STABILITY OF SOIL-STEEL STRUCTURES.

ABDELRAHIM KHALIL MOHAMED. DESSOUKI

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STABILITY OF SOIL-STEEL STRUCTURES

by

Abdelrahim Khalil Mohamed Dessouki

A Dissertation
Submitted to the School of Graduate Studies through
the Department of Civil Engineering in Partial
Fulfillment of the Requirements for the
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1985
Abdelrahim Khalil Mohamed Dessouki
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To my parents and my wife
ABSTRACT

The stability of soil-steel structures is examined in this research. The analysis accounts for the behaviour and the induced stresses in soil and conduit during construction and loading.

An analytical procedure using plane strain finite elements has been developed to study the different aspects of soil-steel structure instability. The conduit is represented by beam-column finite elements with the capability of accommodating plastic hinges when necessary. Interface elements of a spring type are used to represent the linkage between the two materials. Linear and constant strain elements are used to simulate the soil media, depending on the severity of the stress gradient of the structure.

Two soil models are used to model the behaviour of the soil elements, a nonlinear elastic hyperbolic model as well as an elasto-plastic soil model. The parameters defining these models are obtained through triaxial compression tests.

Geometric and material nonlinearities are considered in the analysis. Direct iterations are used to update the material properties. Failure in the soil elements is identified and followed by correction to the stress state in the whole structure.

Instability of the structure, whether initiated by
yielding of the soil mass above the conduit, excessive deflection of the conduit, buckling of the metallic conduit or by instability of the soil-steel structure is predicted.

The results obtained by the computer program are checked with a series of laboratory tests carried out for this purpose. Comparisons with tests reported in other references and with Code Specifications are also made. Reasonable agreement is reached between the analytical and experimental results. Based on this agreement, a parametric study of the variables controlling the soil-steel structure stability is carried out. The height of soil cover, span and shape of the conduit, load type, rigidity of the conduit walls and interface stiffness are among these variables.
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<tr>
<td>A</td>
<td>cross-sectional area per unit length of the conduit wall</td>
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<tr>
<td>a</td>
<td>parameter for elasto-plastic model</td>
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<td>c</td>
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<tr>
<td>D₁</td>
<td>deflection lag factor for the IOWA Formula</td>
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<td>D¹⁺²⁺⁻</td>
<td>variables defined for the incrementalization procedure</td>
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<td>d</td>
<td>crown deflection in the IOWA Formula</td>
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<td>parameter for the hyperbolic volume change</td>
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<td>d₁ - d₉</td>
<td>coefficients of the constitutive matrix</td>
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<td>E</td>
<td>modulus of elasticity of the conduit material</td>
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<td>E'</td>
<td>modulus of soil reaction</td>
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<tr>
<td>Eᵢ</td>
<td>initial tangent modulus</td>
</tr>
<tr>
<td>Eₜ</td>
<td>tangent modulus</td>
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<td>Eᵣᵤᵦ</td>
<td>unloading-reloading modulus</td>
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base for natural logarithm

eccentricity of moving loads

ten-fold increase in confining pressure

Equivalent forces defined for the stress transfer technique

plastic yield surface

buckling stress

plastic collapse yield surface

plastic expansive yield surface

yield stress

value of initial Poisson's ratio at a confining pressure equal to the atmospheric pressure

plastic potential function

plastic collapse potential function

plastic expansive potential function

vertical diameter of conduit

height of soil cover above conduit crown

average cover height

soil height below conduit

cross-section moment of inertia per unit length of conduit

stress invariants

modulus number for hyperbolic soil model

unloading-reloading modulus number

bedding constant for the IOWA Formula
\( k \) number of yield surfaces
\( k_i \) dimensionless stiffness number for interface model
\( k_n \) unit normal stiffness
\( k_s \) unit tangential stiffness
\( k_{si} \) initial unit tangential stiffness
\( k_{sr} \) reduced unit tangential stiffness
\( L \) length of a beam element
\( LR, TP \) parameters for lower and upper triangles
\( L_{11}, L_{22}, L_{12} \) coefficients defining plastic multipliers
\( L' \) length of interface corresponding to one interface element
\( i \) exponent parameter for peak plastic work
\( M \) moment per unit length of conduit
\( M_p \) plastic hinge moment
\( M_{pr} \) reduced plastic hinge moment
\( M_y \) moment at first yield
\( m \) load increment number
\( m_1 \) modulus exponent for hyperbolic soil model
\( m_2 \) modulus exponent for expansive yield function
\( n \) iteration number
\( n_1 = T/T_y \)
\( n_2 \) modulus exponent for elastic strains
\( n_s \) stiffness exponent for interface model
\[ P \text{ coefficient for peak plastic work} \]
\[ P_b \text{ soil pressure on bottom of conduit} \]
\[ P_c \text{ vertical pressure at crown of conduit} \]
\[ P_h \text{ soil pressure on haunches of conduit} \]
\[ P_z \text{ concentrated live load} \]
\[ P_{si} \text{ soil pressure on sides of conduit} \]
\[ P_t \text{ soil pressure on top of conduit} \]
\[ P_1 \text{ equivalent live load pressure} \]
\[ P_2 \text{ soil pressure around the conduit} \]
\[ p \text{ modulus exponent for plastic collapse work} \]
\[ p_a \text{ atmospheric pressure} \]
\[ p_i \text{ uniform live load} \]
\[ p_1 \text{ collapse exponent for plastic collapse function} \]
\[ q \text{ parameter for plastic work-hardening law} \]
\[ R \text{ radius of circular conduit} \]
\[ R_1 \text{ parameter for plastic potential function} \]
\[ R' \text{ radius of curvature} \]
\[ R_b \text{ bottom radius of conduit} \]
\[ R_f \text{ failure ratio parameter for soil} \]
\[ R_h \text{ haunch radius of conduit} \]
\[ R_s \text{ side radius of conduit} \]
\[ R_{sf} \text{ failure ratio parameter for interface} \]
\[ R_t \text{ top radius of conduit} \]
\[ S \text{ span of conduit} \]
$S_1$  parameter for plastic potential function
$S_e$  elastic section modulus
$s$  surface of a body
$T$  thrust in conduit wall
$T_y$  thrust in conduit wall at yield
$T'$  temperature
$T_1, T_2$  coefficients defining the plastic multipliers
$t$  finite element thickness taken as unity
$t'$  time
$t_1$  parameter for plastic potential function
$U$  global displacement
$u$  displacement in the $x$-direction
$V$  volume of a body
$v$  displacement in the $y$-direction
$W$  load per unit length of conduit
$w$  moisture content
$W_c$  total plastic collapse work per unit volume
$W_p$  total plastic expansive work per unit volume
$W_{p,\text{peak}}$  value of $W_p$ at peak
$W_{p,60}$  value of $W_p$ at 60\% of failure-stress level
$X, Y$  global cartesian coordinates
$x'$  anisotropy
$x, y$  local cartesian coordinates
$z$  plastic section modulus
$\theta$  angle measured from the crown of the conduit
\( a_1 \) coefficient for total plastic work
\( a_i \) coefficients of the displacement functions
\( b_1 \) coefficient for total plastic work
\( \gamma \) unit weight of soil
\( \gamma_w \) unit weight of water
\( \gamma_{xy} \) shear strain
\( \dot{\varepsilon}_s \) relative tangential displacement
\( \varepsilon \) strain
\( \varepsilon_a \) axial strain
\( \varepsilon_r \) radial strain
\( \varepsilon_v \) volumetric strain
\( \varepsilon_x \) strain in x-direction
\( \varepsilon_y \) strain in y-direction
\( \varepsilon_{yd} \) yield strain
\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) total strains in principal directions
\( \varepsilon_{1p}, \varepsilon_{2p}, \varepsilon_{3p} \) plastic strains in principal directions
\( \phi \) angle of internal friction for soil
\( \phi' \) angle of friction between soil and metal
\( \phi_0 \) angle of internal friction for a confining pressure equal to the atmospheric pressure
\( \theta \) rotation of a beam element end
\( \theta_h \) relative hinge rotation
\( \theta_0 \) angle separating the conduit into active and passive zones
\( \Delta P \) reduction in \( \theta \) corresponding to a ten-fold increase in confining pressure
\( v \)  

Poisson's ratio

\( v_i \)  

initial Poisson's ratio

\[ v_p = - \frac{\Delta e_3^p}{\Delta e_1^p} \]

\( v_t \)  

tangential Poisson's ratio

\( \sigma \)  

stress

\( \sigma_1 \)  

major principal stress

\( \sigma_2 \)  

intermediate principal stress

\( \sigma_3 \)  

minor principal stress

\( \sigma_{1f} \)  

major principal stress at failure

\( \sigma_{3f} \)  

minor principal stress at failure

\( \sigma_{1i} \)  

major principal stress at the start of an incremental analysis

\( \sigma_{3i} \)  

minor principal stress at the start of an incremental analysis

\( \sigma_{1m} \)  

modified major principal stress after failure

\( \sigma_n \)  

average normal stress at interface element

\( \sigma_x \)  

stress in x-direction

\( \sigma_y \)  

stress in y-direction

\( \sigma_z \)  

stress in z-direction

\( \sigma_{1-3} \)  

stress difference

\( (\sigma_{1-3})_f \)  

stress difference at failure

\( (\sigma_{1-3})_{ult} \)  

ultimate stress difference

\( \tau_f \)  

average shear stress at failure for interface element

\( \tau_s \)  

average shear stress in interface element

\( \tau_{ult} \)  

ultimate shear stress in interface element
\( \tau_{ij} \)  
*shear stress components*

\( \lambda, \lambda \sigma, \lambda p \)  
*plastic multipliers*

\( \eta_p \)  
*total potential energy*

\( \gamma_1 \)  
*value of yield function at failure*

\( \gamma_2 \)  
*parameter for plastic potential function*

\( \{a\} \)  
*vector of interpolation coefficients /

\( \{e\} \)  
*strain vector*

\( \{e^e\} \)  
*elastic strain vector*

\( \{e^p\} \)  
*plastic strain vector*

\( \{e^c\} \)  
*plastic collapse strain vector*

\( \{e^p\} \)  
*plastic expansive strain vector*

\( \{e_{pk}\} \)  
*plastic strain vector associated with k'th yield surface*

\( \{g\} \)  
*stress vector*

\( \{g\}_f \)  
*stress vector at failure*

\( \{b_c\}, \{b_p\} \)  
*vectors defining the elasto-plastic stiffness matrix*

\( \{F\} \)  
*equivalent modal force vector for the stress transfer technique*

\( \{q\} \)  
*nodal displacement vector*

\( \{P\} \)  
*load vector*

\( \{U\} \)  
*global displacement vector*

\( \{u\} \)  
*local displacement vector*

\( \{X\} \)  
*body force vector*
\[ \delta \] interpolation matrix
\[ \delta_s \] interpolation matrix evaluated along the surface of an element
\[ [A] \] matrix containing nodal coordinates terms
\[ [B] \] matrix relating strain vector to vector of interpolation coefficients
\[ [D] \] modulus matrix, or constitutive matrix, or elasticity matrix
\[ [D^e] \] linear elasticity matrix
\[ [D^{ep}] \] elasto-plastic stress-strain matrix
\[ [D_{kk}] \] matrix defined for the derivation of the elasto-plastic stress-strain matrix
\[ [k] \] global element stiffness matrix
\[ [k_e] \] local element stiffness matrix
\[ [k_e^c] \] local stiffness matrix for constant strain triangular element
\[ [k_e^t] \] local stiffness matrix for linear strain triangular element
\[ [k_e^b] \] global elastic stiffness matrix for beam element
\[ [k_g^b] \] global geometric stiffness matrix for beam element
\[ [k_b] \] global stiffness matrix for beam element
\[ [k_e^i] \] local stiffness matrix for interface element
\[ [k_e^q] \] local stiffness matrix for a nine-noded quadrilateral element
\[ [k_{CST}] \] stiffness matrix for constant strain triangular element
\[ [k_{LST}] \] stiffness matrix for linear strain triangular element
[\mathbf{L}] \quad \text{matrix defined for the incrementalization procedure}

[\bar{\mathbf{T}}] \quad \text{surface traction vector}

[\mathbf{V}] \quad \text{Diagonal matrix defined for the incrementalization procedure}
CHAPTER I

INTRODUCTION

1.1 GENERAL

Long span underground steel conduits are currently used in highway and railway bridges as an economic substitute to the conventional bridges. These soil-steel structures are generally easier and faster both in the design and construction processes than the equivalent conventional bridges. Average savings ranging between 30-35% have been reported in Canada, U.S. and Australia, upon using these types of low cost bridges. They have been used for decades as sewers or drainage structures in highway and municipal works. The large number and size of installed structures indicates more confidence in the design and performance of these structures. An example of these structures is shown in Fig. 1.1.

A number of shapes for these long span flexible conduits have been used for different purposes. They are generally made of large diameter and thin walled materials. They may cover the opening totally or sometimes partially. Among the shapes commonly used are circular pipes, vertical and horizontal ellipses, inverted pear type circular low
Figure 1.1 Soil-Steel Bridge Structure.
and high profile arches, or circular pipe arches. Some of these are shown in Fig. 1.2.

Underground metallic structures are usually assembled in situ, of cold formed corrugated sheets (steel or aluminum). The sheets may also be of variable thickness. They are pre-shaped to the required configuration and then assembled by bolts. Longitudinal stiffeners (beams or angles), thrust beams and relieving slabs are sometimes used depending on the necessity. Granular backfill materials which do not experience much changes in their behaviour after construction are generally used rather than cohesive soils.

The steel structure without soil support is usually flexible and resists only a small localized load before it yields or buckles. The interaction of the steel structure with the surrounding backfill material is the only reason for the significant load carrying capacity of the soil-steel structure. It is also evident that the stiffness properties of the soil media completely control the failure mode of the metallic conduit and the corresponding failure loads.

Flexible soil-steel conduits are designed and checked against one or more of the following possible failure modes.

1. Wall crushing, which occurs if the compressive stresses due to thrust exceed the wall's axial strength.
Figure 1.2 Common Shapes for Long Span Flexible Conduits.
2. Separation of the jointed elements at the seams which takes place when the thrust exceeds the seam strength.

3. Local or global elastic buckling of the steel conduit.

4. Inelastic buckling of the conduit with the formation of plastic hinges.

5. Bearing failure of soil which may occur when a relatively small and shallow conduit is subject to heavy loads.

6. Soil failure above the conduit including tension failure, shear failure or plastic yielding and the corresponding large deformations.

In order to predict the behaviour of these soil-steel flexible conduits a number of features therefore have to be exhibited by the analysis procedure, such as elastic or elasto-plastic behaviour at large deflections, local or global types of instability and the development of plastic hinges. The current design practices are addressing the different failure criteria as being independent. However, each of the variables controlling the soil-steel structure instability influences each other and are all interrelated to varying degrees under varying conditions. To date, only empirical formulae and simple approximate theories are available to handle the stability problem of the soil-steel
structures. Simplified simulations such as dividing the structure into two regions, one under active soil pressure and the other under passive pressure, or assuming a particular pressure distribution around a separate ring, are the main approaches used by researchers and codes to analyze the problem. Because of the limitations of the available analyses, and the need to have an analysis that can handle the construction process, compaction loading, soil instability and metallic conduit local or global buckling, the importance of this research is evident.

1.2 OBJECTIVES OF THE RESEARCH

The following are the objectives of the research:

1. Develop a theoretical solution to predict the possible modes of failure by employing an elasto-plastic soil model and by introducing plastic hinges to the conduit walls.

2. Investigate the effect of soil failure on the conduit performance.

3. Study the different types of failure of the conduit (wall crushing, snap-through buckling or global instability).

4. Study the effect of the different parameters on the failure loads, such as the height of soil cover, span of conduit, conduit shape and the conduit stiffness.
5. Define the soil parameters required for the analysis by conventional triaxial tests.

6. Compare the theoretical failure loads with experimental values.
CHAPTER II

LITERATURE SURVEY

The problem of soil-steel structure interaction and stability has been dealt with by many researchers who used simplified theories, empirical formulae or very complicated finite element idealizations. A brief discussion of the available approaches is presented in this Chapter as an introduction to the plane strain finite element approach used in this research.

2.1 FORCE AND LOAD DISTRIBUTION

Marston was the first to investigate the force analysis in a buried conduit (88). Spangler (110, 111) modified and extended Marston's theory to flexible conduits. Their theory is partly an estimation of an effective vertical soil column transferring load directly to the conduit and partly an assumption of variable pressure distribution around that conduit. Figure 2.1 shows the assumed simple pressure distribution used in their method where \( P_c \) is the overburden pressure along with any distributed live load acting at the top of the conduit. Approximate values for thrust and moments at the crown and shoulders are estimated as well and are shown
Figure 2.1 Pressure, Thrust and Moment Distributions Assumed in the Marston-Spangler Method
in the same figure. The crown deflection based on Spangler’s method is given by:

\[ d = \frac{D_1 \cdot X_1 \cdot W \cdot R^3}{EI + 0.061 E'R^3} \]  

(1.1)

where:

- \( D_1 \) = deflection lag factor to compensate for the volume change of the soil with time,
- \( X_1 \) = bedding constant which varies with the angle of bedding,
- \( W \) = load on conduit per unit length,
- \( R \) = radius of the conduit,
- \( EI \) = conduit wall stiffness per unit length in psi units, and
- \( E' \) = modulus of soil reaction.

Equation 1.1 is also known as IOWA Formula which is used in culvert design allowing for 5% of the conduit diameter as maximum crown deflection. Incipient collapse is assumed when the crown deflection reaches 20% of the conduit diameter, which is considered by Okeagu (97) as excessively high. Watkins (116) showed that a conduit wall may fail by ring buckling long before the 20% limit is reached and Bakht (10) showed that for culverts under shallow covers the assumption of a pressure distribution extending over the full span is overconservative.
The ring compression theory, developed by White and Layer (118), assumes the conduit as a thin ring in compression. The pressure distribution and thrust in the conduit walls are shown in Fig. 2.2. If the conduit is of non-circular shape the pressure distribution takes the shape shown in Fig. 2.3.

The different variables appearing in Figs. 2.2, 2.3 are defined as:

$h, h'$ are the soil cover and average soil cover heights,

\[ P_1, P_c, P_2, P_t, P_{si}, P_b, P_h \]

are the equivalent live load, pressure and the soil pressure at crown, all around conduit, top, side, bottom and haunches of the conduit, respectively,

\[ R', R_t, R_s, R_b, R_h \]

are the conduit radius of curvature and radius at top, side, bottom and at haunches, respectively,

$S$ is the conduit span, and

$T$ is the thrust in conduit walls.

Watkins (117) estimated the thrust in the conduit wall due to live load based on a pressure transfer coefficient that accounts for the arching effect of dead loads. The equivalent uniform load acting at the crown level is based on Boussinesq's theory of force effects on an elastic half
Figure 2.2 Pressure Distribution Assumed in Ring Compression Theory for Circular Cross Sections.
Figure 2.3 Pressure Distribution Assumed in Ring Compression Theory for Non-Circular Sections
space, even for large cavities. Because the load is dispersed differently in the horizontal and transverse directions (10) and because of assuming that arching applies to dead and live loads, the analysis is considered inaccurate.

Based on a finite element analysis and equivalent line loads obtained from Boussinesq's theory, the Kaiser Aluminum Method (97) provided expressions for thrusts and bending moments due to live loads. Tests by Bakht (10) showed that live load effects decrease quite rapidly with the depth of cover which is in contradiction with the results estimated by this method.

Burns (16) developed a closed form solution to analyze a soil-steel structure as an elastic cylindrical shell embedded in an isotropic elastic medium of infinite extent. The method can consider or neglect the friction between the conduit and soil around it. Assumptions that over-simplify the problem such as assuming the conduit to be embedded to at least 1 1/2 times its diameter in a weightless, homogeneous, isotropic and linearly elastic medium, limit the range of application of this method.

Dar and Bates (23) and Ray and Krček (66) used Burn's assumptions to develop equations and curves for deformations, bending moments and thrusts in Cartesian and polar coordinates for a soil-steel structure.

Considering the conduit problem as a frame on
elastic supports, which is a more advanced approach, was proposed by Drawsky (31). He replaced the conduit by a segmented ring surrounded by a system of radial springs. Later, Kloppel and Glock (69), assumed the structure to have two interacting zones of earth pressure, an active earth pressure zone defined by the angle \( \theta_0 \) and a passive earth pressure zone. The soil is replaced by discrete elastic springs and the conduit by a two-dimensional polygon as shown in Figure 2.4. The active pressure is then due to the movement of the soil towards the conduit and the passive pressure is activated by the movement of the conduit wall towards the supporting fill. A radial pressure in the form of a cosine wave is assumed to be acting as shown in the figure. Constant values for the coefficient of soil reaction have been used in the analysis, while Okeagu (97) developed an expression for a variable coefficient based on results from a finite element analysis by Hafez (50).

Plane-strain finite elements have been used extensively for the analysis of the buried conduit problem. Brown (15) developed a finite element program for flexible culverts under high fill. Allgood and Takahashi (7), Kirkland and Walker (68), Abel, Mark and Richards (4), recently developed similar programs.

Lebnards and Stetkar (80) compared the available programs and concluded that the most accurate and general is CANDE (Culvert Analysis and Design) developed by
Figure 2.4 Structural System and Loading Assumed by Kloppel and Glock
Katona, Smith, Odello and Allgood (63). This program can handle several conduit materials and uses a Hardin non-linear soil model (54). Programs that employ the hyperbolic stress-strain relationship for the soil were developed by Duncan (35, 36, 37, 38).

The previous programs have various limitations such as specifying the soil cover above the conduit, neglecting the compaction or misrepresenting the sequential construction. A more complete finite element program was presented by Hafez (50) who considered the compaction and construction processes in a more rational way.

2.2 SOIL-STRUCTURE INTERACTION

The phenomena of arching and pressure redistribution in soil-steel structures were identified by many researchers following Marston and Spangler. Experimental work by Luscher and Hoeg (82, 83), showed that the vertical load on a flexible conduit may be reduced or increased as it deflects in the vertical direction. Positive arching was noticed by Hoeg (57) to decrease the vertical pressure on the conduit crown by 30% of the applied burden pressure, which is advantageous. The load reduction, as reported by Allgood and Ciani (6), is dependent on the degree of compaction of the sand; more reduction is observed in dense sand than in loose sand. The unfavourable negative arching is also expected in some situations like the case of a
flexible conduit under a deep cover. This case has been reported by Davis and Bacher (24) where the conduit attracted more load than the overburden pressure applied. The arching effect was also reported by Howard (59), Howard and Selander (60) and Leonards (80).

While testing metallic and plastic tubes buried in sand, Hoeg (57), Howard (59) and Howard and Selander (60), noticed that the pressure on the sides of the conduit was increased beyond the vertical pressure. They noticed that as the structure deflected under vertical loading the sides of the structure pushed into the adjacent soil and the side pressure increased. The sequence of construction is also expected to influence the magnitude and distribution of loads, stresses and deformations in a flexible buried conduit as shown by Selig (106, 107) and Selig and Calabrese (108).

2.3 STABILITY AND STRENGTH ANALYSIS

As mentioned earlier, Spangler (110, 111), based on his experimental investigations, defined the incipient collapse of a circular flexible pipe as occurring when the vertical diameter decreases by 20%. An allowable value of \( \Delta X/R \) of 5% is considered as acceptable for design purposes, where \( \Delta X \) is the horizontal deflection of the pipe defined in a similar way to equation 1.1, and \( R \) is the conduit radius. Meyerhof and Balkie (90)
developed an approximate solution for the buckling of embedded cylindrical shells by incorporating solutions developed by Timoshenko and Gere (114) for the buckling of a flat plate on a Winkler type subgrade.

Watkins (116) noticed experimentally that under certain conditions of the soil and under certain values of conduit wall stiffness, the conduit experienced large ring deformation. However, under other conditions, it failed by ring buckling of the wall before the increase in horizontal diameter reached 20% and is controlled considerably by the conduit radius. Forrestall and Herrmann (44) developed a full analytical solution for the buckling of a shell embedded in an elastic medium, while Duns and Butterfield (40) examined the buckling of a cylindrical shell embedded in smooth contact with an elastic medium of infinite extent.

Chelapati and Allgood (17) suggested different results for the critical buckling stress in terms of a constant modulus of soil reaction and Cheney (19) analyzed the problem of buckling of a cylindrical shell embedded in bonded contact with an elastic medium of infinite extent. Sooy (13) examined the possibility of failure due to unstable conditions in which the conduit walls could develop buckling waves before deformations become sufficiently large to cause collapse.
The values of the modulus of soil reaction used in these different analyses is mainly assumed constant irrespective of any soil-conduit deformations. Equations defining its values were reported by Meyerhof and Baikie (90), Luscher (84), Meyerhof (91), Howard (61) and the Ontario Highway Bridge Design Code (98). Okeagu (97) derived an expression for the nonlinear modulus of subgrade reaction based on results from a finite element analysis. This expression is controlled by the deformation at the interface of the soil-conduit.

A common assumption used in the previously mentioned researches and theories is that the radial boundary pressure is assumed uniform. The model suggested by Kloppel and Glock (69) and shown in Fig. 2.4 is the only deviation and is the basis for other researches (1, 97, 98). They used an energy method to define the snap-through buckling of the elastically supported upper arch of the conduit and gave upper and lower limits for the load carrying capacity of the conduit which took care of the probable formation of yield hinges. Abdel-Sayed (1) reviewed the different analyses and suggested values of the angles that separates the two zones of the conduit, $\theta$, in terms of a flexibility factor $EI/E'R^3$. He also recommended different values of the flexibility factor for the upper arch and the sides and bottom of the conduit. Selvadurai (109) discussed
the techniques that have been used in the analytical study of soil-steel structure interaction analyses. The work of Kloppel and Glock (69), and Abdel-Sayed (1) has been extended recently by Okeagu (97) who applied the variational principle to the total potential energy. He used a variable modulus of subgrade reaction based on the analysis by Hafez (50). Some analysis drawbacks such as the dispersion of load between soil top and conduit, type of loading applied, the simulation and possibility of soil surrounding the conduit to fail, and the lack of comparison with experimental work were noted.
CHAPTER III

ANALYTICAL DEVELOPMENT AND PROCEDURE

3.1 GENERAL

A finite element approach is used as the method of analyzing the soil-steel structure interaction problem.

