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Stress distribution around circular holes in sandwich plates.

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STRESS DISTRIBUTION AROUND CIRCULAR HOLES
IN SANDWICH PLATES

A THESIS

Submitted to the Faculty of Graduate Studies through the Department of Civil Engineering in Partial Fulfilment of the Requirements for the Degree of Master of Applied Science at The University of Windsor

By

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1973

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ABSTRACT

A theoretical solution is presented for the stresses around circular holes in rectangular and circular sandwich plates subjected to uniform tension. The solution is applicable to sandwich plates having an isotropic core and isotropic facings of equal thickness. The core is assumed to be strong enough to resist in-plane stresses. The solution consists in determining face stresses around holes that satisfy the boundary conditions of the plate. Numerical results and graphs are included. The theoretical stresses obtained, compare favorably with the experimental results.
ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

\( x, y, z \) Rectangular coordinates.

\( r, \theta \) Polar coordinates.

\( a \) Radius of hole.

\( b \) Half the width of the rectangular plate; radius of circular plate.

\( E \) Modulus of elasticity.

\( h \) Core thickness.

\( P_r \) Force per unit length in \( r \)-direction.

\( P_x \) Force per unit length in \( x \)-direction.

\( t \) Thickness of one face layer.

\( \sigma_{op} \) Uniform boundary stress in sandwich plate.

\( \sigma_{r\theta}, \sigma_{\theta\theta} \) Radial and tangential normal stresses in polar coordinates.

\( \sigma_{op}, \sigma_{or}, \sigma_{ox}, \sigma_{oc} \) Uniform boundary stresses in faces and cores.

\( \tau_{r\theta} \) Shearing stress in polar coordinates.

\( \psi \) Stress function.

\( \lambda \) Ratio of width of the plate and the diameter of hole.
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CHAPTER I

INTRODUCTION

1.1. **STRESS CONCENTRATION AND SANDWICH CONSTRUCTION.**

When designing structures, various factors have to be taken into consideration. One of these is the "Stress Concentration" around holes. The magnitude of stress concentration around holes in plates used as components of structures has long been an important design consideration. Solutions for the stresses existing on the boundary of various shapes of holes in homogeneous plates have been published. However, no solutions are yet available for sandwich plates with holes subjected to uniaxial load.

Sandwich constructions comprise thin strong facings bonded to each side of a thick core. By themselves, the components have little load-carrying capacity; once bonded together, they produce stiff, lightweight structural members. For sandwich construction to be effective, the adhesive which bonds the facings to the core must be capable of transmitting loads between the two components effecting unit action.

When used in the building industry and if compared
to the common methods of construction, sandwich members reduce both the form work weights and the foundation requirements, whence reduction in cost. Furthermore, sandwich panels are easy to erect and provide a permanent exterior and interior finish.

The work embodied in this thesis consists of:

(a) Theoretical analysis to determine face stresses around holes in sandwich plates subjected to uniaxial tensile load; and,

(b) experimental investigation of several sandwich plates subjected to unidirectional tensile forces.

The sandwich plate is assumed to be a thin plate, consisting of isotropic facings separated by and bonded to an isotropic core. In this analysis, the core is assumed to be strong enough to resist in-plane stresses. Equilibrium and compatibility equations are written for the facings. Relations between core stresses and face stresses are established from the compatibility condition that the displacements of the core and facings are equal at the interface. Thus, by introducing a stress function, the expressions for the face stresses around holes that satisfy the face equilibrium equations as well as the boundary conditions are determined.

The experimental work consisted of tests on sandwich
plates, having aluminum facings and plywood core, and with holes of various diameter-width ratios. These plates were subjected to uniformly distributed tensile load in the length-wise direction. The choice of the different diameter-width ratios was made to justify the application of the expressions for stress components, derived for an infinitely large plate, to a plate of finite width.

Rectangular strain rosettes were installed and strains were obtained by using a Datran Strain Indicator. Poisson's ratio of plywood was determined from the strain measurement in the lateral and longitudinal directions of the plywood specimen subjected to tensile loads.

1.2. RELEVANT PREVIOUS WORK.

The problem of stress distribution in plates with holes has been of interest for many years. Most of this work has been concerned with infinite plates or plates in which the hole diameter is small compared with one of the plate dimensions.

The evaluation of stress concentrations around holes in plates has been a subject of continued theoretical and experimental research for over a century and numerous books 1, 2, 3, and papers have been written dealing with the subject.
Hicks has presented a solution for stress concentration around holes in plates which is restricted to a number of specific problems.

Reissner carried out a rigorous analysis on the finite deflections of rectangular sandwich plates. He developed the basic differential equations for finite transverse deflections of sandwich panels neglecting the transverse normal stresses in the core and the variation of the face stresses over the thickness of the face layers.

Ikeda developed a theory for bending of isotropic flat rectangular sandwich plates taking into account the contributions of the individual stiffnesses of the faces and the core. The solution of the basic differential equations gives the deflection and the stress function which in turn gives the deformation and stresses on facings and the core.

Raville has presented a theoretical solution for the deflection and stresses in a uniformly loaded, simply supported, rectangular sandwich plate which is applicable to plate having an orthotropic core of arbitrary thickness and isotropic facings.

In most of the prior investigations, the face parallel stresses in the core are neglected. A number of papers and books have been published on this subject.

But none of the available solution has any direct bearing on the present solution procedure for stress distribution around circular holes in sandwich plates subjected to uniaxial tensile force.
CHAPTER II

THEORETICAL ANALYSIS

The method of analysis consists of determining expressions for the face stresses around holes that satisfy the face equilibrium equations and the boundary conditions for a sandwich plate subjected to uniform tension on the outer boundary. The face stresses around the hole are evaluated from consideration that the displacements of the core and facings be equal at their mutual interfaces.

The assumptions on which this theoretical investigation is based are as follows:

(a) The linear theory of elasticity is valid.

(b) The facings and the core are of isotropic materials and the facings are of equal thickness.

(c) The core stiffness associated with plane stress components in the plane of the plate are not negligible.

(d) Perfect bonding exists between the facings and core.
FIG. 2.1 DIFFERENTIAL ELEMENT OF UPPER FACING
Figure 2.1 shows an element of the upper facing of the sandwich plate. Summing up forces in the radial direction (including the stresses at the junction of the face layer and core layer, i.e., $T_{z\theta_f}, T_{zrf}$) one obtains the following equation of equilibrium:

$$
\left(\sigma_{zrf} + \frac{\partial \sigma_{zrf}}{\partial \sigma} \right) dx \cdot d\theta \cdot t - \sigma_{zrf} \cdot \sigma \cdot d\theta \cdot d\tau t
$$

$$+ (T_{z\theta_f} + \frac{\partial T_{z\theta_f}}{\partial \sigma} \cdot d\sigma) dx \cdot d\theta \cdot T - T_{z\theta_f} \cdot d\sigma \cdot d\tau t$$

$$- (\sigma_{\theta \theta_f} + \frac{\partial \sigma_{\theta \theta_f}}{\partial \theta} \cdot d\theta \cdot \sin \frac{d\theta}{2} - \sigma_{\theta \theta_f} \cdot \sigma \cdot d\theta \cdot \sin \frac{d\theta}{2}$$

$$- T_{zrf} \cdot \sigma \cdot d\sigma \cdot d\tau = 0$$

Dividing by the elementary area $\pi \sigma \cdot dx \cdot d\tau$ and neglecting small quantities of higher order the above equation becomes:

$$\frac{\partial \sigma_{zrf}}{\partial \sigma} \cdot t + \frac{1}{\sigma} \frac{\partial T_{z\theta_f}}{\partial \theta} \cdot t + (\sigma_{zrf} - \sigma_{\theta \theta_f}) \cdot t - T_{zrf} = 0$$

Hence the eqn. of equilibrium in the radial direction is:

$$\frac{\partial N_{zrf}}{\partial \sigma} + \frac{1}{\sigma} \frac{\partial S_{z\theta_f}}{\partial \theta} + (N_{zrf} - N_{\theta \theta_f}) - T_{zrf} = 0 \quad \cdots \quad (1)$$
Where \[ N_{\theta \phi} = t \cdot \sigma_{\theta \phi} \]

\[ N_{\phi \theta} = t \cdot \sigma_{\phi \theta} \]

and

\[ S_{\phi \theta} = t \cdot T_{\phi \theta} \]

Summing up forces in the circumferential direction, yields the following equation of equilibrium:

\[
\left[ T_{\phi \theta} + \frac{\partial T_{\phi \theta}}{\partial \phi} \right] \frac{dz}{d\phi} + \frac{\partial S_{\phi \theta}}{\partial \phi} \frac{dz}{d\phi} + \frac{\partial N_{\phi \theta}}{\partial \phi} \frac{dz}{d\phi} - T_{\phi \theta} \frac{dz}{d\phi} = 0
\]

Dividing by \( r \cdot dr \cdot d\phi \) and neglecting small quantities of higher order will yield:

\[
\frac{1}{r} \frac{\partial T_{\phi \theta}}{\partial \phi} \frac{dz}{d\phi} + \frac{\partial T_{\phi \theta}}{\partial \phi} \frac{dz}{d\phi} + \frac{\partial S_{\phi \theta}}{\partial \phi} \frac{dz}{d\phi} + T_{\phi \theta} \frac{dz}{d\phi} = 0
\]

