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COMPUTER SIMULATION OF TURNING PROCESSES

By

Liheng Fan

A Thesis
Submitted to the Faculty of Graduate Studies and Research
through the Department of Industrial Engineering in
Partial Fulfilment of the Requirements for
The Degree of Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

1994

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#### THE HUMANITIES AND SOCIAL SCIENCES

<table>
<thead>
<tr>
<th>COMMUNICATIONS AND THE ARTS</th>
<th>( 0779 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Art History</td>
<td>( 0799 )</td>
</tr>
<tr>
<td>Cinema</td>
<td>( 0906 )</td>
</tr>
<tr>
<td>Dance</td>
<td>( 0916 )</td>
</tr>
<tr>
<td>Fine Arts</td>
<td>( 0926 )</td>
</tr>
<tr>
<td>Information Science</td>
<td>( 0723 )</td>
</tr>
<tr>
<td>Journalism</td>
<td>( 0791 )</td>
</tr>
<tr>
<td>Library Science</td>
<td>( 0799 )</td>
</tr>
<tr>
<td>Mass Communications</td>
<td>( 0708 )</td>
</tr>
<tr>
<td>Music</td>
<td>( 0413 )</td>
</tr>
<tr>
<td>Speech Communication</td>
<td>( 0459 )</td>
</tr>
<tr>
<td>Theater</td>
<td>( 0465 )</td>
</tr>
</tbody>
</table>

#### EDUCATION

| General                     | \( 0515 \) |
| Administration              | \( 0514 \) |
| Adult and Continuing        | \( 0516 \) |
| Agricultural                | \( 0517 \) |
| Art                         | \( 0527 \) |
| Bilingual and Multicultural | \( 0282 \) |
| Business                    | \( 0688 \) |
| Community College           | \( 0275 \) |
| Curriculum and Instruction  | \( 0727 \) |
| Early Childhood             | \( 0516 \) |
| Elementary                  | \( 0524 \) |
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| Guidance and Counseling     | \( 0219 \) |
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| History of Education        | \( 0520 \) |
| Home Economics              | \( 0278 \) |
| Industrial Education        | \( 0251 \) |
| Language and Literature     | \( 0279 \) |
| Mathematics                 | \( 0280 \) |
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| Philosophy of Physical      | \( 0998 \) |

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| Religion                    | \( 0422 \) |
| General                     | \( 0576 \) |
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| Management               | \( 0276 \) |
| Marketing                | \( 0338 \) |
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| Economics                | \( 0501 \) |
| General                   | \( 0503 \) |
| Commerce                  | \( 0519 \) |
| Finance                   | \( 0208 \) |
| History                   | \( 0509 \) |
| Theory                    | \( 0511 \) |
| Forestry                  | \( 0253 \) |
| Geography                 | \( 0266 \) |
| Gerontology               | \( 0351 \) |
| History                   | \( 0578 \) |

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| Plant Pathology           | \( 0480 \) |
| Plant Physiology          | \( 0481 \) |
| Range Management          | \( 0777 \) |
| Wood Technology           | \( 0746 \) |
| Biochemistry              | \( 030x \) |
| Anatomy                   | \( 0287 \) |
| Botany                    | \( 030x \) |
| Cell                      | \( 030x \) |
| Ecology                   | \( 0329 \) |
| Entomology                | \( 0353 \) |
| Genetics                  | \( 0379 \) |
| Limnology                 | \( 0791 \) |
| Microbiology              | \( 0410 \) |
| Molecular Biology          | \( 0270 \) |
| Neuroscience              | \( 0317 \) |
| Oceanography              | \( 0416 \) |
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| Astrophysics               | \( 0478 \) |
| Chemistry                  | \( 0269 \) |
| Geology                    | \( 0320 \) |
| Geophysics                 | \( 0323 \) |
| Hydrology                  | \( 0386 \) |
| Mineralogy                 | \( 0286 \) |
| Paleontology               | \( 0345 \) |
| Paleontologist             | \( 0423 \) |
| Paleozoology               | \( 0418 \) |
| Paleosociology             | \( 0437 \) |
| Paleokosmology             | \( 0458 \) |
| Paleogeology               | \( 0464 \) |
| Paleooceanography          | \( 0415 \) |

#### HEALTH AND ENVIRONMENTAL SCIENCES

| Environmental Sciences     | \( 0768 \) |
| Health Sciences            | \( 0366 \) |
| Immunology                 | \( 0393 \) |
| Microbiology               | \( 0364 \) |
| Nanotechnology             | \( 0365 \) |
| Nematology                 | \( 0366 \) |
| Physical Biology           | \( 0479 \) |
| Radiation                  | \( 0275 \) |
| Veterinary Science         | \( 0278 \) |

### PSYCHOLOGY

| General Psychology        | \( 0223 \) |
| Health Psychology         | \( 0223 \) |
| Speech Pathology          | \( 0460 \) |
| Home Economics            | \( 0386 \) |
| Engineering               | \( 0373 \) |
| Aerospace                 | \( 0353 \) |
| Agricultural              | \( 0358 \) |
| Automotive                | \( 0230 \) |
| Biomedical                | \( 0241 \) |
| Chemical                  | \( 0342 \) |
| Civil                     | \( 0349 \) |
| Electromagnetics          | \( 0344 \) |
| Heat and Thermodynamics   | \( 0344 \) |
| Hydraulic                 | \( 0349 \) |
| Industrial                | \( 0348 \) |
| Materials Science         | \( 0379 \) |
| Metallurgy                | \( 0373 \) |
| Metallurgy                | \( 0374 \) |
| Mining                    | \( 0351 \) |
| Nuclear                   | \( 0373 \) |
| Nuclear                    | \( 0375 \) |
| Packaging                 | \( 0359 \) |
| Petroleum                 | \( 0365 \) |
| Sanitary and Municipal    | \( 0354 \) |
| System Science            | \( 0250 \) |
| Geotechnology             | \( 0428 \) |
| Operations Research       | \( 0359 \) |
| Plates Technology         | \( 0795 \) |
| Textile Technology        | \( 0994 \) |

### PSYCHOLOGY

| General Psychology        | \( 0223 \) |
| Health Psychology         | \( 0223 \) |
| Speech Pathology          | \( 0460 \) |
| Home Economics            | \( 0386 \) |
| Engineering               | \( 0373 \) |
| Aerospace                 | \( 0353 \) |
| Agricultural              | \( 0358 \) |
| Automotive                | \( 0230 \) |
| Biomedical                | \( 0241 \) |
| Chemical                  | \( 0342 \) |
| Civil                     | \( 0349 \) |
| Electromagnetics          | \( 0344 \) |
| Heat and Thermodynamics   | \( 0344 \) |
| Hydraulic                 | \( 0349 \) |
| Industrial                | \( 0348 \) |
| Materials Science         | \( 0379 \) |
| Metallurgy                | \( 0373 \) |
| Mining                    | \( 0351 \) |
| Nuclear                   | \( 0373 \) |
| Nuclear                    | \( 0375 \) |
| Packaging                 | \( 0359 \) |
| Petroleum                 | \( 0365 \) |
| Sanitary and Municipal    | \( 0354 \) |
| System Science            | \( 0250 \) |
| Geotechnology             | \( 0428 \) |
| Operations Research       | \( 0359 \) |
| Plates Technology         | \( 0795 \) |
| Textile Technology        | \( 0994 \) |
ABSTRACT

This thesis proposes a new three dimensional cutting process model. The basic features and improvements over previously developed models include: (1) Cutting force calculations based on the concept of equivalent orthogonal cutting process (EOC), which converts the modelling of three dimensional cutting processes into the modelling of orthogonal cutting processes. In the model, both cutting force coefficient and chip load are considered as the functions of cutting conditions, tool geometry, and machine-tool structural vibrations. (2) Microstructure hardness variation of workpiece material has been taken into consideration. (3) The regenerative mechanism and mode coupling effect in machining are included. The structural dynamics equations, which include five vibration modes, are in the form of a set of simultaneous differential equations. The fourth-order Runge-Kutta method is applied to solve these equations numerically.

Based on the proposed model, systematic simulation of turning processes has been conducted. The simulation results show that feed and tool nose radius are the primary cutting parameters in determining surface finish. Surface finish improves with the decrease of feed. The effect of tool nose radius is not monotonic: surface finish improves with the increase of tool nose radius when the tool nose radius is below a certain limit. However, above that limit, surface finish becomes worse with the further increase of tool nose radius. Cutting speed, depth of cut and tool geometrical angles are secondary parameters in determining surface finish, and have much smaller influences.

The simulation results of surface finish are verified experimentally. The simulated surface finish are in agreement with the experimental results.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>x</td>
</tr>
<tr>
<td><strong>CHAPTER</strong></td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. LITERATURE REVIEW</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Static Cutting Force Models</td>
<td>6</td>
</tr>
<tr>
<td>2.2 Dynamic Cutting Process Models</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Surface Finish Mechanisms</td>
<td>17</td>
</tr>
<tr>
<td>2.4 Computer Simulation of Surface Generation</td>
<td>21</td>
</tr>
<tr>
<td>3. THE PROPOSED COMPUTER SIMULATION MODEL</td>
<td>24</td>
</tr>
<tr>
<td>3.1 Structures and Assumptions of the Proposed Model</td>
<td>24</td>
</tr>
<tr>
<td>3.2 The Equivalent Orthogonal Cutting Process</td>
<td>27</td>
</tr>
<tr>
<td>3.3 The Calculation of EOC Parameters</td>
<td>32</td>
</tr>
<tr>
<td>3.4 Cutting Force Equations</td>
<td>46</td>
</tr>
<tr>
<td>3.5 Structural Dynamics Equations</td>
<td>50</td>
</tr>
<tr>
<td>3.6 Surface Generation</td>
<td>53</td>
</tr>
<tr>
<td>4. SIMULATION RESULTS AND EXPERIMENTAL CONFIRMATION</td>
<td>56</td>
</tr>
<tr>
<td>4.1 Simulation Results</td>
<td>56</td>
</tr>
<tr>
<td>4.2 Experimental Confirmation</td>
<td>61</td>
</tr>
<tr>
<td>4.3 A Critical View On surface Finish</td>
<td>66</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Fig. 2.1 Merchant's orthogonal cutting process model [Armarego. 1969]

Fig. 2.2 Usui's three dimensional cutting process model [Usui, 1978]

Fig. 2.3 Illustration of ploughing force

Fig. 2.4 Block diagram of transfer function models [Endres. 1991]

Fig. 2.5 Block diagram of mechanistic models [Endres. 1991]

Fig. 2.6 Calculation of ideal surface roughness [Shaw. 1984]

Fig. 2.7 Calculation of the effect of minimum undeformed chip thickness [Shaw. 1984]

Fig. 2.8 The effect of side flow

Fig. 3.1 Input-output relationship of the proposed model

Fig. 3.2 Block diagram of the proposed computer simulation model

Fig. 3.3 Illustration of inclination angle [Shaw. 1984]

Fig. 3.4 Coordinate systems of the proposed model

Fig. 3.5 The concept of EOC

Fig. 3.6 Determination of chip flow direction [Colwell, 1954]

Fig. 3.7 EOC parameter calculation - Case I

Fig. 3.8 EOC parameters calculation - Case II

Fig. 3.9 Rake and relief angle change due to vibration in y direction

Fig. 3.10 Calculation of effective rake angle
Fig. 3.11 Rake and relief angle change due to vibration in x and z direction

Fig. 3.12 Time shift due to orientation of shear plane

Fig. 3.13 Workpiece microstructure and its effects on cutting tool [Zhang, 1991]

Fig. 3.14 Geometrical shape of chip samples [Zhang, 1991]

Fig. 3.15 Structural dynamics model

Fig. 3.16 Generation of machined surface

Fig. 4.1 Feed and cutting force

Fig. 4.2 Feed and surface finish

Fig. 4.3 Tool nose radius and cutting force

Fig. 4.4 Tool nose radius and surface finish

Fig. 4.5 Depth of cut and cutting force

Fig. 4.6 Depth of cut and surface finish

Fig. 4.7 Cutting speed and cutting force

Fig. 4.8 Cutting speed and surface finish

Fig. 4.9 Rake angle and cutting force

Fig. 4.10 Rake angle and surface finish

Fig. 4.11 Lead angle and cutting force

Fig. 4.12 Lead angle and surface finish

Fig. 4.13 Experimental setup in modal analysis

Fig. 4.14 Inertance transfer function of the workpiece
Fig. 4.15  Compliance transfer function of the workpiece

Fig. 4.16  Experimental setup in surface finish measurement

Fig. 4.17  Surface finish and feed

Fig. 4.18  Surface finish and tool nose radius

Fig. 4.19  Simulated and measured surface profile

Fig. 4.20  Surface finish prediction from different models [El-Wardany, 1991]

Fig. 4.21  Analysis of surface finish with feed

Fig. 4.22  Analysis of surface finish with tool nose radius

Fig. 4.23  Surface finish from different cutting systems
NOMENCLATURE

A three dimensional cutting process chip load

A\textsubscript{or} equivalent orthogonal chip load

b cutting width

C cutting force ratio

C\textsubscript{s} three dimensional cutting process lead angle

C\textsubscript{se} effective lead angle

C\textsubscript{cor} equivalent orthogonal cutting lead angle

CEL cutting edge length

d\textsubscript{0} nominal depth of cut

ECT equivalent chip thickness

f\textsubscript{0} nominal feed rate

F\textsubscript{x}, F\textsubscript{y}, F\textsubscript{z} cutting force in x, y, z direction

F\textsubscript{xw}, F\textsubscript{yw}, F\textsubscript{zw} cutting force without considering ploughing force and microhardness

F\textsubscript{xp}, F\textsubscript{yp}, F\textsubscript{zp} cutting force considering ploughing force

F\textsubscript{p}, F\textsubscript{q} cutting force in orthogonal plane

F\textsubscript{c}, F\textsubscript{dx}, F\textsubscript{dz} ploughing force

h time interval

i inclination angle
$K$  
specific cutting force

$K_\epsilon$  
damping factor in the cutting process

$m$  
Meyer exponent

$m_{1,c_1,k_1}, m_{2,c_2,k_2}$  
tool dynamic characteristics ($i=1,2,3$)

$m_{2,c_2,k_2}$  
workpiece dynamic characteristics ($i=1,2$)

$r$  
chip thickness ratio

$R$  
tool nose radius

$R_t$  
tool sharpness radius

$R_a$  
arithmetic average surface roughness

$R'_a$  
$R_a$ considering minimum undeformed chip thickness

$R_{th}$  
peak to valley surface roughness

$R'_{th}$  
$R_{th}$ considering minimum undeformed chip thickness

$N$  
workpiece rotation speed

$s$  
undeformed chip thickness

$t_m$  
minimum undeformed chip thickness

$t'$  
time lag between two end points of the shear plane

$v$  
cutting velocity

$W(r)$  
geometrical shape function

$x,y,z$  
tool-workpiece relative vibration

$X,Y,Z$  
global coordinates

$\xi$
\( x_i, y_i, z_i \)  tool vibration
\( x_n, y_n, z_n \)  workpiece vibration
\( x_n \)  vibration in the direction normal to cutting edge
\( \tau_s \)  shear stress
\( \beta \)  friction angle on tool face
\( \alpha \)  rake angle
\( \alpha_0 \)  nominal rake angle
\( \alpha_e \)  effective rake angle
\( \alpha_{e0} \)  effective rake angle without vibration
\( \alpha_y \)  normal rake angle considering vibration in y direction
\( \alpha_{ey} \)  effective rake angle considering vibration in y direction
\( \delta \alpha_e \)  effective rake angle due to vibration in x-z plane direction
\( \gamma_e \)  effective relief angle
\( \gamma_{e0} \)  effective relief angle without vibration
\( \gamma_y \)  normal relief angle considering vibration in y direction
\( \gamma_{ey} \)  effective relief angle considering vibration in y direction
\( \delta \gamma_e \)  effective relief angle change due to vibration in x-z plane
\( \phi \)  shear angle
\( \phi_e \)  effective shear angle change
\( \phi_{or} \)  equivalent orthogonal shear angle
\( \delta \phi \) shear angle change due to vibration in previous pass cutting

\( \sigma_y \) yield stress of workpiece material

\( \sigma_v \) variance of workpiece material microhardness

\( \rho(r) \) correlation function for microhardness variance calculation
CHAPTER 1

INTRODUCTION

The study of turning has lasted more than a century, but it still attracts a large amount of research effort. This is because turning is not only the most frequently used machining operation in the modern manufacturing industry, but also because it is a typical single-point machining operation. Other machining operations, such as milling, drilling, and boring are multiple-point machining operations, that can be investigated based on the combinations of single-point machining operations. Thus the study of turning can contribute greatly to the knowledge of metal cutting principles and machining practice.

