EVALUATION OF HOT-WIRE CALIBRATION AND SIGNAL INTERPRETATION METHODS FOR TURBULENCE MEASUREMENTS.

MANGALAM K. SWAMINATHAN

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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L’AVONS RECEUE
EVALUATION OF HOT-WIRE CALIBRATION AND SIGNAL INTERPRETATION METHODS FOR TURBULENCE MEASUREMENTS

A Dissertation
Submitted to the Faculty of Graduate Studies through the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy at the University of Windsor

by

Mangalam K. Swaminathan

Windsor, Ontario, Canada
1985
For
my parents,
my brother Ravi,
'Sudha',
and
Babloo.
ABSTRACT

The calibration of hot-wires and determination of turbulence quantities using time averaged response equations are considered and studied both numerically and experimentally. The calibration study involves the development of a non-linear optimization technique, a comparison of the various heat transfer and effective velocity models, and a determination of the sample size required for accurate calibration. The conventional method of determining turbulence quantities based on a series expansion of the hot-wire response equation is compared with the alternate method based on squaring the response equation before time averaging. In addition, the effects of measurement errors on turbulence measurements are estimated numerically.

The non-linear technique of treating the raw calibration data is based on the iterative procedure of Gauss-Newton. The numerical experiments were carried out using the Monte Carlo simulation technique. This technique was used to simulate the hot-wire response and hence to determine the sample size required for accurate calibration. The effects of measurement error, the range of velocity, the optimization techniques used and the spacing of data points on the sample size required were studied. Using the same technique, pseudo turbulence data and the corresponding hot-wire responses were generated to serve as
a standard, against which the two signal interpretation schemes were compared to evaluate their relative merits. This study included the estimation of truncation errors in the conventional method, the effect of varying the turbulence intensity and the effect of errors in measurement. The experimental programme involved performing velocity and yaw calibrations in the potential core of the jet and the measurement of turbulence in the fully developed turbulent pipe and jet flows.

The results indicate that the non-linear calibration technique is systematic, accurate and easy to use. The yaw calibration obtained using the non-linear technique resulted in a negative value for the yaw factor ($k^2$). This means that the hot-wire, when yawed, responds to a velocity magnitude less than the component normal to the wire. Comparison of the various heat transfer models revealed that the extended power-law model yields the best compromise between low errors in velocity and low uncertainty in the estimated calibration constants. The calibration studies, using Monte Carlo technique, indicate that the sample size required is in the range of 20-30 for practical purposes. The subrange velocity calibration yields more accurate results. Both the numerical and experimental results indicate that the conventional method of determining turbulence quantities with corrections for high turbulence intensity yields reliable results. The improved method
requires that very accurate measurements be performed and is valid for flows with high mean velocities and turbulence intensity levels. The yaw sensitivity factor has negligible effect on the determination of the Reynolds stress. By accurately determining the sensitivity of the hot-wire, the accuracy of turbulence measurements can be improved.
ACKNOWLEDGMENTS

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The technical help rendered by Mr. R. Tattersal in the construction of the experimental facility is gratefully acknowledged. Thanks are also due to Mr. W. Beck for the maintenance and repairs of the instruments. His expert advice proved invaluable in the understanding of the proper use of the instruments. Thanks are also due to Mrs. B. Carr for her patience in typing part of the manuscript and the technical papers. The excellent services provided by 'Miss Apple' will not be forgotten either. The assistance offered by fellow graduate students, Mr. P. Y. Kokate and Mr. V. M. Fernandez, needs a special mention.

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The author sincerely expresses his deep gratitude to his parents and brother without whose support, sacrifice and encouragement this work would not have been possible. His final expression of gratitude goes to the two 'supervisors', Dr. Lakshmi Sridhar and Mrs. Viji Shridhar, for their voluntary involvement in the work.
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NOMENCLATURE

A calibration constant

A₀ initial estimate of A

Aₐ constant in convection loss term (equation 2.9)

a cross-sectional area of the hot-wire

aₐ coefficient in convection loss term (equation 2.9)

a₁, a₂, ... coefficients as defined in equations B.23 and B.24

B calibration constant

B₀ initial estimate of B

Bₐ constant in convection loss term (equation 2.9)

b yaw sensitivity factor as defined in equation 2.29

b₀ coefficient in convection loss term (equation 2.9)

b₁, b₂, ... coefficients as defined in equations B.25, B.26 and B.27

C calibration constant

C calibration constant (exponent)

C₀ initial estimate of C

Cₚ skin friction

Cₚ specific heat at constant pressure

C.T.A. constant temperature anemometer

D calibration constant

D = 1/c

d diameter of hot-wire

(xiv)
E  instantaneous voltage output of C.T.A.
\bar{E}  mean voltage
E_{lin}  instantaneous voltage of the linearized C.T.A.
E_{\theta}  voltage output of C.T.A. at zero velocity
E_{xy}(\alpha)  instantaneous voltage output of hot-wire in the xy plane at a yaw angle of \alpha
\bar{E}_{xy}(\alpha)  mean value of E_{xy}(\alpha)
\bar{E}^{2}_{xy}(\alpha)  mean square value of E_{xy}(\alpha)
e_{xy}(\alpha)  fluctuating component of E_{xy}(\alpha)
e^{2}_{xy}  mean square value of e_{xy}(\alpha)
f  function of its argument
f'  approximate turbulence correction factor for mean velocity
f_{exact}  exact turbulence correction factor
[G]  matrix
Gr  Grashof number
g  acceleration due to gravity
H  heat transfer coefficient
[H]  matrix
H_{g}  ambient static pressure
h  pitch sensitivity factor
h'  manometer reading
I  current passing through the hot-wire
K  thermal conductivity
K_{1}  universal shape factor as defined in equation 2.22
Kn  Knudsen number

(xv)
\( k \)  yaw sensitivity factor as defined in equation 2.30

\( k_0 \)  initial estimate of \( k \)

\( L \)  length between the static pressure tappings in the pipe

\( \text{L.D.A.} \)  Laser Doppler Anemometer

\( l \)  length of the hot-wire

\( M_a \)  Mach number

\( m \)  number of yaw angle settings

\( m_0 \)  yaw sensitivity factor as defined in equation 2.28

\( N \)  number of data points

\( \text{Nu} \)  Nusselt number

\( n \)  number of velocity settings

\( O(\phantom{0}) \)  represents the order of approximation of an equation

\( P \)  coefficient as defined in equation 3.11

\( \text{Pe} \)  Peclet number

\( \text{Pr} \)  Prandtl number

\( p \)  static pressure

\( Q \)  value of a variable

\( q \)  value of the variable \( X \) (equation 2.35)

\( R \)  radius of the pipe

\( R_0 \)  gas constant

\( R' \)  coefficient as defined in equation 3.11

\( \text{Re} \)  Reynolds number

\( r \)  radial distance across the pipe

\( \text{r.m.s.} \)  root mean square
Sensitivity of linearized hot-wire coefficients as defined in equation 3.11

\( S_1, S_2, \ldots \) sum of the errors squared

\( (\text{SES})_U \) sum of the errors squared in velocity

\( \text{SES}(0) \) SES when the step factor is zero

\( \text{SES}(1/2) \) SES when the step factor is 1/2

\( \text{SES}(1) \) SES when the step factor is 1

T local temperature of the wire

\( T_w \) average temperature of the wire

\( T_\infty \) ambient or free stream temperature

\( T \) temperature difference (\( T-T_\infty \))

t time

U instantaneous velocity component in the x direction

\( \bar{U} \) mean velocity in the x direction

\( U_{BN} \) binormal component of the cooling velocity

\( U_c \) calculated velocity from curve fit

\( U_{eff} \) effective cooling velocity of the hot-wire

\( \overline{U}_{eff} \) mean of the effective cooling velocity

\( U_N \) normal component of the cooling velocity

\( U_m \) mean velocity on the centre line of the jet or pipe

\( U_T \) true or measured velocity

\( U_1 \) velocity as defined in equation B.2

\( U_2 \) velocity as defined in equation B.8

\( U_* \) friction velocity

u fluctuating velocity component in x direction
\( u_{\text{eff}} \) fluctuating component of the cooling velocity

\( \bar{uv} \) turbulent shear stress

\( \bar{u}^2 \) mean square value of \( u \)

\( V \) instantaneous velocity component in the \( y \) direction.

\( \bar{V} \) mean velocity in the \( y \) direction

\( \vec{V} \) instantaneous velocity vector

\( v \) fluctuating velocity component in the \( y \) or \( r \) direction

\( \bar{v}^2 \) mean square value of \( v \)

\( W \) instantaneous velocity component in the \( z \) direction

\( \bar{W} \) mean velocity in the \( z \) direction

\( w \) fluctuating velocity component in the \( z \) direction

\( \bar{w}^2 \) mean square value of \( w \)

\( X \) variable in equation 2.35

\( \bar{X} \) mean of the \( X \) values

\( x_1, x_2, \ldots \) variables defined in equation B.1

\( (x, y, z) \) Cartesian coordinate system

\( \alpha \) yaw angle of the hot-wire with respect to the flow

\( \beta \) constant in equation 2.35

\( \beta_1 \) parameter as defined in equation B.9

\( \gamma \) ratio of specific heats

\( \gamma_0 \) temperature coefficient

\( \gamma^* \) step factor as defined in equation 3.14

\( \gamma^1 \) constant in equation 2.5

\( \delta \) temperature coefficient
\( \Delta(\cdot) \) represents uncertainty in the quantity specified within the brackets.

\( \Delta P \) static pressure difference.

\( \Delta \theta_1, \Delta \theta_2 \) (A - A_o), (B - B_o), (c - c_o) and (k - k_o) respectively.

[\( \Delta \theta \)] matrix.

\( \theta \) inclination of the manometer.

\( \lambda \) mean free path.

\( \mu \) absolute viscosity.

\( \nu \) kinematic viscosity.

\( \pi = 22/7 \).

\( \rho \) density of air.

\( \rho_m \) density of manometer fluid.

\( \rho_l \) local resistance of the wire material.

\( \rho_{la} \) resistance of the wire at ambient temperature.

\( \sigma \) standard deviation in velocity.

\( \overline{\sigma} \) mean of the standard deviation.

\( \phi \) pitch angle of the wire.

\( \chi_w \) thermal conductivity of the wire material.

\( \psi \) local convective heat loss rate per unit length.

**Subscripts**

\( o \) denotes standard turbulence data generated.

\( i, j \) integer variable.

\( I, II \) denotes turbulence quantities obtained by methods I and II respectively.

\( 1, 2, 3 \) denotes hot-wire positions when \( \theta = 0, +45 \) and \( -45^\circ \) respectively.

**Note:** an overbar (\( \overline{\cdot} \)) denotes time or ensemble averaged quantity.
CHAPTER I
INTRODUCTION

1.1 Subject of Investigation

This study deals with hot-wire calibrations and signal interpretation procedures for measurement of turbulent flows. Among the approaches available for solving the calibration problem, the direct method of calibrating hot-wires frequently in a known flow is widely used. This method is based on the use of an appropriate analytical expression to relate the hot-wire output to the relevant variables. The present study will, however, be restricted to the expressions relating the response of the hot-wire to velocity and yaw angles. It is further assumed that the mode of heat transfer from the hot-wire is predominantly forced convection.

Signal interpretation for hot-wires in turbulent flows can be carried out in two different ways. The measurement of instantaneous flow using three hot-wire sensors with an on-line data processing system is one of the methods. The second one is based on time averaging technique and the present study will be restricted to this method. Further, the study will be limited to the use of a single normal and a single inclined wire for measuring turbulence quantities. The hot-wire is to be used in the constant temperature mode.

The turbulent flows to be investigated are approximated to be incompressible, isothermal and
statistically steady. The flows chosen are the fully developed turbulent pipe and jet flows. When the fluid is allowed to enter a long circular pipe from a large container, the velocity distribution across the cross-section initially varies with the distance from the inlet. From an initially 'uniform' profile, the velocity distribution changes continuously due to wall friction until a fully developed velocity profile is attained at some downstream cross-section. The flow velocity profile remains constant thereafter. This length, from the inlet to the station where the fully developed profile is attained, is called the inlet length. The inlet length is primarily a function of Reynolds number. The case of fully developed turbulent pipe flow is well understood and offers the advantage that the turbulent shearing stress at the wall can be directly determined by measuring the pressure drop along the pipe. For these reasons, the developed turbulent pipe flow will be one of the flows chosen for evaluation of hot-wire signal interpretation methods.

The other flow investigated will be an axisymmetric, turbulent, submerged jet. Such a jet is formed when fluid exits from a round tube or nozzle into a space filled with the same fluid. The initial profile is dictated by the length of the tube and/or the nozzle design. For a nozzle, the jet velocity profile is generally 'uniform'. At the nozzle exit the difference in velocity between the jet
stream and the surrounding fluid at rest, produces an intense shear region which grows with axial distance. Initially the fluid near the exit, however, remains unsheared. But as the shear region grows, the unsheared region forms a cone in front of the nozzle called the potential core. In the shear region, the mean kinetic energy is converted into turbulent energy by the shearing action, which is then dissipated through viscous action of the fluid. Downstream of the potential core there exists a transition zone where the turbulent flow continues to evolve. The fully developed region begins where the flow variables such as mean velocity and turbulent intensity become self-preserving. The initial and boundary conditions influence the initial flow development and mixing process. Once the large scale structures have evolved, one can expect the flow to achieve independence of the initial conditions and become self-preserving with finite flow length. The nomenclature for the jet and pipe flows are shown in figures 1.1 and 1.2 respectively.

In this study emphasis will be placed on evaluating various calibration and signal interpretation methods available for turbulence measurement under the restrictions outlined above.

1.2 Relevance of the Study

The hot-wire anemometer has been one of the most reliable instruments for measuring transient flows and in particular the dynamic structures of turbulent flows. This
instrument has found wide applications ranging from the measurement of low velocity flows in a coal-mine ventilation shaft to surface heat measurements of the space shuttle. Competitive non-intrusive techniques for flow measurements such as laser doppler anemometers are, however, being developed. The reliability of the laser doppler anemometer in measuring air flows is far from satisfactory. This is because the L.D.A requires the introduction of tracer particles into the medium to scatter the light. Further, the signal obtained from an L.D.A. is not continuous and hence does not result in a good resolution in time to measure the fine structure of turbulence. The advent of microprocessors and computers have revolutionized the data acquisition, processing and display techniques for the hot-wire signal. The aforementioned limitations of the L.D.A. and the added incentive given by modern computers have given a new impetus to the research in the area of hot-wire anemometry. These new approaches in hot-wire anemometry have provided new and/or supplemental information about flows than could not be obtained by other techniques. Further, the limitations of the hot-wires are not necessarily the same as those of the other measuring devices, therefore considerable attention in hot-wire anemometry is being directed to overcome these limitations and to improve and extend the capabilities of this instrument. Hence, despite the progress being made in the L.D.A., hot-wires will continue to be the most reliable
measuring device, especially in turbulent air flows.

Hot-wire anemometry is based on a simple principle, but the use of it is to be approached cautiously if accurate measurements are to be made in complicated flow situations. The ultimate success in using the hot-wire lies in proper interpretation of its signal, which depends on several parameters. This is especially true when the details of the turbulent structure are to be measured. Accurate calibration of hot-wires under different conditions will be the key to correct interpretation and correction of the hot-wire response irrespective of whether a digital or analog evaluation technique is employed. Further, evaluation of the various signal interpretation procedures for turbulence measurements, would help in accurately measuring turbulence. In the present study the emphasis is placed on the calibration procedures and signal interpretation methods for hot-wires in turbulent flows.

1.3 Aims

The aims of the present study are as follows:
1. To develop a simple accurate and systematic method of treating raw hot-wire calibration data for velocity only and combined velocity and yaw calibrations using the analytical calibration approach. To investigate whether a nonlinear calibration technique would serve the above mentioned purpose.
2. To compare the existing analytical calibration equations on a common basis.
3. To develop accurate means of determining the yaw sensitivity of hot-wires and to verify some of the assumptions being made while determining the yaw factor.

4. To develop a method for estimating the number of calibration data points required to determine accurately the calibration constants of the hot-wire. The effects of the various parameters such as the magnitude of errors in velocity, the range of velocity, the spacing of the data within the range on the number of data points required are also to be taken into account.

5. To study the effect of different calibration procedures and yaw sensitivity factor on the measurement of turbulence using single hot-wires in conjunction with analog signal processing equipment.

6. To compare some of the existing methods for analysing hot-wire signals for turbulence measurements.

1.4 Layout of the Material

The information to be presented in this investigation has been arranged systematically as follows: In Chapter II the literature available on the topic is considered and discussed in detail. The development of the nonlinear calibration technique for the combined velocity and yaw, the Monte Carlo simulation and the response equations for turbulence measurements by two methods are presented in Chapter III. The details of the experimental investigation carried out and the associated uncertainties in measurements are described in Chapter IV. The results of
the experimental and numerical investigations are discussed in Chapter V. Finally, a summary of the major conclusions and recommendations are given in Chapter VI.
CHAPTER II.

LITERATURE SURVEY

2.1 General Remarks

The idea of measuring velocities using heated wires dates back to the beginning of this century. According to King (K2, K3) preliminary experiments were carried out by Shakespear at Birmingham as early as 1902. However, King's work was the milestone for both the design of hot-wire anemometers and the theory of convection of heat from hot-wires. Initially the use of the hot-wire was limited to the measurement of mean velocities. But, the invention of Dryden and Keuthe (D4), who introduced electronic compensation for the thermal lag, marked the beginning of the hot-wire anemometer as an instrument capable of measuring rapid velocity fluctuations. With considerable improvements in the electronics of the feedback amplifier, automatic compensation of thermal lag became possible and hence the constant temperature anemometer came into existence. Electronic means of linearizing hot-wire output allowed measurement in flows of high turbulence levels. Digital evaluation techniques of hot-wire responses have not only revolutionized data acquisition, processing and display techniques but have provided additional or new information that cannot be obtained by conventional techniques. Excellent review articles and books tracing the historical development of hot-wire anemometry, its operation and applications are available (B5, C6, C7, C8, H8, R1, S2).
A considerable amount of work is being devoted to understanding the phenomena of heat transfer from the heated element to the surrounding fluid and the effect of the various parameters on which it depends. The approaches to this problem are generally analytical, numerical, empirical and practical in nature. The following sections are devoted to a critical evaluation of the literature available for each of these approaches. In addition, a survey on signal interpretation methods available for turbulence measurements and the Monte Carlo technique will be discussed.

2.2 Analytical Approaches

General analytical or theoretical solutions to the forced convection problem from heated three dimensional wires are difficult to obtain due to the inherent nonlinear nature of the governing equations. At present there appears to be no exact solutions of flow past bodies of finite size. Further, the Reynolds number range in which the hot-wires usually operate, 0.1 to 1000, makes it impossible to apply the approaches of a slow viscous flow or the boundary layer approximations to obtain approximate analytical solutions valid for most of the operating range. Hence, various authors have attempted analytical solutions by making extremely simplifying assumptions.

The well known solution, due to King (K3), avoided the problem by assuming potential flow normal to an infinite wire. King derived the following solutions relating Nusselt number \( \text{Nu} \) and Peclet number \( \text{Pe} \) for the case of
and small Peclet numbers respectively

\[ \nu \hspace{1cm} \text{Nu} = \frac{1}{\pi} + 2 \sqrt{\frac{\text{Pe}}{\pi}} \]  
\hspace{1cm} (2.1)

and

\[ \text{Nu} = \frac{2}{\log(\text{Pe}^{-1}) + 0.423} \]  
\hspace{1cm} (2.2)

The first of these equations was compared with King's empirical formula

\[ H = A \left( 1 + \gamma_0 \left( T_w - T_\infty \right) \right) + B \left( 1 + \delta \left( T_w - T_\infty \right) \right) \sqrt{Ud} \]  
\hspace{1cm} (2.3)

where \( A = 2.50 \times 10^{-4} \) \( (1 + 35d) \)

\[ B = 1.012 \times 10^{-2} \sqrt{d} \]

d is the diameter of the wire in cm, \( H \) is the heat transfer coefficient in W/cm\(^2\)K, \( T_w \) is the wire temperature in °C, \( T_\infty \) is the ambient temperature in °C and the temperature coefficients \( \gamma_0 = 0.0114 \) and \( \delta = 0.00008 \) when \( T_\infty = 17°C \). This comparison showed that the theoretical solution gave an over estimation of the heat transfer by forty percent. The basic assumptions of the theory are unsatisfactory from a practical point of view and thus there is no help in assessing the relative merits of the experimental results.

Another significant piece of information has been derived by Cole and Roshko (C4) using an Oseen approximation for the heat transfer in the limit as \( \text{Pe} \to 0 \). The authors' own experiments failed to validate their approximate
solution

\[ \frac{2}{\text{Nu}} = \log\left( \frac{8}{\text{Pe}} \right) - 0.577 \quad (2.4) \]

The range of validity of their results was limited to a very narrow region. They attributed this to the effect of the aspect ratio of the wire and errors in the experimental determination of the parameters involved. The theoretical Nusselt number evaluated from equations 2.2 and 2.4 tends to zero in the limit as \( \text{Pe} \to 0 \). In reality, however, due to free convection and end conduction losses there is a lower limit for Nusselt number. This and other reasons mentioned place a severe limitation on the use of these equations as a means of calculating the heat transfer characteristics of a hot-wire and hence the corresponding velocity without having to calibrate the hot-wire.

Illingworth (II) extended the above theory to include a second order term in equation 2.4 of the form

\[ \text{Nu} = \left[ 2(\ln\left( \frac{8}{\text{Pe}} \right) - \gamma') \right]^{-1} - \left[ (\text{Pe}/8)^2 \left( 8 + 0.5 \ln^2\left( \frac{8}{\text{Pe}} \right) \right) \right] \]

(2.5)

A comparison with empirical correlations showed discrepancy of the order of 25 percent or more. His iterative method, valid for low Prandtl number (\( \text{Pr} \)), yielded better solutions than Cole and Roshko's (C4) for \( \text{Re} < 0.3 \), beyond which it diverged rapidly from the experimental values of Collis and Williams (C5). Wood (W2) also treated the low Reynolds number problem for the heat transfer from fine hot-wires using Oseen's approximation in conjunction with an
iterative scheme to obtain higher order approximations. Hieber and Gebhart (H6) extended the matched asymptotic expansion method of Kaplun and Lagerstrom (K1) to obtain an equation for Nusselt number for moderate and large Prandtl number flows. Their theory essentially gave the same solution as Wood's for moderate Prandtl numbers. Absence of experimental data for large Prandtl number precluded verification of the theory.

As mentioned earlier, all the above theories neglect free convection and assume two-dimensional flow (uniform surface temperature or no end conduction losses) which are not justified as Re → 0 and hence the use of these theories is limited. Attempts have been made to study the combined effects of free and forced convection. Mahony (M1) studied the heat transfer characteristics of a hot-wire by including buoyancy and aspect ratio effects into the Navier-Stokes equation. He obtained a reasonably good comparison with the experimental results of Collis and Williams (C5) and concluded that the two-dimensional convection solution applies for

\[ \left( \frac{Gr^{1/2}l}{d} \right) > 1.0 \quad (2.6) \]

The problem of mathematically treating the combined effects of free and forced convection was carried out by Wood (W3). The main quantitative results were extracted by elementary arguments regarding orders of magnitude and the conclusion drawn was that free convection in air is only important when

\[ Re < Gr^{1/3} \quad (2.7) \]
This situation exists for typical hot-wires in slow draughts with velocities of a few cm/sec.

These studies were limited by the fact that they were for two-dimensional flows and end conduction was neglected. Further, inherent in all the above theoretical studies is the fact that the fluid was assumed to be a continuum, which is questionable in the low Peclet number range. Molecular effects thus become significant at low Reynolds numbers and high Mach numbers (Ma). The non-dimensional parameter, Knudsen number ($\lambda/\delta$), determines if the molecular effect will dominate the flow and heat transfer characteristics of the wire. Sauer and Drake (S4) analytically treated the problem of slip flow around two-dimensional hot-wires by making approximations about the velocity field around the wire and incorporating a temperature jump boundary condition to solve the energy equation. The comparison of their predicted heat transfer characteristics with experimental data showed that the trend was well predicted. They attributed the discrepancy to the evaluation of the fluid properties used in the experimental data reduction. No limit on the Knudsen was, however, specified to show if molecular effects became dominant.

Attempts have also been made to theoretically treat the case of yawed cylinders. Jones (J1) treated the case of a frictionless fluid flowing past an infinite, yawed cylinder by considering the effects of the stream components perpendicular and parallel to the axis of the cylinder.
separately and superimposed these effects to obtain the solution. Sears (S6) treated laminar boundary layer flow past an infinite, yawed cylinder. It was shown by him that the streamwise flow was given by the same equations as the flow without yaw about the same cylinder and that the axial flow could be calculated by integration of a second order linear differential equation. These results are, however, not useful for practical hot-wire anemometry.

In all the theoretical analysis an infinitely long cylinder was assumed, however, practical hot-wires have a finite length to diameter ratio. As a result, considerable heat loss to the supports occur by conduction and this influences the Nusselt number evaluation. King (K3) was the first to use approximate analytical solutions for the temperature distribution to evaluate the conductive heat loss to the supports. His analytical solution was limited because of the oversimplification of the problem and did not match all the required boundary conditions. Though, solutions are available for flow past a two-dimensional cylinder using Oseen's linearization of the momentum equations, no attempts have been made to use this information and an assumed temperature distribution to obtain results. This is, probably, due to the limited validity of the Oseen approximation near the wire surface, i.e., Re = 1 and the assumed flow being two-dimensional. Betchov (B3) extended King's work by considering a convective loss term which is nonlinear in the
wire to air temperature difference.

A detailed comparison between the theoretical and experimental results of heat transfer from a hot-wire have been carried out by several authors (B4, C5). The variation of Nu with fluid, flow and geometric parameters has been found to be similar in trend but the values are considerably different. Hence, the use of theoretical calibration factors instead of calibrating hot-wires directly will lead to considerable errors even for the simplest case. In addition, experimental determination of the hot-wire geometry, fluid and flow properties required for the evaluation of the theoretical results is an arduous task and considerable errors can occur. Theoretical analysis is therefore of considerable value insofar as determining the important parameters for different flow conditions, predicting trends and providing useful guidelines for other methods of obtaining the heat transfer characteristics of the hot-wire.

The limited scope of the theoretical approach is due to the complicated form of the governing equation which represents the heat transfer from a fine heated wire kept in three-dimensional flow. Further, many of the simplifications developed for the fluid flows are inapplicable over the range of interest for hot-wires. Further, as the analysis is based on the condition of thermal equilibrium in hot-wires, the validity of this assumption needs to be taken into account when investigating turbulent flows. For practical hot-wire anemometry three-dimensional effects, conduction to
the supports, interaction between free and forced convection, compressibility effects at high Mach numbers, etc., are of considerable importance and a theoretical treatment which includes all these effects is not possible with the available mathematical tools. As a consequence of this impasse, attempts have been made to find solutions using numerical and experimental methods to arrive at heat transfer laws for hot-wires.

2.3 Numerical Approaches

Numerical techniques for solving the convection heat transfer problem from hot-wires have been applied with limited success. The numerical solution of even the approximate Navier-Stokes and energy equations given below for the case of \( \frac{1}{d} = 600 \) is involved due to the highly nonlinear and coupled forms of these equations.

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right)
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right)
\]

\[
\frac{\partial W}{\partial t} + U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} = \nu \left( \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right)
\]

\[
\frac{\partial T'}{\partial t} + U \frac{\partial T'}{\partial x} + V \frac{\partial T'}{\partial y} = \frac{K}{\rho c_p} \left( \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} \right)
\]
No further simplification of the above equations can be justified for the problem on hand. This is due to the fact that the usual approximations of boundary layer valid for high Re or the creeping motion valid for low Re are not applicable for the range of interest of Re (10 to 1000). To the author's knowledge, no numerical work has been reported for the case of interest.