Hence, the equation of equilibrium in the circumferential direction is:

\[
\frac{1}{r} \frac{\partial N_{\phi \theta}}{\partial \phi} + \frac{\partial S_{\phi \theta}}{\partial \phi} + \frac{2}{r} S_{\phi \theta} + T_{\phi \theta} = 0 \quad \ldots \ldots \quad (2)
\]

Where \[ N_{\phi \theta} = t \cdot \sigma_{\phi \theta} \] and \[ S_{\phi \theta} = t \cdot T_{\phi \theta} \]

\[ \sigma_{\phi \theta} = \frac{1}{r} T_{\phi \theta} \]
The stress-strain relations for the face are known to be of the following form:

\[
\varepsilon_{\sigma\sigma} = \frac{\partial u_{\sigma}}{\partial \sigma} \quad \Rightarrow \quad \frac{1}{E_f} \frac{\partial (N_{\sigma\sigma} - \nu N_{\sigma\rho})}{\partial \sigma} = \frac{1}{E_f} \frac{\partial (N_{\sigma\rho} - \nu N_{\sigma\sigma})}{\partial \sigma} \quad \text{(3)}
\]

\[
\varepsilon_{\sigma\rho} = \frac{u_{\sigma}}{\sigma} + \frac{1}{\sigma} \frac{\partial v_{\rho}}{\partial \rho} \quad \Rightarrow \quad \frac{1}{E_f} \frac{\partial (N_{\sigma\rho} - \nu N_{\sigma\sigma})}{\partial \sigma} = \frac{1}{E_f} \frac{\partial (N_{\rho\sigma} - \nu N_{\rho\rho})}{\partial \sigma} \quad \text{(4)}
\]

\[
\gamma_{\sigma\rho} = \frac{1}{\sigma} \frac{\partial u_{\rho}}{\partial \rho} + \frac{\partial v_{\rho}}{\partial \sigma} - \frac{v_{\sigma}}{\sigma} \quad \Rightarrow \quad \frac{1}{G_f} \frac{\partial S_{\sigma\rho}}{\partial \sigma} = \frac{2(1 + \nu)}{E_f} \frac{\partial S_{\sigma\rho}}{\partial \sigma} \quad \text{(5)}
\]

Eliminating \( u \) and \( v \) from equations (3), (4), and (5) one obtains the following equation:

\[
\frac{\partial^2 \varepsilon_{\sigma\rho}}{\partial \sigma^2} + \frac{1}{\sigma} \frac{\partial \varepsilon_{\sigma\rho}}{\partial \rho} + \frac{\partial \varepsilon_{\sigma\sigma}}{\partial \sigma} \frac{1}{\sigma} \frac{\partial \varepsilon_{\sigma\rho}}{\partial \sigma} = \frac{1}{\sigma} \frac{\partial \gamma_{\sigma\rho}}{\partial \rho} + \frac{1}{\sigma^2} \frac{\partial \gamma_{\sigma\rho}}{\partial \sigma}
\]

Using the stress-strain relations, this becomes:

\[
\frac{\partial^2 (N_{\sigma\rho} - \nu N_{\sigma\sigma})}{\partial \sigma^2} + \frac{1}{\sigma} \frac{\partial (N_{\sigma\rho} - \nu N_{\sigma\sigma})}{\partial \rho} \nonumber
\]

\[
+ \frac{2}{\sigma} \frac{\partial (N_{\rho\rho} - \nu N_{\rho\rho})}{\partial \sigma} - \frac{1}{\sigma} \frac{\partial (N_{\rho\rho} - \nu N_{\rho\rho})}{\partial \rho} \nonumber
\]

\[
= 2 \left( 1 + \frac{\nu}{2} \right) \frac{1}{\sigma} \frac{\partial^2 S_{\sigma\rho}}{\partial \sigma \partial \rho} + 2 \left( 1 + \frac{\nu}{2} \right) \frac{1}{\sigma^2} \frac{\partial^2 S_{\sigma\rho}}{\partial \sigma \partial \rho} \quad \text{(6)}
\]

Identical equations to equations (1), (2), (3), (4), (5), and (6) are found for the bottom face layer.
FIG. 2.2 DIFFERENTIAL ELEMENT OF THE CORE
For the core layer (figure 2.2), the equation of equilibrium in the radial direction is:

\[
\left( \sigma_{\theta r} + \frac{\partial \sigma_{\theta r}}{\partial \theta} \right) (r + dr) d\theta \cdot h - \sigma_{\theta r} \cdot r \cdot d\theta \cdot h \\
+ \left( \frac{T_{\theta r}}{r} + \frac{\partial T_{\theta r}}{\partial \theta} \right) dr \cdot h - T_{\theta r} \cdot dr \cdot h \\
- \left( \sigma_{\theta \theta} + \frac{\partial \sigma_{\theta \theta}}{\partial \theta} \right) dx \cdot h \cdot \sin \frac{\theta}{2} - \sigma_{\theta \theta} \cdot dx \cdot h \cdot \sin \frac{\theta}{2} \\
+ T_{\theta r} \cdot \theta \cdot dx \cdot dr + T_{\theta r} \cdot \theta \cdot dx \cdot dr \right) = 0
\]

Neglecting squares of small quantities and dividing by \(r \cdot dr \cdot d\theta\) and rearranging yield:

\[
\frac{\partial N_{\theta r}}{\partial \theta} + \frac{1}{r} \frac{\partial S_{\theta r}}{\partial \theta} + \left( \frac{N_{\theta r} - N_{\theta \theta}}{r} \right) + 2T_{\theta r} = 0 \quad (7)
\]

Similarly, the equation of equilibrium in the circumferential direction can be shown to be:

\[
\frac{1}{r} \frac{\partial \sigma_{\theta \theta}}{\partial \theta} \cdot h + \frac{\partial T_{\theta \theta}}{\partial \theta} \cdot h + 2 \frac{T_{\theta \theta}}{r} = 0
\]

or

\[
\frac{1}{r} \frac{\partial N_{\theta \theta}}{\partial \theta} + \frac{\partial S_{\theta \theta}}{\partial \theta} + 2 \frac{S_{\theta \theta}}{r} - 2T_{\theta \theta} = 0 \quad (8)
\]
The stress-strain relations for the core layer may be written as,

$$
\varepsilon_{\text{rrc}} = \frac{\partial u_c}{\partial r} = \frac{1}{E_c} (N_{\text{rrc}} - \nu_c N_{\text{oooc}})
$$

(9)

$$
\varepsilon_{\text{oooc}} = \frac{u_c}{r} + \frac{1}{r} \frac{\partial v_c}{\partial \theta} = \frac{1}{E_{c\theta}} (N_{\text{oooc}} - \nu_c N_{\text{rrc}})
$$

(10)

$$
\gamma_{\text{rocc}} = \frac{1}{r} \frac{\partial u_c}{\partial \theta} + \frac{\partial v_c}{\partial r} = \frac{S_{\text{rocc}}}{G_{c\theta}}
$$

(11)

Eliminating $u$ and $v$ from equations (9), (10), and (11), the compatibility condition for the core layer becomes:

$$
\frac{\partial^2 \varepsilon_{\text{oooc}}}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \varepsilon_{\text{rrc}}}{\partial \theta^2} + \frac{2}{r} \frac{\partial \varepsilon_{\text{oooc}}}{\partial r} - \frac{1}{r} \frac{\partial \varepsilon_{\text{rrc}}}{\partial \theta} = \frac{1}{r^2} \frac{\partial^2 \gamma_{\text{rocc}}}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial \gamma_{\text{rocc}}}{\partial \theta}
$$

Using the stress-strain relations, this becomes:

$$
\frac{\partial^2 (N_{\text{oooc}} - \nu_c N_{\text{rrc}})}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 (N_{\text{rrc}} - \nu_c N_{\text{oooc}})}{\partial \theta^2}
$$

$$
+ \frac{2}{r} \frac{\partial (N_{\text{oooc}} - \nu_c N_{\text{rrc}})}{\partial r} - \frac{1}{r} \frac{\partial (N_{\text{rrc}} - \nu_c N_{\text{oooc}})}{\partial \theta} = \frac{E_c}{G_{c\theta}} \frac{1}{r} \frac{\partial S_{\text{rocc}}}{\partial \theta} + \frac{E_{c\theta}}{r^2} \frac{\partial S_{\text{rocc}}}{\partial \theta}
$$

(12)

From equation (7):

$$
\frac{1}{r_{\text{rrc}}} = - \frac{1}{2} \frac{\partial N_{\text{rrc}}}{\partial r} - \frac{1}{2r} \frac{\partial S_{\text{rocc}}}{\partial \theta} - \frac{1}{2} \left( \frac{N_{\text{rrc}} - N_{\text{oooc}}}{r} \right)
$$

(13)
From equation (14):

\[
\frac{1}{T_{zc}} = \frac{1}{2} \sigma \frac{\partial N_{osc}}{\partial \sigma} + \frac{1}{2} \frac{\partial S_{osc}}{\partial \sigma} + \frac{1}{2} \frac{2S_{osc}}{\sigma} \quad \ldots \ldots (14)
\]

Since \( \frac{1}{T_{zc}} = \frac{1}{T_{zrc}} \) substituting for \( \frac{1}{T_{zrc}} \) in equation (1) by using equation (13), equation (1) becomes:

\[
\frac{\partial (N_{osc} + \frac{1}{2} N_{osc})}{\partial \sigma} + \frac{1}{\sigma} \frac{\partial (S_{osc} + \frac{1}{2} S_{osc})}{\partial \sigma} + \frac{(N_{osc} + \frac{1}{2} N_{osc}) - \frac{1}{2} \frac{N_{osc}}{\sigma}}{g} = 0 \quad (\ldots (15)
\]

