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THOMAS NORMAN. MOORE

University of Windsor

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THE EFFECT OF NON-UNIFORM INSERT PITCH ON NOISE GENERATION DURING FACE MILLING OPERATIONS

by

Thomas Norman Moore

A Dissertation submitted to the Faculty of Graduate Studies and Research through the Department of Mechanical Engineering in Partial Fulfillment of the requirements for the Degree of Doctor of Philosophy at the University of Windsor

(C)

Windsor, Ontario, Canada 1985
ABSTRACT

Excessive noise generation often occurs during the face-milling of thin-walled aluminum workpieces. It has been suggested that such noise can be reduced by utilizing milling cutters which employ non-uniform insert pitch.

This report presents the results of a test series undertaken to determine the noise reduction potential of four different non-uniform insert pitch cutters used to machine the engine mounting face of an aluminum transmission housing. The tests were performed using the actual high volume, transfer-line machinery.

It is shown that the use of non-uniform insert pitch does not necessarily reduce overall noise generation. There is, however, often a reduction in noisiness.

To better understand the structural response of the workpiece a complete modal analysis is performed. In addition, noise frequency response functions are generated and are shown to form a basis on which to predict the noise radiation characteristics of the workpiece.

A hybrid digital-analog computing system is developed and then used to simulate the workpiece response and to demonstrate the influence of various parameters such as forcing function definition, natural frequency, damping, and width of cut on system response.

A more realistic forcing function definition, than the often used force impulse model, is developed. It is
subsequently shown that significant differences in workpiece response occur for excitations based on these two models. It is also demonstrated that variation in width of cut has a significant influence on the forcing function definition.

Several techniques for improving "quiet cutter" design strategies are illustrated. In particular, a technique of sectoring the workface into sections with constant "effective" width of cut is described. Such an approach permits the width of cut variation to be accounted for in a relatively simple manner. It is also shown that noise frequency response functions, when used in conjunction with a realistic model of insert engagement forces, allow prediction of workpiece noise emission characteristics as a function of cutter location.
DEDICATION

To "My Girls":
GEORGINA, LAURA and SARAH

- vi -
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NOMENCLATURE

a - experimentally determined exponent in cutting force model

\( a_n \) - Fourier coefficient of cos terms

B - number of inserts in the cutter

b - experimentally determined exponent in cutting force model

\( b_n \) - Fourier coefficient of sin terms

c - experimentally determined exponent in cutting force model

\( c_1, c_2 \) - complex amplitudes

\( C_n \) - phasor of the nth harmonic component

d - depth of cut (mm)

d\(_B\) - effective diameter of cutter (mm)

d\(_{BA}\) - decibel

D\(_{BA}\) - decibel, A-weighted

D\(_{FT}\) - discrete Fourier transform

F - forcing function (N)

\( F_{max} \) - maximum sampling frequency (Hz)

\( f_0 \) - feed per tooth for equally spaced insert cutter (mm)

\( f_m \) - machine feed rate (mm/min.)

\( f_1(t) \) - non-dimensional cutting force produced by the ith insert

\( f(t) \) - total non-dimensional cutting force acting on the workpiece

\( f_{in} \) - actual frequency component present in signal (Hz)
\( f_s \) - sampling frequency (Hz)
\( f_i \) - feed for the ith tooth of a cutter (mm)
FRF - frequency response function
\( G \) - gap width between any two adjacent pulses (seconds)
\( G_{xx} \) - auto power spectrum of input to system
\( G_{yy} \) - auto power spectrum of system response
\( G_{yx} \) - cross power spectrum
\( \overline{C_{nx}} \) - ensemble average of the cross power spectrum between noise and input signal to a system
\( H(f) \) - frequency response function
\( H(s) \) - transfer function
\( [H(s)] \) - transfer function matrix
\( H_A \) - frequency response function based on input as surface movement and output as sound level at a point in space
\( H_N \) - frequency response function based on input forcing function and output as sound level at a point in space
\( H_s \) - frequency response function of a system based on forcing function input and surface movement output
\( Hz \) - Hertz (cycles per second)
\( h_{rc}(s) \) - an element of the transfer function matrix
\( I \) - time interval between pulses (seconds)
\( j \) - complex operator \( \sqrt{-1} \)
\( K \) - experimentally determined cutting force constant
\( \) - spring constant (N/m)
\( K_1 \) - experimentally determined cutting force constant

- xxi -
\( S_y(f) \) - linear Fourier spectrum of \( y(t) \)

\( s \) - Laplace operator

- scale factor for analog computer

\( T \) - period of one cutter revolution (seconds)

- period of interest in \( L_q \) determination (seconds)

- time record length in DFT (seconds)

\( T_n \) - period of the system at its undamped natural frequency (seconds)

\( t \) - time (seconds)

\( \Delta t \) - time interval between samples in DFT (seconds)

\( \{ U_k \} \) - mode shape vector

\( \{ U_k \}^T \) - transpose of mode shape vector

\( u(t) \) - response of system to arbitrary forcing function (m)

\( W \) - width of workpiece (mm)

\( X_n \) - phasor of the \( n \)th harmonic component of the system response

\( x \) - displacement from datum (m)

\( x(t) \) - time domain input to a system

\( Y \) - displacement of tool from datum during cut (m)

\( Y_o \) - deviation of previously cut surface from datum (m)

\( y(t) \) - time domain response of a system

\( z \) - number of blades in cutter

\( z_c \) - fraction of blades cutting simultaneously

\( \alpha_n \) - phase angle (radians)

- xxiii -
\( \gamma^2 \) - coherence function
\( \Delta \) - effective workpiece width (mm)
\( \zeta \) - damping ratio
\( \theta_i \) - the angular location of tooth i on the cutter (degrees)
\( \lambda \) - spring constant (N/m)
\( \nu \) - cutting speed (m/s)
\( \sigma \) - damping factor (rad/s)
\( \tau \) - time (seconds)
\( \phi \) - angle between adjacent inserts (radians)
\( \phi_n \) - phase angle (radians)
\( \phi(\omega) \) - structural cross receptance (m/N)
\( \psi \) - stability phase angle (radians)
\( \Omega \) - angular velocity of cutter (rad/s)
\( \omega \) - circular frequency of chatter (rad/s)
\( \omega \) - circular frequency (rad/s)
\( \omega_d \) - damped natural frequency (rad/s)
\( \omega_n \) - undamped natural frequency (rad/s)
\( ^* \) - denotes complex conjugate
\( ^\text{—} \) - denotes the ensemble average of the quantity indicated
Chapter I

INTRODUCTION AND OBJECTIVES

The manufacture and sale of machine tools forms a significant portion of Canada's multi-billion dollar metalworking industry. One particular area of importance in the machine tool industry is the design and build of high volume, multi-station, automatic machine tools, often referred to as "transfer lines".

These machines consist of many automatic machine tools which are interconnected by a central transfer spine. The workpieces, generally mounted on pallets, which act as the machining fixtures, are moved sequentially from one tool, or "station", to the next. This process is intended to meet high production requirements and, as such, is used extensively in the transportation industry.

The relatively recent thrust made by the transportation industry toward lighter, more fuel efficient vehicles, has meant that the machine tool industry must often machine thin-walled, light-weight castings on high-volume, transfer-lines. In addition, the need to reduce unit costs has led to demands for substantially increased production rates.
Thus, manufacturers of transfer lines are presently confronted with the need to produce machines which cut more metal faster, from relatively more compliant parts, than has previously been the case. Unfortunately, this often results in the generation of excessive noise levels during the machining cycle.

The generation of such noise levels makes it extremely difficult for manufacturers to meet existing occupational noise exposure limits. The fact that many jurisdictions are presently contemplating a further tightening of noise regulations, will only compound these difficulties. Due to the highly competitive nature of the machine tool industry, particularly as a result of increasingly sophisticated foreign competition, the inability to produce machines which meet present and expected future noise limits could well cripple the domestic industry.

Experience has shown that face-milling of relatively compliant workpieces is a particularly serious noise source. Under such circumstances it is the workpiece itself, and not the machine structure or the milling cutter body, which is the primary radiator of noise.

Although it is theoretically possible to reduce the noise generated during the milling process by employing damping pads, enclosures, or other peripheral methods, the practical problems of additional cost, space limitations,
maintenance difficulties, workpiece deflections, etc. make these solutions generally unattractive.

The reduction of workpiece vibration, and hence noise, at its source is, in every respect, the most desirable solution. This can best be achieved by modifying the milling cutter in such a manner as to minimize the workpiece response to the excitation imparted by the milling cutter insert engagements.

Studies by various researchers have indicated that the non-uniform spacing of events in a multi-event cycle has the potential to reduce the resultant vibration and hence noise. Applied to face-milling cutters, this technique results in an irregular cutting insert pitch. Such an approach is highly attractive since it attacks the source of the noise generation without recourse to expensive and difficult to maintain control methodologies. There would be essentially no cost penalty associated with the production of a face mill with non-constant insert pitch relative to the familiar cutter with uniformly spaced inserts.

Unfortunately, the presently available studies describing the use of non-constant insert pitch in reducing noise generation have been concerned with workpieces of simple geometry (bars, box-beams, etc.) using small, single-purpose, milling machines. No studies exist which investigate the effect of non-uniform insert pitch on noise generation during the face-milling of a geometrically
complex workpiece using actual, high volume, production machinery.

It is, therefore, the purpose of the present study to provide just such information. In addition, based on the results of these tests, means of improving the effectiveness of this methodology will also be proposed.

The General Motors "125" transmission case was selected as the focus of this study since it was geometrically complex and was known to produce excessive noise levels during the face milling of its engine mounting face. Additionally, the actual high-volume transfer line used to machine it was available for test purposes.

The study itself was broken up into three distinct phases and, as such, the results are reported in three main sections:

i) The first section describes the methodology and results of the actual machining tests employing a variety of non-uniform insert pitch milling cutters.

ii) In the next section the workpiece's structural response characteristics are defined using modal analysis and noise frequency response function methods. These characteristics are shown to be important in determining how the workpiece will respond under the influence of the cutter's insert engagement forcing function.
iii) Finally, the results of system simulation studies using a specially built hybrid digital-analog computing system are reviewed. An improved model of the cutting insert forcing function is employed in these simulations. In addition, means of predicting the noise radiation characteristics of the workpiece are illustrated and the results compared to actual measured values.
Chapter II

LITERATURE SURVEY

The typical uses of non-constant insert pitch in face-milling cutters will now be reviewed. This review of the published literature will attempt to define both the present state of knowledge in this particular area and the deficiencies that exist when attempting to apply such knowledge to the design of milling cutters which must be integrated into the actual production process.

However, to better understand the significance of a particular application of non-constant pitch, it is necessary to appreciate the fact that two major types of vibration are encountered in a machine tool system. The following section will review the important features of both types of vibration.

2.1 GENERAL VIBRATION CATEGORIES

In general, machine tool vibration can be classified into two main types: self-excited vibration and forced vibration.

Self-excited or regenerative vibration (also often referred to as "regenerative chatter") arises through instabilities of the machine-tool-workpiece cutting process.
system resulting from dynamic fluctuations in the cutting force during metal removal. It should be noted that instability initially has no vibration forces associated with it, but such forces build up over a period of time. Common controls for self-excited vibration include increasing the static and dynamic stiffness of a machine structure or reducing the cutting force at the tool by selecting (or changing) the direction and magnitude of the cutting force through proper tool design. In other words, chatter can be stopped at two places along its self-excited path: the machine structure or the tooling.

Forced vibrations arise from moving or rotating sources that produce periodic forces on the machine tool. When these sources are located within the machine tool itself, they are often called internal-force vibrations. Those not associated with the machine tool are termed external-force vibrations or transmitted vibration.

Examples of typical transmitted vibration sources include other nearby machinery, such as operating machine tools, which transmit vibration through the building foundation. Control of these vibrations is normally accomplished by either removing the source of vibrations or isolating the source of vibrations from the equipment being affected.

Typical sources of internal-force vibration include milling cutter impacts from interrupted cuts,
gear-drive irregularities, hydraulic or coolant pump motors, spindle unbalance, etc. Control is generally achieved by removing, or more practically speaking, minimizing, the imbalance or eccentricities associated with such sources. In the case of interrupted cuts, as often encountered with face-milling, the interrupted nature of the cut is always present. The forces associated with the cutting action cannot be removed, since they are a necessary part of the metal removal process. Even attempting to significantly reduce the magnitude of the cutting forces would result in unacceptably low metal removal rates, since it is apparent that large force reductions could only be achieved by consequent large reductions in depth of cut and feed rate. Therefore, cutting forces must be dealt with in such a manner as to minimize the response of the excited system and in this way reduce the resulting vibration and, hence, noise radiation. Methods that could be used to implement such a strategy will now be reviewed.

2.2 REVIEW OF METHODS USED TO CONTROL VIBRATION IN MILLING

One method of controlling vibration in metal cutting, and as a direct consequence the associated noise radiation, is through improved design of the machine tool structures and workpiece fixtures.

As an example, it is theoretically possible to reduce noise generation by applying, during the machining
cycle, damping pads at appropriate positions on the workpiece [15]. However, the cost of extra equipment, space limitations, and possible workpiece deflections due to pad pressure, ultimately make such a solution unattractive.

The more basic structural redesigns that would provide some vibration reduction certainly have potential for future machines, but due to the large number of older machine tools presently in use, such an approach will have little effect in the industrial environment for many years since few companies have either the resources or the incentive to rebuild or replace their entire manufacturing facilities. It should be noted that for the type of transfer line milling applications envisaged in this study, the majority of the noise produced is radiated by the workpiece, under the influence of the forced excitation imparted by the engagement of the cutter inserts [21]. Since the machine structure is a minor noise contributor, changes to the structure of the machine tool would not be particularly effective in reducing noise from this type of operation.

Thus, the most attractive procedure, in every respect, is to reduce the workpiece vibrations at their source. This can be best achieved by modifying the milling cutter in such a manner as to minimize the vibration response, and hence noise, of the workpiece under the action of the forces generated by cutter insert engagements.

It has been recognized that unequal blade spacing
in a face-milling cutter can reduce vibration. Commercial cutters with unequal blade spacing have been available for over 30 years. However, the design of these cutters has been, generally, on a trial and error basis with all the attendant difficulties of such a procedure.

The literature published on the selection of optimum milling cutter blade spacing using analytical techniques has focused on both the regenerative and forced vibration areas. As these are two quite different applications, each will now be reviewed in turn.

2.2.1 Regenerative Chatter

The basic theory of regenerative chatter is due to Tlusty et. al. [31] and Tobias and Fishwick [35]. Although this theory has formed the basis for a large volume of theoretical work on the specific mechanisms of chatter, when applied to actual machining conditions the theory has been found to be deficient in many instances. As a result, certain of the assumptions associated with this theory have come under suspicion [19]. The question of deficiencies, however, is far from clear and certainly much more work remains to be done before the definitive theory is promulgated. Therefore, since the specific work involving the use of non-uniform insert spacing in milling cutters has, in most cases, been based upon the theory of Tlusty and Tobias, the details of this theory will now be discussed without further concern for the deficiencies known to exist.
To begin, Tlusty writes the variable cutting force component due to chip thickness variation as,

\[ P = r(Y - Y_0) \quad 2.1 \]

where \( r \) is a coupling coefficient relating the cutting process to the system response, \( Y_0 \) the undulations on the previously cut surface and \( Y \) the vibration of the tool from a fixed reference. The vibration which results from the variable cutting force is given by,

\[ Y = P\phi(\omega) \quad 2.2 \]

where \( P \) has the direction of the cutting force and \( \phi(\omega) \) is the cross receptance of the structure. The limit of stability is then written as,

\[ \left| \frac{Y}{Y_0} \right| = 1 \quad 2.3 \]

A stable phase angle, \( \psi \), is assumed to exist between the subsequent surface undulations which corresponds to a maximum energy of self-excitation.

