1972

The diffractographic method.

Timothy R. Pryor

University of Windsor

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THE DIFFRACTOGRAPHIC METHOD

A Thesis
Submitted to the Faculty of Graduate Studies through the
Department of Mechanical Engineering in Partial Fulfilment
of the Requirements for the Degree of
Doctor of Philosophy at the
University of Windsor

by
Timothy R. Pryor

Windsor, Ontario
1972
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ABSTRACT

This thesis concerns the development of a new dimensional measurement method, including several applications thereof. The disclosed "Diffractographic" Method utilizes changes in a slit type diffraction pattern formed between two members to determine changes in separation of the members as well as quantities related to said changes. Many advantages result, among them accuracy, range, long term stability, and a unique "line" displacement sensing capability.
ACKNOWLEDGEMENT

Thanks are first due to Dr. Walter North, without whose support and assistance most of this work would not have been possible. In addition, the invaluable assistance of Omer Hageniers is gratefully acknowledged, along with the support of Dr. Klaus Mielenz during the crucial early stages. Finally, thanks are due to my wife, Margaret, who typed the manuscript and offered moral encouragement at every turn.

Gratefully acknowledged is the financial support provided by the National Research Council of Canada (Grant No. 3360) and the U. S. National Bureau of Standards.
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NOMENCLATURE

Å Angstrom Unit (10^{-10} meters)
∆ Displacement
ε Strain
F Frequency (Hz)
f Spatial Frequency (Cycles/mm)
G Shear Modulus
γ "Z-Factor", Pattern Location Angle
Hp Horsepower
K Spring Constant of Lead Cell
k Spring Constant of Shaft
l Slit Length
L Effective Shaft Length (for angle of twist measurement)
Lg Gage Length
λ Wavelength
Δλ Wavelength Spread
n Diffraction Pattern Order Number
n_{max} Maximum number of orders observable using a given wave source
N Number of fringes passing a given angular position during displacement
N_r Rotational Speed (RPM)
N_{we} Number of fringes passing a given angular location for a given
N_{z} change in w_e (or z) with z (or w_e) held constant
P Force
Ø Angle of Incidence
Ø_t Angle of Twist
J Polar Moment of Inertia
Q  Beam Length
R  Distance From the Slit at Which the Diffraction Pattern is Observed
R₁  Shaft Centerline to Detector Radius
r  Shaft Radius
ρ  Distance From Shaft Centerline to Illuminated Region of Slit
s  Diffraction Pattern Fringe Spacing
Δs  Change in Diffraction Pattern Spacing
S  Moire Pattern Fringe Spacing
T  Torque
Θ  Diffraction Pattern Fringe Location Angle
u  Distance from Beam Fixed End
w  Slit Width
Δw  Change in Slit Width
w₀  Effective Slit Width in x Direction
w_max  Maximum Usable Slit Width
x  Distance from Diffraction Pattern Centerline
z  Spacing of Slit Boundaries in Direction of Laser Beam
1. INTRODUCTION AND HISTORY

1.1 Background

This thesis concerns an original concept for the measuring of
strain which has become a generalized displacement measuring technique
applicable in many diverse fields. The word "Diffractographic" has
been coined because it utilizes the changing diffraction pattern pro-
duced by changes in the width of a slit-type aperture irradiated by
electromagnetic waves.

Since the ideas presented are new, there is no directly applicable
literature. Accordingly, the chapter provides a history of the method's
development and expansion in scope, referring where apropos to related
work by others. This history also affords an insight into the various
applications explored to date for the technique. Finally, those who
have helped bring the method to fruition are mentioned in turn.

1.2 The Early Days

While an undergraduate student at The Johns Hopkins University, the
author became familiar with the work of James F. Bell of that university,
particularly his Diffraction Grating Strain Gage.\(^1\) Thus, while working
with gas lasers on a research assistantship at the University of Illinois
in the period 1962-63, it was realized that such devices could improve
Bell's method. Accordingly in 1964, a patent\(^2\) was filed covering improve-
ments and disclosing a related two-slit technique for strain measurement.
All further work on these ideas, however, ceased in January, 1965, when
the author went on active military duty.
1.3 The Basement Period

Upon return "from the war" in 1967, it was desired to determine whether the strain gage idea was indeed worthwhile. Not able to find anyone who would support such work, it became necessary to buy a laser, some lenses and other small items for experimentation in the basement after work selling computers.

A comprehensive search was begun of all conceivable techniques for measuring strain using light interference effects between two or more scratches on a specimen surface. While doing so, it was found that simple single slit-type diffraction from a single scratch could also be used. This technique had two advantages—it was the simplest, and it had the shortest gage length.

In the process of experimentation considerable difficulty was experienced in making scratches capable of producing intelligible diffraction patterns, and to simulate their effect, two expedients were tried. First, a wire stretched from the ceiling using barbell weights was illuminated by the laser and the effect on the wire's diffraction pattern due to contraction of its diameter observed—at least until the wire broke and the weights fell on the floor.

The second technique used was to produce a slit-type aperture using the edges of two separate objects, varying their relative positions to cause changes in slit width. A photograph of this experiment is shown in fig. 1, and its implications have formed the basis for the ideas represented in this thesis.

By early spring of 1968, it was evident that the new diffraction ideas were worth pursuing in earnest, and required better equipment than one could hope to have in a basement laboratory. Accordingly, inquiries were made as to possible backing for such work.
The only offer of unequivocal support found was that of Dr. Walter North of the University of Windsor. A friend from University of Illinois days, he believed in the idea. The author, therefore, arrived in Windsor in September, 1968, having filed a patent on the ideas shortly before with the help of Walter Gillis and Andrew Taylor of Larson, Taylor and Hinds, attorneys in Washington, D.C. This patent application, now granted, covered measurement of dimensional changes in wires and other arbitrary apertures, as well as the displacement of two members.

1.4 The First School Year

This first period at the University of Windsor was rather non-productive (at least in the light of future developments), perhaps due to an over-concentration on "scratch type" diffractographic strain gages, which present considerable difficulties in both manufacture and measurement.

1.5 Summer, 1969

In April, 1969, a photomultiplier system ordered months previously to measure minute diffraction pattern changes (characteristic of strained scratches) had not been received. Since the manufacturer was very vague on delivery, it appeared advantageous to look for an alternative location to do the summer's research.

After considerable searching, the author obtained summer employment at the U.S. National Bureau of Standards, working with Dr. Klaus Mielenz's group on Laser Wavelength Standards. The summer at NBS proved to be extremely profitable. Dr. Mielenz allowed work to proceed on the idea with a free hand, and Messrs. Art Funkhouser and John Mikesell helped in many ways to set up the very adequate equipment made available. In addition, access to the excellent library proved extremely helpful and allowed a thorough search of the literature for diffraction-based and other displacement measuring techniques.
The actual experiment conducted was a strain gage test using "multiplier" arms and edges formed by razor blades (see Chapter 7). The success of this two separate-member type transducer experiment proved a major milestone. Furthermore, the atmosphere of working in a building devoted to measurement of dimension and physical quantities inspired extension of the basic displacement technique to the measurement of such variables. In addition, the "z-factor" and "bounce-off" techniques (Chapter 2) were discovered. It was a very interesting summer indeed.

1.6 Fall, 1969

After the author returned to Windsor in the fall, Dr. North concurred that the displacement measurement aspects of the technique represented by far the most promising variation and that all future work would be directed toward the development of displacement-based diffractographic transducers.

Before any more novel transducer devices could be built, however, a new development occurred. In discussing techniques for experimentally verifying the theoretical results obtained by a colleague, Omer Hageniers (in his thesis work on cantilever plates), it became apparent that the results obtained during the summer could be applied to displacements of an edge or surface, with the diffraction pattern yielding information along the whole line at once. An experiment soon confirmed that the diffractographic technique indeed formed "worlds-only line displacement transducer." From this point on, Omer has played an important role in diffractographic development, invaluably assisting the author in most experiments conducted to-date.

During this fall period, the line detection capability was further explored, and a particularly interesting "time averaged" diffraction pattern version was demonstrated, using changes in slit width caused by
vibrating members relative to a fixed edge.

1.7 Winter-Spring, 1970

This period was one of intense development, and the following was accomplished:

1) Moire fringes were demonstrated using both "live" and "double exposure" patterns.

2) The first diffractographic load cell was constructed and successfully tested.

3) The first diffractographic torque transducer was also tested successfully.

4) Diffractograms were played back (presumably for the first time ever) as diffraction gratings, and superposed patterns of different spatial frequencies and rotational orientations were demonstrated.

5) The first test of diffractographic profile analysis was accomplished.

6) A two-slit ring type load cell was constructed and briefly evaluated.

1.8 Summer, 1970 through Winter, 1970

This period was spent evaluating data recording and processing effects of diffractograms. In addition, considerable effort was expended in a series of almost fruitless attempts to enlist government and industry support for diffractographic work.

Significant new technological developments unearthed during this period are:

1) Analog data record production and playback

2) Histogram, Probability Distribution Function and Displayed Trace aspects of analog diffractograms

3) Multi-channel correlation possibilities using electrically and mechanically driven slit arrays and diffractogram masks
4) Possible arithmetic computational capabilities revealed.

1.9 Spring and Summer, 1971

During this period a large number of papers were written and submitted to various journals. In the process of writing them up, many new profile measurement techniques were devised and a totally new type of "crossed fringe" bounce-off pattern was discovered (with Omer Hageniers). Also during this period, the first two contracts awarded on the basis of this work were obtained.

1.10 Fall and Winter, 1971

This period witnessed the first publications in Experimental Mechanics and Applied Optics, and the first long-term test of the "Diffracto" Strain Gage, which is still in progress at this writing (results look good, so far). New effects demonstrated during this period were the transmission of diffraction and interference patterns through dielectric waveguides of the SELFOC type, construction of the first waveguide interferometer, and new edge wave phenomena still under investigation.
2. THEORY

2.1 Background

As mentioned in Chapter 1, this work concerns the use of slit-type diffraction phenomena to measure the changes in separation between two edges. While this has included investigations into strain measurement using edges of reflective strips scratched or deposited onto members, the main thrust has been displacement measurement of one member relative to another. Virtually all practical uses fall into this category, which is emphasized in this thesis.

The theoretical basis of this work can be sufficiently described in simple terms and this simplicity is very largely responsible for its success. In the course of developing applications based on the simple theory, however, several new types of diffraction were found for which some uncomplicated explanations are offered (see sections 2.4 to 2.7).

2.2 The Basic Equations

fig. 2 shows the measurement of change in separation of two objects by a change in the far field of "Fraunhofer" diffraction pattern resulting from changes in the slit aperture formed between the objects. For monochromatic plane waves of wavelength \( \lambda \) (usually produced by a laser) incident on a slit of width \( w \) and length \( l \) \((l \gg w \gg \lambda)\), the far field \((R \gg w)\) diffraction pattern intensity distribution in the plane \( R \theta \) is given by the well known equation

\[
I = I_0 \frac{\sin^2 \beta}{\beta^2}
\]

(2.1)
where $I = \frac{\pi w^2 \sin \theta}{R^2}$ and $I_0$ is the intensity on the $\theta = 0$ axis.

The expression is therefore a rapidly decaying sine squared function with zeros or "minima" everywhere $\theta = \pi, 2\pi, 3\pi, \ldots$ i.e. when

$$w \sin \theta = n \lambda \quad (n = 1, 2, 3, \ldots)$$  \hspace{1cm} (2.2)

Since one is primarily interested in determining displacements, $\Delta w$, equation (2.1) is rewritten in terms of $w$ as 4

$$I = \frac{I_i w^2}{R^2} \frac{\sin^2 (\frac{\pi w \sin \theta}{\lambda})}{(\frac{\pi w \sin \theta}{\lambda})^2}$$  \hspace{1cm} (2.3)

where $I_i$ = incident light intensity. For any choice of $\theta$ and $\lambda$, we may define a constant $c = \frac{\pi}{\lambda} \sin \theta$. Using this, equation (2.3) becomes

$$I = \frac{I_i}{R^2} \frac{\sin^2 c w}{c^2}$$  \hspace{1cm} (2.4)

Intensity at a fixed $\theta$ value is therefore a sine squared function of $w$, and changes in this intensity can be related to displacements. To determine these changes, equation (2.4) is differentiated with respect to $w$, yielding

$$\frac{\partial I}{\partial w} = \frac{I_i}{R^2} \sin (2cw)$$  \hspace{1cm} (2.5)

From equation (2.5) several relationships are apparent:

A. For a given choice of $\lambda$ and $\theta$, $c$ is fixed and the change in intensity \( \Delta I \) for a displacement $\Delta w$ will be the same for all equal values of $\sin (2cw)$. 