However, turning is a complex process. It involves friction, plastic flow and fracture of materials under more extreme conditions than those normally found in other manufacturing processes, such as stamping and drawing. As a result, high temperature and high strain rate are characteristic. It is important to understand how the process "inputs", such as cutting conditions (depth of cut, feed, cutting speed), tool geometry (nose radius, rake angle, relief angle, lead angle, etc.) affect the process behaviour or "output", such as cutting stability, tool wear, surface finish, cutting force, and cutting power. The understanding will enable engineers to optimize cutting operations, which is the motivation in developing models to describe the input-output relationship of a cutting process. Due to great advances in computers, computer simulation has become a powerful tool in studying machining processes. Computer simulation
has a number of advantages over analytical and traditional experimental methods: it can be run without expensive experimental setup, requires much less cutting tests, and more importantly, it can reveal detailed information and mechanisms in metal cutting processes, such as the basic non-linearity in chatter vibration [Tlusty and Ismail, 1981].

In this thesis, modeling of vibrations and cutting forces is first investigated. It is known that dimensional accuracy, surface finish, and productivity are the three major requirements in turning, especially in finish turning. They are all affected by vibration, which is a function of system dynamic characteristics and cutting forces. Poor dynamic characteristics and large cutting forces can induce excessive vibration or chatter, which would result in poor surface finish, early cutter failure, and noise. Vibration in cutting process, involves very complex mechanisms and is difficult to control. Vibration during turning can be classified into two categories: forced vibration and regenerative vibration. One kind of forced vibration may be caused by excitation forces outside the cutting zone, such as from a vibrating base, or from the unbalance of the workpiece being machined. Another kind of forced vibration may be initiated by the cutting process, such as the interrupted cutting. The characteristic of the forced vibration is that the predominant vibration frequency component coincides with the excitation force.

Self-excited vibration in turning may take two forms. The first form is the regenerative effect [Hahn, 1954]. This is due to the fact that vibration in the previous revolution leaves behind an undulation of workpiece surface. This undulation causes the changes of cutting
conditions which brings about the variation of cutting force. The variation of cutting force keeps the vibration maintained or amplified, which in turn leaves behind surface undulation for cutting in the subsequent revolution, and so forth. The second form is the effect of mode coupling [Tlusty and Polacek, 1963], whereby the variation of cutting condition in one direction may induce cutting force change as well as vibration in the perpendicular direction.

Generally speaking, turning is a three dimensional cutting process, i.e. the cutting edge involved in cutting is not a straight edge, instead, tool nose radius takes a substantial part of cutting load. This is particularly true in finish turning, where both feed and depth of cut are small. There are two problems with the current three dimensional cutting process models. The first problem is the assumption that cutting force coefficient is a constant, which results in two shortcomings: (1) the number of cutting tests required to calibrate a model is enormous, due to the combinations of cutting conditions; (2) the omission of cutting force variation caused by the variation of cutting force coefficient. The second problem is that chip load is oversimplified as the product of depth of cut and feed, which neglects the effect of tool nose radius. In order to overcome the above-mentioned shortcomings, the proposed model has made the following improvements:

1. The cutting force coefficient is calculated based on the principles of cutting mechanics, which can include different cutting conditions and the effect of machine-tool vibration in cutting processes. This is done with the aid of the concept of equivalent orthogonal cutting process (EOC), which converts a three dimensional cutting process into its equivalent
orthogonal cutting process.

(2) Chip load is accurately calculated, which accommodates the effect of tool nose radius and machine-tool vibration.

(3) Workpiece microstructure hardness is taken into consideration. The effect of microstructure hardness variation in the light cutting conditions can be severe.

Another major concern of the thesis is the generation mechanisms of surface finish. It is known that there are four effects that can contribute to surface finish [Shaw, 1984]. The first one is the feed mark of tool tip, which reproduces tool tip on the machined surface as a series of arcs equally spaced with the centre points all in the same level. The second is the effect of minimum undeformed chip thickness, which causes a small portion of workpiece material left behind, which should be removed by cutting. The third is the effect of vibration. The fourth is the effect of side flow, which is basically a workpiece material property. The effect of side flow tends to make surface roughness larger. All the effects listed above are considered in the thesis, except the effect of side flow. This is due to the fact that the effect of side flow is small compared to other effects under normal cutting conditions.

The main objectives of the research include:

(1) The development of a new comprehensive computer simulation model, that is capable of simulating cutting force, vibration, and surface finish with a higher accuracy and less cutting test calibration.

(2) Through computer simulation, study of the surface finish generation process under
different cutting conditions and tool geometry.

(3) Investigation of the relationships of cutting force and surface finish with cutting
conditions and tool geometry.

The thesis is organized into five chapters. Chapter 2 reviews the models of metal cutting
processes and discusses the mechanisms of machined surface finish. Chapter 3 describes the
proposed simulation model in detail. Chapter 4 presents the simulation results and
experimental verification. Chapter 5 summarizes the proposed model and the simulation
results, and discusses future work.
CHAPTER 2

LITERATURE REVIEW

For years, researchers in the area of metal cutting have attempted to develop models of cutting processes that describe the mechanisms involved and predict the important behaviours in the process without requiring a large amount of cutting tests. Various models have been developed for this purpose. In this chapter, previous publications relating to the theoretical models are reviewed. The reviewed topics are organized as follows:

(1) Static cutting force models;
(2) Dynamic cutting process models;
(3) Surface finish mechanisms; and
(4) Computer simulation of surface generation.

2.1 Static Cutting Force Models

Static cutting force models deal with situations where machine-tool vibration can be omitted. They can be divided into three groups: three dimensional cutting force models; oblique cutting force models; and orthogonal cutting force models. Three dimensional cutting process is the general form of cutting process, where the cutting edge is a curved edge with cutting
velocity inclined to it. Oblique cutting is the case when cutting edge is a straight line with cutting velocity inclined to it. Orthogonal cutting is the case when cutting edge is a straight line with cutting velocity perpendicular to it. In this section, orthogonal cutting and three dimensional cutting force models will be discussed. The detailed analysis of oblique cutting force models can be found in [Armarego, 1969] [Shaw, 1984].

2.1.1 Orthogonal Cutting Force Models

Orthogonal cutting process modeling forms the foundation of all metal cutting modeling. Although it has taken an inordinate amount of research effort, models which can accurately predict cutting behaviours, like cutting force, temperature, vibration, etc. have not yet been developed. Below some representative works will be briefly reviewed.

2.1.1.1 Merchant’s Model

An early and still the most frequently used work in orthogonal cutting force modeling is that of Merchant [1945]. In this model, the relationships among cutting force, process geometry, and cutting conditions were developed. The resultant force \( R \) may be related to other important forces, such as friction force \( F_\text{f} \) alone the rake face or the power force \( F_p \) in the direction of cutting velocity (see Figure 2.1). Since the resultant cutting force can change in
both direction and magnitude, it is more convenient to consider the two force components $F_p$ and $F_q$ alone and perpendicular to the cutting velocity. These forces are given by the equations below:

\[ F_p = \frac{t \tau_s \cos(\beta - \alpha)}{\sin \phi \cos(\phi + \beta - \alpha)} \]  

(2.1)

\[ F_q = \frac{t \tau_s \sin(\beta - \alpha)}{\sin \phi \cos(\phi + \beta - \alpha)} \]  

(2.2)

where: $t$ is undeformed chip thickness; $b$ is cutting width; $\tau_s$ is shear stress; $\beta$ is friction angle; $\phi$ is shear angle; and $\alpha$ is rake angle. Merchant considered that $\tau_s$ would have the value of the yield stress for the workpiece material, and $\beta$ would have the normal value for dry sliding friction. To determine $\phi$, he used $dF_p/d\phi = 0$, and find that:

\[ \phi = \frac{\pi}{4} - \frac{1}{2} (\beta - \alpha) \]  

(2.3)

Equations (2.1) and (2.2) are based on idealized thin shear plane models, and two assumptions are made. First they involve the minimum-energy principle, which is intuitively appealing, but lacks experimental support. Secondly, $\beta$ is assumed to be a constant, independent of $\phi$. Later research [Drucker and Ekinstein, 1950], [Nigm and Tobias, 1977] found that $\beta$ is related to $\phi$. Experiments by Merchant and other researchers showed that the above model describes the cutting forces satisfactorily. However, the shear angle relationship was found to be inaccurate, and the shear stress and friction value were higher than those of
normal tensile and friction tests.

2.1.1.2 Oxley’s Shear Angle Relationship

Oxley and Palmer [1959] applied slip-line analysis and delivered at the following shear angle equation:

\[ \phi = 50 - 0.8(\beta - \alpha) \] (2.4)

Experimentally, this shear angle equation is found to be more accurate than Merchant’s shear angle equation.

2.1.1.3 Dimensional Analysis of Orthogonal Cutting Process

Drucker and Ekinstein [1950] criticized Merchant’s shear angle equation discussed above as incomplete for not including the effect of feed and cutting speed. They proposed six non-dimensional groups, but did not carry out any experiment. Nigm and Tobias [1977] followed the work of Drucker and Ekinstein, but proposed two different non-dimensional groups. The model analyzed is the same as that of Merchant, which is based on the concept of idealized shear plane. Using dimensional analysis, explicit mathematical expressions are evolved for the chip thickness ratio \( r \) and force ratio \( C \) as functions of the cutting parameters.
From \( r \) and \( C \), shear angle \( \phi \) and cutting force \( F_p \) and \( F_q \) can be calculated. It is shown that [Nigm and Tobias, 1977]:

\[
\beta = \alpha + \tan^{-1} C \tag{2.5}
\]

\[
\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha} \tag{2.6}
\]

When shear angle \( \phi \), friction angle \( \beta \), and shear stress \( \tau \), are known, cutting forces can be calculated as [Armarego, 1969]:

\[
F_p = \frac{b \tau_x}{\sin \phi (\cos \phi - C \sin \phi)} \tag{2.7a}
\]

\[
F_q = C F_p \tag{2.7b}
\]

Nigm and Tobias proposed that the chip thickness ratio and the cutting force ratio take the following forms:

\[
r = F_1 [H(\alpha)s^{a_1}v^{b_1}] \tag{2.8a}
\]

\[
C = F_2 [H(\alpha)s^{a_1'}v^{b_1'}] \tag{2.8b}
\]

where: \( s = \) undeformed chip thickness

\( v = \) cutting speed

\( a_1, b_1, a_1', b_1' = \) material constants

\( H(\alpha) = \) a dimensional function of \( \alpha \)

The explicit forms of Equation (2.8a) and (2.8b) are determined by cutting tests in which
a series of values is assigned to the independent variables α, t, and v; to calculate the chip thickness ratio r and cutting force ratio C. After analyzing cutting test results, the chip thickness ratio and cutting force ratio are given as:

\[ r = e \sin \alpha - (1 - f \sin \alpha) \left[ g + \frac{h}{(10^3 s)^k} \ln(sv''/Q) \right] \]  

(2.9a)

\[ C = -e' \sin \alpha - (1 + f' \sin \alpha) \left[ g' - \frac{h'}{(10^3 s)^k'} \ln(sv''/Q') \right] \]  

(2.9b)

The coefficients contained (e.g., g, h, k, n, Q, e, f, g', h', k', Q') in the above equations are the material characteristics of the workpiece, and for the particular material En2. (British specification, the American equivalent is AISI 1015), they take the value provided in Table 2.1.

<table>
<thead>
<tr>
<th>g</th>
<th>h</th>
<th>k</th>
<th>Q</th>
<th>g'</th>
<th>h'</th>
<th>k'</th>
<th>Q'</th>
<th>e</th>
<th>f</th>
<th>c'</th>
<th>f'</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.147</td>
<td>0.1</td>
<td>0.1</td>
<td>0.075</td>
<td>0.142</td>
<td>0.10</td>
<td>100</td>
<td>0.56</td>
<td>0.83</td>
<td>0.92</td>
<td>0.73</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The shear stress \( \tau_s \) is found to be a material constant, and is not affected by the change of cutting conditions. For the material En2, \( \tau_s \) has a value of 95x10^3 psi.
2.1.2 Three Dimensional Cutting Force Model

In three dimensional cutting, the cutting edge is a curve with cutting velocity inclined to it. Two methods for modeling three dimensional cutting processes have been developed. One is to directly analyze three dimensional cutting mechanics, which can be classified into the analysis of oblique cutting [Merchant, 1945], and double cutting edge cutting [Brown and Armarego, 1964] [Zover, 1966]. However, these efforts are merely of qualitative nature. The other method is based on the equivalent orthogonal cutting process (EOC). The basic idea is that although plastic deformation does not take place in the plane strain condition, as in orthogonal cutting, the deformation can be viewed as a modification of plane plastic deformation, and orthogonal cutting data can be used for prediction. This idea was first proposed by M.C.Shaw [1952], and was later developed into a double edge cutting model by Usui [1978]. Usui’s model is shown in Figure 2.2. The effective rake angle $\alpha_e$ is calculated from the geometric relationship as:

$$\alpha_e = \sin^{-1}(\sin \alpha_n \cos i \cos \eta_e + \sin \eta_e \sin i)$$

(2.10)

where:

- $i = $ inclination angle
- $\alpha_n = $ normal rake angle
- $\eta_e = $ chip flow angle

If $\eta_e$ is known, $\alpha_e$ can be calculated from Equation (2.10). Then the effective shear angle
\( \phi_c \) can be calculated using Equation (2.3). \( \eta_c \) is determined by an minimal energy approach.

and the total cutting energy can be calculated as:

\[
U = U_s - U_f
\]  

(2.11)

where:  
\( U_s = \) shear energy  
\( U_f = \) friction energy on the tool face

Assuming that the chip flows in the direction that minimizes the total cutting energy \( U \). \( \eta_c \) is obtained by numerically minimizing \( U \) with respect to \( \eta_c \). Furthermore, the principle cutting force \( F_N \) can be obtained by solving \( VF_N = U_{\min} \). Thus the three dimensional cutting process is converted to an orthogonal cutting process in the plane containing the chip flow direction.

