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REDUCTION OF FAN NOISE BY UNEQUAL SLOT SPACING

by

Ming Sang Lai

A thesis presented to the University of Windsor in partial fulfillment of the requirements for the degree of Master of Applied Science—in MECHANICAL ENGINEERING

Windsor, Ontario, Canada 1983
TO MY BROTHER
JEFFREY KIT SANG
AND
MY PARENTS
ABSTRACT

The main objective of this study was the analysis of a computer-aided noise reduction technique which is directed primarily at the reduction of milling cutter noise but is also useful in the areas of tire tread and fan design. Because of the excessive costs involved with the design of any one of these components, an alternate method was chosen for verification of the validity of the existing mathematical procedures. A notched rotating disc was used to simulate equivalent unequally spaced force events by its fan action. Seven unevenly spaced and one evenly spaced discs were tested. Where appropriate, theoretical results were verified by the corresponding experimental measurements.

The mathematical analysis indicates that the fundamental tone of the modulated disc can be reduced and redistributed to lower harmonics. The total sound energy of these components is less than that of the fundamental tone, which is produced by the evenly spaced rotor.

The experimental results indicate that the sound energy generated by the unevenly spaced discs is much higher than that of the evenly spaced disc. The major cause of this significant difference is found to be the sound pressure
buildup due to the interaction between the pressure pulses from the blades with closer spacing on the modulated disc. A graphical method is also introduced to predict the sound pressure variations on the unevenly spaced discs.
ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

ABSTRACT ........................................ iv
ACKNOWLEDGEMENTS ............................... vi
TABLE OF CONTENTS .............................. vii
LIST OF FIGURES ................................... x
LIST OF TABLES ................................... xiv
NOMENCLATURE ................................... xv

Chapter ............................................ Page
I. INTRODUCTION ................................. 1
II. LITERATURE SURVEY ......................... 4
State of Trial and Error Approach ............. 4
State of Optimization by Nonlinear Regression Technique ....... 9
III. THEORY ......................................... 11
Analytical ........................................ 11
Fourier Series Analysis ........................ 11
Nonlinear Regression Technique ............... 13
Experimental Noise Measurement ............... 17
Disc Noise .................................... 17
Background Noise .............................. 18
Manipulation of Data .......................... 18
IV. EXPERIMENTAL APPARATUS AND INSTRUMENTATION ........ 24
Apparatus ........................................ 24
Brüel and Kjær 4230 Sound Level Calibrator ... 25
Brüel and Kjær 2204 Sound Level Meter and Its Accessories ....... 26
36K 2204 Impulse Precision Sound Level Meter 26
Brüel and Kjær Microphone Stand UA0049 .... 26
Brüel and Kjær Microphone Extension Cable A00027 .... 27
Brüel and Kjær Flexible Extension Rod UA0196 27
Brual and Kjaer 4165 1/2" Condenser Microphone Cartridge.................. 27
Brual and Kjaer Type UA0237 Windscreen.................. 28
Spectral Dynamics SD 375 Dynamic Analyzer II........... 28
Spectral Dynamics SD422 Video Printer.................. 29
Dodge Model 25 SCR Control and SCR Drive Motor........... 29
General Radio Electronic Stroboscope Type 1538-A........... 29
Gould Advance 33400 Digital Storage Oscilloscope........... 30
Hewlett Packard 7045A X-Y Recorder.................. 30
IBM 370 ........................................ 31

V. PROCEDURE AND RESULTS ........................................... 32

Computational Procedure and Theoretical Results.................. 32
Procedure.............................................. 32
Results and Discussion........................................... 35
Experimental Procedure and Results............................ 36
Procedure.............................................. 36
Discussion of Results........................................... 36
Comparison of Theoretical and Experimental Results........... 38
Recording of Sound Pressure Variation.......................... 40
Discussion of Sound Pressure Variation.......................... 40
Graphical Sound Pressure Wave................................. 45

VI. CONCLUSIONS AND RECOMMENDATIONS.............................. 49

Conclusions.................................................. 49
Recommendations.................................................. 50

APPENDIX Page
A. ILLUSTRATION OF ONE ITERATION ON NONLINEAR REGRESSION TECHNIQUE........... 51

One Illustrative Iteration of Nonlinear Regression Technique........... 51

B. COMPUTER PROGRAM FOR DESIGNING OPTIMAL SPACING DISCS.................. 57

C. SAMPLE CALCULATION........................................... 60

Theoretical Data Refinement........................................... 60
Refinement of Experimental Data........................................... 61

REFERENCES ........................................... 63

FIGURES ........................................... 65

- viii -
LIST OF FIGURES

Figure
1. Even Spaced and Disc 01 ............................................. 65
2. Disc 02 and 001an ...................................................... 65
3. Sinusoidal and Random-6 .............................................. 66
4. Close-together-b and Random-5 ..................................... 66
5. Experimental Apparatus Used in Tests .............................. 67
6. Blades Spacing Resulting from Input Data Set # 1 ............... 68
7. Blades Spacing Resulting from Input Data Set # 2 ............... 68
8. Blades Spacing Resulting from Input Data Set # 3 ............... 69
9. Blades Spacing Resulting from Input Data Set # 4 ............... 69
10. Blades Spacing Resulting from Input Data Set # 5 ............... 70
11. Blades Spacing Resulting from Input Data Set # 6 ............... 70
12. Blades Spacing Resulting from Input Data Set # 7 ............... 71
13. Blades Spacing Resulting from Input Data Set # 8 ............... 71
14. Blades Spacing Resulting from Input Data Set # 9 ............... 72
15. Blades Spacing Resulting from Input Data Set # 10 .............. 72
16. Layout of Even Spaced ................................................... 73
17. Layout of Disc 01 ....................................................... 73
18. Layout of Disc 02 ....................................................... 74
19. Layout of 001an ....................................................... 74
20. Layout of Sinusoidal .................................................. 75
21. Layout of Random-6 ................................................... 75
22. Layout of Close-together-6 ......... 76
23. Layout of Random-5 ............ 76
25. Analytical Harmonic Spectrum of Disc 01 ........ 77
26. Analytical Harmonic Spectrum of Disc 02 ........ 78
27. Analytical Harmonic Spectrum of Random ........ 79
28. Analytical Harmonic Spectrum of Sinusoidal .......... 79
29. Analytical Harmonic Spectrum of Random-6 ........ 79
30. Analytical Harmonic Spectrum of Close-together-6 .... 80
31. Analytical Harmonic Spectrum of Random-5 .......... 80
32. Frequency Spectrum of Plain Disc and Amplitudes of Corresponding Harmonics at 2100 rpm .......... 81
33. Frequency Spectrum of Even Spaced and Amplitudes of Corresponding Harmonics at 2100 rpm .......... 82
34. Frequency Spectrum of Disc 01 and Amplitudes of Corresponding Harmonics at 2100 rpm .......... 83
35. Measured Harmonic Spectrum of Even Spaced at 840 rpm 84
36. Measured Harmonic Spectrum of Even Spaced at 1320 rpm ........ 85
37. Measured Harmonic Spectrum of Even Spaced at 1680 rpm ........ 86
38. Measured Harmonic Spectrum of Even Spaced at 2100 rpm ........ 87
39. Measured Harmonic Spectrum of Disc 01 at 840 rpm 88
40. Measured Harmonic Spectrum of Disc 01 at 1320 rpm 89
41. Measured Harmonic Spectrum of Disc 01 at 1680 rpm 90
42. Measured Harmonic Spectrum of Disc 01 at 2100 rpm 91
43. Measured Harmonic Spectrum of Disc 02 at 840 rpm 92
44. Measured Harmonic Spectrum of Disc 02 at 1320 rpm 93
45. Measured Harmonic Spectrum of Disc 02 at 1680 rpm 94
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Input and Output Data of the Computer Program</td>
<td>132</td>
</tr>
<tr>
<td>2.</td>
<td>Analytical Harmonic Amplitudes and Relative Spectrum Power of Tested Discs</td>
<td>133</td>
</tr>
<tr>
<td>3.</td>
<td>Normalised Analytical Harmonic Amplitudes and Relative Spectrum Power of Tested Discs</td>
<td>134</td>
</tr>
<tr>
<td>4.</td>
<td>Normalised Harmonic Amplitudes and Relative Spectrum Power of Tested Discs at 840rpm</td>
<td>135</td>
</tr>
<tr>
<td>5.</td>
<td>Normalised Harmonic Amplitudes and Relative Spectrum Power of Tested Discs at 1320 rpm</td>
<td>136</td>
</tr>
<tr>
<td>6.</td>
<td>Normalised Harmonic Amplitudes and Relative Spectrum Power of Tested Discs at 1680 rpm</td>
<td>137</td>
</tr>
<tr>
<td>7.</td>
<td>Normalised Harmonic Amplitudes and Relative Spectrum Power of Tested Discs at 2100 rpm</td>
<td>138</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( a_{jj} \)  \hspace{1cm} \text{element of matrix } A

\( a_{jj}^* \)  \hspace{1cm} \text{element of matrix } A^*

\( A \)  \hspace{1cm} \text{product matrix of } X'X

\( A^* \)  \hspace{1cm} \text{scaled matrix of } A

\( AC \)  \hspace{1cm} \text{alternating current (ampere)}

\( A_n \)  \hspace{1cm} \text{amplitude of the forcing function } f(\theta) \text{ at the } i\text{th harmonic}

\( A_{6e} \)  \hspace{1cm} \text{calculated amplitude of the sixth harmonic of Even Spaced}

\( A_i \)  \hspace{1cm} \text{calculated amplitude of the } i\text{th harmonic of any disc}

\( A_i^* \)  \hspace{1cm} \text{normalised amplitude of the } i\text{th harmonic of any rotating disc}

\( A_i^{**} \)  \hspace{1cm} \text{normalised calculated amplitude of the } i\text{th harmonic of any disc}

\( A_{mi}^* \)  \hspace{1cm} \text{normalised amplitude at } i\text{th harmonic of the calculated harmonic spectrum}

\( D \)  \hspace{1cm} \text{diagonal matrix of } X'X

\( D^{-\frac{1}{2}} \)  \hspace{1cm} \text{inverse of the square root matrix of } D

\( DC \)  \hspace{1cm} \text{direct current (ampere)}

\( E(y) \)  \hspace{1cm} \text{expected value of } y

\( E_n \)  \hspace{1cm} \text{power of the forcing function } f(\theta) \text{ at the } n\text{th harmonic}

\( f(t) \)  \hspace{1cm} \text{excitation function}

\( f(\theta) \)  \hspace{1cm} \text{excitation function}

- xv -
$f_0$ expected value of $\langle Y \rangle$ before the correction on $\Theta$-space

$q$ product of $X'$ and $-f_0$

$q^*$ scaled matrix of $q$

$H$ height (cm or mm)

$I$ number of events (blades) or identity matrix

$L$ length (cm or mm)

$L_p$ sound pressure level (dB)

$L_{pi}$ sound pressure level due to the blade spacing at the $i$th harmonic of the rotating disc (dB)

$bL_{pi}$ baseline noise level (dB)

$tL_p$ total sound pressure level of the system (dB)

$tL_{pi}$ total sound pressure level of the system at $i$th harmonic of the rotating disc (dB)

$M(\omega)$ system's amplification factor

$P_i$ root mean square sound pressure of different sound source ($P_i$)

$P_0$ reference sound pressure ($2 \times 10^{-5}$ Pa)

$P_{6e}$ root mean square sound pressure at 6th harmonic of the Even Spaced

$\text{RP}$ relative spectrum power of any disc

$(\Gamma)$ $\Gamma$ iteration of the nonlinear regression technique

$S$ vector of harmonic amplitude $S_n$

$S_n(\omega)$ amplitude of resulting noise of the system at nth harmonic

$w$ width (cm or mm)

$X$ derivative matrix of $f(\Theta)$

$X^T, X'$ transposition matrix of $X$

$x_{ij}$ element of derivative matrix $X$

$y_i$ ideal expected value at $i$th data point
\( \hat{Y}_i \)  
Calculated expected value at \( i \)th data point

\( \langle Y_i \rangle \)  
Calculated expected value at \( i \)th data point obtained by the linearised model

\( Y \)  
Vector of \( Y_i \)

\( \hat{Y} \)  
Vector of \( \hat{Y}_i \)

\( \langle Y \rangle \)  
Vector of \( \langle Y_i \rangle \)

\( Z \)  
Error matrix

\( Z_i \)  
Element of matrix \( Z \)

\( Z^T \)  
Transposition matrix of error matrix \( Z \)

\( \delta_0 \)  
Small correction vector on \( \theta \)-space

\( \delta_t \)  
Small correction vector on \( \theta \)-space calculated by Taylor Series

\( \delta_0^* \)  
Scaled vector of \( \delta_0 \)

\( \delta_t^* \)  
Scaled vector of \( \delta_t \)

\( \varepsilon \)  
Specified small value (0.0001)

\( \Theta_i \)  
Position of the \( i \)th blade

\( \Theta \)  
Blade position vector

\( \Theta_n \)  
New blade position vector

\( \Theta^o \)  
Old blade position vector

\( \lambda \)  
Step size for changing the correction vector

\( \Phi \)  
Sum of squares of error

\( \langle \Phi \rangle \)  
Sum of squares of error calculated by linearised model

\( \omega \)  
Angular frequency (rad/sec)
Chapter I
INTRODUCTION

Milling is a machining process to remove metal by feeding the workpiece past a rotating cutter with a multitooth configuration. It differs from drilling, turning and broaching. On a conventional miller, there are more than two teeth, or inserts, which have cutting edges. The distance between two consecutive teeth is called the pitch. In general, all milling cutters have constant pitches. Because of its high rotating rate and many cutting edges, the machining surface can be machined faster than with single-point tools and often with a better finish. Also it can produce flat or formed surfaces of different varieties and with high accuracy.