The few numerical works that have been reported determine the temperature distribution along the wire for the case of steady state heat loss with the idea of obtaining the end conduction losses (Cl, D2). The differential equation governing the wire temperature distribution for a wire in a uniform steady flow is given by

\[ a \frac{d}{dz} \left( \chi_w(T') \frac{dT'}{dz} \right) + Nu I^2 \rho_1(T') - \psi(U, T') = 0 \]  

(2.9)

where \( a = \pi d^2/4 \), the cross-sectional area of the wire, 
\( T' = T - T_\infty \),
\( \chi_w \) = the thermal conductivity of the wire material
\( I \) = current through the wire,
\( \rho_1 \) = local resistance of the wire per unit length
and \( \psi \) = local convective heat loss per unit length.

The assumptions involved in the above equation are (i) negligible radiation heat loss (ii) negligible radial temperature gradient and (iii) \( l/d \) is large so that the local convective heat loss depends only on the local wire to air temperatures. Davies and Fisher (D2) obtained a
numerical solution to the above equation by replacing \( \psi(U,T') \) by \( \pi \partial \hat{H}(T_w - T_m) \). Results were obtained using two different procedures involving the basic approximation to \( H \) in conjunction with a trial and error approach until the solutions matched the boundary conditions. The temperature distribution obtained could not be compared with experimental results as none were available at that time. Subsequently, Champagne (Cl) also numerically solved equation 2.9 by replacing the term \( \psi(U,T') \) by \( \rho_1 a'(1 + a_o T') + B_o(1 + b_o T') U^{0.45} T' \), i.e., using the empirical heat loss equation of Collis and Williams (C5). The nonlinear second order equation was split into two first order differential equations, which were numerically integrated using a fourth order Runge-Kutta scheme. Again a trial and error procedure was used since an initial guess of the temperature gradient had to be made. The numerical procedure diverged for the case of \( l/d=400 \). Hence, a new variable was defined and a solution obtained using a sixteenth order power series expansion. Comparison of the numerical results with his own measurements of temperature profile showed that the trend was well predicted but the temperatures were overestimated. The values, especially at the wire ends, were in considerable error.

The numerical approach has been hampered by the fact that experimental information on the thermal properties of materials, such as the variation of the thermal conductivity with temperature, resistivity, etc., is
required to initiate the solution process. Other major effects such as the prong interference, yaw effects due to inclination of the wire, etc., have to be incorporated in the analysis. These would make the task more difficult. With the present state of art in numerical techniques it may be possible to obtain useful results for the heat transfer from hot-wires.

2.4 **Empirical Heat Transfer Laws**

As a consequence of the limitations of the theoretical and numerical approaches, for practical applications to hot-wire anemometers, a considerable variety and number of experimental investigations covering the heat transfer from heated cylinders have been carried out. The studies lead to the recognition of important parameters which govern the heat transfer from a hot-wire. For a complete specification of heat transfer from fine hot-wires, a relation of the following type is sought between the various parameters influencing the heat transfer

\[
Nu = f(Re, Gr, Pr, Ma, Kn, \alpha, T_u/T_w, \text{probe geometry})
\]  (2.15)

The formulation of such a relation from experiments is impractical, and even if available would be unwieldy and inaccurate for specific practical application. Hence, the approach has been to obtain heat transfer correlations under limited operating conditions such as continuum flow, and forced convection so that the effects of certain parameters
can be neglected or held constant. McAdams (M3) was the first to collate the early heat transfer measurements of various authors for forced convection from cylinders set normal to the flow for the incompressible case. A wide range of Re, wire diameter, temperature ratio and atmospheric pressure was covered. He recommended a correlation for a wider Reynolds number range of the form

\[ \text{Nu} = A + B(\text{Re})^C \]  \hspace{1cm} (2.11)

where the coefficients A, B and C are given by

<table>
<thead>
<tr>
<th>Range</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 - 1000</td>
<td>0.32</td>
<td>0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>1000 - 50,000</td>
<td>0.0</td>
<td>0.24</td>
<td>0.60</td>
</tr>
</tbody>
</table>

The properties of the fluid, \( \mu \) and \( K \), were evaluated at the film temperature while \( \rho \) was evaluated at the free stream temperature in order to determine Re and Nu. The correlation showed considerable scatter especially at high Reynolds numbers. For measurement of the details of the flow, such as turbulence, McAdams correlation would lead to inaccurate results. It was thought that a more careful investigation covering a restricted range of the variables would yield a satisfactory correlation of the heat transfer which would allow more accurate measurement with the hot-wire. Collis and Williams (C5) carried out an experimental investigation to determine a heat transfer correlation for the Reynolds number range of 0.01 - 140, i.e., covering both free and forced convection ranges. They used a range of high aspect
ratio wires to reduce three dimensional effects ( \( 1/d > 2000 \) ) and end conduction losses. Based on their experiments, the \( \text{Nu} \) was correlated to both \( \text{Re} \) and the temperature loading factor ( \( \frac{T_w}{T_\infty} \) ) by an expression of the form

\[
\text{Nu} \left( \frac{T_w}{T_\infty} \right)^{-0.17} = A + B \text{Re}^C \quad (2.12')
\]

The values of \( A, B \) and \( C \) depended on whether the \( \text{Re} \) was above or below the \( \text{Re} \) for the onset of vortex shedding from cylinders ( \( \text{Re} = 44 \) ). The values recommended were

<table>
<thead>
<tr>
<th>( \text{Re} )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 - 44</td>
<td>0.24</td>
<td>0.56</td>
<td>0.45</td>
</tr>
<tr>
<td>44 - 140</td>
<td>0.00</td>
<td>0.48</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Unlike McAdam's correlation, all the fluid properties were evaluated at the film temperature. It was also concluded that free convection effects could be neglected provided \( \text{Re} > \text{Gr}^{1/3} \). Laurence and Sandborn (L4) also gave a summary of existing data specifically related to hot-wire anemometry in air flows.

The above two correlations, equations 2.11 and 2.12, were obtained by measurements on heated circular cylinders in air and hence in the correlations the Prandtl number has been left out. In general, for forced convection alone

\[
\text{Nu} = f(\text{Re, Pr}) \quad (2.13)
\]

Kramers (K11) compiled the results of many authors who had
carried out experiments with liquids such as water, paraffin oil, transformer oil, etc., with a Prandtl number range of 5.7 to 1240. He suggested the following correlation

\[ \text{Nu} = 0.42 \text{Pr}^{0.2} + 0.57 \text{Pr}^{0.33} \text{Re}^{0.5} \]  \hspace{1cm} (2.14)

valid up to \( \text{Re} = 10^4 \). At higher values of Reynolds number it predicted values of Nusselt number which were too low. For the case of air (\( \text{Pr} = 0.71 \)), the above correlation reduces to

\[ \text{Nu} = 0.39 + 0.51 \text{Re}^{0.5} \]  \hspace{1cm} (2.15)

Comparison of equation 2.15 with equation 2.12 shows significant discrepancy in the prediction of \( \text{Nu} \). Further, equation 2.12 unsatisfactorily compares with the results of McAdams for Reynolds numbers above \( 10^4 \). Hence, van der Hegge Zijnen (V1) modified equation 2.14 by adding a single linear term in \( \text{Re} \) as follows

\[ \text{Nu} = 0.38 \text{Pr}^{0.2} + (0.56 \text{Re}^{0.5} + 0.001 \text{Re}) \text{Pr}^{1/3} \]  \hspace{1cm} (2.16)

which is valid in the range 0.01 \( \leq \text{Re} \leq 500000 \). Davies and Fisher (D2) developed a simple method for obtaining the heat transfer from an electrically heated cylinder by assuming Reynolds analogy to be valid. Their idea was to use the vast amount of accurate theoretical and experimental information available for the momentum transfer of a two-dimensional cylinder set normal to the flow. Evaluating the thermal conductivity \( K \) at the wire temperature and the viscosity and density at the ambient temperature they arrived at the
following heat transfer relationship

\[ \text{Nu} = C_f \left( \text{Re Pr} / \pi \gamma \right) \]  \hspace{1cm} (2.17)

where

- \( \gamma \) = the ratio of specific heats
- \( C_f \) = skin friction
  - \( = 2.6 \text{Re}^{-2/3} \) for \( 0 < \text{Re} < 50 \)
  - \( = 1.4 \text{Re}^{-1/2} \) for \( 40 < \text{Re} < 1000 \)

They demonstrated good agreement between their model and experiments and attributed the large scatter in McAdams (M3) correlation to the fact that \( k \) was evaluated at the film temperature and not the wire temperature. They also argued that this practice leads to the necessity of including the temperature loading factor as was done by Collis and Williams (C5). However, they also reported that the principle source of experimental error was in the surface temperature of the wire which led to a larger scatter in the Nusselt number values calculated by different authors. Another important observation is the value of the power of \( \text{Re} \) in the equation 2.17 which is 1/3 for the range \( 0 < \text{Re} < 50 \) as against Kings' value of 1/2 and 0.45 suggested by Collis and Williams. Davies and Fisher (D2) point out that their proposed model for low heat transfer rates breaks down in the region of a stagnation point, flow separation or reattachment, but dismisses these effects as not being appreciable. By measuring the end conduction losses and performing heat transfer experiments in a vacuum, they were able to determine the radiation loss. Based on these
measurements they concluded an expression of the form \( \text{Nu} = A + B \text{Re}^C \) is not valid since the experimental value of \( A \) far exceeds the sum of the end conduction, bouyancy and radiation losses.

Comparing the heat transfer laws suggested by Collis and Williams and Davies and Fisher, equations 2.12 and 2.17 respectively, it can be seen that they are very different in form. Bradbury and Castro (B4) investigated this discrepancy and supported the relationship of the form of equation 2.12 as it was the more representative of the two for a wide range of wire temperature even though this may not be the best compromise for an empirical heat transfer law. Halton et al (H2) fitted their experimental points to an equation of the form used by Collis and Williams (C5)

\[
\text{Nu} \left( \frac{T_f}{T_\infty} \right)^{-0.154} = 0.384 + 0.581 \text{Re}^{0.439} \quad (2.18)
\]

They reported that their data for forced convection agreed best with the correlation of Hilpert (H7) who had put forth the following correlation

\[
\text{Nu} = A \left( \frac{\text{Re}}{T_f/T_\infty} \right)^{0.25} \quad (2.19)
\]

where

<table>
<thead>
<tr>
<th>Re</th>
<th>A</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 4</td>
<td>0.891</td>
<td>0.33</td>
</tr>
<tr>
<td>4 - 40</td>
<td>0.821</td>
<td>0.385</td>
</tr>
<tr>
<td>40 - 4000</td>
<td>0.615</td>
<td>0.466</td>
</tr>
</tbody>
</table>

Heat transfer equations have also been suggested by other
authors such as Grant and Kronauer (G3), Bearman (B2) and Koch and Gartshore (K9). In all their works the experimental data was fitted to the form

\[ \text{Nu} = A + B \text{Re}^c \]  \hspace{1cm} (2.20)

with the values of A, B and c appropriately evaluated to account for variation in velocity, temperature loading, etc. By far the most comprehensive review article on convective heat transfer from cylinders is by Morgan (M6). In this review the various correlations available in the literature were compared. It was found that the Nusselt number for the experimental data showed a discrepancy anywhere between 10% to 40% depending on the Reynolds number. Apart from the effect of aspect ratio, other possible reasons for this large discrepancy were the wake and blockage effects in the wind tunnel, turbulence in the flow and the temperature loading. The author's proposed correlation, which takes account of all known errors, has a maximum uncertainty of ± 5%. The author, however, recommends more experimental work to study these effects separately and to quantify them.

From the above, even for the simple case of forced convection from cylinders, we see that the correlation between \( \text{Nu} \) and \( \text{Re} \) depends strongly on several factors such as the reference temperature, the temperature loading factor, the Reynolds number range, etc. If one were to however, come up with an empirical form which includes
effects such as compressibility (Ma), non-continuum flow (Kn) and wire inclination, the problem would become more complicated. The work of Bruun (B7, B8) has clearly shown that no accurate and general heat transfer relationship for forced convection can be derived from these experimental data. This explains why attempts to collapse the experimental data have been futile despite correcting the data for end conduction heat loss, radiation, wake blockage corrections for experiments carried out in wind tunnels, aerodynamic interference, etc.

For practical hot-wire anemometry there is little use for these empirical relations. This is because the use requires an accurate value of the wire operating temperature to evaluate the flow properties. Further, working with the non-dimensional forms of Nu and Re is cumbersome. The use of these correlations leads to a considerable loss of accuracy in turbulence measurements. This factor limits the usefulness of the empirical approach. Such studies are, however, useful when the hot-wire cannot be calibrated under the same conditions of temperature and pressure as the flow to be studied. This is particularly true when measurements have to be made for combustion studies which involve a variety of gaseous mixtures and a wide range of pressure. Under such conditions it is advantageous to have the greatest possible generality in calibration as these relations afford. Of course, the analytical expressions used in practical anemometry are based on these correlations.
2.5 Practical Hot-Wire Anemometry

For the reasons mentioned in the previous section, the expressions in which physical variables appear are preferred in practical hot-wire anemometry. Hence, the output from the anemometer, i.e., the unbalance voltage \( E \) is related directly to the instantaneous velocity \( V \), the temperature of the wire \( T_w \), the ambient temperature \( T_\infty \) and the density. Restricting the analysis to the commonly encountered case of isothermal, incompressible flows, the relation can be represented as

\[
E = f(V, T_w/T_\infty)
\]  

(2.21)

In addition, the above relationship is influenced by both the geometry of the hot-wire and the anemometer design. The calibration generally consists of simultaneously obtaining a series of hot-wire outputs and the velocity vector for a fixed value of the temperature and overheat ratio. This raw calibration data contains uncertainty and they are not evenly spaced, hence, there is a need to apply a smoothing process to make the data usable. Several techniques are in use depending on the accuracies required, the type of measurement being planned, i.e., analog or digital measurement. The available techniques will now be discussed.

1. The first method to be described is due to Cheesewright (C3). In this method, the raw data \( (E_i, V_i) \) is transformed into a smooth look-up table with the data being equi-spaced in the \( E \) direction. Consequently, it is easy to transform an acquired \( E \) value into the corresponding velocity value in
just one operation. This approach is of practical importance when handling large amounts of data using digital computers. The method, however, requires an accurate initial calibration and becomes cumbersome when yaw and pitch effects are to be included. The process of generating the smooth look-up table is difficult and considerable error can creep in at this stage. The most important point is that any drift in the wire calibration due to ageing, oxidation of the probe or drift in atmospheric conditions requires that the whole calibration procedure be repeated.

2. To overcome the drawbacks of the above method Bruun (B10) introduced the concept of universal shape function. For the case of probes operated at the same overheat ratio, Bruun expressed the relation between $E$ and $U$, by the universal calibration function

$$E^2 - E_o^2 = K_1 f(U)$$

(2.22)

where $K_1$ is an individual constant for each probe, $E_o$ is the voltage at zero velocity and $f(U)$ is the universal shape function for all standard probes operated at the same overheat. The function depends on the probe type and the support orientation once the hot-wire anemometer equipment has been standardized. This method also requires that $f(U)$ be determined from an accurate initial calibration as described by Bruun. Once the function is known it only requires measurements of $E_o$ and one calibration point $(E, U)$ to determine $K_1$. Hence, subsequent calibrations can easily be carried out. The disadvantage of this method is also that
accurate initial calibration is required. The calibration becomes complicated if yaw and pitch effects are to be included. Further, the function shows a slight velocity dependence.

3. The most frequently used calibration procedure is to smooth the raw calibration data by fitting, generally by the least squares method, a suitable analytical expression (Bruun and Tropea [B9]). The main advantage is that the calibration can be performed readily any number of times. The availability of minicomputers and automatic data acquisition equipment affords efficient performance of such tasks. In practice, each investigator prefers to calibrate the wire as often as possible to accurately interpret the signal and to detect drifts in calibration. This procedure, though seemingly simple, needs to be carried out with caution if accurate results are required. The use of this method is, however, restricted to the simple cases of velocity and yaw calibrations which can be readily performed without much difficulty. Part of the emphasis of the present study will be on the use of analytical expressions for the purpose of calibration. The study will be restricted to the case of incompressible, isothermal flows with the hot-wire operated at constant overheat ratio. Further, only the cases of velocity and yaw calibrations will be considered. A detailed survey of the literature in these areas follows.

2.5.1 Hot-wire Velocity Calibration

In velocity calibration the free stream velocity is
normal to the wire axis. The velocity calibration usually consists of simultaneous measurements of the velocity \( U \) and the anemometer voltage \( E \), and the determination of the calibration constants from an assumed voltage-velocity relationship using a suitable optimization technique. The most common procedure to obtain a voltage-velocity relationship is to use one of the empirical relationships given in the previous section and to replace the \( N_u \) and \( Re \) with the respective expressions containing \( E \) and \( U \). Since the true response of the hot-wire is not known these expressions are only approximate. This together with the fact that the data contains measurement errors in both \( E \) and \( U \), requires that more attention be given to the process of choosing the appropriate heat transfer law and the method of curve fitting if good accuracy is to be obtained.

King (K2) was the first to obtain an analytical expression which governs the cooling of the electrically heated hot-wire based on a theoretical investigation of idealized flow around an infinitely long wire. This law can be expressed as

\[
E^2 = A + BU^{0.5}
\]

(2.23)

where \( A \) and \( B \) are constants to be determined from calibration. Davies and Bruun (D1), based on their experiments, concluded that the exponent of \( U \) was a strong function of velocity and hence the values of \( A \) and \( B \) found for the low velocity range were not appropriate for the high velocity range. They suggested the use of the exponent power
law

\[ E^2 = A + BU^C \]  \hspace{1cm} (2.24)  

following the work of Collins and Williams (C5). This equation, though seemingly simple, considerably increases the computational efforts. This is because the coefficient \( c \) occurs non-linearly. A trial and error approach was adopted to determine all the constants. Further, the above equation is restricted to a limited velocity range (Bruun (B7)). In order to extend the range of validity an additional term was added to King's law. Following the work of van der Hegge Zijnen (V1), Siddall and Davies (S7) suggested the form

\[ E^2 = A + BU^{0.5} + CU \]  \hspace{1cm} (2.25)  

and concluded that this equation was especially suited for representing calibration data over a wide velocity range and was less complicated to fit than the exponent power law. The main drawback of using the above equation is that it is a time consuming process to calculate the velocities from the measured voltages. Richardson (R2) suggested a more complicated form for an extended power law, which is

\[ E^2 = A + BU^{1/2} + CU^{2/3} \]  \hspace{1cm} (2.26)  

Due to the disadvantages mentioned in the use of any of the above forms, George et al (G2) introduced the polynomial heat transfer law

\[ U = A + BE + CE^2 + DE^3 + \ldots \]  \hspace{1cm} (2.27)  

The principal advantages of using the above law are twofold. Firstly, a linear least squares technique can be applied
since the coefficients occur linearly. Secondly, determination of velocity involves only recursive multiplication. It should be borne in mind that a higher order polynomial series will produce a curve which may follow the measured points closely but deviate considerably from the true calibration curve at the off measured points.

The next step in the calibration procedure is to determine the calibration constants from the raw data \( E_i, U_i \) using a suitable optimization technique. The use of equation 2.23 involves a regression of \( E^2 \) on \( U^{0.5} \) instead of \( E \) on \( U \) in order that linear least squares can be used. The use of the power law expression is complicated. Determination of \( A \) is not as simple as measuring the hot-wire output at zero velocity as natural convection dominates the heat transfer and affects the lower limit of the range of applicability of the calibration equation. These have been discussed in some detail by Gebhart and Pera (G1).

Therefore, the conventional method of determining these constants has been to convert the raw data obtained, by plotting \( E^2 \) against \( U^c \) for various values of \( c \). The transformed data are then either graphically analysed by determining the line of best fit or a least squares technique is employed to fit a straight line through the points (Perry and Morrison (P2)). In this latter method it is usual to plot the sum of errors in \( E^2 \) against \( c \). The constants corresponding to the minimum value of the sum of the errors squared are then chosen for use in the
calibration expression. The criticism against the conventional methods mentioned above can be divided into three categories as follows. Firstly, the method deals with \( E^2 \) and \( U^{0.5} \) or \( U^C \) instead of the measured values of \( E \) and \( U \) directly, and hence there is a reason to believe that the calibration can be improved. Secondly, the variable selected for minimization of the sum of the errors squared, i.e., the criterion for least square fit is in \( E^2 \) rather than in \( U \). The third point is that the method is based on a trial and error procedure of varying \( c \) to obtain the required constants.

It is only appropriate that the sum of the errors squared in \( U \) be minimized in view of the fact that the purpose of the hot-wire is to measure the velocity of the fluid. This is true, also, because the largest uncertainties in measurement are usually in \( U \) and not in \( E \). This concept has been used by Larsen and Busch (L2) to calibrate the hot-wire for atmospheric measurements. The use of equation 2.25 or 2.27 involves using polynomial curvefitting, however, the criterion for minimizing the errors is in \( E^2 \) for equation 2.25 and in \( U \) for equation 2.27.

It can now be seen that the problem of velocity calibration needs more attention so that the correct choice of heat transfer law, the suitable curvefitting technique and the proper least square criterion is made. This additional effort is required since careful measurement of turbulence requires that not only the mean velocity be
accurately measured from the calibration curve but also the value of the slope ($\frac{\partial E}{\partial U}$) which is the sensitivity of the hot-wire to velocity perturbation. Another observation which has a bearing on the calibration method to be used is the fact that these constants do not produce the desired linear output when tested on a standard linearizer. There is always a need for trial and error adjustment of the linearizer controls to obtain an output to the desired accuracy. Apart from the inherent limitations, it is believed that the performance of the linearizer can be enhanced by the use of a better calibration procedure.

The availability of various heat transfer laws for the hot-wire makes it difficult for the experimenter to choose the appropriate one for his purpose. Some authors have, therefore, tried to evaluate the relative performances of these analytical expressions. Siddal and Davies (S7) compared the performance of King's law and the extended power law in the high velocity range of 0 to 160 m/s. Bruun and Tropea (B9) compared the performance of some of these expressions in the low velocity range of 2 to 40 m/s. It can be seen that these investigations are incomplete and inconclusive in some aspects. Firstly, Siddal and Davies compared two of the models only in the high velocity range. Further, the criterion for minimizing the sum of the errors squared was in $E^2$. Though Bruun and Tropea stipulated the criterion in $U$, their comparison was restricted to the low velocity range. None of the above mentioned authors have
evaluated the performance of the polynomial heat transfer law. In view of these arguments and the fact that these investigations did not include an estimate of the uncertainties in the parameters evaluated, it is evident that there is a need to compare these laws systematically in order to draw valid conclusions. Further, when comparisons are made between models based on the results of curve fitting, a statistical approach must be taken to verify whether the differences, if any, are significant.

2.5.2 Yaw Calibration

Accurate measurement of the turbulent flow field using a yawed hot-wire requires that the directional sensitivity of the hot-wire be known precisely. Essentially, there are two approaches to determine the yaw sensitivity which is related to the slope evaluation. The first approach is to use a static calibration and the other is a dynamic calibration technique. The dynamic calibration method developed by Perry and Morrison (P2) involves shaking the wire at low frequencies in a uniform flow. This technique was further investigated by Kirchhoff and Safarik (K5). They proved that the dynamic calibration technique is more accurate and consistent than the conventional static calibration procedure. Here, uncertainties related to slope evaluations are avoided since the sensitivity is obtained directly from the measured velocity and voltage fluctuations. The dynamic calibration method, however, requires a sophisticated experimental set-up. Further, Kinns
(K4) has shown that the sensitivity of a yawed hot-wire can be accurately computed even from coarsely spaced static calibration data provided certain criteria recommended by him are observed. In this study only the static calibration method of obtaining the yaw sensitivity will be considered.

The determination of the yaw sensitivity by static calibration can in turn be carried out in two ways. One method is to obtain it by direct differentiation of the anemometer output for independent changes in the two velocity components, i.e., by independently changing the velocities U and v. This method requires exceptional skill on the part of the experimenter and the performance of the calibration equipment. It is also much less accurate. The second method, the one more frequently used and the subject of investigation in this study, is to calibrate for changes in the hot-wire output with U alone and to deduce the transverse sensitivities using the wire inclination and an assumed law for the deviation from the cosine law of cooling or to prescribe an analytical expression for the effective cooling velocity. This procedure involves less calibration work even though it requires that the inclination be accurately measured. It has to be carefully carried out if good accuracy is to be obtained. There is, however, an inherent inaccuracy involved with the assumption of an effective cooling law. One is faced with the problem of choosing the appropriate analytical expression for the effective cooling velocity and determining the constants
involved using only the measured data points obtained from a calibration of the hot-wire in a known flow field.

For a hot-wire inclined normal to the free stream velocity vector, well known studies indicate that the amount of heat transferred from the wire is determined by the stream velocity. Similar conclusive results for a yawed hot-wire are not yet available. The work of Sears (S6), for laminar flows past an infinitely long cylinder, indicates that the effect of the normal component of velocity on heat transfer and that of the tangential component can be considered separately. These results were directly used for a finite wire at an angle of yaw by assuming that the normal component is the effective velocity which determines the rate of heat loss from the wire. Experiments designed to test the validity of this cosine-law were conducted by several authors as reported in reference (S3). The results indicate that this law is applicable at least up to a yaw angle equal to 30°. At greater yaw angles deviations from the cosine-law have been reported and various alternate expressions have been suggested with the idea of accounting for the deviation. Newman and Leary (N1) and Friehe and Schwartz (F1) suggested the use of the following expressions for the effective velocity respectively.

\[ U_{\text{eff}} = U \cos^m \alpha \] (2.28)

\[ U_{\text{eff}} = U (1-b(1-\cos^{1/2} \alpha))^2 \] (2.29)
Hinze (H8), Webster (W1), Champagne (C1) and Jorgensen (J2) have recommended the use of

$$u_{\text{eff}}^2 = u^2 ( \cos^2 \alpha + k^2 \sin^2 \alpha ) \quad (2.30)$$

Here $m_o$, $b$ and $k$ are the yaw sensitivity factors of the hot-wire. The expression given by equation 2.30 has been widely used. Webster was the first to show that the contribution of the term $k^2 \sin^2 \alpha$ was non-zero. Champagne, with the help of careful heat transfer experiments at subsonic speeds, showed that his data correlated well with the above expression. He also concluded that the factor $k$ was primarily a function of the $l/d$ ratio of the wire. A more complicated expression has been suggested by Sanborn and Laurence (S3). However, they have indicated a need for a better equation.