B. Measurement of changes in \( w \) using the changes in \( I \) predicted by equation (2.5) is subject to fluctuations in source intensity and small changes in \( \Theta \) caused by mechanical rotations of the source relative to the detector.

C. Except over small regions, changes in \( I \) are non-linear with changes in \( w \).

2.3 Measurement of Pattern Fringe Movements

While none of the difficulties inherent in measurement of displacement from \( \Delta I \) mentioned in B. and C. above are insurmountable, a generally more attractive method is to utilize spatial changes between pattern reference points. Such spatial changes are linearly related to displacement and if slope reversal points are chosen (the maxima or minima) measurement is effectively independent of source intensity or detector circuit fluctuations.

When visual detection is used, the only practical technique is to use the minima positions as reference points. Such visual measurement is simple, inexpensive, and well suited to the "line" measurement techniques discussed in Chapter 8. For these reasons, it has been extensively used to date and will be emphasized in the following discussion.

A convenient way of measuring changes in \( w \) is to measure changes between two minima of the same order, i.e., located at \( \beta = \pm \pi \).

Doing so, automatically accounts for centerline shifts as well as small rotations of the incident illumination axis relative to the slit normal.

In the small angle region where \( \sin \Theta \approx \tan \Theta \), displacement, \( \delta \), can be expressed using equation (2.2) as,

\[
\delta = w - w_0 = n \lambda R \left( \frac{x}{x_0} - \frac{1}{x_0} \right)
\]
where \( w \) and \( w_0 \) are the initial and final slit widths and \( \alpha \) and \( \alpha_0 \) represent one-half the corresponding order separation distances for a given order \( n \). If a lens is used to form the pattern, \( R \) becomes its focal length.

An alternative relation can be used which is also independent of small center axis rotations. If a fringe frequency (e.g. fringes/mm) is defined as \( f = \frac{1}{\lambda} \) for the case of \( n = 1 \) in equation (2.6) above, then equation (2.6) in terms of \( f \) is

\[
S = \lambda R (f - f_0)
\]  

(2.7)

From this expression, it is apparent that displacement is linearly proportional to fringe frequency in the small angle region.

In virtually all practical cases, \( w \gg \lambda \) and most radiation is therefore concentrated in the region about \( \Theta = 0 \) (see equation 2.1). Thus most diffractographic experiments can be considered to have negligibly small non-linearity.

2.4 Range

In the basic fig. 2 apparatus, the maximum measurable slit width determined the maximum displacement, since minimum slit width is usually around 0.0001" because of mechanical positioning considerations. Several criteria are involved, the first two of which are obvious--i.e., that the laser or other wave source be of sufficient size to cover the slit (thereby posing experimental difficulties if ordinary gas lasers are used with \( w > .060" \)) and that a Fraunhofer type diffraction pattern exists (this latter criterion can always be obtained if a lens is used to form the pattern in its focal plane).
The second pair of criteria are wave source band width and the fringe resolution ability of the detection system. In the first case, the maximum number of fringes which can be unambiguously observed depends on the percentage wavelength spread of the source or, $\Delta \lambda / \lambda$. For gas lasers this value is around $10^{-6}$, implying a theoretical capability of producing one million fringes before any loss of fringe visibility occurs. For diode lasers, the value is around $10^{-3}$ and for light emitting diodes $5 \times 10^{-2}$.

The number of fringes producible $n_{\text{max}}$ is usually important only in count type detection systems using a detector located at a fixed $\Theta$ angle. In this case the maximum slit width is (from equation 2.2):

$$w_{\text{max}} = \frac{n_{\text{max}} \lambda}{\sin \Theta} \quad (2.8)$$

From the above expression, it is apparent that increased values of $w_{\text{max}}$ can be obtained for a given value of $n_{\text{max}}$, merely by decreasing $\sin \Theta$. However, the sensitivity of a count type system can be expressed by

$$N = (n' - n_0) = (w' - w_0) \frac{\sin \Theta}{\lambda} \quad (2.9)$$

where $N$ is the number of fringes (integer plus fractional) passing a photodetector located at the angle $\Theta$ when the slit width changes from $w_0$ to $w'$. Thus sensitivity decreases with $\sin \Theta$, and therefore with any attempt to increase range in this manner.

The major range limitation in most practical laser-based systems is fringe resolution. For example, if we hypothesize a typical laboratory
experiment using a He-Ne gas laser ($\lambda = 25 \times 10^{-6}$ Inches) and

$R = 100$ inches, then the fringe spacing $s$ produced by a slit of width

0.1 inches is, in the small angle region,

$$s = \frac{R \lambda}{w} = 0.025''$$

(2.10)

which is about the limit conveniently discernible by eye. More

sophisticated detection means can, of course, allow considerably higher

values of $w$ to be utilized.

To improve the usable range a longer wavelength (such as the $\lambda = 10.6\mu$

available from a CO$_2$ laser) can be employed and/or the oblique incidence

configuration used as shown in fig. 5. In this latter case one finds

from texts(4) the relation

$$\beta = \frac{\pi}{\lambda} w (\sin \Theta - \sin \Phi)$$

(2.11)

and for small regions about $\Theta = \Phi$ this reduces to

$$\beta = \frac{\pi}{\lambda} w \cos \Phi (\Theta - \Phi)$$

(2.12)

In this case the fringe spacing is multiplied by the value of $1/\cos \Phi$,

and larger ranges are therefore resolvable--at the cost of reduced sensi-
tivity. Several diffractograms taken at various $\Phi$ values are also shown

in fig. 5.

Equation (2.12) is related to the angular alignment tolerances

available in diffractographic experiments. In most cases we assume normal

incidence, i.e. $\Phi = 0$. From the equation we see that this assumption is
valid as long as $\cos \phi = 1$ - a condition reasonably satisfied over an angular spread of $\pm 5$ degrees. Thus the diffractographic system requires no particular alignment of light source relative to the slit, and the small incidence angles often used in deflection measurements (e.g. see section 8.2) can usually be neglected in calculation.

2.5 "Z-Factor"

In the process of doing the above oblique incidence experiment, it was noted that the fringe pattern becomes quite asymmetric and non-linearly spaced when $\phi$ becomes large - i.e., the simple textbook case of equation (2.12) no longer holds. Equation (2.11) however, can be rewritten using the notation of figure 4 as:

$$
\beta = \frac{\pi}{\lambda} \left[ w_0 \sin \gamma + z \left( 1 - \cos \gamma \right) \right]
$$

From this equation it is seen that $\beta$ is really a function of two variables, $w_0$ and $z$, and is of differing magnitude for equal (+) and (-) angles $\gamma$.

Some typical "Z-Factor" patterns are shown in figure 5. By utilizing equation (2.13), analysis of these patterns can give both of the orthogonal co-ordinates, $w_0$ and $z$, representing the position of one point relative to another in a plane.

2.6 "Bounce-Off" Patterns

In figure 6, another type of diffracting aperture is formed between edge and a surface. This arrangement allows one to gage changes in the separation between the two with either serving as a reference. The diffraction pattern produced by such an arrangement appears similar to that of a slit of twice the physical separation, since the edge can be considered as both
a real and a virtual diffracting object boundary.

There are several interesting aspects of such "bounce-off" patterns which are here discussed without recourse to sophisticated diffraction theory. The following simple explanation (due also to Omer Hageniers) appears to adequately predict experimental results and for practical measurement seems satisfactory.

Assuming the surface to be flat and sufficiently reflecting over the region where reflection is occurring, a virtual slit of width 2w is hypothesized, irradiated by plane waves incident at an angle $\Theta$. The resulting diffraction pattern minima are located at $\Theta$ values predicted by a variation of equation (2.11):

$$\pi \theta = \pi \frac{2w}{\chi} (\sin \Theta - \sin \Theta)$$  \hspace{1cm} (2.14)

For angles $\Theta$ of roughly 15 degrees or more, the equation is sufficient to allow displacement information to be calculated, and when $\Theta \lesssim \frac{1}{2}$, the problem reduces to consideration of a slit of width $2w\cos\Theta$ whose center line is $\Theta = \Theta$. However, as $\Theta$ is decreased, another effect becomes increasingly visible for which a tentative explanation is as follows.

2.7 Crossed Fringes

The "virtual slit" illustrated in figure 6 produces fringes at angles $\Theta < 0$ which cannot exist in the real case. However, if they were reflected or "folded back" into the region $\Theta > 0$ as shown in fig. 7, then in this case the two sets of fringes, a "normal" one containing fringes extending from $\beta = +\infty$ to $\beta = \frac{2w}{\chi} \sin \Theta$ at the $\Theta = 0$ surface, and a "reflected" set containing fringes in the range $\beta = \frac{2w}{\chi} \sin \Theta$ to $\beta = -\infty$, would occur in the same $\Theta$ locations and therefore interfere.
The hypothesis above predicts that in the case (fig. 7) where the surface \( \Theta = 0 \) corresponds to a minima of the "normal" pattern, the fringe systems are 180° out of phase at all points \( \Theta < \Phi \) and this produces distinct dark fringes perpendicular to the surface in the region where the intensities of the two systems are comparable. At those points \( \Theta > \Phi \), constructive interference occurs due to the 180° phase shift across the central maximum. The effect of these "crossed" fringes becomes increasingly visible as \( \Phi \) is reduced, since the reflected set progressively includes the regions closer to \( \Theta = 0 \), which contain the most diffracted intensity (see equation 2.1). For a decrease in \( w \), the intensity relative to the "normal" set increases as well.

Measurement of displacement in the presence of crossed fringes is discussed in a plate deflection example (chapter 8). Where the edge and surface remain parallel, being displaced perpendicular to themselves, the whole pattern becomes a crossed fringe, with the intensity of the total pattern on each side of \( \Theta = 0 \) alternately becoming lighter and darker for every displacement of \( 2w \sin \Theta \). The effect is quite noticeable for small angles \( \Theta \).

While the above equations were obtained by considering the surface to be perfectly flat and reflecting over the region contributing to the diffraction pattern, it is obviously of interest to consider relaxation of these requirements. More specifically, the requirement is that the surface be sufficiently reflecting to produce adequately visible fringes, and flat enough to allow the fraunhofer approximation to be used. With regard to the former, reflectivity is a function of both \( \Theta \) and surface roughness and material, and limits the maximum value of \( w \) which can be used. For example
it was determined that for a surface ground steel plate of roughness between 8 and 16 microinches CLA (determined by comparison with standard roughness samples), that \( w \) was limited to roughly 0.015 inches when an angle \( \varnothing \) of 3 degrees was employed.

The effect of surface curvature in the \( xz \) plane is to cause the portions of the incident wavefront striking the edge to lag or lead in phase the center portion, just as in the case of slit illumination by a diverging or converging wavefront. Fresnel type diffraction effects result, which have been observed. It appears that by using a convex lens, a Fraunhofer type pattern should still be attainable under these conditions.
3. WAVE SOURCE CONSIDERATIONS IN DIFFRACTOGRAPHIC DISPLACEMENT SENSORS

3.1 Background

Three basic criteria must be met by any wave source used in a
diffactographic device.

1. The emitted wavefront must be sufficiently space coherent across
the diffracting aperture such that recognizable pattern changes
can indicate edge displacements.

2. Its wavelength must be sufficiently monochromatic to produce
fringe patterns whose movements can be related to displacement.
In addition, where fringe-count systems are used, the number of
counts is limited by the source band width (i.e., wavelength
purity).

3. Source power must be adequate for the detection system employed.

Generally speaking, the success of the diffractographic technique
has resulted from the development of the gas laser, which fulfills all
the criteria above at moderate cost. Wavelength stability is extremely
high (deviation under .001%) and forms a solid basis for stable measure-
ment, for if a gas laser (He-Ne) operates at all, it does so to this
precision.

3.2 Diode Laser Considerations

If a semiconducting diode laser source is employed, immediate cost,
weight, space, power and presumably reliability advantages accrue. How-
ever, several additional problems must be considered:

1) If pulsed (as all commercial devices are at present), the use
of counting type detection systems is limited.

2) For an ordinary device, band width is at best 10 Å, or about
0.2% of wavelength, creating an upper limit to the number of
detectable counts.
3) Wavelength changes with temperature, as discussed in Chapter 6.

4) A lens must sometimes be used to collimate the diode laser beam—a rather easy task, at least to the extent required.

5) Pulsed diode lasers shift wavelength about \(8 \AA\) during each pulse. (Presumably the newly developed room temperature CW types\(^5\) will not exhibit such shifts).