In recent years, productivity has become very important in industries, so much so that increasing the rate of production is now often the main task for engineers. This trend has also influenced the milling tool industry. Therefore, stronger and harder inserts have been produced which can be incorporated into a high speed cutter and which can allow an increase in the depth of cut to generate a faster finish. As a result, high speed milling induces higher excitation forces on the workpiece and the machining
system. Hence larger vibration levels and more noise are generated. Furthermore, a regenerative chatter would be produced by the indented surface which is formed by the previous periodic cutting edges. This results in a poor finish and may damage as well as shorten the life of the tool.

The automotive industry is one of the largest business entities in North America. In recent years the trend has been towards designing lighter vehicles to meet the demands for energy conservation. Some parts of the automobile made with the traditional heavy metal, steel, have been replaced by components made of lighter materials such as aluminum alloy. Most of these parts, such as the transmission casing and the engine block, need milling to trim mating surfaces before installation. Since aluminum is lighter and the product cross sections are thinner, the excitational forces generated during the milling process can induce more vibrations and noise on the aluminum workpiece than on a similar steel workpiece. In addition to a poor surface finish and reduced tool life very high noise levels are produced. These are generally far in excess of limits which are imposed by government regulations.

Because these problems arise most often on high production volume, automated machine tools, where the environment in the cutting area is hostile, standard methods of vibration and noise control cannot be applied. The
solution depends almost exclusively upon the reduction of the excitation force which is generated by the cutter. The most effective method, by far, is using unevenly spaced inserts on the cutter. The earlier cutter design used an irregular pitch cutter with a linear pitch variation. The pitch between every two consecutive pairs of teeth was the same. The nonlinear pitch cutter is a later development. In this case, the distance difference between any two consecutive pairs of teeth varies. In the latest development, a computer aided design technique is used for searching the optimal spacings of the nonlinear pitch inserts.

Since the cost for producing a milling cutter ring is very high, metal-sheet discs have been used as a simulation in the experiment. These discs were notched along their periphery and slightly bent to produce a multibladed fan effect. The noise produced during the rotation is dependent upon the spacings of the notches. After appropriate analysis, the noise measurements were used for comparisons with theoretical results. Two computer designed discs with optimal spacing, in evenly spaced disc and five other unevenly spaced discs have been made and tested.
Chapter II

LITERATURE SURVEY

This literature survey illustrates the historical background and the development of the noise and vibration reduction techniques on the multitooth (multiblade) rotating object (milling cutter or fan) by irregularizing the blade spacing. Basically, this literature survey is divided into two sections. First, the trial and error state, for which no particular technique is used to decide the spacing of the blades. The degree of uneven spacing is optimized by the trial and error method. Secondly, mathematical methods are used for improving the optimization of the blade spacing. The acyclic regression technique, carried out on a digital computer, appears to be the best for reducing the noise and vibration on multitooth (multiblade) rotating disc (object).

2.1 STATE OF TRIAL-AND-ERROR APPROACH

K. D. Kryter and K. S. Pearson [1] found that the perceived noisiness of band-limited random noise is quieter than sound composed of pure tone superimposed on band-limited random noise, if the overall sound pressure levels (Lp) are the same. They have also shown that for equal Lp, sounds of

* Numbers in brackets refer to publications listed in References.
larger tone-to-noise ratio are judged to be noisier.

It might be inferred that sound comprised of a pure tone superimposed on random background noise can be made quieter by dispersing or redistributing the energy of the tone over a number of discrete frequencies without reducing the overall level.

Slavicek [2] suggested that the non-constant pitch milling cutter can be employed in the milling process to reduce workplace vibration and hence cutting noise. In his experiment, a conventional milling cutter and three non-constant pitch milling cutters with a periodic alternation of one greater and one smaller pitch (pitch period of two) were used. The diameters of the face milling cutters were 315 mm with sixteen cemented carbide tipped teeth on each.

Slavicek conducted a series of tests. First, the limiting depth of cut of the conventional milling cutter was found with different cutting speeds. Secondly, three different pitch irregularities were run with the same cutting speed as the conventional one. Although the three irregular pitch milling cutters had different results in the stability of cutting, all of them produced a remarkable increase in cutting stability over the conventional cutter.

Opitz et al. [3] proposed changing the spacing of inserts in a sinusoidal manner and extended Slavicek's experiment in a larger range of spindle speeds. Their findings show that in some speed ranges, there was no
stability gain above the constant pitch arrangement. Within a considerable speed range, which was determined by a mathematical procedure to establish the borderline of stability, a remarkable improvement in the dynamic stability was attained.

Slavicek and Oplitz have proved that the use of a non-constant tooth pitch milling cutter can lead to a significant increase of cutting stability: but they limited their theory to the pitch period of two cutting edges. For this reason, the mathematical and experimental procedures of Slavicek and Oplitz were extended by Vanherck [4]. He used five more cutters with pitch periods of three, four, six, nine and eighteen.

He concluded that a higher pitch period will widen the zone of stability. Also the results indicate that it is possible to design a milling cutter, which makes stable cutting possible, with a depth of cut up to four times greater than the critical value for a cutter with constant pitch. This applies for almost all natural frequencies of a well-defined class of machines. He also proposed and verified a mathematical development of the nonlinear or sinusoidal principle for the design of more stable cutters.

Comparing tests of cutters with nonlinear pitch to linear pitch, in the zone of the smallest stability gain, the nonlinear case always gives better results. But, on the other hand, in the maximum stability gain zone, the linear case is better.
In 1970 Varterasian [5] employed the single period phase modulation of spacing blades for milling cutters to reshape the excitation spectrum during operation. This procedure was repeated until a 'white noise' spectrum was obtained. The 'optimal' design was applied in his experiments. Two sets of thirty-nine-tooth milling cutters were used for the total of one hundred and fifty runs. One set was for rough cutting and the other for finishing. Each set consisted of one conventional and one modulated cutter.

On rough cuts, fifty percent of runs with the modulated cutter were better than with the conventional one and the rest had no stability gain. The finish cutters produced slightly different results. In fifty percent of runs, the modulated cutter performed better than the conventional one. Forty-five percent of runs produced the same result with both units and in five percent of tests, the conventional cutter proved better. Most of the five percent results were not repeatable and, therefore, were hard to assess.

Ewald et al. [5] used a similar approach as Varterasian in an application involving the design of fan blades of an induction motor for the purpose of reducing its noise. Using a sinusoidal modulation, three noise prediction methods and a semi-graphical design using with Bessel functions, Fourier analyses with impulse and sinusoidal wave approximations were carried out. The Bessel function method was used for a continuous phase-modulated
function, while the actual frequency spectrum of the fan is produced by a number of more nearly discrete events. The amplitudes in the frequency spectrum obtained from the Bessel series therefore differ somewhat from those obtained from the fan. The Bessel series, however, approximates more closely the actual fan spectrum when the number of blades is large. If the number of blades of the fan is small, a more realistic result may be estimated by Fourier analysis of the pressure waveform produced by the fan blades.

All three methods provided good estimates of the resampled frequency spectrum, which was produced by the modulated fan. This is because the large number of blades approximates a continuous system. The proportionately small degree of modulation prevented the higher harmonics in the Fourier analysis of the impulse train from appearing in the low-frequency ranges. It was also shown that the one-fold unevenly spaced modulation of the fan blades did not result in an undesirable mechanical unbalance or aerodynamic loss.

Krishnappa [7] used Ewald's modulation principle with tests on a centrifugal fan. A one-fold modulation technique was applied to a twelve-blade centrifugal fan with a modulation amplitude of fifteen degrees. This fan was tested and compared with a similar fan with evenly spaced blades. As a result, at the design region (high pressure rise region), there was a significant reduction in the sound pressure level. When the fan was run at extreme flow
condition (low pressure rise region), although the level of the tone at the blade passing frequency (blade number times the shaft rotating speed) was reduced by the modulation, the side bands increased to levels even higher than those of the blade passing frequency with even spacings.

2.2 STATE OF OPTIMIZATION BY NONLINEAR REGRESSION TECHNIQUE

All the methods, which were described previously, were based on the use of the trial and error method to reach optimum conditions. Although the results are promising, further improvements can be achieved. Also the time required for each trial was lengthy and the procedure was tedious. Doolin et al. [8],[9] derived the insert distribution by means of Marquardt's algorithm for nonlinear regression [10]. This technique has the advantage of eliminating the trial and error procedure for optimizing the shape of the spectrum generated by the milling cutter. If the dynamic frequency-response function of a particular milling-machine-cutter-workpiece system is known, a special cutter can be designed. If not, a flat frequency-response function can be assumed for the system when designing a general cutter. The theoretical results obtained show that for the special design cutter, the vibration power is thirty-five times smaller than for the cutter with even spacings and the general design cutter has the vibration power about fourteen times smaller than the conventional one when both were under
the same cutting condition. Burney, Pandit and Wu [11] designed an eight-blade unevenly spaced cutter based on Doolan's development. Good results were obtained.

Burney and Wu [12] formulated the nonlinear regression technique in mathematical format and illustrated the methodology of this technique on the design of a face milling cutter. This mathematical formulation has been used for the optimal design of the 6-blade discs in this report.
Chapter III
THEORY

3.1 ANALYTICAL

3.1.1 Fourier Series Analysis

The harmonic analysis is commonly used to analyse the excitation spectrum of rotating machines. Because of the periodic characteristic of the exciting force over one cycle of a group of events, the Fourier Series method is generally used for its harmonic analysis. If \( f(t) \) is the excitation function, it can be expressed as an infinite sum of orthogonal trigonometric terms.

\[
f(t) = \bar{f} + \sum_{n=1}^{\infty} B_n \sin(n\omega t) + \sum_{n=1}^{\infty} C_n \cos(n\omega t) \quad (3.1)
\]

where

\[
\bar{f} = \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} f(t) \, dt
\]

\[
B_n = \frac{\omega}{\pi} \int_{0}^{2\pi/\omega} f(t) \sin(n\omega t) \, dt
\]

\[
C_n = \frac{\omega}{\pi} \int_{0}^{2\pi/\omega} f(t) \cos(n\omega t) \, dt
\]

and \( \omega \) is the angular frequency in rad/s

Since \( \Theta = \omega t \) and the events, in this case, are discrete, \( B_n \) and \( C_n \) can be written as

\[
B_n = \frac{1}{\pi} \sum_{i=0}^{I-1} f(\theta_i) \sin(n\theta_i)
\]
\[ C_n = \frac{1}{\pi} \sum_{i=0}^{I-1} f(\theta) \cos(n \theta_i) \] 

where \( f(\theta) \) depends on the waveform of the forcing function which must be known and \( I \) is the number of events.

The power of the forcing function \( f(\theta) \) at the \( n \)th harmonic, \( \delta_n \), is given by

\[ \delta_n = (B_n^2 + C_n^2) \] 

and the amplitude of the corresponding harmonic, \( A_n \), is given by

\[ A_n = (B_n^2 + C_n^2)^{\frac{1}{2}} \]

Thus the excitation spectrum can be estimated. Note that once \( f(\theta) \) is known (or assumed), the amplitude, \( A_n \), is a function of only the blade spacings, \( \theta_i \).

Let \( \theta_0, \theta_1, \theta_2, \ldots, \theta_{I-1} \) be the angular positions of the blades in a cutter having \( I \) blades. \( \theta_0 \) may arbitrarily be taken to be zero. To simplify the analysis, the following assumptions are made:

The impact of each blade on the workpiece is instantaneous. The impact amplitude is the same for each blade and is unaffected by changes in blade spacing. For simplicity, the waveform of the forcing function \( f(\theta) \) is assumed to be impulsive in nature with magnitude equal to one.

