Design and analysis of reinforced earth retaining wall under vertical and horizontal strip load.

Oliver King Tai. Ng

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DESIGN AND ANALYSIS OF REINFORCED EARTH RETAINING WALL
UNDER VERTICAL AND HORIZONTAL STRIP LOAD

by

Oliver King Tai Ng

A Thesis
submitted to the Faculty of Graduate Studies
through the Department of
Civil Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
The University of Windsor

Windsor, Ontario, Canada 1985
C  Oliver King Tai Ng
To My Parents and Sisters.
ABSTRACT

This thesis investigates the behaviour of a reinforced earth retaining wall system under vertical and horizontal surcharge strip loading. Experimental results from a previous author (Peter Seymour) were analysed. A theoretical method, which takes into account the force redistribution characteristic of reinforced earth retaining wall system, was developed to approximate the magnitude and distribution pattern of traction forces. A finite element analysis based on incremental and iteration procedures was developed and used to simulate the behaviour of a reinforced earth retaining wall system under various loading conditions. This includes comparisons involving magnitude and distribution pattern between experimental results and classical rigid retaining wall theory; comparisons between experimental results and redistribution method; and comparisons between experimental results and finite element analysis.
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NOMENCLATURE

\[ [A] \] matrix containing the element nodal coordinates

\[ [A^{-1}] \] inverse of \([A]\]

\(A\) cross-sectional area of strip

\(A_s\) cross-sectional area of strip

\(A_e\) equivalent cross-sectional area of plate

\(b_1\) width of reinforcing strip

\(b\) width of strip footing

\(b'\) actual width of strip footing

\([B]\) constitutive matrix

\(c\) cohesion

\(d\) perpendicular distance from footing to wall

\(d_2\) perpendicular distance between wall and centre line of footing

\(d_1\) perpendicular distance between the wall face and line load \(P\)

\(D_{60}\) diameter at which 60\% of soil is finer

\(D_{10}\) diameter at which 10\% of soil is finer

\(D_{50}\) diameter at which 50\% of soil is finer

\([D]\) constitutive matrix containing the elastic properties of element

\(E\) modulus of elasticity

\(e\) eccentricity

\(E_e\) equivalent stiffness of plate

\(E_i\) initial tangent modulus of soil
Es  modulus of elasticity of reinforcement  
Et  tangent modulus of soil  
f  friction coefficient  
f0  friction coefficient from direct shear test  
F1  horizontal force per unit length of footing  
F  horizontal force per unit length of footing  
FSbond  factor of safety against slippage  
FS  factor of safety against tensile failure  
FT  total frictional resistance  
FN  net frictional resistance  
Fy  yielding load  
FE  excess force  
FET  total excess force  
F  redistributed force  
Fr  frictional resistance against pull out  
Fu  ultimate load of reinforcement  
h  depth below the soil surface  
h0  depth of horizontal stress penetration  
h1  depth of horizontal stress penetration  
H  horizontal strip load intensity  
Hw  height of wall  
k  number of strip level below point of 'zero' difference  
Ka  active pressure coefficient  
Ko  soil coefficient at rest  
K  soil coefficient
[ $K_e$ ] local stiffness matrix of element

[ $K$ ] global stiffness matrix of element

[ $K_c$ ] diagonal material property matrix of interface element

$K_s$ modulus number of soil

$K_{sf}$ unit tangential stiffness of interface element

$K_{nn}$ unit normal stiffness of interface element

$K_r$ stiffness number of interface element

$K_{si}$ initial shear stiffness

$L$ length of truss or interface element

$le$ length of strip beyond failure plane

$l$ width of strip footing

$l'$ actual width of strip footing

$L'$ length of strip beyond failure plane

$L_1$ length of strip between failure plane and 2 : 1 dispersion line

$L_2$ length of reinforcing strip

$L_e$ equivalent length of plate

$L_s$ length of reinforcement

$L_w$ length of wall

$M_f$ reduction factor

$m$ number of strip level where $T > F_w$

$n$ modulus exponent of soil

$n_s$ stiffness exponent of interface element

$N$ number of strips in each level

$P$ vertical line load
\textbf{Pa} \quad \text{soil pressure force} \\
\textbf{P_s} \quad \text{tangential force} \\
\textbf{P_n} \quad \text{normal force} \\
( \textbf{P} \text{ )} \quad \text{force vector} \\
\textbf{P_{at}} \quad \text{atmospheric pressure} \\
( \textbf{q} \text{ )} \quad \text{vector of nodal displacement} \\
\textbf{Q} \quad \text{vertical strip load intensity} \\
\textbf{R} \quad \text{reduction factor depending on the stiffness of soil and reinforcement} \\
\textbf{R_1} \quad \text{distance from point 0 to the far end of footing} \\
\textbf{R_2} \quad \text{distance from point 0 to the near end of footing} \\
\textbf{R_f} \quad \text{failure ratio of soil} \\
\textbf{R_{f_3}} \quad \text{failure ratio of interface element} \\
\textbf{S_e} \quad \text{equivalent stiffness of plate} \\
\textbf{S} \quad \text{total axial stiffness of reinforcements} \\
\bar{s} \quad \text{surface of element} \\
\textbf{T_{max}} \quad \text{maximum traction force} \\
( \textbf{T} \text{ )} \quad \text{transformation matrix} \\
\textbf{T} \quad \text{traction force} \\
\textbf{T(t)} \quad \text{temperature} \\
\textbf{t} \quad \text{time} \\
( \textbf{\bar{T}} \text{ )} \quad \text{surface traction vector} \\
\textbf{U_s} \quad \text{strain energy} \\
( \textbf{u} \text{ )} \quad \text{a vector of displacement at any point} \\
\textbf{u_i} \quad \text{displacement of node i in X-direction} \\
\textbf{v_i} \quad \text{displacement of node i in Y-direction}
\( V \)  
volume of element

\( W_s^t \)  
tangential displacement at top of element

\( W_s^b \)  
tangential displacement at bottom of element

\( \{ W \} \)  
displacement matrix

\( W_n^t \)  
normal displacement at top of element

\( W_n^b \)  
normal displacement at bottom of element

\( w(t) \)  
water content

\( W \)  
width of reinforcing element

\( \{ \bar{x} \} \)  
body force vector

\( x(t) \)  
anisotropy

\( \{ \alpha \} \)  
a vector of coefficients of the assumed interpolation function

\( \alpha_i \)  
coefficients of the displacement functions

\( \alpha' \)  
constant

\( \alpha \)  
angle between vertical and bisector of

\( \beta' \)  
constant

\( \beta \)  
angle of visibility

\( \phi \)  
angle of internal friction

\( \{ \phi \} \)  
the interpolation matrix

\( \{ \phi_s \} \)  
\( \{ \phi \} \) evaluated along surface points only

\( \phi_i \)  
angle of friction between soil and strip

\( \psi \)  
angle of friction between soil and strip

\( \gamma \)  
unit weight of soil

\( \gamma_w \)  
unit weight of water

\( \gamma_{xy} \)  
shear strain
\( \delta \) the angle between the vertical and the line connecting point 0 to the near end of footing.

\( \delta_n \) average relative normal displacement of interface element

\( \delta_s \) average relative tangential displacement of interface element

\( \nu \) Poisson's ratio

\( \theta \) angle of inclination

\( \Pi_p \) total potential energy

\( \Delta s \) horizontal spacing of strip.

\( \Delta h \) vertical spacing of strip

\( \Delta T \) additional tension in reinforcing strip

\( \Delta \sigma_h \) change in horizontal stress

\( \Delta \sigma \) horizontal stress due to horizontal force \( F \)

\( ( \cdot )^T \) transpose of vector

\( \Delta \sigma_s \) change of stress in strip

\( \tau_{xy} \) shear stress

\( \tau_s \) average shear stress.

\( \tau_f \) failure shear of interface element

\( \tau_{ult} \) asymptotic value of shear stress of interface element

\( \{ \varepsilon \} \) strain vector

\( \varepsilon_a \) axial strain

\( \varepsilon(t) \) strain conditions

\( \varepsilon_x \) strain in X-direction

\( \varepsilon_y \) strain in Y-direction

\( \{ \sigma \} \) stress vector
$\sigma_1$  major principal stress
$\sigma_3$  minor principal stress
$\sigma_{i1}$ initial major principal stress
$\sigma_{i3}$ initial minor principal stress
$\sigma_{f1}$ major principal stress at failure
$\sigma_{f3}$ minor principal stress at failure
$\sigma_{i1a}$ major principal stress beyond failure
$\sigma_{i3a}$ minor principal stress beyond failure
$\sigma_v$ vertical stress
$\sigma_{vf}$ vertical stress at failure
$\sigma_{wi}$ increase in normal stress due to surcharge load
$\sigma_h$ lateral pressure
$\sigma_x$ stress in X-direction
$\sigma_y$ stress in Y-direction
$\sigma_{yd}$ yield stress
$\sigma_H$ horizontal stress
$\sigma_n$ average normal stress
$\sigma_1 - \sigma_3$ stress difference
$(\sigma_1 - \sigma_3)_{ult}$ asymptotic stress difference
$(\sigma_1 - \sigma_3)_f$ stress difference at failure
CHAPTER I
INTRODUCTION

1.1 General

Throughout the ages, the concept of incorporating tensile stress resisting material such as fibres, bars or boards into soil fill, to improve the load carrying capacity of the soil, has been practiced. An example is the corduroy roads which are constructed on a foundation of tree trunks and branches in the swampy area where the soil bearing capacity is low.

Though this concept was being used, it was not until the 1960's when this concept began to be scientifically analysed and practiced. Henri Vidal (82), a French engineer, initiated the study of this concept. He coined the term "Reinforced Earth", a cohesive material from associating reinforcing elements and sand. Since then, reinforced earth structures have been completed in all parts of the world in a variety of environmental settings where it has been subjected to a variety of loading conditions. Some of the applications are: foundation improvement; slope stability improvement; industrial structures for material processing and storage facilities; containment dikes for oil storage; and as foundation slabs and hydraulic structures such as sea walls and dams. The most widely used and best known application is in a retaining wall and bridge abutment situation.
Most of the research conducted focused on one behaviour and mechanism of a reinforced earth retaining wall structure. The existing design procedures are based on the data obtained from model studies and full scale structures measurements.

1.2 Objective

The main theme of this research is the study of the behaviour of a reinforced earth retaining wall model. Particularly, this model will be studied while being subjected to a combination of vertical and horizontal surcharge strip loads running parallel to the wall head of the structure. This situation occurs when the reinforced earth retaining wall is used as a bridge abutment. The problem of vertical surcharge strip load arises when highway pavements, railway tracks and continuous wall footings are built on top of the reinforced earth structure.

In order to investigate the behaviour of a reinforced earth retaining wall, Seymour's (75) extensive experimental testings performed on the model are reviewed. The model consisted of Lake Erie sand backfill reinforced with roughened shim steel reinforcing strips. Some of the strips were instrumented with strain gauges to measure the strain developed in the strip during loading. The reinforcing strips were connected to the wall face which were made of thin steel channels connected to each other. The movements of the wall were being measured by dial gauges. The
experimental results were being analysed. The present research is a continuation of Seymour's work where more emphasis will be placed on the development of a theoretical method to approximate the maximum traction distribution developed in the reinforcing strips in a flexible reinforced earth retaining wall system. Comparison will be made among the experimental results, results from existing theory and the results from the new theoretical method. Also a computer model based on finite element method will be developed to simulate the behaviour of the experimental model. The computer results will be compared with experimental results.
CHAPTER II

BACKGROUND AND LITERATURE REVIEW

2.1 General

In this chapter, the basic principle of reinforced earth will be introduced and explained. In addition, the theoretical and experimental developments in explaining the mechanism of reinforced earth retaining wall with and without surcharge load is summarized. Also, the current design method used by the Reinforced Earth company will be included. This is an empirical method based on experimental results from model and full-scale structures. Finally, the development of computer analysis in soil mechanics and reinforced earth retaining wall will be discussed.

2.2 Basic Principle

Reinforced earth is a composite medium. The basic mechanics of reinforced earth were developed and explained by Vidal (82). The idea of introducing reinforcements into a soil medium, usually cohesionless granular material, is to strengthen or to improve the load carrying capacity of the soil medium. Usually the reinforcements are aligned in such a way that they will resist the largest tensile strain. The friction between the soil and the reinforcement, which allows the stress developed in the soil medium to be transmitted to the reinforcement, plays an essential role in the concept of reinforced earth.

The basic mechanics of reinforced earth can be
illustrated by Figure 2.1. If a sample of unreinforced dense granular material is subjected to an axial compressive load $\sigma_1$ (Fig. 2.1a), lateral expansion of the granular material will result. As a result of dilation, the lateral strain is more than one-half the axial strain. The soil mass is stable only if a lateral compressive stress $\sigma_3$, at least equal to $K_a\sigma_1$, is applied to the soil sample. $K_a$ is the active earth coefficient. The Mohr's circle then becomes tangent or falls below the line of rupture. This confining stress $\sigma_3$ has to be applied externally as granular soil without reinforcements cannot withstand unconfined compression.

However, if inextensible horizontal reinforcing elements are introduced to the soil mass (Fig. 2.1b), the reinforcements will prevent the lateral expansion or strain because of friction between the reinforcing elements and the soil. This behaviour will be similar to that of an external load, at least equal to $K_a\sigma_1$, which has been imposed on the element. While each element of the soil mass is acted upon by a lateral stress, and as the vertical stresses increase, the lateral restraining stresses will increase in proportion. Therefore the stress circle is always tangent to or below the rupture line all the time. Failure can occur when the friction between soil and reinforcement is lost or when the reinforcement tensile strength is exceeded.

From the above discussion, it is clear that soil-strip friction plays a major role in the mechanism of reinforced
earth. Schlosser and Vidal (72) performed direct shear tests on metal strips with different surface roughness. It was found that the presence of ribs or grooves on the strip would enable the internal friction of soil to be fully utilized. Currently, this type of strips is being used in practice.

In order to determine the soil-reinforcement friction coefficient, Bacot et al. (4) performed tests on reduced scale reinforced earth retaining model. It was found that the friction coefficient \( f_0 \) obtained from direct shear test was of minimum value. Depending on the characteristics of the reinforcements, he proposed the following equation for determining the friction coefficient \( f \) between soil and reinforcing strip.

\[
f = f_0 + \alpha' L_2 \frac{\beta'}{b}
\]

where \( \alpha' \) and \( \beta' \) are constants.

Tests performed by Schlosser and Elias (70) suggested the friction coefficient was dependent on the angle of internal friction of sand and the density of the soil. At high density the friction coefficient \( f \) was higher than friction coefficient obtained from direct shear test. The reverse was observed at low density. They also suggested the angle of internal friction of sand used be not less than 25°. They also observed that the friction coefficient decreased as the overburden increased.

Kennedy et al. (42), proposed the following formula
for determining the friction coefficient.

\[ f = \left( \frac{\sigma_v}{\sigma_f} \right) \tan \psi \]  

(2.2)

where \( \sigma_v \) is the existing vertical stress; \( \sigma_f \) is the vertical stress at failure, and \( \psi \) is the angle of friction between soil and reinforcement. The ratio \( \left( \frac{\sigma_v}{\sigma_f} \right) \) can be taken as \( \left\lfloor 1 / \text{(factor of safety)} \right\rfloor \).

Currently, the values of \( f \) used in design by Reinforced Earth \( \mathbb{R} \) company, as outlined by McKittrick (53) are: (Fig. 2.2)

- for smooth strips \( f = 0.4 \)
- for ribbed strips \( f = \tan \phi \), for \( h \geq 6 \text{ m} \) (2.3a)
  \[ f = f_1 + \left( \frac{h}{6} \right) (\tan \phi - f_1) \]
  for \( h \leq 6 \text{ m} \) (2.3b)

where \( \phi \) is the angle of internal friction of soil; \( h \) is the depth below the surface, and \( f_1 \) is defined as:

\[ f_1 = 1.2 + \log \left( \frac{D_{60}}{D_{10}} \right) \]  

(2.4)

where \( D_{60} \) and \( D_{10} \) are the diameters at which 60% and 10% respectively of the soil is finer, by weight.

2.3 Reinforced Earth in Retaining Walls

The idea of the retaining wall can be explained by first considering a semi-infinite soil mass with a horizontal surface (Fig. 2.3a). The vertical stress \( \sigma_1 \) at any depth is equal to:

\[ \sigma_1 = \gamma h \]  

(2.5)

where \( \gamma \) is the unit weight of soil, and \( h \) is the depth below the surface. The horizontal stress is equal to:
\[ \sigma_3 = K_0 \gamma h \]  
(2.6)

where \( K_0 \) is the coefficient of earth pressure at rest.