6) Wavelength is presently in the infrared, making visual detection impossible. In addition, the 9100 \(\AA\) wavelength devices (commercially available ones at present all fall in this category) are poorly matched to available films.

An alternative to the diode laser is its non-lasing brother, the light emitting diode (LED). These operate CW with ease, but have band widths of 250–400 \(\AA\). The most severe restriction, however, is the shape of these devices which is usually a square, .015 X .015 inches or so. Since the LED is spatially incoherent, a point type source is required which in this case could be a very narrow line source parallel to the slit. Thus for most efficient use of these devices, it would be desirable to fabricate LEDs of dimension 0.001\" X 0.5\" say, which could be collimated into a usably space coherent wave front.

Another light source arrangement which shows promise is the use of fiber optic guides to convey laser light to (and/or diffraction pattern information from) the slit aperture.

3.3 Other Wave Sources

The visible and near infrared light sources discussed above are by far the most useful, due to their short wavelength, directionality (power/unit area), visibility, cost, etc. This is not to say, however, that other wave sources—both sonic and electromagnetic—cannot be employed.
The difficulty of propagating short wavelength ultrasonic waves in air severely limits the use of these sources in diffractography. X-rays and electron beams also exhibit diffraction effects and this could prove a fruitful area, though fraught with experimental difficulty. Perhaps the most useful of all the alternative sources will be the CO$_2$ laser operating at a wavelength of 10.6 microns.
4. EDGE AND SURFACE CONSIDERATIONS

4.1 Edges

Most of the diffractographic applications to date have utilized two edges to form a slit type aperture. Accordingly, it is only reasonable to ask what constitutes a suitable edge. The experiences to-date are described below:

4.1.1 Material

The edges may be of virtually any material. By far the most common to-date have been those materials which are opaque to the source radiation used. However, this is not a requirement. For example, one could use two semi-opaque object edges which would allow a coherent background wave to pass through.

4.1.2 Geometric Shape and Size

A large variety of edge geometrics can be used, as was demonstrated by Bennett and Harris for the single edge diffraction case. For example, they can come to a sharp point (as in a razor blade) or may possess large radii of curvature (as in a tin can surface). In the latter case, where large radii are used, unwanted coherent light is reflected by the surface into one side of the diffraction pattern. Such reflected "noise" may be reduced by painting the surface black (or the equivalent) and, in any case, meaningful information can usually be extracted from the unaffected portions of the pattern.

Regarding the size of the edges, they may be the natural corners of the largest of members or may be the tip of the thinnest of foils. Particularly convenient is the ability to use an actual edge of a small elastic member employed to transduce a variable by its displacement.
4.1.3 Edge Placement

When the line connecting the two edges is not substantially perpendicular to the incident wavefront (i.e., \( \cos \theta \) not approximately equal to unity) oblique incidence or "\( z \)-factor" effects result, as was previously discussed (section 2.5). Therefore, if \( \theta \) is assumed zero in such a case, errors may result. This effect seldom causes trouble except where very small slit widths (say under 0.001") are utilized. A particular example where it must be accounted for is in "time-averaged" vibration experiments (see section 8.5).

4.1.4 Edge Parallelism

Another alignment problem is edge parallelism. For example, if the right side of the illuminated part of the slit was of width \( w_r \) and the left, \( w_l = .99 w_r \), then the 99th diffraction fringe on the left would correspond to the 100th on the right, and this could prove confusing. This same effect occurs in a random fashion—when the edges are nicked.

One often used solution to the question of parallelism is to focus incident radiation onto the slit by means of a cylindrical lens, such that only one position is irradiated (which is presumably "clean").

4.1.5 Edge Movement

Virtually no force is required to displace one of the edges relative to the other, and this allows devices to be built utilizing mechanical multiplication. In addition, little or no reaction with a measured force can occur.

4.2 Surfaces

The "simple" version of the bounce-off technique discussed in section 2.6 assumed a surface which was flat and sufficiently reflecting
in the region of reflection and at the angle of incidence (\(\theta\)) employed. In effect, this restricts its use (in this form) to surfaces with very large radii of curvature and reasonably small surface roughness. (These two criteria are in fact somewhat dependent since the grazing angle required for rough surfaces require the largest radii of curvature).

Smaller radii of curvature have been observed to produce Fresnel-type diffraction bounce-off fringes, which one would expect due to the wavefront curvature caused by the convex mirror-like surface. This complicates the situation considerably and has not been studied in detail.

Local surface bulges or pits etc., produce distortions in the resulting diffraction pattern in much the same way as do nicks in the blade edges. The checking of surface depressions in this manner has been considered, but not shown to be effective, however. Particularly bothersome are the ubiquitous dust particles which act as phase and amplitude objects in the light path.
5. DETECTION OF DIFFRACTION PATTERN CHANGES

5.1 Background

Detection of pattern changes can be very simple, requiring only one's eye and a measuring scale with which to measure fringe positions. In general however, photoelectric detection is more desirable, and this subject has consumed at least one-half of effort expended to date. Literally hundreds of different detection techniques have been conceived, and a generalized discussion follows.

Once one tries to automate the eye's detection process, a myriad of decisions and trade-offs must be made, some of which are:

1) Speed of response. Can a mechanical adaptation of the eye-ruler technique (using a detector-LVDT combination, say) be made, or must detection occur at television speeds, for example?

2) Size. Can the detection apparatus be laboratory usable or must it be compressed into a small transducer case?

3) Reliability and environmental stability. Laboratory use is one thing, field use in transducers is quite another.

4) Type of response. The diffractographic displacement transducer may operate either analog or quasidigital modes, and each has its advantages.

5) Precision and cost. These are self explanatory and almost always trade off.

Once the detection criteria for a given job have been established, there are several categories of specific detection means which may be considered. While many have been used with other optical devices in the past, virtually all have had to be redesigned for the new situations encountered in diffracography.
5.2 Analog Systems

This category really should be labeled "null-balance" devices, since these appear to be the only practical fully-analog techniques. The term implies that one of the variables in the diffraction equation for fringe frequency or location (i.e., either $\lambda$, $f$, $x$, or $R$) is varied a known amount to cause an initial detector reading to return (after $w$ has undergone a change). Such a system can be very sensitive, just as wheatstone bridges are in the electrical case.

5.3 Quasidigital Systems

This category is quite promising and includes these two basic techniques:

5.3.1 Scanning.

In this case a detector is scanned through the pattern, or conversely the pattern is swept past a detector. The detector output frequency is then the product of fringe frequency ($f$) and a constant sweep rate (which can be divided out to yield $f$ alone). Either static or dynamic values of $w$ can be measured in this manner.

5.3.2 Counting.

This very simple case measures by counting the fringes (or portions thereof) which pass a fixed angular detector location. Since counts are produced only when a change in $w$ occurs, this technique is suitable only for changes in $w$ from a known value (usually zero). The main problem with pure counting types is the necessity for a large number of fringes to obtain adequate resolution. Not only must the source power and detector sensitivity be sufficient, but the maximum change in $w$ must be enough to allow the required number of fringes to pass by. Some fringe
"multiplication" techniques are currently under development.

5.3.3 Combination Scanning-Counting

A technique which combines the characteristics of both of the above methods utilizes a scanning device to sweep the diffraction fringes passing through an aperture extending to $\pm \theta$ past a detector. The output signal is then in counts representing the number of fringes (or maxima and minima) which could "fit" into the fixed angular aperture. Everything about this technique resembles the counting one previously described, except that the scanning means has allowed static values of $w$ to be determined (i.e., a change in $w$ is no longer necessary to produce counts).

5.4 Hybrid Systems

There are several promising techniques which can combine analog and digital methods. One example employs a single detector located at a fixed angle as in 5.3.2 above, using a slope-reversal count to determine the first two digits say, of a displacement display. The last digit, however, obtains from an A-D conversion of the detector voltage.

In another case, a detector is scanned across the pattern until 2, 4, 6 etc. minima are read, and the distance scanned gives the value of $x$ (or $s$) at a distance $R$. This can be analog (if measured with LVDT, say) or quasidigital (if a shaft encoder is used).

5.5 Recording Systems

A "delayed action" detection system can be constructed by directly recording the diffraction pattern on film and replaying it later. This situation will be discussed in more detail (section 7.5.2, Chapter 11). However, suffice it to say that the fringe frequency information can be recovered very neatly in an analogous manner to FM coded voltage signals.
on magnetic tape. These recording systems are therefore quasidigital in nature, and can yield pure analog or semidigital output information depending on the playback technique chosen.

5.6 Totalizing Systems

This is a variation on the quasidigital counting schemes above, whereby the count value is stored and the next value added to it such that after a period of time a certain total count signifies the quantity desired. There are two basic types—Type I which produces counts only when the variable itself changes (thereby producing a total count proportional to the multiple of change cycles times the change per cycle), and Type II in which the pattern is scanned to produce count outputs at given time intervals (or rather intervals controlled by an external source).

5.7 Two Variable Multipliers

The operation of the Type I totalizer in effect gives the product of two variables, number of incidents times average value per incident. However, a more general non-totalizing device can be built in the manner of the scanning detection devices where the scan rate is a variable as well as the slit width.

An excellent example of this is the Diffractographic Dynamometer which the scan rate is rpm dependent and slit width represents torque. The frequency output of the detector is then proportional to the product of slit width x rpm or horsepower—a desired quantity (see section 7.4).
6. SOURCES OF ERROR IN DIFRACTOGRAPHIC MEASUREMENT

6.1 Background

Generally speaking, the diffractographic technique provides one of the most stable and error-free high resolution transduction systems known. This can be deduced from equation (2.7) which can be written as

\[ w = R \lambda f \]  \hspace{1cm} (6.1)

where we note that given proper fraunhofer conditions, small angles \( \theta \) and normal incidence irradiation of the slit (no z-factor), there are only three variables involved. Discussed first are possible errors in these, followed by discussion of errors due to imperfect assumptions concerning edges or diffraction conditions, as well as assorted other causes.

6.2 Wavelength

In almost all diffractographic experiments, wavelength is assumed constant, and for the case of a gas laser source it effectively is (to within 3 parts in \( 10^6 \)). A diode laser however, exhibits positive wavelength shifts of approximately 2.5\( \AA/\degree C \) which for GaAs diodes operating at 9000 \( \AA \) means that a .028% error in slit width results for every degree centigrade difference between the temperature assumed for the diode and actual temperature. For most experiments this would be of little consequence. However, if employed in a practical transducer (eg. on a load cell), one would wish such errors to be automatically compensated for by the measuring system, (just as it is common practice in strain gage cells to account for temperature induced resistance changes).
Some compensation schemes which can be used are:

1) The linear thermal expansion of the edges.

2) Changes in elastic modulus with temperature of the elastic member to which the edge or edges are attached.

3) A reference slit whose width is effectively not a function of temperature can be used to monitor wavelength changes.

4) Thermal expansion in the member determining the slit to detection means distance, R.

Another error is the change in effective wavelength caused by index of refraction changes due to air pressure or temperature fluctuations. These are, however, extremely small and the effect is effectively negligible.

6.3 Errors in R

From equation (6.1) one sees that for a constant $\lambda$, changes in $w$ can be determined by changing $R$ to maintain the same $f$ or by determining $f$ with a constant $R$ assumed. Almost all experiments to date have utilized the latter technique and to do so, one must put a value on just how constant $R$ is.

While thermal expansion effects can change $R$ by a few parts in $10^5$ per degree centigrade, such errors are effectively negligible, relative to the percentage change in slit width measured. Therefore, the major error in $R$ is in its initial determination. Without much trouble it should be possible to attain 0.1% accuracy (1/100 inch in 10 inches) in such measurement, and much higher values can be obtained by sophisticated
techniques. In addition, most diffractographic transducer devices would initially be calibrated using known values of the measured variable, and such calibration would remove errors in R measurement.

6.4 Spatial Frequency and Other Pattern Changes

For gas laser sources in calibrated transducers, it is apparent from the previous discussion that changes in slit width directly result in fringe frequency changes, with \( R \) and \( \lambda \) remaining effectively constant, even over temperature excursions of 50°C or more. The question then is, with what accuracy can pattern changes be measured? Several diffractograms taken at different values of \( R \) and \( w \) are shown in fig. 8, illustrating the problem facing the experimentalist using this technique. (The different exposure levels simulate different laser power levels).

Direct visual determination of diffraction pattern changes implies measurement of minima movements (i.e. changes in \( x \) or \( s \)) since these are the only easily observed features. Accuracy therefore depends on two factors— the accuracy of linear measurement of \( \Delta x \) (or \( \Delta s \)), and \( \Delta x \) or \( \Delta s \) resolution ability. It is thus quite dependent on the individual situation, and a specific experiment is now described to give some idea of what can be accomplished.

