Studies on the effects of shear reinforcement in rib-stiffened reinforced concrete slabs.

Jamal Al-Aref

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/3640

This online database contains the full-text of PhD dissertations and Masters’ theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000ext. 3208.
INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films
the text directly from the original or copy submitted. Thus, some thesis and
dissertation copies are in typewriter face, while others may be from any type of
computer printer.

The quality of this reproduction is dependent upon the quality of the
copy submitted. Broken or indistinct print, colored or poor quality illustrations
and photographs, print bleedthrough, substandard margins, and improper
alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript
and there are missing pages, these will be noted. Also, if unauthorized
copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by
sectioning the original, beginning at the upper left-hand corner and continuing
from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced
xerographically in this copy. Higher quality 6" x 9" black and white
photographic prints are available for any photographs or illustrations appearing
in this copy for an additional charge. Contact UMI directly to order.

Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI®
STUDIES ON THE EFFECTS OF
SHEAR REINFORCEMENT IN
RIB-STIFFENED REINFORCED
CONCRETE SLABS

By

Jamal Al-Aref

A Thesis
Submitted to the College of Graduate Studies and
Research Through Civil Engineering Program in
Partial Fulfillment Of the Requirements for the
Degree of Master of Applied Science at
The University of Windsor

Windsor, Ontario, Canada
1998
The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author’s permission.
© 1998, J. Al-Aref
I hereby declare that I am the sole author of this document.

I authorize the University of Windsor to lend this document to other institutions or individuals for the purpose of scholarly research.

Jamal Al-Aref

I further authorize the University of Windsor to reproduce the document by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Jamal Al-Aref
THE UNIVERSITY OF WINDSOR requires the signatures of all persons using or photocopying this document.

Please sign below, and give address and date.
ABSTRACT

In this study, simple expressions for the anistropic rigidities of orthogonally rib-stiffened reinforced concrete slab structures are presented. These expressions are valid for both, the pre-cracking and post-cracking stages of concrete, and are, therefore, applicable to reinforced as well as prestressed concrete structures. Bending and twisting tests on several orthogonally rib-stiffened reinforced concrete slab structures were conducted. The experimental results of twisting tests show that using shear reinforcement (stirrups) has a significant effect on increasing the pre-cracking and post-cracking torsional rigidities, so proposed equations for estimating these torsional rigidities were developed to properly account for the presence of stirrups. The experimental results of bending tests verify that using stirrups has no effect on both the pre-cracking and post-cracking flexural rigidities, so the existing equations for determining these flexural rigidities were not modified.

The structural response of orthogonally rib-stiffened reinforced concrete slabs to loads was observed. It shows that using stirrups leads to ductile behavior of such structures.

The experimental procedure and the necessary precautions that must be taken to help ensuring accurate results were discussed. The use of realistic estimates for the torsional constants of rib-stiffened reinforced concrete slab structures when using shear reinforcement will lead to better design as well as economy.
TO MY FAMILY
ACKNOWLEDGEMENTS

The author wishes to express his thanks and gratitude to his advisor Dr. J. B. Kennedy, Emeritus and University Professor, for his guidance, valuable suggestions, effort and encouragement during the development of this research. Dr. Kennedy devoted his time and effort to make this study a success. His constant and valuable supervision is greatly appreciated.

Finally, the author wishes to thank his wife Nada for her great support, encouragement and patience throughout the course of this study.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>viii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>xvi</td>
</tr>
<tr>
<td>CHAPTER I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 General</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Objective</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Scope</td>
<td>2</td>
</tr>
<tr>
<td>CHAPTER II HISTORICAL REVIEW</td>
<td>3</td>
</tr>
<tr>
<td>CHAPTER III THEORETICAL FORMULATION</td>
<td>7</td>
</tr>
<tr>
<td>3.1 General Concept</td>
<td>7</td>
</tr>
<tr>
<td>3.2 Assumptions</td>
<td>7</td>
</tr>
<tr>
<td>3.3 Governing Differential Equation</td>
<td>9</td>
</tr>
<tr>
<td>3.4 Rigidities of Orthogonally Rib-Stiffened Uncracked</td>
<td>10</td>
</tr>
<tr>
<td>Sections</td>
<td></td>
</tr>
<tr>
<td>3.4.1 Flexural Rigidity</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>ix</td>
</tr>
</tbody>
</table>
3.4.2 Torsional Rigidity ................................................................. 12

3.5 Rigidities of Orthogonally Rib-Stiffened Cracked Sections........16

3.5.1 Flexural Rigidity ............................................................... 16

3.5.2 Torsional Rigidity ............................................................... 18

CHAPTER IV MATHEMATICAL FORMULATION FOR THE ..............21

EXPERIMENTAL STUDIES

4.1 Rigidities of Orthogonally Rib-Stiffened Slabs .........................21

4.1.1 Flexural Rigidity ............................................................... 21

4.1.2 Torsional Rigidity ............................................................... 23

CHAPTER V EXPERIMENTAL INVESTIGATION .................................25

5.1 Scope of the Experimental Program ........................................25

5.2 Materials ..............................................................................25

5.2.1 Concrete ............................................................................25

5.2.2 Reinforcement ....................................................................26

5.3 Description of the Specimens ..................................................26

5.4 Casting of the Specimens .......................................................28

5.5 Instrumentation .....................................................................28

5.6 Experimental Set-Up and Test Procedure ...............................30

5.7 Experimental Results .............................................................31

CHAPTER VI DISCUSSION OF RESULTS .................................32

6.1 Flexural and Torsional Rigidities ..........................................32

6.2 Structural Response of Specimens to Loads .........................36

x
6.3 Sources of Error ................................................................. 39

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS ......................... 40

7.1 Conclusions ........................................................................ 40

7.2 Recommendations for Future Research ........................... 41

REFERENCES ........................................................................... 43

BIBLIOGRAPHY ....................................................................... 47

TABLES .................................................................................. 49

FIGURES ................................................................................ 57

APPENDIX A: CONCRETE MIX DESIGN ............................... 100

VITA AUCTORIS ...................................................................... 103
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Geometries and Material Properties of Test Specimens</td>
<td>50</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison of Theoretical and Experimental Flexural Rigidities</td>
<td>51</td>
</tr>
<tr>
<td>5.3</td>
<td>Theoretical and Experimental Torsional Rigidities</td>
<td>52</td>
</tr>
<tr>
<td>6.1</td>
<td>Pre-cracking Linear Deflections of Bending Specimens</td>
<td>53</td>
</tr>
<tr>
<td>6.2</td>
<td>Post-cracking Linear Deflections of Bending Specimens</td>
<td>54</td>
</tr>
<tr>
<td>6.3</td>
<td>Pre-cracking Linear Deflections of Twisting Specimens</td>
<td>55</td>
</tr>
<tr>
<td>6.4</td>
<td>Post-cracking Linear Deflections of Twisting Specimens</td>
<td>56</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Isometric View of Test Specimen with Reference Axes</td>
<td>58</td>
</tr>
<tr>
<td>3.2</td>
<td>Cross Section and Method of Partitioning</td>
<td>59</td>
</tr>
<tr>
<td>4.1</td>
<td>Scheme of Loading and Support System for Test Specimens</td>
<td>60</td>
</tr>
<tr>
<td>5.1</td>
<td>Preparation of Some Specimens</td>
<td>61</td>
</tr>
<tr>
<td>5.2</td>
<td>Locations of Dial Gauges for Test Specimens</td>
<td>62</td>
</tr>
<tr>
<td>5.3</td>
<td>Locations of Concrete Strain Gauges for Test Specimens</td>
<td>63</td>
</tr>
<tr>
<td>5.4</td>
<td>Locations of Steel Strain Gauges for Test Specimens</td>
<td>64</td>
</tr>
<tr>
<td>5.5</td>
<td>Preparation of Steel Strain Gauges for Test Specimens</td>
<td>65</td>
</tr>
<tr>
<td>5.6</td>
<td>Curing of Concrete Cylinders</td>
<td>66</td>
</tr>
<tr>
<td>5.7</td>
<td>Compression testing Of Concrete Cylinder</td>
<td>67</td>
</tr>
<tr>
<td>5.8</td>
<td>Loading Frame for Bending Specimen</td>
<td>68</td>
</tr>
<tr>
<td>5.9</td>
<td>Loading Frame for Twisting Specimen</td>
<td>69</td>
</tr>
<tr>
<td>5.10</td>
<td>Calibration of the Load Cell</td>
<td>70</td>
</tr>
<tr>
<td>5.11a</td>
<td>Top Isometric View of Three-point Loading and Three-point Support Arrangement for Bending Specimen</td>
<td>71</td>
</tr>
<tr>
<td>5.11b</td>
<td>Bottom Isometric View of Three-point Loading and Three-point Support Arrangement for Bending Specimen</td>
<td>72</td>
</tr>
<tr>
<td>5.12a</td>
<td>Top Isometric View of Loading and Support Arrangement for Twisting Specimen</td>
<td>73</td>
</tr>
</tbody>
</table>

xiii
5.12b Side Isometric View of Loading and Support Arrangement for Twisting Specimen 74
5.13 Test Set-up for Bending Specimens 75
5.14 Test Set-up for Twisting specimens 76
5.15 Arrangement of Dial Gauges for Bending Specimens 77
5.16 Arrangement of Dial Gauges for Twisting Specimens 78
6.1 Load vs. Deflection for Bending Specimens 79
6.2 Load vs. Deflection for Twisting Specimens 80
6.3 Load-Concrete Strain Relationship for Bending Specimens 81
6.4 Load-Concrete Strain Relationship for Twisting Specimens 82
6.5 Load-Steel Strain Relationship for Bending Specimens 83
6.6 Load-Steel Strain Relationship for Twisting Specimens 84
6.7 Pre-cracking Torsional Rigidity Due to Stirrups vs. Volume Of Stirrups 85
6.8 Post-cracking Torsional Rigidity Due to Stirrups vs. Volume of Stirrups 86
6.9 Pre-cracking Torsional Constant Due to Stirrups vs. Volume of Stirrups 87
6.10 Post-cracking Torsional Constant Due to Stirrups vs. Volume of Stirrups 88
6.11 View of the Transverse Cracks over Support Lines in Bending Specimen A-1 89
6.12 View of the Transverse Cracks over Support Lines in Bending Specimen A-2 90
6.13 View of the Longitudinal Crack in Bending Specimen A-2 91
6.14 View of the Transverse Cracks over Support Lines in Bending Specimen A-3 92
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.15</td>
<td>View of the Longitudinal Crack in Bending Specimen A-3</td>
<td>93</td>
</tr>
<tr>
<td>6.16</td>
<td>View of the Cracks along the Support Line in Twisting Specimen B-1</td>
<td>94</td>
</tr>
<tr>
<td>6.17</td>
<td>View of the Inclined Cracks on Side of Twisting Specimen B-1</td>
<td>95</td>
</tr>
<tr>
<td>6.18</td>
<td>View of the Cracks along the Support Line in Twisting Specimen B-2</td>
<td>96</td>
</tr>
<tr>
<td>6.19</td>
<td>View of the Inclined Cracks on Side of Twisting Specimen B-2</td>
<td>97</td>
</tr>
<tr>
<td>6.20</td>
<td>View of the Cracks along the Support Line in Twisting Specimen B-3</td>
<td>98</td>
</tr>
<tr>
<td>6.21</td>
<td>View of the Inclined Crack on side of Twisting Specimen B-3</td>
<td>99</td>
</tr>
</tbody>
</table>
**NOMENCLATURE**