If a vertical cut is made through the soil mass and the right hand portion is removed, the left hand side can be maintained at rest condition by applying a pressure \( \sigma_3 \), as shown in Fig. 2.3b. This pressure can be exerted externally by building a rigid retaining wall or can be exerted internally by applying the theory of reinforced earth. If horizontal reinforcements are introduced into the soil mass, the friction between soil and reinforcements will restrict the soil mass from moving. The reinforcements are thus under tension and this is the tension which maintains the soil mass without external applied loads.

In order to prevent any erosion and spillage of soil, a cover termed 'skin' is provided at the vertical cut. Usually the skin is affixed to the end of the reinforcements.

From the above discussion, it appears that the reinforced earth retaining wall is made up of three essential elements: namely, the skin, the soil and the reinforcements.

2.3.1 The Reinforcements

Theoretically, any materials possessing the necessary tensile strength and friction surface, may be used as reinforcements. The materials must also have good corrosion resistance. Thus, strips, wire meshes, steel cables and
plastic fabric may be used. Although polyester fabric has no problem with corrosion, the uncertainty of long-term creep strength of the fabric discourages its use. Currently, ribbed galvanized steel strips are used as reinforcements in retaining structures. The ribs increase soil-strip friction and the zinc cover provides corrosion resistance. The problem of corrosion has been studied by Darbin et al. (19). The design yield stress is taken as two thirds of the experimentally determined yield stress. The reinforcing strips are bolted to the skin and laid horizontally over the backfill at a predetermined vertical and horizontal spacings.

2.3.2. The Soil

The soil is dumped and compacted between two horizontal layers of reinforcing strips. The type of soil used is granular cohesionless material with good draining property. With well drained soil, there will be immediate effective normal stress transfer between soil backfill and strips. The soil-strip friction will be developed effectively. The shear strength of the soil will not be reduced due to build up of pore water pressure.

The current minimum specification, adopted by the U. S. Federal Highway Administration, is that the maximum percentage by weight of backfill passing through No. 200 sieve is 25% and that the plasticity index (P. I.) is less than six. If percent passing No. 200 sieve is greater than 25% and percent finer than 15% is less than 15%, the
material is acceptable if $\gamma = 30$ and P.I. < 6. Essentially, only non-plastic soil will be used.

2.3.3 The Skin
In most instances, the skin is viewed as only part of a reinforced earth structure. Although it is not important in the theory of reinforced earth, its main role is to restrain the soil grains located near the exterior between two layers of reinforcements. As well, it prevents local erosion of soil backfill. The skin can be made up of any material, but it must be strong enough to support all the forces. It is especially important that it be durable during the construction stage when all the construction machinery are moving. Presently, interlocking precast concrete panels are being used. The advantages they offer are that they are attractive and easier to connect.

2.4 Reinforced Earth Walls without Surcharge
As with other soil retaining structures, the internal and external stability of a reinforced earth retaining wall must also be checked to ensure that a sound and safe structure has been built.

In the problem of external stability, the reinforced earth retaining wall is considered as a rigid body. Depending on the loading condition, the whole system must be checked against sliding, overturning, foundation bearing failure, settlement, etc. similar to other retaining
structures.

The problem of internal stability has been studied extensively by many researchers. Generally, a reinforced earth retaining wall may fail under one of the following failure modes:

1. Failure due to strip pull out, i.e. insufficient soil-strip friction.
2. Tensile failure of the reinforcing strips.
3. Shear failure of granular material between the reinforcements.

The third kind of failure has been studied by Shen et al. (76).

In order to prevent any of the above failure modes, different hypotheses were proposed to explain the behaviour of reinforced earth. Thus, the force developed in the strip can be determined and the strip size, spacing and length of strips could be selected accordingly.

The first approach was suggested by Schlosser and Vidal (72). Classical theory for a rigid retaining wall was applied. It is assumed that the earth around the strip behind the skin of the wall is in active state, the wedge is in equilibrium under its own weight, soil reaction along the failure plane and the resultant horizontal force in the strips. This method assumed the traction force distribution along the strip to be constant with an exception for a certain length at the free end on which friction is mobilized. This approach was later abandoned because it was
unsuitable to explain the progressive nature of failure.

The second method was proposed by Lee et al. (51). Lee et al. conducted a model wall test. They concluded that the failure plane was a straight line, with an angle of inclination of \((45^\circ + \phi / 2)\) with the horizontal at the wall toe. The Rankine active earth theory was applied behind the wall. The maximum tie tension \(T_{\text{max}}\) at any depth below the surface is given by:

\[
T_{\text{max}} = K a Y h \Delta s \Delta h
\]  

(2.7)

where \(\Delta s\) and \(\Delta h\) are vertical and horizontal spacing. The frictional resistance \(F_r\) against pull out is provided by the soil mass behind the potential failure plane and is given:

\[
F_r = 2 b l_e \sigma_v h f
\]  

(2.8)

where \(l_e\) is the length of strip beyond the failure plane. This method assumes that the maximum tension in strip occurs at the wall face.

A later study conducted by Schlosser and Long (71) shows that the traction force distribution along the strip is varied as shown in Fig. 2.4. This distribution has the following characteristic:

1. The points of maximum tension in any reinforcement layers lie on a curve.

2. The strip tension is not maximum at the wall. It is only fraction of the maximum value measured at the failure plane.

The above distribution has been confirmed by results from
instrumented full scale structures and model studies (3), (41), (55), (71). The parabolic curve divides the reinforced earth retaining wall system into active and resistant zones (Fig. 2.4). In the active zone, the tangential stresses exerted by the soil on the reinforcements are directed to the wall face. In the resistant zone, the tangential stresses are directed away from the wall and in which the soil tends to retain the reinforcements. Juran and Schlosser (40) suggested a logarithmic spiral as the configuration for the parabolic curve. Bassett and Last (6) suggested the failure plane was vertical and parallel to the wall face down to 80% of depth, and then joining the base of wall in a straight line. The current design method suggests one similar to that of Bassett and Last, see Fig. 2.5.

Based on the results of instrumented full scale structures, Shen et al. (76), Chang and Forsyth (11) suggested the state of stress in the backfill of a reinforced earth retaining wall was at rest condition. More extensive studies conducted by Baquelin (5), and Juran et al. (41) indicate that the earth pressure coefficient, K, was equal to Ko at the surface and reduced to Ka at a depth of 6 m, as shown in Fig. 2.6.

2.5 Reinforced Earth Walls with Surcharge

The use of reinforced earth structures as supporting structures (e.g. railway, highway) or as bridge abutments...
has necessitated the study of a reinforced earth retaining wall subjected to imposed loads.

2.5.1 Vertical Load

Schlosser and Long's (71) experimental and theoretical works focused on a reinforced earth retaining wall subjected to a line load running parallel to the wall face. They assumed that the additional vertical stress due to the line load, \( P \), was bounded by the 2 : 1 diffusion lines, as shown in Fig. 2.7. They suggested the additional tension \( \Delta T \) in the strip would be:

\[
\Delta T = K_a \cdot \Delta s \cdot \Delta h \left( P \left/ \left[ d_i \cdot h/2 \right] \right. \right) \quad (2.9)
\]

where \( d_i \) is the perpendicular distance between the wall face and the line load \( P \).

The assumption of 2 : 1 dispersion for vertical stress was confirmed by Baquelin (5) and Mossaad (55), based on measurements on a full scale structures and a model respectively.

Al-Hussaini and Perry (3) carried out a field test on a instrumented reinforced earth retaining wall subjected to a uniform distributed load on the surface. They found that the theoretical results from Rankle earth theory cannot correlate completely with the experimental results.

Kennedy et al. (42) suggested the following equation for estimating the maximum tension force developed in the reinforcing strip due to a strip surcharge load running parallel to the wall. It is a modification of the equation proposed by Terzaghi on calculating the lateral pressure
acting on a rigid retaining wall as shown in Fig. (2.8). The Kennedy's equation is:

\[ T_{\text{max}} = (2-R) Q \left( \beta - \sin \beta \cos 2\alpha \right) \Delta s \Delta h / \pi \]

(2.10)

where \( R \) is reduction factor depending on the stiffness of soil and reinforcements; \( Q \) is the vertical load intensity; \( \beta \) is the angle of visibility at point 0, in radians; and \( \alpha \) is the angle between the vertical and bisector of \( \beta \).

2.5.2 Horizontal Load

Very limited research has been carried out on studying the effect on horizontal load applied on the soil surface. When the horizontal load is applied towards the wall face, the French Ministry of Transport (F. M. T.) suggests that the change in horizontal stress \( \Delta \sigma_h \), as shown in Fig. 2.9, may be determined as:

\[ \Delta \sigma_h = F \left( 1 - \frac{h}{h_0} \right) / \left( 1' + d \right) \]

(2.11)

where \( F \) is the horizontal force per unit length of footing; \( 1' = 1 - 2e \) represents effective width of the eccentrically loaded footing; \( h_0 = 2 \left( 1' + d \right) \) gives the depth of horizontal stress penetration; \( 1 \) is the actual width of the footing; \( e \) is the eccentricity of the resultant force acting on the footing base; and \( d \) is the perpendicular distance from the footing to the vertical wall face. Nevertheless, there have been no data published to support this proposition.

Laban et al. (48) implemented tests on a model reinforced earth retaining wall. The system was first
loaded with vertical strip load and then incremental horizontal strip loads were applied. Experimental results suggested the change in horizontal stress was different than that of the French Ministry of Transport. Details of their results will be discussed in a later chapter.

2.6 Summary of Design Procedures

Current design method suggests that in designing against slippage, a factor of safety should be \( \geq 1.5 \). The factor of safety \( F S_{Bond} \) is determined by:

\[
F S_{Bond} = \frac{2 b f l e}{K \Delta s \Delta h} \quad (2.12)
\]

values of \( K \), \( f \), and \( l e \) are defined in Fig. 2.6, Fig. 2.2 and Fig. 2.5.

In designing against tensile failure, a factor of safety 3.5 should be used. The factor of safety \( F S \) is:

\[
F S = \frac{K \sigma_v \Delta s \Delta h}{F_u} \quad (2.13)
\]

where \( F_u \) is the ultimate load of reinforcements.

These two factors have to be checked for each reinforcement level. The use of different factor of safety will ensure slippage occurs before sudden tensile failure.

2.7 Numerical Methods of Analysis

The finite element method, due to its flexibility, has been utilized widely for the analysis of soil-structure interaction problems. The method was applied by modelling the soil and structure by an assemblage of finite elements
in a two-dimensional plane strain problem.

Important finite element programs were developed by Duncan (26, 27, 28) for the analysis of rigid retaining wall and embankment. Close agreement between experimental results and numerical results were observed. These programs were based on the hyperbolic stress-strain relationship for the soil. Non-linear interface elements were introduced between different materials. The nonlinear, stress dependent properties of the soil and interface behaviour are approximated by performing incremental analyses. Al-Hussaini et al. (2) took the same approach in analysing a reinforced earth retaining wall. Additional interface elements were used to accommodate slippage between and separation of soil and reinforcements of the reinforced earth wall. It was found that the boundary conditions of the wall played a significant role in the finite element results. Thus, some agreements between field results and finite element analyses were obtained. Compaction of soil was introduced by Hafez (37) in the analysis of soil-culvert system.

Hermann and Al-Yassin (38) utilized a different approach, discarding the use of discrete representing of one constituents. They modelled the reinforced earth system as a composite material. The properties of the composite material depends on the properties of the matrix material and the reinforcing members, and their composite interaction. The advantage of this method is the economy of
analysis. This is achieved by not having to discretely represent each and every reinforcing member. However, the disadvantage is that the analysis does not directly yield information about the stress and strain state at the interface of the soil and the reinforcing members nor about localized deformations near the edge of the reinforced mass. The same approach was taken by Naylor (56) in constructing a model for analysing the effects of allowing strip slip, fixity at the face (whether the strips are rigidly or loosely fixed to the facing units), relative longitudinal stiffness of strips and soil, and stiffness of the foundation. But in his analysis, linear soil properties (constant $E$) and one stage analysis were used. This did not take into account the non-linear behaviour of soil.

A modified finite difference solution, based upon the wave equation proposed by Smith (77) for soil-pile system, was developed by Salomone et al. (69) for soil-reinforcement interaction of reinforced earth. This method predicted within 10% of full scale pull out test.

2.8 Other Application

Reinforced earth has other applications other than a retaining wall structure. It has been used to improve the load carrying capacity of weak foundations. Model test results by Binquet and Lee (7) showed that the ultimate bearing capacity and settlement characteristic of sand reinforced with horizontal strip would be improved by a factor of two to four times that of unreinforced soil.
Other materials were utilized instead of metal strips as reinforcement. For instance, Akinmusurn and Akinbolade (1) used rope fibre material and Rea and Mitchell (63) used a grid of interconnected paper cells to reinforced soil. The tests from these authors also showed that the bearing capacity of the soil would be greatly improved. Also Basset and Last (6) performed tests on sand-reinforced with rods aligned with the principal tensile strain direction under a footing. Significant improvement in ultimate bearing capacity was observed. The same improvement in bearing capacity would be achieved by laying the rods horizontally below the footing. The traction force distribution along the strips of a reinforced earth foundation under a strip footing was presented by Schlosser and Long (71) as shown in Fig. (2.10).

The problem of excessive shear stresses induced at the base of a high embankment by the tendency of the embankment to spread have been successfully dealt with by reinforcing the critical area with woven nets of synthetic resin. Wager and Holtz (83) dealt with this problem by using beams or channel sections tied together by a tie rod at the base of the embankment.
CHAPTER III
EXPERIMENTAL INVESTIGATION

3.1 General

The experimental work described in this chapter was carried out by Seymour (75). To aid in complete comprehension of this thesis, the details of experimental set up will be discussed in detail.

The goal of the experimental work was to investigate the behaviour of the model reinforced earth retaining wall under different combinations of vertical and horizontal strip loads. These behaviours provide a base where, theoretical work is developed to predict the response of a full-scale structure subjected to the same kind of loadings.

3.2 Basic Elements of Reinforced Earth Model

3.2.1 Backfill Soil

The backfill soil used in the experiment was Lake Erie sand. It had a mean grain size \( D_{50} = 0.31 \) mm and a coefficient of uniformity of 2.3. The percentage by weight of sand passing through No. 200 sieve was 1%. This was well below the 25% limit suggested by McKittrick (53). The angle of internal friction obtained from direct shear test was 40°. The specific gravity of the sand was 2.65 and the unit weight was kept constant at 17.1 kN/m³. This density represented a medium dense to dense condition and thus ensuring dilation of sand would occur during shear.
3.2.2 Skip Elements

The skin elements were made of galvanized steel channels measuring 0.5 mm thick, 50 mm high and with a flange width of 12.5 mm. The channels spanned the width of the box with a small clearance at either side to avoid friction with the side walls of the box. Foam rubbers were glued at the ends of the channels to prevent sand from spilling out of the clearance.

Ten channels were used to build up the wall. The wall was built in such a way that the flanges of the channels were facing out from the sand. This provided a vertical flat surface in contact with sand backfill. The channels were connected to each other at the flanges by using 3.22 mm diameter bolts. The channels were connected at the points where the strips were attached to the channels. Foam rubbers were glued along the flanges of the channels to prevent sand from coming out between the channels.

3.2.3 Reinforcing Strips

The reinforcing strips were made of smooth shim steel manufactured by Paxam Metals Ltd. They measured 25 mm wide, 0.025 mm thick and 665 mm long. In order to increase the load carrying capacity of the system, sand was glued to the top and bottom surfaces of the strips to increase the soil-strip friction. The strip lengths were selected to be greater than 0.8 times the height of the wall, the minimum ratio in design.
The strips were glued to small pieces of galvanized steel which were attached to the skin elements with 3.22 diameter bolts, as shown in Fig. 3.1. The horizontal spacing of the strips were arranged 185 mm apart, centre to centre, as shown in Fig. 3.2.

In the top channel, 3 additional strips were attached at mid-height of the channel, Fig. 3.3, to prevent excessive movement of the channel due to external load.

Seven of the ten strips located at the centre of the wall were instrumented with strain gauges. The centre strips were chosen because they were least affected by the friction between side wall and sand mass. The gauges were affixed to the strips prior to sand being glued.

The yield stress of the shim steel was found to be $4.5 \times 10^5$ kN/m$^2$ and the modulus of elasticity $E$ was $208.9 \times 10^3$ GPa.

3.2.4 Containment Box

A containment box measuring 0.556 m wide, 0.749 m long and 0.57 m high was made of 12.5 mm thick plexiglass, as shown in Fig. 3.3. The actual reinforced earth retaining wall system measured 0.706 m long, 0.53 m high and 0.556 m wide was constructed in the box. To reduce friction between the sand and the side of the wall, plexiglass was used because of its smooth surface. The dimensions of the box were selected to provide sufficient width to minimize the effect of side wall friction on the instrumented strips at the centre of the box, and sufficient height to study the
stress conditions at different horizontal levels. A steel frame was wrapped around the mid-height of the box to prevent excessive arching of the side walls. The containment box was placed on a metal frame bolted with an electric vibrator for compacting the soil.