An experiment was conducted using a 9 milliwatt He-Ne laser source and the naked eye to detect the 8th minima (\( n = 8 \)). The slit pattern was projected onto a ground glass screen and \( x \) measured using vernier calipers to an average of 5.660 ± 0.003 inches using 15 readings by two observers. This effectively establishes ± 0.003" as the error in \( x \) when observing a small change in slit width. For example, a slit change of .0001" (as indicated by a high resolution micrometer to which the edges were attached)
produced an x change of .047" which we can therefore say is resolved to ± .006". This implies visual measurement to 13 micro inches. A further discussion of pattern detection and measurement is included in reference 7.

6.5 Miscellaneous Error Sources

1) Alignment

If sin θ is not negligible in equation (2.11) and is assumed so, an error results. This is not to say that situations with appreciable values of sin θ cannot be utilized, as the z factor experiments demonstrate. Such errors usually occur when very small slit widths are employed and are a "necessary evil" in the time-averaged vibration experiment.

2) Vibration

This really implies a change of alignment due to vibratory causes or a change in slit width due to transient forces other than the variable measured. Care in construction or setup should eliminate its effects.

3) Corrosion, Wear, Dust and Dirt

As has been shown, the diffractographic method is a very sensitive measurement technique. Small corrosive pits or wear with time can therefore produce error signals, as can dust or dirt on the edges. This latter problem is particularly important in the bounce-off technique, but even in this case, proper design should eliminate its effects.
The following are small errors which must be accounted for in exacting experiments.

1) Local variations in index of refraction due to large thermal gradients cause a random lensing effect which produces fringe motions and overall pattern distortions. The effect is usually noticeable only with electronic detection, particularly when large R and w values are used.

2) Errors due to centerline shifts. When one edge is fixed with the other moving toward or away from it, the pattern centerline ($\Theta = 0$) is translated by an amount equal to half the maximum change $\Delta w$. In most cases this shift is quite small in comparison with the change $\Delta x$ measured at a large distance R, and in these cases the effect may be neglected. In addition, it plays no part at all if measurement of fringe spacing (i.e. the width of a particular order) or the distance 2x between fringes of like order is measured. Finally, if a lens is used to form the pattern, said pattern remains stationary in the focal plane regardless of slit position.

3) Failures in the Fraunhofer Equation Assumptions

This takes several forms. An obvious case is assumption of the small angle $\Theta$ approximation when it doesn't hold. Not so obvious are small changes due to the transition from Fraunhofer to Fresnel
diffraction conditions when R is small and no lens is employed. A final case is one unearthed in this research and concerns the effect of non-uniform illumination across the slit width (assumed uniform in the diffraction equations previously presented). This is a common occurrence when laser sources are used and the case of $\text{TEM}_{00}$ gas laser sources (by far the most common) is discussed in Appendix B.
7. SENSORS OF PHYSICAL QUANTITIES

7.1 Background

A large application area for any displacement transducer is the sensing of variables which can be converted to a displacement. Many types of restoring forces may be used to oppose the action of the variable, among them electrostatic and magnetic forces, gas pressures, centrifugal forces and gravity. Of this group, the latter offers perhaps the best potential for diffractographic applications as a replacement for the sensing means in analytical balances and other "no springs" type scales.

The predominant type of restoring force used in practical transducers today is that furnished by an elastic member, and now discussed are methods by which these may be employed with diffractographic displacement sensing means, following which several examples of experimental prototype transducers are presented.

The forces to be measured can be externally acting (such as a pressure applied to an elastic diaphragm by a liquid), or internally generated by strains due to temperature, applied voltage or the like. Due to the nature of the diffractographic technique, the elastic member may be of any material, thickness and size, and this lends a great deal of flexibility in the design of transducers.

7.2 A Diffractographic Strain Gage

7.2.1 Introduction

The original strain gage comprised of a diffracting strip or scratch on the specimen surface is difficult to apply and even if properly emplaced lacks the sensitivity needed to become a useful tool (except
in those rare instances where one must use its ultra-short gage length or lack of adhesive bonding). To improve both deficiencies, the version shown in figure 9 was devised—where gage length has been essentially "traded" for sensitivity. The effective linear gage factor of such an arrangement can be quite high, since a given strain changes the slit width \( w \) by the same value as it changes the gage length \( L_3 i.e. \Delta L_3 = \Delta w \). Defining gage factor in the traditional manner used for electrical strain gages, we see that

\[
\text{Gage Factor} = \frac{\Delta w/w}{\varepsilon} = \frac{1}{\varepsilon} 
\]

(7.1)

This strain multiplier effect is virtually the same as employed in capacitance gages, and for \( L_3 = 40 \text{ mm}, w = 0.04 \text{ mm} \), the diffractographic gage obtains an intrinsic linear gage factor of 1000—before any further multiplication by mechanical or optical means. From this exercise one also may visualize the sensitivity problem inherent in the diffractographic gage without the multiplier as well as the Bell gage and its derivatives—i.e. they have a linear gage factor of one.

Besides the large increase in sensitivity, another advantage is apparent from figure 9. If the "blades" are suitably affixed, we can therefore accurately measure strain under very unusual temperature conditions.

7.2.2 Experimental Arrangement

In order to experimentally evaluate the technique, a simple tensile test was used in which the naked eye and a ruler formed the readout means. Two diffracting edges were mounted using conventional strain gage adhesives to an aluminum specimen having a 76 mm long test section and 12.7 mm x 3.17 mm crosssection. While many types of edges are suitable for this purpose, razor
blades were used for their convenience and could have been attached to
the specimen by knife edges, welding, or any other suitable means. In
this case the edges were fixed such that a roughly parallel aperture of
some 0.145 mm was formed on either side of the model as the blades ex-
tended beyond the edge of the specimen on both sides. Due to the dimensions
of the razor blades, the gage length was 38.5 mm and the frosted glass
screen on which the diffraction pattern changes were observed was some 2.78
meters from the tensile specimen. A 5 milliwatt He-Ne laser was used as the
light source and an electrical resistance strain gage was mounted on the
specimen for comparison purposes.

7.2.3 Experimental Procedures and Results

At zero strain (as indicated by the electrical strain gage) an initial
measurement of the distance, 2x, between the two 8th order minima was taken
using a clear plastic ruler graduated in millimeters and taped to the
frosted glass screen. After loading the specimen to a given strain level
determined by the electrical gage (say 500 microstrain) the separation
between the 8th order minima was again measured. The resulting edge
displacement calculated using equation (2.6) was then related to strain by
equation (7.1). Figure 10 shows the average results of four tests performed
in this manner.

7.2.4 Discussion

Application of the razor blades to the specimen can be done by eye
with no special jigs or the like, to obtain apertures of 0.1 mm or less.
Though no attempt was made to optimize the detection method in this
experiment, it was nevertheless possible to observe changes of 20
microstrain by eye while agreeing at all times within 5% of the electrical resistance gage. If properly applied, the diffractographyc gage should be unaffected by humidity, temperature, radiation, transverse strain, time effects and most other variables affecting the output of electrical gages.

7.3 A Torque Sensor

7.3.1 Introduction

If the necessary slit edges are attached to rotors connected to opposite ends of an elastic shaft of known characteristics, then from equation (2.7),

\[ (f_1 - f_0) = \frac{1}{\lambda R} (w_1 - w_0) \]  

(7.2)

where the subscripts 0 and 1, respectively, represent initial (no torque) and final (with applied torque) values.

As is apparent from figure 11 the change in slit width \((w_1 - w_0)\) is expressable (using the small angle approximation) as \(\theta / \hat{\theta}\) where \(\hat{\theta}\) is the shaft angle of twist under torque and \(\theta\) is the mean distance from the shaft axis to the illuminated region of the slit. Using the expression for angle of twist in a circular shaft

\[ \hat{\theta} = \frac{TL}{GJ} = \frac{2TL}{\pi G r} \]  

(7.3)

where

\(L\) effective shaft length (i.e. between slit edge rotor connection point)

\(T\) applied torque

\(r\) shaft radius

\(G\) shear modulus of shaft

we can express equation (7.2) as
\[(f_1 - f_0) = \frac{\frac{pL}{4}}{kR} \]
\[(f_1 - f_0) = \frac{2pL}{GAR\pi R} \]

The change in fringe frequency measurable from the diffraction pattern is therefore linearly proportional to torque (as long as the small angle approximations remain valid).

7.3.2 Experimental Tests on a Simple Prototype

Figure 12 illustrates an operational device based on the above result, which reads once per rotational cycle, during the time the slit is in the laser beam. For convenience, two mirrors are used to bring the light in and out at right angles to the shaft. The dimensions and materials chosen resulted in the calibration curve of fig. 13, which is obviously linear.

The prototype was experimentally compared in the apparatus of figure 14 to a commercially available torque transducer of the electrical resistance strain gage type whose static calibration curve is shown in figure 15. Just as in the diffractographic static calibration test, a 1 milliwatt gas laser source was used and fringe frequency visually measured from a screen \((R = 75.1 \text{ inches})\) with vernier calipers. Fringe frequency change was then converted to slit width change and to torque through the calibration curve (fig. 13). Fringe patterns were recorded (over 20 seconds of operation at 1000 rpm) for different torques and in every case the resulting diffraction pattern is stationary–appearing flicker-free to the eye as well at this speed.

Readings of the two torque transducers at the same brake voltages
and at two different rpm values are plotted against each other in figure 16. The offset is no doubt due to friction in the bearings between the two, since the electrical resistance strain gage transducer was closest to the hydraulic motor. Any other disagreement between the two is explained by small percentage errors in reading the various parameters of each, together with small torsional vibrations occurring in the system.

7.3.5 Discussion

From the results above we see that the diffractographic technique produces nearly the same torque readings as one example of the commonly used electrical resistance torque transducer. There is no further conclusion which can be drawn here concerning absolute accuracy of the diffractographic sensor since no rotating torque standard is presently available. It is expected to be quite high however, since the device is effectively linear, and all variables (such as $\lambda$, $R$, $G$, etc.) used to determine the calibration curve as essentially constant within one part in $10^4$ (at least when a gas laser light source is used). Dynamic range is so great that very little difficulty in achieving high results is foreseen.

7.4 A Dynamometer Readout

7.4.1 Introduction

An important modification to the Diffractographic Torque Sensor results by utilizing the device shown in figure 17 to sweep the diffraction pattern through an arc (using a mirror attached to the shaft as shown). In this manner, a detector output frequency (in Hz) of

$$F = 2\pi R_1 \frac{Nc}{60} f$$  \hspace{1cm} (7.5)
may be generated, where

\[ N_r \quad \text{rotational speed in rpm} \]

\[ R_1 \quad \text{distance from the shaft axis to detector} \]

\[ f \quad \text{fringe frequency} \]

If the initial (no torque) slit width is set to zero, instantaneous torque will be proportional to slit width (which now represents change in slit width). Using equations 6.1 and 7.5, detector frequency becomes

\[ F = \left( \frac{2\pi R_1}{60} \right) \frac{T N}{R k} \quad (7.6) \]

where all terms in the parentheses are presumed constant in any given device, and the spring constant, \( k \), equals \( T/w \).

Horsepower, \( H_p \), can be expressed as

\[ H_p = 2.21 \times 10^{-6} \frac{2\pi T N}{R} \quad (7.7) \]

where \( T \) is in kg - cm

Combining equations (7.6) and (7.7) yields

\[ H_p = \left[ \frac{(2.21 \times 10^{-6}) \frac{60 R k}{\lambda}}{R_1} \right] F \quad (7.8) \]

In the device of figure 17 horsepower is therefore linearly proportional to detector output frequency in Hertz. This result is quite advantageous, since horsepower readings are therefore producable once per revolution as a frequency signal which can be easily transmitted and/or converted to digital code.

In the above analysis, it is assumed the detector is sensitive enough to detect, small enough to spatially resolve and fast enough
to respond to the frequency generated. In addition, it is recognized that the central maxima is twice the period of the other fringes, and this must be considered if a frequency counter is used (rather than the oscilloscope employed in the experiment to be described here).

