1986

Postwithdrawal drainage of viscoelastic suspensions.

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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RÉCU
POSTWITHDRAWAL DRAINAGE OF
VISCOELASTIC SUSPENSIONS

by

Matthew Nicholas Godo

A Thesis
submitted to the
Faculty of Graduate Studies and Research
through the Department of
Chemical Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
the University of Windsor

Windsor, Ontario, Canada
1986
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This work is dedicated to my father, Nick Godo, who failed to see me matriculate.
ACKNOWLEDGEMENTS

I would like to express my thanks to Dr. De Kee, who formally introduced me to the field of graduate research. His continual strive for perfection of content and form in this work was both inspiring and inspired. The opportunity to attend a number of conferences, made possible through his financial assistance, proved to be quite informative and is greatly appreciated.

I would also like to express my thanks to Dr. Schlesinger for his very timely, enlightened contributions and financial support. In the physics department, thanks are also due to Mr. Bernard Masse, who constructed the hardware to interface the photodetectors to the microcomputer, and Mr. Rick Buzzeo for his invaluable assistance in the preparation of the figures contained in this work.

Special thanks are extended to John, Tom and Henry Marentette who allowed me to use their time and facilities in the construction of the plate withdrawal support and optical component support. Additionally, minor modifications to the experimental apparatus suggested by John Marentette led to significant reductions in vibration transmission.

The author would also like to thank Mr. Tom Rindlisbacher, who satisfied all the photographic and photocopying requirements of this work.
ABSTRACT

A new experimental method, based upon laser interferometry, was developed to measure the transient film thickness profiles in the postwithdrawal drainage of viscoelastic suspensions. The interference patterns, produced by a Mach-Zender interferometric arrangement, were recorded and subsequently analyzed using a microcomputer which was interfaced to a number of photodetectors. Experimental verification of the new method was established by examining Newtonian mixtures of glycerine and water. Expressions for the transient film thickness profiles formed during the postwithdrawal drainage of the Ellis, three parameter De Kee fluid and Herschel-Bulkley fluid models have been reported. Of these, expressions based upon the Ellis and De Kee fluid models were considered to be in reasonable agreement with experimental data.

Deviations of experimental data below theory were observed at short drain times for all classes of fluids studied. To explain this behavior, the transition drain time theory was developed. In effect, it was suggested that during the withdrawal stage of postwithdrawal drainage, a transition from unsteady to continuous withdrawal occurred. Transition drain time expressions, used to predict the time required for agreement with postwithdrawal drainage theories, were developed and are considered to be in reasonable agreement with experimental data.

Yield stress approximations were obtained using direct measurement, non-linear regression of rheological data and
analysis of postwithdrawal drainage data. Extrapolations using the De Kee model were observed to be superior when compared with the conventional Casson and Herschel-Bulkley models. The failure of postwithdrawal drainage data to predict yield stress was attributed to the presence of high shear rates in the draining film over the range of drain times studied.
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NOMENCLATURE

\(a\) = capillary length, [m]

\(C_0\) = capillary number, see equation (II-75), dimensionless

\(C_0\) = dimensionless parameter, see equation (II-108)

\(C_1\) = dimensionless parameter, see equation (II-109)

\(D\) = dimensionless parameter, see equation (II-77)

\(F_b\) = buoyant force, [N]

\(F_g\) = gravitational force, [N]

\(F_p\) = dimensionless parameter, see equation (II-78)

\(F_r\) = recorded force in yield stress measurement, [N]

\(g_z\) = gravitational acceleration, [m/s^2]

\(h\) = film thickness, [m]

\(h_0\) = constant film thickness or film thickness at the onset of drainage, [m]

\(h_p\) = plane at which plug flow occurs, see equation (II-71), [m]

\(L\) = length of drawn film, [m]

\(m\) = parameter, defined in equation (II-38)

\(m\) = Power Law or Herschel-Bulkley fluid parameter, [Pa·s^n]

\(n\) = Power Law or Herschel-Bulkley fluid parameter, dim.

\(n\) = refractive index, dimensionless

\(P\) = fluid pressure, [Pa]

\(q\) = volumetric flow rate, [m³/s]

\(Q\) = dimensionless volumetric flow rate, see equation (II-80)

\(Re\) = reynolds number, dimensionless, see equation (II-76)

\(S\) = optical path length, see equation (III-1), [m]

\(S\) = surface area of plate in yield stress measurement, [m²]
T = dimensionless parameter, see equation (II-79)
T_0 = dimensionless parameter, variously defined
T_1 = dimensionless parameter, variously defined
T_2 = dimensionless parameter, see equation (IV-13)
T_3 = dimensionless parameter, see equation (IV-14)
t = time, [s]
t_{crit} = critical drain time, see equation (II-40), [s]
t_d = drainage time, [s]
t_d^* = critical drain time, see equation (II-47), [s]
t_d^{**} = transition drain time, see equation (IV-5), [s]
t_w = withdrawal time, [s]
t_1 = De Kee fluid parameter, [s]
v_j = critical junction velocity, [m/s]
v_w = withdrawal speed, [m/s]
v_x = fluid velocity in x-direction, [m/s]
\langle v_z \rangle = fluid velocity in z-direction, [m/s]
v_z = average fluid velocity, [m/s]
W = width of draining film (normal to illustrations), [m]
x = rectangular coordinate
Z = distance upward from bath surface to plane of observation, [m]
z = rectangular coordinate
z = distance downward from top of film to plane of observation [m]
(z/L)_j = critical junction, variously defined, dimensionless
(z/L)_{obs} = dimensionless distance from top of film to plane of observation

(xiv)
\( \alpha \) = Ellis fluid parameter, dimensionless
\( \beta \) = dimensionless function, numerical solution to differential equation, see Tallmadge and Gutfinger [T-3]
\( \Gamma \) = path difference, [m]
\( \dot{\gamma}_{ij} \) = shear rate, [s\(^{-1}\)]
\( \Delta_{ij} \) = rate of deformation, [s\(^{-1}\)]
\( \delta \) = experimental film thickness, [m]
\( \eta \) = non-Newtonian viscosity, [Pa\cdot s]
\( \eta_0 \) = Ellis fluid model parameter, [Pa\cdot s]
\( \eta_1 \) = De Kee fluid model parameter, [Pa\cdot s]
\( \theta \) = parameter, defined in equation (II-15)
\( \theta_1 \) = parameter, defined in equation (II-16)
\( \lambda \) = wavelength, [m]
\( \lambda_0 \) = Maxwell relaxation time, [s]
\( \lambda_1 \) = Oldroyd fluid model time constant, [s]
\( \lambda_2 \) = Oldroyd fluid model time constant, [s]
\( \mu \) = Newtonian viscosity, [Pa s]
\( \mu^* \) = Maxwell viscosity, [Pa s]
\( \mu_0 \) = Bingham plastic viscosity, [Pa s]
\( \mu_1 \) = Casson model viscosity, [Pa s]
\( \pi \) = 3.14159…..
\( \rho \) = fluid density, [kg/m\(^3\)]
\( \sigma \) = surface tension, [N/m]
\( \tau_{ij} \) = shear stress, [Pa]
\( \tau_{ii} \) = normal stress, [Pa]
\( \tau_0 \) = yield stress, [Pa]
$\tau_{1/2} =$ Ellis fluid model parameter, [Pa]

$\psi =$ function, defined in equation (II-14)

$\phi_0 =$ Ellis fluid model parameter, [Pa$^{-1}$s$^{-1}$]

$\phi_1 =$ Ellis fluid model parameter, [Pa$^{-\alpha}$s$^{-1}$]
I. INTRODUCTION

The problem of the entrainment of a liquid onto a solid object being withdrawn from a liquid bath has fundamental and practical applications in the dipcoating, lubricating, paper and photographic film industries.

In practice, fluids found in drainage and/or withdrawal situations are homogeneous solid-liquid mixtures which exhibit complex rheological behavior. Notably, coating liquids are typically suspensions which demonstrate a yield stress.

The objectives of this work are to:

i) Design and fabricate an apparatus capable of making transient film thickness measurements in postwithdrawal drainage studies of liquids containing particulate matter.

ii) Experimentally verify the accuracy of the new apparatus by investigating the postwithdrawal drainage behavior of Newtonian and non-Newtonian fluids.

iii) Experimentally and theoretically examine the postwithdrawal drainage behavior of a series of viscoelastic suspensions.

iv) Examine the yield stress phenomena of the viscoelastic suspensions using the following techniques:

   a) Direct measurement with a proven experimental method.

   b) Linear regression of rheological data based upon the two parameter Casson model.

   c) Non-linear regression of rheological data.
based upon the three parameter Herschel-Bulkley model.

d) Non-linear regression of rheological data based upon the three parameter DeKee model.

e) Interpretation of postwithdrawal drainage data obtained for viscoelastic suspensions.
II. LITERATURE SURVEY

In a comprehensive review by Tallmadge and Gutfinger [T-1], three related problems, notably drainage, withdrawal and removal were defined as follows. Drainage occurs when a liquid flows down a stationary support under the influence of gravity. Withdrawal occurs when the upward motion of a solid object from a liquid bath causes entrainment of the wetting liquid. This classification is limited to the case in which the immersed solid remains in contact with the liquid bath. If contact between the bath and solid is broken, the situation is referred to as removal. Figure (II-1) illustrates the nomenclature for the drainage of a fluid film down a vertical plane surface, while Figure (II-2) illustrates the entrainment of a fluid onto a vertical plane solid object being continuously withdrawn at a constant speed, \( v_w \). The subdivision of the film formed by the continuous withdrawal process will be discussed later.

In subsequent discussions regarding the general drainage and unsteady state withdrawal problems, the z-axis will be considered to be parallel to the vertical surface, the x-axis normal to it and the point of origin taken as the point where the upper portion of the fluid film meets the solid vertical surface. For the drainage problem, the film thickness at any given point will be denoted by 'h' and will be considered to be a function of the axial position, \( z \), and time, \( t \). The distance from the origin to the plane of observation will be defined as 'z' while the distance from the plane of observation to the bath surface is termed 'Z' and the distance from the origin to the
FIGURE (II-1) : The Drainage Process
   (Postwithdrawal or Postflowering)
FIGURE (II-2) : The Continuous Withdrawal Process
Constant Thickness Region (1)

Dynamic Meniscus Region (2)

Static Meniscus Region (3)
bath surface is termed 'L'.

When considering the unsteady state withdrawal problem, the coordinate system is established with respect to the moving solid object, the origin being maintained at the point where the upper portion of the film meets the solid. For continuous withdrawal problems however, the origin of the coordinate system is frequently considered to be such that the waterline of the bath surface is simply an extension of the x-axis and the z-axis is oriented positively upwards. Also, at a given height above the bath surface, the film thickness is considered to be constant and is denoted by \( h_0 \).

A variable used for the classification of withdrawal problems is the volumetric flow rate, \( q \), which is typically not known. Therefore, unsteady withdrawal refers to a situation in which the volumetric flow rate of a liquid is variable for objects of finite length. Thus,

\[
\frac{\partial q}{\partial t} \neq 0 \quad \text{and} \quad \frac{\partial q}{\partial z} \neq 0
\]  

(II-1,2)

Conversely, steady state or continuous withdrawal refers to a situation in which the flow of a liquid is constant for an object of infinite length. Thus,

\[
\frac{\partial q}{\partial t} = \frac{\partial q}{\partial z} = 0
\]  

(II-3,4)

Since the transition in the withdrawal of objects of finite length to objects of infinite length is not well defined, a condition of constant flux may be imposed in certain instances. Thus,
\[ \frac{\partial q}{\partial t} = \frac{\partial q}{\partial z} = 0 \quad \text{for} \quad z_1 < z < z_2 \quad (\text{II-5,6}) \]

Drainage has been characterized by limiting physical situations in the current literature \([T-1]\). For example, free drainage is an ideal model where the effect of the curvature of the film (well above the free surface of the liquid) is considered to be negligible. A more realistic approach is to consider restricted drainage in which curvature effects are not negligible. Experimental results for non-Newtonian and viscoelastic fluids suggest that drainage is of the restricted type \([T-2],[T-3]\). The general drainage problem as posed by Jeffrey's \([J-1]\) in 1930 is concerned with the drainage of a film formed upon the walls of an emptying vessel. Specific classes of drainage include postlowering, postwithdrawal and postremoval. Postlowering drainage describes the drainage of a film formed on the walls of emptying vessel after the downward progress of the contained liquid has ceased. Postwithdrawal drainage describes the drainage of a liquid formed by an initial unsteady withdrawal phase and postremoval drainage describes the drainage of a liquid film subsequently formed by the complete removal of a solid object from a liquid bath.

Some useful observations concerning the classification of drainage and withdrawal problems may be stated as follows:

i) Unsteady withdrawal and Jeffrey's drainage are identical processes related by a simple coordinate transformation. This fact was first recognized by Van Rossum \([V-1]\) in 1958.
ii) The drainage problem illustrated in Figure (II-1) is actually a depiction of either postlowering or post-withdrawal drainage; it should be noted that there is no relative motion between the solid and the free surface of the fluid bath.

iii) The continuous withdrawal problem illustrated in Figure (II-2) is considered to be a case of steady state withdrawal.

A. Mathematical Classification of the Problem

The forces to be considered in the two dimensional problem include accelerational, gravitational, inertial, interfacial (surface tension effects) and viscous forces. According to Whitaker [W-1], the x- and z- components of the momentum balance for an isothermal, incompressible fluid can be written as:

\[ \rho \left( \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_x}{\partial z} \right) = -\frac{\partial p}{\partial x} - \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} \right) \]  \hspace{1cm} (II-7)

\[ \rho \left( \frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} - \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right) + \rho g_z \]  \hspace{1cm} (II-8)

The cartesian form of the continuity equation is:

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \]  \hspace{1cm} (II-9)

To simplify the problem, some experimental restrictions which are consistently imposed include:
1. Only simple axial objects are considered.
   (e.g. flat plate)
2. Consider vertical withdrawal only.
3. Flow is unobstructed (no doctoring blades).
4. Use a homogeneous fluid.
5. Maintain isothermal and isobaric conditions.

Further, three main assumptions made in the drainage and withdrawal problems state that:
1. The fluid is wavefree and in laminar flow.
2. Complete wetting and no slip at the solid-liquid interface.
3. No shear at the liquid-gas interface.

The boundary conditions for both the drainage and withdrawal cases will be discussed separately.

1. Boundary Conditions for the Drainage Problem

It is useful to note that all the analytical studies concerned with drainage have assumed the following:

1. One dimensional flow.
2. Normal stresses are negligible for non-Newtonian fluids.
3. Convective inertial terms are small compared to accelerational terms and may be neglected.

\[
\left( \frac{\partial v_z}{\partial t} - v_x \frac{\partial v_z}{\partial x} - v_z \frac{\partial v_z}{\partial z} \right)
\]

4. Surface curvature is small and thus interfacial tension effects may be neglected.
5. Pressure gradients are negligible compared to viscous
and gravitational forces. \( \left( \frac{\partial P}{\partial z} \approx 0 \right) \)

For the condition of no slip along the solid-liquid interface, note that:

\[ B.C. \quad v_z = 0 \quad \text{when} \quad x = 0 \quad \text{(II-10)} \]

and for negligible shear at the gas-liquid interface,

\[ B.C. \quad \tau_z = 0 \quad \text{when} \quad x = h \quad \text{(II-11)} \]

The necessary initial condition is the specification of the film thickness prior to the onset of drainage. For Jeffrey's drainage of a Newtonian fluid, it was implied by Bird et al. [B-1] that:

\[ I.C. \quad h = \infty \quad \text{when} \quad t = 0 \quad \text{(II-12)} \]

For the postwithdrawal drainage of a Newtonian fluid, Tallmadge [T-4] used the film thickness profile for unsteady withdrawal where:

\[ I.C. \quad h = \sqrt{\frac{\mu v_w z}{\rho g_z L}} \quad \text{when} \quad t = 0 \quad \text{(II-13)} \]

In the study of the drainage of a Newtonian liquid, Denson [D-1] considered the behavior of a sessile drop formed on a horizontal flat surface. Drainage occurred when the flat support was rotated through 90 degrees to assume a vertical orientation. The appropriate initial condition was given by
\[ \psi(h) = 2a(\cos \theta_1 - \cos \theta) + a \ln \left( \frac{\tan \theta_1/2}{\tan \theta/2} \right) \tag{II-14} \]

when \( t = 0 \).

where \( \theta = \sin^{-1}\left( \frac{h-h_0}{2a} \right) \tag{II-15} \)

\( \theta_1 = \sin^{-1}\left( \frac{h_0}{2a} \right) \tag{II-16} \).

Note that Denson considered \( h_0 \) to represent the film thickness of the drop far away from the junction between the fluid, solid and surrounding environment.

Other more complicated initial conditions for non-Newtonian fluids will be discussed in greater detail subsequently.

2. Boundary Conditions for the Withdrawal Problem

It has already been recognized that Jeffrey's drainage is related to unsteady withdrawal; consequently the analysis of the latter problem parallels that of the former. For negligible shear at the gas-liquid interface equation (II-11) may be used and for the condition of no-slip along the solid-liquid interface, use is made of the following condition:

\[ v_z = -v_w \quad \text{when} \quad x = 0 \tag{II-17} \]

The necessary initial condition is obtained from equation (II-12).

The continuous withdrawal problem (illustrated in Figure (II-2)) is more complicated and requires additional analysis. In 1965, Tallmadge and Gutfinger [T-2] and later in 1974,
Spiers, Subbaraman and Wilkinson [S-1], found it convenient to subdivide the drawn film into three distinct regions. The first region, termed the constant thickness region, concerns the part of the film in which the thickness profile is flat (or film thickness is constant). This region is located far above the bath surface (constant flux conditions apply) and as a result of the flat film thickness profile, only viscous and gravitational forces are considered. The flow equation, given by:

\[ \frac{\partial \tau_{xz}}{\partial x} + \rho g = 0 \]  \hspace{1cm} (II-18)

is solved using the boundary condition given in equation (II-11) and,

\[ v_x = v_w \quad \text{when} \quad x = 0 \]  \hspace{1cm} (II-19)

The intermediate region is referred to as the dynamic meniscus region. Here, the film thickness is a function of the axial position, \( z \), and since the film profile is no longer flat, surface tension forces must be considered along with viscous and gravitational forces. The flow equation is:

\[ \sigma \frac{d^3 h}{dz^3} + \frac{\partial \tau_{xz}}{\partial x} + \rho g = 0 \]  \hspace{1cm} (II-20)

Using the boundary conditions, equations (II-11) and (II-19), and by making use of the steady state condition,

\[ q_1 = q_2 \]  \hspace{1cm} (II-21)

a third order differential equation will be obtained which can
be solved for the following boundary conditions:

\[
\text{B.C. } h = h_0, \frac{dh}{dz} = 0, \frac{d^2h}{dz^2} = 0 \quad \text{at} \quad z = z_0
\]  

\[\text{(II-22,23,24)}\]

Here, the plane \( z = z_0 \) refers to the point at which region 1 meets region 2; since selection of the intersection of regions 1 and 2 is arbitrary, it is convenient to apply a transformation to the coordinate system such that \( z_0 \) will become zero.

The third region, referred to as the static meniscus region, is a region in which flow effects are considered to be negligible. The differential equation describing the free surface is based upon capillary statics and was originally reported by Levich and Landau [L-1] to be:

\[
\frac{d^2h}{dz^2} = \frac{\rho gz}{\sigma} \left[ 1 + \left( \frac{dh}{dz} \right)^2 \right]^{3/2}
\]

\[\text{(II-25)}\]

The above expression can be integrated, making use of the following boundary condition,

\[
\text{B.C. } h = \infty \quad \text{at} \quad z = 0
\]

\[\text{(II-25)}\]

To obtain the overall expression which is used to predict the constant film thickness, the integrated form of equation (II-25) is matched with an intermediate form of equation (II-20) (which depends upon the type of fluid being studied) such that the curvature of the film profile computed using either expression is identical at the intersection between regions 2
and 3. It is essential to recognize that a combination of all three equations governing each region is necessary to obtain a solution to the continuous withdrawal problem.

B. Analytical and Experimental Drainage Studies

The first investigator to consider the problem of drainage of a fluid film upon the emptying of a vessel was Jeffreys [J-1] in 1930. He modelled the drainage of a Newtonian liquid from a vertical surface in the emptying of a vessel where only viscous and gravitational forces were considered significant. The simplified momentum balance (from equation (II-8)) was given by:

$$0 = \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z$$

Using the normal boundary conditions (equations (II-10,11,12)), the momentum balance was solved to obtain the following expressions for the velocity profile, volumetric flow rate per unit width and film thickness profile respectively for long drain times.

$$v_z = \frac{\rho g_z}{\mu} \left( h x - \frac{x^2}{2} \right)$$

$$q = \frac{\rho g_z h^3}{3\mu}$$

$$h = \sqrt{\frac{\mu^2}{\rho g_z t}}$$
The first experimental work on the drainage problem was carried out in the early 1930's by Satterly et al [S-2],[S-3]. The film thickness measurement technique was based upon an examination of interference fringes formed in the plane of the fluid film by the reflection of light from a high energy sodium lamp. The Newtonian fluids studied included glycerine, olive oil and mixtures of water, alcohol and glycerine which exhibited kinematic viscosities ranging from 0.000001 to 0.000500 m²/s. The experiments performed covered drain times ranging from 10 to 7500 seconds and drain lengths ranging from .01 to .15 m. The film thicknesses being measured were in the range of 0.0001 to 0.001 meters.

The most significant conclusions to come from Satterly and his coworkers' investigations was that the Jeffrey's expression was valid within experimental error over a wide range of drain times (15-1000 seconds). In particular, the experimental results were within 20% of values predicted by the Jeffrey's expression and the error was attributed to evaporation losses. Additionally, it was concluded that surface tension effects were negligible over the range of drain times observed. It is interesting to note that the types of drainage studied included both postwithdrawal and postremoval; no observations concerning the initial withdrawal or removal stages were recorded however and as a result it is impossible to estimate the error associated with the inappropriate use of the Jeffrey's equation which is applicable only to unsteady withdrawal or Jeffrey's
drainage.

The first person to experimentally investigate Jeffrey's drainage expression correctly was Van Rossum [V-1] in 1958. To establish the film thickness profiles, side view photographs were taken in succession as a vertical plate was withdrawn from a liquid bath (i.e. unsteady withdrawal). Van Rossum investigated the behavior of two Newtonian fluids with densities ranging from 895.0 to 950.0 kg/m³, viscosities ranging from 2.5 to 30.0 Pa·s and surface tensions ranging from 0.0335 to 0.035 N/m. The withdrawal speeds used were .0133 and .0382 m/s and the withdrawal length was varied from 0.09 to 0.18 m. To analyze the experimental data, Van Rossum employed the speed explicit expression of Jeffrey's film thickness profile,

\[ h = \sqrt{\frac{\mu v_w}{\rho g z}} \]  \hspace{1cm} (II-31)

The results obtained were considered to be within experimental error, however deviations from theory were not negligible. In particular, it was noted that above a critical value of \( z/L \), a significant departure from equation (II-31) was observed; the departure became more pronounced as the withdrawal height was increased. Van Rossum however, did not provide any comments or explanations to explain the observed behavior.