- $A_s (A'_s)$: Area of tension steel in longitudinal (transverse) rib ($\text{mm}^2$)
- $A_i$: Area of one leg of stirrups ($\text{mm}^2$)
- $b_x (b_y)$: Width of Longitudinal (transverse) rib (mm)
- $D_x (D_y)$: Flexural rigidity of the orthogonally rib-stiffened slab per unit width in the x- (y-) direction ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D$: Flexural rigidity of the orthogonally rib-stiffened slab per unit width when $D_x = D_y$ ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D_{fp}$: Flexural rigidity of the flange plate per unit width with respect to its middle plane ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D_x' (D_y')$: Flexural rigidity of the flange plate per unit width with respect to the neutral plane of the slab in the x- (y-) direction ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D_1 (D_2)$: Coupling rigidities of the slab per unit width in the x- (y-) direction due to Poisson's effect ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D_{xy} (D_{yx})$: Torsional rigidity of the orthogonally rib-stiffened slab ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
- $D_{xy'} (D_{yx'})$: Torsional rigidity of the orthogonally rib-stiffened slab due to stirrups ($\text{N} \cdot \text{mm}^2 / \text{mm}$)
\( d_x \) (\( d_y \)) \hspace{1cm} \text{Depth of the longitudinal (transverse) rib (mm)}

\( d' \) (\( d'' \)) \hspace{1cm} \text{Concrete cover to the centre of longitudinal (transverse) reinforcement (mm)}

\( E, E_x, E_y \) \hspace{1cm} \text{Modulus of Elasticity of concrete (N/mm\(^2\))}

\( e_x \) (\( e_y \)) \hspace{1cm} \text{Depth of neutral plane of the uncracked section from extreme compression fibre for bending in the x- (y-) direction (mm)}

\( f_c' \) \hspace{1cm} \text{28-day compressive strength of concrete (N/mm\(^2\))}

\( G \) \hspace{1cm} \text{Shear modulus of concrete (N/mm\(^2\))}

\( h \) \hspace{1cm} \text{Thickness of flange plate (mm)}

\( I_x' \) (\( I_y' \)) \hspace{1cm} \text{Moment of inertia of longitudinal (transverse) rib in the x- (y-) direction with respect to the neutral axis (mm}^4\)

\( I_{cx} \) (\( I_{cy} \)) \hspace{1cm} \text{Moment of inertia of concrete in compression and (n-1) times area of compression steel about the neutral axis of the cracked section in the x- (y-) direction (mm}^4\)

\( I_c \) \hspace{1cm} \text{Moment of inertia of concrete in compression and (n-1) times area of compression steel about the neutral axis of the cracked section when } I_{cx} = I_{cy} \text{ (mm}^4\)

\( I_{sx} \) (\( I_{sy} \)) \hspace{1cm} \text{Moment of inertia of the transformed tension steel section about the neutral axis of the cracked section in the x- (y-) direction (mm}^4\)

\( I_s \) \hspace{1cm} \text{Moment of inertia of the transformed tension steel section}
about the neutral axis of the cracked section when \( I_{xx} = I_{xy} \) (mm\(^4\))

\( J \)  
Torsional constant (mm\(^4\))

\( K \)  
Torsional parameter

\( K_1 \)  
Constant for rectangular sections in tension

\( Kd_x \) (\( Kd_y \))  
Depth of neutral plane of the cracked section from extreme compression fibre for bending in the x- (y-) direction (mm)

\( M_x \), \( M_y \), \( M_{xy} \)  
Bending and twisting moments per unit width associated with the x- and y- axes (N.mm/mm)

\( n \)  
Modular ratio

\( q_{(x,y)} \)  
Intensity of lateral load on the stiffened slab (N/mm\(^2\))

\( S_x \) (\( S_y \))  
Spacing of the longitudinal (transverse ) ribs in the x- (y-) direction (mm)

\( S \)  
Spacing of the ribs when \( S_x = S_y \) (mm)

\( S_t \)  
Spacing of stirrups in x- and y- direction (mm)

\( W \)  
Deflection in the z- direction (mm)

\( W_{xx} \), \( W_{yy} \), \( W_{xy} \)  
Curvatures and twist of the surface associated with the x and y- axes (rad/mm)

\( W_{r} \), \( W_{t} \)  
Curvatures associated with r- and t- axes (rad/mm)

\( x, y, z \)  
Axes in the rectangular co-ordinate system

\( x_l, y_l \)  
Width and height of the stirrup (mm)

\( \mu, \mu_{xy} \)  
Poisson's ratio of concrete

xviii
CHAPTER I

INTRODUCTION

1.1 General

Thin plates stiffened by a system of orthogonal ribs have found wide application for aircraft, bridge, building and ship bottom structures as well as in many other branches of contemporary structural engineering. These stiffened elements, representing a relatively small part of the total weight of the structure, substantially influence its strength, stiffness, and stability leading to economy and other advantages. Studies of orthogonally stiffened slabs have been of particular interest and practical importance in bridge structures constructed for reasons of economy and structural efficiency. It becomes essential to use realistic estimates of the rigidity constants of such structures in order to predict accurately this behavior under an applied load.

1.2 Objective

The overall objectives of this study are:

a) To develop simple and rational expressions to predict flexural and torsional rigidities of orthogonally rib-stiffened reinforced concrete slabs for both the pre-cracking and post-cracking stages when using shear reinforcement in the form of stirrups.
b) To observe structural response of orthogonally rib-stiffened reinforced concrete slabs to loads when using stirrups in such structures.

1.3 Scope

Simple expressions were proposed for estimating the pre-cracking and post-cracking torsional rigidities of orthogonally rib-stiffened reinforced concrete slabs by utilizing the experimental results of twisting test.

The theoretical values of the pre-cracking and post-cracking flexural rigidities of orthogonally rib-stiffened reinforced concrete slabs obtained from the existing equations were compared to the experimental results of bending test.

From bending and twisting tests, the ductile behavior of orthogonally rib-stiffened reinforced concrete slabs because of the presence of stirrups was observed.
CHAPTER II

HISTORICAL REVIEW

Historically, the development of stiffened structural elements is one of slow growth. In the early stages of development, man probably learned of the existence of such forms from nature. Sea shells, trees, leaves, and vegetables are all in fact stiffened structures. The wide use of stiffened structural form in engineering began in the nineteenth century, mainly with the application of steel plates for hulls of ships and with the development of steel bridges and aircraft structures.

The study of elastic constants for plates by bending and twisting tests began in 1927 when Bergstrasser (5) used a procedure suggested by Nadai for applying pure bending and twisting moments on a plate of constant thickness. To apply bending moments, he used a rectangular plate supported at three points and loaded at three other points. A twisting moment was applied on a square plate supported at two diagonally opposite corners and loaded at the other two corners. Bergstrasser (5) assumed that the shape of the deflected surface for an isotropic plate due to a pure bending moment \( M_x \) could be expressed as.

\[
W = \frac{6M_x}{Eh^3} (x^2 - \mu y^2)
\]  

(1.2.1)

Where \( x \) and \( y \) are the rectangular co-ordinates.

Thielemann (24) and Hearmon and Adams (14) used essentially the same procedure. The principal difference between these various investigators is in the manner in which they measured the displacements caused by the applied bending and twisting moments or in the
Equations used to determine the elastic constants. Bergstrasser (5) and Hearmon and Adams (14) measured their displacements by placing a measuring device on the deflected plate and thus calculated relative displacements. Thielemann (24) and Hearmon and Adams (14) assumed that the deflected shape for a “specially orthotropic” plate (principal axes parallel to the sides of the plate) had the form,

\[ W = \frac{6M_y}{h^3} (S_{11}x^2 + S_{12}y^2) \]  \hspace{1cm} (1.2.2)

Where \( S_{11} \) and \( S_{12} \) are the elastic constants which can be expressed as \( 1/E_x \) and \( -\mu_y/E_x \), respectively. For the case of a homogeneous isotropic plate, \( E_x = E \) and \( \mu_y = \mu \). Both equations (1.2.1) and (1.2.2) describe a hyperbolic-paraboloid surface.

Witt, Hoppmann and Buxbaum (28) gave the theoretical basis and an experimental method for determining the anisotropic elastic constants of a material by measuring deflections of a thin plate subjected to couples on its boundary. They measured the displacements relative to a fixed plane and described the shape of the deflected surface of the plate by,

\[ W = \frac{6M_y}{h^3} (S_{ii}x^2 + S_{ij}y^2) + A.X + B.Y + C \]  \hspace{1cm} (1.2.3)

where \( S_{ii} \) and \( S_{ij} \) are the elastic constants. The constants \( A \), \( B \), and \( C \) were determined from the boundary conditions with \( W = 0 \) at three support points.

In 1956, Huffington (15) investigated theoretically and experimentally the method for the determination of rigidities for metallic rib-reinforced deck structures. It was applied to the case of equally spaced stiffeners, of rectangular cross-section, and symmetrically placed with respect to its middle plane.

Beckett et al. (4) presented a curvature method for the experimental determination
of elastic constants of orthogonally stiffened plates. The method utilized the fact that pure
twisting moments can be expressed in terms of the curvature and twist of a surface, respectively.

In 1968, Jackson (16) proposed a method to estimate the torsional rigidities of concrete bridge decks using membrane analogy and accounting for the junction effect. The effect of the continuity of the slab on the flange plate was not accounted for. Cardenas et al. (6) investigated the in-plane and flexural stiffnesses of isotropically and non-isotropically reinforced concrete plates. Results indicated that the stiffness of such plates was related quantitatively to the relative orientation of the reinforcement with respect to the applied forces, to the combinations of the applied forces and the amounts of the reinforcement in the two orthogonal directions. Kennedy and Gupta (18) used the elastic constants of the equivalent structure which closely resembled the twisting and bending behavior of an orthogonally stiffened plate structure to analyze the original structure by means of the orthotropic plate theory.

Cusens et al. (11) presented an elastic analysis for plates reinforced with an orthogonal system of rectangular ribs to determinate the rigidities of the structure in flexure and torsion. The analysis did not account for the stiffening effect of the orthogonal ribs on the torsional rigidity of the flange plate; also the method is not applicable for reinforced concrete bridge decks where some cracking is expected or for structures with relatively thick flange plates. Little information is available as to how these rigidities might be assessed for the cracked sections of a concrete structure.

Desayi and Kulkarni (12) suggested empirical reduction factors for the flexural
rigidities of reinforced concrete slabs up to the yield load.