Tobias represents the dynamic system characteristics by an equivalent single degree-of-freedom system as,
\[ m \frac{d^2x}{dt^2} + \frac{cdx}{dt} + \lambda x = -dp \]  

2.4

The forcing function, \(-dp\), consists of a chip thickness variation effect and a penetration rate effect as:

\[ dp = 2 \ Z_c \ K_1 |x(t) - x(t-T/Z)| + Z_c \ \Omega \frac{dx}{dt} \]  

2.5

where \( Z \) is the number of blades in the cutter, \( Z_c \) is the fraction of blades cutting simultaneously, \( K_1 \) and \( \Omega \) are cutting force constants, \( T \) is the time for one cutter revolution and \( \Omega \) is the angular velocity of the cutter. Thus Equation 2.4 contains damping terms on both sides. When the total damping is zero the process is on the threshold of stability, with net positive damping giving a stable mode of operation.

Slavicek [29] considered the effect of unequal insert spacing on milling stability by extending Tlusty's theory. This extension is based upon the fact that for milling, the phase angle \( \psi \) may be written as,

\[ \psi = \frac{\omega l}{v} \]  

2.6

where \( \omega \) is the circular frequency of chatter, \( l \) is the pitch of the inserts and \( v \) is the cutting speed. Equation 2.6 now imposes a geometric condition on the phase angle which was not present in the development by Tlusty.
By assuming that the motion of the cutter is rectilinear and the width and depth of cut are constant, that the inserts vibrate simultaneously with respect to the workpiece and they have equal amplitudes of vibration, that the inserts are set alternately at one larger pitch and one smaller pitch than that of even spacing, and finally, that an equal number of teeth of both pitches cut simultaneously, Slavicek was able to develop a graphical method of stability analysis.

This method was then used to determine the frequencies of unstable vibration and the corresponding limiting depth of cut. Also, knowing the natural frequency of vibration and cutting speed, an expression was determined for an optimum pitch difference.

Experimental tests were run using a vertical milling machine and a face-milling cutter of 3.15 mm diameter containing sixteen cemented carbide inserts. The workpieces were very flexible hollow cast iron boxes. The results showed a significant increase in stability over a narrow speed-frequency band for which the cutters were designed.

Unfortunately this type of approach, that is, limited to pitch periods of two cutting edges, means that for each combination of cutting speed and natural frequency another pitch ratio must be used for optimum performance. Of course for transfer machines, where the dedicated tooling is used at a particular location for a single machining
task, and thus the cutting speed and natural frequencies are well known and constant, such a restriction is not particularly critical. Under such conditions Slavicek's approach to stability enhancement could be quite useful.

Vanherck [36],[37] demonstrated that increasing the number of blades in a pitch period effectively increased the range and level of stability of a cutter with unequal blade pitch. The insert spacing was chosen by arbitrarily selecting increments to be employed in either a linear or sinusoidal based mathematical evolution. The results of a given blade evolution were checked using Thusty's regenerative chatter theory by employing a "machine equivalent" single degree-of-freedom system with a damping ratio of 0.03. All simulations were then carried out on a digital computer. For the case of linear pitch evolution the increase in stability over the constant pitch case was found to be on the order of 400%. For sinusoidal pitch evolution the average increase in stability was always less than that for linear pitch evolution, yet always greater than that for the constant pitch case. Unfortunately, no experimental evidence was presented to substantiate such claims.

A subsequent article by Vanherck [24] again reviews the stability gains available using non-constant insert pitch, and also introduces some graphical aids which are useful in calculating stability limits. Again, although
it is mentioned that the results presented had been experimentally confirmed, nowhere in the paper is such an experimental technique described. Certainly no results from actual cutting tests are reported.

Opitz et. al. [23] extended the theory developed by Tobias to include simultaneous cutting of several milling cutter blades, directional factors for the cutting force, and time lag for phase shifts between force and vibration. The resulting differential-difference equations with lag and time varying coefficients had no mathematical model available to determine stability. Therefore, to analyze this equation an analogue computer was employed. When considering unequal spacing only two different spacings were considered, an even number of blades were assumed to cut simultaneously, and time averages of the directional coefficients were used. A single degree-of-freedom system model was utilized for the analysis. The computations revealed that by using a cutter with irregular tooth pitch, the borderline of stability was shifted toward a higher width of cut within a considerable speed range. However, outside this particular range (which is shown to be dependent upon the system's natural frequency, the number of inserts and the pitch ratio) no improvement was obtained. It was therefore noted that since an important limitation for the use of cutters with irregular tooth pitch is the dependency on the system's natural frequency, then the
application of such cutters is mainly of interest when a
single mode dominates the chatter behaviour. However, it
should be noted that this type of response is not untypical
of many machine tools exhibiting poor chatter
characteristics.

Varterasian [39] [40] used a "mechanical frequency
modulation" method to choose the blade spacing to produce a
"white noise" cutter. The input due to each milling cutter
insert was assumed to be a constant amplitude impulse. In
this manner the excitation force spectrum could be
calculated using Fourier analysis. The concept was to vary
the spacing sufficiently to produce a broad, flat spectrum.
The modulation consisted of an initial single period
sinusoidal modulation, calculation and examination of the
resultant spectrum and subsequently an interactive trial and
error attempt to modulate the blades to smooth the spectrum
further. For the two cutters examined (a "rougger" and
"finisher") "conservative" modulation angles were used due
to the constraints placed on the allowable variations. For
this reason the excitation spectra were not as well
dispersed as the "ideal" spectra. The final spectra had
reduced fundamental amplitude with several of the sidebands
near the blade impacting frequency also containing some of
the vibrational energy.

The cutting tests were run on cast iron
(Mechanite) bars of simple rectangular geometry, with zero
offset between cutter and bar. The results of the tests
showed a reduction in vibration in nearly half of the test
runs, with little change in the remainder.

In several tests the evenly spaced mill actually
performed better than the modulated cutters. It was noted
that since these cases were few, and were "generally not
repeatable, it was difficult to assess their importance.

A subsequent paper by Varterasian [41] discussed
the application of "white noise" design principles to
reamers, again in an effort to reduce chatter. Recognizing
that any real tool has limitations on the actual spacing
variations possible (due to physical size of the tool and
metal removal limitations on a given blade), a method of
quantifying the degree of "whiteness" for a given
arrangement was developed. In this manner the best
practical spacing could be selected, based on a quantitative
evaluation. Three examples of the "white noise" principle
applied to reamers with six, eight, and ten flutes, were
presented. The significant improvement in performance over
their equally spaced counterparts was documented.

Varterasian noted that one means of improving the "white
noise" technique would be to incorporate the machine-cutter
transfer function into the spectrum calculation procedure.
Although some reservations were expressed concerning the
time required to obtain such transfer functions, it might be
noted that present technology now allows rapid acquisition
of such information.
Thusty et al. [32] [33] investigated the performance of "special" milling cutters in controlling chatter. In this case, "special" was taken to mean any cutter which did not have constant pitch between subsequent teeth. Although, as noted in this review, similar work had already been done by other researchers [29] [36] [23], Thusty wished to improve on their "simple" analyses by removing, or making more realistic, assumptions. In this manner, he felt it was possible to provide a more realistic evaluation of special cutter performance.

The most significant simplifying assumption did away with was the requirement that all the cutter teeth have the same directional orientation. This would only be true for a cutter of infinite diameter, or for a completely different class of tools such as broaches. Also, most of the earlier analyses represented the machine tool structure by a single degree-of-freedom system. Thusty modelled the structure using two mutually perpendicular degrees-of-freedom.

Four different cutter types were investigated: non-uniform pitch (linear variation), alternating helix, "Strasmann Cutters" (serrated and sinusoidal teeth), and "Crest Kut" edges (proprietary design). All showed a general improvement in stability against chatter when compared to the regular cutter.
Although Thusty's improved analysis approach gave almost the same results as the simplified analysis in terms of the cutting speed range in which a given cutter would be most effective, he suggested that his analysis provided a more accurate assessment of the actual stability improvement. No experimental verification of the computed stability limits was presented.

In summary then, to indicate the importance of non-uniform insert spacing in chatter suppression consider a quote from Solaja and Kalajdzic [30] in their review of papers dealing with this topic: "it has been proved that with any mode of uneven spacing the stability (against self-excited vibration) cannot be lower than with even edge spacing".

Since regenerative chatter is a problem only in cases where the stability limit is exceeded and since, as just noted, it is generally accepted that any unequal blade spacing will increase stability against these vibrations, therefore the focus of interest in this investigation is the use of non-uniform pitch to reduce forced vibrations caused by the interrupted nature of the cut in the face-milling process. Unlike regenerative chatter, these forced vibrations are always present and must thus be dealt with under all normal machining conditions.

2.2.2 Forced Vibrations

Unfortunately, relative to the volume of work
published in chatter applications there is little available literature exploring the efficacy of non-uniform insert pitch in reducing forced vibration. To some degree this situation is a reflection of the priorities held by the machine tool industry. Until recently forced vibration (and the resultant noise emission) has generally not been looked upon as a serious problem. However, factors such as the introduction of strict noise control regulations, the increasing demand for high quality parts, the requirements for longer tool life, and the emphasis placed on improving machine tool maintainability, have begun to alter previous conceptions. These changes in attitude among machine tool builders will certainly result in substantial interest in the researching and application of non-uniform insert pitch to face-milling cutters and very likely to other processes as well.

Up to this point in time, the problem of reducing forced vibration has been approached by calculating the frequency spectrum of the blade impacts on the workpiece and applying modulation principles to choose blade spacing which will "smoothen" the excitation spectrum.

Doolan et al. [7] described a method of designing a face-milling cutter specifically to minimize forced vibrations. The procedure employed assumed that the force excitation due to the engagement of each cutting insert was equivalent to a constant level impulse which was
unaffected by blade spacing. From this time domain forcing function definition, the equivalent frequency spectrum was derived using Fourier analysis. Also knowing the frequency response function (FRF) of the particular milling machine-cutter-workpiece system, the resulting response spectrum could then be calculated. The "total power" (the sum of the squares of the response spectrum components) of the system response was minimized using Marquardt's [20] compromise algorithm for non-linear least squares and a subsequent random search to account for the multimodality of the objective function. The search methodology was coded to run automatically on a digital computer. Recognizing the practical restraints imposed on milling cutter design, limitations were also placed on both the maximum and minimum allowable insert spacing.

Doolan notes that in the instance where the FRF of the system may not be known, the method proposed could still be used by assuming a frequency response function of unity across the spectrum. Of course, such a procedure then leads to the design of a "white noise" cutter as proposed by Varterasian, since, in effect, the excitation spectrum would be made as broad and flat as possible.

A six-bladed milling cutter was designed using the proposed methodology. However, the system's frequency response function (FRF) was not obtained from the actual milling machine used in the trials, but was a "smoothed"
version of an experimentally determined FRF obtained by Tobias [34] for, what appears to be, a substantially different machine. No justification for this choice was proffered by Doolan. In addition, the geometry and material composition of the workpiece were not described.

During the test sequence, the performance of the non-uniform insert cutter and an identical cutter with equal blade spacing were compared. Noise measurements were made using the 31.5 Hz octave band since the fundamental insert engagement frequency was 36.5 Hz. During the cutting process the conventional cutter produced a level of 28.5 dB while the unequal spacing cutter produced a level "too small to measure within this band". The authors conclude that their unequally spaced design therefore showed a significant reduction in acoustical levels.

It might, however, be argued that such a conclusion is rather tenuous. Measuring only in the 31.5 Hz octave band is not a particularly representative evaluation of the practical value of the noise reduction. In an industrial environment the noise measurements would be A-weighted. As such, the results presented in this paper would have no noise reduction significance, since the A-weighting curve itself provides an effective 39 dB attenuation at the frequencies considered. In addition, complete removal of all energy in the 31.5 Hz octave band would not necessarily have any impact on the overall noise.
level generated during machining. Since no such overall level comparison is provided, the true significance of the noise reduction reported in this paper is impossible to establish.

A subsequent paper [8] by the same authors introduced an "improved" estimate of the time domain representation of the forcing function. In this case the insert engagements were represented by rectangular "pulses" rather than "impulses". The method of insert spacing determination then followed the same method described previously. However, no experimental tests were performed to substantiate actual milling cutter performance.

Burney and Wu [3] provided a simple, step-by-step example of the techniques described in the two previous papers. The method described makes use of matrix algebra to implement Marquardt's algorithm. No new information is presented on design strategies for non-uniform insert spacing, the paper being essentially a tutorial.

2.2.3 Other Applications of Non-Uniform Pitch

Although the present study is concerned with face-milling operations, it is interesting to note that modulation principles have also been applied to the problem of reducing noise from other sources.

Ewald et. al. [10] applied a sinusoidal modulation principle to the blades of an electric motor cooling fan to break up the pure tone noise produced by such a fan. The
resultant frequency spectrum was calculated using Bessel functions and showed a reduction in the amplitude of the blade passing frequency with select sidebands containing the additional energy. Experiments with a fan built by this method showed a noise spectrum close to that predicted. The sinusoidal phase modulation resulted in a series of tones with amplitudes at least 3 dB below that of the unmodulated tone, while reasonable blade spacing was maintained.

Krishnappa [16] examined the effects of modulated blade spacing over a range of operating conditions for a centrifugal fan. He was able to show in his experimental results that modulated blade spacings appear to offer some promise in reducing the peak tone level at the blade passing frequency by distributing this energy to the side bands. It was noted, however, that at extreme flow conditions, the level of some side-band tones closely approached, or in fact exceeded, the magnitude of the blade passing frequency for the evenly spaced fan. This phenomenon was attributed to the presence of dissimilar flows from blade passages.

A five horsepower drip-proof induction motor was re-designed by Pavlovic and Bollinger [25] for quiet operation. The blade spacing of the integral fan was selected using the phase modulation principles described by Ewald [10]. The modulation frequency and phase deviation were selected to eliminate the fundamental blade passing tone and place the energy in sidebands. Sound level
measurements showed the modified fan to be 12 dB quieter than the unmodified fan. However, much of this reduction may have been due to modifications other than the non-uniform blade spacing. In particular, modifications to provide low pressure losses, low leakage around the fan, and the use of minimum air flow rates to maintain desired motor temperature, may well have made significant contributions to the overall noise reduction attained.

Duncan and Dawson [9] employed a small axial-flow fan to demonstrate the use of a non-uniform blade spacing which produced minimum overall noise coupled with the subjectively desirable property of not exciting predominant discrete tones.

Experiments with two fifteen bladed fans, one evenly spaced and the other a sinusoidally modulated configuration, showed that the non-uniform spacing resulted in harmonic levels significantly below those for even spacing, and that the spread of tones was such that no single discrete tone was predominant. The blade passing frequency component was reduced by 10 dB using the non-uniform spacing, while the overall level was reduced by approximately 2 dB. The authors also note that the "whiter" sound field produced by the unevenly spaced blading was much more acceptable than that from the evenly spaced configuration, while the small blade re-locations required made negligible difference to aerodynamic performance.
In an article by Neise [22] numerous methods for reducing noise associated with centrifugal fans were reviewed. He noted that by spacing the blades in an irregular pattern around the impeller, the blade passage sound energy would be spread over a wider frequency band. It was noted however, that although this may be desirable from a subjective point of view, the total sound intensity generated remained unchanged.