Depending upon the type of hot-wire, several approaches are available for obtaining the yaw factor by this method. All these methods are based on the use of the modified form of King's law, given in the following equation, together with one of the above expressions for the effective velocity.

$$E^2 = A + B u_{\text{eff}}^c \quad (2.31)$$

The most common way of obtaining these constants is to perform a velocity calibration at zero yaw angle to obtain the constants $A$, $B$ and $c$. A yaw calibration is then performed over the required range at the selected velocities. The hot-wire outputs are converted to effective
velocities using equation 2.31 by assuming that the constants A, B and c are independent of the range of yaw angle chosen. Then, the use of one of the expressions for effective velocity determines the yaw sensitivity factors \( m_o \), \( b \) or \( k \), at each yaw angle and the stream velocity. Some of the recommendations put forth by various authors are as follows. Hinze (H8) recommends a value for \( k \) between 0.1 to 0.3 depending on the magnitude of velocity. The value of \( k \) increases as velocity decreases. Webster (W1) obtained a value of \( k \) at each velocity in a least square sense. From the plot of \( k \) versus velocity he suggested a value of 0.2 for \( k \) in spite of considerable scatter in the plot. Jorgensen (J2) conducted an experimental study to obtain the overall directional characteristics of the hot-wire. He recommends the use of equation 2.30 in the velocity range of 10-30 m/s and the value of \( k \) obtained at \( \alpha = 90^\circ \) over the full range of yaw angle. By far, the most extensive study on calibration methods for normal, inclined and X-array hot-wires has been reported by Bruun and Tropea (B9). The authors carried out yaw calibration by two methods, each requiring different assumptions to obtain the constants. The first method essentially follows the approach adopted by all the previous authors. In the second method, to investigate the validity of the assumption that the parameters A, B and c were independent of the yaw angle, the authors carried out a velocity calibration for each value of yaw angle covering an angle range of 0-90°. A least square fit then gave A, B
and \( c \) as functions of the yaw angle. These authors used all the three effective velocity models. Their study conclusively proved that the assumption of constant \( A', B \) and \( c \) with \( \alpha \) is not justified. Secondly, they report a negative value for \( k^2 \) as a consequence of relaxing the above assumption. Based on their results, Bruun and Tropea indicated the need for a better calibration and interpretation methods for hot-wire signals.

The combined velocity and yaw calibration of hot-wires has posed a formidable problem due to the highly nonlinear forms of the response equation. Hence, the problem of obtaining the optimized values of the constants \( A, B, c \) and \( k \) simultaneously in a least square sense over the required range of velocity and yaw angle poses severe numerical problems. These have prompted research workers to make simplifying assumptions and adopt various procedures to obtain the constants. In so doing, a certain amount of error and uncertainty was tolerated for lack of a simple, accurate and easy alternative. Furthermore, the procedure is time consuming when one has to calibrate two and three sensor probes. Such calibration procedures yield reliable results when the mean flow direction is known and the turbulence intensities are between 10 to 15 per cent. In an unknown flow field of high turbulence intensity, these methods can yield large errors in both the mean velocities and in the turbulence quantities (B11). Typical examples of such flows are three dimensional boundary layers and turbomachinery.
flows (L1). Hence, the need for better calibration methods cannot be overemphasized. Further, there is a need to compare the various models available for the effective cooling velocity on a common basis to determine their relative merits.

Having made a survey of the literature in the area of practical anemometry, some general comments are in order regarding the present state of calibration methods for both velocity and yaw. One is faced with several decisions even at the early stage of obtaining the raw calibration data for the hot-wire. The ultimate accuracy obtainable from the resulting calibration curve of the hot-wire is dictated by the proper decision at this stage. One of the important decisions to be made is with regard to the sample size or the number of calibration points to be taken so that the result is a true representation of the population and yields the required accuracy. The sample size, in turn, could depend on several factors such as the accuracies of the measuring instruments used during calibration, the range of velocity over which the hot-wire is to be calibrated, the spacing of the data points over the range and the analytical expression to be used. To the knowledge of the author, no work has been reported in the literature which addresses these important questions and provides useful guidelines to hot-wire users so that the overall performance of the hot-wire may be enhanced. The procedure adopted thus far seems to be ad hoc and generally no mention of the sample size
required for calibration is made in the literature. A study in this direction would prove beneficial and would ensure that accurate and consistent measurements are made using hot-wires. It can be easily seen that it is not, however, feasible to carry out an experimental investigation to provide this information. This is because an enormous number of calibrations would have to be carried out if valid conclusions are to be drawn. Factors such as the number of data points, the spacing and the standard deviations of the errors are not always easily controlled in the laboratory.

There is a need to compare the relative performances of the various analytical expressions for velocity and yaw calibrations on a common and sound basis. Further, there is a need to determine if factors such as linear or nonlinear optimization techniques or the criterion chosen for minimizing the sum of the errors squared in U or E affect calibration. All the existing methods seem to involve a trial and error approach. Hence, if a simple, fast, accurate and direct method can be developed, it would reduce the computational effort. In the case of yaw calibration, the assumptions made in deducing the yaw factor need to be verified. This would help to explain the negative value obtained by Bruun and Tropea (B9) for the yaw factor. There has been no study to access the effect that the yaw parameter, obtained by different methods, has on turbulence measurement. Finally, studies reported have not considered an error analysis of their results. Conclusions cannot be
made with certainty without such an analysis.

2.6 Signal Interpretation Methods for Turbulence Measurements

The basic problem in using a hot-wire anemometer for turbulence measurements is in developing the response equation relating the measured hot-wire voltage to the velocity components. The common approach to this problem has been to use the empirical heat transfer law put forth by King (K2) in the form

\[ E^2 = A + BU_{\text{eff}}^C \]  

(2.32)

Here \( E \) is the anemometer output voltage, \( U_{\text{eff}} \) is the effective cooling velocity and \( A, B \) and \( C \) are the calibration constants. The use of this equation raises problems due to the nonlinearity of the equation and due to the dependence of the constants \( B \) and \( C \) on the velocity. In practice, to overcome these problems linearizing circuits are generally used to reduce the form of the equation to

\[ E = S U_{\text{eff}} \]  

(2.33)

where \( E \) and \( S \) are now the linearized voltage output and calibration constant respectively. However, the use of commercially available linearizers requires an estimate of the constants \( A, B \) and \( C \) beforehand. The next part of the problem is to arrive at an analytical expression for \( U_{\text{eff}} \) in terms of the components of velocity in the wire oriented coordinates. This involves the knowledge of the directional
sensitivities of the hot wire as well. Though there are several expressions in use for $U_{\text{eff}}$, the most commonly used one is due to Jorgensen (J2) and is given by

$$U_{\text{eff}} = \left[ U_N^2 + k^2 U_T^2 + h^2 U_{BN}^2 \right]^{1/2} \quad (2.34)$$

where $U_N$, $U_T$ and $U_{BN}$ are the normal, tangential and binormal components of the velocity with respect to the wire and $k$ and $h$ are the yaw and pitch sensitivities of the hot-wire. In a turbulent flow, the components $U_N$, $U_T$ and $U_{BN}$ can be written in terms of the mean and fluctuating velocity components in the three orthogonal directions, the yaw angle ($\alpha$) and the pitch angle ($\phi$) (see figure 2.1). $U_{\text{eff}}$ would then represent the instantaneous effective cooling velocity and $E$ the instantaneous voltage output. The constants involved ($S$, $k$ and $h$) are generally obtained from calibrations.

Essentially there are two methods in use for the measurement of mean velocities and Reynolds stresses of a turbulent flow field. In the first method, a 3 sensor probe is employed together with online data processing of the instantaneous flow field. In principle the method involves no further assumption other than that involved in arriving at equations 2.33 and 2.34. Implementing the method, however, is very expensive and hence beyond the reach of most of the hot-wire users. Further, instantaneous flow field data is not in a very usable form for an engineer. The other commonly used method involves time averaging the
response equation, equations 2.33 and 2.34, to derive expressions for the mean velocities and Reynolds stresses in terms of the mean and r.m.s. voltages of the hot-wire. This method can be used in conjunction with a single wire, a two wire sensor, or a triple wire sensor and a digital or analog data processing system. This study is concerned only with the time averaging scheme used in conjunction with stationary single normal and inclined wires. There are other techniques of measuring turbulence using a single rotated hot-wire as described by Fujita and Kovasznay (F2), which will not be dealt with. Further, the signal analysis will be restricted to the case of linearized output from the C.T.A. It will also be assumed that the mean flow direction is known.

The time averaging technique can in turn be carried out in two different ways, basically. From equations 2.33 and 2.34, we can see that the hot-wire output is directly proportional to the effective velocity, which in turn is composed of the vector components of the velocity field. This implies that the components occur under a square root sign and a simple time averaging of such an expression would lead to no solution of the problem at hand. Hence to derive the required expressions in the conventional method (see Hinze (H8) and Champagne and Sleicher (C2)), the equation for $U_{eff}$ is first expanded in a series. To make the problem tractable, the series is truncated assuming that the third and higher order terms are negligible compared to the second
order terms. The resulting response equation, after truncation, is then time averaged. Manipulation of this equation then leads to a closed form solution for the mean velocities and Reynolds stresses. This will be called method I. Since all the difficulty is caused by the square root sign, the other time-averaging method (see Rodi (R3) and Acirvlellis (A2, A3)) logically involves squaring of equations 2.33 and 2.34 before combining them to obtain the instantaneous response equation. This equation is then time-averaged and an exact solution for the mean velocities and Reynolds stresses obtained. These equations are expressed in terms of the time average of the voltage squares as opposed to the mean and rms voltages of method I. The second method described will be referred to as method II.

The usual criticism of method I is as follows. The method is limited to the investigation of flow fields of low turbulence intensity because of the series expansion and consequent truncation of the response equation. In extreme cases, the expansion itself is mathematically invalid. Several authors such as Guitton (G4) and Heskestad (H5), have attempted to correct the results of this method by including higher order terms. They measured some of these terms and made simplifying assumptions regarding the other terms to arrive at a correction formula. Such corrections are not, however, universal and their validity is doubtful. Method II has come to be accepted as a better
approach for the reason that the time averaged response expression is provided without neglecting any terms. This method too is, however, not without criticism. Since the expressions are based on squared voltages, an additional 'squerer' is required in the measurement system in the case of analog data analysis. The other point of contention is that the mean and the fluctuating velocity fields cannot be separated from each other in the resulting equation. The implication is that, the mean field be either measured separately by another device or recourse be taken to method I. Sampath et al (S1) suggested the use of a Pitot-static probe to obtain the mean velocities. This is not advantageous in view of the time factor, limitations of the probe at low speeds and the correction required for the mean velocity due to turbulence. Furthermore, the advantage obtained through the use of a hot-wire is forfeited. Rodi (R3) developed a hybrid version of methods I and II, in which the mean velocities are determined from the mean voltage signal and the fluctuating velocity components from the squared signal. This procedure not only introduces approximation at this stage but also requires additional measurements of the mean voltages as well and hence it is time consuming. The mean velocities in a three dimensional flow field can also be determined using an inclined hot-wire following the method outlined by Moussa and Eskinazi (M7). This would, however, require extensive calibrations and hence, obviously, much more time. It is also thought that as
the mean and fluctuating fields are coupled, this method is
done suited for highly turbulent flow fields.

Attempts have been made to compare these two
methods of turbulence measurements by Rodi and Acrivellis
(A1) in flow fields of different turbulence intensities.
Rodi evaluated these two methods in the developed region of
a free, round, air jet. From his experiments, Rodi
concluded that his results, obtained using method II, were
more reliable and consistent than those obtained by method
I. His measurements and those of Wygnanski and Fiedler (W4),
both using method I, showed considerable difference and he
attributed this to the thermal wake interference of the x-
wire used by the latter. Rodi's results for method II were
10-15\% higher than method I. However, the results of
Sampath et al (S1) for shear stress, also obtained in the
free jet using method II, were lower than those obtained by
Wygnanski and Fiedler (W4) using method I. In other words,
if the results of Rodi (R3) and Sampath et al (S1), for the
shear stress obtained by method II, were compared they would
show a discrepancy between 20-25\%. It should be pointed out
that Rodi has not compared these two methods on a common
basis since his method II required additional and extensive
calibrations, which were not required in method I.

Acrivellis (A1) evaluated these methods in a fully
developed turbulent pipe flow. He first gives a correct
system of equations to determine the mean velocity and
Reynolds stresses by method II (equations 15, 18, 19 and 20 of reference (A1)). Later, in order to overcome the disadvantage of using a 'squarer' and to separate the mean and fluctuating flow fields, he arrives at yet another, though incorrect, system of equations (equations 29-33 of reference (A1)) as was pointed out by Bartenwerfer (B1). Since, Acrivlellis' results were based on incorrect equations no conclusions could be drawn regarding the comparison in the low turbulence case.

From the foregoing discussion, it can be seen that attempts to compare the methods have not been carried out properly and the comparison is incomplete in many aspects. The comparison was not proper because different calibration procedures were used and an uncertainty analysis was not included in the final analysis to conclude whether the differences, if any, were significant. What is lacking in the comparison is an independent standard with which to compare these methods on a common basis in order to decide without doubt as to which method is more suitable and under what circumstances. The comparison is not complete for the following reasons. The effect of turbulence intensity needs to be studied thoroughly for this limits the use of these methods. At present there is no conclusive study outlining the limitation and use of these methods. Further, there is a need to evaluate the effects of measurement errors in voltages and uncertainty in the calibration constants while using these methods. This would help in determining the
accuracy in calibration parameters required to obtain the desired uncertainty in the computed results of the two methods.

It is, however, not feasible to carry out an experimental investigation to compare these two methods taking care to satisfy all of the above mentioned conditions. For example, it is not experimentally possible to generate data to serve as a standard for comparison with another measuring device since each device has its own limitations and associated measurement errors. At present, however, the hot-wires are still believed to be the most reliable device for turbulence measurement. This precludes the availability of an experimental standard. Further, many of the factors such as turbulence intensity and errors in measurements are not easily controlled in a laboratory situation. Hence, it is not easy to study the effects of these. Therefore, the purpose of this study is to come up with a suitable numerical approach to study the problem elucidated. The scheme would include the generation of a pseudo standard for comparing the two methods. Items of particular interest are an estimate of the range of validity of the methods with regards to turbulence intensity, an estimate of the truncation errors in method I, the effects of measurement errors and uncertainty in calibration constants. This information may possibly be useful in explaining some of the discrepancies found in the experimental comparison of these methods by other authors.
Such a study is believed to provide a useful guideline for turbulence measurement with hot-wires. Experimenters would be in a better position to choose a method depending on their requirements.

2.7 Monte Carlo Technique

Generally, analytical and numerical methods are available for problem solution. Analytical solution methods are well known but are highly limited in solving the current engineering problems. Numerical methods, which substitute numbers for the independent variables and parameters of the model and manipulate these numbers to obtain solution, have a wide scope. One of the special numerical techniques is the Monte Carlo method (B6, K7, K10).

The Monte Carlo method uses random numbers or pseudo random numbers, which are stochastic variables, for the solution of a model. This technique has found applications in three areas. They are in the solution of deterministic problems, model sampling and simulation.

(1) The first area concerns the solution of deterministic problems using random numbers for the evaluation of integrals such as

$$\int_{q}^{\infty} \left( \frac{1}{X} \right)^{\beta} e^{-\beta X} \, dX \quad (q, \beta > 0) \quad (2.35)$$

The value of this integral can be estimated if we realise that $\beta \exp(-\beta X)$ for $X > 0$ is an exponential density function. Hence we can sample $\bar{X}$ from this density function and substitute the sampled value of $X$ into $f(X)$ defined in
the following equation

\[ f(\bar{x}) = \begin{cases} 0 & \text{if } \bar{x} < q \\ 1/\bar{x} & \text{if } \bar{x} > q \end{cases} \quad (2.36) \]

The expected value of the \( f(\bar{x}) \) is equal to the integral in equation 2.35. Such applications are well known in solving problems in mathematics, physics, etc. Halton (H1) gives a very extensive survey of the application in this area.

(2) The second application area is formed by distribution sampling. The purpose is to find the distribution or some of the parameters of the distribution of a stochastic variable. This stochastic variable, that we call the output variable, is a known function of one or more other stochastic input variables which have known distributions. To estimate the output variable we draw a value for each of the input variables from their distributions and calculate the resulting value of the output variable. Such sampling is then repeated many times and this yields an estimate of the distribution. The distribution sampling method has found wide applications in evaluating the performance of various regression analysis techniques (Krutchkoff (K12)). In these studies a model is used to generate data; to these data various regression techniques are applied to estimate the parameters of the assumed model and the distribution of the resulting parameters of the model.

(3) The third application area of the Monte Carlo technique is in simulation, which means any experimentation with a
model. Simulation, in general, can be deterministic or stochastic in nature. The stochastic simulation involves the use of random numbers and hence is sometimes termed Monte Carlo simulation.

The Monte Carlo simulation has found wide applications in various fields such as economics, management, and engineering. This technique, being repetitive in nature, is well suited for modern computers even though it may take a long time to execute one run of a Monte Carlo simulation. This technique has not found a wide application in the area of fluid mechanics and especially in the area of turbulence, which can be considered stochastic in nature. This technique appears well suited to tackle some of the problems addressed in the previous section in connection with the basic problems of calibration, comparisons of the various analytical models, etc. This is because many factors of interest can be easily controlled on a computer such as the random errors in velocity and voltage. Further, experimental validation of some of the existing problems would require an enormous amount of work and hence will be time consuming and expensive. It would be worthwhile to investigate turbulence on a computer using Monte Carlo simulation procedure. In doing so, one can gain a new insight into turbulence and its measurements. To the knowledge of the author, not much work has been done in this area of application.
CHAPTER III

RESPONSE EQUATIONS AND NUMERICAL EXPERIMENTS

In this chapter the developments of the basic system of equations for the two methods of determining the mean velocity, shear stresses and the turbulence intensities from the measured voltages are outlined. The nonlinear optimization technique developed for treating the velocity and yaw calibration data is then described. Also, the Monte Carlo numerical procedure that has been developed is elaborated upon. This method is used to evaluate the hot-wire response equation, simulate turbulence on the computer and compare the methods of interpreting the hot-wire signals. Finally, the numerical programme carried out, using the techniques developed, is described.

3.1 Response Equations for Turbulence Measurements

The basic equations for determining the mean velocity and the turbulence quantities for method I and method II are developed below for the time averaging scheme.

The present analysis is restricted to mean flow in one direction only. The turbulence is, however, three dimensional in nature. The hot-wire probe axis is assumed to be aligned with the mean flow direction in order to simplify the problem. The instantaneous velocity vector is split into components described with respect to the wire oriented coordinate system (see figure 2.1). It is to be noted that the instantaneous velocity is composed of the mean velocity \( \mathbf{U} \) in the \( x \)-direction and the fluctuating velocity
components \((u, v, w)\) in the \(x\), \(y\) and \(z\) directions respectively. In figure 2.1, the hot-wire is located in the \(x-y\) plane with the normal of the wire forming a yaw angle \(\alpha\) with the mean flow direction, which is along the \(x\)-axis. From the geometry it can be seen that

\[
    U_N = (\bar{U}+u)\cos\alpha + v \sin\alpha
\]

\[
    U_T = -(\bar{U}+u)\sin\alpha + v \cos\alpha
\]

\[
    U_{BN} = w
\]

Introducing equation 3.1 into equation 2.34, the effective cooling velocity is obtained as

\[
    U_{\text{eff}} = \left[ \left( (\bar{U}+u)\cos\alpha + v \sin\alpha \right)^2 + h^2 w^2 + k^2 \left( v \cos\alpha - (\bar{U}+u)\sin\alpha \right)^2 \right]^{1/2} \tag{3.2}
\]

The equation which describes the response of the hot-wire to the assumed turbulent flow field is then obtained by substituting equation 3.2 into equation 2.33 to get

\[
    E_{xy}(\alpha)/S = \left[ \left( (\bar{U}+u)\cos\alpha + v \sin\alpha \right)^2 + h^2 w^2 + k^2 (v \cos\alpha - (\bar{U}+u)\sin\alpha)^2 \right]^{1/2} \tag{3.3}
\]

Here, \(E_{xy}(\alpha)\) is the response of the hot-wire in the \(x-y\) plane at a yaw angle \(\alpha\). It is a simple matter to obtain the corresponding expression for the hot-wire response in the \(x-z\) plane by replacing \(v\) by \(w\) and \(w\) by \(v\) in equation 3.3. Now, equation 3.3 is time averaged to derive expressions for the mean velocity and the Reynolds stresses. As indicated earlier, to obtain the equations for method I, the right hand side of equation 3.3 is expanded in a series and the
resulting equation is time averaged to acquire the response of the hot-wire. For method II, the equation is first squared and then time averaged. As many equations as required by the number of unknowns are generated by orienting the hot-wire at different yaw angles in different planes and using the time averaged response equation. The expression for the mean velocity and the following Reynolds stresses $\bar{u}^2$, $\bar{v}^2$, $\bar{w}^2$ and $\bar{uv}$ will be indicated below for both the methods. The details of deriving these equations can be found in Rodi and Acrivellis (R3, A1).

3.1.1 Method I

Mean Velocity:

$$\bar{E}_{xy}(\alpha=0)/S = \bar{U}(1 + k^2\bar{v}^2/2D^2 + h^2\bar{w}^2/2D^2 + O(3))$$

Reynolds Stresses:

$$\bar{u}^2 = e_{xy}^{2}(\alpha=0)/S^2$$

$$\bar{v}^2 = \left[ e_{xy}^{2}(\alpha=45) + e_{xy}^{2}(\alpha=-45) - e_{xy}^{2}(\alpha=0)(1+k^2) \right] / [S^2(1-3k^2)]$$

$$\bar{w}^2 = \left[ e_{xz}^{2}(\alpha=45) + e_{xz}^{2}(\alpha=-45) - e_{xy}^{2}(\alpha=0)(1+k^2) \right] / [S^2(1-3k^2)]$$

$$\bar{uv} = \left[ e_{xy}^{2}(\alpha=45) - e_{xy}^{2}(\alpha=-45) \right] / [2S^2(1-k^2)]$$

(3.4)

Here $e_{xy}^{2}(\alpha=0)$, $e_{xy}^{2}(\alpha=45)$ etc. are the mean square voltages of the fluctuating component in the planes and at the yaw orientations indicated. The above systems of equations have been derived on the assumption that third and higher order correlations are negligible compared to the second order correlations and $k^2 = 0$. From the system of equations it can be seen that six measurements are to be
made: four in the xy plane consisting of a mean voltage at 
\( \alpha = 0^\circ \) and three mean square values of the fluctuating 
voltage at \( \alpha = 0^\circ, 45^\circ \) and \(-45^\circ\) and two mean square values 
at \( \alpha = +45^\circ \) and \(-45^\circ\) in the x-z plane. The expression for 
the cross correlation \( \overline{uw} \) can also be written in terms of 
these measurements. It has not been included in this 
work.

3.1.2) **Method II**

The system of equations for method II for the same 
hot-wire orientations in the corresponding planes is

\[
\begin{align*}
\{U^2+u^2\} + k^2v^2 + h^2w^2 &= \overline{E}^2_{xy}(\alpha=0)S^2 \\quad (3.5) \\
\{U^2+u^2\} + \left[2h^2/(1+k^2)\right]v^2 + \overline{w}^2 &= \overline{E}^2_{xz}(\alpha=45) \\
&\quad + \overline{E}^2_{xz}(\alpha=-45) \quad \text{[}/(1+k^2)S^2\text{]} \\
\{U^2+u^2\} + v^2 + \left[2h^2/(1+k^2)\right]w^2 &= \overline{E}^2_{xy}(\alpha=45) \\
&\quad + \overline{E}^2_{xy}(\alpha=-45) \quad \text{[}/(1+k^2)S^2\text{]} \\
\overline{uv} &= \frac{\overline{E}^2_{xy}(\alpha=45) - \overline{E}^2_{xy}(\alpha=-45)}{2S^2(1-k^2)} \\
\end{align*}
\]

where \( \overline{E}^2_{xy}(\alpha=0) \), \( \overline{E}^2_{xy}(\alpha=45) \) etc. are the mean square of 
the hot-wire instantaneous output voltage in the 
respective planes and at the orientations indicated. This 
system requires measurement of five squared voltages at a 
point to solve for the unknowns \( \{U^2+u^2\}, v^2, w^2, \overline{uv} \) and \( \overline{uw} \). 
The expression for \( \overline{uw} \) has not been included. It is obvious
that one more measurement is required to separate the terms \( U^2 \) and \( u^2 \). This is the inherent problem in this method. It is to be noted here that the expressions are in terms of the time average of the squared voltage \( U \) against the mean square values of the fluctuating voltage and the mean voltage in method I.

3.2 Improved Calibration Method

In the present study a new method has been developed for calibrating hot-wires using the raw calibration data. The objective is to develop a simple, easy and fast means of obtaining the calibration constants of a normal or inclined hot-wire. The method is based on the Gauss-Newton technique (W5). The nonlinear response equation of the hot-wire is expanded in terms of the independent variables in a Taylor series about initial estimates of the constants. Using the least squares principle, a set of normalized equations are set up and solved for the corrections to the initial estimates of the constants. A suitable method, such as matrix inversion procedure, is used to solve for these corrections. An iterative process is established in which the values converge towards the optimum set of constants. The objective is to minimize the value of the sum of the errors squared in \( U \). To ensure convergence of the non-linear regression technique, the modification to the Gauss-Newton method suggested by Hartley (H4) is incorporated.
3.2.1 Yaw Calibration

The response of the hot-wire expressed using the effective velocity model given by equation 2.30 and the modified King's law given by equation 2.31 is

\[ E^2 = A + BU^C ( \cos^2 \alpha + k^2 \sin^2 \alpha )^{c/2} \quad (3.6) \]

The errors in \( U \) are to be minimized and hence we rearrange the above equation to read

\[
U = \left( \frac{(E^2 - A) / B}{(C \cos^2 \alpha + k^2 \sin^2 \alpha)^{1/2}} \right)^{1/c} \quad (3.7)
\]

or

\[ U = f(E, \alpha; A, B, C, k) \quad (3.8) \]

Given \( n \times m \) responses, \( E \), of the hot-wire for \( n \) settings of the velocity and \( m \) settings of the yaw angle, it is required to obtain an optimum set of the parameters \( A, B, C \) and \( k \) such that the sum of the errors squared in \( U \) is minimized, i.e.,

\[
(SES)_U = \sum_{i=1}^{n} \left( \sum_{j=1}^{m} f_{ij}(E_{ij}, \alpha_j; A, B, C, k) \right)^2 \quad (3.9)
\]

is minimized.

This is a classical least squares problem. One of the standard methods for solving this problem, which leads to non-linear equations, depends upon reduction of the residuals in equation 3.9 to linear form by a first order Taylor series approximation taken about an initial or trial solution for the parameters. This is the standard Gauss-Newton approach to solving non-linear regression problems. Hence, initial estimates of the unknowns \( A, B, C \) and \( k \) are
made. The function \( f \) in equation 3.8 is linearized by expanding it about the initial estimates \( A_0, B_0, C_0 \), and \( k_0 \). Retaining only the first order terms, \( f_{ij} \) can be written as

\[
\frac{\partial f_{ij}}{\partial A} \bigg|_{A_0} (A-A_0) + \frac{\partial f_{ij}}{\partial B} \bigg|_{B_0} (B-B_0) + \frac{\partial f_{ij}}{\partial C} \bigg|_{C_0} (C-C_0) + \frac{\partial f_{ij}}{\partial k} \bigg|_{k_0} (k-k_0) + \ldots \tag{3.10}
\]

The subscript \( o \) refers to the evaluation of the function \( f \) and its derivatives at \( (A_0, B_0, C_0, k_0) \). The approximation to the function \( f_{ij} \) given in equation 3.10 is substituted into equation 3.9. Now the normal equations corresponding to the least squares problem in equation 3.9 are obtained by differentiating this equation with respect to the constants \( A, B, C \), and \( k \) and the setting each of them equal to zero. Performing the operations indicated using equations 3.7, 3.9 and 3.10, the four normal equations involving the four unknown corrections to the initial estimates, i.e., \( (A-A_0) \), \( (B-B_0) \), \( (C-C_0) \), and \( (k-k_0) \) are given in matrix form as follows.

\[
\begin{bmatrix}
S_2^2 & S_2S_3 & S_2S_4 & S_2S_5 \\
S_2S_3 & S_3^2 & S_3S_4 & S_3S_5 \\
S_2S_4 & S_3S_4 & S_4^2 & S_4S_5 \\
S_2S_5 & S_3S_5 & S_4S_5 & S_5^2
\end{bmatrix}
\begin{bmatrix}
\Delta^e_1 \\
\Delta^e_2 \\
\Delta^e_3 \\
\Delta^e_4
\end{bmatrix} =
\begin{bmatrix}
S_1S_2 \\
S_1S_3 \\
S_1S_4 \\
S_1S_5
\end{bmatrix} \tag{3.11}
\]
where
\[
S_1 = \Sigma \left[ -m U_i + \sum_{j=1}^{m} \frac{(P_{ij})^{D'}}{\sqrt{R_j'}} \right]
\]
\[
S_2 = \frac{D'}{B} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(P_{ij})^{D'}}{\sqrt{R_j'}}
\]
\[
S_3 = \frac{D'}{B} \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(R_i')^{D'}}{\sqrt{R_j'}} \ln(P_{ij})
\]
\[
S_4 = D'^2 \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{(k \sin \gamma_{ij}(P_{ij})^{D'})}{(\sqrt{R_j'})^3}
\]
\[
P_{ij} = \frac{(E_{ij} - A_0)}{B_i'}; \quad R_i' = \cos^2 \alpha_j + k^2 \sin^2 \alpha_j; \quad D' = 1/c_0,
\]
\[
\Delta \theta_2 = (B - B_0), \quad \Delta \theta_3 = (c - c_0), \quad \Delta \theta_1 = (A - A_0) \text{ and } \Delta \theta_4 = (k - k_0)
\]

The method of solution adopted to solve the above matrix equation for the unknowns is as follows. Let equation 3.11 be represented by

\[
[G] \begin{bmatrix} \xi \end{bmatrix} = [H]
\]

where
\([G]\) is a 4x4 matrix,
\([\xi]\) is a 4x1 matrix

and
\([H]\) is a 4x1 matrix

These four linear equations are solved for the four unknowns, \(\xi_1, \xi_2, \xi_3\) and \(\xi_4\), using the matrix inversion given below

\[
[\xi \xi] = [G]^{-1} [H]
\]
A second approximation of the required constants is obtained by using \( A = A_0 + \Delta \theta_1 \), \( B = B_0 + \Delta \theta_2 \), etc. These are now used instead of the initial estimates \( A_0, B_0, C_0 \) and \( k_0 \). The procedure is iterative and is repeated until changes in the parameters become negligible.