In 1972, the Concrete Reinforcing Steel Institute (10) recommended that the average gross moment of inertia to be used for two-way joist and waffle slabs. Lampert (20) derived theoretical expressions for the post-cracking stiffness of rectangular reinforced concrete beams in torsion and bending using a space truss model. Jofriet and Mcneice (17) used empirical bilinear moment-curvature relationships to incorporate the influence of cracking in the finite element analysis of slab structures. Clark and White (7) carried out tests to determine the torsional stiffness of flexurally cracked slab elements. Recently, Collins and Mitchell (8) suggested a procedure for the pre-cracking and post-cracking torsional stiffnesses of reinforced concrete beams by using the Equivalent Tube method. This method did not account for the contribution of stirrups in the pre-cracking torsional stiffness, and considered only the area enclosed by the centerline of stirrup for the post-cracking torsional stiffness.
CHAPTER III

THEORETICAL FORMULATION

3.1 General Concept

In this chapter, proposed equations for estimating the pre-cracking and post-cracking torsional rigidities of orthogonally rib-stiffened slabs reinforced with stirrups are developed. If a homogeneous material has three mutually perpendicular planes of symmetry with respect to its elastic properties, it is called orthotropic, i.e. materials which are orthogonally anisotropic. For instance, two-way reinforced concrete slabs are intrinsically anisotropic. Other materials with natural anisotropy are plywood, fibre-reinforced plastics, and wood. In certain cases, structural anisotropy is introduced by means of corrugations or ribs such as decks of steel bridges, corrugated slabs, composite beam grid frame works, concrete slabs reinforced with closely spaced ribs, etc. These structures can be analyzed using orthotropic plate theory that assumes that the orthotropy of the structure may be replaced by the orthotropy of the constituent material. Although the actual structural behavior of a stiffened slab cannot be entirely replaced by that of an equivalent orthotropic slab, previous theoretical and experimental investigations indicate good agreement (13).

3.2 Assumptions

In dealing herein with orthotropic concrete structures, it is assumed that orthotropy
is a result of geometry and not of material. Except for the assumptions made with respect to the rib-stiffened construction, the assumptions for orthotropic slabs are based on the same assumptions used in the analysis of isotropic slabs, and they are outlined as follows:

a) The material of the slab is considered to be continuous and homogeneous. by transforming the reinforcing steel area into an equivalent area of concrete.

b) The slab thickness is uniform and small compared with the other dimensions of the slab. Thus, the shearing stresses and normal stresses in the direction transverse to the plate are small and can be neglected.

c) The deflections of the loaded slab are small in comparison to its thickness so that membrane stresses in the slab can be neglected.

d) The material of the slab is elastic, i.e. the stress-strain relationship is governed by Hooke’s Law.

e) Straight lines normal to the middle surface of the slab remain straight and normal to the middle surface of the slab after loading.

f) The ratio of stiffener spacing to slab boundary dimensions is small enough for the real structure to be replaced by an idealized one with continuous properties.

g) Flexural and torsional rigidities do not depend on the boundary conditions of the slab or on the distribution of the vertical load.

h) The neutral plane in each of the two orthogonal directions coincides with the centre of gravity of the total section in the corresponding direction.

i) The area of the flange plate (deck) is magnified by the factor $1/ (1-\mu^2)$ to allow for the effect of Poisson’s ratio $\mu$. 

8
3.3 Governing Differential Equations

Adopting the assumptions outlined in 3.2, the following expressions are obtained for the bending and torsional moments (26),

\[ M_x = - (D_x W_{xx} + D_y W_{yy}) \]
\[ M_y = - (D_y W_{yy} + D_x W_{xx}) \]
\[ M_{xy} = D_{xy} W_{xy} \]
\[ M_{yx} = D_{yx} W_{yx} \]

in which,

\( M_x \) = bending moment vector in the y-direction;
\( M_y \) = bending moment vector in the x-direction;
\( M_{xy} \) = twisting moment vector in the x- and y-directions;
\( M_{yx} \) = twisting moment vector in the y- and x-directions;
\( W_{xx}, W_{yy} \) = curvatures associated with the bending moments \( M_x \) and \( M_y \), respectively;
\( W_{xy} \) = twisting curvature associated with the twisting moment \( M_{xy} \);
\( W_{yx} \) = twisting curvature associated with the twisting moment \( M_{yx} \);
\( W \) = lateral deflection in the Z-direction of a point on the structure (Fig. 3.1);
\( D_x, D_y \) = flexural rigidities of the slab per unit width in x and y directions, respectively;
\( D_1, D_2 \) = coupling rigidities of the slab per unit width due to Poisson’s ratio, and
\( D_{xy}, D_{yx} \) = torsional rigidities of the slab.

The equation of equilibrium of the moments, after neglecting the shearing forces acting on the element, is given by (26),

\[ M_{x,xx} + 2M_{xy,xy} + M_{y,yy} = - q_{(x,y)} \]
By substituting expressions (3.3.1) in the moment Eq. (3.3.2), a fourth order
differential equation governing the deflection of the orthotropic slab be obtained in
rectangular co-ordinates (26),

\[ D_x W_{xxxx} + 2HW_{xyxy} + D_y W_{yyyy} = q_{(x,y)} \]  \hspace{1cm} (3.3.3)

Where,

\[ 2H = D_1 + D_2 + D_{xy} + D_{yx}, \text{ which is the effective torsional rigidity of the orthotropic} \]
\[ \text{slab.} \]

### 3.4 Rigidities of Orthogonally Rib-Stiffened Uncracked Sections

#### 3.4.1 Flexural Rigidity

Figure 3.1 shows a typical section of a rib-stiffened slab. Based on the
assumptions made earlier, the orthotropic flexural rigidities, \( D_x \) and \( D_y \), as well as the
coupling rigidities \( D_1 \) and \( D_2 \) due to Poisson's effect, of an uncracked concrete section
can be expressed as (27),

\[
D_x = D_{fp} + \frac{[Eh(e_x - h/2)^2 / (1 - \mu^2)] + EI_x' / S_x}{12(1 - \mu^2)}
\]

\[
D_y = D_{fp} + \frac{[Eh(e_y - h/2)^2 / (1 - \mu^2)] + EI_y' / S_y}{12(1 - \mu^2)}
\]

\[
D_1 = \mu D_x'
\]

\[
D_2 = \mu D_y'
\]

Where,

\[ D_{fp} = \text{flexural rigidity of the flange plate (deck) per unit width with respect to its middle} \]
\[ \text{plane,} \]
\[ = \frac{Eh'}{12(1 - \mu^2)}; \]

10
E = modulus of elasticity of concrete, which can be taken as (2).

\[ E = 33 \, w_c^{1.5} \left( f'_c \right)^{1/2}; \]

\[ f'_c \] = 28-day concrete cylinder strength in psi;

\[ h = \text{thickness of the flange plate (deck)}; \]

\[ \mu = \text{Poisson's ratio of concrete which can be taken as (19).} \]

\[ \mu = \left( f'_c \right)^{1/2} / 350; \]

\[ S_x (S_y) = \text{spacing of the longitudinal (transverse) ribs}; \]

\[ e_x (e_y) = \text{depth of neutral plane from the top fibre for bending in the x- (y-) direction. i.e.,} \]

\[ e_x = \left\{ b_x d_x (h + d_x/2) + (n - 1)A_x(h + d_x - d') + S_x h^2/2(1 - \mu^2) \right\} / \left\{ b_x d_x + (n - 1)A_x + S_x h/(1 - \mu^2) \right\} \]

\[ e_y = \left\{ b_y d_y (h + d_y/2) + (n - 1)A_y(h + d_y - d'') + S_y h^2/2(1 - \mu^2) \right\} / \left\{ b_y d_y + (n - 1)A_y + S_y h/(1 - \mu^2) \right\} \]

\[ A_x' + S_x h/(1 - \mu^2) \] \hspace{1cm} (3.4.2)

\[ I_x' (I_y') = \text{moment of inertia of longitudinal (transverse) rib with respect to the assumed neutral axis, i.e.,} \]

\[ I_x' = (b_x d_x^3/12) + b_x d_x \left\{ (h + d_x/2) - e_x \right\}^2 + (n - 1)A_x \left\{ (h + d_x - d') - e_x \right\}^2 \]

\[ I_y' = (b_y d_y^3/12) + b_y d_y \left\{ (h + d_y/2) - e_y \right\}^2 + (n - 1)A_y \left\{ (h + d_y - d'') - e_y \right\}^2 \] \hspace{1cm} (3.4.3)

in which,

\[ n = \text{modular ratio} = E_x / E_c; \]

\[ b_x (b_y) = \text{width of longitudinal (transverse) rib}; \]

\[ d_x (d_y) = \text{depth of the longitudinal (transverse) rib}; \]

\[ d' (d'') = \text{concrete cover to the centre of the longitudinal (transverse) reinforcements}; \]

\[ A_x (A_y') = \text{area of tension steel in the longitudinal (transverse) rib}; \]
$D_x' (D_y') = \text{flexural rigidity of the flange plate (deck) per unit width with respect to the}$

neutral plane of the gross cross section associated with bending in the x- (y-) direction.

It should be noted that in this research no term was added to the existing equation to account for the presence of stirrups since it has an insignificant effect on the pre-cracking flexural rigidity as was confirmed experimentally.

In the specimens tested, the cross section has the same properties in the x- and y-directions. Thus,

$$D_x = D_y = D$$

$$D_x' = D_y', \text{ and}$$

$$D_1 = D_2 \quad (3.4.1.4)$$

3.4.2 Torsional Rigidity

The twisting rigidity of an uncracked section of an orthogonally rib-stiffened concrete slab is estimated by using the membrane analogy method as proposed by Timoshenko and Goodier (25). The stiffening effect afforded by rib stiffeners (beam stems) in the orthogonal direction to the one under consideration is taken into account (3).

Referring to the tee-section shown in Figs. 3.2a and 3.2b, then considering the geometry of the deflected membrane, the torsional constants for the rectangular sections 1, 2, and 3 are calculated and added to yield the torsional constant of the cross-section. Thus for a section normal to the y-axis (Figs. 3.1 and 3.2b), (25),

$$J_y = J_1 + J_2 + J_3 \quad (3.4.2.1)$$

where,
$J_1$, $J_2$ and $J_3$ are the contributions of areas 1, 2, and 3 respectively, defined as,

\[
J_1 = \frac{1}{2} \left( K_i S_i h^3 \right)
\]

\[
J_2 = K_i d_y b_y^3 \quad \text{for} \quad d_y \geq b_y
\]

\[
J_2 = K_i b_y d_y^3 \quad \text{for} \quad b_y \geq d_y
\]

\[
J_3 = 4K_i(n - 1)(A_y')^2 / \pi
\]

in which, $K_i$ is the constant for rectangular sections in torsion (25). The contribution $J_3$ of the reinforcing steel is calculated by transforming the area of steel into equivalent area of concrete. The contribution $J_3$, being relatively small, can be ignored in a practical design.