In reality, the higher harmonics have progressively less amplitude in the excitation spectrum; therefore, the number of harmonics calculated in the Fourier Series analysis is restricted to \( I+1 \). Where \( I \) is the number of events of one cycle.
If the machine tool system's amplification factor, \( M(\omega) \), is known the amplitude of the resulting vibration or noise, \( S_\alpha(\omega) \), is given by

\[
S_\alpha(\omega) = A_n \times M(\omega) \quad \text{(3.5)}
\]

Since \( M(\omega) \) is not known, it can be taken to be one as a general case. Now the objective is to minimize the values of \( S_\alpha(\omega) \). The only parameter of the objective function \( S_\alpha(\omega) \) is the blade position \( \theta_i \). Hence, a nonlinear regression routine based on the principle of Marquardt's compromise [10] is used for determining those blade spacings which minimize the values of \( S_\alpha(\omega) \).

3.1.2 Nonlinear Regression Technique

Let the mathematical model be defined by:

\[
\mathcal{L}(\theta) = f(S_1, S_2, S_3, \ldots, S_n; \theta_1, \theta_2, \theta_3, \ldots, \theta_k)
\]

\[
= f(S, \theta) \quad \text{(3.6)}
\]

where \( S_1, S_2, S_3, \ldots, S_n \) are independent variables, \( \theta_1, \theta_2, \theta_3, \ldots, \theta_k \) are the population values of \( k \) parameters, and \( \mathcal{L}(\theta) \) is the expected value of the dependent variable \( y \).

Let the data points be denoted by

\[
(Y_i, S_1, S_2, S_3, \ldots, S_n; i = 1, 2, 3, \ldots, n) \quad \text{(3.7)}
\]

where \( Y_i \) is the ideal expected value at the \( i \)th data point.

The objective is to compute those estimates of the parameters which will minimize

\[
\Phi = \sum_{i=1}^{n} [Y_i - \hat{Y}_i]^2
\]
\[ \| \hat{y} - \hat{y} \| ^2 \] 

where \( \hat{y} \) is the value of \( y \) predicted by equation (3.6), at the input data point.

Ideally, \( y \) should be equal to zero, hence equation (3.6) becomes

\[ \Phi = \| y - \hat{y} \| ^2 \]

For the convenience of computation, the actual nonlinear model has to be linearised. The method used is based upon expanding \( f \) in a Taylor series. The linear terms are as follows:

\[ \langle Y(S_i, \theta + \delta_t) \rangle = f(S_i, \theta) + \sum_{j=1}^{k} \left( \frac{\partial f}{\partial \delta_j} \right) \langle \delta_t \rangle_j \]

or

\[ \langle Y \rangle = f\theta + X\delta_t \]

(3.9)

The vector \( \delta_t \) is a small correction to \( \theta \), with the subscript \( t \) used to designate \( \theta \) as calculated by this Taylor series method. The brackets \( \langle \rangle \) are used to distinguish predictions based upon the linearised model from those upon the actual nonlinear model. Thus, the value of \( \Phi \) predicted by equation (3.9) is

\[ \langle \Phi \rangle = \sum_{i=1}^{n} \left( -\langle y_i \rangle \right)^2 \]

(3.10)

Now, \( \delta_t \) appears linearly in equation (3.9), and can therefore be found by the standard least-squares method of setting \( \partial \langle \Phi \rangle / \partial \theta_j = 0 \), for all \( j \). Thus \( \delta_t \) is found by solving

\[ A \delta_t = q \]

(3.11)
where

\[
\begin{align*}
A_{[k \times k]} &= X^T X \\
X_{[n \times k]} &= \left( \frac{\partial f_i}{\partial \theta_j} \right) \\
q_{[k \times 1]} &= \left[ \sum_{i=1}^{n} (-f_i) \frac{\partial f_i}{\partial \theta_j} \right] \\
&= X^T (-f o)
\end{align*}
\]

\[i = 1, 2, 3, \ldots, n \quad j = 1, 2, 3, \ldots, k\]

From equation (3.11), the algorithm can be formulated as follows:

\[
(A + \lambda I) \delta_0 = q
\]

(3.15)

The relevant properties of the solution, \( \delta_0 \), of equation (3.11) are invariant under linear transformations of the \( \theta \)-space. But the properties of the derivative matrix, \( X \), are not scale invariant. It becomes necessary to scale the \( \theta \)-space in some convenient manner. The manner chosen is scale the \( \theta \)-space in units of the standard deviations of the derivatives \( \partial f_i / \partial \theta_j \), taken over the sample points \( i = 1, 2, \ldots, n \). Since these derivatives depend, in general, on the \( \theta_j \) themselves, the current trial values of the \( \theta_j \) are necessary in the evaluation of the derivatives. This choice of scale causes the \( A \) matrix to be transformed into the matrix of simple correlation coefficients among the \( \partial f_i / \partial \theta_j \). This choice of scale is, in fact, a device for improving the numerical aspects of computing procedures.

The superscript \( T \) denotes matrix transposition.
Thus a scaled matrix $A^*$ and a scaled vector $q^*$ are defined:

$$A^* = \begin{pmatrix} a_{jj}^* \end{pmatrix} = \begin{pmatrix} a_{jj}' / \sqrt{a_{jj} a_{jj}'} \end{pmatrix} \quad \text{(3.16)}$$

$$q^* = \begin{pmatrix} q_j^* \end{pmatrix} = \begin{pmatrix} q_j / \sqrt{a_{jj}} \end{pmatrix} \quad \text{(3.17)}$$

and solve for the Taylor Series correction using

$$A^* s_j^* = q^* \quad \text{(3.18)}$$

$$s_j = s_j^* / \sqrt{a_{jj}} \quad \text{(3.19)}$$

The broad outline of the appropriate algorithm is now clear. Specifically, at $r$th iteration the equation

$$(A^{*(r)} + \lambda^{(r)} I) \delta^{*(r)} = q^{*(r)} \quad \text{(3.20)}$$

is constructed. This equation is then solved for $\delta^{*(r)}$. Then equation (3.19) is used to obtain $s^{(r)}$. The new trial vector

$$\theta^{(r+1)} = \theta^{(r)} + s^{(r)} \quad \text{(3.21)}$$

will lead to a new sum of squares $\phi^{(r+1)}$.

When $\phi^{(r)} - \phi^{(r+1)} \leq \varepsilon \quad \text{(3.22)}$

The iteration procedure is terminated.

where $\varepsilon$ is a specified small value.

It may be remarked that when using floating point arithmetic it is often desirable to accumulate the sum of the squares of equation (3.10) in double precision, after computing $Y_i$ to single precision. Failure to do this can lead to erratic behaviour of $\phi$ near the minimum due only to rounding errors. In extreme cases it may also be necessary to compute $Y_i$ to double precision.
3.2 EXPERIMENTAL NOISE MEASUREMENT

3.2.1 Disc Noise

Metal-sheet discs were employed during the tests. These discs were notched along their periphery and bent slightly to produce a multibladed fan effect. The noise of these rotating discs has two main components. First, the discrete tones, which are integer multiples of the motor rotational speed and are caused by the transfer of energy from the blade to the air. In fact, the discrete frequency noise has the character of pure tones and is generated mainly by rotating equipment.

In the case of evenly spaced discs, the blade passing tone, which has the frequency of the product of the number of blades and the rotational speed (rev/sec) of the motor shaft, is the predominant discrete tone component. For the unevenly spaced discs, the amplitude of the discrete tones is dependent upon the spacing of the blades.

The second main component is broad band noise caused by vortex shedding from the blades. This part is essentially continuous and lacking any pure tones, with random amplitude and phase components. The characteristic of this component can be illustrated as that of a small radial segment of blade near the centre of the disc rotating through the air. A vortex tone would be produced, whose frequency relies upon the velocity of that segment. An adjacent segment farther out radially would also produce a pure tone, but at a
slightly higher frequency due to the higher linear velocity. Continuing this reasoning over the entire blade, and for each blade of fan, it is easy to see that the resultant noise is broad band in character.

3.2.2 Background Noise
This noise is composed of the drive motor noise and the room noise. The motor noise may be caused by such phenomena as rotational unbalance, rotor/stator interaction and slot harmonics. It is also produced by excitation of the natural vibration frequency of the motor structure, air movement and resonance of air chambers. The room noise is generated mainly from the ventilation ductings, gas and water pipes in the laboratory room. This duct noise is due to the ventilation fan noise and the duct wall vibration caused by the flow inside the ducts.

3.3 Manipulation of Data
The motor noise and room noise are not produced by the rotating disk. These undesired components partially mask the experimental results. Meanwhile, the broad band noise component will also obscure the analysis of the harmonic spectrum of the disks. For these reasons, it is necessary to eliminate or minimize the effect of the undesired noise from the measured result. Theoretically, such separation can be achieved by the decibel subtraction, if the sum of the total $L_p$ and the $L_p$ of individual sources are known.
basically, the total $L_p$ can be expressed as:

$$L_p = 10 \log \left( \frac{P_1^2}{P_0^2} + \frac{P_2^2}{P_0^2} + \frac{P_3^2}{P_0^2} + \cdots + \frac{P_i^2}{P_0^2} \right) \cdots (3.23)$$

where $L_p$ is the total sound pressure level (dB), $P_1, P_2, P_3, \ldots, P_i$ are the root mean square sound pressure of different sound sources (Pa), $P_0$ is the reference sound pressure ($2 \times 10^{-5}$ Pa).

Because of the arithmetic property of this expression, the sound pressure level of any individual sound source can similarly be expressed by rearranging equation (3.23). This applies not only to the overall noise spectrum but also to discrete frequencies. In the harmonic analysis, the discrete frequency tones, or the harmonic tones, of the discrete noise are the point of interest. Therefore, the decibel subtraction is applied to sound pressure levels at those harmonic frequencies.

The expression for the decibel subtraction is shown as below:

$$L_{i} = 10 \log \left( \frac{10^{L_{pi}/10} - 10^{bL_{pi}/10}}{10} \right) \cdots (3.24)$$

where $L_{pi}$ is the sound pressure level due to the blade spacing at $i$th harmonic frequency of the rotating disc.

$T_{pi}$ is the total sound pressure level of the system at $i$th harmonic frequency of the rotating disc.
is the total sound pressure level of the system at the ith harmonic frequency of the rotating disc without the harmonic tone (base line noise level).

According to equation (3.24), the discrete tones produced by the rotating disc can be obtained by subtracting the base line noise spectrum from the total sound pressure spectrum. In practice, it is hard to separate two noise components from one single sound source. In other words, the broad band noise will combine with the discrete tone noise in the harmonic spectrum. For the base line noise, a plain disc with the same dimensions and the same running speed as the notched discs was used and its noise spectrum was recorded. Hence the sound pressure level of the discrete tones with the broad band noise could be obtained by applying equation (3.24). Although the broad band noise components combine with the discrete tones, most of the time the discrete tones have higher amplitudes than the broad band noise and therefore, the effect of the interference of the broad band noise in the harmonic spectrum procurement is insignificant.

The experimental harmonic spectra are obtained in sound pressure level (dB), which is proportional to the sound power and differs from the theoretical spectra whose amplitudes represent relative force. Furthermore, different rotating speeds are tested on the same disc. Therefore,
higher amplitude are obtained at a higher rotating speed. The comparisons between the spectra of the same disc running at different speeds are difficult. For these reasons, more modifications have to be imposed on the experimental data. On the other hand, the theoretical results are only based on the relative amplitude of excitation force. As there is no datum for the reference value, further modification are also required here.

First, for the experimental harmonic spectra, the amplitudes in sound pressure level have to be changed to the relative sound pressure. This procedure is shown as follows:

\[ L_{pi} = 10 \log \left( \frac{P_i}{P_0} \right)^2 \]  \hspace{1cm} (3.25)

where

- \( L_{pi} \) is the sound pressure level at ith harmonic of a rotating disc.
- \( P_i \) is the root mean square sound pressure at ith harmonic of a rotating disc.
- \( P_0 \) is the reference pressure (2x10^{-5} Pa).

By rearranging equation (3.25), the relative sound pressure at ith harmonic can be obtained as

\[ \frac{P_i}{P_0} = \left( \frac{10L_{pi}}{10} \right)^{\frac{1}{2}} = 10^{L_{pi}/20} \]  \hspace{1cm} (3.26)

Secondly, these relative sound pressures have to be normalised. Because the comparisons will be made between the evenly spaced disc and the unevenly spaced discs, the amplitude of the relative sound pressure at the sixth
harmonic of the evenly spaced disc is used as the reference. The normalisation procedure is developed as follows:

\[ A_i^* = \frac{(P_i/P_o)}{(P_{6e}/P_o)} \quad \ldots \quad (3.27) \]

where

- \( A_i^* \) is the normalised amplitude of the \( i \)th harmonic of any rotating disc.
- \( P_i/P_o \) is the relative sound pressure of the \( i \)th harmonic of any rotating disc.
- \( P_{6e}/P_o \) is the relative sound pressure of the sixth harmonic of the evenly spaced disc running at the same speed as the comparison disc.