Stephenson and Wu [1988] proposed a computer model for the mechanics of the three dimensional cutting process which is based on a variational energy method. The total cutting energy is expressed as a function of chip flow angle \( \eta_c \), chip thickness \( t_c \), and chip radius of curl \( R_c \) which are found by minimizing the cutting energy. Based on these parameters \( (\eta_c, t_c, R_c) \), an equivalent orthogonal cutting process was determined.

Rubenstein [1990] proposed an EOC process for oblique cutting based on the following assumptions: (a) The two processes have the same depth of workpiece material extruded below the cutting edge, and; (b) The two processes have the same area of contact below the flank face. Based on these assumptions, an EOC process is set up and the cutting force equations are derived.
2.2 Dynamic Cutting Process Models

If vibration is taken into consideration, the cutting force model is called a dynamic cutting force model. However, when a model has dynamic cutting force as an output, it should also have vibration as an output. Therefore these models are more often referred to as dynamic cutting process models. From the knowledge of the cutting process under steady-state conditions, it is clear that any fluctuation of cutting conditions will cause fluctuation in the cutting force. Depending on how these fluctuations are directed and phased (relative to the vibration), they may either damp or excite the vibration. If the system is subjected to some sudden force pulse, it vibrates, but the damping of the cutting system may eliminate the vibration after a few oscillations. If, however, the cutting force fluctuations are large and are opposing the damping effect, they cause the vibration to grow and maintained, with the damping and exciting forces just in balance. This is self-excited vibration.

In turning operations, the cutting tool moves over the surfaces it has previously machined. If vibration was present on the previous pass, both the surface and cutting forces are undulated. Depending on the phase relationships, this may increase the cutting surface undulations. This effect cannot be explained as a forced vibration, but rather an effect inherent to the cutting process itself. This kind of self-excited vibration is known as regenerative vibration.

A feature of vibration in cutting is the possibility of mode-coupling. A vibration
initiated in one direction may lead to the variations of cutting conditions, which causes force fluctuations not only in the direction of vibration, but also in the direction perpendicular to it. Machining vibrations are usually considered in the following four categories [Armarego, 1969]:

(1) Free vibration. Free vibration may be initiated by some impact or shock condition, and will decay under the damping action of the structure.

(2) Forced vibration caused by a source other than cutting. This may be initiated by unbalance in the machine-tool drive or by some dynamic load external to the machine.

(3) Forced vibration initiated by the cutting process. The cutting process may have an inherent periodicity which can lead to a forced vibration, such as discontinuous chip formation.

(4) Self-excited vibration. A phenomenon in which the vibration movement caused the cutting process to provide energy to maintain the vibration, such as regenerative vibration, and mode coupling effect.

The difficulty in analyzing of vibration in cutting process lies in the fact that the cutting process itself may interact with the machine-tool structure, as found in the damping effect of relief angle. The damping effect of relief angle takes the form of a ploughing force through cutting edge sharpness radius. It is known that the cutting edge of the tool is never perfectly "sharp", as shown in Figure 2.3. There is a radius between rake plane and flank plane. Because of the existence of this radius, some workpiece material may be deformed and removed as part of the chip, some material may be trapped under the tool, which forms a contact force at the tool tip region and tool flank plane between the tool and workpiece. The latter phenomenon
is known as ploughing force. Experiments have shown that the relief angle of the tool has a strong damping effect on the cutting process [Sisson and Kegg, 1969] which is due to the ploughing force. Wu [1989] derived a set of equations to calculate ploughing force.

Dynamic cutting process models can be further classified into finite element models, transfer function models, and mechanistic models. Finite element models are based on finite element theory. Transfer function models usually analyze the transfer functions of cutting processes to study chatter and are used to determine the cutting stability limit. The transfer function models usually take the form shown in Figure 2.4, where the primary feedback path represents the effect of current vibration, and the regenerative feedback path represents the effects of vibration in the previous pass. Tlusty and Polachek [1963] derived an equation to calculate the stability limit:

\[
   b_{\text{lim}} = \frac{1}{2K_d \text{Re}(G)_{\text{min}}} \tag{2.12}
\]

where:
- \( b_{\text{lim}} \) = limit width of undeformed chip thickness
- \( \text{Re}(G)_{\text{min}} \) = magnitude of the minimum real part of transfer function of the machine-tool structure.
- \( K_d \) = dynamic cutting force coefficient

Mechanistic models are usually implemented for computer simulation [Zhang and Kappor, 1991] [Endres, 1991], and usually take the form shown in Figure 2.5. In these models, cutting forces are calculated as functions of the cutting force coefficient, which is often taken
as a constant, and the chip load is calculated by geometrical correlations between the tool and the workpiece, and is capable of integrating time variant effects caused by vibration. The two feedbacks in this case are the same as those in the transfer function models. Numerical methods are required to solve the vibration equations of the model in the time domain.

Compared to transfer function models, mechanistic models can account for more complex geometry involved in oblique, single-point, and multiple-point tool cutting. Another difference is that the experimental work required to calibrate mechanistic models over a wide range of applications is much less than that for transfer function models.

2.3. Surface Finish Mechanisms

Among the various factors which affect the surface finish of turned workpiece, the major factors include cutting tool geometry, cutting conditions, and workpiece material. They take effect in four forms. The first form is the deterministic portion of surface roughness formed by the ideal tool and workpiece geometrical motion. Under ideal conditions, surface roughness profiles are formed by the repetition of tool tip profile at the interval of feed. The second form is the effect of minimum undeformed chip thickness. This effect causes some workpiece material which should be removed remain on the machined surface. The third form is the modulation of the surface roughness profile caused by vibration. The fourth form is the effect of side flow, which tends to make the peak to valley value of the roughness profile larger than that of the
ideal surface roughness profile.

2.3.1 The Effect of Feed and Tool Geometry

In an ideal cutting situation, where a continuous chip is formed with no built-up edge, and the cutting system is free from vibration, the tool profile will be reproduced on the workpiece surface in the form of feed marks. Equations can be derived from the tool and process geometry as shown in Figure 2.6. This is the situation where the feed is small, so the feed mark left is essentially the tool nose radius. The condition for this situation, which is usually met in the finish turning, is as follows [Shaw, 1984]:

\[ f \leq 2R \sin C_e \]  \hspace{1cm} (2.13)

where: \( f = \text{feed} \)

\( R = \text{tool nose radius} \)

\( C_e = \text{end cutting edge angle} \)

and, the peak to valley value of the roughness profile \( R_{\text{tv}} \) can be calculated as:

\[ R_{\text{tv}} = R - \frac{1}{2} \sqrt{4R^2 - f^2} = \frac{f^2}{8R} \]  \hspace{1cm} (2.14)

The \( R_s \) value in this case is:
\[ R_a = \frac{f^2}{32R} \]  

(2.15)

2.3.2 The Effect of Minimum Undeformed Chip Thickness

When the undeformed chip thickness is below a certain value, chip will not be formed; instead, rubbing take place between the tool flank face and the workpiece [Shaw, 1984]. This effect can leave behind a small triangular portion of material on the machined surface, as shown in Figure 2.7. The theoretical surface finish peak-valley value, which considers this effect can be formulated as [Shaw, 1984]:

\[ R'_{a_m} = \frac{f^2}{8R} + \frac{t_m}{2} \left( 1 + \frac{R_{f_m}}{2} \right) \]  

(2.16)

where, \( t_m \) is the minimum undeformed chip thickness, and surface finish \( R_a \) value can be formulated as:

\[ R'_{a} = \frac{f^2}{32R} + \frac{t_m}{8} \left( 1 + \frac{R_{f_m}}{2} \right) \]  

(2.17)

In Equation (2.16) and Equation (2.17), the first term stands for the effect of feed and tool geometry and the second term stands for the effect of minimum undeformed chip thickness.
2.3.3 The Effect of Vibration

Vibration is another important factor that affects the roughness profile of the machined surface. Vibration in cutting process can be classified into forced vibration and self-excited vibration. Although various kinds of exciting force can cause forced vibration, the vibration frequency of the forced vibration often coincides with a natural frequency in some part of machine tool structure. Self-excited vibration has a much more complicated mechanism, as explained in Section 2.2. Vibration can cause displacements between the workpiece and the tool, which is transferred to the machined surface spirally, while the roughness profile is measured in the axial direction. In other words, the roughness profile is modulated by the displacement in such a way that the tool tip is offset by values of the displacement in every rotation of the spindle.

2.3.4 The Effect of Side Flow

The metal left behind by the secondary cutting edge is subject to sufficient high pressure to cause the metal to flow to the side as shown in Figure 2.8 [Shaw, 1984]. Figure 2.8(a) shows the profile left behind in the absence of side flow. Figure 2.8(b) shows the profile with side flow. The peak to valley distance is seen to be larger in the presence of side flow. The amount of side flow depends on the properties of the workpiece material. In general, the softer and more
ductile the workpiece material, the higher the side flow of the tool mark.

2.4 Computer Simulation of Surface Generation

Current computer simulation models for surface generation usually consider two factors. One is the periodic components, which are basically the feed marks of cutting tool determined by the geometry of the cutting tool and cutting parameters. The second is cutting system vibration, which is in the form of modulation of feed marks.

Jang and Series [1989] presented a simulation model in predicting surface roughness in turning. The basic features are:

(1) Only vibration in the x-direction is considered

(2) Chip load is roughly calculated as the product of feed and depth of cut. The tool nose radius portion is completely omitted.

(3) The cutting force coefficient is taken as a constant. It is generally accepted that cutting force $F$ can be calculated as the product of cutting force coefficient $K$ and the chip load $A$, which is:

$$F = KA \quad (2.18)$$

However, cutting force coefficient $K$ is directly related to cutting conditions. Taking $K$ as a constant means that only one cutting condition is included. The change of cutting condition
requires the re-calibration of $K$. In practice, this would require monumental cutting tests. Also, in the dynamic cutting process, with the change of cutting conditions due to vibration, $K$ cannot be kept as a constant. Although the change is small, it is the variable part of the cutting force that can cause vibration in the cutting system, while the invariable part of the cutting force can only cause the static deflection in the cutting system, and will not contribute to the generation of surface roughness.

(4) A regression analysis is used to develop predictive equations from the simulation results, which includes the natural frequencies of the cutting system, but not the damping ratio.

Zhang and Kappor [1991] developed a computer simulation model for the surface generation of bored parts. The basic features include:

(1) Vibration in two directions is considered.

(2) The cutting coefficient is taken as a constant.

(3) Workpiece material microstructure hardness variations are considered. Many research workers attributed the random vibration of the cutting system and the random variation of cutting force to the variation of the workpiece material microstructure hardness. This is the first work to mathematically express this phenomenon. This topic will be discussed in detail in Section 3.4.

Endres [1991] developed a simulation model for turning surface generation, the basic features of which are:

(1) Vibration in three directions is considered; the structural dynamics model used is
distributed parameter model.

(2) Chip load is accurately calculated.

(3) The cutting force coefficient is considered as a variable, but determined by empirical equations, which means that more cutting tests are needed to calibrate the model before it can be applied.

In summary, in current models, the cutting force coefficients are either taken as a constant or calculated using empirical equations. Chip load in some models is roughly calculated. In the next chapter, a new computer simulation model will be introduced, in which the cutting force coefficient is calculated based on cutting mechanics and the chip load is accurately calculated.
CHAPTER 3

THE PROPOSED COMPUTER SIMULATION MODEL

In this chapter, a new mechanistic computer simulation model for three dimensional cutting processes is presented. The discussions are divided into six sections as follows:

1. The structures and basic assumptions of the proposed model;
2. The equivalent orthogonal cutting (EOC) process;
3. The calculation of EOC parameters;
4. The proposed cutting force equation;
5. Structural dynamics equation; and

3.1 The Structures and Basic Assumptions of the Proposed Model

3.1.1 Block Diagram of the Proposed Model

The proposed computer simulation model is aimed at simulating (a) cutting forces, (b) machine-tool structural vibration, and (c) surface finish. These can be considered as the "output" of the simulation model, as shown in Figure 3.1. The "input" variables include tool geometry and cutting conditions, while the "constraints" are workpiece material and machine-
tool structural dynamic characteristics.

Figure 3.2 illustrates the structural composition of the proposed simulation model. As can be seen the entire model is composed of the following equations:

1. Effective shear angle equation;
2. Effective rake angle equation;
3. Effective relief angle equation;
4. Chip load equation;
5. Effective lead angle equation;
6. Specific cutting force equation;
7. Cutting force equation;
8. Workpiece-machine-tool structural vibration equation;
9. Cutting power and stability equations; and
10. Surface generation equation.

There are also two feedback paths in the model. One is from the output of the structural dynamics equation to the first four equations without a time delay, which describes the effect of present vibration on the four variables (i.e., effective shear angle, effective rake angle, effective relief angle, effective lead angle and chip load). The other one is from the output of the structural dynamics equation to the first four equations with a time delay, which describes the effect of vibration in the preceding pass of cutting on the four variables. This is the regenerative effect as discussed in Section 2.2.
3.1.2 Basic Assumptions of the Proposed Model

The basic assumptions of the proposed model include:

(1) The inclination angle of the cutting tool is zero. The inclination angle is the angle between the main cutting edge and the plane perpendicular to the cutting speed, as shown in Figure 3.3. In practice, the inclination angle may be positive, negative, or zero. It controls the chip flow direction. Zero inclination angle is often used in practical turning operations. Assuming zero inclination angle enables much easier calculation of the EOC parameters.

(2) Continuous chip is produced. Continuous chip is a very common type of chip form, especially in turning steel without a chip breaker. Assuming continuous chip simplifies the model in that no periodic force fluctuations occur due to chip breakage.

(3) The machined surface is generated by incorporating the relative motion between the tool and workpiece with the effect of minimum undeformed chip thickness. The relative motion between the tool and workpiece includes the ideal geometrical motion and vibration. With this assumption, other factors, such as side flow effect and spindle rotational error (which are also able to contribute to surface roughness, are omitted).