For the calculation of $J_1$, a reduction factor $1/2$ is used which accounts for the top flange (deck) of an isolated tee-section. From Bali (3), since the slabs considered herein are stiffened by ribs in two orthogonal directions, the torsional contribution $J_1$ of the slab in one direction (see hatched area in Fig. 3.1) is augmented by the stiffening rib in the orthogonal direction in the following manner:

Considering the section normal to the $x$-axis (Figs. 3.1 and 3.2a), the presence of the transverse rib "W" will increase the torsional contribution $J_x$ of the slab $S$ (deck) to a value denoted by $J_{(s \cdot w)}$ (3). Thus, the modified value of $J_1$ for the hatched area (normal to the $y$-axis) becomes (3),

\[
(J_1)_{\text{modified}} = (J_1) \left( J_{(s \cdot w)} / J_x \right)
\]  

Then, Eq. (3.4.2.1) becomes (3),

\[
J_x = (J_1)_{\text{modified}} + J_2 + J_3
\]

where,

$J_x$ is the torsional constant of the cross-section normal to the $y$-axis.
Then, the torsional rigidity $D_{yx}$ can be calculated from (25),

$$D_{yx} = GJ_y/S_y$$  \hspace{1cm} (3.4.2.5)

where, $G =$ shear modulus $= E / (2(1 + \mu))$

Similarly, the torsional rigidity, $D_{xy}$, of a section normal to the x-axis (Figs. 3.1 and 3.2a) can also be calculated from,

$$D_{xy} = GJ_x/S_x$$  \hspace{1cm} (3.4.2.6)

where,

$J_x$ is the torsional constant of the cross-section normal to the x-axis.

In the specimens tested, the section properties were the same in x- and y-directions. Thus,

$$J_x = J_y = J; \text{ and,}$$

$$D_{xy} = D_{yx}$$

So, Eq. (3.4.2.4) becomes,

$$J = (J_1)_{\text{modified}} + J_2 + J_3$$  \hspace{1cm} (3.4.2.7)

Also, either of Eqs. (3.4.2.5), (3.4.2.6) can be written as,

$$D_{xy} = GJ/S$$  \hspace{1cm} (3.4.2.8)

To account for the contribution of stirrups in increasing the pre-cracking torsional rigidity, the existing equation (3.4.2.8.) is modified to,

$$D_{xy} = GJ/S + D_{xy}'$$  \hspace{1cm} (3.4.2.9)

where,

$D_{xy}'$ is a new term developed in this research to account for the pre-cracking torsional rigidity due to stirrups. This term can be calculated from,
\[ D_{xy} = \frac{GJ_4}{S} \]  

(3.4.2.10)

Where, \( J_4 \) can be found from,

\[ J_4 = \alpha [2(x_1 + y_1)A_i / S_i]^{\beta} \]  

(3.4.2.11)

in which,

\( J_4 \) is the pre-cracking torsional constant due to stirrups to represent the contribution of stirrups to the pre-cracking torsional rigidity (\( \text{mm}^4 \));

\( \alpha \) is a constant found experimentally from Fig. 6.9, and

\[ = 108 \times 10^3 \text{ (mm}^2) \];

\( [2(x_1 + y_1)A_i / S_i] \) is the volume of stirrups per unit length (\( \text{mm}^3/\text{mm} \)), in which \( x_1 \) and \( y_1 \) are the width and height of the stirrup, respectively, \( A_i \) is the area of one leg of stirrup, and \( S_i \) is the spacing of stirrups;

\( \beta \) is a dimensionless constant found experimentally from Fig. 6.9, and

\[ = 1.2 \]

Thus, the pre-cracking torsional rigidity \( D_{xy} \) can be determined from Eq. (3.4.2.9), in which,

\( J \) is estimated from Eq. (3.4.2.7); and

\( D_{xy} \) is estimated from Eq. (3.4.2.10).
3.5 Rigidities of Orthogonally Rib-Stiffened Cracked Sections

3.5.1 Flexural Rigidity

After cracking of the concrete section, the structure continues to behave elastically, provided that the stress in the steel is below the yield point and the compressive stress in the concrete does not exceed 0.5 \( f'_c \) (21). In addition to the assumptions made in section 3.2, it is assumed that the tension cracks have progressed to the neutral axis (assumed to be in the flange plate (deck), which is generally the case). For the computation of the rigidities, the transformed section consisting of concrete in compression and \((n - 1)\) times area of the compression steel (if provided) and \(n\) times the area of the tension steel is used. Based on the above simplifying assumptions, the rigidities of the cracked section can be expressed as (3).

\[
D_x = E \{ I_{cx}/(1 - \mu^2) + I_{cx}\}/S_x \\
D'_y = E \{ I_{cy}/(1 - \mu^2) + I_{sy}\}/S_y \\
D_1 = \mu \ D'_x \\\nD_2 = \mu \ D'_y
\]

(3.5.1.1)

where,

\( I_{cx} \) (\( I_{cy} \)) = moment of inertia of the concrete in compression and \((n - 1)\) times area of compression steel (if any) about the neutral axis in the \(x\) (\(y\)) direction, respectively, i.e., if the neutral axis lies in the flange plate (deck) which is generally the case (3),

\[
I_{cx} = S_x (kd_x)^3/3 \\
I_{cy} = S_y (kd_y)^3/3
\]

(3.5.1.2)
$I_{xx} (I_{yy}) = \text{moment of inertia of the transformed tension steel section about the neutral axis in the x- (y-) direction respectively (3), i.e.,}$

\[ I_{xx} = nA_s \{(h + d_s - d') - kd_s\}^2 \]
\[ I_{yy} = nA_s' \{(h + d_s - d'') - kd_s\}^2 \]  \hspace{1cm} (3.5.1.3)

$D_x' (D_y') = \text{flexural rigidity of the flange plate (deck) with respect to the neutral plane of the cracked section associated with bending in the x- (y-) direction.}$

The location of the neutral axis $kd_s$ or $kd_d$ is determined by equating the tension force to the compression force of the section. Thus, by assuming that the neutral axis lies in the flange plate (deck), $kd_s$ is calculated from, (3).

\[ nA_s \{(h + d_s - d') - kd_s\} - S_x (kd_s)^2 / 2 (1 - \mu_s^2) = 0 \]

and $kd_d$, from, \hspace{1cm} (3.5.1.4)

\[ nA_s' \{(h + d_s - d'') - kd_d\} - S_y (kd_d)^2 / 2 (1 - \mu_d^2) = 0 \]

It should be noted that in this research no term was added to the existing equation to account for the presence of stirrups since it has an insignificant effect on the post-cracking flexural rigidity as was confirmed experimentally.

Since in the specimens tested,

$D_x = D_y = D$

$D_x' = D_y' = D'$, and

$D_1 = D_2$

Eqs. (3.5.1.1) reduce to,

$D = E \{I_x'(1 - \mu_s^2) + I_s\} / S$

$D_1 = \mu D'$ \hspace{1cm} (3.5.1.5)
where,

\[ I_{sx} = I_{sy} = I_s \]
\[ I_{cx} = I_{cy} = I_c \]
\[ S_x = S_y = S \]

### 3.5.2 Torsional Rigidity

For a rib stiffened slab construction, it can be shown (22) that,

\[ (D_{sx} + D_{sy} + D_1 + D_2) = 2K(D_sD_y)^{1/2} \]  \hspace{1cm} (3.5.2.1)

or,

\[ K = (D_{sx} + D_{sy} + D_1 + D_2) / 2(D_sD_y)^{1/2} \]  \hspace{1cm} (3.5.2.2)

Experimental studies have shown that for this type of slabs, \( D_{sx} = D_{sy} \), (3).

Thus, Eq. (3.5.2.2) reduces to (3),

\[ K = (2D_{sx} + D_1 + D_2) / 2(D_sD_y)^{1/2} \]  \hspace{1cm} (3.5.2.3)

Hence,

\[ D_{sx} = K (D_sD_y)^{1/2} - (D_1 + D_2)/2 \]  \hspace{1cm} (3.5.2.4)

The co-efficient \( K \) is a torsional parameter with a lower limit of zero (for a grillage with members having no torsional rigidity) and an upper limit of one (for a true orthotropically reinforced slab) (3).

In the specimens tested,

\[ D_x = D_y = D, \text{ and} \]
\[ D_1 = D_2 \]

Thus, Eq. (3.5.2.3) reduces to,
\[ K = \frac{D_x + D_t}{D} \]  

(3.5.2.5)

It is reasonable to assume that the co-efficient \( K \) remains the same before and after cracking of the concrete (3). Thus, the co-efficient \( K \) can be calculated from Eq. (3.5.2.5) by using pre-cracking flexural and torsional rigidities from Eqs. (3.4.1.1), and (3.4.2.9), respectively.

Also, Eq. (3.5.2.4) reduces to,

\[ D_x = KD - D_t \]  

(3.5.2.6)

To account for the contribution of stirrups in increasing the post-cracking torsional rigidity, Eq. (3.5.2.6) which based on the existing equation (3.5.2.4) is modified to,

\[ D_x = KD - D_t + D_{xy}' \]  

(3.5.2.7)

where,

\( D_{xy}' \) is a new term developed in this research to account for the post-cracking torsional rigidity due to stirrups. This term can be calculated from,

\[ D_{xy}' = GJ'/S \]  

(3.5.2.8)

where,

\[ J' = \alpha' [2(x_x + y_y)A_v/S_v]^{\nu} \]  

(3.5.2.9)

in which,

\( J' \) is the post-cracking torsional constant due to stirrups to represent the contribution of stirrups to the post-cracking torsional rigidity (mm\(^4\));

\( \alpha' \) is a constant found experimentally from Fig. 6.10, and

\[ = 664 \times 10^3 \text{ (mm}^3\text{)}; \]

\[ [2(x_x + y_y)A_v/S_v] \text{ is the volume of stirrups per unit length (mm}^3\text{/mm), in which } x_x \text{ and } y_y \text{ are} \]
the width and height of the stirrup, respectively, $A_r$ is the area of one leg of stirrup, and $S_r$ is the spacing of stirrups;

$\beta'$ is a dimensionless constant found experimentally from Fig. 6.10, and

$= 0.6$

Thus, the post-cracking torsional rigidity $D_{xy}$ can be determined from Eq. (3.5.2.7) in which,

$K$ is estimated from Eq. (3.5.2.5);

$D_r, D_1$ are estimated from Eqs. (3.5.1.5); and

$D_{xy}$ is estimated from Eq. (3.5.2.8).
CHAPTER IV

MATHEMATICAL FORMULATION FOR THE
EXPERIMENTAL STUDIES

4.1 Rigidities of Orthogonally Rib-Stiffened Slabs

The bending and twisting moments associated with an orthotropic slab structure can be expressed as (26),

\[ M_x = - (D_x W_{xx} + D_1 W_{yy}) \]
\[ M_y = - (D_y W_{yy} + D_2 W_{xx}) \]
\[ M_{xy} = D_{xy} W_{xy} \]  \hspace{1cm} (4.1.1)

4.1.1 Flexural Rigidity

Assuming that in laboratory test arrangements (3), it is possible to apply pure bending moments \( M_x \), \( M_y \), and a pure twisting moment \( M_{xy} \), then for the case when only a pure bending moment \( M_x \) is applied to a slab specimen and \( M_y = M_{xy} = 0 \), Eqs. (4.1.1) yield,

\[ W_{xx} = S_{xx} M_x \]
\[ W_{yy} = - S_{xy} M_x \]  \hspace{1cm} (4.1.2)

in which,

\[ S_{xx} = D_y / (D_1 D_2 - D_x D_y) \]
\[ S_{xy} = D_2 / (D_1 D_2 - D_x D_y) \]  \hspace{1cm} (4.1.3)