Similarly, the normalisation technique can be extended to the theoretical results. The expression for the normalised result is then:

\[ A_i^{**} = \frac{A_i}{A_{6e}} \quad \ldots \quad (3.28) \]

where

- \( A_i^{**} \) is the normalised calculated amplitude of the \( i \)th harmonic of any disc.
- \( A_i \) is the calculated amplitude of the \( i \)th harmonic of any disc.
- \( A_{6e} \) is the calculated amplitude of the sixth harmonic of the evenly spaced disc.

The relative power of the spectrum can be calculated by using the following formula:

\[ \hat{P} = \sum_{i=1}^{n} (A_{i^*})^2 \quad \ldots \quad (3.29) \]
where

\[ R_P \text{ is the relative power of harmonic spectrum.} \]

\[ A_m \] \[ * \text{ is the normalised amplitude at } i \text{th harmonic of the calculated harmonic spectrum.} \]

\[ n \] \[ \text{ is the number of blades on the disc plus one.} \]
Chapter IV

EXPERIMENTAL APPARATUS AND INSTRUMENTATION

In this chapter, the experimental apparatus and instrumentation employed in this study will be discussed. Generally speaking, the description will be brief, noting basic features of the instruments, the calibration procedure required for the instrument and any special aspects which may have been utilized.

4.1 APPARATUS

Nine discs were cut from 1 mm galvanized steel sheet with a diameter of 254 mm. One of the discs was left plain to provide the baseline rotational noise. The others had radial lines cut at different angular distributions. A 5 degree segment was then bent back at each cut to form in effect a very simple blade. The discs are shown in Figures 1 to 4.

The experimental set-up consisted of a Dodge model 25 SCM control and a SCM motor, which was used to drive the discs. The motor was mounted on a small table 10 inches above the base. Beneath the disc, a one and one half inch glass floor pad was placed on the base to reduce the sound wave and wind noise reflected by the hard surface of the
base. These reflected sound wave and wind induced noise may affect the accuracy of data procurement. A microphone cartridge, which was mounted on a tripod, was set at the same level as the centre of the disc and was located 25 cm away from the disc edge with a facing angle of 45°. A wind screen was put onto the microphone to minimize the wind induced noise effect. A 3m extension cable was connected between the microphone cartridge and the sound level meter. This ensured that the recording equipment was located well away from the microphone position. Hence, the effects of reflected sound were minimized. The FFT analyser was connected to the AC output of the sound level meter and its video output was connected to the SD 422 video printer. This set-up is shown in Figure 5.

4.2 DRUEL AND KJÆER 4230 SOUND LEVEL CALIBRATOR

The 36 x 4230 sound level calibrator is a pocket sized unit (11 mm long x 44 mm in diameter and 250 g in weight) powered by a 9-volt battery. The unit is mounted directly on the 1 inch or 1/2 inch microphone of the instrument to be calibrated. It provides a 94 dB signal (re. 2x10⁻⁵ Pa) at 1000 Hz (±1.3%). The accuracy of the sound pressure level is ±0.5 dB from 0 to 50°C. The influence of static pressure is ±0.15 dB/100 mbar between 500 mbar and 1100 mbar.
4.3 BRUEL AND KJAER 2204 SOUND LEVEL METER AND ITS ACCESSORIES

4.3.1 B&K 2204 Impulse Precision Sound Level Meter

The impulse precision sound level meter is a compact and portable instrument for precision sound and vibration measurements. The instrument conforms to IEC 179 for precision sound level meters, the proposed IEC recommendation for impulse precision sound level meters and DIN 45633 parts 1 and 2. The instrument is powered by three standard 1.5-volt batteries (size D), giving up to 3 hours of continuous operation. There are four weighting networks, 'C', 'B', 'A' and 'D'. The dynamic range with 1/2 inch microphone at the 'lin' setting between 2-40 KHz is from 60-154 dB. The frequency range with an 1/2" microphone Type 4113 is 0-20 KHz (±1 dB). The operating temperature range of the instrument is -20°C to 50°C. It has dimensions of 33(L) x 12(W) x 9(H) cm and has weight 2.7 Kg.

4.3.2 BRUEL and KJAER Microphone Stand UA0049

The stand is a lightweight portable tripod of rugged construction. It has rigid telescopic leg and can be used not only for supporting the B&K microphone but also for the somewhat heavier sound level meters.
4.3.3 Bruel and Kjaer Microphone Extension Cable AO0027
It is a 3 m long multi-core shield cable supplied with BSK microphone connectors at both ends. The diameter is 6 mm. The capacitance to ground of the signal conductor is approximately 100 pF/m. Each conductor is insulated with polyethylene and the outside of the cable has a grey PVC covering. The cable is specially designed for low leakage between the cores.

4.3.4 Bruel and Kjaer Flexible Extension Rod UA0196
It fits between the input stage of an impulse precision sound level meter Type 2204 and the half-inch condenser microphone. It is 21 cm long.

4.3.5 Bruel and Kjaer 4165 1/2" Condenser Microphone Cartridge
The Bruel and Kjaer Type 4165 1/2" microphone cartridge is a freefield condenser microphone which outputs a voltage proportional to the sound pressure level. It has a frequency response range from 3 Hz (-3dB) to 20 kHz(±2dB) for freefield 0° incidence. The total harmonic distortion remains less than 1% for the sound pressure level below 140 dB.
4.3.6 Brueil and Kjaer Type UA0237 Windscreen

The windscreen type UA0237 is a ball of specially prepared porous polyurethane sponge. It has a diameter of 9 cm and when in use, it is simply pushed on as far as it will go over the microphone (with protecting grid). It gives an effective reduction of the order of 10 dB or higher of wind induced noise at wind velocity of 20 mph or lower.

4.4 Spectral Dynamics SD 375 Dynamic Analyzer II

The SD 375 is a microprocessor-based FFT analyzer and signal processor that analyzes frequencies up to 100 KHz with 400-line per channel resolution. The output is a representation of the magnitude of the frequency components of an input signal. Analysis may be performed in 21 ranges from 1 Hz full scale to 100 KHz full scale in 1-2-4-5 sequence. The frequency accuracy is ±0.0025% of full scale with frequency response of ±0.5 dB at filter centers over entire frequency range up to 50 KHz and 50 KHz and ±1.0 dB at filter centers from 50 KHz to 100 KHz. The spectrum amplitude linearity is ±1 dB or ±0.05% of full scale to 70 dB below full scale, whichever is greater. A averager for signal-to-noise enhancement in the frequency domain is built-in. The number of averages can be specified by the number of ensembles (N). It has dimensions of 432(W) x 483(L) x 267(H) mm and has weight of 25 Kg. The operating temperature range is 5°C to 45°C.
4.5 SPECTRAL-DYNAMICS SD422 VIDEO PRINTER

The SD422 video printer produces archival-quality hard copies of spectral data directly from the raster scan CRT of SD375 FFT analyzer without the need for hardware or software interfacing. Only one BNC cable is needed. The unit weighs 7 kg and has dimensions of 285(W) x 400(L) x 110(H) mm. The normal picture size is 96(W) x 128(L) mm with a normal printing speed of 13.5 seconds per screen. The operating temperature is 0–40°C and the required power input is 120 volts ±10% at 50/60 cycle.

4.6 DODGE MODEL 25-SCR CONTROL AND SCR DRIVE MOTOR

The SCR control is a unit to control the speed of theSCR drive motor. This unit connects with a one-quarter hp. output DC motor. The required power input is 115 volts with one phase of 50/60 cycle. The SCR drive motor has continuous output of one-quarter hp. The DC field voltage and DC armature voltage are 100 and 90 volts respectively. The base rpm is 1750 with the armature current 3 amperes.

4.7 GENERAL RADIO ELECTRONIC STROBOSCOPE TYPE 1538-A

This instrument was used to measure the speed of the motor. The unit has the range of 110 to 150,000 flashes per minute in four direct-reading ranges of 110 to 690, 670 to 4170, 4000 to 25,000 and 24,000 to 150,000 rpm. The accuracy of this stroboscope is ±1% of reading on all ranges after
calibration against line frequency. The required power input is 100 to 125 or 195 to 250 volts, 50 to 400 Hz, 15W or 20 to 30VDC, 12W. It has dimensions of 270(W) x 170(H) x 160(L) mm and weighs 3.3 Kg.

4.8 **ADVANCE DS4000 DIGITAL STORAGE OSCILLOSCOPE**

The DS4000 digital storage oscilloscope provides real time display and storage of the pulse signal. It has vertical sensitivity of 5mV/cm to 20V/cm in 12 ranges with accuracy of ±3% in calibrated positions. The horizontal sensitivity is 1μs/cm to 20 sec/cm in 23 ranges (±3%). Power supply is 115 volts, 220 volts or 240 volts ±10% and 45-400 Hz. It has dimensions of 174(H) x 312(W) x 417(L) mm with weight 11 Kg. The operating temperature range is 0 to 50°C.

4.9 **HEWLETT PACKARD 7045A X-Y RECORDER**

The X-Y recorder is used to plot cartesian co-ordinate graphs from DC electrical information. The unit is portable and has dimensions of 400(L) x 456(H) x 165(W) mm. The 7045A instrument features high speed capacity and rapid acceleration to accurately record high frequency and fast moving input signals. The recorder includes 10 calibrated DC input ranges in each axis from 0.2mV/cm to 4V/cm. Accuracy is ±0.1% of full scale. The required input power is 115 volts ±10% and 50-4000 Hz range.
4.10 **IBM 370**

All theoretical results were determined by means of a digital computer (IBM 370), located in the Computer Centre, Essex Hall, University of Windsor.
Chapter V

PROCEDURE AND RESULTS

In this chapter, the computational and experimental procedures are described and the refined results are discussed. The first section is comprised of the computational procedures, computer applications and theoretical results. The second section deals with the experimental procedures and results. Comparisons between the results of regular and irregular samples are also included. Finally, comparisons between the refined experimental and theoretical results, sound pressure spectra of tested specimens and the sound pressure wave prediction method by graphics are discussed.

5.1 COMPUTATIONAL PROCEDURE AND THEORETICAL RESULTS

5.1.1 Procedure

The theoretical results were derived predominantly by means of a digital computer. The design of the optimal 6-blade disc and prediction of the spectrum amplitudes of harmonics of different discs are included.

The computational procedure of the optimal 6-blade disc is based on the formulations derived from Marquardt's algorithm for the nonlinear regression method. The design
procedure is shown in Chapter 3.1 and one iteration of the nonlinear regression technique is illustrated in Appendix A. In order to ensure that the result does not converge to the local minimum, an initially random search was applied and ten sets of random input data had been used to feed into the computer for the calculation. To prevent round off error, the software was written in double-precision arithmetic and is shown in Appendix B. The results included the ten sets of output data that were obtained. When the layouts of these findings were drawn, it was noted that, in general, only two distinctive blade orientation sets were attained. The computer results are shown in Table 1 and the layouts are also shown in Figures 6 to 15. The two distinctive designs are called Disc 01 and Disc 02. These discs will be used for comparison with the evenly spaced disc in terms of the harmonic spectrum. In papers previously discussed, different design techniques were applied and consequently different results were obtained. Discs based on these are named Doolan, Sinusoidal, Random-6, Close-together-6 and Random-5 and will be used for comparison with the computer designed and the evenly-spaced discs. The blade arrangement of Doolan is based on Doolan's 6-blade milling cutter design. The blade spacing of Sinusoidal has been calculated by using Donald Ewald's sinusoidal modulation principle [3]. The maximum blade angle change (the modulation amplitude, in this case is thirty degrees and the
one-fold modulation technique is used. The blade spacings of Random-6, Close-together-6 and Random-5 were randomly chosen by the author. The Random-6 has six blades scattered on the circular disc whereas Random-5 has only five. The Close-together-6 has all blades clustered on a semi-circular segment of the disc. The blade positions of these discs are illustrated in Figures 15 to 23. Using the same assumptions as applied to the 'optimal discs' (see Chapter 3.1), the harmonic spectra of these discs were derived by means of the Fourier Analysis and were subsequently evaluated by means of the digital computer. It was found that the higher harmonics generally contributed relatively little to the frequency spectra, and were often insignificant. Therefore, in the computational procedure, for all six-blade discs, only the amplitudes of the first seven harmonics of the motor shaft rotating speed were calculated. In the case of Random-5, the amplitudes of the first six harmonics were obtained. The spectrum power of the discs were also obtained by summing up the squares of the harmonic amplitudes (see equation 3.3).