3.1.3 The Coordinate Systems

In the proposed model, four coordinate systems are used. Figure 3.4 shows three of the
coordinate systems. The first is the global coordinate system denoted as X-Y-Z with its origin at the right end of the workpiece centre. The second describes the vibration of the cutter, denoted as \( x_1\)-\( y_1\)-\( z_1\), with its origin at the centre of tool nose radius. The third describes the vibration of the workpiece, denoted as \( x_2\)-\( y_2\)-\( z_2\), with its origin at workpiece centre. The fourth is a relative system representing the relative vibration between the tool and the workpiece in the three directions. It is denoted as (x,y,z), and defined as:

\[
\begin{align*}
x &= x_1 - x_2 \\
y &= y_1 - y_2 \\
z &= z_1 - z_2
\end{align*}
\] (3.1)

3.2 The Equivalent Orthogonal Cutting (EOC) Process

The basic idea of using EOC is to convert the modeling of three dimensional cutting processes into the modeling of orthogonal cutting processes.

3.2.1 The Ideal Orthogonal Cutting Process

The basic assumptions of ideal orthogonal cutting processes include the following [Shaw, 1984]:

1. The tool is perfectly sharp and there is no contact along the clearance face:
(2) The shear surface is a plane extending upward from the cutting edge:

(3) The cutting edge is a straight line extending perpendicularly to the direction of motion and generates a plane surface as the work moves past it:

(4) The chip does not flow to either side:

(5) The depth of cut is constant:

(6) The width of the tool is greater than that of the workpiece:

(7) The work moves relative to tool with a constant speed:

(8) A continuous chip is produced with no built-up-edge; and

(9) The shear and normal stresses in the shear plane and tool are uniform.

The orthogonal cutting process represents a good approximation of the behaviour of the major cutting edge, where the straight line portion of the edge takes most cutting. In machining practice, planing and turning the end of a tube are typical orthogonal cutting operations. A large number of models have been developed to describe orthogonal cutting process, such as Merchant's model, Oxley's shear angle relationship, and the dimensional analysis reviewed in Section 2.1. The critical parameter in orthogonal process models is shear angle. Once shear angle is determined, cutting forces can be calculated using Equations (2.1) and (2.2).
3.2.2 The Concept of Equivalent Orthogonal Cutting Process

Most cutting operations are three dimensional cutting processes. The major difference between a three dimensional cutting process and an orthogonal cutting process is that the cutting edge involved in cutting is not a straight line. Rather it is a curved edge. In other words, the tool nose (arc portion) takes a significant part of cutting task. This is particularly true when the depth of cut and feed are both small, as in finish turning. Another difference is that the primary deformation zone is not a simple plane, but is a curved plane. Thus the primary deformation will not be in the plane strain condition. As a result, cutting force analysis should be a three dimensional problem, which is a very difficult task, due to the change of shear stress in both magnitude and orientation with different cutting conditions.

In order to analyze cutting forces in three dimensional cutting processes, two methods can be employed. They are the direct method and the equivalent method. The former is to directly model the three dimensional cutting mechanics, and due to its complexity, it is rarely used. The latter is to model a three dimensional cutting process, based on its EOC process. Due to its simplicity and reasonable accuracy, this method is more widely used and is adopted in this thesis.

The model of the three dimensional cutting process developed in the thesis is as shown in Figure 2.2, follows the Usui’s model. But the present model differs from Usui’s model in three aspects: (1) Usui’s model is a static model, while the present one is a dynamic model (i.e.
the structural vibration is considered); (2) The calculation of EOC parameters is different. Usui applied the minimum energy method to determine the chip flow direction, while the present one employed the method proposed by Colwell [Colwell, 1954]; and (3) The shear angle calculations are different. Usui used empirical equation to determine the shear angle, while the present one employs the dimensional analysis technique.

The EOC process is assumed to have the following properties:

1. Its chip load is the same as the actual cutting. That is, its cross cutting area perpendicular to the cutting velocity is the same as that of the actual cutting.

2. Its chip flow direction is the same as the actual cutting. Thus the equivalent lead angle is the effective lead angle of the actual cutting.

3. The equivalent rake and relief angles are the respective effective angles of the actual cutting process measured in the effective plane which contains both cutting velocity and chip flow direction.

4. The cutting edge length is the same as the actual cutting, thus the equivalent chip thickness is the cutting area divided by the length involved in cutting.

Based on the above assumptions, analysis of the actual cutting process is converted into the analysis of its equivalent orthogonal cutting process. As shown in Figure 3.5, the relationship between these two processes can be mathematically expressed as:
\[
A_{or} = A \\
CEL = b_0 \\
ECT = \frac{A_{or}}{CEL} \\
C_{or} = C_{ce} \\
\phi_{or} = \phi_e \\
\alpha_{or} = \alpha_e \\
\gamma_{or} = \gamma_e
\]  

(3.2)

3.2.3 Determination of Chip Flow Direction

In order to calculate the EOC parameters, the chip flow direction \( V_c \) must be determined first. The chip flow direction is an important practical parameter related to the deformation process. Colwell [1954] developed a geometrical relationship for predicting the chip flow direction in turning. By considering the plan view of cutter as shown in Figure 3.6, he reasoned that the chip flow direction would be perpendicular to the line joining points \( A \) and \( B \) on the cutting edges. By changing the plane geometry and cutting conditions over a wide range, he showed that the experimental results correlated well with the predicted values. Stabler [1966] suggested that the chip flow direction in turning could be found by adding vectorially the velocity directions at the two cutting edges. In the proposed model, Cowell’s method is adopted, due to it’s simplicity and effectiveness.
3.3 The Calculation of EOC Parameters

The EOC parameter calculation is described in three sections. Section 3.3.1 includes the calculation of Chip Load $A$, Cutting Edge Length $CEL$, Equivalent Chip Thickness $ECT$, and Effective Lead Angle $C_e$. Section 3.3.2 includes the calculation of Equivalent Rake Angle $\alpha_e$ and Equivalent Relief Angle $\gamma_e$. Section 3.3.3 includes the calculation of Equivalent Shear Angle $\phi_e$ and Equivalent Friction Angle $\beta$.

3.3.1 The Calculation of $A$, $CEL$, $ECT$, $C_e$

As shown in Figure 3.5, chip load is the area of the cross section of undeformed chip in the direction of cutting velocity. Cutting edge length is the portion of cutting edge in contact with workpiece surface, and may include both the arc portion and straight line portion. Equivalent chip thickness is the chip thickness of EOC and is calculated using chip load divided by cutting edge length. The effective lead angle is determined by the chip flow direction on the tool face. It should be pointed out that all these four parameters are time-variant due to structural vibration. So the calculations must be able to accommodate the time variant cutting condition and cutter geometry.

In order to calculate these parameters, two separate cases have been considered. In the first case, both the arc portion and straight line portion of the cutting edge are involved in
cutting. In the second case, only the arc portion of cutting edge is involved in cutting.

Case I — Both the Arc and Straight Line Portion Engaged in Cutting

As shown in Figure 3.7(a), the cross section of the undeformed chip is composed of two adjacent cutting edge profiles. The present one is $ABCD$, and the previous one is $FGEA$. In order to facilitate the calculation, a local coordinate system is setup as shown in Figure 3.7(a). The origin of the coordinate system is $D$. Figure 3.7(b) shows part of the magnified graph of Figure 3.7(a). The chip load is the area of $ABCLDFHE$. The calculation of this area is divided into two parts $S_{ABCHE}$ and $S_{CLDFH}$. $S_{ABCHE}$ is calculated using the following relationship:

$$ S_{ABCHE} = S_{AOI} - S_{AOE} + S_{AQO} - S_{EHPQ} - S_{FOI} $$  \hspace{1cm} (3.3)

$S_{CLDFH}$ is calculated using the following relationship:

$$ S_{CLDFH} = S_{CLB} + S_{LDH} $$  \hspace{1cm} (3.4)

The effective lead angle can be calculated as:

$$ C_{se} = \tan^{-1} \frac{u_A}{v_A} $$  \hspace{1cm} (3.5)

where, $(u_A, v_A)$ is the local coordinate of point $A$. The coordinates of other point are denoted in the same way, such as $(u_c, v_c)$, $(u_E, v_E)$. The cutting edge length can be calculated as:
In order to realize these calculations, it is necessary to know the coordinates of the following points $A, C, E, Q, O_1, O_2$. Suppose that the centre of $ABC$ arc $O_1$ is $(u_1, v_1)$, and the centre of $AE$ arc $O_2$ is $(u_2, v_2)$. Then $(u_1, v_1)$ can be calculated as:

\[
\begin{align*}
v_1 &= d_0 - (x_1 - x_2) - R \\
u_1 &= R \cos C_s + R \tan C_s (R \sin C_s + v_1)
\end{align*}
\]

Similarly, the coordinates of $O_2$ can be calculated as:

\[
\begin{align*}
v_2 &= d_0 - (x_1' - x_2') - R \\
u_2 &= f_0 + z_1' + u_1' - z_1
\end{align*}
\]

The coordinates of point $A$ are calculated as the intersection of arc $ABC$ and arc $AE$.

The equation of arc $ABC$ can be written as:

\[
(u - u_1)^2 + (v - v_1)^2 = R^2
\]

The equation of arc $AE$ can be written as:

\[
(u - u_2)^2 + (v - v_2)^2 = R^2
\]

Solving these two equations simultaneously, the coordinate $(u_A, v_A)$ of the cross point $A$ can be obtained as the solutions of equations. Point $C(u_c, v_c)$, tangent to arc $ABC$ on the line $CD$, can be calculated as follows:
\[ v_e = v_1 + R \sin C_s \]
\[ u_e = v_c \tan C_s \]  \hspace{1cm} (3.11)

Point \( E(u_e, v_e) \), intersection of arc \( AE \) and line \( O_2E \) can be calculated as follows:

\[ v_e = v_2 R \sin C_s \]
\[ u_e = v_2 \tan C_s + (u_2 - u_1) \]  \hspace{1cm} (3.12)

The coordinate of the cross point \( Q \) of line \( O_rA \) and line \( O_2E \) can be calculated as the intersection of line \( O_2E \) and line \( O_rA \). The equation of line \( O_2E \) can be expressed as:

\[ \frac{u - u_e}{v - v_e} = - \cot C_s \]  \hspace{1cm} (3.13)

The equation of line \( O_rA \) can be expressed as:

\[ \frac{u - u_A}{v - v_A} = \frac{u - u_1}{v - v_1} \]  \hspace{1cm} (3.14)

Solving Equation (3.13) and Equation (3.14) simultaneously, the coordinates of point \( Q \) can be calculated as follows:

\[ v_Q = \frac{u_2 (v_A - v_1) - u_1 v_A + u_A v_1 + v_2 \cot C_s (v_A - v_1)}{\cot C_s (v_A - v_1) - u_1 + u_A} \]  \hspace{1cm} (3.15)
\[ u_Q = u_2 - \cot C_s (v_Q - v_2) \]

Also, the distances between point \( A \) and point \( C \) can be calculated as:

35
\[
\overline{AC} = \sqrt{(u_A - u_C)^2 + (v_A - v_C)^2}
\]  
(3.16)

Similarly, the distances \(AE, AG, AQ, O_2Q, EQ\) can be calculated using the end point coordinates.

The distance between point \(A\) and point \(G\) can be calculated as:

\[
\overline{AG} = R \sin \left[ \tan^{-1} \frac{\nu_A - \nu_2}{u_2 - u_A} \right]
\]  
(3.17)

The angle \(AO_1C\) can be calculated as follows:

\[
\angle AO_1C = 2 \sin^{-1} \frac{\overline{AC}}{2R}
\]  
(3.18)

The angle \(AO_2E\) can be calculated as:

\[
\angle AO_2E = 2 \sin^{-1} \frac{\overline{AE}}{2R}
\]  
(3.19)

The angle \(AO_2F\) can be calculated as:

\[
\angle AO_2F = -\tan^{-1} \frac{\nu_A - \nu_2}{u_A - u_2}
\]  
(3.20)

The angle \(\alpha_2\) can be calculated as:

\[
\alpha_2 = \cos^{-1} \frac{\overline{AG}}{\overline{AQ}}
\]  
(3.21)

So the distance between point \(P\) and point \(Q\) can be calculated as:
$$\overline{PQ} = \overline{O_1Q} \cos \alpha_2$$  \hspace{1cm} (3.22)

The area $S_{CLDFH}$ can be calculated as follows:

$$S_1 = (u_2 - u_1)[v_c - (u_2 - u_1) \cos C_s \sin C_s] - \frac{1}{2} (u_2 - u_1)^2 \cos C_s \sin C_s$$  \hspace{1cm} (3.23)

In Equation (3.23), the first part is $S_{LDFF}$ and the second part is $S_{CLFH}$.

The area of triangle $AQO_2$ can be calculated as:

$$S_{AQO_2} = \frac{1}{2} \overline{AGO_2Q}$$  \hspace{1cm} (3.24)

The area of $EHQP$ can be calculated as:

$$S_{EHQP} = \overline{EQPQ}$$  \hspace{1cm} (3.25)

The area of triangle $PO_1Q$ can be calculated as:

$$S_{PO_1Q} = \frac{1}{2} \overline{PQ}^2 \tan \alpha_2$$  \hspace{1cm} (3.26)

Thus the total chip load can be calculated as:

$$A = \frac{1}{2} R^2 [LAO_1C - LAO_2E] + S_{AQO_2} - S_{EHQP} - S_{PO_1Q} + S_{CLDFH}$$  \hspace{1cm} (3.27)

The cutting edge length CEL can be calculated using Equation (3.6), and the effective lead angle can be calculated using Equation (3.5).
Case II — Only the Arc Portion of the Cutting Edge Engaged in Cutting

As shown in Figure 3.8, the origin of the local coordinate system is point \( B \). The chip load is \( S_{ABC} \), which can be calculated as:

\[
A = S_{AO_1B} - S_{AO_2E} - S_{EO_1B} - S_{EO_1Q} + S_{AO_2Q}
\]  

(3.28)

The cutting edge length can be calculated as:

\[
CEL = RL_{AO_1B}
\]  

(3.29)

The effective lead angle can be calculated as:

\[
C_{SE} = \tan^{-1} \frac{u_A}{v_A}
\]  

(3.30)

In order to realize these calculations, it is necessary to know the coordinates of the following points \( A, E, Q, O_1, O_2 \). Suppose that the centre of \( ABC \) arc \( O_1 \) is \((u_1, v_1)\), and the centre of \( AE \) arc \( O_2 \) is \((u_2, v_2)\). Then \((u_1, v_1)\) can be calculated as:

\[
v_1 = \frac{d_0 - R}{R}(x_1 - x_2)
\]

(3.31)

\[
u_1 = \sqrt{R^2 - v_1^2}
\]

Similarly, the coordinate \((u_2, v_2)\) can be calculated as:

\[
v_2 = \frac{d_0 - R}{R}(x_1' - x_2')
\]

(3.32)

\[
u_2 = \sqrt{R^2 - v_2^2} + u_1 - z_1
\]

The coordinates of point \( A \) are calculated as the intersection of arc \( ABC \) and arc \( AE \).

