2000

Investigation of damping treatments for propeller shaft vibration.

Jennifer Lynn. Durfy
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/2502

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000ext. 3208.
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600
INVESTIGATION OF DAMPING TREATMENTS FOR PROPELLER SHAFT VIBRATION

by

Jennifer L. Durfy

A Thesis
Submitted to the Faculty of Graduate Studies and Research through the Department of Mechanical, Automotive and Materials Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

Windsor, Ontario, Canada
2000
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author’s permission.

L’auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L’auteur conserve la propriété du droit d’auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-65288-2
ABSTRACT

This study was performed to evaluate the dynamic behaviour of four different propeller shafts on the basis of mode shape, frequency and damping in response to an excitation in the frequency range of 275-400 Hz. The differences among the four shafts were characterized by the damping treatment. One shaft had no damping treatment, another shaft had an old style of cardboard liner, another shaft had a new style of cardboard liner and the last shaft had a foam damping treatment.

The modal testing of the propeller shafts was conducted at the University of Windsor/DaimlerChrysler Automotive Research and Development Centre. Two shakers were used to excite each shaft while it was supported in the free-free condition. Four measurements were taken around the circumference of each shaft at five different axial locations using modal accelerometers. Data was collected using the LMS Roadrunner 32 system and analyzed using the LMS Modal Analysis module.

Theoretical calculations were completed and a finite element model was developed in order to compare these results with experimental findings. Validation tools were used to support the modal model and to validate the measurement system.

It was concluded that the propeller shaft with the foam damping treatment was the only shaft that did not exhibit resonant behaviour in the 275-400 Hz and demonstrated higher damping values than the other three shafts. Good correlation was obtained between finite element results and experimental results.
DEDICATION

This work is dedicated to my parents for providing me with a foundation based on self-discipline, perseverance and love and to my husband Bruce for providing me with a future filled with promise, laughter and love.
ACKNOWLEDGEMENTS

The author of this thesis would like to express her sincere gratitude to the many people who offered time, resources and knowledge towards this project. Special thanks are extended to my advisor, Dr. P. R. Frise, for his support and his never waiving faith in me. Special recognition is extended to Mr. G. Andrews and the University of Windsor/DaimlerChrysler Automotive Research and Development Centre for without them this project would never have seen the light of day. Appreciation is extended to Mr. P. Haggerty for defining the project and providing resources.

The author would also like to thank Dr. R. Gaspar for his time, knowledge and dedication. Thanks are extended to Mr. L. Martin and Mr. A. Metelka for their support and willingness to lend a helping hand. Special consideration is given to Mr. W. Robitaille for his aid and resourcefulness. Acknowledgement and thanks are also given to Dana Spicer Driveshaft Division for their timely assistance in providing testing materials.

Special thanks go to N. Chana for his valuable assistance in any matter that was presented. I would also like to thank all of my family members for their encouragement throughout this period of study.
# TABLE OF CONTENTS

**ABSTRACT** ........................................................................................................................................ iv

**DEDICATION** .................................................................................................................................... v

**ACKNOWLEDGEMENTS** .................................................................................................................. vi

**LIST OF TABLES** .............................................................................................................................. x

**LIST OF FIGURES** .......................................................................................................................... xii

I. **INTRODUCTION** .......................................................................................................................... 1
   1.0 General ........................................................................................................................................ 1
   1.1 An Overview of Noise/Vibration/Harshness ............................................................................. 1
   1.2 The Present Project ..................................................................................................................... 2
   1.3 Previous Efforts ............................................................................................................................ 3
   1.4 Objectives ..................................................................................................................................... 5

II. **REVIEW OF LITERATURE** ......................................................................................................... 6
   2.0 Introduction ................................................................................................................................. 6
   2.1 Driveline Disturbances ............................................................................................................... 6
       2.1.0 Identification of Driveline Disturbances ............................................................................ 6
       2.1.1 Control of Driveline Disturbances ..................................................................................... 9
   2.2 Axle Gear Mesh ............................................................................................................................ 14
       2.2.0 Description of Axle Gear Mesh ......................................................................................... 14
       2.2.1 Control of Axle Gear Mesh ............................................................................................... 14
   2.3 Modal Testing ............................................................................................................................... 17
       2.3.0 General ............................................................................................................................... 17
       2.3.1 Structural Dynamics .......................................................................................................... 18
       2.3.2 The Fourier Transform ...................................................................................................... 19
       2.3.3 Modal Test Preparation .................................................................................................... 21
       2.3.4 Data Acquisition ............................................................................................................... 24
       2.3.5 Analysis of Data ................................................................................................................ 25
   2.4 Damping Treatments .................................................................................................................... 28
   2.5 Theory ......................................................................................................................................... 29

III. **THEORETICAL MODELLING OF PROPELLER SHAFT** ......................................................... 30
   3.0 Introduction ................................................................................................................................. 30
   3.1 Shell and Beam Theory .............................................................................................................. 30
       3.1.1 Bending Beam Theory ..................................................................................................... 31
       3.1.2 Shell Theory ....................................................................................................................... 33
   3.2 Finite Element Analysis .............................................................................................................. 37

IV. **EXPERIMENTAL DETAILS** ....................................................................................................... 41
   4.0 General ....................................................................................................................................... 41
A.4.2 Graphical Results

B. EXPERIMENTAL EQUIPMENT SPECIFICATIONS ........................................... 113

C. CALCULATIONS FOR CANTILEVER BEAM ...................................................... 118
   C.1 Physical Data
   C.1.1 Beam Dimensions ........................................................................ 119
   C.1.2 Material Properties ...................................................................... 119
   C.2 Preliminary Calculations .................................................................. 119
   C.3 Calculation of Beam Frequencies ...................................................... 120

D. MODE SHAPES FOR SHAFT WITH NO DAMPING TREATMENT .............. 121

E. MODAL MASS CORRECTION ...................................................................... 126

VITA AUCTORIS ............................................................................................... 129
LIST OF TABLES

Table 3.1  Natural Frequencies of Shell Mode Calculations ........................................ 37
Table 3.2: Results from ANSYS analysis of Propeller Shaft........................................... 40
Table 5.1: Modal Assurance Criterion Table for Propeller Shaft with No Damping
  Treatment ..................................................................................................................... 68
Table 6.1: Summary of Frequency and Mode Shape For Tested Propeller Shafts ....... 74
Table 6.2: Modal Assurance Criterion Table for Propeller Shaft with No Damping
  Treatment ..................................................................................................................... 78
Table 6.3: Modal Assurance Criterion Table for Propeller Shaft with Old Style
  Cardboard Liner .......................................................................................................... 79
Table 6.4: Modal Assurance Criterion Table for Propeller Shaft with New Style
  Cardboard Liner .......................................................................................................... 79
Table 6.5 Modal Assurance Criterion Table for Propeller Shaft with Foam Damping
  Treatment ..................................................................................................................... 80
Table 6.6: Modal Phase Collinearity Table for Propeller Shaft with No Damping
  Treatment ..................................................................................................................... 80
Table 6.7: Modal Phase Collinearity Table for Propeller Shaft with Old Style
  Cardboard Liner .......................................................................................................... 81
Table 6.8: Modal Phase Collinearity Table for Propeller Shaft with New Style
  Cardboard Liner .......................................................................................................... 81
Table 6.9: Modal Phase Collinearity Table for Propeller Shaft with Foam............... 81
Table 6.10: Comparison between Propeller Shaft with No Damping Treatment (A)
  and Propeller Shaft with Old Style Cardboard Liner (B) ........................................... 82
Table 6.11: Comparison between Propeller Shaft with No Damping Treatment (A)
  and Propeller Shaft with New Style Cardboard Liner (B) ........................................... 83
Table 6.12: Comparison between Propeller Shaft with No Damping Treatment (A)
  and Propeller Shaft with Foam Damping Treatment (B) ........................................... 83
Table 6.13: Comparison of Beam and Shell Theory Results with Experimental
  Results ........................................................................................................................... 85
Table 6.14: Comparison of Finite Element Results with Experimental Results........... 86
Table 6.15: Comparison of Theoretical Results and Experimental Results for
Cantilever Beam

Table E.1: Comparison of Modal Models based on Mass Subtraction for Propeller
Shaft with No Damping Treatment

Table E.2: Comparison of Modal Models based on Mass Subtraction for Propeller
Shaft with Old Style Cardboard Liner

Table E.3: Comparison of Modal Models based on Mass Subtraction for Propeller
Shaft with New Style Cardboard Liner

Table E.4: Comparison of Modal Models based on Mass Subtraction for Propeller
Shaft with Foam Style Damping Treatment
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Current Propeller Shaft</td>
<td>3</td>
</tr>
<tr>
<td>1.2</td>
<td>Modes of Vibration for Propeller Shaft</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>Hooke or Cardan Joint, {31}</td>
<td>11</td>
</tr>
<tr>
<td>2.2</td>
<td>Typical Cardboard Liner and Assembly, {43}</td>
<td>12</td>
</tr>
<tr>
<td>2.3</td>
<td>Tuned Absorber and Internal Vibration Absorber, {43}</td>
<td>16</td>
</tr>
<tr>
<td>2.4</td>
<td>Physical Representation of the Influence of Other Modes on a Single Mode {23}</td>
<td>27</td>
</tr>
<tr>
<td>3.1</td>
<td>Radial and Axial Nodal Patterns for a Cylinder, {13}</td>
<td>35</td>
</tr>
<tr>
<td>3.2</td>
<td>Shaft Model Created in ANSYS</td>
<td>38</td>
</tr>
<tr>
<td>3.3</td>
<td>Elements used in Modeling of Shaft</td>
<td>39</td>
</tr>
<tr>
<td>4.1</td>
<td>Support Fixture for Propeller Shaft</td>
<td>41</td>
</tr>
<tr>
<td>4.2</td>
<td>Points 1-20 Represent Measurement Locations and Points 21-40 are Used Only to Define the Shaft Geometry</td>
<td>43</td>
</tr>
<tr>
<td>4.3</td>
<td>Screen Layout for Calibration Procedure</td>
<td>47</td>
</tr>
<tr>
<td>4.4</td>
<td>Ratio Calibration of Force Transducer</td>
<td>48</td>
</tr>
<tr>
<td>4.5</td>
<td>Plot of Decay of Accelerometer Response and of Hammer Impact</td>
<td>50</td>
</tr>
<tr>
<td>4.6</td>
<td>Comparison of Frequency Response Function of End, Quarter and Middle of Shaft</td>
<td>51</td>
</tr>
<tr>
<td>4.7</td>
<td>Experimental Setup of Shakers, Stingers and Force Transducers</td>
<td>53</td>
</tr>
<tr>
<td>4.8</td>
<td>Experimental Setup for Modal Testing</td>
<td>54</td>
</tr>
<tr>
<td>4.9</td>
<td>Roadrunner software screen for Acquisition Parameters</td>
<td>56</td>
</tr>
<tr>
<td>4.10</td>
<td>Roadrunner acquisition screen display</td>
<td>57</td>
</tr>
<tr>
<td>4.11</td>
<td>Input Sensitivity and Gain Control Screen Display</td>
<td>58</td>
</tr>
<tr>
<td>5.1</td>
<td>Display showing Summation of FRFs and Mode Indicator Functions</td>
<td>62</td>
</tr>
<tr>
<td>5.2</td>
<td>Stabilization Diagram for Time Domain MDOF method</td>
<td>64</td>
</tr>
<tr>
<td>5.3</td>
<td>Synthesized and Measured FRF for Point #1 on Shaft with No Damping Treatment</td>
<td>66</td>
</tr>
<tr>
<td>5.4</td>
<td>First Bending Mode of Propeller Shaft with No Damping Treatment</td>
<td>67</td>
</tr>
<tr>
<td>5.5</td>
<td>Experimental Set-up for Cantilever Beam Test</td>
<td>70</td>
</tr>
</tbody>
</table>
Figure 6.1: Frequency Response and Coherence for Point #1 on Propeller Shaft with
No Damping........................................................................................................ 72

Figure 6.2: Summation of FRFs for Propeller Shaft with No Damping Treatment ...... 75
Figure 6.3: Summation of FRFs for Propeller Shaft with Old Damping Treatment ...... 76
Figure 6.4: Summation of FRFs for Propeller Shaft with New Damping Treatment .... 76
Figure 6.5: Summation of FRFs for Propeller Shaft with Foam Damping Treatment.... 77
Figure 6.6: First Bending Mode of Cantilever Beam at 60.94 Hz and 5.12%
Damping............................................................................................................ 87

Figure 6.7: Second Bending Mode of Cantilever Beam at 318.07 Hz and 0.98%
Damping............................................................................................................ 88

Figure A.1: Table from Blevins {13} containing values for the dimensionless
frequency parameter .......................................................................................... 102

Figure A.2: Table from Blevins {13} with values for α₁ and α₂............................... 104

Figure A.3: First Bending Mode (First Root).......................................................... 109

Figure A.4: First Bending Mode (Second Root).................................................... 109

Figure A.5: First Breathing and First Bending Mode ............................................. 110

Figure A.6: First Breathing and Second Bending Mode....................................... 110

Figure A.7: Second Bending Mode (First Root).................................................... 111

Figure A.8: Second Bending Mode (Second Root)................................................ 111

Figure A.9: First Breathing and Third Bending Mode........................................... 112

Figure B.1: Specifications for the Endevco Model 61A-500 Accelerometer .......... 114

Figure B.2: Specifications for the PCB Model 208C02 Force Sensor.................... 115

Figure B.3: Specifications for Ling Dynamic Systems V203 Shaker..................... 116

Figure B.4: Specifications for Bruel & Kjaer Calibration Exciter.......................... 117

Figure D.1: First Bending Mode (First Root) for Propeller Shaft with No Damping
Treatment .......................................................................................................... 122

Figure D.2: First Bending Mode (Second Root) for Propeller Shaft with No Damping
Treatment .......................................................................................................... 122

Figure D.3: First Breathing and First Bending Mode of Propeller Shaft with No
Damping Treatment ............................................................................................ 123
Figure D.4: First Breathing and Second Bending Mode of Propeller Shaft with No Damping.......................................................... 123

Figure D.5: Second Bending Mode (First Root) of Propeller Shaft with No Damping.......................................................... 124

Figure D.6: Second Bending Mode (Second Root) of Propeller Shaft with No Damping.......................................................... 124

Figure D.7: First Breathing and Third Bending Mode of Propeller Shaft with No Damping.......................................................... 125
I. INTRODUCTION

1.0 General

The research for this thesis has been conducted under the supervision of Dr. P. R. Frise at the University of Windsor with the support of DaimlerChrysler Canada Inc. A unique relationship exists between the University of Windsor and DaimlerChrysler Canada at a research facility known as the Automotive Research and Development Centre (ARDC). One aspect of this relationship is the development of projects that will enable graduate students to pursue a Master's degree while providing industry with applied research including viable solutions to a real world problem.

1.1 An Overview of Noise/Vibration/Harshness

The area of research that is the focus of this graduate thesis is known as Noise/Vibration/Harshness (NVH). The automotive industry has realized the increasing importance of this field in terms of customer satisfaction and global competition. This branch of study has evolved from a reactive approach, where attempts were made to mask unwanted noise and vibration, to a proactive approach that incorporates NVH solutions in the beginning design phase of the vehicle. Automotive NVH methodology is extremely equipment-intensive and highly technical in nature.

A branch of study that stems from NVH methodology is Modal Analysis. Modal Analysis begins with the identification of an undesirable disturbance. This disturbance can be anything from wind noise to a vibrating component such as a steering column. The next step is to design a test procedure that will provide data relating to the behaviour
of the excited structure. The methods for data collection vary depending on the type of structure and the associated disturbance. Digital signal processing is used to acquire and analyze the data. In most situations, this means translating the data from the time domain into the frequency domain. The time domain represents a given signal as an amplitude as a function of time. This signal can be broken down into series of sine waves, each sine wave having unique amplitude and phase characteristics. These sine waves can be represented in the frequency domain as an amplitude as a function of frequency by means of the Fourier Transform. Different tools can then be utilized to gain valuable information from the data. By creating an experimental model, changes can be made to the design and the relative difference can be quantified. This is an important factor when dealing with automotive production and launch issues. Changes to the vehicle design at these late stages can be very costly.

1.2 The Present Project

Testing has shown that there is room for improvement in the noise and vibration characteristics of the propeller shaft from a 2-wheel drive truck. The vibration/noise presents itself while driving between the range of 1500 – 2200 RPM which corresponds to 60 – 84 km/h (38 – 52 mph). The vibration/noise is only evident when the vehicle is in coast, not while it is in drive. This turns the focus of the problem investigation from one of powertrain induced vibration to drivetrain induced vibration.

Shown in Figure 1.1 is the current production propeller shaft which is a one-piece aluminum shaft that has a length of 1605 mm (63.2”), an outside diameter of 114.8 mm
(4.5") and wall thickness of 1.89 mm (0.074"). The damping treatment for this shaft is a corrugated piece of cardboard tube, approximately 1-2 ft. long, that is pressed into each end of the shaft.

![Current Propeller Shaft](image)

Figure 1.1: Current Propeller Shaft

The geometry of the shaft makes it susceptible to NVH type issues. This large cylinder of aluminum is a good amplifier of axle noise, specifically pinion-gear mesh. The benefits of using such a shaft include the elimination of launch shudder (shaking forces that result from the rearmost universal joint being under maximum torque and angle conditions) that is associated with two-piece propeller shafts, lightweight characteristics and fewer parts due to the elimination of the centre bearing.