Varterasian's work in the development of a "white noise" milling cutter, previously described in this survey, was an offshoot from earlier work undertaken to quiet snow tires [38]. The principle is the same as that described earlier for milling cutters, with the tread pitch adjusted in such a manner as to result in a flat excitation spectrum (i.e. a "white noise" spectrum).

Tests carried out on tires with standard commercial tread spacing and those with "white noise" spacing, showed that the latter provided a definite reduction in the sound levels associated with the tread frequency and its harmonics. The reductions were on the order of 5 to 8 dB, with significant spreading of energy to the sidebands. However, overall noise level reduction due to the "white noise" spacing was only on the order of 1 dB.

The possibility of reducing noise from circular saws used in woodcutting was investigated by De Vries and Wu [6]. The technique employed was essentially that developed
by Doolan [7]. It was assumed that the impact amplitude of individual teeth was the same for each tooth and was not affected by blade spacing. Also, the system's frequency response function was assumed to be unity. Unfortunately, this approach is often taken when using Doolan's method, perhaps because the investigators either have difficulty making the necessary FRF measurements on the system of interest, or perhaps, because they have difficulties defining exactly what the "system of interest" is supposed to be. This is, to some extent, a failing with the general methodology, since the original work by Doolan seems to suggest a system frequency response function definition which is more in keeping with chatter considerations than with noise emission.

In any case, the method is generally executed with a FRF assumed to be unity. This simply means that Varterasian's "white noise" technique [39] is actually being employed, since the forcing function spectrum is made as "flat" as possible. This inability to utilize the appropriate system frequency response function to determine the system's ultimate response, and then re-order the inserts to minimize the response spectrum (and not the input spectrum), confines this method to reductions in the tonal character of the noise with little likelihood of reducing the overall noise level.

A nine-tooth saw was used to test the design
methodology. The non-uniform spacing produced about a 20 dB reduction in noise level (relative to the evenly spaced saw blade) in the 500 Hz octave band, which contained the fundamental tooth engagement frequency. However, the overall A-weighted noise levels for each saw were essentially the same. Thus, as expected, although a reduction in noisiness occurred there was no reduction in the overall noise level.

A second paper by De Vries [5] again described the design of a circular saw for wood cutting using Doolan's methodology. In this case it was a sixty-tooth saw. Again the FRF of the system was assumed to be unity. The resulting saw design was not tested experimentally.
Chapter III

INSTRUMENTATION

This chapter will review the instrumentation employed in this study. Since there were five well defined "systems" of equipment used, the chapter will be separated into five main sections.

Each section will provide details of the configuration of a given system, together with a discussion of the use to which it was put. Following this discussion, a brief description of each instrument that made up the system will be given. In those instances where a particular instrument was used in more than one system, its description will be included only in the first section in which it appears.

3.1 SYSTEM 1 - CUTTING NOISE DATA ACQUISITION

The cutting noise data acquisition system was used to obtain a record of the noise levels generated during each of the milling cutter tests to be described in Chapter IV. The system is shown schematically in Figure 3.1.

The microphone and sound level meter performed the actual measuring task while the tape recorder was used to generate a permanent record of each test and thus permit later analysis of the sound level records in the laboratory.
The sound level calibrator was used to calibrate the system before and after each test sequence.

3.1.1 **Bruel & Kjaer 4165 Microphone**

The 4165 is a free-field, 1/2" condenser microphone used for general and low sound level measurements. The polarization voltage is 200 V DC. The open circuit frequency response for free-field, 0° incidence is generally 2.6 to 18.5 kHz (individually calibrated). Open circuit sensitivity is generally 50 mV/Pa (individually calibrated). The influence of vibration, for a 9.81 m/s² acceleration in the axial direction, is 88 dB. The influence of static pressure is -1.8 dB/atm, while relative humidity will cause less than 0.1 dB change in the absence of condensation.

3.1.2 **Bruel & Kjaer 2209 Sound Level Meter**

The B&K 2209 is a portable, hand held precision sound level meter. In conjunction with the 4165, this sound level meter has a measuring range from 24 to 140 dB. The low frequency cut-off is adjustable to either 2 Hz or 10 Hz, with an upper cut-off of 70 kHz. The RMS detector has a crest factor of up to 40, while the peak detector has a 20s rise time and a "hold" facility. The "A", "B", "C", "D" and "Lin" weighting networks are built into the instrument. The meter has "fast", "slow" and "impulse" response settings. The instrument is also equipped with AC and DC outputs for connection of level recorders, tape recorders,
etc. The built-in power supply delivers stabilized voltages to the amplifier circuits and polarization voltage for the microphone. Battery life is 8 hours in continuous operation, using three D-size standard batteries. The sound level meter has overall dimensions of 90 x 120 x 550 mm.

3.1.3 Uber 4000 Report-IC Tape Recorder

This is a two-track, direct record, portable, reel-to-reel tape recorder. For all measurements taken in this study, the tape speed used was 7 1/2 ips which gives a frequency response of 35 to 20,000 Hz, a dynamic range greater than 56 dB and wow and flutter limit of ± 0.2%. Power is supplied by five D-size batteries, or from mains supply using a power adaptor.

3.1.4 Brüel & Kjaer 4230 Sound Level Calibrator

The B & K 4230 sound level calibrator is a pocket size (110 mm long x 44 mm diameter) battery operated unit. This unit is mounted directly on the microphone of the instrument to be calibrated. It provides a 94 dB signal at 1000 Hz. The accuracy of the sound pressure level is ± 0.5 dB from 0 to 50°C and the accuracy of the frequency is ± 1.5%. The sound pressure generated is independent of the microphone's volume and can thus be used to calibrate both 1" and 1/2" microphones. The influence of static pressure is quite small being ± 0.05 dB/100 mbar from 500 to 1100 mbar. Power is supplied by a single 9V battery.
3.2 SYSTEM 2 - CUTTING NOISE ANALYSIS

The cutting noise analysis system was used to analyze the data obtained during the milling cutter noise tests. Figure 3.2 shows a schematic diagram of the system.

The tape recorder permitted playback of the noise levels generated during each cutting test. This signal was then fed into one of three instruments, depending on the type of analysis desired.

The FFT analyzer provided calibrated spectral analysis of the cutting noise signals. These spectra were displayed on the built-in display screen for review. The digital plotter provided a means of obtaining hard copies of the displayed spectra for later study and presentation purposes.

The ability of the FFT analyzer to perform "waveform calculations" permitted the spectra to be "corrected" for ambient noise sources. In this technique, the cutting noise spectrum (containing cutting noise plus ambient noise) was stored in memory, together with the appropriate ambient spectrum (obtained with spindle operating but tool not cutting). A simple subtraction command then performed a "decibel subtraction", resulting in the "cutting noise only" spectrum. It is this type of spectrum which is presented in Chapter IV.

The level recorder provided a strip chart history of the sound level versus time for each cutting noise test.
The time scale could be adjusted in calibrated steps to produce chart lengths that were both relatively easy to handle and yet detailed enough for study. The writing speed of the pen could also be adjusted in calibrated steps. For all charts shown in this report the writing speed was 200 mm/sec. Such a speed produces charts which approximate the "Fast" response of a sound level meter.

The sound level analyzer provides both the \( L_{eq} \) value of a signal and also its statistical distribution in 1 dB increments.

3.2.1 Hewlett Packard 5423A Structural Dynamics Analyzer

This is a dual channel, FFT analyzer with built-in software for performing structural modal analyses. The analyzer can perform both baseband and band selectable measurements over a 0 to 25 KHz frequency range. Frequency resolution is 4 mHz anywhere in this range. The dynamic range is 75 dB. All displayed data may be stored for later retrieval on a built-in digital cassette storage system.

The analyzer is capable of both stimulus response and response only analysis, including measurements such as transfer function, coherence function, auto spectrum, cross spectrum, linear spectrum, time record, auto correlation and cross correlation. In addition, a built-in "waveform calculator" allows further processing of measured or synthesized data.

Standard driver routines permit all displayed data
to be plotted on a digital plotter for display and/or "hard copy" storage.

The built-in software provides complete support for modal analysis including, model generation, parameter extraction, and model animation.

Power is mains supplied, 120 V AC.

3.2.2 Hewlett Packard 9872A Digital Plotter

This instrument is a four pen, flatbed, digital plotter driven over an IEEE 488 interface. The platten has electrostatic hold-down and will accommodate paper sizes up to 297 x 420 mm. The pens may be changed automatically under programme control.

Addressable resolution is 0.025 mm with a repeatability for a given pen of 0.1 mm (pen-to-pen of 0.2 mm) and a plotting accuracy of ± 0.2% of deflection. Pen down velocity is programmable from 1 to 36 cm/s in increments of 1 cm/s. Power requirement is 120 V AC.

3.2.3 Brüel & Kjær 2307 Level Recorder

The B & K 2307 can accurately record the rms, Average or Peak level of an AC signal in the frequency range from 2 Hz to 200 kHz and can also record DC signals. The dynamic range of the level recorder is determined by which of six available potentiometers is used. Various paper and writing speeds are available by setting the appropriate controls. Writing speeds range from 4 mm/s up to 2000 mm/s. Full scale response time can be as small as 50 ms. Power
supply is 120 V AC.

3.2.4 Metrosonics dB 601 Sound Level Analyzer

The sound level analyzer is an environmentally sealed unit designed for both laboratory and in-field use. The unit provides the $L_{eq}$ value of a signal, the duration of operation, and both "% time present" and "% time exceeded" statistical distributions of the sound level in 1 dB increments. The dynamic range is 100 dB. In addition, the $L_n$ value may be called up at any time, where $n$ is an integer from 0 to 99. The analyzer has a built-in LED display of the instantaneous sound level using either a "fast" or "slow" display time. The sampling rate of the analyzer is variable in 10 steps. Power is supplied from either an internal rechargeable battery or from 120 V AC mains.

3.3 SYSTEM 3 - MODAL ANALYSIS

This system was used to perform modal analyses on the subject transmission case. A schematic diagram of the system is shown in Figure 3.3.

The instrumented hammer provided the force stimulus in the form of an impact force. The hammer has a force transducer mounted on its impact surface to provide a measure of the input force. This signal was then fed into channel #1 of the structural dynamics analyzer.

The accelerometer was mounted at various points on the transmission case to obtain the response to the force stimulus. This surface acceleration signal was fed back to
channel #2 of the analyzer. Bee's wax was used to mount the accelerometer.

The power units supply constant current to the force transducer and the accelerometer and also condition (bias removal) the signal for supply to the analyzer.

The structural dynamics analyzer calculated the necessary frequency response functions to permit modal parameter extraction. These parameters could then be used to animate the geometric model of the transmission case which was developed using the analyzer's model generation software. The animated model was then studied in detail using both the built-in display screen and hard copy plots produced on the digital plotter.

The accelerometer calibrator was used to supply calibration signals both before and after a test series.

3.3.1 PCB #303A02 Accelerometer

This is a piezoelectric element accelerometer with built-in amplifier. Range for ± 5 V output is ± 500 g. Resonant frequency is 76 kHz with a frequency range (± 5%) of 1 to 10,000 Hz. Voltage sensitivity is 9.18 mV/g. Transverse sensitivity is 1.6% and output bias level is 8.3 V. Mass of the accelerometer is 2 gm. Excitation requirement is 2 mA at +18 V DC. Overall dimensions are a height of 12 mm and diameter of 7 mm.

3.3.2 PCB #886A04 Instrumented Hammer

The force transducer used in this hammer is a PCB
#208A04 with a sensitivity of 1.15 mV/N. The actual hammer sensitivity is 0.96 mV/N, with no hammer extender. The natural frequency of the force transducer is 70 kHz with a rise time of 10 μs. The hammer weight is 1.3 N or 2.6 N with extender. Range for 5 V output is 4400 N. Three interchangeable hammer tips are available. All tests reported in this study used the aluminum tip. Excitation for the force transducer is 2mA at +18 V DC.

3.3.3 PCB #480A Power Supply

This power unit supplies a constant 2 mA excitation current from a +18 V battery and diode source. Power is supplied by two series-connected 9 V batteries. Frequency response is essentially that of the transducer and the transducer signal is neither amplified nor attenuated. A de-coupling capacitor connected between the input and output terminals eliminates bias on the output and blocks any D.C. signal transmission. A self-test meter circuit indicates normal or faulty operation. Battery life is in excess of 80 hours. Overall dimensions are 74 x 102 x 46 mm.

3.3.4 Bruehl & Kjaer 4291 Accelerometer Calibrator

The accelerometer calibrator is a fully portable vibration reference source for field and laboratory calibration. It consists of a built-in, stable 79.6 Hz (+ 0.5 Hz) sinusoidal generator, a compact electromagnetic vibrator mechanically isolated from the case and a velocity-monitoring coil feeding a built-in measuring
amplifier and coil. The transducer to be calibrated is attached to the vibration table located in the centre of the front panel. The peak acceleration amplitude output is 10.0 + 0.2 m/s². Distortion is less than 1% at temperatures higher than 10°C. Transverse axis amplitude is less than 10% of main axis amplitude at 79.6 Hz. Power is from 4 D-size batteries with a 20 hour life in intermittent use, or an external 28 V DC source. Overall dimensions are 140 x 133 x 200 mm.

3.4 SYSTEM 4 - NOISE FREQUENCY RESPONSE FUNCTION

ACQUISITION

This system was used to obtain noise frequency response functions from the transmission case used in this study. Figure 3.4 shows a schematic diagram of this system.

As can be seen from Figure 3.4, this system is very similar in configuration to that of the Modal Analysis System. The only difference being the replacement of the accelerometer and its power supply with a sound level meter. In this manner, the response variable measured was not surface acceleration, but sound pressure at a discrete point in space.

The FFT analyzer was used to obtain the frequency response functions with the input being force and the output being sound level. This data was not used for modal analysis, but analyzed and manipulated "as is".

The sound level calibrator was used before and
after each test sequence to calibrate the system.

3.5 **SYSTEM 5 - HYBRID DIGITAL - ANALOG COMPUTER**

The hybrid digital-analog computer (DAC) system was used to simulate the response of a single degree-of-freedom system to an arbitrary series of rectangular force pulses. A schematic diagram of the system is shown in Figure 3.5.

Since Chapter VI provides significant detail on the DAC system developement concepts and operation, only a cursory review will be given here.

The digital computer, in conjunction with the A/D converter, provided the simulated force pulse train, in the form of rectangular voltage pulses between ± 5 V.

The analog computer permitted simulation of the single degree-of-freedom system through patching of amplifiers and appropriate potentiometer setting. The system output was in the form of a calibrated voltage, directly related to the system's displacement (or velocity, if desired).

The digital oscilloscope was used to observe and capture the system response signal, and also permitted generation of a "hard copy" of the traces using the analog plotter.

The dynamics analyzer was used to obtain spectral analyses of the system output signals, and also transfer functions and coherence measurements, as desired. Permanent
copies of these measurement displays were made using the digital plotter.

The Prowler Data Acquisition System was used to initially determine the timing accuracy of the pulse trains used as the forcing function to the analogue computer and, subsequently, to "fine tune" the machine language programme used for the generation of these pulses. Once the programme was producing pulses of acceptable accuracy (interval errors on the order of 0.07%) the Prowler was no longer used in the system.

3.5.1 Apple Computer Inc., APPLE II+, Microcomputer

This is a desk top, 8-bit, microcomputer with a built-in keyboard, separate monitor, two 5 1/4" disc drives, the standard DOS 3.3 disk operating system and the "Applesoft" BASIC programming language. The total memory space available (RAM and ROM) was 64KB. The processor was a 6502 running at 1.023 MHz.