Using equation (15), equation (2) becomes:

\[
\left\{ \frac{1}{2} \frac{\partial (N_{osc} + \frac{1}{2} N_{osc})}{\partial \sigma} + \frac{\partial (S_{osc} + \frac{1}{2} S_{osc})}{\partial \sigma} \right\} + \frac{2}{\sigma} \left( S_{osc} + \frac{1}{2} S_{osc} \right) = 0 \quad (\ldots (16)
\]

Hence, the two equations of equilibrium and compatibility equation for the face layer in terms of face and core
stresses become, respectively:

\[
\frac{\partial}{\partial \sigma}\left(\frac{N_{\sigma x f} + \frac{1}{2} N_{\sigma y c}}{2} + \frac{1}{\sigma} \partial\left(S_{\sigma y f} + \frac{1}{2} S_{\sigma y c}\right)\right) + \frac{\partial}{\partial \theta}\left(\frac{N_{\sigma y f} + \frac{1}{2} N_{\sigma x c}}{2}\right) - \frac{\partial}{\partial \sigma}\left(N_{\sigma y f} + \frac{1}{2} N_{\sigma x c}\right) = 0 \quad \text{[15]}
\]

\[
\frac{1}{2} \frac{\partial}{\partial \theta}\left(N_{\sigma y f} + \frac{1}{2} N_{\sigma x c}\right) + \frac{\partial}{\partial \sigma}\left(S_{\sigma y f} + \frac{1}{2} S_{\sigma y c}\right) + \frac{\partial}{\partial \sigma}\left(N_{\sigma y f} + \frac{1}{2} N_{\sigma x c}\right) = 0 \quad \text{[16]}
\]

and,

\[
\frac{\partial^2}{\partial \sigma^2}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) + \frac{1}{\sigma^2} \frac{\partial^2}{\partial \theta^2}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) + \frac{\partial}{\partial \theta}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) - \frac{\partial}{\partial \sigma}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) = 0
\]

\[
= 2(1 + \nu) \frac{1}{k^2} \frac{\partial^2}{\partial \sigma^2} \frac{\partial}{\partial \theta}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) + 2(1 + \nu) \frac{1}{k^2} \frac{\partial}{\partial \theta}\left(N_{\sigma y f} - \frac{1}{2} N_{\sigma x f}\right) \quad \text{[16]}
\]

Assuming strains are equal at the interface between core and face, one can write in \( r \)-direction, \( E_c = E_r \).

i.e.,

\[
\frac{1}{E_c} \left(N_{\sigma x e} - \nu_c N_{\sigma y e}\right) = \frac{1}{E_r} \left(N_{\sigma x f} - \nu \frac{1}{2} N_{\sigma y f}\right)
\]

or,

\[
N_{\sigma x e} - \nu_c N_{\sigma y e} = \frac{E_c}{E_r} \frac{k}{t} \left(N_{\sigma x f} - \nu \frac{1}{2} N_{\sigma y f}\right) \quad \text{(17.1)}
\]
Again, assuming that $\varepsilon_{\text{soc}} = \varepsilon_{\text{sof}}$ (in the $\theta$-direction)
i.e.,
$$\frac{1}{\varepsilon_{\text{c}}} (N_{\text{soc}} - V N_{\text{sr}}) = \frac{1}{E_{\text{f}}} (N_{\text{sof}} - V_{\text{f}} N_{\text{sr}})$$
or,
$$N_{\text{soc}} - V_{\text{c}} N_{\text{sr}} = \frac{E_{\text{c}}}{E_{\text{f}}} \left( N_{\text{sof}} - V_{\text{f}} N_{\text{sr}} \right) \quad \cdots \cdots \quad (17. b)$$

Again, assuming that $\gamma_{\text{soc}} = \gamma_{\text{sof}}$, then
$$\frac{S_{\text{soc}}}{E_{\text{c}}} = \frac{2}{E_{\text{f}}} \left( 1 + \frac{V_{\text{f}}}{E_{\text{f}}} \right) S_{\text{sof}} \quad \cdots \cdots \quad (17. c)$$

Multiplying equation (17.b) by $V_{\text{c}}$ and adding equation (17.a),
this becomes:
$$N_{\text{sr}} (1 - V_{\text{c}}^2) = \frac{E_{\text{c}}}{E_{\text{f}}} \left[ V_{\text{c}} N_{\text{sr}} - V_{\text{c}} V_{\text{f}} N_{\text{sr}} + V_{\text{c}} N_{\text{sof}} - V_{\text{f}} N_{\text{sof}} \right]$$

Therefore,
$$N_{\text{sr}} = D_{1} \left[ N_{\text{sr}} - V_{\text{c}} V_{\text{f}} N_{\text{sr}} + V_{\text{c}} N_{\text{sof}} - V_{\text{f}} N_{\text{sof}} \right] \quad \cdots \cdots \quad (18)$$

where
$$D_{1} = \frac{1}{1 - V_{\text{c}}^2} \cdot \frac{E_{\text{c}}}{E_{\text{f}}} \cdot \frac{h}{t}$$
Multiplying equation (17.a) by $\nu_c$ and adding equation (17.b), this becomes:

$$N_{ooc} (1 - \nu_c^2) = \frac{E_c}{E_p} \frac{h}{t} \left( N_{oof} - \nu_c \frac{N_{oof}}{E_p} + \nu_c \frac{N_{oof}}{E_p} - \nu_c \frac{N_{oof}}{E_p} \right)$$

Therefore,

$$N_{ooc} = D_1 \left[ N_{oof} - \nu_c \frac{N_{oof}}{E_p} + \nu_c \frac{N_{oof}}{E_p} - \nu_c \frac{N_{oof}}{E_p} \right] \quad \ldots \quad (19)$$

From equation (17.c)

$$\dot{S}_{ooc} = \frac{E_c}{E_p} \frac{h}{t} \left( 1 + \nu_c \right) S_{oof}$$

Therefore,

$$\dot{S}_{ooc} = D_2 \dot{S}_{oof} \quad \ldots \quad \ldots \quad (20)$$

where

$$D_1 = \frac{1}{1 - \nu_c^2} \frac{E_c}{E_p} \frac{h}{t} \quad \ldots \quad (a)$$

and

$$D_2 = \left( 1 + \nu_c \right) \frac{E_c}{E_p} \frac{h}{t} \quad \ldots \quad (b)$$
Using equations (18), (19) and (20); equation (15) can be written as:

\[
\left\{ \begin{array}{l}
(1 + \frac{1}{2} D_i - \frac{1}{2} D_i \frac{U_c U_f}{c} ) \frac{\partial N_{x rf}}{\partial \eta} + \frac{1}{2} D_i (U_c - U_f) \frac{\partial N_{oof}}{\partial \eta} \\
+ (1 + \frac{1}{2} D_e) \frac{1}{\eta} \frac{\partial S_{x rf}}{\partial \eta} \\
+ (1 + \frac{1}{2} D_i - \frac{1}{2} D_i \frac{U_c U_f}{c} - \frac{1}{2} D_i \frac{U_c + U_f}{c} ) \frac{\partial (N_{x rf} - N_{oof})}{\partial \eta} = 0
\end{array} \right. 
\]

Or,

\[
\left\{ \begin{array}{l}
(2 + D_i - D_i \frac{U_c U_f}{c} ) \frac{\partial N_{x rf}}{\partial \eta} + D_i (U_c - U_f) \frac{\partial N_{oof}}{\partial \eta} \\
+ (2 + D_e) \frac{1}{\eta} \frac{\partial S_{x rf}}{\partial \eta} \\
+ (2 + D_i - D_i \frac{U_c U_f}{c} - D_i \frac{U_c + D_i U_f}{c} ) \frac{\partial (N_{x rf} - N_{oof})}{\partial \eta} = 0 
\end{array} \right. 
\]  \quad (2.7)

With equations (18), (19) and (20), equation (16) can be written as:

\[
\left\{ \begin{array}{l}
\frac{1}{\eta} \frac{\partial \sqrt{N_{oof} + \frac{1}{2} D_i (N_{oof} \frac{U_c U_f}{c} + U_c N_{x rf} - U_f N_{x rf})}}{\partial \eta} \\
+ \frac{\partial [S_{x rf} + \frac{1}{2} D_e S_{oof}]}{\partial \eta} \\
+ \frac{2}{\eta} [S_{x rf} + \frac{1}{2} D_e S_{oof}] \\
\end{array} \right. 
\]  \quad (2.8)
Multiplying by two and rearranging, this becomes:

\[
\begin{align*}
(z + D_1 - D_f \frac{V_c}{V_f}) \frac{1}{\sigma} \frac{\partial N_{sof}}{\partial \sigma} + D_1 \left( V_c - V_f \right) \frac{1}{\sigma} \frac{\partial N_{x rf}}{\partial \sigma} \\
+ \left( z + D_2 \right) \frac{\partial S_{sof}}{\partial \sigma} + \left( z + D_2 \right) \frac{2 \cdot S_{sof}}{\sigma}
\end{align*}
\]

\[= 0. \quad (2.3)\]