1.3 Previous Efforts

The vehicle manufacturer has significant experience regarding the influence of the propeller shaft on NVH issues. Numerous techniques have been investigated in order to
alleviate or minimize the vibration. Some of these methods include using an Internal Vibration Absorber (IVA) propeller shaft that serves as a low cost dynamic absorber, aluminum graphite wrap on the exterior of the shaft and the addition of fins to the inside of the shaft. The vehicle manufacturer has determined that the noise is evident between 60-84 km/h (38-52 mph) and that pinion-gear mesh is the suspect for the source of the excitation. In this situation, the pinion has 11 teeth and the gear has 39 teeth and the gear mesh frequency can be calculated from Warring {52}:

\[ f_g = \frac{nN}{60} \]  

where:

- \( N \) = is the number of teeth on the pinion
- \( n \) = is the relative speed in rev/min

Using this equation, the frequency range of interest becomes 275 Hz to 400 Hz based on the 1500 – 2200 rev/min noise/vibration range. For testing purposes, the frequency range of interest has been expanded to 150-600 Hz. It has been suggested by the vehicle manufacturer that the torsion of the shaft is not a contributor due to the first torsion mode lying outside the frequency range of interest and that the focus should be on the first two bending modes and the first breathing mode as shown below in Figure 1.2.

![Figure 1.2: Modes of Vibration for Propeller Shaft](image)
1.4 Objectives

The objectives of this work are:

1. To determine if the propeller shaft exhibits resonance behaviour in the specified range of interest,

2. To evaluate the differences between a propeller shaft with no damping treatment and three shafts with different styles of damping treatments (old style of cardboard liner, new style of cardboard liner and foam damping treatment) on the basis of frequency, mode shape and damping,

3. To develop a robust modal model that can predict changes to a structure without having to make physical modifications,

4. To compare theoretical calculations and finite element results with experimental findings, and

5. To determine if any one propeller shaft would be better suited for vehicle operation than the current production propeller shaft.
II. REVIEW OF LITERATURE

2.0 Introduction

This chapter will present a review of the literature that describes the different types of disturbances that are associated with vehicle drivelines. Common methods that are used to control these respective disturbances are also presented including material on axle noise and a more detailed survey of the methods that are employed to reduce or eliminate axle noise. The section entitled Modal Analysis will review literature that deals with the concepts and strategies associated with this form of testing including instrumentation, digital signal processing parameters and experimental setup. Some of the latest developments in damping materials will be reviewed along with theoretical concepts that will be applied for the present program.

2.1 Driveline Disturbances

2.1.0 Identification of Driveline Disturbances

Automotive driveline disturbances have been well documented through the years. In most cases, the disturbance is classified according to the source of excitation. For a rear wheel drive vehicle with a front located engine, the SAE publication \{43\} has divided these disturbances into 4 categories: powerplant (engine coupled with transmission), universal joints, driveshaft and rear axle.

For powerplant induced disturbances there are five major contributors which include gear rattle, shudder, clunk, boom, and tip-in-moan. Gear rattle occurs with
manual transmission equipped vehicles during low speed acceleration in high gear.

Essentially what is happening is the engine firing impulses cause all gearing in mesh to
oscillate in a torsional mode through the lash limits of the mating gears. The striking of
the gear teeth that results in an audible sensation is known as gear rattle. Shudder is a
periodic disturbance that can be either engine induced (which means the shudder occurs
at low engine speed and high torque under hard acceleration with a fully engaged clutch)
or clutch induced (which results from self-excited vibration of the clutch plate due to
improper clutch linkage modulation). Driveline clunk resembles an impact type of
disturbance which is, for the most part, associated with backlash in transmission or axle
gearing and/or sudden engagement of automatic transmission friction elements. Boom is
an audible disturbance that is usually experienced during low to medium speed
acceleration. Boom is linked with the vehicle structural resonance and/or drivetrain first
mode vertical bending resonance. The final disturbance associated with powerplant
induced disturbances is tip-in-moan. This disturbance is a response of the vehicle to
engine firing. The primary subsystem is the drivetrain which is set into a vibration
motion of relatively large amplitudes when the engine firing frequency coincides with the
drivetrain second mode vertical bending natural frequency (the physical representation
resembles that of one period of a sine wave). This response is transmitted to the vehicle
passenger via the rear engine mount or the rear suspension, \{43\}.

Pickford \{41\} discusses how nonconstant velocity universal joints (also known as
Hooke joints) will induce disturbances with frequencies that are twice the driveshaft
rotational speed. These disturbances can be torsional in nature which present themselves
as a result of the joint angle being excessive or joint yoke phasing is incorrect which is
essentially a design or assembly error. The disturbance can also be from inertia vibration. This vibration can be produced by oscillating torque loads resulting from the driveshaft inertia being accelerated through nonuniform motion due to excessive joint angles. A double Cardan joint, which may be either a near-constant or a constant velocity joint depending on its design, may cause clicking or buzzing disturbances caused by the unseating of the centering ball in the joint. The clicking noise is more commonly noticed in high torque vehicle start operations whereas the buzzing noise may occur in vehicles operating at relatively high speed.

There are three types of disturbances that are associated with driveshaft induced disturbances, two of which are related to the design of the driveshaft: driveshaft ringing and spline grunt. Driveshaft ringing can occur in a hollow driveshaft responding to diverse driveline inputs in the area of 100 Hz. Spline grunt can reveal itself when the spline is required to slip axially under torque loads as the suspension travels vertically. This more frequently occurs when the vehicle is started forward or when the vehicle suddenly brakes. The third disturbance is drivetrain critical speed and that is a result of driveshaft unbalance which is presented in detail in Nunney [35]. The disturbance occurs when the rotational frequency of the driveshaft is equivalent to a bending resonant frequency of the shaft. The theoretical formula for calculating critical speed can be expressed as follows:

\[ N_c = 30\pi \left( \frac{EIg}{WL^3} \right)^{\frac{1}{2}} \]  \hspace{1cm} 2.1
where:

\[ N_c = \text{Revolutions per minute, \text{rpm}} \]
\[ E = \text{Modulus of Elasticity, Pa (psi)} \]
\[ I = \text{Area moment of Inertia, m}^4 (\text{in}^4) \]
\[ g = \text{Acceleration due to gravity, m/s}^2 (\text{in/s}^2) \]
\[ W = \text{Total weight of shaft, N (lb.)} \]
\[ L = \text{Shaft length between supports (universal joint centres), m (in)} \]

Rear axle induced disturbances include axle noise, start-up groan, axle chuck and driveline rumble. Axle noise is described in detail in the following section. Start-up groan occurs when the vehicle starts moving in forward or reverse gear. The disturbance is a result of the slipping and sticking of the mating tooth surfaces of the drive pinion and ring gear. Axle chuck is due to excessive clearance between the hub of the differential side gear and the bore of the differential case. This disturbance is most commonly experienced when the vehicle is coasting from high speed to a slow crawl. The root cause of driveline rumble is axle gear runout. The runout itself causes frequencies that are very low and do not contribute to axle noise except that these frequencies are often in the range of rear suspension natural frequencies. These resonances occur at certain speeds that can result in driveline rumble {43}.

2.1.1 Control of Driveline Disturbances

Some of the common methods of controlling these disturbances can also be divided into the same four categories: powerplant (engine coupled with transmission), universal joints, driveshaft and rear axle. There are a number of techniques for dealing with the signals associated with engine and transmission induced disturbances. Clutch tuning can reduce or eliminate engine induced boom or gear rattle by reducing the rate or
stiffness of the coil springs that are circumferentially located on the clutch hub and reducing the clutch friction damping. These are design changes to the clutch and/or change in friction material and these changes may have undesirable side effects. Increasing the rotational inertia of the engine flywheel is another method for reducing or eliminating gear rattle or boom. A body boom tuned absorber is a rubber bushing isolated mass attached to the pinion nose of the rear axle carrier by means of a bracket. The tuned frequency of this mass is usually 40-50 Hz. Another type of tuned absorber is a transmission-mounted tuned absorber consisting of a weight attached to the transmission extension housing via an absorber arm. The most common use for this absorber is as a tip-in-moan attenuator and thus it would be tuned to the drivetrain second mode vertical bending natural frequency {43}.

For universal joint disturbances, there are limits that should be followed to minimize the disturbance as explained in Pickford {41}. A typical Hooke or Cardan joint is displayed in Figure 2.1 and described in Martin {31}. The limit associated with torsional excitation is 400 rad/s² angular acceleration and is comparable to a joint angle of 3 degrees operating at 3600 rpm. Design considerations can be made to decrease the joint angle which in turn will lower the torsional excitation. Inertia excitation has a limit of 1000 rad/s² angular acceleration which corresponds to an equivalent joint angle of 4.75 degrees operating at 3600 rpm. Inertia vibrations can be reduced by decreasing the joint angle and/or the size of the driveshaft. These limits are ideal; in practice higher limits may be acceptable in certain situations. For double Cardan joint click, proper design of the centering device components (to eliminate unseating of the centering ball) or restricting the maximum operation joint angle can attenuate the disturbance.
Figure 2.1: Hooke or Cardan Joint \{31\}

The other disturbance caused by the unseating of the centering ball in a double Cardan joint, the buzz disturbance, can be attenuated by shifting or eliminating the excitation frequency that is causing the joint to respond. Mazziotti \{32\} investigated the use of rubber springs between universal joint centres to increase the torsional flexibility thereby reducing disturbances that result from joint angles operating past the optimum value.

For driveshaft induced disturbances, the most common attenuator is a liner inside the hollow tube shaft as shown in Figure 2.2. Berker and Hoover \{12\} describe a liner as an oversized tube of spiral wound corrugated cardboard paper that can be cut to the length of the shaft or cut in smaller sections and inserted into each end of the shaft. This technique reduces driveshaft ringing. When the shaft is assembled, the spiral wound corrugation distorts and provides an interference fit between the cardboard tube and the inside wall of the shaft so that the cardboard does not move relative to the shaft during operation. Press fitting the liner into the shaft is another method used to assemble this
combination. The liner produces only a minimal effect on the critical speed and dynamic balance of the shaft assembly. The control of critical speed vibration requires a sufficiently high driveshaft critical speed and/or a low unbalance.

![Diagram of cardboard liner and assembly](image)

**Figure 2.2:** Typical Cardboard Liner and Assembly \[43\]

Rear axle induced disturbances can be controlled to a certain degree and the methods for doing so will be presented in the following section. For start-up groan, a tuned absorber is tuned to a frequency of 100-150 Hz and mounted to a slip yoke. A tuned absorber is also used to attenuate driveline rumble. This absorber is tuned to 40-60 Hz and bolted to the rear axle carrier and nestled along the driveshaft. Subjective methods for determining vibration acceptability levels were investigated by Berker and Hoover \[12\].

There has been extensive research done on the response characteristics of
drivelines under various forms of excitation. Hodgetts and Parkins {25} investigated the vibration modes of a driveline between 10-200 Hz. The results of this investigation indicated that there were eleven modes of vibration in this range and the first bending mode of the shaft was recorded at 116 Hz. A similar study was performed by Hajduk {21} but was confined to the propeller shaft and the frequency range of interest was 335-710 Hz. The bending modes of interest in this study were the second and third bending modes. Changing the mass, stiffness or end conditions of the shaft will alter the frequency response and resultant noise periods. This effect was demonstrated in this study by inserting a rubber coupling into the shaft, adding ring masses to the outside of the shaft and changing the design of the constant velocity joints.

Different evaluation methods have been developed to determine the forces acting on a propeller shaft. Ono {36} used an influential coefficient matrix to separate the sum of forces located at the ends of the propeller shaft. The results of this matrix method were compared to the data obtained from propeller shaft set up on a rigid bench tester. One end of the shaft was attached to a drive spindle while the other end was attached to a driven spindle. The correlation between the two methods was found to be good.

Another method that is gaining in popularity is the finite element approach. Menday et al. {34} used finite element modeling to evaluate "clonk" (a short duration jerk) on a two-piece driveshaft. Donley et al. {15} used finite elements to predict operating responses in a driveline system. A modal test of an aluminum propeller shaft is performed and the resulting FRFs are displayed from 0-2000 Hz. The majority of the results obtained for the finite element model fell within 1% of the modal test frequencies.
2.2 Axle Gear Mesh

2.2.0 Description of Axle Gear Mesh

Axle gear mesh or axle noise, is a major contributor to driveline vibration. An SAE publication {43} describes axle noise as an audible disturbance that occurs in the 350-650 Hz frequency range. Axle gears are designed to operate at a constant velocity ratio but due to minute variations in manufacturing, this may not be the case. This variation is known as conjugation deviation and is generally present at a frequency equal to or twice the tooth contact frequency. The deviation causes the drive pinion to speed up and slow down with respect to the ring gear for each tooth engagement, which in turn generates drivetrain vibrations, usually with small amplitudes. The problem occurs when the forcing frequency, at a given vehicle speed, coincides with a bending or torsional natural frequency of the drivetrain causing an amplification of the existing motion of the drivetrain component. The signal is then transmitted to the passenger compartment primarily through the rear suspension and secondarily through an airborne path under the vehicle.

2.2.1 Control of Axle Gear Mesh

Axle noise can be interpreted as a torsional or bending excitation although the excitation may both be torsional and bending since the two modes are coupled through the rear axle assembly. However, the primary cause of a specific resonance is related either to the torsional or bending properties of the drivetrain. Torsional attenuators
include tuned absorbers, inertia disks and rubber element driveshafts. Bending attenuators include untuned internal absorbers and driveshafts with low bending compliance (i.e. high stiffness in bending), \{43\}.

Efforts by Esser \{18\} and Schwibinger et al. \{45\} have been made to reduce torsional vibration effects by using tuned absorbers. A tuned absorber consists of a thin elastomer ring mounted concentrically between a steel or cast iron inertia ring and a cylindrical section of the drivetrain. The absorber may be mounted on the universal joint slip yoke hub, driveshaft tube, companion flange hub or axle housing tubes. This device can also be of the internal vibration absorber (IVA) construction where a tubular inertia mass is located by means of two elastomeric rings inside a driveshaft tube as displayed in Figure 2.3. Both devices are based on the concept of the damped vibration absorber, where the amplitude of the main mass, excited by a disturbing force at a given frequency, is reduced by a smaller mass tuned to the disturbed frequency. The ideal location for the absorber is at the antinode of the torsional system. This point corresponds to the point of maximum angular acceleration or highest amplitude of oscillation. The damper will have no effect at a nodal point (point of zero motion). Further study has been performed by Couderc et al. \{14\} on torsional vibrations caused by gear mesh. The study involved an experimental setup where the inputs to the system were modeled to simulate the gear meshing. The experiment enabled a reasonable model to be developed to represent torsional disturbances.
The SAE publication \cite{43} provides a good description of inertia disk and rubber element tube driveshaft used to reduce or eliminate torsional disturbances caused by axle noise. The concept of an inertia disk is identical to the tuned absorber except that it does not incorporate an elastomer ring. The disk is either pressed onto the machined hub diameter of the slip yoke or companion flange, or cast together with these components. The rubber element tube driveshaft consists of four to eight rubber rings bonded by an epoxy type adhesive and pressed into a long outer tube. This design enables the drive torque to be entirely transmitted through the rubber element. The normal position for the rubber element is at the rear of the driveshaft adjacent to the differential.

An untuned internal absorber is used to control drivetrain-bending resonances that result from axle noise. The configuration of the absorber is identical to the torsional tuned absorber mounted internally on the driveshaft except that the elastomer rings are relatively stiffer. The tuning frequency of this device is in the range of 650-750 Hz, which is above the normal 350-650 Hz frequency range of axle noise and this is why the
absorber is referred to as untuned. The function of this absorber is to add inertia to the system which in turn modifies its bending characteristics. The other method used to control bending resonances is to design a shaft that has a shorter length, thinner wall tubing and larger outside diameter. This design would change the bending characteristics of the shaft and create a shaft with low bending compliance.

Ideally, a method or solution to eliminate gear mesh would eliminate this noise and vibration. Unfortunately, the meshing of metal surfaces will always produce a disturbance to some degree. Remmers \(\{42\}\) has examined the dynamics of gear mesh so that a source can be identified by the amplitude at resonance at the corresponding gear mesh frequency. Quality and tolerances are important factors in the production of gears as well as in the meshing of gears and this was studied by Lehmann \(\{27\}\) who examined the effects of axle gear noise on the whole vehicle. The aim of this study was to provide a quality index in the production of gears.

2.3 Modal Testing

2.3.0 General

Modal testing is a branch of experimental work that examines the dynamic behaviour and properties of a structure. These properties are mode shapes, frequency content and damping. Avitabile \(\{4, 6\}\) and Singleton \(\{48\}\) explain that the modes of vibration of a structure are similar to a wave-like motions that travel back and forth through the structure until they decay or are dissipated by damping. A mode shape is a unique displacement vector that exists for each distinct frequency and damping. The
vibration of any structure can be expressed as a sum of that structure’s vibration modes. These modes operate at a specific frequency and when the structure is excited at the same frequency, the amplitude of the specific mode increases causing an increase in vibration levels. The weight and stiffness of the structure determine where these natural frequencies and mode shapes will exist. It is important to know how a structure will vibrate when excited so that efforts can be made to design better structures. The modal testing process begins with an understanding of the time and frequency domain along with structural dynamics. The next consideration in the process is the experimental setup. The acquisition phase which includes digital signal processing is followed by the analysis phase.

2.3.1 Structural Dynamics

Singleton [48] explains that the motion of many types of complex mechanical structures can be described by a time varying waveform using Newton’s Second Law, \( F=ma \). The following equation is an extension of this principle:

\[
mx''(t) + cx'(t) + kx(t) = f(t)
\]  \( 2.2 \)

where:

- \( m \) = mass
- \( c \) = damping
- \( k \) = stiffness
- \( x''(t) \) = acceleration
- \( x'(t) \) = velocity
- \( x(t) \) = displacement

This mathematical model represents a single degree of freedom (SDOF) system. A degree of freedom, in modal analysis, is a measurement point and direction defined on a
structure. In order to obtain a complete “picture” of the response of a structure there has to be a model that has multiple degrees of freedom (MDOF). In the time domain, the MDOF waveform is one waveform but Hewlett {24} explains that any waveform that exists in the real world can be generated by adding sine waves (single degree of freedom system signals) and can be represented by only one combination of sine waves. This is where the frequency domain can be useful. In this domain, each of the sine waves is represented by the amplitude of the sine wave and the frequency at which it occurs. This enables a clearer representation of the signal content.