More often than not the non-linear least squares problem leads to a \([G]\) matrix such that the inverse of it contains some very large off diagonal elements. This is an example of an ill-conditioned system of equations and hence matrix. This is a result of the non-linear forms of the function and poor initial guesses. Therefore, the resultant matrix tends to be singular. The solution of such a system of equations yields new values of the parameters which are not sufficiently close to the initial estimates and may result in a larger value of the sum of errors squared.

Several standard techniques for treating such problems are available (H4, M2, M4) to ensure convergence and to obtain accurate and reliable computational results. The modification to the Gauss-Newton method suggested by Hartley has been used. The iterative procedure suggested by Hartley is as follows.

1. Guess values of the parameters \( A_0, B_0, C_0 \) and \( k_0 \).
2. Determine the steps \( \Delta \theta_1, \Delta \theta_2, \Delta \theta_3 \) and \( \Delta \theta_4 \) by solving the system given by equation 3.11.
3. Determine a step length factor \( \gamma \) such that SES at \( A = A_0 + \gamma \Delta \theta_1 \), etc., is an approximate minimum for \( 0 < \gamma \) < 1.
4. Use \( A = A_0 + \gamma \Delta \theta_1 \), \( B = B_0 + \gamma \Delta \theta_2 \), etc., and perform
The next iteration.

A convenient method for obtaining the required step factor $\gamma^*$, recommended by Hartley (H4), is to use

$$\gamma^* = \left( \frac{1}{2} \right) + \left[ \left( \frac{1}{4} \right) \left( \text{SES}(0) - \text{SES}(1) \right) / (\text{SES}(1) - 2\text{SES}(1/2) + \text{SES}(0)) \right] \quad (3.14)$$

where SES(0), SES(1/2) and SES(1) are the sum of the errors squared in U using a step length factor of 0, 1/2, and 1 respectively. Hence to initiate the iterative scheme, the problem is to be solved with three different step factors.

Using the modifications suggested above, a numerical scheme to solve equation 3.11 has been developed and the logic used is indicated in figure 3.1. A listing of the computer programme based on this logic is given in Appendix A.1.

3.2.2 Velocity Calibration

Proceeding in the same way, as in the previous section, the velocity calibration equation 2.24 is rearranged to read

$$U = \left( \frac{E^2 - A}{B} \right)^{1/c} \quad (3.15)$$

Performing all the operations outlined earlier would lead to the following set of three normal equations to be solved for the unknowns $\ell_1, \ell_2$ and $\ell_3$:

$$-\sum_{i=1}^{n} \left( P_i^D - U \right) (P_i) D_i^{i-1} + \sum_{i=1}^{n} P_i (P_i')^{i-1} \Delta \ell_i c_i B_i o_i + \sum_{i=1}^{n} \left( P_i (2D_i - 1) \right) \Delta \ell_2 c_i B_i o_i$$

$$+ \sum_{i=1}^{n} P_i (2D_i - 1) \ln(P_i) \frac{\Delta \ell_3}{c_i o_i} = 0$$
\[ - \sum_{i=1}^{n} \left( P_{i}^{D} - U_{i} \right) \frac{P_{i}^{D} - 1}{c_{o}B_{o}} + \sum_{i=1}^{n} \left( P_{i}^{2D} - 1 \right) \frac{\Delta E_{1}}{c_{o}^{2}B_{o}^{2}} + \sum_{i=1}^{n} \frac{P_{i}^{2D}}{c_{o}B_{o}} \Delta E_{2} + \sum_{i=1}^{n} \frac{P_{i}^{2D}}{c_{o}B_{o}} \Delta E_{3} = 0 \]

where \( P_{i} = (E_{i}^{2} - A_{o})/B_{o} \)

In the case of velocity calibration, only three equations are to be solved simultaneously, and hence Cramer's rule was used to reduce computer time. The rest of the procedure was exactly the same as in the yaw calibration case. The computer programme is provided in Appendix A.2.

3.3 Monte Carlo Testing Procedure

This section describes the Monte Carlo technique used to solve the problems connected with calibration and turbulence measuring techniques. The technique adopted involves both simulation and model sampling. The procedure is essentially that followed by Krutchkoff (K12) for evaluating the direct and indirect regression methods.

3.3.1 Evaluation of the Basic Systems of Equation for Turbulence Measurements

The procedure followed here can be divided into two major parts. The first part deals with the generation of pseudo turbulence using random number generators.
calculation of the average stress field from the generated turbulence and the use of equation 3.3 to obtain the voltage information. The second part deals with the comparison of the two methods and a study of the effects of measurement errors. The voltage information obtained in the first part forms an input to the second part. The stress field obtained in the first part serves as a standard to compare the Reynolds stresses obtained by using the systems of equations for method I and method II. These will be elaborated on in the following paragraphs.

At the outset, the constants occurring in equation 3.3, viz., \( S, k, h \), and the mean velocity \( (U) \) are assigned typical values without any loss of generality (see Krutchkoff (K12)). Now, using the random number generator a set of \( u, v \) and \( w \) values are picked up with preassigned mean and standard deviation. The mean was, of course, set at zero and the standard deviation was varied to change the pseudo turbulence level. The set of \( u, v \) and \( w \) values was used in equation 3.3 and the corresponding equation in the \( x-z \) plane at the orientations mentioned earlier to obtain five values of the instantaneous voltages (\( E_{xy}(\alpha=0), E_{xy}(\alpha=+45), E_{xy}(\alpha=-45), E_{xz}(\alpha=45) \) and \( E_{xz}(\alpha=-45) \)). From the \( u, v \) and \( w \) values the instantaneous values of the following stresses \( u^2, v^2, w^2 \) and \( uv \) values can be computed. The random number generator is used again to choose another set of \( (u, v, w) \) and the above procedure is repeated over a large number of trials. The ensemble averages of the Reynolds
stress \( \bar{u}^2 \), \( \bar{v}^2 \), \( \bar{w}^2 \) and \( \bar{u}\bar{v} \) are obtained using the instantaneous voltage values, obtained over a large number of trials. It is a simple matter to determine the ensemble averages of the following voltages \( \bar{E}^2 \), \( \bar{E} \) and \( \bar{e}^2 \) for different orientations in different planes. Here, the ergodic hypothesis is assumed to apply. Thus, the standard values for the Reynolds stresses and the corresponding time-averaged voltage values are generated.

The second part consists of introducing errors into the values of \( \bar{U} \), \( \bar{E}^2 \), \( \bar{E} \) and \( \bar{e}^2 \) and the calibration constants \( S \) and \( k \). It should be pointed out that \( \alpha \) and \( h \) are assumed error free for the sake of simplicity. Random number generators are then used to choose random errors in these quantities with a preassigned standard deviation of the errors about their mean values. Using these perturbed values and the systems of equations for method I and method II, the mean velocity and the Reynolds stresses are obtained for both the methods. These values are then compared with the standard values generated for the stresses in part I. The procedure is repeated over a large number of trials to estimate the effects of errors in the aforementioned quantities and compare the two methods. The turbulence levels can be varied by changing the standard deviations in \( u,v \) and \( w \) keeping \( \bar{U} \) fixed. The whole procedure is then repeated to study their effects. The comparisons are based on the normalized standard deviations in the Reynolds stresses expressed as percentages, viz., \( \left[ \frac{u^2}{\bar{u}^2} \right] \).
\( u_o^2 \times 100 \), etc. Here \( u_o^2 \), etc., are the stresses obtained using the corresponding method and \( u_o^2 \), etc., are the standard values generated. The logic described in Figure 3.2 and a listing of the program based on this logic can be found in Appendix A.3.

3.3.2 Calibration Studies

The model chosen for generating the calibration data of a hot-wire is

\[
E^2 = A + BU^{0.4}
\quad (3.17)
\]

where \( E \) is the voltage output of the bridge, \( U \) is the velocity of the free stream and \( A \) and \( B \) are the calibration constants. Krutchkoff has shown that the constants in this type of equation can be assigned typical values without loss of generality in a numerical simulation. The values assigned were \( A = 54 \) (volts\(^2\)) and \( B = 6.0 \) (volts\(^2\))/(m/s\(^{0.4}\)) based on actual calibrations. The range over which \( U \) is to be varied is arbitrarily fixed. The standard deviation of the error in \( U \) is then specified as a percentage of the range. Random numbers are used to generate a predetermined number of velocities in the range and the corresponding error values. With the values of velocities and the values of the constants \( A \) and \( B \), the corresponding output voltages, \( E \) can be calculated. Once the voltages have been determined, the error values generated are introduced into the velocities. It should be pointed out that these errors are independent and had a Gaussian distribution with zero mean. Then any of
the standard curve fitting techniques can be employed to smooth the simulated calibration data and obtain the constants of the assumed expression and the standard deviation. For a given set of conditions, the numerical experiment is repeated over a large number of trials to obtain average values of the constants and the standard deviation. The conditions such as the sample size and spacing of the data points can then be systematically varied and the experiment repeated to study their effects. The logic diagram is provided in figure 3.3 and the listing of the programme is given in Appendix A.4 for the case of curve fitting using the equation suggested by Collis and Williams (C5).

3.4 Numerical Experiment

All the numerical experiments were carried out on an IBM 3031 computer based on the logic indicated. The IBM package 'RANDU' was used for generating the random numbers.

3.4.1 Calibration Studies

The simulated calibration data were curve fitted using the following equation suggested by Collis and Williams

$$E^2 = A + BU^{0.45}$$  \hspace{1cm} (3.18)

The curve fitting method consisted of performing a linear regression of $E^2$ on $U^{0.45}$. This method will henceforth be called the linear calibration method.

The floating exponent power-law model

$$E^2 = A + BU^c$$  \hspace{1cm} (2.24)
together with the nonlinear technique developed in Section 3.2.2 was also considered and compared with equation 3.18. This procedure will be referred to as the nonlinear calibration method.

At the outset, experiments were carried out to determine the number of trials beyond which the averages of the standard deviations are independent of the number of trials. The standard deviation of the curve fit was used in a similar manner in what follows to aid the analysis and draw conclusions from the results. Having determined the number of trials required, experiments were conducted to study the effects of changes in the various parameters, mentioned earlier, on the number of samples required. The method of choosing the velocities was carried out in two different ways. One was to choose the required number of samples spaced randomly in the range and the other way was to choose them uniformly spaced over the range. The standard deviation of the independent random errors introduced into the velocity chosen were assigned as a percentage of the range used except when studying the effects of varying the range of velocity. In this case the absolute error was fixed so that the effects of changing the range could be isolated and studied. The sample size was varied between 5 and 50. Higher sample sizes were not tried because of practical implications. A summary of the numerical experiment carried out is given in Table 3.1.
3.4.2 Evaluation of Turbulence Equations

The numerical experiment consisted of testing the random number generator, determination of the number of trials required to obtain reliable results and finally carrying out the required experiments.

The random number generator produces numbers, 'the mean' and standard deviation of which differ slightly from the preset values. This would introduce errors in the final result. Hence the random numbers generated were modified by adding a fixed quantity to exactly obtain the desired mean. The fixed quantity added was the difference between the desired mean and the actual mean obtained. The numbers thus obtained were multiplied by a constant factor such that the required standard deviation was also exactly obtained. This constant factor was equal to the ratio of the desired standard deviation to the actual standard deviation obtained after the first modification of the random numbers.

Further, one more condition was stipulated on u and v to obtain a nonzero cross product term uv. The condition imposed was that whenever u was positive, v was assigned a negative value and vice versa. This condition amounted to assuming the sign of the slope of the mean velocity at the hypothetical point of measurement based on the mixing length model (see Schlichting (§5)). These operations can be shown not to affect the characteristics of the generated random numbers (see Hamming (H3)). The indicated logic was incorporated in the software as can be seen from...
the logic diagram in figure 3.2.

The second step was to determine the number of trials required to generate a reliable pseudo turbulence field. The criterion chosen is that the ensemble average of the shear stress ($\overline{uv}_o$) computed from the field generated become independent of the number of trials. Since modification of the random numbers, described earlier, yielded a constant value for $\overline{u}^2_o$, $\overline{v}^2_o$ and $\overline{w}^2_o$ independent of the number of trials, the criterion was fixed based on the $\overline{uv}_o$ value. The effect of the seed value, required to initiate the random number generator, was also investigated.

To determine the number of trials required to generate a reliable pseudo turbulence field, the mean velocity and the constants occurring in equation 3.3 need to be prescribed. The typical values assigned were $\overline{U} = 25$ m/s, $S = 0.3$ volts/m/s, $k = 0.2$ and $h = 1.0$. The standard deviations of the errors in $S$, $k$, $\overline{U}$ and the voltages were set at zero. The standard deviations of the fluctuating velocity components $u$, $v$ and $w$ were set at 10% of the mean velocity.

Keeping these factors constant, the number of trials was varied between 5 to 40,000 to determine the number of trials at which $(\overline{uv}_o)$ attained a constant value. The number of trials required for the second part, when measurement errors were introduced, was arbitrarily set at 5000 trials.

The numerical experiments were then carried out to estimate the approximation involved in method I and its variation with turbulence intensity, to study the effects of
measurement errors in $S$, $k$, $\bar{U}$, $\bar{E}^2$ and $\bar{e}^2$, and finally, to evaluate the two methods. To estimate the truncation error in method I, the standard deviations in measurement errors were set at zero. Under these conditions, method II should yield the same results for the Reynolds stresses as the standard since no approximations are involved. However, method I is likely to produce deviations from the stress field generated due to truncation and the consequent approximation involved. The difference in the Reynolds stresses between these two methods, would then give an estimate of the truncation error in method I. The effect of truncation on the mean velocity, determined using method I, is described below. From the expression for mean velocity in equation 3.4, we can write

$$\bar{E}_{xy}(\alpha=0)/\bar{SU} = 1 + k^2 \bar{v}^2/2 \bar{U}^2 + h^2 \bar{w}^2/2 \bar{U}^2 + O(3) \quad (3.19)$$

The product $\bar{S}\bar{U}$ can be determined from the assumed values of $S$ and $\bar{U}$. This represents the mean voltage output, $E_0$, of the hot-wire at $\alpha = 0$ and with no turbulence present in the stream. $\bar{E}_{xy}(\alpha=0)$ represents the mean voltage generated using random numbers when turbulence is present. From the turbulence field generated the quantities $\bar{v}^2$ and $\bar{w}^2$ are known. Hence the right hand side of the above equation can be independently evaluated to the third order of accuracy and is the commonly used turbulence factor, $f$. If the right hand side is evaluated without any truncation it may be considered as an exact factor, $f_{exact}$. In physical
experiments, $f_{\text{exact}}$ cannot be obtained because $U$ is not known a priori and the difficulty in measuring the higher order terms. In this numerical experiment, however, $f_{\text{exact}}$ can be determined from the values of $S$, $\bar{U}$ and $E_{xy}$. Equation 3.19 may be rewritten as

$$f_{\text{exact}} = f' + O(3) \quad (3.20)$$

The variations of $f'$ and $f_{\text{exact}}$ with turbulence intensity were studied by changing the levels between 0 and 50% keeping all the other factors constant.

To study the effects of measurement errors two separate experiments were carried out. In the first, a uniform error of 1% was introduced in all the quantities $\bar{U}$, $S$, $k$, $E^2$ and $e^2$ and the normalized standard deviation in the Reynolds stresses were obtained. The error in only one of the above quantities was then held at 1% and the errors in other quantities were set at zero. The experiment was repeated to estimate the individual contribution of the various quantitites to the total error. This experiment was carried out for different turbulence intensities in the range of 10\% to 50\%. In the second experiment, the errors in these quantities were set at the worst possible combination based on past experience. The standard deviations of the errors chosen were 1\% in $\bar{U}$, 0.5\% in $E^2$ and $e^2$, 10\% in $S$ and 20\% in $k$. For this combination of errors, the normalized standard deviations in the Reynolds stresses were obtained for both the methods. Then, the error in each quantity was varied
between zero and the limit chosen while keeping the errors in the other quantities fixed at the values chosen. This experiment was carried out for only two values of turbulence intensity, 10% and 50%. In addition the experimental program carried out is given in table 3.2.
CHAPTER IV
EXPERIMENTS

4.1 Objectives

The aims of the present experimental programme are to provide additional information not available in the literature and to verify some of the numerical results obtained. The objectives of the experiments carried out with regards to hot-wire calibration are:

1. to evaluate some of the existing analytical expressions available for velocity and yaw calibrations,
2. to see if the nonlinear optimization technique yields better results,
3. to carry out careful yaw calibrations and to see if relaxing the existing assumptions in the analysis yields better results.

The experiments to compare methods I and II of interpreting hot-wire signals for turbulence measurements were carried out in fully developed jet and pipe flows. The experiment was so designed that the input remained the same for both the methods. The need for an electrical unit to square the signal, i.e., the squarer, in the case of method II has been avoided. The aims of this part of the experimental programme are:

1. to compare the two methods and to evaluate their performance in a flow with low and high turbulence intensity,
2. to study the effects of yaw sensitivity factor \( (k^2) \) on turbulence measurements.

4.2 Experimental Facilities

4.2.1 Turbulent Air Jet Facility

The schematic diagram of the turbulent air jet experimental set-up is shown in figure 4.1.

Air from the laboratory compressed air supply passed through a 3-micron filter, pressure regulator, a valve and a rotameter before entering the large settling chamber which contained a set of flow straighteners followed by three screens. The settling chamber was fitted with a 2.54cm nozzle, through which the axisymmetric jet issued. The jet exit velocity was controlled by operating the valve.

The traverse mechanism shown in figure 4.1 was capable of moving in the \( x, y \) and \( z \) directions as indicated. The hot-wire or Pitot tube could be mounted on the probe mount. The mounting was such that the hot-wire could be set at any yaw angle and at the same point in the flow field. The yaw setting was measured using a protractor with a vernier attached to the probe mount rod. Figure 4.2 gives a photograph of the yaw mechanism. The hot-wire with the probe holder could be rotated about its axis and set at any orientation desired. The probe mount and the support rod mechanism was designed such that it was rigid and practically has no deflection under aerodynamic loading. It also had low aerodynamic resistance. The probe mount mechanism was rigidly fastened to the traverse on rollers.
This whole system was then fixed to a carriage which travels on tracks along the axis of the jet (x-direction). The tracks were made of cold rolled steel bars and hence provided a sturdy track that deflects a negligible amount under the load of the carriage. The carriage was prevented from moving in a lateral direction to the tracks by roller bearings adjusted to give zero backlash. The tracks were fastened onto the top of a hydraulic table which allowed positioning of the probe in the vertical y-direction. The hydraulic table was bolted to the floor after being positioned relative to the jet. The traverse mechanism was motorised and the hot-wire can be set in the radial and axial direction with a scale having a precision of 0.003 mm. The air jet along with the traverse mechanism were enclosed in a 1.3x1.3 mm² wire mesh cage to minimize the effect of room drafts. An overview of the experimental set-up is presented in figure 4.3.

The calibration tests were performed in the uniform flow field of the potential core of the jet exhausting from a 2.54 cm vortex filament nozzle. These tests were performed with the hot-wire probe mounted on the probe mount mechanism and the Pitot-static probe mounted separately on a magnetic stand. The Pitot-static probe was placed beside the hot-wire in the potential core of the jet as shown in the photograph in figure 4.4. Thus, both velocity and yaw calibrations were carried out. The turbulence level in the potential core was measured to be 0.3%.
To perform turbulence measurements in the fully developed region of an air jet, the 2.54 cm nozzle was replaced with a 1 cm ASME nozzle. The measuring plane was located 75 diameters downstream of the nozzle exit.

4.2.2 Turbulent Pipe Flow Facility

The determination of the yaw factor and the turbulence measurements in fully developed pipe flow were carried out in the facility shown schematically in figure 4.5. It consisted of a straight commercial aluminium pipe of 10.8 cm inside diameter and 15.24 m long connected to a wooden settling chamber of length 1.65 m and cross-section 0.91 m x 0.91 m. The settling chamber, having flow straighteners and screens inside, received air from a blower through a flexible pipe. The inside surface of the pipe was polished using glasswool to give a mirror finish. This aided in the alignment of the hot-wire probes. The outlet of the pipe was traversed using a traverse mechanism fitted with a vernier scale capable of a precision of 0.03 cm (see figure 4.6).

The static pressure variation along the pipe was measured using the ten static pressure taps located, as shown in the figure, near the exit of the pipe.

4.2.3 Equipment Used

All hot-wire measurements were carried out with the DISA 55M system. This system consisted of a constant temperature anemometer (type 55M10), a linearizer (type 55D10), a signal conditioner (type 55D26), an rms meter
(type 55D35) and a digital voltmeter (type 55D31). In addition, a Fluke multimeter (type 8050A) was also used. DISA miniature probes (type 55PL1) with a 5 micron diameter tungsten normal wire were used for measuring the mean axial velocity and the axial turbulence intensity. The shear stress and the turbulent intensities in the other directions were measured using the normal wire and a single 5 micron tungsten slanting wire (type 55PL2). A Metemaster, type 65601, oscilloscope was used as visual check of the hot-wire output and for optimization of the frequency response of the system.

For calibration purposes, in the potential core of the jet, a Pitot-static tube of outer diameter 3 mm was used. The mean velocities in the turbulent flows of the pipe and the jet were also measured by traversing a total pressure tube of outside diameter 1.8 mm. The measurements with the Pitot-static tube or the Pitot tube were accomplished by using an inclined manometer (Lambrecht Type 655). The equipment described is shown in the photograph of figure 4.7.

4.3 Experimental Precautions

The following experimental precautions were taken to ensure that the data obtained had minimum error and to detect changes in experimental conditions so that such data could be rejected.

1. All instruments were operated as recommended by the manufacturer.
2. Hot-wires were periodically cleaned in an ultrasonic bath of acetone wash and inspected under a microscope to check for accumulation of dust.

3. During calibration the hot-wire and the Pitot-static probes were mounted side by side such that the probes did not interfere with each other.

4. For calibration the alignment of the normal or inclined wires was carried out by setting the wire at different yaw angles and locating the position where the hot-wire gave the maximum output.

5. For turbulence measurement, the calibration of the wire was checked at the beginning and the end of each series of results for one traverse. If the slope of the curve had changed by more than 5%, the whole series of results for that traverse were rejected and new readings were taken.

6. Turbulence measurement at a reference location was frequently checked to detect drifts in calibration and in the analog instruments.

7. As the experiments were carried out over long intervals of time, any changes in the atmospheric conditions, were compensated by adequately changing the overheat ratio of the C.T.A.

8. When the slanting wire was used, the alignment was adjusted until there was no variation in the anemometer mean voltage on rotation of the probe about its axis.

9. When measuring the mean and the rms voltages in the turbulent flow fields, large time constants were used (100
sec) in regions of high shear. Readings were taken after a settling time of at least 4 minutes. At each station, observations were repeated to ensure that stable averages had been obtained.

4.4 Measurements

The actual experimental programme carried out is given in Table 4.1. As can be seen in all hot-wire measurements the overheat ratio was set at 0.8. Only normal wires were used in the experiments carried out to compare the optimization techniques and the heat transfer models. The velocity ranges chosen were as indicated in Table 4.1. Yaw calibrations were, however, conducted for both normal and inclined wires. Before carrying out the turbulence measurements additional experiments involving Pitot probe surveys were carried out in the pipe as well as in the jet to ensure that these flows were axisymmetric and fully developed near the intended plane of measurement. To measure the flow symmetry Pitot probe surveys were carried out along two different diameters in the same plane. The fully developed nature of the flow was checked by measuring the mean velocities in two different planes in the same direction. The Reynolds number indicated for the pipe flow is based on the internal diameter of the pipe and the average velocity over the cross-section. For the jet case it is based on the exit velocity and the nozzle diameter. The required calibration results were deduced using the programmes developed. Using the raw data and the equations
developed for methods I and II, the turbulence quantities were computed.

Since the experiments were conducted over long time periods, the temperature of the medium changed. The changes in the jet were less than 1°C and hence no attempts were made to incorporate corrections for these changes. However, in the case of the pipe experiment, the blower pumping the air through the pipe tended to heat the air by 5-8°C over that at which the calibrations were performed. Hence, this change was accounted for by increasing the overheat accordingly. In the case of the pipe experiment, no wall corrections were applied to the hot wire data taken close to the pipe walls. Other forms of corrections such as end conduction loss, radiation heat loss, etc., were also not applied. In the case of jet flow it was assumed that the flow was parallel to the jet axis. Hence, no correction was applied to account for the inclination of the flow with respect to the wire. The turbulence measurement in the pipe and jet flows were carried out several times under the same conditions to check for repeatability.

4.5 Uncertainty Analysis

The objective of an experimental programme is to obtain true values of the variable of interest. However, all experiments are subjected to errors of fixed and random nature due to the scatter in the raw data used in calculating the result. As most experiments in engineering, such as the present one, are single-sample
experiments, statistics cannot be applied to assure reliability of the results. Hence, an uncertainty analysis has to be carried out on the measured set of data. Such an analysis is required to properly interpret one's experimental results and to ensure that proper comparisons are made with other results.

In the present work different approaches available for uncertainty analysis have been used to evaluate the uncertainties in the experimental values reported. For the purposes of uncertainty analysis, the following assumptions will be made.

1. Fixed or systematic errors are negligible.
2. Probe is properly aligned with respect to the mean velocity.
3. The angle of inclination of the slanted hot-wire, specified by the manufacturer, is error free.
4. Linear distances are error free.
5. The instrument errors are assumed to be one half of the smallest division to which a value can be accurately measured.
6. The uncertainties in the input variables are specified at 20/1 odds. The result is also valid for 20/1 odds.