In 1936, Green [G-1] attempted to solve the one dimensional drainage problem in which accelerational effects were considered for a Newtonian fluid. The momentum balance (from equation II-8) was given by:
\[ \rho \frac{\partial v_z}{\partial t} = \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z \]  

(II-32)

Specifically, Green considered both Jeffrey's drainage and a special type of postwithdrawal drainage. In the former case, use was made of the boundary condition of no slip (equation (II-10)). Additionally, it was suggested that the fluid was initially at rest and after a given time, the fluid would descend solely under the influence of gravity far away from the plate. Mathematically,

I.C. \( v_z = 0 \) when \( t = 0 \)  

(II-33)

B.C. \( v_z = g_z t \) when \( x = \infty \)  

(II-34)

The resulting expression for the film thickness profile however, did not converge to the Jeffrey's solution for long drain times and was consequently of little value.

In his analysis of postwithdrawal drainage, Green considered an infinitely long film of constant thickness. Using the boundary conditions expressed in equations (II-10,11) and (II-33), an expression for the drainage of the film was obtained. The main error in Greens' analysis was the suggestion that as the film drained, it retained its uniform thickness. Also, the initial film thickness was difficult to establish experimentally.

In 1945, Wyllie [W-2] argued that Greens' solution was substantially correct for an initial drainage period up to some
time, $t_0$, for the postwithdrawal drainage situation. Thus, a new expression which included Greens' solution as an initial condition in the general solution to equation (II-32) was reported. The new expression however contained two constants which had to be determined experimentally and it was later correctly recognized by Denson [D-1], that neither of the expressions developed by Wyllie or Green satisfied the continuity equation. Thus, while the theoretical work of Green and Wyllie was shown to contain serious errors, it should be recognized that these authors were the first to give consideration to the initial behavior of draining films.

In 1964, Tallmadge and Gutfinger [T-5], presented a solution to equation (II-32) which did not suffer from either Greens' or Wyllies' shortcomings. The velocity profile, volumetric flow rate per unit width and implicit film thickness profile for a Newtonian fluid were reported as:

$$v_x = \frac{\rho g_z}{\mu} \left( h x - \frac{h^2}{2} - \frac{2h^2}{\pi^3} \sum_{n=1}^{\infty} \frac{e^{-m^2 \nu t} \sin nx}{(n-1/2)^3} \right)$$  \hspace{1cm} (II-35)

$$q = \frac{\rho g_z h^3}{3\mu} \left( 1 - \frac{6}{\pi^4} \sum_{n=1}^{\infty} \frac{e^{-m^2 \nu t}}{(n-1/2)^4} \right)$$ \hspace{1cm} (II-36)

$$z = \frac{\rho g_z h^2 t}{\mu} + \frac{2h^4 g_z}{\pi^6 \nu^2} \sum_{n=1}^{\infty} \frac{e^{-m^2 \nu t} (5 + 2m^2 \nu t)}{(n-1/2)^6}$$ \hspace{1cm} (II-37)

where $m = (n-1/2) \pi / h$. \hspace{1cm} (II-38)

To obtain equation (II-37), Tallmadge and Gutfinger made use
of the following boundary condition:

\[ B.C. \quad h = 0 \quad \text{at} \quad z = 0 \quad (II-39) \]

A direct comparison of equation (II-37) to equation (II-30) suggested that after a critical time, the simpler Jeffrey's equation could be used to predict drainage behavior. For a difference of 1%, Tallmadge and Gutfinger reported this critical time to be given by:

\[ t_{\text{crit}} = 1.95 \sqrt{\frac{z}{g_z}} \quad (II-40) \]

and noted that this time was independant of fluid properties. In 1970, Denson [D-1] noted that two initial conditions were required to solve equation (II-32); Tallmadge and Gutfinger had only used one. Hence the suggestion that the critical time was independant of fluid properties was considered to be erroneous.

To substantiate his arguments concerning the dependance of the critical time upon fluid properties as well as axial position and initial film thickness, Denson [Q-1] studied the drainage of a sessile drop, which was previously discussed. Denson's solution to the momentum balance given in equation (II-27) was reported as:

\[ z = \frac{\rho g_z h^2 t}{\mu} + \psi(h) \quad (II-41) \]

where the function \( \psi(h) \) was considered to be an implicit function representative of the initial film thickness profile.
For his sessile drop experiments the implicit function has been given in equation (II-14). The film thickness measurement technique was based upon the principle of light absorption. The fluids studied had kinematic viscosities ranging from 0.0000585 to 0.0261 m²/s. Drain times covered a range of approximately 10 to 1000 seconds and the film thicknesses measured ranged from roughly 0.00001 to 0.0001 meters. The agreement between theory and experiment was reported to be within 5% and was considered to be good.

The most significant conclusion to come from Denson's work on the drainage of Newtonian fluids was that the critical time required for Jeffrey's equation to become valid varied directly with the viscosity of the liquid, the axial position along the solid and the capillary length (which suggested some dependance on surface tension of the liquid) and varied indirectly with the initial film thickness. Denson also suggested that after properly establishing the initial film thickness profile for any drainage process, the critical drain time required for Jeffrey's drainage expression to become valid could be established.

To this end, in 1971, Tallmadge and Lang [T-4] presented an analysis of the postwithdrawal drainage problem for Newtonian fluids. In their approach, the expression describing unsteady withdrawal (equation (II-30) or (II-31)) was used as an initial condition to obtain the film thickness profile for the postwithdrawal drainage stage. The speed explicit and time explicit film thickness profiles obtained for the postwithdrawal drainage of a Newtonian fluid were respectively given by:
\[ h = \sqrt{\frac{\mu z v_w}{\rho g z (L + v_w t_d)}} \]  \hspace{1cm} (II-42)

\[ h = \sqrt{\frac{\mu z}{\rho g z (t_w + t_d)}} \]  \hspace{1cm} (II-43)

To validate their expressions, postwithdrawal drainage experiments were carried out using an oil possessing a viscosity of 1.9 Pa.s, a density of 887.0 kg/m$^3$ and a surface tension of 0.0326 N/m. The withdrawal length was approximately 0.18 m, withdrawal speeds varied from 0.0002 to 0.022 m/s and total times (withdrawal plus drainage) covered a range of 20 to 200 seconds. The plane of observation was located very close to the upper edge of the film, ranging from 0.01 to 0.04 m. Using a light absorption technique, film thickness measurements ranging from 0.0002 to 0.0003 meters were made. In general, agreement between theory and experiment was considered to be quite good.

Tallmadge and Lang further suggested that after a sufficient length of time, the Jeffrey's expression (equation (II-30)) could be used with little error to describe the postwithdrawal drainage process; this observation was consistent with those results obtained by Sattéry et al [S-2],[S-3]. Notably, the dependance of the critical time required for Jeffrey's expression to become valid was not established with respect to fluid properties and it is very important to note that the experimental studies examined only the top portion of the film formed in the withdrawal stage.
In 1971, Tallmadge [T-6], [T-7] and Groenveld [G-2], suggested that Jeffrey's drainage expression had limited validity in the unsteady withdrawal process where surface tension effects were not negligible. Specifically, they considered a critical plane of observation above which the Jeffrey's expression held and below which, a continuous withdrawal theory could be applied. Thus,

\[ h = \sqrt{\frac{\mu z}{\rho g_z} \frac{1}{T_0}} \quad \text{for} \quad \left( \frac{z}{L} \right)_j < T_0^2 \]  \hspace{1cm} (II-44)

\[ h = T_0 \sqrt{\frac{\mu v_w}{\rho g_z}} \quad \text{for} \quad T_0^2 < \left( \frac{z}{L} \right)_j < 1 \]  \hspace{1cm} (II-45)

The variable \( T_0 \) is a function of fluid properties and withdrawal speed and can be obtained from expressions describing the continuous withdrawal theories which will be discussed in the next section.

Tallmadge also extended the critical junction concept to describe the postwithdrawal drainage behavior of Newtonian fluids. Thus, at the start of postwithdrawal drainage, the critical junction would begin to move downwards towards the bath surface with a propagation velocity given by:

\[ v_j = T_0^2 v_w \]  \hspace{1cm} (II-46)

Then, the time required for the critical junction to travel the distance to the bath surface at the constant propagation
velocity was reported as:

\[ t^*_d = t_w \left( \frac{1 - \frac{t^2}{t_0^2}}{\frac{t^2}{t_0^2}} \right) \quad (\text{II-47}) \]

Thus, after a critical drain time, \( t^*_d \), the entire film thickness profile could be considered to be parabolic; the film thickness profile developed for postwithdrawal drainage (equation (II-42) or (II-43)) could be applied. As the critical junction moved down the solid, its location and the film thickness profile were described by the following expressions:

\[ \left( \frac{Z}{L} \right)_j = t_0^2 \left( 1 + \frac{t_d}{t_w} \right) \quad \text{for} \quad t_d < t^*_d \quad (\text{II-48}) \]

\[ h = \sqrt{\frac{\mu z}{\rho g_z (t_w - t_d)}} \quad \text{for} \quad \left( \frac{z}{L} \right) < \left( \frac{z}{L} \right)_j \quad (\text{II-49}) \]

\[ h = t_0 \sqrt{\frac{\mu v_w}{\rho g_z}} \quad \text{for} \quad \left( \frac{z}{L} \right) > \left( \frac{z}{L} \right)_j \quad (\text{II-50}) \]

A great deal of analytical work regarding the drainage of non-Newtonian fluids has been prepared to date. In general, researchers have solved the momentum balance given by:

\[ 0 = \frac{\partial \tau_{xz}}{\partial x} + \rho g_z \quad (\text{II-51}) \]

using the boundary conditions given in equations (II-10,11). To solve the unsteady state mass balance and thus obtain an
expression for the film thickness profile, use is made of the relationship,

$$\frac{dz}{dt} = -\frac{\partial h/\partial t}{\partial h/\partial z}$$  \hspace{1cm} (II-52)$$

The correct initial condition is obtained from equation (II-12). Thus, most of the analytical work concerning the drainage of non-Newtonian fluids involves the simple extension of Jeffrey's expression (equation (II-30)). The first such extension was provided by Tallmadge and Gutfinger [T-3] who in 1965, investigated the Ellis and Ostwald-de Waele fluid models. The rheological expressions describing these two models are given by:

**Ellis Fluid**  \hspace{1cm} \eta = \frac{\eta_0}{1 + \frac{T_{xz}}{T_{1/2}} \alpha^{-1}} \hspace{1cm} (II-53)

**Ostwald-de Waele (Power Law) Fluid**  \hspace{1cm} \eta = m \left| \dot{\gamma}_{xz} \right|^{n-1} \hspace{1cm} (II-54)

The expressions for the velocity distribution, volumetric flow rate per unit width and implicit film thickness profile for the Ellis Model (analogous to equations (II-28,29,30) for the Newtonian case) were respectively reported as:

$$v_z = \varphi_0 \rho g z \left( h_x - \frac{x^2}{2} \right) + \frac{\varphi_1 (\rho g_z)^\alpha}{\alpha + 1} \left[ h^{\alpha+1} - (h-x)^{\alpha+1} \right]$$  \hspace{1cm} (II-55)
\[ q = \frac{\varphi_o \rho g_z h^3}{3} + \frac{\varphi_1 (\rho g_z)^{\alpha} h^{\alpha+2}}{\alpha + 2} \]  
(II - 56)

\[ z = (\varphi_o \rho g_z h^2 + \varphi_1 (\rho g_z)^{\alpha} h^{\alpha+1}) t \]  
(II - 57)

where

\[ \varphi_o = \frac{1}{\eta_0} \]  
(II - 58)

\[ \varphi_1 = \frac{1}{\eta_0 \bar{\gamma}_{1/2}^{\alpha+1}} \]  
(II - 59)

The corresponding expressions for the Ostwald-de Waele model were reported as:

\[ v_z = \left(\frac{\rho g_z}{m}\right)^{1/n} \left(\frac{n}{n+1}\right) \left(h^{(n+1)/n} - (h-x)^{(n+1)/n}\right) \]  
(II - 60)

\[ q = \left(\frac{\rho g_z}{m}\right)^{1/n} \left(\frac{n}{2n+1}\right) h^{(2n+1)/n} \]  
(II - 61)

\[ h = \left(\frac{m}{\rho g_z}\right)^{1/(n+1)} \left(\frac{Z}{t}\right)^{n/(n+1)} \]  
(II - 62)

To study the drainage behavior of fluids possessing a yield stress, expressions were developed for the Bingham Plastic and the Casson model by Gutfinger [G-3] and Pendergrass [P-1], respectively. The empirical relationships used to describe the
rheological behavior of the yield stress models are given by:

**Bingham Plastic**

\[
\tau_{xz} = \pm \tau_o - \mu_o \dot{\gamma}_{xz}, \quad |\tau_{xz}| \leq \tau_o \quad (II - 63a)
\]

\[
\dot{\gamma}_{xz} = 0, \quad |\tau_{xz}| < \tau_o \quad (II - 63b)
\]

**Casson Model**

\[
\sqrt{\frac{\tau_{xz}}{\tau_0}} = \sqrt{\frac{\tau_0}{\mu_1}} \sqrt{\dot{\gamma}_{xz}}, \quad |\tau_{xz}| \leq \tau_o \quad (II - 64a)
\]

\[
\dot{\gamma}_{xz} = 0, \quad |\tau_{xz}| < \tau_o \quad (II - 64b)
\]

The velocity profile for both regions of the draining film, volumetric flow rate per unit width and film thickness profile for the Bingham Plastic were reported as:

\[
v_z = \frac{\rho g_z}{\mu_o} \left( h x - \frac{x^2}{2} \right) - \frac{\tau_0}{\mu_o} x, \quad |\tau_{xz}| \leq \tau_o \quad (II - 65a)
\]

\[
v_{zp} = \rho g_z h_p^2, \quad |\tau_{xz}| < \tau_o \quad (II - 65b)
\]

\[
q = \frac{\rho g_z h^3}{3 \mu_o} \left[ \frac{3}{2} \left( \frac{h_p}{h} \right)^2 - \frac{1}{2} \left( \frac{h_p}{h} \right)^3 \right] \quad (II - 66)
\]

\[
h = \frac{\tau_0}{2 \rho g_z} + \sqrt{\left( \frac{\tau_o}{2 \rho g_z} \right)^2 + \frac{\mu_o^2}{\rho g_z}} \quad (II - 67)
\]
The expressions similar to equations (II-65a, 65b, 66 and 67) for the Casson model were originally reported in dimensionless form by Pendergrass; in non-dimensional form they may be written as:

\[ v_z = \frac{\rho g_z}{\mu_1} \left( h_x - \frac{x^2}{2} \right) + \frac{\tau_o}{\mu_1} x \]

\[ \frac{4}{3} \sqrt{\rho g_z \tau_o} \left[ \left( \frac{\sqrt{h}}{x} \right)^3 - \left( \frac{\sqrt{h} - x}{x} \right)^3 \right], \quad |\tau_{xz}| \approx \tau_o \quad (II - 68a) \]

\[ v_{zp} = \frac{4 \rho g_z h^2}{3 \mu_1} \left[ 1 - \frac{1}{2} \left( \frac{h_P}{h} \right) - \frac{1}{8} \left( \frac{h_P}{h} \right)^2 - \sqrt{1 - \frac{h_P}{h}} \right], \quad |\tau_{xz}| < \tau_o \quad (II - 68b) \]

\[ q = \frac{4 \rho g_z h^3}{3 \mu_1} \left[ \frac{3}{5} \frac{h_P}{h} - \frac{3}{40} \left( \frac{h_P}{h} \right)^2 + \frac{1}{40} \left( \frac{h_P}{h} \right)^3 - \frac{3}{5} \sqrt{1 - \frac{h_P}{h}} \right] (II - 69) \]

\[ h \left( \sqrt{h - \sqrt{\frac{\tau_o}{\rho g_z}}} \right)^2 - \frac{\mu_1 z}{\rho g_z} = 0 \quad (II - 70) \]

where \( h_p = h - \frac{\tau_o}{\rho g_z} \quad (II - 71) \)

The analysis of simple drainage for viscoelastic fluids has also been attempted. In 1970, Wilkinson [W-3] solved equation (II-51) for the Maxwell fluid, whose rheological behavior is described by:

\[ \tau_{xz} + \lambda_0 \frac{\partial \tau_{xz}}{\partial t} = - \mu^* \frac{\partial v_z}{\partial x} \quad (II - 72) \]

The expression he obtained for the film thickness profile
incorrectly predicts that viscoelastic effects will lead to a thickening in the draining film over that which would be observed in the draining of a Newtonian fluid possessing the same steady shear viscosity.

Later in 1980, Ghosh, Mitra and Roy [G-4] attempted to solve the non-Newtonian form of equation (II-32) for a viscoelastic fluid which was rheologically characterized by an integral representation suggested by Walters [W-4]. Unfortunately, the expression that was developed to predict the film thickness profile reduced to Wyllie's expression for a Newtonian fluid as a limiting case. Consequently, the film thickness profile cannot be used since it does not satisfy the continuity equation.

Experimental studies investigating the drainage behavior of non-Newtonian fluids to date has been limited. In 1972, Denson [D-2] used his sessile drop technique to study the drainage of non-Newtonian fluids. Employing the Ellis model, Denson generated the following expression for the implicit film thickness profile:

$$z = \left( \phi_0 \rho g_z h^2 + \phi_1 (\rho g_z)^{\alpha h^{\alpha-1}} \right) t + \psi(h)$$  \hspace{1cm} (II-73)

The function $\psi(h)$ has previously been given in equation (II-14). The fluids that were examined included a polystyrene mixture and a polyisobutylene mixture which were characterized well using the Ellis fluid model and two polyacrilamide solutions containing glycerine and water which were described
well by the Power Law model over the range of shear rates investigated. Drain times ranged from 10 to 1000 seconds and film thicknesses, which were measured using the previously described light absorption technique, covered a range of roughly 0.00001 to 0.0001 meters. Denson noted that agreement between the data and theory was excellent when the fluid being examined exhibited Newtonian behavior over the observed shear rate range. However, if the fluid exhibited non-Newtonian behavior for the shear rate being studied, it was observed that at low drain times, the theory predicted greater thicknesses than those recorded and at longer drain times, the opposite was observed. Thus, it was concluded that the drainage behavior of non-Newtonian fluids depended largely upon whether the shear stresses in the film were greater or less than the shear stress at which the viscosity of the given fluid exhibited dependancy on the shear stress (or shear rate).

Most recently in 1983, Tallmadge, Weinberger and Khilar [T-2] investigated the postwithdrawal drainage behavior of viscoelastic polyacrilamide solutions containing glycerine and water similar to those used by Denson [D-2]. The fluids were rheologically characterized using the Power Law model (equation (II-53)). Withdrawal speeds covered a range of 0.00125 to 0.00334 m/s, withdrawal lengths ranged from 0.14 to 0.18 m and observation heights were varied from 0.006 to 0.046 m. The film thicknesses being measured were on the order of 0.0001 to 0.001 meters. It was observed that the film thicknesses measured during the initial drainage stages were notably below those
predicted by the postwithdrawal drainage expression for Power Law fluids which was reported in time explicit form as:

\[ h = \left( \frac{m}{\rho g z} \right)^{1/(n-1)} \left( \frac{z}{t_w + t_d} \right)^{n/(n-1)} \]  \hspace{1cm} (II-74)

It should be noted that this observation is consistent with the work performed by Denson [D-2] on non-Newtonian fluids. Additionally, Tallmadge observed after sufficient drain times that agreement between the theory and the experimental data improved; this phenomena was typically observed to be very abrupt and the time at which it occurred was termed the transition time. Thus, Tallmadge et al. concluded that in the postwithdrawal drainage of a viscoelastic solution, elastic effects were present during unsteady withdrawal and initial drainage and became negligible for long drain times. It was also suggested that the transition time was dependant upon the rheological properties of the fluid being considered but was independant of both the speed and time of withdrawal.

C. Analytical and Experimental Withdrawal Studies

There have been many theories presented in withdrawal studies, yet they may be divided into three general classes. The first class consists of solving equation (II-18) using the boundary conditions given in equations (II-11) and (II-19) to yield a film thickness profile for the entire film (e.g. unsteady withdrawal). Alternately, the equations can be solved to determine the steady state film thickness associated with the
continuous withdrawal problem.

The second class of studies concerns the continuous withdrawal case only, with the emphasis being placed upon the prediction of the steady state film thickness for various fluid regimes. To obtain a solution to the problem, the differential equations used to describe each region of the film formed in the continuous withdrawal process (illustrated in Figure (II-2)) are linearized subject to various additional constraints.

The third class of studies involves the simplified solution of equations (II-7, 8 and 9) subject to constraints obtained from class II type solutions. This class of studies is frequently referred to as the 'Non-Linear Theory of Free Coating'.

The flow regime parameters of importance are the capillary number, Ca, and the Reynolds number, Re, defined by:

\[
\text{Ca} = \frac{\text{viscous force}}{\text{surface tension}} = \frac{\mu v_w}{\sigma} \quad (\text{II}-75)
\]

\[
\text{Re} = \frac{\text{inertial force}}{\text{viscous force}} = \frac{h(v_z)_{av} \rho}{\mu} \quad (\text{II}-76)
\]

Additional dimensionless variables which are frequently used include:

\[
\text{D} = \frac{\text{gravitational force}}{\text{surface tension}} = \frac{h \left( \rho g_z \right)^{1/2}}{\sigma} \quad (\text{II}-77)
\]

\[
\text{Fp} = \frac{\text{viscous force}}{\text{surface tension}} = \mu \left( \frac{g_z}{\rho \sigma^3} \right)^{1/4} \quad (\text{II}-78)
\]
\[ T = \frac{\text{gravitational force}}{\text{viscous force}} = h \left( \frac{\rho g_z}{\mu v_w} \right)^{1/2} \quad (11-79) \]
\[ Q = \frac{\text{gravitational force}}{\text{viscous force}} = q \left( \frac{\rho g_z}{\mu v_w^3} \right)^{1/2} \quad (11-80) \]

1. Class I Solutions to the Withdrawal Problem

The first analytical and experimental investigation of the continuous withdrawal problem was supplied by Goucher and Ward [6-5] in 1922. The velocity profile, obtained by solving equation (II-18) for a Newtonian fluid, and applying the boundary condition given in equation (II-19) was reported as:

\[ v_z = \frac{\rho g_z}{\mu} \left( h x - \frac{x^2}{2} \right) + v_w \quad (11-81) \]

To obtain an expression for the film thickness, it was assumed that the velocity at the free surface of the film was zero. Mathematically,

\[ \text{B.C. } v_z = 0 \quad \text{at } x = h \quad (11-82) \]

Making use of the above assumption, the film thickness was reported as:

\[ h = \sqrt{\frac{2 \mu v_w}{\rho g_z}} \quad (11-83) \]

The fluids which Goucher and Ward used in their investigations were waxes with melting points above room temperature; thus, shortly after a film was formed by the
withdrawal process, it would solidify thereby satisfying the boundary condition presented in equation (II-82). In the liquid state, the viscosity and density of the waxes covered a range of 0.0122 to 0.032 Pa·s and 800 to 1200 kg/m³, respectively. Withdrawal speeds were varied from 0.0246 to 0.0785 m/s. The experimental error in the prediction of film thickness was roughly 10% at low withdrawal speeds and 25% at high withdrawal speeds.