2.3.2 The Fourier Transform

The means of going from the time domain to the frequency domain and back again is accomplished by the Fourier Transform and its inverse and is presented in Hewlett {24}. This Fourier Transform pair is defined as:

\[ S_x(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, dt \quad \text{(Forward Transform)} \] \hspace{1cm} (2.3)

\[ x(t) = \int_{-\infty}^{\infty} S_x(f) e^{j2\pi ft} \, df \quad \text{(Inverse Transform)} \] \hspace{1cm} (2.4)

where:

- \( x(t) \) = time domain representation of the signal \( x \)
- \( S_x(f) \) = frequency domain representation of the signal \( x \)
- \( j = \sqrt{-1} \)

The Fourier Transform is valid for periodic and non-periodic signals. This transform is
cumbersome to calculate by hand so the utility of this transform increased with the introduction of digital computers. To compute a transform digitally, a numerical integration must be performed. This integration gives an approximation to a true Fourier Transform and is called the Discrete Fourier Transform. There are three difficulties associated with computing the Fourier Transform. The first of the difficulties is that the desired result is a continuous function but the function can only be calculated at discrete points. The transform becomes,

\[ S_x(m\Delta f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi m\Delta f} dt \]  

where:

\[ m = 0, \pm 1, \pm 2 \text{ and } \Delta f = \text{frequency spacing of lines} \]

Evaluating an integral is the same as computing the area under a curve. This presents the second problem since a curve can be defined by an infinite number of points so in order to calculate this area, areas of narrow rectangles under the curve are added together. The last problem is that integration should go from minus infinity to plus infinity and thus would take an infinite time to obtain a result. Therefore, the transform must be computed over a finite time interval. The formula for the Discrete Fourier Transform is then:

\[ S'_x(m\Delta f) \approx \frac{T}{N} \sum_{n=0}^{N-1} x(n\Delta t)e^{-j2\pi mn/N} \]  

where:

\[ T = \text{is the finite time interval (s)} \]
\[ N = \text{is the total number of rectangles} \]
The Fast Fourier Transform (FFT) is an algorithm for computing the Discrete Fourier Transform (DFT). The FFT requires that N be a multiple of 2, this allows symmetries to be used to reduce the number of calculations required. It is important to remember that the FFT only gives samples of the Fourier Transform and it is only a transform of a finite time record of the input. It should also be noted that no information is lost going from the time domain to the frequency domain, the transform is just another way to represent the signal.

2.3.3 Modal Test Preparation

There has been research published by Harris (22), Inman (26), McConnell (33) and Patrick (37) that dealt with the modal testing process as a whole. This includes the necessary steps that are required in setting up a modal test. The first consideration is how the structure will be supported during testing. Analytically, boundary conditions can be specified in a completely free or completely constrained sense. However, as Hewlett (23) clarifies, in testing practice it is not possible to achieve these conditions. A free-free boundary condition implies that the structure is floating in space with no attachments while completely constrained boundary condition requires that all motion be set to zero. Neither of these situations can be fully achieved in practice. However, an approximation to the free-free condition can be to suspend the structure from very flexible supports or place it on a very soft cushion while keeping in mind that the highest rigid body mode frequency must be less than one tenth of the first flexible mode of the support. For very large structures, free-free boundary condition is not practical and the structure should
therefore be fixed, taking care to measure the frequency response of the base at the
attachment points to determine if the amplitude of the response is lower than the response
of the structure.

Exciting the structure is the next step in setting up a modal test and there are
several different ways to accomplish this. The most common methods that are used
today are excitation by means of an impact hammer or by a modal shaker. The
advantages and disadvantages of each method are presented by Avitabile \{10\}, Peterson
\{38, 39\} and Stable \{50\}. The impact hammer is less expensive but is more time
consuming and does not allow for complete excitation of a multiple input – multiple
output (MIMO) system. While a modal shaker is more expensive, it does allow for
constant excitation input which enables simultaneous data collection from multiple
accelerometers. Other issues are discussed by Avitabile \{8, 9, 11\} such as which type of
excitation should be used with a shaker and the best way to conduct an impact test. More
detailed research has been conducted in different areas of modal testing. A paper by Foss
\{20\} describes an experimental technique to eliminate the effect of the suspension on
acquired frequency response functions. In a free-free boundary condition test, the
structure is suspended by very flexible supports which have a very low frequency but can
still have an effect on acquired data. The technique involves supporting the structure
from spring preloaded shaker armatures. All the forces applied to the structure are then
measured as excitation, including the suspension forces.

Once it has been determined how the structure is to be excited the transducers for
sensing force and motion need to be selected. The most common type of transducer is
the piezoelectric type and is the most widely used in modal testing. Hewlett \{23\} defines
this transducer as an electromechanical sensor that generates an electrical output when subjected to vibration. This is accomplished with a crystal element that creates an electrical charge when mechanically strained. The transducer that generates a charge proportional to the applied force is known as a load cell or force transducer. The response transducer has an internal mass that applies a force to the crystal element which is proportional to acceleration and is known as an accelerometer. The optimum accelerometer should have high sensitivity, wide frequency range and low mass. Depending on the structure and type of test trade-offs may need to be made in the selection process of transducers. The Accredited Standards Committee S2 of the American National Standards Institute {1} provides standards on transducer mountings, calibration and transducer operation.

Accelerometer placement is the next issue that concerns the set-up phase of modal testing. Pickeral (40) and Shih et al. (47) discuss this topic in detail. The keypoints are understanding the relationship between degrees of freedom (DOF) and measurements and the difference between a driving point measurement and a transfer measurement. The mobility matrix described in Dossing (17) defines the relationship between degrees of freedom and the measurements. Each row of the matrix contains measurements with a common response DOF (accelerometer) and each column contains measurements with a common excitation DOF (force transducer). Avitabile (3) summarized that all of the information, for a linear mechanical system, needed for a complete set of modal parameters is contained in one row or one column of this matrix. This is true based on a fundamental assumption of modal testing that a mode of vibration can be excited at any point on a structure, except at nodes of vibration where it has no motion. Therefore, the
minimum number of accelerometers and transducers needed is equal to the number of specified DOFs. The diagonal of the matrix contains a group of measurements for which the response and excitation DOFs are the same. This is called a driving point measurement. A set of scaled mode shapes along with modal mass and stiffness cannot be extracted from a set of measurements that does not contain a driving point. A driving point is also helpful in checking measurement quality and locating the excitation placement. A transfer measurement is all of the measurements off of the diagonal line of the matrix.

2.3.4 Data Acquisition

The acquisition phase of modal testing involves selecting the measurement system that will be used to acquire the frequency response functions (FRFs). A FRF is defined by Dossing \cite{16} as:

\[ H(\omega) = \frac{X(\omega)}{F(\omega)} \]  \hspace{1cm} 2.7

where:

- \( H \) = frequency response function
- \( X \) = output spectrum
- \( Y \) = input spectrum

This function represents the complex ratio between input and output as a function of frequency \((\omega)\). Complex ratio means that the function has a magnitude and a phase. Hewlett \cite{23} explains that the measurement system that is used to acquire FRFs is a dynamic signal analyzer which is a Fourier transform based instrument. This system enables the simultaneous measurement of input and response as a result of a given signal or excitation. In impact testing the excitation is a short impulse. For shaker testing, there
are a variety of signals that can be used but the most common is a burst random signal. The differences between the excitation signals are presented clearly in Avitabile {11}, Hewlett {23} and LMS {28}.

Digital signal processing covers the topics that are associated with how the data is acquired. The most fundamental concept is known as the Shannon Sampling Theorem, {24}, {28} and {30}:

\[ F_s > 2f_{\text{max}} \]  

where:

\[ F_s \quad = \quad \text{sampling frequency} = 1/\Delta t \]
\[ f_{\text{max}} \quad = \quad \text{maximum frequency of interest} \]

This means that the sampling frequency must be at least twice the maximum frequency of the analog signal. There are numerous other factors that influence the data such as aliasing, quantization, leakage, averaging and window functions. These topics are covered in detail in {24} and {29}.

2.3.5 Analysis of Data

Parameter estimation techniques are used to identify the modal parameters of the physical system by using the acquired frequency response functions. These techniques are commonly referred to as curve fitters and are mathematically involved algorithms. The concepts, however, are much easier to identify with as well as, computers handle the cumbersome mathematics associated with these techniques.

Avitabile {5, 7}, Dossing {17} and Hewlett {23} cover the topic of curve fitting
in the general sense. Curve fitters can be categorized into single mode versus multiple mode classification. In general, single mode methods are less accurate but require less computational resources. Single mode methods have the basic assumption that in the vicinity of a resonance, the response is primarily due to that single mode. There are four key factors in identifying a resonance: magnitude of the frequency response is a maximum, the imaginary part of the frequency response is a maximum or a minimum, the real part of the frequency response is zero and the response lags the input by 90° phase. The two common single mode methods are quadrature and circle fit methods. For the quadrature method, the magnitude of the modal coefficient is taken as the value of the imaginary part at resonance. The circle fit method is based on the idea that the frequency response of a mode traces out a circle in the imaginary plane. The method fits a circle to the real and imaginary part of the frequency response data by minimizing the error between the radius of the fitted circle and the measured data {23}. The modal coefficient is the diameter of the fitted circle.

Multiple mode methods can be separated into two different forms: the partial fraction form and the polynomial form {17, 28}. The partial fraction form involves a least squares error approach which results in a set of linear equations that must be solved for the modal coefficient and a set that must be solved for frequency and damping. This is an iterative type of solution so there may result convergence problems and long computation times. The polynomial form uses a root finding solution to determine the modal parameters. This method is usually much quicker and results in minimal convergence problems. The multiple mode methods should be used when it is uncertain if a specific resonance has contribution from the modes surrounding it. These methods
account for the residual effects of these out of band modes. A physical description of this effect can be seen in Figure 2.4.

![Diagram showing the influence of other modes on a single mode.]

**Figure 2.4:** Physical Representation of the Influence of Other Modes on a Single Mode (23). The mode in the middle is influenced by the lower and upper modes. The lower mode has mass-like behaviour and the upper mode has stiffness-like behaviour. These influences can be quantified by calculating upper and lower residue values.

There are a number of validation tools that are used to verify the modal
parameters. Dossing {17} explains that the one row or column that was measured during a test can be used to synthesize a non-measured FRF. By measuring the FRF on the test object and comparing it to the predicted FRF, a confidence level can be obtained regarding the accuracy of the modal model. This technique is described by Hewlett {23} and is called the modal assurance criterion (MAC). This is a correlation coefficient between two mode shapes (the experimental mode shape versus the mode shape synthesized from modal parameters). If the coefficient is equal to 1.0 then the two shapes are perfectly correlated, if less than 0.9 there are significant inconsistencies between the two shapes. Once confidence has been ascertained in the modal model then structural changes (i.e. mass and stiffness properties) can be made and quantitative changes can be evaluated.

2.4 Damping Treatments

Technological advances in damping materials have made the development of new damping treatments applicable to many different vibration problems. A spray applied damping material has been introduced by Sophiea and Xiao {49} and compared with existing anti-chip coatings. This spray-on material was proven to be low in cost, easily applied and have superior damping attributes. Another new innovation evaluated by Ahmadian et al. {2} is a smart damping material. This material is a piezoceramic material that acts as a passive vibration suppression tool. The material can be tuned to a resonance frequency, which in turn enables the dissipation of energy.

Other research in damping techniques has been applied directly to shafts or cylindrical rotating objects. The method of constrained layer damping applied to shafts
was presented by Schwartz {44}. A viscoelastic material is sandwiched between two stiff materials and applied to a shaft by using heat shrink tubing. A paper by Sekhar and Kumar {46} presents a comparison of shafts made out of boron epoxy, carbon epoxy and graphite epoxy. Overall, the boron epoxy provided the best overall damping characteristics and the lowest amplitude response of all three materials.

2.5 Theory

Blevins {13} presents the formulas for natural frequency and mode shape on different shaped structures. Chapter 8 deals with the vibration of straight beams which results in the determination of bending modes of vibration. Thomson {51} also deals with the vibration of straight beams. Chapter 12 in Blevins {13} focuses on shell theory and is relevant in the calculation of breathing modes. A paper by Forsberg {19} presents a study that compares theories by Flugge and Donnell for different boundary conditions. The comparisons were done on the basis of natural frequency and mode shape.
III. THEORETICAL MODELLING OF PROPELLER SHAFT

3.0 Introduction

Two approaches have been investigated in this chapter to determine the natural frequencies of the propeller shaft. The first approach involves the use of bending beam and shell theory while the second approach utilizes finite element methods.

3.1 Shell and Beam Theory

A key advantage of this study is the relatively simple design of a propeller shaft which is essentially a thin walled cylinder. A theoretical solution can be obtained by utilizing shell theory and beam theory. Blevins [13] provides formulas in his book for calculating frequencies and mode shapes of structures. Shell theory will be used for determining the breathing modes of the propeller shaft. The bending modes of the propeller shaft will be determined using free-free beam mode calculations.

The boundary condition that has been selected for the propeller shaft is the free-free condition. This boundary condition represents the shaft floating in space with no attachments to any other object and is relatively easy to approximate in a testing environment. The structure is suspended from a rigid frame with flexible supports. This arrangement works because the frequency of the supports (bungey cords, surgical tubing etc.) will be much lower than the frequency range of interest. It is important to understand that this condition does not reflect the conditions under which the propeller shaft operates in a vehicle. The goal is to select support conditions that are experimentally repeatable so that the results of the dynamic measurements reflect the
properties of the structure. The boundary condition is an assumption or approximation that enables a simplified approach to solving a problem.

Since damping is experimentally determined it will not be possible to arrive at a theoretical model of a propeller shaft with a specific damping treatment applied to it. The only theoretical model that will be investigated will be that for the undamped propeller shaft. This model will serve to provide a bounding value when evaluating the data obtained from testing an undamped propeller shaft.

3.1.1 Bending Beam Theory

When a beam experiences transverse vibration it flexes perpendicular to its own axis to alternately store potential energy in the elastic bending of the beam and then releases this energy as kinetic energy for the transverse motion. There are assumptions that have to be made before any of the bending beam equations are used and they are:

- The beams are uniform along the span.
- The beams are composed of a linear, homogeneous, isotropic elastic material.
- The beams are slender.
- Only deformations normal to the undeformed beam axis are considered.
- No axial loads are applied to the beam.
- The shear centre of the beam cross section coincides with the centre of mass, so that rotation and translation of the beam are uncoupled.

The shaft has a change in diameter for a small section and when an effective diameter was calculated to accommodate this it was found to vary from the major diameter by only
0.69%. For the purpose of this project the shaft is considered uniform and the shaft
diameter that will be used is the major diameter. The shaft material is Al-6061 and for all
intends and purposes can be assumed to be linear, homogeneous, isotropic and elastic.

The shaft is slender since its cross sectional dimensions are a magnitude smaller than the
shaft length. No loads will be applied to the shaft and the symmetry of the shaft supports
that the shear centre of the beam cross section is the same as the centre of mass.

The natural frequency of a tubular beam can be calculated from:

\[
f_i = \frac{\lambda_i^2}{2\pi L^2} \left(\frac{EI}{m}\right)^{\frac{1}{2}}
\]

where:

\[
\begin{align*}
\lambda & = \text{a dimensionless parameter which is a function of the boundary} \\
& \quad \text{conditions applied to the beam} \\
L & = \text{the length of the shaft} \\
E & = \text{the modulus of elasticity} \\
I & = \text{area moment of inertia of the beam cross section} \\
m & = \text{beam mass per unit length}
\end{align*}
\]

The values obtained from this equation for an undamped propeller shaft (see properties
and dimensions in Appendix A) are as follows:

\[
\begin{align*}
f_1 & = 235 \text{ Hz} \\
f_2 & = 647 \text{ Hz}
\end{align*}
\]
where:

\[ f_1 \] = the first bending natural frequency
\[ f_2 \] = the second bending natural frequency

The calculations for these values are located in A.2 Bending Beam Calculations in Appendix A. There was one limitation that was observed in calculating the bending natural frequencies and that was the end caps on the shaft. The caps have appreciable mass and cannot be ignored yet the shape of them interferes with the uniform shaft concept. For this situation, the length of the shaft that was used was the uniform length of shaft but the mass of the shaft was taken to be the shaft including the end caps.

3.1.2. Shell Theory

Breathing modes of vibration are mathematically known as shell modes of vibration. A shell is defined by Blevins (13) as “a sheet of elastic material which conforms to a curved surface, the midsurface of the shell”. This includes objects such as a cylinder or something more complicated like a wine glass. The difficulty in analyzing such objects is that the curvature of the shell couples the objects’ flexural and extensional vibrations.

There are a few assumptions that are employed in the analysis of shells:

- The shell must have a constant thickness and have thin walls. The propeller shaft has a uniform wall thickness and has thin walls, the general rule being that the wall thickness must be less than 10% of the shell radius.

- The shell must be composed of a linear, elastic, homogeneous isotropic material.

- No loads have been applied to the shell.

- The deformations of the shell are small in comparison with the radius of the
shell.

- Rotary inertia and shear deformations are neglected.