3.5.2 Electronic Associates Inc. TR20 Analogue Computer

The TR20 is a desk top, general purpose analogue computer composed of solid-state computing components. The computing components in the TR20 are divided into three sections: the first section contains the coefficient attenuators; the next section houses the integrator networks, comparators, and other non-linear computing components; the last section provides space for twenty operational amplifiers. The computing components are interconnected by using patch cords between the appropriate
input and output terminations. These connections are made on a pre-patch panel which allows problem patching away from the computer. A built-in voltmeter allows input and output voltage levels to be monitored. The voltmeter range is switchable in six ranges between 0.1 and 30 volts. Overload indicators signal any output overloads on the amplifiers. A mode control switch allows selection of "reset", "hold" or "operate" conditions. Power is supplied from mains at 120 V AC.

3.5.3 Gould OS 4020 Digital Oscilloscope

The OS 4020 is a dual-channel oscilloscope which provides both real time and digital storage measurement capabilities. The digital storage facility allows both pre-trigger viewing, and simultaneous display of both real time and stored signals. An interface is incorporated to provide analogue output to an X-Y recorder. Display is on an 8 x 10 cm CRT with illuminated graticule. The vertical sensitivity is 5 mV/cm to 20 V/cm in 12 ranges with an accuracy of ± 3% and a bandwidth of DC to 10 MHz. The timebase is adjustable from 1 µs/cm to 20 s/cm in 23 ranges with an accuracy of ± 3%. Power is supplied from 120 V AC mains. Overall dimensions are 178 x 312 x 417 mm.

3.5.4 Hewlett Packard 7044 Analog Plotter

The 7044 is an analog, X-Y recorder, with a single pen and electrostatic paper hold-down. Maximum paper size is 11 x 17 in. input ranges are 0.5 to 500 mV/in in ten
steps and also 1 to 10 V/in in four steps, with continuous vernier between ranges. Accuracy is \( \pm 2\% \) of full scale. Slewing speed is 20 in/s with peak acceleration in the Y-axis of 1000 in/s\(^2\) and X-axis of 500 in/s\(^2\). Overshoot is 2\% of full scale, maximum. Power supply is 120 V AC. Overall dimensions are 400 x 483 x 165 mm.

3.5.5 Norland Corp. N4008/4005 "Prowler", Data Acquisition System

The Prowler is a microcomputer-based, desktop, two-channel, data acquisition system. Each channel may be either an N4008 module (maximum digitization rate of 20 MHz/50 ns) or an N4005 module (maximum digitization rate of 100 kHz/10 μs). The system used in this study included one each of these units, but only the N4005 unit was used for the timing measurements. Thus the specifications for this unit only will be presented here. The digitizing resolution is 12 bits (0.025%) with a DC accuracy of \( \pm 1\% \) of full range. The bandwidth is (-3dB point) DC to 300 kHz with a flatness of \( \pm 2.5\% \) at 50 kHz. The maximum allowable voltage at input is 200 V DC. The input voltage range is adjustable in eight steps from \( \pm 0.1 \) V to \( \pm 2.0 \) V. Linearity is \( \pm 0.018\% \) of full range. The timebase is adjustable from 2 s to 50 ns, crystal controlled to \( \pm 150 \) ppm. Screen resolution is 512 x 253 pixels. Overall dimensions are 400 x 424 x 211 mm. Power is 120 V AC mains supply.
Chapter IV
CUTTING NOISE MEASUREMENTS

4.1 INTRODUCTION AND OBJECTIVES

As noted earlier, up to this time the theoretical and experimental work published on the application of non-uniform insert pitch has been restricted to very simple workpiece geometries. Generally, these workpieces are not representative of the type encountered in high-volume machining operations, as for example, in automotive manufacturing operations. Also, the literature available on the practical application of non-uniform insert pitch in reducing noise generation (as contrasted to its use in improving chatter resistance) during face milling, is quite minimal.

For these reasons it was decided that a test series be undertaken to determine the effect of non-uniform insert pitch on noise generation during the face milling of a geometrically complex workpiece. Further, it was decided that the tests be performed on a workpiece which was actually in high volume production and also that the machining conditions approximate, as closely as possible, those of the actual "in-plant" process.

It was expected that such tests would serve two
very important purposes:

i) They would permit an evaluation of the effectiveness of present methodologies for determining non-uniform insert pitch when applied to workpieces of complex geometry which are machined under "real-life", high-volume conditions.

ii) They would highlight those variables which may be important in determining the noise generated during a particular cut, yet which are not adequately accounted for in present "quiet cutter" design.

4.2 METHODOLOGY

The workpiece selected for study was the General Motors "125" Transmission Case. See Figure 4.1. This part was chosen since the case is geometrically complex, it produces excessive noise levels during the face milling of the engine mounting face, and the actual high volume transfer line used to machine it was available for test purposes. This transmission case is of cast aluminum (G.M. #EMS-2E) construction.

All test results reported here were obtained while the workpiece was being machined in the actual multistation pallet transfer machine (Lamb #L-2789) to be used subsequently for high-volume production. In this type of machine the workpiece is firmly clamped to a pallet which
acts as a travelling fixture. With each cycle of the machine, both the workpiece and the pallet are moved on rails to the next machining station. Figure 4.2 shows the transmission case clamped to the transfer pallet.

4.2.1 Cutter Geometry

A total of five different cutter configurations, all with eighteen cutting inserts, were employed in this study.

The tooling consisted of the following three cutter bodies:

i) A Valenite 406 mm diameter ring-type cutter with equally spaced insert pockets.

ii) A Valenite 406 mm diameter ring-type cutter with a 1° staggered pocket configuration. This cutter employed "standard" tungsten carbide inserts (SEC-631-J). For reference purposes this cutter configuration will be referred to as "1° STAG". See Figure 4.3.

iii) A Valenite 406 mm diameter ring-type cutter with a 1/2° staggered pocket configuration. The cutter employed standard inserts. This cutter configuration is designated "1/2° STAG". See Figure 4.4.

Insert pockets in all cutter bodies provided a 10° positive axial and radial rake for the standard inserts. In two cases the standard inserts used in the "equal spacing"
cutter body are modified to provide a total of three configurations from this single cutter body:

1) Standard inserts set into the equally spaced pockets. This configuration is designated "EQ SP".

2) Five "standard" inserts were modified by grinding back 1.5 mm and installed at randomly chosen positions. The remainder of the pockets employed "standard" inserts. This configuration is designated "EQ. SP. 5 GR". See Figure 4.5.

3) Five "standard" inserts were modified by grinding with 1.5 mm smaller inscribed circle (I.C.) and installed at the same positions as in (ii) above. The remainder of the pockets employed standard inserts. This configuration is designated "EQ. SP. 5 IC". See Figure 4.5.

The "1° STAG" and "1/2° STAG" configurations were chosen as representative of the design approach suggested by Vanherck [37] and Varterasian [39]. Basically, the insert spacing varies sinusoidally around the cutter. This can be seen quite clearly in Figure 4.6. In this figure the ratio of the actual pitch in degrees to the average pitch in degrees is plotted for each tooth. For the case of the "1/2° STAG" cutter the variation is reasonably smooth. For
the "10 S7AC" cutter the variation is much coarser with multiple crossings of the EQ. SP. line. Obviously, on such plots a cutter with equally spaced inserts will be represented by a horizontal straight line, since each insert will have the same L/L\text{avg} value. Note that the solid lines joining the points on these plots are used only to enhance the pattern formed by the plotted points and do not signify the existence of a continuous relationship between these points.

The cutter configurations "EQ. SP. 5GR" and "EQ. SP. 5IC" were based on techniques employed by Reif [27] so successfully in a previous field application. In the present case it was hoped to determine whether small random perturbations of the cutting edges guarantee a reduction in noise relative to the evenly spaced configuration. It is generally agreed [30] that any change from even spacing improves the chatter resistance of a face milling cutter; however, it has not been shown that simple, random variations in insert pitch always result in noise reductions. These cutters were expected to help clarify the situation.

The "EQ. SP." configuration was employed as the "control" against which the results from the other cutters would be compared. This configuration is the industry standard, although there are some non-uniform pitch cutters presently in volume production, primarily to increase
chatter resistance.

The cutter spindle rotated at 967 rpm for all tests. This translates into a surface speed of 21 m/s and a feed per tooth of approximately 0.2 mm for the equally spaced cutter. These are quite conservative values and would not be expected to put undue stress on the cutting inserts. They are, in fact, indicative of values which might be chosen for the production situation where reasonable tooth life would be desired.

4.2.2 Noise Measurements

The noise measurements were made during the face-milling of the engine mounting face of the transmission case. See Figure 4.7. The total cutting cycle time was 9.4 seconds.

To begin a test sequence, the pallet and transmission case were manually advanced into Station 4LH of Lamb Machine #L-2789. Once clamped, the milling head was manually advanced so that only this cutting operation occurred during the measurement period.

The sound level meter was positioned at a height 1 m above the floor and at a distance of approximately 1.5 m from the workpiece along an unobstructed line-of-sight. The sound level measured during the cutting sequence was tape recorded for later analysis. Samples of both the background and idle (spindle rotating but not cutting) noise levels were recorded for later use in correcting the cutting noise.
levels. All sound levels were A-weighted since it is the A-weighted noise levels which are used for determining an employee's noise exposure values in industrial environments. Calibration signals were recorded both at the beginning and end of each test sequence.

Measurements were made for each milling cutter configuration at seven depths of cut, from 2.1 mm to 3.9 mm. Due to the practical constraints of relatively lengthy set-up time for each depth of cut, the limited number of castings available for test machining, and the need to minimize the time the machine was tied up with test runs, only one casting was machined at each depth of cut for each cutter configuration.

To permit simple reference to these cutting tests, each combination of cutter configuration and depth of cut was given a test number. The designations are summarized in Table 4.1. As an example, the test shown as (20) in Table 4.1 is referred to in this discussion as Test #20 and designates a "1° STAG" cutter used at a 3.4 mm depth of cut. The blanks in the table indicate measurements which for various reasons (equipment failure, set-up difficulties, etc.) were not ultimately obtained, although they were initially planned.

4.3 RESULTS AND DISCUSSION

The primary objective of this series of tests was to determine whether any of the non-uniform insert pitch
configurations provided a reduction in noise generation over the period of a cutting cycle, relative to that produced by the "EQ. SP." cutter. It would also be useful to be able to rank each cutter as to its relative efficiency in reducing noise generation. To meet such objectives it is first necessary to decide what type of descriptor will be used to compare the results from each test.

It would certainly be possible to simply compare the overall noise levels produced during each cutting sequence and make some judgement on this basis. Such a comparison of two tests is shown in Figure 4.8. It seems reasonable, on the basis of this figure, to state that on the "average" the "EQ. SP." cutter generates less noise over the cutting cycle than does the "1/2° STAG" cutter. However, in many cases, the differences in sound level are much closer, with first one cutter and then the other producing higher noise levels. In these instances, a simple comparison of noise level history does not provide a firm basis upon which to make a judgement of the "quieter" cutter.

In those cases where it is desired to compare, or rank, noise sources it has been found that a particularly useful single number descriptor is $L_{eq}$. This is described as the constant sound level which contains the same energy as the actual time-varying sound level, over the time interval of interest. Mathematically it is written as:
\[ L_{eq} = 10 \log \left[ \frac{1}{T} \int_{0}^{T} \left( \frac{P}{P_r} \right)^2 dt \right] \]

4.1

It is this single number descriptor which will be used to rank the cutters employed in this test series. Table 4.2 summarizes the \( L_{eq} \) values obtained during each cutting test. As noted earlier the blanks indicate measurements which for various reasons were not obtained.

A review of the data contained in Table 4.2 indicates that the four milling cutters employing non-uniform insert spacing generated noise levels similar to, or higher than, those of the equally spaced cutter. As expected, for all five cutter designs the noise generated increased with increasing depth of cut, although both the "1° STAG" and "EQ. SP. 51C" configurations seem to indicate less sensitivity to this variable over the range studied. See Figure 4.9.

The question of why no reduction in cutting noise level was achieved by the non-uniform pitch cutters still remains to be answered.

To this end, it is instructive to review the spectral data from the cutting noise tests. As an example, consider Test #24 and Test #25. Figure 4.10 shows a comparison of the spectra generated during each of these tests. In this particular case the data is presented with the magnitude of the components in the form of the dimensionless ratio \( \frac{p}{p_r} \), where \( p \) is the r.m.s sound
pressure and $p_r$ is the standardized reference pressure of $2 \times 10^{-5}$ Pa. This form of display was chosen, rather than the more conventional decibel amplitude descriptor of $20 \log \frac{p}{p_r}$, to enhance, for comparison purposes, the differences which exist in the two spectra.

Review of Figure 4.10 does indeed indicate that the non-uniform insert pitch caused redistribution of the sound energy over a wider bandwidth when compared with the equally spaced cutter. Both Test #24 and Test #25 produced the same $L_{eq}$ value, yet the redistribution of energy due to the non-uniform pitch employed in Test #24 is readily apparent. The cursor shown marks the "EQ. SP." cutter's fundamental "insert engagement frequency" (frequency of cutter rotation x number of inserts) while the open circles mark the positions of the first fifteen higher harmonics of this fundamental frequency.

It is significant that although no reduction in overall $L_{eq}$ value was achieved, Test #24 indicates a substantial reduction in "noisiness" relative to Test #25 due to the significant reduction of sound energy concentrated at discrete, harmonically related components [18]. Such a phenomenon was not restricted to this particular test grouping but was found to be generally valid. This reduction in "noisiness" is, in itself, an important step toward providing an acceptable noise environment near milling operations since such a reduction in "noisiness"
provides a subsequent reduction in "annoyance" for those personnel working nearby. However, it must be remembered that the noise exposure limits of regulations are based on measurement of overall noise levels and thus a reduction in noisiness (a rearrangement of the tonal character of the noise) does not necessarily translate into a reduction in measured noise exposure. For this reason it is important to gain an understanding of how an absolute reduction in noise level (as indicated, say, by a reduction in \( L_{eq} \)) can be achieved.

A review of all the sound level-versus-time readings, which were generated during the cutting tests for each milling configuration, indicates that, although unique, they do exhibit remarkable similarities. Figures 4.11 and 4.12 show the sound level-time histories for the various cutter configurations and for depths of cut of 2.4 mm and 2.9 mm, respectively. The five cutter configurations being compared are significantly different yet it is apparent that various common regimes of noise generation (characterized by changes in overall sound level and the predominant spectral components) are excited as the cutter moves across the part. It would seem that the changing geometry of the part (and its associated structural properties such as stiffness, etc.) and the changing geometry of the cut itself (entrance angle, exit angle, width of cut, etc.) are the dominant factors in producing these regimes.
These similarities are again highlighted if the sound level produced by the cutting process is presented in a slightly different manner from that of Figures 4.11 and 4.12.

Figure 4.13 shows the time a particular sound level was present during a given cutting cycle as a fraction of the total cutting time. Note that these distributions are "multimodal". This again indicates the existence of various regimes of vibration and thus noise generation.

Figure 4.14 demonstrates how the regimes are defined by the changes in geometry encountered by the cutter as it moves over the part profile. It can be seen that the sound level varies over a range of approximately 10 dB during the cutting sequence. This corresponds to more than a tenfold increase in absolute acoustic energy.

Figure 4.10 shows a comparison of two spectra from two different test cuts. These spectra were obtained by ensemble averaging over the available cutting period. For the bandwidth chosen, this resulted in 18 averages being used to produce the spectra shown. Thus these spectra are representations of the "typical" spectral content of the noise signal generated during the whole cutting process.