In 1958, Van Rossum [V-1] critically examined the expression for the volumetric flowrate of a Newtonian fluid being continuously withdrawn. In dimensionless form, the expression was reported as:

\[ T^3 - 3T + 3Q = 0 \]  \hspace{1cm} (II-84)

It was shown that for a given positive flowrate, \( Q \), there could be two possible positive values of film thickness, \( T \). In addition, it was noted that the maximum dimensionless volumetric flow rate and corresponding film thickness were given by:

\[ T = 1 \quad ; \quad Q = 0.66 \]  \hspace{1cm} (II-85,86)

It was also noted that the expression developed by Goucher and Ward (equation (II-83)) could be obtained from equation (II-84) and in dimensionless form could be represented by:

\[ T = 1.41 \quad ; \quad Q = 0.471 \]  \hspace{1cm} (II-87,88)

To establish which particular film thickness was valid for a
given flowrate, Van Rossum performed continuous withdrawal studies on two different oils. The viscosities were reported as 0.3 and 2.4 Pa.s, densities as 880 and 894 kg/m³ and surface tensions as 0.032 and 0.0335 N/m, respectively. To generate the film, Van Rossum developed an apparatus which consisted of a brass belt whose ends were carefully soldered together to form a continuous loop. The ascending section of the belt was drawn through a fluid bath. Film thickness measurements were made using a micrometer and the volumetric flow rate was measured by scraping the liquid off the descending section of the belt. This apparatus, with slight modifications, has been used by almost all investigators studying the continuous withdrawal problem. The withdrawal velocity was varied between 0.00387 and 0.0323 m/s. The capillary numbers ranged from roughly 0.035 to 2.34. Van Rossum observed that all data generally agreed with equation (II-84). In addition, it was noted that at higher withdrawal velocities (or higher capillary numbers), the dimensionless thickness and dimensionless flowrate approached the following values:

\[ T = 0.68, \quad Q = 0.54 \]  

(II-89,90)

In 1970, Groenveld [G-6],[G-7] attempted to further investigate equation (II-84) experimentally. The apparatus used by Groenveld employed a rotating horizontal glass cylinder which was partially submerged in a fluid bath. Film thickness measurements were made using a compensating light absorption technique and volumetric flowrates were established by removing
the adhering film from the descending side of the glass cylinder (identical to the technique used by Van Rossum). The fluids which were investigated were Newtonian mixtures of glycerine and water. Viscosities ranged from 0.13 to 8.4 Pa·s, densities ranged from 1215 to 1385 kg/m³ and surface tensions ranged from 0.0629 to 0.080 N/m. Withdrawal speeds were varied from 0.0024 to 0.44 m/s. The capillary number range investigated was roughly from 0.00831 to 6.10. Based upon his experimental results, Groenveld concluded that for a capillary number of the order of unity,

\[ T = 0.66; Q = 0.56 \]  \hspace{1cm} (11-91,92)

It should be noted that this result is consistent with observations made earlier by Van Rossum. It was also concluded that when inertial forces were appreciable and the Reynolds number was close to unity,

\[ T = 0.52; Q = 0.47 \]  \hspace{1cm} (11-93,94)

2. Class II Solutions to the Withdrawal Problem

The first investigators to employ the previously mentioned matching technique were Landau and Lévich [L-1] in 1942 and Derjaguin [D-3] in 1943. For low capillary numbers, the steady state film thickness for a Newtonian fluid in dimensionless form was reported as:

\[ T_0 = 0.944 \text{ Ca}^{1/6} \]  \hspace{1cm} (11-95)
The above expression is frequently referred to as the 'Low Speed Theory of Withdrawal'. In 1970, experiments performed by Groenveld [G-8] established that the capillary numbers over which equation (II-95) was valid ranged from 0.0001 to 0.001. Groenveld also noted that surface tension lowering effects resulting from impurities would lead to an increase in the experimentally measured film thickness.

For high capillary number withdrawal of Newtonian fluids, the steady state thickness was reported by Landau and Levich [L-1] to be:

$$ T_0 = 1 $$  \hspace{1cm} (II-96)

Later in 1965, Tallmadge and White [T-8] developed an improvement to the Landau-Levich expressions by changing the method in which the differential equations describing the fluid motion (equations (II-18,20,25)) were linearized. The implicit expression for the steady state film thickness which is also referred to as the 'Medium Speed Theory of Withdrawal' is given by:

$$ T_0 = 0.944 \, \text{Ca}^{1/6} \, (1-T_0^2)^{2/3} $$  \hspace{1cm} (II-97)

Using the data of Morey [M-1] and Van Rossum [V-1], equation (II-97) was tested over a capillary number range of 0.0001 to 2.0. Based upon these calculations, Tallmadge and White concluded that the 'Medium Speed Theory of Withdrawal' represented a significant improvement over the previously
developed theories.

In 1971, Tallmadge and Soroka [T-9] modified equation (II-20) to account for inertial effects. The new expression was given by:

\[ \sigma \frac{d^3h}{dz^3} + \frac{\partial \tau_{xz}}{\partial x} + \rho g_z = \rho \nu_w \frac{\partial \nu_x}{\partial x} \]  
(II - 98)

The solution of the four-force equation (i.e. surface tension, viscous, gravitational and inertial) is referred to as the 'Inertial Theory of Plate Withdrawal'. The appropriate analytical solution is given by:

\[ Ca = 1.09 D_o^{3/2} + D_o^2 + 0.50 \zeta D_o^2 \]  
(II-99)

where \( \zeta = \exp\left(-\frac{5.13 Fp^2}{Ca^{4/3} D_o}\right) \)  
(II-100)

Continuous withdrawal experiments were carried out on three Newtonian oils using an apparatus similar to the one developed by Van Rossum [V-1]. Viscosities ranged from 0.0533 to 19.3 Pa.s, densities ranged from 869 to 885 kg/m³ and surface tensions ranged from 0.031 to 0.0326 N/m. The range of capillary numbers examined was from 0.00006 to 44 and equation (II-99) was considered to be the most general expression which could be used to describe the continuous withdrawal behavior of Newtonian fluids.

Unlike class I solutions, class II solutions were further
developed to handle the behavior of non-Newtonian fluids. In 1965, Tallmadge and Gutfinger [T-3] presented an expression applicable to the continuous withdrawal behavior of Power Law fluids (see equation (II-54)) which was analogous to equation (II-97) or the 'Medium Speed Theory of Withdrawal' for Newtonian fluids. Thus,

\[ \text{Ca} = \frac{8}{p^n (2n+1)^{4}} \frac{T^{(4-n)/(n+1)/n}}{(1-T^{(n+1)/n})^{4}} \]  \hspace{1cm} (II-101)

Here, the dimensionless variables, Ca, and T are defined as follows:

\[ \text{Ca} = \frac{m v^n h^{1-n}}{\sigma} \] \hspace{1cm} (II-102)

\[ T_0 = h_0 \left( \frac{\rho g z}{m v^n} \right)^{1/(n+1)} \] \hspace{1cm} (II-103)

To examine the validity of equation (II-100) experiments were performed on inelastic solutions of Carbopol and viscoelastic solutions of carboxymethyl cellulose (CMC) whose surface tensions ranged 0.0517 to 0.0711 N/m. Subsequent analysis of the experimental results revealed that equation (II-100) provided only an upper limit to the film thickness. It was also observed in the case of the viscoelastic fluids that elastic effects were significant in the withdrawal process.

The first theory of withdrawal for Ellis fluids (see
equation (II-53)) was also presented by Tallmadge [T-10] in 1966. The expression, (analogous to equation (II-97) for Newtonian fluids), was reported as:

\[ D_0 = 0.944 \left[ \frac{C_0 (1 - T_0^2 - T_1^{\alpha+1})}{1 - T_2} \right]^{2/3} \quad \text{(II-104)} \]

Owing to the increased rheological complexity of the Ellis fluid model, five dimensionless variables were used in the above expression:

\[ T_0 = h_0 \left( \frac{\varphi_0 \rho g z}{v_w} \right)^{1/2} \quad \text{(II-105)} \]
\[ T_1 = h_0 \left[ \frac{\varphi_1 (\rho g z) \alpha}{v_w} \right]^{1/(\alpha+1)} \quad \text{(II-106)} \]
\[ T_2 = \left( \frac{3 \alpha}{\alpha + 2} \right) \left( \frac{C_0}{C_1} \right) T_1^{(\alpha^2 - 1)/\alpha} \quad \text{(II-107)} \]
\[ C_0 = \frac{v_w}{\varphi_0 \sigma} \quad \text{(II-108)} \]
\[ C_1 = \left( \frac{h_0^{\alpha-1} v_w}{\varphi_1} \right)^{1/\alpha} \frac{1}{\sigma} \quad \text{(II-109)} \]

In order to test the withdrawal theory for Ellis fluids, Tallmadge and Hildebrand [T-11] studied the behavior of a Carbopol solution which had a density of 1000 kg/m³ and a
surface tension of 0.0577 N/m. Over withdrawal speeds ranging from 0.01 to 0.90 m/s, it was noted that equation (II-104) predicted film thicknesses to within 40%.

Using equation (II-104), Tallmadge and Hildebrand [T-11] generated an improved expression to predict the withdrawal behavior of Power Law fluids. This expression was reported as:

\[
D_0 = 0.944 \left[ \frac{(n+2)Ca(1 - T_0^{n-1})}{3nT_0^{(n^2-1)/n}} \right] 
\]

(II-110)

The dimensionless parameters, Ca and T_0, were previously defined in equations (II-102,103). Experimental analysis revealed that the errors associated with the use of equation (II-101) were on the order of 240% to 500% whereas errors associated with the use of equation (II-110) to predict film thickness were on the order of 40% to 90%. It was concluded that equation (II-110) represented a substantial improvement over the previously developed expression.

In 1978, Spiers, Adachi and Wilkinson [S-4] attempted to use a four constant Oldroyd model to predict the steady state thickness in the free coating of a vertical plate by a viscoelastic liquid. The constitutive equation describing the rheological behavior was given by:

\[
\tau_{ij} + \lambda_1 \frac{\delta \tau_{ij}}{\delta t} + \mu_0 \tau^n_n \Delta_{ij} = 2\eta_0 \left( \Delta_{ij} + \lambda_2 \frac{\delta \Delta_{ij}}{\delta t} \right) 
\]

(II-111)

The differential equations modelling the system were solved numerically using the Runge-Kutta-Merson algorithm and were compared with experimental results obtained from aqueous
mixtures of polyacrilamide and CMC. It was observed that experimental film thicknesses were much lower than those predicted and although this observation was consistent with the findings of Tallmadge and Hildebrand [T-11], no quantitative explanations were offered to explain the deviations from theory.

3. Class III Solutions to the Withdrawal Problem

The non-linear theory of free coating was first suggested by Esmail and Hummel [E-1] in 1975 and later [E-2] in 1978. The general approach is to solve the two dimensional mathematical statement of the problem (see equations (II-7,8,9)) subject to the following constraints for the meniscus region of the fluid film:

\[
\frac{\partial v'_z}{\partial x} + \frac{\partial v_x}{\partial z} + 2 \frac{dh/dz}{1-(dh/dz)^2} \left( \frac{\partial v_x}{\partial x} - \frac{\partial v_z}{\partial z} \right) = 0 \quad (II-112)
\]

\[
p + \sigma \frac{d^2h/dz^2}{[1+(dh/dz)^2]^{3/2}} + \mu \frac{dh}{dz} \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right) + 2\mu \frac{\partial v_x}{\partial x} = \rho_0 \quad (II-113)
\]

Using the direct method of Galerkin, Esmail and Hummel developed a differential equation whose solution gives not a unique relationship between the dimensionless thickness, \(T\), and Capillary number, \(Ca\), but a family of curves relating the two variables. The agreement between the new theory and experimental data for the withdrawal and drainage of Newtonian fluids is considered to be excellent. According to the authors, the new theory is also capable of predicting the upper meniscus
profile accurately where so many previous theories have failed.

Recently in 1981, Frederickson and Watts [5-1] used the direct method of Galerkin to predict the film thickness profiles of various Newtonian fluids for various stages of drainage. The results of this study have not yet been compared with actual experimental data.
III. EXPERIMENTAL PROCEDURE

A. Experimental Objectives and Apparatus

The main objective of the experimental program is to continuously measure the thickness of suspensions in post-withdrawal drainage situations. The experimental set-up is similar to that used by Tallmadge et al. [T-2] with one significant difference: The film thickness measurement technique is based upon the (well established) principle of laser interferometry. This technique was chosen over the more convenient light absorption technique employed by Tallmadge et al. because of its ability to handle liquid films containing particulate matter. The actual apparatus used in the experimental program consists of the following major components.

i) Vibration Damping Support System

ii) Optical Component Support System

iii) Plate Withdrawal Support System and Variable Speed Motor

iv) Microcomputer

Each component will now be discussed in detail.

1. Vibration Damping Support System

The design of the vibration damping support system was very critical since the interferometer was known to be sensitive to forced vibrations. In particular, the interferometer had to be isolated from vibrations produced by heating and air conditioning machinery and heavy traffic. However, it was recognized that a high frequency disturbing force would not produce significant vibrations in a system of low natural frequency. Thus, in order to lower the natural frequency of the
system, attempts were made to both increase the mass and decrease the 'elastic' spring constants in the table supports.

The chosen vibration damping support system is illustrated in Figure (III-1). Note that the table top was made of a heavy marble slab weighing approximately 300 kg. In addition, large styrofoam blocks and two rubber pads were used in each table support; these materials were used because of their relatively low spring constants.

2. Optical Component Support System

The optical component support system is illustrated in Figure (III-2). Fastened to this support is a laser (35mW HeNe by Spectra-Physics), three plane mirrors (Melles-Griot), two beam splitters (roughly 50% transmittance), a magnifying lens and an arrangement of 10 photodetectors (time constant @ 0.0001 seconds). Note that the vertical elevation of the laser and all optical components may be easily varied to study the film from a variety of positions.

The optical configuration is commonly referred to as the Mach-Zender [M-2],[Z-1] interferometer which is schematically illustrated in Figure (III-3). It was expected that by mounting all the optical components on one common support, relative vibrations within each arm of the interferometer would be reduced.

It is important to realize that while both rays in the interferometer will be sent through a vertical glass plate on which the fluid film is being produced, only one ray in the interferometer will be sent through the fluid film. As a
FIGURE (III-1) : Vibration Damping Support System
Note: All dimensions in meters

Marble Slab
Plywood Spacers

1.00
1.98

0.05
0.01
0.20
0.01
0.45

Rubber Pads
Styrofoam
Concrete Slabs

30

48
FIGURE (III-2) : Optical Component Support System
Note: All dimensions in meters (except as shown)

9/16" Hexagonal Nuts
3/4" Hexagonal Nuts

Turning Plane Mirror Support
Upper Beam Splitter Support
Upper Plane Mirror Support
Magnifying Lens Support
Photodetector Support
Lower Plane Mirror Support
Lower Beam Splitter Support
Forward Laser Support
Rear Laser Support

1/2" Threaded Rod
3/8"-Threaded Rod
FIGURE (III-3) : Mach-Zender Interferometer
result, the measured path difference will be directly related to the change in film thickness alone.

One final comment is concerned with the applicability of the Mach-Zender interferometer. In effect, this optical configuration is capable of measuring relative changes in film thickness only. Clearly, it is not possible to obtain absolute film thickness measurements directly.

3. Plate Withdrawal Support and Variable Speed Motor

The plate withdrawal support, shown in Figure (III-4), consists primarily of a steel framework, on the top of which is mounted a gear. Since the gear will be moving during the unsteady withdrawal period of the experiment, vibrations will be present. To minimize the effect of these operating vibrations, the natural frequency of the steel framework was lowered by adding very heavy faceplates on each side. Also shown in Figure (III-4) is the sample container (rectangular plexiglass box) in which the vertical glass plate rests prior to withdrawal. Note that plexiglass guides have been mounted inside the box to ensure that as the plate is withdrawn, movement will be constrained to the vertical direction only.

A chain fastened to the glass plate, is led around the gear positioned at the top of the framework, to a wall mounted axle which is driven by a variable speed motor. The chain is used to eliminate slippage and is considered to be inextensible.

The variable speed motor arrangement actually consists of an AC motor (1/4 Hp) connected to a worm gear speed reducer (maximum reduction 1750:1). The whole assembly is controlled
FIGURE (III-4): Plate Withdrawal Support System
manually, using a wall mounted panel. The variable speed motor arrangement has a speed range covering roughly 0.0015 to 0.0700 m/s. The technique used to calibrate the plate withdrawal speed and subsequent results obtained are presented in Appendix I. Both the AC motor and worm gear speed reducer were mounted on an adjacent wall to help minimize operating vibrations.

4. The Microcomputer and its Use in Film Thickness Measurements

In order to understand the role of the microcomputer in the measurement of the fluid film thickness, it is necessary to understand the principle of interferometry. To begin, note that the Mach-Zender interferometer is capable of producing a field of uniform light and dark lines known as fringes. Next, recognize that a fringe shift will result if:

i) the sample, inserted in one arm of the interferometer has local variations in thickness or,

ii) the sample, inserted in one arm of the interferometer has local variations in refractive index.

To develop the simple expression which relates a shift of fringes to a change in local film thickness, one has the following definitions:

Optical Path Length

\[ S = L \eta_0 \]  

(III-1)

Path Difference

\[ \Gamma = S_2 - S_1 \]  

(III-2)
As the film drains, the optical path length may be continuously represented by:

\[ \Gamma = (n_{\text{fluid}} - n_{\text{air}})(L_2 - L_1) \]  

\[ (\text{III-3}) \]

From optics, one recognizes that:

\[ \Gamma = m\lambda \]  

\[ (\text{III-4}) \]

Thus, one can write:

\[ m = \frac{(n_{\text{fluid}} - n_{\text{air}})(L_2 - L_1)}{\lambda} \]  

\[ (\text{III-5}) \]

Therefore, a given path difference will cause additional wavelengths to be present within the thicker medium and a fringe shift of 'm' fringes will result.

Now, since both sides of the vertical glass plate will be coated with a film, we have:

\[ \delta = \frac{(L_2 - L_1)}{2} \]  

\[ (\text{III-6}) \]

Substitution now leads to the following expression which may be used to predict the film thickness in both unsteady withdrawal and postwithdrawal drainage:

\[ \text{Experimental Film Thickness} \quad \delta = \frac{m\lambda}{2(n_{\text{fluid}} - 1)} \]  

\[ (\text{III-7}) \]
As the film thickness changes with time, the photodetectors which are interfaced with the microcomputer (64K TRS-80 Color Computer by Radio Shack) will detect the movement of fringes in the interference pattern. In addition, since the clockspeed of the computer is accurately known, the time dependance of the transient film thickness may be easily determined by examining the fringe displacement.

B. Materials and Measurement of Physical Properties

The postwithdrawal drainage behavior was studied for the following list of fluids:

<table>
<thead>
<tr>
<th>CLASS</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>Glycerine/Water (80/20 wt%)</td>
</tr>
<tr>
<td>Viscoelastic</td>
<td>Aqueous PAA (1.0 wt%) (Separan NP-10)</td>
</tr>
<tr>
<td>Viscoelastic</td>
<td>Silica @ 5-50 nm dia. (1/2/3 wt%)</td>
</tr>
<tr>
<td>Suspensions</td>
<td>in Glyc./Water (75/25 wt%) containing 1.0 wt% PAA (Separan AP-30)</td>
</tr>
</tbody>
</table>

An anti-bacterial agent (0.01 wt% phenylmercuric acetate) was added to each polymer solution to prevent biological degradation.

In order to accurately characterize each fluid, a number of physical properties were obtained. Specifically, the relationship between viscosity and shear rate was established to determine the rheological parameters. This was accomplished using the Contraves Rheomat-30 viscometer. In addition, the yield stress of the suspensions was determined using the modified Fisher-Autotensiomat Surface Tension Analyzer. Surface
tensions of the Newtonian and viscoelastic fluids were measured using the Du Nuoy ring method. Other fluid properties of interest which were determined for all fluids included the density and index of refraction which were established using pycnometers and an Abbe refractometer, respectively. Appendix II provides complete details on experimental technique and summarizes all physical properties, including rheological model parameters.

C. Experimental Method

First, the computer, motor, and laser power supply were turned on 2 or 3 hours before the experiment was to start. Next, one of the outer faceplates of the plate withdrawal support system was detached to facilitate removal of the sample container and vertical glass plate. The former was then cleaned and rinsed with distilled water and the latter was cleaned with distilled water and nitric acid followed by liberal rinsing with acetone (Note that the glass plate was allowed to dry under ambient air conditions). At this point, the sample container was charged with the liquid sample (roughly 8.0 liter capacity) carefully, attempting to minimize the entrainment of air bubbles. The vertical glass plate was then fitted into the sample container and both components were returned to the plate withdrawal support system. Next, the lower beam of the interferometer was positioned at the desired level above the surface of the liquid in the sample container. This height was then recorded as '2', which represents the distance of the plane of observation above the liquid bath. After this was
accomplished, the upper arm of the interferometer was adjusted to provide a satisfactory interference pattern. Immediately thereafter, the faceplate was bolted back into position and the chain was fastened to the glass plate and led around the drive shaft.

Before proceeding with any experiments, all ventilation and exhaust fans in the lab were turned off by the maintenance personnel. This step was essential since interferometric measurement was very sensitive to external disturbances. Once all preparatory steps were completed, one of two data acquisition programs was loaded into the microcomputer. The program selection depended upon whether an initial investigation or complete experimental record of postwithdrawal drainage behavior was required. Both programs were written using assembly language and a small basic program was written to access the machine language routines.

In the case of an initial investigation, the simple data acquisition program was employed. The primary function of this program was to provide details concerning the speed of movement of interferometric fringes. Trial and error procedures were used to determine the sampling rates necessary to distinguish fringe displacement during postwithdrawal drainage experiments. Features of the program included the storage and retrieval from cassette tape and display (hardcopy or screen) of up to 12000 data points obtained by sampling any one of the ten available photodetectors. To provide the user with maximum flexibility, the data acquisition procedure was divided into four distinct
sequences. Within each sequence, a selection from a total of 17 sampling rates ranging from 50 to 7000 readings/second could be made. In addition, the number of samples to be collected at each sampling rate had to be established. Taken together, this information defined what is referred to as the 'sample rate profile'.

When a complete experimental record of postwithdrawal drainage was required, the advanced data acquisition program was employed. This program was a modification of the simple data acquisition program. Its improved features included the storage and retrieval from cassette tape and graphical display (hardcopy or screen) of up to 24200 data points obtained by sampling any two of the ten available photodetectors. The four sequence sample rate profile was increased to handle eight sequences and a total of 27 sampling rates ranging from \( \frac{1}{2} \) to .2500 readings/second were made available.

Before starting the plate withdrawal motor, the photodetector(s) to be sampled and the gear number and dial setting (motor control panel) had to recorded in the computer program for future reference and documentation. Once this was accomplished the program provided the operator with a 10 second preparatory countdown. At the end of this countdown, the computer would immediately begin sampling and the motor would be manually started. After a suitable withdrawal distance was obtained, the motor was turned off and the computer would continue to take readings until it completed the sample rate profile. At the end of the experiment, the withdrawal distance,
L, was determined by referring a calibration mark on the chain connected to the glass plate with a conveniently mounted ruling device.

A listing of the assembly language computer programs used in this work is prohibitive due to their length; a lab manual complete with actual photographs of the experimental set-up and the computer program listings used for plate speed calibration, data acquisition and data analysis has been prepared to provide additional details.
IV. RESULTS AND DISCUSSION

A summary of the data obtained in postwithdrawal drainage studies has been presented in Appendix III. Prior to a discussion regarding the behavior of each class of fluids, the techniques used to reduce and analyze the data will be reviewed.