3.3 Measuring Instruments

3.3.1 Strain Gauges

The strain gauges used to measure the strain along the reinforcing strips were of type EA-06-250BG-120 of 120 ohms (0.3%) resistance and a gauge factor of 2.03 (0.5%) at 75°F. The strain limit of them were 5%. The strain gauges were connected to a Vishay Ellis V/E 20-A electronic strain gauge indicator and V/E 22 automatic printer, by eight V/E 21 switch. The strain indicator was capable to read up to 80 gauges simultaneously. In this experiment, 39 strain gauges were used.

The strain gauges were installed at locations as shown in Fig. 3.4. No gauges were placed on level 7 and level 9, as these levels were not considered critical under the conditions being investigated. The gauges were placed along an assumed failure plane for load location II. This failure plane was a line joining the base of the wall to the inner edge of the loading block. The remaining gauges were evenly distributed along the length of the strip.

3.3.2 Dial Gauges
Dial gauges were set up along the vertical centre line of the wall and the middle of the channel to measure the horizontal deformations of the wall, as shown in Fig. 3.5. These gauges could be read to an accuracy of 0.001 of an inch (0.0254 mm).

3.4 Construction Procedure

The reinforced earth wall model was constructed as follows: the first channel was placed directly on the floor of the plexiglass box. Two wooden posts were used to hold the wall element (channel) vertical during construction. The first bed of sand of predetermined weight (to ensure a unit weight of 17.1 kN/m$^3$) was then placed behind the wall element, levelled using horizontal guide lines drawn on the sides of the box, and then hand compacted by a trowel. Three reinforcing strips were placed along the compacted sand surface, each with the strengthened end over the flange of the channel. The next channel was placed on top, and the two channels and the reinforcing strips bolted together. More sand was placed on the top of the first layer, and the procedure was repeated until the ten channel wall elements were in place. Then the wooden posts were removed from the front of the wall. The dial gauges and strain gauges were all set to zero and the model was ready for loading.

3.5 Loading System

The loading system used to apply the external pressure
consisted of a steel block 100 mm x 100 mm cross-section spanning the full width of the box, parallel to the wall face. A leverage system, Fig. 3.6, acting against the metal frame which was bolted to the floor, was used to apply the vertical load on the block. Dead weights were suspended from the end of a metal beam by a hanger. The load was applied centrally on the steel block through a 25 mm diameter steel rod with its longitudinal axis lying directly above the central axis of the block.

Horizontal forces were applied to mid-height of the same steel block using a separate loading system. In order that the horizontal force on the block would be fully transmitted to the soil mass without frictional interference from the vertical loading system, frictionless runners were placed on the top of the loading block, directly under the steel rod transmitting vertical loads. Coarse abrading paper was glued to the base of the smooth loading block, to make its roughness more in line with the roughness of the concrete footing.

3.6 Test Procedure

After construction of the model, and prior to the application of a strip surcharge load on the soil surface, all measuring instruments were turned down to zero. Since the experimental programme was formulated to investigate the effects of vertical and horizontal loads on a reinforced earth retaining wall, stresses in the reinforcing elements due to the soil's weight were not included in this
These stresses were studied and discussed by previous investigators (Romstad et al. (67); Schlosser and Long (71)).

The strip surcharge load was applied at four different locations, I to IV, as shown in Fig. 3.7. At location I, the load is at the wall face, this represents the most critical position, which could produce a local failure near the top. At location II, the load was at the middle of Rankle's active failure zone. At location III, the load was just inside the Rankle's failure zone. At location IV, the load was just outside the Rankle's failure zone.

At each location, a vertical strip load of intensity, ranging from 60 kN/m² to 100 kN/m², was applied first. These surcharge load magnitudes are in line with actual loads applied to prototype structures investigated by others (Juran et al. (40); Al-Hussaini and Perru (2)). Once the vertical strip load was applied and the resulting strain readings and wall deflection were recorded, the model was subjected to a horizontal surcharge load applied in several increments. Strain and wall deflection readings were taken at each increment. The horizontal load was applied in both directions, towards and away from the wall surface. Preliminary testings indicated local shear failure occurred in the immediate vicinity of the loading block when the ratio of horizontal load to vertical load was in the region of 0.42. In order to prevent shear failure, the maximum value of the ratio was limited to 0.33.
CHAPTER IV

DISCUSSION OF EXPERIMENTAL RESULTS AND DESIGN METHOD BASED
ON THEORY OF ELASTICITY

4.1 Effect of Vertical Strip Load

Based on the experimental work done by Terzaghi (80), it was found that the lateral pressure, \( \sigma_h \), at any depth against a conventional rigid retaining wall due to a vertical surcharge strip load running parallel to the retaining wall face can be expressed as (79):

\[
\sigma_h = \frac{2 Q (\beta - \sin \beta \cos 2\alpha)}{T} \tag{4.1}
\]

where \( Q \) is the intensity of the vertical strip load; \( \beta \) is the angle of visibility at \( O \), in radians; and \( \alpha \) is the angle between the vertical and bisector of (see Fig. 2.7).

Assuming reinforcing elements are introduced into the soil mass and it is the reinforcing elements resisting the lateral pressure, due to the vertical surcharge strip load, the traction force in reinforcing element can be obtained as, (Fig. 4.1):

\[
T = \sigma_h (\Delta s) (\Delta h) \tag{4.2}
\]

where \( \Delta s \) and \( \Delta h \) are horizontal and vertical spacings between reinforcing elements.

In a rigid retaining wall, the lateral pressure induced by the vertical strip load acts against the wall. The maximum lateral pressure at any depth occurs at the wall face. In a flexible reinforced earth retaining wall system,
Kennedy et al. (42) and Laba et al. (48) have shown that the maximum lateral pressure or traction force in reinforcing element at any depth would be developed at points very close to the potential failure plane obtained by Culmann's method.

Experimental results obtained from the flexible reinforced earth retaining wall model (curve B) versus theoretical rigid retaining wall results based on Eq. 4.2 (curve A) are shown in Fig. 4.2 to Fig. 4.5. These four figures show the results when the vertical surcharge strip load is placed at load location I, II, III and IV respectively, as shown in Fig. 3.7. When the vertical surcharge strip load is placed inside the Rankine active wedge, i.e. at load location I, II, and III, a common trend is observed. It can be seen that the critical or optimal traction force in a flexible reinforced earth retaining wall (curve B) occurs at a level below the one calculated for the rigid retaining wall (curve A), (see Fig. 4.2, 4.3 and 4.4). In the upper quarter of the structure, curve A shows a larger force magnitudes than curve B. However, in the lower portion of the structure, the reverse is observed.

When the vertical surcharge strip load is moving away from the wall face, the difference in force magnitude between curve A and curve B is reduced. At load location IV (Fig. 4.5) where the vertical surcharge strip load is placed outside the Rankine active wedge, the magnitude of the traction force at any depth in curve B is about the same or similar to that of curve A.
The deflections of the wall under various load locations are shown in Fig. 4.6 and Fig. 4.7. It has been observed at load location I that there is a very large deflection at the top of the wall. This large deflection may suggest that slippage occurs in the top region. While the vertical surcharge strip load is moved away from the wall face to load location II, III and IV, the magnitude of the deflection is reduced. The location of the largest deflection moves downward simultaneously. The load location IV has almost negligible deflection of the wall. The reinforced earth wall behaves more or less like a rigid retaining wall.

The frictional resistance that can be developed in each level of reinforcing element can be determined by the method given by Kennedy et al. (42). If the frictional resistance at each reinforcing level is known, it may be determined if slippage occurs or not. The basic assumptions of Kennedy et al.'s method are: the potential failure plane will be the one obtained by Culmann's method; the increase of vertical pressure in soil due to the vertical surcharge strip load follows a 2:1 dispersion (see Fig. 4.8). The total frictional resistance, \( F_T \), that can be developed at any level behind the potential failure plane can be calculated as:

\[
F_T = (2L'b) (\gamma h) \tan \phi_1 + (2L_1 b) \sigma_{v1} \tan \phi_1 \tag{4.3}
\]

where

- \( b \) = width of reinforcing element
- \( \phi_1 \) = angle of friction between soil and reinforcing
element

\( Y = \) unit weight of soil

\( h = \) depth measured from soil surface

\( \sigma_{v_1} = \) increase in vertical stress intensity due to surcharge load

\( L' = \) length of reinforcing element behind the potential failure plane

\( L_1 = \) length of reinforcing element between the potential failure plane and 2 : 1 dispersion line in resistant zone.

The increase in vertical stress intensity, \( \sigma_{v_1} \), can be obtained as:

\[
\sigma_{v_1} = Q \left[ \frac{1}{1 + h} \right] \quad (4.4)
\]

when \( h \leq 2d \)

or

\[
\sigma_{v_1} = Q \left[ \frac{1}{1 + d + h / 2} \right] \quad (4.5)
\]

when \( h > 2d \)

The force, \( P_a \), developed in the reinforcing element due to the active pressure of the soil can be calculated as proposed by Lee et al. (51); Schlosser and Long (71); Vidal (82) as:

\[
P_a = (Ka)(Yh)(\Delta s)(\Delta h) \quad (4.6)
\]

where \( Ka = \) active pressure coefficient

\[ = \tan^2 \left( 45^\circ - \phi / 2 \right) \]

\( \phi = \) angle of internal friction of soil

The net frictional resistance, \( F_N \), that is available to resist the lateral pressure, due to vertical surcharge
load, will be the difference between Eq. (4.3) and Eq. (4.6):

\[ F_N = F_T - Pa \]  \hspace{1cm} (4.7)

Equation 4.7 is applicable if the stress developed in the reinforcing strip is below the yielding point of the strip material. The yield strength, \( \sigma_{yd} \), of the reinforcing strip, obtained from a tensile test, was \( 4.5 \times 10^5 \) \( \text{kN/m}^2 \).

The cross-sectional area, \( A \), of the strip was \( (0.025 \text{ m wide} \times 25 \text{ m thick}) = 6.25 \times 10^{-2} \text{ m}^2 \). The yielding load, \( F_y \), of the strip can be obtained as:

\[ F_y = \sigma_{yd} A \]  \hspace{1cm} (4.8)

The yielding load, \( F_y \), of the strip used in the experiment was 281.25 N.

One test on the model by Seymour (75) has shown that the first yielding of strip was recorded when the applied strip load intensity, at load location II, was about 173 \( \text{kN/m}^2 \). The vertical surcharge strip load was limited to be 100 \( \text{kN/m}^2 \) in the experiment, to prevent the reinforcing elements from reaching the yield point. Under the applied load intensity in the experiment, the frictional resistance, \( F_n \), calculated from Eq. 4.7 (curve C) and results from Eq. 4.8 are shown in Fig. 4.9 to Fig. 4.12 for different load locations. In general, the magnitude of \( F_n \) is smaller than the yielding load in the upper half of the structure, and the reverse is observed in the lower half. Experimental results (curve B) are also shown in Fig. 4.9 to Fig. 4.12. Figure 4.9 shows that
when the vertical surcharge strip load is placed at load location I, the frictional capacity at strip level 2 and 3 are fully utilized. The induced traction forces in the remaining reinforcing strips are well below the frictional capacity and yielding load. Load location II (see Fig. 4.10) has the induced traction forces at level 2 and 3 almost reached to the frictional capacity. The remaining induced traction forces are below the yielding load and frictional capacity. At load locations III and IV (see Fig. 4.11 and Fig. 4.12), all induced traction forces in reinforcing strips are well below the frictional capacity and yielding load.

The behaviour of a reinforced earth retaining wall system can be explained as follows: the low overburden soil in the upper level causes the frictional resistance, that may be developed in those reinforcing strips, to resist the lateral pressure which is induced by loads will be small. When large lateral pressure is applied in the top region, slippage or ultimate utilization of the frictional resistance may occur. The wall deflection in Fig. 4.6 and curves in Fig. 4.9 for the case of load location I confirm this point. Figure 4.13 and Fig. 4.14 show that when the vertical surcharge strip load is at load location I or II, the theoretical forces calculated from Eq. 4.2 (curve A) are much larger than the frictional resistances calculated from Eq. 4.7 (curve C) in the upper region of the structure. In order for a reinforced earth retaining wall system to remain
in equilibrium, the excess lateral force in the upper region has to be redistributed to the lower level strips. At load location III (see Fig. 4.15), though the calculated forces from Eq. 4.2 are smaller than the frictional resistance provided at every reinforcing strip level, small force redistribution are still possible due to the flexibility of the wall. When the surcharge load is placed at load location IV (see Fig. 4.16), the calculated forces from Eq. 4.2 are much smaller than the frictional capacity. The large distance from the wall face reduces the likelihood of redistribution of lateral forces due to flexibility of wall. Force distribution in a reinforced earth retaining wall system along the height of the wall behaves similar to a rigid retaining wall.

4.1.1 Proposed Theoretical Method for Determining Traction Force Distribution

The following method is proposed to find the stress or traction force distribution in a reinforced earth retaining wall system subjected to vertical surcharge strip load. This method takes into account the force redistribution characteristic of reinforced earth system. This method is suggested to be used in case where the calculated forces from Eq. 4.2 are larger than the frictional capacities of the strips in the upper level of the structure. In this case, it will be applicable for load location I and II. The maximum tension force that can be developed in the reinforcing element is limited to be equal to the yielding
load, \( F_y \). When the calculated forces from Eq. 4.2 are smaller than the frictional capacity at any depth, the force distribution of the rigid retaining wall may be used to represent that of a reinforced earth retaining wall.

In the upper region of Fig. 4.13 and Fig. 4.14, the forces calculated from Eq. 4.2 (curve A) are larger than the frictional capacities calculated from Eq. 4.7 (curve C). The difference in force magnitude between curve A and curve C is considered as excess force, \( F_E \), in that strip level. Total excess force, \( F_{ET} \), will be the summation of all the excess forces in every strip level where the value of curve A is larger than the value of curve C. The total excess force, \( F_{ET} \), will be redistributed to the strips below the intersection point of curve A and curve C. It is assumed that the maximum traction force that can be developed at any level will be equal to the smaller of the frictional resistance capacity or yielding load. One more assumption is that the total excess force, \( F_{ET} \), will be equally divided among the strips below the intersection point as \( F_D \). In order to obtain the theoretical traction force distribution in the flexible reinforced earth retaining wall, force \( F_D \) is added to the value of curve A at every level below the intersection point of curve A and curve C. In the strip level where the theoretical force is larger than the net frictional resistance, the force value is limited to be the value of net frictional resistance. The excess force will be equally distributed to the strips below
that level.

The final result is shown as curve D in Fig. 4.13 and Fig. 4.14. The load location I (see Fig. 4.13) has the theoretical force distribution in very good agreement with the experimental results, especially in the upper quarter of the structure. The theoretical force distribution curve value is larger in the upper half and smaller in the lower half at load location II. This may be explained as the method used to assume ultimate utilization of friction in the top levels. The total excess force, \( F_{ET} \), that is going to be redistributed will be smaller than the actual difference between experimental results and results from Eq. 4.2. Therefore, smaller force distribution in the lower level will be obtained. The theoretical critical traction force occurs at the same strip level as that of the experiment. The magnitude of the theoretical critical traction force is equal to or larger than that obtained from experiment. For design purposes, it is on the safe side.

4.1.2 Summary of Design Method

The design procedure mentioned in the above section is outlined step by step in the following:

1. Calculate the traction force, \( T \), in the reinforcing strips, for a rigid retaining by using:

\[
\sigma_h = \frac{2Q}{\pi} \left( \beta - \sin \beta \cos 2\alpha \right) \quad (4.1)
\]

\[
T = \sigma_h (\Delta s)(\Delta h) \quad (4.2)
\]
2. In order to determine the net frictional resistance, \( F_N \), first calculate the total frictional resistance, \( F_T \), as:

\[
F_T = (2L' \cdot b) (\gamma b) \tan \phi_t + (2L_1 \cdot b) \sigma_{vi} \tan \phi_t
\]

(4.3)

\[
\sigma_{vi} = \frac{Q}{\left( \frac{1}{1 + h} \right)}, \quad h < 2d \quad (4.4)
\]

\[
\sigma_{vi} = \frac{Q}{\left( \frac{1}{1 + d + h / 2} \right)}, \quad h > 2d \quad (4.5)
\]

Then determine the active soil pressure force,

\[
Pa \quad \text{as:}
\]

\[
Pa = (K_a)(\gamma h)(\Delta s)(\Delta h)
\]

(4.6)

The net frictional resistance, \( F_N \), can be obtained as:

\[
F_N = F_T - Pa
\]

(4.7)

3. Check the yielding load, \( F_Y \), of the reinforcing strip by:

\[
F_Y = \sigma_{yd} \cdot A
\]

(4.8)

4. In each strip level, check whether Eq. 4.7 or Eq. 4.8 represents the ultimate resistance that can be developed.

5. Calculate excess force, \( F_E \), in strip levels where forces calculated from Eq. 4.2 are larger than those calculated from Eq. 4.7 as

\[
F_E = T - F_N
\]

(4.9)

6. Total excess force, \( F_{ET} \), will be:

\[
F_{ET} = \sum_{n=1}^{m} (F_E)_n
\]

\( n = 1, 2, \ldots, m \quad (4.10)\)
where \( m \) = number of strip level where \( T > F_N \)

7. The total excess force, \( F_{ET} \), is then equally divided among the strips below the intersection point of curve A and curve C as force \( F_D \) by using:

\[
F_D = \frac{F_{ET}}{k}
\]

(4.11)

where \( k \) = number of strip level below point of intersection of curve A and curve C (point of zero difference)

8. The theoretical force distribution of a flexible reinforced earth retaining wall will be:

a. Above the point of zero difference, traction force in strip will be equal to \( F_N \)

b. Below the point of zero difference, traction force in strip will be equal to \( T + F_D \), where \( T + F_D < F_N \)

c. In the case where \( T + F_D > F_N \) in a strip level has occurred, the difference between \( T + F_D \) and \( F_N \) (i.e., \( T + F_D - F_N \)) is equally distributed to the strips below the level where \( T + F_D > F_N \).