The equation of arc \( ABC \) can be written as:
\[(u-u_1)^2+(v-v_1)^2=R^2\]  \hspace{1cm} (3.33)

Equation of arc \(AE\) can be written as:

\[(u-u_2)^2+(v-v_2)^2=R^2\]  \hspace{1cm} (3.34)

Solving these two equations simultaneously, the coordinate \((u, v)\) can be obtained as the solutions of equations. The coordinate of point \(E\) can be calculated as:

\[
\begin{align*}
  v_E &= 0 \\
  u_E &= u_2 - u_1 
\end{align*}
\]  \hspace{1cm} (3.35)

The coordinate of the cross point \(Q\) can be calculated in the similar manner to Case I as:

\[
\begin{align*}
  v_Q &= \frac{u_2(v_A-v_1)-u_1v_A+u_Av_1-v_2\cot S_c(v_A-v_1)}{\cot S_c(v_A-v_1)-u_1+u_A} \\
  u_Q &= u_2 - \cot S_c(v_Q-v_2)
\end{align*}
\]  \hspace{1cm} (3.36)

where, the angle \(S_c\) can be calculated as:

\[
S_c = \tan^{-1} \frac{v_E-v_2}{u_2-u_E}
\]  \hspace{1cm} (3.37)

The angle of \(AO_1B\) can be calculated as:

\[
\angle AO_1B = 2\sin^{-1} \frac{\overline{AB}}{2R}
\]  \hspace{1cm} (3.38)

The angle of \(AO_2E\) can be calculated as:
\[ \angle AO_2E = 2\sin^{-1}\frac{AC}{2R} \]  

(3.39)

Similarly, the following angles can be calculated:

\[ \angle AO_2M = \tan^{-1}\frac{v_1 - v_2}{u_2 - u_A} \]

\[ \angle AMO_2 = \tan^{-1}\frac{v_1 - v_1}{u_A - u_1} \]

(3.40)

\[ \angle EO_1N = \tan^{-1}\frac{u_1 - u_A}{-v_1} \]

\[ \angle AO_1N = \frac{\pi}{2} - \angle AMO_2 \]

\[ \angle AO_1E = \angle EO_1N + \angle AO_1N \]

The distance between point \( M \) and \( O_2 \) can be calculated as:

\[ \overline{MO_2} = (v_1 - v_2)(\tan \angle AO_2M - \tan \angle AMO_2) \]  

(3.41)

The area of triangle \( AQO_2 \) can be calculated as:

\[ S_{AQO_2} = \frac{1}{2} \overline{MO_2}(v_a - v_Q) \]  

(3.42)

The distance between point \( O_1 \) and point \( E \) can be calculated as:

\[ \overline{O_1E} = \sqrt{(u_1 - u_I)^2 + v_1^2} \]  

(3.43)

The area of the triangle \( EO_1Q \) can be calculated as:
\[ S_{EO, Q} = \frac{1}{2} O_i E^2 \sin(\angle AO_i E) \cos(\angle AO_i E) \]  

(3.44)

Having calculated all the sub-areas, the total chip load \( A \) can be calculated using Equation (3.28). The effective lead angle \( C_{se} \) can be calculated using Equation (3.30). The cutting edge length \( CEL \) can be calculated using Equation (3.29).

Note the conditions by which these two cases are divided are as follows:

\[ \text{if} \quad R \sin C_e \geq -v_1 \quad \text{then CASE I} \]
\[ \text{if} \quad R \sin C_e < -v_1 \quad \text{then CASE II} \]  

(3.45)

3.3.2 The Calculation of \( \alpha_{cr} \) and \( \gamma_{cr} \)

The effective rake \( \alpha_{cr} \) and relief angles \( \gamma_{cr} \) of the three dimensional cutting process are taken as the EOC rake and relief angles respectively, as assumed in Section 3.2.2. These two angles are measured in the plane containing both cutting velocity and chip flow direction in the tool face. They are composed of three components: (1) the rake and relief angle change caused by vibration in \( y \)-direction; (2) nominal rake and relief angles, which are the respective angles when the system is free from vibration; and (3) the rake and relief angle change caused by vibration in \( x-z \) plane.

Due to the vibration in \( y \)-direction, both the rake and relief angle will change, as shown in Figure 3.9. Assuming the vibration caused the nominal position of workpiece "O(0,0)" is
changed to "O'(x,y)". Then the rake and relief angles become:

\[
\begin{align*}
\alpha_y &= \alpha_0 - \Delta \alpha = \alpha_0 - \tan^{-1} \frac{y}{R - d_0} \\
\gamma_y &= \gamma_0 + \Delta \gamma = \gamma_0 + \tan^{-1} \frac{y}{R - d_0} 
\end{align*}
\] (3.46)

Note in the above equations, vibration in x-direction is omitted, due to the fact that its contribution is very small.

Based on geometrical description as shown in Figure 3.10, the nominal effective rake angle can be calculated. In a similar manner, the nominal effective relief angle can be calculated. The results are as follows:

\[
\begin{align*}
\alpha_{\alpha_0} &= \sin^{-1} [\sin \alpha_0 \cos (C_{\alpha e} - C_{\beta})] \\
\gamma_{\alpha_0} &= \sin^{-1} [\sin \gamma_0 \cos (C_{\alpha e} - C_{\beta})] 
\end{align*}
\] (3.47)

Considering the changes caused by vibrations in y-direction, the nominal rake and relief angle in the Equation (3.47) are replaced by the angles calculated by Equation (3.46). That is:

\[
\begin{align*}
\alpha_{\gamma} &= \sin^{-1} [\sin \alpha_0 \cos (C_{\alpha e} - C_{\beta})] \\
\gamma_{\gamma} &= \sin^{-1} [\sin \gamma_0 \cos (C_{\alpha e} - C_{\beta})] 
\end{align*}
\] (3.48)

The vibrations in x-z plane on the changes are mainly caused by the cutting velocity directional change, shown in Figure 3.11. Note that only the present vibration can contribute to these changes. Surface undulation due to vibration in the previous pass does not cause the rake and relief angle change directly. This vibration may cause shear angle change, which will
be discussed in Section 3.3.3. The effective rake and relief angle changes caused by vibration in x-z plane are relatively complex, because the changes are not same alone the cutting edge, but vary at different positions of the cutting edge. In order to calculate rake and relief angle change due to the vibration in x and z direction respectively, the method employed is through the study of the EOC, because the angle changes alone cutting edge in orthogonal cutting process are the same. To do this, a new coordinate $x_n$ (chip flow direction in tool face) is introduced as shown in Figure 3.5, and the relationship between $x_n$ coordinate and x-z coordinates is

$$x_n = x \sin C_{sc} + z \cos C_{se} \quad (3.49)$$

Accordingly, as shown in Figure 3.11, the changes of rake and relief angle, are as follows:

$$\delta \alpha_e = -\tan^{-1} \frac{\dot{x}_n}{V + \dot{y}}$$

$$\delta \gamma_e = \tan^{-1} \frac{\dot{x}_n}{V + \dot{y}} \quad (3.50)$$

Finally, the effective rake and relief angles can be calculated as:

$$\alpha_e = \alpha_{ey} - \tan^{-1} \frac{\dot{x}_n}{V + \dot{y}}$$

$$\gamma_e = \gamma_{ey} + \tan^{-1} \frac{\dot{x}_n}{V + \dot{y}} \quad (3.51)$$

43
3.3.3 The Calculation of $\phi_\alpha$ and $\beta$

Theoretical models, such as Merchant's model and Oxley's shear angle relationship, that do not need cutting tests have the fatal drawback of low accuracy. Usually they are only of qualitative natures. The shear angle relationship from the dimensional analysis combined both theoretical analysis and cutting tests: the forms of the chip thickness ratio and cutting force ratio are determined by theory and the constants in the formula are determined by cutting tests. Although there are many constants in the formula, these constants are deemed to be workpiece material constants that will not change as cutting conditions change. As a result, the dimensional analysis technique not only greatly simplifies the problem, but also has the advantage of improved accuracy. As a result, the effective shear angle $\phi_\alpha$ and friction angle $\beta$ are both calculated based on the dimensional analysis technique [Nigm and Tobias, 1977]. The chip thickness ratio is calculated as:

$$r = e \sin \alpha + (1 - f \sin \alpha) \frac{h}{(10^3 s)^k} \ln(Q'/Q)$$  \hspace{1cm} (3.52)

The cutting force ratio is calculated as:

$$C = -e' \sin \alpha + (1 + f' \sin \alpha) \frac{h'}{(10^3 s')^{k'}} \ln(Q'/Q')$$  \hspace{1cm} (3.53)

In Equations (3.52) and (3.53), the coefficients ($e$, $f$, $h$, $k$, $Q$, $e'$, $f'$, $h'$, $k'$, $Q'$, $e'$) are the characteristics of workpiece material. Table (2.1) lists the values for the particular
material (En2). Rake angle \( \alpha \) is calculated using Equation (3.51). Undeformed chip thickness is the equivalent chip thickness in EOC. That is:

\[
\delta = ECT
\]  

(3.54)

and the cutting speed is:

\[
v = v_0 + \dot{y}(t)
\]  

(3.55)

So the friction angle can be calculated as [Nigm and Tobias, 1977]:

\[
\beta = \alpha + \tan^{-1}C
\]  

(3.56)

and the shear angle is calculated as:

\[
\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}
\]

(3.57)

The change of shear angle due to undulation on the machined surface of the previous pass is calculated as follows [Lin and Weng, 1990]:

\[
\delta \phi(t) = \tan^{-1} \left( \frac{\dot{x} (t - 1/N - t')}{V + \dot{y}(t - 1/N - t')} \right)
\]

(3.58)

where, \( 1/N \) is the time interval for one revolution of workpiece. Note a time shift \( t' \) due to the orientation of shear plane has been considered. The time shift is used to compensate for the time lag between the tool point and end point of the shear plane on the machine surface of the previous pass, as shown in Figure 3.12. It is calculated as:
\[ t' = \frac{EC T \cot(\phi)}{V} \quad (3.59) \]

Therefore, the final effective shear angle can be written as:

\[ \phi_e(t) = \phi + \tan^{-1} \frac{x_r(t - 1/N - t')}{V + y(t - 1/N - t')} \quad (3.60) \]

where \( \phi \) is shear angle calculated using Equation (3.57). As the rake angle, undeformed chip thickness, and cutting speed calculations have included vibration calculation, the calculated shear angle and friction angle are dynamic in nature.

### 3.4 Cutting Force Equations

The cutting force calculation includes three steps:

1. Calculation of the cutting force from EOC;

2. Incorporation of ploughing force into the force model; and

3. Modification of the force model to incorporate workpiece material microstructure hardness variation.

#### 3.4.1 Calculation of the Cutting Force Based on the Merchant’s Model

Cutting forces in orthogonal cutting process can be obtained using Merchant’s diagram.
As shown in Figure 2.1, the cutting force components $F_p$, $F_q$ [Armarego, 1969] can be determined as follows:

$$F_q = \frac{\tau_s \sin(\beta - \alpha_c)}{\sin \phi_e \cos(\phi_e - \beta - \alpha_c)} A \quad (3.61)$$

$$F_p = \frac{\tau_s \cos(\beta - \alpha_c)}{\sin \phi_e \cos(\phi_e + \beta - \alpha_c)} A \quad (3.62)$$

The cutting forces are basically calculated by two parts: chip load $A$ and the expression before $A$, which is cutting force coefficient $K$. $K$ is a function of shear stress, shear angle, friction angle, and rake angle. Shear stress is independent of cutting conditions [Nigm and Tobias, 1977], and for material En2, it takes the value of $\tau_s = 95 \times 10^3$ psi. The shear angle, friction angle, and rake angle, however, are functions of vibration. Therefore, the calculated $K$ is not a constant during cutting process. Furthermore, $F_p$ and $F_q$ can be projected into x-y-z coordinates. Thus the cutting forces in the turning process, namely radial cutting force $F_x$, main cutting force $F_y$, feed force $F_z$ can be determined as follows:

$$F_x = F_q \sin \alpha_x$$

$$F_y = F_p$$

$$F_z = F_q \cos \alpha_y$$

\[ (3.63) \]
3.4.2 Ploughing Force Equation

No literature has been found that deals with ploughing force in three dimensional cutting. In the proposed model, ploughing force is estimated by EOC. Ploughing force is known to be proportional to the relative vibration between workpiece and cutting tool, and inversely proportional to cutting speed [Sisson and Kegg, 1969]. The direction of ploughing force is in the x-z plane, since vibration in the y-direction cannot contribute to ploughing force. The ploughing force can be expressed as:

\[ F_d = \frac{K_a \dot{x}_n}{V} \]  \hspace{1cm} (3.64)

This force can be projected into the x and z direction as:

\[ F_{dx} = \frac{K_a \dot{x}}{V} \]  \hspace{1cm} (3.65)

\[ F_{dz} = \frac{K_a \dot{z}}{V} \]  \hspace{1cm} (3.66)

where, \( K_a \) is analytically derived by Sisson and Kegg [1969] as:

\[ K_a = \frac{wR_1 b \sigma_y}{\gamma^2 e} \]  \hspace{1cm} (3.67)

where: \( b = 1/4 \)

\( w = \text{CEL} \)

48
After ploughing force is considered, the cutting forces can be expressed as:

\[
\begin{align*}
F_{xp} &= F_{r} + F_{d} \\
F_{yp} &= F_{a} \\
F_{zp} &= F_{x} + F_{d}
\end{align*}
\] (3.68)

3.4.3 The Workpiece Microstructure Hardness

It has long been speculated that the random variation of cutting force present in turning processes can be caused by the nonhomogeneous distribution of microhardness in the workpiece material [Merchant, 1945]. This random variation of cutting force can cause additional vibration in the cutting process, which in turn will contribute to the formation of the random portion of the surface profile. Recently, Zhang and Kappor [1991] developed a mathematical model to describe this phenomenon. Figure 3.13(a) shows a micrograph of ferrite and pearlite structures present in mild carbon steel at room temperature. Figure 3.13(b) indicates the cutting edge experiences different microstructural portions. Since the pearlite structure is harder than the ferrite structure, the generated cutting force varies.