It is instructive to consider spectra generated during particular sections of the total cutting time. Figure 4.15 indicates the variation in spectral content for three arbitrary "regimes" defined for Test #1 results. Such
results are representative of those obtained from all test sequences. It is apparent that the spectral content of the noise generated changes significantly during the cutting process. This phenomenon is quite apparent simply from listening to the cutting process.

If frequency spectra are obtained for very short periods throughout the cutting process, the variation in frequency content is very noticeable. Figure 4.16 shows spectra obtained at seven points during the cutting cycle of Test #4. Each spectrum represents two ensemble averages with a total acquisition time of approximately 0.75 seconds.

The significance of this finding lies in the fact that the vibration (and hence noise) response of the casing is changing with time. This indicates that either the forcing function is changing with time or the point of application of the forcing function is a significant factor in determining the response of the transmission casing. In fact, it is certain that both factors are influencing the noise measured in this test series. It is well known in structural analysis that, for a given forcing input, the point of application of the force will influence the response of the structure. Also, knowing that the "width of cut" is directly related to the duration of the cutting force generated by an insert, it is easy to see (refer to Figure 4.7) that the forcing function itself is changing throughout the cutting cycle. Although these facts seem
simple enough, it should be noted that they are either ignored or poorly handled in the existing "quiet cutter" design approaches. Even in one of the better "quiet cutter" design methodologies, that by Doolan and Wu [7], the structural responses used to characterize the problem are those of the Machine-Tool-Workpiece System. Such a choice is a very poor one for the lightweight, geometrically complex workpieces considered here. With reference to Figure 4.15, it can be seen that the sound level spectra consist predominantly of discrete components at frequencies of the higher harmonics of the cutting insert impacts. This indicates that the noise, and hence vibration, are predominantly excited by the action of the cutter on the workpiece. Also, when it is recognized that the compliance of the transmission case is many times greater than that of the milling cutter and the machine tool structure, it must be concluded that the cause of the noise generated is almost exclusively the vibration of the workpiece and not that of the machine tool structure.

It is apparent that the choice of the Machine-Tool-Workpiece System response properties by Doolan and Wu was a vestige from studies of non-uniform insert pitch used to improve chatter resistance. In the case of chatter analysis the use of Machine-Tool-Workpiece System response properties is quite appropriate. It is not appropriate when noise generation from lightweight
geometrically complex workpieces is being considered. In this case, only the structural response properties of the workpiece itself are important.

4.4 SUMMARY

At the beginning of this chapter it was stated that the cutting noise tests were undertaken in an effort to provide answers (since none could be found in the literature) to the following two questions:

i) How effective are present methodologies for determining a non-uniform insert pitch which will reduce noise generation during the face-milling of geometrically complex workpieces in "real life", high volume production conditions?

ii) Which variables can be identified as important in determining the noise generated during the face-milling process, yet which are not adequately accounted for in present "quiet cutter" design?

With reference to the first question, it is apparent that the present methodologies for determining non-uniform insert pitch are not particularly effective and do not guarantee a reduction in noise generation during face-milling - in many cases the noise level actually increases!

As far as an answer to the second question, it is
obvious that both the definition of the forcing function and the response properties of the workpiece itself are important factors in the noise generation during the cutting process, yet they have either been ignored or poorly handled in existing design approaches.

These deficiencies could well explain the poor results obtained from non-uniform insert pitch cutter designs. For this reason, significant effort was expended on the determination of the transmission case's response properties and also an evaluation of the importance of forcing function definition on workpiece response. The methods of approach and the results obtained in these two areas of investigation will now be detailed in Chapters V and VI, respectively.
Chapter V

WORKPIECE RESPONSE CHARACTERISTICS

5.1 INTRODUCTION AND OBJECTIVES

The primary objective of this chapter is to provide detailed information on the structural response of the transmission case which was the workpiece for the milling tests described in Chapter IV. This type of information is absolutely necessary if it is ever hoped to improve on "quiet cutter" design, since the noise generated during the milling operation is directly related to the structural response of the transmission case under the forcing function generated by the cutting inserts' engagements.

Simply recognizing that structural response data is required, however, does not answer the question: What kind of response data is useful for our particular needs and why? These issues will be dealt with for the remainder of this chapter.

5.2 MODAL ANALYSIS

One of the more powerful techniques for obtaining structural response data is the modal analysis method. It was applied to the subject transmission case because of its ability to determine "global" structural properties such as
modal mass, stiffness and damping. However, before reporting on the results of this analysis it is important to review the fundamentals of modal analysis to be certain that a common ground for terminology is defined and the limitations and errors associated with modal analysis are clearly recognized.

5.2.1 Background Discussion

Simply stated, modal analysis is the process of characterizing the dynamic properties of an elastic structure by identifying its modes of vibration through physical measurements. Physically speaking, modes of vibration are the so-called "natural" frequencies at which a structure's predominant motion is a well-defined waveform. Mathematically speaking, modes of vibration are defined by particular parameters of a linear dynamic model. As shown in Figure 5.1, each mode is defined in terms of a resonant frequency, a damping factor and a mode shape (its "modal parameters"). A structural dynamic model can be completely represented (in either the time or frequency domain) in terms of these parameters.

5.2.1.1 Normal Mode Method vs. Transfer Function Method

From the practical hardware consideration of how modal parameters might be extracted from a particular structure there are presently two basic methods in use.

The first is the Normal Mode Method. This is the "original" method of experimentally determining modal
parameters and basically requires a large number of exciters "tuned" in such a manner that the structure is excited in a single mode of vibration. The forcing function is sinusoidal and, obviously, only one mode of vibration is excited at a given time. The instrumentation used in this type of test is predominantly of the analog type. The Normal Mode Method of testing is generally accepted as being the more expensive and time consuming of the two methods. The aircraft and aerospace industries are strong advocates of normal mode testing.

The Transfer Function Method has been gaining increasing recognition in recent years, particularly since the advent of powerful microcomputers has permitted the development of digital Fourier analysers. These machines have the ability to quickly and accurately provide the frequency spectrum of a time-domain signal. This ability has made the Transfer Function Method a viable alternative to the Normal Mode Method. Simply put, the Transfer Function Method requires that "frequency response functions" between points of interest on the structure be obtained. Once these are available they may be operated on to obtain the modal parameters; i.e. the natural frequency, damping factor, and characteristic mode shape. The great advantage of the Transfer Function Method is that the modal responses of many modes can be measured simultaneously. In addition,
only a single point excitation is required in most cases.

Although the debate as to which method is the more accurate, reliable, least sensitive to non-linear effects, etc. continues (and, it is safe to say, will continue for some time to come), the Transfer Function Method was chosen for use in this study for the simple expedient that only equipment to perform this type of analysis was available. With careful experimental procedures it is expected to provide highly reliable results. Since it is the Transfer Function Method that is utilized in this work, the remaining discussion will focus on the various characteristics of this method.

5.2.1.2 Structural Dynamics Model

In order to gain some insight into the dynamics of structures, some groundwork upon which to build a structural model must be laid down. It is generally assumed that certain criteria are observed by the model:

i) The structure is linear.

ii) The structure is time invariant.

iii) The analytic equations used to describe the structure must use parameters which are measurable.

Consider the idealized single degree of freedom system shown in Figure 5.2. Under the action of the external force shown, the equation of motion of the mass is

\[ M\ddot{x} + C\dot{x} + Kx = F(t) \]
Taking the Laplace transform of Equation 5.1 gives,

\[(Ms^2 + Cs + K) X(s) = F(s)\]  

Letting,

\[B(s) = (Ms^2 + Cs + K)\]  

then,

\[B(s) X(s) = F(s)\]  

Rearranging Equation 5.4 gives,

\[X(s) = \frac{1}{B(s)} F(s)\]  

where,

\[H(s) = \frac{1}{B(s)}\]  

and \(H(s)\) is called the system "transfer function". Note that the transfer function is, by definition, a function of the Laplace operator \(s\). Combining Equation 5.5 and 5.6,

\[X(s) = H(s) F(s)\]  

\[\text{5.7} \]
Combining Equation 5.7 and 5.3,

\[ H(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + K} \]  

and rearranging,

\[ H(s) = \frac{1/M}{S^2 + Cs/M + K/M} \]  

The "characteristic equation" for the system is defined as,

\[ s^2 + Cs/M + K/M = 0 \]  

The roots of Equation 5.10 can be shown to be,

\[ s_{1,2} = -C/2M \pm \sqrt{(C/2M)^2 - (K/M)} \]  

The case when \((C/2M)^2 - (K/M)\) is negative is of particular interest. In this instance the system is underdamped (oscillatory in nature) which best represents the majority of structural systems.

Using well known definitions, it can be shown that,

\[ s_{1,2} = (-\xi \pm \sqrt{\xi^2 - 1}) \omega_n \]  

Now "damping factor" is defined as the real part of each root,
\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \quad 5.14 \]

Thus,

\[ s_{1,2} = -\sigma \pm j\omega_d \quad 5.15 \]

Now let,

\[ p = -\sigma + j\omega_d \quad 5.16 \]

\[ p^* = -\sigma - j\omega_d \quad 5.17 \]

The system transfer function can now be written,

\[ H(s) = \frac{1/M}{(s-p)(s-p^*)} \quad 5.18 \]

The fundamental definitions and relationships for the single degree-of-freedom system have now been established. However, these properties must be linked to measurable parameters if they are to be of any use.
A partial fraction expansion of Equation 5.13 yields,

\[ H(s) = \frac{C_1}{(s-P)} + \frac{C_2}{(s-P^*)} \quad 5.19 \]

Solving for \( C_1 \) and \( C_2 \) by evaluating Equation 5.19 first at \( s = P \) and then \( s = P^* \) yields the "complex amplitudes",

\[ C_1 = \frac{1/M}{2j\omega_d} \quad 5.20 \]
\[ C_2 = \frac{1/M}{-2j\omega_d} \quad 5.21 \]

Letting \( C_1 = A \) and \( C_2 = A^* \) gives,

\[ A = \frac{1/M}{2j\omega_d} \quad 5.22 \]
\[ A^* = \frac{1/M}{-2j\omega_d} \quad 5.23 \]

Therefore Equation 5.19 may be written as,

\[ H(s) = \frac{A}{(s-P)} + \frac{A^*}{(s-P^*)} \quad 5.24 \]

For convenience a slightly different form of the "complex amplitudes" called "complex residues" are defined as,

\[ R = 2jA \quad R^* = -2jA^* \quad 5.25 \]
or,

\[
R = \frac{1}{M \omega_d} \quad R^* = \frac{1}{M \omega_d} \tag{5.26}
\]

The standard form of the single degree-of-freedom transfer function becomes,

\[
H(s) = \frac{R}{2j(s-P)} - \frac{R^*}{2j(s-P^*)} \tag{5.27}
\]

At this point in the development it should be noted that the Laplace domain does not lend itself well to hardware implementation. However, Fourier analysis, with its many varieties of hardware implementation, can be used to extract the necessary frequency and amplitude information. Damping can then be extracted by a variety of different types of curve fitting techniques.

The "system frequency response function" may be defined as the system transfer function evaluated along the \(jh\) axis. Note that this is the same as arbitrarily assigning the system damping to be zero.

Thus,

\[
H(j\omega) = H(s) \bigg|_{s=j\omega} = \frac{R}{2j(j\omega-P)} - \frac{R^*}{2j(j\omega-P^*)} \tag{5.28}
\]

or,

\[
H(j\omega) = \frac{1}{2} \left( \frac{R}{(\omega_d-\omega)+j\sigma} - \frac{R}{(-\omega_d-\omega)+j\sigma} \right) \tag{5.29}
\]
There are very few realizable systems that can be accurately modelled by a single degree-of-freedom system. For the case of an n-degree of freedom system it can be shown that the following transfer function matrix results,

\[ [H(s)] = \sum_{k=1}^{n} \left( \frac{[R_k]}{2j(s-P_k)} - \frac{[R_k^*]}{2j(s-P_k^*)} \right) \]  

The "mode shape" (or eigenvector) of a system can be defined as a vector which represents the motion of a given structure when excited at a resonance (natural mode of vibration). This vector consists of a series of complex displacements which describe the amplitude and direction of deflections at each point on the given structure. In fact, just such a quantity has already been described, that is, the system residues. We can now form the expression,

\[ [R]_k = Q_k \{U_k\} \{U_k\}^T \]  

where, \( k \), represents the kth mode of the system and \( Q_k \) is an arbitrary scale factor. Obviously, the mode shape vector, \( \{U_k\} \), can have different magnitude for a given residue matrix simply depending on the value given to \( Q_k \). The scale factor \( Q_k \) is usually chosen according to one of several standard scaling methods such as Unit Scale Factor (\( Q_k = 1 \)), Modal Mass = 1 and \( Q_k = 1/\omega \), etc.
At this point a distinction must be made between the concept of "degrees-of-freedom" and "modes". Since it is not usually required to make measurements on single degree-of-freedom systems, it is generally necessary to handle the situation where an arbitrary number of points and directions on a structure are selected for study, and only a limited number of modes are of concern. As an example, it may be desired to observe the motion at 20 points on a structure in the x-direction and study the mode shapes for the first three bending modes. In this case a total of 20 degrees-of-freedom and 3 mode shape vectors would be required. Each mode shape vector would have 20 elements. Thus Equation 5.30 may be rewritten,

$$h_{rc}(s) = \sum_{k=1}^{n} \left[ \frac{R_{rc}(k)}{2j(s-P_k)} - \frac{R_{rc}^*(k)}{2j(s-P_k^*)} \right]$$  \hspace{1cm} (5.32)

where,

- $r$ - row # of transfer matrix $H(s)$
- $c$ - column 3 of transfer matrix $H(s)$
- $n$ - total number of modes considered
- $k$ - current mode number
- $P_k$ - pole location for the kth mode
- $R_{rc}$ - residue of $h_{rc}(s)$ for the kth mode

Note that each term in the summation expression, Equation 5.32, is an $n \times n$ matrix which represents the contribution of each mode, $k$, to the transfer matrix.
It is worthwhile noting here that for each shape vector identified, a hypothetical single degree-of-freedom system can be constructed out of parameters derived from the modal parameters. These include the modal mass,

\[ M_k = \frac{1}{\bar{Q}_k \bar{\omega}_k} \]

the modal stiffness,

\[ K_k = (\bar{\sigma}_k^2 + \bar{\omega}_k^2) M_k \]

and the modal damping,

\[ C_k = 2\bar{\sigma}_k M_k \]

5.33
5.34
5.35

5.2.1.3 Parameter Extraction

So far the discussion has centered on how modal parameters such as natural frequency, damping and residues can be combined to determine spatial deflections of structures. It would now be useful to note how these parameters are obtained from measurements made on the structure.

Generally the natural frequencies can be determined simply by noting the location of peaks in the frequency response function of the system under test.

The damping ratio can be obtained by finding the frequencies \( (\bar{\omega}_u, \bar{\omega}_l) \) on either side of the damped natural
frequency which display amplitudes 3dB below the peak of the frequency response magnitude. Then the expression,

$$\zeta = \frac{1}{2} \left( \frac{\omega_u}{\omega_d} \right)$$  \hspace{1cm} 5.36

can be used to obtain the damping ratio.

The residue can now be determined using the following approach.