For calibration purposes the following procedure was adopted for estimating the uncertainties in the parameters of the heat transfer models. Based on the constant odds procedure given by Kline and McClintock (K8) the uncertainty in the velocities measured using a Pitot-
Static probe was estimated to be less than ± 1%. The accuracy with which voltages can be measured is specified by the manufacturer of the voltmeter. This value, ± 0.3%, has been assumed to be the uncertainty in the measurement of voltages. Using these uncertainties, ΔE and ΔU, the uncertainties in the parameter estimates of the models were carried out using a procedure similar to that given by Moffat (M5). This method is valid irrespective of whether the approach is linear or non-linear. In this method, if

\[ E = f( U; A, B, C ) \]  

(4.1)

and the uncertainties in E and U are known as ΔE and ΔU, it is a simple matter to carry out uncertainty analysis on the computer based data reduction program. In principle, using the raw calibration data, \((E_i, U_i)\) with \(i = 1\) to \(N\), the best estimate \(A_o, B_o, C_o\) is determined using the program. Then one of the variables, say \(E_i\), is indexed by the amount ΔE and the new values \(A_1, B_1, C_1\) are calculated along with the differences \((A_1 - A_o), (B_1 - B_o)\) and \((C_1 - C_o)\). This process is repeated for every value of each variable and the contributions squared and accumulated to yield the square of the required uncertainties in these parameters. The uncertainty estimates in these parameters are presented along with the results in the next chapter. The uncertainty in the value of yaw factor \(k^2\), obtained using the non-linear yaw calibration technique, has been estimated to be ± 13%. The uncertainty obtained in \(k^2\) obtained from the measurement of the shear stress profile in fully developed
pipe is estimated to be within ±11%. These uncertainty estimates are based on the procedure of Kline and McClintock (K8).

The uncertainties in the turbulence measurements were estimated by following the systematic procedure developed by Yavuzkurt (Y1) for hot-wire measurements. The details of the uncertainty analysis carried out for turbulence measurements and the estimated uncertainties are presented in Appendix B.
CHAPTER V

RESULTS AND DISCUSSION

In this chapter the experimental and numerical results of the present investigations are presented and discussed. The first part of the numerical results pertain to the studies on velocity and yaw calibrations using the Monte Carlo technique. The second part deals with the comparison of two signal interpretation methods used to measure turbulence. The results of this comparison has been obtained using the Monte Carlo technique. The experimental programme was carried out to complement the numerical study. Hence, it was also concerned with the velocity and yaw calibrations carried out in the potential core of the jet and turbulence measurements in the pipe and jet flows. These results are presented and compared with the numerical results.

5.1 Numerical Experiments

Results of the numerical experiments are presented in this section. The results pertain to the calibration studies and the comparison of the two signal interpretation methods for turbulence measurement.

5.1.1 Calibration Studies

5.1.1.1 Number of Trials Required

The number of trials required for Monte Carlo simulation was decided by carrying out an experiment, the results of which are given in table 5.1. In principle the results become more reliable as the number of trials tend to
infinity. However, based on the results given in the table and as a compromise between computer time and reliability of the end results, the number of trials was fixed at 500 for a given set of conditions in each experiment. This number was assumed to be independent of the changes in the variables of interest to be studied.

5.1.1.2 Effect of Sample Spacing

The effect of changing the spacing of the samples over the velocity range was investigated for the conditions indicated in figure 5.1. It can be seen that the choice of random or uniform spacing of the calibration data has no effect on the outcome of the results. Hence, in all the experiments discussed below, the random sampling technique was adopted.

5.1.1.3 Linear and Non-linear Methods of Calibration

The non-linear method of velocity calibration, which has been developed, is compared with the conventional linear method in figures 5.2 and 5.3 based on Monte Carlo testing. From figure 5.2 it can be seen that the non-linear method not only yields lower errors but also shows practically no effect with change in sample size. The testing shows that the non-linear method consistently yields lower errors in \( U \). A plot of the square of the relative error against the sample size is shown in figure 5.3 for true velocities (\( U_T \)) of 2 m/s and 20 m/s. The calculated velocities (\( U_C \)) were obtained in the following way. The voltage corresponding to the true velocity is calculated...
from equation 3.17 with the assumed values of A and B. This value of voltage together with the average values of the calibration constants, obtained using the Monte Carlo technique for equations 3.18 and 2.24, then yields the calculated velocities \( U_c \) for the linear and nonlinear method respectively. It is seen that the nonlinear method yields significantly lower errors than the linear method in the lower end of the velocity range chosen. The errors are comparable in the higher end of the range. However, the linear calibration method was used to treat the simulated hot wire data in the other cases to reduce computer time. It is believed that the conclusions drawn would be valid qualitatively.

5.1.1.4 Effect of the Magnitude of Error in Velocity

The effect of the magnitude of error in velocity on the sample size required for calibration is shown in figure 5.4. It can be seen that the standard error can be reduced considerably by properly choosing the number of samples taken. The number of data required appears to be independent of the magnitude of errors in velocity measurement in the range of 1-3%. The sample size required to accurately determine the fit would be in the range 20 to 30. The conclusion is based on the limiting trends indicated by the plot of standard error with the sample size.

5.1.1.5 Effect of the Range of Velocity

The effect of varying the range of velocity is
shown in figure 5.5. For the same absolute error and sample size, the effect of widening the range is to increase the standard error of the estimate. The contribution to the standard error of estimate comes from scatter in the data which is dependent upon the velocity due to the nonlinear nature of the curve. The effect of increasing the range by four fold increases the standard error by two folds. A slightly higher number of observations is required (approximately forty) to accurately determine the fit in this case. This is due to the fact that the absolute error assigned is greater than 3 per cent on a relative scale. This shows that a higher sample size would be required when greater errors are expected in the velocity. For a range of velocity that is large, Bruun (B9) has experimentally shown that subrange velocity calibration yields improved results. As his results are based on limited experimental results, it was decided to investigate this point further.

The results of subrange velocity calibration are shown in figure 5.6. For a fixed absolute error, the subrange velocity calibration is seen to improve the accuracy considerably. It can also be seen that for subranges of 20 m/s, the errors are less at higher velocities.

Finally, it should be pointed out that the quantitative results presented here pertain to the constants A, B and c assumed. However, should these values change, it is a simple matter to establish the sample size required using the
procedure indicated. Even though this study is directed to hot-wire users, the procedure indicated can be used for providing useful guidelines to general experiments of this nature.

5.1.1.6 Yaw Calibration

The method of determining the calibration constants (A, B, and c) and the yaw factor (k), based on the non-linear optimization technique developed, is compared with the method recommended by Jorgensen (J2) using Monte Carlo simulation of the calibration data. The calibration data was simulated using equation 3.6 with A=5.4, B=6.0, c=0.4 and k=0.1. The values of U, θ and the error in U were selected from the range 0-20 m/s, -30° to +30° and 0-5% respectively using a random number generator. The results are summarized in table 5.2. Comparing the sum of the error squared in U for both the methods, it is evident that the present method produces better calibration constants.

5.1.2 Evaluation of the Systems of Equations for Turbulence Measurement

5.1.2.1 Number of Trials Required

The number of trials required to generate a reliable pseudo turbulence field is based on the result indicated in figure 5.7. This figure gives the variation of $\overline{uv}_o$ with the number of trials. It is seen that the Reynolds shear stress attains a constant value beyond 10,000 trials. However, the number of trials was set at 15,000 allowing a factor of safety. This number was held constant for all
subsequent experiments. Holding this constant, the seed value required to initiate the random number generator was varied to ensure that no significant changes were observed.

5.1.2.2 Truncation Errors in Method I.

The results of the variation of \( f' \) and \( f_{\text{exact}} \) with turbulence intensity is given in figure 5.8. The percentage variation between the two is in the range of 0 to 1.5% as the intensity is changed from 0 to 50%, indicating that the second order corrections for the mean velocity yield accurate results for the range studied. The common assumption that \( f' = 1 \) is seen to be justified only up to a 10% turbulence intensity beyond which the errors in the mean velocity are significant. The variation of the errors introduced in the Reynolds stresses, calculated using method I, due to the truncation of the series expansion with turbulence intensity can be seen in figure 5.9 for the values of \( S, k \) and \( \bar{U} \) chosen. As expected method II produces insignificant errors in \( \bar{u}^2 \), \( \bar{v}^2 \) and \( \bar{uv} \). It is believed that these insignificant errors are due to roundoff errors. From this figure, it is seen that beyond 20% turbulence intensity, the errors due to truncation increase considerably. Errors in \( \bar{v}^2 \) and \( \bar{uv} \) due to truncation are seen to be higher compared to errors in \( \bar{u}^2 \). The error in \( \bar{w}^2 \) and \( \bar{v}^2 \) are almost identical and hence has not been included in the figure. For turbulence intensities below 20%, the truncation errors in Reynolds stresses can be held below 10% for the values of the constants chosen. By repeating the
numerical experiment; it is a simple matter to estimate the
truncation error should the values of $S$, $k$ and $\bar{u}$ change.

5.1.2.3 Effects of Measurement Errors.

The results obtained by choosing a uniform 1% measurement error in the quantities $S$, $k$, $\bar{u}$, $\bar{E}^2$ and $\bar{e}^2$ for the case of low and high turbulence intensities of 10% and 50% are given in tables 5.3 and 5.4. Comparison of the tables indicates that method II yields lower errors in the ensemble-averaged variables at high turbulence intensity and method I at low levels. From table 5.3 for low turbulence levels it is seen that for method I a 1% error in all the measurable quantities produces errors in Reynolds stresses which are comparable to the truncation errors (compare values in row one to the values in row seven of table 5.3). At low turbulence levels, method II yields considerable errors, especially in $\bar{u}^2$. For this case it is seen that method I is preferable. The system of equations for method II is ill-conditioned for the case of low turbulence and hence the results are sensitive to measurement errors. The major contributing factors to the overall error are the errors in $S$, the voltages and mean velocity in the case of method II. Table 5.4 gives the results for the high turbulence case. For method I, it is seen that the truncation errors are so high (values in the last row of the table) that the errors introduced in the various quantities have no effect on the final result. Method II, however, yields reasonable overall results for the high turbulence
case. Here again the major contributing factors are the errors in $S$, $\bar{U}$ and the voltages. It is consistently seen that effects of errors in $k$ are insignificant and that, for method II, measurement of $\bar{U}$ significantly affects the determination of $u^2$ especially for the low turbulence case.

The effect of turbulence intensity for the case of 1% uniform error in all the quantities is shown in figure 5.10. It can be observed that for method I the error in $\bar{u}^2$ is lower compared to $v^2$ and $\bar{uv}$ throughout the range of turbulence intensity studied. The opposite is true for method II. Below 20% turbulence level, method I is recommended and above 40-50% method II is recommended. It should be pointed out that the values of standard deviation in errors chosen may be unrealistic. Should a more reliable estimate be available, it is a simple matter to generate the results and obtain the range of validity of these methods with regards to the turbulence intensity. If the standard deviations of the errors in the measured quantities are known, then the Monte Carlo procedure, described earlier, can be used to estimate the uncertainties in the measured quantities. The method suggested here is an extension of the method proposed by Moffat (M5) for determining the uncertainties in the measurements. The method proposed here includes the use of a random number generator to simulate random errors within the limits specified and averaging the effects over a large number of trials. It is believed that this would yield a more reliable estimate of the
uncertainties in the measured quantities. The logic is simple and should pose no problem to incorporate it in the data reduction routine.

In the second experiment to determine the effects of errors, the standard deviations of the errors in \( S, k, \bar{U} \) and voltages were set at 10\%, 20\%, 1\% and 0.5\% respectively. These values correspond to the worst possible combination of the errors based on past experience. The results obtained by varying the errors in \( k \) between 0-20\% of the mean value of 0.2, keeping all the other errors at the chosen value, are shown in figure 5.11 for the case of both low and high turbulence intensities. It is seen for the values chosen, that the errors in \( k \) have an insignificant effect on the final results. The error values obtained in Reynolds stresses are high indicating that the errors assigned may be unrealistic. The effects of the variation of errors in \( S \) and voltages are shown in figures 5.12 and 5.13 respectively. From these plots, only the errors in \( S \) seem to have a significant effect on the accurate determination of the Reynolds stresses. No particular trend can, however, be discerned from this plot. For the combination of errors prescribed here, it can be seen from figures 5.10 to 5.12 that method 1 is recommended for both the low as well as the high turbulence level cases. A more rigorous procedure would be to repeat the numerical experiment for different combinations of the errors in \( S, \bar{U}, k \) and the voltages and to study the effects of variation of these errors.
5.2 Experiments

Now, the experimental results of calibrations and turbulence measurements will be presented and discussed.

5.2.1 Velocity Calibration

The results and discussion presented in this section pertain to velocity calibration.

5.2.1.1 Experimental Evaluation of the Non-linear Calibration Method

The velocity calibrations were carried out to verify the numerical results obtained. The objective was to see if the non-linear method yielded low errors in velocity for experimentally obtained calibration data in the low and high velocity ranges. Only one hot-wire was used for calibration in the low velocity range while three, henceforth termed Probe 1, Probe 2 and Probe 3, were used in the high velocity range. The calibration of Probe 2 was repeated three times in the high velocity range. The calibration was carried out using both the conventional and the non-linear methods on each data set. A typical result obtained by the conventional method is shown in figure 5.14 for the low velocity range along with the constants A, B and C obtained. Using these constants the sum of the errors squared in $U$ was calculated. The computer programme developed for the new method was also used to generate another set of constants and the SES in $U$ for each data set. All of the results obtained by both the methods for the two
velocity ranges have been summarized in tables 5.5, 5.6 and 5.7.

Comparing the SES in U obtained by both the methods (see the above mentioned tables), it is seen, that the new method gives about 4-14% smaller errors in U. The percentage difference in the sum of the errors squared between the two methods, is seen to be higher in the high velocity range from tables 5.5 and 5.6. The new constants A, B and C differ by about 3-28% from the constants obtained by the conventional method. The percentage difference in the velocities, predicted using these constants is about 1% at the lowest velocity and about 0.2% at the highest velocity. Table 5.7 shows that the calibration constants drift for the same probe obtained on different days although under almost identical conditions. The drift is believed to be due to factors such as dust particles settling on the hot wire. In order to minimise the drifts which may affect a proper comparison of the linearizer outputs, the calibration (number 2) was performed, the constants immediately determined and set on the linearizer output. From the results reported, it can be safely concluded that the new method consistently yields lower sum of the errors squared in U for the experimental calibration data as well. The results also prove that the nonlinear optimization technique can be successfully used to obtain reliable values of the constants. Further, the method provided a more systematic way of determining these constants.
The constants obtained by both the methods, for Probe 2 in the high velocity range, were set independently on a DISA linearizer. The linearizer output, $E_{lin}$, was recorded by varying the velocity in the range required. Figures 5.15a and 5.15b show the plots of the linearized outputs compared as a function of the velocity for both the velocity ranges. The SES in $U$, obtained for each case is indicated therein. The improvements in the linearizer output, using the constants obtained by the new method for the Probe 2, is evident from the figure. Considering the linearized outputs, the sum of the errors in $U$ of the new method is smaller than that obtained by the conventional method by 12.9%.

5.2.1.2 Comparison of the Heat Transfer Models

Now, the results of the comparisons of the various heat transfer models available for the velocity calibration are presented in tables 5.8 and 5.9 for the high and low velocity ranges, respectively. The tables also show the pertinent factors that were considered for the study such as the models used, the curve fitting technique adopted, etc.. The uncertainty in the parameter estimates are also presented along with the constants obtained in each case. The standard deviation reported is in velocity in all the cases.

In the high velocity range (table 5.8), comparison of the $\sigma$ for the various models indicate that they differ from one another by less than one standard
deviation. This should indicate that the differences in $\sigma$ are not significant in a statistical sense. For the three parameter estimate, however, the exponent and extended power law models yield the lowest $\sigma$. Comparison of the uncertainties in the estimated constants shows that King's law yields the lowest uncertainties in the calibration constants. In the polynomial heat transfer law, equation 2.27, the independent variable $E$ has been replaced by $E^2$. This change seems to provide lower errors in velocity for a given order of the polynomial. The reason for this can also be seen by rewriting King's law, equation 2.23, as

$$U^{0.5} = \frac{1}{B}E^2 - \frac{1}{B}A$$

$$U = \frac{1}{B^2}E^4 - \frac{2A}{B^2}E^2 + \frac{A}{B}$$

Equation 5.1 has the form of equation 2.27, the polynomial heat transfer model. In the low velocity range (table 5.9) the sum of the errors squared of the King's law model differs by at least one standard deviation from the rest. Hence the difference can be considered to be significant. The other observations made are valid in the low range as well.

The decision as to which model is to be preferred should be based on both the $\sigma$ as well as the uncertainty in the estimation of the parameters. In addition, further considerations such as the data processing instruments available, the ease of data reduction for turbulence measurement needs to be taken into account. The
use of the polynomial heat transfer law requires that higher order voltage correlations be measured. If analogue linearization of the hot-wire is to be carried out using the standard commercially available linearizer, such as the DISA 55D10, then use of the extended or polynomial laws lead to errors. Even though the criteria that errors in velocity should be minimized yields lower errors in an absolute sense, the differences are insignificant. This leads to considerable errors in the parameter estimated as can be seen from the errors in the estimates of the polynomial approach. The polynomial law shows the highest errors in the parameter estimates. In a statistical sense the nonlinear technique seems to offer no particular advantage other than the determination of all the constants simultaneously without trial and error.

In view of the results presented, it is seen that the extended power law yields a compromise between low standard deviation in \( U \) and low uncertainties in \( A, B \) and \( c \) for both the velocity ranges. The added advantage of using a polynomial curve fitting technique can also be seen. However, if one prefers to look at the problem of choosing a heat transfer model from a somewhat physical reasoning based on King's law rather than as a regression problem, the exponent power law comes closer to the actual heat transfer phenomena.

5.2.2 Yaw Calibration

The results of yaw calibrations are presented and discussed in this section.
5.2.2.1 Evaluation of the Non-linear Yaw Calibration Method

The nonlinear method outlined to obtain the optimised values of \( A, B, c \) and \( k \) simultaneously using the entire yaw calibration data, obtained over the required ranges of velocity and yaw angles, has been used. The results are presented in Table 5.10. The results pertain to a yaw angle range of \(-50^\circ\) to \(+50^\circ\) and a velocity range of 2-35 m/s. A normal wire was used in the calibration. Both negative and positive angles were covered to take into account any asymmetry in the probe construction. The results of four repeated calibrations of the same probe are presented in the table.

The nonlinear method adopted in the present study shows numerical convergence for the experimental data. Kjellstrom and Hedberg (K6), however, indicate that they were unsuccessful in such an attempt. The present study clearly indicates that reliable results can be obtained. In all the existing methods it has been implicitly assumed that the sign of \( k^2 \) is positive based on an oversimplified physical reasoning. As can be observed, the present study yields a negative value for \( k^2 \). Bruun and Tropea (B9) reported a similar result, though their approach was different. They, as mentioned earlier, made a simplifying assumption in the calibration method that \( A, B \) and \( c \) were invariant with yaw angle \( \alpha \) and obtained the result of a negative \( k^2 \). The present method yields the same result by treating the complete set of data, taken together and
obtained an optimum set of values for A, B, c and k valid
for the entire range of experimentation. The negative value
of $k^2$ implies that the velocity to which the hot-wire
responds is less than that of the normal component to the
wire, i.e., $U \cos \alpha$. This is contrary to the popular notion.
The variations of A, B and c with $\alpha$ are sufficient to change
from a positive value of $k^2$ to a negative value. There is a
need, however, for a careful experiment to investigate this
phenomena. The heat transfer experiments of the type
conducted by Champagne (Cl) needs to be carried out but with
the prong interference and its heat conduction charateristics simulated.

The advantages of the present method are as
follows. The constants are obtained in a systematic manner
by minimising the errors in the required variable. The ad
hoc method of determining the directional sensitivities and
the associated assumptions required to be made are
eliminated. The algorithm developed is suitable for use on
an on-line computer.

5.2.2.2 Accuracy of the Present Method

The error in mean velocity for the yaw calibration
is plotted as a function of the yaw angle in figures 5.16
and 5.17. The error in the mean velocity is the difference
between the measured value and that computed using the
equations, obtained from the curve fit, and the voltage
values. The results of the errors in mean velocity obtained
using $k=0.0$, i.e., the cosine law and $k=0.2$, the value
recommended by Champagne (C1), has also been presented in the same figures. The constants A, B and c used for the case of \( k=0.0 \) or \( k=0.2 \), were obtained from the velocity calibration performed at \( \alpha = 0^\circ \). Figure 5.16, for the low velocity case of 8 m/s, indicates that the error in \( U \) in using any one of the \( k \) values is comparable. At the high velocity of 30 m/s (figure 5.17), however, the present method yields lower errors in velocity over the entire range of \( \alpha \) tested. The use of \( k=0.2 \) leads to an error of more than 2% in the mean velocity beyond \( \alpha = 40^\circ \). The use of \( k=0 \) is justified in the range tested and yields results closer to the present ones.

5.2.2.3 Comparison of Different \( U_{\text{eff}} \) Models

The various models for the effective velocity have been compared in table 5.11 based on the experimental data. The non-linear method was used as this has been shown to yield reliable results. The software was modified to accommodate several models. All the models used give comparable SES in \( U \) and hence compare favourably.

5.2.2.4 Measurement of \( k \) in the Fully Developed Pipe Flow

The relationship for the shear stress, \( \bar{\tau} \), from the system of equations for method I, equation 3.4 is

\[
\bar{\tau} = \frac{[e_{xy}(\alpha=45) - e_{xy}(\alpha=-45)]}{[2s^2(1-k^2)]} \quad (5.2)
\]

This equation can be written as

\[
k^2 = 1 - \left( \frac{e_{xy}(\alpha=45) - e_{xy}(\alpha=-45)}{s^2\bar{\tau}} \right) \quad (5.3)
\]

By measuring the pressure drop, \( \Delta P \), at the pipe wall over an axial distance \( L \) in the fully developed zone, the shear
stress at the wall can be computed using the approximate expression

$$\overline{uv} = \frac{(\Delta P \cdot R)}{2L}$$ (5.4)

where $R$ is the radius of the pipe. It is known that the shear stress drops linearly from this value at the wall to zero at the centre of the pipe (reference S5). Hence, equation 5.3 was used along with the predicted value of $uv$ at the appropriate position $r$, the sensitivity of the hot-wire $S$ and the measured values of the mean square voltages to calculate $k^2$. The results obtained for two different slant hot-wires are presented in table 5.12. The calibration of probe #1 was repeated twice. The results indicate that the yaw factor is a weak function of the position in the flowfield. Comparison of the values for probes 1 and 2 shows that the difference is more than 100%. Negative value was also obtained as can be seen. The discrepancies observed are likely to be due to the accurate initial orientation of the wire, with respect to the mean velocity, required and any asymmetry present in the flow. There is a need for a better traverse mechanism capable of micro-manipulation of the pitch and yaw angles of the hot-wire to obtain good alignment. With the present experimental set-up this was not possible and further this approach is only restricted to inclined wires. In general it can be stated that accurate determination of the yaw factor is a difficult task.

5.2.3 Turbulence Measurements in Pipe and Jet Flows

The results of the turbulence measurements in the pipe and jet flows are presented in this section.
5.2.3.1 Pitot Traverse

The results of a Pitot traverse, to determine the velocity profiles in the pipe and jet flows and hence to ascertain whether these flows can be considered fully developed and axisymmetric, are presented in figures 5.18 and 5.19. The results of the mean velocity profiles obtained along two orthogonal diameters and in two different x-planes are plotted on a non-dimensional scale. The results indicate that these flows can be safely considered to be axisymmetric and fully developed. The pipe flow shows a slight asymmetry on one side of the pipe. Measurements in the pipe were, however, made along the half that had a better agreement.

5.2.3.2 Mean Velocity Measurements

The mean velocity profiles measured in the fully developed zone of the pipe and the jet are shown in figures 5.20 and 5.21 respectively. The velocities measured using hot-wire and Pitot probe are compared. The hot-wire data have not been corrected for turbulence intensity.

These measurements show good agreement for the pipe case. In the case of the jet flow, the difference between the two measurements varies between 0% at the centre of the jet to as high as 30% at y/x of 0.14. This indicates the need for corrections due to high turbulence intensities. The Pitot probe measurements are also subjected to similar errors. These corrections, if incorporated, will move these curves closer. Further, results obtained at large values of
y/x are unreliable because of intermittency and eddies present near the jet boundaries.

5.2.3.3 Comparison of Turbulence Measurements

In this section the turbulence quantities measured in the pipe and jet flows, by the two methods, are compared. The present results have also been compared with the results available in the literature.

5.2.3.3.1 Pipe Flow

The turbulence measurements in the pipe flow by method 1 have been compared with the results of Islam (I2), Patel (Pl), Tucker (T1), Laufer (L3) and Acritelli (A1) in the figures 5.22 to 5.25. These authors have also measured turbulence using a similar procedure based on the binomial expansion of the response equation. Except the profile of $\sqrt{\langle u'^2 \rangle}$, all the other turbulence profiles obtained in the present study compare favourably with others' results. The two trials of the present study show good repeatability. The scatter in the results shown is considerable for all the quantities, for example it is about 15% for $\sqrt{\langle u'^2 \rangle}$ and $\langle uv \rangle$ at an r/R of 0.6. The uncertainty in the measurement of these quantities is about 5% (see figure B.3 in Appendix B) and hence can only partly explain the scatter. The unexplained scatter can be due to other effects such as the Reynolds number, different geometric and kinematic conditions, equipment used, empirical corrections applied, etc. There is a need for a higher order uncertainty analysis and better control of these experiments in order to improve the
results. From the comparisons presented two observations are in order. The scatter in the $\sqrt{u^2}$ measurements is comparable to other quantities even though $\sqrt{u^2}$ is believed to be measured more accurately than the other quantities. The other observation of this study is that the $\sqrt{v^2}$ data alone do not compare well with those obtained by others, in spite of the fact that the same raw data were used in determining all the turbulence quantities. These observations point to the possibility that some quantities are more susceptible to measurement errors than others.

Measurements of turbulence in the pipe, using method II, has been carried out by Acrivellesis (A1). His results, however, are in error due the approximate equations used. Acrivellesis assumed that

$$E^2_{xy}(\alpha) = s^2 U^2(\cos^2\alpha + k^2 \sin^2\alpha) \quad (5.5)$$

and arrived at a system of equations (equations 30 - 32 of reference (A1)) which is valid only for the case of

$$\left(\frac{v^2}{U^2}\right) \text{ and } \left(\frac{w^2}{U^2}\right) < 1$$

as was shown by Bartenwerfer (B1). Hence Acrivellesis' results are subjected to a systematic error, the magnitude of which is dependent on the turbulence intensity. Since turbulence in the pipe is low, these errors are likely to be small. Even, accounting for this error, the results of Acrivellesis showed considerable discrepancy in the two methods of measurements. Rodi (R3), with the help of a simple error analysis, has shown that the new method is inapplicable for low intensity flows. The error analysis
presented in Appendix B and the numerical results presented earlier, more rigorously, confirm this point. The experimental results of the present study for the quantities $\bar{u}^2$ and $\sqrt{\bar{v}^2}$ using method II are given in Table 5.13 and figure 5.26 respectively. This further substantiates the inapplicability of method II for the case of low intensity flows. The results of the $\bar{uv}$ profiles, however, compare well as can be seen from the figure 5.27. This is because, the equations for $\bar{uv}$ for the two methods (equations 3.4 and 3.5), are identical for the case of $E^2(\alpha=45) = E^2(\alpha=-45)$. This is very nearly satisfied in the case of the pipe flow due to the moderate turbulence levels present in such flows. Further comments and discussion regarding method II will be presented in the next section in connection with the jet flow.