Constant strain triangular elements are used to represent the soil media below and away from the conduit, while linear strain quadrilateral elements are used in regions of high stress gradient above and around the conduit. Both a hyperbolic stress-strain model and an elasto-plastic model are used to update the soil properties in the analysis. Beam elements which can accommodate the nonlinearity and possibility of plastic hinge formation are used for the conduit wall. A spring type interface element simulating the sliding or separation between the soil media and culvert wall, is used.

A load increment procedure accompanied by an iterative analysis technique is performed to represent the construction process and follow the failure of soil elements or conduit collapse. The analysis procedure carried out to handle this problem can be summarized as explained below.
The complete soil-steel structure is divided into soil layers and elements and the conduit into beam finite elements. Each layer of soil elements below the steel conduit is generated and analyzed according to an iterative procedure. The whole steel conduit is added along with another row of soil elements and interface elements between the conduit and soil and the analysis procedure is repeated. Additional layers of soil elements and interface elements are then introduced.

The iterative analysis procedure carried out when each new layer of elements is introduced (gravity and compaction forces are applied during construction) and when live load increment is applied to the completed structure can be summarized as follows:

Stresses in interface elements are calculated and stresses in failing elements are corrected. Forces in beam elements are calculated and plastic hinges are identified. Stresses in soil elements are calculated. If any element fails the excess stresses are transferred to the structure and the element properties are updated simultaneously.

Details of the stiffness matrices development and the iterative procedures followed are given within the contents of this chapter.
3.2  **FINITE ELEMENT MESH AND CHARACTERISTICS**

The most complete idealization to the soil-steel interaction problem using a finite element analysis is a three-dimensional analysis to take any variable soil or loading pattern in the longitudinal conduit direction, and the time dependent properties into account. To keep the computational time and costs reasonable, a plane-strain condition is assumed for the cross-section under consideration where a unit thickness is assumed for all elements in the mesh.

In the analysis described herein, the soil elements are taken as membrane elements while the conduit walls are taken as conventional beam-column elements. The finite element mesh shown in Fig. 3.1 describes the simulation of the interaction problem for ranges of soil cover up to one half of the conduit diameter. When the soil cover exceeds one half of the conduit diameter the finite element mesh shown in Fig. 3.2 is used where a reasonable height to width ratio of the elements is observed. The number of elements above the conduit is reduced to one layer of elements when the soil cover is extremely shallow, again to keep the height/width ratio at a reasonable value.

In both cases, the triangular elements are constant
Figure 3.1 Finite Element Mesh for a Circular Cross-Section.
Figure 3.2 Finite Element Mesh for a Deep Soil Cover.
strain elements which are a good approximation to the soil behaviour in regions where sharp changes in stresses do not exist. Basically these are below and away from the conduit. Linear strain quadrilateral elements are used in the regions of high stress gradient above and around the conduit walls. They are much superior in their behaviour prediction to the constant strain elements and they give a better and more accurate picture for the initiation and propagation of cracks and failure in the soil media. Comparison of results using constant strain triangular elements against higher order elements are given by Brebbia and Connor (14). The experience of Finn and Miller (43) has been that one linear strain triangular element is about equivalent to 6-10 constant strain elements. When an elliptically shaped conduit is to be analyzed the same described mesh can be used.

3.3 **FINITE ELEMENT FORMULATION**

The variational principle of minimum potential energy where the displacements are used as primary unknowns, is used to formulate the element stiffness matrices in this study.

In the variational analysis a displacement function, usually a polynomial, describing uniquely the state of
displacements within an element is assumed. The total potential energy of a linear structural system can then be expressed in a matrix form, as:

\[ \mathcal{V}_p = \frac{1}{2} \int (\varepsilon)^T \{c\} \, dV - \int (\mathcal{N})^T \{\bar{X}\} \, dV - (\{u\}^T \{\bar{T}\} \, dS) \]  

(3.1)

where,

- \( \mathcal{V}_p \) = the total potential energy,
- \( \{\varepsilon\} \) = the strain vector,
- \( \{\sigma\} \) = the stress vector,
- \( V \) = the volume of the element,
- \( \{u\} \) = the displacement vector,
- \( \{\bar{X}\} \) = the body force vector,
- \( \{\bar{T}\} \) = the surface traction vector,
- \( S \) = the surface of the element, and
- \( \{\cdot\}^T \) = the transpose of a vector.

The displacements, strains, stresses and loads are related to each other through a set of equations as:

\[ \{u\} = [I]\{u\} \]  

(3.2)

\[ \{q\} = [A]\{u\} \]  

(3.3)

\[ \{\varepsilon\} = [B]\{x\} = [B][A]^{-1}\{q\} \]  

(3.4)

\[ \{\sigma\} = [D]\{\varepsilon\} = [D][B][A]^{-1}\{q\} \]  

(3.5)

where,

- \([\cdot]\) = the interpolation matrix,
- \(\{x\} = the\, generalized\, coordinates\, vector,\)
\{q\} = the vector of nodal displacements,
\{A\} = the matrix containing the element nodal coordinates/terms,
\{B\} = the constitutive matrix obtained by differentiating \{\phi\}, and
\{D\} = the constitutive matrix or the stress-strain law for a given element.

The set of equations 3.2 to 3.5 can now be substituted into the total potential energy expression of the system given by equation 3.1 and this results in:

\[ \psi = \frac{1}{2} \int \{q\}^T [A^{-1}]^T [B]^T [D] [A]^{-1} \{q\} d\psi \]
\[ - \int \{q\}^T [A^{-1}]^T \{\phi\} \frac{d\psi}{d\psi} \]
\[ - \int \{q\}^T [A^{-1}]^T \{\gamma\} \frac{d\psi}{d\psi} \]

where,

\[ \{\gamma\} = \{\phi\} \] evaluated along surface points only.

The previous expression which is a scalar function of the discretized unknown nodal displacements \{q\} can be differentiated with respect to the displacements and the result equated to zero. The element stiffness matrix can be expressed as:

\[ \{k\} = \int [A^{-1}]^T [B]^T [D] [B] [A]^{-1} d\psi \]  \hspace{1cm} (3.7)

The previous procedure is applicable equally well to both the local or the global coordinate systems. It has been used to derive the stiffness matrices for the
different types of elements except the beam-column finite elements, where a nonlinear strain-displacement relation is assumed to allow investigating the nonlinear deformations. This will be discussed in the following section.

3.4 BEAM COLUMN FINITE ELEMENTS

Conventional beam-column finite elements are used to represent the conduit walls and predict the buckling loads for these walls. The beams are straight elements with three degrees of freedom at each node and are shown in Fig. 3.3.

The displacement interpolation functions of the element with respect to its local axes are given by:

\[ u = a_1 + a_2 x \]  
\[ v = a_3 + a_4 x + a_5 x^2 + a_6 x^3 \]  
\[ \theta = \frac{\partial v}{\partial x} \]  

where \( \theta \) is the rotation at the end. The coefficients \( a_1 \) to \( a_6 \) are given by:

\[ a_1 = u_1 \]  
\[ a_2 = \frac{u_2 - u_1}{L} \]  
\[ a_3 = v_1 \]  
\[ a_4 = \theta_1 \]
Figure 3.3 Beam-Column Element.
\[ a_5 = \frac{1}{L^2} (3v_2 - 3v_1 - 2L\theta_1 - L\theta_2) \]  \hspace{1cm} (3.15)

\[ a_6 = \frac{1}{L^3} (2v_1 - 2v_2 + L\theta_1 + L\theta_2) \]  \hspace{1cm} (3.16)

where \( L \) is the beam element length.

A nonlinear strain-displacement relationship that takes the form

\[ \epsilon_x = \frac{du}{dx} - \nu \frac{d^2v}{dx^2} + \frac{1}{2} \left( \frac{dv}{dx} \right)^2 \]  \hspace{1cm} (3.17)

is used to take the geometric nonlinearity into consideration. This changes the form of the element stiffness matrix as shown by Gallagher (46) to be divisible into:

\[ [k]_b = [k_e]_b + [k_g]_b \]  \hspace{1cm} (3.18)

where,

- \([k_e]_b\) is the elastic element stiffness matrix, and
- \([k_g]_b\) is the geometric element stiffness matrix. The details of these matrices are given in Appendix A.

### 3.4.1 Iterative Procedure for Beam-Column Element Forces

The thrust in a beam element is one of the variables defining the geometric stiffness matrix. The thrust value at the end of the load increment should be known before starting the analysis due to this load increment.

Iterations are thus required to reach the value of the element forces. Direct iterations have been used where the problem is solved based on the element forces.
from previous loading cases and the new forces are calculated. These are used to get another value of the force vector in another iteration which are compared with the previous value. If the percentage error in their value is less than 2%, the solution is considered converged and the iterations stop, otherwise the iterations continue until convergence is reached. Because the contribution of the beam elements to the master stiffness matrix is generally small, only one or two iterations are generally required for convergence.

3.4.2 **Plastic Hinges in Beam Elements**

An important cause of nonlinearity in the behaviour of structures is due to the development of material plasticity. Theoretically, plastic hinges may develop at the conduit nodes as the load level is increased. The simplest way of dealing with the effect of plasticity in simple structures is to assume that the material behaves in a rigid-plastic manner and then use the simple plastic theory to evaluate the collapse load of the structure. A more realistic approach suitable for finite element analysis would be to assume the material to behave in an elastic-plastic manner. The overall non-linearity of the load-deflection relationship when neglecting the axial
loads is purely due to the development of plastic hinges at discrete sections in the structure.

By introducing the effects of axial loads in the members of a structure a piece-wise linear elastic-plastic response is modified to a non-linear response, forcing the structure to collapse at a load factor below that given by the simple plastic theory and before the formation of sufficient hinges to convert the structure into a mechanism. The different expected load deflection curves are compared in Figure 3.4, for an example structure given by Majid (87) where the failure load is represented by point F on the elastic-plastic curve. The axial load effect in the conduit walls is considered in the present analysis.

It is noticed that as the axial load in a member of a structure increases, a plastic hinge in that member withstands smaller moments and the plastic hinge moment of the section is reduced due to the effect of the axial load. If the axial load in a member is increased excessively, especially in case of conduits under very deep cover, it is possible to induce a plastic hinge to act as a real hinge by reducing the plastic hinge moment of the section to zero.
Figure 3.4 Nonlinear Load-Deflection Curves of Structures
3.4.3 Plastic Hinges Introduction

The introduction of plastic hinges at the conduit nodes starts by assigning inactive degrees of freedom corresponding to each relative rotation of the two elements meeting at the node. The inactive degrees of freedom have a number equal to the number of beam elements. The band width of the structure becomes excessively large if the numbering of the inactive degrees of freedom follows the numbering of the total degrees of freedom. To reduce the band width effectively, the inactive relative rotations numbering follow those of the rigid joint rotations. This leaves the stiffness matrix of the structure with zeros in the corresponding rows and columns. Any arbitrary value is then assigned to the pivoting elements to avoid changing the stiffness matrix into a singular one. When the moment at either end reach the plastic hinge moment, \( M_p \), a plastic hinge is inserted at that end of the element which changes the element stiffness matrix of the element; the additional degree of freedom (the relative hinge rotation) enters the formulation. Correspondingly, the load vector is modified to accommodate a plastic moment \( M_p \) applied at the plastic hinge. The modified element stiffness matrix and the load vector are defined in Appendix A. The same
changes may apply at the other end of the element (87).

The criterion for the plastic hinge insertion is that the bending moment at either end of the element reaches the value of the plastic moment. Because of the existence of the axial force, a reduced value of the plastic moment, $M_{pr}$, is used instead. The following interaction equation developed by Kloppel and Glocker (69), and shown diagrammatically in Figure 3.5, defines the value of $M_{pr}$ for corrugated sheets as:

$$M_{pr} = M_p \left(1 - n_1^2\right)$$

where,

- $M_{pr}$ = the reduced plastic hinge moment,
- $M_p = f_y Z$ = plastic hinge moment,
- $n_1 = T/T_y$,
- $T_y = f_y A$,
- $T$ = axial load in the member, and
- $Z$ = plastic section modulus.

At the end of a given load increment, a check is made as to whether the end moment of any member has reached the value, $M_{pr}$. If so, the contribution to the column of the relative hinge rotation $\dot{\theta}_H$ is added and the analysis is repeated for the same load increment and the modified geometry.
Figure 3.5 Diagramatic Representation of the Interaction Equation.
3.5 **INTERFACE FINITE ELEMENTS**

Interface finite elements are introduced between the nodes of different materials to account for the possible relative movements between these materials. The interface insertion is essentially achieved by assigning a different nodal point number to each side of the interface. Researchers have basically dealt with this subject using two approaches, the method of constraints and the method of stiffness. Katona et. al. (63) gave a complete finite element formulation using the method of constraints which is based on constraint equations to model interfaces. Although this method has proven to be more powerful in the comparison made by Leonards (80), it requires a large stiffness matrix, more computer time and more iterations than the method of stiffness which is employed in the present analysis. Ngo and Scordellis (95) developed this concept of adding the stiffness of a linkage element to the total stiffness matrix to analyze a reinforced concrete beam. Other researchers followed by adopting a one dimensional finite element to represent the interface behaviour in jointed rock. Goodman (48) and Ghaboussi et. al. (47) came up with basically the same interface element stiffness matrix using two different approaches. The disadvantage of their linear interface
element is that it represents complete contact or total separation along the interface length. Possible separation at any point along this length cannot be taken into account in particular when dealing with curved conduits.

The spring type interface element used by Hafez (50), and which represents the behaviour of a point not a line, gives a more consistent representation of the state of normal and shearing stresses between the soil media and the conduit wall. It has only two nodes as shown in Figure 3.6 which are initially coinciding when no stresses are existing between the two materials. It represents a length along which the two materials are in contact but it actually has no physical dimension.

The derivation of its element stiffness matrix which is based on defining the strain energy of the element in terms of the relative displacements between its nodes is given by Hafez (50) as:

$$\left[ \begin{array}{cccc} k_s & 0 & -k_s & 0 \\ 0 & k_n & 0 & -k_n \\ -k_s & 0 & k_s & 0 \\ 0 & -k_n & 0 & k_n \end{array} \right]$$

$$\left[ k_e \right]_i = L'$$

(3.20)
Figure 3.6 Spring Type Interface Elements.
where,

\[ L' = \text{length along the interface corresponding to the element}, \]

\[ k_s = \text{unit tangential stiffness for the element, and} \]

\[ k_n = \text{unit normal stiffness}. \]

3.5.1 Interface Model

The two unit stiffnesses \( k_s \) and \( k_n \) defined in equation 3.20 should be evaluated for the analysis to be complete. The unit normal stiffness, \( k_n \), increases with the increase in the normal compressive stress and decreases to zero if tension is developed. It can be obtained from a direct shear box test and is shown in Figure 3.7 to be constant with increasing normal stress in the range of working stresses.

On the other hand, the unit shear stiffness, \( k_s \), is not constant. It increases with an increase in the normal stress and decreases with increasing the shear stress. Its variation is nonlinear as shown in Figure 3.7. As indicated by Clough and Duncan (21), this nonlinear shear stress-shear displacement relationship can be given in a hyperbolic form as:

\[
\delta_s = \frac{\delta_s}{a_1 + b_1 \delta_s} \quad (3.21)
\]
Figure 3.7 Stress-Deformation Curves for an Interface Element.

Figure 3.8 Transformed Shear Stress-Shear Displacement Relationship
where,
\[ \delta_s = \text{the interface shear displacement, and} \]
\[ a_1, b_1 = \text{empirical constants to be determined experimentally.} \]

The relationship given by equation 3.21 can be transformed into a straight line as shown in Figure 3.8 and it takes the form:
\[ \frac{\delta_s}{\tau_s} = a_1 + b_1 \delta_s \quad (3.22) \]

where,
\[ \frac{1}{a_1} = \text{the initial unit shear stiffness, } k_{si}, \text{ and} \]
\[ \frac{1}{b_1} = \text{the asymptotic value of shear stress, } \tau_{ult}. \]

The initial shear stiffness, \( k_{si} \), is related to the normal stress at the interface by the relationship given by Janbu (62),
\[ k_{si} = k_i \cdot \gamma_w \cdot \left( \frac{n}{p_a} \right)^n_s \quad (3.23) \]

where,
\[ k_i = \text{a dimensionless stiffness number,} \]
\[ \gamma_w = \text{unit weight of water expressed in the same units as } k_{si}, \text{ and} \]
\[ n_s = \text{stiffness exponent.} \]

The constants \( k_i \) and \( n_s \) are determined experimentally by plotting \( (k_{si}/\gamma_w) \) versus \( (n/p_a) \) in a log-log scale as
shown in Figure 3.9.

The ultimate shear, $\tau_{ult}$, is related to the shear failure, $\tau_f$, by the relation:

$$\tau_f = R_{sf} \cdot \tau_{ult} \quad (3.24)$$

where $R_{sf}$ is a failure ratio.

The shear strength, or the shear failure $\tau_f$ is also related to the normal stress and the angle of friction between the soil and the wall material, $\phi'$, as follows:

$$\tau_f = \gamma_n \cdot \tan \phi' \quad (3.25)$$

The hyperbolic equation 3.21 can be expressed in another form by substituting equations 3.23 to 3.25 into 3.21 to get:

$$\tau_s = \frac{\frac{\gamma_n}{k_i} \cdot \frac{\gamma_n}{P_a}^{n_s} \cdot \frac{\gamma_n}{P_a}^{n_s} + \frac{R_{sf} \cdot \delta_s}{\gamma_n \cdot \tan \phi'}}{1} \quad (3.26)$$

The unit shear stiffness, $k_s$, which is the slope of the tangent to the shear stress-displacement curve, is obtained by differentiating equation 3.26 with respect to $\delta_s$, and this gives:

$$k_s = k_i \cdot \gamma_w \left(\frac{\gamma_n}{P_a}\right)^{n_s} \left[1 - \frac{R_{sf} \cdot \tau_s}{\gamma_n \cdot \tan \phi'}\right]^2 \quad (3.27)$$
Figure 3.9 Variation of Initial Unit Tangential Stiffness.
3.5.2 Iterative Procedure for Interface Element Properties

The value of the unit shear stiffness, which is obtained and updated by equation 3.27, and defined as the tangential to the shear stress-displacement curve, is obtained by iterations. The direct iterations, or successive substitution technique, is employed herein. In this technique a given load increment \( \{ \Delta P \}_m \) is applied to the structure with the stiffness matrix and the unit shear stiffnesses based on initial displacements \( U_0 \) as shown in Figure 3.10.a., and is used to compute a new set of displacements \( U_1 \). The displacements \( U_1 \) are then used to update the stiffness \( k_s \) and the stiffness matrix as well as to compute \( U_2 \). The scheme is represented by the following incremental equation:

\[
\{ \Delta U^{n+1} \}_m = [k]_n^T \{ \Delta P \}_m
\]

where,

\( n \) = the iteration number, \\
\( m \) = the load increment number, \\
\( \{ U \} \) = the displacement vector, \\
\( \{ P \} \) = the load vector, and \\
\( [k]_n \) = the updated stiffness matrix after iteration \( n \).
Figure 3.10.a Load-Deflection Curve in a Direct Iterative Technique.

Figure 3.10.b Load-Deflection Curve in a One Step Iterative Technique
The iterations terminate when the percentage error in the
value of the unit shear stiffness in two consecutive
iterations does not exceed 1%. Because this method tends
to be expensive in terms of computer time, and because
the stiffness of the interface element is small compared
to that of the structure a one step method has also been
used. In a particular incremental step the stiffness
matrix, derived based on the displacement \( U_{0,i} \) as shown
in Figure 3.10.b, and the incremental load, yields the
incremental deflections and point 'b' is reached at the
end of the \( n \)th step. The values of the displacement at
\( U_{0} \) and \( (U_{0} + \Delta U) \) are then averaged and the \( k_{s} \) value and
the stiffness matrix are updated based on this average
value. The computations of the step \( n \) is continued
starting from 'a', using the loads \( \Delta P \) and the updated
stiffness matrix. The one step iteration proved successful in all cases studied except when the conduit is very
flexible where more iterations are then required.

3.5.3 Iterative Procedure for Failure in Interface
Elements

Tension or shear failure may take place in the inter-
face element during any loading increment.

Tensile stresses may be observed in a given load
increment in any element; this indicates a possible tension failure. Theoretically, this situation is equivalent to zero values for both the element's unit normal and tangential stiffnesses. This, however, would lead to complete separation between the two nodes connected by the interface element and the possibility of a singular stiffness matrix.

In order to avoid this possibility, the following initial conditions are assumed for the interface element in tension: the two nodes coincide, the total normal and shear stresses are set to zero, and the \( k_s \) and \( k_n \) values are given their original values. Direct iterations then start with the updated initial conditions. The iterations may indicate that a) the interface element has attracted compression and the iterations terminate, or b) the interface element continues to be in tension in which case the iterations are carried on until the tension failure is eliminated.

If the shear stress in an interface element exceeds the shear strength defined by equation 3.25 and shown in Figure 3.11, shear failure takes place. If failure occurs in any loading increment, the unit shear stiffness is reduced to:

\[
k_{sr} = k_s \cdot \frac{\tau_f}{\tau_s} = k_s \cdot \frac{n \tan \psi'}{\tau_s}. \tag{3.29}
\]
Figure 3.11 Shear Stress-Normal Stress Relationship for an Interface Element.
and iterations resume. The iterations terminate when, by further reduction of $k_s$, the shear stress does not exceed the shear strength.

3.6 **SOIL FINITE ELEMENTS**

As mentioned previously, constant strain and linear strain elements are used to simulate the soil in the finite element analysis. A description of their element stiffness matrix formulation, the soil models chosen to update their parameters and the failure of these elements follow in the next sections.

3.6.1 **Constant Strain Triangular Finite Elements**

The constant strain elements are used to simulate the soil media below and away from the conduit. A triangular membrane element which has three corner nodes with two degrees of freedom (the displacements $u$ and $v$) at each of these nodes is shown in Figure 3.12. The linear interpolation function defining the displacements at any point within the element and satisfying the boundary conditions is given by:

\[
\begin{align*}
    u &= a_1 + a_2 x + a_3 y \\
    v &= a_4 + a_5 x + a_6 y
\end{align*}
\]  

(3.30)  

(3.31)
Figure 3.12 Constant Strain Triangular Element.
where,

\[ u = \text{displacement in the x-direction;} \]
\[ v = \text{displacement in the y-direction, and} \]
\[ a_i = \text{coefficients in the displacement function.} \]

The strain-displacement relationships for these two dimensional elements are given by:

\[
\{\varepsilon\} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} \tag{3.32}
\]

where,

\[ \varepsilon_x = \text{strain in the x-direction,} \]
\[ \varepsilon_y = \text{strain in the y-direction, and} \]
\[ \gamma_{xy} = \text{shear strain.} \]

The constitutive equation relating stresses to strains for a soil model in a plane strain condition can be expressed as:

\[
\{\sigma\} = [ D ] \{\varepsilon\} \tag{3.33}
\]

where,

\[
\{\sigma\}^T = \{\sigma_x, \sigma_y, \tau_{xy}\} \tag{3.34}
\]
and

\[
[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1 - \nu & \nu & 0 \\
\nu & 1 - \nu & 0 \\
0 & 0 & \frac{1 - 2\nu}{2}
\end{bmatrix}
\] (3.35)

In general, to take the expected unsymmetry of the elasto-plastic model into account, it can be assumed to take the form:

\[
[D] = \begin{bmatrix}
d_1 & d_2 & d_3 \\
d_4 & d_5 & d_6 \\
d_7 & d_8 & d_9
\end{bmatrix}
\] (3.36)

Using the previously assumed displacement function and the variational principle of minimum potential energy, the element stiffness matrix for the plane-strain constant-strain triangular element in the local system of axes is generated and given in Appendix B.

3.6.2 Linear Strain Quadrilateral Finite Elements

The stiffness matrix of the linear strain element is derived either by assuming a displacement function or by making use of the similarity of the two elements. Figure 3.13 shows a nine-noded quadrilateral element with respect to its local and global cartesian coordinates. It is composed of two 6-noded linear strain triangular elements related by common degrees of freedom at the diagonal line.
Figure 3.13 Nine-Noded Linear Strain Quadrilateral Element.

Figure 3.14 Eight-Noded Linear Strain Quadrilateral Element.
A quadratic polynomial is assumed to describe the displacements within the triangular element. The function is as follows:

\[ u = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 y^2 + a_6 xy \]  
\[ v = a_7 + a_8 x + a_9 y + a_{10} x^2 + a_{11} y^2 + a_{12} xy \]  

(3.37) \hspace{1cm} (3.38)

The similarity between the constant strain and linear strain elements explained by Pederson (101) and given in Appendix C is used to derive the element stiffness matrix for a linear strain triangular element. By combining the stiffness matrices of the two triangular elements shown in Figure 3.13, the stiffness matrix for the nine-noded quadrilateral element is derived and is given in Appendix D. A condensation technique outlined by Gallagher (46) and Cook (22) is then used to eliminate the two interior degrees of freedom. This changes the element to an eight-noded quadrilateral element as shown in Figure 3.14, which results in an efficient storage scheme and a reduction in the total number of degrees of freedom. The condensation has been carried out numerically inside the computer program. Again, because of the assumed unsymmetric elasticity matrix (equation 3.36) an unsymmetric element stiffness matrix is obtained.
3.7 NON-LINEAR SOIL MODELS

A general equation for stresses in a soil media can be expressed as a function of numerous variables as follows:

\[ \sigma = f (\varepsilon(t'), T'(t'), w(t'), x(t'), \ldots, t') \]  (3.39)

where \( \varepsilon, T', w, x', t' \) are strains, temperature, moisture content, anisotropy and time respectively. Because of the complexity of the state of stress there has not been available any single soil model to consider all the variables together. Some of the variables have been considered in available nonlinear soil models. Considering time-independent non-linear soil models, these can be divided into two groups, nonlinear or variable modulus models and plasticity or elasto-plastic models. In contrast to linear models, the nonlinear variable modulus models assume that the soil stiffness increases with a confining pressure increase and decreases with an increase in shear strain. Complete descriptions of these models are given with comparisons of their ease or difficulties and accuracy in Ref. (25, 28, 30, 34, 37, 38, 54, 67, 70, 119).

Elasto-plastic models which simulate constitutive laws for the soil in a more consistant way than the variable modulus models, especially at and after failure are
given in further details in section 3.8.