4.1.3 Failure Behaviour of Reinforced Earth Retaining Wall

An experimental test was performed by Seymour (75) on the reinforced earth model. The vertical surcharge strip load intensity, at load location II, increased from 0 kN/m\(^2\), until the structure failed. The strain in the
strips and the deflections of the wall were recorded. It was found that the reinforcing strip at level 4 started to yield when the applied strip load intensity was approximately 173 kN/m². Subsequently, as the intensity of the vertical strip load was increased further, yielding of strips at level 5, 6, 2 and 3 were recorded. Finally, the reinforced earth wall structure failed as the strip load intensity reached around 314 kN/m². The wall failed suddenly while the next load increment was applied. The failure of the structure was caused by the breakage of reinforcing elements in level 3 to level 6 when the wall was allowed to move 25 mm. All the middle strips from level 3 to level 6 were broken around the failure plane that was obtained from Culmann's method (81). Some of the side strips (i.e. strips closer to the side of the box) were broken around the wall face in those levels.

Figure 4.17 shows the displacement of the wall as the first yielding of reinforcing element as well as the displacement of the wall prior to failure. Figure 4.18 shows the curves of experimental traction forces, frictional resistance calculated from Eq. 4.7 and the yielding load, $F_y$, when yielding of strip was observed. The shape of the deflection of the wall in Fig. 4.17 can be explained by first studying Fig. 4.18. In Fig. 4.18, under a vertical strip load intensity of 173.7 kN/m², the yielding load, $F_y$, in the reinforcing strips is smaller than the frictional resistance, $F_N$, of the strips below
level 2. First yielding of the strip is observed in strip level 4. At strip level 2, traction force developed in the strip reaches the frictional capacity. Therefore, slippage is possible in this level. The smaller traction force developed in level 3 may suggest slippage occurred before the applied load intensity (i.e. 173.7 kN/m²), stress relaxation causes extra stress being transfer to lower level. The slippage of the top three strips, subjected to the largest traction forces caused by strip load, may explain why the second wall channel has the largest displacement, as shown in Fig. 4.17. When the intensity of the strip load is increased, the stress redistribution causes the strips in level 5 and level 6 to reach yield point. Therefore, larger displacement of the wall in this region may occur. Simultaneously, the large strip load intensity causes the strip footing to sink into the sand mass and the large displacement of wall channel causes the footing to rotate. The combined effect of these factors produces the final shape of the displacement of the wall before failure, as shown in Fig. 4.17.

4.2 Effect of Horizontal Strip Load

Experimental work done by Laba et al. (48) on a flexible reinforced earth retaining wall has shown that when the system was subjected to the application of horizontal strip load in either direction (towards or away from the wall face), the relationship between change in the reinforcing strip stress and the horizontal load intensity,
H, was virtually linear in most cases, Figure 4.19 and Fig. 4.20 show that this linear relationship is also independent of the magnitude of the vertical strip load.

4.2.1 Horizontal Strip Load Acting Towards the Wall

As mentioned in Section 2.5.2, the French Ministry of Transport (F. M. T.) proposed the following formula for calculating the horizontal stress distribution, $\Delta \sigma_H$, within a soil mass, when the reinforced earth retaining wall is subjected to a horizontal force, $F$, acting towards the wall face (see Fig. 2.9). The formula is expressed as:

$$\Delta \sigma_H = \frac{F}{1' + d} \left[ 1 - \frac{h}{h_0} \right]$$

Figure 2.9 shows the stress distribution calculated from Eq. 2.11. It can be seen that the largest horizontal stress occurs at the top of the soil surface. The horizontal stress intensity reduces linearly to zero at a depth $h_0$ when the distance between the footing and the wall face increases, i.e. the strip footing moves away from the wall, the horizontal stress intensity decreases. The depth of penetration, $h_0$, will increase.

Tensar, a company producing reinforcing grids, proposed a formula similar to that of French Ministry of Transport as:

$$\Delta \sigma = \frac{2 F_i}{h_i} \left[ 1 - \frac{h}{h_i} \right]$$

$$h_i < H_W$$

(4.12)
\[ \Delta \sigma = \frac{2F_1}{H_W} \left[ 1 - \frac{h}{H_W} \right] \]

where \( \Delta \sigma \) = horizontal stress due to \( F_1 \)

\( F_1 \) = horizontal force per unit length of wall

\( h_i \) = depth of influence

\[ h_i > H_W \]  \( (4.13) \)

\[ b_i + 2d_2 \]

\( d_2 \) = perpendicular distance between the wall and the centre-line of footing

\( \phi \) = angle of internal friction of backfill soil

\( b_i \) = actual width of the footing, in case where there is eccentricity, \( e \), developed,

\( b' = b_i - 2e \) will be used in Eq. \( 4.14 \)

Instead of \( b_i \).

\( H_W \) = height of the wall.

Figure 4.21 shows the dispersal of horizontal load of Eq. 4.12 to Eq. 4.14. Similar to the F. M. T. equation, the largest horizontal stress occurs at the top of the soil surface and then reduces linearly to zero at a depth \( h_i \). The depth of influence is a function of the angle of internal friction, \( \phi \), of the backfill soil. Thus, the depth of influence will be deeper for well compacted soil. The depth of influence, \( h_i \), will increase and the horizontal stress intensity will decrease when the horizontal load is moved away from the wall face.

Another expression of horizontal stress, \( \sigma_H \), induced in a semi-infinite elastic medium under a horizontal strip
load intensity, \( H \), was suggested by Scott (74) as:

\[
\sigma_h = (H / \pi) \left[ 1 - \frac{R_1^2}{R_2^2} - \sin \beta \sin (\beta + 2\delta) \right]
\]

(4.15)

where

- \( H \) = horizontal strip load intensity
- \( \beta \) = angle of visibility at point 0
- \( R_1 \) = distance from point 0 to the far end of footing
- \( R_2 \) = distance from point 0 to the near end of footing
- \( \delta \) = the angle between the vertical and the line connecting point 0 to near end of footing.

The various symbols used in Eq. 4.15 are shown in Fig. 4.22a and the solution of Eq. 4.15 is shown in Fig. 4.22b in dimensionless form \((\sigma_h / H)\). It can be seen that zero stress occurs along a vertical line, passing through the middle of the strip footing. The largest increase and decrease in stress values occur in the medium under the edges of the loaded area. The sign of change of stress in the medium depends on the direction of the applied strip load.

Figure 4.23 illustrates contour diagrams of the ratio \( \Delta \sigma_s / H \) plotted on the side view of the reinforced earth model with the horizontal load at load location II, applying towards the wall face. \( \Delta \sigma_s \) is defined as change of reinforcing element stress, due to the application of
horizontal load of intensity, \( H \). Fig. 4.23 indicates that the largest increase in stress in reinforcing strip or in soil occurs at a certain distance from the wall and at a certain level below the soil surface. With the exception at load location I, where maximum element stresses were found at wall face, at load location's II and III, maximum stresses were found in the region midway between the wall face and the potential failure plane (i.e., represented by dash line in Fig. 4.23).

In order to compare the experimental results with the stress distribution, proposed by F. M. T. and Tensar, the experimental results are transformed into a dimensionless ratio \( \Delta \sigma_H / H \). The change in the soil stress distribution, \( \Delta \sigma_H \), due to the horizontal load intensity, \( H \), can be deduced from \( \Delta \sigma_s \) (the change in the reinforcing strip stress distribution) as:

\[
\Delta \sigma_H ( \Delta s ) ( \Delta h ) = \frac{\Delta \sigma_s ( A )}{( \Delta s ) ( \Delta h )} \tag{4.16}
\]

Equation 4.5 can be expressed in dimensionless form as:

\[
\frac{\Delta \sigma_H}{H} = \frac{\Delta \sigma_s}{H} \cdot \frac{A}{( \Delta s ) ( \Delta h )} \tag{4.17}
\]

where \( \Delta s \) and \( \Delta h \) are the horizontal and vertical spacings of reinforcing elements; \( A \) is the cross-sectional area of reinforcing element; \( \Delta \sigma_H / H \) is the ratio of change in soil stress to applied horizontal load intensity \( H \), and \( \Delta \sigma_s / H \) is the ratio of change in reinforcing element stress to the same applied horizontal load intensity \( H \). Figure 4.24 shows the comparisons between values of
\( \Delta \sigma_H / H \) obtained from Eq. 2.9, values obtained from Eq. 4.12 and value measured on the reinforced wall retaining wall model at load location II. The applied vertical load intensity, \( Q \), was equal to 100 kN/m² and the applied horizontal load intensity, \( H \), was equal to 19.5 kN/m². It can be seen that the maximum horizontal stress ratio occurs at a certain level below the soil surface which is different from the results suggested by F. M. T. and Tensar. The depth of stress penetration measured in the experiment is deeper than those calculated from F. M. T. and Tensar. Laba et al. (49) suggested the difference between theoretical methods and the experimental values is caused by stress redistribution in the reinforced earth backfill resulting from the deflection of the flexible wall face.

4.2.2 Horizontal Strip Load Acting Away From the Wall

Figure 4.25 is a contour diagram of the ratio \( \Delta \sigma_s / H \). It is plotted on the side view of the reinforced earth wall, with the horizontal load at load location III, acting away from the wall. As shown in Fig. 4.25, the two sensitive areas are developed where the largest change of \( \Delta \sigma_s / H \) occurs. The first region is located in the active zone and the second region is located in the resisting zone. These two regions are roughly divided by the potential failure plane (in dash line). It was found that the change in reinforcing element stress was the smallest along this potential failure plane. The negative sign shown by the contours in Fig. 4.25 indicates that the applied horizontal
strip load caused an overall decrease in the existing traction forces in the elements, induced initially by the vertical strip load. The largest stress change is not located along the uppermost reinforcing element, but some depth below it. Laba et al. (48) suggested this phenomenon can be explained as deflection of the wall facing. This is due to initial application of the vertical strip load which caused a redistribution of stresses in this region from the very top to the lower reinforcing element levels.

4.3 Effect of Combined-Vertical and Horizontal Strip Load

In an actual situation, the horizontal strip load cannot be applied alone to the flexible reinforced earth retaining wall system without the presence of the vertical surcharge strip load. Of all the experimental work performed on the model, a vertical surcharge strip load was applied first, then different magnitudes of horizontal strip loads were applied to the system. The horizontal load was applied in both directions, towards and away from the wall face. Different combinations of vertical strip load (Q) and horizontal strip load (H) were applied to all four load locations as shown in Fig. 3.7. The ratio of horizontal strip load to vertical strip load was limited to 0.33 to avoid local soil failure in the vicinity of the load block.

4.3.1 Horizontal Component Acting Towards the Wall

The deflection of the wall, under combined vertical and horizontal strip load, at various load locations, are shown
In Fig. 4.26 to Fig. 4.29, the horizontal load was acting towards the wall face. When the vertical load component is constant, the deflection of the wall increases as the magnitude of the horizontal load component is increased. The largest increase in wall deflection occurs in the upper part of the structure. When the location of the horizontal strip load is shifted away from the wall face, the magnitude of the deflection of the wall decreases. The load location's I, II and III (see Fig. 4.26 to Fig. 4.28) have large deflection of the wall at the top. The shape of the deflection may suggest that slippage occurs in the top region of the structure.

Figure 4.30 to 4.33 show the maximum traction forces developed in every level for various load locations. The maximum traction force is determined from the maximum strain along the length of the strip without specifying on a specific strain gauge. A frictional capacity curve based on Eq. 4.7 is also shown in these figures. At load location I (see Fig. 4.30), traction forces developed in levels 2 and 3 are larger than the frictional capacity when the magnitude of the horizontal strip load is increased from 0 to 10 kN/m. The same is observed in level 2 and 3, at load location II (see Fig. 4.31), when the magnitude of the horizontal strip load is increased from 0 to 26 kN/m². Load location's III and IV (see Fig. 4.32 and Fig. 4.33), show that the traction forces developed in every strip level are within the frictional capacity based on Eq. 4.7 in the
tested range of horizontal strip load magnitude, it is observed that at all investigated load locations, maximum traction forces developed in every strip level will increase as the magnitude of the horizontal strip load is increased.

Theoretically, the traction force developed in the strip cannot be larger than the frictional resistance provided. However, it is observed that at load location I and II (see Fig. 4.30 and Fig. 4.31), traction forces larger than frictional resistance are developed in upper strip levels. In order to explain this phenomenon, the stress distribution along the length of the strip as shown in Fig. 4.34 to Fig. 4.36 will be studied first. Figure 4.36 shows the stress distribution of strip level 2 at load location III. This is a typical shape where maximum stress remains at the same location when the magnitude of the horizontal load is increased. However, at load location I strip level 2 (see Fig. 4.34), the location of maximum stress or potential failure plane developed when vertical strip load was first applied and shifted towards the wall faces. This occurred as the magnitude of the horizontal load was increased to 10 kN/m². The same was observed for strip level 3, at load location II (see Fig. 4.35), when the magnitude of the horizontal strip load was increased to 26 kN/m².

The shift of maximum stress location may be explained as follow: when the vertical surcharge strip load is applied alone, formation of a potential failure plane (location of maximum stress in each strip level) similar to
that of Culmann's method is obtained. As mentioned in Section 4.2.1, when horizontal surcharge strip load is applied, the largest increase in strain or stress along the length of the strip, occurs in a location about midway between the wall face and Culmann's potential failure plane. But it occurs at the wall face at load location 1. Therefore, when the magnitude of the horizontal strip load is increased to a certain value, under the combined effect of the vertical and horizontal strip load, maximum strain or stress will develop at a location closer to the wall face. In another way, the potential failure plane is moved towards the wall face. The movement of the failure plane towards the wall face provides a longer resisting zone behind the failure plane. Thus, larger traction force may be developed in the strip before slippage occurs. Also the frictional resistance calculated from Eq. 4.7 is based on the assumption that Culmann's failure plane has developed. Therefore, traction force may be developed in the strip larger than frictional resistance, as shown in Fig. 4.30 and 4.31.

4.3.2. Proposed Theoretical Method for Determining Traction Force Distribution With Horizontal Load Acting Towards the Wall

In order to calculate the traction force distribution along the depth of the wall, under the combination of vertical and horizontal surcharge strip load, a method
similar to that proposed in Section 4.1.1 for vertical strip load, will be utilized. Some modification of that method is needed before it is applied in this situation. The first step, outlined in Section 4.1.2 (Summary of Design Method), will be changed to the following:

1) Calculate the traction force, $T$, in the reinforcing strips for a rigid retaining wall subjected to vertical and horizontal strip load by using:

$$T = (\sigma_h + \Delta \sigma_h) (\Delta s) (\Delta h)$$  \hspace{1cm} (4.18)

where:

$$\sigma_h = 2Q (\beta - \sin \beta \cos 2\alpha) / \pi$$  \hspace{1cm} (4.1)

$$\Delta \sigma_h = F (1 - h / ho) / (1 + d)$$  \hspace{1cm} (2.11)

In fact, traction force, $T$, in Eq. 4.18 is the combination of equations proposed by Terzaghi and French Ministry of Transport. The remaining steps in Section 4.1.2 (i.e. step 2 to step 8) will be followed to complete the calculation.

The results calculated from the proposed method for load location I to location III are shown in Fig. 4.37 to Fig. 4.41. It is found for load location I (see Fig. 4.37 and Fig. 4.38), that the theoretical results are in adequate agreement with the experimental results. At load location II (see Fig. 4.39 and Fig. 4.40), the theoretical results are larger than the experimental results in level 4 and 5, and it is also in adequate agreement with the experimental results in the remaining strips. It is found that a better
agreement between the theoretical results and experimental results may be achieved if there is no movement of the potential failure plane, i.e., Culmann's potential failure plane holds. The theoretical results obtained for load location III (see Fig. 4.41) are satisfactory and the theoretical results are near to that calculated from Eq. 4.18. At load location IV (see Fig. 4.42), it can be seen that results obtained from Eq. 4.18 may be used to represent the traction force distribution in the flexible reinforced earth retaining wall system. In general, the design method is suggested to be used for cases where calculated forces from Eq. 4.18 are larger than the frictional capacities (calculated from Eq. 4.7) of the strips in the upper level of the structure and that there is no movement of potential failure plane.