5.2.3.3.2 Jet Flow

The results of the turbulence measurement in the jet flow obtained using both the methods are presented and compared with the results of Wygnanski (W4), Rodi (R3) and Sampath et al (S1) in figures 5.28 to 5.31. The results of Wygnanski were obtained by method I and those of Rodi and Sampath by using method II.

The following discussion pertains to the comparison of the results of the present study, obtained by method I, with those of Wygnanski. The comparison of the $\sqrt{\bar{u}^2}$ profile, given in figure 5.28, shows a discrepancy of nearly 20% near the centre of the jet. This is likely to be due to the
poor low frequency response of the anemometer system used. More than 23% of the turbulent energy is contained below the frequency of 5 Hz and hence an improperly set anemometer for low frequency will act as a filter introducing errors in the measurement (reference (W4)). The uncertainty estimate in \( \sqrt{u^2} \) decreases from 12% at the centre to 4% at \( y/x \) of 0.12, figure B.4, conforming with the trend shown in figure 5.28. The trend shows a good agreement at the jet boundary and a poor one at the centre. From figure 5.29 the difference in the \( uv \) value at the peak, \( y/x \) of 0.05, is about 20% and the uncertainty in measurement is about 10% excluding the error due to the high turbulence intensity. If this value of 10% is also added, the total error will be 20%. Hence from a statistical point of view the apparent discrepancy is not significant. The profiles of \( \sqrt{v^2} \) and \( \sqrt{w^2} \) show good agreement over the whole region (figures 5.30 and 5.31) and the differences found can be explained.

The comparison of the results obtained by method II and those obtained by Rodi (R3) and Sampath et al (S1) are discussed. In general, the present results show poor agreement, the worst being the result for \( \sqrt{u^2} \) (figure 5.26). The profiles of \( \sqrt{v^2} \) and \( \sqrt{w^2} \) show good agreement only in the outer region (\( 0.06 < y/x < 0.16 \)). An important observation is that the results of Rodi and Sampath agree extremely well for all the quantities except the shear stress. This value shows a difference of anywhere between 20-50% over the region of measurement. In addition, with
respect to Wygnanski's results for $\overline{uv}$ Rodi's results shows an increase. Rodi justified this increase numerically using an integrated momentum equation for $\overline{uv}$. On the contrary, Sampath's result shows a decrease. This points to either a basic inconsistency in the measurement of Sampath or the method itself. As pointed out earlier in connection with the numerical results, method II is represented by a system of ill-conditioned equations. The system is well described for the case of high turbulence levels. Hence, the present results for $\sqrt{v^2}$ and $\sqrt{w^2}$ show good agreement close to the jet boundary where the turbulence is high. The opposite is true for the region near the centre where low turbulence exists. The reason for the wider discrepancy in the results of $\sqrt{u^2}$, both in magnitude and shape, is the additional constraint on the accuracy of the mean velocity required, besides the turbulence level. At the centre of the jet the turbulence is relatively low, 25%, and close to the jet boundary the mean velocities are low. Further, near the boundary the turbulence is high and eddies are present, which causes a large error in mean velocity measurement with any instrument.

Rodi used a graphical solution method for the system of equations representing method II. This required an elaborate yaw calibration at each point in the flow field. He also used a squarer in the measurement section and long integration times to obtain stable averages. The present study was limited by the instruments in that only integration time constant of 100 s was available. Another
important difference is that Rodi determined $\bar{U}$ more accurately by assuming that the third order correlations to be small compared to the mean velocity. In the present study, the use of Pitot probe was investigated. The exact method followed by Sampath et al in solving the equations and the instruments used have not been mentioned. Hence, no specific comments can be made about their results.

The following reasons are likely to have contributed to the wide discrepancy in the present measurements of turbulence using the approach of method II.

1. The ill-conditioned system of equations used.

2. The errors in the positioning and rotation of the slant hot-wires. There was no monitor to ensure that these conditions are met in the present experimental set-up.

3. The use of Pitot probe to measure mean velocity. There is a limitation on its use in highly turbulent flows.

4. Errors due to the assumption of $\bar{V}=\bar{W}=0$ which is strictly not true in a jet.

5. Magnification of the errors in the voltages since the values of $\bar{E^2}$ required for method II are obtained by adding $\bar{E^2}$ and $e^2$. That is, $E$ values are squared to obtain $\bar{E^2}$.

5.2.3.4 Effect of Yaw Sensitivity Factor

The effect of the factor $k^2$ on turbulence measurement is shown in figures 5.26 and 5.27 for the pipe case and in figures 5.32 to 5.34 for the jet case. The effect of using the value of $k^2$ obtained by the non-linear yaw calibration method, the value zero for the cosine law
and the value 0.04 recommended by the Champagne (Cl) has been compared in these figures. It can be observed that method I is insensitive to the value being used. The differences observed can be shown to be statistically insignificant. For the case of pipe flow, the profile of $\sqrt{v^2}$ in figure 5.26 obtained by method II is sensitive to the value of $k^2$ used. The profiles of $\sqrt{u^2}$ and $\sqrt{w^2}$ are also expected to follow the same trend. The yaw factor has, however, effect on $uv$ for method II as can be seen from figure 5.27. In the case of the jet flow the results are not affected by the value of the yaw factor in both the methods. This is not surprising since the stress levels are high. The experimental results complement the numerical results in this respect. The value of the yaw factor exerts no influence on the determination of turbulence.

From the foregoing discussion it can be seen that method I is robust and yields reliable results. This approach is insensitive to several factors and with provision to correct for high turbulence intensity, is likely to provide better results. The uncertainty analysis also points to this fact. Method II requires careful measurements. In any flow, the local turbulence level varies over a wide range. The method II being extremely sensitive to the turbulence level will prove to be unreliable. Even the elaborate approach of Rodi (R3) showed insignificant differences with the conventionally obtained results based on the uncertainty analysis. Hence, the tremendous care and
the need for additional instrument is truly not worth the effort. The results of the present study indicate why method II is not used even though the idea was introduced a decade ago.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The Monte Carlo procedure has been successfully used to solve some problems existing with hot-wire measurements by simulating the hot-wire response and turbulence data on a computer. This procedure has been used to address some problems of calibration and to evaluate the relative merits of two methods of obtaining the time averaged turbulence quantities. Based on the results presented for calibration studies, a sample size of 20-30 is recommended for practical purposes as a compromise between accuracy requirements and time consumption. When hot-wires are to be used over a large velocity range, subrange velocity calibration leads to a considerable improvement in the accuracy of velocity measurement. The spacing within a subrange has negligible effect on the accuracy of the calibration constants obtained.

The comparison of the series expansion and the squaring methods of turbulence measurement also included an estimation of the truncation error in the conventional series expansion method, the effects of varying the turbulence intensity and the measurement errors on the results. The results indicate that the truncation errors in method I for Reynolds stresses are less than 10% when the turbulence levels are below 20%. For higher values of turbulence levels, the error increases rapidly. For mean
velocity measurement by the series expansion method (method I), the truncation errors are below 1.5% for turbulence intensities as high as 50%. The choice of the method should be based on both the turbulence levels and the measurement errors expected. If the measurement errors are of the order of 1% in all quantities, then method I is recommended for turbulence levels below 20% and the squaring method (method II) for levels above 40-50%. Between 20-40% turbulence level either method is suitable. If measurement errors are expected to be high (i.e., of the order 10-20% in S and k.) method I proves to be relatively better. To obtain accurate results, the hot-wire sensitivity (S) and the voltages have to be determined as accurately as possible. The error in the yaw sensitivity (k) has negligible effect on the results obtained.

From the experimental results presented, the following conclusions can be drawn. The non-linear velocity and yaw calibration methods have been tested for reliability and accuracy. These methods are systematic and easier to use compared to the existing methods. The results of the yaw calibrations indicate that the yaw factor can assume a negative value. This implies that a yawed hot-wire responds to a velocity component which is less than that of the velocity component normal to the wire.

A comparison of the various heat transfer models shows that for proper comparison both the (SES)_y and the uncertainty in the estimated parameters have to be
considered. The results indicate that the extended power law yields the best compromise between low standard deviation in velocity and low uncertainty in the estimated parameters. The polynomial model yields the lowest standard deviation in velocity, but the uncertainties in the parameter estimations are considerable. From the statistical point of view, the differences in the values of the standard deviations for the various models are not significant. All the effective velocity models considered show favourable comparison.

The analysis of the measurements made in the pipe and the jet using the two methods of signal interpretation, clearly indicates that method I is robust and yields reliable results. This method is relatively insensitive to several factors such as the errors in calibration, hot-wire settings, etc. The use of method II requires careful measurements and is only suitable for flows with high mean velocity and turbulence levels. Method I with provision to correct for high intensity of turbulence will provide better results. Except in the case of method II for low turbulence, the yaw sensitivity factor has negligible effect on the determination of turbulence. The experimental results of the comparison of the two methods complements the numerical results, substantiating the conclusions drawn in this regard.

Finally, the study indicates the need to include uncertainty analysis in turbulence measurements to make proper comparisons and draw valid conclusions. A new
approach based on the Monte Carlo technique has been introduced to estimate the uncertainties in measurements.

6.2 Recommendations

The present study reveals that the Monte Carlo technique can provide a viable means of generating a set of turbulence data that could be used as a standard. This new approach can provide an insight into the various problems existing in the area of turbulence measurement and possibly in the other areas of turbulence as well. A detailed study on the use of the Monte Carlo approach in simulating the characteristics of turbulence on a computer should be carried out. Some of the effects that could be incorporated to simulate turbulence more realistically are the skewness in the distributions of the fluctuating quantities, the flatness factor, etc.
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R1 Rasmussen, C.G. and Madsen, B.B., "Hot-Wire and Hot-Film Anemometry: An Introduction to the Theory and Application of DISA Constant-Temperature Anemometer", Manual Published by DISA Electronik.
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Figure 1.1 Nomenclature for Jet Flow

(See Reference (12))

Figure 1.2 Nomenclature for Pipe Flow
Figure 2.1 Schematic of Wire-Oriented Velocity Components
Figure 3.1 Logic Diagram: Non-Linear Yaw Calibration
Figure 3.2 Logic Diagram: Comparison of the Two Methods of Signal Analysis Using Monte Carlo Technique
A

Is J = 1?

Yes

Calculate $u^2, v^2, w^2$
Determine Voltages $E_0, E_1, E_2$, $E_3$ & $E_4$ Using Equation 3.15
Keep the Sum: $u, v, w, u^2$, $v^2, w^2, \ldots, E_0, E_1, \text{etc.}$

Yes

Determine Mean Voltages

$
A_{AE} = \frac{E_0}{N_1}
$

$B_{B1} = \frac{E_1}{N_1}, B_{B2} = \ldots, \text{etc.}
$

Yes

Is J = 3?

Yes

Calculate Mean Values of $u, v, w$

$E_{P1} = \frac{u}{N_1}, E_{P2} = \frac{v}{N_1}, E_{P3} = \frac{w}{N_1}$

No

Is J = 2?

Yes

Calculate Standard Deviations in $u, v, w$:

Determine: $Z_1 = SU/STDU, Z_2 = SV/STDV, Z_3 = SW/STDW$

Modification of Random Number Generator Ending

Figure 3.2 Continued
Figure 3.2 Continued
Use Random Number Generator and
33,3K, SUB & SE and Introduce Errors
S=S, UB-UB SUB, etc.,

Determine Stresses by Method I Using
Equation 3.16:
\[ u_1^2 = U21, \ldots, v_1^2 = V21, \ldots, \text{ etc.} \]

Determine Stresses by Method II Using
Equation 3.17:
\[ U22, V22, \text{ etc.} \]

Determine Average Values of the Stresses
\[ u_1^2 = \frac{U21}{N2} = \frac{SU1}{N2}, \ldots \]
\[ u_2^2 = \frac{U22}{N2} = \frac{SU2}{N2}, \text{ etc.} \]

Determine Deviations from the
Standard Values:
\[ u_1^2 = \frac{[(u_1^2 - u_0^2) \times 100]}{u_0^2} = \frac{[(SU1 - U20) \times 100]}{U20}, \text{ etc.} \]

Print All Deviations
Stop

Conclusion of Figure 3.2
Figure 3.3 Logic Diagram: To Determine the Sample Size Using Monte Carlo Technique.
Figure 4.2 Yaw-Mechanism Used in Calibration
Figure 4.3 Overview of the Air Jet Facility
Figure 4.4 Experimental Set-Up for Calibration of Hot-Wires
Figure 4.5 Schematic of the Pipe Flow Facility

- **Blower**
- **Flexible Tube**
- **Settling Chamber**
- **Supports**
- **Static Pressure Taps**
- **Hot-Wire Probe**
- **Pipe**
  - Length: 15.24 m
  - Diameter: 10.8 cm
- **Dimensions**: 1.65 m x 0.91 m x 0.91 m
- **C.T.A System**

Flow direction indicated by arrows.
Figure 4.6 Traverse Mechanism at the Exit of the Pipe Flow Facility
Figure 4.7  C.T.A. System and Other Auxiliary Measuring Units
Figure 5.1 Effect of Sample Spacing

Range: 0-20 m/s
Error: 2% of range
Method: Linear
Spacing: Uniform, Random
Figure 5.2 Comparison of Linear and Non-Linear Methods of Calibration
Figure 5.3 Variation of Relative Error in Velocity with Sample Size
Method: Linear
Range: 0-20 m/s
Spacing: Random
Error: • 1% of Range
     ▲ 2% of Range
     ■ 3% of Range

Figure 5.4 Effect of the Magnitude of Error in Velocity
Method: Linear
Absolute Error: 0.8 m/s
Spacing: Random

Figure 5.5 Effect of Velocity Range
Figure 5.6 Effect of Subrange Velocity Calibration
Figure 5.7. Effect of Number of Trials on $\overline{uv}$

Standard Deviations in $u$, $v$ and $w = 2.5 \text{ m/s}$

- $k = 0.2$
- $S = 0.3 \text{ Volts/m/s}$
- $h = 1.0$
- $\overline{U} = 25 \text{ m/s}$
Figure 5.8 Variation of Turbulence Correction Factor for Mean Velocity with Turbulence Intensity
Figure 5.9 Effect of Turbulence Intensity on the Measurement of Reynolds Stresses (0% Error)
Figure 5.10 Effect of Turbulence Intensity on the Measurement of Reynolds Stresses (1% Error)
Error in $S = 10\%$
\[ \frac{e^2}{\bar{u}} = 1\% \]

and $E^2$ and $e^2 = 0.5\%$

---

Figure 5.11 Effect of Variation of Errors in $k$ on Reynolds Stresses
Figure 5.12 Effect of Variation of Errors in S on Reynolds Stresses

Error in $k = 20\%$

$\bar{U} = 1\%$

and $\bar{E}^2$ and $\bar{e}^2 = 0.5\%$

Percent Standard Deviation in $\bar{u}$ and $uv$

Percent Error in S

Method I  Method II  Quantity

Low Turbulence Case

High Turbulence Case

$\bar{u}^2$  $\bar{u}v$  $\bar{u}^2$  $\bar{uv}$
Figure 5.14 Plot of SES in $E^2$ Versus $c$

At SES Minimum

$A = 5.20$

$B = 5.98$

$c = 0.385$
Figure 5.15 Outputs of a Linearizer for High Velocity Range

- **a.** Setting Conventional Constants
- **b.** Setting New Constants
Figure 5.16  Effect of Yaw Sensitivity Factor on Mean Velocity Measurements: Low Velocity
Figure 5.17 Effect on Yaw Sensitivity Factor on Mean Velocity Measurement: High Velocity

- Present ($k^2 = -0.05$)
- Champagne ($k^2 = 0.04$)
- Cosine Law ($k^2 = 0.00$)

$U = 30.38 \text{ m/s}$
Figure 5.19 Mean Velocity Measurements in Jet Flow Using Pitot Probe
Figure 5.20  Pipe Flow: Comparison of Mean Velocity Profiles
Figure 4.21 Jet Flow: Comparison of Mean Velocity Profiles

x/D = 75

- Hot-Wire
- Pitot Probe
Figure 5.22 Distribution of Longitudinal Turbulence Intensity Across the Pipe: Comparison of the Results
Figure 5.23: Distribution of Shear Stress Across the Pipe: Comparison of the Results.
Figure 5.24 Distribution of the Transverse Turbulence Intensity Across the Pipe: Comparison of the Results
Figure 5.25 Distribution of Binormal Turbulence Intensity Across the Pipe: Comparison of the Results
Figure 5.26 Pipe Flow: Comparison of the Data Analysis Methods and the Effect of $k (\sqrt{\kappa} / U_e$ Versus $r/R$)
Figure 5.27  Pipe Flow: Comparison of the Data Analysis Methods and the Effect of $k$

$\frac{u v}{U_2} \text{ Versus } \frac{R}{R}$
Figure 5.28 Jet Flow: Comparison of the Data Analysis Methods ($\sqrt{\frac{u'^2}{U_m^2}}$ Versus $y/x$)
Figure 5.29 Jet Flow: Comparison of the Data

Analysis Methods $\frac{u' v'}{U'^2_m}$ Versus $y/x$
Figure 5.30 Jet Flow: Comparison of the Data Analysis Methods ($\frac{\sqrt{y^2}}{U_m}$ Versus $y/x$)
Figure 5.31 Jet Flow: Comparison of the Data Analysis Methods
\( \sqrt{\frac{\nu^2}{U_m^2}} \) Versus \( \frac{y}{x} \)
Figure 5.32 Jet Flow: Effect of Yaw Factor on Longitudinal Turbulence Intensity
Figure 5.33 Jet Flow: Effect of Yaw Factor on Traverse Turbulence Intensity
Figure 5.34 Jet Flow: Effect of Yaw Factor on Shear Stress
<table>
<thead>
<tr>
<th>No.</th>
<th>Effect Studied</th>
<th>Sample Size</th>
<th>Standard Deviation of U</th>
<th>Range of U (m/s)</th>
<th>Curve Fitting</th>
<th>Sample Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5, 7, 10, 15, 20, 30, 40 &amp; 50</td>
<td>1%, 2% &amp; 3% of Range</td>
<td>0-20</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>1</td>
<td>Magnitude of Errors in U</td>
<td>Same as above</td>
<td>Absolute: 2% of U</td>
<td>0-20, 0-40, 0-80</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>Range of U</td>
<td>Same as above</td>
<td>Same as above</td>
<td>0-20, 20-40, 40-60</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>Subrange Velocity Calibration</td>
<td>Same as above</td>
<td>Same as above</td>
<td>0-20, 20-40, 40-60</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>Curve Fitting Technique</td>
<td>Same as above</td>
<td>2% of Range</td>
<td>0-20, 0-20</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>5</td>
<td>Sample Spacing</td>
<td>Same as above</td>
<td>2% of Range</td>
<td>0-20</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

x Indicates Work Done

Table 3.1 Calibration Studies: Summary of Numerical Experiments
<table>
<thead>
<tr>
<th>No.</th>
<th>Effect Studied</th>
<th>Errors Introduced in $S$, $k$, $U$, $E$, $\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Truncation Error in Method I</td>
<td>$0%$ $0%$ $0%$ $0%$ $0%$</td>
</tr>
<tr>
<td>2</td>
<td>Effect of Turbulence on Mean Velocity Measurement</td>
<td>$0%$ $0%$ $0%$ $0%$ $0%$</td>
</tr>
<tr>
<td></td>
<td>Contribution of $S$</td>
<td>$1%$ $0%$ $0%$ $0%$ $0%$</td>
</tr>
<tr>
<td></td>
<td>Contribution of $k$</td>
<td>$0%$ $1%$ $0%$ $0%$ $0%$</td>
</tr>
<tr>
<td></td>
<td>Contribution of $U$</td>
<td>$0%$ $0%$ $1%$ $0%$ $0%$</td>
</tr>
<tr>
<td></td>
<td>Contribution of Voltages</td>
<td>$0%$ $0%$ $0%$ $1%$ $1%$</td>
</tr>
<tr>
<td>3</td>
<td>Uniform Errors in All the Quantities</td>
<td>$1%$ $1%$ $1%$ $1%$ $1%$</td>
</tr>
<tr>
<td></td>
<td>Variation of Errors in the Range</td>
<td>$0-10%$ $0-20%$ $0-5%$ $0-0.5%$ $0-0.5%$</td>
</tr>
</tbody>
</table>

Note: In all the cases turbulence intensity was varied between 10-50%.

Table 3.2 Comparison of the Two Methods of Signal Analysis: Summary of Numerical Experiments
<table>
<thead>
<tr>
<th>No.</th>
<th>Experiment Conducted</th>
<th>Conditions of Experiment</th>
<th>Facility Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comparison of Optimization Techniques</td>
<td>Over Heat Ratio = 0.8</td>
<td>2.54cm Jet</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Velocity Ranges:</td>
<td>DISA C.T.A. System/Probe</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 - 10 m/s</td>
<td>Pitot-Static Probe</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 - 37 m/s</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Comparison of Heat Transfer Models</td>
<td>Over Heat Ratio = 0.8</td>
<td>Same as Above</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Velocity Ranges:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 - 35 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 - 100 m/s</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Yaw Calibration and Comparison of Ueff Models</td>
<td>Velocity Range:</td>
<td>Same as Above Plus Yaw Mechanism</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3 - 30 m/s</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Yaw Angle Ranges:</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-90 = +90 deg.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-50 = +50 deg.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Yaw Calibration in Turbulent Pipe Flow</td>
<td>Re = 1.86x10^5</td>
<td>Turbulent Pipe Flow Facility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fully Developed Flow</td>
<td>DISA C.T.A. System</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plane of Measurement</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x/2R = 140</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Turbulence Measurement in Fully Developed Pipe Flow</td>
<td>Same as Above</td>
<td>Same as Above</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Turbulence Measurement in Fully Developed Free Jet</td>
<td>Re = 10^5</td>
<td>DISA C.T.A. System</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ma = 0.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Plane of Measurement</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>z/d = 75</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 Details of the Experimental Programme
<table>
<thead>
<tr>
<th>No.</th>
<th>No. of Trials</th>
<th>( \bar{v} ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>0.52</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>0.54</td>
</tr>
<tr>
<td>3</td>
<td>5000</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>10000</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Sample Size: 5  
Error: 2% Of the Range  
Range: 0-20 m/s  
Method: Linear  
Spacing: Random

Table 5.1 Effect of the Number of Trials on the Standard Error of Estimate
<table>
<thead>
<tr>
<th>Method</th>
<th>A</th>
<th>B</th>
<th>c</th>
<th>k</th>
<th>(SES)_U</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jorgensen</td>
<td>5.54</td>
<td>5.91</td>
<td>0.404</td>
<td>0.052</td>
<td>0.425</td>
</tr>
<tr>
<td>Present</td>
<td>5.39</td>
<td>6.03</td>
<td>0.400</td>
<td>0.100</td>
<td>0.308</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>2.70</td>
<td>-2.01</td>
<td>0.991</td>
<td>-92.3</td>
<td>27.5</td>
</tr>
</tbody>
</table>

Table 5.2 Comparison of Yaw Calibration Methods Using Simulated Data
<table>
<thead>
<tr>
<th>Remarks</th>
<th>Method 1</th>
<th>Method II</th>
<th>Method II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\bar{u}^2}{\bar{v}^2}$</td>
<td>$\frac{\bar{u}^2}{\bar{v}^2}$</td>
<td>$\frac{\bar{u}^2}{\bar{v}^2}$</td>
</tr>
<tr>
<td>$1%$ Error in $S$, $k$, $U$, $\bar{k}^2$ &amp; $\bar{e}^2$</td>
<td>2.3</td>
<td>3.3</td>
<td>3.4</td>
</tr>
<tr>
<td>$1%$ Error in $S$</td>
<td>2.0</td>
<td>2.4</td>
<td>2.7</td>
</tr>
<tr>
<td>$1%$ Error in $k$</td>
<td>0.5</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>$1%$ Error in $U$</td>
<td>0.5</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>$1%$ Error in $\bar{k}^2$</td>
<td>0.5</td>
<td>1.4</td>
<td>1.9</td>
</tr>
<tr>
<td>$1%$ Error in $\bar{e}^2$</td>
<td>1.1</td>
<td>2.6</td>
<td>2.8</td>
</tr>
<tr>
<td>$0%$ Error in all the Quantities</td>
<td>0.5</td>
<td>1.4</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 5.3 Contribution to Errors in Reynolds Stresses by Various Parameters (10% Turbulence Intensity)
<table>
<thead>
<tr>
<th>Remarks</th>
<th>Method I</th>
<th></th>
<th></th>
<th>Method II</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\frac{u^2}{u}$</td>
<td>$\frac{v^2}{v}$</td>
<td>$\frac{w^2}{w}$</td>
<td>$\frac{uv}{uv}$</td>
<td>$\frac{u^2}{u}$</td>
</tr>
<tr>
<td>1% Error in $S, k, u, \bar{e}^2$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>17.3</td>
<td>4.8</td>
</tr>
<tr>
<td>1% Error in $S$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>10.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1% Error in $k$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>1% Error in $\bar{U}$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>8.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1% Error in $\bar{e}^2$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>11.6</td>
<td>4.4</td>
</tr>
<tr>
<td>1% Error in $\bar{e}^{-2}$</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>0% Error in all the Quantities</td>
<td>18.0</td>
<td>46.7</td>
<td>45.2</td>
<td>48.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 5.4 Contribution to Errors in Reynolds Stresses by Various Parameters (50% Turbulence Intensity)
<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>(SES)\text{U}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional</td>
<td>5.20</td>
<td>5.98</td>
<td>0.385</td>
<td>0.0769</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>New</td>
<td>6.13</td>
<td>5.19</td>
<td>0.418</td>
<td>0.0734</td>
</tr>
<tr>
<td>Method</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent</td>
<td>17.8</td>
<td>-13.2</td>
<td>8.57</td>
<td>-4.55</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 5.5 Low Velocity Range: Comparison of the Calibration Constants Obtained by the New and Conventional Methods
<table>
<thead>
<tr>
<th>Probe #1</th>
<th>Probe #2</th>
<th>Probe #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>SES</td>
<td>SES</td>
<td>SES</td>
</tr>
<tr>
<td>Conventional Method</td>
<td>6.57</td>
<td>5.57</td>
</tr>
<tr>
<td>New Method</td>
<td>5.52</td>
<td>6.31</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>16.0</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Table 5.6 High Velocity Range: Comparison of the Calibration Constants for Three Different Probes
<table>
<thead>
<tr>
<th></th>
<th>Calibration #1</th>
<th>Calibration #2</th>
<th>Calibration #3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Conventional Method</td>
<td>6.64</td>
<td>6.31</td>
<td>0.386</td>
</tr>
<tr>
<td>New Method</td>
<td>5.83</td>
<td>6.88</td>
<td>0.371</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>-12.2</td>
<td>9.00</td>
<td>-3.89</td>
</tr>
</tbody>
</table>