Figure 3.14 shows that in a cutting process, the chip volume removed during each revolution of the workpiece can be thought of as a series of blocks of identical geometrical shape. The geometrical shape of each block has a length of \( \pi D/n_1 \) and a cross sectional area equal to the product of depth of cut and feed. To approximate the nonhomogeneous
distribution of microhardness in each block, it is assumed that the mean of the sample hardness conforms to a normal distribution with a mean of $\mu_z$ and a variance of $\sigma_z$. For material En2, $\mu_z$ is experimentally found to be 126 (BHN), and $\sigma_z$ is derived as:

$$\sigma_z^2 = 4\pi \int_0^\infty r^2 \rho(r) W(r) dr$$

(3.69)

where, $W(r)$ is geometrical shape function and $\rho(r)$=correlation function. The detailed analysis and determination of $W(r)$ and $\rho(r)$ can be found in [Zhang and Kappor, 1991]. After introducing a ratio $(h_i/h_{avg})^m$, where, $h_i$ is instantaneous sample mean hardness, $h_{avg}$ is average sample mean hardness, and $m$ is the Meyer exponent, (for carbon steel, $m=0.454$), the instantaneous cutting force can be modified as:

$$F_x = F_{xp} \times (\frac{h_i}{h_{avg}})^m$$

$$F_y = F_{yp} \times (\frac{h_i}{h_{avg}})^m$$

$$F_z = F_{zp} \times (\frac{h_i}{h_{avg}})^m$$

(3.70)

3.5 Structural Dynamics Equation

In a turning operation, the following are the possible modes of vibration

[Armarego, 1969]:

50
(1) Tool shank bending in both vertical plane and horizontal plane;

(2) Tool post slide bending in vertical plane;

(3) Workpiece bending between centres in both vertical and horizontal plane;

(4) Workpiece in torsion about its axis;

(6) Lathe bed bending in vertical plane and horizontal plane; and

This list is by no means exhaustive. It is known that x-direction is the sensitive direction for vibration to contribute to surface roughness. It is accurate enough to model the cutting system as a two-degrees-of-freedom system for simulating surface roughness [Jang and Serig, 1989]. In order to simulate the turning process comprehensively, however, the cutting system is approximated as a five-degrees-of-freedom system, as shown in Figure 3.15. The equations of vibration are:

\[
\begin{align*}
\frac{m_{x_1}}{x_1} + c_{x_2} \dot{x}_1 + k_{x_1} x_1 &= F_x \\
\frac{m_{y_1}}{y_1} + c_{y_2} \dot{y}_1 + k_{y_1} y_1 &= F_y \\
\frac{m_{z_1}}{z_1} + c_{z_2} \dot{z}_1 + k_{z_1} z_1 &= F_z \\
\frac{m_{x_2}}{x_2} + c_{x_2} \dot{x}_2 + k_{x_2} x_2 &= -F_x \\
\frac{m_{y_2}}{y_2} + c_{y_2} \dot{y}_2 + k_{y_2} y_2 &= -F_y
\end{align*}
\]  

(3.71)

where the first three equations denote the dynamics of cutter and last two equations denote the dynamics of machine tool and workpiece. In these equations, the dynamic characteristics (m's, c's, and k's) can be determined by modal analysis. Hence when cutting forces are determined as discussed in previous sections, Equation (3.71) can be solved numerically. The numerical method used is the fourth order Runge-Kutta method. First, Equation (3.71) is
transformed into a set of first-order differential equations as follows:

\[ \dot{u}_i = f(t, u_1, u_2, \ldots, u_{10}), \quad i = 1, 2, \ldots, 10 \] (3.72)

or:

\[ \dot{u}_1 = \frac{1}{m_1}(F_x - c_{11}u_1 - k_{11}u_2) \]

\[ \dot{u}_2 = u_1 \]

\[ \dot{u}_3 = \frac{1}{m_1}(F_y - c_{12}u_3 - k_{12}u_4) \]

\[ \dot{u}_4 = u_3 \]

\[ \dot{u}_5 = \frac{1}{m_1}(F_z - c_{13}u_5 - k_{13}u_6) \]

\[ \dot{u}_6 = u_5 \]

\[ \dot{u}_7 = \frac{1}{m_2}(-F_y - c_{21}u_7 - k_{21}u_8) \]

\[ \dot{u}_8 = u_7 \]

\[ \dot{u}_9 = \frac{1}{m_2}(-F_x - c_{22}u_9 - k_{22}u_{10}) \]

\[ \dot{u}_{10} = u_9 \]

where:

\[ \dot{x}_1 = u_1 \quad x_1 = u_2 \]

\[ \dot{y}_1 = u_3 \quad y_1 = u_4 \]

\[ \dot{z}_1 = u_5 \quad z_1 = u_6 \]

\[ \dot{x}_2 = u_7 \quad x_2 = u_8 \]

\[ \dot{y}_2 = u_9 \quad y_2 = u_{10} \] (3.74)

The fourth-order Runge-Kutta Method employs the following formula:
\[ u_{i,m+1} = u_{im} + \frac{h}{6} (k_{ii} + 2k_{i2} + 2k_{i3} - k_{id}) \]  

(3.75)

where,

\[ k_{ii} = f_i(t_m, u_1, u_2, \ldots, u_{10}) \]
\[ k_{i2} = f_i(t_m + \frac{h}{2}, u_{1+}, u_{1+} + \frac{h}{2}, u_{10+}, u_{10+} + \frac{h}{2}) \]
\[ k_{i3} = f_i(t_m + \frac{h}{2}, u_{1+}, \frac{h}{2}, k_{i2+}, \frac{h}{2}, k_{i3+}, \frac{h}{2}) \]
\[ k_{id} = f_i(t_m + h, u_{1+}, h k_{i3+}, \ldots, u_{10+}, h k_{i3+}) \]  

(3.76)

where, \( i=1,2,\ldots,10 \), \( m=0,1,\ldots,25 \times 3600 \) (the simulation points), and \( h \) is the simulation step or time interval.

3.6 Surface Generation

In the proposed model, the machined surface is generated by considering: (1) the relative motion between work-piece and cutting tool; (2) vibration; (3) the effect of the minimum undeformed chip thickness. The procedures of modeling the surface generation is presented below.

**Step 1.** Considering first relative motion as shown in Figure 3.16(a). It results in a series of arcs, equi-spaced, with the centre points all at the same level. Figure 3.16(b) shows the profiles when the vibration is presented. As a result, the centre points in both X and Z direction are varying. The coordinates of centre points in the ith circle \((C_{ix}, C_{iz})\) are as follows.
\[ C_u = x_1(t) - x_2(t) - R \]
\[ C_u = \frac{1}{2} f_0 (2i - 1) + z_i(t) \]  

(3.77)

Furthermore, the intersections of adjacent arcs are determined by solving the following simultaneous equations:

\[ (X_k - C_u)^2 + (Z_k - C_u)^2 = R^2 \]
\[ (X_k - C_{(i-1)u})^2 + (Z_k - C_{(i-1)u})^2 = R^2 \]

(3.78)

where, \((X_i, Z_i)\) is the surface profile. Having each arc equation and intersection coordinates, the machined surface profile can be determined. The surface roughness \(R_a\) is calculated as:

\[ R_a = \frac{1}{n} \sum_{i=1}^{n} |X_k - X_a| \]

(3.79)

where: \(n\) is the number of points in the profile calculated, \(X_a\) is the average profile height and is calculated as:

\[ X_a = \frac{1}{n} \sum_{k=1}^{n} X_k \]

(3.80)

**Step 2.** Taking into account the effect of minimum undeformed chip thickness, the \(R_a\) calculation is modified as:

\[ R_a' = R_a + \frac{f_m}{8} \left(1 + \frac{R_t}{m}\right) \]

(3.81)

The second term in Equation (3.81) stands for the effect of the minimum undeformed chip thickness. The surface profile is generated by using an equivalent tool nose radius, which gives the same \(R_a\) values when this the minimum effect is not taken into consideration, that is
\[ R_e = \frac{f^2}{32 R_a} \quad (3.82) \]

**Step 3.** Goto step 1 and plot surface profile with the tool nose radius being \( R_e \).

Based on the above proposed model, the simulation results will be presented in the next chapter.
CHAPTER 4

SIMULATION RESULTS AND EXPERIMENTAL CONFIRMATION

4.1 SIMULATION RESULTS

Using the proposed model, systematic simulation of turning process has been conducted. The input, output, and constraint variables of the model are as shown in Figure 3.1. During the entire simulation, the constraints are the same. That is, the workpiece material is AISI 1015 steel, and the structural dynamics parameters are as shown in Table 4.1 (the results of modal analysis in Section 4.2.1).

In each simulation, only one of input variables is to be changed. All of the output variables in Figure 3.1 can be given. For example, in simulating the effect of feed, all other input variables are kept constant, while the feed is changed in the range 0.06-0.24 mm/rev. The output variables are cutting forces, vibration, and surface finish. All the cutting forces shown are the average cutting forces; cutting force variations are not shown.

In this section, the simulation results of cutting forces and surface finish are plotted out. A typical simulated surface profile is compared to the measured surface profile in Section 4.2.
Table 4.1 Structural dynamics parameters

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping ratio (%)</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_1$</td>
<td>344</td>
<td>0.7</td>
<td>43</td>
</tr>
<tr>
<td>$y_1$</td>
<td>167</td>
<td>2.1</td>
<td>66</td>
</tr>
<tr>
<td>$z_1$</td>
<td>338</td>
<td>0.73</td>
<td>73</td>
</tr>
<tr>
<td>Workpiece</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_2$</td>
<td>370</td>
<td>7</td>
<td>4.5</td>
</tr>
<tr>
<td>$y_2$</td>
<td>370</td>
<td>7</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 4.2. shows a set of input variables that every simulation has as an input point. In other words, only one of variables in the set is varied in each simulation, while others are kept unchanged.

Table 4.2 Typical input variables

<table>
<thead>
<tr>
<th>Workpiece diameter (mm)</th>
<th>Spindle speed (rpm)</th>
<th>Depth of cut (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>1250</td>
<td>1.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feed (mm/rev)</th>
<th>Sharpness radius (mm)</th>
<th>Tip radius (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.012</td>
<td>0.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rake angle (Deg.)</th>
<th>Relief angle (Deg.)</th>
<th>Lead angle (Deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>

57
4.1.1 The Effect of Feed

Feed is varied in the range 0.06-0.24 mm/rev. while all other input variable are kept constant. Figure 4.1 shows the relationship of cutting forces with the feed. As can be seen, the three cutting force components increase almost linearly with the increase of feed. This is because chip load is increased when feed increases. Figure 4.2 shows the relationship of surface finish with the feed. As can be seen, $R_a$ increases very rapidly with the increase of feed.

4.1.2 The Effect of Tool Nose Radius

The tool nose radius is varied in the range 0.1-1.2 mm. Figure 4.3 shows the relationship of cutting force with tool nose radius. As can be seen, two cutting force components $F_x$ and $F_y$ almost increase linearly with the increase of tool nose radius, while $F_z$ component remains largely unchanged. There are two effects that contribute to the constant result of $F_z$. One is that with the increase in tool nose radius, the chip load is increased, which tends to increase $F_z$. This effect is reflected in the increase of $F_y$. Another effect is that with the increase in tool nose radius, the chip flows more in the x-direction, so the orientation of cutting force in the horizontal plane is more in x-direction, which tends to decrease $F_z$. Due to the balance of the two effects, $F_z$ remains fairly constant.

Figure 4.4 shows the relationship of surface finish $R_a$ with the tool nose radius. As can
be seen, surface finish improves continually with the increase of tool nose radius until nose radius reached 0.4 mm, after which point the surface finish \( R_z \) becomes slightly larger with further increase of nose radius. This curve can be explained by three effects. The first effect is the geometrical contribution of nose radius, which tends to make \( R_z \) smaller when nose radius is increased. This effect dominant smaller range of tool nose radius. So \( R_z \) decreases with the increase of tool nose radius in this range. The second effect is vibration, which is more severe when tool nose radius is larger. This is because both \( F_x \) and \( F_y \) are increased, when the nose radius is increased. The third effect is the minimum undeformed chip thickness, which tends to increase when nose radius is increased. These simulation results prove that surface finish will not infinitely improve as tool nose radius is increased, as would be predicted from ideal surface finish equations.

4.1.3 The Effect of Depth of Cut

Depth of cut is varied in the range 0.8-3 mm. Figure 4.5 shows the relationship of cutting force with depth of cut. As can be seen, the three cutting force components all increase almost linearly with the increase of depth of cut: The phenomenon that \( F_z \) increases faster than \( F_x \) can be explained by the fact that chip flow is more in z-direction when depth of cut is increased, which means a smaller effective lead angle.

Figure 4.6 shows the relationship of surface finish with depth of cut. As can be seen,
surface finish $R_a$ increases slightly with the increase of depth of cut. This is because larger depths of cut would result in larger cutting forces, which in turn would cause larger vibration, and hence larger $R_a$ values.

4.1.4 The Effect of Cutting Speed

Cutting speed is varied in the range 24-116 m/min. Figure 4.7 shows the relationship of cutting force with cutting speed. As can be seen, cutting forces first decrease with the increase in cutting speed, then decrease slowly with the further increase in cutting speed. This is due to the effect of built-up edge, which has been taken into consideration in the dimensional analysis technique.

Figure 4.8 shows the relationship of surface finish with the cutting speed. As can be seen, with the increase in cutting speed, $R_a$ decreases with the increase in cutting speed. This effect is because cutting force becomes small when cutting speed is increased. Thus, smaller vibration and better surface finish are obtained.

4.1.5 The Effect of Rake Angle

Rake angle is varied in the range $1^\circ$-$14^\circ$. Figure 4.9 shows the relationship of cutting force with rake angle. As can be seen, the three cutting force components all decrease with the
increase in rake angle.

Figure 4.10 shows the relationship of surface finish with the rake angle. As can be seen, the $R_a$ value slightly decreases with the increase of rake angle. This is due to the fact that cutting force decreases with the increase of rake angle, resulting in less vibration and better surface finish.

4.1.6 The Effect of Lead Angle

The lead angle is varied in the range $11^\circ$-$65^\circ$. Figure 4.11 shows the relationship of cutting force with the lead angle. As can be seen, $F_x$ increases and $F_z$ decreases with the increase in lead angle. This phenomenon can be explained by chip flow direction: when the lead angle increases, chip flows more in $x$-direction, thus $F_x$ is increased, while $F_z$ is decreased. Figure 4.12 shows the relationship of surface finish with lead angle. As can be seen, $R_a$ increases slightly with the increase in lead angle. This is because with the increase in lead angle, $F_x$ is increased; so vibration in $x$-direction is increased. This additional vibration contributes to surface roughness.

4.2 EXPERIMENTAL CONFIRMATION

In order to confirm the validity of the proposed model, experiments have been
conducted to compare the predicted surface finish and the measured surface finish from actual cutting. It is well known that feed and tool nose radius are the primary factors that influence surface finish. As a matter of fact, from geometrical motion perspective, surface finish is entirely determined by feed and tool nose radius as expressed by Equation (2.16). When vibration and minimum undeformed chip thickness effect are taken into consideration, the relationships of surface finish with feed and tool nose radius are quite complicated, as shown in Figure 4.2 and 4.4. As a result, the relationships of surface finish with feed and the relationship of surface finish with tool nose radius were the focus of the tests.

The experiments were conducted in two parts. The first part was modal analysis designed to evaluate the dynamic characteristics of the cutting system. The second part was the actual cutting tests.