As will be explained later, the hyperbolic model is chosen to represent the behaviour of the triangular elements, while the elasto-plastic model is modelling the quadrilateral elements behaviour of the finite element mesh shown in Figures 3.1, and 3.2.

3.7.1 **Hyperbolic Stress-Strain Model**

The mathematical function used to define the stress-strain relationship in a nonlinear model can be assumed as a parabola, a hyperbola or a polynomial. The hyperbolic soil model has proved to be the most accurate, appropriate and the easiest nonlinear model to simulate the soil-steel structure interaction problem (Leonards 80). Its parameters which can be obtained from a triaxial test are known and classified for most soils (119).

An equation expressing the nonlinear stress-strain relationship for most soils from a triaxial test is given in the work by Konder et. al. (70). The hyperbola suggested takes the form given by the equation:

\[
(s_1 - s_3) = \frac{\varepsilon_a}{1 + \frac{E_i}{E_a} + \frac{s_a}{(s_1 - s_3)_{ult}}}
\]  

(3.40)
where,

\( \sigma_1 \) = major principal stress,

\( \sigma_3 \) = minor principal stress,

\( E_i \) = the initial tangent modulus, and

\( \varepsilon_a \) = the axial strain.

The stress difference \( (\sigma_1 - \sigma_3) \) at failure will always be smaller than the asymptotic value \( (\sigma_1 - \sigma_3)_{ult} \).

If the previous hyperbolic equation is transformed into a straight line, it takes the form:

\[
\frac{\varepsilon_a}{\sigma_1 - \sigma_3} = \frac{1}{E_i} + \frac{\varepsilon_a}{(\sigma_1 - \sigma_3)_{ult}} \tag{3.41}
\]

which is the best straight line fit for the hyperbola. Both the hyperbola and the straight line equations are shown in Figure 3.15.

It has been suggested by Janbu (62) that the initial tangent modulus is related to the confining pressure by the relation:

\[
E_i = K_s \frac{\sigma_a}{\sigma_a} \left( \frac{\sigma_3}{\sigma_a} \right)^{m_1} \tag{3.42}
\]

where,

\( K_s \) = a dimensionless modulus number determined experimentally,

\( m_1 \) = a dimensionless modulus exponent determined experimentally, and
Figure 3.15.a Hyperbolic Stress Strain Relationship.

Figure 3.15.b Transformed Hyperbolic Stress-Strain Relationship.
\( P_a \) = the atmospheric pressure given in the same units as \( E_1 \) and \( z_3 \).

If equation 3.42 is transformed into a straight line on a log scale, the constants \( K_s \) and \( m_1 \) can be obtained as shown in Figure 3.16.

A Mohr-Coulomb failure criteria (119), is adopted to define the stress differences at failure as follows:

\[
\left( \sigma_1 - \sigma_3 \right)_{\text{f}} = \frac{2 \sigma_3 \sin \phi + 2 c \cos \phi}{1 - \sin \phi} \tag{3.43}
\]

where,

\( \phi \) = the angle of internal friction, and

\( c \) = the cohesion intercept (Figure 3.17).

The same stress difference at failure (or the compressive strength) is related to the asymptotic values as suggested by Duncan et al. (34), as follows:

\[
\left( \sigma_1 - \sigma_3 \right)_{\text{f}} = R_f \left( \sigma_1 - \sigma_3 \right)_{\text{ult}} \tag{3.44}
\]

where,

\( R_f \) = a failure ratio smaller than unity.

The tangent modulus for the given soil which is given by:

\[
E_t = \frac{d}{d_a} \left( \sigma_1 - \sigma_3 \right) \tag{3.45}
\]

can also be represented by the use of equations 3.40 through
Figure 3.16 Variation of Initial Tangent Modulus.
Figure 3.17 Mohr-Coulomb Strength Parameters.
3.45 as follows:

\[
E_t = K_s \cdot \frac{\sigma_a}{P_a} \left( \frac{\sigma_3}{P_a} \right)^m \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3) (1 - \sin \theta)}{2 \sigma_3 \sin \theta + 2 \sigma_3 \cos \theta} \right]^2
\]  

(3.46)

The previous equation is used to update the tangent modulus in an incremental finite element analysis under any stress combinations. The parameters defined by this equation are obtained from a triaxial test as will be discussed later in Chapter IV.

3.7.2 **Hyperbolic Nonlinear Volume Change**

The second independent stress-strain coefficient, which is the tangential Poisson's ratio, is determined by measuring the volume change in a triaxial test. The volumetric strain \( \varepsilon_v \) is related to the axial strain \( \varepsilon_a \) in the triaxial test by the relation:

\[
\varepsilon_v = \varepsilon_a + 2 \varepsilon_r
\]  

(3.47)

where \( \varepsilon_r \) is the radial strain. The relation between \( \varepsilon_a \) and \( \varepsilon_r \), shown in Figure 3.18.a, is approximated by the hyperbolic equation suggested by Wong and Duncan (119):

\[
\varepsilon_a = \frac{\varepsilon_r}{\nu_i - d'. \varepsilon_r}
\]  

(3.48)

where,

\( \nu_i \) = the initial Poisson's ratio, and
Figure 3.18.a Hyperbolic Axial Strain-Radial Strain Relationship.

Figure 3.18.b Transformed Hyperbolic Axial Strain-Radial Strain Relationship.
\[ \nu_t = \frac{G - F \cdot \log_{10} \left( \frac{c_3}{P_a} \right)}{\left[ 1 - \frac{d' (\sigma_1 - \sigma_3)}{K_s \cdot P_a \cdot \left( \frac{c_3}{P_a} \right)^n} \left[ 1 - \frac{R_f (\sigma_1 - \sigma_3) (1 - \sin \theta)}{2 \sigma_3 \sin \theta + 2 c \cos \theta} \right] \right]^{2}} \]

where,

- \( G = \) value of \( \nu_i \) at a confining pressure equal to the atmospheric pressure, and
- \( F = \) the reduction in \( \nu_i \) for a ten-fold increase in \( \sigma_3 \).

3.7.3 Iterative Procedure for Soil Properties

The previously mentioned equations 3.46 and 3.49 are used to update the soil properties in the incremental finite element analysis. Iterations are carried out in each increment to improve the accuracy of the soil properties. The iterations follow the same procedure described previously for the interface elements in section 2.5.2. The direct iteration technique, and the one step
Figure 3.19 Variation of Initial Poisson's Ratio
method are both built into a computer program. When the direct iteration method is used, the number of iterations is limited to the number of iterations required to deal with the failure of the soil elements as will be discussed in the following section.

3.7.4 Failure in Soil Elements

Based on the hyperbolic model and the Mohr-Coulomb failure criterion, failure may be predicted in soil elements. Tension failure may develop in an element if it is not subject to pressure while shear failure may develop because of the relative movement between soil particles along a plane of shear.

Tension failure is detected in an element if any of its principal stresses indicates a tensile value which is unacceptable and unrealistic in cohesionless soils.

A soil element is considered to have failed in shear when during the analysis its maximum shear stress exceeds the shear strength defined by the equation:

\[ \tau_f = c + \sigma_n \tan \phi \]  \hspace{1cm} (3.50)

The two parameters \( c \) and \( \phi \) described by the equation can be obtained from a triaxial test. The angle of internal friction \( \phi \) decreases as the confining pressure increases (Wong and Duncan 119). The equation relating
these is:

\[ \phi = \phi_o - \Delta \phi \log_{10} \left( \frac{\sigma_3}{p_a} \right) \]  \hspace{1cm} (3.51)

where,

\( \phi_o \) = the angle of internal friction for a confining pressure equal to the atmospheric pressure, and

\( \Delta \phi \) = the reduction in \( \phi \) corresponding to a 10-fold increase in \( \sigma_3 \).

3.7.5 Iterative Procedure for Soil Failure

If soil tension failure is detected, as explained in the previous section, a stress transfer or stress release technique, discussed later in section 3.7.6, is applied to dissipate these excess element stresses into neighbouring elements without violating equilibrium. An initial tangent modulus for the element is assumed based on \( \sigma_3 \) equal to 0.05 of the atmospheric pressure (50) and an initial value of Poisson's ratio is taken as 0.49, a slightly smaller than 0.5 which would cause the determinant of the elasticity matrix to be infinity. Iterations are carried out with the updated element properties and the equivalent forces from the stress release at its ends. Further iterations may attract compression to the element and the iterations terminate, or the element
may continue to be in tension. The stress transfer is continued until the tension inside the element is eliminated.

If the stress circle intersects the strength envelope of a soil element as shown in Figure 3.20, which indicates shear failure, it is suggested that the maximum principal stress \( \sigma_{1f} \) be reduced to 0.95 \( \sigma_{1m} \) so as to fall within the limits. The reduction in the maximum principal stress \( \sigma_{1f} - 0.95 \sigma_{1m} \) is then transferred to neighbouring elements. In subsequent load increments the tangent modulus is calculated based on the new stress level instead of the stress level at failure, otherwise equation 3.46 would give a zero value, Poisson's ratio is limited to 0.49, else equation 3.49 would indicate infinity.

3.7.6 Stress Transfer Technique

The excess stresses developed in an element which fails in shear or tension are transferred to the neighbouring elements. This is achieved by applying the technique suggested by Zienkiewicz et al. (120), and which does not disturb equilibrium. If the stress vector in the failing element reaches the value \( \tau \), while the failure stress vector is \( \tau \), the difference
Figure 3.20 Shear Failure and Correction Criterion in a Soil Element.
\( \{ \sigma \} = \{ \sigma \}_e - \{ \sigma \}_f \) is to be transferred to the neighbouring elements. The following steps are followed:

1. The equivalent nodal forces due to the stress vector \( \{ \sigma \} \) are evaluated according to the equation:

\[
\{ F \} = [B][A]^{-1} \{ \sigma \}
\]  

(3.52)

This set of forces is shown for an example triangular element in Figure 3.21.a.

2. Two equal and opposite sets of these forces are assumed to be applied at the element nodes. One set will eliminate the excess stresses in the element \( \{ \sigma \} \) as shown in Figure 3.21.b. The other set is applied to the whole structure and the iterations start.

During the iterations, the applied force vector will be dissipated in the structure and the failing element may attract a part of the applied forces.

This attracted portion of the stresses is less in value than the original \( \{ \sigma \} \) value. It is again transformed into a force vector and another iteration is carried out. The iterations end after the failing element ceases to be in a failing condition.
Figure 3.21.a  Excess Stresses and Equivalent Forces Acting on a Failing Element.

Figure 3.21.b. Equilibrating and Excess Sets of Applied Forces.
3. During this process, other elements may attract stresses that cause failure in these elements. The correction to the stresses in these elements is carried out simultaneously inside the same iterations.

3.7.7 Limitations of the Hyperbolic Model

Hyperbolic elastic stress-strain relationship of the type developed by Duncan and Chang (34) have proven useful for finite element analyses of a number of types of geotechnical engineering problems. These simple relationships are capable of modelling nonlinear stress-strain behaviour quite effectively, and, for conditions not too close to failure, they provide fairly accurate and quite useful answers for a variety of practical problems (33, 35, 36, 37, 50, 78, 85, 96).

The simple relationships have a number of limitations, however. These include the following as reported by Ozawa and Duncan (100):

1. Real soils have the capability of resisting any type of deformations, at and after failure, except the one which has brought the soil to failure (100).
This is not accurately modelled by the hyperbolic relationship. The transfer of stresses after failure does not induce reduction in the resistance of the element. Reducing the value of the shear modulus or both the bulk and shear moduli, are other ways of reducing the resistance after failure. These would cause the soil to behave like a liquid or a gas (34,71) which is not realistic in cohesionless soils.

2. Soil is a dilatant material as shown from tests. Using the generalized Hooke's law as a deformation model in the hyperbolic stress-strain relationship, implies that the soil is not dilatant, or changes in shear stress do not cause changes in volume. This contradicts actual soil behaviour.

3. Experimental evidence shows that the intermediate principal stress, 3, has an appreciable influence on the strength of many soils. In the hyperbolic relationship, the Mohr-Coulomb failure criterion is used to model the strength of soil. This implies that the intermediate principal stress has no effect on the strength.

4. Experiments on real soils have shown that
principal axes of strain increment coincide with the principal axes of stress increment at low stress levels (103). At stress levels approaching failure, the strain increment axes coincide with the principal axes of stress (and not of stress increment). The hyperbolic relationship predicts only elastic strains: the principal axes of strain increment always coincide with the principal axes of stress increment.

Because of these limitations, an elasto-plastic soil model is employed in the analysis and is explained in the following sections.

3.8 ELASTO-PLASTIC SOIL MODEL

Employing an elasto-plastic stress-strain relationship instead of the hyperbolic relationship can overcome the previous limitations. Proper modelling of the behaviour of soils at and after failure, dilatancy, the dependency of the strength on the intermediate principal stress, the coincidence of strain increments and stress increment axes at low stress levels, and the coincidence of strain increment and stress axes at higher stress levels, are some of the capabilities of an elasto-plastic stress-strain relationship. The use of an elasto-plastic soil
model also allows the analysis to account for the interaction between soil failure and the behaviour up to instability of the steel conduit.

An elasto-plastic soil model has thus been chosen to represent the behaviour of the quadrilateral elements around and above the conduit where the stress gradient is sharp and failure in soil elements is expected to take place. Because an elasto-plastic model is more complicated, requires more computer time and calculations but is more accurate, the hyperbolic model is used to represent the behaviour of the triangular elements below and away from the conduit.

Various constitutive equations for soils have been reported in the literature. These include elasto-plastic, incremental and rate type models. They are advanced to meet with the aspects of soil behavior under more and more complex stress or strain paths. The theories available are able to model different aspects of soil behaviour considered as limitations in the variable modulus models mentioned in the previous section. Among these the Lade-theory (75) seems to be a good compromise between an improvement to an older constitutive equations such as the Duncan law and very sophisticated models requiring much sophisticated experimental tests to identify its
parameters. The Lade-theory model only requires conventional triaxial compression tests to obtain the parameters involved. Several researches have been carried out by implementing the Lade-theory. Among these, Haussler (56) who applied it in grain bins problems, and Aubry and Des Croix (9) who tested the theory in foundation problems and described some of its limitations while Leonards compared it to other elasto-plastic models in his report (80). The assumptions required from plasticity theory and the description of the adopted Lade model are given in the following sections.

3.8.1 Assumptions from Plasticity Theory

The following are the assumptions from the classical theory of plasticity which are used to derive the incremental elasto-plastic stiffness matrix (89, 100, 104).

1. Yield Function. The yield function, \( f \), defines a surface in the stress coordinates known as the yield surface. The yield surface defines the boundary that separates the states of stress that cause only elastic strains (which falls inside the yield surface) and the states of stress that cause both elastic and plastic strains (which
falls outside the yield surface). When the material is loaded and experiences hardening, the yield surface expands and the stress level increases in such a way that the current state of stress falls on the current yield surface. When the material is unloaded, and the stress level decreases, the current state of stress falls inside the yield surface which remains where it was, and all strains are elastic. During reloading, the strains are all elastic until the state of stress goes back to the yield surface by increasing the stress level. Following that stage, further loading yields elastic and plastic strains.

The yield function for any work-hardening or softening material may be expressed as a function of the stresses and plastic strains:

$$f(\sigma, \varepsilon^P) = 0$$  \hspace{1cm} (3.53)

2. Plastic Potential Function. The ratios of the components of plastic strain increment can be derived by defining a plastic potential function, $g$. This plastic potential function may have the same form as the yield function (as is usually assumed in the classical plasticity theory when
dealing with metals). This assumption is not necessary true for soils, however. The two surfaces are shown in many researches to be different for soils (27, 29, 73, 74, 75, 86, 100, 121, 122). The plastic potential function may be expressed as a function of the stresses:

\[ g = g(\{\sigma\}) \]  

(3.54)

3. Flow Rule. The values of the plastic strain increments can be found by making use of the normality rule. The normality rule of the plastic strain increment directions states that the plastic strain increments are related to the plastic potential function as:

\[ (\varepsilon^p) = \lambda \cdot \frac{\partial g}{\partial \{\sigma\}} \]  

(3.55)

where,

- \( \lambda \) is a plastic multiplier,
- \( g \) is the plastic potential function, and
- \( \frac{\partial g}{\partial \{\sigma\}} \) defines the outward normal to the plastic potential function.

From the normality rule definition, if the principal plastic strain increments \( \varepsilon^p_1 \), \( \varepsilon^p_2 \) and \( \varepsilon^p_3 \) are plotted on a set of axes which coincide with the principal stress axes \( \sigma_1, \sigma_2 \)
and \( \sigma_3 \), the resultant vector of plastic strain increment has an outward direction normal to the plastic potential surface at that point.

4. Incremental Elastic and Plastic Strains. During an infinitesimal change in stress, the total strain increment, \( \{d\varepsilon\} \), is assumed to be divisible into elastic component, \( \{d\varepsilon^e\} \), and plastic component, \( \{d\varepsilon^p\} \). That is:

\[
\{d\varepsilon\} = \{d\varepsilon^e\} + \{d\varepsilon^p\} \tag{3.56}
\]

5. Relationship Between Stress Increments and Elastic Strain Increments. The increments of stress are assumed to be related to the increments of elastic strain by means of a symmetrical elasticity matrix, \([D^e]\):

\[
\{d\tau\} = [D^e]\{d\varepsilon^e\} \tag{3.57}
\]

Using equations 3.53 through 3.57, and the incrementalization procedure described in section 3.9.1, the incremental elastoplastic stress-strain relationship for work-hardening or softening materials may be derived. The final form of the incremental stress-strain relationship may be written as:

\[
\{d\tau\} = [D^{ep}] \{d\varepsilon\} \tag{3.58}
\]
in which \( \mathbf{D}^{ep} \) is the elasto-plastic stress-strain matrix.

3.8.2 Model Description

The constitutive law described by Lade (75) employs two yield surfaces which may be activated simultaneously, one at a time, or not at all. The total strain increments, \( \{ d \varepsilon \} \), are divided into an elastic component, \( \{ d \varepsilon^e \} \), a plastic collapse component, \( \{ d \varepsilon^c \} \), and a plastic expansive component, \( \{ d \varepsilon^p \} \), such that:

\[
\{ d \varepsilon \} = \{ d \varepsilon^e \} + \{ d \varepsilon^c \} + \{ d \varepsilon^p \}.
\]

These strain increments are calculated separately; the elastic strains by Hooke's law, the plastic collapse strains and plastic expansive strains by plastic stress-strain theories that involve, respectively, a cap-type yield surface and a conical yield surface with apex at the origin of the stress space.

Figure 3.22 from Ref. 75 shows the parts of the total strain that are considered to be elastic, plastic collapse, and plastic expansive components of strain in a drained triaxial compression test. Variations of stress difference, \( (\sigma_1 - \sigma_3) \), and volumetric strain, \( \varepsilon_v \), with axial strain, \( \varepsilon_a \), are shown in this figure for a typical test performed
Figure 3.22 Schematic Illustration of the Strain Components in Drained Triaxial Compression Test.
with constant value of confining pressure, $c_3$. Elastic (recoverable) and plastic (irrecoverable) deformations occur from the initiation of loading in cohesionless soils. This behaviour is highly nonlinear, and a decrease in strength may follow peak failure. The volumetric strain is initially compressive but may be followed by expansion as shown in the figure or by continued compression. The plastic strains are smaller than the elastic strains at the initiation of loading, but at higher values of stress difference the plastic strains dominate the deformations.

3.8.2.1 Elastic Strains

The recoverable part of the strain increment or the elastic strain increment, is defined from Hooke's law based on the unloading-reloading behaviour (75). The unloading-reloading modulus defined as (34);

$$ E_{ur} = K_{ur} \cdot P_a \cdot \left( \frac{c_3}{P_a} \right)^{n_2} \quad (3.60) $$

is used to calculate this value, where,

$K_{ur} = $ dimensionless constant modulus number, and

$n_2 = $ modulus exponent.

The values of the two constants are derived from a triaxial compression test. The modulus exponent, $n_2$, has been found as equal to the modulus $m_1$ of equation 3.42 (34). The value of Poisson's ratio is supposed to be very small.
and is taken 0.2 for the elastic parts of the unloading-reloading stress path (75).

3.8.2.2 Plastic Expansive Strains

Figure 3.23 shows the yield surface for the plastic strain components on the triaxial plane. A conical yield surface, curved in this plane, is used to describe the plastic expansive behaviour. It is defined in terms of the first and the third stress invariants, $I_1$ and $I_3$, as

\[ f_p = (I_1^3/I_3 - 27)(I_1/p_a)^{m_2} \]  
\[ f_p = \eta_1 \text{ at failure.} \]

where,

\[ I_1 = \sigma_x + \sigma_y + \sigma_z \]  
\[ I_3 = \sigma_x \sigma_y \sigma_z + \tau_{xy} \tau_{yz} \tau_{zx} + \tau_{yz} \tau_{zx} \tau_{xy} - (\sigma_x \tau_{yz} \tau_{zx} + \sigma_y \tau_{zx} \tau_{xy} + \sigma_z \tau_{xy} \tau_{zx}) \]

\[ (3.63) \]

\[ (3.64) \]

The values of $\eta_1$ and $m_2$ can be determined by plotting $(I_1^3/I_3 - 27)$ vs. $(p_a/I_1)$ at failure in a log-log diagram, as shown in Figure 3.24. On this diagram $\eta_1$ is the intercept with $(p_a/I_1) = 1$, and $m_2$ is the slope of the straight line.
Figure 3.23 Schematic Illustration of Conical and Spherical Cap Yield Surfaces in Triaxial Plane.
Figure 3.24 Determination of the Conical Yield Surface Parameters.
The yield and failure surfaces in the principal stress space are shaped like asymmetric conoids with their pointed apices at the origin of the stress space as shown in Figure 3.25:a. At failure the apex angle increases with the value of \( \gamma_1 \), and the curvature of the failure surface increases with the value of \( m_2 \). The cross sections of the yield surfaces, whose traces in the octahedral plane are shown in Figure 3.25:b, model the experimentally determined three-dimensional strength of sands with good accuracy (73, 75, 100).

The plastic potential function is modeled on the yield function as follows:

\[
q_p = I_1^3 - (27 + \gamma_2 \cdot (p_a/I_1)^{m_2}) \cdot I_3
\]

(3.65)

where \( \gamma_2 \) is a constant for given values of \( f_p \) and the confining pressure. It is given by (75):

\[
\gamma_2 = \frac{3 (1 + \nu^p) \cdot I_1^2 - 27 \nu^p (\sigma_1 + \nu^p \sigma_3)}{(p_a/I_1)^{m_2} \left[ \nu^p (\sigma_1 + \nu^p \sigma_3) - \frac{m_2 (1 + \nu^p) \cdot I_1^2}{f_p (p_a/I_1)^{m_2} + 27} \right]}
\]

(3.66)

where,

\[
\nu^p = -\frac{\Delta \varepsilon_3}{\Delta \varepsilon_1}
\]

(3.67)

The variation of \( \gamma_2 \) with \( f_p \) and \( \sigma_3 \) is shown in Figure 3.26, which can be modeled by a simple expression of the
Figure 3.25 Traces of Yield and Failure Surfaces in
a) Triaxial Plane
b) Octahedral Plane
Figure 3.26 Variation of $n_2$ with Yield Function.

Figure 3.27 Variation of Intercept with Confining Pressure.
form:

\[ n_2 = S_1 f_p + R_1 \sqrt{\frac{\sigma_3}{\sigma_a}} + t_1 \]  \hspace{1cm} (3.68)

where,

\[ S_1 = \text{slope of the straight line, and} \]

\[ R_1 \] and \[ t_1 \] = two terms that model the variation of the intercept and are determined as shown in Figure 3.27.

The expression for \( f_p \) indicates that the corresponding flow rule is nonassociated, i.e., the plastic expansive strain increment vectors are superimposed on the stress space from angles with the conical yield surfaces different from 90°, as shown in Figure 3.28.

The work-hardening and softening law employed with the conical yield surface is assumed to take the form:

\[ f_p = a \cdot e^{-b \cdot \frac{W_p}{P_a} \left( \frac{W_p}{P_a} \right)^{1/q}}, \quad q > 0 \]  \hspace{1cm} (3.69)

where,

\[ W_p = \text{the total plastic work per unit volume}, \text{ and} \]

\[ a, b \text{ and } q = \text{constants for a given value of the confining pressure and are defined by the following equations and from Figures 3.29 to 3.31:} \]

\[ a = \eta_1 \cdot \left( \frac{e \cdot P_a}{W_{p_{\text{peak}}}} \right)^{1/q} \]  \hspace{1cm} (3.70)
Figure 3.28 Schematic Diagram of Yielding Process with Plastic Strain Components Superimposed in Triaxial Plane.
Figure 3.29 Variation of Plastic Work with Yield Function.

Figure 3.30 Variation of Peak Plastic Work with Confining Pressure.

Figure 3.31 Variation of \( q \) with Confining Pressure.
\[ b = \frac{1}{q \cdot \frac{W_{p\text{peak}}}{W_{p60}}} \]  
\[ q = \frac{\log \left( \frac{W_{p\text{peak}}}{W_{p60}} \right)}{1 - \frac{W_{p60}}{W_{p\text{peak}}}} \cdot \log e \]  
\[ \log \left( \frac{n_1}{f_{p60}} \right) \]  
\[ W_{p\text{peak}} = \frac{P \cdot P_a \cdot \sigma^3}{P_a} \]  
\[ q = a_1 + b_1 \cdot \frac{\sigma^3}{P_a} \]  

3.8.2.3 Plastic Collapse Strains

The yield surface corresponding to the plastic collapse strains forms a cap on the open end of the conical yield surface, as was shown in Figure 3.23. The collapse yield surface is shaped as a sphere with center at the origin of the principal stress space. This yield surface is described in terms of the first and the second stress invariants, \( I_1 \) and \( I_2 \) as (75):

\[ f_c = I_1^2 + 2I_2 \]  
where,

\[ I_1 \]  
is defined by equation 3.63, and

\[ I_2 = \tau_{xy} \cdot \tau_{yx} + \tau_{yz} \cdot \tau_{zy} + \tau_{zx} \cdot \tau_{xz} - (\sigma_x \cdot \sigma_y + \sigma_y \cdot \sigma_z + \sigma_z \cdot \sigma_x) \]  

It is important to note that yielding resulting from outward movement of the cap does not result in eventual
failure. Failure is entirely controlled by the conical yield surface.