These assumptions are reasonable in order to determine the theoretical mode shapes of the propeller shaft. Since the radius of the shaft is 57.4mm and the wall thickness is 1.89mm, it is clear to see that the wall thickness is indeed less than 10% of the shaft radius. The shaft material is Al-6061 and can be considered homogeneous, linear and isotropic. The propeller shaft is free of all external loads and the deformation of the shaft will be less than 57.4mm at all times.

There are many different shell theories proposed by different people with the main difference being the assumptions that are made about the form of small terms and the order of terms retained in the analysis. Each shell theory discussed in Blevins \{13\} describes the motion of the shell in terms of an eighth-order differential equation. Inertia terms for each of the three mutually orthogonal displacements are retained in the analysis. If the spatial dependence of each of the deformations can be estimated, then the natural frequencies of the shell can be reduced to the solution of a cubic characteristic polynomial and the relative amplitude of the three displacements can be found from a three by three matrix of simultaneous linear equations \{13\}. A simplified general solution will be used to calculate the shell modes.

Shell modes have two nodal patterns, one being the circumferential nodal pattern and the other being the axial nodal pattern. By looking at Figure 3.1 it is clear to see that circumferential nodal pattern of i = 2 corresponds to the first breathing mode. The nodal pattern of i=1 is the configuration that is assumed in bending beam theory. Therefore for the shell mode calculations i=2 will be used. The axial node pattern has divisions along
the length of the shaft. Since the shaft has a finite length, the solution pertaining to \( i=2 \) and \( j=0 \) is not relevant. The calculations that are pertinent are \( i=2 \) and \( j=1 \), \( i=2 \) and \( j=2 \) and \( i=2 \) and \( j=3 \). Figure 3.1 shows that \( j=3 \) is associated with the third bending mode and is technically not one of the modes of interest but has been included to facilitate a clear understanding during the testing of the structure.

![Circumferential Nodal Pattern](image)

![Axial Nodal Pattern](image)

Figure 3.1 Radial and Axial Nodal Patterns for a Cylinder \{13\}
The equation for calculating the natural frequencies of the shell modes is as follows:

\[ f_{ij} = \frac{\lambda_{ij}}{2\pi R} \left[ \frac{E}{\mu(1 - \nu^2)} \right]^{\frac{1}{2}} \]

where:

- \( \lambda_{ij} \) = dimensionless frequency parameter
- \( R \) = radius of cylinder (m)
- \( E \) = modulus of elasticity (Pa)
- \( \mu \) = density of shell material (kg/m³)
- \( \nu \) = Poisson’s ratio

The dimensionless frequency parameter can be expressed as the following equation:

\[ \lambda_{ij}^2 = \beta_j^4 + \frac{k t^2}{\beta_j^2 \beta_j^2 \beta_j^2 \beta_j^2 + 2 \nu t^2 (t^2 - 1) \alpha_1 + 2 (1 - \nu) (t^2 - 1)^2 \alpha_2} + \frac{k t^4 (t^2 - 1)^2}{\beta_j^2 \beta_j^2 \beta_j^2 \beta_j^2 + 2 \nu t^2 (t^2 - 1)^2 \alpha_2 + k t^4 (t^2 - 1)^2} \]

where:

- \( \beta_j = \frac{\lambda_{ij} R}{L} \), \( \lambda_{ij} \) = dimensionless parameter from Figure A.1
- \( k = \frac{t^2}{12 R^2} \)

- \( \alpha_1 \) and \( \alpha_2 \) = integrals of the mode shape, values are located in Appendix A
- \( R \) = outer radius of shaft
- \( L \) = length of shaft
- \( t \) = wall thickness of shaft
- \( i \) = number of circumferential divisions
- \( j \) = number of axial divisions

36
The results obtained for the shell modes with \( i=2 \) and \( j=1, 2 \) and 3 are presented in Table 3.1.

<table>
<thead>
<tr>
<th>First Breathing and First Bending ( i=2 ) and ( j=1 )</th>
<th>First Breathing and Second Bending ( i=2 ) and ( j=2 )</th>
<th>First Breathing and Third Bending ( i=2 ) and ( j=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>394 Hz</td>
<td>467 Hz</td>
<td>645 Hz</td>
</tr>
</tbody>
</table>

Table 3.1 Natural Frequencies of Shell Mode Calculations.

The calculations for these results can be view in A.3 Shell Mode Calculations in Appendix A.

The shell mode calculations were also inflexible in terms of accounting for the shape and mass of the shaft end caps. Another disadvantage of using the shell theory formulae was the difficulty in obtaining mode shape information. This is a result of the mode coupling effect mentioned earlier in this section.

3.2 Finite Element Analysis

Some limitations were observed in the using of the equations in the previous section. The first limitation was the shaft end caps and how to account for that mass as well as the location of that mass. Another limitation was the inability to obtain a physical representation of the shell modes. In an attempt to overcome these limitations, a simple finite element model was constructed in order to better represent the deformation of the shaft as well as obtain more accurate natural frequencies. By creating a model, the shaft end caps can be approximated as a point mass at each end of the shaft connected to the
shaft by links with very high stiffness.

The software that was used for this analysis is ANSYS, (©1994, Version 5.6.1). Using elastic 8 node shell elements with a thickness of 0.00189 mm created the shaft. The material properties were imported from the software library for Aluminum 6061. The shaft in Figure 3.2 was divided into 40 sections along the z-axis and 16 sections radially, the shaft was then meshed using the quad mapped mesh.

![Shaft Model Created in ANSYS](image)

**Figure 3.2:** Shaft Model Created in ANSYS

Two more element types were then created, the first being a mass element of 0.691 kg (see Appendix A.4 for calculation) and then a 3-D link. The mass element was created on a node in the middle of each end of the shaft. A link element was created from the mass element to every circumferential node at the shaft ends. The link was given a small cross-section and a large stiffness value. This mass and link combination was an
approximation of the end caps on the shaft. Figure 3.3 illustrates the type and location of each element used in the model.

![Diagram of shaft elements]

**Figure 3.3: Elements used in Modeling of Shaft**

Since the shaft is to be tested in the free-free condition, no loads or constraints were applied to the shaft. The modal analysis was performed using the Block Lanczos method for the frequency range of 50 to 800 Hz. The results were viewed in the General Postprocessor and animated so that a clear representation of the mode shape could be viewed. Figures A.3 through A.9 in Appendix A.4 provide a picture of each mode shape and an animation file is located on CD. Table 3.2 summarizes the mode shape and natural frequency of the results obtained.
<table>
<thead>
<tr>
<th>MODE SHAPE</th>
<th>FREQUENCY</th>
<th>LOCATION OF GRAPHIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending (first root)</td>
<td>176 Hz</td>
<td>Figure A.3</td>
</tr>
<tr>
<td>First Bending (second root)</td>
<td>176 Hz</td>
<td>Figure A.4</td>
</tr>
<tr>
<td>First Breathing and First Bending Mode</td>
<td>392 Hz</td>
<td>Figure A.5</td>
</tr>
<tr>
<td>First Breathing and Second Bending Mode</td>
<td>425 Hz</td>
<td>Figure A.6</td>
</tr>
<tr>
<td>Second Bending Mode (first root)</td>
<td>523 Hz</td>
<td>Figure A.7</td>
</tr>
<tr>
<td>Second Bending Mode (second root)</td>
<td>524 Hz</td>
<td>Figure A.8</td>
</tr>
<tr>
<td>First Breathing and Third Bending Mode</td>
<td>533 Hz</td>
<td>Figure A.9</td>
</tr>
</tbody>
</table>

Table 3.2: Results from ANSYS analysis of Propeller Shaft
IV. EXPERIMENTAL DETAILS

4.0 General

The testing was conducted at the University of Windsor/DaimlerChrysler Canada ARDC. Shown in Figure 4.1 is the test stand that was constructed from 1” square tubing to which surgical tubing was attached to support the propeller shaft. The stand has been designed so that the propeller shaft can be accessed from all directions. This enables flexibility when choosing the location for the excitation points.

Figure 4.1: Support Fixture for Propeller Shaft. Note the surgical tubing which is used to suspend the shaft. The tubing is very flexible and has low mass, thus the suspension simulates a free-free boundary condition.

A phenomenon known as repeated roots exists when testing symmetrical structures. By definition, this means that there are two mode shapes operating at the
same frequency. In order to detect these repeated roots, one of two excitation methods must be employed. The first method involves using an impact hammer. While this option is time consuming, it is very easy to perform a roving input to the system. The structure can be excited at different points circumferentially and longitudinally. The second method requires the use of two shakers. The first shaker is capable of exciting the first root and the other shaker is positioned so as to excite the second root. For this project, two shakers were used to excite the system.

A second issue of equal importance stems from the accelerometer selection process. The problem arises from the fact that the propeller shaft is made out of aluminum, making it a very lightweight structure. If the accelerometer has an appreciable mass then it can alter the modal characteristics of the structure. This concept is known as mass loading and it can have the effect of lowering the measured natural frequency of the structure to be less than its true value. In order to prevent this undesirable effect, two methods can be utilized. The first method involves using lightweight accelerometers. Though this option doesn’t guarantee the elimination of mass loading effects, it lowers the added mass to the system to an acceptable level so that the effect can be neglected. The other option is to account for the mass loading effect by means of modal mass correction methods. One of the advantages of the analysis software package that has been chosen to analyze the data, is the ability it has to subtract mass values from measurement locations. A combination of both methods has enabled the mass loading effect to be accounted for in this present project.
4.1 Equipment

The propeller shaft was divided into sections around the circumference and down the length of the shaft as demonstrated in Figure 4.2. It was determined that four measurement locations would be needed around the circumference of the shaft and that five measurement locations would be needed along the length of the shaft. The first bending mode has peak deformation at the mid-point of the shaft making this location a necessary measurement. The second bending mode has peak deformation at the quarter-point of the shaft which is another necessary measurement. The shell modes need at least four measurement locations to describe the motion in that plane. One assumption that has been made is that if the excitation sources are perpendicular to one another than the repeated modes will also be perpendicular to one another.

Figure 4.2: Points 1-20 Represent Measurement Locations and Points 21-40 are Used Only to Define the Shaft Geometry
For this project, it was determined that twenty response measurements and two excitation sources would provide enough data to perform a modal analysis on the propeller shaft. The type of accelerometer that was selected to capture the structure's response was the Endevco Isotron 61A-500 accelerometer. This is a uniaxial accelerometer where the 500 number refers to the sensor's voltage sensitivity in mV/g. Other specifications on this sensor can be found in Figure B.1 in Appendix B. The advantages of using this accelerometer include lightweight characteristics (~6.1 gm), good phase response and rugged design. Though twenty measurements were needed, only 10 accelerometers were used due to channel limitations on the acquisition system. This resulted in testing the shaft in two phases: first half and then the second half. The half of the shaft with no accelerometers on it had dummy masses placed on the empty measurement locations. As long as the shaft was uniformly loaded, the mass value can be removed during the data analysis.

Two vibration shakers were required to excite the structure in two different locations. Both of these shakers are electro-dynamic vibration generators and operate like a loud speaker. There is movement of a coil which is produced by a current in the coil which in turn produces a magnetic field opposing a static magnetic field. The static magnetic field is produced by a permanent magnet and the force that the coil can produce is proportional to the current flowing through the coil. The power is supplied to the vibrator's armature coil by means of an amplifier. One vibrator had a 17.8 N capacity and the other vibrator had a 222.5 N capacity. Specifications for the 17.8 N vibrator can be found in Figure B.3 in Appendix B. A suitable amplifier was selected to operate with the 17.8 N vibrator. The vehicle manufacturer supplied the 222.5 N vibrator with an
amplifier specifically designed for that vibrator.

The excitation force produced by the vibrator was measured by using a quartz force transducer. The force transducer used for this project was the PCB 208C02 with a sensitivity of 11 mV/N. The dynamic range for this transducer is rated at 450 N compression and 450 N tension. Specifications for this sensor can be found in Figure B.2 in Appendix B. The force transducer and vibrator are connected together by an excitation stinger that consists of a thin flexible rod with an attachment mean at both ends. The stinger transmits forces in the stiff axial direction and flexes laterally to reduce input side loads to the structure. Reducing side loading is important because the lateral structural inputs are not measured by a uniaxial reference force sensor.

The analog signals from the transducers were powered, amplified, filtered and converted to a digital output by the LMS Roadrunner 32 system. This system has twelve input channels, one eight channel module and one four channel module. The sample rate for the eight channel input module is 50 kHz per channel and 100 kHz for the four channel module. Both modules are digitized by a sigma-delta 16 bit analog to digital converter (ADC). Also provided by the system was a four channel output card needed to provide the signals to the shakers. The Roadrunner has a built-in signal generator that is capable of creating tone and swept sine, random and burst random signals. The system software controls the data acquisition process and allows the user to input parameters such as triggering options, sampling rates, frequency range and other test related items. The software is designed for easy transfer of Roadrunner files to LMS Modal Analysis module, the chosen data analysis package for this project.

Careful consideration was given to calibration and placement of shakers and this
required special equipment. One important tool that was required to calibrate the transducers was a calibration exciter. The topic of calibration is covered in the following section and the specifications for the calibration exciter can be found in Figure B.4 in Appendix B. In order to determine the optimum location for exciting the propeller shaft a modal survey was performed. For this survey, a PCB 086C03 impact hammer was used with a sensitivity of 2.36 mV/N to excite all of the points to determine the best driving point. The modal survey was completed on an ACE FFT Analyzer since the LMS Roadrunner system was not available at that time. The ACE analyzer is a 32-bit floating point DSP processor operating at 50 MHz coupled with two 16-bit input channels and two 16-bit signal sources. The maximum bandwidth is 20 kHz and has resolution capabilities of 25-1600 frequency lines.

4.2 Calibration

4.2.1 Calibration of Accelerometers

The calibration of accelerometers was completed using the Brueul &Kjaer calibration exciter. The accelerometer was attached to the exciter with beeswax and then attached to an input channel via a low-noise cable. The LMS Roadrunner software helped facilitate the calibration process by providing different calibration options and by automatically updating calibrated values in the test set-up. The type of calibration used was an absolute root mean square (rms) technique. The exciting frequency was 159.2 Hz and corresponded to a reference value of 1g. The exciter was turned on and the signal was recorded for 5 seconds. The voltage level at 1g was recorded as the sensitivity value
of the transducer. Figure 4.3 shows the screen layout used during the calibration process.

![Screen Layout](image)

**Figure 4.3:** Screen Layout for Calibration Procedure

All ten accelerometers were calibrated before and after testing as well as compared to the manufacturer's sensitivity value. All accelerometers were within 5% of the manufacturer's value and were within 2% of the before and after calibration.

**4.2.2 Calibration of Force Transducers**

The calibration of the force transducer was not as straightforward as calibrating the accelerometers. The ratio calibration method was employed and is based on the force
equals mass multiplied by acceleration principle. The force term is represented by the force transducer which is to be calibrated. The acceleration term is a calibrated accelerometer while the mass term is a mass of known quantity. A four pound mass was suspended with an accelerometer attached to one side and the force transducer attached to the other side as seen in Figure 4.4.

Figure 4.4: Ratio Calibration of Force Transducer

The ratio value was calculated as the inverse of the mass value since:

\[
\frac{a}{F} = \frac{1}{m}
\]  

4.1
The calibrated force value can then be obtained since there is only one unknown in the equation. The sampling frequency was 5000 Hz and the frequency range was 200 Hz to 500 Hz. The results obtained were within an acceptable level but since the methods of the manufacturer for calibrating transducers are much more advanced and reliable it was decided that the manufacturer's value should be used for testing purposes.

4.3 Modal Survey

A modal survey was performed in order to determine the optimum location to excite the structure, physically this refers to where the shakers should be located relative to the structure. The equipment used for this survey was an impact hammer and one calibrated Endevco 61A-500 accelerometer. Each of the four shafts was laid out and numbered in the exact same manner. The shaft selected for the modal survey was the shaft with the current damping treatment. The shaft was suspended from the support structure and it's levelness was checked with a driveline inclinometer to within 0.3°.

The sensitivities of the accelerometer and the hammer were input into the channel descriptions of the ACE analyzer. A frequency span of 750 Hz and 1600 spectral lines was chosen which according to the Nyquist Sampling Criterion results in a time period of 2.56 seconds. To avoid leakage (where the signal is not periodic within the time period) the structure's response to the hammer impact must decay to zero within the 2.56 second time frame. Five impacts were averaged for each measurement location and each impact was screened before it was saved to ensure a sharp peak for the impact and that there were no double hits. Figure 4.5 demonstrates the signal characteristics observed during data collection.
A ten percent delay was used in acquiring the response signal so that the entire response signal was captured. A gain of ten was applied to the hammer and a gain of unity was applied to the accelerometer.

Each of the twenty measurement locations were impacted and the response was recorded. Some observations were made such as the similarity in response between measurements having the same axial location but varying radial location. This held true for all axial locations down the shaft. Another observation made was the symmetry of the shaft at the axial midpoint of the shaft. This means the first half of the shaft responded in the same manner as the second half of the shaft. These observations helped in comparing the different measurement locations. This meant that only three locations needed to be compared: one measurement at the end, one at the one-quarter distance and one at the middle distance. The comparison criterion was based on the sharpness of the
resonance peaks, the number of resonance peaks and the steepness of the anti-resonance drop after a resonance peak.

![Graphs showing frequency response function for different locations of a shaft](image)

**Figure 4.6:** Comparison of Frequency Response Function of End, Quarter and Middle of Shaft

From Figure 4.6 it is clear to see that the end of the shaft exhibited the best resonance behaviour. From this analysis it was decided that the ends of the shaft would be the optimum location to excite the structure. The undamped propeller shaft was also tested to increase confidence level in selecting the locations for excitation.
4.4 Experimental Setup

Each of the four shafts was divided into four sections lengthwise and four sections radially with scribed lines. The intersections of these lines represent one measurement location and each intersection was labeled. Each shaft was to be suspended from the universal joint hole located on the shaft's end caps and was check to make sure it was level before testing commenced.