4.2.1 Modal Analysis

The experimental setup was as shown in Figure 4.13. The parameters used in the experiment were as follows:

- **Machine tool:** Harrison M400
- **Cutter:** AR8-C5 (nose radius = 0.8mm, rake angle = 10°, relief angle = 6°)
- **Workpiece:** 9.5" (length) x 3/4" (diameter), AISI 1015 steel
Accelerometer: PCB 303 A02
Hammer: PCB 086 B01
Analyzer: HP 35660A
Computer HP 236, with software (ENTEK EASY)

In the experiment, the hammer provided the input excitation force to the structures and the accelerometer were used to pick up the structural vibration signals. The analyzer provided the signal acquisition and transfer function calculations. The transfer functions were analyzed in the computer to produce the dynamic characteristics of the structures. The tested vibration modes of the workpiece include two directions (i.e. in the horizontal plane and vertical plane). Due to the symmetric nature of the workpiece and its clamping apparatus (the chuck and the tailstock), the dynamic characteristics of the workpiece in the two planes were assumed to be the same. As a result, only the vertical plane was tested.

The mounting position of the accelerometer and the impacting position of the hammer were all in the centre of the workpiece. The accelerometer was mounted at the bottom of the workpiece with wax and the impacting position of the hammer was at the top of the workpiece. The transfer function of the workpiece is shown in Figure 4.14. This transfer function is in terms of inertance. However, as the major concern in this research is vibration in terms of displacement, the inertance transfer function was converted into a compliance transfer function by the analyzer as shown in Figure 4.15. The inertance transfer function was then analyzed by
the computer in the frequency range where the compliance transfer function showed the highest peak. In the same way, the dynamic characteristics of the tool in three directions were also analyzed. The results are as shown in Table 4.1 and were subsequently used for computer simulation.

4.2.2 Cutting Tests

In the cutting test, the same experimental setup as in modal analysis was used. All the cutting tests was without coolant. The cutting conditions were as follows:

- Cutting speed: 74 (m/min)
- Depth of cut: 1.6 (mm)
- Feed: 0.063, 0.08, 0.1, 0.125, 0.16, 0.2, 0.23 (mm/rev)
- Tool nose radius: 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 (mm)

Note that when feed was changed, all other cutting conditions were kept unchanged. The tool nose radius was kept as 0.8 mm; when tool nose radius was changed, all other cutting conditions were kept unchanged. The feed was kept as 0.08 mm/rev.

After the cutting, surface finish $R_a$ and surface profile of the workpiece were measured by a Mitituyo stylus (Surftec-II) with a 0.8 mm cut-off length and 500 Hz sampling rate. The
experimental setup for the surface finish measurement is shown in Figure 4.16. The $R_s$ value was given by the stylus instrument itself, while the surface profile was A/D converted into a computer by a Snap-Shot Storage system. The $R_s$ values were measured at three different positions on the workpiece and then the average value was taken as the experimental result.

The measured $R_s$ values with different feed are plotted in Figure 4.17. The results of measured $R_s$ values with different tool nose radius are plotted in Figure 4.18. A simulated surface profile was compared with the measured profile in Figure 4.19. The cutting conditions for these two profiles are the same. The feeds were 0.12 and the tool nose radius were 0.8 mm. Note the equivalent nose radius for the simulation was 0.24 mm.

4.2.3 Analysis of Results

Comparing the experimental results and the simulation, the following can be seen:

(1) The average relative prediction error for the surface finish with feed relationship (Figure 4.17) is 5.49%.

(2) The average relative prediction error for the surface finish with tool nose radius relationship (Figure 4.18) is 16.13%.

(3) The simulated surface profile was matched with the measured surface profile in both feed pitch and amplitude (Figure 4.19).
4.3 A Critical View On Surface Finish

As an important requirement in finish turning, surface finish has attracted many research efforts. From the surface finish formation mechanism perspective, the following effects are commonly accepted to be the cause of surface finish. These are (1) the feed marks left by the ideal geometrical motion between tool and workpiece; (2) the minimum undeformed chip thickness; (3) vibration; and (4) side flow. Under certain circumstances, other factors may also contribute additional surface finish, such as tool wear and machine tool accuracy. From the cutting condition perspective, surface finish is mainly determined by the primary cutting parameters, that is feed, tool nose radius and cutting speed, although the secondary parameters (i.e. depth of cut, tool geometrical angles) may have some influence on surface finish. The trends of the surface finish are governed by the primary parameters.

Over the years, researchers have developed many models to establish the relationships of surface finish with cutting conditions. These models are mostly in the form of empirical equations. However, the investigation done by El-Wardany and El-Bastawi [1991] has shown that there are considerable variations in the calculated surface finish from different models, even when the cutting parameters are chosen to be the same. This is shown in Figure 4.20. One of the main reasons for the discrepancies is the ignoring the dynamic effect of the machine tool-workpiece system as well as the tool holder assembly [El-Wardany and Elbastawi, 1991].

In Section 4.1, systematic simulation of surface finish was conducted and confirmed
experimentally. However, these simulations and experiments were conducted using the cutting parameters as the input variables to the model. No attempt has been made to account for the mechanisms involved, which is to be discussed in this section. As mentioned in Chapter 1, this is one of the advantages of theoretical models as well as of computer simulation. In this chapter, first, the simulation results of surface finish in Section 4.1 are analyzed in terms of the mechanisms involved. Then it is attempted to explain the considerable variation of surface finish prediction from different models, which was shown in Figure 4.20.

4.3.1 The Explanation of Simulation Results

The simulation results of the relationship of surface finish with feed are analyzed in terms of (1) feed marks; (2) feedmarks and minimum undeformed chip thickness; and (3) vibration. The results are shown in Figure 4.21. As can be seen, of the three effects, the minimum undeformed chip thickness has the largest effect, the feed mark effect is next largest and vibration has the smallest effect. This is reasonable considering that the first vibration modes responsible to the formation of surface finish are due to the machine tool structure. The machine tool (Harrison M400) we used is very stiff. Therefore, the vibration effect is the smallest. The same analysis also applies to the relationship of surface finish with tool nose radius, as shown in Figure 4.22.
4.3.2 Explanation of the Result Variation of Different Models

In order to verify that the variation of different model predictions can be caused by the
dynamic effects of the cutting systems, another two cutting systems are assumed to have the
dynamics characteristics shown in Table 4.3. Note these parameters are the dynamic
characteristics (in both the horizontal plane and the vertical plane) of the workpiece assembly
(including chuck and tailstock) only. The dynamic characteristics of the tool assembly (tool and
tool post) are the same as given in Table 4.1. As can be seen, the only difference in the three
systems are in the natural frequencies for vibration modes of the workpiece.

Table 4.3 Structural dynamics parameters for different systems

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping ratio (%)</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>system 1: x_2</td>
<td>100</td>
<td>7</td>
<td>4.5</td>
</tr>
<tr>
<td>system 2: x_2</td>
<td>150</td>
<td>7</td>
<td>4.5</td>
</tr>
<tr>
<td>system 3: x_2</td>
<td>370</td>
<td>7</td>
<td>4.5</td>
</tr>
</tbody>
</table>

The simulation results of the relationship of surface finish with feed for the three cutting
systems are shown in Figure 4.23. These relationships have exactly the same input cutting parameters as the simulation in Section 4.1. As can be seen, the surface finishes from the three cutting systems are quite different. This in fact can explain why large variations exists in the prediction results from different models for the same cutting parameters. That is, the variations of surface finish prediction of different models can be caused by different cutting system dynamics characteristics.
CHAPTER 5

SUMMARY AND FUTURE WORK

5.1 Summary of The Proposed Model

In this thesis a three dimensional cutting process model is developed based on the concept of the equivalent orthogonal cutting process (EOC). In the model, both the cutting force coefficient and chip load are calculated incorporating the varying cutting conditions and tool geometry due to vibration. The vibrations are considered to have five degrees of freedom. The structural dynamics model is in the form of five simultaneous differential equations which are solved numerically by the fourth-order Runge-Kutta method. The machined surface is simulated by incorporating tool and workpiece relative geometrical motion, vibration and the minimum undeformed chip thickness effect.

5.2 Summary of Simulation and Experimental Confirmation

Through computer simulation in the given conditions, the following results have been obtained:

(1) The primary cutting parameters in the formation of surface roughness are feed and tool nose radius. When tool nose radius reaches a certain value, surface finish will not improve
further with the increase in tool nose radius. Surface finish improves with the decrease in feed.

The secondary cutting parameters are cutting speed, depth of cut and tool geometrical angles. Their effects are small.

(2) For the present setup, it was found that the minimum undeformed chip thickness is the largest contribution to surface roughness, the next was tool feed mark, while the vibration effect was the smallest. However, this order may be different for other setups.

(3) The simulated surface profile is found to be in agreement with the measured surface finish profile, in terms of feed pitch and amplitude.

5.3 Recommendations for Future Work

It is recommended that future work be conducted in the following aspects:

(1) Workpiece material properties in terms of side flow should be considered.

(2) Machine tool accuracy, especially the spindle bearing accuracy should be considered.

(3) More cutting tests should be carried out to verify the model in term of cutting force and vibration.
REFERENCES


Komvopoulos, K. 1990 "Finite Element Prediction of Chip Geometry and Tool/Workpiece


Fig. 2.1 Merchant's orthogonal cutting process model [Armarego, 1969]
Fig. 2.2 Usui’s three dimensional cutting process model [Usui, 1978]
Fig. 2.3 Illustration of ploughing force
Fig. 2.4 Block diagram of transfer function models [Endres, 1991]

Fig. 2.5 Block diagram of mechanistic models [Endres, 1991]
Fig. 2.6 Calculation of ideal surface roughness [Shaw, 1984]

Fig. 2.7 Calculation of the effect of minimum undeformed chip thickness [Shaw, 1984]
Fig. 2.8 The effect of side flow
Fig. 3.1 Input-output relationship of the proposed model
Fig. 3.2 Block diagram of the proposed computer simulation model
inclination angle = $\angle ACB$

Fig. 3.3 Illustration of inclination angle [Shaw, 1984]
Fig. 3.4 Coordinate systems in the proposed model
Fig. 3.5 The concept of EOC
Fig. 3.6 Determination of chip flow direction [Colwell, 1954]
Fig. 3.7 EOC parameter calculation - case 1
Fig. 3.8 EOC parameter calculation - case II
Fig. 3.9 Rake and relief angle change due to vibration in y direction
Fig. 3.10 Calculation of effective rake angle
Fig. 3.11 Rake and relief angle change due to vibration in x and z direction
Fig. 3.12 Time shift due to orientation of shear plane
Fig. 3.13 Workpiece microstructure and its effect on cutting tool [Zhang, 1991]

Fig. 3.14 Geometrical shape of chip samples [Zhang, 1991]
Fig. 3.15 Structural dynamics model
Fig. 3.16 Generation of machined surface
Fig. 4.1 Feed and cutting force

Fig. 4.2 Feed and surface finish
Fig. 4.3 Tool nose radius and cutting force

Fig. 4.4 Tool nose radius and surface finish
Fig. 4.5 Depth of cut and cutting force

Fig. 4.6 Depth of cut and surface finish
Fig. 4.7 Cutting speed and cutting force

Fig. 4.8 Cutting speed and surface finish
Fig. 4.9 Rake angle and cutting force

Fig. 4.10 Rake angle and surface finish
Fig. 4.11 Lead angle and cutting force

Fig. 4.12 Lead angle and surface finish
Fig. 4.13 Experimental setup in modal analysis
Fig. 4.14 Inertance transfer function of workpiece
Fig. 4.15 Compliance transfer function of workpiece
Fig. 4.16 Experimental setup in surface finish measurement
Fig. 4.17 Surface finish and feed
Fig. 4.18 Surface finish and tool nose radius
Fig. 4.19 Simulated and measured surface profile
Fig. 4.20 Surface finish prediction from different models [El-Wardany, 1991]

* Reference no. in [El-Wardany, 1991]
Fig. 4.21 Analysis of surface finish with feed
Fig. 4.22 Analysis of surface finish with tool nose radius
Fig. 4.23 Surface finish from different cutting systems
APPENDIX

Computer Simulation Program
C MAIN PROGRAM FOR COMPUTER SIMULATION OF TURNING PROCESS

DIMENSION CX(36,100), CZ(36,100), C(5), X(5), ZZZ(100), Y(10,2), Y1(3600,10), Y2(3600,11)
1 KK(10,4), ZNZ(36,200), ZZZZ(100), XXZ(100),
CHARACTER*10 PRO1, PRO2, PRO3, PRO4
CHARACTER*2 PI(10)
REAL M11, M12, M13, M2, K, CK

DATA(PI(i),i=1,10)='1', '2', '3', '4', '5', '6', '7', '8', '9', '10'
DATA D, R, D0, F0, R0, AFA, GMA, CS, R1/19.125, 8.0, 1.0, 0.8, 0.17, 0.01, 0.57, 0.01/
DATA EP1, FP1, GP1, HP1, PN1, PQ1, PK1/56.0, 83.0, 0.0, 147.0, 0.9, 0.1, 0.1/
DATA EPPI, FPPI, GPPI, HPPI, PNPI, PQPI, PKPI/0.92, 0.73, 0.075, 0.142, 100.0, 0.1/

C INPUT CUTTING PARAMETERS

PRINT *, "PLEASE INPUT THE FOLLOWING PARAMETERS"

6 PRINT *, "WORKPIECE DIAMETER (MM)"
READ *, D
PRINT *, "TURNING SPEED (R/Min)"
READ *, N
PRINT *, "DEPTH OF CUT (MM)"
READ *, D0
PRINT *, "FEED RATE (MM)"
READ *, F0

C INPUT CUTTER PARAMETERS

PRINT *, "TOOL'S NOSE RAIDIUS (MM)"
READ *, R0
PRINT *, "RAKE ANGLE"
READ *, AFA
AFA=AFA*3.14/180
PRINT *, "RELIEF ANGLE"
READ *, GMA
GMA=GMA*3.14/180
PRINT *, "LEAD ANGLE"
READ *, CS
CS=CS*3.14/180
PRINT *, "TOOL SHARPNESS RADIUS R1"
READ *, R1

C INPUT DYNAMIC PARAMETERS

PRINT *, "DO YOU WANT TO CHANGE PARAMETERS? (1/0)"
READ 4, NY
4 FORMAT (I2)
IF (NY.EQ.1) GOTO 6
PRINT *, "PLEASE WAIT"
DATA M11, M12, M13, M2/43.0, 66.7, 4.5/
DATA(C(i),i=1,5)/0.909, 0.0, 0.045, 0.045/
DATA(K(i),i=1,5)/1.8, 1.8, 1.8, 1.8, 1.8/
K(1)=(2*3.14*K(1))/2*M11/1000