The plastic potential function which corresponds to an associated flow rule is identical to the yield function:

\[ q_c = I_1^2 + 2 I_2 \]  \hspace{1cm} (3.77)

The plastic collapse strain increment vectors are thus perpendicular to the spherical yield surface, as was shown in Figure 3.28.

The work-hardening law used with the spherical yield surface is expressed as (75):

\[ W_c = C \cdot p_a \left( \frac{f_c}{p_a^2} \right)^{p_{1}} \]  \hspace{1cm} (3.78)

where,

- \( W_c \) = the total plastic work required to produce collapse strains per unit volume, and
- \( C \) and \( p_{1} \) = dimensionless constants that can be estimated by plotting \( (W_c/p_a) \) vs. \( (f_c/p_a^2) \) in a log-log scale as shown in Figure 3.32.

3.9 **ANALYTICAL FORMULATION**

The basic concepts from classical plasticity theory mentioned before are employed in the derivation of the elasto-plastic stress-strain relationship.

The total strain increment, \( \delta c \), is defined as:
Figure 3.32 Variation of Plastic Collapse Work and Yield Function.
\begin{equation}
\{ \text{d} \varepsilon \} = \{ \text{d} \varepsilon^e \} + \{ \text{d} \varepsilon^p_1 \} + \{ \text{d} \varepsilon^p_2 \} + \ldots + \{ \text{d} \varepsilon^p_k \} + \ldots + \{ \text{d} \varepsilon^p_n \}
\end{equation}

where,

\( \{ \text{d} \varepsilon^e \} = \) the elastic strain increment, and

\( \{ \text{d} \varepsilon^p_k \} = \) the plastic strain increment associated with the k'th surface \( (1 < k < 5) \).

The strain increments are expressed in the following vector form:

\begin{equation}
\{ \text{d} \varepsilon \}^T = \{ \varepsilon_x', \varepsilon_y', \varepsilon_z', 2 \varepsilon_{xy}', 2 \varepsilon_{xz}', 2 \varepsilon_{yx}' \}
\end{equation}

and the stress increment vector is defined as:

\begin{equation}
\{ \text{d} \sigma \}^T = \{ \sigma_x', \sigma_y', \sigma_z', \sigma_{xy}', \sigma_{xz}', \sigma_{yx}' \}
\end{equation}

The stress increments and the elastic strain increments are related by means of a symmetrical elasticity matrix, \( [D^e] \), as given before by equation 3.57. The elasticity matrix is defined as:

\begin{equation}
[D^e] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix}
1-\nu & \nu & \nu & 0 & 0 & 0 \\
1-\nu & \nu & 0 & 0 & 0 & 0 \\
1-\nu & 0 & 0 & 0 & 0 & 0 \\
\frac{1-2\nu}{2} & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\
\text{sym.} & \frac{1-2\nu}{2} & 0 & \frac{1-2\nu}{2}
\end{bmatrix}
\end{equation}
where,

\[ E = \text{Young's modulus, and} \]

\[ \nu = \text{Poisson's ratio of the material.} \]

3.9.1 Incrementalization Procedure for the Derivation of the Elasto-Plastic Stress-Strain Matrix

The basic concept in the incrementalization procedure is that all the yield functions, \( f_k \), which are zero, i.e., which indicate yielding, remain equal to zero during any increments in stress or strain. Thus, for each \( f_k = 0 \):

\[
\frac{\partial f_k}{\partial \sigma} \{ \Delta \sigma \} + \frac{\partial f_k}{\partial \varepsilon^{pl}} \{ \Delta \varepsilon^{pl} \} = 0
\]

(3.83)

where,

\[
\frac{\partial f_k}{\partial \sigma} = \begin{pmatrix}
\frac{\partial f_k}{\partial \sigma_x}, & \frac{\partial f_k}{\partial \sigma_y}, & \frac{\partial f_k}{\partial \sigma_z}, & \frac{\partial f_k}{\partial \tau_{xy}}, & \frac{\partial f_k}{\partial \tau_{xz}}, & \frac{\partial f_k}{\partial \tau_{yz}}
\end{pmatrix}
\]

(3.84)

\[
\frac{\partial f_k}{\partial \varepsilon^{pl}} = \begin{pmatrix}
\frac{\partial f_k}{\partial \varepsilon^{pl}_x}, & \frac{\partial f_k}{\partial \varepsilon^{pl}_y}, & \frac{\partial f_k}{\partial \varepsilon^{pl}_z}, & \frac{\partial f_k}{\partial \varepsilon^{pl}_{xy}}, & \frac{\partial f_k}{\partial \varepsilon^{pl}_{xz}}, & \frac{\partial f_k}{\partial \varepsilon^{pl}_{yz}}
\end{pmatrix}
\]

(3.85)
The expression for \( \{d_\varepsilon\} \) in equation 3.57 is substituted into equation 3.83 giving:

\[
\frac{3f_k}{\varepsilon_p} T \left[ \frac{\varepsilon_p}{3} \right] \{d_\varepsilon\} + \frac{3f_k}{\varepsilon_p} T \{d_\varepsilon P_k\} = 0 \quad (3.86)
\]

The expression for \( \{d_\varepsilon^e\} \) obtained from equation 3.79 is substituted into equation 3.86 giving:

\[
\frac{3f_k}{\varepsilon_p} T \left[ \frac{\varepsilon_p}{3} \right] \{d_\varepsilon\} - \{d_\varepsilon P_1\} - \cdots - \{d_\varepsilon P_k\} - \cdots \{d_\varepsilon P_n\} + \frac{3f_k}{\varepsilon_p} T \{d_\varepsilon P_k\} = 0 \quad k = 1, 2, \ldots, n \quad (3.87)
\]

The flow rule associated with \( f_k \) is then substituted into equation 3.87, giving:

\[
\frac{3f_k}{\varepsilon_p} T \left[ \frac{\varepsilon_p}{3} \right] \{d_\varepsilon\} - \lambda_1 \frac{3g_1}{\varepsilon_p} - \cdots - \lambda_k \frac{3g_k}{\varepsilon_p} - \cdots - \lambda_n \frac{3g_n}{\varepsilon_p} + \lambda_k \frac{3f_k}{\varepsilon_p} T \{3g_k\} = 0 \quad k = 1, 2, \ldots, n \quad (3.88)
\]

These are \( n \) equations that must be solved to obtain the \( n \) unknowns, \( \lambda_k \), \( k = 1, 2, \ldots, n \). Equation 3.88 may be written in a matrix form as:

\[
\frac{3f}{\varepsilon_p} T \left[ \frac{\varepsilon_p}{3} \right] \{d_\varepsilon\} - \frac{3g}{\varepsilon_p} \{\lambda\} + [V] \{\lambda\} = \{0\} \quad (3.89)
\]
where,

\[
\frac{3f}{3\sigma} = \text{a } 6\times n \text{ matrix, the } k'\text{th column of which is } \left( \frac{3f_k}{3\sigma} \right),
\]

\[
\frac{3g}{3\sigma} = \text{a } 6\times n \text{ matrix, the } k'\text{th column of which is } \left( \frac{3g_k}{3\sigma} \right),
\]

\[
[V] = \text{a diagonal } n\times n \text{ matrix with the diagonal element } \left( \frac{3f_k}{3\sigma} \right), \text{ and } \left( \frac{3g_k}{3\sigma} \right), \text{ and } \left( \frac{3z^p_k}{3\sigma} \right)
\]

\[
\{\lambda\} = \text{the unknown } n\times 1 \text{ vector.}
\]

Equation 3.89 may be transformed to the form:

\[
(\left( \frac{3f}{3\sigma} \right)^T [D^e] \left( \frac{3g}{3\sigma} \right) - [V] ) \{\lambda\} = \left( \frac{3f}{3\sigma} \right)^T [D^e] \{d\sigma\}
\]

\[
(3.90)
\]

Denoting:

\[
[L] = (\left( \frac{3f}{3\sigma} \right)^T [D^e] \left( \frac{3g}{3\sigma} \right) - [V] )
\]

\[
(3.91)
\]

and its inverse as \([L]^{-1}\), then:

\[
\{\lambda\} = [L]^{-1} \left( \frac{3f}{3\sigma} \right)^T [D^e] \{d\sigma\}
\]

\[
(3.92)
\]

But from equation 3.57 and 3.79:

\[
\{d\sigma\} = [D^e] \left( \{d\sigma\} - \{d\sigma^p_1\} - \ldots - \{d\sigma^p_{k} \} - \ldots - \{d\sigma^p_n\} \right)
\]

\[
(3.93)
\]
\[
\{d\sigma\} = \left[D^e\right] \left\{d\varepsilon\right\} = \left[\frac{\partial \sigma}{\partial \varepsilon}\right] \{\lambda\} \quad (3.94)
\]

Using equation 3.92, equation 3.94 changes to:
\[
\{d\sigma\} = \left(D^e - \left[D^e\right] \left[\frac{\partial \sigma}{\partial \varepsilon}\right] [L]^{-1} \left[\frac{\partial \varepsilon^p}{\partial \sigma}\right]^T \left[D^e\right]\right) \{d\varepsilon\}
\]
(3.95)

Thus,
\[
[D^{ep}] = \left[D^e\right] - \left[D^e\right] \left[\frac{\partial \sigma}{\partial \varepsilon}\right] [L]^{-1} \left[\frac{\partial \varepsilon^p}{\partial \sigma}\right]^T \left[D^e\right]
\]
(3.96)

which is the required elasto-plastic stiffness matrix. This will be symmetric only if \(\left[\frac{\partial \varepsilon^p}{\partial \sigma}\right] = \left[\frac{\partial \sigma}{\partial \varepsilon}\right]\), i.e., if associated flow occurs on all active yield surfaces. In this case, \([L]\) is symmetric as is its inverse, thereby resulting in a symmetric \([D^{ep}]\) matrix.

Equation 3.96 is the general expression for an elasto-plastic stiffness matrix with multiple and simultaneous yield surfaces. The special case of the incremental matrix with two yield surfaces described in section 3.8.2, is summarized in section 3.9.2. Also, the condensation of the matrix to handle the plane strain condition assumed for the present analysis is given in detail in section 3.9.3.
3.9.2 Incremental Constitutive Law with Two Yield Surfaces

To obtain the incremental elasto-plastic stiffness matrix for the special case of a constitutive law with two yield surfaces, two equations similar to equation 3.90 are solved:

\[
\lambda_c \cdot L_{11} + \lambda_p \cdot L_{12} = T_1 \tag{3.97}
\]

\[
\lambda_c \cdot L_{21} + \lambda_p \cdot L_{22} = T_2 \tag{3.98}
\]

from which,

\[
\lambda_c = \frac{L_{22} \cdot T_1 - L_{12} \cdot T_2}{L_{11} \cdot L_{22} - L_{12} \cdot L_{21}} \tag{3.99}
\]

\[
\lambda_p = \frac{L_{22} \cdot T_2 - L_{21} \cdot T_1}{L_{11} \cdot L_{22} - L_{12} \cdot L_{21}} \tag{3.100}
\]

where,

\[
L_{11} = \frac{\partial f_c}{\partial \sigma} \left[ D^e \right] \frac{\partial g_C}{\partial \sigma} + \frac{\partial f_c}{\partial \omega_c} \left[ \sigma \right]^T \frac{\partial g_C}{\partial \sigma} \tag{3.101}
\]

\[
L_{22} = \frac{\partial f_p}{\partial \sigma} \left[ D^e \right] \frac{\partial g_p}{\partial \sigma} + \frac{\partial f_p}{\partial \omega_p} \left[ \sigma \right]^T \frac{\partial g_p}{\partial \sigma} \tag{3.102}
\]

\[
L_{12} = \frac{\partial f_c}{\partial \sigma} \left[ D^e \right] \frac{\partial g_p}{\partial \sigma} \tag{3.103}
\]

\[
L_{21} = \frac{\partial f_p}{\partial \sigma} \left[ D^e \right] \frac{\partial g_c}{\partial \sigma} \tag{3.104}
\]
\[ T_1 = \left( \frac{3f_c}{\partial c} \right) [D^e] \{ dc \} \]  \hspace{1cm} (3.105) \\
\[ T_2 = \left( \frac{3f_{pc}}{\partial c} \right) [D^e] \{ dc \} \]  \hspace{1cm} (3.106) 

The elasticity matrix is given in equation 3.82, while other components involved in equations 3.101 to 3.106 are summarized below.

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{3f_c}{\partial c} \\
\frac{3g_c}{\partial c}
\end{bmatrix} = 2 \begin{bmatrix}
2 \gamma_{yz} \\
2 \gamma_{zx} \\
2 \gamma_{xy}
\end{bmatrix}
\]  \hspace{1cm} (3.107) 

\[
\frac{3f_c}{\partial W_c} = \frac{P_a}{C \cdot p_1} \left( \frac{f_c}{p_a} \right)^2 1 - p_1 
\]  \hspace{1cm} (3.108) 

where, \( f_c \) is given by equation 3.75.

\[
\{ \varepsilon \}^T \left( \frac{3g_c}{\partial c} \right) = 2 \cdot I_1^2 + 4 \cdot I_2 = 2 \cdot g_c 
\]  \hspace{1cm} (3.109) 

where, \( g_c \) is given by equation 3.77.
\[
\begin{align*}
\frac{\partial f}{\partial \sigma_1} &= I_1^2 \cdot \left( \frac{I_1}{I_3} \right)^{m_2} \\
\frac{\partial f}{\partial \sigma_2} &= 2 (a_x - 27) \cdot \left( \frac{I_1}{I_3} \right)^{m_2} \\
\frac{\partial f}{\partial \sigma_3} &= 2 (a_y - 27) \cdot \left( \frac{I_1}{I_3} \right)^{m_2} \\
\frac{\partial f}{\partial \sigma_4} &= 2 (a_z - 27) \cdot \left( \frac{I_1}{I_3} \right)^{m_2} \\
\end{align*}
\]
\[
\frac{\partial^2 \mathbf{P}}{\partial \mathbf{w}^2} = \frac{f_p}{p_a} \left[ \frac{p_a}{q \cdot w_p} - b \cdot p_a \right]
\]  
(3.112)

where, \(f_p\) is given by equation 3.61.

\[
\begin{align*}
(\sigma)_{\text{T}} \frac{\partial^2 \mathbf{P}}{\partial \mathbf{c}^2} &= 3 \mathbf{g}_p + m_2 \frac{p_a}{I_1} \mathbf{m}_2 \mathbf{I}_3
\end{align*}
\]  
(3.113)

The incremental material stiffness matrix \([D^e]\) for the special case of a constitutive law with two yield surfaces is thus given by:

\[
[D^e_{\text{ep}}] = [D^e] - \frac{[D^e]}{A_1} \left( \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2} \right)_{\mathbf{b}_c} \mathbf{T} + \left( \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2} \right)_{\mathbf{b}_p} [D^e]
\]  
(3.114)

where,

\[
A_1 = L_{11} L_{22} - L_{12} L_{21}
\]  
(3.115)

\[
\begin{align*}
\{b_c\} &= L_{22} \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2} - L_{12} \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2} \\
\{b_p\} &= L_{11} \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2} - L_{21} \frac{\partial^2 \mathbf{c}}{\partial \mathbf{c}^2}
\end{align*}
\]  
(3.116)

3.9.3 Plane Strain Constitutive Law with Two Yield Surfaces

If \(z\) is chosen as the direction normal to the plane, then:

\[
\tau_{yz} = \tau_{zx} = 0
\]  
(3.118)

and the appropriate columns may be deleted from the relationship. The rows corresponding to these stress components cease to be of interest in the two dimensional anal-
ysis. The normal stress $q_z$ is however not zero and the condition that:

$$c_z = 0$$  \hspace{1cm} (3.119)

has to be imposed. Since from equations 3.55 to 3.57:

$$\{dc\} = \left[D^e\right]^{-1} \{d\sigma\} + \lambda_k \left(\frac{\partial \pi}{\partial \sigma}\right)$$  \hspace{1cm} (3.120)

$$0 = \left(\frac{3f_k}{3\sigma}\right)_T \{d\sigma\} + \left(\frac{3f_k}{3\varepsilon}\right)_T \{d\varepsilon^P\}$$  \hspace{1cm} (3.121)

or,

$$0 = \left(\frac{3f_k}{3\sigma}\right)_T \{d\sigma\} + A_2 \cdot \lambda$$  \hspace{1cm} (3.122)

where

$$A_2 = \left(\frac{3f_k}{3\varepsilon}\right)_T \{d\varepsilon^P\} \cdot \frac{1}{\lambda}$$  \hspace{1cm} (3.123)

For the particular case with two simultaneous yield surfaces, equation 3.122 breaks into:

$$0 = \left(\frac{3f_c}{3\sigma}\right)_T \{d\sigma\} + D_{kkc} \lambda_c$$  \hspace{1cm} (3.124)

$$0 = \left(\frac{3f_p}{3\sigma}\right)_T \{d\sigma\} + D_{kkp} \lambda_p$$  \hspace{1cm} (3.125)

where,

$\lambda_c$ and $\lambda_p$ = plastic multipliers

$$D_{kkc} = \left(\frac{3f_c}{3\varepsilon}\right)_T \left(\frac{3\pi_c}{3\sigma}\right)$$  \hspace{1cm} (3.126)

$$D_{kkp} = \left(\frac{3f_p}{3\varepsilon}\right)_T \left(\frac{3\pi_p}{3\sigma}\right)$$  \hspace{1cm} (3.127)
Equations 3.120, 3.124, and 3.125 can be written as:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 \\
-\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 \\
-\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 \\
0 & 0 & 0 & 2(1+\nu)
\end{pmatrix}
\begin{pmatrix}
\frac{\partial q_c}{\partial \sigma_x} & \frac{\partial q_p}{\partial \sigma_x} \\
\frac{\partial q_c}{\partial \sigma_y} & \frac{\partial q_p}{\partial \sigma_y} \\
\frac{\partial q_c}{\partial \sigma_z} & \frac{\partial q_p}{\partial \sigma_z} \\
\partial \tau_{xy} & \partial \tau_{xy}
\end{pmatrix}
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy}
\end{pmatrix}
\]

(3.128)
Imposing the condition $\epsilon_z = 0$ gives:

$$\sigma_z = \nu \sigma_x + \nu \sigma_y - E \frac{3 \sigma_c}{3 \sigma_z} \lambda_c - E \frac{3 \sigma_p}{3 \sigma_z} \lambda_p$$  \hspace{1cm} (3.129)

By back substitution in the $1^{st}$ of 3.128:

$$\epsilon_x = \frac{\sigma_x}{\nu} (1-\nu^2) - \frac{\sigma_y}{\nu} (1+\nu) + \left( \frac{3 \sigma_c}{3 \sigma_x} + \nu \frac{3 \sigma_c}{3 \sigma_z} \right) \lambda_c$$

$$+ \left( \frac{3 \sigma_p}{3 \sigma_x} + \nu \frac{3 \sigma_p}{3 \sigma_z} \right) \lambda_p$$  \hspace{1cm} (3.130)

The same can be applied for $\epsilon_y$, $\gamma_{xy}$. From the $5^{th}$ of 3.128:

$$0 = \left( \frac{3 \sigma_c}{3 \sigma_x} + \nu \frac{3 \sigma_c}{3 \sigma_z} \right) \sigma_x + \left( \frac{3 \sigma_c}{3 \sigma_y} + \nu \frac{3 \sigma_c}{3 \sigma_z} \right) \sigma_y + \frac{3 \sigma_c}{3 \sigma_{xy}} \gamma_{xy}$$

$$+ \left( D_{kkc} - E \frac{3 \sigma_c}{3 \sigma_z} \frac{3 \sigma_c}{3 \sigma_z} \right) \lambda_c + \left( -E \frac{3 \sigma_p}{3 \sigma_z} \frac{3 \sigma_p}{3 \sigma_z} \right) \lambda_p$$  \hspace{1cm} (3.131)
Similar expression is obtained by substituting in the 6th of 3.128. Equation 3.128 can now be reduced to:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix}
= \begin{pmatrix}
\frac{1-\nu^2}{E} & \frac{\nu(1+\nu)}{E} & 0 \\
-\frac{\nu(1+\nu)}{E} & \frac{1-\nu^2}{E} & 0 \\
0 & 0 & \frac{2(1+\nu)}{E}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial f_c}{\partial \sigma_x} + \nu \frac{\partial f_c}{\partial \sigma_z} \\
\frac{\partial f_c}{\partial \sigma_y} + \nu \frac{\partial f_c}{\partial \sigma_z} \\
\frac{\partial f_c}{\partial \tau_{xy}} + \nu \frac{\partial f_c}{\partial \tau_{xy}}
\end{pmatrix}
+ \begin{pmatrix}
\frac{\partial f_p}{\partial \sigma_x} + \nu \frac{\partial f_p}{\partial \sigma_z} \\
\frac{\partial f_p}{\partial \sigma_y} + \nu \frac{\partial f_p}{\partial \sigma_z} \\
\frac{\partial f_p}{\partial \tau_{xy}} + \nu \frac{\partial f_p}{\partial \tau_{xy}}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial g_c}{\partial \sigma_x} + \nu \frac{\partial g_c}{\partial \sigma_z} \\
\frac{\partial g_c}{\partial \sigma_y} + \nu \frac{\partial g_c}{\partial \sigma_z} \\
\frac{\partial g_p}{\partial \sigma_x} + \nu \frac{\partial g_p}{\partial \sigma_z}
\end{pmatrix}
= \sigma_x \\
\sigma_y \\
\tau_{xy}
\end{pmatrix}
\]

These can be put in a different form as:

\[
\{\varepsilon\} = \left[D^e\right]^{-1} \{\sigma\} + \left[\frac{\partial G}{\partial \lambda}\right] \{\lambda\}
\]

(3.133)
and \( \{0\} = \left( \frac{3FF}{\partial \sigma} \right) \{d\sigma\} + [D_{kk}] \{\lambda\} \) \hspace{1cm} (3.134)

But \( \{d\sigma\} = [D^e] \{d\varepsilon^e\} \)

\[
\{0\} = \left( \frac{3FF}{\partial \sigma} \right) [D^e] \{d\varepsilon^e\} + [D_{kk}] \{\lambda\} \hspace{1cm} (3.135)
\]

\[
\{0\} = \left( \frac{3FF}{\partial \sigma} \right) [D^e] \{d\varepsilon^e\} - \left( \frac{3GG}{\partial \sigma} \right) \{\lambda\} + [D_{kk}] \{\lambda\} \hspace{1cm} (3.136)
\]

which represents a set of two simultaneous equations that can be solved to get \( \lambda_c \) and \( \lambda_p \) in the following procedure.

\[
(\frac{3FF}{\partial \sigma}) [D^e] \left( \frac{3GG}{\partial \sigma} - [D_{kk}] \right) \{\lambda\} = \left( \frac{3FF}{\partial \sigma} \right) [D^e] \{d\varepsilon\} \hspace{1cm} (3.137)
\]

where

\[
\left( \frac{3FF}{\partial \sigma} \right) = \begin{bmatrix}
\frac{3FF_c}{\partial \sigma} \\
\frac{3FF_p}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
\frac{3f_c}{\partial \sigma} + \nu \frac{3f_c}{\partial \sigma} \\
\frac{3f_p}{\partial \sigma} + \nu \frac{3f_p}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
\frac{3f_c}{\partial \sigma} + \nu \frac{3f_c}{\partial \sigma} \\
\frac{3f_p}{\partial \sigma} + \nu \frac{3f_p}{\partial \sigma}
\end{bmatrix}
\]

\[
\left( \frac{3GG}{\partial \sigma} \right) = \begin{bmatrix}
\frac{3GG_c}{\partial \sigma} \\
\frac{3GG_p}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
\frac{3g_c}{\partial \sigma} + \nu \frac{3g_c}{\partial \sigma} \\
\frac{3g_p}{\partial \sigma} + \nu \frac{3g_p}{\partial \sigma}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{3f_c}{\partial \sigma} + \nu \frac{3f_c}{\partial \sigma} \\
\frac{3f_p}{\partial \sigma} + \nu \frac{3f_p}{\partial \sigma}
\end{bmatrix} \begin{bmatrix}
\frac{3g_c}{\partial \sigma} + \nu \frac{3g_c}{\partial \sigma} \\
\frac{3g_p}{\partial \sigma} + \nu \frac{3g_p}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
\frac{3f_c}{\partial \sigma} + \nu \frac{3f_c}{\partial \sigma} \\
\frac{3f_p}{\partial \sigma} + \nu \frac{3f_p}{\partial \sigma}
\end{bmatrix}
\]

\[
(3.138)
\]

\[
\begin{bmatrix}
\frac{3g_c}{\partial \sigma} + \nu \frac{3g_c}{\partial \sigma} \\
\frac{3g_p}{\partial \sigma} + \nu \frac{3g_p}{\partial \sigma}
\end{bmatrix} = \begin{bmatrix}
\frac{3g_c}{\partial \sigma} + \nu \frac{3g_c}{\partial \sigma} \\
\frac{3g_p}{\partial \sigma} + \nu \frac{3g_p}{\partial \sigma}
\end{bmatrix}
\]

\[
(3.139)
\]
\[
[D_{kk}] = \begin{bmatrix}
D_{kcc} & D_{kcp} \\
D_{kpc} & D_{kpp}
\end{bmatrix} = \begin{bmatrix}
(D_{kcc} - E \frac{\partial f_c}{\partial \sigma_z} \frac{\partial g_c}{\partial \sigma_z}) & (-E \frac{\partial f_c}{\partial \sigma_z} \frac{\partial g_p}{\partial \sigma_z}) \\
(-E \frac{\partial f_p}{\partial \sigma_z} \frac{\partial g_c}{\partial \sigma_z}) & (D_{kpp} - E \frac{\partial f_p}{\partial \sigma_z} \frac{\partial g_p}{\partial \sigma_z})
\end{bmatrix}
\]

and \[
\{\lambda\} = \begin{bmatrix}
\lambda_c \\
\lambda_p
\end{bmatrix}
\]