A. Data Reduction and Analysis

The Mach-Zender interferometer is an instrument capable of measuring very small changes in thickness (1 x 10^-7 m), however it is incapable of making absolute thickness measurements. By sampling only one photodetector, it is impossible to determine whether the film thickness is increasing or decreasing. Further, waves or vibrations cannot be detected. To solve the latter problem, two adjacent photodetectors were sampled nearly simultaneously. Segments of a typical interferogram produced by the advanced data acquisition program (described in Chapter III) are illustrated in Figure (IV-1). The ordinate represents the intensity of light being detected by the photodetectors (where 1.00 refers to lowest intensity and 0.00 refers to highest intensity), and the abscissa represents the elapsed time since the motor was switched on. Segment (a) refers to the time period during which the plate was being withdrawn. Vibrations and a very rapidly shifting interference pattern were frequently observed during the unsteady withdrawal stage of the postwithdrawal drainage process. Vibrations were portrayed in the interferogram as regions in which changes in the phase shift between detector outputs were frequent and random. Such
FIGURE (IV-1) : Segments of a Typical Interferogram
circumstances made further analysis of such data obsolete. Note that up to 10 seconds, the sampling rate used was 10 readings/second, since data collection during withdrawal was impractical. After 10 seconds, the sampling rate was increased to 250 readings/second to detect the drainage of the film at the earliest possible moment.

In segment (b), the presence of a wave in the draining film is indicated by the single change in phase shift between detector outputs. After the peak of the wave (in the draining film) has passed the plane of observation, another change in phase shift is expected. Note that the sampling rate over the entire segment is 200 readings/second. Waves were distinguished from vibrations by noting the time required for a change in phase shift to occur; waves required longer periods of time.

Segment (c) illustrates a well-established interference pattern which is characterized by a constant phase shift between detector outputs. Note that at 76 seconds (1' 16") , the sampling rate was reduced from 50 readings/second to 20 readings/second. This is clearly indicated by the change in the width of the peaks.

To determine the transient film thickness profiles, only portions of the interferogram similar to segment (c) were analyzed. Since no waves were present in such segments, the output from only one detector was needed for further analysis. Each fringe was represented by a peak (or valley) in the interferogram. To determine the time at which each peak maxima occurred, the natural cubic spline interpolation algorithm [8-3]
was used first to smooth raw data. This technique was chosen over the trigonometric polynomial approximation scheme (Fast Fourier Transform algorithm [C-1]) because of its relative simplicity and ability to easily handle data collected at different sampling rates. Peak maxima were established by determining the point at which the sign of the slope of intensity versus 'elapsed' time curve changed from positive to negative. Usually, 10000 to 12000 data points covering several different sampling rates would be processed from each experiment to produce enough peak maxima to establish the postwithdrawal drainage behavior for drain times ranging from 10 to 250 seconds. The number of peaks resolved for an individual experiment ranged from 108 to 764 and the change in film thickness was determined using equation (III-7). To obtain a convenient representation of the experimental data, a datum thickness was chosen to minimize the average error between the experimental results and theory.

B. Newtonian Postwithdrawal Drainage

The results of nine experiments covering three different withdrawal speeds and three different observation heights is presented in Figure (IV-2). Note that not all experiments were performed at the same temperature. It has been observed in all but one of the experiments that there exists a significant deviation between the parabolic film thickness profile (equation (II-43)) and the actual data at low drain times; otherwise agreement between theory and experiment is good. To explain
FIGURE (IV-2): Newtonian Postwithdrawal Drainage.

(a) \( v_w = 24.3 \times 10^{-3} \text{ m/s} \)
- \( \square: (z/L)_{obs} = 0.860 \)
- \( \bigcirc: (z/L)_{obs} = 0.561 \)
- \( \bullet: (z/L)_{obs} = 0.402 \)

(b) \( v_w = 16.2 \times 10^{-3} \text{ m/s} \)
- \( \square: (z/L)_{obs} = 0.855 \)
- \( \bigcirc: (z/L)_{obs} = 0.552 \)
- \( \bullet: (z/L)_{obs} = 0.396 \)

(c) \( v_w = 8.7 \times 10^{-3} \text{ m/s} \)
- \( \square: (z/L)_{obs} = 0.833 \)
- \( \bigcirc: (z/L)_{obs} = 0.555 \)
- \( \bullet: (z/L)_{obs} = 0.375 \)

Note: Solid line represents equation (II-43).
this deviation, an extension of the discussion provided by Tallmadge [T-6],[T-7] and Groenveld [G-2] is presented.

Tallmadge postulated the existence of a critical junction during the withdrawal stage of the postwithdrawal drainage process above which Jeffrey's [J-1] expression held and below which a continuous withdrawal theory could be applied. Further, once postwithdrawal drainage was initiated, the critical junction would propagate downward towards the bath surface at some constant velocity (equation (II-46)). The time required for the critical junction to reach the bath surface is termed the critical drain time, $t^*_d$, and can be predicted using equation (II-47). Since the experimental set-up currently being employed is used to measure the film thickness at a particular observation plane, it will be useful to define a transition drain time, $t^{**}_d$, as the time required for the critical junction to pass below the observation plane. Thus, after this transition drain time, agreement between the parabolic theory (equation (II-43)) and experimental results is expected. The transition drain time expression for the postwithdrawal drainage of any fluid is given by:

$$t^{**}_d = t_w \left( \frac{(z/L)_{obs} - (z/L)_j}{(z/L)_j} \right)$$  \hspace{1cm} (IV-1)

The location of the critical junction, $(z/L)_j$, corresponds to a zero transition drain time. For a Newtonian fluid, the critical junction can be established using,


\[(z/L)_{j} = T_{0}^{2}\]  \hspace{1cm} (IV-2)

Based upon calculation of the capillary number (equation (II-75)), the medium speed theory of withdrawal (equation (II-97)) developed by Tallmadge [T-8] was used to calculate the dimensionless film thickness, \(T_{0}\). The theoretical drain time is easily calculated using equation (IV-1). The experimental transition drain time however, was much more difficult to establish since plots of film thickness versus drain time were non-linear. Estimations were made by determining the time at which the error between the experimental and theoretical film thickness became less than or equal to the average error. This technique was subsequently used to establish the experimental drain times for all postwithdrawal drainage experiments. A summary of the results appear in Table (IV-1). Comparison of the transition drain times with Figure (IV-2) suggest that equation (IV-1) adequately predicts the time required for the parabolic film thickness profile (equation (II-43)) to be valid. Based upon these results, it is clear that the transition drain time is directly proportional to the dimensionless observation height, \((z/L)_{obs}\), which is consistent with observations made by Denson [D-1]. Additionally, the transition drain time is directly proportional to the withdrawal time. To determine the effect of other variables, note that the transition drain time is inversely proportional to the critical junction, \((z/L)_{j}\). Then, from equation (II-79), the transition drain time will increase if the viscosity or withdrawal speed increases or the
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_o$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $Ca \times 10^1$, $T_0$</th>
<th>Initial Film Thickness $h_o \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^{<strong>}$ (pred.), $t_d^{</strong>}$ (exp.) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.860</td>
<td>0.262</td>
<td>0.861, 0.512</td>
<td>323</td>
<td>18.0, 17.5</td>
</tr>
<tr>
<td>24.3</td>
<td>0.561</td>
<td>0.288</td>
<td>1.306, 0.489</td>
<td>416</td>
<td>6.9, 8.0</td>
</tr>
<tr>
<td>24.3</td>
<td>0.402</td>
<td>0.239</td>
<td>0.575, 0.489</td>
<td>252</td>
<td>3.4, 7.3</td>
</tr>
<tr>
<td>16.2</td>
<td>0.855</td>
<td>0.239</td>
<td>0.575, 0.489</td>
<td>251</td>
<td>28.7, 12.1</td>
</tr>
<tr>
<td>16.2</td>
<td>0.552</td>
<td>0.263</td>
<td>0.871, 0.513</td>
<td>325</td>
<td>11.3, 13.8</td>
</tr>
<tr>
<td>16.2</td>
<td>0.396</td>
<td>0.213</td>
<td>0.355, 0.462</td>
<td>187</td>
<td>5.9, 8.8</td>
</tr>
<tr>
<td>8.7</td>
<td>0.833</td>
<td>0.206</td>
<td>0.308, 0.454</td>
<td>171</td>
<td>53.7, 61.1</td>
</tr>
<tr>
<td>8.7</td>
<td>0.555</td>
<td>0.228</td>
<td>0.467, 0.477</td>
<td>221</td>
<td>26.6, 29.2</td>
</tr>
<tr>
<td>8.7</td>
<td>0.375</td>
<td>0.182</td>
<td>0.190, 0.427</td>
<td>126</td>
<td>12.5, 12.7</td>
</tr>
</tbody>
</table>

TABLE (IV-I): Summary of Newtonian Transition Drain Time Calculations 
(data from the current work)
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_{obs}$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $Ca \times 10^3$ $T_0$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^{<strong>}$ (pred.) $t_d^{</strong>}$ (exp.) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.0</td>
<td>0.238</td>
<td>0.187</td>
<td>0.208 0.432</td>
<td>147</td>
<td>1.8</td>
</tr>
<tr>
<td>28.0</td>
<td>0.096</td>
<td>0.187</td>
<td>0.208 0.432</td>
<td>147</td>
<td>-</td>
</tr>
<tr>
<td>3.1</td>
<td>0.237</td>
<td>0.213</td>
<td>0.357 0.462</td>
<td>65</td>
<td>13.5</td>
</tr>
<tr>
<td>3.1</td>
<td>0.211</td>
<td>0.213</td>
<td>0.357 0.462</td>
<td>65</td>
<td>-</td>
</tr>
<tr>
<td>3.1</td>
<td>0.114</td>
<td>0.213</td>
<td>0.357 0.462</td>
<td>65</td>
<td>-</td>
</tr>
</tbody>
</table>

**TABLE (IV-2): Summary of Newtonian Transition Drain Time Calculations (data from Tailmadge [T-2])**
density decreases.

It is interesting to note that postwithdrawal drainage studies of Newtonian fluids obtained by Tallmadge et al. [T-2] do not demonstrate deviations from theory at low drain times. To investigate this further, a summary of the data obtained by Tallmadge has been presented in Table (IV-2). In three of the five experiments investigated, the plane of observation lies above the critical junction and as a result, no transition drain time exists. The remaining two experiments were carried out very near the critical junction; the transition drain time is short and no experimental data were presented prior to these times.

Thus, it is suggested that the transition drain time expression (equation (IV-1)) is a reasonable attempt to account for deviations at low drain times in the postwithdrawal drainage of a Newtonian fluid.

C. Non-Newtonian Postwithdrawal Drainage

1. Establishment of Rheological Parameters

Postwithdrawal drainage experiments were carried out on a viscoelastic fluid and a series of viscoelastic suspensions. The rheological behavior of the viscoelastic fluid was characterized using the Ellis model (equation (II-53)). To simplify establishment of the rheological parameters, non-linear regression was performed on the following expression, suggested by Bird et al. [B-2]:
\[
\frac{1}{\eta} = \varphi_0 + \varphi_1 |\tau_{xz}|^{\alpha-1}
\]

(IV-3)

The relationship between the parameters in equations (IV-3) and (II-53) has been given in equations (II-58 and 59). A plot of the viscosity-shear rate data for the viscoelastic solution in Figure (IV-3) illustrates that agreement with equation (IV-3) is good.

The rheological behavior of the viscoelastic suspensions were modelled using two three parameter expressions. The first, suggested by De Kee and Turcotte [D-4] is given as:

\[
\tau_{xz} = \pm \tau_0 - \eta_1 \dot{\gamma}_{xz} e^{-t_1 \dot{\gamma}_{xz}} \quad |\tau_{xz}| \leq \tau_0 \quad (IV-4a)
\]

\[
\dot{\gamma}_{xz} = 0 \quad |\tau_{xz}| < \tau_0 \quad (IV-4b)
\]

The second, which is a simple generalization of the Bingham plastic (equation (II-63a,b)), is the well known Herschel-Bulkley expression:

\[
\tau_{xz} = \pm \tau_0 - m |\dot{\gamma}_{xz}|^{n-1} \dot{\gamma}_{xz} \quad |\tau_{xz}| \leq \tau_0 \quad (IV-5a)
\]

\[
\dot{\gamma}_{xz} = 0 \quad |\tau_{xz}| < \tau_0 \quad (IV-5b)
\]

To establish values for the three parameters in the De Kee model, three different numerical techniques were employed. The first technique was used to obtain estimates for \(\eta_1\) and \(t_1\). This was accomplished by letting \(\tau_0 = 0\), and fitting the best
FIGURE (IV-3): Viscosity versus Shear Rate Data for the Viscoelastic Solution

\[ \varphi_0 = 9.54 \times 10^{-2} \quad \text{l/Pa} \cdot \text{s} \]

\[ \varphi_1 = 1.38 \times 10^{-2} \quad \text{l/Pa} \cdot \text{s} \]

\[ \alpha = 2.24 \]

\[ \eta_0 = 10.4 \quad \text{Pa} \cdot \text{s} \]

\[ \tau_{1/2} = 4.77 \quad \text{Pa} \]

Note: Solid line represents equation (IV-3).
straight line through a semi-logarithmic plot of the viscosity-shear rate data. Low shear rate data were not used in this technique since yield stress behavior would affect the prediction of the other parameters. The parameter values were subsequently used in attempts to predict yield stress values from post-withdrawal drainage data; these results will be discussed later.

The second technique used non-linear regression of the shear stress-shear rate data to generate values for all three parameters. In this effort, the yield stress was considered to be unknown.

The third technique also used non-linear regression to generate values for all three parameters, again based on rheological data. The difference between this technique and the second was that the experimentally observed yield stress was included as a data point. This technique was considered to be the most representative of fluid behavior and the parameters obtained were subsequently used to analyze the post-withdrawal drainage data. A comparison of the results obtained using each technique has been presented in Figure (IV-4) for one of the viscoelastic suspensions. From this, agreement between equation (IV-4a) and the rheological data is considered to be acceptable.

To establish values for the three parameters in the Herschel-Bulkley model, three numerical techniques similar to the one's used for the De Kee model were employed. The first technique, used to obtain estimates for \( m_i \) and \( n_i \), involved setting \( T_0 = 0 \) and fitting the best straight line through a
FIGURE (IV-4): Viscosity versus Shear Rate data for a Typical Viscoelastic Suspension.

(1% solids)

--- : semi-logarithmic approximation

--- : non-linear regression of rheological data
(unknown yield stress)

----- : non-linear regression of rheological data
(known yield stress)

Note: All solids lines represent equation (IV-4a) for different parameter values.
log-log plot of the viscosity-shear rate data. The parameters obtained from non-linear regression of rheological data, which included the measured yield stress as a data point, were considered to be most representative of fluid behavior and were subsequently used in postwithdrawal drainage studies.

Appendix II contains the results of calculations performed on each viscoelastic suspension.

2. Ellis Fluid Data

The results of nine experiments covering three withdrawal speeds and three observation heights is illustrated in Figure (IV-5). The expression used to predict the film thickness for the postwithdrawal drainage of an Ellis fluid is given by:

\[ \varphi_0 \rho g z h^2 + \varphi_1 (\rho g z)^\alpha h^{\alpha + 1} \frac{z}{t_d + t_w} = 0 \quad (IV-6) \]

Agreement of experimental data with equation (IV-6) is considered to be good, however similar to the Newtonian postwithdrawal drainage results, significant deviations from theory were observed at low drain times. The expression used to describe the critical junction for an Ellis fluid (analogous to equation (IV-2)) has been developed (see Appendix IV, section 3) and is reported as:

\[ (z/L)_j = T_0^2 + T_1^{\alpha + 1} \quad (IV-7) \]

The dimensionless groups, \( T_0 \) and \( T_1 \), have been defined in equations (II-105) and (II-106). Using the continuous
FIGURE (IV-5) : Postwithdrawal Drainage for an Ellis Fluid

(a) $v_w = 24.3 \times 10^{-3}$ m/s

- : $(z/L)_{obs} = 0.813$
- : $(z/L)_{obs} = 0.477$
- : $(z/L)_{obs} = 0.312$

(b) $v_w = 16.2 \times 10^{-3}$ m/s

- : $(z/L)_{obs} = 0.817$
- : $(z/L)_{obs} = 0.488$
- : $(z/L)_{obs} = 0.398$

(c) $v_w = 8.7 \times 10^{-3}$ m/s

- : $(z/L)_{obs} = 0.798$
- : $(z/L)_{obs} = 0.484$
- : $(z/L)_{obs} = 0.346$

Note: Solid line represents equation (IV-6)
withdrawal theory for an Ellis fluid (equation (II-104)), developed by Tallmadge [T-10] to predict the film thickness at the onset of drainage, transition drain times were calculated by substituting equation (IV-7) into equation (IV-1). A summary of all relevant calculations is presented in Table (IV-3). Although the transition drain times were qualitatively correct, they did not quantitatively agree with experimentally observed values. Additionally, film thickness predictions at the onset of drainage were not in good agreement with extrapolations of the experimental data. Since the transition drain time depends upon the film thickness at the moment drainage begins, estimated values were used to see if better agreement between experimental data and equation (IV-7) could be obtained. Zero drain time film thickness estimations were obtained by linear extrapolation of the low drain time data. A summary of the calculations are presented in Table (IV-4). Here, the transition drain time is in much better agreement with experimental data. Comparison of Tables (IV-3) and (IV-4) suggest that the transition drain time is inversely proportional to the film thickness at the onset of drainage. This observation is consistent with results obtained by Denson [D-2].

In general, predicted transition drain times are lower than those experimentally observed. However, if one assumes that the actual film thickness at the onset of drainage is lower than that predicted by linear extrapolation, then a longer transition drain time will result.

Tallmadge et al. [T-2], who studied the postwithdrawal
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_o$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $C_0, C_1, T_0, T_1, T_2$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^{<strong>}$ (pred.) $t_d^{</strong>}$ (exp.) [s]</th>
</tr>
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<tbody>
<tr>
<td>24.3</td>
<td>0.813</td>
<td>0.305</td>
<td>3.68 0.45 0.25 0.64 5.90</td>
<td>1190</td>
<td>10.4 41.5</td>
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<td>0.471</td>
<td>0.305</td>
<td>3.68 0.45 0.25 0.64 5.90</td>
<td>1190</td>
<td>3.4 29.8</td>
</tr>
<tr>
<td>24.3</td>
<td>0.312</td>
<td>0.305</td>
<td>3.68 0.45 0.25 0.64 5.90</td>
<td>1190</td>
<td>0.1 25.7</td>
</tr>
<tr>
<td>16.2</td>
<td>0.817</td>
<td>0.291</td>
<td>-2.45 0.34 0.27 0.63 4.89</td>
<td>1020</td>
<td>16.4 52.9</td>
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<tr>
<td>16.2</td>
<td>0.488</td>
<td>0.291</td>
<td>2.45 0.34 0.27 0.63 4.89</td>
<td>1020</td>
<td>6.1 34.0</td>
</tr>
<tr>
<td>16.2</td>
<td>0.398</td>
<td>0.291</td>
<td>2.45 0.34 0.27 0.63 4.89</td>
<td>1020</td>
<td>2.1 25.1</td>
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<tr>
<td>8.7</td>
<td>0.798</td>
<td>0.272</td>
<td>1.32 0.23 0.29 0.60 3.64</td>
<td>804</td>
<td>28.6 116.8</td>
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<td>8.7</td>
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<td>0.272</td>
<td>1.32 0.23 0.29 0.60 3.64</td>
<td>804</td>
<td>12.4 59.7</td>
</tr>
<tr>
<td>8.7</td>
<td>0.346</td>
<td>0.272</td>
<td>1.32 0.23 0.29 0.60 3.64</td>
<td>804</td>
<td>3.0 31.2</td>
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</table>

TABLE (IV-3): Summary of Ellis Fluid Transition Drain Time Calculations (Initial film thickness predicted using equation (II-104)).
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_\text{obs}$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $C_0$ $C_1$ $T_0$ $T_1$ $T_2$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^{**}$ (pred.) [s]</th>
<th>$t_d^{**}$ (exp.) [s]</th>
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</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.813</td>
<td>0.109</td>
<td>3.68 0.37 0.18 0.45 3.81</td>
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<td>41.5</td>
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<td>3.68 0.35 0.17 0.42 3.46</td>
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<td>24.3</td>
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<td>20.3</td>
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<td>0.112</td>
<td>2.45 0.29 0.19 0.45 3.25</td>
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<td>2.45 0.27 0.17 0.40 2.81</td>
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<td>24.7</td>
<td>25.1</td>
</tr>
<tr>
<td>8.7</td>
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<td>0.112</td>
<td>1.32 0.19 0.21 0.44 2.46</td>
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<td>90.6</td>
<td>116.8</td>
</tr>
<tr>
<td>8.7</td>
<td>0.484</td>
<td>0.096</td>
<td>1.32 0.19 0.20 0.41 2.29</td>
<td>553</td>
<td>64.8</td>
<td>59.7</td>
</tr>
<tr>
<td>8.7</td>
<td>0.346</td>
<td>0.094</td>
<td>1.32 0.19 0.20 0.41 2.27</td>
<td>550</td>
<td>29.3</td>
<td>31.2</td>
</tr>
</tbody>
</table>

**TABLE (IV-4):** Summary of Ellis Fluid Transition Drain Time Calculations (initial film thickness predicted using linear extrapolation)
drainage behavior of a viscoelastic solution, also noted the deviation of low drain time data from theory. Based upon their results, they concluded that there was no appreciable influence of either time of withdrawal or speed of withdrawal. In the present work however, the critical drain time expression for the Ellis fluid (equation (IV-7)) may be used to suggest that the transition drain time is proportional to withdrawal speed and the rheological parameter, $\alpha$, and is inversely proportional to the rheological parameters, $\phi_0$ and $\phi_1$, and the fluid density. Also, from equation (IV-1), the transition drain time is proportional to the withdrawal time and observation plane. A closer examination of the data provided by Tallmadge et al, does indeed suggest that the transition drain time is directly proportional to the observation height in most cases. It is suspected however that the effect of increased withdrawal time on the transition drain time was compensated by a decrease in the observation height. An analysis using the transition drain time expression should probably be employed. Insufficient data from this reference prevents presentation of the appropriate calculations.

3. Viscoelastic Suspension Data

A total of eighteen experiments, covering three withdrawal speeds and two observation heights for a series of three viscoelastic suspensions, have been analyzed using the postwithdrawal drainage film thickness profiles developed for the De Kee and Herschel-Bulkley fluid models (see Appendix IV, sections 1 and 2). The expressions are respectively reported
as:

\[
\left(\rho g z \right)^{2} \left( t_{1} \right)^{3} + \left( \rho g z \frac{2 \rho g z t_{1}}{\eta_{1}} \right) h^{2} + \left( \frac{\tau_{0}}{\eta_{1}} \right) t_{1} \left( t_{1} - \tau_{0} \right) h \right)
- \frac{z}{t_{d} + t_{w}} = 0 \quad (IV-8)
\]

\[
h^{n+1} - h^{n} \left( \frac{\tau_{0}}{\rho g z} \right) - \left( \frac{m}{\rho g z} \right) \left( \frac{z}{t_{d} + t_{w}} \right)^{n} = 0 \quad (IV-9)
\]

The results of analysis using equation (IV-8) have been presented in Figures (IV-6, 7 and 8) and the results using equation (IV-9) have been presented in Figures (IV-9, 10 and 11). Two separate sets of figures were required to display essentially the same data since the datum thickness was different for each fluid model. The expected short drain time deviation was again observed using both models however, a long drain time deviation was observed in the case of the Herschel-Bulkley model only. By comparison, the postwithdrawal drainage expression based upon the De Kee model was observed to be more representative of the experimental data over a wider range of drain times. It is suspected that the inability of equation (IV-9) to adequately represent postwithdrawal drainage data at long drain times is attributable to a deficiency of the Herschel-Bulkley model. Specifically, for the type of fluids studied, low shear rate behavior which is manifested in the later stages of postwithdrawal drainage, is not predicted well using equation (IV-5a).