Before presenting experimental results, comments on two limitations of the readout technique are in order. First, since slit width must be zero with no torque applied to obtain relation (7.6), sensing can only begin when the first diffraction minima on each side of $\theta = 0$ can reliably be detected. This implies a minimum usable slit width and therefore minimum detectable torque value dependent on laser power, mirror width, and detector sensitivity. Obviously, the larger full torque slit width is, the less of a percentage this "dead" region becomes. Optimum values might be .015 mm minimum slit width, 1.5 mm slit width at full torque, yielding a 1% of full torque dead zone. For most applications however, the resulting dead zone in terms of a percentage of full horsepower would be considerably less, since for maximum sensitivity such sensors are seldom operated near the bottom of their operating torque range.

The second limitation concerns detector response and its limit on horsepower. For example, if the detector/amplifier combination is limited to one MHz in its resolution of the sine-squared-like diffraction fringes, then at a full torque slit width of 1.5 mm, rotational speed is limited to a value found by combining equations (6.1) and (7.5) and solving for $N$. For the case of $R_1 = R$, this results in approximately 10,000 rpm maximum at full-torque. Obviously, this can be improved by using faster detectors, smaller slit widths or the like.
7.4.2 Experimental Tests

Figure 18 schematically illustrates the experimental apparatus used to produce and verify the horsepower readings discussed above. With this arrangement, torque and speed were determined using presently available transducers and simultaneously compared with readings obtained with the diffractographic device (which was in the actual experiment, a modified version of fig. 12). From the calibration curve (fig. 13) we find that $k = 544$, for $w$ in cm, $T$ in kg-cm. Substituting this value into equation (7.8) together with experimental values of $R = 235$ cm, $R_1 = 232$ cm, and $\lambda = 6.328 \times 10^{-5}$ cm, gives

$$Hp = 4.57 \times 10^{-6} F$$  \hspace{1cm} (7.9)

Sample oscilloscope traces shown in figures 19 and 20 illustrate the general nature of the signals obtained. Period (sec) readings taken from the scope were inverted to obtain frequency (HZ) and then converted to horsepower using the relation (7.9) above. The results obtained are compared in figure 21 with those obtained using torque and rotational speed values measured experimentally by other means. One must conclude the diffractographic device represents a valid method for measuring horsepower, though no absolute accuracy determination results from the experiment.

7.5 A Diffractographic, Self-Recording Load Cell

7.5.1 Introduction

One of the most common transducers utilizing an elastic member is the "electronic" load cell, which is finding ever-increasing use in the large (160 million dollars/year) weighing and batching industry. For
these purposes, a device using the quasi-digital techniques discussed in chapters 5 and 7 would no doubt be apropos. However, discussed here is an experimental version designed for the long term monitoring of loads, such as might be encountered in structural applications.

If the load cell has a spring constant $K$, then the force $P$ required to change the no load aperture width $w_o$ to some new value $w_L$ is given by

$$P = K(w_L - w_o) \tag{7.10}$$

This may be written in terms of the fringe frequency $f$, using equation 2.7, as:

$$P = KR \Delta (f_L - f_o) \tag{7.11}$$

Thus a change in fringe frequency $(f_L - f_o)$ gives the force.

### 7.5.2 Film Recording

A diffractographic load cell employing elastic deflections in its top and bottom plates is illustrated in fig. 22. To provide a self contained data taking capability, the load cell was constructed utilizing a small internal laser source together with a film recording "pack" or "cassette". Employed was a Laser Diode Laboratories model LP-23 pulser and their LC-23 converter which drives the LD-23 lasing diode with 135 nanosecond pulses, 500 pps, from a 28 volt battery.

The resulting 10 watt laser pulses are easily observed on a Kodak IR Phosphor Viewing Screen. Observation of the diffracted fringes requires more sensitivity which, in the cases presented here, is furnished by Kodak high speed infrared film (which for the 0.9000 μm wavelength of this laser is anything but "high speed").

The film "cassette" used was a gutted and lensless 35 mm camera,
whose operation is straightforward. The 35 mm film width used was far larger than necessary for adequate fringe pattern recording (as is 8 mm, for that matter). At the present time however, 16 mm IR film appears the narrowest available.

7.5.3 Discussion of Results

A sequence of Diffractogram sections corresponding to various loads are shown in Figure 29, along with a plot of the results as load vs. fringe frequency. The linear nature of the device is self-evident, though the accuracy represented is limited by the testing machine and the line frequency readout method employed (visual, using the enlarged load diffractograms and a vernier calipers graduated in 0.001" increments). In all cases, the device returned to the original fringe frequency \( f_0 \) when load was removed, indicating that no measurable changes in diode wavelength had occurred.

As mentioned previously, measurement accuracy of the device depends on the precision used to measure \( (f_L - f_0) \), since proper compensation can remove the effects of thermal changes on the other three factors of equation 7.11. Using semi or full-automatic electro-optical equipment (e.g. a microdensitometer), this value can be obtained much more accurately and expeditiously than the visual method used here.

The exposure was quite long (1 sec. per load record) and higher power, lower wavelength diode lasers are eagerly awaited so that a better match can be made with available films (the recently developed cw diode laser seems almost ideal). Note that the diode laser's bandwidth of 20-30 \( \AA \) seems to cause no fringe degradation problems.
A final note concerning the rationale behind recording load data as grating patterns on film. This device is one of many such diffractographic sensors which utilize a diffraction pattern to indicate a physical quantity in an advantageous manner. Since the information exists as a diffraction pattern in the first place, recording it as such is a one-step process, while conversion to an electrical signal and thence to magnetic tape complicates the situation. Film does have a disadvantage in that recording must be done at the device, although it is hoped to lessen this restriction by utilizing fiber optic tubes to carry the pattern information to a central recording location (and perhaps even a laser beam to the slit).
8. DISPLACEMENT MEASUREMENT ALONG-A-LINE

8.1 Introduction

An absolutely unique feature of the Diffractographic method is its ability to make highly accurate displacement measurements along a whole line *at once*. This is accomplished by forming the required slit-type aperture between a reference edge (usually straight) and a line of the object—either an edge, or a line region of the surface opposite the reference edge.

In most cases, a cylinder lens is used to illuminate the whole aperture simultaneously and two simple examples are now given to illustrate the method.

8.2 The Beam Experiment

The first along-a-line experiment was to determine deflection of simply supported beams for a central point load, since load application is simple, and the deflection equation is well known for this problem. The beam loading arrangement and fixed edge are shown in fig. 24. The first beam tried was C1020 cold rolled steel \( (E = 28 \times 10^6) \), 8" long, with loading and support pins \( \frac{1}{8} \) inch in diameter. The only surface preparation done was to paint the beam surface flat black to prevent unwanted reflections, thereby improving the quality of the diffractograms produced.

Fig. 25 shows the experimental arrangement. A 9 milliwatt He-Ne gas laser was used, and its output radiation expanded into a sheet by a cylinder lens. This diverging *fan* of light was collected by a collimating lens, and the slit between beam and fixed edge illuminated by the sheet of parallel light so formed. No collimating lens is necessary, but was used here to allow the whole beam diffractogram to be displayed.
and photographed on the ground glass screen.

An initial aperture width of .0138 inches was used, and diffractograms were recorded at three different loadings. (Figures 26-28). It is immediately evident from these diffractograms where the maximum deflection occurs. The small irregularities in the patterns are due to imperfections (e.g. paint globules) in the reference edge or beam which locally change the slit width.

Fig. 29 shows the theoretical deflection curve as well as experimental deflection values calculated by visually determining fringe locations on the diffractograms and measuring the x values with a vernier caliper. The points were chosen arbitrarily and any others along the beam could have been used. At the centre of the beam, where the deflection is a maximum, there is a difference of about 3% between theory and experiment. Such a difference is to be expected, since the sample elastic modulus was estimated from tables, and all weights and dimensions involved (including x and R) were known only to between 0.2 and 2%.

Diffractographic applications were further investigated by a series of tests on 5" long cement paste beams. While the diffractograms produced were not as "clean" as those produced by the steel beam (due to irregularities in the cement) they were still quite usable for calculation. A comparison of the diffractographically determined deflected beam shape and that theoretically predicted is shown in fig. 30. Here the elastic modulus has been chosen to produce agreement with experimental deflection directly under the load. As can be seen, a non-uniform deflected shape occurred, probably due to inhomogenities in the cement paste (which was assumed to have a unique elastic modulus).
8.3 Plate Deflection Using the Bounce-off Method

Fig. 31 is a schematic of an apparatus utilizing the bounce-off technique to measure deflection of an end loaded cantilevered steel plate, 20 x 15 x 0.62 cm. A whole line of the plate was illuminated simultaneously and diffractograms at various loads are shown in figures 32 and 33. Since grazing incidence was used (θ = 2.4 degrees) to obtain good reflection off the steel surface, the "crossed" interference fringes discussed in Chapter 2 are very much in evidence.

Plate deflection measurements were made in two ways. First, the central maximum width was determined and used in equation (2.6), as the intensity ratio of central maximum \( B = -π \) to reflected fringes \( B = \left( \frac{2n \sin \delta + 1}{\lambda} \right) \pi \) was felt to be large enough to make interference effects caused by the latter effectively negligible.

The second measurement technique involved the use of the crossed interference fringes themselves. In a manner reminiscent of photoelasticity and holography, deflection at a position along the slit is obtained by counting the number of crossed fringes contained between the position in question and the undeflected base of the plate. Each fringe is proportional to the deflection necessary to cause another full diffraction fringe to appear at the θ = 0 line, and this deflection can be expressed using equation (2-14) as,

\[
\delta = w_n - w_{n-1} = \frac{\lambda}{2 \sin \delta} \tag{8.1}
\]
Results of the experiment are shown in Fig. 34 which also includes a precision dial gage reading (at one point) and a theoretical curve for comparison. The departure from this latter curve is not considered significant since end conditions due to the clamping arrangement were not "perfect" (as assumed and no exact figures for the material properties were available.

Since we are concerned here with measurement of deflection, we note that such a system is twice as sensitive to changes in $w$ as the previous version. In general however, a decreased range results since one must resolve twice the fringe frequency as well. A further consideration regarding slit width range is that usually one would prefer $w$ to remain small, restricting the reflection region (of approximate length $2w \cot \theta$) in order to retain flatness and uniform reflectance.

While this plate experiment does not conclusively establish the limiting accuracy of the bounce-off technique, it does at least indicate that results obtained are within a few percent of what one would obtain by other means. This being the case, there are many advantages gained, primarily because of the ease in obtaining non-contacting information along a whole line at once.

8.4 A Spatial Filter/Moire Technique for Line Displacement Determination

8.4.1 The Formation of Moire Fringes

Those familiar with moire methods will recognize that the "before" Diffractogram shown in Fig. 26, looks very much like a grille, and indeed it is—though not of a pure square-wave variety as is commonly used in moire work. From equation (2.10) the minima lines of the diffractogram grille are, within the small angle approximation, equally spaced and
proportional to \( \Delta R \). Thus two diffractogram grilles overlaid can be expected to show moire fringes if they were produced by different slit widths \( w \) (\( \lambda \), \( R \), and other photographic variables held constant or varied by known amounts).

Perhaps more useful however, are two other ways of forming the fringes. The first results when a diffractogram negative (i.e. "spatial filter") is replaced in the same position it was taken. In this situation, the "live" diffractogram existing at a later time is superimposed upon it, with moire fringes immediately visible and proportional to the instantaneous slit width of the aperture itself. The second type results when a double exposure diffractogram is taken, corresponding to the beam aperture width at two different positions. Before presenting the results of a cantilever beam deflection experiment, the rules for location of such moire fringes are briefly reviewed.

8.4.2 Moire Fringe Interpretation:

Two grilles of spacing \( s_0 \) and \( s \) upon superposition produce the well known "vernier" moire fringes of spacing \( S \), which can be expressed as: (8)

\[
S = \left| \frac{s}{s - s_0} \right| \tag{8.1}
\]

Since equation (2.6) can be written in terms of fringe spacing as:

\[
\delta = R \left( \frac{1}{\lambda} - \frac{1}{s_0} \right) \tag{8.2}
\]

\[
\delta = R \left( \frac{s_0 - s}{s - s_0} \right) \tag{8.2}
\]

It is seen that

\[
\delta = \frac{\Delta R}{S} \tag{8.3}
\]

Equation (8.3) can now be used to calculate beam deflections if the wave length (\( \lambda \)), slit to screen distance (\( R \)) and moire fringe spacing (\( S \)) are known. Since \( R \lambda \) is typically greater than \( 10^{-3} \) inches\(^2\), a
deflection of \( .001 ^\circ \pm .00001 ^\circ \) can be measured if \( S \) can be measured as \( 1 ^\circ \pm 0.01 ^\circ \) (usually achievable in practice).