4.3.3 Horizontal Component Acting Away From the Wall

Figure 4.43 shows the stress distribution along the length of the strip in level 4 at load location III, when the horizontal strip load, \( H \), is acting away from the wall. The combination of vertical and horizontal surcharge strip load indicate that the stress developed in the reinforcing strip will decrease when the magnitude of the horizontal component is increased. The smallest decrease in stress in the strip occurs at the location where potential failure plane is formed by vertical surcharge strip load. This trend holds true for the other load locations. For design purposes, a flexible reinforced earth retaining wall which
is subjected to a combination of vertical and horizontal strip loads. The horizontal component acting away from the wall, it is satisfactory to design the system as though it is subjected to vertical strip load only. The method outlined in Section 4.1.2 will be applicable.
CHAPTER V

MATHEMATICAL FORMULATION

5.1 General

The composite system of soil, reinforcing material and skin element in reinforced earth system is treated as a continuum discretized by a finite number of elements. Linear strain rectangular elements are used to represent the soil media. The reinforcing strips are treated as truss members which are subjected to compression or tension. Since the skin is mainly used to prevent the spillage of soil and erosion, its presence does not contribute much strength to the whole system. It is treated as a truss member too. In order to simulate the behaviour of frictional sliding, separation and rebounding at the common interface between different materials, a four noded interface element is employed. The stress-strain relationship for the soil and the load-displacement relationship for the interface element are assumed to be nonlinear. The stress-strain relationship for the reinforcing strip and skin are assumed to be linear elastic.

Construction simulation which consists of adding additional layer of soil and compaction effects is described. Iteration processes are applied to improve the theoretical results from finite element analysis.
5.2 Finite Element Formulation

The formulation of finite element method can be based on direct approach and variational principle. The variational principle is used here because it is convenient in formulating the equations of different kinds of elements. The variational principle of minimum potential energy is outlined below.

For each type of elements, a displacement function which is usually in polynomial form, is chosen in terms of a generalized coordinates is defined in matrix form as:

\[
\{ u \} = \{ \Phi \} ( \alpha ) \tag{5.1}
\]

where,

\[
\{ u \} \text{ is a vector of displacement at any point.}
\]

\[
( \alpha ) \text{ is a vector of coefficients of the assumed interpolation function}
\]

\[
\{ \Phi \} \text{ is the interpolation matrix}
\]

Substituting the coordinates of the nodes of the element into Equation 5.1 yields:

\[
\{ q \} = [ A ] ( \alpha )' \tag{5.2}
\]

where,

\[
\{ q \} \text{ is the vector of nodal displacement}
\]

\[
[ A ] \text{ is the matrix containing the element nodal coordinates}
\]

By combining Equations 5.1 and 5.2 displacement can be expressed in terms of nodal displacements as:

\[
\{ u \} = \{ \Phi \} [ A^{-1} ] \{ q \} \tag{5.3}
\]

where,
\[
\begin{bmatrix} A^{-1} \end{bmatrix} \text{ is inverse of } \begin{bmatrix} A \end{bmatrix}
\]

By differentiating Equation 5.3, the strain at any point within the element is:
\[
\{\xi\} = \{B\} \{A^{-1}\} \{q\} \quad (5.4)
\]

where,
\[
\{\xi\} \text{ is the strain vector}
\]
\[
\{B\} \text{ is the constitutive matrix (obtained by differentiating } \{\phi\}).
\]

From the theory of elasticity, the stress-strain relationship can be expressed as:
\[
\{\sigma\} = \{D\} \{\xi\} \quad (5.5)
\]

where,
\[
\{D\} \text{ is the constitutive matrix (stress-strain law),}
\]
\[
\text{which contains the elastic properties of the element.}
\]

The total potential energy of the element can be expressed in matrix form as:
\[
\Pi_p = \frac{1}{2} \int \{\xi\}^T \{\sigma\} d\gamma - \int \{u\}^T \{\bar{x}\} d\gamma - \int \{u\}^T \{\bar{T}\} d\bar{s} \quad (5.6)
\]

where,
\[
\Pi_p \text{ is the total potential energy}
\]
\[
\{\bar{T}\} \text{ is the surface traction vector}
\]
\[
\{\bar{x}\} \text{ is the body force vector}
\]
\[
\bar{s} \text{ is the surface of the element}
\]
\[
\gamma \text{ is the volume of the element}
\]
\[
\{\cdot\}^T \text{ is the transpose of the vector.}
\]

Substituting Equations 5.3 through 5.5 into Equation
5.6 Yield:

\[ \pi_p = \frac{1}{2} \int (q^T \{ A^{-1} \}^T \{ B \} \{ D \} \{ B \} \{ A^{-1} \} (q) \, dV \]

\[ - \int (q^T \{ A^{-1} \}^T \{ \phi \} \{ \bar{x} \} \, dV \]

\[ - \int (q^T \{ A^{-1} \}^T \{ \phi_s \} \{ \bar{\tau} \} \, ds \] 

\begin{equation}
(5.7)
\end{equation}

where,

\[ \{ \phi_s \} = \{ \phi \} \] evaluated along surface points only.

The total potential energy of the whole system is the algebraic summation of the total potential energy of individual elements. By taking the variation of Equation 5.7 with respect to displacement and equating the result to zero, the stiffness matrix of an element can be given as:

\[ \{ K_e \} = \int \{ A^{-1} \}^T \{ B \} \{ D \} \{ B \} \{ A^{-1} \} \, dV \]

\begin{equation}
(5.8)
\end{equation}

In order to transfer the stiffness matrix from local coordinates to global coordinates, a transformation matrix can be used as follows:

\[ \{ K \} = \{ T \}^T \{ K_e \} \{ T \] 

\begin{equation}
(5.9)
\end{equation}

where,

\[ \{ K \} \] is the element stiffness matrix in global coordinates system,

\[ \{ T \} \] is the transformation matrix containing direction cosines of the local axes with respect to global axes.

5.3 Linear Strain Rectangular Element
In this analysis, linear strain rectangular element is used to represent soil. The formulation of the element is briefly described below.

Since the element, shown in Fig. 5.1 has eight degrees of freedom, eight unknown coefficients must be involved in the polynomial representing the displacement pattern. These functions are assumed as:

\[ u = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy \]  
\[ v = \alpha_5 + \alpha_6 x + \alpha_7 y + \alpha_8 xy \]  

where,

- \( u \) is the displacement in \( x \)-direction,
- \( v \) is the displacement in \( y \)-direction,
- \( \alpha_i \) is the coefficient of the displacement functions.

The relationships between strains and displacements for a plane element are given as:

\[ \epsilon_x = \frac{\delta u}{\delta x} \]  
\[ \epsilon_y = \frac{\delta v}{\delta y} \]  
\[ \gamma_{xy} = \frac{\delta u}{\delta y} + \frac{\delta v}{\delta x} \]  

where,

- \( \epsilon_x \) is the strain in the \( x \)-direction
- \( \epsilon_y \) is the strain in the \( y \)-direction
- \( \gamma_{xy} \) is the shear strain

Substituting Equation 5.10 into Equation 5.11, the following expressions are obtained for the strains at any
point within the element.

\[ \varepsilon_x = \alpha_2 + \alpha_4 y \] (5.12a)

\[ \varepsilon_y = \alpha_7 + \alpha_8 x \] (5.12b)

\[ \gamma_{xy} = \alpha_3 + \alpha_4 x + \alpha_5 + \alpha_8 y \] (5.12c)

For a plane strain condition, the stress-strain relationships are:

\[ \sigma_x = \frac{E}{(1+v)(1-2v)} \left[ (1-v)\varepsilon_x + v\varepsilon_y \right] \] (5.13a)

\[ \sigma_y = \frac{E}{(1+v)(1-2v)} \left[ v\varepsilon_x + (1-v)\varepsilon_y \right] \] (5.13b)

\[ \tau_{xy} = \frac{E}{2(1+v)} \gamma_{xy} \] (5.13c)

where,

\( \sigma_x \) is the stress in the \( x \)-direction

\( \sigma_y \) is the stress in the \( y \)-direction

\( \tau_{xy} \) is the shear stress

Rewriting Equations 5.10, 5.12 and 5.13 in matrix form, matrices \([ A ]\), \([ B ]\), \([ D ]\) are obtained. By substituting the matrices in Equation 5.8 and performing the integration, the element stiffness matrix \([ Ke ]\) is generated. Appendix B shows the details of matrices \([ A ]\), \([ B ]\), \([ D ]\) and \([ Ke ]\).

5.4 Time-Independent Nonlinear Stress-Strain Models for Soil

In general, the stress condition in soil depends on many different conditions and can be expressed as a function as follow:
\[ \sigma = F(\varepsilon(t), T(t), w(t), x(t), \ldots, t) \]  

(5.14)

where,

- \( \varepsilon(t) \) is the strain conditions
- \( T(t) \) is the temperature
- \( w(t) \) is the water content
- \( x(t) \) is the anisotropy
- \( t \) is the time

In order to reduce the complexity of the problem, soil models must include those important variables while unessential variables will be excluded. In this case, time and anisotropy variables are discarded.

The time-independent nonlinear soil model can be divided as two groups: variable modulus models and elasto-plastic models. The elasto-plastic models relate the rate of stress to the rate of strain. They require a yield criterion, a hardening rule, and a flow rule. They are capable of simulating better constitutive laws for soil than the variable modulus models and also take into account the unloading condition. But the drawback for these models is it is difficult to determine parameters that can correlate to laboratory test. Details of the elasto-plastic models are referenced in 49, 58, 59, 60, 68, 73, 84 and 87.

There are many variable modulus models available. The details for these models can be obtained from references 9, 18, 21, 22, 23, 24, 25, 26, 36, 43, 44, 45, 62, 64, and 85. The major differences among these models are: the mathematical function used to relate stress and strain,
the method used to set up the modulus. The mathematical function is assumed to be one of the following: a parabola, a hyperbola, a polynomial, or a spline. The method for setting up modulus is either a secant method or tangent method. But these models show a common trend: the stiffness of soil is increased with the increase of confining pressure, or with the decrease of shear strain.

The hyperbolic soil model has been used widely and successfully in earth retaining structures. Its parameters can be obtained easily from triaxial tests and it is relatively easy to program in finite element analysis. This model is used for the present study.

5.4.1 Hyperbolic Stress-Strain Model

Based on experimental work, Kondner et al. (44, 45, 46) had shown that the stress-strain relationship of a number of soil in a conventional triaxial test can be approximated by a rectangular hyperbola. The hyperbolic curve, shown in Fig. 5.2, can be represented by the equation:

\[
\sigma_i = \frac{\epsilon_a}{1 + \frac{\epsilon_a}{E_i} + \frac{\epsilon_a^p}{(\sigma_i - \sigma_3) \text{ult.}}}
\]  \hspace{1cm} (5.15)

where:

- \( \sigma_i \) is the major principal stress
- \( \sigma_3 \) is the minor principal stress
- \( \epsilon_a \) is the axial strain
- \( E_i \) is the initial tangent modulus
( \( \sigma_1 - \sigma_3 \) )_{ult} is the ultimate stress difference which is the asymptotic value of stress difference.

Equation 5.15 can be transformed to a linear form, as shown in Fig. 5.3, as:

\[
\frac{\varepsilon_a}{\sigma_1 - \sigma_3} = \frac{1}{E_i} + \frac{\varepsilon_a}{(\sigma_1 - \sigma_3)_{ult}} \quad (5.16)
\]

where,

\[
\frac{1}{E_i}
\]

is the intercept of the straight line.

\[
\frac{1}{(\sigma_1 - \sigma_3)_{ult}}
\]

is the slope of the line.

For all soil, except fully saturated soils, tested under unconsolidated undrained conditions show that the value of \( E_i \) and \( (\sigma_1 - \sigma_3)_{ult} \) increase as \( \sigma_3 \) is increased. From experimental work, Janbu (39) suggested the following empirical relationship between initial tangent modulus and confining pressure.

\[
E_i = K_s \cdot \frac{\sigma_3}{P_at} \left( \frac{\sigma_3}{P_at} \right)^n \quad (5.17)
\]

where,

\( K_s \) is an experimental parameter called the modulus number;

\( n \) is an experimental parameter called the modulus exponent;

\( P_at \) is the atmospheric pressure.

The parameters \( K_s \) and \( n \) can be determined experimentally by plotting Equation 5.17, as a straight line in log scale, shown in Fig. 5.4. Equation 5.17 as
valid for any unit systems as long as the units of $\sigma_1$, $E_1$ and $\sigma_3$ are the same.

Using the Mohr-Coulomb failure criterion, the relationship between compressive strength and confining pressure can be shown as (26):

$$\left( \sigma_1 - \sigma_3 \right)_f = \frac{2 \sigma_3 \sin \phi + 2 c \cos \phi}{1 - \sin \phi} \tag{5.18}$$

where,

$\left( \sigma_1 - \sigma_3 \right)_f$ is the compressive strength, or stress difference at failure.

c is the cohesion intercept.

$\phi$ is the angle of internal friction.

Duncan et al. (26) related the stress difference at failure to the asymptotic stress difference as:

$$\left( \sigma_1 - \sigma_3 \right)_f = R_f \left( \sigma_1 - \sigma_3 \right)_{ult} \tag{5.19}$$

where $R_f$ is the failure ratio and it varies between 0.5 and 0.9 for most soils.

The tangent modulus $E_t$ can be expressed as:

$$E_t = \frac{\delta}{\delta \varepsilon_a} \left( \sigma_1 - \sigma_3 \right) \tag{5.20}$$

By differentiating Equation 5.15 with respect to $\varepsilon_a$ and substituting Equations 5.17, 5.18 and 5.19 into the resulting expression, the following equation for the tangent modulus is obtained:

$$E_t = K_s P_a \left( \frac{\sigma_3}{P_a} \right)^H_1 \left[ 1 - \frac{R_f \left( \sigma_1 - \sigma_3 \right) (1 - \sin \phi)}{2 c \cos \phi + 2 \sigma_3 \sin \phi} \right]^2 \tag{5.21}$$
Equation 5.21 can be used to calculate the tangent modulus for any stress states provided the parameters $K_s$, $n$, $R_f$, $c$, and $\phi$ are known. These five parameters can be obtained by performing standard triaxial tests. Several reports (47, 85) have compiled those parameters for a wide range of soil.

5.5 Truss Finite Element for Reinforcing Strip and Skin

The truss element is defined by two nodes, as shown in Fig. 5.5, and each node has one degree of freedom. It has constant properties along the longitudinal axis. The displacement function with respect to local coordinates system can be defined as:

$$u = \alpha_1 + \alpha_2 x$$  \hspace{1cm} (5.22)

The coefficients $\alpha_1$ and $\alpha_2$ can be found as:

$$\alpha_1 = u_1$$ \hspace{1cm} (5.23)

$$\alpha_2 = \frac{u_2 - u_1}{L}$$ \hspace{1cm} (5.24)

where $L$ is the length of the truss element.

The stress-strain relationship is defined as:

$$\sigma = E\epsilon$$

where $E$ is the modulus of elasticity.

By substituting the appropriate matrices in Equation 5.8, the local stiffness matrix is given as:

$$[ K_e ] = \frac{E A}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ \hspace{1cm} (5.25)
where \( A \) is the cross-sectional area of the element.

The following transformation matrix is used to transform the local element stiffness matrix (Equation 5.25) to the global matrix:

\[
[T] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 \\
0 & 0 & \cos \theta & \sin \theta
\end{bmatrix}
\] (5.26)

where \( \theta \) is the angle of inclination between the local coordinates and the global coordinates.

5.6 Interface Finite Element

Before the development of interface model, in the conventional element analysis, the contact surface between two different materials has the same common nodes. This implies that both material undergo the same displacement under loading. Therefore, possible relative movement between the two materials due to shearing behaviour is not accounted for. In order to overcome this shortcoming in the analysis, interface finite element is introduced into the contact surface between two different materials. The element is designed to handle the interface behaviour, and allow independent movements of the two materials in contact. This is basically done by assigning a different nodal point on each side of the interface.

Either the method of constraints or the method of stiffness can be used to treat the interface condition in a finite element formulation. The method of constraints has been addressed elsewhere (10, 17). This method is based on
constraint equations to model interface. This method has disadvantages of requiring a larger stiffness matrix, more computer time, and an iteration technique which does not always converge.