21
Similarly, when only $M_y$ is applied and $M_x = M_{xy} = 0$,

$$W_{yy} = S_{yy}M_y$$

$$W_{xx} = - S_{yx}M_y$$  \hspace{1cm} (4.1.4)

in which,

$$S_{yy} = D_x / (D_1D_2 - D_2D_y)$$

$$S_{yx} = D_1 / (D_1D_2 - D_2D_y)$$  \hspace{1cm} (4.1.5)

Solving Eqs. (4.1.3) and (4.1.5) yields (3),

$$D_x = S_{yy} / (S_{xy}S_{yx} - S_{xx}S_{yy})$$

$$D_y = S_{xx} / (S_{xy}S_{yx} - S_{xx}S_{yy})$$  \hspace{1cm} (4.1.6)

Since in the specimens tested, the cross section has the same properties in $x$- and $y$- directions. So, from Eqs. (3.4.1.4),

$$D_1 = D_2$$  \hspace{1cm} (4.1.7)

Substituting Eq. (4.1.7) in Eqs. (4.1.3) and (4.1.5) yields,

$$S_{xx} = S_{yy} = S_1$$

$$S_{xy} = S_{yx} = S_2$$  \hspace{1cm} (4.1.8)

Substituting Eqs. (4.1.8) in Eqs. (4.1.2) and (4.1.4) yields,

$$W_{xx} = S_1M_x$$

$$W_{yy} = - S_2M_x$$  \hspace{1cm} (4.1.9)

and,

$$W_{yy} = S_1M_y$$

$$W_{xx} = -S_2M_y$$  \hspace{1cm} (4.1.10)
Also, substituting Eqs. (4.1.8) in Eqs. (4.1.6), the flexural rigidity can now be written as,

\[ D = \frac{S_1}{(S_2^2 - S_1^2)} \]  

(4.1.11)

Thus, by applying a known moment \( M_x \) (\( M_y \)) to the slab specimen and measuring the deflections of the surface of the slab at the grid points shown in Fig. 5.2a, the curvatures can be calculated as will be seen in Eqs (6.1.1) and (6.1.2). Hence, the values for \( S_1 \) and \( S_2 \) can be calculated from Eqs. (4.1.9) (Eqs. (4.1.10)). Then, the flexural rigidity \( D \) can be determined from Eq. (4.1.11).

### 4.1.2 Torsional Rigidity

To deduce the torsional rigidity \( D_{xy} \) (3), the specimen is subjected to a known twisting moment, \( M_{xy} \). Since the twist of the surface \( W_{xy} \) cannot be determined directly, Mohr’s circle for curvature is used in this determination. Consider a set of rectangular axes, \( r- \) and \( t- \) inclined to the \( x- \) and \( y- \) axes by an angle \( \theta \) (Figs. 3.1 and 5.2b). The curvature in the \( r \) and \( t \) direction at a point on the specimen can be expressed in terms of the known curvatures and twist at the same point along \( x- \) and \( y- \) axes. These relationships are given by (4),

\[ W_{rr} = W_{xx} \cos^2 \theta + W_{xy} \sin 2\theta + W_{yy} \sin^2 \theta \]

\[ W_{tt} = W_{xx} \sin^2 \theta - W_{xy} \sin 2\theta + W_{yy} \cos^2 \theta \]  

(4.1.12)

where,

\( W_{rr}, W_{tt} \) are the curvatures in the \( r- \) and \( t- \) directions.

If \( \theta = 45^0 \), Eqs. (4.1.12) yields,

\[ W_{xy} = \frac{(W_{rr} - W_{tt})}{2} \]  

(4.1.13)
Thus, assuming that a pure twisting moment $M_{xy}$ is applied to the slab specimen, the deflections at the grid points shown in Fig. 5.2b of the deformed surface of the slab are measured. From that, the curvatures in the r, and t directions can be calculated, then, the value of the surface twist $W_{xy}$ can be found as will be seen in Eq. (6.1.3). Hence, the torsional rigidity $D_{xy}$ can be determined from the last of Eqs. (4.1.1).
CHAPTER V

EXPERIMENTAL INVESTIGATION

5.1 Scope Of the Experimental Program

The experimental program consisted of tests on six orthogonally rib-stiffened reinforced concrete slabs. These tests were classified into two series of test specimens. The first series consisted of three rectangular in plan specimens for bending tests where the spacing of stirrups was varied. The second series consisted of three squares in plan specimens for pure twisting tests with varying spacing of stirrups.

The above tests were intended to predict the flexural and torsional rigidities during the pre-cracking and post-cracking stages when using shear reinforcement (stirrups), and to observe the structural response of bending and twisting specimens to loads. The mathematical formulations developed in Chapter IV were used to calculate the flexural and torsional rigidities from the experimented results.

5.2 Materials

5.2.1 Concrete

High Early Strength Portland cement CSA, type 30, manufactured by Canada Cement Company was used in all the slab specimens. The maximum size of the aggregate was restricted to 6 mm due to the narrow dimensions between the sides of the formwork.
The combined aggregate was prepared according to (1). This gave a well-graded combined aggregate with a fineness modulus equal to 2.56. A concrete mix design example is given in Appendix A. Natural tap water was used in the concrete mix. Water-cement ratio of 0.4 was selected to achieve the required 28-day concrete strength of 42 N/mm². Mixing of the concrete was performed in an Eerich Counter Current Concrete Mixer, Model EA2 (2W) with a capacity of five cubic feet. The concrete cylinder strengths of the specimens tested are given in Table 5.1.

5.2.2 Reinforcement

Plain mild steel wires of 6 mm diameter were used as main reinforcement for flexure. The wires were cut, cleaned from rust at some spots wherever that was necessary by using a metal brush in such a way to keep the bond forces be developed on the interface between concrete and steel. These bond forces are essential to prevent slip from occurring at the interface (21). Then, the wires were fixed in the formwork. Plain mild steel wires of 3 mm diameter were used for stirrups and for temperature reinforcement. The modulus of elasticity was 200,000 N/mm².

5.3 Description Of The specimens

A total of six specimens were tested for the bending and twisting tests. All specimens had a continuous slab with a thickness of 38 mm and the depth of the ribs was 115 mm. The width of the ribs was 50 mm except the external ribs where it was 75 mm. The ribs were placed at 280 mm spacing from center to center in both directions, and 290 mm to the
external ribs.

In each case, the first layer of steel was placed at 10 mm clear cover, and the second layer orthogonal to the first, was placed just over the first layer. Geometric description and properties of these specimens are given in Table 5.1. These specimens were divided into two series A and B. Each series had three specimens.

In series A for the bending tests, the specimens were 2340 mm x 1500 mm in plan. Closed stirrups were used with the following spacing:

For specimen A-1: no stirrups were used.

For specimen A-2: $S_t = 140$ mm.

For specimen A-3: $S_t = 70$ mm.

In series B for the twisting test, the specimens were 1500 mm x 1500 mm in plan. Closed stirrups were used with the following spacing:

For specimen B-1: no stirrups were used.

For specimen B-2: $S_t = 140$ mm.

For specimen B-3: $S_t = 70$ mm.

The following steel reinforcement was used:

Two wires ($\phi = 6$ mm) were placed at the lower corners of the ribs in both directions for all specimens. Two wires ($\phi = 3$ mm) were placed at the upper corners of the ribs in the slab deck for all specimens to serve as temperature reinforcement as well as to hold the stirrups. Closed stirrups ($\phi = 3$ mm) were used for specimens A-2, A-3, B-2, and B-3.
5.4 Casting Of the Specimens

All the specimens were cast in forms made of 19 mm thick plywood. The voids between the ribs were made by gluing styrofoam blocks to the plywood form following a marked pattern on the wood. The reinforcement was then placed between the styrofoam blocks in both directions. For the bending specimens the reinforcement in the longer direction was placed first on small wooden blocks to keep it 10 mm from the bottom of the forms. This provided for the minimum cover required for the steel. The steel in the shorter direction was then placed on the top of the reinforcement in the longer direction. Both layers of reinforcement were tied together with thin wire for stability during casting of the concrete. The stirrups were formed to the required shape and placed in specimens A-2, A-3, B-2, and B-3. The wire mesh was then placed and fixed properly. Fig. 5.1 shows the preparation of some specimens.

The concrete was then cast, tamped, and vibrated. After that, the top surface of the concrete was given a smooth final finish by hand trowel. All the specimens were moist cured for a period of 14 days. To determine the compressive strength of the concrete, three 152 x 305 mm (6 x 12 inches) concrete cylinders were cast with each specimen. These cylinders were subjected to a compression test at the time of testing of the test specimen.

5.5 Instrumentation

The deflections of the concrete specimens for bending and twisting tests were measured by means of mechanical dial gauges with a sensitivity of 0.025 mm. All the dial gauges were placed at the surface of the ribs, and supported by a steel frame clamped unto
the laboratory test floor. The locations of the dial gauges for each series is shown in Fig. 5.2.

A T-shaped loading frame was made for the bending tests and a straight loading frame was fabricated for the twisting tests. A bearing plate was welded to the legs of the T-shaped frame to transmit the load to the specimen in a predetermined position. The loading frames used for the bending and twisting tests are shown in Figs. 5.8 and 5.9, respectively.

A Universal flat load cell having a capacity of 113500 N was used for all models to measure the vertical load applied by means of a hydraulic jack at the center of loading frame. The calibration of the load cell is given in Fig. 5.10.

Concrete strains were measured on the surface of the deck slab by using electric strain gauges. Each 30 mm in length, type KFG-30-120-C1-11 with an average resistance of 120.2 ohms and a gauge factor of 2.11. The concrete surface at the gauge locations was prefinished to mount the gauges in accordance with the requirements for concrete strain gauge application. After soldering the wires to the gauges, the latter were moisture-proofed. The gauges were then connected to the strain indicator, balanced and made ready for testing.

Strains in the reinforcing steel were measured in the longitudinal direction for the bending specimens and in both directions for the twisting specimens by using electric strain gauges. Each strain gauge was 2 mm in length type N11-FA-2-120-11 with an average resistance of 119.8 ohms and gauge factor of 2.12. The gauge locations on the steel wires were first made smooth by using fine sandpaper, cleaned by an acid cleaner and a base neutralizer, and furnished by a liquid accelerator, then, the electric strain gauges were glued
to the steel wires. The connecting wires were then connected to the strain gauges by using a light weight lead-based solder, the entire connections and the strain gauges were then coated with a quick drying latex coating and covered to protect them from moisture in the concrete mix as shown in Fig 5.5. An automatic strain indicator manufactured by Vishay Intertechnology was used to record the strains during the application of the vertical loading.

5.6 Experimental Set-up and Procedure

A three-point loading and a three-point support arrangement (Figs 5.11a and 5.11b) was adopted for applying a pure bending moment to the bending specimens. To apply a pure twisting moment to the concrete specimens, the latter were supported at two diagonally opposite corners with downward concentrated equal loads applied at the other two corners as shown in Figs 5.12a and 5.12b.