8.4.3 Experimental Verification:

The aluminum cantilever beam used in this experiment was 3.50 inches long, 0.125\( ^\circ \) thick and 0.750\( ^\circ \) wide. Its theoretical deflection with a point end load is given by

\[
\delta = \frac{P}{6EI} \left( \mu^3 - 3\alpha^2 + 2\alpha^3 \right)
\]  

(8.4)

A double exposure "before and after" moire pattern for such a cantilever is shown in fig. 35. While very easy to take, such photographs require considerable optimization for good contrast results.

Figures 36 and 37 present moire patterns of the "live-fringe" type which allow immediate readout. These are produced using the apparatus of fig. 38 (by passing the diffraction pattern representing the loaded beam through a negative or "filter" pattern recorded in the original unloaded condition.) A dowel placed on the pattern centreline (\( \theta = 0 \)) is used to block most of the very intense central maximum, preventing "splattering" of the camera film emulsion due to gross over-exposure.

Five different loadings from 0.2 to 1.0 lbs were used in all, and the normalized beam shapes are plotted in fig. 39, together with the curve predicted by equation (8.4) and a.0001\( ^\circ \) resolution dial gage reading for the tip deflection at the 1 lb. load.

The deviation from the theoretical curve is not considered important since it was calculated assuming an elastic modulus of \( 10^7 \) psi, exact dimensions and a perfect point end loading—none of which were completely true. More to the point, is the comparison with the curve obtained by the "regular" diffractographic method. The two versions agree within
a few percent for the 1 lb. load at which both were done, and this is all
that can be expected in this experiment due to loading and fringe measure-
ment inaccuracies (fringe positions were measured directly from the film
by eye with a vernier caliper). Both regular and moiré diffractographic
techniques are seen to agree with the dial gage result within a few
percent as well.

Several observations can be made concerning the nature of the
moiré fringes and their use. First, the fringes indicate displacement
values from whatever the filter pattern represents, and the same fringe
movements can be created by moving the beam toward or away from the fixed
edge. (i.e. a positive or negative sign of P).

Secondly, from equation (8.3) it can be seen that the moiré fringe
spacing depends only on deflection and is independent of the value chosen
for initial slit width \( w_0 \). This is a considerably different result from
the "regular" technique and produces a unique advantage in that neither
the reference edge or the tested object need be straight in order to
produce a simply interpretable fringe pattern. Furthermore, added
clearance between the object and reference edges can be used (within the
resolution limits of the film, lens or ground glass screen used to
record the fine spaced diffraction fringes which result).

8.5 Time-Averaged Vibration Analysis

The moiré fringe formation described above is not the only useful
diffractographic adaptation of holographic techniques. Time-averaged
vibration fringes may also be formed, and represent a very inexpensive
and reasonably accurate method for calibrating accelerometer displacements
or determining deflection of members along a line. This latter case is
here illustrated, applied to the simple case of a cantilever beam. The
patterns are produced by using the straight edge to create a zero slit width situation for no excitation of the vibrating member (fig. 40).

When the vibrating member is excited, the negative amplitudes are cut off and the only diffraction produced is that representing positive vibration amplitudes. Since the vibration is of sine wave character, spending most of its time near maximum amplitude, the degraded contrast integrated diffraction fringes are those corresponding to a slit width equal to the maximum amplitude (fig. 41).

A few comments on the vibration experiment are now in order. First, the slit edge arrangement of fig. 41 is the cause of certain intrinsic errors which are especially troublesome where small amplitude vibrations are encountered. For example, the condition of "zero" slit width always means ± some value, and this unknown value is an error in measurement of the maximum displacement.

Another error results due to the necessity for overlapping the two edges (to eliminate negative amplitudes) which causes "z-factor" to occur in the zero slit width region. As a rule-of-thumb, equation (2.13) predicts that \( w_e \) must be greater than \( 3z \) (where \( z \) is now the gap between the overlapping edges) if errors of less than 5% are to be incurred by assuming \( w = w_e \). If one assumes a minimum z-clearance of .001", this means that \( w_{\text{max}} \) is limited to .003" (min) at the 5% level if no account of z factor is to be made.

Before closing this discussion of vibration measurement mention should be made of the unique suitability of stroboscopic techniques in diffractographic work. By controlling the phase and frequency of the laser one can essentially freeze a vibrating member's deflection curve at any
point in the cycle, watching it's movements in slow motion if desired. Thus phase, as well as amplitude, information concerning the vibration studied may be obtained.
9. PROFILE MEASUREMENT

9.1 Introduction

Profile measurement is utilized in many manufacturing industries for part inspection and optically speaking, standard imaging devices such as contour projectors or microscopes are almost universally used for this purpose. To improve the performance of such devices, Lanseroux and Birch, have described a diffractive technique employing a spatial filter to remove the D.C. term (and in some cases higher order terms) from the edge image of a coherently illuminated object profile. An improved edge image results, though the technique does not appear to have been industrially used, perhaps because of the limitation noted by Dew.

A quite different diffractive technique is that used by Rogers and Armitage to monitor film strip edge straightness. In their device, a straight reference edge is placed next to the film edge to form an aperture which diffracts light from two sources whose separation can be adjusted such that slit width at a point can be determined. By successively performing this operation at all positions along the slit, profile may be obtained. Their technique especially in regards to use of a reference edge, is similar to that employed here and curiously does not appear to have been developed further. Perhaps the specialized nature, slow speed and limited accuracy of the device they describe was responsible.

The "Diffractographic" technique previously discussed in terms of displacement measurement may also be used to produce a projected profile diffraction pattern, which can yield the same information as a profile image of an object edge. In most instances, this pattern has been generated along a line of the slit-type aperture formed between straight
reference edge (or surface) and an edge (or line upon the surface) of a test object. While visual observation of the profile patterns has been extensively used to date, the technique seems most suited to automated inspection using television scans or spatial filtering by "master" diffraction patterns. Before discussing several applications, a brief review of the simple theory utilized will be presented.

9.2 Theory

As has already been discussed in Chapter 2, the spacing of the single slit diffraction fringes can be represented as (within the small angle approximation)

\[ s = \frac{\Delta R}{w} \]  (2.10)

The difference in slit width between two position 1 and 2 along the slit can therefore be expressed by

\[ w_2 - w_1 = R \lambda \left( \frac{1}{s_1} - \frac{1}{s_2} \right) \]  (9.1)

If the positions of points on one edge (or surface) are known, then variations in slit width can be related to the other edge (or surface) completing the slit. In other words, the "test" object profile can be obtained from its contribution to the diffraction pattern produced by the slit formed between it and the "reference" object.

The considerations above are similar to those utilized in Chapter 9 and the "bounce off" technique (using the aperture between an edge and a slit surface) described therein can be used here as well. In this case, slit width \( w \) in the equations above becomes twice the aperture width and for non-grazing incidence a cosine correction factor is required.

9.3 General Considerations

Because of the large number of possible profiles, many of which
require specialized reference edge or surface arrangements, it is difficult to imagine one, universal device (such as a contour projector). Some factors are however, common to all types and a few of these are now discussed.

Fig. 42 illustrates perhaps the simplest situation whereby a straight reference edge is placed next to a wavy object edge whose profile we wish to check. Several things can be noted. First, over the length $l$, slit width $w$ varies in a manner indicating profile, as well as any difference in the separation of the two end positions $F$ and $2$. For simplicity, we assume that $w_1$ has been adjusted equal to $w_2$ such that no linear slit width variation remains. Obviously two other such points could have been used for this purpose as well.

In the region "A" we notice that slit width varies rapidly with $y$ in what would be described more as a "roughness" than a change in gross profile. Its effects appear in virtually all diffractograms (see for example fig. 27) and may also be caused by particles on the surface or edge. Surface roughness (or particle sizes) could conceivably be extracted from the diffraction pattern though this more complicated situation will not be treated here.

Region "B" illustrates a section of test edge whose slope varies significantly in the region being illuminated. A wide variety of local geometries are included in this class, most producing various unwanted diffraction effects in many different directions when an ordinary laser source is used.

While several means for obtaining usable and easily interpretable patterns are under investigation, only those slit-type apertures which
have slowly changing curvatures in the region illuminated are discussed here.

A final general characteristic is that of the diffraction patterns produced and their detection. Besides the large magnification of small profile changes, there are two other advantages of projected patterns. First, very high speed readout of profile information encoded as fringe frequency can take place using television cameras or laser scanner/detector combinations, allowing a whole profile length to be effectively quantified in "real time". Secondly, the projected profile diffraction pattern, representing one or more positions on the length L, can be spatially filtered, giving instantaneous "error" signals. Several applications will now be discussed.

9.4 Checking of Straight Edges with a Flat Mirror Reference

In the arrangement of fig. 42 one might logically question how the reference edge was determined straight in the first place. Fig. 43 illustrates an application of the "bounce-off" technique (2.5) to just this problem. The reference in this case is a 1/8 wave flat mirror (assumed "perfectly flat") and a diffractogram representing a razor blade edge tested is also shown. Note the obvious curvature and roughness magnification present, using the experimental values \( w = 30 \mu m \), and \( \theta = 15^\circ \).

The use of a cylinder lens has allowed production of a diffractogram representing undulations along the whole blade edge. In the same apparatus, the incident beam may be focused to a line such that a very small region of the slit is illuminated. The region of the focal line is therefore "blown up" on the screen and effective magnification obtainable at the screen using a 10mw gas laser source (\( \lambda = 0.6328 \mu m \)) can be several thousand in both x and y directions.
9.5 Reference Edge Applications

Once the profile of an edge is known (using for example, the technique of fig. 43) we can use the edge as mentioned previously to check object profiles. A first example would be to reverse the experiment above and use the edge to check surface profiles. As mentioned in 8.3, grazing incidence is usually required to achieve sufficient reflectance and the object surface must be quasi-flat in the region of the edge. Curved objects can be used, but fresnel-type diffraction often results, complicating interpretation.

When the curved objects mentioned above have sufficient curvature they may be used as edges, relative to a reference edge or surface. Several industrially important objects such as roller bearings, shafts and millrollers fall into this category. Thin sheet material passing over a tension roller could also be similarly profiled.

A particular example is shown in fig. 44 together with the diffractogram produced. Here portions of the surface of two camshaft lobes are being compared to a tool maker's straight edge. The surfaces (of a reject camshaft) show significant and uneven, "crowning".

9.6 A Non-Straight Reference Edge

In the previous examples the reference edge (or surface) has been straight (or flat), thereby implying a limit on object profile variations (i.e., slit width changes) of around 1.5 mm or less, due to diffraction considerations at visible wavelengths. While oblique incidence techniques could improve this (with an accompanying loss of resolution—see 2.4), such limitations effectively restrict application in this basic form to quasi-straight or flat test objects.
A further class of objects which could be gaged in much the same way are those curved objects to which an arc-like reference edge could be compared. In this case a nearly constant slit width is utilized and the laser/detector array pair scanned around the slit. Many other arrangements are of course possible within the framework of this example.

Perhaps more interesting is the experiment of fig. 45. Here an object with a jagged edge profile (in this case a house key) is being compared to a maser edge profile. Were the aperture formed to be illuminated simultaneously, or scanned by a raw laser beam, diffraction would occur in many directions, since the aperture axis of symmetry follows the jagged profile itself. For present purposes, such a complex pattern is useless, since easy extraction of quantitative data is almost impossible.

In the case shown however, a cylinder lens is used to focus the radiation into a line upon the aperture, forcing an axis of symmetry in the horizontal direction as shown. To further facilitate observation of the pattern, the master can be made such that \( w_{\text{vertical}} \) is constant at all positions if a "perfect" test object is used. Therefore, as the keymaster aperture is scanned past the cylinder lens focal line (or visa-versa) deviations are immediately detected as diffraction pattern changes from the standard.

9.7 Spatially Filtered Patterns and Moires

A particularly useful characteristic of the projected fringe patterns produced by the Diffractographic technique, is that the information concerning profile is encoded as spatial fringe frequency which may be filtered to obtain photoelectric signals signifying a "match" with the
pattern produced by a known edge profile. Alternatively, moire fringes proportional to a deviation from the filter profile may be produced. In the latter case, the moires can be produced "live" through a filter, by double exposure on film, or by simple overlay of two diffractograms.

Two examples utilizing these filterable characteristics are presented. In the first, an apparatus arrangement similar to that of fig. 43 was used to check double edge razor blade shelf-to-edge tolerances as well as general edge straightness. Here, the blade shelf rested on blocks of equal height above a flat mirror, and the diffraction pattern produced using a designated "master" blade was recorded on a photographic plate placed at plane A-A. Thirty other blades of various manufacture were then tested in the same arrangement, projecting their patterns through the negative master pattern replaced at plane A-A. The moire fringes produced by several such blade edge patterns are shown in fig. 46 (the central portion, $\Theta = 0.00325$, has been blocked to prevent film "splatter"). The undesirable effects of dust on the mirror are quite noticeable.