Both shakers were located at one end of the shaft, the large shaker on the ground (in the vertical direction) and the small shaker on a table (in the horizontal direction). The shaker bodies were check to make sure they were level and tightened so as not to move during testing. Each shaker was connected to the proper amplifier and the amplifiers were connected to channel 1 and channel 2 on the Roadrunner output module. A stinger was secured to each of the shakers and a force transducer was attached to the free end of the stinger. The cables for the force transducers were connected to channel 1 and channel 2 on the Roadrunner input module. The small shaker corresponded to channel 1 on both input and output modules and the big shaker was channel 2. Each force transducer cable was labeled with the force transducer serial number and the corresponding channel ID. The force transducer was attached to an aluminum mounting disk that was specially made with one side radiused to follow the curvature of the shaft. The shakers were then located and secured to the shaft with beeswax as demonstrated in Figure 4.7.

The accelerometers were attached to the shaft with beeswax and then connected by a cable to the data acquisition system. The cable was labeled with the accelerometer serial number and corresponding channel ID. The first measurement point was connected
to the first available channel which was channel 3 (since channel 1 and 2 were occupied

Figure 4.7:  Experimental Setup of Shakers, Stingers and Force Transducers

by the force transducers). Since there are only ten available channels for accelerometers
and there are twenty measurement locations, the remainder of the locations had masses
secured to the shaft with beeswax. These masses were constructed to be the same weight
and approximate size of the accelerometers. Once data had been collected for the first
half of the shaft, the location of the accelerometers and masses were reversed.

Each shaft was setup in the exact same manner and careful consideration was
given to the removal and placement of the transducers. The experimental setup is displayed in Figure 4.8.

![Experimental Setup for Modal Testing](image)

**Figure 4.8:** Experimental Setup for Modal Testing

### 4.5 Experimental Procedure

The testing procedure for each of the four propeller shafts was identical. The order of testing for the shafts was as follows:

1. Propeller shaft with no damping treatment (NO)
2. Propeller shaft with old style of cardboard liner (OLD)

3. Propeller shaft with new style of cardboard liner (NEW)

4. Propeller shaft filled with foam (FOAM)

The first step required was the defining and creating of the test in the software. This included selecting the frequency bands for each channel, channel descriptions, acquisition parameters and trigger functions.

In choosing the frequency bands for each channel, the maximum frequency of interest must be known. Frequency components that occur at a frequency greater then $\frac{f}{2}$ the sampling frequency in the analog time history will cause amplitude and frequency errors in the computed frequency domain. These errors are known as aliasing and have been avoided for this testing by choosing the maximum frequency to be 625 Hz for each channel and using a system that samples at 2.5 times the maximum frequency. A block size of 4096 was selected in the acquisition parameters and this value corresponds to a time period of 2.62 seconds and a frequency resolution of 0.3815 Hz.

The next phase in creating the test was defining all of the channels. A geometry was created in the analysis software with specific numbers for each measurement location. This format was used in defining the channels in order to facilitate a smooth transfer of data from acquisition to analysis. The measurement direction was specified as positive $r$ for the accelerometers and negative $r$ for the force transducers since the coordinate system used was cylindrical ($r=$radius, $\theta=$angle and $z=$length). An increment number of 10 was applied to all accelerometer channels in order to collect the data from the second half of the shaft and merge it with the first half. Once the channels were defined, the hardware information for each channel was input. This included the voltage
sensitivity (which was automatically filled in during the calibration procedure), the channel gain and sensor coupling. The channel gain was selectable for ±0.1, ±1 and ±10. The force transducers were set to 0.1 and the accelerometers were set to 1. The sensor coupling for all sensors used in this test was ICP (integrated circuit piezoelectric) which means that the sensors did not require additional external power since the cards on the Roadrunner supplied the excitation power (2 – 10 mA constant current).

The acquisition parameters selected for this test can be viewed in Figure 4.9.

![Figure 4.9: Roadrunner software screen for Acquisition Parameters](image)

Figure 4.9: Roadrunner software screen for Acquisition Parameters

The save mode refers to what will be saved when the measurement is complete. The FRF selection means that the FRF and the coherence for each measurement will be saved. A 5% pretrigger was selected to ensure that the signal is zero at the start of the time period and the trigger level was set to half a Newton. A rectangular or uniform window was
selected and is the same as not applying a window to the data.

The signal generator in Roadrunner is what provides the signal through the output module to the amplifiers and then to the shakers. The type of signal selected for both shakers was burst random. The burst time was selected to be 1.6 seconds and the dead time was selected to be 1.02 seconds with an amplitude of 2 volts.

Once all of the parameters were defined, the system was ready to acquire data. With the amplifiers turned down to zero, the signal generator was started and two independent signals were fed into the amplifiers. The amplifiers were turned up slowly until the structure was adequately excited. The force transducer signal was checked to make sure the signal stopped and started within the prescribed time period. Figure 4.10 shows the Roadrunner acquisition window.

Figure 4.10: Roadrunner acquisition screen display
By pressing the start button located on the acquisition display window, a real time display of the inputs and outputs could be viewed. This was necessary for a number of reasons. It was important to verify that the response of each accelerometer decayed to zero, ensuring that no leakage would occur in the measurements. The arrow buttons on the display allowed the user to quickly move from channel to channel during the real time display and also during acquisition. Another consideration that had to be made was whether or not the channels were utilizing a good range in the analog to digital converter. This was accomplished by pressing the button with the green status bar and the following display (Figure 4.11) would appear.

Figure 4.11: Input Sensitivity and Gain Control Screen Display
As stated before, the input and output modules have a 16 bit ADC which corresponds to an ADC range of 96 dB. Since an overhead of 12 dB was selected, this leaves a dynamic range of 84 dB. The largest dB value noted in the above display is −14 dB which refers to there being 14 dB not being utilized in the conversion process. This still implies that 12 out of the 16 bits are being used to convert the analog data to digital form and was considered to be more than sufficient. The 2 bits reserved for overhead lend confidence towards not receiving any channel overloads.

Three trial runs were then performed before every acquisition. One reason for doing this was to assess whether or not the experiment was repeatable and in all cases this held firm. The trial runs were also done so that the coherence (fractional portion of the output that is linearly related to the input) could be assessed and any changes that needed to be made could be made before the data was collected. A coherence value of 0.8 or greater was considered to be good.

When all concerns were addressed, measurements were acquired for the first half of the shaft. By pressing the INCR (increment) button on the acquisition display the measurement points were updated for the next ten measurements (the second half of the shaft). At this point, the accelerometers and masses were interchanged. The same procedure was followed in acquiring the measurements from the second half of the shaft. When all of the shaft data had been collected it was merged and saved in one file. Each of the four shaft files was then brought into the LMS Modal Analysis software package to be analyzed.
V. ANALYSIS OF DATA

5.0 General

The software package that was chosen for this project was the LMS Cada-X system (Version 3.5C), specifically the Geometry, Modal Analysis and Modal Design modules. The goal of modal analysis is to reduce measured frequency response functions (FRFs) to a limited number of modal parameters like resonant frequencies, damping and mode shapes which describe the measurements in the best possible way. In the chosen software, this was done in a five-step process:

1. Create a geometry
2. Gather FRF measurements
3. Estimation of poles
4. Estimation of residues
5. Validation

The following sections describe each step in detail along with a section describing the procedure used to validate the entire measurement system.

5.1 Geometry

Prior to analysis, a model was created for the structure that was tested. This model was created in the Geometry module under Analysis options located in the Cada-X Kernel. The model was constructed using the same dimensions located in Appendix A that were used for the theoretical calculations. A wireframe of the model was first created using the twenty measurement locations. Since there were only four points
around the circumference of the shaft, four more points had to be added to each radial location because the original four points only defined a square (see Figure 4.2). These additional points are known as slave degrees of freedom and the value for that location was an interpolation between two measurement locations. The geometry of shaft was saved to the project database and was used for each of the four shaft analyses.

5.2 Gathering of Frequency Response Functions

A project database was created along with four different tests for each of the four shafts tested. In the main Cada-X window, the Roadrunner files were imported into each of the respective four tests. Each test was analyzed separately and followed the exact same format. After the files were loaded into the appropriate test, the test data was sifted in the Modal Analysis module. As mentioned in the previous chapter, the Roadrunner acquisition system saved the FRF and the coherence for each measurement location. For modal parameter estimation only FRFs are required, by sifting the data the FRFs can be separated from the coherence functions. Once the FRFs have been gathered together, an index table was created.

Within the index table two valuable tools were utilized. The first of which was the summation option. This option calculated the summed FRF of all the FRFs located in the test. The summation of FRFs helped to detect the resonance peaks because it showed all excited modes and diminished the noise level. The second tool calculated the mode indicator functions (MIFs) which are functions that exhibit minima at the resonance frequencies. The number of MIFs that can be calculated for a data set corresponded with the number of references, so for this project two MIFs were calculated (two shakers).
The MIFs exhibit a minimum at each of the resonance frequencies of the system. Figure 5.1 shows the summed FRF and MIF functions.

![Graph showing FRFs and MIFs]

**Figure 5.1:** Display showing Summation of FRFs and Mode Indicator Functions

### 5.3 Estimation of Poles

The estimation of poles was completed in an Application Specific Monitor (ASM). There are four different ASMs: Single Degree of Freedom (SDOF), Complex MIF, Time Domain Multiple Degree of Freedom (MDOF) and Frequency Domain MDOF. Each of these ASMs refer to the theoretical method that is used for estimating modal parameters and is also known as curvefitting. The ASM selected for the analysis of the propeller shaft was the Time Domain MDOF because this method works for a multiple degree of freedom system and is more suited to structures with low damping.
This ASM is based on the least squares complex exponential method that fits decaying exponentials to response data. The solutions or roots to the fitted equation (also known as the transfer function) are the poles.

The first step in this procedure involved the creation of a stabilization diagram. This was created using the summed FRFs and is an incremental approach whereby the function is first described with a system of 2, 3, 4,... modes. For a model size of m, m values of frequency and damping can be found. If the model size is augmented by 1, so is the number of found resonances. The whole idea is that solutions with equal values for frequency and damping correspond with the true resonances, the ones which correspond to the physical system. The process starts from a model with size m which gives m solutions for frequency, damping and participation vectors. In the next step, the model with m+1 gives m+1 solutions. Each pole from the m+1 model is compared with the ones from the m model. Different situations can arise and have different symbols on the stabilization diagram (Figure 5.2):

- It is the first time the solution is found. This is a new pole.
  ➢ original or o
- If the frequencies of the solution are compared and the frequency value difference is smaller than a tolerance value then the pole of the m+1 model is stable in frequency
  ➢ frequency or f
- If the damping values of both solutions differ more than a certain tolerance value and so do the vector properties. The solution is stable in frequency.
  ➢ frequency or f
- The damping values exceed the tolerance range but the vector properties don't. The solution is stable in pole vector.
  ➢ vector or v
- The damping values do not differ a certain tolerance range but the vector
properties do. The solution is stable in damping.

Both damping and vector properties differences are smaller than the tolerance.
The solution is stable all over.

The stable poles are the ones that correspond to the real resonance phenomena from the physical system.

**Figure 5.2: Stabilization Diagram for Time Domain MDOF method**

When the software is set to “select poles” mode, a dialogue window appears that displays information about each symbol once the symbol is highlighted on the diagram.

The poles were selected by double clicking on the stable poles (two or three letter “s” characters in a column).
5.4 Estimation of Residues

Curvefitting methods are applied to a selected frequency range but a mode of vibration participates over the entire frequency range. Residues account for the effect of the participation of “out of band” modes in the selected frequency range. The lower residual term accounts for mass effects and the upper residual accounts for flexibility effects. Residues are a result of partial fraction expansion of the transfer function and are an indication of the strength of the pole. The significance of the residues in modal analysis is that without calculating the residues, scaled mode shapes cannot be obtained. The math behind this concept is documented well in LMS [28].

Estimating the residues in the software is straightforward and involves two options. The first option requires selecting whether complex or real residue values are required. The real residue value option was selected since lightly damped structures should have mode shapes that are real. The other option allowed the user to specify which residues to calculate, upper and/or lower. Both upper and lower residual terms were calculated during the analysis.

5.5 Validation

The LMS software package provides validation tools that were useful in assessing the results obtained from analysis. One of these validation tools is the synthesis of FRFs. With the resonance frequencies, damping values and residues obtained during the parameter estimation, a recreation of the measured FRF was obtained. This comparison was done for every measurement location and a sample plot can be seen in Figure 5.3. Along with the visual display, additional information was given on error and correlation
values between the measured and synthesized FRF. The error value is the least squares difference normalized to the synthesized values as shown below:

\[
LSerror = \frac{\sum_{i}(S_i - M_i) \times (S_i - M_i)^*}{\sum_{i}(S_i \times S_i^*)}
\]

5.1

where:
- \(S\) = synthesized value
- \(M\) = measured value

The correlation value is the normalized complex product of the synthesized and measured values as shown below:

\[
correlation = \frac{\left| \sum_{i}(S_i \times M_i^*) \right|^2}{\left( \sum_{i}(S_i \times S_i^*) \right) \left( \sum_{i}(M_i \times M_i^*) \right)}
\]

5.2

---

**Figure 5.3** Synthesized and Measured FRF for Point #1 on Shaft with No Damping Treatment
The animated display feature in the Modal Analysis module enabled each mode shape to be visualized as displayed in Figure 5.4. This included the deformed shape of the structure and how the structure moved for a given mode shape. An animation file was created for each mode shape and a graphics file was created for the deformed shape of the structure for each of the four tests. This representation was used to validate the measurement points in terms of direction and magnitude.

Figure 5.4: First Bending Mode of Propeller Shaft with No Damping Treatment

Another validation tool that was employed for this analysis was the Modal Assurance Criterion (MAC). This criterion evaluates the geometric correlation between two mode shapes. It is defined as the normalized product of two mode shapes, which
means that the value is zero when the two mode shapes are orthogonal and equal to one when they are parallel or proportional to each other. The MAC value between modes of structures should be low, this means the mode shapes are unique. Table 5.1 shows that each mode is 100% equal to itself and unique with respect to the other modes.

<table>
<thead>
<tr>
<th>No</th>
<th>mode no</th>
<th>freq Hz</th>
<th>mode_1</th>
<th>mode_2</th>
<th>mode_3</th>
<th>mode_4</th>
<th>mode_5</th>
<th>mode_6</th>
<th>mode_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>170.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100.0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>172.03</td>
<td>6.0</td>
<td></td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>394.53</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>438.16</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td></td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>496.83</td>
<td>0.3</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>498.40</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>9.0</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>551.34</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>1.3</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 5.1: Modal Assurance Criterion Table for Propeller Shaft with No Damping Treatment

To validate whether or not the mode shapes were real or complex, the Mode Phase Collinearity (MPC) was evaluated for each mode. For the modes to be real, the phase angle between two mode shape coefficients of the same mode should be around 0, +180 or −180 degrees. A high MPC value indicates the mode is real.

To account for the effects of mass loading that were described in Chapter 4, an option in the Modal Design module was used to validate the procedures implemented to account for these effects. This option allowed the subtraction of a mass value from all of the measurement locations and then recalculated the modal parameters for the structure. The difference in frequency and damping could then be quantified and evaluated to
determine if the modal model was affected by mass loading effects.

Once all of the modal models for each shaft were validated, the models were compared on the basis of frequency, damping and mode shape. Special consideration was made in the comparison of FRFs and coherence, the synthesis of FRFs and the animated mode shapes.

5.6 Validation of Measurement System

A validation was performed on the measurement system by acquiring data on a cantilever beam. A cantilever beam was chosen since the vibration modes of a cantilever beam are straightforward. The beam was supplied by the University of Windsor and was constructed for the purpose of modal testing. The physical dimensions and theoretical calculations for this beam can be found in Appendix C.

The same acquisition parameters were applied to the beam test as were used for the testing of the shafts. The only differences were that only one shaker was required to excite the beam and only eight accelerometers were used to gather response data. The data collected from the beam test was analyzed with respect to frequency and mode shape since these parameters can be compared to the theoretical results. Figure 5.5 shows the experimental set-up for the testing of the cantilever beam.
Figure 5.5: Experimental Set-up for Cantilever Beam Test
VI. RESULTS AND DISCUSSION

6.0 General

This chapter presents the results for the modal analysis of the four different shafts. The frequency response and coherence functions are discussed as well as the synthesized frequency response functions and the animated mode shapes. The modal models for the propeller shafts are compared and validation results are presented. The measurement system validation results are presented and discussed.

6.1 Frequency Response and Coherence Functions

Before any analysis was initiated, the frequency response and coherence functions were plotted for each measurement point of every propeller shaft. This was done to determine the quality of the measurements obtained and to make observations before analyzing the data.

The coherence function is defined as the fractional portion of the output that is linearly related to the input. If the coherence function is less than 1.0 then either:

- The system relating the input and output is not linear,
- There are unmeasured inputs acting on the system,
- There is no output from the system,

or
- There are bias errors such as noise or leakage in the measurement.

The coherence function varied from point to point but in general the value was above 0.8 and the dips in the function occurred where there is no output from the system (anti-resonances). Figure 6.1 displays the frequency response and coherence function for point
number 1 on the propeller shaft with no damping treatment.

Figure 6.1: Frequency Response and Coherence for Point #1 on Propeller Shaft with No Damping

A few measurement locations had the coherence function fall below 0.8 at one particular frequency range but maintained itself above 0.8 for the majority of the frequency range of interest. This behaviour was most noticeable for the propeller shaft with no damping treatment and the propeller shaft with the old style of cardboard liner. These shafts may have been more susceptible to external acoustics such as fans and machinery operating in the vicinity of the test area.