To examine the short drain time deviations, the transition drain time theory (equation (IV-1)) was applied. The critical
FIGURE (IV-6): Postwithdrawal Drainage for a De Kee Fluid
(viscoelastic suspension - 1% solids)

(a) $v_w = 24.3 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.733$
   - ○: $(z/L)_{obs} = 0.576$

(b) $v_w = 16.2 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.778$
   - ○: $(z/L)_{obs} = 0.593$

(c) $v_w = 8.7 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.787$
   - ○: $(z/L)_{obs} = 0.582$

Note: Solid line represents equation (IV-8).
FIGURE (IV-7): Postwithdrawal Drainage for a De Kee Fluid
(viscoelastic suspension – 2% solids)

(a) $v_w = 24.3 \times 10^{-3}$ m/s
   \( \square: (z/L)_{\text{obs}} = 0.779 \)
   \( \bigcirc: (z/L)_{\text{obs}} = 0.445 \)

(b) $v_w = 16.2 \times 10^{-3}$ m/s
   \( \square: (z/L)_{\text{obs}} = 0.778 \)
   \( \bigcirc: (z/L)_{\text{obs}} = 0.437 \)

(c) $v_w = 8.7 \times 10^{-3}$ m/s
   \( \square: (z/L)_{\text{obs}} = 0.773 \)
   \( \bigcirc: (z/L)_{\text{obs}} = 0.437 \)

Note: Solid line represents equation (IV-8).
FIGURE (IV-8) : Postwithdrawal Drainage for a De Kee Fluid
(viscoelastic suspension - 3 % solids)

(a) $v_w = 24.3 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.833$
   - ○: $(z/L)_{obs} = 0.468$

(b) $v_w = 16.2 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.820$
   - ○: $(z/L)_{obs} = 0.440$

(c) $v_w = 8.7 \times 10^{-3}$ m/s
   - □: $(z/L)_{obs} = 0.821$
   - ○: $(z/L)_{obs} = 0.409$

Note: Solid line represents equation (IV-8).
FIGURE (IV-9): Postwithdrawal Drainage for a Herschel-Bulkley Fluid
(viscoelastic suspension - 1% solids)

(a) $v_w = 24.3 \times 10^{-3}$ m/s
   
   □ : $(z/L)_{obs} = 0.733$
   ○ : $(z/L)_{obs} = 0.576$

(b) $v_w = 16.2 \times 10^{-3}$ m/s
   
   □ : $(z/L)_{obs} = 0.778$
   ○ : $(z/L)_{obs} = 0.593$

(c) $v_w = 8.7 \times 10^{-3}$ m/s
   
   □ : $(z/L)_{obs} = 0.787$
   ○ : $(z/L)_{obs} = 0.582$

Note: Solid line represents equation (IV-9).
FIGURE (IV-10): Postwithdrawal Drainage for a Herschel-Bulkley Fluid,
(viscoelastic suspension - 2% solids)

(a) $v_w = 24.3 \times 10^{-3}$ m/s
   $\square: (z/L)_{obs} = 0.779$
   $\circ: (z/L)_{obs} = 0.445$

(b) $v_w = 16.2 \times 10^{-3}$ m/s
   $\square: (z/L)_{obs} = 0.778$
   $\circ: (z/L)_{obs} = 0.437$

(c) $v_w = 8.7 \times 10^{-3}$ m/s
   $\square: (z/L)_{obs} = 0.773$
   $\circ: (z/L)_{obs} = 0.437$

Note: Solid line represents equation (IV-9).
FIGURE (IV-11): Postwithdrawal Drainage for a Herschel-Bulkley Fluid (viscoelastic suspension - 3 % solids)

(a) \( v_w = 24.3 \times 10^{-3} \) m/s
   □ : \((z/L)_{obs} = 0.833\)
   ○ : \((z/L)_{obs} = 0.468\)

(b) \( v_w = 16.2 \times 10^{-3} \) m/s
   □ : \((z/L)_{obs} = 0.820\)
   ○ : \((z/L)_{obs} = 0.440\)

(c) \( v_w = 8.7 \times 10^{-3} \) m/s
   □ : \((z/L)_{obs} = 0.821\)
   ○ : \((z/L)_{obs} = 0.409\)

Note: Solid line represents equation (IV-9).
junction expressions for the De Kee and Hershel-Bulkley models are respectively reported as:

\[
\frac{(z/L)_j}{T_0^2 (1 - 2T_2 + T_3) + T_1 (T_2 - 1)}
\]  
\[(IV-10)\]

where

\[T_0 = h_0 \sqrt{\frac{\rho g_z}{\eta_1 v_w}}\]  
\[(IV-11)\]

\[T_1 = \frac{h_0 \tau_0}{\eta_1 v_w}\]  
\[(IV-12)\]

\[T_2 = \frac{\tau_0 t_1}{\eta_1}\]  
\[(IV-13)\]

\[T_3 = \frac{h_0 \rho g_z t_1}{\eta_1}\]  
\[(IV-14)\]

and

\[
\frac{(z/L)_j}{n+1} = T_0^n - T_1^n
\]  
\[(IV-15)\]

where

\[T_0 = h_0 \left(\frac{\rho g_z}{m v_w^n}\right)^{1/(n+1)}\]  
\[(IV-16)\]

\[T_1 = \frac{h_0 (\tau_0)^{1/n}}{v_w (m)}\]  
\[(IV-17)\]

Film thickness predictions at the onset of drainage were
obtained from linear extrapolation of low drain time data. A summary of all pertinent results using the De Kee model expression for three viscoelastic suspensions appear in Tables (IV-5,6 and 7). The analogous results, using the Herschel-Bulkley expression appear in Tables (IV-8,9 and 10). In general, transition drain times for the De Kee model were both observed and predicted to be longer than those for the Herschel-Bulkley model. In addition, predictions based upon the De Kee model were in better agreement with experimentally observed values. Film thickness predictions at the onset of drainage were typically lower using the Herschel-Bulkley model. Also, transition drain times were much lower than those experimentally observed in all cases. This is unexpected since the transition drain time is inversely proportional to initial film thickness. To explain this situation, one must reconsider the determination of the datum thickness. Attempts to fit the experimental data are strongly affected by the inability of the Herschel-Bulkley postwithdrawal drainage expression to handle long drain times (or low shear rate phenomena). If one shifts the datum downward such that better agreement with long drain time data is obtained, the experimental transition drain time will increase rapidly (as the slope of the film thickness curve becomes smaller) and the theoretical transition drain time will also increase slightly (since the extrapolated initial film thickness will be lowered slightly). By shifting the datum thickness upward such that better short drain time agreement is obtained, the experimental transition drain time will decrease
<table>
<thead>
<tr>
<th>Withdrawal Speed ( v_w \times 10^3 ) [m/s]</th>
<th>Observation Plane ( \eta/\epsilon )</th>
<th>Critical Junction ( \eta/\epsilon )</th>
<th>Dim. Parameters</th>
<th>Initial Film Thickness ( h_0 \times 10^6 ) [m]</th>
<th>Transition Drain Time ( \tau_0 ) (exp.)</th>
<th>Transition Drain Time ( \tau_0 ) (exp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3 ( \times 10^3 )</td>
<td>0.733</td>
<td>0.278</td>
<td>0.496 x 10^3</td>
<td>7.63 x 10^6</td>
<td>0.136</td>
<td>650</td>
</tr>
<tr>
<td>24.1 ( \times 10^3 )</td>
<td>0.576</td>
<td>0.262</td>
<td>0.486 x 10^3</td>
<td>7.36 x 10^6</td>
<td>0.133</td>
<td>633</td>
</tr>
<tr>
<td>16.2 ( \times 10^3 )</td>
<td>0.778</td>
<td>0.273</td>
<td>0.497 x 10^3</td>
<td>7.63 x 10^6</td>
<td>0.111</td>
<td>532</td>
</tr>
<tr>
<td>16.2 ( \times 10^3 )</td>
<td>0.593</td>
<td>0.240</td>
<td>0.467 x 10^3</td>
<td>7.63 x 10^6</td>
<td>0.105</td>
<td>500</td>
</tr>
<tr>
<td>8.7 ( \times 10^3 )</td>
<td>0.787</td>
<td>0.240</td>
<td>0.474 x 10^3</td>
<td>7.63 x 10^6</td>
<td>0.078</td>
<td>372</td>
</tr>
<tr>
<td>8.7 ( \times 10^3 )</td>
<td>0.582</td>
<td>0.238</td>
<td>0.472 x 10^3</td>
<td>7.63 x 10^6</td>
<td>0.077</td>
<td>370</td>
</tr>
</tbody>
</table>

**TABLE (IV-5)**: Summary of De Kee Fluid Transition Drain Time Calculations (viscoelastic suspension - 1% solids)

\[ \tau_0 = 4.2 \times 10^{-2} \text{ Pa} \]
\[ t_1 = 1.48 \times 10^{-2} \text{ s} \]
\[ \eta_1 = 8.28 \times 10^{-1} \text{ Pa\cdot s} \]
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_{obs}$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $T_0$, $T_1 \times 10^3$, $T_2 \times 10^6$, $T_3$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d$ (pred.) [s]</th>
<th>$t_d$ (exp.) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.779</td>
<td>0.229</td>
<td>0.456, 1.53, 8.11, 0.110</td>
<td>573</td>
<td>16.2</td>
<td>19.6</td>
</tr>
<tr>
<td>24.3</td>
<td>0.445</td>
<td>0.261</td>
<td>0.485, 1.63, 8.11, 0.117</td>
<td>610</td>
<td>4.8</td>
<td>7.3</td>
</tr>
<tr>
<td>16.2</td>
<td>0.778</td>
<td>0.306</td>
<td>0.528, 2.17, 8.11, 0.104</td>
<td>543</td>
<td>14.9</td>
<td>15.6</td>
</tr>
<tr>
<td>16.2</td>
<td>0.437</td>
<td>0.236</td>
<td>0.466, 1.92, 8.11, 0.092</td>
<td>479</td>
<td>8.2</td>
<td>12.3</td>
</tr>
<tr>
<td>8.7</td>
<td>0.773</td>
<td>0.306</td>
<td>0.536, 3.01, 8.11, 0.077</td>
<td>403</td>
<td>25.7</td>
<td>38.9</td>
</tr>
<tr>
<td>8.7</td>
<td>0.437</td>
<td>0.235</td>
<td>0.471, 2.65, 8.11, 0.066</td>
<td>355</td>
<td>14.9</td>
<td>15.7</td>
</tr>
</tbody>
</table>

**TABLE (IV-6):** Summary of De Kee Fluid Transition Drain Time Calculations (viscoelastic suspension - 2% solids)

$T_0 = 5.0 \times 10^{-2}$ Pa

$\dot{t}_1 = 1.25 \times 10^{-2}$ s

$\eta_1 = 7.67 \times 10^{-1}$ Pa·s
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_{obs}$</th>
<th>Critical Junction $(z/L)_{j}$</th>
<th>Dim. Parameters</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain $t_d^*$ Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.833</td>
<td>0.377</td>
<td>0.573 3.52 1.69 0.158</td>
<td>737</td>
<td>8.0 6.5</td>
</tr>
<tr>
<td>24.3</td>
<td>0.468</td>
<td>0.286</td>
<td>0.504 3.09 1.69 0.139</td>
<td>648</td>
<td>4.4 6.1</td>
</tr>
<tr>
<td>16.2</td>
<td>0.820</td>
<td>0.337</td>
<td>0.551 4.14 1.69 0.124</td>
<td>578</td>
<td>12.7 11.2</td>
</tr>
<tr>
<td>16.2</td>
<td>0.440</td>
<td>0.269</td>
<td>0.496 3.73 1.69 0.111</td>
<td>520</td>
<td>6.1 7.2</td>
</tr>
<tr>
<td>8.7</td>
<td>0.821</td>
<td>0.377</td>
<td>0.591 6.07 1.69 0.097</td>
<td>455</td>
<td>18.6 29.3</td>
</tr>
<tr>
<td>8.7</td>
<td>0.409</td>
<td>0.235</td>
<td>0.472 4.85 1.69 0.077</td>
<td>363</td>
<td>12.0 12.7</td>
</tr>
</tbody>
</table>

**TABLE (IV-7): Summary of De Kee Fluid Transition Drain Time Calculations**

- $\tau_0 = 9.4 \times 10^{-2}$ Pa
- $t_1 = 1.46 \times 10^{-2}$ s
- $\eta_1 = 8.10 \times 10^{-1}$ Pa.s
<table>
<thead>
<tr>
<th>Withdrawal Speed $\nu_w \times 10^3$ [m/s]</th>
<th>Observation Plane $\frac{z}{L}_{obs}$</th>
<th>Critical Junction $\frac{z}{L}_c$</th>
<th>Dim. Parameters $T_0$, $T_1 \times 10^6$</th>
<th>Initial Film Thickness $h_0' \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^\ast$ (pred.), $t_d^{**}$ (exp.) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.773</td>
<td>0.371</td>
<td>0.583, 5.92</td>
<td>628</td>
<td>7.1, 8.4</td>
</tr>
<tr>
<td>24.3</td>
<td>0.577</td>
<td>0.357</td>
<td>0.571, 5.79</td>
<td>615</td>
<td>4.2, 6.3</td>
</tr>
<tr>
<td>16.2</td>
<td>0.778</td>
<td>0.366</td>
<td>0.580, 7.34</td>
<td>520</td>
<td>10.3, 14.7</td>
</tr>
<tr>
<td>16.2</td>
<td>0.393</td>
<td>0.328</td>
<td>0.546, 6.91</td>
<td>490</td>
<td>8.1, 12.3</td>
</tr>
<tr>
<td>8.7</td>
<td>0.787</td>
<td>0.323</td>
<td>0.542, 9.64</td>
<td>366</td>
<td>25.7, 39.2</td>
</tr>
<tr>
<td>8.7</td>
<td>0.582</td>
<td>0.322</td>
<td>0.541, 9.63</td>
<td>366</td>
<td>14.4, 21.3</td>
</tr>
</tbody>
</table>

**TABLE IV-8:** Summary of Herschel-Bulkley Fluid Transition Drain Time Calculations (viscoelastic suspension - 1% solids)

- $\tau_0 = 4.1 \times 10^{-2}$ Pa
- $m = 9.48 \times 10^{-1}$
- $n = 8.30 \times 10^{-1}$
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_\text{obs}$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $T_0$, $T_1 \times 10^3$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d^{<strong>}$ (pred.) $t_d^{</strong>}$ (exp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.779</td>
<td>0.301</td>
<td>0.531, 0.94</td>
<td>572</td>
<td>10.7, 12.4</td>
</tr>
<tr>
<td>24.3</td>
<td>0.445</td>
<td>0.325</td>
<td>0.553, 0.98</td>
<td>596</td>
<td>2.5, 4.2</td>
</tr>
<tr>
<td>16.2</td>
<td>0.778</td>
<td>0.371</td>
<td>0.593, 1.30</td>
<td>529</td>
<td>10.6, 13.4</td>
</tr>
<tr>
<td>16.2</td>
<td>0.437</td>
<td>0.302</td>
<td>0.532, 1.17</td>
<td>474</td>
<td>4.3, 7.7</td>
</tr>
<tr>
<td>8.7</td>
<td>0.773</td>
<td>0.375</td>
<td>0.597, 1.83</td>
<td>397</td>
<td>17.9, 32.1</td>
</tr>
<tr>
<td>8.7</td>
<td>0.437</td>
<td>0.296</td>
<td>0.527, 1.61</td>
<td>351</td>
<td>8.2, 11.6</td>
</tr>
</tbody>
</table>

**TABLE (IV-9): Summary of Herschel-Bulkley Fluid Transition Drain Time Calculations**

(viscous fluid - 2% solids)

- $T_0 = 4.7 \times 10^{-2}$ Pa
- $M = 8.11 \times 10^{-1}$
- $N = 8.83 \times 10^{-1}$
<table>
<thead>
<tr>
<th>Withdrawal Speed $v_w \times 10^3$ [m/s]</th>
<th>Observation Plane $(z/L)_o$</th>
<th>Critical Junction $(z/L)_j$</th>
<th>Dim. Parameters $T_0$ $T_1 \times 10^3$</th>
<th>Initial Film Thickness $h_0 \times 10^6$ [m]</th>
<th>Transition Drain Time $t_d$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.3</td>
<td>0.833</td>
<td>0.446</td>
<td>0.651 1.88</td>
<td>704</td>
<td>5.7</td>
</tr>
<tr>
<td>24.3</td>
<td>0.468</td>
<td>0.366</td>
<td>0.586 1.69</td>
<td>633</td>
<td>1.9</td>
</tr>
<tr>
<td>16.2</td>
<td>0.820</td>
<td>0.415</td>
<td>0.627 2.26</td>
<td>562</td>
<td>8.6</td>
</tr>
<tr>
<td>16.2</td>
<td>0.440</td>
<td>0.345</td>
<td>0.568 2.04</td>
<td>510</td>
<td>2.6</td>
</tr>
<tr>
<td>8.7</td>
<td>0.821</td>
<td>0.458</td>
<td>0.662 3.33</td>
<td>446</td>
<td>12.6</td>
</tr>
<tr>
<td>8.7</td>
<td>0.409</td>
<td>0.309</td>
<td>0.537 2.70</td>
<td>361</td>
<td>5.2</td>
</tr>
</tbody>
</table>

*TABLE (IV-10): Summary of Herschel-Bulkley Fluid Transition Drain Time Calculations (viscoelastic suspension - 3% solids)*

\[
\begin{align*}
T_0 &= 8.6 \times 10^{-2} \text{ Pa} \\
\eta &= 8.92 \times 10^{-1} \\
\eta &= 8.58 \times 10^{-1} \\
\end{align*}
\]
more rapidly than the predicted transition drain time, which will also decrease. Shifting the datum upward however, will also bring initial film thickness estimates into better agreement with those obtained using the De Kee model. Hence, better agreement with equations (IV-1) and (IV-9) may be obtained by shifting the datum thickness upwards, thus placing more emphasis on the short drain time data. Clearly, the Herschel-Bulkley model must be considered to be notably less accurate at long drain times (or low shear rate phenomena) when compared to the De Kee model.

One final note regarding transition drain time expressions for the De Kee and Herschel-Bulkley models is concerned with their dependance on physical properties. The transition drain time is proportional to the withdrawal speed and yield stress for both cases. For the De Kee fluid, the transition drain time is proportional to \( \eta_1 \) and is inversely proportional to \( t_1 \). For the Herschel-Bulkley model, it is proportional to \( m \) and \( n \).

### D. Yield Stress Determination

The yield stress of each of the viscoelastic suspensions was determined using the five following methods:

1. Direct measurement.
2. Linear regression of rheological data.
   (Casson Model)
3. Non-linear regression of rheological data.
   (Herschel-Bulkley Model)
5. Interpretation of postwithdrawal drainage data.

Table (IV-11) summarizes the results of yield stress determination for each of the above methods. Each method will now be discussed in detail.

1. Direct Measurement

Yield stress measurements were made using the direct method developed by De Kee et al. [D-5]. The technique, which used the modified Fischer Surface Tension Analyzer to withdraw a flat plate from a fluid sample, was reported to possess the ability to measure yield stresses ranging from 2.0 to 90.0 Pa. It was also suggested that if the plate could be completely supported by the sample, a yield stress was present. In the measurement of much lower yield stresses (0.01 to 0.1 Pa), this phenomena was not observed. It is now suggested that while observation of support of the plate by the sample implies the presence of a yield stress, this condition is not necessary. Proof of this statement is related to the well known principle of flotation. In effect, an object will float if the buoyant force, $F_b$, acting upwards is greater than the force of gravity, $F_g$, acting on the object itself. Thus,

$$F_g < F_b \quad \text{(object floats)} \quad \text{(IV-18)}$$

$$F_g > F_b \quad \text{(object sinks)} \quad \text{(IV-19)}$$

When a plate can be suspended in a sample, this observation
<table>
<thead>
<tr>
<th>Method of Determination</th>
<th>Fluid 1 % Solids</th>
<th>Fluid 2 % Solids</th>
<th>Fluid 3 % Solids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Measurement</td>
<td>0.042</td>
<td>0.049</td>
<td>0.087</td>
</tr>
<tr>
<td>Casson Model</td>
<td>0.091</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>Herschel-Bulkley Model</td>
<td>-0.22</td>
<td>-0.078</td>
<td>0.005</td>
</tr>
<tr>
<td>DeKee Model</td>
<td>0.022</td>
<td>0.092</td>
<td>0.19</td>
</tr>
<tr>
<td>Postwithdrawal Drainage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_w = 24.3 \times 10^{-3}$ m/s</td>
<td>0.42</td>
<td>1.22</td>
<td>0.44</td>
</tr>
<tr>
<td>$v_w = 16.2 \times 10^{-3}$ m/s</td>
<td>0.37</td>
<td>1.37</td>
<td>0.83</td>
</tr>
<tr>
<td>$v_w = 8.7 \times 10^{-3}$ m/s</td>
<td>0.51</td>
<td>0.33</td>
<td>0.82</td>
</tr>
<tr>
<td>Average</td>
<td>0.43</td>
<td>0.97</td>
<td>0.70</td>
</tr>
</tbody>
</table>

*TABLE (IV-11): Yield Stress [Pa] Determination Summary*
suggests simply that the buoyant force plus the product of the yield stress and plate area, \( \tau_0 \) and \( S \) respectively, is greater than the force of gravity acting on the plate. Thus, the high yield stress situation may be represented by:

\[
F_g \ll F_b + \tau_0 S \quad \text{(high yield stress)} \quad (IV-20)
\]

The corresponding low yield stress inequality is expressed by:

\[
F_g \gg F_b + \tau_0 S \quad \text{(low yield stress)} \quad (IV-21)
\]

When the low yield stress condition applies, calculation of the yield stress is simplified since the calculation of the buoyant force is no longer necessary. Specifically, when the high yield stress condition applies, De Kee et al. [D-5] reported that the yield stress could be calculated from:

\[
\tau_0 = \frac{(F_r + F_b)}{S} \quad (IV-22)
\]

For the low yield stress condition however, the yield stress may be calculated from:

\[
\tau_0 = \frac{F_r}{S} \quad (IV-23)
\]

Measurements made on the series of viscoelastic suspensions suggested that the solids concentration was directly proportional to the yield stress.
2. Analysis of Rheological Data

Yield stress estimations can be obtained from extrapolation of the shear stress-shear rate data. Using pseudo-linear regression, data can be fitted to the Casson model (equation (II-64a)); the results in Table (IV-11) are qualitatively correct, but the difference between the yield stress obtained by direct measurement is large.