The method of stiffness is basically a simple concept of using an element with different normal and tangential stiffness across the interface. The concept of adding a linkage element stiffness to the total structural stiffness matrix was developed by Ngo and Scordellis (57). Goodman (34) developed one-dimensional finite element to represent the interface behavior for the jointed rock. In his formulation, the interface between two adjacent locations at a distance L apart may be represented by an element having two pairs of nodes as shown in Fig. 5.6. This interface model was successfully used by Duncan et al. (26) in the analysis of a rigid retaining wall and by Al-Hussaini, et al. (2) in the analysis of a reinforced earth retaining wall. Ellison (29) simplified the solution by using a two nodes linkage element. Ghaboussi, et al. (31) presented a different approach to develop a one-dimensional interface finite element by using relative displacements between opposite sides of the slip surface as degree of freedom. In this study, the four noded linkage proposed by Goodman (34) is used. The deviation of the stiffness matrix is included in the following subsection.

5.6.1 Four-Nodes Interface Elements

The interface finite element is used here to reflect
the state of normal and shear stresses between the soil media and the wall face, and between the soil media and the reinforcing elements. The element representing the interface between different materials has zero thickness. The interface element cannot resist any tension, if any tension develops normal to the interface, separation between two materials will take place. The interface element can also resist high compression with negligible deformation. The shear strength of the element comes from the friction between the soil and the wall, and between the soil and the reinforcing elements. The relative displacement of different materials depends on the shear behaviour of the element.

The four-nodes interface element is shown on Fig. 5.6. It has 4 nodes, 1, 2, 3, 4 and a local coordinate system with the x-axis along the length. The origin is at the centre. The element has length L and the nodal pairs (1, 4) and (2, 3) initially have the same coordinates. The strain energy, Us, stored in the element over the unit width can be given as:

\[ Us = \frac{1}{2} \int_{-L/2}^{L/2} W_1 \, P_i \, \text{d}x \]  
(5.27)

In matrix notation,

\[ Us = \frac{1}{2} \int_{-L/2}^{L/2} (W^T \, [P]) \, \text{d}x \]  
(5.28)

where,
(W) = \begin{bmatrix} \dot{W}_s - \dot{W}_b \\ \dot{W}_n - \dot{W}_n \end{bmatrix} \quad (5.29)

(P) = \begin{bmatrix} P_s \\ P_n \end{bmatrix} \quad (5.30)

\dot{W}_s = \text{the tangential displacement at top of the element;}
\dot{W}_b = \text{the tangential displacement at bottom of the element;}
\dot{W}_n = \text{the normal displacement at top of the element;}
\dot{W}_n = \text{the normal displacement at bottom of the element;}
P_s = \text{the tangential force;}
P_n = \text{the normal force.}

The force-displacement relationship can be expressed as:

(P) = [Kc](W) \quad (5.31)

where

\[ [Kc] = \begin{bmatrix} K_{sf} & 0 \\ 0 & K_n \end{bmatrix} \quad (5.32) \]

where \( K_{sf} = \text{unit tangential stiffness for the element,} \)
\( K_n = \text{unit normal stiffness.} \)

The variables \( K_s \) and \( K_n \) are determined experimentally as explained in Section 5.6.2 and Section 5.6.3.

Substituting Equation 5.31 into Equation 5.28 yields:
Through a linear interpolation formula, the displacement, \( \{ \mathbf{W} \} \), may be expressed in terms of the nodal point displacements, \( \{ \mathbf{q} \} \). Let \( u_1 \) and \( v_1 \) be displacements in the tangential and normal directions, respectively, at nodal point 1. It is found that:

\[
\begin{align*}
\mathbf{W}_b^b &= \frac{1}{2} \left[ u_1 \left( 1 - \frac{2x}{L} \right) + u_2 \left( 1 + \frac{2x}{L} \right) \right] \\
\mathbf{W}_n^b &= \frac{1}{2} \left[ v_1 \left( 1 - \frac{2x}{L} \right) + v_2 \left( 1 + \frac{2x}{L} \right) \right] \\
\mathbf{W}_b^c &= \frac{1}{2} \left[ u_3 \left( 1 - \frac{2x}{L} \right) + u_4 \left( 1 + \frac{2x}{L} \right) \right] \\
\mathbf{W}_n^c &= \frac{1}{2} \left[ v_3 \left( 1 - \frac{2x}{L} \right) + v_4 \left( 1 + \frac{2x}{L} \right) \right]
\end{align*}
\]

or in matrix notation,

\[
\{ \mathbf{W} \} = \frac{1}{2} \{ \mathbf{D} \} \{ \mathbf{q} \}
\]

where,

\[
\{ \mathbf{q} \}^T = \{ u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3 \quad u_4 \quad v_4 \}
\]
\[
[D] = \begin{bmatrix}
-1+\frac{2x}{L} & 0 & -1-\frac{2x}{L} & 0 & 1+\frac{2x}{L} & 0 & 1-\frac{2x}{L} & 0 \\
0 & -1+\frac{2x}{L} & 0 & -1-\frac{2x}{L} & 0 & 1+\frac{2x}{L} & 0 & 1-\frac{2x}{L}
\end{bmatrix}
\]

(5.37)

Substituting Equation 5.35 into Equation 5.33, yields:

\[
U_{s} = \frac{1}{8} \int_{-L/2}^{L/2} \{q\}^T \{D\}^T \{K_e\} \{D\} \{q\} \, dx
\]

(5.38)

Taking the variation of Equation (5.38) integrating over the length of the element, the local element stiffness matrix, \([K_e]\), in its local coordinate system is given as:

\[
[K_e] = \frac{L}{6} \begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(5.39)

The following transformation matrix is used to transform the local element stiffness matrix, given by Equation 5.39, to
a Global matrix:

\[
[T] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-sin \theta & \cos \theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cos \theta & \sin \theta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sin \theta & \cos \theta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos \theta & \sin \theta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\sin \theta & \cos \theta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\
\end{bmatrix}
\]

(5.40)

where \( \theta \) is the angle of inclination of the local coordinates with respect to the global coordinates.

When loads are applied to the reinforced earth structure, every pair of nodes of the interface element, initially at the same position, will be displaced. The stresses developed in the interface element can be obtained as:

\[
\sigma_n = K_n \cdot \delta_n \quad (5.41)
\]

\[
\tau_s = K_{sf} \cdot \delta_s \quad (5.42)
\]

where \( \sigma_n \) = average normal stress

\( \tau_s \) = average shear stress

\( \delta_n \) = average relative normal displacement

\( \delta_s \) = average relative shear displacement.

5.6.2 Unit Normal Stiffness at the Interface

The value of the interface unit normal stiffness, \( K_n \),
is expected to increase with an increase in the normal stress, and to diminish to zero once tension is developed. The unit normal stiffness, $K_n$, can be determined from a direct shear box test on composite specimens. Each specimen consists of partly granular soil and partly galvanized steel or partly roughened shim steel similar to those in model test. When normal stress is applied, the specimen shortens. The total deformation of the specimen consists of three parts: normal deformation of the interface, elastic shortening of galvanized steel or roughen shim steel, and the deformation of the granular soil. By plotting the normal deformation of the interface against the normal stress, for different normal stress values, a graph as shown in Fig. 5.7 can be obtained. The slope of the curve gives the unit normal stiffness. The curve is linear under working stress conditions which indicating that $K_n$ is constant.

In the absence of accurate experimental data, other authors (37, 52, 2) suggest to assign a high unit normal stiffness value to prevent significant overlap between the two materials. By using a value of $10^6$ lb/in$^3$ ($2.7123 \times 10^{11}$ kN/m$^3$), Hafez (37) obtained satisfactory results in his analysis of soil-steel structure. This value will be used in the analysis.

If tensile stress is developed in the normal direction of the interface element, the unit normal stiffness is adjusted to zero. Physically, it means that the two pairs
of nodes are not connected to each other and separation between two different materials is developed.

5.6.3 unit Tangential Stiffness at the Interface

The unit shear stiffness, \( K_s \), is expected to increase with an increase in the compressive normal pressure and to decrease with the increase in shear stress. The shear stress-shear deformation curve for a direct shear box test under a constant normal pressure is shown in Fig. 5.7. The unit tangential stiffness is not constant as a unit normal stiffness. It is a non-linear function of normal stress, shear stress and angle of friction between soil and steel. Clough and Duncan (14) show that the non-linear shear stress-displacement relationship can be approximated by a hyperbola of the form:

\[
\tau_s = \frac{\delta_s}{a + b \cdot \delta_s} \quad (5.43)
\]

where \( \delta_s \) is the interface shear displacement; 'a' and 'b' are empirical constant determined experimentally. The stress-displacement relationship given by Equation 5.43 can be transformed into a straight line in another set of axes as shown in Fig. 5.8 of the form:

\[
\frac{\delta_s}{\tau_s} = a + b \cdot \delta_s \quad (5.44)
\]

Consequently, coefficient 'a' is the intercept and coefficient 'b' is the slope of the best fit straight line for the experimental values of \( \delta_s / \tau_s \) plotted against values of \( \delta_s \). The reciprocal of 'a' is the initial
slope of the shear stress-displacement curve, which is called the initial shear stiffness $K_{si}$. The reciprocal of 'b' is the asymptotic value of shear stress denoted by $\tau_{ult}$. The failure shear, $\tau_f$, is always smaller than the ultimate shear and they can be related by the formula:

$$\tau_f = Rfs \cdot \tau_{ult} \quad (5.45)$$

where $Rfs$ is the failure ratio which is always smaller than unity. The shear strength is directly proportional to the normal stress and the angle of friction, $\phi$, between the soil and reinforcing element and it can be expressed as:

$$\tau_f = \sigma_n \cdot \tan \phi \quad (5.46)$$

The initial shear stiffness, $K_{si}$, is related to the normal stress as suggested by Janbu (39) as (see Fig. 5.9):

$$K_{si} = K_I \cdot Y_w \cdot \left( \frac{\sigma_n}{\gamma_w} \right)^{\eta_s} \quad (5.47)$$

where, $K_I$ is the dimensionless stiffness number.

$\eta_s$ is the stiffness exponent.

$Y_w$ is the unit weight of water.

Substituting Equations 5.45 through 5.47 into Equation 5.43 yields:

$$\tau_s = \frac{\delta_s}{1 + \frac{Rfs \cdot \delta_s}{K_I \cdot Y_w \cdot \left( \frac{\sigma_n}{\gamma_w} \right)^{\eta_s} \cdot \sigma_n \cdot \tan \phi}} \quad (5.48)$$

The shear stiffness, $K_s$, is defined as the slope of the tangent to shear-stress-displacement curve. By
differentiating Equation 5.48 with respect to \( \delta_s \), the tangent shear stiffness can be obtained as:

\[
K_{sf} = K_I \gamma_w \left( \frac{\sigma_n}{\text{Pat}} \right)^n_s \left( 1 - \frac{Rfs \tau_s}{\sigma_n \tan \phi_i} \right)^2
\]

(5.49)

The four parameters \( K_I \), \( n_s \), \( Rfs \) and \( \phi_i \) in Equation 5.49 can be obtained from shear box tests on composite specimens.

5.7 Two-Dimensional Idealization of Reinforced Earth System

Since the reinforcing elements are placed at known vertical and horizontal intervals in the soil mass of a reinforced earth system, an accurate analysis of the system required that the system be treated as a three-dimensional (3D) problem. This requires a complex modelling of the system and needs lots of computing time. A method used by Al-Hussaini, et al. (2) is adopted here so that the 3D problem can be approximated by a structurally equivalent two-dimensional system.

5.7.1 Reinforcing Elements

In order to idealize the reinforced earth system as a two-dimensional plane strain problem, the reinforcing elements at each level are replaced by a plate extended to the full width and breath of the wall. The major response of the reinforcing elements is to provide axial stiffness and the total axial stiffness, \( S \), of each level is:
\[ S = N \frac{A_s E_s}{L_s} \]  

where,  
\( N \) = total number of strips in each level  
\( A_s \) = cross-sectional area of the reinforcing element  
\( E_s \) = modulus of elasticity of the reinforcing element  
\( L_s \) = length of the reinforcing element.

By replacing the reinforcing elements as a plate, the equivalent stiffness of the plate, \( S_e \), is defined as:

\[ S_e = \frac{A_e E_e}{L_e} \]

where,  
\( A_e \) = equivalent cross-sectional area of the plate  
\( E_e \) = equivalent modulus of the plate  
\( L_e \) = equivalent length of the plate

Since the stiffness \( S \) and the equivalent stiffness \( S_e \) should be the same and also the length \( L_s \) and \( L_e \) are the same, the equivalent modulus \( E_e \) can be obtained as:

\[ E_e = \frac{N A_s E_s}{A_e} \]

5.7.2 Soil-Strip Interface Element

As discussed in Section 5.7.1, in order to obtain the finite element plane strain solution, the reinforcing elements are assumed to be replaced by a plate of equivalent stiffness. This assumption increases the shear resistance between the reinforcing element and the sand by a factor \( m_f \), where \( m_f \) is defined as the ratio of the
horizontal cross-sectional area of the wall to the total area of the reinforcing elements at that level. Thus,

$$M_f = \frac{L_2 \times L_f}{N \times L_2 \times W}$$  \hspace{1cm} (5.53)

where \(L_f\) = the length of the wall

\(L_2\) = the length of the reinforcing element

\(W\) = the width of the reinforcing element

\(N\) = number of reinforcing elements

The increase in shear stress at the interface element can be compensated for by reducing the tangent shear stiffness in Eq. 5.49 by \(M_f\). Thus, the constitutive equation for the interface element used in the actual finite element analysis is:

$$Ksf = \frac{K_1}{M_f} \gamma_w \left( \frac{\sigma_n}{\sigma_n + \gamma D_s} \right) \left( 1 - \frac{Rfs \tau_s}{\sigma_n \tan \phi} \right)^2$$  \hspace{1cm} (5.54)

5.8 Sequential Construction and Compaction Simulation

In order to calculate the dead load stresses, incremental analysis procedure is applied. The finite element mesh shown in Fig. 5.10 is broken into series of small construction layers. Simulating placement of new layer is done by applying nodal forces equal to the weight of the added layer. The effects on the existing layers are analysed. Each element of the added layer is assigned initial stresses consistent with the overburden, at the centre. The element tangent modulus according to the hyperbolic
model can be calculated. Then, a finite element analysis is carried out on the existing mesh and the added layer. The incremental displacements and stresses are then added to the accumulated displacements and stresses of the existing nodes and elements to yield the final value after construction of a new layer. The initial assigned stresses in the newly placed elements are replaced by resulting incremental stresses at the end of the analysis. In order to measure the movements due to subsequent construction, as a reference position, the top nodes of the newly placed layer are assigned zero displacement. The procedure is repeated as new layer is added until the required height of the system is reached.

Simulation of compaction is done by applying a specified surface load acting along the top of the most recently added incremental layer during the sequential construction simulation. Equivalent nodal forces that produce the same work done by the assumed surface load are applied in the incremental analysis. In order to complete the compaction simulation, the surface loads must be removed after the compacting of the added layer of soil. Therefore, during an intermediate incremental finite element analysis that simulates the construction process, the nodal forces acting on the newly added layer will be the summation of three components. The first component is the weight of the newly applied layer. It acts at all the nodes of the elements in the layer. The second component is the
compaction load on the new layer. It acts at the top surface nodes only. The third component of the nodal forces acts at the bottom surface nodes only, simulating removal of the compaction loads which were applied earlier on the lower layer. After the analysis, the resulting incremental displacements and stresses are treated as described before.

Although the compaction loads are removed during the simulation of the newest layer, their effects in densifying the soil is locked in the finite element. Stiffer soil properties will result.

5.9 Iterative Procedure for Accurate Representation of Material Properties

In an incremental finite element analysis, element properties remain constant. So the stress-strain path will be a straight line. The repeat use of this technique will lend to an increasing divergence between the piecewise linear element properties used in the analysis and the assumed nonlinear properties for the element. Figure 5.11 shows the difference between the stress-strain curve (curve a' b' c') obtained from the incremental analysis and the assumed, stress-strain curve (curve a b c). So an iterative procedure is coupled with incremental analysis to bring the piecewise linear stress-strain path much closer to the assumed curve.

5.9.1 Iteration for Soil Element

The following iterative procedure is used to improve
the soil properties:

1. Under certain loading condition, an initial element properties are assigned to the elements.

2. At the end of the analysis, the resulting stresses are used to calculate final element properties as given by Equation 5.2.1.

3. The final and initial properties are averaged and the analysis is repeated under the same loading condition using the averaged properties.

4. At the end of iterative procedure, the final element stresses will be used to calculate the initial element properties in the subsequent finite element analysis.

5.9.2 Iteration for Interface Elements

The following iterative procedure is used to obtain the value of unit tangential stiffness for an interface element:

1. Under a certain loading condition, the normal and tangential stresses are determined for the interface element. Initial element properties can be determined for the interface element and used in the primary analysis.

2. The resulting normal and tangential stresses in the element are substituted into Equation 5.54 to calculate the corresponding unit tangential stiffness.