115
K(2)=(2*3.14*K(2))**2*512/1000
K(3)=(2*3.14*K(3))**2*513/1000
DO 7 I=4,5
K(I)=(2*3.14*K(I))**2*512/1000
7 CONTINUE
C(1)=2*(C(1)**(K(1)**M11/1000))**.5
C(2)=2*(C(2)**(K(2)**M12/1000))**.5
C(3)=2*(C(3)**(K(3)**M13/1000))**.5
DO 11 I=4,5
C(I)=2*(C(I)**(K(I)**M2/1000))**.5
11 CONTINUE
PRINT *,K(1),C(1),K(5),C(5)
C CALCULATE V,KCR,TAO, ETC.
TAOS=655
H=1/(60.*RN)
OPEN (1,FILE='FF61.DAT')
OPEN (5,FILE='SF61.DAT')
DO 207 JJ=1,20
IF (JJ.GT.1) FO=FO-0.02
V=3.14*D*RN/60
KCR=DO**R1**GMAY(4*V*COS(CS))
FAIK=2**((0.47-DO/R0)**3)
DO 190 JJ=1,20
51 DO 180 J=1,3600
N1=(J(J-1)**F0-J**F0/360)/F2
XX1=(J(J-1)**F0-J**F0/360-F2*N1
XX=XX1+F0
IF (XX.GT.F2)GOTO 56
YY=XX1+F0
56 XX=F0-F2->XX1
C CALCULATE FAINT
58 FAINT=FA10
NN=DO*COS(FAINT)/(V*H*SIN(FAINT))
JJ=1,NN
IF (JJ.LE.0) JJ=3600-JJ
DO 160 KI=1,4
C EOC PAREMETER CALCULATION CASE I
AFANT=AFA-ATAN(Y(4,1)-Y(10,1))/(D/2-D0))
GMANT=GMAS-(AFA-AFANT)
C PRINT *,YY=-.Y(2,1),Y(8,1)
V1=DO-(Y(2,1)-Y(8,1))/R0
IF (V1.LT.-R0*SIN(CS)) GOTO 33
U1=R0*COS(CS)-TAN(CS)**(R0*SIN(CS)-V1)
U2=(Y(2,1)-FO-U1-Y(6,1))
V2=DO-(Y(2,1)-Y(8,1))/R0
VC=V1-R0*SIN(CS)
UC=VC*TAN(CS)

116
S1=(U2-U1)*(V1*(U2-U1)*COS(CS)*SIN(CS))-.5*(U2-U1)**2*COS(CS)*SIN(CS)
UA=ZZ(R0,U1,U2,V1,V2)
VA=V2-(R0**2*(U2-UA)**2)**.5
AC=SQR((VA-VC)**2-(UA-UC)**2)
AO1C=2*ASIN(AC/(2*R0))
VE=V2-R0*SIN(CS)
UE=VE*TAN(CS)-U2-U1
AE=SQR((VA-VE)**2-(UA-UE)**2)
AO2E=2*ASIN(AE/(2*R0))
VQ=(U2*(VA-V1)-U1*VA-UA-V1-V2*CTAN(CS)*(VA-V1))/(CTAN(CS)*(VA-V1)-U1-UA)
UQ=U2-CTAN(CS)*(VQ-V2)
AG=RA*SIN(ATAN((VA-V2)/(U2-UA))-CS)
OQ=SQR((VQ-V2)**2-(UQ-U2)**2)
AQ=SQR((VA-VQ)**2-(UA-UQ)**2)
SAOQ2=5*AG*OQ
AFA2=ACOS(AQ/AG)
O1Q=SQR((V1-VQ)**2-(U1-UQ)**2)
SEHPQ=EQ*PQ
PQ=O1Q*COS(AFA2)
SPO1Q=5*PQ**2*TAN(AFA2)
S23=5*R0**2*(AO1C-AO2E)-SAOQ2-SEHPQ-SPO1Q
AST=S1-S23
CEL=V1*COS(CS)-R0*AO1C
CSE=ATAN(UA/VA)
PRINT **AST=**AST,S23,S1
GOTO 59

C EOC PARAMETER CALCULATION CASE II

33  V1=(R0-D0-Y(1,1)-Y(1,3))
U1=SQR((R0**2-V1)**2)
V2=(R0-D0-Y(1,2)-Y(1,8))
U2=FO-Y(1,6)+U1-Y(6,1)
UA=ZZ(R0,U1,U2,V1,V2)
VA=V2-(R0**2*(U2-UA)**2)**.5
AB=SQR((UA**2-VA)**2)
AO1B=2*ASIN(AB/(2*R0))
AO2C=2*ASIN(AC/(2*R0))
VE=0
UE=U2-U1
SC=ATAN((VE-V2)/(2-UE))
VQ=(U2*(VA-V1)-U1*VA-UA-V1-V2*CTAN(SC)*(VA-V1))/(CTAN(SC)*(VA-V1)-U1-UA)
UQ=U2-CTAN(SC)*(VQ-V2)
AMO2=ATAN((VA-V1)/(UA-U1))
A02M=ATAN((VA-V2)/(U2-UA))
Q2M=(VA-V2)*(CTAN(AO2M)-CTAN(AMO2))
SAOQ2=5*O2M*(VA-VQ)
EO1N=ATAN(U1-UE)-(V1))
AO1N=3.142-AMO2
AO1E=EO1N-AO1N
O1E=SQR((UE-U1)**2-V1)**2
SEO1Q=5*O1E*SIN(AO1E)*COS(AO1E)
SBE01 = -5*V1*UE
AST = 5*R0**2*(AOI1.B-AO2E)+SAQ02.SEO1Q.SBE01
CEL = R0*AO1B
CSE = ATAN(UA/VA)
PRINT *,AST,CSE\+180/3.14,SAQ02.MO2.VQ.AO2M.AMO2.VA.V2
59 AFAET = ASIN(SIN(AFANT)**COS(CSE-CS))
ECT = AST/CEL
CFF = EP1**SIN(AFAET)-(1-EP1**SIN(AFAET))/((GPI1-HP1**LOG(ECT)*0.04*(0.2*V)*FNGP1))((40.*ECT)**PK11)
P = EP1**SIN(AFAET)-(1-EP1**SIN(AFAET))/((GPI1-HP1**ALOG(ECT)*0.04*(0.2*V)**FNGP1))((40.*ECT)**PK11)
BATA = AFANT - ATAN(CFF)
FAINT = ATAN(P**COS(AFANT)/(1-P**SIN(AFANT)))

C FORCE CALCULATION

IF (*.1)K1,E1) MM=100
FUNNY1 = FUNNY1 = 1
IF (FUNNY1.LT.FUNNY-43) GOTO 61
RIAN = 0.9690D0=144/DO-35.1/DO**2
IF (D0.EQ.-4) RIAN = RIAN
IF (D0.EQ.-3) RIAN = RIAN-30
IF (D0.EQ.-2) RIAN = RIAN-33
IF (D0.EQ.-1) RIAN = RIAN-35
FK = (GAUS(MM,126.RIAN)/126.)**0.454
61 FUNNY = FUNNY1
FX = TAOS**SIN(BATA-AFANT)**SIN(CSE)**AST**SIN(AFANT)**COS(AFANT-BATA-AFANT)**FK
FY = FX**COS(BATA-AFANT)**SIN(CSE)**SIN(BATA-AFANT))
FZ = FX**COS(CSE)**SIN(CSE)
FDX = KCR**Y(1.1)**Y(7.1)/GMANT**2
FDZ = KCR**Y(5.1)/GMANT**2
FXT = FX-FDX
80 FZT = FZ-FDZ

C SOLVING DIFFERENTIAL EQUATION

DO 150 I=1,10
IF (LEQ.1) K1(L1) = 1000*(FXT-C(1)**Y(1.1)-K(1)**Y(2.1))/M11
IF (LEQ.2) K1(L1) = Y(1.1)
IF (LEQ.3) K1(L1) = 1000*(FY-C(2)**Y(3.1)-K(2)**Y(4.1))/M12
IF (LEQ.4) K1(L1) = Y(1.1)
IF (LEQ.5) K1(L1) = 1000*(FZT-C(3)**Y(5.1)-K(3)**Y(6.1))/M13
IF (LEQ.6) K1(L1) = Y(1.1)
IF (LEQ.7) K1(L1) = 1000*(1-FXT-C(4)**Y(7.1)-K(4)**Y(8.1))/M2
90 IF (LEQ.8) K1(L1) = Y(7.1)
IF (LEQ.9) K1(L1) = 1000*(1-FY-C(5)**Y(9.1)-K(5)**Y(10.1))/M2
IF (LEQ.10) K1(L1) = Y(9.1)
150 CONTINUE
DO 160 I=1,10
Y(1.1) = Y(L2)+H*K1(L1)/2
IF (K1.EQ.1) Y(L1) = Y(L2)+H*K1(L1)
160 CONTINUE
DO 170 I=1,10
Y(L2) = Y(L2)+H*(K1(L1)-2*K1(L2)-2*K1(L3)-K1(L4))/6

118
Y(1:)=Y(L2)
CONTINUE
IF(JJ.EQ.10) GOTO 171
J=*J/10
SS=REAL(Y1.10:0,J)
IF(SS.LE.0) GOTO 171
X=(Y(2.1)-Y(1.1))*1000
FORMAT(15.5X,F8.3)
DO 183 I=1,10
Y(I,J)=Y(I,1)
CONTINUE
N1=J/100
R1=REAL(J1.10:0

CONTINUE
CZ(36,1J)=F0*REAL(C2*JJ-1),2,-Y1(340.6)
CX(36,1J)=Y1(340.2)-Y1(340.8)
IF(JJ.EQ.1) GOTO 177
ZZZ(IJ)=ZZZ(RJ,CZ(36,1J),CZ(36,1J),CX(36,1J),CZ(36,1J))
PRINT *,ZZZ(IJ),CZ(36,1J),CX(36,1J),"GG"
DO 175 I=1,20
ZZZ(1)=ZZZ(1)-REAL(J)*ZZZ(1J)-ZZZ(1J-1)/20.
XXX(1)=XXX(1)+REAL(J)*(RJ)**2+(ZZZ(1J)-RJ)**2+5-RJ
PRINT *,CX(36,1J),"cz"",CZ(36,1J-1),ZZZ(1),"GOOD"
ZZZ(I)=1000*ZZZ(I)
IN=IN+1
ZM(36,1N)=XXX(I)
PRINT *,"PROFILE",ZZZ(I),XXX(I)
IF(I.EQ.1)PRINT *,F0,FX,FY,FZ,AST
WRITE(4,176) DO,111,ZZZ(I),XXX(I)
176 FORMAT(F10.3,5X,F10.3,5X,F10.3)
CONTINUE
AFX=AFX-FX
AFY=AFY-FY
AFZ=AFZ-FZ
AASF=AASF-AST
INDEX=INDEX+1
DO 190 J=1,3600
XN=(Y1(1.1)-Y1(1.1))*SIN(CS)+Y1(1.6)*COS(CS)
IF(XN.GT.Y2(J,11)) F0*COS(CS) GOTO 179
Y2(J,11)=XN
DO 173 I=1,10
Y2(J,1)=Y1(I.1)
CONTINUE
GOTO 190
179 Y2(J,11)=Y2(J,11)+F0*COS(CS)
CONTINUE
Z0=0
DO 202 I=20,400
Z0=ZN(34,I)+Z0
CONTINUE
Z0=Z0/151
119
RA=0
DO 201 I=250,400
RA=RA-ABS(ZN(36,1)-ZA)
PRINT *,"ZN(36,1):",ZN(36,1)
201 CONTINUE
RA=RA/150
RMAX=0
RMIN=0
DO 220 I=2,14,3
RMAX1=0
RMIN1=1000
DO 210 J=1,20
IF (ZN(36,I*20-J).GT.RMAX1) RMAX1=ZN(36,I*20-J)
IF (ZN(36,I*20-J).LT.RMIN1) RMIN1=ZN(36,I*20-J)
210 CONTINUE
RMAX=RMAX-RMAX1
RMIN=RMIN-RMIN1
220 CONTINUE
PP=(RMAX-RMIN)/5
PRINT *,"RA=",’RA,’ RZ=",’PPEX,TCL
WRITE (5,221) F0,RA
221 FORMAT(F10.3,5X,F10.3)
DO 208 I=1,10
DO 208 J=1,2
Y(LJ)=0
208 CONTINUE
DO 209 I=1,13600
DO 209 J=1,10
Y1(LJ)=0
209 CONTINUE
DO 211 I=1,13600
DO 211 J=1,11
Y2(LJ)=0
211 CONTINUE
IN=0
AFX=AFX/INDEX
AFY=AFY/INDEX
AFZ=AFZ/INDEX
AAST=AAST/INDEX
WRITE (1,181) F0,FX,FY,FZ
181 FORMAT(F10.3,5X,F10.3,F10.3)
AFX=0
AFY=0
AFZ=0
AAST=0
INDEX=0
207 CONTINUE
238 FORMAT(10I)
CLOSE (1)
CLOSE (5)
STOP
END
FUNCTION ZZ(R0,C22,CZ1,CX1,CX1)
C
C THIS FUNCTION CALCULATE INTERSECTION OF TWO CIRCLES
C
A=CX2-CX1
B=2*(CZ2-CZ1)
D=CZ1**2-CZ2**2
E=A**2-D
H=B**2-4*A**2
F=2*E*B-3*A**2*CZ2
G=2*E**2*A**2*(R0**2-2*CZ2**2)
Q=F**2.4*H**2
IF (Q.LT.0) Q=0
ZZ=(Q**.5 -F)/(2*H)
RETURN
END

FUNCTION CTAN(X)
CTAN=1./TAN(X))
RETURN
END

FUNCTION GAUS(M,MEAN,VAR)
C
C THIS FUNCTION GENERATES GAUSSIAN DISTRIBUTION VARIABLE N(M.V)
C CREATE NO.1 FIRST; THEN N(M.V)
C
GRN=SQRT(-2*ALOG(RANDOM(M)))*COS(2*3.14159265*RANDOM(M))
GAUS=SQRT(VAR)*GRN+MEAN
RETURN
END

FUNCTION RANDOM(M)
C
C THIS FUNCTION GENERATES RANDOM VARIABLES DISTRIBUTED
C UNIFROMLY IN THE INTERVAL [0,1].
C
M=2048*M+1
M=M(M/1048576)*1048576
RANDOM=FLOAT(M+1)/1048577.0
RETURN
END

C CALCULATION OF SAMPLE VARIANCE CGMAS AUG. 15

FUNCTION AR(D0,F0,N0)
C
C THIS FUNCTION CALCULATES SAMPLE VARIANCE
C
L=3.14*50/33.
DR=.0075
SUM=0
S=0
PR=0
WR=0
DO 20 J=1,60
RR=REAL(J)
IF (RR.LE.3) PR=1.2*RR
IF ((RR.GT.3).AND.(RR.LE.8.)) PR=-4.05*(RR-3.)
IF ((RR.GT.8).AND.(RR.LE.13.)) PR=15-0.01*(RR-8.)
IF ((RR.GT.14).AND.(RR.LE.60.)) PR=1.01/46*(RR-13.)
RR=RR*.0075
AC=(.0075**10)**2
WR=AC/RR**2(2*3.14)**(1-RR/0)
IF (WR.LT.0) WR=0
SUM=SUM-WR*PR*RR**2*DR
20 CONTINUE
NX=INT(DO*FR/AC)
IF (NX.EQ.0) NX=1
21 AR=4*3.14**7409/(AC*L)*SUM
RETURN
END
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