Table 5.7 High Velocity Range: Repeated Calibration of Probe #2
<table>
<thead>
<tr>
<th>No.</th>
<th>HEAT TRANSFER LAW</th>
<th>CRITERIA FOR LEAST SQUARES FIT</th>
<th>METHOD OF CURVE FITTING</th>
<th>NO. OF PARAMETERS</th>
<th>PARAMETERS</th>
<th>ESTIMATED PERCENTAGE UNCERTAINTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KING'S LAW ( e^2 = a + b/u )</td>
<td>MINIMUM SES IN ( e^2 )</td>
<td>LINEAR</td>
<td>2</td>
<td>8.18 2.01 - - 0.74</td>
<td>0.49% 0.50% - - 47.70%</td>
</tr>
<tr>
<td>2</td>
<td>FLOATING EXPONENT POWER LAW ( e^2 = a + b/u )</td>
<td>MINIMUM SES IN ( e^2 )</td>
<td>LINEAR - TRIAL AND ERROR</td>
<td>3</td>
<td>4.49 3.72 0.40 - 0.64</td>
<td>14.70% 9.14% 2.50% - 57.81%</td>
</tr>
<tr>
<td></td>
<td>( e^2 = a + b/u + c/u^2 )</td>
<td>MINIMUM SES IN ( u )</td>
<td>NON-LINEAR - ITERATION</td>
<td>3</td>
<td>4.07 3.93 0.39 - 0.64</td>
<td>16.71% 7.60% 2.56% - 56.25%</td>
</tr>
<tr>
<td>3</td>
<td>EXTENDED POWER LAW ( e^2 = a + b/u + c/u^2 )</td>
<td>MINIMUM SES IN ( e^2 )</td>
<td>POLYNOMIAL FIT</td>
<td>3</td>
<td>6.61 2.42 -0.02 - 0.64</td>
<td>3.30% 2.47% 14.80% - 56.25%</td>
</tr>
<tr>
<td></td>
<td>QUADRATIC ( u = a + b(e^2) + c(e^2)^2 )</td>
<td>MINIMUM SES IN ( u )</td>
<td>POLYNOMIAL FIT</td>
<td>3</td>
<td>42.34 -6.29 0.30 - 0.66</td>
<td>16.84% 10.20% 4.70% - 64.13%</td>
</tr>
<tr>
<td>4</td>
<td>3rd Order ( u = a + b(e^2) + c(e^2)^2 + d(e^2)^3 )</td>
<td>MINIMUM SES IN ( u )</td>
<td>POLYNOMIAL FIT</td>
<td>4</td>
<td>52.78 6.22 -0.25 0.0074 0.66</td>
<td>110.71% 126.03% 140.00% 65.00% 68.07%</td>
</tr>
</tbody>
</table>

**TABLE 5.8** COMPARISON OF THE HEAT TRANSFER LAWS IN HIGH VELOCITY RANGE
<table>
<thead>
<tr>
<th>No.</th>
<th>HEAT TRANSFER LAW</th>
<th>CRITERIA FOR LEAST SQUARES FIT</th>
<th>METHOD OF CURVE FITTING</th>
<th>NO. OF PARAMETERS</th>
<th>PARAMETERS</th>
<th>ESTIMATED PERCENTAGE UNCERTAINTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>King's Law (E^2 = A + B/U)</td>
<td>Minimum Sess in (E^2)</td>
<td>Linear</td>
<td>2</td>
<td>5.68 2.27 - -</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
<td>Floating Exponent Power Law (E^2 = A + B U^C)</td>
<td>Minimum Sess in (E^2)</td>
<td>Linear-Trial and Error</td>
<td>3</td>
<td>4.10 3.27 0.43 -</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Minimum Sess in (U)</td>
<td>Non-Linear-Iteration</td>
<td>3</td>
<td>3.97 3.35 0.42 -</td>
<td>0.09</td>
</tr>
<tr>
<td>3</td>
<td>Extended Power Law (E^2 = A + B U^C)</td>
<td>Minimum Sess in (E^2)</td>
<td>Polynomial Fit</td>
<td>3</td>
<td>4.97 2.62 -0.04 -</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>Quadratic (U = A + B E^2 + C E^4)</td>
<td>Minimum Sess in (U)</td>
<td>Polynomial Fit</td>
<td>3</td>
<td>12.68 -3.07 0.22 -</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>3rd Order (U = A + B E^2 + C E^4 + D E^6)</td>
<td>Minimum Sess in (U)</td>
<td>Polynomial Fit</td>
<td>4</td>
<td>-2.36 0.037 0.022 0.0044</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 5.9 Comparison of Heat Transfer Laws in Low Velocity Range
<table>
<thead>
<tr>
<th>Trial #</th>
<th>A</th>
<th>B</th>
<th>c</th>
<th>$k^2$</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.70</td>
<td>4.78</td>
<td>0.412</td>
<td>-0.0576</td>
<td>0.315</td>
</tr>
<tr>
<td>2</td>
<td>6.41</td>
<td>4.23</td>
<td>0.431</td>
<td>-0.0511</td>
<td>0.192</td>
</tr>
<tr>
<td>3</td>
<td>5.25</td>
<td>4.96</td>
<td>0.401</td>
<td>-0.0532</td>
<td>0.221</td>
</tr>
<tr>
<td>4</td>
<td>6.13</td>
<td>4.24</td>
<td>0.433</td>
<td>-0.0441</td>
<td>0.227</td>
</tr>
</tbody>
</table>

Velocity Range: 2 - 35 m/s

Yaw Angle Range: -50° - +50°

Hot-Wire Used: Inclined Type

Table 5.10 Repeated Yaw Calibration Using the New Method
<table>
<thead>
<tr>
<th>Data Set #</th>
<th>Model Used for $u_{\text{eff}}$</th>
<th>A</th>
<th>B</th>
<th>c</th>
<th>$k^2/m_e/b$</th>
<th>(SES)$_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$u_{\cos a}$</td>
<td>7.63</td>
<td>4.78</td>
<td>0.409</td>
<td>-</td>
<td>5.23</td>
</tr>
<tr>
<td></td>
<td>$u[\cos^2 a + k^2\sin^2 a]^{1/2}$</td>
<td>7.56</td>
<td>4.84</td>
<td>0.406</td>
<td>-0.0171</td>
<td>4.21</td>
</tr>
<tr>
<td></td>
<td>$u_{\cos^m o a}$</td>
<td>7.55</td>
<td>4.85</td>
<td>0.406</td>
<td>1.02</td>
<td>5.21</td>
</tr>
<tr>
<td></td>
<td>$u[1 - b(1 - \cos^{1/2} a)]^2$</td>
<td>7.55</td>
<td>4.85</td>
<td>0.405</td>
<td>1.02</td>
<td>5.21</td>
</tr>
<tr>
<td>2</td>
<td>$u_{\cos a}$</td>
<td>4.73</td>
<td>4.22</td>
<td>0.378</td>
<td>-</td>
<td>2.96</td>
</tr>
<tr>
<td></td>
<td>$u[\cos^2 a + k^2\sin^2 a]^{1/2}$</td>
<td>4.33</td>
<td>4.57</td>
<td>0.361</td>
<td>-0.0252</td>
<td>2.31</td>
</tr>
<tr>
<td></td>
<td>$u_{\cos^m o a}$</td>
<td>4.24</td>
<td>4.65</td>
<td>0.358</td>
<td>1.05</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>$u[1 - b(1 - \cos^{1/2} a)]^2$</td>
<td>4.24</td>
<td>4.65</td>
<td>0.357</td>
<td>1.04</td>
<td>2.19</td>
</tr>
</tbody>
</table>

Table 5.11 Comparison of Different Models for $u_{\text{eff}}$ using Experimental Data
<table>
<thead>
<tr>
<th>Position (y/R)</th>
<th>Yaw Sensitivity Factor (k²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probe #1</td>
</tr>
<tr>
<td>0.19</td>
<td>-0.04</td>
</tr>
<tr>
<td>0.28</td>
<td>0.03</td>
</tr>
<tr>
<td>0.38</td>
<td>0.02</td>
</tr>
<tr>
<td>0.47</td>
<td>0.06</td>
</tr>
<tr>
<td>0.56</td>
<td>0.09</td>
</tr>
<tr>
<td>0.66</td>
<td>0.09</td>
</tr>
<tr>
<td>0.75</td>
<td>0.04</td>
</tr>
<tr>
<td>0.85</td>
<td>0.08</td>
</tr>
<tr>
<td>0.94</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Hot-Wire Used: Inclined Type

Table 5.12 Measurement of Yaw Sensitivity Factor In Fully Developed Pipe Flow
<table>
<thead>
<tr>
<th>Position ((y/R))</th>
<th>( \frac{u^2}{U^2} ) Method I</th>
<th>( \frac{u^2}{U^2} ) Method II</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>0.922</td>
<td>-48.7</td>
</tr>
<tr>
<td>0.19</td>
<td>1.02</td>
<td>-44.2</td>
</tr>
<tr>
<td>0.28</td>
<td>1.16</td>
<td>-36.8</td>
</tr>
<tr>
<td>0.38</td>
<td>1.28</td>
<td>-31.1</td>
</tr>
<tr>
<td>0.47</td>
<td>1.40</td>
<td>-24.1</td>
</tr>
<tr>
<td>0.56</td>
<td>1.54</td>
<td>-32.1</td>
</tr>
<tr>
<td>0.66</td>
<td>1.71</td>
<td>-26.4</td>
</tr>
<tr>
<td>0.75</td>
<td>1.85</td>
<td>-24.2</td>
</tr>
<tr>
<td>0.85</td>
<td>2.00</td>
<td>-13.1</td>
</tr>
<tr>
<td>0.94</td>
<td>2.14</td>
<td>5.06</td>
</tr>
</tbody>
</table>

Table 5.13 Pipe Case: Comparison of the Two Methods of Turbulence Measurements
APPENDIX A.1

COMPUTER PROGRAMME: YAW CALIBRATION

******************************************************************************
C PURPOSE
C TO DETERMINE THE CALIBRATION CONSTANTS A, B, C AND THE YAW
C SENSITIVITY FACTOR (K) SIMULTANEOUSLY USING NONLINEAR
C OPTIMISATION TECHNIQUE FOR HOT-WIRES
C
METHOD
C THE NONLINEAR METHOD OF GAUSS-NEWTON IS USED TO SET UP LEAST
C SQUARES NORMAL EQUATIONS. THESE EQUATIONS ARE SOLVED USING
C MATRIX INVERSION. HARTLEY'S METHOD IS USED TO ENSURE
C CONVERGENCE. THE ERRORS IN VELOCITY ARE MINIMISED.
C
CALIBRATION EQUATION
C KINN'S LAW: E\(^2\) = A + B U\(^\text{eff}\)
C
EFFECTIVE VELOCITY MODEL:
C
U\(^\text{eff}\) = U (\cos^2(\alpha) + k \sin^2(\alpha))\(^{1/2}\)
C
DESCRIPTION OF PARAMETERS
C N = NO. OF CALIBRATION CONSTANTS
C M = NO. OF YAW SETTINGS
C L = NO. OF VELOCITY SETTINGS
C SH = AMBIENT PRESSURE IN mm OF HG
C SGL = SPECIFIC GRAVITY OF MANOMETER FLUID
C SL1/SL2 = SLOPE OF THE MANOMETER
C E = HOT-WIRE OUTPUT VOLTAGE IN VOLTS
C H = MANOMETER READING IN mm OF FLUID
C A, B, C, K = CALIBRATION CONSTANTS
C G = TERMINATION CRITERIA FOR ITERATION
C
******************************************************************************

REAL*8 E(15,15),V(15),AL(15),PM(15,15),VP(15,15),H(15)
1,VD(15,15),P(15,15),RR(15),R(15),Z(4,4),T(4,4),IR(4),IC(4),
2 U(4)
REAL*8 K,A,B,C,SES,DET,SM
U(1)=U(2)=U(3)=U(4)=0.0.
C *** I N P U T D A T A ***
READ,N,M,L,SH,TEM,SGL,SL1,SL2
READ,(AL(I),I=1,L)
READ,(H(I),I=1,M)
READ,((E(I,J),J=1,L),I=1,M)
PR=13.6*SH
DEN=PR*1000.0/(29.2*(TEM+273.16))
SIN=SL1/SL2
DENR=SGL/DEN-1.
C *** CALCULATION OF VELOCITY ***
DO 1 I=1,M
   V(I)=DSQRT(0.01962*SIN*DNR*H(I))
1 CONTINUE
DO 2 J=1,L
   AL(J)=AL(J)/57.2958
2 CONTINUE
G=0.00001
C *** INITIAL GUESSES FOR THE CONSTANTS ***
A=3.5; B=3.5; C=0.4; K=-0.025
10 SES=0.0
C *** REDUCING THE STEP SIZE ***
DO 12 I=1,4
   U(I)=U(I)*0.92
12 CONTINUE
   A=A+U(1); B=B+U(2); C=C+U(4); K=K+U(3)
D=1./C
C *** CALCULATION OF THE SUM OF THE ERROR SquARED IN U ***
DO 20 I=1,M
DO 30 J=1,L
   VP(I,J)=(((E(I,J)**2-A)/B)**D)/DSQRT(DOCS(AL(J))**2+(K)*
   DSIN(AL(J))**2)
   VD(I,J)=VP(I,J)-V(I)
   SES=SES+VD(I,J)**2
30 CONTINUE
20 CONTINUE
PRINT 31,A,B,C,K,SES
Z(1)=Z(2)=Z(3)=Z(4)=0.0
C *** DETERMINATION OF THE COEFFICIENTS OF THE MATRIX ***
DO 25 I=1,4
DO 26 J=1,4
   T(I,J)=0.0
26 CONTINUE
25 CONTINUE
DO 40 I=1,M
DO 50 J=1,L
   P(I,J)=(E(I,J)**2-A)/B
   RR(J)=(DOCS(AL(J))**2)+(K)*(DSIN(AL(J))**2)
   R(J)=1.0/DSQRT(RR(J))
   F=D/B
   S1=(P(I,J)**D)*R(J)
   S2=V(I)
   S3=-1.*F*(P(I,J)**(D-1))*R(J)
   S4=-1.*F*S1
   S5=-1.*S1*(D**2)*DLOG(P(I,J)).
   S6=-0.5*(DSIN(AL(J))**2)*(P(I,J)**D)*R(J)**3
   T(1,1)=T(1,1)+S3*S3
   T(1,2)=T(1,2)+S4*S3
   T(1,4)=T(1,4)+S5*S3
   T(1,3)=T(1,3)+S6*S3
   T(2,1)=T(2,1)+S3*S4
T(2,2) = T(2,2) + S4*S4
T(2,4) = T(2,4) + S5*S4
T(2,3) = T(2,3) + S6*S4
T(4,1) = T(4,1) + S3*S5
T(4,2) = T(4,2) + S4*S5
T(4,4) = T(4,4) + S5*S5
T(4,3) = T(4,3) + S6*S5
T(3,1) = T(3,1) + S3*S6
T(3,2) = T(3,2) + S4*S6
T(3,4) = T(3,4) + S5*S6
T(3,3) = T(3,3) + S6*S6
Z(1) = Z(1) + (S2-S1)*S3
Z(2) = Z(2) + (S2-S1)*S4
Z(4) = Z(4) + (S2-S1)*S5
Z(3) = Z(3) + (S2-S1)*S6

50 CONTINUE
40 CONTINUE

C *** SOLUTION OF THE MATRIX ***
CALL MINVRD(T,N,N,DET,IER,IR,IC)
DO 60 I = 1, N
   U(I) = 0.0
   DO 70 J = 1, 4
      SUM = T(I,J)*Z(J)
      U(I) = U(I) + SUM
    CONTINUE
60 CONTINUE

IF (DABS(U(3)).GT.G) GO TO 10
STOP
END
APPENDIX A.2

COMPUTER PROGRAMME: VELOCITY CALIBRATION

********************************************************************************
C C PURPOSE C C
C TO OBTAIN CALIBRATION CONSTANTS FOR HOT-WIRES BY NONLINEAR C C
C CALIBRATION TECHNIQUE. THIS PROGRAM IS RESTRICTED TO VELOCITY CALIBRATION C C
C C METHOD C C
C USES GAUSS-NEWTON PROCEDURE TO SET UP NORMAL EQUATIONS TO BE SOLVED BY LEAST SQUARE METHOD. THESE EQUATIONS ARE SOLVED ITERATIVELY USING CRAMER'S RULE, HARTLEY'S METHOD IS USED TO ENSURE COVERAGECE. THE METHOD MINIMISES THE ERRORS SQUARED IN VELOCITY.
C C CALIBRATION EQUATION USED: POWER-LAW MODEL
C C DESCRIPTION OF PARAMETERS
C N - NUMBER OF DATA POINTS
C SH - AMBIENT PRESSURE IN mm OF HG.
C SGL - SPECIFIC GRAVITY OF MANOMETER FLUID.
C SL1/SL2 - SLOPE ON THE INCLINED MANOMETER
C E - HOT-WIRE OUTPUT VOLTAGE IN VOLTS
C H - MANOMETER READINGS IN mm OF FLUID
C TEMP - AMBIENT TEMPERATURE IN DEGREE CELCIUS
C A,B,C - CALIBRATION CONSTANTS
C G - TERMINATION CRITERIA FOR ITERATION
C U,V,W - CORRECTIONS FOR THE CALIBRATION CONSTANTS
C
********************************************************************************
DIMENSION E(31),VO(31),VN(31),P(31),R(31),H(31)
C *** INPUT DATA ********************************************************************************
U=V=W=0.0
G=0.00001
READ,N,SH,TEM,SGL,SL1,SL2
READ,(E(I),I=1,N)
READ,(H(I),I=1,N)
PR=13.6*SH
DEN=PR*1000.0/(29.2*(TEM+273.16))
SIN=SL1/SL2
DENR=SGL/DEN-1.
ZZ=SQR(0.01962*SIN*DENR)
PRINT,FR,DEN,ZZ
C *** DETERMINATION OF VELOCITY FROM MANOMETER READINGS ********
DO 100 I=1,N.
VO(I)=SQR(0.01962*SIN*DENR*H(I))
100 CONTINUE
PRINT,(VO(I),I=1,N)
PRINT 41
FORMAT(1,1).
C *** INITIAL GUESSES FOR CALIBRATION CONSTANTS **************
C = .5; A = 5.0; B = 3.0
21 SES=Q.
C *** REDUCING THE STEP SIZE **************************************
U = U*0.7.
V = V*0.7
W = W*0.7.
A = A+U; B = B+V; C = C+W
D = 1.0/C
C *** DETERMINATION OF THE SUM OF THE ERRORS SQUARED *************
DO 3 J = 1,N
VN(J) = ((E(J)**2-A)/B)**D
3 SES=SES+(VO(J)-VN(J))**2
PRINT 40, A, B, C, SES
40 FORMAT(2X, 4(E12.9))
C *** COMPUTING THE COEFFICIENTS OF NORMAL EQUATIONS *************
C *** AND SOLUTION USING CRAMER'S RULE **************
S1=S2=S3=S4=S5=S6=S7=S8=S9=0.0
DO 4 I = 1,N
P(I) = (E(I)**2-A)/B
R(I) = P(I)/(C*B)
SM3 = ((P(I)**D-VO(I))*P(I)**(D-1))
S1 = S1+SM3
SM1 = P(I)**(2*D-1)*ALOG(P(I))
S2 = S2+SM1
SM2 = R(I)*P(I)**(2*(D-1))
S3 = S3+SM2*R(I)
S4 = S4+SM2
S5 = S5+SM1*R(I)
S6 = S6+SM3*R(I)
S7 = S7+SM2*R(I)
S8 = S8+(P(I)**D-VO(I))*P(I)**D*ALOG(P(I))
S9 = S9+(P(I)**D*ALOG(P(I)))*2
W = (-S1*(S2*S3-S4*S5)-S6*(S7*S5-S2*S4)-S8*(S4**2-S7*S3))/
1 (1/C**2*(S7*(S3*S9-S5**2)+S4*(S5*S2-S4*S9)+S2*(S4*S5-
2 S3*S2)))
V = (S4*S1-S6*S7+1/C**2*(S7*S5-S4*S2))*/(S4**2-S7*S3)
U = (S1-S4*V-(S2/C**2)*W)/(S7/(C*B)).
PRINT, U, V, W
IF (ABS(U).GT.G) GO TO 21
IF (ABS(V).GT.G) GO TO 21
IF (ABS(W).GT.G) GO TO 21
STOP
END
APPENDIX A.3

COMPUTER PROGRAMME: EVALUATION OF THE SYSTEMS OF EQUATIONS FOR TURBULENCE MEASUREMENT

*******************************

PURPOSE

TO GENERATE PSEUDO TURBULENCE AND TO COMPARE THE PERFORMANCE OF THE TWO METHODS OF TURBULENCE MEASUREMENTS. ALSO TO STUDY THE EFFECT OF MEASUREMENT ERRORS AND TO ESTIMATE THE TRUNCATION ERROR IN METHOD I.

METHOD

RANDOM NUMBER GENERATORS ARE USED TO GENERATE TURBULENCE AND TO INTRODUCE MEASUREMENT ERRORS. THE PERFORMANCE IS EVALUATED BY MONTE CARLO TESTING. PLEASE SEE LOGIC DIAGRAM FOR FURTHER DETAILS.

REMARKS

a. METHOD I INVOLVES BINOMIAL EXPANSION OF THE RESPONSE EQUATION AND THEN TIME-AVERAGING TO DETERMINE EXPRESSION FOR THE STRESSES

METHOD II INVOLVES SQUARES AND SUBSEQUENTLY TIME-AVERAGING.

b. THE PROGRAM CONSISTS OF THREE PARTS

1. TESTING AND MODIFICATION OF RANDOM NUMBERS
2. DETERMINATION OF THE STANDARD VALUES FOR THE STRESSES AND THE CORRESPONDING VOLTAGES
3. INTRODUCTION OF ERRORS AND COMPARISON OF THE TWO METHODS

SUBROUTINES USED

RANDU - RANDOM NUMBER GENERATOR ON IBM

DESCRIPTION OF THE INPUT VARIABLES

S - SENSITIVITY OF HOT-WIRE
K - YAW SENSITIVITY FACTOR
H - PITCH SENSITIVITY FACTOR
ALP - INCLINATION OF HOT-WIRE
UB - MEAN VELOCITY
IX - SEED VALUE TO INITIATE THE RANDOM NUMBER GENERATOR
N1 - NO. OF TRIALS FOR GENERATING TURBULENCE
N2 - NO. OF TRIALS FOR MONTE CARLO TESTING
SU,SV,SW - STANDARD DEVIATION IN THE FLUCTUATING VELOCITY
SS,SK,SE,SUB,SSE - STANDARD DEVIATION OF THE ERRORS IN S,K,
MEAN VOLTAGE, MEAN VELOCITY AND FLUCTUATING VOLTAGES

*******************************
REAL*8 S,K,H,ALP,UB,SU,SV,SW,SS,SK,SE,SUB,AN2,CA,SA,K2,AN1,
1UEFF, BE, S1, S2, S3, S4, S5, S6, S7, S8, S9, S10, S11, S12, S13, S14, S15
2, S16, S17, U, V, W, U2, V2, W2, E0, E1, E2, E3, E4, P0, P1, P2, P3, P4, P5,
3U0, A0, AE, AE2, AE3, AE4, BE0, BE1, BE2, BE3, BE4, BE5, BE6, BE7
4, CE1, CE2, CE3, CE4, CE5, CE6, CE7, CE8, CE9, CE10, CE11, CE12, CE13, CE14
5, CE15, CE16, CE17, CE18, CE19, CE20, CE21, CE22, UME, WME, UME, WME,
6, STDV, STDV
7, AM, AS, T0, T1, T2, T3, T4, AAE, BB1, BB2, BB3, BB4, BB5, BB6, BB7,
8, T8, T9, T10, T11, T12, T13, H2, AE, S16, S17, V20, W20, BE8, BE9, CE, SV2,
9, SW2, R1, R2, R3, STDH, SSE, EP2, EP3
C *** INPUT DATA ***
  READ 200, S, K, H, ALP, UB
200  FORMAT(3F3.1,E8.6,F4.1)
   PRINT 100, S, K, H, ALP, UB
100  FORMAT(2X,'S=', F3.1, 1X, 'K=', F3.1, 1X, 'H=', F3.1, 1X, 'ALP=', F4.1, 1X, 'MEAN VELOCITY(UB)=' , F4.1)
   READ 201, IX, N1, N2
201  FORMAT(19,215)
   PRINT 101, IX, N1, N2
101  FORMAT(2X, 'SEED # (IX)=', I9, 1X, '# OF TRIALS(TURB)=', I5, 1X, '
   # OF TRIALS(ERF)=', I4)
   AN1=N1
   AN2=N2
   READ 202, SU, SV, SW
202  FORMAT(3F5.2)
   PRINT 102, SU, SV, SW
102  FORMAT(2X, 'STD. DEV. IN U=', F5.2, 1X, 'STD DEV IN V=', F5.2, 1X, 'STDDEV IN W=', F5.2)
   READ 203, SS, SK, SE, SUB, SSE
203  FORMAT(5F6.4)
   PRINT 103, SS, SK, SE, SUB, SSE
103  FORMAT(2X, 'ERR IN S=', F6.4, 1X, 'ERR IN K=', F6.4, 1X, 'ERR IN L=', F6.4, 1X, 'ERR IN MEAN VELOCITY=', F6.4, 1X, 'ERR IN FLUCT. V=', F6.4)
   H2=H**2
   K2=K**2
   CA=DCOS(ALP)
   SA=DSIN(ALP)
   AE=S*UB
   UEFF=UB*DSQRT(CA*CA+K2*SA*SA)
   BE=S*UEFF
   PRINT 104, AE, BE
104  FORMAT(2X, 'MEAN VOLTAGE @ ALP=0=', F4.2, 1X, 'MEAN VOLTAGE AN ALP=', F4.2)
C *** TESTING AND MODIFICATION OF RANDOM NUMBERS AND DETERMINATION OF MEAN VOLTAGES ***
DO 40 J=1,3
 IX=911967579
 T0=0.0
 T1=0.0
 T2=0.0
 T3=0.0
T4=0.0
T5=0.0
T6=0.0
T7=0.0
T8=0.0
T9=0.0
T10=0.0
DO 30 I=1,N1
AM=0.0
CALL GAUSS(IX, SU, AM, U)
CALL GAUSS(IX, SV, AM, V)
IF((U.GT.0.0).AND. (V.GT.0.0)) V=-1.*V
IF((U.LT.0.0).AND. (V.LT.0.0)) V=-1.*V
CALL GAUSS(IX, SW, AM, W)
IF (J.EQ.1) GO TO 50
U=U-EP1
V=V-EP2
W=W-EP3
IF (J.EQ.2) GO TO 50
U=Z1*U
V=Z2*V
W=Z3*W
50
V2=V*V
U2=U*U
W2=W*W
E0=S*DSQRT((UB+U)**2+H2*W2+K2*V2)
T0=T0+E0
T1=T1+E1
T2=T2+E2
T3=T3+E3
T4=T4+E4
T5=T5+U
T6=T6+V
T7=T7+W
T8=T8+U2
T9=T9+V2
T10=T10+W2
-30 CONTINUE
AAE=T0/N1
BB1=T1/N1
BB2=T2/N1
BB3=T3/N1
BB4=T4/N1
PRINT 114, AAE, BB1, BB2, BB3, BB4
114 FORMAT(5(2X,F5.3))
IF (J.EQ.3) GO TO 40
IF (J.EQ.2) GO TO 60
EP1=T5/N1
EP2=T6/N1
EP3=T7/N1
PRINT 113, EP1, EP2, EP3
60 STDU=DSQRT((T8-T5*EP1)/(N1-1))
STDV=DSQRT((T9-T6*EP2)/(N1-1))
STDW=DSQRT((T10-T7*EP3)/(N1-1))
PRINT 113,STDU,STDV,STDW
Z1=(12.5/STDU)
Z2=(12.5/STDV)
Z3=(12.5/STDW)
PRINT 113,Z1,Z2,Z3
40 CONTINUE
C *** GENERATION OF TURBULENCE AND CORRESPONDING VOLTAGES ***
S1=0.0
S2=0.0
S3=0.0
S4=0.0
S5=0.0
S6=0.0
S7=0.0
S8=0.0
S9=0.0
S10=0.0
S11=0.0
S12=0.0
S13=0.0
S14=0.0
S15=0.0
S16=0.0
S17=0.0
S18=0.0
S19=0.0
S20=0.0
S21=0.0
S22=0.0
S23=0.0
IX=911967579
DO 10 I=1,N1
AM=0.0
CALL GAUSS(IX,SU,AM,U)
CALL GAUSS(IX,SV,AM,V)
IF((U.GT.0.0). AND. (V.GT.0.0)) V=-1.*V
IF((U.LT.0.0). AND. (V.LT.0.0)) V=-1.*V
U=U-EP1
V=V-EP2
CALL GAUSS(IX,SW,AM,W)
W=W-EP3
U=Z1*U
V=Z2*V
W=Z3*W
U2=U*U
V2=V*V
W2=W*W
S1=S1+U2
S2=S2+V2
S3=S3+W2
S4=S4-U*V
S20=S20+U
S21=S21+V
S22=S22+W
EO=S*DSQRT((UB+U)**2+H2*W2+K2*V2)
E1=S*DSQRT(((UB+U)*CA+V*SA)**2+H2*W2+(K2*(V*CA-(UB+U)*SA)**2))
E2=S*DSQRT(((UB+U)*CA-V*SA)**2+H2*W2+(K2*(V*CA+(UB+U)*SA)**2))
E3=S*DSQRT(((UB+U)*CA+W*SA)**2+H2*W2+(K2*(W*CA-(UB+U)*SA)**2))
E4=S*DSQRT(((UB+U)*CA-W*SA)**2+H2*W2+(K2*(W*CA+(UB+U)*SA)**2))
F0=E0-AAE
F1=E1-BB1
F2=E2-BB2
F3=E3-BB3
F4=E4-BB4
S5=S5+E0
S6=S6+E1
S7=S7+E2
S8=S8+E3
S9=S9+E4
S10=S10+E0*E0
S11=S11+E1*E1
S12=S12+E2*E2
S13=S13+E3*E3
S14=S14+E4*E4
S15=S15+F0*F0
S16=S16+F1*F1
S17=S17+F2*F2
S18=S18+F3*F3
S19=S19+F4*F4
S23=S23+F0