Recalling Equation 5.29, we have

$$H(j\omega) = \frac{1}{2} \left( \frac{R}{\omega_d - \omega} + j\sigma \right) - \frac{R^*}{\left(-\omega_d - \omega\right) + j\sigma}$$

Consider $H(j\omega)$ at resonance ($\omega = \omega_d$),

$$H(j\omega_d) = \frac{1}{2} \left( \frac{R}{j\sigma} - \frac{R^*}{-2\omega_d + j\sigma} \right)$$  \hspace{1cm} 5.37

A convenient approximation can be made in cases where the damped natural frequency is much larger than the damping factor,

$$H(j\omega_d) \approx \frac{1}{2} \left( \frac{R}{j\sigma} \right)$$  \hspace{1cm} 5.38

Equation 5.38 is valid for lightly damped modes, which exhibit damped natural frequencies that are much greater than their associated damping factors.

Note from Equation 5.38 that $H(j\omega_d)$ is purely
imaginary,

\[ \text{IMAG} \{H(j\omega_d)\} \approx \frac{R}{2\sigma} \]  \hspace{1cm} 5.39

\[ \text{REAL} \{H(j\omega_d)\} \approx 0 \]  \hspace{1cm} 5.40

Rearranging Equation 5.39 gives,

\[ R = 2\sigma \text{ IMAG} \{H(j\omega_d)\} \]  \hspace{1cm} 5.41

Recalling Equation 5.13,

\[ \sigma = \zeta \omega_n \]

and recognizing that for \( \zeta \ll 1 \),

\[ \omega_n \approx \omega_d \]

Then,

\[ \sigma \approx \omega_d \zeta \]  \hspace{1cm} 5.42

It is now apparent that Equation 5.36 permits \( \zeta \) to be determined, and then Equation 5.42 can be used to determine \( \sigma \). The residue (and ultimately mode shape) can then be obtained from Equation 5.41 since the value of the imaginary part of the frequency response function, at \( \omega_d \), can easily
be determined from the Fourier analyser.

This technique is referred to as the "quadrature peak-peaking" technique and is suitable for application to frequency response functions that are essentially noise-free, display light damping, have enough data points to accurately estimate the damped natural frequency, the 3dB down points and the magnitude of the imaginary part of the frequency response function. Modes should also be widely spaced or the skirt of one mode might distort an adjacent mode and cause significant errors in the parameter estimates.

5.2.1.4 Frequency Response Functions

If it is desired to use the so-called "Transfer Function Method" to perform a modal analysis then it is apparent from the previous description that transfer functions must be obtained. It is therefore prudent to review how these measurements are actually performed by the Discrete Fourier Transform analyser and the types of errors that can occur in this process.

To begin, it is instructive to note that there is not a single transfer function that must be measured, but there are actually six transfer functions that could be measured. These are listed in Table 5.1. In general all the transfer functions contain the same information and the choice of which to use is usually based on the equipment available, the frequency range of interest, etc. In this
study, measurements were made using an accelerometer and thus "accelerance" transfer functions were obtained. The advantages of using accelerometers include high sensitivity, small size and mass, good frequency response and absence of external-conditioning amplifiers.

At this point it is important to make a distinction between "transfer functions" and "frequency response functions". Transfer functions are mathematical abstractions which are functions of the Laplace operator, s. Unfortunately, the Laplace domain is not presently implemented in hardware and it is therefore necessary that measurements be made in the frequency domain (amplitude and frequency). Such measurements are termed frequency response functions and are, in fact, merely the Laplace transform evaluated along the jω, or frequency, axis of the complex Laplace plane.

One additional condition must be put on our use of the term "frequency response function" (FRF). Since the FRF is, for our purposes, obtained from actual hardware measurements, it is understood that the FRF includes system noise, measurement errors, etc. How such errors may be minimized is discussed later.

How exactly are the FRFs obtained? To answer this question it is first necessary to gain a firm grasp of the measurement process.

It must first be understood that digital
instrumentation is employed. Thus all measurements must be
discrete and of finite duration. This means that in order
to implement the Fourier transform digitally, it must be
changed to a finite form known as the Discrete Fourier
Transform (DFT). Consequently all continuous time wave
forms which must be transformed must be sampled at discrete
intervals of time, uniformly separated by an interval Δt.
This also means that only a finite number of samples, N, can
be taken and stored. The record length, T, is then,

\[ T = NΔt \quad 5.43 \]

The result of implementing the DFT in a digital memory is
that it no longer contains magnitude and phase information
at all frequencies as would be the case for the continuous
Fourier transform, but describes the spectrum at discrete
frequencies and with finite resolutions up to some maximum
frequency. According to Shannon's sampling theorem the
maximum frequency is given by,

\[ f_{\text{max}} = \frac{1}{2Δt} \quad 5.44 \]

Also, as a consequence of inherent characteristics of the
Fourier transform (to completely describe a given frequency
two values are required) we have,
\[ \frac{N}{2} \Delta f = F_{\text{max}} \]

By combining Equations 5.43, 5.44 and 5.45 the physical law which defines the maximum frequency resolution obtainable for a sampled record of length, \( T \), is determined. Namely,

\[ \Delta f = \frac{1}{T} \]

With these relationships in mind, consider the general case for a system frequency response function measurement as shown in Figure 5.3.

In this figure \( x(t) \) represents the time domain input to the system, \( y(t) \) the time domain response of the system, \( S_x(f) \) the linear Fourier spectrum of \( x(t) \), and \( S_y(f) \) the linear Fourier spectrum of \( y(f) \). \( H(f) \) is the system frequency response function and \( h(t) \) is the system impulse response.

It is well known that for a linear system the time domain output is given by,

\[ y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) \, d\tau \]

Applying the Fourier transform to the above "convolution integral" gives,

\[ S_y(f) = S_x(f) \cdot H(f) \]
Rearranging (and dropping the frequency dependence notation) gives,

\[ H = \frac{S_y}{S_x} \quad 5.49 \]

Unfortunately, Equation 5.49 has limited practical application because of severe restrictions on the type of exciting force (input) that can be used, and also difficulties associated with reduction of measurement noise.

The actual hardware implementation is therefore based on the following. Consider power spectra to be defined as,

\[ G_{xx} = S_x S_x^* \quad 5.50 \]

where, \( G_{xx} \), represents the auto power spectrum of the input \( x(t) \) and, \( ^* \), denotes the complex conjugate. In a similar manner then,

\[ G_{yy} = S_y S_y^* \quad 5.51 \]

where, \( G_{yy} \), represents the auto power spectrum of the output \( y(t) \).

The cross power spectrum is defined as,

\[ G_{yx} = S_y S_x^* \quad 5.52 \]
Now multiplying the numerator and denominator of Equation 5.49 by $S_x^*$ results in the expression,

$$H = \frac{S_y}{S_x} \cdot \frac{S_x^*}{S_x^*} = \frac{G_{yx}}{G_{xx}} \quad 5.53$$

Definition of the frequency response function in this manner provides three important benefits:

i) This technique measures magnitude and phase since the cross power spectrum contains phase information.

ii) This formulation is not limited to sinusoids, but may be used by any arbitrary waveform that is Fourier transformable (as most physically realizable time functions are).

iii) Averaging can be applied to the measurement. This alone is a major consideration because of the large variance in the transfer function estimate when only one measurement is used.

As a result of point (iii), the frequency response function is actually implemented as,

$$H = \frac{\overline{G_{yx}}}{\overline{G_{xx}}} \quad 5.54$$

where, $\overline{G_{yx}}$ denotes the ensemble average of the cross power spectrum and $\overline{G_{xx}}$ represents the ensemble average of the
input auto power spectrum.

The importance of averaging can be shown if the frequency response function model is modified to include noise as shown in Figure 5.4. In this case, $S_x(f)$ is the linear Fourier spectrum of the measured input signal, $S_d(f)$ is the linear Fourier spectrum of the desired response measurement, $S_y(f)$ is the linear Fourier spectrum of the measured response, $N(f)$ is the linear Fourier spectrum of the noise and $H(f)$ is the system frequency response function.

Without repeating the mathematics for the general model in the presence of noise, it will simply be stated that the result obtained would be,

$$H = \frac{\overline{G_{yx}} - \overline{G_{nx}}}{\overline{G_{xx}}}$$

where, $\overline{G_{nx}}$ would be the ensemble average of the cross power spectrum between the noise and the input, and the remainder of the symbols are as previously defined.

If it is assumed that the noise has zero mean value and is incoherent with the measured input signal, then it is easy to see that as the number of averages increases the term $\overline{G_{nx}}$ becomes smaller and the frequency response function comes closer to the desired value of $\frac{\overline{G_{yx}}}{\overline{G_{xx}}}$.  

5.2.1.5 Coherence

Frequency response functions associated with most
Mechanical systems are so complex in nature that it is virtually impossible to judge their validity simply by inspection. Some physical measure of the degree to which the output of a system is the result of a given input, would be desirable.

One means of measuring the influence of noise and/or non-linearities, and thus the degree to which the frequency response function is contaminated, is by calculating the coherence function, \( \gamma^2 \).

In general the coherence function can be defined as,

\[
\gamma^2 = \frac{\text{Response power caused by applied input}}{\text{Measured response power}}
\]

When implemented on a digital Fourier analyzer which calculates frequency response functions the coherence is obtained using the expression,

\[
\gamma^2 = \frac{|G_{yx}|^2}{G_{xx} G_{yy}} \quad \text{where} \quad 0 \leq \gamma^2 \leq 1
\]

If the coherence is equal to 1 at any specific frequency, then the system is said to have perfect causality at that frequency. That is, the total response power is completely a result of the measured input power (or, perhaps, by sources which are coherent with the measured input power). In the case where the coherence is less than one, then the measured response power is greater than that
which would result from the measured input power alone. This would indicate that some extraneous noise was also contributing to the output power. In the extreme, where the coherence is zero at a particular frequency, the output is caused totally by sources other than the measured input.

Since the coherence function indicates the degree of causality in a frequency response measurement, it has two important uses:

i) It can be used qualitatively to determine how much averaging is required to reduce measurement noise.

ii) It can serve as a monitor on the quality of the frequency response function measurements.

5.2.1.6 Excitation Techniques

One consideration that has yet to be addressed is the type of forcing (input) function used to excite the test structure. This is an important consideration since the type of excitation force employed has a significant influence on the quality of the measured frequency response functions.

Generally the available types of excitation can be classified into three broad categories with some sub-categories. These are:

I) Random Excitation Techniques
   i) Pure Random
   ii) Pseudo-Random
iii) Periodic Random

II) Sinusoidal Excitation

III) Transient Testing

i) Impact

ii) Step Relaxation

To review all these techniques, and to note their relative strengths and weaknesses is beyond the scope of this review. As such, remarks will be limited to the Impact Testing Method, since this is the method used in the present work. Reference [26] contains an excellent review of all the methods listed.

In the Impact Testing Method a hand-held hammer with a load-cell mounted to its tip is used to impact the structure. The load cell then measures the input force and a transducer mounted at a point on the structure measures the response.

In general, impact testing has several important advantages [26]:

i) No elaborate fixturing is required to hold the structure under test.

ii) No electro-mechanical exciters are required.

iii) The method is extremely fast — often as much as 100 times as fast as an analog swept-sine test.

Of course, this method also has its drawbacks. The most serious is that the power spectrum of the input force is not as easily controlled as it is when a mechanical
shaker is used. This results in the excitation of non-linearities and consequently some variability between successive measurements. This is a direct consequence of the shape and amplitude of the input force signal.

The impact force can be altered by using a softer or harder hammer tip. The wider the width of the pulse, the lower the frequency range of excitation. Thus a hammer with a hard tip can be used to excite higher frequency modes, whereas a softer tip can be used to provide more energy at lower frequencies.

Since the total energy supplied by a pulse is distributed over a broad frequency range, the actual excitation energy available at a particular frequency can be quite low. This can cause problems when testing large, heavily damped structures, because the frequency response function estimate will suffer as a result of the poor signal-to-noise ratios.

Also in cases where the transient response signal does not decay to zero within the measurement window, severe leakage can occur. The use of an appropriate response window can significantly limit errors due to this effect. The problem of leakage will be discussed in more detail in the following section.

In spite of these problems, impact testing can produce excellent results when utilized carefully, fully recognizing where measurement errors are likely to occur,
and taking appropriate precautionary measures.

5.2.1.7 Measurement Errors Resulting from Use of Digital Techniques

As a result of the use of digital techniques, which require discrete measurements and finite sampling periods, a number of errors can be introduced into the calculated Fourier transforms and, consequently, the frequency response functions. This section will review the major types of errors introduced and how they can be minimized.

a) ALIASING

The first source of error to be discussed is aliasing. Two signals are said to alias if the absolute value of the difference of their frequencies falls in the frequency range of interest. This difference frequency is always generated in the process of sampling. Figure 5.5(a) shows an input frequency which is slightly higher than the sampling frequency, and so a low frequency alias term is generated. Note that it would appear to anyone viewing the display of the analyzer that a spectral component with frequency \( f_{in} - f_s \) is present in the input signal. This is obviously not the case. However it is impossible to tell whether a given component is actually present in the input signal or is simply an alias, once the processing has occurred. It is therefore imperative that aliasing be avoided under all allowable measuring conditions. One means
of assuring that no aliasing can occur is by sampling at a rate which is greater than twice the highest frequency present in the input signal. Figure 5.5(b) illustrates this case. This situation is realized in hardware implementations by using a low-pass filter on the input to limit the highest frequency component to a known value and utilizing a sampling rate greater than twice this frequency. Because of transition band considerations associated with real low-pass filters, the sampling rate is often set at from 2.5 to 4 times the maximum desired input frequency.

b) LEAKAGE.

A second source of error inherent in the Fourier analyser is "leakage". This is a rather more important source of error than aliasing, since it is so much more difficult to limit.

The basic causes of leakage are the use of a sample of finite duration and the inherent characteristics of the Discrete Fourier Transform (DFT). Since it is undesirable to wait an infinitely long time to perform a measurement (and because only a finite memory storage capability is available in the analyzer), a sample of finite duration called a "time record" is utilized. Now, the DFT is premised upon the assumption that this time record is repeated throughout time as illustrated in Figure 5.6(a). This causes no particular difficulty in the case of the
transient signal shown. However, consider the case where the signal is continuous, as is the sine wave shown in Figure 5.6(b). In this particular case, if the time record contains an integral number of cycles (the signal is then said to be "periodic" in the time record) then the assumed input exactly matches the actual input and no error occurs. However, if, as illustrated in Figure 5.6(c), the input is not periodic in the time record, then the DFT actually operates on a signal very different from the actual signal. The result is a smearing of energy throughout the frequency domain, which is called "leakage" (i.e., energy actually associated with a single frequency "leaks" into all other frequencies present in the analysis range).

Generally, the signals to be measured are not periodic in the time record and thus leakage will be present. One way to minimize leakage is by using "windows". Basically these are weighting functions which force the input data near the beginning and end of the time record to zero. Note in Figure 5.6(c) that it is the data near the edges of the time record which causes the greatest problem. By forcing this data to zero, the DFT is made to "ignore" the ends and concentrate on the middle of the time record where the data is more representative of the actual input signal. Figure 5.7 illustrates such a process. Note that the assumed input is still not a match of the actual input data and some leakage will occur. However, the effect will
be very much smaller than for the unwindowed case. Thus, choice of the appropriate window can significantly reduce leakage but will never completely remove its effects.

c) QUANTIZATION

Since the DFT is being used, it is necessary to obtain digitized samples of the continuous input signals. This process is accomplished by the Analog to Digital Converter (ADC). The "quantization" of the input signals has associated with it a number of possible error sources. In particular, such factors as actual versus effective word size, the ADC dynamic range, bit drop out, overload recovery, conversion error, differential non-linearity, aperture error and digitizer noise could be considered.