3. The initial and final stiffness are averaged and
the finite element analysis is repeated under the same loading condition using the averaged stiffness.

4. At the end of the iterative procedure, the final normal and tangential stresses are used to calculate the corresponding unit tangential stiffness, which will be used as initial stiffness of the interface element for the next incremental analysis.

5.10 Soil Failure

In the course of finite element analysis, the soil element can fail under either tension or shear. If tensile stress is detected in the soil element at the end of an incremental analysis, the element is considered to be failed in tension. A stress transfer method used by Hafez (37) (Section 5.11) will be used to reduce the stresses in the failing element to zero without violating equilibrium. The stresses in the failing element will be dissipated to the neighbouring element at the end of the transfer process. In the following incremental analysis, a Poisson's ratio equals to 0.49 and a initial tangent modulus computed by assuming a value equal to 0.1 atmospheric pressure will be used for the failing element. Under the incremental loading, the failing element may attract compressive stresses and the analysis can be continued, or it may remain in tension. The stress transfer technique will be applied again. Local failure in the soil is assumed if development of tensile
stresses in several elements along a line during consecutive loading increments are observed.

If at the end of an incremental analysis, the maximum shear stress in an element exceeds the soil strength. The soil element fails in shear. The stress transfer technique is used to reduce the stresses $\sigma_{fz}$ and $\sigma_{3a}$ to $\sigma_{f}$ and $\sigma_{3f}$, respectively, as shown in Fig. 5.12. $\sigma_{f}$ and $\sigma_{3f}$ can be determined from the initial stresses $\sigma_{11}$ and $\sigma_{33}$ by using the following assumption:

$$\frac{\sigma_{3f} - \sigma_{3i}}{\sigma_{1f} - \sigma_{1i}} = \frac{\sigma_{3a} - \sigma_{3i}}{\sigma_{1a} - \sigma_{1i}} \quad (5.55)$$

Equations 5.18 and 5.55 are used to determine the limits within which the principal stresses are kept during the stress transfer process. During the following incremental analysis, a modulus calculated on the basis of stress level equal to 0.95 of the stress level at failure and a Poisson's ratio equal to 0.49 are used for the failing element. If shear failure still exists in the element or propagates to adjacent elements along a line, local failure is assumed to occur.

5.11 Stress Transfer Method

The followings steps used in stress transfer method are extracted from the works of Hafez (37):

1. Determine the equivalent nodal forces required to change the stresses in a certain element.

2. Equal and opposite pairs of nodal forces are
imagined to act at the element nodes. This will not disturb equilibrium.

3. One set of the forces vanishes when the stresses are reduced to the desired level.

4. An analysis is carried out by applying the other set of nodal forces as external loading to the system. The same element properties for each element in the incremental analysis are used. The resulting stresses and displacements are added to the corresponding stresses and displacements calculated at failure.

5. If the final stresses obtained in step 4 still indicate failure, the procedure from step 1 to step 4 will be repeated until the deviation between the final stresses and $\sigma_{1f}$ and $\sigma_{3f}$ becomes negligible.

Local failure is assumed if the stresses in the element do not converge to the limiting values $\sigma_{1f}$ and $\sigma_{3f}$ or if other neighbouring elements fail under the stress transfer method.

5.12. Interface Element Failure

Since neither the soil and the reinforcing strips nor the soil and the wall channels are bonded together, no tensile stress can exist between the two materials. No tensile stress can develop in the interface element. If,
under certain loading condition, tensile stress is detected in an interface element, the incremental analysis will be repeated after changing the interface element properties. The unit tangential and unit normal stiffness are each set to a small residual value of 0.5 kN/m$^3$. The results of the repeated analysis will show that the normal and shear stresses at the location under consideration have negligible value. The failure of the particular element causes the neighbouring elements to take up the extra stresses.

During incremental analysis, shear failure results if shear stress is larger than the shear strength of the interface element. If shear failure occurs, incremental analysis will be repeated by using a reduced unit tangential stiffness value of 0.5 kN/m$^3$. This will redistribute the excess stress to neighbouring elements.

5.13 Computer Program

A computer program was developed to handle the mathematical computations for the incremental analyses described in this chapter. The flow chart of this program is shown in Appendix C.
CHAPTER VI

ANALYTICAL RESULTS BASED ON FINITE ELEMENT METHOD

6.1 General

Experimentally obtained deflections of wall surface and tension forces in reinforcing strips of the reinforced earth wall model are compared with the analytical results based on finite element method. All four load locations with vertical strip load only will be investigated.

The analytical results are based on incremental analysis in which the non-linear behaviour of the dense soil elements is reflected by using the following parameters (2): \( K_s = 580 \), \( R_f = 0.85 \), \( n = 0.50 \), \( \phi = 40^\circ \), \( v = 0.4 \), \( V_{\text{failure}} = 0.49 \), \( c = 0 \text{ kN/m}^2 \) and \( \gamma = 17.1 \text{ kN/m}^3 \). The experimental parameters used to represent the non-linear behaviour of the interface elements between soil and reinforcing strips are (37): \( K_i = 43070 \), \( R_{fs} = 0.834 \), \( n_s = 0.6 \), and \( \phi_i = 40^\circ \). The parameters used to represent the soil-base interface elements are (2): \( K_L = 3280 \), \( R_{fs} = 1.0 \), \( n_s = 1.1 \) and \( \phi_i = 18^\circ \).

6.2 Deflection of Wall Face

The deflections of the wall face, due to applied vertical strip load, obtained by finite element analysis are plotted on Figs. 6.1 and 6.2. For comparison, experimental wall face deflections are also plotted in those figures. The smaller scale in Figs. 6.1 and 6.2 applies to the
analytical deflections of wall face.

At load location I (Fig. 6.1), the analytical result indicates a large outward movement from the vertical position of the wall occurred in the top channel. This agrees with the trend observed experimentally. But at load location II (Fig. 6.1), the analytical result shows that the top channel will rotate inwards. This movement is completely different from that observed from experiment. At these two locations, the bulge at the bottom shows the roller boundary condition at that point is not satisfactory to simulate the real behaviour. At load location III and IV (Fig. 6.2) the shape of the analytical deflection is very close to that of experimental deflection.

In general it is found that the numerical value of the analytical results in all four load locations are only fractions of the experimental results. The ratio of maximum analytical deflection to maximum experimental deflection varies from around one-tenth at load location I to around one-third at load location IV.

6.3 Location of Maximum Strain

Figure 6.3 to Fig. 6.6 show the analytical result of the location of maximum strain or potential failure plane of every strip level in all four load locations. At load location I (Fig. 6.3), the location of maximum strain in the first reinforcing strip developed at about the quarter point from the outside corner of the strip footing. This analytical potential failure plane extends vertically down
to the tenth strip. At load location II (Fig. 6.4), the analytical potential failure plane starts below the inside corner of the strip footing and extends vertically down to the eighth strip. This potential failure plane then moves towards the wall face at the ninth strip and extends vertically to the tenth strip. At load location III and IV (Fig. 6.5 and Fig. 6.6), the analytical potential failure plane starts about the quarter point from the inside corner of the base of the strip footing and extends downwards. This potential failure plane moves away from the wall and outside the strip footing at the second strip level and then extends vertically down to the tenth level.

As mentioned in Chapter IV, it was found that experimentally the potential failure plane due to vertical strip load applied inside the Rankine failure wedge can be approximated by Culmann's potential failure plane. It can be seen that at load location I to III, the analytical potential failure planes are completely different from the Culmann's potential failure planes (see Fig. 6.3 to Fig. 6.5).

At load location I (Fig. 6.3), the analytical potential failure plane develops much closer to the wall than the one observed experimentally. At load II and III (Fig. 6.4 and Fig. 6.5), the location of the analytical potential failure plane and Culmann's potential failure plane is about the same at the first strip level. But below the first level, the analytical potential failure plane locates much further
away from the wall and behind the Culmann's potential failure plane. It is found that the lower the strip level, the larger the distance between the two locations of potential failure plane. Since Culmann's potential failure plane is valid for load location I to III, no comparison of potential failure plane can be made for load location IV.

6.4 Maximum Traction Force

The analytical maximum traction forces developed in every strip level in Fig. 6.7 to Fig. 6.10 are computed based on the strain value obtained from the analytical potential failure plane shown in Fig. 6.3 to Fig. 6.6. At load location I (Fig. 6.7), unlike the experimental result where optimal traction force occurred at a depth of 0.15 m from top, the analytical traction forces developed in the reinforcing strip are quite uniform from top level to bottom level. The analytical optimal traction force value is only about half of the experimental value. The smaller traction force at level I may indicate that the analytical method is able to simulate the stress redistribution behaviour of reinforced earth retaining wall. The same smaller traction force at level I is observed at load location II to IV.

At load location II (Fig. 6.8), the optimal analytical maximum traction force is observed to be at a depth of 0.05 m while the experimental result shows a depth of 0.15 m. The optimal analytical maximum traction force is not significantly much larger than the other traction force.
Overall, the analytical traction force in every strip is only about half the experimental value. Similar results are observed for load location III (Fig. 6.9). At this load location, the difference between analytical and experimental results in each strip level is smaller. At the lower third of the structure, good agreement between analytical result and experimental result is observed. At load location IV (Fig. 6.10), results similar to that of load location III are observed. The difference between analytical result and experimental result is even smaller, but is still significantly large at certain levels. Very good agreement between experimental values and analytical values in the lower third of the wall are observed.

6.5 Stress Along the Reinforcing Strip

The analytical stress distribution along the length of the reinforcing strip is shown in Fig. 6.11 to Fig. 6.13 for load location I to III. Only one strip level is shown at each load location. It is found that the shape of the stress distribution curve at that strip level can be used to represent the general behaviour of the remaining strips.

At load location I (see Fig. 6.11), the analytical result shows that the maximum stress occurs very close to the wall face and diminishes to a very very small value or zero value at the far end of the strip. The experimental stress distribution along the strip is similar to the analytical stress distribution except that the maximum experimental stress occurs at a distance further away from
the wall face and the stress values along the length of the strip are different. At load locations II and III (Fig. 6.12 and Fig. 6.13), the analytical stress curve and experimental stress curve are similar in shape with the exception of different stress values and different locations of maximum stress. Overall, the analytical results do agree with the experimental results in that maximum stress does not occur at the wall face but at a distance away from the wall face and also the stress at the end of the strip is close to zero.

6.6 Conclusion

The discrepancy between the experimental results and analytical results may be due to the use of rectangular finite element to represent soil element. The rectangular element cannot fully represent the soil element as under incremental loading, large deformation of soil element will occur and the soil element will no longer be rectangular in shape. The local axes of individual finite element will move. Also the free roller boundary condition at the base of the wall cannot simulate the real behaviour as there is friction between wall base and the container. Finally the flexibility of the wall elements may also have to take into account in the analysis.
CHAPTER VII
CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

Based on the experimental work done by Seymour (75) and the analytical work presented in this thesis, the following conclusions can be made:

1. The experimental results indicate that the potential failure plane developed in the reinforced earth system, due to vertical strip load placing inside Rankine failure wedge, can well be approximated by the failure plane obtained by Culmann's method.

2. The proposed theoretical method, the 'redistribution method', which takes into account the force redistribution characteristic of reinforced earth system, is satisfactory to determine the magnitude and distribution pattern of the traction forces in a reinforced earth retaining wall system subjected to vertical strip load acting inside the Rankine failure wedge. The maximum traction force that can be developed in the reinforcing element is limited to either frictional capacity between soil and steel reinforcing element or the yielding load of steel, whichever is smaller.

3. When the vertical strip load is placed outside the Rankine failure wedge, Terzaghi's equation developed for rigid retaining wall subjected to vertical strip
load will provide rather conservative values of the traction force distribution in the system.

4. When horizontal strip load is applied towards the wall face, the magnitude of stresses developed in the reinforcing elements calculated from the French Ministry of Transport method and the Tensor method are not comparable to the stresses obtained from the experimental results.

5. The redistribution method can be used to determine the traction forces developed in reinforcing strips when the reinforced earth retaining wall is subjected to combined action of the vertical and horizontal strip loads. When the horizontal strip load is acting towards the wall, reasonable traction force distribution can be obtained by the redistribution method provided that there is no movement of the potential failure plane which originally formed by vertical strip load.

6. The equation obtained by combining the Terzaghi's equation and the French Ministry of Transport formula can provide rather conservative force distribution in a reinforced earth system when the combined vertical and horizontal strip load is located outside the Rankine failure wedge and the horizontal component is acting towards the wall.

7. When the horizontal component of a combined vertical and horizontal loading system is acting away from
the wall, it is safe to design the reinforced earth system as if it was subjected to the action of the vertical load only.

8. Although the finite element method can provide more information such as deflection of wall, stress state in the soil and the stress distribution along the length of the reinforcing strip, other than maximum traction force distribution in reinforcing strips, in this study it does not provide satisfactory results that are comparable to experimental results.

7.2 Recommendations for Further Research

1. It is suggested to investigate the response and behaviour of a reinforced earth retaining wall system when subjected to dynamic loading in the case when the system is acting as a bridge abutment.

2. For finite element analysis, it is suggested that higher degree soil element, beam element for wall channel and different boundary condition can be used.

3. It is suggested to investigate the behaviour of a reinforced earth retaining wall system if reinforcing strips of different length are used or using wire mesh instead of individual strips.
Fig. 2.1: State of Stress in Reinforced Earth
Fig. 2.2: Design Values of Friction as a Function of Depth

Fig. 2.3: Vertical Cut in an Earth Mass
Fig. 2.4: Stresses Exerted on the Reinforcement by the Soil.

Fig. 2.5: Effective Length of the Reinforcement.
Fig. 2.6: Design Values of Coefficient of Lateral Earth Pressure as a Function of Depth.

\[ \Delta \sigma_v = \frac{P}{b + \frac{h}{2}} \]

Fig. 2.7: Distribution of Vertical Stress due to Surcharge Line Load.
Fig. 2.8: Distribution of Horizontal Stresses Against a Rigid, Vertical Retaining Wall due to a Surcharge Strip Load

\[ \Delta \sigma_H = \frac{F}{l^+d} \left[ 1 - \frac{h}{h_o} \right] \]

where

\[ l' = l - 2e \]

\[ h_o = 2(l' + d) \]

Fig. 2.9: Horizontal Stress Distribution Proposed by the French Ministry of Transport
Fig. 2.10: Traction Force Distribution Along the Strips of a Raft Foundation (After Schlosser and Long)
Fig. 3.1: Vertical Section Through the Skin Elements and Reinforcing Strips (After Seymour)

Fig. 3.2: View in Plan of the Skin Elements and Reinforcing Strips (After Seymour)
Fig. 3.4 : Location of the Strain Gauges on the Reinforcing Strips (After Seymour)
Fig. 3.5: Location of Dial Gauges

Fig. 3.6: Loading System
Fig. 3.7: Locations of the Strip Surcharge Load (After Seymour)
Fig. 4.1: Determining Traction Forces from Terzaghi's Equation
Fig. 4.2: Comparison of Maximum Traction Forces Values Between Terzaghi's Equation and Experimental Value at Location I
Fig. 4.3: Comparison of Maximum Traction Force Values Between Terzaghi’s Equation and Experimental Value at Location II

Load Location II
Q = 100 kN/m²
Fig. 4.4: Comparison of Maximum Traction Force Values Between Terzaghi's Equation and Experimental Value at Location III
Fig. 4.5: Comparison of Maximum Traction Force Values Between Terzaghi's Equation and Experimental Value at Location IV

Load Location IV

\( q = 100 \text{kN/m}^2 \)
Fig. 4.6: Wall Deformation Under a Vertical Load at Load Location I and II
Fig. 4.7: Wall Deformation Under a Vertical Load at Load Location III and IV.
Fig. 4.8a: Partition of the Active and Resistant Zones, and the 2:1 Load Dispersion Configuration

4.8b: Free Body Diagram of Strip Portion
Fig. 4.9: Frictional Capacities of Strips with Vertical Load at Location I
Fig. 4.10: Frictional Capacities of Strips with Vertical Load at Location II
Fig. 4.11: Frictional Capacities of Strips with Vertical Load at Location III
Fig. 4.12: Frictional Capacities of Strips with Vertical Load at Location IV
Fig. 4.13: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location I.
Fig. 4.14: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location II
Fig. 4.15: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location III
Fig. 4.16: Maximum Traction Forces Distribution at Location IV

Load Location IV

\[ Q = 100 \text{kN/m}^2 \]
Fig. 4.17: Wall Deformations at First Yielding and Prior to Failure With Surcharge Load at Load Location II

- Q = 173.7 kN/m² for first yielding
- Q = 300 kN/m² before failure

Channel no. vs. deflection (mm)
Fig. 4.18: Maximum Traction Forces Distribution at First Yielding
Fig. 4.19: Change in Reinforcing Element Stress with Horizontal Load Intensity $H$ Acting Towards Wall Face (After Seymour)
Fig. 4.20: Change in Reinforcing Element Stress with Horizontal Load Intensity $H$ Acting Away from Wall Face (After Seymour)
Fig. 4.21: Horizontal Stress Distribution Proposed by Tensar
Fig. 4.22a: Horizontal Strip Load on Semi-Infinite Elastic Medium