Interpretation of the moire patterns is easy and quick. Using equation (2.10) one can write an equation similar to equation (911).

\[
\text{Interpretation: } w_t - w_m = R \lambda \left( \frac{s_m - s_t}{s_t} \right)
\]  \hspace{1cm} (9.2)

where $s_t$ and $s_m$ are the diffraction pattern fringe spacings produced at a given position along the edge using test and master blades respectively. Since moire fringe spacing is defined as

\[
S = \left| \frac{s_m s_t}{s_m - s_t} \right|
\]  \hspace{1cm} (9.3)
then using (9.2)

\[ |w_t - w_m| = \frac{\lambda f}{S} \]  

(9.4)

The difference or "error" (+ or -) in the dimension of the test blade relative to the master blade at the position in question is therefore inversely proportional to the moire fringe spacing. For the purposes of industrial gaging, such moires could very quickly indicate whether or not a certain tolerance level had been exceeded. In addition, a very advantageous property of such patterns is that they correspond only to changes in fringe frequency from the filter pattern.

A second example of spatial filtering is shown in fig. 47, where a lens is used to image the test pattern passing through the filter onto a photodetector. This "optical null balance" technique produces a distinct drop in signal when a fringe frequency "match" is obtained, indicating that the test blade and master are equivalent at the position(s) in question. In a basic test of the technique, an adjustable edge was uniformly moved through a range of slit widths monitored by an LVDT connected to the x-axis of an x-y recorder. Photodetector voltage was monitored on the y axis, and a typical plot is shown in fig. 48. Not only does a 50% modulation occur at the "match", but the curve is quite symmetric indicating that equivalent positive or negative departures from the master pattern will produce nearly the same detector signals.

Fig. 49 is a similar plot over a much larger range of slit widths. As can be seen, within a considerable distance on either side of the "match" there are no regions which could produce similar (and therefore erroneous) output signals. It would seem possible to extend this null balance system to the more general case of variable profile patterns.
In this case a go/no/go signal could be produced whenever an average fringe frequency difference occurred along the filter. Different rejection criteria could be used as needed, with perhaps multiple detectors looking at different sections of the filtered pattern.
10 THE DIFFRACTOGRAM AS A DATA RECORD

10.1 Introduction

During the course of these investigations, photographic negatives were often made of the resulting diffractions if for no other reason than to allow their presentation or reproduction. In the load cell application of 7.5 however, the film recording means formed an integral part of the system, producing a series of diffractogram records relatable to the instantaneous loads at various times (fig. 25). The properties and means of employing such records are now discussed.

10.2 Background

The encoding of information in the form of a spatial frequency is hardly new. For example, a recent series of papers and patents by Lamberts, Higgins, Baldwin, and Wankerkhove have described in detail a means for encoding digital information using a set of fixed frequency superposed gratings whose collective presence or absence represents a binary number. Their latest system produces the gratings through a fixed angle hologram-type arrangement and appears to work quite well. It has the following advantages.

1. Photographic film has a much greater information density than magnetic tape. This allows very high frequency recording at moderate film transport speeds.

2. The grating records are redundant, greatly reducing scratch and dust problems.

3. Playback can be accomplished with spatially coherent, parallel illumination (just like any other diffraction grating). This gives a freedom from precise head spacings, track positions etc.
4. When superposed, the gratings take up no more space than a non-redundant spot-coded binary number.

Despite the advantages, their system appears to be the only one seriously proposed for practical use, and there furthermore has apparently been no consideration of analog data recording with gratings. Even if the same has been proposed in the past, there are two very good reasons why it hasn't been pursued—The ready availability of magnetic tape, and the difficulty in making analog records of varying grating spacing.

10.3 Diffractogram Records

It has been found here that a slit diffraction pattern recorded on film (or presumably other media as well) can act as a diffraction grating possessing the same desirable qualities described above. Already noted is the linear fringe spacing of such diffractograms (equation 2.10), and fig. 50 is a playback of load cell diffractograms similar to those of fig 23 though produced by a gas laser. Interestingly, a nearly sine-wave grating-like performance was obtained (i.e. a zero and first orders with virtually no higher orders present). In these particularly "noisy" diffractograms, no filtering of the pattern was used, requiring the playback beam to illuminate only the outer fringes, where the ratio of fringe intensity over the illuminated region was on the order of 5 or 10 to 1. For the same reason, the diffractograms of fig 23 were made off-axis in order not to overdrive the film.

10.4 "Line" Patterns

Another type of diffractogram which can produce grating order representations is that resulting from slits formed by deflecting members relative to straight edges. In these experiments the y-axis represents position along the length of a member and the diffractogram fringe
frequency becomes a continuous function of y.

Fig. 38 illustrates typical apparatus for production of such diff-
ractions and fig. 51, illustrates some typical diffractiongrams and
their playback orders. With such a system, a magnified "first order
image" of the beam deflection curve may be projected and information
taken directly therefrom. Diffractionographic representation of object
profiles could also be projected in this manner.

The beam patterns shown were not recorded in the same way, as the
load ones. In this case a graded filter was placed at the position shown
in fig. 38 to multiply the diffraction pattern by a rough approximation
of the inverse of the mean pattern envelope. This produced more or less
evenly exposed fringes and good diffraction orders on playback.

10.5 Superposed Diffractiongrams

The Lamberts and Higgins superposition technique can be applied
here as well, though for a different purpose. In this case it is desired
to have one of two situations.

1) N superposed channels, each operating in a
different spatial frequency range. If second order prob-
lems arise due to cross products etc., the total
frequency range would have to be under one octave,
say 40-75 lines/mm.

2) N separate values taken at different times and
superposed. This saves film but in general is not par-
icularly desirable since time information is lost.
A superposed "line" diffractogram and its playback orders are shown in fig. 51 (bottom), while playback of a superposition of four loadcell diffractograms is shown at the bottom of fig. 50. Despite the apparent success, it has not yet been possible to intelligibly record over 6 such superpositions unless azimuthal coding is used (in which case 13 superpositions have been achieved).

10.6 **Continuous Data Records**

The diffractogram producing devices above do so by their very nature, providing the unique, redundant, data record as a sort of "by-product". Now described is a recording device capable of converting electrical signals from any source into such data records. This device has as its primary purpose the production of continuous analog records and serves in much the same capacity as a magnetic tape recorder. It is a unique advantage of film recording, that either discrete (as in the load cell case) or continuous records can be made with equal facility, since the recording medium velocity in no way is used for the recording process.

The key item in such a device is of course some sort of slit whose width is electrically changeable, and a very reliable mechanical device is shown in fig. 52. Here an electrically actuated piezoelectric bimorph deflects to increase or decrease the slit width in response to positive or negative voltage signals of up to about 100 volts maximum. The changing slit pattern is then focused onto a small filmplane aperture and recorded on the moving film. While the light source shown in this example is a visible laser whose beam has been transported to the slit via a single mode fiber optic tube, we recognize that any source of
suitable coherence monochromaticity and intensity could have been used—providing suitable recording materials could be found. (The only really serious contenders at this writing are the aforementioned diode laser sources and other laser visible lasers).

10.7 Electrical Reproduction of Diffractogram Records

Unlike the digital devices of Lamberts et al, the orders produced by the analog patterns recorded here do not occur at predetermined \( \Theta \) locations. Since the \( \Theta \) location is indeed the value of the variable desired, some means must be employed to determine where the order falls. Many different techniques can be employed and a particularly simple one follows.

Fig. 53 illustrates a device using a split photodetector producing a voltage signal proportional to a deviation from a known \( \Theta \) location corresponding to the photocell centerline. When diffractogram records are played through the system, an output trace similar to Fig. 54 is produced. (Note, this figure was made without the very desirable automatic gain control shown in figure 53.)

10.8 Advantages of Diffractogram Records

A first question to be answered is why would anyone record diffractograms on film when well proven easily available magnetic tape techniques exist (especially at the low frequencies implied by mechanical slit moving devices)? Some possible reasons are given below.

11. 1 Packing Density

Presently available magnetic tape systems can record (in wideband FM mode) signals of 0 - 200 Hz at about 2.5 cm/sec, placing 4 tracks on a 6 mm wide tape. A comparable diffractographic
recorder should be capable (using a 2 μm film recording aperture) of achieving the same results at roughly the transport speed and the film width and with enhanced dynamic range. If superposed patterns were used (implying less dynamic range), 20 or more channels should be possible on "super-8" film cartridges.

2. Easier and More Accurate Playback

The Diffractogram record is highly redundant and carries information perpendicular to the direction of film travel in the form of a fringe frequency. Several problems of magnetic tape systems do not therefore apply, i.e. signal amplitude does not change if speed fluctuates, and scratches tears, etc. do not cause total loss of signal amplitude. In addition, no precise head-to-tape location need be maintained. Records can be visually observed and optically processed if desired, a characteristic under investigation.

Advantage (1), in particular, lends itself to construction of recorders for long term data monitoring, such as in pollution, seismic, or aircraft vehicular and structural performance studies. However, much more work needs to be done, particularly regarding practical apparatus for getting the small diffractograms onto film. A fully electro-optical, rather than electro-mechanical, slit actuating system would be highly desirable and could conceivable allow competition with high speed magnetic tape recorders as well.
11. CONCLUSION, SUMMARY, AND PROJECTION

A new displacement and profile measurement technique has been described which has several useful characteristics, in particular:

- unparalleled stability over time, together with quasi-digital outputs
- unique "line" sensing capability
- visual representation of information if desired
- direct film recording capability producing a unique optically processable data record
- easily obtainable accuracy of less than 10 microinches with a range in excess of 0.1 inch, in a non-contacting manner.

Also described herein has been three new types of diffraction patterns together with several applications:

- "z-factor" patterns allowing large ranges at reduced sensitivity as well as displacement measurement in two orthogonal directions simultaneously

- "Bounce-off" patterns generated between a flat surface and an edge which allow more accurate measurement of such object profiles as well as twice the displacement sensitivity (of ordinary edge-edge patterns).

- Time-averaged patterns which allow measurement of maximum vibrational amplitudes using stationary diffraction fringes.

Many different applications have been used to illustrate the various features and it is interesting to speculate on future utilization of the diffractograph method. At this writing one may reasonably predict the following:

**Virtually Sure Applications**

1) Long Term Displacement Transduction: for example, diffractograph- graphic strain gages on concrete structures.
2) Calibration Type Transducers: such as proving rings etc.
3) Research Apparatus for Studies of Structural Displacements.
   micro profiles of linear objects, and the like
4) Industrial Checking of Edge Type Object Profiles Using
   the Bounce-off Technique

Quite Probable Applications

1) "Along-a-Line" Type Measurement in Industry, for thickness profiling, flaw detection and the like--preferably on-line.
2) Large Scale Diffractographic Transducer: Employment in Commerce and Industry, particularly regarding quasidigital versions.
3) Educational Uses resulting from the techniques ability to present visual displacement data of high accuracy

Significant Possible Applications

Diffractographic data recording, transmission and processing occupies this category along with Diffractographic computers, character readers and the like. While all are possible, the existing competition is so stiff that any development effort might be more cost-effective if channeled into the categories above. Many factors such as improvements in light recording materials, can alter this assessment, however.
12. FUTURE INVESTIGATIONS

What follows is a rather exhaustive, if unorganized, list of desirable diffractographic experiments to be conducted which could open up new uses or offer new information.

1. Electromagnetic wave sources at other wavelengths should be checked out, in particular, soft x-ray, millimeter wave, and CO$_2$ or other powerful infrared devices.

2. Investigations should be continued into the possibility of spatially varying with time diffraction patterns as communication media, particularly in conjunction with SELFOC or other fiber optic waveguides.

3. Non-laser (incoherent) devices should be tried, in particular the light emitting diode. These offer a real opportunity for low cost, high efficiency light sources for non-critical applications.

4. Many lasers can emit multiple wavelengths, and the technique of exact fringes used in gage block interferometry (for splitting fringes, in effect) should be tried in the diffraction case.

5. Because of their importance to future widespread utilization of the technique, every apropos characteristic of diode lasers must be explored.

6. The time-averaged vibration technique seems amenable to new spatial filtering methods capable of eliminating the necessity of overlapping the two edges. Suffice it to say however, that moire fringes formed between maximum and minimum slit widths (corresponding to positive and negative maximum vibration amplitudes, respectively) do not have sufficient contrast to be usable.