Equation (22) may be written as:

\[
\begin{align*}
K_1 \frac{\partial N_{sof}}{\partial \sigma} + K_2 \frac{\partial N_{sof}}{\partial \sigma} + K_3 \frac{1}{\sigma} \frac{\partial S_{sof}}{\partial \sigma}
+ (K_1 - K_2) \frac{(N_{x rf} - N_{sof})}{\sigma}
\end{align*}
\]

\[= 0. \quad (2.4)\]

and, equation (23) as:

\[
\begin{align*}
K_1 \frac{1}{\sigma} \frac{\partial N_{sof}}{\partial \sigma} + K_2 \frac{1}{\sigma} \frac{\partial N_{sof}}{\partial \sigma} + K_3 \frac{\partial S_{sof}}{\partial \sigma}
+ K_3 \frac{2 \cdot S_{sof}}{\sigma}
\end{align*}
\]

\[= 0. \quad (2.5)\]

in which:

\[
\begin{align*}
K_1 &= z + D_1 - D_f \left( \frac{V_c}{V_f} \right) \\
K_2 &= D_1 \left( \frac{V_c}{V_f} \right) \\
K_3 &= z + D_2
\end{align*}
\]

\[= (2.6)\]

It can be readily shown from equations (21) and (26) that:

\[
\frac{K_1 - K_2}{K_3} = 1 \quad (2.6a)
\]
Rearranging equations (24) and (25) yields:

\[
\frac{\partial}{\partial \sigma} \left( k_1 N_{\sigma \rho} + k_2 N_{\theta \phi} \right) + \frac{1}{\sigma} \frac{\partial}{\partial \theta} \left( k_3 S_{\rho \phi} \right) + \frac{(k_1 - k_2)}{\sigma} \left( N_{\sigma \rho} - N_{\theta \phi} \right) = 0 \tag{27}
\]

and,

\[
\frac{1}{\sigma} \frac{\partial}{\partial \theta} \left( k_1 N_{\theta \phi} + k_2 N_{\sigma \rho} \right) + \frac{\partial}{\partial \sigma} \left( k_3 S_{\rho \phi} \right) + \frac{2}{\sigma} \left( k_3 S_{\theta \phi} \right) = 0 \tag{28}
\]

Let,

\[
\sigma_1 = k_1 N_{\sigma \rho} + k_2 N_{\theta \phi} = \frac{1}{\sigma} \frac{\partial \phi}{\partial x} + \frac{1}{\sigma^2} \frac{\partial^2 \phi}{\partial y^2} \tag{29}
\]

\[
\sigma_2 = k_1 N_{\theta \phi} + k_2 N_{\sigma \rho} = \frac{\partial^2 \phi}{\partial x^2} \tag{30}
\]

\[
T = k_3 S_{\rho \phi} = \frac{1}{\sigma} \frac{\partial \phi}{\partial x} - \frac{1}{\sigma} \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial}{\partial y} \left( \frac{1}{\sigma \theta} \frac{\partial \phi}{\partial x} \right) \tag{31}
\]

Solving for \( N_{\sigma \rho} \) and \( N_{\theta \phi} \) from equations (29) and (30) yields:

\[
N_{\theta \phi} = \frac{1}{(k_1 - k_2)} \left( k_1 \frac{1}{\sigma} \frac{\partial \phi}{\partial x} + k_2 \frac{1}{\sigma^2} \frac{\partial^2 \phi}{\partial x^2} - k_2 \frac{\partial^2 \phi}{\partial y^2} \right) \tag{32}
\]
and,
\[ N_{\theta \phi} = - \frac{1}{(k_1 - k_2^2)} \left( k_1 \frac{\partial \phi}{\partial \gamma} + k_2 \frac{\partial^2 \phi}{\partial \gamma^2} + \kappa_1 \gamma \frac{\partial \phi}{\partial \gamma} \right). \tag{33} \]

From equation (31):
\[ S_{\theta \phi} = \frac{1}{k_3} \left( \frac{1}{\gamma^2} \frac{\partial \phi}{\partial \phi} - \frac{1}{\gamma} \frac{\partial^2 \phi}{\partial \gamma \partial \phi} \right) = - \frac{1}{k_3} \frac{\partial}{\partial \gamma} \left( \frac{1}{\gamma} \frac{\partial \phi}{\partial \phi} \right). \tag{34} \]

It can be readily shown that equations (27) and (28) will be identically satisfied by the function \( \phi \) as defined by equations (32), (33), and (34).

From equations (29) and (30):
\[ \sigma_{ij} - \sigma_{ij}^0 = N_{\theta \phi} \left( k_1 - \phi k_z \right) + N_{\gamma \phi} \left( k_2 - \phi k_z \right) \]
\[ = (k_1 - \phi k_z) \left[ N_{\theta \phi} + \frac{k_2 - \phi k_z}{k_1 - \phi k_z} \cdot N_{\gamma \phi} \right] \]

Let \( \nu = \frac{k_2 + \nu \frac{k_1}{k_1 + \nu k_z}} {k_1 + \nu k_z} \)

Then,
\[ \sigma_{ij} - \sigma_{ij}^0 = (k_1 - \phi k_z) \left[ N_{\theta \phi} - \frac{\nu}{\nu} N_{\gamma \phi} \right] \]

Therefore,
\[ N_{\theta \phi} - \frac{\nu}{\nu} N_{\gamma \phi} = \frac{1}{k_1 - \phi k_z} (\sigma_{ij} - \sigma_{ij}^0) \tag{35} \]
Similarly, from equations (29) and (30), one can write:

\[
N_{\tau_{\rho}} - \nu N_{\phi} = \frac{1}{\kappa_1 - \nu \kappa_2} (\sigma_1 - \nu \sigma_2) \quad \ldots \ldots \quad (36)
\]

where

\[
\nu = \frac{K_2 + \nu K_1}{K_1 + \nu^2 K_2} \quad \ldots \ldots \quad (37)
\]

From equation (31):

\[
S_{\phi \phi} = \frac{T}{\kappa_3}
\]

Multiplying this by \(2(1+\nu)\) and substituting the value of \(\nu\) from equation (37) and rearranging; and after using equation (26)a, the above equation becomes:

\[
2(1+\nu) S_{\phi \phi} = \frac{1}{\kappa_1 - \nu \kappa_2} \cdot 2(1+\nu) T \quad \ldots \ldots \quad (38)
\]

By examining equations (35), (36), (38) and (6); one can observe that equation (6) is identical to the compatibility equation (4.22) of Wang.

In terms of the stress function \(\phi\) as defined by equations (32), (33), and (34), the compatibility equation (6) becomes:

\[
\left(\frac{\partial^2}{\partial x^2} + \frac{1}{\gamma} \frac{\partial}{\partial x} + \frac{1}{\gamma^2} \frac{\partial^2}{\partial \gamma^2}\right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{\gamma} \frac{\partial \phi}{\partial x} + \frac{1}{\gamma^2} \frac{\partial^2 \phi}{\partial \gamma^2}\right) = 0 \quad (39)
\]
FIG. 2.3 CIRCULAR RING OF RADIUS 'b' OF SANDWICH PLATE SUBJECTED TO UNIFORM NORMAL FORCE ACTING AROUND THE OUTSIDE OF THE RING
Consider a circular sandwich plate of radius "b" with a small circular hole of radius "a" at center. Figure 2.3. If the sandwich plate is subjected to uniform normal force on the outer boundary in the radial direction, the force carried by the face can be found as follows:

$$
\sigma_{x} = \sigma_{o_{x}} + 2t + \sigma_{o_{c_{x}}} \cdot h
$$

From equation (18):

$$
\sigma_{o_{x}} = \frac{1}{1 - \frac{v_{c}}{v_{t}}} \frac{E_{c}}{E_{f}} \left[ \sigma_{o_{x}} \left( 1 - \frac{v_{c}}{v_{t}} \right) + \sigma_{o_{x}} \left( \frac{v_{c}}{v_{t}} \right) \right]
$$

(41)

Since the external force is applied only in the radial direction, equation (41) becomes, at the boundary:

$$
\sigma_{x_{c}} = \frac{1}{1 - \frac{v_{c}}{v_{t}}} \frac{E_{c}}{E_{f}} \sigma_{o_{x}}
$$

(42)

I.e. $$\sigma_{x_{c}} = c \cdot \sigma_{o_{x}}$$ at the boundary.

(43)

Where

$$
c = \frac{1 - \frac{v_{c}}{v_{t}}}{1 - \frac{v_{c}}{v_{t}}} \frac{E_{c}}{E_{f}}
$$

(44)
Or, by defining \( \overline{\sigma_{cc \gamma}} = \overline{\sigma_{rr \gamma}} \) and \( \overline{\sigma_{0f \gamma}} = \overline{\sigma_{xx \gamma}} \) at the boundary, one could write by virtue of equation (43):

\[
\overline{\sigma_{cc \gamma}} = C. \overline{\sigma_{0f \gamma}} \quad \cdots \cdots \quad (4-5)
\]

Substituting equation (45) in equation (40):

\[
\overline{P_\gamma} = \overline{\sigma_{0f \gamma}} \cdot 2 \ell + C. \overline{\sigma_{0f \gamma}} \cdot h \quad \cdots \cdots \quad (4-6)
\]

\[
= \overline{\sigma_{0f \gamma}} \ (2 \ell + Ch)
\]

Therefore,

\[
\overline{\sigma_{0f \gamma}} = \frac{\overline{P_\gamma}}{(2 \ell + Ch)} \quad \cdots \cdots \quad (4-7)
\]

The stress distribution is symmetrical with respect to the axis through the center of the plate perpendicular to the XY-plane. Thus the stress function \( \Phi \) does not depend on \( \Theta \) and is a function of \( r \) only.