Realizing that, in reality, little can be done about any of the errors listed above after the purchase of the equipment itself, it is sufficient to state that the higher the quality of the ADC the smaller will be the contribution of the errors noted. That is, the strategy for reducing such errors is simply to purchase quality instrumentation.

Perhaps one area that should be discussed, however, is the concept of ADC dynamic range since, in fact, errors associated with this parameter can be controlled by proper measurement technique.

Any ADC has a given dynamic range over which it is designed to operate, say, \( \pm 1 \) V. It is important, then, that
the input signal level, through appropriate setting of the variable input ranges which are standard with all Fourier analyzers, be adjusted to utilize as much of this range as possible.

As a simple example of why this is important, consider a 4 bit ADC with a range of ±1 V. Assuming bit #3 is used as a sign bit, then such a converter can uniquely represent 15 different signed numbers (0 to 7 and -1 to -7). Thus a given value can be represented to one part in fifteen. For the bi-polar range of ±1 V, the ADC would represent the converted analog voltage in 1/7 volt increments. An ADC value of 0010 would represent a value of 2/7 = 0.2857 V while a value of 1111 would represent a value of -7/7 = -1 V. Thus it can easily be seen that an input signal in the range of ±1 V would effectively utilize all 4 bits of the ADC.

Now, suppose that the input signal is actually in the ±0.1 V range while the ADC range remains at ±1 V. In this case only 2 bits of the 4 bit ADC would be used to represent the analog voltages in this range since each ADC increment represents 0.1429 V. Thus instead of having a total of 15 different values that could be used to represent the bi-polar signal, there is now effectively only three. The result is very poor accuracy in measuring the incoming signal.

Similar difficulties can occur even with the 12
bit ADCs which are typically in use on Fourier analysers. It is therefore of paramount importance that the full operating range of the ADC be used. This is typically accomplished by acquiring representative test signals and adjusting the input attenuators until the ADC just indicates an overload condition. The input is then reduced in level by one adjustment increment.

5.2.2 Methodology

One particularly important question to answer through modal analysis of the transmission case is whether the response properties of the case change significantly when mounted in the transfer pallet which is used to move it from station to station in the transfer machine. The significance of this question will be recognized when it is remembered that any structural response data needed for use in design of a "quiet" face-milling cutter must usually be obtained well before the transfer machine itself is built, due to the long lead times necessary for such tooling. This means that the transfer pallets are not usually available for testing purposes unless extraordinary efforts are made to expedite their design and manufacture.

Also, modal data for the transmission case in the "free-free" condition would actually be used to design the transfer pallets in the first place. It would therefore be ideal if this one modal analysis would provide sufficient information for design of both the pallet and the "quiet" cutter.
For these reasons it was determined that two modal analyses would be performed on the G.M. "125" transmission case; one for the case in the "free-free" condition (freely suspended) and the other for the case clamped in the actual transfer pallet. In this manner it was expected that a determination could be made as to the significance of any changes in response properties.

5.2.2.1 Geometrical Definition of the Transmission Case

The first step in performing a modal analysis is defining the location of the points of interest on the structure being tested. For the transmission case it was desired to use sufficient points to assure a reasonable display of the mode shapes during animation, and also guarantee that a significant number of points were included from those areas expected to be important to the noise generation during the face milling operation. However, too many points makes the measurements unnecessarily complex and time consuming. As a compromise, a total of 86 coordinate points (actual measuring locations) were used. Of these, eleven points were given two degrees-of-freedom (the eleven coordinate points on the engine mounting face) so that a total of 97 frequency response function measurements would have to be performed in each data acquisition phase.

The location of each coordinate point had to be determined with reference to arbitrary coordinate axes (the "global" reference axes) and the coordinates entered into
the analyser. Table 5.2 shows an excerpt from the measurement point coordinate table. The required dimensions were obtained from part print drawings and, as necessary, a coordinate measuring machine. In fact, all measurements did not have to be with reference to a single coordinate system but could, where helpful, be referenced to local coordinate systems. In these cases it was only necessary to define where the origin of the local coordinate system was located relative to the "global" coordinate system and how it was oriented relative to it. Table 5.3 shows the specification of a local coordinate system.

As the coordinate points were located, their positions were permanently marked on the test transmission housing and each point was given an arbitrary numerical designation, starting with 1 and ending with 86. These are the same numbers that appear in the first column of Table 5.2. This "tagging" of coordinate points helps prevent confusion during data acquisition and permits easy reference to a given location.

Once the coordinate points had been specified it remained only to tell the analyzer how the points should be joined up so that the displayed model of the transmission case reasonably approximated the actual part. The result of this step is shown in Figure 5.8. A photograph of the actual part is included in this figure for comparison purposes. Once the display model is in the analyzer's
memory it can be rotated about any axis, viewed from
different directions, or viewed from different distances,
all as desired. However, the display model can not yet be
animated since no modal shape data is available.

5.2.2.2 Measurement of Frequency Response Functions

The next major step in obtaining the modal
analysis of the transmission case is measurement of the
required frequency response functions.

Fortunately, it is not necessary to measure all
the frequency response functions in the input - output
degrees-of-freedom matrix. (In this instance it would mean
97 x 97 = 9,409 measurements!) It can be shown that it is
only necessary to measure one row or one column in this
matrix as long as the driving point frequency response
function measurement is included. The driving point is that
location where both the input and output measurements are
made at the same time. This requirement can be satisfied by
inputting a force at a given point and measuring outputs at
that point and all the other degrees-of-freedom on the
casing or, by applying the input force at each
degree-of-freedom and measuring the output at only one of
these points. It was deemed to be easier, and less error
prone, to input the forcing function at a single point and
measure the output at all degrees-of-freedom. The input
force was applied to Point #49 which was located on the
engine mounting face of the transmission housing. Figure
5.9 shows the location of Point #49 and the direction of force application.

As noted earlier, the Impact Testing Method was used to obtain the necessary frequency response function measurements. Before this method can be used effectively it is often necessary to apply both a "force window" to the input signal and a "response (or exponential) window" to the output signal.

A typical force pulse is shown in Figure 5.10(a). Note that, in general, the force pulse is completely contained in the time record and is therefore said to be "self-windowing" and leakage will not be a problem on this channel. However, movement of the hammer after striking the structure and stray electrical noise can cause the time record to have non-zero values after the end of the force pulse, thus introducing measurement errors. To avoid this, a force window is applied to the input signal. The window is set to unity where the impact data is valid and zero everywhere else in the time record.

If the response of the structure dies out within the time record then it is also self-windowing. However, in many cases the response does not die out within the time record, as shown in Figure 5.10(b). In this case leakage will occur. To minimize this error, a response window is applied to the output signal to force it to zero by the end of the time record. Note that unlike the windows discussed earlier, the "response window" is not zero at both ends of
the time record. Since it is known that the response of the
structure will be zero at the beginning of the time record
(just before the hammer blow) there is no need for the
window function to be zero there. Also, most of the
information about the structural response is contained at
the beginning of the time signal so it is advantageous to be
sure that this section is weighted most heavily by the
response window.

For measurements on the transmission case both a
force window and response window were applied to the data.
The values for these windows were obtained by observing the
input and response signals for each measurement point. The
windows were adjusted as necessary. Figure 5.11 shows the
results of using the response window on an actual
measurement.

When generating the frequency response function
measurements it must be decided what frequency range should
be used to acquire the data. This is important since a wide
frequency setting means a loss in frequency resolution, and
therefore errors in modal parameter estimation, while a
narrow frequency setting usually means a large number of
measurements to cover the overall range of interest.

The first step in selecting appropriate
measurement frequency ranges is to decide on the most likely
overall range of interest. With the cutting noise
information of Chapter IV available, it is apparent that
structural modes much above 3,000 Hz would not be of interest, since these result in only minor noise generation during the cutting process. This is demonstrated in Figure 5.12 which shows a selection of noise spectra from various cutting tests. Each spectrum has indicated on it a 10 dB band which defines the spectral components which contribute significantly to overall noise level. Note that seldom are components above 3,000 Hz included in this band.

Since the analyzer in use had a frequency range setting of 0 to 3,200 Hz available, then such measurements could, in fact, have been made. However, review of typical frequency response functions, as shown on Figure 5.13, indicated the presence of numerous modes, many of which were quite closely coupled. Therefore to avoid resultant large parameter estimation errors, it was decided to break up the overall range of interest (i.e., 0 to 3,200 Hz) into three smaller ranges, thus effectively "separating" the modes and increasing the inherent frequency resolution of the measurements.

When using "Co-Quad" plots to identify modes, it should be remembered that at resonance the imaginary part of the frequency response function "peaks" while the real part goes to zero.

The three measurement ranges utilized were: 0 to 1,600 Hz, 1,550 to 2,350 Hz, and 2,400 to 3,200 Hz. This combination of ranges provided a reasonable balance between
frequency resolution and the number of measurements required to perform the modal analysis. It should be pointed out here that a complete set of frequency response functions (97 in this case) has to be generated for each measurement range used. Thus as the number of measurement ranges increases, the time and cost involved in making the necessary measurements rises significantly.

Once frequency ranges have been determined it is necessary to determine the type of hammer and tip that are to be used for exciting the structure. This is important since, as noted earlier, the pulse shape, and thus the frequency content, is affected by the hammer mass and the hardness of its tip. It is critical that the input spectrum contain energy over the whole frequency range of interest and that this energy be of a reasonable magnitude, so as to be sure all relevant modes are excited and to preclude significant signal-to-noise ratio problems. This does not mean that the energy content must be constant over the range of interest, since the frequency response measurements determine the output acceleration per unit input force. Figure 5.14 shows a typical averaged auto-spectrum of the input force generated by a "medium" weight hammer with an aluminum tip. This was the combination subsequently used for the generation of all frequency response functions. It can be seen that the spectral content is continuous throughout the range of interest and that, although varying,
a reasonable amount of excitation energy is present up to 3,200 Hz.

Having guaranteed excitation energy is present up to the limiting frequency of interest, it is then important to be sure that the response of the casing is properly measured. The first step here is to be sure that the accelerometer used has a linear frequency response over the frequency range of interest. In this case, the accelerometer had a linear response from almost D.C. to 10,000 Hz, which is more than adequate. Also, since the mass of the transducer can locally "load" the transmission case, it is important to keep the accelerometer's mass to a minimum. The small mass (2 gm) of the accelerometer employed guaranteed minimum localized loading effects. Finally, the transducer must be mounted to the transmission case in such a manner as to prevent damage to the case and also permit easy removal of the accelerometer from each measuring point. Also the mounting method should not degrade the high end frequency response limit of the transducer sufficiently that it falls below the upper frequency of interest in the tests. To meet all of these requirements it was decided that the accelerometer would be mounted using bee's wax.

As noted earlier a modal analysis was to be performed on the transmission case for two different mounting conditions. In one set of tests the case was
freely suspended as shown in Figure 5.15. The suspension elements were made sufficiently compliant to ensure the rigid body modes of the case would be well below the first elastic body modes of the transmission case. In the second modal analysis the case was clamped in its transfer pallet as shown in Figure 5.16. In this instance all transfer pallet clamps were tightened to operational levels and the pallet itself was mounted on a rigid support structure. This support was of sufficient height above the floor plant to permit reasonable access to the case. Between the support structure and the pallet, a layer of neoprene rubber was used to lower rigid body modes and to isolate the pallet/case assembly from transient floor vibrations during measurements.

For a given frequency range, and a particular suspension method, the following steps were used to obtain the necessary frequency response functions:

1) Turn on all equipment and allow a five minute "warm-up" period.

2) Connect the impact hammer (force signal) to Channel #1 of the analyzer. Connect the accelerometer (response signal) to Channel #2 of the analyzer.

3) Enter the calibration coefficients for the load cell of the impact hammer and the accelerometer into the analyzer. This allows properly scaled response plots to be generated.
4) Mount the accelerometer at the response point of interest.

5) Arm the analyzer.

6) Strike the input point (in this case always Point #49) with the hammer.

7) Watch for any overloads on the input or output channels. If none, increase the sensitivity of the channels and repeat (6) until overloads occur on each channel; then decrease sensitivity on each channel by one adjustment level. This assures that optimum use is made of the ADC range.

8) Having adjusted the ADC range, arm the analyzer and strike Point #49. The frequency response function is then calculated and displayed by the analyzer.

9) Observe the coherence function (it is automatically calculated for each frequency response function) and if unsatisfactory, check for loose or broken cables, low power supplies, improper accelerometer mounting, etc. Make necessary adjustments.

10) Repeat steps (8) and (9) until five acceptable frequency response functions have been obtained. These five measurements are then averaged to provide the frequency response
function to be used for parameter extraction for this particular input-output combination.

11) For each subsequent degree-of-freedom steps (4) through (10) are repeated until all 97 frequency response functions have been obtained. Each of these frequency response functions is stored on a digital cassette tape for future recall during the parameter extraction process.

5.2.2.3 Parameter Extraction

Having obtained all the necessary frequency response measurements, it now remains to extract the necessary modal parameters from this data.

Fortunately the majority of the work involved from this point on can be handled by the analyzer itself. The only major exception is the identification of the resonant (or "natural") frequencies. In this case, the frequency response curves must be manually reviewed to identify the resonant frequencies. As an example, Figure 5.17 shows a typical frequency response function for the "free-free" condition between points #49 and #16. Point #16 is in the "bell" of the transmission case. See Figure 5.16. Now in Figure 5.17, to identify a resonant frequency, it is necessary to locate peaks in the imaginary data and a zero crossing in the real data. Such a location (frequency) has been noted on this figure and identified as Mode #1. This
numerical designation is purely arbitrary and is used to simplify reference to a given mode (i.e. "Mode #1" instead of "Mode at 243.75 Hz"). Each additional frequency, at which resonance is identified, would be designated with a mode number, usually in order of ascending frequency. The frequency associated with each mode is entered into the analyzer for later use. This entry is somewhat simplified since a cursor can be used to designate a mode and a simple key entry then transfers the appropriate frequency data to the analyzer.

It is important that a reasonable number of the frequency response function measurements be reviewed to identify modal frequencies, this, in spite of the fact that modal frequency is a "global" parameter. A "global" parameter is one which is the same for a given mode anywhere on the structure. Thus modal frequency is a global parameter since the frequency associated with any particular mode is the same anywhere on the structure of interest. Also damping is a global parameter since, for a given mode, it has the same value at any point on the structure. The reason for reviewing more than one frequency response function can be demonstrated by referring to Figure 5.18. Here again Mode #1 has been located (243.75 Hz); however, the very much smaller magnitude of the peak in the imaginary data is evident. This results from the fact that, although frequency (and damping) are global properties and thus are
theoretically obtainable from any point on the structure, the magnitude of the response of any point (the "residue" in the mathematical formulation) varies from point to point. Therefore the response of a particular point in a given mode of vibration may be quite small, thus making it difficult to identify the peak in the imaginary part of the frequency response function. In the extreme, a point may, in fact, be a "node" for a given mode of vibration and thus not respond at all. In this instance, no indication of this mode would show up on the frequency response function. Thus it is necessary to review numerous frequency response functions to be sure that all modes of vibration have been identified.

Also, as each mode (resonant frequency) is identified using the cursor on the display screen of the analyzer, the damping value for that mode is automatically extracted using the technique described earlier. This means that it is important that well defined (i.e. reasonable response level) examples of the modes be used for damping extraction. Thus the frequency response functions must be reviewed to identify the "best" example of a given mode so that errors in damping estimation are minimized.