Using non-linear regression, shear stress-shear rate data can be fitted to the three parameter Herschel-Bulkley model (equation (IV-5a)). Although the rheological data can be described well, yield stress predictions are negative in two cases which is physically impossible. Thus, yield stress predictions from the Herschel-Bulkley model are clearly unreliable.

Finally, the shear stress-shear rate data can be fit to the three parameter De Kee model (equation (IV-4a)) using non-linear regression. Using this model, the predicted yield stress values are qualitatively correct and better agreement with direct measurements is observed in two out of three cases when compared to other extrapolation techniques. It has previously been recognized by De Kee and Turcotte [D-4] that the De Kee model can advantageously be used to replace the commonly used Casson model and the Herschel-Bulkley model in establishing yield stress estimations using extrapolation of rheological data. Our results are in agreement with this conclusion. The actual predicted yield stress values are however quite different from those experimentally observed; it is suspected that this
difference is primarily due to the absence of shear stress-shear rate data at much lower shear rates. Appendix II summarizes all the results obtained from both linear and non-linear regression of rheological data.

3. Interpretation of Postwithdrawal Drainage Data

In order to obtain estimates of yield stress, postwithdrawal drainage data were analyzed using two different approaches. Before outlining each approach in detail, note that experimental data only establishes the change in film thickness, \( \delta \), between two successive time intervals. Since the previous postwithdrawal drainage results were established using estimates for the yield stress (based upon non-linear regression of rheological data) absolute film thickness measurements could not be used. Further, the lack of an explicit film thickness expression necessitated the use of trial and error methods to establish yield stress values.

The first approach attempted to solve the following expression:

\[
\delta = h_a - h_b \quad \text{(IV-24)}
\]

where

\[
h_a = h \bigg| _{t = t_a, \tau_0 \text{ (est.)}} \quad \text{(IV-25)}
\]

\[
h_b = h \bigg| _{t = t_b, \tau_0 \text{ (est.)}} \quad \text{(IV-26)}
\]

(Note: \( t_a < t_b \))
Note that the film thickness, $h$, was obtained by solving the cubic expression for film thickness (equation (IV-8)) based upon the De Kee model. Analysis using the postwithdrawal drainage expression developed for the Herschel-Bulkley model was not carried out due to its failure to adequately characterize experimental results, especially at long drain times. The viscosity parameters which were used were obtained from the semi-logarithmic representation of the viscosity-shear rate data. The next approach manipulated the cubic expression for film thickness by expressing it as a quadratic expression in terms of yield stress as follows:

$$
\tau_0 = \rho g z h + \frac{\eta_1}{2t_1} \left( 1 - \sqrt{1 + \frac{4zt_1}{ht}} \right)
$$

(IV-27)

To obtain an estimate for the yield stress, the following expression was solved:

$$
\rho g z \delta = \frac{\eta_1}{2t_1} \left( \sqrt{1 + \frac{4zt_1}{h_a t_a}} - \sqrt{1 + \frac{4zt_1}{h_b t_b}} \right)
$$

(IV-28)

Preliminary attempts to solve equations (IV-24) and (IV-28) considered small changes in film thickness which occurred over very short periods of time. These investigations led to situations in which it became mathematically impossible to generate estimates for the yield stress. Specifically, the right hand side of equations (IV-24) and (IV-28) were minimized at $T_0 = 0$. If the experimental change in thickness was less than this quantity, no realistic yield stress estimate could be
obtained. To overcome this problem, the largest drain time range was used. Unfortunately, deviations of short drain time data from equation (IV-8) could only be predicted if the yield stress was known a priori by using equations (IV-1) and (IV-10). Thus, the maximum drain time range was successively reduced (by increasing \( t_q \)) until it again became impossible to determine the yield stress. In general, yield stress estimates decreased in magnitude as the drain time range decreased. Also, as the drain time range decreased, the change in successive yield stress estimations decreased. Nearly identical results were obtained using equations (IV-24) and (IV-28). An average of the two techniques for 2 or 3 readings corresponding to the shortest drain time ranges were reported for the three withdrawal speeds studied. It was observed that withdrawal speed had no influence on yield stress estimates; in addition, the results are not quantitatively correct and the difference between direct measurements is considerable.

To explain the failure of the postwithdrawal drainage data to estimate yield stress, it is useful to consider the shear stress distribution in the film which is reported as:

\[
\tau_{xz} = -\rho g_z (h - x) \tag{IV-29}
\]

Since the above expression represents a straight line, the magnitude of the average shear stress may be expressed as:

\[
|\tau_{xz}| = \frac{\rho g_z h}{2} \tag{IV-30}
\]
To obtain some idea of the influence of the yield stress, the average shear stress was calculated for the largest and smallest film thickness measurements obtained for each viscoelastic suspension. The fraction of yield stress determined by direct measurement divided by the average yield stress was also established. The results of these calculations are presented in Table (IV-12). Examination of the calculations suggest that yield stress phenomena are not manifested in the postwithdrawal drainage experiments which were performed. In particular, it should be noted that the yield stress is typically at least one order of magnitude lower than the average shear stress after drain times exceeding four minutes. Two important observations may be obtained from the results presented in Table (IV-12). First, when considering a number of viscoelastic suspensions, as the yield stress increases, its relative importance with respect to the shear rate increases. Thus, one would expect that a fluid possessing a high yield stress would influence postwithdrawal drainage results more than a similar fluid possessing a low yield stress. Additionally, it is noted that the size of the "plug flow" region of the draining film will increase as the yield stress of the fluid increases. Second, the magnitude of the shear stress decelerates rapidly as the film drains. In all cases, the yield stress-average shear stress ratio increased notably over the drain times studied. This suggests that after long drain times, the shear stress will be less than the yield stress in much of the film. As a result,
<table>
<thead>
<tr>
<th>Yield Stress $\tau_0 \times 10^2$ [Pa]</th>
<th>Film Thickness $h \times 10^5$ [m]</th>
<th>Average Shear Stress $\gamma$ [Pa]</th>
<th>Yield/Average Shear Stress [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>632 (max)</td>
<td>3.7</td>
<td>1.1</td>
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<td>4.8</td>
<td>565 (max)</td>
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<td>123 (min)</td>
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<td>8.7</td>
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<tr>
<td></td>
<td>119 (min)</td>
<td>0.71</td>
<td>12.3</td>
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</tbody>
</table>

TABLE (IV-12): Yield Stress-Average Shear Stress Comparison
the yield stress phenomena will be expected to influence postwithdrawal drainage behavior. Thus, it is suggested that failure of the postwithdrawal drainage data to produce satisfactory estimates for the yield stress is due to the fact that yield stress phenomena are not manifested by the viscoelastic suspensions prepared over the range of drain times observed.
V. CONCLUSIONS

An experimental method, based upon laser interferometry, was developed to investigate transient film thickness behavior for postwithdrawal drainage situations. Experimental verification of the new method was accomplished by studying the postwithdrawal drainage behavior of a Newtonian fluid. Postwithdrawal drainage expressions, developed for the Ellis fluid (viscoelastic solution) and De Kee fluid model (viscoelastic suspension) were also experimentally verified. A postwithdrawal drainage expression, developed for the Herschel-Bulkley model, was shown to be inaccurate in predicting long drain time results for viscoelastic suspensions.

Deviations below theory were noted in all experimental investigations at short drain times. To account for this behavior, it has been suggested that during the unsteady withdrawal stage of postwithdrawal drainage, a transition to continuous withdrawal occurs. This transition, which may or may not have been complete in all cases, had the effect of lowering the film thickness at the onset of drainage. A general theory has been developed to predict the time required for agreement of postwithdrawal drainage expressions with experimental data. This transition drain time, has been experimentally verified for Newtonian, Ellis and De Kee fluid models. The transition drain time, shown to be proportional to the withdrawal time and observation plane, \((z/L)_{obs}\), strongly supports the suggestion that a transition from unsteady to continuous withdrawal may be
occuring. The transition drain time was also shown to be inversely proportional to the critical junction, \((z/L)\), which was uniquely defined for each class of fluids.

Postwithdrawal drainage data, used to predict the yield stress of viscoelastic suspensions, was shown to be inferior when compared with direct measurement techniques and regression analysis of rheological data. Non-linear regression based upon the De Kee fluid model proved to be the most accurate method to predict yield stress. The failure of postwithdrawal drainage data in the prediction of yield stress was primarily due to the high shear stresses present in the draining film over the range of drain times studied. Additionally, the viscoelastic fluids used in experimental investigations possessed very low yield stresses.
VI. RECOMMENDATIONS FOR FURTHER WORK

i) The most significant drawback associated with the interferometric measuring system is its inability to make absolute film thickness measurements. To make such measurements, a camera placed at right angles to the vertical glass plate may be employed. Modifications to the existing apparatus will include cutting a viewing hole through one of the 1/2" steel faceplates and beveling the edges of the vertical glass plate to eliminate edge effects. To determine the time at which photographs are taken, a chronometer (activated by starting the plate withdrawal motor) should be placed within the viewing angle of the camera.

ii) One major area which still requires work is in the development of viscoelastic suspensions which demonstrate suitable physical properties. In particular, suspensions must have a measurable yield stress, possess some degree of stability and still permit passage of light without excessive refraction. The ideal way to achieve this combination is to find a solid and liquid which possess nearly identical indices of refraction and densities. The former condition will allow high solid concentrations (which will in turn lead to higher yield stresses) and the latter condition should ensure relatively good stability.

iii) Once suitable viscoelastic suspensions have been developed, post-withdrawal drainage experiments covering longer drain times (up to 10 minutes) should be carried out for fluids possessing
high yield stresses. Subsequent analysis of the data should then be performed to establish whether yield stress predictions can be made using this technique.

iv) Interferometric data should be collected and analyzed for the withdrawal stage of postwithdrawal drainage to investigate the transition from unsteady to continuous withdrawal. To accomplish this, the experimental apparatus will have to be modified in order to further minimize vibrations. Such modifications could include adding weight to the existing optical component support system or using some type of shock absorber on the plate withdrawal support system.

v) The capacity of the experimental apparatus remains greatly unexplored; out of 40 possible withdrawal speeds, only 3 have been examined. Using the current software, wave formation which is expected at higher withdrawal speeds may be investigated during both the withdrawal and drainage stages of postwithdrawal drainage.

vi) Modification of the solid support could lead to the investigation of postwithdrawal drainage from different geometries (i.e. cylinders) for many classes of fluids. Previous studies have been limited to drainage from flat plates (except for drainage from a cylinder for a Newtonian fluid) and the postwithdrawal drainage of complex geometries may have more practical applications in industry.

vii) Postremoval drainage, also important in industry, may be investigated by extending the plate guides in the sample container.
APPENDIX I
Experimental Determination of Plate Withdrawal Speed
To begin, the experimental technique used to establish the plate withdrawal speed will be summarized.

A number of opaque strips were fastened to the glass plate; the strips were not equally spaced but were of equal width (0.00635 m). It should be noted that the plate was inserted into the plate guides mounted on the inside of the rectangular box (see Figure (III-4)). During all plate withdrawal speed experiments, the rectangular box was filled with the Newtonian fluid which was subsequently used in postwithdrawal drainage experiments. It was assumed that the additional force required to withdraw the glass plate from a viscoelastic suspension would have a negligible effect on the speed since the motor was substantially over-sized.

Next, the interferometer was adjusted so that the intensity of only one arm was being measured by the photodetector. Using the advanced data acquisition program, a record similar to the interferograms obtained in the postwithdrawal drainage experiments was generated as the opaque strips (mounted on the glass plate) intermittently obscured the laser beam.

Since the photodetector sampling rate can be established using the computer program, the plate velocity may be determined by counting the number of readings which are taken during the time the laser beam is blocked by each opaque strip.

Typically, the pulse 'shadows' generated were tapered. That is, more samples could be counted at a level of 10% partial obscurity as opposed to a level of 90% partial obscurity. Using this criteria, it will be suggested that the withdrawal speed is
bounded by:

\[ \frac{\phi w}{a} = \frac{v_w}{b} = \frac{\phi w}{b} \]  

(App. I-1)

where  

- \( a \) = no. of samples above 90% obscured intensity 
- \( b \) = no. of samples above 10% obscured intensity 
- \( w \) = width of opaque strip (on plate) [m] 
- \( \phi \) = sampling rate [1/s]

The average withdrawal speed is reported as:

\[ v_{w_{avg}} = \frac{\phi w}{2} \left( \frac{1}{a} + \frac{1}{b} \right) \]  

(App. I-2)

Since there are 10 dial settings and 4 gear selections, a total of 40 different withdrawal speeds are obtainable. In this work, only three withdrawal speeds were used, pertaining to a gear selection of 1 (largest diameter gear) and dial selections of 2, 3 and 4. Using three different sampling rates for each dial setting/gear number, an investigation was carried out to establish:

i) The reproducibility of the plate withdrawal speed.

ii) The time required for the plate to reach constant velocity.

A summary of the results obtained are presented in Tables (App. I-1) to (App. I-9). Also, the average plate velocity was plotted with respect to elapsed time; the results are illustrated in Figure (App. I-1). The plate withdrawal speed was modelled using the following expression:
\[ v_w = v_0 \left( 1 - e^{-t/t_0} \right) \]  

\text{(App. I-3)}

where \( v_0 \) = constant plate speed [m/s]  
\( t_0 \) = time constant [s]

The above expression was used since high speed withdrawal rates exhibited a noticeable time to reach constant velocity in preliminary investigations. Further, since the plate was being withdrawn for a very short time, the period required for the plate to reach constant velocity must be accounted for to accurately determine the time at which withdrawal stops and drainage begins. Using non-linear regression, the constants in equation (App. I-3) were determined and are summarized in the figure caption for Figure (App. I-1).

The computer programs used to establish the plate withdrawal speed and perform the non-linear regression analysis are available in a separate lab manual.
<table>
<thead>
<tr>
<th>Opaque Strip Number</th>
<th>Plate Withdrawal Velocity ($x 10^3$) [m/s]</th>
<th>Time Range [min:sec]</th>
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</thead>
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<tr>
<td></td>
<td>($V_w$)$_{\text{min}}$</td>
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**TABLE (App. I-2):** Plate Withdrawal Speed Calibration

- **Dial Setting**: 4
- **Gear Number**: 1
- **Sampling Rate**: 500 (readings/second)
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<th>Time Range [min:sec]</th>
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**TABLE (App. I-1) : Plate Withdrawal Speed Calibration**

- **Dial Setting**: 4
- **Gear Number**: 1
- **Sampling Rate**: 100 (readings/second)
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<th>Opaque Strip Number</th>
<th>Plate Withdrawal Velocity (x 10^3) [m/s]</th>
<th>Time Range [min:sec]</th>
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**TABLE (App. I-3) : Plate Withdrawal Speed Calibration**

- **Dial Setting** : 4
- **Gear Number** : 1
- **Sampling Rate** : 1000 (readings/second)
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<th>Plate Withdrawal Velocity (x $10^3$) [m/s]</th>
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**TABLE (App. I-4):** Plate Withdrawal Speed Calibration

- **Dial Setting:** 3
- **Gear Number:** 1
- **Sampling Rate:** 100 (readings/second)
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<th>Plate Withdrawal Velocity ( \times 10^3 ) [m/s]</th>
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**TABLE (App. I-5): Plate Withdrawal Speed Calibration**

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<th>Plate Withdrawal Velocity (x 10^3) [m/s]</th>
<th>Time Range [min:sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((V_w)_{min})</td>
<td>((V_w)_{avg})</td>
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<tr>
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<td>15.6</td>
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</table>

**TABLE (App. I-6) : Plate Withdrawal Speed Calibration**

- **Dial Setting** : 3
- **Gear Number** : 1
- **Sampling Rate** : 625 (readings/second)
<table>
<thead>
<tr>
<th>Opaque Strip Number</th>
<th>Plate Withdrawal Velocity (x 10^3) [m/s]</th>
<th>Time Range [min:sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(V_w)min</td>
<td>(V_w)avg</td>
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<tr>
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<td>7.5</td>
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<tr>
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<td>7.2</td>
</tr>
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</table>

**TABLE (App. I-7): Plate Withdrawal Speed Calibration**

- **Dial Setting**: 2
- **Gear Number**: 1
- **Sampling Rate**: 100 (readings/second)
<table>
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<tr>
<th>Opaque Strip Number</th>
<th>( (V_w)_{\text{min}} )</th>
<th>( (V_w)_{\text{avg}} )</th>
<th>( (V_w)_{\text{max}} )</th>
<th>( t_{\text{start}} )</th>
<th>( t_{\text{end}} )</th>
<th>( t_{\text{mid}} )</th>
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**TABLE (App. I-8):** Plate Withdrawal Speed Calibration

- **Dial Setting:** 2
- **Gear Number:** 1
- **Sampling Rate:** 200 (readings/second)
<table>
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<tr>
<th>Opaque Strip Number</th>
<th>Plate Withdrawal Velocity (x $10^3$) [m/s]</th>
<th>Time Range [min:sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($V_{w_{min}}$) ($V_{w_{avg}}$) ($V_{w_{max}}$)</td>
<td>$t_{start}$</td>
</tr>
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</tr>
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<td>0:19.82</td>
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<td>0:20.77</td>
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<td>0:22.90</td>
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</table>

**TABLE (App. I-9):** Plate Withdrawal Speed Calibration

- **Dial Setting**: 2
- **Gear Number**: 1
- **Sampling Rate**: 500 (readings/second)
FIGURE (App. I-1) : Plate Withdrawal Speed vs. Elapsed Time

(a) Dial Setting
   Gear Number : 1
   Constant Plate Speed [m/s] : 0.0243
   Time Constant [s] : 1.14

(b) Dial Setting
   Gear Number : 3
   Constant Plate Speed [m/s] : 0.0161
   Time Constant [s] : 1.28

(c) Dial Setting
   Gear Number : 2
   Constant Plate Speed [m/s] : 0.00868
   Time Constant [s] : 1.66
APPENDIX II

Summary of Physical Properties and
Results of Non-Linear Regression of
Rheological Data
<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Surface Tension $\sigma \times 10^3$ (N/m)</th>
<th>Refractive Index $n_D$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{H}_2\text{O}$</td>
<td>998</td>
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<td>1.3330</td>
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<td>Glycerine</td>
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<td>55.1</td>
<td>-</td>
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<td>Glyc./$\text{H}_2\text{O}$</td>
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<td>54.6</td>
<td>1.4550</td>
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<tr>
<td>PAA/$\text{H}_2\text{O}$</td>
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<td>68.9</td>
<td>1.4240</td>
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<td>Visc. Susp. 1% Solids</td>
<td>1172</td>
<td>-</td>
<td>1.4385</td>
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<tr>
<td>2% Solids</td>
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<td>3% Solids</td>
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<td>-</td>
<td>1.4379</td>
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</table>

*TABLE (APP. II-1): Summary of Physical Properties*
<table>
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<tr>
<th>Parameters</th>
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<th>3</th>
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<tr>
<td>( \eta_1 \times 10^1 )</td>
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<td>7.41</td>
<td>8.28</td>
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</tbody>
</table>

**Rheological Data**

| Shear Rate \([s^{-1}]\) | Shear Stress \((\text{exp.}) [\text{Pa}]\) | Shear Stress \((\text{pred.})[\text{Pa}]\) |
|------------------------|--------------------------------|
| 0.629                  | 0.61                           | 0.53 |
| 0.855                  | 0.81                           | 0.72 |
| 1.05                   | 1.03                           | 0.88 |
| 1.16                   | 1.10                           | 0.98 |
| 1.43                   | 1.30                           | 1.20 |
| 1.58                   | 1.42                           | 1.32 |
| 1.94                   | 1.79                           | 1.61 |
| 2.17                   | 1.83                           | 1.80 |
| 2.63                   | 2.24                           | 2.17 |
| 2.93                   | 2.40                           | 2.40 |
| 3.58                   | 3.02                           | 2.91 |
| 3.97                   | 3.13                           | 3.20 |
| 4.87                   | 3.74                           | 3.89 |
| 5.41                   | 4.02                           | 4.27 |
| 6.62                   | 4.93                           | 5.13 |
| 7.35                   | 5.12                           | 5.63 |
| 9.07                   | 6.05                           | 6.27 |
| 9.96                   | 6.51                           | 7.33 |
| 12.29                  | 7.68                           | 8.73 |
| 13.55                  | 8.37                           | 9.44 |
| 16.63                  | 9.89                           | 11.05 |
| 18.41                  | 10.66                          | 11.91 |
| 22.70                  | 13.80                          | 13.75 |
| 25.00                  | 13.48                          | 14.62 |
| 30.80                  | 15.95                          | 16.49 |
| 34.00                  | 16.97                          | 17.33 |
| 41.70                  | 19.69                          | 18.89 |
| 56.7                   | 25.52                          | 20.43 |

**Statistical Summary**

- Average Error: 0.61, 0.31, 0.51
- Average Variance: 1.28, 0.44, 1.17
- Standard Error: 0.09, 0.05, 0.08
- Correlation Coefficient: 0.98, 0.99

**Table (App. II-2):** Summary of Non-linear Regression of Rheological Data based upon the Do Kee model (1 % solids)
<table>
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<td>2</td>
</tr>
<tr>
<td>$\eta_1 \times 10^1$</td>
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</tr>
</tbody>
</table>

### Rheological Data

<table>
<thead>
<tr>
<th>Shear Rate [s$^{-1}$]</th>
<th>Shear Stress (exp.) [Pa]</th>
<th>Shear Stress (pred.) [Pa]</th>
</tr>
</thead>
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<td>0.340</td>
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<td>0.855</td>
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<tr>
<td>46.3</td>
<td>20.52</td>
<td>0.84</td>
</tr>
</tbody>
</table>

### Statistical Summary

- Average Error: 0.43
- Average Variance: 0.54
- Standard Error: 0.08
- Correlation Coefficient: 0.99

**TABLE (App. II-3): Summary of Non-linear Regression of Rheological Data**

*Based upon the De Kee Model (2% solids)*
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_0 \times 10^1 )</td>
<td>1</td>
</tr>
<tr>
<td>( t_1 \times 10^2 )</td>
<td>2</td>
</tr>
<tr>
<td>( \eta_1 \times 10^1 )</td>
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</tr>
<tr>
<td>0.00</td>
<td>1.9</td>
</tr>
<tr>
<td>1.64</td>
<td>1.12</td>
</tr>
<tr>
<td>9.04</td>
<td>7.45</td>
</tr>
<tr>
<td></td>
<td>8.10</td>
</tr>
</tbody>
</table>

Rheological Data

<table>
<thead>
<tr>
<th>Shear Rate ([s^{-1}])</th>
<th>Shear Stress (\text{exp.}) ([\text{Pa}])</th>
<th>Shear Stress (\text{pred.}) ([\text{Pa}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.340</td>
<td>0.36</td>
<td>0.45</td>
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<tr>
<td>0.403</td>
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<tr>
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<td>1.79</td>
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<td>12.29</td>
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<td>3.77</td>
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</table>

**Statistical Summary**

<table>
<thead>
<tr>
<th></th>
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<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>Average Error</td>
<td>0.60</td>
<td>0.26</td>
<td>0.38</td>
</tr>
<tr>
<td>Average Variance</td>
<td>0.97</td>
<td>0.19</td>
<td>0.47</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.14</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>Correlation Coefficient</td>
<td>0.98</td>
<td>0.99</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE (App. II-4):** Summary of Non-linear Regression of Rheological Data based upon the De Kee model (3% solids)
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_0 \times 10^1$</td>
<td>1</td>
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<tr>
<td>$n$</td>
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<tr>
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<tr>
<td>0.987</td>
<td>1.17</td>
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</tbody>
</table>