Hence, the compatibility equation (39) becomes:

\[
\left( \frac{d^2}{d \gamma^2} + \frac{1}{\gamma} \frac{d}{d \gamma} \right) \left( \frac{d^2 \Phi}{d \gamma^2} + \frac{1}{\gamma} \frac{d \Phi}{d \gamma} \right) = 0
\]
\[ \frac{d^4 \phi}{d \gamma^4} + \frac{2}{\gamma} \frac{d^3 \phi}{d \gamma^3} - \frac{1}{\gamma^2} \frac{d^2 \phi}{d \gamma^2} + \frac{1}{\gamma^3} \frac{d \phi}{d \gamma} = 0 \quad \ldots \quad (28) \]

This is a homogeneous linear differential equation.

The general solution is:

\[ \phi = C_1 \gamma^2 \log \gamma + C_2 \gamma^2 + C_3 \log \gamma + C_4 \quad \ldots \quad (29) \]

where \( C_1, C_2, C_3, \) and \( C_4 \) are constants of integration.

From equation (32), the stress component:

\[ \sigma_{\phi\phi} = \frac{1}{t(K_1^2 - K_2^2)} \left( K_1 \frac{1}{\gamma} \frac{d \phi}{d \gamma} - K_2 \frac{d^2 \phi}{d \gamma^2} \right) \]

Or,

\[ \sigma_{\phi\phi} = \frac{1}{t(K_1^2 - K_2^2)} \left[ C_1(K_1 - 2K_2 \log \gamma - 3K_2 - 2K_2 \log \gamma) \right] + 2 \frac{C_3}{\gamma} (K_1 - K_2) + \frac{C_3}{\gamma^2} (K_1 + K_2) \quad \ldots \quad (30) \]

From equation (33), the stress component:

\[ \sigma_{\phi\phi} = -\frac{1}{t(K_1^2 - K_2^2)} \left( K_1 \frac{1}{\gamma} \frac{d \phi}{d \gamma} - K_2 \frac{d^2 \phi}{d \gamma^2} \right) \]

Or,
\begin{equation}
\frac{\sigma}{\theta_0} = - \frac{1}{t(k_1^2 - k_2^2)} \left[ c_1(k_1 + 2k_1 \log \frac{x}{a} - 3k_2 - 2k_2 \log \frac{x}{a}) \right. \\
\left. + 2c_2(k_1 - k_2) + \frac{c_3}{x^2}(k_1 + k_2) \right] 
\end{equation} \text{ (51)}

From equation (34):

\begin{equation}
T_{\gamma_0, \phi} = 0 
\end{equation} \text{ (52)}

The boundary conditions are,

\begin{equation}
\frac{\sigma}{\gamma_0} = 0 \text{ at } \gamma = a 
\end{equation} \text{ (a)}

and referring to equation (47):

\begin{equation}
\frac{\sigma}{\phi_0} = \frac{P_x}{(2t + c_1)} \text{ at } \gamma = b 
\end{equation} \text{ (b)}

Substituting the boundary conditions (53) in equation (50) yields,

\begin{equation}
\begin{aligned}
c_1(k_1 + 2k_1 \log \frac{a}{x} - 3k_2 - 2k_2 \log \frac{a}{x}) \\
+ 2c_2(k_1 - k_2) + \frac{c_3}{a^2}(k_1 + k_2) &= 0 
\end{aligned} 
\end{equation} \text{ (54)}

\begin{equation}
\begin{aligned}
c_1(k_1 + 2k_1 \log \frac{b}{x} - 3k_2 - 2k_2 \log \frac{b}{x}) \\
+ 2c_2(k_1 - k_2) + \frac{c_3}{b^2}(k_1 + k_2) &= \frac{t(k_1^2 - k_2^2)P_x}{(2t + c_1)} 
\end{aligned} 
\end{equation} \text{ (55)}
Since there is no external load applied at the boundary of the hole, the third boundary condition becomes:

\[ T_{z_a} = 0 \]  \hspace{1cm} (5.6)

Since \( T_{z_a} = T_{z_r} \), equation (13) can be written as:

\[ 2 T_{z_a} = -\frac{\partial N_{z_a}}{\partial \theta} - \frac{1}{\gamma} \frac{\partial S_{z_a}}{\partial \theta} \left( \frac{N_{z_a} - N_{0a}}{\gamma} \right) \]

Using the relations (18), (19) and (20), and rearranging this becomes:

\[ -2 T_{z_a} = D_{1} \frac{\partial (N_{z_a})}{\partial \gamma}(1 - \nu_c \nu_f) + D_{1} \frac{\partial (N_{0a})}{\partial \gamma} (\nu - \nu_f) \]

\[ + \frac{D_{2}}{\gamma} \frac{1}{\partial \theta} \frac{\partial (S_{z_a})}{\partial \theta} \]

\[ + \frac{D_{2}}{\gamma} \left( 1 - \nu_c \nu_f - \nu_c + \nu_f \right) (N_{z_a} - N_{0a}) \]
Making use of equations (32), (33) and (34), and rearranging the above equation, becomes:

\[-2(k_1^2 - k_2^2)I_{x y} = D_1(I - V \cdot U_f)\left[\frac{1}{r} \frac{\partial}{\partial r} \frac{\partial \phi}{\partial r} - k_1 \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + k_1 \frac{1}{r^3} \frac{\partial^3 \phi}{\partial r \partial \theta^2}\right]
\]

\[-k_2 \frac{1}{r^2} \frac{\partial \phi}{\partial r} + k_2 \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} - k_2 \frac{1}{r^2} \frac{\partial^2 \phi}{\partial r \partial \theta}\]

\[-k_2 \frac{1}{r^2} \frac{\partial \phi}{\partial r} + \frac{k_2 - k_1}{k_2} D_2 \left[\frac{1}{r^2} \frac{\partial \phi}{\partial \theta} - \frac{1}{r} \frac{\partial^2 \phi}{\partial r \partial \theta}\right]\]

\[+ D_1(I - V \cdot U_f) - D_1(U_c - U_f)\left[\frac{1}{r} \frac{\partial}{\partial r} \frac{\partial \phi}{\partial r} - k_1 \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2}\right]\]

\[+ k_1 \frac{1}{r^2} \frac{\partial \phi}{\partial r} - k_2 \frac{2}{r^2} \frac{\partial \phi}{\partial r} + k_2 \frac{1}{r} \frac{\partial \phi}{\partial \theta}\]

\[= \left(\frac{k_1^2 - k_2^2}{k_3}\right) = \left(k_1 - k_2\right)\left(k_1 + k_2\right) = k_1 + k_2 \quad \text{(a)}\]

From equation (26a), it can be shown that:

\[\frac{k_1^2 - k_2^2}{k_3} = \frac{(k_1 - k_2)(k_1 + k_2)}{k_3} = k_1 - k_2 \quad \text{(b)}\]

From equation (26b), one can find that:

\[D_1(I - V \cdot U_f) = k_1 - 2 \quad \text{(c)}\]

\[D_1(U_c - U_f) = k_2 \quad \text{(d)}\]

\[D_2 = k_3 - 2 \quad \text{(e)}\]

(57)

(58)
Substituting the relations (53) in equation (57), and rearranging, yields:

\[
-2\left(k_i - k_z\right) T = \frac{1}{\gamma} \frac{\partial^2 \phi}{\partial z^2} \cdot z_k - \frac{1}{\gamma} \frac{3}{\delta^2} \frac{\partial^2 \phi}{\partial \delta^2} (k_i - k_z - k_i k_2 - k_z k_3 + 4 k_e)
\]

\[
- \frac{1}{\gamma} \frac{\partial \phi}{\partial z} \cdot z_k + \frac{3}{\delta^2} (2 k_e)
\]

\[
+ \frac{1}{\gamma} \frac{3}{\delta^2} \frac{\partial^2 \phi}{\partial \xi \partial \delta} (k_i - k_z - k_i k_2 - k_z k_3 + 2 k_e)
\]

It can be shown that

\[
k_i - k_z - k_i k_2 - k_z k_3 = 0.
\]

Using this fact, it follows that:

\[
T \cdot \frac{z \gamma^2}{k_i - k_z} = -\frac{k_e}{k_i - k_z} \left[ \frac{1}{\gamma} \frac{\partial \phi}{\partial \xi} - 2 \frac{1}{\delta^2} \frac{\partial^2 \phi}{\partial \delta^2} \right.
\]

\[
- \frac{1}{\gamma} \frac{\partial \phi}{\partial z} + \frac{1}{\delta^2} \frac{\partial^2 \phi}{\partial \xi \partial \delta} + \frac{3}{\delta^2} \frac{\partial^2 \phi}{\partial \delta^2}
\]

\[
\left. \right] \quad (59)
\]

In the analysis of the circular panel with axial symmetry \( \phi \) is a function of \( r \) only.