10 CONTINUE
UME=S20/N1
VME=S21/N1
WME=S22/N1
PRINT 113,UME,VME,WME

113 FORMAT(3F8.4)
STDU=DSQRT((S1-S20*UME)/(N1-1.))
STDV=DSQRT((S2-S21*VME)/(N1-1.))
STDW=DSQRT((S3-S22*WME)/(N1-1.))
PRINT 113,STDU,STDV,STDW
U20=S1/N1
V20=S2/N1
W20=S3/N1
UVO=S4/N1
PRINT 107,U20,V20,W20,UVO

107 FORMAT(2X,'NORMAL STRESS=',F7.3,1X,'TANGENTIAL STRESS=',
1F7.3,1X,'BINORMAL STRESS=',F7.3,1X,'SHEAR STRESS=',F7.3)
AEO=S5/N1
AE1=S6/N1
AE2=S7/N1
AE3=S8/N1
AE4=S9/N1
PRINT 108, AE0, AE1, AE2, AE3, AE4
108 FORMAT(2X,'AE0=',F6.3,1X,'AE1=',F6.3,1X,'AE2=',F6.3,1X,
1 'AE3=',F6.3,1X,'AE4=',F6.3)
BE0=S10/N1
BE1=S11/N1
BE2=S12/N1
BE3=S13/N1
BE4=S14/N1
PRINT 109, BE0, BE1, BE2, BE3, BE4
109 FORMAT(2X,'BE0=',F7.2,1X,'BE1=',F7.2,1X,'BE2=',F7.2,1X,
1 'BE3=',F7.2,1X,'BE4=',F7.2)
CE0=S15/N1
CE1=S16/N1
CE2=S17/N1
CE3=S18/N1
CE4=S19/N1
PRINT 110, CE0, CE1, CE2, CE3, CE4
110 FORMAT(2X,'CE0=',F8.3,1X,'CE1=',F8.3,1X,'CE2=',F8.3,1X,
1 'CE3=',F8.3,1X,'CE4=',F8.3)
CC=S23/N1
PRINT 150, CC

C *** INTRODUCTION OF ERRORS IN U, S, K & E'S ***
C *** AND COMPARISON OF THE TWO METHODS ***
SU1=0.0
SV1=0.0
SW1=0.0
SUV1=0.0
SU2=0.0
SV2=0.0
SW2=0.0
SUV2=0.0
DO 20 I=1,N2
AM=K
AS=AM*SK
CALL GAUSS(IX,AS,AM,KP)
D1=KP*KP
AM=S
AS=AM*SS
CALL GAUSS(IX,AS,AM,SP)
D2=SP*SP
AM=UB
AS=AM*SUB
CALL GAUSS(IX,AS,AM,UBP)
D3=UBP*UBP
AM=BE0
AS=AM*SE
CALL GAUSS(IX,AS,AM,BE5)
AM=BE1
AS=AM*SE
CALL GAUSS(IX,AS,AM,BE6)
AM=BE2
AS = AM*SE
CALL GAUSS(IX, AS, AM, BE7)
AM = BE3
AS = AM*SE
CALL GAUSS(IX, AS, AM, BE8)
AM = BE4
AS = AM*SE
CALL GAUSS(IX, AS, AM, BE9)
AM = CE1
AS = AM*SE
CALL GAUSS(IX, AS, AM, CE6)
AM = CE0
AS = AM*SE
CALL GAUSS(IX, AS, AM, CE5)
AM = CE2
AS = AM*SE
CALL GAUSS(IX, AS, AM, CE7)
AM = CE3
AS = AM*SE
CALL GAUSS(IX, AS, AM, CE8)
AM = CE4
AS = AM*SE
CALL GAUSS(IX, AS, AM, CE9)
G1 = (1. - 3.*D1)*D2
G2 = (1. - D1)*D2^2
G3 = (1. + D1)*D2
G4 = (2*H2)/(1 + D1)
G5 = 1 + D1
U21 = CE5/D2
V21 = (CE6 + CE7 - CE5*G5)/G1
W21 = (CE8 + CE9 - CE5*G5)/G1
UV1 = (CE7 - CE6)/G2
SU1 = SU1 + (U21 - U20)**2
SV1 = SV1 + (V21 - V20)**2
SW1 = SW1 + (W21 - W20)**2
SU2V1 = SU1V1 + (UV1 - UV0)**2
R1 = (BE5/D2) - D3
R2 = ((BE6 + BE9)/G3) - D3
R3 = ((BE6 + BE7)/G3) - D3
R4 = (R2 - R3)/(G4 - 1)
V22 = (R1 - R2)/(D1 - G4)
W22 = V22 - R4
U22 = R1 - (V22*D1) - W22
UV2 = (BE7 - BE6)/G2
SU2 = SU2 + (U22 - U20)**2
SV2 = SV2 + (V22 - V20)**2
SW2 = SW2 + (W22 - W20)**2
SU2V2 = SU2V2 + (UV2 - UV0)**2
CONTINUE
SU1 = (DSQRT(SU1/AN2)/U20)*100
SV1 = (DSQRT(SV1/AN2)/V20)*100
SW1 = (DSQRT(SW1/AN2)/W20)*100
SU2V1 = (DSQRT(SU2V1/AN2)/UV0)*100
SU2 = (DSQRT(SU2/AN2)/U20)*100
SV2 = (DSQRT(SV2/AN2)/V20)*100
SW2 = (DSQRT(SW2/AN2)/W20)*100
SUV2 = (DSQRT(SUV2/AN2)/UV0)*100
PRINT 112, SU1, SV1, SW1, SUV1
PRINT 112, SU2, SV2, SW2, SUV2

112 FORMAT (4(2X, F9.3))
STOP
END
SUBROUTINE GAUSS (IX, AS, AM, AV)
A = 0.0
DO 100 I = 1, 12
CALL RANDU (IX, IY, Y)
IX = IY
100 A = A + Y
AV = (A - 6.)*AS + AM
RETURN
END
**APPENDIX A.4**

**COMPUTER PROGRAMME: CALIBRATION STUDIES**

*****************************************************************************
C
C PURPOSE
C TO DETERMINE THE SAMPLE SIZE REQUIRED AND THE EFFECTS OF
C SPACING OF DATA, VELOCITY RANGE AND ERRORS IN MEASUREMENTS
C FOR HOT-WIRE VELOCITY CALIBRATION
C
C METHOD
C USES THE DISTRIBUTION SAMPLING TECHNIQUE OF MONTE CARLO
C PROCEDURE TO SIMULATE HOT-WIRE DATA. THE SIMULATED DATA
C IS CURVEFITTED USING LINEAR LEAST SQUARES
C
C EQUATIONS
C FOR DATA SIMULATION: \( E^2 = 5.4 + 6.0U^{0.4} \)
C FOR CURVE FITTING: EXPRESSION OF COLLINS AND WILLIAM
C \[ E^2 = A + BU^{0.45} \]
C
C SUBROUTINES USED
C RANDU - RANDOM NUMBER GENERATOR AVAILABLE ON IBM
C GELPLSD - FOR SOLVING SIMULTANEOUS EQUATIONS, AVAILABLE
C ON IBM
C
C DESCRIPTION OF PARAMETERS
C N - NO. OF CALIBRATION CONSTANTS
C M - NO. OF DATA POINTS
C MM - NO. OF TRIALS FOR MONTE CARLO TESTING
C IX,IX1,IX2 - SEED VALUES TO INITIATE RANDU
C VMAX - MAXIMUM VELOCITY OF THE RANGE
C VMIN - MINIMUM VELOCITY OF THE RANGE
C VR - VELOCITY RANGE
C FERR - ERROR IN VELOCITY
C AA,BB - CALIBRATION CONSTANTS
C SIG - STANDARD DEVIATION IN THE CURVEFIT
C AVG, BVG - MEAN OF AA,BB OVER MM TRIALS
C
*****************************************************************************

REAL*8 WK1(5,5),WK2(5,5),E(50),U(50),COEPS(5),UC(50)
1 ,X(50),Y(50),R,UD(50)

C *** INPUT DATA
READ,N,M,MM,VMAX,VMIN,FERR
PRINT,M,MM,VMIN,VMAX,FERR
VR=VMAX-VMIN
IX=7337
IX1=3895
IX2=19199
AA=BB=SUM=0.0
C *** TRIALS FOR MONTE CARLO TESTING BEGINS
   DO 100 I=1, MM
C *** GENERATION OF M DATA POINTS
   DO 1 I=1, M
      CALL RANDU(X, Y, YFL)
      X=Y
      U(I)=YMIN+VR*YFL
      E(I)=DSQRT(5.4+6.0*U(I)**0.4)
      CALL RANDU(IY1, IY1, YFL1)
      IY1=IY1
      CALL RANDU(IY2, IY2, YFL2)
      IY2=IY2
      IF(YFL1, GE, 0.5) THEN DO
         U(I)=U(I)+((VR*PERR*YFL2)/100.)
      ELSE DO
         U(I)=U(I)-((VR*PERR*YFL2)/100.)
      ELSE END IF
      IF(U(I), LT, 0.0) U(I)=-U(I)
      X(I)=(U(I)**0.45)
      E(I)=E(I)**2
      CONTINUE
C *** DETERMINATION OF CALIBRATION CONSTANTS
   CALL GEPLSD(N, COEFS, M, X, Y, IER, WK1, WK2)
   BB=BB+COEFS(1)
   AA=AA+COEFS(2)
   SES=0.0
   DO 2 I=1, M
      R=(Y(I)-COEFS(2))/COEFS(1)
      UC(I)=R**2(2.22222)
      UD(I)=(UC(I)-U(I))**2
      SES=SES+UD(I)
   CONTINUE
   SUM=SUM+SES
   CONTINUE
C *** END OF TRIALS AND DETERMINATION OF AVERAGE VALUES
   AVG=AA/MM
   BVG=BB/MM
   SUMG=SUM/MM
   SIG=SQRT((SUMG/(M-2)))
   PRINT, AVG, BVG, SIG
   STOP
END
APPENDIX B

UNCERTAINTY ANALYSIS

The uncertainty analysis is based on the constant odds combination form given below in equation (B.1), which has been shown to assess the uncertainty of a variable value $Q$ with good accuracy (Kline and McClintock (K8)).

$$\frac{\Delta Q}{Q} = \left( \frac{a_1}{x_1} \Delta x_1 \right)^2 + \left( \frac{a_2}{x_2} \Delta x_2 \right)^2 + \ldots$$

(B.1)

where $Q$ is a single value of the result to be calculated from a single set of input data $(x_i)$, and $\Delta x$'s are the set of corresponding estimated uncertainty intervals. In this analysis the input variables are specified at 20 to 1 odds, hence, the calculated uncertainty in the result is also given to the same odds. Following the work of Yavuzkurt (Y1), the general equations of uncertainty are developed, the calculation procedure is outlined and results are presented in the sections given below.

B.1 Working Equations to Determine Uncertainties

B.1.1 Uncertainty in the Calculation of Velocity from Manometer Reading

During calibration the linearized anemometer output $E_{lin}$ (in volts) and the micromanometer reading $h'$ (in mm) of manometer fluid is recorded. The $h'$ reading is actually taken in order to calculate the effective velocity ($U_{eff}$). The reading of $h'$, however, will yield a velocity $U_1$ due to uncertainty in the basic measurements of temperature, pressure, etc. $U_1$ can be calculated using
\[ U_1 = \left( \frac{2h' \sin \theta_0 \beta R_o T_\infty \rho_m}{H_g} \right)^{\frac{1}{2}} \quad (B.2) \]

The uncertainty in this velocity, \( \Delta U_1 \), due to uncertainties in manometer reading \( (\pm \Delta h') \), ambient temperature \( (\pm \Delta T_\infty \, ^\circ K) \) and ambient static pressure \( (\pm H_g) \) is given by

\[ \Delta U_1 = \left[ \left( \frac{\partial U_1}{\partial h'} \Delta h' \right)^2 + \left( \frac{\partial U_1}{\partial T_\infty} \Delta T_\infty \right)^2 + \left( \frac{\partial U_1}{\partial H_g} \Delta H_g \right)^2 \right]^{\frac{1}{2}} \quad (B.3) \]

using the constant odds product form. It is further assumed here that uncertainties in acceleration due to gravity \( (g) \), gas constant \( (R_o) \), density of manometer fluid \( (\rho_m) \) and inclination of the manometer given by \( \sin \theta_0 \) are negligible. Equation (B.3) can be put in the form

\[ \Delta U_1 = \frac{1}{2} U_1 \left[ \left( \frac{\Delta h'}{h'} \right)^2 + \left( \frac{\Delta T_\infty}{T_\infty} \right)^2 + \left( \frac{\Delta H_g}{H_g} \right)^2 \right]^{\frac{1}{2}} \quad (B.4) \]

or

\[ \Delta U_1 = \alpha_1 U_{\text{eff}} \quad (B.5) \]

where

\[ \alpha_1 = \frac{1}{2} \left[ \left( \frac{\Delta h'}{h'} \right)^2 + \left( \frac{\Delta T_\infty}{T_\infty} \right)^2 + \left( \frac{\Delta H_g}{H_g} \right)^2 \right]^{\frac{1}{2}} \quad (B.6) \]

B.1.2 Error due to Curvefitting

Since a linearizer was used, the equation used for curvefitting was

\[ U_{\text{eff}} = (1/S) F_{\text{lin}} \quad (B.7) \]
Here, $E_{lin}$ is assumed error free. The curvefit will yield a velocity ($U_2$) different from $U_1$ because of fitting uncertainty ($\pm \Delta U_2$). $\Delta U_2$ can be expressed as

$$\Delta U_2 = \beta_1 U_{eff}$$  \hspace{1cm} (B.8)

where $\beta_1$ can be obtained from curvefit data

$$\beta_1 = \left[ \frac{\sum_{i=1}^{n} \Delta U_2}{n \sum_{i=1}^{n} U_1 i} \right]^{\frac{1}{2}}$$  \hspace{1cm} (B.9)

Hence, the measured instantaneous effective velocity will have an uncertainty of

$$\Delta U_{eff} = \left[ (\Delta U_1)^2 + (\Delta U_2)^2 \right]^{\frac{1}{2}} = U_{eff} (\alpha_1^2 + \beta_1^2)^{\frac{1}{2}}$$  \hspace{1cm} (B.10)

**B.1.3 Uncertainty in $\overline{U}_{eff}$ and $u_{eff}^2$**

Expressing

$$U_{eff} = \overline{U}_{eff} + u_{eff}$$  \hspace{1cm} (B.11)

we have from equation B.10

$$\Delta U_{eff} = (\overline{U}_{eff} + u_{eff}) (\alpha_1^2 + \beta_1^2)^{\frac{1}{2}}$$  \hspace{1cm} (B.12)
Assuming $\alpha_1$ and $\beta_1$ to be independent of time, we obtain upon time-averaging
\[ \Delta U_{\text{eff}} = \overline{U}_{\text{eff}} (\alpha_1^2 + \beta_1^2)^{\frac{1}{2}} \] (B.13)

Using equations B.10 - B.13, it can also be shown that
\[ \Delta u^2_{\text{eff}} = 2(\alpha_1^2 + \beta_1^2)^{\frac{1}{2}} \overline{u^2}_{\text{eff}} \] (B.14)

B.1.4 Errors in Mean Velocity and Turbulence Quantities by Method I

Expressions for determining uncertainties in the mean velocity and turbulence quantities for Method I are derived below:

B.1.4.1 Mean Velocity

From equation 3.16 we have
\[ \overline{E}_{xy}(\alpha=0) / S = \overline{U} + \frac{k^2}{2} \overline{v^2} + \frac{h^2}{2} \overline{w^2} + O(3) \]

which can be written as
\[ \overline{U}_{\text{eff}}(\alpha=0) = \overline{U} + \frac{k^2}{2} \overline{v^2} + \frac{h^2}{2} \overline{w^2} \] (B.15)

since \[ \overline{E}_{xy}(\alpha=0) / S = \overline{U}_{\text{eff}} \]

Generally, one assumes
\[ \overline{U}_{\text{eff}} = \overline{U} \] (B.16)

hence, the error in $\overline{U}$, $\Delta U_3$, due to approximation or turbulence intensity
\[
\Delta U_3 = \frac{k^2}{2} \frac{v^2}{U} + \frac{h^2}{2} \frac{w^2}{U}
\]

Therefore, the total error in \( \bar{U} \), using hot wires, is given by combining equations B.13 and B.17 to yield

\[
\frac{\Delta \bar{U}}{\bar{U}} = (\alpha_1^2 + \beta_1^2) + \frac{k^2}{2} \frac{v^2}{U} + \frac{h^2}{2} \frac{w^2}{U}
\]

**B.1.4.2 Longitudinal Normal Stress \( \bar{u}^2 \)**

The equation for normal longitudinal stress \( \bar{u}^2 \), by method I, written to the fourth order is

\[
\frac{e_{xy}^2(\alpha=0)/S^2}{\bar{u}^2} = \bar{u}^2 + k^2 \frac{uv^2}{U} + h^2 \frac{uw^2}{U} + O(4)
\]

Since \( \frac{e_{xy}^2(\alpha=0)}{S^2} = \bar{u}^2_{\text{eff}}(\alpha=0) \), we can write

\[
\bar{u}^2_{\text{eff}}(\alpha=0) = \bar{u}^2 + k^2 \frac{uv^2}{U} + h^2 \frac{uw^2}{U}
\]

The error in \( \bar{u}^2 \) due to calibration uncertainty and approximation is given by

\[
\frac{\Delta \bar{u}^2}{u^2} = 2(\alpha_1^2 + \beta_1^2) + \frac{k^2}{2} \frac{v^2}{\sqrt{u^2}} + \frac{h^2}{2} \frac{w^2}{\sqrt{u^2}}
\]

where \( \sqrt{uv^2} \) and \( \sqrt{uw^2} \) have been approximated by (see Guitton (G4))

\[
\sqrt{uv^2} = 0.5\sqrt{\bar{u}^2} \cdot \sqrt{v^2}
\]

\[
\sqrt{uw^2} = 0.5\sqrt{\bar{u}^2} \cdot \sqrt{w^2}
\]
B.1.4.3 Other stresses $\overline{v^2}$, $\overline{w^2}$, and $\overline{uv}$

In these cases approximation errors will not be considered since no information is available regarding higher order correlations, i.e., third and higher. However, the errors in $\overline{v^2}$ and $\overline{uv}$ due to turbulence intensity, for method I, has been estimated by Guitton (G4), which will be quoted here for reference. These errors are estimated to be of the order of 12% for free jet. The errors for turbulent pipe flows are significantly lower, 3–5%. From the equations of the system 3.16 we can write

$$
\overline{v^2} = \frac{\left[ \overline{u_{eff}^2(\alpha=45)} + \overline{u_{eff}^2(\alpha=-45)} - \overline{u_{eff}^2(\alpha=0)}(1+k^2) \right]}{(1-3k^2)}
$$

$$
\overline{uv} = \frac{\left[ \overline{u_{eff}^2(\alpha=45)} - \overline{u_{eff}^2(\alpha=-45)} \right]}{2(1-k^2)}
$$

(B.22)

Following the same procedure as before, the expressions for uncertainty in $\overline{v^2}$ and $\overline{uv}$ are given below. The uncertainty expression for $\overline{w^2}$, i.e. for $\Delta \overline{w^2}$, has not been included as it would be similar to that of $\overline{v^2}$.

$$
\Delta \overline{v^2} = \left[ (a_1 \Delta k)^2 + (a_2 \overline{u_{eff}^2(\alpha=0)})^2 + (a_3 \overline{u_{eff}^2(\alpha=45)})^2 \right]^{1/2}

+ \left[ a_4 \overline{u_{eff}^2(\alpha=-45)} \right]^2
$$

(B.23)

where

$$
a_1 = \frac{6k}{(1-3k^2)} \left[ \overline{u_{eff}^2(\alpha=45)} + \overline{u_{eff}^2(\alpha=-45)} \right] \left( \frac{8k}{(1-3k^2)} \overline{u_{eff}^2(\alpha=0)} \right)
$$
\[ a_2 = -\frac{(1+k^2)}{(1-3k^2)} \]

\[ a_3 = a_4 = \frac{1}{(1-3k^2)} \]

\[ \Delta u v = \left[ (a_5 \Delta u_{eff_1}^2 (\alpha=45))^2 + (a_6 \Delta u_{eff_1}^2 (\alpha=-45))^2 + (a_7 \Delta k)^2 \right]^{1/2} \]  
(B.24)

where

\[ a_5 = \frac{1}{2(1-k^2)} \]

\[ a_6 = \frac{-1}{2(1-k^2)} \]

\[ a_7 = \frac{-k}{(1-k^2)} \left[ \frac{2}{u_{eff_1} (\alpha=45)} - \frac{2}{u_{eff_1} (\alpha=-45)} \right] \]

B.1.5 Errors in Mean Velocity and Turbulence Quantities by Method II

Now, the expressions for uncertainties in mean velocity and turbulence quantities by method II are given below:

B.1.5.1 Mean Velocity

In method II, the mean velocity has been measured using a Pitot probe and an inclined micromanometer. Hence, the uncertainty associated with mean velocity can be calculated using equation B.5. However, in this case the error in \( \Delta \beta \) will be considerable due to turbulence present in the flow. If the hot wire data is to be used for mean velocity calculation following method I, then the uncertainty is given by equation B.18.

B.1.5.2 Errors in Reynold Stresses

Using the equations given in the system 3.17 and the uncertainty
procedure adopted, the following equations can be derived to determine
the uncertainties in \( \bar{v} ^ 2 \), \( \bar{w} ^ 2 \) and \( u ^ 2 \). It should be explicit that there
are no approximation errors involved in the expression for the stresses.

\[
\bar{v} ^ 2 = \left[ (b_1 \Delta U_{\text{eff}} (\alpha=0))^2 + (b_2 \Delta u_{\text{eff}} (\alpha=0))^2 + (b_3 \Delta U_{\text{eff}} (\alpha=45))^2 + (b_4 \Delta u_{\text{eff}} (\alpha=45))^2 + (b_5 \Delta U_{\text{eff}} (\alpha=-45))^2 + (b_6 \Delta u_{\text{eff}} (\alpha=-45))^2 \right] \]

where

\[
b_1 = b_2 = 1
\]

\[
b_3 = b_4 = b_5 = b_6 = \frac{-1}{(1+k^2)}
\]

\[
\bar{w} ^ 2 = \left[ (b_7 \Delta \bar{U} ^ 2)^2 + (b_8 \Delta \bar{U}_{\text{eff}} (\alpha=45))^2 + (b_9 \Delta u_{\text{eff}} (\alpha=45))^2 + (b_{10} \Delta \bar{U}_{\text{eff}} (\alpha=-45))^2 + (b_{11} \Delta u_{\text{eff}} (\alpha=-45))^2 + (b_{12} \Delta \bar{U}_{\text{eff}} (\alpha=45))^2 + (b_{13} \Delta u_{\text{eff}} (\alpha=45))^2 + (b_{14} \Delta \bar{U}_{\text{eff}} (\alpha=-45))^2 + (b_{15} \Delta u_{\text{eff}} (\alpha=-45))^2 \right] \]

where

\[
b_7 = 1
\]

\[
b_8 = b_9 = b_{10} = b_{11} = \frac{-1}{(k^2-1)}
\]
\[ b_{12} = b_{13} = b_{14} = b_{15} = \frac{1}{(k^2+1)} \]

And finally,

\[
\Delta u^2 = \left[ (b_{16} \Delta U_{\text{eff}}^2(\alpha=0))^2 + (b_{17} \Delta u_{\text{eff}}^2(\alpha=0))^2 + (b_{18} \Delta \omega^2)^2 \\
+ (b_{19} \Delta v^2)^2 + (b_{20} \Delta U^2)^2 \right]^{\frac{1}{2}} \tag{B.27}
\]

where

\[ b_{16} = b_{17} = 1 \]

\[ b_{18} = b_{20} = -1 \]

\[ b_{19} = -k^2 \]

In this analysis \( \Delta k \) has been taken to be zero. This assumption has been made for two reasons. First is that it makes the analysis simple. The second reason is that most authors use the value of \( k = 0.2 \) for \( \ell/d = 200 \). This implies that any errors in the results, caused by the variations in \( k \), are negligible.

The following equation has been used in the derivation of the above equations

\[
\frac{E^2}{S^2} = \frac{(E+e)^2}{S^2} = \frac{E^2 + e^2}{S^2} = \frac{U^2}{U_{\text{eff}}^2} + \frac{v^2}{v_{\text{eff}}^2} \tag{B.28}
\]

B.2 Results of Uncertainty Analysis

The uncertainties associated with the measurements of mean and
Reynolds stresses were calculated using equations B.18, B.21 and B.22 for method I and using equations B.5, B.25, B.26 and B.27 for method II. The following basic uncertainties were specified:

\[
\begin{align*}
\text{Uncertainty in the measurement of barometer height} & = 763 \pm 0.1 \text{ mm} \\
T_\infty & = 21 \pm 0.5 ^\circ \text{C} \\
\Delta h' & = \pm 0.1 \text{ mm in potential core} \\
\pm 2 \text{ mm in turbulent flow} \\
\Delta k & = 0.2 \pm 0.01 \\
\beta_1 & = 0.005 \\
S & = 0.109 \text{ volt/ms}^{-1} \text{ for jet flow} \\
& = 0.279 \text{ volt/ms}^{-1} \text{ for pipe flow} \\
\end{align*}
\]

The calculations were performed on a computer. The results of the uncertainties in the quantities have been expressed as percentages and have been plotted as a function of the nondimensional position in the flowfield. Figures B.1 through B.4 summarize the results of the uncertainty analysis.
Figure B.2 Jet Flow: Percentage Uncertainty in $U$
Figure B.3 Pipe Flow: Percentage Uncertainty in Reynolds Stresses

\( \triangledown \) \( \overline{v^2} \) (Method II)

\( \square \) \( \overline{uv} \) (Method I or II)

\( \circ \) \( \overline{u^2} \) (Method I)

\( \circ \) \( \overline{u^2} \) (Method I)
Figure B.4 Jet Flow: Percentage Uncertainty in Reynolds Stresses
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