The theoretical results are shown in Table 2. For convenience in comparisons, the results were normalised by dividing the harmonic amplitudes of all discs with the amplitude of the sixth harmonic of the Even Spaced (see equation 3.29). The new relative spectrum powers were also obtained from the normalised results (see equation 3.29).
The sample calculations of these manipulations are shown in Appendix C.1. The results obtained are listed in Table 3 and the harmonic frequency spectra of the discs are illustrated in Figures 24 to 31.

5.1.2 Results and Discussion

All the refined theoretical results are listed in Table 3 and in Figures 24 to 31. In Figure 24, the harmonic spectrum of "Even Spaced" is shown. There is only one component at the sixth harmonic and it is normalised to unity. Figures 25 to 31 show that for all unevenly spaced discs the amplitude of the sixth harmonic has been reduced and its sound energy has been redistributed to the lower harmonics. From the normalised results in Table 3, it is noted that all the unevenly spaced discs have a lower spectrum power than the Even Spaced. The spectrum power of the latter has been normalized to unity by equation 3.29. The corresponding values for Disc 01, Disc 02, Doolan, Sinojial, Random-6, Close-together-6 and Random-5 are 0.2306, 0.2810, 0.5506, 0.5238, 0.7347, 0.4691 and 0.4284 respectively. It can be seen that the computer designed discs (Disc 01 and Disc 02) have almost 72% reduction of sound power in comparison with the Even Spaced. Although the power reduction with the other discs is less, it is still significant. For the worst case, Random-6, the sound power is 26% lower than the Even Spaced.
5.2 EXPERIMENTAL PROCEDURE AND RESULTS

5.2.1 Procedure

Each of the nine discs was attached to the motor shaft to run at four different speeds: 840, 1320, 1680 and 2100 rpm. The noise level was measured with the sound level meter B&K 2204 and the frequency spectrum was obtained by means of the SD 375 FFT analyzer. After every run, the result was printed out by the SD 422 video printer.

In Figure 32 the spectrum of the plain disc represents the background noise at 2100 rpm which includes the room noise (duct noise from ventilation ducts, pipe flow noise from plumbing and steam pipes), motor noise and the aerodynamic noise of the rotating disc. Since the pressure of these noise sources was undesirable and obscured the basic results, they were eliminated by using the mathematical technique which has been discussed in Chapter III. Examples of these manipulations are shown in Appendix C.2. The final, refined data are listed in Table 4 to 7 and the harmonic spectra of different discs are drawn in Figures 35 to 36.

5.2.2 Discussion of Results

Figures 35, 36, 37 and 38 show the experimental harmonic spectrum of even spaced at different speeds. In those figures, it is noted that, the spectra have one main component with magnitude of one at the sixth harmonic and
some minor components probably generated by the unbalance of
the motor and the broad band noise overlapping at the lower
harmonics. Figures 39, 40, 41 and 42 show the harmonic
spectra of Disk 01. The second harmonics have the highest
amplitude around 3. The fourth harmonics also have a
relative high amplitude around 1.4. Figures 43, 44, 45 and
46 show that the harmonic spectra of Disk 32 consist mainly
of the first, second and third harmonics. The amplitudes of
the second harmonics are the highest in this case. They all
have the amplitude above 2. The harmonic spectra of Doolan
are shown in Figures 47 to 50. The second harmonics have
the highest amplitude, of magnitudes greater than 3 and
second highest are the third harmonics. Figures 51, 52, 53
and 54 show the harmonic spectra of Sinusoidal. In this
case, the first harmonic has the highest amplitude and the
fourth harmonic follows. Both harmonics have amplitudes
greater than one. The harmonic spectra of random-6 are
shown in Figures 55, 56, 57 and 58. The third harmonics
have the highest amplitudes in this case with magnitudes
greater than 2. Figures 59 to 62 show the harmonic spectra
of Close-together-6 at different speeds. The first
harmonics in these spectra have an extremely high amplitude.
They all exceed 4.5. The harmonic spectra of Random-5 are
shown in Figures 63 to 66. The second harmonic has the
highest amplitude in these spectra.
In general, each disc has the same spectral signature when it was run at different speeds. In comparison with the Even Spaced, the amplitudes of harmonics of the unevenly spaced discs tend to decrease when the running speed is increased. The amplitude of the sixth harmonics of the unevenly spaced discs are reduced, but some of the lower harmonics become very high. Many of them have amplitudes higher than the sixth harmonic of the Even Spaced. In the case of Close-together-6 (Figures 59 to 62), the first harmonic has relative amplitudes as high as six.

From the Tables 4 to 7, it is noted that the spectrum powers of the Even Spaced are around unity. For the modulated discs, the spectrum powers are much higher than one. The highest one occurred with the Close-together-6, which has spectrum power of 45.6 with running speed of 840 rpm (see Table 4). The lowest one occurred with the Sinusoidal, which has spectrum power of 5.8 with running speed of 1680 rpm (see Table 6). In comparison with the Even Spaced, these higher spectrum powers are caused by the higher amplitudes at the lower harmonic tones.

5.3 **Comparison of Theoretical and Experimental Results**

Theoretical results show that the unevenly spaced discs produce less acoustic power than the Even Spaced. This is due to the modulation effect which reduces the amplitude of the original discrete components and redistributes the
energy to the lower harmonics. Based on the assumptions, every blade generates the same acoustical power when passing through a stationary point during the operation.

The experimental results show that all the unevenly spaced discs give more sound power than the Even Spaced. The least spectrum power occurs with the Sinusoidal at 1680 rpm and has the relative power value of 6.32. The highest was with the close-together-6 at 840 rpm with the relative power value of 45.6. The discrepancy in magnitude between the theoretical and experimental results is very large. The possible reasons are as follows:

1. In the theoretical solution, the excitational signal is assumed to be impulsive and is represented by a rectangular pulse. In the experiment, the real pressure pulse has a continuously changing and complex shape with significantly longer duration.

2. Because of this, interaction between successive pressure pulses is likely to occur, particularly in the case of closely spaced blades.

3. The different width of the blade spacing was assumed to have no effect on the shape of the excitational pulse. This cannot be realised in practice.

To obtain an indication of this phenomena, the acoustic pressure variation with time was recorded and examined.
5.4 RECORDING OF SOUND PRESSURE VARIATION

The procedure used to record the sound pressure was the same as that for the recording of the frequency spectrum and the OS 4100 oscilloscope was also connected. When the running disc was in a steady state, the sound pressure signal was captured on the display of the oscilloscope by using the triggering switch. The pressure variation in the time domain was then printed on the HP 7045-A X-Y recorder. During this operation, all discs were running at 2000 rpm. The sound pressure waves of the different discs are illustrated in Figures 67 to 74.

5.5 DISCUSSION OF SOUND PRESSURE VARIATION

In the graph obtained from the evenly spaced disc (see Figure 57), it is noted that the shape of the pulse is similar to a sine wave with superimposed high frequency noise. The latter is probably caused by air turbulence. Generally speaking, this curve is similar to the noise signal of a conventional 6-blade fan. This causes the highest amplitude at the fundamental frequency of Even Spaced (6 x shaft speed). The corresponding harmonic spectra are shown in Figures 35, 36, 37 and 38.

In the case of Disc 01 (see Figure 68), there are two peaks for every cycle. Each curve consists of two subsidiary peaks before reaching the main crest. These combinations represent the pressure pulses generated by two
clusters of three blades each (see Figure 17). If the time
axis is converted to angular displacement, it can be seen
that the distribution of crests corresponds approximately to
that of the blades. It seems that, because of the close
spacing of the slots and the symmetric geometry, due to
interaction a stepwise pressure buildup takes place
resulting in only two effective pressure pulses. The system
has the general characteristics of a two-slot disc, as can
be seen from Figures 39, 40, 41 and 42. In each, the
fundamental pressure component has the frequency of 2 x
shaft speed. Comparison with the corresponding spectrum of
the calculated results, Figure 25 shows the same qualitative
conditions but very different magnitudes. Whereas the
calculated value of the fundamental component of Disc 01 is
only 0.4 of the magnitude of Even Spaced fundamental, in
experimental results, it is nearly three times greater (see
Figure 42). This significant quantitative difference must
be predominately due to the interaction between successive
pulses and the effects of turbulence in the experiment. The
forcing function has thus completely different
characteristics from those assumed for the theoretical
analysis.

Figure 49 shows the sound wave of Disc 02. One large
and one small peak are clearly evident in every cycle. The
large peak represents the pressure pulses generated by the
cluster of four blades and the small one represents the
pressure pulses generated by the other two close blades (see Figure 19). Noting the magnitudes of these pulses and the subsidiary crests along the curve, the phenomenon of the stepwise pressure buildup due to the close spacing of the slot is more apparent. The corresponding harmonic spectra are shown in Figures 43, 44, 45, and 46. Due to the irregularity of blade spacing, there is no obvious fundamental component. The second and third harmonics have the dominant amplitudes in the spectra. Comparison with the corresponding calculated spectrum, Figure 26 shows the same qualitative conditions but very different magnitudes. Whereas the calculated value of second and third harmonic are 0.3 and 0.4 respectively, in experimental results, both corresponding amplitudes are over 2 (see Figure 46). The cause of this significant quantitative difference is due to the same reason as in the case of Disc 01.

The sound pressure wave of Doolan is shown in Figure 70. There are three peaks in every cycle. These three peaks were generated by the three groups of blades (see Figure 19). The smallest peak was produced by the single-blade. The pressure pulse at the left of the smallest peak was generated by the cluster of three blades and the pulse at the right was generated by the cluster of two blades. The stepwise pressure buildup due to the interaction of successive pulses formed by clusters of blades and the disproportional increase in magnitude can be clearly seen.
The resulting harmonic spectra are shown in Figures 47, 48, 49 and 50. In each, the second and the third harmonics have dominant values. Comparison with the corresponding calculated spectrum in Figure 27 shows the same spectral pattern but differs in magnitudes.

Figure 71 shows the sound pressure variation of Simcoilal. There are four peaks formed by one cluster of three close blades and the three other separate blades in every cycle (see Figure 20). The first peak in the figure represents the sound pressure buildup due to the three close blades. The other three peaks represent the sound pressure pulses generated by the three separate blades. The harmonic spectra obtained from this disc are shown in Figures 51, 52, 53 and 54. The corresponding calculated spectrum is illustrated in Figure 23. It shows the same qualitative conditions as the experimental spectra but differs in magnitudes. For the first and second harmonics, the calculated values are 0.25 and 0.13, whereas the experimental values are 2.31 and 0.68 respectively.

Figure 72 shows the sound pressure variation of Hanlon-6. There are four peaks in every cycle. The first and the fourth peaks in the figure were generated by the two clusters of two blades each. The two other pressure pulses in between were produced by the other two single blades (see Figure 21). The harmonic spectra are shown in Figures 55, 56, 57 and 58. Comparison with the calculated spectrum in
Figure 29, shows again the significant differences in magnitudes. Whereas the experimental amplitudes of the first and third harmonics are 1.5 and 2.2 (see Figure 58), the calculated values are 0.3 and 0.4 respectively.

Figure 73 shows the sound pressure variation of close-together. In one cycle, there is only one large peak produced by the cluster of 6 close blades (see Figure 22). Because of the close spacing of slots, a stepwise pressure buildup takes place resulting in one effective pressure pulse. This system has the general characteristics of a one-slot disc, as can be seen in Figures 59, 60, 61 and 62. In each, the fundamental component has the frequency of the shaft speed. Comparison with the corresponding calculated spectrum, Figure 30 shows the similar qualitative conditions but very different magnitudes. Whereas the calculated value of the fundamental component is 0.5, in experimental results, it has value up to 6.3 (see Figure 59). The cause of this quantitative difference had been discussed in the previous cases.

Figure 74 shows the sound pressure wave of Random-5. Two peaks can be seen in every cycle. Both peaks are almost the same in magnitude. The narrower one represents the pressure pulses generated by the cluster of three blades and the other one represents the pressure pulses generated by the cluster of two blades (see Figure 23). The harmonic spectra of random-5 are shown in Figures 63, 64, 65 and 66.
Comparison with the corresponding calculated spectrum presented in Figure 31, shows the same qualitative conditions but significant differences in magnitudes. The first and second harmonic at the calculated spectrum are 0.25 and 0.42, whereas the experimental results are 2.2 and 3.2 respectively (see Figure 66).

In general, experimentally the unevenly spaced discs have higher harmonic amplitudes than the Even Spaced. This is because the interaction of the successive pulses, due to the close spacing of blades, causes the sound pressure build up which increases the magnitude of the sound pressure pulses. Hence, this results in higher harmonic amplitudes in the cases of unevenly spaced discs.