Hence,

\[
T \cdot \frac{z \gamma^2}{k_i - k_z} = -\frac{k_e}{k_i - k_z} \left[ \frac{1}{\gamma} \frac{\partial \phi}{\partial \xi} - \frac{1}{\delta^2} \frac{\partial \phi}{\partial \delta} + \frac{3}{\delta^2} \frac{\partial^3 \phi}{\partial \delta^3} \right] \quad (60)
\]
which reduces to:

\[ T_{zxf} = - \frac{k_z}{k_1^2 - k_z^2} \left[ \frac{4C_1}{\gamma} \right] \quad \text{(61)} \]

From the boundary condition (56) and from equation (61):

\[ C_1 = 0 \quad \text{(62)} \]

Hence equations (54) and (55) can be written as:

\[ 2C_z(k_1 - k_z) + \frac{C_3}{a^2} (k_1 + k_z) = 0 \]

\[ 2C_z(k_1 - k_z) + \frac{C_3}{b^2} (k_1 + k_z) = -\frac{t(k_1^2 - k_z^2)P_x}{(2t + ch)} \]

Solving these two equations,

\[ C_3 = -\frac{a^2 b^2}{(b^2 - a^2)} \frac{t(k_1 - k_z)P_x}{(2t + ch)} \quad \text{(63)} \]

and:

\[ 2C_z = +\frac{b^2}{b^2 - a^2} \frac{t(k_1 + k_z)P_x}{(2t + ch)} \quad \text{(64)} \]
Substituting the values of constants \( C_1, C_2 \) and \( C_3 \), the stress components become:

\[
\begin{align*}
\sigma_{xx} &= \frac{b^2}{\sigma^2} \frac{P_x}{(2t+ch)} \frac{x^2-a^2}{b^2-a^2} \\
\sigma_{\theta \theta} &= \frac{b^2}{\sigma^2} \frac{P_{\theta}}{(2t+ch)} \frac{x^2+a^2}{b^2-a^2}
\end{align*}
\]

and,

\[\tau_{\phi x} = 0\]
FIG. 2.4 FLAT SANDWICH PLATE WITH CIRCULAR HOLE SUBJECTED TO UNIFORM TENSION IN X-DIRECTION
RECTANGULAR SANDWICH PLATE

Figure 2.4 represents a sandwich plate of thickness \(h+2t\), submitted to a uniform tension of magnitude \(P_x\) in the \(x\)-direction. The tension load is uniform across the thickness of sandwich plate.

If core and faces are well bonded together in such a way that no relative movement occurs between them, and if an external load is then applied to such a plate, the loads carried by the face and core can be determined from considerations of elasticity as follows:

Strains in the core in the \(x\) and \(y\) directions are:

\[
\begin{align*}
\varepsilon_{xc} &= \frac{1}{E_c} \left( \sigma_{xc} - \nu_c \sigma_{yc} \right) \\
\varepsilon_{yc} &= \frac{1}{E_c} \left( \sigma_{yc} - \nu_c \sigma_{xc} \right)
\end{align*}
\]  

\( (66) \)

Strains in the face are:

\[
\begin{align*}
\varepsilon_{xf} &= \frac{1}{E_f} \left( \sigma_{xf} - \nu_f \sigma_{xf} \right) \\
\varepsilon_{yf} &= \frac{1}{E_f} \left( \sigma_{yf} - \nu_f \sigma_{xf} \right)
\end{align*}
\]  

\( (67) \)
Since the faces and core are well bonded together, it is assumed that the strains along the contact surface are equal, i.e.:

\[
\begin{align*}
\epsilon_{xc} &= \epsilon_{xf} \quad (a) \\
\epsilon_{yc} &= \epsilon_{yf} \quad (b)
\end{align*}
\] (68)

Using equations (66) and (67), equation (68) can be written as:

\[
\sigma_{xc} - \nu \sigma_{yc} = \frac{E_c}{E_f} \left( \sigma_{xf} - \nu \sigma_{yf} \right) 
\] (69)

and,

\[
\sigma_{yc} - \nu \sigma_{xc} = \frac{E_c}{E_f} \left( \sigma_{yf} - \nu \sigma_{xf} \right) 
\] (70)

From equations (69) and (70):

\[
\sigma_{xc} = \frac{1}{(1 - \nu_c^2)} \frac{E_c}{E_f} \left( \sigma_{xf} - \nu \sigma_{yf} + \nu \sigma_{yf} - \nu \sigma_{xf} \right)
\]
If the plate is subjected to load in the x-direction only, at boundary, the above equation becomes:

\[
\overline{\sigma}_{\infty x} = \frac{1 - \nu_c^2 \nu_f^2}{1 - \nu_c^2} \frac{E_c}{E_f} \overline{\sigma}_{o f x}
\]

Where \( \overline{\sigma}_{\infty x} = \overline{\sigma}_{x c} \) at the boundary and \( \overline{\sigma}_{o f x} = \overline{\sigma}_{x f} \) at the boundary,
or,

\[
\overline{\sigma}_{\infty x} = K \overline{\sigma}_{o f x} \quad \cdots \cdots \cdots \cdots \cdots \quad (71)
\]

where \( K = \frac{2G_c}{E_f} \frac{1 - \nu_c \nu_f}{1 - \nu_c} \quad \cdots \cdots \cdots \cdots \cdots \quad (72) \)

Since the materials are well bonded together,

\[
\overline{P}_x = 2t \overline{\sigma}_{o f x} + h \overline{\sigma}_{\infty x}
\]

Referring to the relations (71) and (72), this can be written as:

\[
\overline{P}_x = 2t \overline{\sigma}_{o f x} + K \overline{\sigma}_{o f x} \cdot h
\]

Hence,

\[
\overline{\sigma}_{o f x} = \frac{\overline{P}_x}{2t + Kh} \quad \cdots \cdots \cdots \cdots \cdots \quad (73)
\]
If a small circular hole is made in the middle of the plate, the stress distribution in the neighborhood of the hole will be changed, but the change is negligible at distances which are large compared with the radius of the hole (Saint-Venant's principle). Thus, points at such distances may be regarded as at infinity.

Hence the stresses \( \sigma_{x=0} \), \( \sigma_{0y} \), and \( \tau_{x0} \) are found in the normal way for face layers around the hole. Also, by referring to equation (65).

\[
\sigma_{x=0} = \frac{\sigma_{0y}}{2} \left(1 - \frac{a^2}{x^2}\right) + \frac{\sigma_{0y}}{2} \left(1 + \frac{3a^2}{x^2} - \frac{4a^2}{y^2}\right) \cos 2\beta \quad (74)
\]

\[
\sigma_{0y} = \frac{\sigma_{0y}}{2} \left(1 + \frac{a^2}{y^2}\right) - \frac{\sigma_{0y}}{2} \left(1 + \frac{3a^2}{y^2}\right) \cos 2\beta \quad (75)
\]

\[
\tau_{x0} = -\frac{\sigma_{0y}}{2} \left(1 - \frac{3a^2}{x^2} + \frac{2a^2}{y^2}\right) \sin 2\beta \quad (76)
\]

Using equation (73), these become:

\[
\sigma_{x=0} = \frac{P_x}{(2t+kh)^2} \left(1 - \frac{a^2}{x^2}\right) + \frac{P_x}{(2t+kh)^2} \left(1 + \frac{3a^2}{x^2} - \frac{4a^2}{y^2}\right) \cos 2\beta
\]

\[
\sigma_{0y} = \frac{P_x}{(2t+kh)^2} \left(1 + \frac{a^2}{y^2}\right) - \frac{P_x}{(2t+kh)^2} \left(1 + \frac{3a^2}{y^2}\right) \cos 2\beta
\]

\[
\tau_{x0} = -\frac{P_x}{(2t+kh)^2} \left(1 - \frac{3a^2}{x^2} + \frac{2a^2}{y^2}\right) \sin 2\beta
\]
If \( P \) is the total load applied to sandwich plate in the \( x \)-direction, then rearranging these equations:

\[
\sigma_{xx} = \frac{P}{ab(2t + kh)} \left[ (1 - \rho^2) + \left( 1 + 3\rho^2 - 4\rho^4 \right) \cos 2\theta \right] \quad (71)
\]

\[
\sigma_{yy} = \frac{P}{ab(2t + kh)} \left[ (1 + \rho^2) - \left( 1 + 3\rho^2 \right) \cos 2\theta \right] \quad (78)
\]

\[
\tau_{xy} = -\frac{P}{ab(2t + kh)} \left[ (1 - 3\rho^2 + 2\rho^4) \sin 2\theta \right] \quad (79)
\]
CHAPTER III

EXPERIMENTAL STUDY

3.1 Materials and Apparatus.

In order to verify the theory and calculations, an experimental study was performed on different models. The plates were 28 inches x 10 inches but with different hole diameters. All plates were made of 0.025 inch thick aluminum facings with 3/4 inch plywood core, but one plate consisted of a 1/2 inch thick plywood core. Six plates of different hole diameter to width ratio were tested.

Six strain rosette gages, three on one face and three on the other face were installed symmetrically on each plate. In one plate, four linear gages around the hole and six strain rosette gages on a 45° line, three on one side of hole and three on the other side of the hole on the same face were installed. Strain gages were installed symmetrically to each other to minimize the error in applying the axial load or in effecting a uniform distribution of load throughout the width of the plate. All rosette gages were of the three-gage 45° rectangular type. A typical tension-test arrangement is
FIG. 3.1 TYPICAL TENSION ARRANGEMENT
shown in figure 3.1.