7. The profile patterns produced by various non-slit-like apertures appear
amenable to filtering and focusing techniques which will lend an axis of symmetry to the situation, allowing considerably easier interpretation.

8. The many possible z-factor devices need to be investigated, especially those capable of detecting displacements in two or three dimensions simultaneously.

9. The very much related two-slit transducer (and its three-slit variant) should be compared conclusively to the single slit type described here.

10. The index of refraction measurements which have been diffractively made along a line should be further investigated.

11. Large-aperture-lens-formed patterns need to be conclusively delineated as to practical possibilities and uses.

12. Multi-transducer element line arrays should be constructed and tried in conjunction with the many possible filtering techniques.

13. Diffractographic measurement of corrosion, wear, and other related phenomena to be investigated.

14. The vast array of possible optical data processing techniques applicable to diffractogram records (or their equal) must be investigated. In particular is the possibility of optical time integration using a cylinder lens to give the probability distribution function of analog records. Correlations and convolutions may also be possible.

15. Extensive investigation and sheer inventiveness is required to develop inexpensive reliable and accurate fringe splitting techniques for diffractographic transducers, as well as other simple detection schemes which give readily interpretable results.
16. The optical reading of embossed characters needs considerable work. Is it useful?

17. Arithmetic computation and real time correlation using matched diffractogram filter line arrays needs further study.

18. The $z$-factor and bounce-off techniques should be checked out completely from an optical point of view, together with cylinder vs. edge patterns.

19. Thermal coefficient of expansion, concrete shrinkage and plastic curing creep or shrinkage experiments can be performed.

20. The use of Diffractography to determine deflections of unusual structural shapes (e.g., a beam with holes, an arch etc.) should be exploited.

21. Non-destructive testing applications such as flaw determination in honeycomb panels should be investigated.

22. Dynamic Diffractographic measurements (on shock loaded bars, gun barrels, etc.) can be made and should be quite useful.
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Fig. 1: Original Diffractographic Experiment
Fig. 2: Fraunhofer Diffraction Pattern Formation by a Slit Aperture
Fig. 3: Oblique Incidence Illumination. Diffractograms are Actual Size x 0.87, and were produced by a slit of \( w = 9.35 \times 10^{-3} \) inches at a distance \( R \) from the Camera of 80 inches.
Fig. 4: "Z-Factor" Diffraction Pattern Arrangement

\[ w = \sqrt{\left(w_0 z\right)^2 + z^2} \]
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\[ w = 3.35 \times 10^{-3} \text{ inches} \]

\[ z = 0, 0.2, 0.4, 0.6, 0.8, 1.0 \text{ inches} \]

\[ w = 13.3 \times 10^{-3} \text{ inches} \]
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<table>
<thead>
<tr>
<th>R</th>
<th>W</th>
<th>Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>6'</td>
<td>W</td>
<td>1/8 sec.</td>
</tr>
<tr>
<td>6'</td>
<td>2W</td>
<td>1/8 sec.</td>
</tr>
<tr>
<td>3'</td>
<td>W</td>
<td>1/8 sec.</td>
</tr>
<tr>
<td>3'</td>
<td>W</td>
<td>1/16 sec.</td>
</tr>
<tr>
<td>3'</td>
<td>W</td>
<td>1/32 sec.</td>
</tr>
<tr>
<td>6'</td>
<td>W</td>
<td>1/32 sec.</td>
</tr>
<tr>
<td>6'</td>
<td>W</td>
<td>1/16 sec.</td>
</tr>
<tr>
<td>6'</td>
<td>W</td>
<td>1/8 sec.</td>
</tr>
<tr>
<td>6'</td>
<td>4.67x10^{-3}''</td>
<td>Same</td>
</tr>
<tr>
<td>6'</td>
<td>4.37x10^{-3}''</td>
<td></td>
</tr>
<tr>
<td>3'</td>
<td>4.67x10^{-3}''</td>
<td>Same</td>
</tr>
<tr>
<td>3'</td>
<td>4.37x10^{-3}''</td>
<td></td>
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</tbody>
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- Tachogenerator
- Power Supply
- Brake
- Diffracto Transducer
- Strain Gage Transducer
- Recorder
- Hydraulic Motor
- Laser
- Screen
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SPEED = 800 rpm
TIME BASE = 50μ sec/div
SPEED = 800 rpm
TIME BASE = 50μ sec/div

Fig. 19: Detector Output at Constant Speed and Various Torques
Fig. 20: Detector Output at Constant Torque and Various Speeds
Fig. 20: Detector Output at Constant Torque and Various Speeds

TORQUE = 12.1 KG-CM
TIME BASE = 50 µ sec/div

1150 rpm
725 rpm
600 rpm
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SCREEN

REFERENCE

KEY

LENS
Fig. 46: Moire Fringe Photographs and the Change in Various Razor Blade Profiles Calculated Therefrom
Fig. 47: Arrangement Used to Illustrate Spatial Filtering of a Diffracton Pattern
Fig. 48: Detector Output Voltage versus Slit Width Change

$w_0 = 0.362 \text{ mm}$

**Diagram:**

- **X-axis:** $\Delta w$, Slit Width Change ($\mu$m)
- **Y-axis:** Photodetector Output (volts)

The graph shows a sinusoidal relationship between the photodetector output voltage and the slit width change, with $w_0 = 0.362$ mm.
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From Top to Bottom: 25, 125, 225, 325 pounds
Fig. 50: Playback Orders Corresponding to Various Loads
From Top to Bottom: 25, 125, 225, 325 pounds
Fig. 51: Line Diffractograms and Their Playback Orders
Fig. 51: Line Diffractionograms and Their Playback Orders
Fig. 52: Recording Apparatus
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Appendix A: What is Single Slit Diffraction?

Introduction

During the course of this work, the simple Fraunhofer diffraction relationships (equation 1 and 2, text) have been used on countless occasions. These relationships are derived in virtually all intermediate optics texts, always by integrating the effects of an array of coherent point or "secondary" sources filling the aperture.

An alternative and little discussed approach also exists. First proposed by Thomas Young in 1802, it essentially implies that slit diffraction may be explained by interference between a direct undiffracted wave and two boundary diffraction waves, one from each edge. This simple explanation is appealing because it describes diffraction in terms of the cause of its origin—the edges, (or rather boundaries in the general case). It is the purpose of this appendix to add some observations which lend credence to this approach.

A dramatic and often observed demonstration is shown in fig. (A - 1a). Two single edge patterns produced by two adjacent non-parallel edges are gradually brought into parallelism—yielding an obvious interference effect which is called a slit diffraction pattern. When a laser source is used this effect is so obvious as to leave little doubt what is occurring and it is this very approach, interference between diffracting edge waves, which was used to obtain the "Z-factor" relation of Chapter 2.

Another example of edge participation is shown in fig. (A - 1b). In this case a $\text{TEM}_{00}$ mode gaussian (see Appendix B) of 5 milliwatts, and a $\text{TEM}_{01}$ mode of 0.5 milliwatts, both from the same laser, illuminate the same slit, though at different times: In the latter case, the peaks of each lobe (which are 180° out of phase) fell on the slit edges. From the figure it is apparent
that almost as much energy was distributed into the secondary maxima, even though the total output was ten times less.

An interesting practical conclusion results from this experiment. Essentially the reverse of apodization is here desired, i.e., we wish the most intensity in the incident irradiation field to lie in the region of the slit edges. While filters can accomplish this using any incident field, nothing worthwhile is accomplished unless the energy can be redistributed. Perhaps the most efficient approach is to construct a laser which emits only in the TEM$_{01}$ mode, at the same or greater power levels than the emission in the TEM$_{00}$ mode. In a gas laser form, this could probably be done by making the tube diameter just large enough to support the TEM$_{01}$ mode and using a wire (or scratch on a mirror, etc.) to cause high losses in the TEM$_{00}$ and TEM$_{01}$ modes.

A logical follow-on to the preceding TEM$_{01}$ mode experiment conclusively demonstrates the existence of edge waves. As shown in fig. A-2, two diverging phase related beams from a Michelson interferometer each illuminate one edge of a slit. The diffractograms produced illustrate that two individual edge patterns (top), when brought into line, produce fringes in regions away from $\theta = 0$ just like those of a slit illuminated by a single beam. Fringes can be shifted by varying the phase relation between the beams.

Considerable work is in progress concerning their effects. One interesting result is that the often observed variation in overall intensity between the sides of a pattern may very well be explainable by variations in edge wave intensity. The lower diffractogram of fig. A-2 is evidence of this.

It is further noted that the "bounce-off" arrangement discussed in chapters 2 and 8 can be explained as the superposition of four boundary waves; one which is not reflected, two which are reflected once and a fourth which is reflected twice. Since a $\pi$ phase change occurs on reflection, it is
apparent that the first and fourth waves add coherently and are $180^\circ$ out of phase with the once-reflected waves. In the language of chapter 2, these once-reflected waves produce the "normal", and the zero and twice-reflected waves the "reflected" set of fringes.
A. Effect of Edge Non-Parallelism

\[ \xi = 8^\circ \ 2^\circ \ 0^\circ \]

B. Slit Illumination by Various Laser Modes

Exposure: 1/15 sec.
Mode Power: 5 milliwatts

Exposure: 1/15 Sec.
Mode Power: 0.5 milliwatts

Fig. A-1: Boundary Wave Illustrations
Fig. A-2: Illumination of Slit Boundaries by Two Separate Phase-Related Beams from a Michelson Interferometer.
Appendix B: Slit Illumination by TEM\textsubscript{00} Laser Beams

Introduction

To-date, nearly all diffractographic experiments have been performed using gas lasers. Since the common laser of this type is the single transverse mode, or TEM\textsubscript{00}, type it is informative to consider the apropos characteristics of waves emitted by such lasers, and their effect if any on diffractographic measurements.

Fig. A - 1 has illustrated the TEM\textsubscript{00} modes primary characteristics, i.e., a circularly symmetric spot with gaussian distribution of intensity along the mode radius together with a constant phase. Since equation 1 assumes both constant phase and constant amplitude, some effects on diffraction patterns produced with such illumination could be expected. Two examples are illustrated--laser beam and slit axis co-incident, and not co-incident.

Laser Beam and Slit Axis Co-incident

The TEM\textsubscript{00} gaussian laser beam has a circularly symmetric intensity, whose radial distribution is described by

\[ I = I_0 e^{-2r_1^2/r^2} \]  \hspace{1cm} (B - 1)

where \( r_1 \) is the beam radius at which intensity equals \( 1/e^2 \) that at the center, \( I_0 \).

Most diffractographic experiments to-date have been conducted using He - Ne lasers which typically have values of \( r_1 \) under 1mm--even when the laser-slit distance is a meter or more. This implies that when slit widths of 1 millimeter or so are used, intensity at the slit edges is \( \frac{1}{2} \) of that at the center, or less. In other words, as slit width is increased diffracted fringe intensity could be expected to drop. This phenomenon is illustrated in fig. (B - 1a).
Laser Beam not on Slit Axis

If the slit and gaussian laser beam centerlines are not co-incident, differing incident power falls on each edge, and the effects in the diffraction pattern can be quite noticeable. Nicknamed "fuzzy fringes", the effect is often observed in practice when large slit widths (relative to laser beam diameter) are used. In particular, the situation occurs when one slit edge is fixed with the other moving in such a manner as to increase the separation between the laser beam and slit centerlines.

Fig. (B - 1b) presents diffractograms taken for three slit-beam axis orientations. An obvious decrease in contrast of the diffraction minima occurs when the illuminating radiation distribution becomes non-uniform. In the worst cases, virtually no radiation falls on one slit edge and the resulting pattern is very much like a single edge type with a slight trace of modulation due to radiation from the other edge.

Clearly, this condition results in degraded contrast fringes and should be avoided if possible. However, a more important question is whether the fringe spacing one might measure from such a pattern actually changes from that produced using uniform illumination. If so, an error exists whose value is the change. A series of experimental measurements (using the eyeball-vernier caliper technique) was taken on live diffraction patterns and the results indicate that no error in fringe spacing occurs which is detectable in this manner. See reference 7, however, for more information.
A. Slit Width Effects

Laser Beam Diameter Visible: .090"
All Exposures 1/60 sec.

\[ w \text{ (inches x } 10^{-3}) \]

- 4.26
- 9.25
- 15.5
- 39.4
- 56.5
- 70.0
- 87.0

B. Off-Axis Effects

Laser Beam Diameter Visible: .070"
Slit Width, \( w = .012" \)

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Fig. B-1: Gaussian Illumination Effects
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