The Fourier Series prediction method provides a very good qualitative approximation in the harmonic analysis. The differences between the calculated and experimental values are due to the assumption of impulsive rectangular pressure pulses used in the prediction method and the interaction between successive pulses generated by closer spacing blades in the experiment.

5.5 Graphical Sound Pressure Wave

In the last section, it can be seen that the difference in harmonic amplitudes between the calculated and experimental results is mainly due to the interaction of the successive pulses of the blades, which had not been taken into
consideration in the mathematical development. The magnitude of the stepwise pressure buildup depends upon the number and the closeness of blades. Considering the elastic and viscous properties of air, the sound wave variation generated by the discs may be modeled. If a blade turns and passes a stationary point, the air particles are coerced to move. This causes the fluctuation of pressure and hence produces sound waves. When an equally spaced disc is considered, the blades pass a point constantly resulting in a constant fluctuation of sound pressure. Hence no stepwise pressure buildup takes place. In the case of unevenly spaced discs, two pulses are generated if two blades pass through the point of measure. If the blades are closely spaced, the second pulse reinforces the first before it diminishes in its magnitude, and hence increases the sound pressure. Consequently the sound pressure buildup occurs. Continuing this reasoning, a higher pressure buildup will be generated if the number of clustered blades increases. This pressure buildup appears to depend only upon the closeness of the blades spacing. Based on these reasons, an approximate graphical result of sound pressure variations can be made. To start drawing the sound pressure wave, some assumptions have to be followed.

1. All blades generating the pulses have the same amplitude.
2. The shape of the pulse is triangular in general and the span of the pulse or the evenly spaced along the horizontal axis is equal to its blade spacing angle projected on the horizontal axis.

3. The datum is set by drawing the pulses of the biggest cluster of blades. The static pressure line is set at the middle of the magnitude of the highest peak on the graph.

4. If any two consecutive blades have the spacing less than one half of the even spacing, the rising curve of the pulses due to these two blades are partly overlapping. The position of the start and end of the rising curve are determined by the location of the first and the second blade respectively. Application can be extended to multiple blades, if the spacings are close enough.

5. In drawing the declining slope of any pulse, it cannot be drawn to drop below the datum. When it drops down to the datum, it is assumed to reverse upward with the same rising slope.

To verify this graphical technique, the sound pressure wave of evenly spaced running at 2000 rpm was used as reference (see Figure 67). The sound pressure waves of the unevenly spaced discs are drawn and compared with the corresponding experimental results.
The horizontal distance, measured for one cycle (360 degrees) in Figure 57, is 13.2 cm and the average amplitude of the pulses is 4.7 cm. For the cases of 6-blade discs, the rising slope of the pulse is, therefore $4.7 / (13.2/12) = 4.27$ and the inclined angle is $76.82^\circ$ from horizontal. The negative slope of the pulse and the declined angle are $-4.27$ and $113.18^\circ$ respectively. The span of the pulse is 13.2 cm/2 = 2.2 cm. For the case of Random-5, the corresponding rising slope is $4.7 / (13.2/10) = 3.56$ and the inclined and declined angles are $74.31^\circ$ and $105.69^\circ$ respectively. The span of the pulse is 2.64 cm. Based on these data, the sound pressure wave of the eight discs running at 2000 rpm are drawn and illustrated in Figures 75 to 82.

Comparing the drawn pressure spectra (Figures 75 to 82) with the experimental results (Figures 57 to 74), the spectra of the corresponding discs are similar for the presence of high and low frequency noise due to air turbulence, which are superimposed on the pulses generated by the blades. As can be seen in the experimental results, the rising and declining slopes are not constant compared with the graphical result. This may be caused by the variation of air density which changes the pressure gradient. In general the graphical method provides a very good approximation in the description of sound pressure variations for the unevenly spaced discs.
Chapter VI
CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

1. Results obtained from the mathematical analysis indicate the potential for significant reduction of vibrations and noise.

2. Results obtained from experiments reveal that the modulated discs produce considerably higher noise levels than those predicted by theory.

3. It is shown that this is due to the assumption of a rectangular shaped, short duration pressure pulse for the theoretical analysis, which is not even remotely supported by the experimental results.

4. This problem is further exacerbated by the interaction of pressure effects from successive blades.

5. From experimental results, it is clearly evident that the pressure pulses are of relatively long duration and are of approximately sinusoidal shape. When blades are too closely spaced, interaction between successive pressure pulses takes place, resulting in a disproportionate pressure rise.
6. The Fourier analysis shows a good qualitative representation of the harmonic spectrum.
7. The graphical method of sound pressure wave construction shows satisfactory approximation in predicting wave shapes and magnitudes in the presence of the interaction effect.

6.2 RECOMMENDATIONS
1. Further studies should be undertaken to verify the graphical method in sound pressure wave prediction.
2. Additional work should be done to determine the relationship between the magnitude of the pressure pulses generated by the blades and their spacings.
3. Mathematical analyses should be performed to determine the harmonic spectrum of optimal blade spacing by using pulses with variable shapes and variable amplitudes.
Appendix A

ILLUSTRATION OF ONE ITERATION ON NONLINEAR REGRESSION TECHNIQUE

A.1 ONE ILLUSTRATIVE ITERATION OF NONLINEAR REGRESSION TECHNIQUE

Let there be a rotating disc with six blades. If we take the force to be an impulse with magnitude one and also assume that the amplification factor \( M(\omega) \) for the rotating system is unity, the amplitude of the resulting vibration or noise, \( S_n(\omega) \) is given by

\[
S_n(\omega) = \sum_{n=0}^{N} \left( B_n^2 + C_n^2 \right)^{1/2}
\]

where

\[
B_n = \frac{1}{\pi} \sum_{\ell=0}^{N} \sin \theta_{\ell}
\]

and

\[
C_n = \frac{1}{\pi} \sum_{\ell=0}^{N} \cos \theta_{\ell}
\]

Let the initial values for the six blade positions be

\[
\theta_0 = 0^\circ, \quad \theta_1 = 60^\circ, \quad \theta_2 = 119^\circ, \quad \theta_3 = 180^\circ, \quad \theta_4 = 240^\circ, \quad \theta_5 = 300^\circ
\]

Since the first blade can always remain at the zero position, this is essentially a problem of determining optimum values for five parameters viz; \( \theta_1, \theta_2, \theta_3, \theta_4 \) and \( \theta_5 \).
Now

\[ S_n(\omega) = A_n = (\sin^2 + \cos^2)^{1/2} \]

\[ = \left\{ \left( \sin \theta_1 + \sin \theta_2 + \sin \theta_3 + \sin \theta_4 + \sin \theta_5 \right)^2 + \left( \cos \theta_1 + \cos \theta_2 + \cos \theta_3 + \cos \theta_4 + \cos \theta_5 \right)^2 \right\}^{1/2} \]

\[ = \frac{1}{\pi} \left\{ \left( \sum_{i=1}^{5} \sin \theta_i \right)^2 + \left( 1 + \sum_{i=1}^{5} \cos \theta_i \right)^2 \right\}^{1/2} \]

(Note \( \theta_0 = 0° \).

\[ = \frac{\|A_n\|}{\pi}, \text{ say} \]

Let us consider the amplitude spectrum over seven significant harmonics, i.e. \( n = 7 \). For this example the steps of the first iteration are as follows.

**Step 1:** Calculate the values of \( S_i(\omega) \) for \( i = 1, 2, 3, 4, 5, 6, 7 \). They are 0.0056, 0.0111, 0.0167, 0.0222, 0.0278, 1.9084, 0.0389.

Now the error matrix \( Z \) is set up with each element \( Z_i = |J - S_i(\omega)|_{\theta = 0°} \) as follows:

\[
Z = \begin{bmatrix}
0-0.0056 & -0.0056 \\
0.0111 & -0.0111 \\
0.0167 & -0.0167 \\
0.0222 & -0.0222 \\
0.0278 & -0.0278 \\
1.9084 & -1.9084 \\
0.0389 & -0.0389 \\
\end{bmatrix}
\]

The initial sum of squares of the error is \( Z^TZ = 3.6452 \).
Step 2: Set up the derivative matrix $X$; each element $X_{ij}$ of the matrix being given by

$$X_{ij} = \frac{\partial a_i}{\partial \theta_j} \bigg|_{\theta = \theta^0}$$

where $i = 1, 2, 3, 4, 5, 6, 7$ and is the number of harmonics,

$j = 1, 2, 3, 4, 5$ and is the number of parameters and

$\theta^0$ is the vector of initial values of blade positions.

$$X_{ii} = 1/(2\pi [AN]^2)$$

$$X = 2 \left( \sum_{i=1}^{n} \sin \theta_i \right) n \cos \theta_j - 2 \left( 1 + \sum_{i=1}^{n} \cos \theta_i \right) n \sin \theta_j$$

and, therefore, with $\theta^0 = (0^\circ, 60^\circ, 119^\circ, 180^\circ, 240^\circ, 300^\circ)$

$X_{12}$, for example, $= \frac{1}{2\pi} x \frac{1}{0.017+5} x (2x-0.0041567-2x0.013286)$

$= -0.3183$

Similarly, all elements of $X$ are calculated:
Step 3: Obtain $X'X$


Step 4: Obtain the scaled matrix $D^{-\frac{1}{2}}X'XD^{-\frac{1}{2}}$ where $D$ is the diagonal matrix from $X'X$

$$D = \begin{bmatrix} 3.5410 & 0 & 0 & 0 & 0 \\ 0 & 10.5391 & 0 & 0 & 0 \\ 0 & 0 & 3.1219 & 0 & 0 \\ 0 & 0 & 0 & 3.5410 & 0 \\ 0 & 0 & 0 & 0 & 10.5125 \end{bmatrix}$$

and

$$D^{-\frac{1}{2}} = \begin{bmatrix} 0.5314 & 0 & 0 & 0 & 0 \\ 0 & 0.3080 & 0 & 0 & 0 \\ 0 & 0 & 0.5660 & 0 & 0 \\ 0 & 0 & 0 & 0.5314 & 0 \\ 0 & 0 & 0 & 0 & 0.3080 \end{bmatrix}$$

$$D^{-\frac{1}{2}}X'XD^{-\frac{1}{2}} = \begin{bmatrix} 1.0300 & 0.3246 & 0.9905 & -0.6885 & -0.6715 \\ 0.3246 & 1.0000 & 0.3025 & -0.6718 & -0.6145 \\ 0.9905 & 0.3025 & 1.0000 & -0.6864 & -0.6420 \\ -0.6885 & -0.6718 & -0.6864 & 1.0000 & -0.3261 \\ -0.6715 & -0.6145 & -0.6420 & 3.3261 & 1.0000 \end{bmatrix}$$
Step 5: frame $\lambda I$ where $\lambda$ is a constant representing the step size and is chosen as a small value, say 0.6

$$
\begin{align*}
\lambda I &= \begin{pmatrix}
0.6 & 0 & 0 & 0 & 0 \\
0 & 0.6 & 0 & 0 & 0 \\
0 & 0 & 0.6 & 0 & 0 \\
0 & 0 & 0 & 0.6 & 0 \\
0 & 0 & 0 & 0 & 0.6
\end{pmatrix}
\end{align*}
$$

$$
\therefore \quad \left( D^{\frac{1}{2}} X' X D^{\frac{1}{2}} + \lambda I \right)^{-1}
= \begin{pmatrix}
1.6000 & 0.3246 & 0.9905 & -0.5885 & -0.6715 \\
0.3246 & 1.6000 & 0.3025 & -0.5718 & -0.6145 \\
0.9905 & 0.3025 & 1.6000 & -0.5864 & -0.6420 \\
-0.5885 & -0.6718 & -0.6420 & 1.0000 & 0.3261 \\
-0.6715 & -0.6145 & -0.5864 & 0.3261 & 1.6000
\end{pmatrix}
$$

and

$$
\left( D^{\frac{1}{2}} X' X D^{\frac{1}{2}} + \lambda I \right)^{-1}
= \begin{pmatrix}
1.1390 & 0.0643 & -0.5122 & 0.2470 & 0.2468 \\
0.0643 & 3.8721 & 0.0792 & 0.3626 & 0.3198 \\
-0.5122 & 0.0792 & 1.1202 & 0.2498 & 0.2140 \\
0.2470 & 0.3526 & 0.2498 & 0.9607 & 0.1474 \\
0.2468 & 0.3198 & 0.2140 & 0.1474 & 0.9073
\end{pmatrix}
$$
Step 6: Obtain the $X'Z$ matrix

$$
\begin{bmatrix}
0.0983 \\
-0.1341 \\
0.0940 \\
-0.0081 \\
-0.0494
\end{bmatrix}
$$