Equation 3.137 can be expanded to:

\[
\begin{bmatrix}
\frac{\partial f_c}{\partial \sigma} T & \frac{\partial g_c}{\partial \sigma} \\
\frac{\partial f_p}{\partial \sigma} & \frac{\partial g_p}{\partial \sigma}
\end{bmatrix}
\begin{bmatrix}
[D_{kcc}] & D_{kcp} \\
D_{kpc} & D_{kpp}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\lambda_c \\
\lambda_p
\end{bmatrix} = \begin{bmatrix}
\frac{\partial f_c}{\partial \sigma} T [D_{kcc}] \{de\} \\
\frac{\partial f_p}{\partial \sigma} T [D_{kpc}] \{de\}
\end{bmatrix}
\]

If we define

\[
L_{11} = \frac{\partial f_c}{\partial \sigma} T [D_{kcc}] \frac{\partial g_c}{\partial \sigma} - D_{kcc}
\]

\[
L_{22} = \frac{\partial f_p}{\partial \sigma} T [D_{kpp}] \frac{\partial g_p}{\partial \sigma} - D_{kpp}
\]

\[
L_{12} = \frac{\partial f_c}{\partial \sigma} T [D_{kcp}] \frac{\partial g_p}{\partial \sigma} - D_{kcp}
\]

\[
L_{21} = \frac{\partial f_p}{\partial \sigma} T [D_{kpc}] \frac{\partial g_c}{\partial \sigma} - D_{kpc}
\]

\[
T_1 = \frac{\partial f_c}{\partial \sigma} T \{de\}
\]
\[ T_2 = \left( \frac{3FF}{3\sigma} \right)^T \left[ D^e \right] \{ \delta \sigma \} \] (3.148)

equation 3.142 takes the form:

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
\lambda_c \\
\lambda_p
\end{bmatrix} =
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix}
\] (3.149)

\[
\begin{bmatrix}
\lambda_c \\
\lambda_p
\end{bmatrix} =
\begin{bmatrix}
L_{22} & -L_{12} \\
-L_{21} & L_{11}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix}
\]

\[ \lambda = \frac{L_{22} T_1 - L_{12} T_2}{\text{Det}} \] (3.151)

\[ \lambda_p = \frac{L_{11} T_2 - L_{21} T_1}{\text{Det}} \] (3.152)

\[ \text{Det} = L_{11} L_{22} - L_{12} L_{21} \] (3.153)

The components of the vector \{\delta \} are substituted in the equation:

\[ \{ \delta \sigma \} = \left[ D^e \right] \{ \delta \} - \left( \frac{3GG}{3\sigma} \right) \{ \delta \} \] (3.154)

which gives:

\[ \{ \delta \sigma \} = \left[ D^e \right] \{ \delta \} - \frac{1}{\text{Det}} \left[ \left( \frac{3GG}{3\sigma} \right) \left\{ b^c \right\}^T + \left( \frac{3GG}{3\sigma} \right) \left\{ b^P \right\}^T \right] \left[ D^e \right] \{ \delta \} \] (3.155)
where

\[ \{ b_c \} = L_{22} \left( \frac{\partial F}{\partial \sigma} \right) - L_{12} \left( \frac{\partial F}{\partial \sigma} \right) \]

(3.156)

\[ \{ b_p \} = L_{11} \left( \frac{\partial F}{\partial \sigma} \right) - L_{21} \left( \frac{\partial F}{\partial \sigma} \right) \]

(3.157)

Finally, the elasto-plastic stiffness matrix is defined as:

\[ [D^{EP}] = [D^e] - \frac{[D^e]}{\text{Det}} \left( \frac{\partial G}{\partial \sigma} \right) \{ b_c \}^T \]

\[ + \left( \frac{\partial G}{\partial \sigma} \right) \{ b_p \}^T [D^e] \]

(3.158)
3.9.4 Iterative Procedure for Soil Properties

The material nonlinearity of the elasto-plastic model is represented by equation 3.158. It is known as mentioned by Zienkiewicz (122) that the relationship between stress increment and strain increment in an elasto-plastic model is not uniquely defined. Iterations are thus required to reach a converged value of the elasto-plastic stiffness matrix defined by the equation. As the 'incremental deformation' and not the 'total deformation' is used in the analysis, small load increments are applied. In each increment the direct iterations discussed earlier for the interface and other soil elements is used to update the material properties and the stiffness matrix at the end of each iteration. This is carried out simultaneously with the transferring of stresses from the elements experiencing the after peak behaviour to the nonfailing elements as will be discussed in the next section. The number of iterations is thus limited to the number of iterations required for the stress transfer.

3.9.5 Problems, Comments and Modifications to the Soil Model

The following modifications to the original Lade
theory have been implemented in the computer program:

1. After-Peak Behaviour. In the Lade theory, \( f(W_p) \) is a function decreasing asymptotically to zero after the peak and so no use could be made of such an essential parameter away from failure. The function \( f(W_p) \) is modified after the peak so that its asymptotic value or the residual value is related to the stresses at failure, as suggested by Aubry and Des Croix (9). The residual stresses, as shown from the triaxial test results, are approximately 60% of the failure stresses, a value which is considered in the present analysis.

2. Dilatancy. The phenomenon of dilatancy is due to the coupling between deviatoric stress rates and volumetric strain rates. To limit the dilatancy, which may cause troubles in the numerical algorithm especially at low pressures, a limit to the value of \( \tau_2 \) is taken as (9):

\[
\tau_2 \lim = -27 + 9 \frac{I_1^2}{I_2} \tag{3.159}
\]

3. The Stress Ratio. The ratio \( I_1^3/I_3 \) used to define the yield surface is very sensitive to the stress ratios. It has been noticed that under
low pressures, the high stress ratios cause the elements to fail rapidly. If failure is detected at these low pressures, the stresses are modified so as to bring the value of the stress ratio to its original value before failure and this reduces the value of $f_p$ consequently. The excess stresses are then transferred to the neighbouring elements by the procedure described earlier. If in the following iterations, failure occurred, then the element is considered to be in a failing state and the stresses are retained.

4. Symmetry of the Stiffness Matrix. The elasto-plastic model discussed in details earlier employs a plastic potential function, $g$, which is different from the yield function, $f$. This indicates a nonassociated flow rule and consequently, a nonsymmetric stiffness matrix. Although the non-associated flow rule is supported by the experiments carried out by other researchers and is well documented in most of the soil elasto-plastic models, (27, 29, 73, 74, 75, 86, 100, 121, 122), this was modified in the present analysis. The plastic potential function is assumed identical
to the yield function and a symmetric matrix is obtained. Both procedures are built-in in the computer program but only the symmetric matrix is used to get the results in this research. The only reason is to limit the computer storage and computer time requirements. Because the results are reasonably satisfactory, no attempt to compare both procedures is made for the conduit problem, but only for a small test problem where the differences were not clear enough to lead to good comparison.

5. Work-Softening. The work-hardening law defined by equation 3.69 represents also a work-softening behaviour. Different approaches have been suggested to handle the problem of work-softening. Because the material stiffness matrix has a positive slope even after the peak, this leads to a continuous increase in the element stiffness. Besseling (11), Owen, Prakash and Zienkiewicz (99), Hoeg (58), Desai (27), Eisenberg (41), Prevost and Hoeg (102) and Lo and Lee (81) are among those who tried to solve the work-softening problem in soils. Assigning a negative value to the slope of the equivalent stress against the
plastic uniaxial strain accompanied by the Von-Mises and the Prandtl-Reuss relations, is one way of dealing with the problem. Another way is to approximate the post-peak strain-softening slope with small positive linear elastic increments and an initial stress release similar to the one described by Zienkiewicz et. al. (120).

In the present analysis, equilibrium iterations are performed within each load increment together with applying the stress transfer technique described previously. Equilibrium at a given load step is assumed to be achieved when the excess stress in all strain-softening elements are negligible. The numerical algorithm is so arranged that any post-peak positive stiffness brings the stress state back down to the post-peak slope through the stress release. Figure 3.33.a shows schematically how the stress state may vary in an element, if the minor principal stress is kept constant. However, during incremental loading and subsequent stress redistribution, due to some elements going past peak point, the minor principal stress in an
Figure 3.33.a Stress-Strain Curve for an Element with Constant Minor Principal Stress.

Figure 3.33.b Stress-Strain Curve for an Element with Varying Minor Principal Stress.
element does not remain constant. Thus, depending on the structural geometry and the progress of failure, the stress path followed by an element may appear superimposed on top of a set of fundamental stress-strain curves corresponding to various confining pressures, as shown in Figure 3.33.b.

6. Low Stress Levels. When the confining pressure is low, problems in convergence may be observed. In handling the work-softening problem, and as was shown in Figure 3.33, the stresses generated during a given loading increment are observed not to pass the yield surface. This cannot be achieved properly when the stress levels are low, unless very small load increments are applied. Because this leads to an excessive computer time, larger load increments were used. In a given low stress element, the stresses generated by the large load increment may exceed the range for failing the element and keeping the stresses, after the stress transfer, above the residual level. The stresses in this element are thus assumed to be constant after failure and have the values corresponding to the residual level. Any stresses
attracted by this element after that stage are then transferred to the rest of the structure. This problem is shown to be less severe when larger elements close to the free surface of the structure are used, which leads to higher confining pressure.

7. Simultaneous Yield Surfaces. If the plastic loading state prevails, it becomes necessary to determine the value of the two plastic multipliers, \( \lambda_c \) and \( \lambda_p \), as a function of \( \{d\sigma\} \) and \( \{d\varepsilon\} \). At a conical yield surface, where the two yield surfaces meet, either one, two or none of the yield surfaces are activated, i.e., have an associated positive plastic multipliers. It has been noticed by Aubry and Des Croix (9) that on a stress path, for which the two yield surfaces are activated, as positive work-hardening is associated with the first one, only the second yield surface is activated after the peak. When plastic loading occurs, the plastic multipliers are actually the solution of a system of two inequalities:

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
\lambda_c \\
\lambda_p
\end{bmatrix} \geq
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix}
\]

(3.160)
with a positive constraint $\lambda_i > 0$. The coefficients of the system of equations have been defined in section 3.9.3.

The solution of the problem is carried out by assuming the equations as equalities inside the iterations. If any of the plastic multipliers becomes negative, the terms corresponding to it are deleted from the equations. For example, if $\lambda_c > 0$, then $\lambda_c$ is taken as zero and equation 3.160 becomes:

$$ L_{22} \cdot \lambda_p = T_2 \quad (3.161) $$

It has also been noticed that the quantity

$$ \left( \frac{\partial f_p}{\partial \sigma} \right) T \left[ D^e \right] \left( \frac{\partial q_p}{\partial \sigma} \right) $$

of equation 3.102 becomes very small at low confining pressure for dense sand. The quantity $\left( \begin{array}{c} \sigma^T \\ \frac{\partial q_p}{\partial \sigma} \end{array} \right)$ is generally negative or close to zero when the stresses are low and may cause the iterations to diverge as mentioned before.

3.10 CONSTRUCTION AND COMPACTION STAGE

3.10.1 Construction Sequence

The finite element mesh is divided into construction layers. This represents basically what is really done in
the field. The sequence of construction is carried out in the following procedure.

1. The complete soil-steel structure is divided into soil layers and elements and the conduit into beam finite elements as shown in Figures 3.1, 3.2 and 3.34. The layers numbering shown in Figure 3.34 is used to explain the procedure.

2. The soil base is formed by any specified number of layers (referring to the figure, layers 1 to 3 form the base). The depth of layers is taken arbitrary.

3. The complete steel conduit along with one additional layer (layer 4 in the figure) is added and the necessary interface elements are introduced.

4. The two soil layers consisting of only triangular elements (layers 5 and 6 in the figure) and the corresponding interface elements are generated. This completes the mesh up to the conduit spring lines.

5. Three more soil layers, which include both triangular and quadrilateral elements, are added layer by layer. This completes the mesh generation as
Figure 3.4: Finite Element Mesh for an Elliptical Shape Conduit.
shown in Figure 3.34 for a general case of an elliptic conduit.

3.10.2 **Gravity and Compaction Forces**

When a new soil layer is added to the structure, the elements of this layer are added to the existing finite element mesh and the loads corresponding to this layer are applied to the structure. The resultant of three types of loads are considered to accompany the addition of a soil layer:

1. **Elements Gravity Forces.** The gravity force (weight) of a constant strain element is divided equally between the three nodes of that element. For a linear strain element, the weight is divided between its three mid-side nodes. The weight of an element is simply its area times the unit weight of soil.

2. **Compaction Forces.** The compaction is represented by assuming a specified uniform surface load (usually 1.0 psi (6.895 kPa) to 5.0 psi (34.475 kPa) depending on the problem type) to be acting on top of the layer added. Using virtual work, the surface load is divided equally between the two top nodes of constant strain element, while
in a linear strain element, 2/3 of the surface load is applied at the mid-side node and 1/6 of the load is applied to each of the two end nodes as shown in Figure 3.35.

3. Compaction Forces Release. The equipment used to compact the previous layer leaves the site before the adding of a new layer of soil. This is simulated by applying upward forces at the bottom nodes of the new layer and is shown in Figure 3.35.

The three sets of forces are added together at the beginning of the new increment, the material properties for soil and interface elements are updated according to the nonlinear or the elasto-plastic relationships and the incremental analysis then starts.

3.11 MESH GENERATION AND GEOMETRIC NONLINEARITY

Based on the procedure outlined by Hafez (50), a scheme for an automatic generation of the finite element mesh is used. The mesh data is generated from a minimum number of geometric parameters such as the height and span of the conduit and the height of soil cover. It also leads to a minimum band width which reduces the computer time required considerably.
Constant Strain Elements

Type of Loading

Linear Strain Elements

\[ W_1 = \text{triangular element weight}, \quad P_1 = \frac{W_1}{3} \]

\[ W_2 = \text{total compaction force} \]

\[ W_3 = \text{previous compaction force} \]

Figure 3.35: Loads Acting on an Element During Construction Stage.
The geometric nonlinearity is handled by updating the nodal coordinates after each row of elements is added or after each load increment. The areas and geometry of the finite elements in the mesh are all modified accordingly which is essential when a large deformation analysis for stability is required.

3.12 **ANALYSIS PROCEDURE**

Geometric and material nonlinearities have been considered in the analysis as discussed earlier. The nonlinearities of the problem have been handled by an incremental analysis accompanied by an iterative technique. The flow-chart given in Appendix E illustrates the procedure followed. The procedure of analysis consists of the following steps:

1. The completed soil-steel structure is divided into elements and layers and the applied live load is divided into load increments according to sections 3.10 and 3.11.

2. The finite element mesh is updated after each row of elements is added during the construction stage, and the finite element properties are also updated based on the stresses and displacements from previous load increments.
3. For each load increment, iterations are carried out to update the element properties and correct the state of stress in the elements as discussed in the rest of the given steps.

4. The interface elements are checked to identify the elements that fail in tension or shear. The stresses in the failing elements are corrected by applying the technique described in section 3.5.3 and their properties are updated by using the procedure in section 3.5.2.

5. The forces in the beam elements are calculated and formation of any plastic hinges is identified. A converged geometry is then reached by following the procedure given in section 3.4.1.

6. Correcting the stresses in the soil elements follow the interface and beam elements corrections. The excess stresses in the failing soil elements are transferred to the structure according to section 3.7.6 until failure in any of the elements is corrected. The elements properties are updated according to section 3.7.3 simultaneously with the stress transfer.

7. At this stage, the analysis of the structure has
converged and the solution to the given load increment is obtained. The stresses are added to the previous stresses and the displacements of the mesh points are updated. These are used to obtain new element properties and stiffness matrices for the next load increment.

Using fewer elements of the same type as employed in the analysis referred to above, several calibration tests were run to determine the convergence characteristics, the effect of changing the iteration sequence and the uniqueness of the solution. In all cases the procedure employed gave satisfactory results. However, combining the iterations for the interface, beam and soil elements and correcting them simultaneously did not lead to a convergent analysis procedure.
CHAPTER IV
EXPERIMENTS AND VERIFICATION

4.1 GENERAL

The analysis procedure described in Chapter III requires properties of the analyzed soil both for the hyperbolic non-linear and the elasto-plastic models. In the absence of full scale stability tests, a laboratory model for the soil-steel structure was built and tested for different geometry and loading conditions to verify the theoretical analysis. The following are the experimental tests performed for this purpose:

1. A triaxial test to determine the soil properties for the hyperbolic and elasto-plastic models.
2. A model of the conduit embedded in soil and loaded until failure in different loading cases.

4.2 TRIAXIAL TEST

The soil tested was clean, dry, Lake Erie Sand. The bulk density of the sand was found to be 81.4 pcf (1304 kg/m$^3$) before compaction and 104 pcf (1666 kg/m$^3$) after compaction. The moist bulk density was 106 pcf (1698 kg/m$^3$) before compaction and 108 pcf (1730 kg/m$^3$) after compaction.
The specific gravity for the sand was 2.65.

A sieve analysis was conducted for the sand sample and its result is shown in Figure 4.1 and given in Table 4.1. From the figure, it is seen that the sand was fairly uniform and not very well graded. The uniformity coefficient for this type of sand was 1.76.

Two sets of triaxial tests were required to obtain the soil parameters for the analysis. An isotropic compression test was needed for the collapse yield surface while a conventional triaxial compression test was required for the expansive yield surface as well as the hyperbolic model.

In the conventional triaxial test, a constant confining pressure was applied to the sample then the other principal stress was increased up to and after failure. Four isotropic compression tests were run using 4 in. (102 mm) diameter samples while 8 triaxial compression tests were run using 4 in. (102 mm) and 2 in. (51 mm) diameter samples.

The standard procedure for the triaxial test and the modifications to the test results mentioned in Réf. (12, 32, 45), were carried out. The experimental setup for the test is shown in Figure 4.2.
Fig. 4.1 Grain Size Distribution Diagram
TABLE 4.1 Sieve Analysis Results of the Sand Sample

<table>
<thead>
<tr>
<th>Sieve Openings</th>
<th>Sieve Mesh</th>
<th>Weight Retained in Grams</th>
<th>Percent Retained</th>
<th>Percent Finer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches</td>
<td>millimeters</td>
<td>Sieve</td>
<td>Sieve + Soil</td>
<td>Soil</td>
</tr>
<tr>
<td>0.185</td>
<td>4.699</td>
<td>4</td>
<td>562.7</td>
<td>562.7</td>
</tr>
<tr>
<td>0.065</td>
<td>2.00</td>
<td>10</td>
<td>483.0</td>
<td>483.3</td>
</tr>
<tr>
<td>0.0328</td>
<td>0.833</td>
<td>20</td>
<td>469.8</td>
<td>493.4</td>
</tr>
<tr>
<td>0.0164</td>
<td>0.417</td>
<td>40</td>
<td>430.8</td>
<td>525.5</td>
</tr>
<tr>
<td>0.0082</td>
<td>0.250</td>
<td>60</td>
<td>380.9</td>
<td>671.9</td>
</tr>
<tr>
<td>0.0058</td>
<td>0.147</td>
<td>100</td>
<td>368.2</td>
<td>451.3</td>
</tr>
<tr>
<td>0.0029</td>
<td>0.074</td>
<td>200</td>
<td>343.6</td>
<td>352.4</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.046</td>
<td>pan</td>
<td>414.6</td>
<td>417.3</td>
</tr>
</tbody>
</table>

Weight of Sample = 500 grams.
Figure 4.2 Experimental Set-Up for the Triaxial Test.
4.2.1 Parameters Derived From The Triaxial Test

The parameters for the hyperbolic and the elasto-plastic models discussed in Chapter III are derived here from the triaxial test results. The derivation of these parameters is discussed in the following sections.

4.2.1.1 Hyperbolic Model Parameters

A plot of the transformed stress versus strain is shown in Figure 4.3. The best fit for each set of data is also shown in the same figure. The initial modulus $E_i$ of equation 3.42 is the reciprocal of the intercept of each line. The ultimate strength $(\sigma_1 - \sigma_3)_{ult}$ is the reciprocal of the slope of the line. The results plotted are for both the 4 in. (102 mm) and 2 in. (51 mm) diameter samples. Another plot of $(E_i/p_a)$ versus $(\sigma_3/p_a)$ is shown in Figure 4.4 in a log-log scale. From that plot, the values for the modulus number, $K_s$, and the modulus exponent, $m_1$, are defined as shown.

The parameters derived are substituted in equation 3.40 with arbitrary values of the axial strain. The deviatoric stress $(\sigma_2 - \sigma_3)$ is calculated and plotted against the axial strain $(\varepsilon_a)$ together with the test data to check the accuracy of the hyperbolic function. These are shown in Figure 4.5, where very good agreement
Figure 4.3. Experimental Transformed Stress-Strain Relationship.
Figure 4.4 Experimental Variation of Initial Tangent Modulus
Figure 4.5 Experimental and Calculated Stress-Strain Relationship
is noticed between the assumed and the measured values up to the peak or the failure stresses. Less agreement is noticed, however, after the peak.

The parameters of equation 3.49 are another objective of the triaxial test. These parameters are used to determine the value of Poisson's ratio under low or high overburden pressures. As noticed by Mahmoud (85), it is extremely difficult to predict accurate values using the available cell under low values of the confining pressure. Only the first portion of the hyperbolic relation shown in Figure 3.18 could be measured and plotted in Figure 4.6. A straight line is the best fit to the test data. As higher values of the confining pressure are expected in the analysis, dependable values for these parameters are picked up from other References (50, 119).

4.2.1.2 Elastic Parameters

The unloading-reloading modulus is used to define the elastic strains in the analysis. Two additional tests were conducted where the samples were subjected to one or more cycles of unloading and reloading as suggested by Wong and Duncan (119). The results of one of these tests are shown in Figure 4.7. From the figure it is seen that the behaviour is similar for loading and unloading and is
Figure 4.6. Experimental Variation of Lateral Strain with Axial Strain.
Figure 4.7 Experimental Determination of Unloading-Reloading Modulus.
nearly linear and elastic. Using the values of \( E_{ur} \) obtained from the tests, the value of \( m_1 \) obtained in section 4.2.1.1 and the definition of \( E_{ur} \) from equation 3.60, the unloading-reloading modulus \( K_{ur} \) could be obtained.

**4.2.1.3 Plastic Collapse Parameters**

Only two parameters are required to completely define the plastic collapse strains as explained in Chapter III. The isotropic compression triaxial test, which is the only loading condition that does not produce plastic expansive strains is performed for the 4 in. (102 mm) diameter sand sample. Table 4.2 shows a sample of the results obtained from these tests. A plot of \( (W_c/P_a) \) versus \( (f_c/P_a)^2 \) is shown in a log-log scale in Figure 4.8. From the best fit line, the two parameters, the collapse modulus \( C \) and the collapse exponent \( p_1 \) of equation 3.78, are obtained.

**4.2.1.4 Plastic Expansive Parameters**

More parameters are required to define the plastic expansive strains. These were defined previously in Chapter III. A sample of the triaxial test results is given in Table 4.3. The values of \( \eta_1 \) and \( m_2 \) in equations
TABLE 4.2 Sample of the Results of the Isotropic Compression Test

Average sample diameter = 3.97 in.
Average sample height = 7.97 in.

<table>
<thead>
<tr>
<th>$q_3$ (psi)</th>
<th>$\epsilon_v \times 10^3$</th>
<th>$d\epsilon_v \times 10^3$</th>
<th>$f_C$</th>
<th>$W_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2</td>
<td>0.2718</td>
<td>0.2718</td>
<td>81.12</td>
<td>0.0014</td>
</tr>
<tr>
<td>10.1</td>
<td>0.636</td>
<td>0.3643</td>
<td>306.03</td>
<td>0.0005</td>
</tr>
<tr>
<td>14.9</td>
<td>1.0174</td>
<td>0.3812</td>
<td>666.03</td>
<td>0.0070</td>
</tr>
<tr>
<td>20.3</td>
<td>1.4020</td>
<td>0.3846</td>
<td>1236.27</td>
<td>0.0184</td>
</tr>
<tr>
<td>25.2</td>
<td>1.7859</td>
<td>0.3839</td>
<td>1905.12</td>
<td>0.0280</td>
</tr>
<tr>
<td>29.8</td>
<td>2.1676</td>
<td>0.3817</td>
<td>2664.12</td>
<td>0.0395</td>
</tr>
<tr>
<td>35.0</td>
<td>2.5467</td>
<td>0.3790</td>
<td>3675.0</td>
<td>0.0527</td>
</tr>
<tr>
<td>41.1</td>
<td>2.9228</td>
<td>0.3762</td>
<td>4824.03</td>
<td>0.0678</td>
</tr>
<tr>
<td>45.2</td>
<td>3.2961</td>
<td>0.3733</td>
<td>6210.75</td>
<td>0.0846</td>
</tr>
<tr>
<td>50.0</td>
<td>3.6666</td>
<td>0.3705</td>
<td>7500.0</td>
<td>0.1031</td>
</tr>
<tr>
<td>55.7</td>
<td>4.0345</td>
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<tr>
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<td>4.3998</td>
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<tr>
<td>65.1</td>
<td>4.7627</td>
<td>0.3629</td>
<td>12714.03</td>
<td>0.1688</td>
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<tr>
<td>70.3</td>
<td>5.1233</td>
<td>0.3606</td>
<td>14826.27</td>
<td>0.1941</td>
</tr>
</tbody>
</table>
Figure 4.8 Experimental Variation of the Plastic Collapse Work and the Yield Function.
TABLE 4.3 Sample of the Results of the Conventional Triaxial Compression Test

Average sample diameter = 4.02 in.
Average sample height = 7.95 in.
Confining Pressure = 4.00 psi

<table>
<thead>
<tr>
<th>((q_1 - q_3)) psi</th>
<th>(e_a \times 10^3)</th>
<th>(e_v \times 10^3)</th>
<th>(e_r \times 10^3)</th>
<th>(f_p)</th>
<th>(\omega_p \times 10^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.116</td>
<td>-0.0145</td>
<td>-0.0651</td>
<td>0.124</td>
<td>0.000</td>
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<tr>
<td>1.01</td>
<td>0.239</td>
<td>-0.0597</td>
<td>-0.149</td>
<td>0.455</td>
<td>0.000</td>
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<td>2.0</td>
<td>0.513</td>
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<td>-0.383</td>
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<td>0.0000013</td>
</tr>
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<td>3.51</td>
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<td>-0.911</td>
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<td>0.000253</td>
</tr>
<tr>
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<td>-1.134</td>
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</tr>
<tr>
<td>6.51</td>
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</tr>
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<td>-3.259</td>
<td>-3.430</td>
<td>15.275</td>
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<td>-3.961</td>
<td>18.369</td>
<td>0.0285</td>
</tr>
<tr>
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<td>-4.368</td>
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<tr>
<td>11.55</td>
<td>9.226</td>
<td>0.09</td>
<td>-4.567</td>
<td>26.951</td>
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</tr>
<tr>
<td>12.02</td>
<td>10.84</td>
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<td>12.98</td>
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<td>0.2431</td>
</tr>
<tr>
<td>13.51</td>
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<td>21.08</td>
<td>0.724</td>
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</tr>
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<td>13.85</td>
<td>20.702</td>
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<td>35.97</td>
<td>0.6335</td>
</tr>
<tr>
<td>13.22</td>
<td>26.965</td>
<td>165.87</td>
<td>69.46</td>
<td>35.21</td>
<td>1.0235</td>
</tr>
<tr>
<td>12.83</td>
<td>37.56</td>
<td>192.71</td>
<td>77.56</td>
<td>34.43</td>
<td>1.2535</td>
</tr>
</tbody>
</table>
3.61 and 3.62 are determined by plotting \((I_1^3/I_3 - 27)\) versus \((p_a/I_1)\) at failure in a log-log diagram as shown in Figure 4.9. The results in the figure show \(n_1\) as the intercept with \((p_a/I_1) = 1\) and \(m_2\) as the slope of the straight line.