Rheological Data

<table>
<thead>
<tr>
<th>Shear Rate [s$^{-1}$]</th>
<th>Shear Stress (exp.) [Pa]</th>
<th>Shear Stress (pred.) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.629</td>
<td>0.61</td>
<td>0.68</td>
</tr>
<tr>
<td>0.855</td>
<td>0.81</td>
<td>0.87</td>
</tr>
<tr>
<td>1.049</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>1.162</td>
<td>1.10</td>
<td>1.12</td>
</tr>
<tr>
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<td>1.32</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1.79</td>
<td>1.70</td>
</tr>
<tr>
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<tr>
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<td>26.82</td>
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</tbody>
</table>

Statistical Summary

Average Error | 0.19 | 0.06 | 0.22 |
Average Variance | 0.13 | 0.009 | 0.16 |
Standard Error | 0.04 | 0.02 | 0.04 |
Correlation Coefficient | 0.99 | 1.00 | 0.99 |

TABLE (App. II-5) : Summary of Non-linear Regression of Rheological Data based upon the Herschel-Bulkley Model (1% solids)
### Parameters

<table>
<thead>
<tr>
<th>Method</th>
<th>( \tau_0 \times 10^2 )</th>
<th>( n )</th>
<th>( m )</th>
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</thead>
<tbody>
<tr>
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<td>0.883</td>
<td>0.811</td>
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### Rheological Data

<table>
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<tr>
<th>Shear Rate ((s^{-1}))</th>
<th>Shear Stress ((\text{exp.}, \text{Pa}))</th>
<th>Shear Stress ((\text{pred.}, \text{Pa}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.340</td>
<td>0.34</td>
<td>0.34</td>
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<tr>
<td>0.463</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>0.529</td>
<td>0.49</td>
<td>0.56</td>
</tr>
<tr>
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<td>0.76</td>
</tr>
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<td>0.91</td>
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<tr>
<td>46.3</td>
<td>22.36</td>
<td>23.54</td>
</tr>
</tbody>
</table>

### Statistical Summary

- Average Error: 0.19, 0.10
- Average Variance: 0.13, 0.03
- Standard Error: 0.05, 0.05
- Correlation Coefficient: 0.99, 1.00

**TABLE (App. II-6): Summary of Non-linear Regression of Rheological Data**

Based upon the Herschel-Bulkley Model (2% solids)
### Parameters

<table>
<thead>
<tr>
<th></th>
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<th>( n )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
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### Rheological Data

<table>
<thead>
<tr>
<th>Shear Rate ([s^{-1}])</th>
<th>Shear Stress (\text{exp.} ) ([\text{Pa}])</th>
<th>Shear Stress (\text{pred.} ) ([\text{Pa}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.340</td>
<td>0.37</td>
<td>0.41</td>
</tr>
<tr>
<td>0.463</td>
<td>0.61</td>
<td>0.53</td>
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### Statistical Summary

<table>
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<tr>
<th></th>
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<th>( \text{Average Variance} )</th>
<th>( \text{Standard Error} )</th>
<th>( \text{Correlation Coefficient} )</th>
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**TABLE (App. II-7)**: Summary of Non-linear Regression of Rheological Data based upon the Herschel-Bulkley Model (3% solids)
Parameters

\[ \tau_0 \times 10^1 \quad 9.11 \]
\[ \mu_1 \times 10^1 \quad 5.14 \]

Rheological Data

<table>
<thead>
<tr>
<th>Shear Rate [s^{-1}]</th>
<th>Shear Stress [exp.] [Pa]</th>
<th>Shear Stress [prod.] [Pa]</th>
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</thead>
<tbody>
<tr>
<td>0.629</td>
<td>0.61</td>
<td>0.92</td>
</tr>
<tr>
<td>0.855</td>
<td>0.81</td>
<td>1.09</td>
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<tr>
<td>1.049</td>
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<tr>
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Statistical Summary

| Average Error | 0.39 |
| Average Variance | 0.32 |
| Standard Error | 0.14 |
| Correlation Coefficient | 0.99 |

TABLE (App. II-8): Summary of Pseudo-linear Regression of Rheological Data based upon the Casson Model (1% solids)
Parameters

\[ \tau_0 \times 10^1 \]

\[ \mu_1 \times 10^1 \]

Shear Rate \( \mu_s \) Shear Stress \( \tau_s \) \(\tau_{r} \) Shear Stress \( \tau_s \) \(\mu_1 \times 10^1 \) Shear Stress \( \tau_s \)

\begin{tabular}{ccc}
Shear Rate \( \mu_s \) & Shear Stress \( \tau_s \) & Shear Stress \( \tau_1 \) \\
0.340 & 0.32 & 0.51 \\
0.463 & 0.45 & 0.61 \\
0.629 & 0.49 & 0.73 \\
0.855 & 0.81 & 0.89 \\
1.049 & 0.94 & 1.02 \\
1.162 & 0.94 & 1.09 \\
1.425 & 1.14 & 1.26 \\
1.579 & 1.30 & 1.36 \\
1.938 & 1.68 & 1.59 \\
2.165 & 1.66 & 1.71 \\
2.63 & 2.01 & 1.99 \\
2.93 & 2.23 & 2.16 \\
3.58 & 2.86 & 2.53 \\
3.97 & 2.88 & 2.75 \\
4.87 & 3.86 & 3.25 \\
5.41 & 3.86 & 3.54 \\
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11.85 & 8.21 & 7.79 \\
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18.41 & 10.66 & 10.24 \\
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30.8 & 16.23 & 16.37 \\
34.0 & 17.44 & 17.93 \\
41.7 & 20.27 & 21.67 \\
46.3 & 22.36 & 23.89 \\
\end{tabular}

Statistical Summary

- Average Error: 0.32
- Average Variance: 0.21
- Standard Error: 0.17
- Correlation Coefficient: 0.99

TABLE (App. II-9): Summary of Pseudo-linear Regression of Rheological Data based upon the Casson Model (2% solids)
### Parameter

\[
\begin{align*}
\tau_0 &\times 10^1 & 1.2 \\
\mu_1 &\times 10^1 & 4.64
\end{align*}
\]

### Rheological Data

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### Statistical Summary

- Average Error: 0.22
- Average Variance: 0.12
- Standard Error: 0.12
- Correlation Coefficient: 0.99

**TABLE (App. II-10):** Summary of Pseudo-linear Regression of Rheological Data based upon the Casson Model (3% solids)
APPENDIX III
Summary of Postwithdrawal Drainage Experiments
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Average Absolute Error:

Standard Deviation

TABLE: App. III-1: Newtonian Postwithdrawal Drainage

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.024
Withdrawal Height, \( L \) [m] = 0.144
Viscosity, \( \mu \) [Pa s] = 0.19
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Average Absolute Error: 4.45
Standard Deviation: 3.03

**TABLE (App. III-2): Newtonian Postwithdrawal Drainage**

Additional Information:
- Withdrawal Velocity, \( v_w \) [m/s] = 0.0243
- Withdrawal Height, \( L \) [m] = 0.148
- Viscosity, \( \eta \) [Pa s] = 0.29
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Average Absolute Error: 0.55
Standard Deviation: 0.34

TABLE (PP. III-3): Newtonian Postwithdrawal Drainage

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0243
Withdrawal Height, \( L \) [m] = 0.092
Viscosity, \( \mu \) [Pa·s] = 0.12
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Average Absolute Error: 3.91
Standard Deviation: 7.16

TABLE (App. III-1): Newtonian Postwithdrawal Drainage

Additional Information:
Withdrawal Velocity, \( v \) [m/s] = 0.0162
Withdrawal Height, \( h \) [m] = 0.159
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Viscosity, \( \eta \) [Pa s] = 0.19
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Average Absolute Error

\[
\text{Standard Deviation} = 4.60 \text{, 1.94} \]

\[
\text{TABLE (App. III-5): Newtonian Postwithdrawal Drainage} \]

Additional Information:

Withdrawal Velocity, \(v_0\) [m/s] = 0.0162
withdrawal Height, \(L\) [m] = 0.145
Viscosity, \(\mu\) [Pa s] = 0.29

Withdrawal Time, \(t_w\) [s] = 10.2
Observation Height, \(z\) [m] = 0.065
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Average Absolute Error: 2.38
Standard Deviation: 3.44

**TABLE (App. III-6): Newtonian Postwithdrawal Drainage**

Additional Information:

Withdrawal Velocity, \(v_w\) [m/s] = 0.0167
Withdrawal Time, \(t_w\) [s] = 6.9
Withdrawal Height, \(L\) [m] = 0.091
Observation Height, \(\xi\) [m] = 0.055
Viscosity, \(\mu\) [Pa s] = 0.11
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Average Absolute Error: 1.38
Standard Deviation: 3.57

**TABLE (App. III-7): Newtonian Postwithdrawal Drainage**

**Additional Information:**

Withdrawal Velocity, \( \bar{v} \) [m/s] = 0.0067

Withdrawal Time, \( t_w \) [s] = 17.5

Withdrawal Height, \( h \) [m] = 0.138

Observation height, \( l \) [m] = 1.023

Viscosity, \( \eta \) [Pa s] = 0.19
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<th>EXPERIMENTAL [m]</th>
<th>ABSOLUTE ERROR [m]</th>
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Average Absolute Error: 1.88
Standard Deviation: 1.32

**TABLE (app. III-3): Newtonian Postwithdrawal Drainage**

Additional Information:
- Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
- Withdrawal Height, \( L \) [m] = 0.146
- Viscosity, \( \mu \) [Pa s] = 0.29
- Observation Height, \( z \) [m] = 0.065
- Withdrawal Time, \( t_w \) [s] = 18.4
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Average Absolute Error: 3.56
Standard Deviation: 5.27

TABLE (App. III-1): Newtonian Postwithdrawal Drainage

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 3.0087
Withdrawal Height, \( z \) [m] = 3.938
Viscosity, \( \mu \) [Pa·s] = 0.11
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Average Absolute Error 72.71
Standard Deviation 106.66

TABLE (App. III-10): Non-Newtonian Postwithdrawal Drainage
(Ellis Fluid)

Additional Information:

Withdrawal Velocity, \( v_w \) [m/s] = 0.0241
Withdrawal Time, \( t_w \) [s] = 6.2
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Average Absolute Error: 40.96
Standard Deviation: 81.78

TABLE (App. III-11): Non-Newtonian Postwithdrawal Drainage
(Eliss fluid)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0243
Withdrawal Height, \( L \) [m] = 0.121
Withdrawal Time, \( t_w \) [s] = 6.1
Observation Height, \( z \) [m] = 0.064
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Average Absolute Error: 37.40 6.25%
Standard Deviation: 50.09 6.31%

**TABLE (App. III-12): Non-Newtonian Postwithdrawal Drainage (Ellis Fluid)**

Additional Information:

- Withdrawal Velocity, \(v\) [m/s] = 0.0243
- Withdrawal Height, \(L\) [m] = 0.077
- Withdrawal Time, \(t_w\) [s] = 4.2
- Observation Height, \(z\) [m] = 0.05
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Average Absolute Error: 57.07
Standard Deviation: 100.85

TABLE (App. III-13): Non-Newtonian Post-withdrawal Drainage
(Fluid: Ellis Fluid)

Addition Information:
- Withdrawal Velocity, u [m/s] = 0.0162
- Withdrawal Height, L [m] = 0.126
- Withdrawal Time, t [s] = 9.0
- Observation Height, Z [m] = 0.023
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Average Absolute Error: 55.96

Standard Deviation: 92.95

**Table (App. III-14):** Non-Newtonian Postwithdrawal Drainage (Ellis Fluid)

Additional Information:
- Withdrawal Velocity, \( v_w \) [m/s] = 0.0162
- Withdrawal Height, \( L \) [m] = 0.125
- Observation Height, \( Z \) [m] = 0.064
- Withdrawal Time, \( T_w \) [s] = 9.0
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Average Absolute Error: 49.31
Standard Deviation: 67.44

**TABLE (App. III-15)**: Non-Newtonian Postwithdrawal Drainage (Ellis Fluid)

Additional Information:

- Withdrawal Velocity, \( v_w \) [m/s] = 0.0162
- Withdrawal Height, \( L \) [m] = 0.086
- Withdrawal Time, \( t_w \) [s] = 6.5
- Observation Height, \( z \) [m] = 0.053
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Average Absolute Error 14.57 2.61
Standard Deviation 22.28 3.61

TABLE (App. III-16): Non-Newtonian Postwithdrawal Drainage
(Ellis Fluid)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
Withdrawal Height, \( L \) [m] = 0.114
Withdrawal Time, \( t_w \) [s] = 14.7
Observation Height, \( z \) [m] = 0.023
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Average Absolute Error: 37.74
Standard Deviation: 73.85

**TABLE (App. III-17):** Non-Newtonian Post-Wrastow Drainage

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
Withdrawal Height, \( L \) [m] = 0.124
Withdrawal Time, \( t_w \) [s] = 15.9
Observation Height, \( z \) [m] = 0.064
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Average Absolute Error: 37.27
Standard Deviation: 58.49

TABLE (App. III-10): Non-Newtonian Postwithdrawl Drainage (Ellis Fluid)

Additional Information:

Withdrawal Velocity, $v_w$ [m/s] = 0.0087
Withdrawal Height, $L$ [m] = 0.083
Withdrawal Time, $t_w$ [s] = 10.3
Observation Height, $z$ [m] = 0.053
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Average Absolute Error: 14.89
Standard Deviation: 18.15

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Additional Information:

Withdrawal Velocity, \( \nu_w \) [m/s] = 0.0243
Withdrawal Height, \( H \) [m] = 0.139
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Average Absolute Error: 17.38 m
Standard Deviation: 26.76 m

TABLE (App. III-20): Non-Newtonian Postwithdrawal Drainage [De Kee model]
(Viscoelastic Suspension - 1% solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0241
Withdrawal Time, \( t_w \) [s] = 6.5
Withdrawal Height, \( L \) [m] = 0.132
Observation Height, \( z \) [m] = 0.030
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Average Absolute Error: 13.14
Standard Deviation: 18.37

**Table:** App. III-21: Non-Newtonian Post-withdrawal Drainage [De Koe model]
Viscoelastic Suspension - L (solids)

Additional Information:
Withdrawal Velocity, $v_w$ [m/s] = 0.0162
Withdrawal Time, $t_w$ [s] = 10.2
Observation Height, $z$ [m] = 0.059
Withdrawal Height, $L$ [m] = 0.145
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Average Absolute Error 9.01

Standard Deviation 2.11

Table: App. X-21: Non-Newtonian Postwithdrawal Drainage [De Kee model]
Viscoelastic Suspension - 1% solids

Additional Information:
Withdrawal Velocity, \( V_w \) [m/s] = 0.0162
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Withdrawal Time, \( t_w \) [s] = 9.6
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Average Absolute Error

10.57

Standard Deviation

16.15

**TABLE (App. III-23): Non-Newtonian Postwithdrawal Drainage (De Kee model)**

**Additional Information:**

**Withdrawal Velocity, \( v \) [m/s] = 0.0087**

**Withdrawal Time, \( t \) [s] = 17.8**

**Withdrawal Height, \( H \) [m] = 0.141**

**Observation Height, \( z \) [m] = 0.059**
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Average Absolute Error: 6.70
Standard Deviation: 9.39

TABLE (App. III-24): Non-Newtonian Postwithdrawal Drainage (De Veer model)
(Viscoelastic Suspension - L = solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
Withdrawal Time, \( t_w \) [s] = 17.8
Withdrawal Height, L [m] = 0.141
Observation Height, \( z \) [m] = 0.030
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Average Absolute Error 20.21
Standard Deviation 4.08

TABLE (App. III-25): Non-Newtonian Porous Withdrawal Drainage (De Kee model)
(Viscous-shear Suspension - 2% solids)

Additional Information:
Withdrawal Velocity, \(v_w\) [m/s] = 0.024
Withdrawal Time, \(t_w\) [s] = 6.7
Withdrawal Height, \(L\) [m] = 0.136
Observation Height, \(z\) [m] = 0.030
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Average Absolute Error: 6.60 [m], 1.71 [%]

Standard Deviation: 7.40 [m], 1.29 [%]

**Additional Information:**

Withdrawal Velocity, \( v_w \) [m/s] = 0.0243

Withdrawal Time, \( t_w \) [s] = 6.7

Withdrawal Height, \( L \) [m] = 0.137

Observation Height, \( Z \) [m] = 0.076
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Average Absolute Error: 11.29
Standard Deviation: 15.67

**TABLE (App. III-27):** Non-Newtonian Postwithdrawal Drainage [De Kee model] (Viscoelastic Suspension - 2% solids)

Additional Information:

Withdrawal Velocity, \( v_0 \) [m/s] = 0.0162
Withdrawal Height, \( L \) [m] = 0.135
Withdrawal Time, \( t_w \) [s] = 9.6
Observation Height, \( L \) [m] = 0.030
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<th>EXPERIMENTAL THICKNESS [m]</th>
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Average Absolute Error: 11.73
Standard Deviation: 2.96

TABLE (App. III-28) : Non-Newtonian Postwithdrowal Drainage (Le Fee code)
(Viscoelastic Suspension - 2 % solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0162
Withdrawal Time, \( t_w \) [s] = 9.6
Withdrawal Height, \( L \) [m] = 0.135
Observation Height, \( \tilde{L} \) [m] = 0.376
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Average Absolute Error
Standard Deviation

**TABLE (App. III-29): Non-Newtonian Postwithdrawal Drainage [De Kee model]**

(Viscouselastic Suspensions - 2 % solids)

Additional Information:
Withdrawal Velocity, \(V_w\) [m/s] = 0.0087
Withdrawal Height, \(L\) [m] = 0.132
Withdrawal Time, \(t\) [s] = 16.8
Observation Height, \(e\) [m] = 0.03
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Average Absolute Error: 5.77
Standard Deviation: 11.90

TABLE (App. III-30): Non-Newtonian Postwithdrawal Drainage [De Kee model]
(Viscoelastic Suspension = 2 % solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
Withdrawal Height, \( L \) [m] = 0.135
Withdrawal Time, \( t_w \) [s] = 17.1
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Average Absolute Error: 6.28
Standard Deviation: 6.26

**TABLE (App. III-31):** Non-Newtonian Postwithdrawal Drainage (De Kee model)
(Viscoelastic Suspension) % solids

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0241
Withdrawal Height, \( L \) [m] = 0.141
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Average Absolute Error 19.42 4.44

Standard Deviation 12.48 2.03

**TABLE (App. III-12):** Non-Newtonian Postwithdrawal Drainage (De Kee model)
(Viscoelastic Suspension - 2% solids)

Additional Information:
Withdrawal Velocity, \( \nu \) [m/s] = 0.0243
Withdrawal Height, \( L \) [m] = 0.132
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Average Absolute Error: 7.64
Standard Deviation: 10.37

TABLE (App. III-3): Non-Newtonian Postwithdrawal Drainage (C. Yee model)
(Viscouslastic Suspension - 3 % solids)

Additional Information:
Withdrawal Velocity, Vw [m/s] = 0.0162
Withdrawal Height, L [m] = 0.134
Withdrawal Time, t_w [s] = 9.5
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Average Absolute Error: 12.01
Standard Deviation: 11.15

**TABLE (App. III-34): Non-Newtonian Post withdrawal Drainage [De Kébié, 1951]**
(Viscoelastic Suspension - % solid)

Additional Information:
Withdrawal Velocity, \(v_w\) [m/s] = 0.0162
Withdrawal Time, \(t_w\) [s] = 8.8
Withdrawal Height, \(H\) [m] = 0.122
Observation Height, \(h\) [m] = 0.022
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Average Absolute Error

Standard Deviation

TABLE : App. (III-35) : Non-Newtonian Postwithdrawal drainage (De Kee model) (Viscoelastic Suspension = 3 kg solids)

Additional Information:

Withdrawal Velocity, \( V \) [m/s] = 0.0087
Withdrawal Height, \( L \) [m] = 0.127
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Average Absolute Error: 5.00
Standard Deviation: 3.34

TABLE (App. III-36): Non-Newtonian Postwithdrawal Drainage (De Kee model)
(Viscoelastic Suspension - 3 % solids)

Additional Information:
Withdrawal Velocity, \( v \) [m/s] = 0.0087
Withdrawal Height, \( l \) [m] = 0.123
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Average Absolute Error: 15.22  
Standard Deviation: 11.44

TABLE (App. III-37): Non-Newtonian First Withdrawal Drainage [Herschel-Bulkley Model] (Viscoelastic Suspension - 1 % solids)

Additional Information:
Withdrawal Velocity, \( v \) [m/s] = 0.0243  
Withdrawal Time, \( t_w \) [s] = 6.8  
Withdrawal Height, \( L \) [m] = 0.139  
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Average Absolute Error: 19.90
Standard Deviation: 4.63

**TABLE** (App. III-33): Non-Newtonian Postwithdrawal Drainage [Berschel-Bulkley model] (Viscoelastic Suspension - 1% solids)

Additional Information:

Withdrawal Velocity, \( v_w \) [m/s] = 0.024
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Average Absolute Error: 11.99
Standard Deviation: 3.76

TABLE (App. III-39) : Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model)
(Viscoelastic Suspension - 1% solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0162
Withdrawal Height, \( L \) [m] = 0.145
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Average Absolute Error: 11.86
Standard Deviation: 8.17

**TABLE (App. III-40):** Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley Model)
(Viscoelastic Suspension - 14 solids)

Additional Information:

Withdrawal Velocity, $v_0$ [m/s] = 0.0162
Withdrawal Height, L [m] = 0.135
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Average Absolute Error: 11.69
Standard Deviation: 12.13

TABLE (App. III-41): Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model)
(Viscoelastic Suspension - 1% solids)

Additional Information:
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Average Absolute Error: 7.79
Standard Deviation: 7.23

TABLE (App. III-42): Non-Newtonian Postwithdrawal Drainage [Herschel-Bulkley model]
Viscoelastic Suspension - 1% solids

Additional Information:
Withdrawal Velocity, $v_w$ [m/s] = 0.0087
Withdrawal Time, $t_w$ [s] = 17.8
Withdrawal Height, $L$ [m] = 0.141
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Average Absolute Error: 15.45

Standard Deviation: 27.34

TABLE (App. III-43): Non-Newtonian Post-withdrawal Drainage (Berschel-Bulkley model)
(Viscoselastic Suspension - 2 % solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0243
Withdrawal Time, \( t_w \) [s] = 6.7
Withdrawal Height, \( h \) [m] = 0.136
Observation Height, \( z \) [m] = 0.010
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Average Absolute Error: 8.52
Standard Deviation: 4.75

TABLE (App. III-44): Non-Newtonian Postwithdrawal Drainage [Herschel-Bulkley model]
(Viscoelastic Suspension - 2 % solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.024
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Average Absolute Error

Standard Deviation 11.71 2.88

TABLE (App. III-45): Non-Newtonian Postwithdrawal Drainage, Herschel-Bulkley model
(Viscoelastic Suspension - 2% solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0152
Withdrawal Height, \( L \) [m] = 0.135
Withdrawal Time, \( t_w \) [s] = 9.6
Observation Height, \( L \) [m] = 0.030
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<th>ABSOLUTE ERROR [( \mu \text{m} )]</th>
<th>PERCENT ERROR [%]</th>
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Average Absolute Error: \( 8.18 \) \( \mu \text{m} \)
Standard Deviation: \( 10.64 \) \( \mu \text{m} \)