The frequency response and coherence functions did determine that all of the tested shafts exhibited resonance behaviour in the frequency range of interest (150-500 Hz). It was also surmised that the acquired data was sufficient to proceed with the modal
analysis. All of the plots of frequency response and coherence have been saved in GIF format and are located on the CD at the back of this book.

6.2 Animated Mode Shapes

Once it was determined that the measurements were suitable for analysis, the analysis procedure described in Chapter 5 was completed for each of the four shafts. All of the results were saved under the following headings: NO (propeller shaft with no damping treatment), OLD (propeller shaft with old style of cardboard liner), NEW (propeller shaft with new style of cardboard liner) and FOAM (propeller shaft filled with foam composite). This notation will be used to describe each of the four shafts for the remainder of this thesis.

As soon as the analysis was complete the animations for each shaft were viewed which enabled a physical confirmation of the theoretical and finite element results. Each of the modes presented through animation was accounted for theoretically. The NO shaft, OLD shaft and NEW shaft all exhibited the same modes but the frequencies at which these modes occurred varied. The FOAM shaft also had similar mode shapes except the first breathing mode shape was not present. The concept of repeated roots was demonstrated for the first and second bending modes of each shaft except for the FOAM shaft where this was only evident with the first bending mode. The assumption made about the repeated mode shape occurring in the plane of excitation held true. These mode shapes did appear 90 degrees apart, which was the same configuration of the shakers. Table 6.1 summarizes the mode shape and frequency for each of the four shafts.

73
<table>
<thead>
<tr>
<th>MODE SHAPE</th>
<th>NO</th>
<th>OLD</th>
<th>NEW</th>
<th>FOAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending</td>
<td>170 Hz</td>
<td>169 Hz</td>
<td>175 Hz</td>
<td>169 Hz</td>
</tr>
<tr>
<td>First Bending (Repeated)</td>
<td>172 Hz</td>
<td>170 Hz</td>
<td>176 Hz</td>
<td>171 Hz</td>
</tr>
<tr>
<td>First Breathing, First Bending</td>
<td>394 Hz</td>
<td>384 Hz</td>
<td>395 Hz</td>
<td>-</td>
</tr>
<tr>
<td>First Breathing, Second Bending</td>
<td>438 Hz</td>
<td>410 Hz</td>
<td>437 Hz</td>
<td>430 Hz</td>
</tr>
<tr>
<td>Second Bending</td>
<td>497 Hz</td>
<td>460 Hz</td>
<td>496 Hz</td>
<td>499 Hz</td>
</tr>
<tr>
<td>Second Bending (Repeated)</td>
<td>498 Hz</td>
<td>463 Hz</td>
<td>498 Hz</td>
<td>-</td>
</tr>
<tr>
<td>First Breathing, Third Bending</td>
<td>551 Hz</td>
<td>515 Hz</td>
<td>550 Hz</td>
<td>555 Hz</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of Frequency and Mode Shape For Tested Propeller Shafts

The frequencies for the first bending mode of each shaft are very close, varying within 5%. Most of the shafts remain this close for each of the mode shapes except for the propeller shaft with the old style of cardboard liner. While the variation is still less than 10% it does indicate that this shaft is different. This difference may be attributed to from where the shafts were obtained. The NO, NEW and FOAM shafts were all obtained from the propeller shaft manufacturer whereas the OLD shaft was obtained from the
vehicle manufacturer. The OLD shaft was taken from a production vehicle and the other three shafts were built specifically for testing purposes.

To better demonstrate how the FOAM shaft had two less mode shapes in the frequency range of interest it was useful to examine the summed FRF plot of each of the four shafts as shown in Figures 6.2 through 6.5. The NO, OLD and NEW shafts have the same number of resonant peaks (mode shapes) and relatively the same amplitude for the resonant peaks. The FOAM shaft clearly shows a decrease in amplitude in most of the resonant peaks and only four discernable resonant peaks. The repeated roots are not as visible on the summed FRF plot and account for the other missing mode.

![Graph showing sum blocks and resonant frequency peaks](image-url)

**Figure 6.2:** Summation of FRFs for Propeller Shaft with No Damping Treatment
Figure 6.3: Summation of FRFs for Propeller Shaft with Old Damping Treatment

Figure 6.4: Summation of FRFs for Propeller Shaft with New Damping Treatment
Figure 6.5: Summation of FRFs for Propeller Shaft with Foam Damping Treatment

A figure for each mode shape of the shaft with no damping treatment applied to it can be found in Appendix D. The figures for the rest of the shafts along with animation files for every mode shape have been placed on the CD at the back of this book.

6.3 Synthesis of Frequency Response Functions

The results obtained from the synthesizing of FRFs were a clear indication that the modal model described the behaviour of the structure. The majority of the synthesized FRFs had correlation values of 90% or greater and error values less than 10%. Any of the measurement points that deviated away from the majority had one thing in common and that was the small shaker as the reference shaker. It is a possibility that the mass of the small shaker was not large enough in relation to the mass of the propeller shaft to not experience reactionary forces produced by the propeller shaft. Attempts were
made to rigidly secure the small shaker to a large mass but this did not significantly change any of the results. For each propeller shaft tested, the synthesized and measured FRF for each measurement location can be found on the CD at the end of this book.

6.4 Validation and Comparison of Models

To verify that all of the mode shapes were unique and real, the Modal Assurance Criterion and the Mode Phase Collinearity values were calculated. A low value on a scale from zero to one hundred indicates that the mode shape is unique for the MAC and a high value indicates that the mode shape is real for the MPC. Tables 6.2 through 6.5 present the MAC values for each propeller shaft.

<table>
<thead>
<tr>
<th>No</th>
<th>mode</th>
<th>freq (Hz)</th>
<th>mode_1</th>
<th>mode_2</th>
<th>mode_3</th>
<th>mode_4</th>
<th>mode_5</th>
<th>mode_6</th>
<th>mode_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>170.47</td>
<td>100.0</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>172.03</td>
<td>6.0</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>394.53</td>
<td>0.1</td>
<td>0.1</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>438.16</td>
<td>0.0</td>
<td>0.0</td>
<td>0.5</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>496.83</td>
<td>0.3</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>498.40</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>9.0</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>551.34</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>1.3</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6.2: Modal Assurance Criterion Table for Propeller Shaft with No Damping Treatment
### Table 6.3: Modal Assurance Criterion Table for Propeller Shaft with Old Style Cardboard Liner

<table>
<thead>
<tr>
<th>No</th>
<th>mode no</th>
<th>freq Hz</th>
<th>mode_1</th>
<th>mode_2</th>
<th>mode_3</th>
<th>mode_4</th>
<th>mode_5</th>
<th>mode_6</th>
<th>mode_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>168.96</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>170.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100.0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>383.86</td>
<td>0.2</td>
<td>0.3</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>409.89</td>
<td>0.1</td>
<td>0.0</td>
<td>0.4</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>460.50</td>
<td>0.6</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>462.78</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>515.37</td>
<td>0.1</td>
<td>0.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

### Table 6.4: Modal Assurance Criterion Table for Propeller Shaft with New Style Cardboard Liner

<table>
<thead>
<tr>
<th>No</th>
<th>mode no</th>
<th>freq Hz</th>
<th>mode_1</th>
<th>mode_2</th>
<th>mode_3</th>
<th>mode_4</th>
<th>mode_5</th>
<th>mode_6</th>
<th>mode_7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>175.46</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>176.25</td>
<td>2.8</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>395.56</td>
<td>1.7</td>
<td>0.2</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>437.32</td>
<td>0.1</td>
<td>0.1</td>
<td>0.9</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>496.11</td>
<td>6.4</td>
<td>8.2</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>497.95</td>
<td>1.5</td>
<td>7.3</td>
<td>0.0</td>
<td>0.0</td>
<td>6.1</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>550.30</td>
<td>1.6</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>100.0</td>
</tr>
<tr>
<td>No</td>
<td>mode</td>
<td>freq</td>
<td>mode 1</td>
<td>mode 2</td>
<td>mode 3</td>
<td>mode 4</td>
<td>mode 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>163.98</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>170.73</td>
<td>9.1</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>429.52</td>
<td>0.1</td>
<td>0.0</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>498.96</td>
<td>0.0</td>
<td>0.4</td>
<td>0.1</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>555.48</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.7</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5 Modal Assurance Criterion Table for Propeller Shaft with Foam Damping Treatment

The MAC values for each of the shafts were very low indicating that the modes are unique. The only exception to this is in Table 6.3 where the second mode is related to the first mode by 85%. A possibility for this occurrence could be a result of improper pole selection from the stabilization diagram. Tables 6.6 through 6.9 present the MPC values for each propeller shaft.

<p>| Mode Phase Collinearity (MPC) and Deviation (MPD) |</p>
<table>
<thead>
<tr>
<th>No</th>
<th>mode</th>
<th>freq</th>
<th>MPC (%)</th>
<th>MPD phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>170.47</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>172.03</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>394.53</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>438.16</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>496.83</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>498.40</td>
<td>100.0</td>
<td>LOW</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>551.34</td>
<td>100.0</td>
<td>LOW</td>
</tr>
</tbody>
</table>

Table 6.6: Modal Phase Collinearity Table for Propeller Shaft with No Damping Treatment
Table 6.7: Modal Phase Collinearity Table for Propeller Shaft with Old Style Cardboard Liner

<table>
<thead>
<tr>
<th>No</th>
<th>mode</th>
<th>freq (Hz)</th>
<th>MPC (%)</th>
<th>MPD (deg)</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>168.95</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>170.23</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>383.86</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>409.89</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>460.50</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>462.78</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>515.37</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
</tbody>
</table>

Table 6.8: Modal Phase Collinearity Table for Propeller Shaft with New Style Cardboard Liner

<table>
<thead>
<tr>
<th>No</th>
<th>mode</th>
<th>freq (Hz)</th>
<th>MPC (%)</th>
<th>MPD (deg)</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>175.46</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>176.25</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>366.56</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>437.32</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>496.11</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>497.95</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>550.30</td>
<td>100.0</td>
<td>0.00</td>
<td>low</td>
</tr>
</tbody>
</table>

Table 6.9: Modal Phase Collinearity Table for Propeller Shaft with Foam
All of the MPC values for each shaft were 100% indicating that all of the modes are real and not complex.

As discussed in Chapter 5, there is a method in the Modal Design module that allows for the subtraction of mass values from measurement locations. The accelerometer weighed 6.5 gm so this value was subtracted from each measurement point for each shaft. The results of this showed that there was not a significant change in frequency or damping which is another confirmation that the modal model is valid. The results for this comparison have been tabulated and can be viewed in Tables E.1 through E.4 in Appendix E.

A comparison was done between the propeller shaft with no damping treatment and each of the propeller shafts with damping treatment on the basis of frequency and damping. Tables 6.10 through 6.12 present these results.

<table>
<thead>
<tr>
<th>No</th>
<th>mode</th>
<th>mode</th>
<th>frequency</th>
<th>Modal model comparison</th>
<th>damping</th>
<th>(%)</th>
<th>MPC (%)</th>
<th>MAC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
<td>170.47</td>
<td>168.95</td>
<td>-1.52</td>
<td>1.15</td>
<td>0.72</td>
<td>-0.43</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>11</td>
<td>170.47</td>
<td>170.23</td>
<td>-0.24</td>
<td>1.15</td>
<td>1.34</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>11</td>
<td>172.03</td>
<td>170.23</td>
<td>-1.80</td>
<td>1.26</td>
<td>1.34</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>12</td>
<td>394.53</td>
<td>383.86</td>
<td>-10.67</td>
<td>0.20</td>
<td>0.34</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>13</td>
<td>438.16</td>
<td>409.69</td>
<td>-28.27</td>
<td>0.25</td>
<td>0.44</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>14</td>
<td>466.83</td>
<td>460.50</td>
<td>-6.34</td>
<td>0.41</td>
<td>0.61</td>
<td>0.20</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>15</td>
<td>496.40</td>
<td>462.78</td>
<td>-33.63</td>
<td>0.73</td>
<td>0.86</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>16</td>
<td>551.34</td>
<td>515.37</td>
<td>-35.97</td>
<td>0.34</td>
<td>0.42</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6.10: Comparison between Propeller Shaft with No Damping Treatment (A) and Propeller Shaft with Old Style Cardboard Liner (B)
<table>
<thead>
<tr>
<th>No mode</th>
<th>mode</th>
<th>frequency</th>
<th>Modal model comparison</th>
<th>ducting</th>
<th>(%</th>
<th>diff.</th>
<th>MPC (%)</th>
<th>MAC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>20</td>
<td>170.47</td>
<td>175.46</td>
<td>4.99</td>
<td>1.15</td>
<td>0.22</td>
<td>-0.93</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>22</td>
<td>172.05</td>
<td>176.25</td>
<td>4.22</td>
<td>1.26</td>
<td>1.24</td>
<td>-0.02</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>22</td>
<td>394.56</td>
<td>395.56</td>
<td>1.04</td>
<td>0.20</td>
<td>0.25</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>23</td>
<td>438.16</td>
<td>437.32</td>
<td>-0.84</td>
<td>0.26</td>
<td>0.52</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>24</td>
<td>488.44</td>
<td>489.11</td>
<td>0.72</td>
<td>0.41</td>
<td>0.33</td>
<td>-0.08</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>25</td>
<td>498.40</td>
<td>497.95</td>
<td>-0.46</td>
<td>0.73</td>
<td>0.53</td>
<td>-0.19</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>26</td>
<td>551.94</td>
<td>550.30</td>
<td>-1.04</td>
<td>0.34</td>
<td>0.31</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Table 6.11: Comparison between Propeller Shaft with No Damping Treatment (A) and Propeller Shaft with New Style Cardboard Liner (B)

<table>
<thead>
<tr>
<th>No mode</th>
<th>mode</th>
<th>frequency</th>
<th>Modal model comparison</th>
<th>ducting</th>
<th>(%</th>
<th>diff.</th>
<th>MPC (%)</th>
<th>MAC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>30</td>
<td>170.47</td>
<td>168.98</td>
<td>-1.49</td>
<td>1.15</td>
<td>1.13</td>
<td>-0.02</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>31</td>
<td>172.05</td>
<td>170.73</td>
<td>-1.32</td>
<td>1.26</td>
<td>1.44</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>32</td>
<td>394.56</td>
<td>392.52</td>
<td>34.09</td>
<td>0.20</td>
<td>1.78</td>
<td>1.58</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>33</td>
<td>438.16</td>
<td>437.52</td>
<td>-0.64</td>
<td>0.26</td>
<td>1.78</td>
<td>1.53</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>34</td>
<td>488.44</td>
<td>488.96</td>
<td>2.12</td>
<td>0.41</td>
<td>3.76</td>
<td>3.36</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>35</td>
<td>498.40</td>
<td>498.96</td>
<td>0.56</td>
<td>0.73</td>
<td>3.76</td>
<td>3.03</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>34</td>
<td>551.94</td>
<td>550.48</td>
<td>4.14</td>
<td>0.34</td>
<td>1.59</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Table 6.12: Comparison between Propeller Shaft with No Damping Treatment (A) and Propeller Shaft with Foam Damping Treatment (B)

These results clearly show a significant increase in damping for the propeller shaft with the foam damping treatment. In Table 6.12, the third and sixth comparison are meaningless since there was not a mode for the foam shaft at the corresponding frequency, the computer software picks the next closest value to compare with. The differences in frequency were minimal except for the comparison between the NO shaft
and the OLD shaft. These differences were discussed in Section 6.2. Given that the pinion-gear mesh frequency range was 275-400 Hz and the shaft with foam damping treatment had no resonant frequency in that range makes investigation into using this shaft in production a very viable solution.

6.5 Validation of Measurement System

6.5.0 General

The validation of the measurement system can be demonstrated in two different ways. The first way involves the comparison of the theoretical calculations, finite element results and the experimental results. This comparison was done with the propeller shaft with no damping treatment due to the limitations in calculating theoretical damping values. A good correlation between these results builds confidence in the experimental findings. The second means of validation was discussed in Chapter 5 and involved the testing of a cantilever beam. The results of this test along with a comparison between theoretical and experimental results will be presented.

6.5.1 Comparison of Theoretical and Experimental Results

The best way to display and compare the theoretical, finite element and experimental results is in tabular form. Table 6.13 presents these results along with the percent difference between theoretical and experimental values as well as the percent difference between finite element and experimental values.
<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Theoretical Result (Hz)</th>
<th>Experimental Result (Hz)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending (first root)</td>
<td>235</td>
<td>170</td>
<td>27</td>
</tr>
<tr>
<td>First Bending (second root)</td>
<td>235</td>
<td>172</td>
<td>27</td>
</tr>
<tr>
<td>First Breathing, First Bending</td>
<td>394</td>
<td>395</td>
<td>0</td>
</tr>
<tr>
<td>First Breathing, Second Bending</td>
<td>467</td>
<td>438</td>
<td>6</td>
</tr>
<tr>
<td>Second Bending (first root)</td>
<td>647</td>
<td>497</td>
<td>23</td>
</tr>
<tr>
<td>Second Bending (second root)</td>
<td>645</td>
<td>498</td>
<td>23</td>
</tr>
<tr>
<td>First Breathing, Third Bending</td>
<td>645</td>
<td>551</td>
<td>15</td>
</tr>
</tbody>
</table>

**Table 6.13: Comparison of Beam and Shell Theory Results with Experimental Results**

The theoretical results do vary from the experimental results by a substantial amount but can be attributed to the inability to account for the end caps in the theoretical calculations. Considering this restriction the results are meaningful especially when noticing that the only mode of vibration within the predefined frequency range of interest is the First Breathing, First Bending mode and the percent difference for that mode is 0%.
As a means of accounting for the end caps, a finite element solution was completed and the comparison of those results is presented in Table 6.14.