As outlined by Lade (75), the expression to define \(n_2\) can be given by equation 3.66. The test results for a typical confining pressure are plotted as shown in Figure 4.10 between \(n_2\) and \(f_p^p\) defined by equation 3.61. The ratio \(\kappa^p = -\frac{\Delta e_2^p}{\Delta e_1^p}\) is determined from the triaxial test by subtracting the elastic and plastic collapse strains from the total strains according to equation 3.59, to get the plastic expansive strains. The best fit lines for all tests are plotted also, from which the slope \(S_1\) and the intercepts are measured. The constants \(R_1\) and \(t_1\) are determined as shown on the diagram in Figure 4.11.

The plastic work at each stage of the triaxial compression tests is calculated from:

\[
W_p = \int (\sigma_{ij}^p)^T (\varepsilon_{ij})^p dV
\]

in which \((\sigma_{ij}^p)^T (\varepsilon_{ij})^p\) is the plastic expansive work done per unit volume during the strain increment. The diagram in Figure 4.12 for a typical confining pressure shows the measured variation of the total plastic work with the value of \(f_p^p\) and the confining pressure \(c_3\).
Figure 4.9 Experimental Determination of the Yield Surface Parameters.
Figure 4.10 Experimental Variation of $n_2$ with the Yield Function.

- $\sigma_3 = 4$ psi
- $\sigma_3 = 8$ psi
- $\sigma_3 = 12$ psi
- $\sigma_3 = 16$ psi

$S_1 = 0.41$

○ measured ($\sigma_3 = 8$ psi)

- best fit lines
Figure 4.11 Variation of Intercept with Confining Pressure.
Figure 4.12 Experimental and Analytical Variation of Plastic Work with Yield Function.
Equation 3.69 contains three parameters, $a$, $b$ and $q$, to define the relationship between $W_p$ and $f_p$. The variation of $W_{peak}$ or the value of $W_p$ at peak with the confining pressure $\sigma_3$ is shown in the log-log diagram, Figure 4.13 as a straight line. From that diagram the work hardening exponent $i$ and the work hardening constant $P$ are determined. The values of $q$ are calculated from equation 3.72 and are plotted against $\sigma_3/p_a$ as shown in Figure 4.14 to define the two constants $a_1$ and $b_1$. The two other constants of equation 3.69, $a$ and $b$, are then calculated according to equations 3.70 and 3.71.

4.2.1.5 Summary of Triaxial Test Results

From the tests described in the previous sections the following values for the parameters are obtained.

The hyperbolic model parameters:

- $K_s$, modulus number $= 680$
- $m_1$, modulus exponent $= 0.62$

The unloading-reloading parameter:

- $K_{ur}$, unloading-reloading modulus $= 1130$

The plastic collapse parameters:

- $C$, collapse modulus $= 0.00025$
- $P_1$, collapse exponent $= 0.94$
Figure 4.13 Experimental Variation of Peak Plastic Work with Confining Pressure.
Figure 4.14 Variation of $q$ with Confining Pressure.
The plastic expansive parameters:

\[
\begin{align*}
\alpha_1 & \text{, yield constant} = 36 \\
\beta_2 & \text{, yield exponent} = 0.132 \\
S_1 & \text{, work-hardening constant} = 0.41 \\
R_1 & \text{, work-hardening constant} = -1.23 \\
t_1 & \text{, work-hardening constant} = 2.20 \\
P & \text{, work-hardening constant} = 0.19 \\
\alpha_1 & \text{, work-hardening constant} = 3.11 \\
\beta_1 & \text{, work-hardening constant} = -0.041 \\
\gamma & \text{, work-hardening constant} = 1.19
\end{align*}
\]

4.3 **LABORATORY CONDUIT TEST**

A small scale flexible conduit has been used to check the validity of the theoretical investigation. The conduit was tested under several loading conditions and two different heights of soil cover. The strains and deflections of the conduit walls as well as the deflections of the soil top surface were recorded. The tests were run by increasing the load until failure was detected.

4.3.1 **Test Cell Description**

The test was conducted on an isotropic conduit made of mild steel (ASTM 1020), with a diameter of 18 in. (457 mm),
0.075 in. (1.9 mm) (14 gauge) thick and 40 in. (1016 mm) long. Tension test were conducted on three samples of the conduit material to define its properties. From the tests it was found that the modulus of elasticity is 26290 ksi (181240 PPa), Poisson's ratio is 0.33, and the yield stress is 42 ksi (289 MPa). The conduit ends were wrapped with foam rubber to seal the gap between the conduit wall and test cell and to allow the free movement of the conduit walls. Temperature-compensated uniaxial electrical resistance gauges were installed around the conduit mid-section and longitudinally. These strain gauges were of the type N11-FA-5-120-11 with a gauge factor of 2.10 +/-1.0% at 11°C. After installation, the strain gauges were connected to a multichannel automatic digital strain indicator.

A rectangular test box with inner dimensions of 88.5 in. (2248 mm) x 40.5 in. (1028 mm) x 48 in. (1219 mm) was constructed, with an open top, of 3/4 in. (19 mm) thick plywood, 3/4 in. (19 mm) plexiglass and structural angles as shown in Figure 4.15.

4.3.2 Experimental Set-Up

The previously described test box was placed under a loading frame and hydraulic jack. A soil bed 12 in. (228 mm) deep, was laid in several layers and was manu-
Figure 4.15 The Rectangular Test Box.
ally compacted. The conduit was then placed on the soil bed and the soil on both sides of the conduit were placed and compacted in 3 in. (76 mm) layers. Finally, a soil cover of 3 in. (76 mm) was placed over the conduit crown, compacted and levelled to give a reasonably smooth top surface to apply the load. The strain readings were recorded after each layer was added. Three hydraulic jacks of capacity 10 kips (44.5 kN), 25 kips (111 kN) and 50 kips (222 kN) were used to apply the load.

A total of four tests were conducted. In all the tests, three dial gauges were installed on the soil top surface and two dial gauges were installed inside the conduit to take measurements of the vertical deflection along the top soil surface and the conduit crown. The arrangement is shown in Figure 4.16.

4.3.3 Test 1: Procedure and Results
4.3.3.1 Test Procedure

In test 1, with a soil cover of 3 in (76 mm), a wood beam 3.5 in. (89 mm) x 3.5 in. (89 mm) and 38 in. (965 mm) long was placed along a line directly above the conduit crown. Another steel beam S 75x8, 30 in. (762 mm) long was placed on top of the wood beam. The loading beams are shown in Figure 4.17.a. A flat universal
Figure 4.16 Dial Gage Arrangement.
   a) Laboratory Test No.1.  b) Laboratory Test No.3.
a) Loading system for Test 1 and Test 2

b) Loading system for Test 3

c) Loading system for Test 4

Figure 4.17 Loading Arrangements for the Laboratory Tests.
calibrated load cell with maximum capacity of 10 kips (44.5 kN) was then used to measure the load applied by the hydraulic jack. A portable strain indicator, with which the load cell had been calibrated, was used to take these measurements. With this set up, the initial zero load strains and deflections were recorded. The load was then applied through the beams on to the soil in 100 lb. (0.45 kN) increments. At the end of each load increment, the dial gauges and the strain gauges readings were recorded. The procedure was repeated until failure was observed when the load cell reading began to decrease accompanied by a gradual and continuous increase in the measured deflections and strains. At the end, the load was released and the final readings were taken.

4.3.3.2 Experimental Observations

The experimental load-deflection curves for this test are shown in Figure 4.18. The following can be observed from the figure. Once the experiment was set up and the load was ready to be applied, the soil seemed to be loose on top. In the first few load increments, the soil top deflected much faster than the conduit, until it reached a stable compacted shape. Negligible deflections of the conduit crown occurred. Further applied load increments penetrated to the conduit and it started to
Figure 4.18 Experimental Load-Deflection Curves for Laboratory Test No. 1.
deflect with the soil. The difference between the crown and soil deflections increased with an increase in load until the soil apparently failed, which was detected by observing the displacements after each load increment. By further increasing the load the increments of deflection for both soil and conduit were almost equal which indicated that the soil had failed and it was just trapped between the loading beams and the conduit. Most of the load was then transferred to the conduit which deflected faster. By further applying the load, instability of the soil-conduit took place which was detected by the gradual increase of the deflections even with reducing the applied load. Upon unloading, the soil surface did not recover its original level and permanent deformations were observed; an indication of soil failure. The conduit also was subjected to permanent deformations detected after unloading. The strains in the conduit indicated that the yield stresses had been exceeded and that a plastic hinge had formed at the crown level.

4.3.3.3 Comparison of Experimental and Analytical Results

The results obtained from the test for thrusts, moments, and deflections under live loads as well as the failure loads are presented and compared with the analytical
results. The analytical results are obtained through the computer program discussed in detail in Chapter III. The properties of the sand used were presented in section 4.2. The behaviour of the interface elements is represented by the following parameters from a previous research on the same type of sand (85): \( k_i = 43070, R_{sf} = 0.834, n_s = 0.6, \) and \( \psi' = 23^\circ. \)

4.3.3.4 Comparison of Thrusts

The results for thrusts in the conduit walls for both the experimental and analytical investigations are shown and compared in Figure 4.19.

Fair agreement is generally observed from the figure between the analytical and experimental results. The analytical thrusts seem to be more uniform than the experimental ones. Irregularities in soil properties and nonuniformity of compaction are possible reasons for the irregular, nonsymmetric thrusts obtained experimentally. The thrust distribution is shown in the figure to have a maximum value at the shoulders of the upper half of the conduit. A smaller value is detected for the thrust at the conduit crown. The thrust in the lower half decreases more rapidly than in the upper half, possibly because of the soil behaviour in these regions. The soil in the lower half of the conduit is sharing the load
Figure 4.19 Experimental and Analytical Thrusts at failure and at 0.9 kips for Laboratory Test No.1.
with the conduit, consequently, the load taken by the conduit is less in value than in the upper half where most of the load is taken by the conduit.

4.3.3.5 Comparison of Moments

The results for the bending moments in the conduit walls for both the experimental and the analytical investigations are shown and compared in Figure 4.20.

It is noticed that the moment, \( M \), reached experimentally after the yield stress is exceeded is calculated from the following relationship for a rectangular cross-section (105).

\[
\frac{M}{M_y} = \frac{3}{2} - \frac{1}{2 \left( \frac{\varepsilon}{\varepsilon_{yd}} \right)^2}
\]

(4.2)

where,

\( M_y \) = the yield moment = \( f_y \cdot S_e \),

\( S_e \) = the elastic section modulus,

\( \varepsilon_{yd} \) = the strain at yield = \( f_y / E \), and

\( \varepsilon \) = the measured strain greater than \( \varepsilon_{yd} \).

A plastic hinge is assumed to develop experimentally when the strain reaches 0.2% offset from the strain at yield, i.e. when

\[
\varepsilon = \varepsilon_{yd} + 0.002 = \frac{f_y}{E} + 0.002
\]

(4.3)
Figure 4.20 Experimental and Analytical Moments at failure and at 0.9 kips for Laboratory Test No.1.
More irregularities in the bending moments are observed than in the thrust values. The analytical moments follow the same trend as the experimental ones as can be noticed from the figure. The maximum value of the moment is at the conduit crown. Similar to the thrust, the moments are also high at the shoulders of the upper half of the conduit. They are negative compared to positive moments at the crown and horizontal sections of the conduit. The moment deteriorates and becomes negligible much faster in the lower half of the conduit than noticed in the thrust values. This indicates that more uniform soil pressure is activated at the lower half.

Reasonable agreement is observed between the experimental and the analytical results. Only minor discrepancy is observed in the value of the moment at the conduit crown, where the plastic hinge had developed. Generally a larger difference could be expected at the crown because of the plastic hinge assumptions previously discussed (section 3.4.3). The plastic hinge is assumed to resist a constant moment value, \( M_p \), once it has developed. This indicates that in the analysis, the hardening of the conduit material is neglected. In the test, the moment at that particular section increased continuously up to and after the hinge had developed.
4.3.3.6 Comparison of Deflections

Figure 4.21 shows the load-deflection curves for the experimental and analytical tests. The trend of the experimental curves shows a fairly linear increase in deflection for both soil and conduit up to what is believed to be soil failure or slippage, followed by a nonlinear increase in deflection up to instability. In the first portion of the curves, the soil deflection is larger, approximately double the conduit deflection. In the second portion, where the soil is thought to be trapped after failure between the loading beams and the conduit, the change in deflection of the soil becomes almost identical to that of the conduit.

The conduit deflections obtained analytically, however, are generally less than the experimental values. The loss of resistance of the soil after failure is apparently not accurately simulated. The analytical load-deflection curves indicate that the conduit does not attract the load transferred from the soil after failure; instead the load is directed towards the sides. The reason for this is the extremely low level of stresses dealt with in the analysis of this problem and the assumption of constant stresses in the failing elements after failure as was mentioned in section 3.9.5. This lead to constant element properties after failure and
Figure 4.21 Experimental and Analytical Load - Deflection
Curves for Laboratory Test No. 1.
consequently constant deflection increments. Nevertheless, the maximum deflections at instability are approximately 80% of the experimental values.

4.3.3.7 Comparison of Soil Failure and Instability Loads

Failure of the soil above the conduit and instability of the soil-steel structure are determined experimentally and predicted analytically. Soil failure is assumed to take place experimentally when the load-deflection curves change their slope and the conduit starts to resist most of the load. Analytically, soil failure occurs when a series of yielded or failed elements extend from the conduit to the top surface of the soil as shown in Figure 4.22 which shows the spreading of the plastic zone in this test.

Instability of the soil-steel structure occurs experimentally when the deflections increase continuously while the load is released or kept constant. The analytical load that causes instability of the soil-steel structure is the load which causes the determinant of the structure's stiffness matrix to change from positive to negative.

Experimentally, the soil failure occurred under a load of 1.5 kips (6.7 kN), while analytically this was reached at 1.2 kips (5.4 kN). The theoretical
Figure 4.22 Spreading of the Plastic Zone in Test No.1.
instability load for test 1 is 2.3 kips (10.2 kN) compared to 2.0 kips (9.0 kN) obtained experimentally, which indicates a reasonable agreement between experiment and analysis.

4.3.4 Test 2: Procedure and Results
4.3.4.1 Test Procedure and Observations
Prior to test 2, the soil was disturbed and recompacted and the soil cover was increased to be 6 in. (152 mm). In test 2, the same loading beams and cells used in test 1, were used to apply the load in 250 lb (1.10 kN) increments until failure.

The general experimental observations from test 1 mentioned in section 4.3.3.2 are also applicable to test 2. The soil deflected faster than the conduit until it failed then the load was transferred totally to the conduit until instability occurred. A slip, or a jump in the soil deflection occurred when the soil failed. The slip was suspected to be caused by an eccentric load. The test was repeated but the same load-deflection curves were again obtained.

4.3.4.2 Comparison of Experimental and Analytical Results
It should be mentioned here that initial values of
moments or the residual moments and thrusts in the conduit walls, resulting from test 1, were calculated by measuring the strains prior to applying the live loads and were fed as data to the computer program. Only the final thrusts and bending moments are plotted and compared here. Similar measurements for the residual moments and thrusts were taken in the subsequent tests.

The thrust distributions obtained from the analytical and experimental investigations are shown in Figure 4.23. From the figure it can be seen that the thrust distribution for test 2 is similar to test 1. The maximum values are at the shoulders and the thrust decreases at the lower half. Reasonable agreement is concluded from the figure between the two investigations.

Figure 4.24 shows the analytical and experimental moment distributions for this test. Again, they are similar to test 1 but with smaller values because of the increase in the soil cover. The maximum moment value occurs at the crown and then it deteriorates faster in the lower half of the conduit.

The analytical and experimental load-deflection curves for this test are shown in Figure 4.25. The slip obtained experimentally, which occurred because of the high concentration of stresses at the applied load area...
Figure 4.23 Experimental and Analytical thrusts at failure and at 1.0 kip for Laboratory Test No.2.
Figure 4.24  Experimental and Analytical Moments at failure
and at 1.0 kip for Laboratory Test No.2.
Figure 4.25 Experimental and Analytical Load-Deflection Curves for Laboratory Test No. 2.
due to the narrow width of the loading beam, is only an experimental behaviour that cannot be simulated analytically. At failure, the maximum deflections which are plotted show fair agreement.

The progress of the plastic zone in this test is shown in Figure 4.26 indicating that soil failure took place at a load of 1.2 kips (5.4 kN). Experimentally, this occurred at 1.35 kips (6.1 kN). The experimental failure load was 2.1 kips (9.3 kN) and the calculated value was 2.2 kips (9.8 kN) which indicates the reasonable agreement.

4.3.5 Test 3: Procedure and Results
4.3.5.1 Test Procedure and Observations

Plywood loading plates 30 in. (762 mm) long, 18 in. (456 mm) wide and 1 in. (25 mm) thick were used for the third test. Two of these plywood plates were placed on top of the 6 in. (152 mm) soil cover. Three steel I-beams S 75x8; 30 in. (762 mm) long were placed at center and 1 in. (25 mm) away from the plywood edges.

Another rigid W 150x14 beam was placed across the three I-beams and was used to apply the load equally to the beams. The loading arrangement for this test is shown in Figure 4.17.b. A load cell of 25 kips (112 kN) capacity was used and the load was applied in 0.5 kips
\[ P = 1.2 \text{ kips} \quad P' = 0.3 \text{ kips} \]

Figure 4.26 Spreading of the Plastic Zone in Test No.2.
(2.22 kN) increments up to a total load of 5.0 kips (22.2 kN) then the load increment was increased to 1.0 kip (4.45 kN) until failure was reached.

Because the load applied was more uniform than in the first and second tests, more uniformity in the behaviour of the soil-steel structure was observed. The strain gauges and dial gauges readings were increasing slowly until the soil was trapped and then the rate of increase became higher. Cracks in the soil surface starting from the loading plate edges were recognized and indicated the soil failure. By applying more load increments, instability of the system finally occurred and no more load could be sustained by the structure.

4.3.5.2 Comparison of Experimental and Analytical Results

Under more confinement, test 3, the thrusts shown in Figure 4.27 still have the same general distribution, although the lower half attracted more thrust than in test 1 and test 2. The thrust became more uniform but the upper half of the conduit still attracts more thrust than the lower half.

Higher values of moments are reached if a concentrated load is applied (as was shown by tests 1 and 2), and major decrease in their values is detected if the load becomes uniform and distributed over a wider area as shown in
Figure 4.27 Experimental and Analytical Thrusts at Failure and at $p_z = 9.25$ psi for Laboratory Test No. 3.
Figure 4.28 for the results of test 3. Less discrepancy in the value of the crown moment is observed from the figure between the analytical and experimental values and general agreement is also noticed.

The comparison between the analytical and experimental deflections shown in Figure 4.29 seems reasonable for the initial part of the curve and at failure the maximum deflections are in the range of 80-92% of the measured values.

Soil failure was detected at a load of 6 kips (26.7 kN) experimentally while the analysis estimated soil failure to occur at 5.0 kips (22.2 kN).

Instability occurred experimentally when the applied load reached 10.5 kips (46.7 kN), while the analytical load is 10 kips (44.4 kN) which reflects the general agreement.

4.3.6 Test 4: Procedure and Results
4.3.6.1 Test Procedure and Observations

In the fourth test, wider plates were used to test the failure under more uniform load and with a more confined soil to reduce its tendency to fail. The lower plywood plate had dimensions of 30 in. (762 mm) long and 48 in. (1220 mm) wide on top of which other small size
Figure 4.28: Experimental and Analytical Moments at Failure and at $p_i = 9.25$ psi for Laboratory Test No.3.
Figure 4.29 Experimental and Analytical Load-Deflection Curves for Laboratory Test No. 3.
plates were used to force the load to be applied uniformly. The same I-beams and W-beam from test 3 were used again. The loading plates and beams for the fourth test are shown in Figure 4.17.c. The soil cover for this test was 6 in. (152 mm) and the load increment was 2.5 kips (11.1 kN).

Similar behaviour of the soil-steel system was observed in this test as in test 3. Cracks were observed to start from the loading plate edges and spread towards the cell walls at higher load level and this indicated the start of a soil failure. In the next few load increments, a soil wedge could be seen at the surface away from the plates and more deflections were then observed.

4.3.6.2 Comparison of Experimental and Analytical Results

As expected, and seen in Figure 4.30, the values of thrust in the conduit wall increased by the increase of the soil cover and the widening of the loading area. The conduit lower half attracted more thrust than in the previous tests and the thrust distribution is almost having a constant value. This was shown to be accompanied by reduction in the bending moments in the conduit walls as shown in Figure 4.31. The figure indicates much smaller values for the moments than in the previous tests with negligible values at the lower half. It is probable,
Figure 4.30 Experimental and Analytical Thrusts at Failure and at $p_z = 14$ psi for Laboratory Test No. 4.
Figure 4.31 Experimental and Analytical Moments at Failure and at $p_t = 14$ psi for Laboratory Test No. 4.
by further increasing the soil cover that failure will be entirely governed by resistance of the wall to crushing.

The analytical and experimental deflections for this test are compared in Figure 4.32. The calculated deflections at instability are in the range of 60-80% of the measured values. It is also noticed that the agreement is closer when the soil element stresses are higher, as in load case 4, than when the stresses are low, as in load case 1.

Soil failure occurred experimentally at a load of 28 kips (124 kN) while analytically this was detected at 22 kips (98 kN).

The analytical instability load for this test is 36 kips (160 kN) and the experimental value was 44 kips (196 kN); another indication of the general agreement.

4.3.7 Observations from Additional Tests

The main objective of the experiments carried out here was to check the validity and accuracy of the theoretical analysis outlined in Chapter III. The attempt to test all possible failure modes of the conduit was beyond the objectives of the experimental program. However, an attempt was made to reach a snap-through type buckling of the conduit. Trials using set-up of test 3
Figure 4.32: Experimental and Analytical Load-Deflection Curves for Laboratory Test No.4.
and 4, and sometimes with changing the arrangement of the loading beams, or reducing the plate rigidity, did not lead to a snap-through failure mode. Because the loading system of plates and beams deflected excessively after the soil failure, the full load did not penetrate to the conduit directly. The soil wedge which formed at soil failure slipped and most of the load was transferred to the sides away from the conduit.

Snap-through type buckling requires a number of plastic hinges to develop in the conduit walls. Only one hinge developed both analytically and experimentally and the failure always took place before a second hinge could develop. The lack of enough soil support to the conduit, especially at the level of the shoulders, was a reason for the instability of the soil-steel structure to take place before snap-through buckling of the conduit.

Another possible way of achieving this type of failure would be to cover the whole soil area and to load it. This, however, would require excessively high load to achieve the required failure type.

The high stiffness of the conduit material is another explanation to the nonoccurrence of the snap-through type buckling. A 14-gauge steel sheet was used to form the conduit. The conduit was not flexible enough to snap through. A smaller thickness of material or a larger
conduit diameter should be used if that type of failure is to be detected. Aluminum could also have been used instead of the steel to form the conduit.

4.4 COMPARISON WITH REFERENCE TESTS

Similar tests have been reported in other references (42, 50). A brief discussion of these tests and comparison of their results and the analytical results obtained in the present analysis are given in the following sections.

4.4.1 Model Description

The test was conducted on a conduit made of aluminum alloy 6061-T-6 plate having a modulus of elasticity of 10000 ksi (68950 MPa), a Poisson's ratio of 0.33, and a yield stress of 40 ksi (276 MPa). The conduit was 31 in. (787 mm) diameter, 3/16 in. (5.0 mm) thickness and 60 in. (1524 mm) long. The ends were wrapped with foam rubber to seal the gaps between the conduit wall and the test cell. The test cell was a box made of 3/4 in. (19 mm) thick plywood, and 1/2 in. (13 mm) thick plexiglass, and with an open top. The soil tested was clean, dry sand from Lake Erie. The soil bed was 12 in. (304 mm) and the soil cover was 5 in. (127 mm) and 10 in. (254 mm).
4.4.2 Experimental Set-Up

Different loading conditions required different loading plates arrangement. In test 1, the load was a line load applied on a line directly above the conduit crown through an I-beam of 4 in. (102 mm) width. The soil cover was 5 in. (127 mm). In the second test, a 10.5 in. (267 mm) wide plate was placed between the steel beam and the soil to apply a more uniformly distributed load to the structure. In a third test, the height of cover was increased to 10 in. (254 mm) and the loading plate size was increased further. A plate of width equal to the conduit diameter was placed on the soil top to apply the load for the third test. Because failure in the third test was not a conduit failure, a trial was made in the fourth test to reach a conduit failure by confining the top of the soil on the two sides of the conduit and by applying the load to the area in between these two confinements.

4.4.3 Observations and Results

The experimental load-deflection curves for test 1, are shown in Figure 4.33. The curves are almost linear up to the maximum applied load. The conduit deflection was half of the top soil deflection. The maximum applied load was 2.5 kips (11.1 kN) after which the soil failed
Figure 4.33 Experimental and Analytical Load-Deflection Curves for Reference Test No. 1.
and a permanent displacement in the soil top of 0.3 in. (8 mm) was observed and indicated the soil failure. Only elastic stresses were reached by the conduit and were recovered upon unloading.