3.2 Experimental Procedure.

Structural adhesive EC-2216 A and EC-2216 B were mixed in 3:2 ratio and then applied on both sides of plywood core and on the inside surface of aluminum facings. Plywood was placed in between the two aluminum facings and uniform pressure was applied and kept overnight. The sandwich plate was cut to the required dimension and a center line, a 45° line, a 90° line and points for installation of strain gages were marked. A hole of the required diameter was drilled at the center of the plate.

The plate was cleaned and the gage locations on surfaces of the plate were marked precisely and the strain gages were installed following the standard procedure.

The sandwich plate was initially loaded and reloaded to a small load to get a uniform alignment of the bolts. All the gages were "zeroed in" before loading of each plate and the zero readings were recorded for the strains. For each increment of loading similar readings were recorded. The average of the loading was computed and the strain readings were tabulated. Three strain values $\varepsilon_A$, $\varepsilon_B$, and $\varepsilon_C$ were obtained from each rosette gage. From these values, face stresses were calculated by the following formulas:
\[ \sigma_{\tau \tau} = \frac{E_p}{1 - \nu f} \left[ \varepsilon_B (1 - \nu) + \left( \varepsilon_A + \varepsilon_c \right) \nu f \right] \]  \hspace{1cm} (8.0)  \\
\[ \sigma_{\theta \theta} = \frac{E_p}{1 - \nu f} \left[ (\varepsilon_A + \varepsilon_c) - \varepsilon_B (1 - \nu) \right] \]  \hspace{1cm} (8.1)  \\
\[ \tau_{\tau \theta} = \frac{E_p}{2(1 + \nu_f)} \left[ \varepsilon_A - \varepsilon_c \right] \]  \hspace{1cm} (8.2)
4.1 DISCUSSION OF RESULTS

Equations (77), (78), and (79) give the polar components of stress at any point in the plate defined by the coordinate system \( r \) and \( \theta \). Sample program and calculations in Appendix A.

Poisson’s ratio of plywood was found by conducting tension tests on a number of specimens. Douglas Fir Plywood consists of built-up panels of veneers in which the grain of each ply is at right angles to the one adjacent to it. Each panel is constructed with an odd number of plies balanced as to grain direction and thickness about the central ply. The lateral and longitudinal strains of 10 specimens were obtained and averaged. A plot of the averaged values is shown in figure 4.1. Thus Poisson’s ratio of the plywood used in the experiment was calculated to be 0.196 ± 0.002.

The theoretical distribution of the ratios \( \frac{\sigma_{rad}}{\sigma_{opx}} \) and \( \frac{\sigma_{rad}}{\sigma_{opx}} \) are shown as a function of position along the 90° line in figures 4.2 to 4.5 for different ratios of \( \lambda \). An examination of these values clearly indicates that the presence of the hole in a sandwich plate under uniaxial tension increases the \( \sigma_{rad} \) stresses by a factor of approximately 15, that is, in this particular case the stress concentration is approximately 15. A prior work of Hicks suggests that the maximum stress in the isotropic
plate occurs around the hole on a diameter perpendicular to the direction of the applied stress and the stress concentration is 3. The present theoretical solution of maximum stress around holes in the same direction as in the isotropic case is approximately fifteen times the applied stress. Hence a comparison of stress concentration around holes in sandwich plates with that of an isotropic plate shows that this value in the former case is five times the later case. Experimental values are also plotted in figures 4.2 to 4.5. From these figures, one can conclude that, in general, the percentage error between the experimental and theoretical results increases as \( \lambda \) decreases.

From Appendix A, one can observe that \( \sigma_{\theta\phi} \) is greatest when \( \theta = \pi/2 \) or \( 3\pi/2 \), i.e., at the edges of the hole on the line perpendicular to the direction of the load. This was verified experimentally and found that the point of maximum stress occurs at the edges of the hole as seen in figure 4.15. It is observed from figures 4.2 to 4.5 and 4.7 that shear stresses \( \tau_{\theta\phi} \) are present. However, theoretically these stresses should not exist. This discrepancy is due to improper orientation of the strain gages along the 90° line. It can be appreciated that any small misalignment will lead to this error.

From Appendix A, it can be seen that the ratio \( \sigma_{\theta\phi} / \sigma_{\phi\phi} \) at the boundary of hole is equal to
approximately -5 when $\Theta = 0^\circ$. Thus, the influence of the hole not only produces a concentration of stresses but also a change in the sign of the stresses, as in the case of the homogeneous plate with circular hole.

The distribution of the ratio $\sigma_{op}/\sigma_{op}$ around the hole is also plotted and compared to the experimental values in figure 4.8. It can be seen that the maximum value of the ratio occurs at the hole on the y-axis while the minimum value occurs at the hole on the x-axis.

4.2. BEHAVIOR OF SANDWICH PANEL AND PLYWOOD PANEL UNDER TENSILE LOAD (FIGURE 4.13 to 4.23).

In the sandwich panel failure occurs more or less in the core and in the faces simultaneously but the core gets weaker first. The bond between core and faces is perfect until failure occurs. That is, the facings and core are well bonded together in such a way that no relative movement occurs between them. It was observed that in the case of the plywood plate, the failure does not occur at one section, whereas in the case of the sandwich plate it does.

4.3. LIMITATIONS.

The theory is derived for the stresses around circular holes in rectangular and circular sandwich plates.
The theory is derived for an infinitely large plate. This solution is satisfactory, for a plate of finite size provided the width of the plate is not less than, approximately five times the diameter of the hole. The hole is located at the center of the plate. The rectangular sandwich plate is subjected to uniaxial tensile load and the circular sandwich plate is subjected to uniform all-around in-plane tension on the outer boundary.

The plate is a composite plate consisting of two thin faces and a thick core. The core and facings are considered isotropic and the facings are of equal thickness and of same material. The face-parallel stresses in the core are not negligible.

Attention is restricted to linear theories of thin flat plates under static loading. Facings and core are treated by classical thin-plate theory.

Using plywood as a core material for experimental study of the problem, the value of $G$ depends upon the number of layers and the direction of applied load. The theoretical results are placed by using $G=E/2(1+\nu)$. However the other values of $G$ in the range of $E/6$ to $E/10$ were also used in the theoretical calculation.
The results obtained from these latter values of G (not shown plotted) were closer to the experimental values when compared to the results by using value of $G = \frac{E}{2(1+\nu)}$. It should be noted that the theoretical results obtained by treating plywood as isotropic on one hand and the results treating it as non-homogeneous material with G value $E/6$ to $E/10$ were sensibly same. The behaviour of plywood as a core material can be approximated to be isotropic and homogeneous. Experimental results justifies this approximation.

4.4. SOURCE OF ERRORS

The theoretical values calculated from the present study are in fair agreement with the test results. The sources of error in the experiment may include one or more of the following:

(a) Irregular, non-uniform and improper bonding of the faces with core.

(b) Inaccuracy in making the center lines and marking the points for installation of strain gages.

(c) Inaccuracy in drilling the holes.

(d) Stresses induced by drilling holes.

(e) Inaccuracy in orientation of strain gages.

(f) The difficulty in applying pure axial load.

(g) Inaccuracy in the measurement of strain.

(h) Difficulty in obtaining good representative test specimen of plywood.
FIG. 4-1 LOAD-STRAIN CURVE OF PLUYWOOD
(TO DETERMINE POISSON'S RATIO)
FIG. 4.2 DISTRIBUTION OF $\sigma_{yy}/\sigma_{opx}$ AND $\sigma_{xx}/\sigma_{opx}$ ALONG THE Y AXIS
FIG. 4.3 DISTRIBUTION OF $\sigma_{rr}/\sigma_{y}\lambda$ AND $\sigma_{\theta\theta}/\sigma_{y}\lambda$
ALONG THE Y AXIS
FIG. 4: DISTRIBUTION OF $\frac{\sigma_{yy}}{\sigma_{xx}}$ AND $\frac{\sigma_{xx}}{\sigma_{yy}}$ ALONG THE Y AXIS
FIG. 4-5 DISTRIBUTION OF $\sigma_{rr}/\sigma_{opx}$ and $\sigma_{go}/\sigma_{opx}$ ALONG THE Y AXIS
FIG. 4.6 DISTRIBUTION OF $\sigma_{tr}/\sigma_{ox}$, $\sigma_{el}/\sigma_{ox}$ and $T_{rel}/\sigma_{ox}$ ALONG RADIAL LINE $\theta = 45^\circ$
**THEORETICAL EXPERIMENTAL**

\[ \lambda = 10.00 \]

\[ \theta = 90^\circ \]

\[ \text{CORE} = \frac{1}{2} \]

**FIG. 4.7 DISTRIBUTION OF** $\sigma_{rr} / \sigma_{0x}$ **and** $\sigma_{00} / \sigma_{0x}$ **ALONG THE Y AXIS**
FIG. 4.8 DISTRIBUTION OF $\sigma_{0e1}/\sigma_{0e1}$ AROUND HOLES
FIG. 4.9  ULTIMATE TENSILE TEST

PLYWOOD SPECIMEN

ELONGATION (ins x 10^2)

LOAD (lbs x 1000)
Fig. 4.10  Ultimate Tensile Test
A theoretical solution is presented to determine the stress distribution around circular holes in rectangular and circular sandwich plates subjected to uniform tensile load. The method of analysis consists of determining expressions for the face stresses around holes that satisfy the face equilibrium equations and the boundary conditions. The face stresses around the hole are evaluated from consideration that the displacements of the core and facings are equal at their mutual interfaces. The analysis is based on the assumptions that:

(a) the linear theory of elasticity is valid;
(b) the facings and the core are of isotropic materials and the facings are of equal thickness; and
(c) the core stiffness associated with the plane stress components in the plane of the plate are not negligible.
THEORETICAL VALUES OF RECTANGULAR SANDWICH PLATE WITH HOLE

SUBJECTED TO UNIAXIAL PULL

DIMENSION P(20), R(25), C(10), S(10), DEGREE(10)

PUNCH 9

9 FORMAT(/ // 1X, 6DH THEORETICAL VALUES OF RECTANGULAR SANDWICH PLATE
WITH CIRCULAR HOLE)

PUNCH 10

10 FORMAT(23X, 27H SUBJECTED TO UNIAXIAL PULL//)

READ 11, EF, EC, FNU, CNU

11 FORMAT(F10.0, F10.0, F6.0, F8.0)

READ 12, H, T

12, FORMAT(F7.0, F7.0)

READ 13, TV08, DH

13, FORMAT(F7.0, F7.0)

READ 14, K, (DEGREE(1), I=1)K

14 FORMAT(12/ (7F5.0))

PUNCH 15, EF, EC

15 FORMAT(/ // 1X, 31H MODULUS OF ELASTICITY OF FACE = 4X*F14.5,11H LBS,
2/50.0*IN./// 1X, 31H MODULUS OF ELASTICITY OF CORE = 5X*F14.5,11H LBS/
3RS/SQ.IN.)

