilon_a \quad \varepsilon_y = \varepsilon_c \quad \gamma_{xy} = 2 \varepsilon_b - (\varepsilon_a + \varepsilon_c) \]  

(H.7)

**H.3 Principal Stresses and Strains**

**H.3.1 Principal Strains**

The principal strains \( \varepsilon_{1,2} \) in the plane of the rosette are illustrated in a circle shown in Fig. H.3(a), and can be calculated from the following expr
\[ \varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \]  

(H.8)

where the directions of the principal strains from the direction of \( \varepsilon_x \) is \( \theta_{1,2} \) which can be expressed as

\[ \theta_{1,2} = \frac{1}{2} \tan^{-1} \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \]  

(H.9)

Making use of the expressions in Eq. (H.7) the principal strains and their angle of rotation can be calculated from

\[ \varepsilon_{1,2} = \frac{1}{2} \left[ \varepsilon_a + \varepsilon_c \pm \sqrt{(\varepsilon_a - \varepsilon_c)^2 + (2\varepsilon_b - \varepsilon_a - \varepsilon_c)^2} \right] \]

\[ \theta_{1,2} = \frac{1}{2} \tan^{-1} \frac{2\varepsilon_b - \varepsilon_a - \varepsilon_c}{\varepsilon_a - \varepsilon_c} \]  

(H.10)

**H.3.2 Principal Stresses**

The principal stresses \( \sigma_{1,2} \) in the plane of the interconnector are illustrated in Mohr's circle, shown in Fig. H.3(b), and can be calculated from the following expression

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \]  

(H.11)

where \( \sigma_x \) and \( \sigma_y \) are the unit normal stresses in the \( x \) and \( y \) directions, respectively; and \( \tau_{xy} \) is the unit shear stress on the plane perpendicular to the \( x \) axis and parallel
to the \( y \) axis and is equal in magnitude to \( \tau_{yx} \). As this is a plane stress problem the stresses \( \sigma_x \), \( \sigma_y \), and \( \tau_{xy} \), can be expressed in terms of the strains \( e_x \), \( e_y \), and \( \gamma_{xy} \) as

\[
\sigma_x = \frac{E}{(1-v^2)}(e_x + v e_y) \\
\sigma_y = \frac{E}{(1-v^2)}(e_y + v e_x) \\
\tau_{xy} = G\gamma_{xy} = \frac{E}{2(1+v)}\gamma_{xy}
\]  

(H.12)

Substituting these expressions into Eq. (H.11) gives the expression for principal stresses in terms of strains as

\[
\sigma_{1,2} = \frac{E}{2} \left[ \frac{1}{1-v}(e_x + e_y) \pm \frac{1}{1+v} \sqrt{(e_x - e_y)^2 + (e_{xy})^2} \right]
\]  

(H.13)

By using the expressions in Eq. (H.7) the principal stresses can be calculated in terms of the strain gauge readings as follows

\[
\sigma_{1,2} = \frac{E}{2} \left[ \frac{1}{1-v}(e_a + e_c) \pm \frac{1}{1+v} \sqrt{(e_a - e_c)^2 + (2e_b - e_a - e_c)^2} \right]
\]  

(H.14)

Similarly the angle \( \theta_{p1,2} \) of the principal stresses can be found to be the same as that in Eq. (H.10).
FIG. H.1: Configuration of strain rosette
FIG. H.2: Transformation of strain
(a) Mohr's circle for strains

(b) Mohr's circle for stresses

FIG. H.3: Mohr's circle for strains and stresses
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