Analytically, the instability load is 2.9 kips (12.9 kN). This is obtained by using the same soil properties mentioned in section 4.2.1.5.

In the second test, again only elastic strains were detected in the conduit wall irrespective of the nonlinear load deflection curves shown in Figure 4.34. The nonlinearity of the conduit displacement after certain load has been reached, indicated that soil failure took place. The analytical instability load is 3.5 kips (15.8 kN), which is in reasonable agreement with the test result of 4.4 kips (19.6 kN).

The instability load reached in the third test was 29.3 kips (130 kN), while the analytical failure load is 26.4 kips (117 kN).

In the fourth test, confining the soil top helped to reach a higher pressure intensity before failure. Because of the deformation of the posts confining the top sand surface and the deformation of the test cell itself the load was apparently transferred to the sides as the soil deflected. This was the probable reason that the conduit failure could not be reached. A maximum load of 51.8 kips
Figure 4.34 Experimental and Analytical Load-Deflection Curves for Reference Test No.2.
(230 kN) could be sustained by the structure before instability took place. The computer program indicated a load of 43 kips (191 kN) to cause instability.

The previous analytical and experimental results and their comparison indicate the fair agreement of the two investigations in predicting the instability loads of the soil-steel structures.

4.5 **COMPARISON WITH CODES**

The results of the experimental laboratory tests and their analytical evaluations are compared in this section with Code equations. The Ontario Highway Bridge Design Code (OHBDC) and the American Association of State Highway and Transportation Officials (AASHTO) are the available Codes dealing with the stability of soil-steel structures.

The OHBD Code (98) estimates that elastic buckling for the tested conduit dimensions mentioned in section 4.3.1 and for a modulus of soil reaction of 1000 psi (6.9 MPa), will take place when the stresses in the conduit wall reach the value of the buckling stress 2034 psi (14.0 MPa). The stresses are mainly caused by the thrust due to dead and live loads, while no consideration is given to the bending stresses. It is also assumed in the Code that this buckling stress is independent of the loading condition. A comparison of the results of the intensity
of applied load at failure in the analytical and experimental study and the OHBD Code is given in Table 4.4. From the table it is seen that the agreement is better under uniform load than under a line load. The Code overestimates the failure load when a line load is applied. This can be explained by the fact that the line load represents a severe case of loading that causes an early failure in soil which is not accounted for in the Code equations.

The buckling stress calculated from the AASHTO Specifications (8) is 2588 psi (17.8 MPa) and the predicted failure loads are also given in the same Table 4.4. Basically, the same discrepancies noticed in the OHBD Code are applicable to the AASHTO specifications since both are based on the ring compression theory but with different methods of analysis.

It can be concluded that the present study seems to analyze the instability problem in a consistent and realistic way and avoids the shortcomings of both Codes.
### TABLE 4.4 Comparison Between the Experimental, Analytical, the OHBD Code and the AASHTO Specifications for Failure Loads.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Present Study</th>
<th>OHBDC</th>
<th>AASHTO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60.5 lb/in</td>
<td>299 1b/in</td>
<td>381 lb/in</td>
</tr>
<tr>
<td></td>
<td>17.43 psi</td>
<td>27.56 psi</td>
<td>33.46 psi</td>
</tr>
<tr>
<td></td>
<td>25.2 psi</td>
<td>20.67 psi</td>
<td>25.7 psi</td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>52 lb/in</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>19.44 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30.55 psi</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conduit Diameter = 18 in
Wall Thickness = 0.075 in
Soil Cover = 6 in
Chapter V
PARAMETRIC STUDY

5.1 GENERAL

The computer program developed to solve for the stability of buried conduits based on the theoretical analysis described in Chapter III, is verified with experimental tests and compared to other available referenced tests and Codes as outlined in Chapter IV. The computer program predicts the stress state in the soil elements at the various stages of construction, loading and failure, the loads at which soil failure is assumed to occur, the forces in the conduit walls, the formation of plastic hinges, the conduit buckling loads or the loads causing soil-steel structure instability.

From the theoretical investigations it is evident that the soil-steel structure behaviour and failure is dependent on a number of parameters. Some of the geometric parameters affecting the failure are the height of soil cover above the conduit, the size of conduit and the conduit shape. The type of applied load, the rigidity of the conduit walls, the stiffness of the interface elements and the applied load eccentricity are other parameters to be considered.

A study is made to investigate the effects of these parameters on the soil failure loads and the soil-steel structure instability loads using the computer program developed. In each case studied the analysis procedure described earlier
including the construction sequence, the application of compaction and live loads, the material properties updating and the stress correcting iterations, is carried out. The live load is applied to the completed structure in increments until instability occurs.

The investigation is carried out by varying only one parameter while keeping the others constant. A symmetric uniformly distributed live load applied to the soil top surface and covering a span equal to the conduit diameter is used for this purpose as a constant loading type.

The following parameters for the hyperbolic model are used (119): \( K_s = 1200, R_x = 0.85, m_1 = 0.48, \phi_c = 45^\circ, \)
\( \beta = 7^\circ, G = 0.5, F = 0.23, d' = 11.7 \) while the following values for the elasto-plastic model parameters are used (75):
\( K_{ur} = 1680, m_1 = 0.57, \nu = 0.20, C = 0.00023, p_1 = 0.85, \)
\( n_1 = 80, m_2 = 0.23, R_1 = -2.95, S_1 = 0.44, t_1 = 8.45, \gamma_1 = 3.0, \)
\( \beta_1 = 0.060, \psi = 0.12, \varepsilon = 1.16. \) The interface properties are those mentioned in section 4.3.3.3. The conduit material properties are those obtained from the tension tests and mentioned in section 4.3.1.

5.2 HEIGHT OF SOIL COVER

The height of soil cover is a major parameter in analysing a soil-steel structure. A relatively shallow cover of soil leads to rapid soil failure and is a reason that Codes (98) limit its value to a minimum of \( 1/6 \) of the conduit diameter.

The effect of the soil cover height is investigated by
analyzing a series of structures under different covers and with all other parameters being constant. The structures are of a circular cross-section having a diameter of 240 inches (6.10 m) and made of galvanized steel 2x6 corrugated sheets with a thickness of 0.184 inches (5 mm). The soil cover varies between 1/10 to 1.0 of the conduit diameter.

Figure 5.1 shows the ratio of the soil cover height to the conduit diameter (h/S) vs. the intensity of the applied load (p) at soil failure and at instability. As expected, the general trend of the two curves indicates that the load required for failure increases with an increase in the soil height. When the soil cover increases, the load is dispersed to a wider width, thus decreasing the possibility of failure. Similar results are also given by Okeagu (97) for instability loads and by Hafez (50) for soil failure loads.

5.3 CONDUIT SIZE

To study the effect of the conduit size on the failure loads another series of structures are analyzed. The soil cover for this case is constant at 60 inches (1.53 m) while the circular conduit diameter is changed between 100 inches (2.54 m) and 300 inches (7.62 m). The conduit wall thickness is taken as 0.184 inches (5 mm).

Figure 5.2 shows that by increasing the conduit size and keeping the soil cover constant, the intensity of the load required to cause failure decreases considerably and gradually. The failure load is thus inversely proportional to the conduit size.
Figure 5.1 Variation of Height of Soil Cover with Failure Loads
Figure 5.2 Variation of Conduit Size with Failure Loads.
5.4 **CONDUIT SHAPE**

As mentioned earlier, horizontal and vertical ellipses are commonly used in soil-steel structures in addition to the circular shapes. The effect of the conduit shape is investigated by analyzing a set of structures with different shapes and constant other parameters. The soil cover is kept at 60 inches (1.53 m) and the conduit horizontal span is 240 inches (6.10 m). The rise to span ratio in a horizontal ellipse is taken as 0.5 and is changed to 1.5 for a vertical ellipse.

From Figure 5.3 it can be concluded that the vertical ellipse is the stiffest among all shapes in resisting the loads. It deforms less at the crown and this may direct the applied loads towards the sides resulting in higher failure loads. In contrast to the vertical ellipse, the horizontal ellipse deforms much faster at the conduit crown, the shear stresses in the soil media are consequently increased and failure occurs at a lower load. The circular conduit behaves in an intermediate manner between the two elliptic shapes. Its crown deflection and resistance to the applied loads is between the other elliptic conduits which agrees with the results obtained for soil failure by Hafez (50).
Figure 5.3 Variation of Conduit Shape with Failure Loads
5.5 **TYPE OF APPLIED LOAD**

A constant load type has been assumed for the previous investigations; a uniform load applied to the soil top surface covering a span equal to the conduit diameter. The effect of the load type is studied by analyzing a circular structure of 240 inches (6.10 m) diameter, 0.184 inches (5 mm) thickness and a soil cover of 60 inches (1.53 m). The span covered by the applied load is changed as shown in Figure 5.4 for types I, II and III. The load is again applied in increments and increased until instability occurs.

From the figure it is shown that by loading a smaller span (type I) failure occurs faster than by spreading the load to a wider span (type II and type III). For load type I, the soil at the conduit sides is unconfined and thus tends to be uplifted which results in faster failure. By confining more area (load type III) more resistance to failure is noticed than for partially loaded and confined spans (load type II).
Figure 5.4 Variation of Type of Applied Load with Failure Loads.
5.6 RIGIDITY OF CONDUIT WALLS

The rigidity of conduit walls controls the type of failure reached by the structure. With a smaller thickness (i.e., more flexible conduit) larger conduit deformations occur. This may lead to the development of plastic hinges and to an increase of the shear stresses in the soil media which results in faster failure. By increasing the conduit wall thickness and consequently the stiffness, less conduit deformations occur, plastic hinges may not develop and soil above the conduit becomes stiffer in resisting the loads. These conclusions are reached by analyzing three circular conduits 240 inches (6.10 m) diameter with a soil cover of 60 inches (1.53 m). The conduit wall thickness is assumed to be 0.184 inches (5 mm), 0.138 inches (3.5 mm) and 0.249 inches (6 mm). The results for this investigation are shown in Figure 5.5.
Figure 5.5 Variation of Conduit Wall Thickness
with Failure Loads.
5.7 INTERFACE STIFFNESS

The interface element properties may affect the behaviour of the soil-steel structure. A soil-steel structure with a steel conduit of 240 inches (6.10 m) diameter and 0.184 inches (5 mm) thickness and covered with 60 inches (1.53 m) of soil is analyzed with different interface stiffnesses. The two limits for the interface condition are the 'no slip' or the 'bonded' case, the 'full slip' or the frictionless case. The 'no slip' case is simulated by assuming the unit tangential stiffness, \( k_s \) value, of the interface elements to be constant and equal to one half of the \( k_n \) value, 500000 lb./in\(^3\) (135 N/mm\(^3\)). The 'full slip' case is simulated by assuming the \( k_s \) value to be very small and equal to 5 lb/in\(^3\) (0.00136 N/mm\(^3\)). An intermediate value of \( k_s \) as defined in section 4.3.3.3 is also considered for the frictional slip case as a reasonable value between the two limits.

The results for these analyses are compared in Figure 5.6. From the figure it is seen that the bonded case led to a relatively faster soil failure and consequently instability due to the increase in shear stresses in soil around the conduit. This seems to be unrealistic behaviour as compared to the two other cases. Similar behaviour is noticed for both the frictional slip and the frictionless cases. Katona (65) recommended that these are used in the soil-steel structure analysis as they provide a more reasonable representation of the behaviour than the bonded case.
Figure 5.6 Variation of Interface Unit Tangential Stiffness with Failure Loads.
5.8 **ECCENTRICITY OF APPLIED LOAD**

To study the effect of the eccentricity of applied load on the failure loads, a structure of 240 inches \((6.10 \text{ m})\) span has been analyzed. The height of soil cover is 50 inches \((1.53 \text{ m})\). Two concentrated and equal moving loads, each having \(P_2\) value are used to represent the rear tandem of a testing truck.

Figure 5.7 shows the effect of the load eccentricity on the failure loads. From the figure it can be seen that the failure loads decrease by increasing the eccentricity from zero to a specific limit after which the failure loads increase considerably. Under small eccentricity the load case causes faster soil failure than under no eccentricity and consequently the instability load is also reduced. With the loads moving away from the conduit center line, the possibility of soil failure and instability are reduced considerably and a higher load is then required to cause instability.
Figure 5.7 Variation of Load Eccentricity with Failure Loads.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 GENERAL

A plane-strain finite element analysis procedure was used to study the behaviour of soil-steel structures during construction, loading and ultimately to predict the failure loads.

Beam-column finite elements simulating the metallic conduit (which are essential for a stability analysis), spring type interface elements (representing the slip or separation of the two materials) and triangular and quadrilateral elements representing the soil media were used in the soil-steel structure simulation. A hyperbolic model and an elasto-plastic model were used to represent the behaviour of the soil elements. Plastic hinge formation is predicted and is accounted for in the analysis. The construction procedure, including application of compaction loads, was simulated to represent accurately the stress distribution in the structure prior to applying the live loads. Soil failure was predicted and its effect on soil-steel structure instability was studied. The analytical results were compared with experimental tests and governing Codes. A parametric
study was carried out to study the effect of the main parameters related to soil failure and soil-steel structure instability.

From the results of the analysis procedure developed it has been verified that:

1. Although the finite element method is a rather complicated analysis procedure, indications are that it is a much more accurate idealization of the soil-steel structure compared to other available approximate analyses.

2. The ability to include the formation of plastic hinges in the metallic conduit is important in order to predict the stability response.

3. Snap-through type failure could not be reached experimentally with the soil-steel structure tested. A conduit with high ring flexibility under a high depth of soil cover may experience this type of failure.

4. The load carrying capacity of the soil-steel structure is sensitive to the value of the unit shear stiffness of the interface elements and the amount of slip that occurs between the soil and the conduit.

5. Among all the variables, the soil properties have a significant effect on the soil-steel structure behaviour. The sensitivity and accuracy of the analysis is largely governed by the soil properties.

6. The conduit experiences relatively large deflections as plastic hinges are developed and the resulting increase
in flexibility accelerates the instability of the soil-steel structure.

6.2 CONCLUSIONS

From the analytical and experimental results and from the parametric study carried out in this research, the following can be concluded:

1. The analysis procedure developed satisfactorily agrees with the experiments carried out on isotropic conduits under symmetric type loading assuming a plane-strain condition.

2. Under relatively shallow soil covers, the soil tends to fail at relatively low loads resulting in a reduction of the support provided for the metallic conduit. As a result, instability of the soil-steel structure may occur at lower loads than those predicted by procedures that neglect the effect of soil failure.

3. The formation of plastic hinges does not constitute complete failure of the conduit but rather affects the load carrying capacity. Therefore, the design criterion of soil-steel structures should not be based solely on the maximum stresses due to bending moment and axial force but should also consider the stability analysis. This criterion should include the effect of soil failure on the buckling load.
4. The hyperbolic stress-strain model is sufficiently accurate to simulate the behaviour of the soil media remote from the most stressed regions.

5. The use of the elasto-plastic model representing the behaviour of soil in the regions of severe stress gradients accurately predicts soil failure as well as the behaviour of the soil-steel structure in the post-soil failure stage.

6. Generally, the moments and deflections in the lower third of the conduit are small in comparison with those of the upper half.

7. From the symmetrically loaded cases studied, the maximum positive moment always occurs at the conduit crown, while the maximum negative moment occurs at the shoulders of the upper half of the conduit.

8. The maximum thrust in the symmetrically loaded cases occurs at the shoulders in the upper half of the conduit.

9. Negligible moments and fairly constant thrust distribution are obtained if the height of soil cover exceeds one-half of the conduit diameter.

10. Generally, the failure loads increase with an increase of the soil cover.

11. Increasing the conduit size leads to a gradual decrease in the load carrying capacity of the soil-steel structure.
12. The load carrying capacity of the soil-steel structure is increased considerably by using a vertical elliptic conduit, and decreases rapidly if a horizontal ellipse is employed. A circular conduit offers an average load carrying capacity.

13. Applying a given load intensity to a wider area leads to confining the soil and reducing its tendency to fail. The total load carrying capacity is thus improved compared to loading a narrower area.

5.3 RECOMMENDATIONS

The following are recommendations for future research which the author believes is necessary:

1. A simpler elasto-plastic soil model requiring fewer soil parameters and having reasonable accuracy would be more practical.

2. A full scale soil-steel structure should be tested under actual field conditions to monitor the behaviour up to instability.

3. The effect of adjacent conduits on the structural behaviour and their load carrying capacity is a direction for more research.

4. The effect of using longitudinal stiffeners, thrust beams and relieving slabs on soil-steel structure
instability should be assessed.

5. The response of soil-steel structures to dynamic loads and their effect on the load carrying capacity should be investigated.
APPENDIX A

BEAM-COLUMN ELEMENT STIFFNESS MATRIX

A.1 THE GLOBAL ELASTIC ELEMENT STIFFNESS MATRIX

\[
\begin{bmatrix}
k_{11} & k_{12} & k_{13} & -k_{11} & -k_{12} & k_{13} \\
k_{22} & k_{23} & -k_{22} & -k_{12} & k_{23} \\
k_{33} & -k_{33} & k_{33} & -k_{23} & k_{33}/2 \\
k_{11} & k_{12} & -k_{13} & k_{11} & -k_{22} & k_{33}/2 \\
\end{bmatrix}
\]

\[
[k_e]_b = \begin{bmatrix}
k_{11} & k_{12} & k_{13} & -k_{11} & -k_{12} & k_{13} \\
\end{bmatrix}
\]

sym.

where,

\[k_{11} = \cos^2 \theta_b (EA/L) + \sin^2 \theta_b (12 EI/L^3)\]

\[k_{12} = \cos \theta_b \cdot \sin \theta_b (EA/L) - \cos \theta_b \sin \theta_b (12 EI/L^3)\]

\[k_{13} = \sin \theta_b (6 EI/L^2)\]

\[k_{22} = \sin^2 \theta_b (EA/L) + \cos^2 \theta_b (12 EI/L^3)\]

\[k_{23} = \cos \theta_b (6 EI/L^2)\]

\[k_{33} = (4 EI/L)\]

where \(\theta_b\) is angle between the local and global axes.
A.2 THE GLOBAL GEOMETRIC ELEMENT STIFFNESS MATRIX

\[
[k_g]_b = \frac{T}{L} \begin{bmatrix}
  k_{44} & k_{45} & k_{46} & -k_{44} & -k_{45} & k_{46} \\
  k_{55} & k_{56} & -k_{45} & -k_{55} & k_{56} & \text{sym.} \\
  k_{66} & -k_{46} & -k_{56} & k_{66}/4 & k_{44} & k_{45} & -k_{46} & k_{55} & -k_{56} & k_{66}
\end{bmatrix}
\]

where,

\[
k_{44} = \sin^2 \theta_b \frac{6}{5}
\]

\[
k_{45} = \cos \theta_b \sin \theta_b \frac{6}{5}
\]

\[
k_{46} = \sin \theta_b \frac{L}{10}
\]

\[
k_{55} = \cos \theta_b \frac{6}{5}
\]

\[
k_{56} = \cos \theta_b \frac{L}{10}
\]

\[
k_{66} = 2/15 (L^2)
\]
A.3 THE MODIFIED ELEMENT STIFFNESS AND LOAD MATRICES

\[
\{ P \} = \{ k \} \{ U \}
\]

\[
\begin{bmatrix}
H_i \\
P_i \\
M_i \\
\hline
H_j \\
P_j \\
M_j \\
\hline
M_p
\end{bmatrix} =
\begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\
k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\
k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\
k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\
k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\
k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \\
k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66}
\end{bmatrix}
\begin{bmatrix}
U_i \\
V_i \\
\Theta_i \\
U_j \\
V_j \\
\Theta_j \\
\hline
\Theta_H_j
\end{bmatrix}
\]

where \( k_{11}, k_{12}, k_{22}, \ldots \) are the sum of the components of the elastic and geometric global stiffness matrices defined in A.1 and A.2.
APPENDIX B

CONSTANT STRAIN TRIANGULAR ELEMENT STIFFNESS MATRIX

The plane strain condition in a constant strain triangular element leads to the following stiffness matrix in the local axes system where the notations are given in section 3.6.1

\[
[k_e] = \begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\
k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\
k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\
k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\
k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\
k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66}
\end{bmatrix}
\]

and \( t \) is the thickness of the element taken as unity.

\[
k_{11} = d_1 y_1^3 - (d_3 + d_7) y_3 (x_3 - x_2) + d_9 (x_3 - x_2)^2
\]

\[
k_{12} = -d_2 y_3 (x_3 - x_2) + d_9 (x_3 - x_2)^2 + d_3 y_3^2 - d_9 y_3 (x_3 - x_2)
\]

\[
k_{13} = -d_1 y_1^3 + d_7 y_3 (x_3 - x_2) + d_3 x_3 y_3 - d_9 x_3 (x_3 - x_2)
\]

\[
k_{14} = -d_2 x_3 y_3 - d_8 x_3 (x_3 - x_2) - d_3 y_3^2 + d_9 y_3 (x_3 - x_2)
\]

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\[ k_{15} = -d_3 x_2 y_3 + d_9 x_2 (x_3 - x_2) \]

\[ k_{16} = -d_2 x_2 y_3 + d_8 x_2 (x_3 - x_2) \]

\[ k_{21} = d_7 y_3^2 - (d_4 + d_9) y_3 (x_3 - x_2) + d_6 (x_3 - x_2)^2 \]

\[ k_{22} = d_5 (x_3 - x_2)^2 - (d_6 + d_8) y_3 (x_3 - x_2) + d_9 y_3^2 \]

\[ k_{23} = -d_7 y_3^2 + d_4 y_3 (x_3 - x_2) - d_6 x_3 (x_3 - x_2) + d_9 x_3 y_3 \]

\[ k_{24} = d_8 x_3 y_3 - d_5 x_3 (x_3 - x_2) + d_6 y_3 (x_3 - x_2) - d_9 y_3^2 \]

\[ k_{25} = d_6 x_2 (x_3 - x_2) - d_9 x_2 y_3 \]

\[ k_{26} = -d_5 x_2 (x_3 - x_2) - d_8 x_2 y_3 \]

\[ k_{31} = -d_1 y_3^2 + d_7 x_3 y_3 + (d_3 y_3 - d_9 x_3) (x_3 - x_2) \]

\[ k_{32} = -d_3 y_3^2 + d_9 x_3 y_3 + (d_2 y_3 - d_8 x_3) (x_3 - x_2) \]

\[ k_{33} = d_1 y_3^2 - (d_3 + d_7) x_3 y_3 + d_9 x_3^2 \]

\[ k_{34} = d_8 x_3^2 + d_3 y_3^2 - (d_2 + d_9) x_3 y_3, \]

\[ k_{35} = d_3 x_2 y_3 - d_9 x_2 x_3 \]

\[ k_{36} = d_2 x_2 y_3 - d_8 x_2 x_3 \]

\[ k_{41} = -d_7 y_3^2 + d_4 x_3 y_3 + (d_9 y_3 - d_6 x_3) (x_3 - x_2) \]
\[ k_{42} = -d_9 y_3^2 + d_6 x_3 y_3 + (d_8 y_3 - d_5 x_3)(x_3 - x_2) \]
\[ k_{43} = d_7 y_3^2 + d_6 x_3^2 - (d_4 + d_9)x_3 y_3 \]
\[ k_{44} = d_9 y_3^2 + d_5 x_3^2 - (d_6 + d_8)x_3 y_3 \]
\[ k_{45} = d_9 x_2 y_3 - d_6 x_2 x_3 \]
\[ k_{46} = d_8 x_2 y_3 - d_5 x_2 x_3 \]
\[ k_{51} = d_9 x_2 (x_3 - x_2) - d_7 x_2 y_3 \]
\[ k_{52} = d_8 x_2 (x_3 - x_2) - d_9 x_2 y_3 \]
\[ k_{53} = d_7 x_2 y_3 - d_9 x_2 x_3 \]
\[ k_{54} = d_9 x_2 y_3 - d_8 x_2 x_3 \]
\[ k_{55} = d_9 x_2^2 \]
\[ k_{56} = d_8 x_2^2 \]
\[ k_{61} = d_6 x_2 (x_3 - x_2) - d_4 x_2 y_3 \]
\[ k_{62} = d_5 x_2 (x_3 - x_2) - d_6 x_2 y_3 \]
\[ k_{63} = d_4 x_2 y_3 - d_6 x_2 x_3 \]
\[ k_{64} = d_6 x_2 y_3 - d_5 x_2 x_3 \]
\[ k_{65} = d_6 x_2^2 \]
\[ k_{66} = d_5 x_2^2 \]
APPENDIX C

SIMILARITY BETWEEN CONSTANT STRAIN AND LINEAR STRAIN TRIANGULAR ELEMENTS

Based on the previously generated element stiffness matrix \([k]_{\text{CST}}\) of order six for the constant strain triangular element (Appendix B), the submatrices of order two in the upper triangle of the stiffness matrix \([k]_{\text{LST}}\) of order twelve for the corresponding linear strain triangular element are given by:

\[
\begin{align*}
[k]_{\text{LST}}^{\text{I, I}} &= [k]_{\text{CST}}^{\text{I, I}} \\
[k]_{\text{LST}}^{\text{II, II}} &= [k]_{\text{CST}}^{\text{II, II}} \\
[k]_{\text{LST}}^{\text{III, III}} &= [k]_{\text{CST}}^{\text{III, III}} \\
[k]_{\text{LST}}^{\text{I, I}} &= -\frac{1}{3}[k]_{\text{CST}}^{\text{I, I}} \\
[k]_{\text{LST}}^{\text{I, III}} &= [k]_{\text{CST}}^{\text{I, III}} \\
[k]_{\text{LST}}^{\text{II, III}} &= [k]_{\text{CST}}^{\text{II, III}} \\
[k]_{\text{LST}}^{\text{I, VI}} &= [k]_{\text{LST}}^{\text{II, IV}}^T = \frac{4}{3}[k]_{\text{CST}}^{\text{I, II}} \\
[k]_{\text{LST}}^{\text{I, V}} &= [k]_{\text{LST}}^{\text{II, IV}} \\
[k]_{\text{LST}}^{\text{III, VI}} &= 0
\end{align*}
\]
\[ \begin{align*}
[k_{IV, V}]_{LST} &= \frac{4}{3} \left( [k_{I, II}]_{CST} + [k_{I, II}]_{CST}^T \right) \\
&\quad + [k_{II, III}]_{CST} + [k_{II, III}]_{CST}^T
\end{align*} \]
APPENDIX D
NINE NODED QUADRILATERAL ELEMENT STIFFNESS MATRIX