PUNCH 16, FNU, CNU

16 FORMAT(/ // 1X, 31HP, POISSON RATIO OF FACE = 5X*F14.5/// 1X
4, 31HP, POISSON RATIO OF CORE = 6X*F14.5)

PUNCH 17, H, T

17 FORMAT(/ // 1X, 31HT, THICKNESS OF FACEINGS = 7X*F14.5, 5H INCH
5/1/// 1X, 31HT, THICKNESS OF CORE = 8X*F14.5, 5H INCH)

PUNCH 18, DH, TW0D

18 FORMAT(/ /// 1X, 31HD, DIAMETER OF THE HOLE = 6X*F14.5, 5H INCH
6/ /// 1X, 31HW, WIDTH OF THE PLATE = 7X*F14.5, 7H INCHES///
9))

PUNCH 19

19 FORMAT(3X, 4HLOAD, 7X, SHANGLE, 8X, 9HRAD, DIST, 6X, 3HR/A)

PUNCH 20

20 FORMAT(4X, JHLUS, 7X, 6H, DEGREE, 8X, 6H, INCHES/)

READ 21, N, (P(L), L=1)N

APPENDIX A
21 FORMAT(13/(1UF7.0))
  READ 22,M1,(R(J),J=1,M)
22 FORMAT(13/(12F6.0))
  READ 23,K1,(C(I),S(I),I=1,K1)
23 FORMAT(12/(2F10.0))
  XNAREA=TWOd*(H+2.*T)
  A=MH/2.*0
  B=TWOB/2.*0
  EK=((1.+CNU*PNU)/(1.+CNU**2))*EC/EF
  DO 99 L=1:N1
  SIGOFX=P(L)/(2.*B*0.2*T+EK*H))
  DO 99 J=1:M1
  DO 98 J=1:M1
    RHQ=A/R(J)
    A1=1.*RH0**2
    A2=1.*RH0**2
    A3=1.*RH0**2-3.*RH0**4
    A4=1.*RH0**2+3.*RH0**4
    A5=1.*RH0**2
    SIGRR=(SIGOFX*A2*)*(A1+A4*C(I))
    SIGTT=(SIGOFX/2.)*(A2-A5*C(I))
    TAURT=(SIGOFX/2.)*(A3)*S(I)
    TO FIND NC=0 DIMENSIONAL VALUES
    RHOT=RH(J)/N
    SIGNR=SIGJR/(P(L)*XNAREA)
    SIGNT=SIGJT/(P(L)*XNAREA)
    TAUNT=TAURT/(P(L)*XNAREA)
    PUNCH 24*(P,L,DEGREE(1),R(J),RHOT,SIGNR,SIGNT,TAUNT)
24 FORMAT(1X,F8.15,F5.15,F7.4,F8.4,F7.4,F8.4,F7.4,F8.4)
98 CONTINUE
99 CONTINUE
100 CALL EXIT
END

ZZZZ

A-2.
THEORETICAL VALUES OF RECTANGULAR SANDWICH PLATE WITH CIRCULAR HOLE
SUbjected TO UNIAXIAL PULL

MODULUS OF ELASTICITY OF FACE = 10000000. LBS/SQ. IN.

MODULUS OF ELASTICITY OF CORE = 1462500. LBS/SQ. IN.

POISSON RATIO OF FACE = 0.33

POISSON RATIO OF CORE = 0.196

THICKNESS OF FACINGS = 0.025 INCH

THICKNESS OF CORE = 0.75 INCH

DIAMETER OF THE HOLE = 0.75 INCH

WIDTH OF THE PLATE = 10.0 INCHES

Appendix A
<table>
<thead>
<tr>
<th>LOAD (LBS)</th>
<th>ANGLE DEGREE</th>
<th>RAD. DIST (INCHES)</th>
<th>R/A</th>
<th>$\tau_{123}/\sigma_{OPX}$</th>
<th>$\tau_{042}/\sigma_{OPX}$</th>
<th>$\tau_{124}/\sigma_{OPX}$</th>
</tr>
</thead>
<tbody>
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A-4
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C. EXPERIMENTAL VALUES OF RECTANGULAR SANDWICH PLATE WITH HOLE
S\SUBJECTED TO UNIAXIAL PULL\N
DIAMETER R2=RP(3\SUB1)EPSI(2\SUB15\SUB3)R1=RP(2\SUB15\SUB3)EPSI(3\SUB1)
PUNCH 10

13 FORMAT(///I1X,'*H EXPERIMENTAL VALUES OF RECTANGULAR SANDWICH PLATE')
15 'WITH CIRCULAR HOLE')
PUNCH 15

17 FORMAT(23X,'H, SUBJECTED TO UNIAXIAL PULL')
20 I2\T10H
23 FORMAT(2F6\*4)
PUNCH 21, T93JH

21 FORMAT(1X,'LTH OF THE PLATE = 1.30479 INCHES, 22 HOLE DIAMETER OF T')
24 HOLE = 1.484439 INCH/7)
PUNCH 25

25 FORMAT(3X,'H LOAD \XH, \XHANGLE, 1X, 5X, 1YRAJ, 1YST, 16X, 3MR/A)
PUNCH 26

26 FORMAT(4X,'H PLUS 7X, 80DEGREE, 5X, 6HINCHES/)
READ 33, H'T45FNU

30 FORMAT(2F6\*10\T10F5\*0)
READ 33, (R(I), J=1,3)
35 FORMAT(3F6\*4)
READ 40, (P(L), J1, N)
40 FORMAT(13/(10F7\*0),)
A=4H/23,
XYAREA=200*(H4*3)
107 READ 45, DEGREE
45 FORMAT(F4\*0)
1IF(DEGREE=398, 150, 200, 50
C. STRAINS ARE IN MICRO INCH PER INCH
50 READ 55, M\SUB1(\SUB EPSI(1\SUB J\SUB K\SUB K=1,3) J=1, M\SUB1) J=1,2)
55 FORMAT(12/(2F9\*3))
60 I99 L=1\N1
READ 6\SUB0 \SUB ((\SUB EPSI(1\SUB J\SUB K\SUB K=1,3)\N1) J=1, M\SUB) J=1,2)
69 FORMAT(JFB\SUB0)

Appendix B
DO 199 J=1,M+1
DO 190 K=1,J+1
198 EPS(J,K) = ((EPS(1,J,K) - EPS(1,J,K)) + (EPS(2,J,K) - EPS(2,J,K))) / 2.0
C1 = EF*1.0E-06/(1.*FNU**2)
C2 = EF*1.0E-06/(2.*FNU)
SIGRR = C1*(EPS(J+2) - EPS(J+1)) + EPS(J+1) + EPS(J+3) / FNU
SIGTT = C1*(EPS(J+1) + EPS(J+3)) - EPS(J+2) / FNU
TAURT = C2*(EPS(J+1) - EPS(J+3))
RHOF = R(J)/A
SIGRN = SIGTT/(P(L)/XAREA)
SIGTN = SIGTT/(P(L)/XAREA)
TAUN = TAURT/(P(L)/XAREA)
PUNCH 65*P(L),DEGREE,R(J),RHOE,SIGRN,SIGTN,TAUN
65 FORMAT(1X,F8.1,5X,F5.1,5X,F7.4,5X,F9.4,3X,F9.4,3X,F9.4)
199 CONTINUE
GO TO 197
270 CALL EXIT
END
VITA AUCTORIS

1940 Born in DEVAKOTTAI, MADRAS, INDIA, April 19

1956 In July, entered the University of Madras, India

1957 In April, passed Pre-University Course
       In July, entered Annamalai University, India

1962 In April, graduated with Bachelor of Engineering
       in Civil Engineering from Annamalai University

1963 In July, graduated with Master of Engineering
       in Structural Engineering from Annamalai University.
       Joined the Public Works Department of the Government
       of Madras as an Junior Engineer

       In October, resigned from P.W.D., Madras and joined
       the Faculty of Engineering, Annamalai University

1967 Resigned from Annamalai University and
       enrolled at the University of Windsor

1969 Left the University of Windsor

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