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Finite Element Result (Hz)</th>
<th>Experimental Result (Hz)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending (first root)</td>
<td>176</td>
<td>170</td>
<td>3</td>
</tr>
<tr>
<td>First Bending (second root)</td>
<td>176</td>
<td>172</td>
<td>2</td>
</tr>
<tr>
<td>First Breathing, First Bending</td>
<td>392</td>
<td>395</td>
<td>-1</td>
</tr>
<tr>
<td>First Breathing, Second Bending</td>
<td>425</td>
<td>438</td>
<td>-3</td>
</tr>
<tr>
<td>Second Bending (first root)</td>
<td>523</td>
<td>497</td>
<td>5</td>
</tr>
<tr>
<td>Second Bending (second root)</td>
<td>524</td>
<td>498</td>
<td>5</td>
</tr>
<tr>
<td>First Breathing, Third Bending</td>
<td>533</td>
<td>551</td>
<td>-4</td>
</tr>
</tbody>
</table>

Table 6.14: Comparison of Finite Element Results with Experimental Results

In general, the finite element results are within 5% of the experimental results and lead to two observations. The first observation reveals that the end caps on the propeller shaft significantly influence the dynamic behaviour of the shaft and could not be neglected or disregarded. The second observation indicates that the experimental results
obtained from testing are meaningful and sufficient to draw conclusions from.

6.5.3 Cantilever Beam Results

The following results were obtained from the testing of the cantilever beam.

Figure 6.6 displays the first bending mode of the cantilever beam and Figure 6.7 displays the second bending mode of the cantilever beam.

![Figure 6.6: First Bending Mode of Cantilever Beam at 61 Hz and 5.12% Damping](image)

Figure 6.6: First Bending Mode of Cantilever Beam at 61 Hz and 5.12% Damping
Figure 6.7: Second Bending Mode of Cantilever Beam at 318 Hz and 0.98% Damping

Table 6.15 compares the theoretical calculations obtained for the cantilever beam with the experimental results.

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Theoretical Result (Hz)</th>
<th>Experimental Result (Hz)</th>
<th>Percent Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Bending</td>
<td>57</td>
<td>61</td>
<td>-7</td>
</tr>
<tr>
<td>Second Bending</td>
<td>357</td>
<td>318</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 6.15: Comparison of Theoretical Results and Experimental Results for Cantilever Beam

The results from this comparison indicate that the testing methodology and measurement system apparatus were more than able to produce meaningful
measurements. The percent difference between the theoretical calculations and experimental findings can be attributed to factors such as bending of the beam due to repetitive testing and loosening of beam connections to the base. All in all, this exercise was able to demonstrate a good degree of reliability in the measurement system used for this project.
VII. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

Careful analysis of the results yielded the following conclusions:

1. Each of the propeller shafts tested exhibited resonance behaviour in the frequency range of 150-600 Hz.

2. Seven modes of vibration were identified for the propeller shaft with no damping treatment, the propeller shaft with the old style of cardboard liner and the propeller shaft with the new style of cardboard liner. Only five modes of vibration were identified for the propeller shaft with the foam damping treatment. The only shaft that did not exhibit resonance behaviour in the pinion-gear mesh frequency range of 275-400 Hz was the propeller shaft with the foam damping treatment. The resonant frequencies did not vary significantly from shaft to shaft. The damping values for the propeller shaft with the foam damping treatment were noticeably higher than the other three shafts.

3. A robust model was developed that can predict changes to a structure without having to make physical modifications. This was demonstrated through the mass loading correction procedure. Validation techniques were used to substantiate the modal model.

4. A comparison of theoretical and experimental results concluded that the end caps of the propeller shaft could not be disregarded for calculation purposes. A comparison between finite element results and experimental findings showed good correlation maintained within 5 percent.
5. Given that the only propeller shaft that did not show resonant behaviour in the frequency range of 275-400 Hz was the propeller shaft with the foam damping treatment, it would be worthwhile to further investigate the use of this shaft for vehicle production.

6. Consideration was given to the accuracy of the transducers, signal analyzer and other related testing equipment and it was surmised that the measurements obtained in this thesis have an uncertainty value in the range of 5-10%.

7.2 Recommendations

The following recommendations will aid in the continued investigation and validation of damping treatments for propeller shaft vibration.

1. The use of tri-axial accelerometers would provide a more complete modal analysis of the structure and would provide simultaneous information on the motion of the structure in both the radial and axial direction.

2. An increase in measurement locations would provide a finer meshed model that would closer represent the motion of the structure.

3. Investigation into using a larger shaker than the 17.8 N shaker would provide insight into the poor measurement quality at the small shaker measurement locations.

4. Given that a test procedure has been defined and validated, it would be useful to test alternate damping materials and compare the results to the results presented in this thesis.
VIII. CONTENTS OF COMPACT DISC

FREQUENCY RESPONSE AND COHERENCE FUNCTIONS

Shaft with Foam
  Point#1.gif-Point#20.gif
Shaft with New Style Liner
  Point#1.gif-Point#20.gif
Shaft with Old Style Liner
  Point#1.gif-Point#20.gif
Shaft with No Damping
  Point#1.gif-Point#20.gif

MODE SHAPE ANIMATIONS – ANSYS

FBM1.avi  (First Bending Mode)
FBM2.avi  (First Bending Mode, Repeated)
FBBF.B avi (First Breathing, First Bending Mode)
FBSB.avi  (First Breathing, Second Bending Mode)
SBM1.avi  (Second Bending Mode)
SBM2.avi  (Second Bending Mode, Repeated)
FBBT.B avi (First Breathing, Third Bending Mode)

MODE SHAPE ANIMATIONS – EXPERIMENTAL

Shaft with Foam
  foamfirstbending.avi (First Bending Mode)
  foamfirst2.avi      (First Bending Mode, Repeated)
  foamsecondbreathe.avi (First Breathing, Second Bending Mode)
  foamsecondbending.avi (Second Bending Mode)
  foamthirdbreathe.avi (First Breathing, Third Bending Mode)
Shaft with New Style Liner
  newfirstbending.avi (First Bending Mode)
  newfirst2.avi      (First Bending Mode, Repeated)
  newfirstbreath.avi (First Breathing, First Bending Mode)
  newsecondbreathe.avi (First Breathing, Second Bending Mode)
  newsecondbending.avi (Second Bending Mode)
  newsecond2.avi    (Second Bending Mode, Repeated)
  newthirdbreathe.avi (First Breathing, Third Bending Mode)
Shaft with Old Style Liner
  oldfirstbending.avi (First Bending Mode)
  oldfirst2.avi      (First Bending Mode, Repeated)
  oldfirstbreath.avi (First Breathing, First Bending Mode)
oldsecondbreath.avi  (First Breathing, Second Bending Mode)
oldsecondbending.avi  (Second Bending Mode)
oldsecond2.avi  (Second Bending Mode, Repeated)
oldthirdbreath.avi  (First Breathing, Third Bending Mode)

Shaft with No Damping
nofirstbending.avi  (First Bending Mode)
nofirst2.avi  (First Bending Mode, Repeated)
nofirstbreath.avi  (First Breathing, First Bending Mode)
nosecondbreath.avi  (First Breathing, Second Bending Mode)
nosecondbending.avi  (Second Bending Mode)
nosecond2.avi  (Second Bending Mode, Repeated)
nothirdbreath.avi  (First Breathing, Third Bending Mode)

MODE SHAPE FIGURES
Shaft with Foam
foamfirstbending.gif  (First Bending Mode)
foamfirst2.gif  (First Bending Mode, Repeated)
foamsecondbreath.gif  (First Breathing, Second Bending Mode)
foamsecondbending.gif  (Second Bending Mode)
foamthirdbreath.gif  (First Breathing, Third Bending Mode)

Shaft with New Style Liner
newfirstbending.gif  (First Bending Mode)
newfirst2.gif  (First Bending Mode, Repeated)
newfirstbreath.gif  (First Breathing, First Bending Mode)
newsecondbreath.gif  (First Breathing, Second Bending Mode)
newsecondbending.gif  (Second Bending Mode)
newsecond2.gif  (Second Bending Mode, Repeated)
newthirdbreath.gif  (First Breathing, Third Bending Mode)

Shaft with Old Style Liner
oldfirstbending.gif  (First Bending Mode)
oldfirst2.gif  (First Bending Mode, Repeated)
oldfirstbreath.gif  (First Breathing, First Bending Mode)
oldsecondbreath.gif  (First Breathing, Second Bending Mode)
oldsecondbending.gif  (Second Bending Mode)
oldsecond2.gif  (Second Bending Mode, Repeated)
oldthirdbreath.gif  (First Breathing, Third Bending Mode)

Shaft with No Damping
nofirstbending.gif  (First Bending Mode)
nofirst2.gif  (First Bending Mode, Repeated)
nofirstbreath.gif  (First Breathing, First Bending Mode)
nosecondbreath.gif  (First Breathing, Second Bending Mode)
nosecondbending.gif  (Second Bending Mode)
nothirdbreath.gif  (First Breathing, Third Bending Mode)
SYNTHESIZED FREQUENCY RESPONSE FUNCTIONS

Shaft with Foam
  Point#1.gif-Point#20.gif

Shaft with New Style Liner
  Point#1.gif-Point#20.gif

Shaft with Old Style Liner
  Point#1.gif-Point#20.gif

Shaft with No Damping
  Point#1.gif-Point#20.gif
REFERENCES


5. Avitabile, P., “Curvefitting is so Confusing to Me, What Do All the Techniques Mean?”, SEM Experimental Techniques, February, 1999.


APPENDIX A

A. THEORETICAL CALCULATIONS AND MODELLING
A.1 Physical Data

A.1.1 Dimensions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Length of Shaft</td>
<td>1.605 m</td>
</tr>
<tr>
<td>R</td>
<td>Outer Radius of Shaft</td>
<td>0.0574 m</td>
</tr>
<tr>
<td>t</td>
<td>Wall Thickness of Shaft</td>
<td>0.00189 m</td>
</tr>
<tr>
<td>M</td>
<td>Mass of Shaft</td>
<td>4.313 kg</td>
</tr>
</tbody>
</table>

A.1.2 Material Properties

Material Properties for Aluminum 6061 from p. 438 Blevins, {13}.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ν</td>
<td>Poisson's Ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
<td>69 Gpa</td>
</tr>
<tr>
<td>μ</td>
<td>Density of Shell Material</td>
<td>2710 kg/m³</td>
</tr>
</tbody>
</table>

A.1.3 Preliminary Calculations

\[
\frac{4.313 \text{kg}}{m (\text{mass per unit length})} = 1.605m = 2.6872 \text{ kg/m}
\]

\[
I (\text{area moment of inertia}) = \pi R^3 t = \pi(0.0574)^3 (0.00189) = 1.123 \times 10^{-6} \text{ m}^4
\]

A.2 Bending Beam Calculations

\[
f_i = \frac{\lambda_i^2}{2\pi^2} \left( \frac{EI}{m} \right)^{\frac{1}{2}}
\]
where:
\[ \lambda_i = \text{a dimensionless parameter which is a function of the boundary conditions applied to the beam (see Figure A.1)} \]

Notation: 
- \( x \) = distance along span of beam;
- \( m \) = mass per unit length of beam;
- \( E \) = modulus of elasticity;
- \( I \) = area moment of inertia of beam about neutral axis (Table 5-1);
- \( L \) = span of beam;
- see Table 3-1 for consistent sets of units.

**Natural Frequency (Hertz):**
\[ f_i = \frac{\lambda_i^2}{2\pi L} \left( \frac{EI}{m} \right)^{1/2} \quad i=1,2,3,\ldots \]

<table>
<thead>
<tr>
<th>Description</th>
<th>( \lambda_i ): ( i=1,2,3,\ldots )</th>
<th>Mode Shape, ( \gamma_i (x) )</th>
<th>( a_i ): ( i=1,2,3,\ldots )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Free-Free</td>
<td>4.73004074</td>
<td>( \frac{\lambda_i}{L} \cos \frac{\lambda_i}{L} + \frac{\lambda_i}{L} \sin \frac{\lambda_i}{L} )</td>
<td>0.982502315</td>
</tr>
<tr>
<td></td>
<td>7.83329462</td>
<td>( \frac{\lambda_i}{L} \cos \frac{\lambda_i}{L} + \frac{\lambda_i}{L} \sin \frac{\lambda_i}{L} )</td>
<td>1.000777312</td>
</tr>
<tr>
<td></td>
<td>10.9956478</td>
<td>( \frac{\lambda_i}{L} \cos \frac{\lambda_i}{L} + \frac{\lambda_i}{L} \sin \frac{\lambda_i}{L} )</td>
<td>0.999964450</td>
</tr>
<tr>
<td></td>
<td>14.1371655</td>
<td>( \frac{\lambda_i}{L} \cos \frac{\lambda_i}{L} + \frac{\lambda_i}{L} \sin \frac{\lambda_i}{L} )</td>
<td>1.000001450</td>
</tr>
<tr>
<td></td>
<td>17.2387997</td>
<td>( \frac{\lambda_i}{L} \cos \frac{\lambda_i}{L} + \frac{\lambda_i}{L} \sin \frac{\lambda_i}{L} )</td>
<td>0.999999937</td>
</tr>
<tr>
<td></td>
<td>( (2L + 1)^{1/2} \frac{\lambda_i}{L} )</td>
<td>( \gamma_i (x) )</td>
<td>0.1.0 for ( i=5 )</td>
</tr>
</tbody>
</table>

**Figure A.1:** Table from Blevins (13) containing values for the dimensionless frequency parameter

For \( i=1 \), First Bending Mode

\[ \lambda = 4.73004074 \]
\[ L = 1.605 \text{ m} \]
\[ E = 69 \times 10^9 \text{ Pa} \]
\[ I = 1.123 \times 10^{-6} \text{ m}^4 \]
\[ m = 2.6872 \text{ kg/m} \]

\[ f_1 = \frac{(4.73004074)^2}{2\pi(1.605)^2 \left( \frac{69 \times 10^9)(1.123 \times 10^{-6})}{2.6872} \right)^{1/2}} \]

\[ = (1.382292)(169.8104559) \]

\[ = 234.73 \text{ Hz} \]

102
For $i=2$, Second Bending Mode

\[
\lambda = 7.85320462 \\
L = 1.605 \text{ m} \\
E = 69 \times 10^9 \text{ Pa} \\
I = 1.123 \times 10^{-6} \text{ m}^4 \\
m = 2.6872 \text{ kg/m}
\]

\[
f_2 = \frac{(7.85320462)^2}{2\pi(1.605)^2} \left( \frac{(69 \times 10^9)(1.123 \times 10^{-6})}{2.6872} \right)^{\frac{1}{2}}
\]

\[
= (3.81034136)(169.8104559) \\
= 647.04 \text{ Hz}
\]

A.3 Shell Mode Calculations

\[
f_{ij} = \frac{\lambda_{ij}}{2\pi R} \left[ \frac{E}{\mu(1-\nu^2)} \right]^{\frac{1}{2}}
\]

$\lambda_{ij}$ = a dimensionless frequency parameter given by the equation:

\[
\lambda_{ij}^2 = \frac{\beta_j^4 + k^2 \beta_j^2 \left[ \beta_j^2 i^2 + 2\nu i^2 (i^2 - 1) \alpha_1 + 2(1-\nu)(i^2 - 1)^2 \alpha_2 \right] + k^4 (i^2 - 1)^2}{\beta_j^2 \alpha_2 + i^2 (i^2 + 1)}
\]

103
where:

\[ \beta_j = \frac{\lambda_j R}{L}, \quad \lambda_j = \text{dimensionless parameter from Figure A.1} \]

\[ k = \frac{t^2}{12R^2} \]

\[ \alpha_1 \text{ and } \alpha_2 = \text{integrals of the mode shape, values are located in Figure A.2} \]