The mechanical dial gauges were located at twelve grid locations for the bending specimens (Figs 5.2a), and at sixteen grid locations for the twisting specimens (Fig. 5.2b). These deflections were measured at some distance from the loading and supporting disturbance zones. Concrete strain gauges were fixed on the surface of the deck slab at six locations in the longitudinal direction for the bending specimens (Fig. 5.3a), and at eight locations for the twisting specimens (Fig. 5.3b). The steel strain gauges were fixed at twelve locations in the longitudinal direction for the bending specimens (Fig. 5.4a), and at eight locations for the twisting specimens (Fig. 5.4b). The test set-ups for bending and twisting tests are shown in Figs. 5.13 and 5.14, respectively.

Deflections, concrete strains, and steel strains were measured during loading of the
concrete specimens. The loading was carried out in stages. As shown in Figs. 6.1 and 6.2, the abrupt departure from the initial linear load-deflection relationship indicates the cracking of the specimens, so a change in the stiffness properties. Loading was continued until failure of the specimen was reached.

5.7 Experimental Results

The experimental results of the pre-cracking and post-cracking flexural rigidities for the bending specimens are presented in Tables 5.2, where they are compared with the theoretical ones calculated from the existing equations.

Table 5.3 shows the experimental results of the pre-cracking and post-cracking torsional rigidities for the twisting specimens. These results were utilized in developing the proposed equations.
CHAPTER VI

DISCUSSION OF RESULTS

6.1 Flexural and Torsional Rigidities

The theoretical flexural and torsional rigidities of orthogonally rib-stiffened reinforced concrete slab structures are presented in Tables 5.2 and 5.3, respectively. The linear deflections derived from the dial gauge readings for the tested bending specimens are listed in Tables 6.1 and 6.2 for the pre-cracking and post-cracking stages, respectively, and for the tested twisting specimens are listed in Tables 6.3 and 6.4 for the pre-cracking and post-cracking stages, respectively. These linear deflections were obtained statistically by using a HEWLETT PACKARD 48G package. For all tested specimens, two linear regression lines for the pre-cracking and post-cracking stages were fitted to the measured load-deflection experimental results at each dial gauge location in order to obtain representative values where to be used in calculating the experimental rigidities. The load-average deflection of dial gauges at locations 6 and 7 for the bending specimens and that of dial gauge at location 7 for the twisting specimens were plotted and presented in Figs. 6.1 and 6.2, respectively. From these plots, the abrupt change in stiffness after cracking of the concrete is clearly evident. The load-deflection behavior of the uncracked specimen appears to be elastic and linear. Furthermore, it is observed that the rate of change of the load-
deflection behavior of the specimen continues to be almost constant over a considerable range of loading. Deflections were measured up to a load of 43000 N for the bending specimen A-1, to a load of 50000 N for specimen A-2, and to a load of 54000 N for specimen A-3. They were measured up to a load of 14000 N for twisting specimen B-1, and to a load of 16000 N for specimens B-2 and B-3.

The bending curvatures were calculated from the linear deflections, \( w \), at the twelve grid points shown in Fig. 5.2a.

The longitudinal curvature was then calculated along line A-A from,

\[
2[w - (w_1 + w_4)/2]/a^3
\]

where,

\( w \) is the deflection at a particular grid point.

Similarly, the longitudinal curvatures along lines B-B, C-C, and D-D were also calculated.

Thus, the average longitudinal curvature was then calculated from,

\[
W_{yy} = 2[(w_5 + w_6 + w_7 + w_8)/4 - (w_1 + w_2 + w_3 + w_4 + w_9 + w_{10} + w_{11} + w_{12})/8]/a^2
\]

(6.1.1)

The transverse curvature along line E-E was calculated from,

\[
2[(w_2 + w_3)/2 - (w_1 + w_2 + w_3 + w_4)/4]/a^2
\]

Similarly, the transverse curvatures along lines F-F, and G-G were also calculated.

Thus, the average transverse curvature was then calculated from,

\[
w_{xx} = 2[(w_2 + w_3 + w_6 + w_7 + w_{10} + w_{11})/6 - (w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 + w_9 + w_{10} + w_{11} + w_{12})/12]/a^2
\]

(6.1.2)

Thus, by using Eqs. (4.1.10), since \( M_y \) was the applied moment in the tested bending
specimens, the constants $S_1$ and $S_2$ were found. Then, the experimental values of the flexural rigidities were calculated from Eq. (4.1.11).

The twisting curvatures were calculated from the linear deflection, $w$, at the sixteen grid points shown in Fig. 5.2b. For example, the curvature at point 6 along axis $t$ was calculated from,

$$[w_6 - (w_1 + w_{11})/2]/a^2$$

Also, the curvature at point 6 along axis $r$ was calculated from,

$$[w_6 - (w_3 + w_9)/2]/a^2$$

Then, based on Eq. (4.1.13), the twisting curvature at point 6 was determined from,

$$(w_1 + w_{11} - w_3 - w_9)/4a^2$$

Similarly, the twisting curvatures at points 7, 10, and 11 were also calculated.

Thus, the average twisting curvature was calculated from,

$$W_{xy} = [(w_1 + w_3 + w_5 + w_6 + w_{11} + w_{12} + w_{15} + w_{16}) - (w_1 + w_5 + w_6 + w_{11} + w_{15} + w_{16})]/16a^2$$

(6.1.3)

Computing the twisting curvature $W_{xy}$ from Eq. (6.1.3) for the applied torsional moment $M_{xy}$, the experimental torsional rigidity $D_{xy}$ was then calculated from the last of Eqs. (4.1.1).

The experimental values of the flexural rigidities $D_x \ (= D_y)$ for the three bending specimens, given in Table 5.2, show that the presence of stirrups has an insignificant effect on both the pre-cracking and post-cracking flexural rigidities. They were also compared with the theoretical ones calculated from the existing equations (3.4.1.1) and (3.5.1.1) for the pre-cracking and post-cracking flexural rigidities, respectively, where these existing equations
take no account for the presence of stirrups. From the comparison in Table 5.2, the close agreement between the theoretical values and the experimental ones is clear where the discrepancy between them ranges from 3% to 7%. Based on that, the existing equations for the pre-cracking and post-cracking flexural rigidities were not modified.

The experimental values of the torsional rigidities $D_{xy} (= D_{yx})$ for the three twisting specimens are given in Table 5.3. They show that the pre-cracking torsional rigidity for specimen B-2, with spacing of stirrups $S_i = 70$ mm, increased by 34% and the post-cracking torsional rigidity increased by 91% when compared to those for specimen B-1 where no stirrups used. Also, the pre-cracking torsional rigidity for specimen B-3, where $S_i = 70$ mm, increased by about 100% and the post-cracking torsional rigidity increased by about 200% than those for specimen B-1. This indicates that the presence of stirrups has significant effect in increasing the torsional rigidity of the structure.

The experimental results, shown in Figs 6.7 and 6.8, of the twisting specimens represent the pre-cracking and post-cracking torsional rigidity ($D_{xy}^{'})$ due to stirrups with respect to the volume of the stirrups per unit length. The results show the significant influence of stirrups in increasing the torsional rigidity. That is also evident in studying load-deflection, load-concrete strain, and load-steel strain, which are presented in Figs 6.2, 6.4, and 6.6, respectively. From these results, one can observe that the presence of stirrups reduced the deflections and strains.

For the bending specimen, Figs. 6.1, 6.3, and 6.5 represent load-deflection, load-concrete strain, and load-steel strain, respectively. They indicate that presence of stirrups has an insignificant effect.
The experimental results shown in Figs. 6.9 and 6.10 represent the torsional constant due to stirrups versus volume of stirrups per unit length for pre-cracking and post-cracking stages, respectively. Based on these experimental results, new expressions “shown in Eqs. (3.4.2.11) and (3.5.2.9)” were developed in order to account for the torsional rigidity due to stirrups. The proposed equations (3.4.2.9) and (3.5.2.7) were utilized to replace the existing equations in calculating the theoretical values of the pre-cracking and post-cracking torsional rigidities by considering the presence of stirrups. These theoretical values are shown in Table 5.3.

It should be noted that creep affects the rigidities of concrete structures. The influence of creep can be taken into account by adjusting the modulus of elasticity of the concrete $E_c$ to a time-dependent modulus of deformation $E_e$ (23). Furthermore, creep of concrete also influences the modular ratio $n$. To allow for this long-term effect, it has been suggested (9) that a value of $n=10$ be used. Analytical studies (13) of using the theoretical expressions for bending and torsional rigidities reveal that there is a good agreement between the theoretical and experimental results for deflections and moments for tests on orthogonally rib-stiffened reinforced and prestressed concrete slabs.

### 6.2 Structural Response of Specimens to Loads

For all specimens, the loading was continued until failure of the specimen was reached.

In bending specimen A-1 where no stirrups used, flexural cracks appeared on the
surface of ribs in the transverse direction over and between lines of supports at a load of 18000N (Fig. 6.11). Then, a longitudinal crack was observed at a load of 27000 N in the deck slab under the concentrated load applied at the middle of the edge, where it extended longitudinally up to the middle support. It is clear that this longitudinal crack appeared in a location where both bending moment and shear force are large. Upon increasing the load further, a sudden failure occurred at a load of 45000 N along the initial longitudinal crack.

Bending specimen A-2, with stirrups spaced by $S_s = 140$ mm, behaved initially as specimen A-1 for the transverse cracks (Fig. 6.12). The longitudinal crack was also observed (Fig. 6.13). This figure shows that the flexural tension crack was formed first, then it followed an inclined direction. This crack is known as a flexural-shear crack. The loading was continued until an ultimate load of 52000 N was reached. The failure was ductile due to the shear resistance from the stirrups. This kind of desirable failure mode, where large values of shear and moment are present, is mainly due to the presence of stirrups. The stirrups appear to restrain the growth of inclined cracks leading to their yield, thereby creating ductility.

Bending specimen A-3, with stirrups spaced by $S_s = 70$ mm, behaved almost the same as specimen A-2 (Fig. 6.14). However, the longitudinal crack under the concentrated load and its inclination was not pronounced as that in specimen A-2 (Fig. 6.15). Its penetration into the compression zone was reduced considerably, with proportional uncracked concrete available at the head of the crack for resisting the combined action of shear and compression. That kind of behavior is reflected in attaining a higher ultimate load of 56000 N as shown in Figs 6.3 and 6.5.
In twisting specimen B-1 where no stirrups used, cracks appeared along the support line at the surface of the ribs at a load of 9000 N (Fig. 6.16). Also, cracks inclined about 45° were observed at the sides of the specimen (Fig. 6.17). These inclined cracks were formed after the diagonal tension stresses exceeded the tension resistance of the concrete. The loading was continued until a sudden failure occurred at a load of 15000 N at the same inclined crack (Fig. 6.17). It is clear that the inclined failure crack shown on the side of the specimen is due to the diagonal tension stresses resulting from the applied twisting moment. That sudden failure is more likely due to the absence of any shear reinforcement (in the form of stirrups) in the specimen.