4.22b: Dimensionless Solution to Equation 4.15
Test no. II
Load Location II

Wall Face

$\Delta \sigma_s / H = 6000$
$5000$
$4000$
$3000$
$2000$
$1000$

Strain Gauge No

Fig. 4.23: Contour Diagram of Ratio $\Delta \sigma_s / H$ (Horizontal Load Acting Towards Wall Face). (After Laba et. al)
Test no. 11
Load Location II
\[ Q = 100 \text{kN/m}^2; \ H = 19.5 \text{kN/m}^2 \]

Fig. 4.24: Comparison Between Values of \( \Delta \sigma_H / H \) Obtained from French Ministry of Transport Equation, Values Obtained from Tensar Equation and Values Measured on the Model; (Load at Location II Acting Towards Wall Face). (After Laba et. al)
Test no. 12
Load Location III

Wall Face

$\Delta \sigma_s / H = -3000$

-2000

-1000

0

Strain Gauge

100 mm Scale

Fig. 4.25: Contour Diagram of Ratio $\Delta \sigma_s / H$ (Horizontal Load Acting Away from Wall Face). (After Løba et. al)
Fig. 4.26: Wall Deformation Under Vertical and Horizontal Load Components at Location I (After Seymour)
Fig. 4.27: Wall Deformation Under Vertical and Horizontal Load Components at Location II (After Seymour)
Fig. 4.28: Wall Deformation Under Vertical and Horizontal Load Components at Location III (After Seymour)

Fig. 4.29: Wall Deformation Under Vertical and Horizontal Load Components at Location IV (After Seymour)
Fig. 4.30: Effect of Horizontal Component on Distribution of Maximum Traction Force with Depth at Location I

Load Location I

$Q = 60 \text{ kN/m}^2$

$H = 0.5 \text{ kN/m}^2$

Eq. [4.7]
Fig. 4.31: Effect of Horizontal Component on Distribution of Maximum Traction Force with Depth at Location II.
Fig. 4.32: Effect of Horizontal Component on Distribution of Maximum Traction Force with Depth at Location III
Fig. 4.33: Effect of Horizontal Component on Distribution of Maximum Traction Force with Depth at Location IV
Fig. 4.34: Change in Strip Stress Due to Horizontal Load Component Acting Towards Wall Face

Load Location I
Reinforcing Strip No. 2
Q = 60 kN/m²
H = 0.5, 10 kN/m²

Fig. 4.35: Change in Strip Stress Due to Horizontal Load Component Acting Towards Wall Face

Load Location II
Reinforcing Strip No. 3
Q = 100 kN/m²
H = 0, 5, 13, 19.5, 26 kN/m²
Fig. 4.36: Change in Strip Stress Due to Horizontal Load Component Acting Towards Wall Face

Load Location III
Reinforcing Strip No. 2
$Q = 100 \text{ kN/m}^2$
$H^* = 0, 5, 10, 15, 17.6 \text{ kN/m}^2$
Fig. 4.37: Comparison of Maximum Traction Force Values Between Redistributio Method and Experimental Values at Location I

Load Location I
Q = 60 kN/m$^2$
H = 5 kN/m$^2$
Fig. 4.38: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location I

Load Location I

\[ Q = 60 \text{ kN/m}^2 \]

\[ H = 10 \text{ kN/m}^2 \]
Fig. 4.39: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location II.

Load Location II
Q = 100 kN/m²
H = 19.5 kN/m²
Fig. 4.40: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location II
Fig. 4.41: Comparison of Maximum Traction Force Values Between Redistribution Method and Experimental Values at Location III

Load Location III

\( Q = 100 \text{ kN/m}^2 \)

\( H = 17.6 \text{ kN/m}^2 \)
Fig. 4.42: Comparison of Maximum Traction Force Values Between Values Obtained from Eq. 4.18 and Experimental Values at Location IV.
Test no. 14
Load Location III
Reinforcing Element No. 4
$Q = 100 \text{kN/m}^2$
$H = 0, 5, 10, 15, 20, 23.3 \text{kN/m}^2$

Fig. 4.43: Change in Strain Stress Due to Horizontal Load Component Acting Away from Wall Face
Fig. 5.1: Linear Strain Rectangular Element

Fig. 5.2: Hyperbolic Stress-Strain Path
Fig. 5.3: Transformed Hyperbolic Stress-Strain Relationship

\[ \frac{\varepsilon_a}{\sigma_1 - \sigma_3} \]

Fig. 5.4: Variation of Initial Tangent Modulus

\[ \log \left( \frac{E_i}{P_a} \right) \]

\[ \log K_S \]

\[ \log \left( \frac{\sigma_3}{P_a} \right) \]
Fig. 5.5: Truss Element

Fig. 5.6: One Dimensional Interface Element
Fig. 5.7: Stress-Deformation Curves for the Interface Element

Fig. 5.8: Transformed Hyperbolic Shear Stress-Shear Deformation Relationship
Fig. 5.9: Variation of Initial Unit Tangential Stiffness
Fig. 5.11: Nonlinear Analysis Using Incremental Technique

Fig. 5.12: Shear Failure Consideration in Soil Element (After Hafez)
Fig. 6.1: Comparison of Wall Deformation Between Finite Element Values and Experimental Values at Location I and II
Fig. 6.2: Comparison of Wall Deformation Between Finite Element Values and Experimental Values at Location III and IV
Load Location I $Q = 80 \text{ kN/m}^2$

Fig. 6.3: Comparison of Potential Failure Plane Between Finite Element Method and Culmann's Method at Location I
Load Location II  $Q = 100 \text{ kN/m}^2$

Culmann's Potential Failure Plane

Analytical Potential Failure Plane

Fig. 6.4: Comparison of Potential Failure Plane Between Finite Element Method and Culmann's Method at Location II
Fig. 6.5: Comparison of Potential Failure Plane Between Finite Element Method and Culmann's Method at Location III
Load Location IV \[ Q = 100 \text{ kN/m}^2 \]

Analytical Potential Failure Plane

**Fig. 6.6**: Location of Potential Failure Plane Obtained from Finite Element Method
Fig. 6.7: Comparison Between Values of Maximum Traction Forces Obtained from Finite Element Method and Experimental Results at Location I
Fig. 6.8: Comparison Between Values of Maximum Traction Forces Obtained from Finite Element Method and Experimental Results at Location II.
Fig. 6.9: Comparison Between Values of Maximum Traction Forces Obtained from Finite Element Method and Experimental Results at Location III.
Fig. 6.10: Comparison between values of maximum traction forces obtained from finite element method and experimental results at location IV.

Load Location IV

\[ Q = 100 \text{ kN/m}^2 \]
Fig. 6.11: Comparison of Stress Distribution Along the Strip Between Finite Element Method and Experimental Results

Load Location I
Reinforcing Strip No. 2
$Q = 60 \text{ kN/m}^2$

Experimental
Analytical

DISTANCE FROM WALL (mm)

Fig. 6.12: Comparison of Stress Distribution Along the Strip Between Finite Element Method and Experimental Results

Load Location II
Reinforcing Strip No. 3
$Q = 100 \text{ kN/m}^2$

Experimental
Analytical

DISTANCE FROM WALL (mm)
Fig. 6.13: Comparison of Stress Distribution Along the Strip Between Finite Element Method and Experimental Results
APPENDIX B

STIFFNESS MATRICES FOR RECTANGULAR ELEMENTS
The polynomial functions for displacements at any point within the rectangular element written in matrix form is given in Equations 5.1 as:

\[
\{ u \} = \{ \phi \} \{ \alpha \} \tag{B.1}
\]

where,

\[
\{ u \}^T = \{ u, v \} \tag{B.2}
\]

\[
\{ \phi \} = \begin{bmatrix}
1 & x & y & xy & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & x & y & xy
\end{bmatrix} \tag{B.3}
\]

\[
\{ \alpha \}^T = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8) \tag{B.4}
\]

The vector of interpolation coefficient \( \{ \alpha \} \) is related to the vector of nodal displacements \( \{ q \} \) through Equation 5.2:

\[
\{ q \} = \{ A \} \{ \alpha \} \tag{B.5}
\]

where,

\[
\{ q \}^T = \{ u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4 \} \tag{B.6}
\]

\[
\{ A \} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & b & 0 & 0 & 0 & 0 & 0 \\
1 & a & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & a & b & ab \\
0 & 0 & 0 & 0 & 0 & 1 & a & b \end{bmatrix} \tag{B.7}
\]
a, b are sides of the rectangular element (see Fig. 5.1).

Solving Equation B.5 for \( \{ \alpha \} \), yields:

\[
\{ \alpha \} = [ A ]^{-1} \{ q \}
\]  \hspace{1cm} (B.8)

where,

\[
[A]^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1/a & 0 & 0 & 0 & 1/a & 0 & 0 & 0 \\
-1/b & 0 & 1/b & 0 & 0 & 0 & 0 & 0 \\
1/ab & 0 & -1/ab & 0 & -1/ab & 0 & 1/ab & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1/a & 0 & 0 & 0 & 1/a & 0 & 0 \\
0 & -1/b & 0 & 1/b & 0 & 0 & 0 & 0 \\
0 & 1/ab & 0 & -1/ab & 0 & -1/ab & 0 & -1/ab \\
\end{bmatrix}
\]  \hspace{1cm} (B.9)

The relationship between strains and displacements at any point within the element are given by Equations and can be written in matrix form as:

\[
\{ \varepsilon \} = [ B ] \{ \alpha \}
\]  \hspace{1cm} (B.10)

where,

\[
\{ \varepsilon \} = \{ \varepsilon_x \, \varepsilon_y \, \varepsilon_{xy} \}
\]  \hspace{1cm} (B.11)

\[
[B] = \begin{bmatrix}
0 & 1 & 0 & y & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & x & 0 & 1 & 0 & y \\
\end{bmatrix}
\]  \hspace{1cm} (B.12)

For plane strain condition, the stress-strain relationships can be shown in matrix form as:

\[
\{ \sigma \} = [ D ] \{ \varepsilon \}
\]  \hspace{1cm} (B.13)
where,

\[
\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix}
\]

\[D = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} 1 - \nu & \nu & 0 \\ \nu & 1 - \nu & 0 \\ 0 & 0 & \frac{1 - 2\nu}{2} \end{bmatrix}
\]

Substituting Equation B.9, B.12 and B.15 into Equation 5.6 and performing the integration over the volume of the element, the element stiffness matrix is generated as:

\[
[K_e] = \frac{E_t}{12 (1 + \nu)(1 - 2\nu)} \begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} & K_{17} & K_{18} \\
K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} & K_{27} & K_{28} \\
K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} & K_{37} & K_{38} \\
K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} & K_{47} & K_{48} \\
K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} & K_{57} & K_{58} \\
K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} & K_{67} & K_{68} \\
K_{71} & K_{72} & K_{73} & K_{74} & K_{75} & K_{76} & K_{77} & K_{78} \\
K_{81} & K_{82} & K_{83} & K_{84} & K_{85} & K_{86} & K_{87} & K_{88}
\end{bmatrix}
\]

where,

\[t\] is the thickness of the element.

\[K_{11} = 4 \left[ (1 - \nu) \frac{b}{a} + (1 - 2\nu) \frac{a}{2b} \right]\]

\[K_{21} = 3 \left[ \nu + (1 - 2\nu) / 2 \right]\]

\[K_{22} = 4 \left[ (1 - \nu) \frac{a}{b} + (1 - 2\nu) \frac{b}{2a} \right]\]

\[K_{31} = 2 \left[ (1 - \nu) \frac{b}{a} - (1 - 2\nu) \frac{a}{b} \right]\]
$K_{32} = 3 \left[ v - \left( 1 - 2v \right)/2 \right]$

$K_{33} = K_{11}$

$K_{41} = 3 \left[ -v + \left( 1 - 2v \right)/2 \right]$

$K_{42} = 2 \left[ -2 \left( 1 - v \right)a/b + \left( 1 - 2v \right)b/2a \right]$

$K_{43} = -K_{21}$

$K_{44} = K_{22}$

$K_{51} = 2 \left[ -2 \left( 1 - v \right)b/a + \left( 1 - 2v \right)a/2b \right]$

$K_{52} = K_{41}$

$K_{53} = 2 \left[ -\left( 1 - v \right)b/a - \left( 1 - 2v \right)a/2b \right]$

$K_{54} = K_{21}$

$K_{55} = K_{11}$

$K_{56} = K_{32}$

$K_{61} = 2 \left[ \left( 1 - v \right)a/b - \left( 1 - 2v \right)b/a \right]$

$K_{62} = K_{21}$

$K_{63} = 2 \left[ -\left( 1 - v \right)a/b - \left( 1 - 2v \right)b/2a \right]$

$K_{64} = K_{43}$

$K_{65} = K_{22}$

$K_{66} = K_{53}$

$K_{71} = K_{53}$

$K_{72} = K_{43}$

$K_{73} = K_{51}$

$K_{74} = K_{32}$

$K_{75} = K_{31}$

$K_{76} = K_{41}$

$K_{77} = K_{11}$

$K_{81} = K_{43}$

$K_{82} = K_{64}$

$K_{83} = K_{41}$
\[ K_{84} = K_{62} \]
\[ K_{85} = K_{32} \]
\[ K_{86} = K_{42} \]
\[ K_{87} = K_{21} \]
\[ K_{88} = K_{22} \]

Remaining terms are obtained by symmetry.
APPENDIX C
FLOW CHART
NN = Number of nodes
NSM = Number of soil elements
NRM = Number of reinforcements
NSKM = Number of skin elements
NDF = Number of degree of freedom
NLAYER = Total number of incremental analyses
NL = Number of soil layer
\( \sigma_n \) = Normal stress at interface
\( \tau_s \) = Shear stress at interface
NTL = Total number of finite elements in each incremental analysis
\( \phi_f \) = Angle of friction

START

Read Input Data

Generate Nodal Coordinates, Soil Element, Interface Element and Skin Element

Set Stress Vectors for Interface = 0

A
DO 1 I=1, NLayer

DO 2 J=1, 2

Set Load Vector = 0

Set Stiffness Matrix = 0

DO 3 K=1, NTL

IF IT(K)=1 THEN YES Add Soil Element Stiffness

IF IT(K)=2 THEN YES Add Interface Stiffness

IF IT(K)=3 THEN YES Add Reinforcement or Skin Stiffness

3

B
IT(K) = 1

Calculate Soil Stresses and Check Failure

Soil Failure

YES

Solve for Displacements

Add Stress Transfer Loads

Set Load Vector = 0

NO

Modify Soil and Interface Properties

Calculate Total Stresses and Displacement

Calculate New Properties for Soil

Calculate New Interface Shear Stiffness
Print
- Element Stresses
- Nodal Displacement

STOP
By following the steps outlined in Section 4.12, the following calculations show how the redistribution method works. If a vertical strip load of intensity of 100 kN/m is placed at load location II (see Fig. 4.8a), at strip level 3 (i.e., $h = 0.10 \text{ m}$), from Equation 4.2, traction force $T = 292.67 \text{ N}$. The length of strip $L_1$, between potential failure plane and 2 : 1 dispersion line, is 0.0848 m. The length of strip $L$, between potential failure plane and end of strip, is 0.5258 m. The increase in normal pressure due to vertical strip load is 50 kN/m$^2$. The soil normal pressure at this level is 1.72 kN/m$^2$. From Eq. 4.3, the total frictional capacity $F_T$ is calculated to be 215.7 N. By subtracting the soil lateral pressure force (Eq. 4.6) which is equal to 3.46 N, the net frictional capacity $F_N$ (Eq. 4.7) is 212.24 N. The excess force $F_E$ at level 3 will be the difference between $T$ and $F_N$ and is equal to 80.43 kN. By adding together the excess force at levels 1, 2, and 3, the total excess force $F_{ET}$ is equal to 264.46 N. The total excess force is equally distributed to the 7 strips below. So from level 4 to level 10, each level has to take up extra 37.78 N. At level 3, the traction force is limited to be equal to the net frictional capacity which is equal to 212.24 N. The same applies to level 1 and 2. Then the traction force distribution is obtained as shown in Fig. 4.14.
BIBLIOGRAPHY


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The author was born on February 12, 1957 in Hong Kong. In June of 1975, he finished his secondary school education, obtaining a diploma in Mechanical Engineering from Aberdeen Technical School in Hong Kong. In September of 1976, he entered into the Civil Engineering program in the University of Windsor, Windsor, Ontario, Canada. In June of 1980, he graduated from the University of Windsor with a degree of Bachelor of Applied Science in Civil Engineering (Honours with Distinction). In September of the same year, he enrolled in a Master's programme in the Department of Civil Engineering of University of Windsor.