Step 7: Obtain the new vector of blade positions $\theta_n$ as

$$\theta_n = \theta^0 + D^{-\frac{1}{2}}(D^{-\frac{1}{2}}X'X0^{-\frac{1}{2}} + \lambda I)^{-1}D^{-\frac{1}{2}}X'Z$$

For this $\theta_n$ the value of $\Delta n(\omega)$ is calculated and its sum of squares over $n$ is compared with the original one. The iterative scheme is carried on until the difference between any two successive sums of squares is less than a specified value ($0.001$ is chosen). This implies that there is no significant reduction in the sum of squares of the spectra amplitudes. The corresponding $\theta_n$, therefore, gives the optimum blade positions.
Appendix B

COMPUTER PROGRAM FOR DESIGNING OPTIMAL SPACING DISCS

C*******************************************************************************
C        WNER=1000.0 IS THE INITIAL 'GEZZ ERROR' VALUE
C        NUBLD=NUMBER OF BLADES CONSIDERED IN THE PROBLEM
C        NUSIHA=NUMBER OF SIGNIFICANT HARMONICS CONSIDERED
C        j(j)=AMPLITUDE OF HARMONICS
C        Z=ERROR MATRIX
C        T=TRANSPOSE OF Z
C        A=PRODUCT OF SUBROUTINE FOR MATRIX MULTIPLICATION
C        M=TRANSPOSE OF MATRIX INVERSION
C        D=DERIVATIVE MATRIX
C        X=TRANSPOSE OF X
C        D=DIAGONAL MATRIX OF D
C        DINV=INVERSE OF SQUARE ROOT OF D
C*******************************************************************************

DIMENSION IR(5),IC(5)
REAL S(10),AN(10),THETA,PHA(10),Z(10,1),Z(1,10)
REAL SSUM(10),XT(10,10),CSUM(10),PIL,D(10,10)
REAL DT,W(20),X(10,10),DINV(10,10)
WER=100.0000
PI=3.141592653500
NUBLD=6
NUSIHA=7
NUPDF=NUBLD-1
REAL, (PHA(K),K=1,NUPDF)
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REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
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REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)
REAL, (PHA(K),K=1,NUPDF)}
5 SSUM (J) = SIN(SUM J)
    AN (J) = COS(SUM **) 2 + SIN(SUM **) 2
    S (J) = DSQR T (AN (J) ) / PI
    4 (J, J1) = - S (J)
    ZT (1, J) = - S (J)
    CONTINUE

C********************************************************************
CALL MAMP1D (ZT, 1, 10, Z, 10, 1, 1, NUSIHA, 1, W, 20)
C********************************************************************
C DO LOOP FOR CALCULATING DERIVATIVE MATRIX X AND ITS
C TRANSPOSE
C********************************************************************
    DO 7 K = 1, NUSIHA
    DO 6 KK = 1, NUMDF
    PHI = K * PHI (K) * PI / 180.0000
    X (K, KK) = (0.5) * PI * (1.00 / DSQR T (AN (K) ) ) * ( (2.00 * SSUM (K)
    * K * COS (PHI) ) * - 2.00 * SSUM (K) * K * DSIN4 (PHI) )
    AT (KK, K) = X (K, KK)
    CONTINUE
    CONTINUE

C********************************************************************
CALL MAMP2D (XT, 10, 10, X, 10, 10, NUMDF, NUSIHA, NUMDF, W, 20)
DO 9 JJ = 1, NUMDF
DO 10 JJJ = 1, NUMDF
B (JJ, JJJ) = 3.0000
VINHA (JJ, JJJ) = 0.0000
CONTINUE
   9 CONTINUE
   10 CONTINUE
    DO 11 MM = 1, NJMDF
    DO (NN, MM) = X (MM, MM)
    VINHA (MM, MM) = DSQR T (1.0000 / X (MM, MM))
    CONTINUE

C********************************************************************
C OBTAIN THE NEW VECTOR OF BLADE POSITIONS
C********************************************************************
CALL MAMP2D (VINHA, 10, 10, X, 10, 10, NUMDF, NUMDF, NUMDF, W, 20)
CALL MAMP1D (X, 10, 10, VINHA, 10, 10, NUMDF, NUMDF, NUMDF, W, 20)
DO 12 NN = 1, NJMDF
X (NN, NN) = X (NN, NN) + 0.0000
CONTINUE
CALL MAMP2D (VINHA, 10, 10, X, 10, 10, NUMDF, NUMDF, NUMDF, W, 20)
CALL MAMP1D (X, 10, 10, VINHA, 10, 10, NUMDF, NUMDF, NUMDF, W, 20)
CALL MAMP2D (X, 10, 10, VINHA, 10, 10, NUMDF, NUMDF, NUMDF, W, 20)
CALL MAMP1D (X, 10, 10, XT, 10, 10, NUMDF, NUMDF, 1, W, 20)
DO 13 JJ = 1, NJMDF
PHA (IJ) = PHA (IJ) + X (IJ, 1)
CONTINUE

C********************************************************************
PRINT *, 'ZT=' , ZT (1, 1)
IF (DABS (SINEI - ZT (1, 1)) .LT. 0.00000000) GO TO 14
Appendix C

SAMPLE CALCULATION

The results obtained from the theoretical development and the primary experiment cannot be compared directly. Therefore, some modifications and normalisation have to be done on these data to attain valuable results for further analysis.

C.1 THEORETICAL DATA REFINEMENT

To normalise the theoretical results listed in Table 2, the following procedure is used. Consider the second harmonic of the Disc 01, its calculated amplitude is 0.7675 and the calculated amplitude of the sixth harmonic of the even spaced is 1.9099. Applying equation 3.28, the amplitude of the second harmonic of Disc 01 becomes

\[ A_2 = \frac{0.7675}{1.9099} = 0.4019 \]

This result is listed in Table 3.

Similarly, the rest of the data can be normalised by using the same procedure as shown above. To develop the relative spectrum power of the discs, equation 3.29 is applied when all the harmonic amplitudes of different discs at different speeds have been obtained. For instance, the relative spectrum power of Disc 01 is calculated as:

\[ WP = \sum_{i=1}^{i+1} A_i^2 \]
= (0.0006)^2 + (0.4019)^2 + (0.0017)^2
+(0.3192)^2 + (0.0029)^2 + (0.1307)^2 + (0.0941)^2
= 0.2806

This result is listed in Table 3.

C.2 REFINEMENT OF EXPERIMENTAL DATA

To eliminate the baseline noise in the experiment, equation 3.24 is applied. Considering the sixth harmonic of plain disc, Even Spaced and Disc 01 running at 2100 rpm and thereafter, the sound pressure levels of those discs are as follows:

\[ L_p \text{ of plain disc} = 38.6 \text{ dB} \quad \text{(see Figure 32)} \]

\[ L_p \text{ of Even Spaced} = 83.4 \text{ dB} \quad \text{(see Figure 33)} \]

\[ L_p \text{ of Disc 01} = 59.7 \text{ dB} \quad \text{(see Figure 34)} \]

After application of equation 3.24, \( L_p \) of Even Spaced becomes

\[ 10 \log \left(10^{83.4/10} - 10^{38.6/10}\right) = 83.4 \text{ dB} \]

Similarly, \( L_p \) of Disc 01 becomes 59.7 dB.

To obtain the normalised results, the following procedure is performed. From equation 3.26, the relative sound pressure of Disc 01 is calculated as:

\[ \frac{p_6}{p_0} = 10^{59.7/10} = 10^{59.7/20} \]

The relative sound pressure of Even Spaced is shown as:

\[ \frac{p_6}{p_0} = 10^{83.4/10} = 10^{83.4/20} \]
Applying equation 3.27, the normalised amplitude of Even Spaced is obtained as:

\[ A_6 \text{ of Even Spaced} = 10^{ \frac{83.4}{20} } \frac{xPo}{10} \frac{83.4}{20} xPo \]

\[ = 1.0 \]

and the result from Disc 01 becomes

\[ A_6 \text{ of Disc } 01 = 10^{ \frac{59.7}{20} } \frac{xPo}{10} \frac{83.4}{20} xPo \]

\[ = 0.0657 \]

The relative spectrum power can be obtained by equation 3.29 (see list section). Same procedures were being applied to the rest of the experimental data, and the results obtained are listed in Table 4 to 7.
REFERENCES


Figure 1: Even Spaced and Disc 01

Figure 2: Disc 02 and Doolan
Figure 3: Sinusoidal and Random-6

Figure 4: Close-together-6 and Random-5
Figure 5: Experimental apparatus used in tests
Figure 6: Blades spacing resulting from input data set #1

Figure 7: Blades spacing resulting from input data set #2
Figure 8: Blades spacing resulting from input data set #3

Figure 9: Blades spacing resulting from input data set #4
Figure 10: Blades spacing resulting from input data set #5

Figure 11: Blades spacing resulting from input data set #6
Figure 12: Blades spacing resulting from input data set # 7

Figure 13: Blades spacing resulting from input data set # 8
Figure 14: Blades spacing resulting from input data set #9

Figure 15: Blades spacing resulting from input data set #10
Figure 16: Layout of Even Spaced

Figure 17: Layout of Disc 01
Figure 18: Layout of Disc 02

Figure 19: Layout of Doolan
Figure 20: Layout of Sinusoidal

Figure 21: Layout of Random-6
Figure 22: Layout of Close-together-6

Figure 23: Layout of Random-5
Figure 24: Analytical harmonic spectrum of Even Spaced

Figure 25: Analytical harmonic spectrum of Disc 01
Figure 26: Analytical harmonic spectrum of Disc 02

Figure 27: Analytical harmonic spectrum of Doolan
Figure 28: Analytical harmonic spectrum of Sinusoidal

Figure 29: Analytical harmonic spectrum of Random-6
Figure 30: Analytical harmonic spectrum of Close-together-6