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( 4 )</th>
<th>( 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Free-free</td>
<td>0.5499</td>
<td>0.7467</td>
<td>0.8180</td>
<td>0.8585</td>
<td>0.8843</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.2116</td>
<td>1.7662</td>
<td>1.5456</td>
<td>1.4244</td>
<td>1.3473</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Free-pinned</td>
<td>0.7467</td>
<td>0.8585</td>
<td>0.9021</td>
<td>0.9251</td>
<td>0.9394</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.7662</td>
<td>1.4244</td>
<td>1.2938</td>
<td>1.2247</td>
<td>1.1819</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Clamped-free</td>
<td>-0.2441</td>
<td>0.6033</td>
<td>0.7440</td>
<td>0.8182</td>
<td>0.8585</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.3219</td>
<td>1.4712</td>
<td>1.2529</td>
<td>1.1820</td>
<td>1.1615</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Clamped-pinned</td>
<td>0.7467</td>
<td>0.8585</td>
<td>0.9021</td>
<td>0.9251</td>
<td>0.9394</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7467</td>
<td>0.8585</td>
<td>0.9021</td>
<td>0.9251</td>
<td>0.9394</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Clamped-clamped</td>
<td>0.5499</td>
<td>0.7467</td>
<td>0.8180</td>
<td>0.8585</td>
<td>0.8843</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5499</td>
<td>0.7467</td>
<td>0.8180</td>
<td>0.8585</td>
<td>0.8843</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Pinned-pinned,</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sliding-sliding,</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sliding-pinned</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure A.2: Table from Blevins {13} with values for \( \alpha_1 \) and \( \alpha_2 \)
For $i=2$ and $j=1$, First Breathing and First Bending

\[
\lambda_1 = 4.73004074
\]
\[
R = 0.0574 \text{ m}
\]
\[
L = 1.605 \text{ m}
\]
\[
t = 0.00189 \text{ m}
\]
\[
E = 69 \times 10^9 \text{ Pa}
\]
\[
\mu = 2710 \text{ kg/m}^3
\]
\[
\nu = 0.33
\]
\[
\alpha_1 = 0.5499
\]
\[
\alpha_2 = 2.2116
\]

\[
\beta_1 = \frac{(4.73004074)(0.0574)}{1.605}
\]
\[
= 0.1691616
\]

\[
k = \frac{(0.00189)^2}{12(0.0574)^2}
\]
\[
= 9.0348 \times 10^{-5}
\]

\[
\lambda_{21}^2 = 0.000705319
\]

\[
\lambda_{21} = 0.026557843
\]

\[
f_{21} = \frac{(0.026557843)\left[\frac{69 \times 10^9}{2710(1-0.33^2)}\right]}{2\pi(0.0574)\left[\frac{69 \times 10^9}{2710(1-0.33^2)}\right]^{1/2}}
\]
\[
= 393.62 \text{ Hz}
\]
For \( i=2 \) and \( j=2 \), First Breathing and Second Bending

\[
\begin{align*}
\lambda_2 &= 7.85320462 \\
R &= 0.0574 \text{ m} \\
L &= 1.605 \text{ m} \\
t &= 0.00189 \text{ m} \\
E &= 69 \times 10^9 \text{ Pa} \\
\mu &= 2710 \text{ kg/m}^3 \\
\nu &= 0.33 \\
\alpha_1 &= 0.7467 \\
\alpha_2 &= 1.7662
\end{align*}
\]

\[
\beta_2 = \frac{(7.85320462)(0.0574)}{1.605} \\
= 0.280856041
\]

\[
k = \frac{(0.00189)^2}{12(0.0574)^2} \\
= 9.0348 \times 10^{-5}
\]

\[
\lambda_2^2 = 0.000993925
\]

\[
\lambda_2 = 0.031526572
\]

\[
f_{22} = \frac{(0.031526572)^{\frac{1}{2}}}{2\pi(0.0574)} \left[ \frac{69 \times 10^9}{2710(1 - 0.33^2)} \right]^{\frac{1}{2}} \\
= 467.26 \text{ Hz}
\]
For $i=2$ and $j=3$, First Breathing and Third Bending

$\lambda_3 = 10.9956078$

$R = 0.0574 \text{ m}$

$L = 1.605 \text{ m}$

$t = 0.00189 \text{ m}$

$E = 69 \times 10^9 \text{ Pa}$

$\mu = 2710 \text{ kg/m}^3$

$v = 0.33$

$\alpha_1 = 0.818$

$\alpha_2 = 1.5456$

$$
\beta_3 = \frac{(10.9956078)(0.0574)}{1.605}
= 0.393238559
$$

$$
k = \frac{(0.00189)^2}{12(0.0574)^2}
= 9.0348 \times 10^{-5}
$$

$\lambda_2^2 = 0.001895393$

$\lambda_{23} = 0.043536117$

$$
f_{23} = \frac{(0.043536117)\left[\frac{69 \times 10^9}{2710(1-0.33^2)}\right]}{2\pi(0.0574)}
= 645.26 \text{ Hz}
$$
A.4 ANSYS Results

A.4.1 Weight of End Caps

Volume of shaft not including end caps:

\[ V = \pi (R_o^2 - R_i^2)L \]

\[ V = \pi ((0.0574)^2 - (0.0555)^2) (1.605) \]

\[ V = 0.0010816 \, m^3 \]

\[ m = V \mu \]

\[ m = (0.0010816 \, m^3) \times (2710 \, \frac{kg}{m^3}) \]

\[ m = 2.9312 \, kg \]

Total weight of shaft minus weight of shaft without end caps:

\[ m_{EC} = m_T - m_S \]

\[ m_{EC} = 4.313 - 2.9312 = 1.3818 \, kg \]

Since there are two end caps:

\[ m_{EC} = \frac{1.3818}{2} = 0.691 \, kg \]
A.4.2 Graphical Results

Figure A.3: First Bending Mode (First Root)

Figure A.4: First Bending Mode (Second Root)
Figure A.5: First Breathing and First Bending Mode

Figure A.6: First Breathing and Second Bending Mode
Figure A.7:  Second Bending Mode (First Root)

Figure A.8:  Second Bending Mode (Second Root)
First Breathing and Third Bending
i=2, j=3

Figure A.9: First Breathing and Third Bending Mode
APPENDIX B

B. EXPERIMENTAL EQUIPMENT SPECIFICATIONS
### Endevco Isotron Accelerometer

<table>
<thead>
<tr>
<th><strong>Model Number:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>61A-500</td>
</tr>
</tbody>
</table>

#### Dynamic Performance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Sensitivity</td>
<td>mV/g</td>
<td>500</td>
</tr>
<tr>
<td>Measurement Range</td>
<td>g</td>
<td>±10</td>
</tr>
<tr>
<td>Frequency Range (±5%)</td>
<td>Hz</td>
<td>1-2000</td>
</tr>
<tr>
<td>Mounted Resonant Frequency</td>
<td>Hz</td>
<td>15 000</td>
</tr>
<tr>
<td>Amplitude Nonlinearity</td>
<td>%</td>
<td>less than or equal to 1</td>
</tr>
<tr>
<td>Transverse Sensitivity</td>
<td>%</td>
<td>less than or equal to 3</td>
</tr>
</tbody>
</table>

#### Environmental

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shock Limit (max.)</td>
<td>g pk.</td>
<td>8000</td>
</tr>
<tr>
<td>Operating Temperature Range</td>
<td>°C</td>
<td>-20 to 85</td>
</tr>
<tr>
<td>Strain Sensitivity</td>
<td>g/µstrain</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>@250µstrain</td>
<td></td>
</tr>
</tbody>
</table>

#### Electrical

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation Voltage/Constant Current</td>
<td>Vdc/mA</td>
<td>21-24/2-10</td>
</tr>
<tr>
<td>Warm-up time (to reach 90% bias)</td>
<td>sec</td>
<td>&lt;5</td>
</tr>
</tbody>
</table>

#### Mechanical

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing Material</td>
<td>Aluminum Alloy, Anodized</td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>mm x mm x mm</td>
<td>14.27 x 14.27 x 14.27</td>
</tr>
<tr>
<td>Weight</td>
<td>gm</td>
<td>6.1</td>
</tr>
<tr>
<td>Electrical Connector</td>
<td></td>
<td>10-32 coax.</td>
</tr>
</tbody>
</table>

**Figure B.1:** Specifications for the Endevco Model 61A-500 Accelerometer
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PCB Quartz Force Sensor</strong></td>
<td><strong>Model Number:</strong></td>
<td><strong>208C02</strong></td>
<td></td>
</tr>
<tr>
<td><strong>DYNAMIC PERFORMANCE</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Voltage Sensitivity (±15%)</td>
<td>mV/N</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Compression/Tension Range</td>
<td>N</td>
<td>450/450</td>
<td></td>
</tr>
<tr>
<td>Frequency Range (±5%)</td>
<td>Hz</td>
<td>0.001 to 36k</td>
<td></td>
</tr>
<tr>
<td>Maximum Compression</td>
<td>N</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>Maximum Tension</td>
<td>N</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Resolution (broadband)</td>
<td>N rms</td>
<td>0.0045</td>
<td></td>
</tr>
<tr>
<td><strong>ENVIRONMENTAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature Coefficient</td>
<td>%/°C</td>
<td>Less than or equal to 0.09</td>
<td></td>
</tr>
<tr>
<td>Operating Temperature Range</td>
<td>°C</td>
<td>-54 to +121</td>
<td></td>
</tr>
<tr>
<td><strong>ELECTRICAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excitation Voltage/Constant Current</td>
<td>Vdc/mA</td>
<td>20-30/2-20</td>
<td></td>
</tr>
<tr>
<td>Discharge Time Constant</td>
<td>sec</td>
<td>Greater than or equal to 500</td>
<td></td>
</tr>
<tr>
<td>Output Impedance</td>
<td>ohm</td>
<td>Less than or equal to 100</td>
<td></td>
</tr>
<tr>
<td><strong>MECHANICAL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing material</td>
<td>Stainless Steel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>mm x mm x mm</td>
<td>22.7</td>
<td></td>
</tr>
<tr>
<td>Electrical Connector</td>
<td>gm</td>
<td>10-32 coax.</td>
<td></td>
</tr>
</tbody>
</table>

**Figure B.2:** Specifications for the PCB Model 208C02 Force Sensor
<table>
<thead>
<tr>
<th><strong>Ling Dynamic Systems Shaker</strong></th>
<th><strong>Model Number:</strong> V203</th>
</tr>
</thead>
</table>

**DYNAMIC PERFORMANCE**
- **Sine Force, Peak** N 17.8
- **Armature Resonance Frequency** Hz 13000
- **Useful Frequency Range** Hz 5-13000
- **Effective Mass of Moving Element** kg 0.02
- **Maximum Displacement** mm 5
- **Amplifier Rating** kVA 0.048

**ENVIRONMENTAL**
- **Max. Working Ambient Temperature** °C 30
- **Heat Rejected to Air** W 48

**ELECTRICAL**
- **Electrical Requirement - Amplifier** kVA 0.13
- **Impedance at 500 Hz** ohm 2

**MECHANICAL**
- **Size (H x W x L)** mm x mm x mm 128 x 102 x 117
- **Weight** kg 3.17

*Figure B.3: Specifications for Ling Dynamic Systems V203 Shaker*
<table>
<thead>
<tr>
<th>DYNAMIC PERFORMANCE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>Acceleration</td>
<td>10 ms(^{-2}) (RMS)</td>
</tr>
<tr>
<td>Transverse Amplitude</td>
<td>%</td>
</tr>
<tr>
<td>Maximum Load</td>
<td>gm</td>
</tr>
<tr>
<td>Signal Duration</td>
<td>sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ENVIRONMENTAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature Range</td>
<td>°C</td>
</tr>
<tr>
<td>Humidity</td>
<td>@ 30°C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ELECTRICAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Requirements</td>
<td>V</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MECHANICAL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Size (Dia. x L)</td>
<td>mm x mm</td>
</tr>
<tr>
<td>Weight</td>
<td>gm</td>
</tr>
</tbody>
</table>

Figure B.4: Specifications for Bruel & Kjaer Calibration Exciter
APPENDIX C

C.  CALCULATIONS FOR CANTILEVER BEAM
C.1 Physical Data

C.1.1 Beam Dimensions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>Length of Beam</td>
<td>0.381 m</td>
</tr>
<tr>
<td>W</td>
<td>Width of Beam</td>
<td>0.0381 m</td>
</tr>
<tr>
<td>H</td>
<td>Height of Beam</td>
<td>0.009525 m</td>
</tr>
<tr>
<td>M</td>
<td>Mass of Beam</td>
<td>1.0802 kg</td>
</tr>
</tbody>
</table>

C.1.2 Material Properties

Material Properties for Carbon Steel

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
<td>207 Gpa</td>
</tr>
<tr>
<td>μ</td>
<td>Density</td>
<td>76.5 kN/m³</td>
</tr>
</tbody>
</table>

C.2 Preliminary Calculations

Calculate the mass per unit length (ρ):

\[ \rho = \frac{M}{L} = \frac{1.0802}{0.381} = 2.8352 \frac{kg}{m} \]

Calculate the Area Moment of Inertia (I):

\[ I = \frac{WH^3}{12} = \frac{(0.0381)(0.009525)^3}{12} = 3.0 \times 10^{-9} m^4 \]
C.3 Calculation of Beam Frequencies

\[ f_n = \frac{(\beta_n L)^2}{2\pi} \sqrt{\frac{EI}{\rho L^4}} \]

where:

- \( f_n \) = the frequency of the fundamental, second … mode shape
- \((\beta_n L)^2\) = is an end condition value given by page 223 of Thomson, \{51\}

For \((\beta_1 L)^2 = 3.52\)

\[ f_1 = \frac{3.52}{2\pi} \sqrt{\frac{(207e9)(3e-9)}{(2.8352)(0.381)^4}} = 57\,\text{Hz} \]

For \((\beta_2 L)^2 = 22.0\)

\[ f_2 = \frac{22.0}{2\pi} \sqrt{\frac{(207e9)(3e-9)}{(2.8352)(0.381)^4}} = 357\,\text{Hz} \]
APPENDIX D

D. MODE SHAPES FOR SHAFT WITH NO DAMPING TREATMENT
Figure D.1: First Bending Mode (First Root) for Propeller Shaft with No Damping Treatment

Figure D.2: First Bending Mode (Second Root) for Propeller Shaft with No Damping Treatment
Figure D.3:  First Breathing and First Bending Mode of Propeller Shaft with N+0 Damping Treatment

Figure D.4:  First Breathing and Second Bending Mode of Propeller Shaft with No Damping
Figure D.5: Second Bending Mode (First Root) of Propeller Shaft with No Damping

Figure D.6: Second Bending Mode (Second Root) of Propeller Shaft with No Damping
Figure D.7: First Breathing and Third Bending Mode of Propeller Shaft with No Damping
APPENDIX E

E. MODAL MASS CORRECTION
### Modal Mass Correction Comparison
Propeller Shaft with No Damping Treatment

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Percent Difference</th>
<th>Damping (%)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Corrected</td>
<td>Original</td>
<td>Corrected</td>
</tr>
<tr>
<td>1</td>
<td>170</td>
<td>172</td>
<td>-1</td>
<td>1.15</td>
</tr>
<tr>
<td>2</td>
<td>172</td>
<td>173</td>
<td>-1</td>
<td>1.26</td>
</tr>
<tr>
<td>3</td>
<td>395</td>
<td>400</td>
<td>-1</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>438</td>
<td>444</td>
<td>-1</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>497</td>
<td>502</td>
<td>-1</td>
<td>0.41</td>
</tr>
<tr>
<td>6</td>
<td>498</td>
<td>506</td>
<td>-2</td>
<td>0.73</td>
</tr>
<tr>
<td>7</td>
<td>551</td>
<td>559</td>
<td>-1</td>
<td>0.34</td>
</tr>
</tbody>
</table>

**Table E.1:** Comparison of Modal Models based on Mass Subtraction for Propeller Shaft with No Damping Treatment

### Modal Mass Correction Comparison
Propeller Shaft with Old Style of Cardboard Liner

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Percent Difference</th>
<th>Damping (%)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Corrected</td>
<td>Original</td>
<td>Corrected</td>
</tr>
<tr>
<td>1</td>
<td>169</td>
<td>170</td>
<td>0</td>
<td>0.72</td>
</tr>
<tr>
<td>2</td>
<td>170</td>
<td>172</td>
<td>-1</td>
<td>1.34</td>
</tr>
<tr>
<td>3</td>
<td>384</td>
<td>387</td>
<td>-1</td>
<td>0.34</td>
</tr>
<tr>
<td>4</td>
<td>410</td>
<td>417</td>
<td>-2</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>461</td>
<td>464</td>
<td>-1</td>
<td>0.61</td>
</tr>
<tr>
<td>6</td>
<td>463</td>
<td>466</td>
<td>-1</td>
<td>0.86</td>
</tr>
<tr>
<td>7</td>
<td>515</td>
<td>524</td>
<td>-2</td>
<td>0.42</td>
</tr>
</tbody>
</table>

**Table E.2:** Comparison of Modal Models based on Mass Subtraction for Propeller Shaft with Old Style Cardboard Liner
### Modal Mass Correction Comparison
Propeller Shaft with New Style of Cardboard Liner

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Percent Difference</th>
<th>Damping (%)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Corrected</td>
<td></td>
<td>Original</td>
</tr>
<tr>
<td>1</td>
<td>175</td>
<td>176</td>
<td>-1</td>
<td>0.22</td>
</tr>
<tr>
<td>2</td>
<td>176</td>
<td>177</td>
<td>-1</td>
<td>1.24</td>
</tr>
<tr>
<td>3</td>
<td>396</td>
<td>401</td>
<td>-1</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>437</td>
<td>444</td>
<td>-2</td>
<td>0.52</td>
</tr>
<tr>
<td>5</td>
<td>496</td>
<td>500</td>
<td>-1</td>
<td>0.33</td>
</tr>
<tr>
<td>6</td>
<td>498</td>
<td>502</td>
<td>-1</td>
<td>0.53</td>
</tr>
<tr>
<td>7</td>
<td>550</td>
<td>559</td>
<td>-2</td>
<td>0.31</td>
</tr>
</tbody>
</table>

#### Table E.3: Comparison of Modal Models based on Mass Subtraction for Propeller Shaft with New Style Cardboard Liner

### Modal Mass Correction Comparison
Propeller Shaft with Foam Damping

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Percent Difference</th>
<th>Damping (%)</th>
<th>Percent Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Corrected</td>
<td></td>
<td>Original</td>
</tr>
<tr>
<td>1</td>
<td>169</td>
<td>170</td>
<td>-1</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>171</td>
<td>172</td>
<td>-1</td>
<td>1.44</td>
</tr>
<tr>
<td>3</td>
<td>430</td>
<td>439</td>
<td>-2</td>
<td>1.78</td>
</tr>
<tr>
<td>4</td>
<td>499</td>
<td>505</td>
<td>-1</td>
<td>3.76</td>
</tr>
<tr>
<td>5</td>
<td>555</td>
<td>568</td>
<td>-2</td>
<td>1.59</td>
</tr>
</tbody>
</table>

#### Table E.4: Comparison of Modal Models based on Mass Subtraction for Propeller Shaft with Foam Style Damping Treatment
VITA AUCTORIS

Jennifer L. Durfy was born on March 20, 1973 in Windsor, Ontario. She graduated from Walkerville Collegiate Institute in 1992. After working for two years she went to the University of Windsor, Ontario where she received the degree of Bachelor of Applied Science in Mechanical Engineering in 1998. Jennifer is currently a candidate for the Master’s degree of Applied Science in Mechanical Engineering at the University of Windsor and is expected to graduate in the Fall of 2000.