**TABLE (App. III-46):** Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model) (Viscoelastic Suspension - 2% solids)

Additional Information:
Withdrawal Velocity, \( v_w [\text{m/s}] = 0.0162 \)
Withdrawal Height, \( L [\text{m}] = 0.135 \)
Withdrawal Time, \( t_w [\text{s}] = 9.6 \)
Observation Height, \( z [\text{m}] = 0.076 \)
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<th>ABSOLUTE ERROR [m]</th>
<th>PERCENT ERROR [%]</th>
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Average Absolute Error: 4.96
Standard Deviation: 2.51

TABLE (App. III-47): Non-Newtonian Postwithdrawal Drainage [Herschel-Bulkley model]
(Viscoelastic Suspension - 2 % solids)

Additional Information:
Withdraval Velocity, \( \nu \) [m/s] = 0.0087
Withdrawal Height, \( L \) [m] = 0.132

\( \text{Withdrawal Time, } t_w \) [s] = 16.8
\( \text{Observation Height, } \hat{z} \) [m] = 0.010
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Average Absolute Error: 7.35
Standard Deviation: 8.59

TABLE (App. III-48) : Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model)
(Viscoelastic Suspension - 2 % solids)

Additional Information:
Withdrawal Velocity, \( \nu \) [m/s] = 0.0087
Withdrawal Time, \( t_w \) [s] = 17.2
Observation Height, \( \zeta \) [m] = 0.076
Withdrawal Height, \( L \) [m] = 0.135
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<th>Percent Error [%]</th>
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Average Absolute Error: 10.52
Standard Deviation: 5.47

TABLE (App. III-49): Non-Newtonian Postwithdrawal Drainage [Herschel-Bulkley model] (Viscoelastic Suspension - 3% solids)

Additional Information:

Withdrawal Velocity, v [m/s] = 0.0243
Withdrawal Height, L [m] = 0.141
Withdrawal Time, t [s] = 5.9
Observation Height, z [m] = 0.075
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Average Absolute Error

Standard Deviation

| TABLE (App. III-50) : Non-Newtonian Postwithdrawal Drainage [Berschel-Bulkley model]
| (Viscoelastic Suspension - 3 % solids) |

Additional Information:

Withdrawal Velocity, \( v \) [m/s] = 0.024
Withdrawal Time, \( t_w \) [s] = 6.5
Withdrawal Height, \( h_w \) [m] = 0.132
Observation Height, \( \gamma \) [m] = 0.022
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Average Absolute Error: 8.62
Standard Deviation: 6.24

TABLE (App. III-51): Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model)
(Viscoelastic Suspension - 3 % solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0162
Withdrawal Height, \( L \) [m] = 0.124
Withdrawal Time, \( t_w \) [s] = 9.5
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Average Absolute Error: 16.94
Standard Deviation: 10.77

Additional Information:
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Observation Height, z [m] = 0.022
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Average Absolute Error: 7.06
Standard Deviation: 6.38

TABLE (App. III-53): Non-Newtonian Postwithdrawal Drainage (Herschel-Bulkley model)
(Viscoelastic Suspension - 3% solids)

Additional Information:
Withdrawal Velocity, \( v_w \) [m/s] = 0.0087
Withdrawal Time, \( t_w \) [s] = 16.2
Withdrawal Height, \( L \) [m] = 0.127
Observation Height, \( z \) [m] = 0.075
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Average Absolute Error: 7.53; Standard Deviation: 4.36

TABLE (App. III-34): Non-Newtonian Postwithdrawal Drainage [Berschel-Bulkey model]
(Viscoelastic Suspension - 3 % solids)

Additional Information:
Withdrawal Velocity, $v_w$ [m/s] = 0.0087
Withdrawal Time, $t_w$ [s] = 15.8
Withdrawal Height, $L$ [m] = 0.123
Observation Height, $z$ [m] = 0.022
APPENDIX IV
Detailed Calculations for
Postwithdrawal Drainage Film
Thickness Profiles and
Transition Drain Times
Consider the postwithdrawal drainage problem from a vertical flat plate, which consists of an initial unsteady withdrawal process, followed by unrestricted drainage of the entrained film (see Figure (II-1)). To develop the film thickness profile, the following assumptions have been made:

1. Local film thickness is a function of \( z \) and \( t \).
2. Laminar flow, free from waves.
3. One-dimensional, incompressible flow.
4. Consider only viscous and gravitational forces; fluid acceleration is negligible.
5. Shear stress at the gas-liquid interface is negligible.
6. There is no slip at the solid-liquid interface.
7. Normal stresses are negligible.

The unsteady state mass balance may be developed by noting that accumulation within the control volume can only occur if the size of the control volume changes (i.e. fluid is incompressible). Then,

\[
\Delta t \left( \rho \langle v_z \rangle W h_z \bigg|_z - \rho \langle v_z \rangle W h_z \bigg|_{z + \Delta z} \right) = -\rho W \Delta z \left( h_t \bigg|_t - h_t \bigg|_{t + \Delta t} \right) \quad \text{(App. IV-0.1)}
\]

Note that \( \rho \) and \( W \) are constant; dividing by \( \rho W \Delta t \Delta z \) and taking the limit as \( \Delta t \) and \( \Delta z \) approach zero yields,

\[
\lim_{\Delta z \to 0} \left( \frac{\langle v_z \rangle h_z \bigg|_z - \langle v_z \rangle h_z \bigg|_{z + \Delta z}}{\Delta z} \right) = -\lim_{\Delta t \to 0} \left( \frac{h_t \bigg|_t - h_t \bigg|_{t + \Delta t}}{\Delta t} \right) \quad \text{(App. IV-0.2)}
\]
\[
\frac{\partial (\nu z)}{\partial z} + \frac{\partial h}{\partial t} = 0 \quad \text{(App. IV.0.3)}
\]

To determine the film thickness profile as a function of \(z\) and \(t\), an expression for the average velocity must be developed. The overall momentum balance, which can be simplified from equation (II-8) using the assumptions, is given as:

\[
\frac{\partial \tau_{xz}}{\partial x} = \rho g_z \quad \text{(App. IV-0.4)}
\]

Integration of the above expression leads to:

\[
\tau_{xz} = \rho g_z + C_1 \quad \text{(App. IV-0.5)}
\]

To evaluate the constant of integration, use is made of the following boundary condition:

\[
\text{B.C. 1} \quad \tau_{xz} = 0 \quad \text{at} \quad x = h \quad \text{(App. IV-0.6)}
\]

Hence, the momentum flux distribution is:

\[
\tau_{xz} = -\rho g_z (h - x) \quad \text{(App. IV-0.7)}
\]

To establish the velocity profile, a rheological expression describing the shear stress-shear rate behavior is required. In this study, the De Kee fluid, Herschel-Bulkley fluid and Ellis fluid models will be studied.
1. De Kee Fluid Model

The De Kee fluid model is given by:

\[
\tau_{xz} = \tau_0 - \eta_1 \dot{\gamma}_{xz} e^{-t_1 \dot{\gamma}_{xz}}, \quad |\tau_{xz}| \geq \tau_0 \quad \text{(App. IV-1.1)}
\]
\[
\dot{\gamma}_{xz} = 0, \quad |\tau_{xz}| < \tau_0
\]

According to (App. IV-1.1), the velocity gradient is zero when the momentum flux is less than the yield stress. Hence, one expects a "plug flow" region in the outer part of the film, as sketched in Figure (App. IV-1). Between the plug flow region and the vertical flat plate, equations (App. IV-0.7) and (App. IV-1.1) are related as follows:

\[
-\tau_0 - \eta_1 \dot{\gamma}_{xz} e^{-t_1 \dot{\gamma}_{xz}} = -\rho g_z (h-x) \quad \text{(App. IV-1.2)}
\]

Rearranging and using the Taylor series expansion of the exponential function, one may write,

\[
\dot{\gamma}_{xz} - t_1 \dot{\gamma}_{xz}^2 + \frac{t_1^2}{2} \dot{\gamma}_{xz}^3 - \ldots = \frac{\rho g_z (h-x) - \tau_0}{\eta_1} \quad \text{(App. IV-1.3)}
\]

Now, using the well established series reversion technique, an explicit expression for the shear rate may be obtained. As a useful approximation, consider,
FIGURE (App. IV-1) : Momentum Flux and Velocity Distribution in Draining Films
For a section of constant $z$

Velocity Distribution

Momentum Flux Distribution

$v_z(x)$

$\tau_{xz,\text{max}}$

$\tau_{xz}(x)$

$\tau_{xz} = \tau_0$

$v_z$

$v_{zp}$

plug flow region

$x = h$

$x = h_p$
\[ \dot{\gamma}_xz = \frac{d\nu_z}{dx} = \frac{\rho g_z (h - x) - \tau_0}{\eta_1} + t_1 \left( \frac{\rho g_z (h - x) - \tau_0}{\eta_1} \right)^2 \]  
(App. IV-1.4)

Integration of the above expression leads to:

\[ \nu_z = \frac{\rho g_z}{\eta_1} \left( h x - \frac{x^2}{2} \right) - \frac{\tau_0}{\eta_1} x + t_1 \left[ \left( \frac{\rho g_z}{\eta_1} \right)^2 \left( h^2 x - h x^2 + \frac{x^3}{3} \right) \right] - \frac{2 \rho g_z \tau_0}{\eta_1^2} \left( h x - \frac{x^2}{2} \right) - \left( \frac{\tau_0}{\eta_1} \right)^2 x \]  
+ C_2  
(App. IV-1.5)

To evaluate the constant of integration, use is made of assumption #6. Mathematically,

**BC. 2**  \[ \nu_z = 0 \text{ at } x = 0 \]  
(App. IV-1.6)

Evaluation of the constant of integration leads to the velocity profile in the film between the plug flow region and the solid surface. Hence,

\[ \nu_z = \frac{\rho g_z}{\eta_1} \left( h x - \frac{x^2}{2} \right) - \frac{\tau_0}{\eta_1} x + t_1 \left[ \left( \frac{\rho g_z}{\eta_1} \right)^2 \left( h^2 x - h x^2 + \frac{x^3}{3} \right) \right] - \frac{2 \rho g_z \tau_0}{\eta_1^2} \left( h x - \frac{x^2}{2} \right) - \left( \frac{\tau_0}{\eta_1} \right)^2 x \]  
, \[ |\tau_{xz}| \equiv \tau_0 \]  
(App. IV-1.7)

For the plug flow region, we will define a critical thickness, \( h \) such that:

\[ -\tau_0 - \eta_1 \dot{\gamma}_{xz} e_{z} \dot{t}_1 \dot{\gamma}_{xz} = -\rho g_z (h - h_p) \]

\[ : h_p = h - \frac{\tau_0}{\rho g_z} \]  
(App. IV-1.8)
Appropriate substitution of the critical thickness into equation (App. IV-1.7) leads to the following expression for the plug flow velocity:

\[ v_{zp} = \frac{\rho g z h_p^2}{2\eta_1} \left[ 1 + \frac{2\rho g z h_p \tau_1}{3\eta_1} \right], \quad |\tau_{xz}| < \tau_0 \quad (\text{App. IV-1.9}) \]

The average velocity over a cross-section of the film is determined as follows:

\[
\langle v_z \rangle = \frac{1}{h} \left\{ \int_0^{h_p} v_z \, dx + \int_{h_p}^{h} v_{zp} \, dx \right\}
\]

\[
= \frac{\rho g z h^2}{3\eta_1} \left[ \frac{3(h_p)^2}{2(h)} - \frac{1}{2} \left( \frac{h_p}{h} \right)^3 + \frac{\rho g z h \tau_1}{\eta_1} \left( \frac{h_p}{h} \right)^3 - \frac{1}{4} \left( \frac{h_p}{h} \right)^4 \right] \quad (\text{App. IV-1.10})
\]

Replacing the critical thickness with equation (App. IV-1.8) and substituting into the momentum balance equation (App. IV-0.3) leads to:

\[
\frac{\rho g z h}{\eta_1} \left[ \left( h - \frac{\tau_0}{\rho g z} \right) + \frac{\rho g z \tau_1}{\eta_1} \left( h - \frac{\tau_0}{\rho g z} \right)^2 \right] \frac{\partial h}{\partial z} + \frac{\partial h}{\partial l} = 0 \quad (\text{App. IV-1.11})
\]

To solve the first order p.d.e., use is made of the subsidiary equation:

\[
\frac{dz}{\frac{\rho g z h}{\eta_1} \left[ \left( h - \frac{\tau_0}{\rho g z} \right) + \frac{\rho g z \tau_1}{\eta_1} \left( h - \frac{\tau_0}{\rho g z} \right)^2 \right]} = \frac{dl}{1} = \frac{dh}{0} \quad (\text{App. IV-1.12})
\]
Essentially,

\[ \frac{\partial h}{\partial z} dz + \frac{\partial h}{\partial t} dt = 0 \]  

(App. IV-1.13)

Making use of equation (App. IV-1.13) in equation (App. IV-1.11) leads to:

\[ \frac{\rho g_z h}{\eta_1} \left[ (h - \tau_0) + \frac{\partial g_z t_1}{\rho g_z} \left( h - \tau_0 \right)^2 \right] = \frac{Z}{t} + C_3 \]  

(App. IV-1.14)

To solve the constant of integration, use will be made of the following initial condition, which is valid for unsteady withdrawal only:

\[ \text{I.C. 1} \quad h = \infty \quad \text{at} \quad t = 0 \]  

(App. IV-1.15)

Thus, for unsteady withdrawal or Jeffreys' drainage, the film thickness profile is reported as:

\[ \left( \frac{\rho g_z}{\eta_1} \right)^2 t_1 h^3 + \left( \frac{\rho g_z}{\eta_1} - \frac{2 \rho g_z \tau_0}{\eta_1^2} \right) h^2 + \left( \frac{\tau_0}{\eta_1} \right)^2 t_1 \frac{\eta_1}{\eta_1} h = \frac{Z}{t} \]  

(App. IV-1.16)

To obtain an expression for the film thickness profile for a postwithdrawal drainage situation, equation (App. IV-1.16) is used as an initial condition for drainage. That is,

\[ \text{I.C. 2} \quad \left( \frac{\rho g_z}{\eta_1} \right)^2 t_1 h^3 + \left( \frac{\rho g_z}{\eta_1} - \frac{2 \rho g_z \tau_0}{\eta_1^2} \right) h^2 + \left( \frac{\tau_0}{\eta_1} \right)^2 t_1 \frac{\eta_1}{\eta_1} h = \frac{Z}{t_w} \]  

when \( t_d = 0 \)  

(App. IV-1.17)

The expression which can be used to describe the postwithdrawal
drainage behavior of a De Kee fluid is given by:

\[
\left(\frac{\rho g z}{\eta_1}\right) l_1 h^3 + \left(\frac{\rho g z}{\eta_1} - \frac{2\rho g z \tau_0 l_1}{\eta_1^2}\right) h^2 + \left(\frac{\tau_0}{\eta_1}\right) l_1 - \frac{\tau_0}{\eta_1} h = \frac{z}{l_d + l_w}
\]  

(App. IV-1.18)

In withdrawal problems, the transition from the unsteady case to the continuous case is not well defined. Indeed, at the beginning of postwithdrawal drainage, there may not be immediate agreement with equation (App. IV-1.18). To develop the expression which allows us to predict the time required for the agreement of (App. IV-1.18) with typical experimental data, we will define \( h_0 \) as the film thickness at the critical junction separating unsteady withdrawal from continuous withdrawal. To determine the location of the critical junction, use will be made of the following dimensionless variables:

\[
T_0 = h_0 \sqrt{\frac{\rho g z}{\eta_1 \nu_w}} \tag{App. IV-1.19}
\]

\[
T_1 = \frac{h_0 \tau_0}{\eta_1 \nu_w} \tag{App. IV-1.20}
\]

\[
T_2 = \frac{\tau_0 l_1}{\eta_1} \tag{App. IV-1.21}
\]

\[
T_3 = \frac{\rho g z h_0 l_1}{\eta_1} \tag{App. IV-1.22}
\]

Equation (App. IV-1.16) may now be rewritten, defining the
critical junction as:

\[
\frac{z}{t} = \frac{h_0^2 \rho g_z}{\eta_1} \frac{\rho g_z h_o t_1}{\eta_1} + h_0 \frac{\rho g_z}{\eta_1} - 2 \frac{\rho g_z h_o^2 T_0 t_1}{\eta_1} + \frac{\tau_0 t_1 h_o T_0}{\eta_1} - \frac{h_o T_0}{\eta_1}
\]

\[
\frac{z}{L t_w} = v_w \frac{T_0^2 t_1}{L} + v_w \frac{T_0^2}{L} - 2 v_w T_0^2 t_2 + v_w T_2 t_1 - v_w t_1
\]

\[
\left(\frac{z}{L}\right)_j = T_0^2 (1 - 2T_2 + T_3) + T_1 (T_2 - 1)
\]

(App. IV-1.23)

When withdrawal stops, the critical junction will immediately propagate downward towards the bath surface at what is assumed to be a constant velocity. Thus,

\[
v_j = \frac{d z}{d t} = \frac{\rho g_z h}{\eta_1} \left[ \left( h - \frac{T_0}{\rho g_z} \right) + \frac{\rho g_z t_1}{\eta_1} \left( h - \frac{T_0}{\rho g_z} \right)^2 \right]
\]

\[
= \left(\frac{z}{L}\right)_j v_w
\]

(App. IV-1.24)

The time required for the critical junction to travel below a specified plane of observation, say \((z/L)_{obs}\), is termed the transition drain time and may be determined by:

\[
t_d = \frac{z_{obs} - z_j}{v_j}
\]

\[
= \frac{L \left( (z/L)_{obs} - (z/L)_j \right)}{[ T_0^2 (1 - 2T_2 + T_3) + T_1 (T_2 - 1)] v_w}
\]
\[
\begin{align*}
  t_d &= \frac{L}{v_w} \left[ \frac{(z/L)_{\text{obs}} - (z/L)_j}{(z/L)_j} \right] \\
  t_d^{*} &= t_w \left[ \frac{(z/L)_{\text{obs}} - (z/L)_j}{(z/L)_j} \right] \quad \text{(App. IV-1.25)}
\end{align*}
\]

A special case of the transition drain time is recognized when the plane of observation corresponds to the bath surface. This is referred to as the critical drain time:

\[
  t_d^{*} = t_w \left[ \frac{1 - (z/L)_j}{(z/L)_j} \right] \quad \text{(App. IV-1.26)}
\]

Some special limiting cases can be recognized in equations (App. IV-1.7, 1.10, 1.16, 1.18, 1.19, 1.20, 1.21, 1.22, 1.23, 1.24, 1.25 and 1.26). For a Newtonian fluid, let

\[
\begin{align*}
  \tau_{0} &= 0 \\
  t_1 &= 0 \\
  \eta_1 &= \mu 
\end{align*}
\]

(App. IV-1.27)

For a Bingham plastic, let

\[
\begin{align*}
  t_1 &= 0 \\
  \eta_1 &= \mu_0 
\end{align*}
\]

(App. IV-1.28)

2. Herschel-Bulkley Fluid Model
The Herschel-Bulkley model is given by:

\[
\tau_{xz} = \tau_0 - m |\dot{\gamma}_{xz}|^{n+1} \dot{\gamma}_{xz}, \quad |\tau_{xz}| \geq \tau_0 \quad \text{(App. IV-2.1)}
\]

\[
\dot{\gamma}_{xz} = 0, \quad |\tau_{xz}| < \tau_0.
\]

To develop the film thickness profile expression for postwithdrawal drainage, the momentum balance (equation (App. IV-0.7)) is combined with equation (App. IV-2.1). The boundary conditions which were used for the De Kee fluid model are still valid; since the solution is directly patterned after that used for the De Kee model, equations analogous to (App. IV-1.7, 1.9, 1.10, 1.16 and 1.18) will be summarized:

\[
v_z = \frac{m}{\rho g_z} \frac{n}{n+1} \left[ \left( \frac{\rho g_z h - \tau_0}{m} \right)^{\frac{n+1}{n}} - \left( \frac{\rho g_z (h-x) - \tau_0}{m} \right)^{\frac{n+1}{n}} \right] \quad |\tau_{xz}| \geq \tau_0 \quad \text{(App. IV-2.2)}
\]

\[
v_{zp} = \frac{n}{n+1} \left( \frac{\rho g_z}{m} \right)^{\frac{1}{n}} h_p^{\frac{n+1}{n}} \quad |\tau_{xz}| = \tau_0 \quad \text{(App. IV-2.3)}
\]

\[
\langle v_z \rangle = \left( \frac{\rho g_z}{m} \right)^{\frac{n}{2n+1}} h p^{\frac{n+1}{n}} \left[ \frac{2n+1}{n+1} \left( \frac{h_p}{h} \right)^{\frac{n+1}{n}} - \frac{n}{n+1} \left( \frac{h_p}{h} \right)^{\frac{2n+1}{n}} \right] \quad \text{(App. IV-24)}
\]

\[
h^{n+1} - \frac{h^n (\tau_0)}{\rho g_z} - \frac{m}{\rho g_z} \left( \frac{z}{t} \right)^n = 0 \quad \text{(App. IV-2.5)}
\]

\[
h^{n+1} - \frac{h^n (\tau_0)}{\rho g_z} - \frac{m}{\rho g_z} \left( \frac{z}{t_{d+w}} \right)^n = 0 \quad \text{(App. IV-2.6)}
\]
To develop the transition drain time expression, use will be made of the following dimensionless expressions:

\[ T_0 = h_0 \sqrt{\frac{\rho g_z}{m v_w^n}} \]  
(App. IV-2.7)

\[ T_1 = \frac{h_0}{v_w} \left( \frac{\tau_0}{m} \right)^{1/n} \]  
(App. IV-2.8)

The critical junction expression, similar to equation (App. IV-1.23) is given as:

\[ \frac{z}{L} = T_0^{n+1} - T_1^n \]  
(App. IV-2.9)

Note that expressions for the propagation velocity, transition drain time and critical drain time are general and depend only upon the critical junction, which is unique for each fluid model.

Again, special limiting cases can be recognized in equations (App. IV-2.2 and 2.4 through 2.9). For a Newtonian fluid, let

\[
\begin{align*}
\tau_0 &= 0 \\
n &= 1 \\
m &= \mu
\end{align*}
\]  
(App. IV-2.10)

For a power law fluid, let
\[ \tau_0 = 0 \quad \text{(App. IV-2.11)} \]

For a Bingham plastic, let

\[
\begin{align*}
    n &= 1 \\
    m &= \mu_0
\end{align*} \quad \text{(App. IV-2.12)}
\]

3. Ellis Fluid Model

The expressions describing the Ellis fluid have previously been presented (see equations (II-55 through 59)). To develop the critical junction expression, use is made of the dimensionless groups below:

\[ T_0 = h_0 \sqrt{\frac{\varphi_0 \rho g z}{v_w}} \quad \text{(App. IV-3.1)} \]

\[ T_0 = h_0 \left[ \frac{\varphi_0 (\rho g z)^{\alpha}}{v_w} \right]^{1/(\alpha + 1)} \quad \text{(App. IV-3.2)} \]

Using the implicit film thickness profile for the unsteady withdrawal of an Ellis fluid, the critical junction expression is given by:

\[ (z/L)_j = \tau_0^2 + \tau_1^{\alpha + 1} \quad \text{(App. IV-3.3)} \]
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