In twisting specimen B-2 with stirrups spaced by $S_t = 140$ mm, cracks as in specimen B-1 appeared along the support line (Fig. 6.18). Also, inclined cracks at the sides of the specimen were also observed (Fig. 6.19). The loading was continued until failure of the specimen occurred at an ultimate load of 17000 N. The failure of specimen B-2 was ductile since large deformations were observed at the ultimate load (Fig 6.19). Such desirable ductile behavior is due to the presence of the closed stirrups, which actually yielded before failure of the specimen.

Twisting specimen B-3, with stirrups spaced by $S_t = 70$ mm, behaved as specimen B-2 where the cracks shown in Figs. 6.20 and 6.21. However, the specimen failed at an ultimate load of 18000 N. The failure was ductile due to yielding of the stirrups.
6.3 **Sources of Error**

Tables 5.2 and 5.3 show the standard deviations (S.D.) of the experimental data for the flexural and torsional rigidities, respectively.

From table 5.2, the discrepancies between the experimental and the theoretical values for the flexural rigidity can be attributed to the following sources of error:

1. The assumptions made in the theory to formulate the theoretical expressions.
2. Positioning of the reinforcement in the ribs.
3. Estimates of the strength of concrete from the tests on 6 x 12 inches cylinders.
5. Deformation and inaccurate positioning of the loading frame.
6. Inaccuracies in the reading of deflections from the dial gauges.
7. The calibration of the load cell.

However, notwithstanding the above discrepancies, the comparison between the theoretical values based on the existing equations and the experimental results for the flexural rigidity can be considered good enough from an engineering point of view.

In table 5.3, the experimental results were utilized for developing the proposed equations for the pre-cracking and post-cracking torsional rigidities.
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The overall objectives of this study were to conduct studies on the effects of shear reinforcement (stirrups) on the rigidities of orthogonally rib-stiffened reinforced concrete slab systems, and on their structural behavior. Experiments were carried out to predict flexural and torsional rigidities of orthogonally rib-stiffened reinforced concrete slabs, and to observe structural response of such structures to loads.

To this end, on the basis of an idealization of integrally rib-stiffened slabs to orthotropic slabs of uniform thickness, proposed equations for the pre-cracking and post-cracking torsional rigidities were derived to account for the presence of stirrups, and the ductile behavior of such structural systems when using stirrups was observed.

Based on the results obtained from the theoretical and experimental studies, the following conclusions are drawn:

1. The presence of stirrups has an insignificant effect on increasing the flexural rigidity of orthogonally rib-stiffened reinforced concrete slabs as expected.

2. Based on the experimental results of bending tests, the existing equations for determining the pre-cracking and post-cracking flexural rigidities of orthogonally rib-stiffened reinforced concrete slabs were not modified.

3. The presence of stirrups has a significant effect on increasing the pre-cracking and
post-cracking torsional rigidities of orthogonally rib-stiffened reinforced concrete slabs, thus enhancing the gross stiffness of such structures subjected to eccentric loading conditions.

4. Based on the experimental results of twisting specimens, proposed equations for estimating the pre-cracking and post-cracking torsional rigidities of orthogonally rib-stiffened reinforced concrete slabs were developed to account for the presence of stirrups.

5. The complete absence of shear reinforcement (stirrups) in the ribs can result in a sudden failure of the structure without warning. That is especially when the structure is subjected to large values of shear and moment or shear stresses due to eccentric loading.

Therefore, it is a good practice to provide a minimum amount of shear reinforcement (stirrups) in rib-stiffened reinforced concrete slab structures even if the strength calculation does not require it. Such reinforcement restrains growth of the inclined cracks which leads to yielding of the stirrups, thereby creating ductility of the structure and providing warning in the case of impending failure. The amount of minimum shear reinforcement is to be used according to the code of practice for beams and girders.

7.2 Recommendations for Future Research

The following suggestions are recommended for future research as an extension of this investigation:

1. Using shear reinforcement in non-orthogonally rib-stiffened reinforced concrete slab
structures.

2. Derive expressions for calculating the pre-cracking and post-cracking torsional rigidities for different diameters of stirrups.

3. For the flexural tests, length of the specimen should be more than twice its width in order to insure sufficient test area between the supports for the measurement of deflections as well as a better distribution of loads.
REFERENCES

1. American Concrete Institute, “ACI Manual of Concrete Practice (1989)”. Detroit, Michigan.


BIBLIOGRAPHY


TABLES
### TABLE 5.1 GEOMETRIES AND MATERIAL PROPERTIES OF TEST SPECIMENS

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>$f'_e$ (N/mm²)</th>
<th>Plan Dimension ** (mm x mm)</th>
<th>Sectional Details* (mm)</th>
<th>Spacing of Ribs*** (mm)</th>
<th>Volume of Stirrups per Unit Length*** (mm³/mm)</th>
<th>Slab Details** (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>-47</td>
<td>2340 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>0</td>
<td>1500 2260 40 40</td>
</tr>
<tr>
<td>A-2</td>
<td>50</td>
<td>2340 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>19</td>
<td>1500 2260 40 40</td>
</tr>
<tr>
<td>A-3</td>
<td>-46</td>
<td>2340 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>38</td>
<td>1500 2260 40 40</td>
</tr>
<tr>
<td>B-1</td>
<td>59</td>
<td>1500 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>0</td>
<td>1500 N/A 40 40</td>
</tr>
<tr>
<td>B-2</td>
<td>-46</td>
<td>1500 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>19</td>
<td>1500 N/A 440 40</td>
</tr>
<tr>
<td>B-3</td>
<td>55</td>
<td>1500 x 1500</td>
<td>50 115 38</td>
<td>280</td>
<td>38</td>
<td>1500 N/A 40 40</td>
</tr>
</tbody>
</table>

* Refer to Figure 3.1. For the external ribs: $b_x = b_y = 76$ mm

** Refer to Figure 4.1

***$x$, $y$: width and height of stirrup, respectively.

$A_x$: area of one leg of stirrup.

$S_x$: Spacing of stirrups.
### TABLE 5.2 COMPARISON OF THEORETICAL AND EXPERIMENTAL FLEXURAL RIGIDITIES

<table>
<thead>
<tr>
<th>Bending Specimen</th>
<th>Pre-cracking Rigidity ($\times 10^7$ N.mm$^2$/mm)</th>
<th>Post-cracking Rigidity ($\times 10^7$ N.mm$^2$/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_x = D_y$ Theoretical*</td>
<td>$D_x = D_y$ Theoretical*</td>
</tr>
<tr>
<td>A-1</td>
<td>297</td>
<td>287</td>
</tr>
<tr>
<td>A-2</td>
<td>309</td>
<td>292</td>
</tr>
<tr>
<td>A-3</td>
<td>294</td>
<td>307</td>
</tr>
</tbody>
</table>

* No term to account for the presence of stirrups has been developed since the effect of stirrups on flexural rigidity is insignificant as is clear from the experimental values.

** S.D. = Standard Deviation
### TABLE 5.3 THEORETICAL AND EXPERIMENTAL TORSIONAL RIGIDITIES

<table>
<thead>
<tr>
<th>Torsional Specimen</th>
<th>Pre-cracking Rigidity $(x10^7 \text{ N.mm}^2/\text{mm})$</th>
<th>Post-cracking Rigidity $(x10^7 \text{ N.mm}^2/\text{mm})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{xy} = D_{yx}$</td>
<td>$D_{xy} = D_{yx}$</td>
</tr>
<tr>
<td></td>
<td>Proposed Equation</td>
<td>Experimental</td>
</tr>
<tr>
<td>B-1</td>
<td>43</td>
<td>41</td>
</tr>
<tr>
<td>B-2</td>
<td>55</td>
<td>57</td>
</tr>
<tr>
<td>B-3</td>
<td>84</td>
<td>85</td>
</tr>
</tbody>
</table>

* S.D. = Standard Deviation
### TABLE 6.1 PRE-CRACKING LINEAR DEFLECTIONS OF BENDING SPECIMENS

Pre-cracking Linear Deflection (x10^{-5} mm/N)

<table>
<thead>
<tr>
<th>Location</th>
<th>Specimen Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A-1</td>
</tr>
<tr>
<td>1</td>
<td>-1.175</td>
</tr>
<tr>
<td>2</td>
<td>-1.393</td>
</tr>
<tr>
<td>3</td>
<td>-1.248</td>
</tr>
<tr>
<td>4</td>
<td>-1.102</td>
</tr>
<tr>
<td>5</td>
<td>-0.733</td>
</tr>
<tr>
<td>6</td>
<td>0.291</td>
</tr>
<tr>
<td>7</td>
<td>0.442</td>
</tr>
<tr>
<td>8</td>
<td>-0.878</td>
</tr>
<tr>
<td>9</td>
<td>-0.145</td>
</tr>
<tr>
<td>10</td>
<td>0.218</td>
</tr>
<tr>
<td>11</td>
<td>0.145</td>
</tr>
<tr>
<td>12</td>
<td>-0.218</td>
</tr>
</tbody>
</table>

Note: Upward deflections are considered positive.
**TABLE 6.2 POST-CRACKING LINEAR DEFLECTIONS OF BENDING SPECIMENS**

Post-cracking Linear Deflection (x10^-5 mm/N)

<table>
<thead>
<tr>
<th>Location</th>
<th>Specimen Number</th>
<th>A-1</th>
<th>A-2</th>
<th>A-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>-2.383</td>
<td>-2.092</td>
<td>-2.355</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>-3.597</td>
<td>-3.709</td>
<td>-3.558</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>-3.340</td>
<td>-3.200</td>
<td>-3.732</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>-2.199</td>
<td>-1.941</td>
<td>-1.611</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1.136</td>
<td>1.102</td>
<td>1.432</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>2.171</td>
<td>2.568</td>
<td>1.981</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>2.199</td>
<td>1.326</td>
<td>1.511</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>1.097</td>
<td>0.990</td>
<td>1.360</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>-1.141</td>
<td>-1.141</td>
<td>-0.587</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>1.326</td>
<td>0.850</td>
<td>0.778</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>0.923</td>
<td>0.067</td>
<td>1.108</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>-0.224</td>
<td>0.476</td>
<td>1.354</td>
</tr>
</tbody>
</table>

Note: Upward deflections are considered positive.
**TABLE 6.3 PRE-CRACKING LINEAR DEFLECTIONS OF TWISTING SPECIMENS**