The plane strain condition in a linear strain quadrilateral element with nine nodes described in section 3.6.2 takes the form:

\[
[k_e]_q = \begin{bmatrix}
 k_{11} & k_{12} & \ldots & \ldots & k_{118} \\
 k_{21} & k_{22} & \ldots & \ldots & k_{218} \\
 \vdots & \vdots & \ddots & \vdots & \vdots \\
 k_{181} & k_{182} & \ldots & \ldots & k_{1818}
\end{bmatrix}
\]

If \( TP = t/2 \times 3 \times y_4 \),
\[ LR = t/2 \times 3 \times y_2, \]
and \( t \) is the thickness of the element taken as unity, then all the stiffness matrix elements are zeros, except those mentioned below.

\[
k_{11} = TP \left[ d_1 y_4^2 + d_9 (x_4 - x_3)^2 - (d_3 + d_7)y_4(x_4 - x_3) \right]
\]
\[
+ LR \left[ d_1 y_2^2 + d_9 (x_2 - x_3)^2 - (d_3 + d_7)y_2(x_2 - x_3) \right]
\]
\[ k_{12} = \text{TP} \left[ d_3 y_4^2 + d_8 (x_4 - x_3)^2 - (d_2 + d_9) y_4 (x_4 - x_3) \right] \]
\[-\text{LR} \left[ d_3 y_2^2 + d_8 (x_2 - x_3)^2 + (d_2 + d_9) y_2 (x_2 - x_3) \right] \]
\[ k_{13} = -\text{LR} \left[ d_3 x_3 y_2 + d_9 x_2 (x_2 - x_3) \right] \]
\[ k_{14} = \frac{\text{LR}}{3} \left[ d_2 x_3 y_2 + d_8 x_3 (x_2 - x_3) \right] \]
\[ k_{15} = \frac{\text{TP}}{3} \left[ d_1 y_4^2 - d_3 x_4 y_4 + (d_9 x_4 - d_7 y_4) (x_4 - x_3) \right] \]
\[ + \frac{\text{LR}}{3} \left[ d_1 y_2^2 + d_3 x_2 y_2 + (d_9 x_2 + d_7 y_2) (x_2 - x_3) \right] \]
\[ k_{16} = \frac{\text{TP}}{3} \left[ d_3 y_4^2 - d_2 x_4 y_4 + (d_8 x_4 - d_9 y_4) (x_4 - x_3) \right] \]
\[ - \frac{\text{LR}}{3} \left[ d_3 y_2^2 + d_2 x_2 y_2 + (d_8 x_2 + d_9 y_2) (x_2 - x_3) \right] \]
\[ k_{17} = \frac{\text{TP}}{3} \left[ d_3 x_3 y_4 - d_9 x_3 (x_4 - x_3) \right] \]
\[ k_{18} = \frac{\text{TP}}{3} \left[ d_2 x_3 y_4 - d_8 x_3 (x_4' - x_3) \right] \]
\[ k_{19} = -4 k_{13} \]
\[ k_{110} = -4 k_{14} \]
\[ k_{115} = -4 k_{17} \]
\[ k_{116} = -4 k_{18} \]
\[ k_{117} = -4 k_{15} \]
\[ k_{118} = -4 \, k_{16} \]

\[ k_{21} = \text{TP} \left[ d_7 \, y_4^2 + d_6 (x_4 - x_3)^2 - (d_4 + d_9) y_4 (x_4 - x_3) \right] \]

\[ - \text{LR} \left[ d_7 \, y_2^2 + d_6 (x_2 - x_3)^2 + (d_4 + d_9) y_2 (x_2 - x_3) \right] \]

\[ k_{22} = \text{TP} \left[ d_9 \, y_4^2 + d_5 (x_4 - x_3)^2 - (d_6 + d_8) y_4 (x_4 - x_3) \right] \]

\[ + \text{LR} \left[ d_9 \, y_2^2 + d_5 (x_2 - x_3)^2 + (d_6 + d_8) y_2 (x_2 - x_3) \right] \]

\[ k_{23} = \text{LR}/3 \left[ d_9 \, x_3 \, y_2 + d_6 \, x_3 \, (x_2 - x_3) \right] \]

\[ k_{24} = -\text{LR}/3 \left[ d_8 \, x_3 \, y_2 + d_5 \, x_3 \, (x_2 - x_3) \right] \]

\[ k_{25} = \text{TP}/3 \left[ d_7 \, y_4^2 - d_9 \, x_4 \, y_4 + (d_6 \, x_4 - d_4 \, y_4) (x_4 - x_3) \right] \]

\[ -\text{LR}/3 \left[ d_7 \, y_2^2 + d_9 \, x_2 \, y_2 + (d_6 \, x_2 + d_4 \, y_2) (x_2 - x_3) \right] \]

\[ k_{26} = \text{TP}/3 \left[ d_9 \, y_2^2 - d_8 \, x_4 \, y_4 + (d_5 \, x_4 - d_6 \, y_4) (x_4 - x_3) \right] \]

\[ + \text{LR}/3 \left[ d_9 \, y_2^2 - d_8 \, x_2 \, y_2 + (d_5 \, x_2 + d_6 \, y_2) (x_2 - x_3) \right] \]

\[ k_{27} = \text{TP}/3 \left[ d_9 \, x_3 \, y_4 - d_6 \, x_3 \, (x_4 - x_4) \right] \]

\[ k_{28} = \text{TP}/3 \left[ d_8 \, x_3 \, y_4 - d_5 \, x_3 \, (x_4 - x_3) \right] \]

\[ k_{29} = -4 \, k_{23} \]

\[ k_{210} = -4 \, k_{24} \]

\[ k_{215} = -4 \, k_{27} \]
\[ k_{216} = -4 \, k_{28} \]
\[ k_{217} = -4 \, k_{25} \]
\[ k_{218} = -4 \, k_{26} \]
\[ k_{31} = -LR/3 \left[ d_7 \, x_3 \, y_2 + d_9 \, x_3 \, (x_2 - x_3) \right] \]
\[ k_{32} = LR/3 \left[ d_9 \, x_3 \, y_2 + d_8 \, x_3 \, (x_2 - x_3) \right] \]
\[ k_{33} = LR \, d_9 \, x_3^2 \]
\[ k_{34} = -LR \, d_8 \, x_3^2 \]
\[ k_{35} = LR/3 \left[ d_7 \, x_3 \, y_2 + d_9 \, x_2 \, x_3 \right] \]
\[ k_{36} = -LR/3 \left[ d_9 \, x_3 \, y_2 + d_8 \, x_2 \, x_3 \right] \]
\[ k_{39} = k_{19} \]
\[ k_{310} = k_{29} \]
\[ k_{311} = -4 \, LR/3 \left[ d_3 \, x_3 \, y_2 + d_9 \, x_2 \, x_3 \right] \]
\[ k_{312} = 4 \, LR/3 \left[ d_9 \, x_3 \, y_2 + d_6 \, x_2 \, x_3 \right] \]
\[ k_{41} = LR/3 \left[ d_4 \, x_3 \, y_2 + d_6 \, x_3 \,(x_2 - x_3) \right] \]
\[ k_{42} = -LR/3 \left[ d_6 \, x_3 \, y_2 + d_5 \, x_3 \,(x_2 - x_3) \right] \]
\[ k_{43} = -L \, d_6 \, x_3^2 \]
\[ k_{44} = LR \ d_5 \ x_3^2 \]

\[ k_{45} = - LR/3 \{ d_4 \ x_3 \ y_2 + d_6 \ x_2 \ x_3 \} \]

\[ k_{46} = LR/3 \{ d_6 \ x_3 \ y_2 + d_5 \ x_2 \ x_3 \} \]

\[ k_{49} = k_{110} \]

\[ k_{410} = k_{210} \]

\[ k_{411} = 4LR/3 \{ d_2 \ x_3 \ y_2 + d_8 \ x_2 \ x_3 \} \]

\[ k_{412} = -4LR/3 \{ d_8 \ x_3 \ y_2 + d_5 \ x_2 \ x_3 \} \]

\[ k_{51} = TP/3 \{ d_1 \ y_4^2 - d_7 \ x_4 \ y_4 + (d_9 \ x_4 - d_3 \ y_4) (x_4 - x_3) \} \]

\[ + LR/3 \{ d_1 \ y_2^2 + d_7 \ x_2 \ y_2 + (d_9 \ x_2 + d_3 \ y_2) (x_2 - x_3) \} \]

\[ k_{52} = TP/3 \{ d_3 \ y_4^2 - d_9 \ x_4 \ y_4 + (d_8 \ x_4 - d_2 \ y_4) (x_4 - x_3) \} \]

\[ - LR/3 \{ d_3 \ y_2^2 + d_9 \ x_2 \ y_2 + (d_5 \ x_4 + d_2 \ y_2) (x_2 - x_3) \} \]

\[ k_{53} = -k_{311}/4 \]

\[ k_{54} = -k_{411}/4 \]

\[ k_{55} = TP \{ d_1 \ y_4^2 + d_9 \ x_4^2 - (d_3 + d_7) x_4 \ y_4 \} \]

\[ + LR \{ d_1 \ y_2^2 + d_9 \ x_2^2 + (d_3 + d_7) x_2 \ y_2 \} \]

\[ k_{56} = TP \{ d_3 \ y_4^2 + d_8 \ x_4^2 - (d_2 + d_9) x_4 \ y_4 \} \]

\[ - LR \{ d_3 \ y_2^2 + d_8 \ x_2^2 + (d_2 + d_9) x_2 \ y_2 \} \]
\[ k_{57} = \frac{TP}{3} \left[ -d_3 x_3 y_4 + d_9 x_3 x_4 \right] \]
\[ k_{58} = \frac{TP}{3} \left[ -d_2 x_3 y_4 + d_8 x_3 x_4 \right] \]
\[ k_{511} = k_{311} \]
\[ k_{512} = k_{411} \]
\[ k_{513} = -4k_{57} \]
\[ k_{514} = -4k_{58} \]
\[ k_{517} = k_{117} \]
\[ k_{518} = k_{217} \]
\[ k_{61} = \frac{TP}{3} \left[ d_7 y_4^2 - d_4 x_4 y_4 + (d_6 x_4 - d_9 y_4)(x_4 - x_3) \right] \]
\[ -\frac{LR}{3} \left[ d_7 y_2^2 + d_4 x_2 y_2 + (d_6 x_2 + d_9 y_2)(x_2 - x_3) \right] \]
\[ k_{62} = \frac{TP}{3} \left[ d_9 y_4^2 - d_6 x_4 y_4 + (d_5 x_4 - d_8 y_4)(x_4 - x_3) \right] \]
\[ +\frac{LR}{3} \left[ d_9 y_2^2 + d_6 x_2 y_2 + (d_5 x_2 + d_8 y_2)(x_2 - x_3) \right] \]
\[ k_{63} = -k_{312}/4 \]
\[ k_{64} = -k_{412}/4 \]
\[ k_{65} = TP \left[ d_7 y_4^2 + d_6 x_4^2 - (d_4 + d_9) x_4 y_4 \right] \]
\[ -LR \left[ d_7 y_2^2 + d_6 x_2^2 + (d_4 + d_9) x_2 y_2 \right] \]
\[ k_{66} = TP \left[ d_9 y_4^2 + d_5 x_4^2 - (d_6 + d_8) x_4 y_4 \right] \\
+ LR \left[ d_9 y_2^2 + d_5 x_2^2 + (d_6 + d_8) x_2 y_2 \right] \\
k_{67} = TP/3 \left[ -d_9 x_3 y_4 + d_6 x_3 x_4 \right] \\
k_{68} = TP/3 \left[ -d_8 x_3 y_4 + d_5 x_3 x_4 \right] \\
k_{611} = k_{312} \\
k_{612} = k_{412} \\
k_{613} = -4 k_{67} \\
k_{614} = -4 k_{68} \\
k_{617} = k_{118} \\
k_{618} = k_{218} \\
k_{71} = TP/3 \left[ d_7 x_3 y_4 - d_9 x_3 \left( x_4 - x_3 \right) \right] \\
k_{72} = TP/3 \left[ d_9 x_3 y_4 - d_8 x_3 \left( x_4 - x_3 \right) \right] \\
k_{75} = TP/3 \left[ -d_7 x_3 y_4 + d_9 x_3 x_4 \right] \\
k_{76} = TP/3 \left[ -d_8 x_3 y_4 + d_8 x_3 x_4 \right] \\
k_{77} = TP d_9 x_3^2 \\
k_{78} = TP d_8 x_3^2 \]
\[ k_{713} = k_{513} \]
\[ k_{714} = k_{613} \]
\[ k_{715} = k_{115} \]
\[ k_{716} = k_{215} \]
\[ k_{81} = TP/3 \left[ d_4 x_3 y_4 - d_6 x_3 \left( x_4 - x_3 \right) \right] \]
\[ k_{82} = TP/3 \left[ d_6 x_3 y_4 - d_5 x_3 \left( x_4 - x_3 \right) \right] \]
\[ k_{85} = TP/3 \left[ -d_4 x_3 y_4 + d_6 x_3 x_4 \right] \]
\[ k_{86} = TP/3 \left[ -d_6 x_3 y_4 + d_5 x_3 x_4 \right] \]
\[ k_{87} = TP \ d_6 x_3^2 \]
\[ k_{88} = TP \ d_5 x_3^2 \]
\[ k_{813} = k_{514} \]
\[ k_{814} = k_{614} \]
\[ k_{815} = k_{116} \]
\[ k_{816} = k_{216} \]
\[ k_{91} = -4 \ k_{31} \]
\[ k_{92} = -4 \ k_{32} \]
\[ k_{93} = + k_{91} \]
\[ k_{94} = k_{101} \]
\[ k_{99} = + \frac{8 LR}{3} \left[ d_1 y_2^2 + d_7 y_2 (x_2 - x_3) + d_9 (x_2 - x_3)^2 + d_9 x_2 x_3 + d_3 x_2 y_3 \right] \]
\[ k_{910} = -\frac{4LR}{3} \left[ (d_3 + d_7) y_2^2 + (d_4 + d_9) y_2 (x_2 - x_3) + (d_2 + d_9) x_2 y_2 + (d_6 + d_8) (x_2 - x_3)^2 + (d_6 + d_8) x_2 x_3 \right] \]
\[ k_{911} = -\frac{8LR}{3} \left[ d_1 y_2^2 + d_3 y_2 (x_2 - x_3) + d_7 x_2 y_2 + d_9 x_2 \right] \]
\[ k_{912} = \frac{4L}{3} \left[ (d_3 + d_7) y_2^2 + (d_2 + d_9) y_2 (x_2 - x_3) + (d_4 + d_9) x_2 y_2 + (d_6 + d_8) x_2 (x_2 - x_3) \right] \]
\[ k_{917} = -2 k_{311} \]
\[ k_{918} = \frac{4LR}{3} \left[ (d_2 + d_9) x_3 y_2 + (d_6 + d_8) x_2 x_3 \right] \]
\[ k_{101} = -4 k_{41} \]
\[ k_{102} = -4 k_{42} \]
\[ k_{103} = k_{912} \]
\[ k_{104} = k_{102} \]

\[ k_{109} = k_{910} \]

\[ k_{1010} = \frac{8LR}{3} \left[ d_9 y_2^2 + d_6 y_2 (x_2 - x_3) + d_5 (x_2 - x_3)^2 \right. \]
\[ \left. + d_5 x_2 x_3 + d_8 x_2 y_2 \right] \]

\[ k_{1011} = k_{912} \]

\[ k_{1012} = -\frac{8LR}{3} \left[ d_9 y_2^2 + d_8 y_2 (x_2 - x_3) + d_5 x_2 (x_2 - x_3) \right. \]
\[ \left. + d_6 x_2 y_2 \right] \]

\[ k_{1017} = k_{918} \]

\[ k_{1018} = 2k_{412} \]

\[ k_{113} = -4 k_{35} \]

\[ k_{114} = -4 k_{45} \]

\[ k_{115} = k_{113} \]

\[ k_{116} = -4 k_{36} \]

\[ k_{119} = -\frac{8LR}{3} \left[ d_1 y_2^2 + d_7 y_2 (x_2 - x_3) + d_9 x_2 (x_2 - x_3) \right. \]
\[ \left. + d_3 x_2 y_2 \right] \]

\[ k_{1110} = k_{912} \]
\[k_{1111} = k_{99}\]

\[k_{1112} = k_{910}\]

\[k_{1117} = 2k_{19}\]

\[k_{1118} = -\frac{4LR}{3} \left[ (d_2 + d_9)x_3 y_2 + (d_6 + d_8)x_3(x_2 - x_3) \right]\]

\[k_{123} = k_{116}\]

\[k_{124} = -4k_{46}\]

\[k_{125} = k_{114}\]

\[k_{126} = k_{124}\]

\[k_{129} = k_{1110}\]

\[k_{1210} = -\frac{8LR}{3} \left[ d_9 y^2_2 + d_6 y_2(x_2 - x_3) + d_5 x_2(x_2 - x_3) + d_8 x_2 y_2 \right]\]

\[k_{1211} = k_{109}\]

\[k_{1212} = k_{1010}\]

\[k_{1217} = k_{1118}\]

\[k_{1218} = 2k_{210}\]

\[k_{135} = -4k_{75}\]
\[ k_{136} = -4 \, k_{76} \]

\[ k_{137} = k_{135} \]

\[ k_{138} = -4 \, k_{85} \]

\[ k_{1313} = \frac{8TP}{3} \left[ \frac{d_1}{4} \left( x_4 - x_3 \right)^2 - d_7 \cdot y_4 \right] + d_9 \cdot \left( x_4 - x_3 \right)^2 \\
+ d_9 \cdot x_3 \cdot x_4 - d_3 \cdot x_4 \cdot y_4 \] \]

\[ k_{1314} = \frac{4TP}{3} \left[ (d_3 + d_7) \cdot y_4^2 - (d_4 + d_9) \cdot y_4 \cdot (x_4 - x_3) + \\
(d_6 + d_8) \cdot (x_4 - x_3)^2 - (d_2 + d_9) \cdot x_4 \cdot y_4 + (d_6 + d_8) \cdot x_3 \cdot x_4 \right] \]

\[ k_{1315} = \frac{8TP}{3} \left[ -d_1 \cdot y_4^2 + d_7 \cdot y_4 \cdot (x_4 - x_3) - d_9 \cdot x_4 \cdot (x_4 - x_3) \\
+ d_3 \cdot x_4 \cdot y_4 \right] \]

\[ k_{1316} = \frac{4TP}{3} \left[ - (d_3 + d_7) \cdot y_4^2 + (d_4 + d_9) \cdot y_4 \cdot (x_4 - x_3) - (d_6 + d_8) \cdot x_4 \cdot (x_4 - x_3) + (d_2 + d_9) \cdot x_4 \cdot y_4 \right] \]

\[ k_{1317} = 2 \, k_{115} \]

\[ k_{1318} = \frac{4TP}{3} \left[ -(d_2 + d_9) \cdot x_3 \cdot y_4 + (d_6 + d_8) \cdot x_3 \cdot (x_4 - x_3) \right] \]

\[ k_{145} = k_{138} \]

\[ k_{146} = -4 \, k_{86} \]
\[ k_{147} = k_{136} \]
\[ k_{148} = k_{146} \]
\[ k_{1413} = k_{1314} \]
\[ k_{1414} = \frac{8TP}{3} \left[ d_9 y_4^2 - d_6 y_4 (x_4 - x_3) + d_5 (x_4 - x_3)^2 + d_5 x_4 x_3 - d_8 x_4 y_4 \right] \]
\[ k_{1415} = k_{1316} \]
\[ k_{1416} = -k_{1414} \]
\[ k_{1417} = k_{1318} \]
\[ k_{1418} = 2k_{216} \]
\[ k_{151} = -4k_{71} \]
\[ k_{152} = -4k_{72} \]
\[ k_{157} = k_{151} \]
\[ k_{158} = -4k_{81} \]
\[ k_{1513} = \frac{8TP}{3} \left[ -d_9 y_4^2 + d_3 y_4 (x_4 - x_3) - d_9 x_4 (x_4 - x_3) + d_7 x_4 y_4 \right] \]
\[ k_{1514} = k_{1316} \]
\[ k_{1515} = k_{1313} \]
\[ k_{1516} = k_{1314} \]
\[ k_{1517} = 2k_{513} \]
\[ k_{1518} = \frac{4TP}{3} \left[ (d_2 + d_9)x_3y_4 - (d_5 + d_8)x_3x_4 \right] \]
\[ k_{161} = -4k_{81} \]
\[ k_{162} = -4k_{82} \]
\[ k_{167} = k_{152} \]
\[ k_{168} = k_{162} \]
\[ k_{1613} = k_{1316} \]
\[ k_{1614} = \frac{8TP}{3} \left[ -d_9y_4^2 + d_8y_4(x_4 - x_3) - d_5x_4(x_4 - x_3) \right. \]
\[ \left. + d_6x_4y_4 \right] \]
\[ k_{1615} = k_{1413} \]
\[ k_{1616} = k_{1414} \]
\[ k_{1617} = k_{1518} \]
\[ k_{1618} = 2k_{614} \]
\[ k_{171} = -4 \ k_{51} \]
\[ k_{172} = -4 \ k_{52} \]
\[ k_{175} = k_{171} \]
\[ k_{176} = -4 \ k_{61} \]
\[ k_{179} = 2 \ k_{113} \]
\[ k_{1710} = \frac{4LR}{3} \left( (d_4 + d_9)x_3 \ y_2 + (d_6 + d_8)x_2 \ x_3 \right) \]
\[ k_{1711} = 2 \ k_{91} \]
\[ k_{1712} = -4 \ \frac{LR}{3} \left( (d_4 + d_9)x_3 \ y_2 + (d_6 + d_8)x_3(x_2 - x_3) \right) \]
\[ k_{1713} = 2 \ k_{135} \]
\[ k_{1714} = \frac{4TP}{3} \left[ -(d_4 + d_9)x_3 \ y_4 + (d_6 + d_8)x_3(x_4 - x_3) \right] \]
\[ k_{1715} = 2k_{135} \]
\[ k_{1716} = \frac{4TP}{3} \left( (d_4 + d_9)x_3 \ y_4 - (d_6 + d_8)x_3 \ x_4 \right) \]
\[ k_{1717} = k_{99} + k_{1313} \]
\[ k_{1718} = k_{910} + k_{1314} \]
\[ k_{181} = -4 \ k_{61} \]
\[ k_{182} = -4 \ k_{62} \]
\[ k_{185} = k_{172} \]
\[ k_{186} = k_{182} \]
\[ k_{189} = k_{1710} \]
\[ k_{1810} = 2k_{124} \]
\[ k_{1811} = k_{1712} \]
\[ k_{1812} = 2k_{102} \]
\[ k_{1813} = k_{1714} \]
\[ k_{1814} = 2k_{162} \]
\[ k_{1815} = k_{1716} \]
\[ k_{1816} = 2k_{146} \]
\[ k_{1817} = k_{1718} \]
\[ k_{1818} = k_{1010} + k_{1414} \]
APPENDIX E
FLOW CHART

START

H = Height of conduit
S = Span of conduit
NLB = Number of soil layers below conduit
NLC = Number of soil layers up to the crown level
NLT = Total number of soil layers
KI = Number of interface elements in the incremental analysis
KIT = Total number of interface elements
KQ = Number of quadrilateral elements in the incremental analysis
KQT = Total number of quadrilateral elements
ITE = Iteration number for soil properties
NITE = Number of iterations specified for the incremental analysis
IFS = Stress transfer iteration number
NIFS = Number of iterations specified for the stress transfer
NT = Total number of incremental analyses
A

Read Input Data

S=H

Yes

Calculate Equal Lengths for Interface Elements

Generate Fixed Nodal Coordinates for Base

1

Do 1 I = 1, NT

ITE = 0
IFS = 0

Yes

I > NLT

Generate Nodal Coordinates and Degrees of Freedom for a new layer

B
B

I = NLB

Yes

Generate All Beam Elements

No

NLC > I ≥ NLB

Yes

Generate New Interface Elements

No

Generate New Triangular Elements

I > NLC

Yes

Generate New Triangular and Quadrilateral Elements

No

Set Load Vector = 0

IFS > 0

Yes

Add Stress Transfer Loads

No

2

3

C
Set Stiffness Matrix = 0

Yes

I > NLB

Add Beam Stiffness
Add Interface Stiffness

No

Add Stiffness of Triangular Elements

Yes

I > NLC

Add Stiffness of Quadrilateral Elements

No

IFS > 0

Yes

Add Weights of New Soil Elements

No

I ≤ NLT

Yes

Add Compaction Load, Release Previous Compaction

No

Read External Loads

D
Solve for Nodal Displacements

- IF $I > 0$
  - Yes: Proceed to step 5
  - No: Proceed to step 4

- IF $I > NLB$
  - Yes: Check Failure of Interface Elements
    - Normal Stress $> 0$
      - Yes: Modify Interface Properties
      - No: Proceed to step 3
    - Shear Stress $> Shear Failure$
      - Yes: Proceed to step 3
      - No: Calculate Forces in Beam Elements
  - No: Proceed to step 3
Calculate Total Stresses and Displacements

Update Soil and Interface Properties.

Update Structure Coordinates

Print
- Element Stresses and Properties
- Nodal Coordinates and Displacements

STOP
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VITAE AUCTORIS

1950  Born, December 4, in Cairo, Egypt.

1968  Joined the Faculty of Engineering, University of Ain Shams, Cairo, Egypt.

1973  Graduated with a Bachelor of Science Degree (honours) in Civil Engineering (Structural Division) and ranked first in the Civil class. Appointed as Teaching Assistant in the Department of Civil Engineering, Faculty of Engineering, Ain Shams University.

1977  Graduated with a Master of Science Degree in Engineering (Structural) from Ain Shams University, Cairo, Egypt.

1979  Joined the Faculty of Engineering (Civil Department), University of Windsor, Windsor, Ontario, Canada. Awarded a Teaching and Research Assistantship for Graduate Study at the University of Windsor. Also Awarded the University of Windsor Scholarship.

1981  Transferred to the Ph.D. Program in Civil Engineering, Faculty of Engineering, University of Windsor.