Figure 31: Analytical harmonic spectrum of Random-5
Figure 32: Frequency spectrum of plain disc and amplitudes of corresponding harmonics at 2100 rpm
Figure 33: Frequency spectrum of Even Spaced and amplitudes of corresponding harmonics at 2100 rpm
<table>
<thead>
<tr>
<th>X</th>
<th>Frequency (Hz)</th>
<th>Y Amplitude (Y(A))</th>
<th>DBR</th>
<th>AVG N</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>76.3</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>92.7</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
<td>63.2</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>85.6</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>5</td>
<td>175</td>
<td>64.7</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>6</td>
<td>210</td>
<td>59.7</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>7</td>
<td>245</td>
<td>52.1</td>
<td>DBR</td>
<td>AVG N 300</td>
</tr>
<tr>
<td>8</td>
<td>280</td>
<td>68.7</td>
<td>DBR</td>
<td>AVG N 300</td>
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Figure 34: Frequency spectrum of Disc 01 and amplitudes of corresponding harmonics at 2100 rpm
Figure 35: Measured harmonic spectrum of Even Spaced at 340 rpm
Figure 36: Measured harmonic spectrum of Even Spaced at 1320 rpm
Figure 37: Measured harmonic spectrum of Even Spaced at 1680 rpm
Figure 38: Measured harmonic spectrum of Even Spaced at 2100 rpm
Figure 39: Measured harmonic spectrum of Disc 01 at 340 rpm
Figure 40: Measured harmonic spectrum of Disc 01 at 1320 rpm
Figure 41: Measured harmonic spectrum of Disc 01 at 1680 rpm
Figure 42: Measured harmonic spectrum of Disc 01 at 2100 rpm
Figure 43: Measured harmonic spectrum of Disc 02 at 840 rpm
Figure 44: Measured harmonic spectrum of Disc 02 at 1320 rpm
Figure 45: Measured harmonic spectrum of Disc 02 at 1680 rpm
Figure 46: Measured harmonic spectrum of Disc 02 at 2100 rpm
Figure 47: Measured harmonic spectrum of Doolan at 340 rpm
Figure 48: Measured harmonic spectrum of Doolan at 1320 rpm
Figure 49: Measured harmonic spectrum of Doolan at 1680 rpm
Figure 50: Measured harmonic spectrum of Doolan at 2100 rpm
Figure 51: Measured harmonic spectrum of sinusoidal at 840 rpm
Figure 52: Measured harmonic spectrum of Sinusoidal at 1320 rpm
Figure 53: Measured harmonic spectrum of Sinusoidal at 1680 rpm
Figure 54: Measured harmonic spectrum of Sinusoidal at 2100 rpm
Figure 55: Measured harmonic spectrum of Random-6 at 340 rpm
Figure 56: Measured harmonic spectrum of Random-6 at 1320 rpm
Figure 57: Measured harmonic spectrum of Random-6 at 1680 rpm
Figure 58: Measured harmonic spectrum of Random-6 at 2100 rpm
Figure 59: Measured harmonic spectrum of Close-together-6 at 840 rpm
Figure 60: Measured harmonic spectrum of Close-together-6 at 1320 rpm
Figure 61: Measured harmonic spectrum of Close-together-6 at 1680 rpm
Figure 62: Measured harmonic spectrum of Close-together-6 at 2100 rpm
Figure 63: Measured harmonic spectrum of Random-5 at 340 rpm
Figure 64: Measured harmonic spectrum of Random-5 at 1320 rpm
Figure 65: Measured harmonic spectrum of Random-5 at 1680 rpm
Figure 66: Measured harmonic spectrum of Random-5 at 2100 rpm
Figure 67: Experimental sound pressure wave of Even Spaced
Figure 69: Experimental sound pressure wave of Disc 02
Figure 71: Experimental sound pressure wave of Sinusoidal
Figure 72: Experimental sound pressure wave of Random-6
Figure 73: Experimental sound pressure wave of Close-together-6
Figure 75: Graphic sound pressure wave of Even Spaced
Figure 76: Graphic sound pressure wave of Disc 01
Figure 77: Graphic sound pressure wave of Disc 02.
Figure 78: Graphic sound pressure wave of Doolan
Figure 79: Graphic sound pressure wave of sinusoidal
Figure 81: Graphic sound pressure wave of Close-together-6
Figure 82: Graphic sound pressure wave of Random-5
<table>
<thead>
<tr>
<th>SET#</th>
<th>INPUT</th>
<th>DATA</th>
<th>OUTPUT</th>
</tr>
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<td></td>
<td>INITIAL</td>
<td>SPACINGS</td>
<td>(IN DEGREE)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>90.0</td>
<td>150.0</td>
<td>230.0</td>
</tr>
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<td>2</td>
<td>30.0</td>
<td>60.0</td>
<td>120.0</td>
</tr>
<tr>
<td>3</td>
<td>62.0</td>
<td>119.0</td>
<td>185.0</td>
</tr>
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<td>50.0</td>
<td>129.0</td>
<td>180.0</td>
</tr>
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<td>5</td>
<td>60.0</td>
<td>120.0</td>
<td>180.0</td>
</tr>
<tr>
<td>6</td>
<td>50.0</td>
<td>119.0</td>
<td>180.0</td>
</tr>
<tr>
<td>7</td>
<td>48.0</td>
<td>98.0</td>
<td>169.0</td>
</tr>
<tr>
<td>8</td>
<td>50.0</td>
<td>119.0</td>
<td>180.0</td>
</tr>
<tr>
<td>9</td>
<td>60.0</td>
<td>119.0</td>
<td>180.0</td>
</tr>
<tr>
<td>10</td>
<td>49.0</td>
<td>100.0</td>
<td>170.0</td>
</tr>
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Table 1: Input and output data of the computer program
<table>
<thead>
<tr>
<th>NO. OF HARMONICS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>SPECTRUM POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN SPACED</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.910</td>
<td>0.000</td>
<td>3.648</td>
</tr>
<tr>
<td>DISC 01</td>
<td>0.001</td>
<td>0.768</td>
<td>0.003</td>
<td>0.610</td>
<td>0.006</td>
<td>0.250</td>
<td>0.008</td>
<td>1.023</td>
</tr>
<tr>
<td>DISC 02</td>
<td>0.274</td>
<td>0.561</td>
<td>0.710</td>
<td>0.056</td>
<td>0.316</td>
<td>0.154</td>
<td>0.069</td>
<td>1.025</td>
</tr>
<tr>
<td>DOOLAN</td>
<td>0.203</td>
<td>0.746</td>
<td>0.761</td>
<td>0.603</td>
<td>0.494</td>
<td>0.227</td>
<td>0.417</td>
<td>2.009</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td>0.483</td>
<td>0.245</td>
<td>0.000</td>
<td>0.791</td>
<td>0.845</td>
<td>0.527</td>
<td>0.039</td>
<td>1.911</td>
</tr>
<tr>
<td>RANDOM-6</td>
<td>0.311</td>
<td>0.210</td>
<td>0.806</td>
<td>0.511</td>
<td>0.897</td>
<td>0.457</td>
<td>0.784</td>
<td>2.680</td>
</tr>
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<td>CLOSE-TOGETHER-6</td>
<td>1.013</td>
<td>0.378</td>
<td>0.354</td>
<td>0.386</td>
<td>0.165</td>
<td>0.479</td>
<td>0.106</td>
<td>1.711</td>
</tr>
<tr>
<td>RANDOM-5</td>
<td>0.481</td>
<td>0.799</td>
<td>0.318</td>
<td>0.245</td>
<td>0.682</td>
<td>0.259</td>
<td>-----</td>
<td>1.563</td>
</tr>
</tbody>
</table>

* Refer to rotating speed of the motor.

Table 2: Analytical harmonic amplitudes and relative spectrum power of tested discs
<table>
<thead>
<tr>
<th>NO. OF HARMONICS *</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>RELATIVE SPECTRUM POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN SPACED</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>DISC '01</td>
<td>0.001</td>
<td>0.402</td>
<td>0.002</td>
<td>0.319</td>
<td>0.003</td>
<td>0.131</td>
<td>0.004</td>
<td>0.281</td>
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<tr>
<td>DISC 02</td>
<td>0.144</td>
<td>0.294</td>
<td>0.372</td>
<td>0.030</td>
<td>0.166</td>
<td>0.081</td>
<td>0.036</td>
<td>0.281</td>
</tr>
<tr>
<td>DOOLAN</td>
<td>0.106</td>
<td>0.390</td>
<td>0.398</td>
<td>0.316</td>
<td>0.259</td>
<td>0.119</td>
<td>0.218</td>
<td>0.551</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td>0.253</td>
<td>0.128</td>
<td>0.000</td>
<td>0.414</td>
<td>0.442</td>
<td>0.276</td>
<td>0.020</td>
<td>0.524</td>
</tr>
<tr>
<td>RANDOM-6</td>
<td>0.163</td>
<td>0.110</td>
<td>0.422</td>
<td>0.268</td>
<td>0.470</td>
<td>0.239</td>
<td>0.411</td>
<td>0.735</td>
</tr>
<tr>
<td>CLOSE-TOGETHER-6</td>
<td>0.530</td>
<td>0.198</td>
<td>0.186</td>
<td>0.202</td>
<td>0.087</td>
<td>0.251</td>
<td>0.056</td>
<td>0.469</td>
</tr>
<tr>
<td>RANDOM-5</td>
<td>0.252</td>
<td>0.418</td>
<td>0.167</td>
<td>0.128</td>
<td>0.357</td>
<td>0.136</td>
<td>-----</td>
<td>0.428</td>
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</table>

* Refer to rotating speed of the motor.

Table 3: Normalised analytical harmonic amplitudes and relative spectrum power of tested discs.
<table>
<thead>
<tr>
<th>NO. OF HARMONICS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN SPACED</td>
<td>0.474</td>
<td>0.218</td>
<td>0.164</td>
<td>0.107</td>
<td>0.054</td>
<td>0.030</td>
<td>1.315</td>
</tr>
<tr>
<td>DISC 01</td>
<td>0.550</td>
<td>3.661</td>
<td>1.78</td>
<td>1.412</td>
<td>0.125</td>
<td>0.051</td>
<td>15.83</td>
</tr>
<tr>
<td>DISC 02</td>
<td>1.326</td>
<td>2.611</td>
<td>2.483</td>
<td>2.222</td>
<td>2.022</td>
<td>0.556</td>
<td>15.18</td>
</tr>
<tr>
<td>DOOLAN</td>
<td>1.347</td>
<td>3.790</td>
<td>2.918</td>
<td>2.022</td>
<td>0.603</td>
<td>0.151</td>
<td>26.55</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td>2.960</td>
<td>3.790</td>
<td>2.918</td>
<td>2.022</td>
<td>0.603</td>
<td>0.151</td>
<td>12.68</td>
</tr>
<tr>
<td>RANDOM-6</td>
<td>1.652</td>
<td>1.790</td>
<td>1.212</td>
<td>0.748</td>
<td>0.236</td>
<td>0.292</td>
<td>13.39</td>
</tr>
<tr>
<td>CLOSE-TOGETHER-6</td>
<td>6.342</td>
<td>4.064</td>
<td>1.171</td>
<td>0.493</td>
<td>0.676</td>
<td>0.175</td>
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</tr>
<tr>
<td>RANDOM-5</td>
<td>2.681</td>
<td>4.064</td>
<td>1.171</td>
<td>0.493</td>
<td>0.676</td>
<td>0.175</td>
<td>25.87</td>
</tr>
</tbody>
</table>

Refer to the rotating speed of the motor.

Table 4: Normalised harmonic amplitudes and relative spectrum power of tested discs at 840 rpm.
<table>
<thead>
<tr>
<th>NO. OF HARMONICS</th>
<th>NORMALISED AMPLITUDES</th>
<th>RELATIVE SPECTRUM POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN SPACED</td>
<td>0.160 0.237 0.121 0.077 0.059 1.000 0.055</td>
<td>1.106</td>
</tr>
<tr>
<td>DISC 01</td>
<td>0.510 3.467 0.164 1.413 0.124 0.121 0.060</td>
<td>14.14</td>
</tr>
<tr>
<td>DISC 02</td>
<td>1.174 1.882 1.513 0.127 0.121 0.035 0.022</td>
<td>7.260</td>
</tr>
<tr>
<td>DOOLAN</td>
<td>1.260 3.715 3.020 1.202 0.646 0.139 0.159</td>
<td>26.42</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td>2.654 0.827 0.107 1.585 0.933 0.347 0.061</td>
<td>11.26</td>
</tr>
<tr>
<td>RANDOM-6</td>
<td>1.611 1.300 2.630 1.084 1.000 0.229 0.217</td>
<td>13.48</td>
</tr>
<tr>
<td>CLOSE-TOGETHER-6</td>
<td>5.886 1.797 1.188 0.733 0.232 0.275 0.095</td>
<td>39.96</td>
</tr>
<tr>
<td>RANDOM-5</td>
<td>2.654 4.027 1.135 0.436 0.646 0.159 0.230</td>
<td>25.23</td>
</tr>
</tbody>
</table>

* Refer to the rotating speed of the motor.

Table 5: Normalised harmonic amplitudes and relative spectrum power of tested discs at 1320 rpm
<table>
<thead>
<tr>
<th>NO. OF HARMONICS</th>
<th>NORMALISED AMPLITUDES</th>
<th>RELATIVE SPECTRUM POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td>EVEN SPACED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.219</td>
</tr>
<tr>
<td>DISC 01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>2.919</td>
</tr>
<tr>
<td>DISC 02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.746</td>
<td>2.189</td>
</tr>
<tr>
<td>DOOLAN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.772</td>
<td>3.128</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.879</td>
<td>0.700</td>
</tr>
<tr>
<td>RANDOM-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.098</td>
<td>1.084</td>
</tr>
<tr>
<td>CLOSE-TOGETHER-6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.705</td>
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</tr>
<tr>
<td>1</td>
<td>2.023</td>
<td>3.313</td>
</tr>
</tbody>
</table>

* Refer to the rotating speed of the motor.

Table 6: Normalised harmonic amplitudes and relative spectrum power of tested discs at 1680 rpm
<table>
<thead>
<tr>
<th>NO. OF HARMONICS</th>
<th>NORMALISED AMPLITUDES</th>
<th>RELATIVE SPECTRUM POWER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>EVEN SPACED</td>
<td>0.156</td>
<td>0.175</td>
</tr>
<tr>
<td>DISC 01</td>
<td>0.440</td>
<td>2.917</td>
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<tr>
<td>DISC 02</td>
<td>0.357</td>
<td>2.138</td>
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<tr>
<td>DOOLAN</td>
<td>1.011</td>
<td>3.020</td>
</tr>
<tr>
<td>SINUSOIDAL</td>
<td>2.317</td>
<td>0.683</td>
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<tr>
<td>RANDOM-6</td>
<td>1.531</td>
<td>1.083</td>
</tr>
<tr>
<td>CLOSE-TOGETHER-6</td>
<td>5.012</td>
<td>1.333</td>
</tr>
<tr>
<td>RANDOM-5</td>
<td>2.238</td>
<td>3.162</td>
</tr>
</tbody>
</table>

* Refer to the rotating speed of the motor.

Table 7: Normalised harmonic amplitudes and relative spectrum power of tested discs at 2100 rpm
VITA AUCTORIS

1956
Born in Kowloon, Hong Kong on January 30.

1976
Completed high school at Central Peel Secondary School, Brampton, Ontario, Canada in June.

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