Pre-cracking Linear Deflection \( \times 10^{-5} \) mm/N

<table>
<thead>
<tr>
<th>Location</th>
<th>B-1</th>
<th>B-2</th>
<th>B-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.273</td>
<td>-7.245</td>
<td>-0.375</td>
</tr>
<tr>
<td>2</td>
<td>-22.026</td>
<td>-13.371</td>
<td>-4.627</td>
</tr>
<tr>
<td>3</td>
<td>-13.657</td>
<td>-17.534</td>
<td>-7.928</td>
</tr>
<tr>
<td>4</td>
<td>-18.284</td>
<td>-20.857</td>
<td>-10.350</td>
</tr>
<tr>
<td>5</td>
<td>-14.099</td>
<td>-11.167</td>
<td>-1.785</td>
</tr>
<tr>
<td>6</td>
<td>-10.350</td>
<td>13.215</td>
<td>-2.423</td>
</tr>
<tr>
<td>7</td>
<td>-16.521</td>
<td>-10.792</td>
<td>-5.729</td>
</tr>
<tr>
<td>8</td>
<td>-19.386</td>
<td>-16.544</td>
<td>-8.151</td>
</tr>
<tr>
<td>10</td>
<td>-15.419</td>
<td>-14.451</td>
<td>2.204</td>
</tr>
<tr>
<td>11</td>
<td>17.181</td>
<td>-11.609</td>
<td>-1.320</td>
</tr>
<tr>
<td>12</td>
<td>-14.099</td>
<td>-13.942</td>
<td>-3.967</td>
</tr>
<tr>
<td>13</td>
<td>-20.046</td>
<td>-13.019</td>
<td>-11.452</td>
</tr>
<tr>
<td>14</td>
<td>-18.060</td>
<td>-14.406</td>
<td>-10.132</td>
</tr>
<tr>
<td>15</td>
<td>-5.947</td>
<td>-11.385</td>
<td>-4.185</td>
</tr>
<tr>
<td>16</td>
<td>-5.729</td>
<td>-9.712</td>
<td>-0.878</td>
</tr>
</tbody>
</table>

Note: Upward deflections are considered positive.
TABLE 6.4 POST-TCRACKING LINEAR DEFLECTIONS OF TWISTING SPECIMENS

Post-cracking Linear Deflection (x10^{-5} mm/N)

<table>
<thead>
<tr>
<th>Location</th>
<th>B-1</th>
<th>B-2</th>
<th>B-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-29.736</td>
<td>-18.899</td>
<td>-8.767</td>
</tr>
<tr>
<td>2</td>
<td>-44.931</td>
<td>-24.449</td>
<td>-12.057</td>
</tr>
<tr>
<td>3</td>
<td>-64.535</td>
<td>-30.575</td>
<td>-25.808</td>
</tr>
<tr>
<td>4</td>
<td>-83.921</td>
<td>-39.297</td>
<td>-41.351</td>
</tr>
<tr>
<td>5</td>
<td>-45.155</td>
<td>-30.396</td>
<td>-17.332</td>
</tr>
<tr>
<td>6</td>
<td>-37.887</td>
<td>30.967</td>
<td>-14.675</td>
</tr>
<tr>
<td>7</td>
<td>-53.306</td>
<td>-31.280</td>
<td>-25.808</td>
</tr>
<tr>
<td>8</td>
<td>-79.736</td>
<td>-33.831</td>
<td>-26.239</td>
</tr>
<tr>
<td>9</td>
<td>-64.759</td>
<td>-42.598</td>
<td>-33.719</td>
</tr>
<tr>
<td>10</td>
<td>-53.743</td>
<td>-36.830</td>
<td>20.751</td>
</tr>
<tr>
<td>11</td>
<td>43.393</td>
<td>-31.851</td>
<td>-19.738</td>
</tr>
<tr>
<td>12</td>
<td>-42.072</td>
<td>-27.621</td>
<td>-15.990</td>
</tr>
<tr>
<td>13</td>
<td>-66.566</td>
<td>-58.017</td>
<td>-43.751</td>
</tr>
<tr>
<td>14</td>
<td>-59.030</td>
<td>-43.258</td>
<td>-37.501</td>
</tr>
<tr>
<td>15</td>
<td>-46.475</td>
<td>-32.863</td>
<td>-13.791</td>
</tr>
</tbody>
</table>

Note: Upward deflections are considered positive.
FIGURES
FIG. 3.2 CROSS SECTION AND METHOD OF PARTITIONING
FIG. 4.1 SCHEME OF LOADING AND SUPPORT SYSTEM FOR TEST SPECIMENS
FIG. 5.2 LOCATIONS OF DIAL GAUGES FOR TEST SPECIMENS

KEY:
- Support Points
- Loading Points
- Dial Gauge Locations
- a = 280 mm

a) Bending Specimen
b) Twisting Specimen
FIG. 5.3 LOCATIONS OF CONCRETE STRAIN GAUGES FOR TEST SPECIMENS

KEY:
- Support Points
- Loading Points
- Concrete Strain Gauges
  - $a = 280 \text{ mm}$

a) Bending Specimen
b) Twisting Specimen
FIG. 5.5 PREPARATION OF STEEL STRAIN GAUGES FOR TEST SPECIMENS
FIG. 5.6 CURING OF CONCRETE CYLINDERS
FIGURE 5.8 LOADING FRAME FOR BENDING SPECIMEN
FIGURE 5.9 LOADING FRAME FOR TWISTING SPECIMEN
FIG. 5.10 CALIBRATION OF THE LOAD CELL
FIGURE 5.11a TOP ISOMETRIC VIEW OF THREE-POINT LOADING AND THREE-POINT SUPPORT ARRANGEMENT FOR BENDING SPECIMEN
FIGURE 5.11b BOTTOM ISOMETRIC VIEW OF THREE-POINT LOADING AND THREE-POINT SUPPORT ARRANGEMENT FOR BENDING SPECIMEN
FIGURE 5.12a  TOP ISOMETRIC VIEW OF LOADING AND SUPPORT ARRANGEMENT FOR TWISTING SPECIMENT
FIGURE 5.12b SIDE ISOMETRIC VIEW OF LOADING AND SUPPORT ARRANGEMENT FOR TWISTING SPECIMEN
FIG. 5.14 TEST SETUP FOR TWISTING SPECIMENS

Scale: 1:20
FIG. 5.15 ARRANGEMENT OF DIAL GAUGES FOR BENDING SPECIMENS
FIG. 6.1  LOAD vs. DEFLECTION FOR BENDING SPECIMENS
FIG. 6.2 LOAD vs. DEFLECTION FOR TWISTING SPECIMENS
FIG. 6.3 LOAD-CONCRETE STRAIN RELATIONSHIP FOR BENDING SPICEMENS
FIG. 6.4 LOAD-CONCRETE STRAIN RELATIONSHIP FOR TWISTING SPECIMENS
FIG. 6.5 LOAD-STEEL STRAIN RELATIONSHIP FOR BENDING SPECIMENS
FIG. 6.6 LOAD-STEEL STRAIN RELATIONSHIP FOR TWISTING SPECIMENS
FIG. 6.7 PRE-CRACKING TORSIONAL RIGIDITY DUE TO STIRRUPS vs. VOLUME OF STIRRUPS
FIG. 6.8 POST-CRACKING TORSIONAL RIGIDITY DUE TO STIRRUPS vs. VOLUME OF STIRRUPS
FIG. 6.9 PRE-CRACKING TORSIONAL CONSTANT DUE TO STIRRUPS vs. VOLUME OF STIRRUPS
FIG. 6.10 POST-CRACKING TORSIONAL CONSTANT DUE TO STIRRUPS vs. VOLUME OF STIRRUPS
FIG. 6.11 VIEW OF THE TRANSVERSE CRACKS OVER SUPPORT LINES IN BENDING SPECIMEN A-1
FIG. 6.12  VIEW OF THE TRANSVERSE CRACKS OVER SUPPORT LINES IN BENDING SPECIMEN A-2
FIG. 6.14 VIEW OF THE TRANSVERSE CRACKS OVER SUPPORT LINES IN BENDING SPECIMEN A-3
FIG. 6.16 VIEW OF THE CRACKS ALONG SUPPORT LINE IN TWISTING SPECIMEN B-1
FIG. 6.17 VIEW OF THE INCLINED CRACKS ON SIDE OF TWISTING SPECIMEN B-1
FIG. 6.19 VIEW OF THE INCLINED CRACKS ON SIDE OF TWISTING SPECIMEN
FIG. 6.20 VIEW OF THE CRACKS ALONG SUPPORT LINE IN TWISTING SPECIMEN B-3
FIG. 6.21 VIEW OF THE INCLINED CRACKS ON SIDE OF TWISTING SPECIMEN B-3

Scale: 1/13
CONCRETE MIX DESIGN

A.4.1 General

Concrete mix design for the tested specimens was based on the guidelines stated in (1).

A.4.2 Design Parameters

- High Early Strength Portland Cement (Type 30)
- Fineness modulus for fine aggregate = 2.56
- Compressive strength $f'_c = 6000$ psi
- Slump = 3 in.
- Air Content = 0.0 %

A.4.3 Design Steps

Step (1) - Max. Size of Aggregate

- Max. Size < $b/5 < 2.5/5 < 0.5$ in. (13 mm)

Take max. Size = 1/4 in. (6 mm)

Step (2) - Estimation of Mixing Water Content
According to Table 5.3.3, (1), for slump = 3 - 4 in., nominal max. Size of aggregate = 1/4 in.;

\[ W = 385 \text{ lb/yd}^3 = 228 \text{ kg/m}^3 \]

**Step (3) - Selection of Water-Cement Ratio**

According to Table 5.3.4(a), (1), W/C = 0.4

**Step (4) - Calculation of Cement Content**

Cement = \( \frac{228}{0.4} = 570 \text{ kg/m}^3 \) (962.5 lb/yd^3)

**Step (5) - Estimation of Coarse Aggregate Content**

According to Table 5.3.6, (1), for fineness modulus of 2.56 and max. Size of 1/4 in. (6 mm), the coarse aggregate content = 0.484. As the rodded- weight of coarse aggregate = 1600 kg/m\(^3\) (2700 lb/yd\(^3\)), the dry weight of coarse aggregate = 0.484 x 1600 = 775 kg/m\(^3\) (1308 lb/yd\(^3\)).

**Step (6) - Estimation of Fine Aggregate Content:**

From Table 5.3.7.1, (1), the estimated weight of fresh concrete = 3840 lb/yd\(^3\) (2278 kg/m\(^3\)). Hence, fine aggregate = 2278 - (228 + 570 + 775) = 705 kg/m\(^3\)

**Step (7) - The Final Estimated Batch**

a) Water = 228 kg/m\(^3\) (14.3 lb/ft\(^3\))

b) Cement = 570 kg/m\(^3\) (35.6 lb/ft\(^3\))

c) Coarse aggregate = 775 kg/m\(^3\) (48.4 lb/ft\(^3\))

d) Fine aggregate = 705 kg/m\(^3\) (44.0 lb/ft\(^3\))

This leads to the following mix proportion,

\( W: C: CA: FA = (1: 2.5: 3.4: 3.1) \)
VITA AUCTORIS

JAMAL AL-AREF

The author was born on 01 July 1958, in Damascus, Syria. In 1975, he completed his high school education at “Ibn Alamied school”, Damascus, Syria. In May of 1981, he graduated from Damascus University with a degree of Bachelor of Civil Engineering.

From July 1981 to June 1995, he worked in several engineering companies in Damascus, Syria, as Structural Design Engineer, Project Engineer, and Project Manager.

In January 1996, he enrolled in M.A.Sc. Program in the Department of Civil and Environmental Engineering at the University of Windsor, Windsor, Ontario, Canada, and worked as a teaching assistant and research assistant. He prepared this thesis in partial fulfillment of the requirements for the degree of Master of Applied Science at the University of Windsor.