Having obtained all the necessary frequency response functions, and having used the most appropriate of these to identify the modes of vibration and their associated damping, all that remains is to determine the residues. These are then utilized to generate the mode
shapes. Although there is, in fact, a substantial amount of calculation associated with these tasks, most are "transparent" to the user since the analyzer can be programmed to automatically manipulate the frequency response functions and extract the necessary data.

Once the residue table has been obtained (see Table 5.4 for an excerpt from a typical residue table) the analyzer can be used to generate the mode shape vector for each mode. All that is required is the choice of a "scaling factor": This is the Q term shown in Equation 5.31. Typically, this scale factor was set to unity. Having set the scale factor, the shape vector amplitudes can be calculated. An excerpt from a typical mode shape table is shown in Table 5.5. Once the mode shape vector data is available we can animate the geometrical model generated earlier.

5.2.3 Results

At this point all modal parameters are now available for the modes identified in the frequency ranges of interest. This means that the animated mode shapes can be observed on the analyzer's screen, if desired, and the various modal parameters available for each mode can be used for comparison purposes.

5.2.3.1 Transmission Case Freely Suspended

Over the complete range of interest (0 to 3,200 Hz) a total of 39 modes of vibration were identified for
"freely" suspended transmission case. These are summarized in Table 5.6. Each of these modes could be animated to investigate areas of greatest movement for a given mode and to compare the differences in movement between modes. This type of comparison helps to identify those areas which are potentially the greatest noise generators.

It is difficult to demonstrate modal animation properly "on paper", however an attempt will be made to indicate the type of display provided and the information to be obtained from such displays.

Figure 5.19 attempts to convey an impression of the animated mode display. Figure 5.19(a) shows the undeformed display of the transmission case. During animation, the various measuring points move from one extreme of travel to the other, with some points being in phase with one another while the remainder are 180° out of phase (characteristic of a normal mode). The magnitude of this motion for a given point is purely arbitrary and can be changed as desired to enhance the display. However, the relative motion of the points is fixed and is a result of the values calculated for the mode shape vector. In Figure 5.19 the upper diagram shows the transmission casing at one extreme of its "travel" during animation, while the lower diagram shows the other extreme. The smoothly animated motion between these extremes must be imagined. The speed at which travel between extremes occurs can be adjusted as
desired. As mentioned earlier, the display can be rotated, viewed from different positions or distances, and the animation "frozen" at any time.

Another useful feature for study of the mode shapes is the ability to display various components which make up the complete structure. A "component" in this sense is an arbitrarily designated set of measuring points. A point is designated as belonging to a given component during the coordinate entry phase of the display model generation. Although component designations are purely arbitrary, they are usually based on some recognition of "physical distinction" in the actual test structure. As an example, a box-like structure might be perceived as being built-up of a number of "plates" each of which is to be considered a separate component. The reasons for utilizing such a "component-part" approach might include the desire to study only the mode shapes of a particular area of a structure, or the recognition that the wire-frame diagram display is difficult to "sort-out" for complex structures and therefore display of only certain sections at any one time allows a better comprehension of exactly how the structure is moving in a given mode. An example for the three components defined for the transmission case model is shown in Figure 5.20. The measuring points for each of these components were colour coded on the actual transmission case, as may be observed in Figures 5.15 and 5.16.
When, during the following discussion, it is desired to illustrate a given mode shape only a single extreme from the undeformed position will be shown.

If all the freely suspended modes of the transmission case are reviewed with the intention of determining whether it is the "bell housing" or "shaft housing" which might contribute more to noise generation, it becomes apparent that both are significant contributors to vibrational energy (and thus noise). However, the bell housing is generally the more structurally active and thus would be expected to be the greater of the contributors. This is not surprising if one makes note of the geometry associated with each of these components. Figure 5.21 shows a selection from the 39 modes identified for the freely suspended case. Notice that, in general, the bell housing is the more active component and that the modes tend to take on increasingly complex shapes for higher frequencies.

The large number of modes over the range of interest (0 to 34200 Hz) and their increasing geometrical complexity indicate that attempting to reduce noise generation during face milling by increasing the number of clamping points on the pallet (i.e. shift modes by changing the effective stiffness and damping of the case) or by adding additional external damping pads to the pallet would all be for naught.

Increasing the number, or perhaps type, of clamps
on the pallet would certainly change the effective stiffness and damping of the transmission case and thus "shift" its modes of vibration. However, such a strategy can only be really successful if it can be guaranteed that the modes are shifted away from frequencies associated with the forcing function. Unfortunately, for the case of a face milling operation, the large number of frequency components associated with the forcing function almost guarantees that numerous modes will still be excited, with consequent significant noise generation. Also it should be remembered that a clamp is only effective in increasing the stiffness of a workpiece if it restrains a point which would otherwise undergo large movement relative to the fixture, in this case, the pallet. Of course, the difficulty in performing such a feat with a part exhibiting as many modes as a transmission housing, lies in the fact that a point having significant relative movement for one, or even several modes will have almost no relative movement in a great number of the remaining modes. As a result, the influence of the clamp is greatly reduced since it has a significant effect only on the motion associated with a few modes, and thus the resultant vibration reduction will often be far less than desired.

5.2.3.2 Transmission Case Clamped in Pallet

Over the complete frequency range of interest a total of 31 modes of vibration were identified when the
transmission case was clamped in its transfer pallet.

A review of the various mode shapes again indicated that although both the bell housing and shaft housing were significant contributors to vibrational energy, the bell housing was the structurally more active of the two components. Thus it would generally be expected to be the greater noise contributor.

The results from the "clamped" tests emphasize the futility of attempting to substantially reduce noise generation by the addition of clamping points and/or damping pads to the pallet.

The modal density has not changed significantly from the "unclamped" case. Thus there are still a significant number of modes to be excited by the harmonically rich forcing function.

The mode shapes are still very complex, and the addition of one, or perhaps two (the practical limit) clamps cannot be expected to produce dramatic results. Especially in light of the fact that going from no clamps at all (freely suspended) to numerous clamping points (see Figure 5.16) has not significantly reduced the modal density in the frequency range of interest, the mode shape complexity, nor has it been effective in reducing the noise levels generated during face milling of the engine mounting face to acceptable levels (recall results in Chapter IV). To expect that the addition of one or two clamps could produce
significant improvements in this situation is rash.

If the modal parameters for the first 23 modes of vibration are compared as shown in Table 5.7, it is apparent that clamping the part in the pallet has significantly increased the effective damping of the transmission case and yet, based on the measurements of Chapter IV, the problem of excessive noise generation during milling still persists. To expect that the addition of, at best, a few damping pads will provide a significant noise reduction is unrealistic.

Thus it is clear that these tests show the impracticality of employing additional clamps or damping pads to reduce noise generation during face milling of the engine mounting face. This indicates that only changes to the forcing function, which is exerted by the engagements of the cutting inserts, has practical potential for workpiece vibration (and noise) reduction.

It was noted earlier in this chapter that one of the reasons for performing modal analyses on both the freely suspended and the clamped transmission housing was to determine whether the data from the freely suspended type of test might be used for "quiet cutter" design. It will be remembered that this possibility has a number of practical advantages.
Unfortunately, review of the data shown in Table 5.7 indicates that this possibility is rather remote. Such a procedure could only be justified if the response of the transmission case is relatively unchanged by clamping in the pallet or that the response parameters change in an easily predictable manner. It is apparent that neither of these requirements is met. The response parameters are significantly different and they do not change in an easily determinable manner.

Analytical procedures are available [11] [13], which will permit prediction of the changes in the response characteristics of a structure if the change in mass, stiffness or damping is known for all locations on the part. However, their practical value when applied to the present situation is questionable, since the change in effective mass, damping and stiffness at a given point due to the addition of a clamp or rest pad is particularly difficult to quantify. It is certain that the effort expended in attempting to effectively employ such methods would far exceed that of a second modal analysis, and be nowhere near as reliable.

5.3 **FREQUENCY RESPONSE FUNCTIONS**

Additional knowledge concerning the response characteristics of the subject transmission case can be obtained from a review of the frequency response functions obtained from particular points on the transmission case.
under various test conditions. Also, these tests can help to clarify what type of information may be most useful in the design of "quiet cutters".

It must be remembered that it is desirable to be able to predict, through simple test procedures, certain characteristics of the noise generated by the transmission case under the influence of the forces exerted by the engagement of the milling cutter inserts during the actual machining process. This information could then be used to help design a "quiet cutter" without trial and error procedures with expensive production-type tooling.

First it would be desirable to be able to predict the frequency content of the noise that would be generated by the actual milling process.

Secondly, it would be desirable to be able to determine how the response of the workpiece changes as the point of application of the forcing function changes. In particular, the change in predominant frequencies of noise generated and some indication of the overall magnitude of noise generation for particular locations of the forcing function impact, would be very valuable information.

Since FRF data is easily obtained and contains much information concerning the response of a structure to a given forcing function, it was decided to perform a series of such tests on the transmission case. These FRF's would then be compared with the data obtained during actual
machining of the case to determine if useful information concerning noise generation could be inferred from them.

5.3.1 **Accelerance Frequency Response Function - Freely Suspended Transmission Case**

5.3.1.1 **Methodology**

The first series of frequency response functions (FRF) was obtained with the transmission case freely suspended as shown in Figure 5.15. The input force was applied, using the instrumented hammer, at various points along the milled engine mounting face. The force was applied in the direction shown in Figure 5.22. Basically, the direction chosen approximated the path of a cutting insert as it would first engage the transmission case at the point designated. In some instances this had to be modified slightly since the hammer blow had to be applied "squarely" on a flat surface to preclude difficulties with multiple hits and poor coherence. In these instances the direction of application would be changed sufficiently to obtain the desired "normal" hammer blow. The point designations (i.e., 44, 45 ... etc.) shown in Figure 5.22 are the same as those used for these positions during the modal analysis described earlier. However, during the modal analysis the response transducer was mounted at these points in such a manner that its measuring axis was coincident with one of the axes of the designated coordinate systems. In the present instance the forcing function is applied along axes which generally
do not coincide with the coordinate axes. Thus, the information contained in this series of tests is unique and not available through any manipulation of the earlier modal analysis data.

The response transducer (accelerometer) was mounted, using wax, at two locations, Points #32 and 71. These points were taken to be representative of locations on the "bell housing" and "shaft housing" components of the transmission case, respectively. Of course, it is known that each point on these components will have unique responses to a given input force. However, since it is the "general trends" which are of interest, it was felt that this simplification in procedure would still permit a reasonable evaluation of the type of information to be obtained from such tests. If these preliminary results show promise, then future work could be undertaken to incorporate averaging or weighting techniques in the utilization of FRF data from numerous points on the transmission case.

The frequency response functions themselves were obtained in the same manner as described under modal analysis and therefore the details will not be repeated.

A check was kept on the validity of each measured FRF by inspecting its associated coherence function. Figure 5.23 shows a typical example of the coherence functions obtained. The coherence is generally close to a value of one over the frequency range of interest with significant
excursions from unity occurring only at locations of anti-resonances. This is quite acceptable. In those instances where extremely poor coherence was evident, checks were made for loose or broken cables, poor transducer mounting, etc. and any problems were corrected before proceeding with measurements.

The frequency response functions illustrated in this section will typically be presented in a "magnitude" format, where the magnitude value at each frequency is obtained by taking the square root of the sum of the squares of the imaginary and real parts of the FRF at each frequency (i.e., \( \sqrt{(\text{REAL})^2 + (\text{IMAG})^2} \)). The phase information which is also available for this FRF data is generally not shown here because of its limited use for present purposes. The combined presentation of magnitude and phase information as a function of frequency is often referred to as a "Bode Diagram".

The FRF will typically be illustrated for the frequency range 0 to 6,000 Hz. Although it is known for the workpiece used in this study that the noise generation of interest is typically below 3,000 Hz, this type of knowledge would not be available "a priori" in the general case, since no noise measurements could have been performed on the workpiece during machining. Thus it is necessary to be able to determine from a wide frequency test range the actual
range of interest (largest noise contribution) for the particular workpiece under test. This is one of the abilities which will be watched for as the following test results are reviewed.

5.3.1.2 Discussion of Results

Figure 5.24 shows examples of the FRF data obtained for output Point #32 while Figure 5.25 provides examples for output Point #71.

The first observation to be made is that the FRF data indicates the frequency range of interest to be 3,000 Hz to 6,000 Hz (and perhaps higher). This is at odds with the known result of approximately 200 to 3,000 Hz. That is, the range of frequencies in which the greatest noise is generated during milling.

Secondly, if the relative strength of each FRF is qualitatively ranked, say be taking note of the magnitude and number of peaks present, then it is apparent that there is, in general, a direct relationship between this "strength" and the measured overall noise level produced as the milling cutter passes each of the inputs points on the engine mounting face. Review of Figures 5.24 and 5.25 together with Figure 4.14 will illustrate this relationship.

Thus it would appear that acceleration FRF data can provide some indication of the relative magnitude of the overall noise levels likely to be produced during the milling process. However, this type of data does not, on
its own, provide a reasonable indication of the frequency range which would be associated with this noise generation.

Of course, it will be remembered that the actual noise generation during the milling process occurs while the workpiece is clamped in a pallet. Therefore the usefulness of the frequency response function data may be enhanced if the measurements are made with the transmission case clamped in its transfer pallet.

5.3.2 Accelerance Frequency Response Functions - Transmission Case Clamped in Pallet

This series of tests was undertaken with the transmission case clamped in its transfer pallet as shown in Figure 5.16.

The procedure was as described in the previous section, the only difference being the clamping of the transmission case.

The measured FRF data is shown in Figure 5.26 and Figure 5.27 for Points #32 and #71, respectively.

The most striking change between this FRF data and that for the freely suspended transmission (Figures 5.24 and 5.25) is the significant drop in the magnitudes associated with the response functions for the clamped transmission casting. This is predominantly due to the significant increase in the effective damping of the casing resulting from the clamping process. This increase in damping was clearly demonstrated in the modal analysis data presented earlier.
It is also apparent that although the clamping process has altered the frequency response functions, as evidenced in Figure 5.28, they still indicate the frequency range of significance to be approximately 3,000 to 6,000 Hz. This is not in keeping with the significant milling noise generation range which is known to be in the interval of 200 to 3,000 Hz.

As before, a qualitative review of the FRF data's "strength" seems to correlate well with changes in overall noise level generation during the actual milling process. However, the usefulness of this data in "quiet cutter" design would be quite limited.

It may be that accelerance is not the best FRF measurement to use if it is desired to correlate structural vibration to noise generation. It may well be that Mobility or Dynamic Compliance frequency response functions may be more appropriate in defining the range of frequencies over which noise generation is important.

Figure 5.29 shows a Mobility plot for input at Point #49 and output at Point #32. Notice that the frequency range of interest, although shifted somewhat, still indicates 3,000 Hz and above to be most significant. The Dynamic Compliance plot of Figure 5.30 shows further spreading of the spectrum to lower frequencies but the high frequencies are still given too much emphasis, and the very low frequencies are far too heavily weighted. These are
typical of the results obtained for the other points.

Thus, changing to measurement of Mobility or Dynamic Compliance does not enhance the usefulness of our structural response functions as means of obtaining an insight into the characteristics of the noise generated during milling.

5.3.3 Noise Frequency Response Functions

Up to this point it has been attempted to relate the noise generation of the transmission case to its various structural frequency response functions. Figure 5.31(a) diagramatically illustrates this type of measurement. In this diagram $H_s$ represents the frequency response function for the transmission case. It is apparent from this diagram that $H_s$ is not necessarily directly related to the noise output of the transmission case when excited by the forcing function. Certainly the frequency response function $H_s$ will indicate at which frequencies the surface movement is largest, but it is the complex interaction of the structure's surface and the surrounding air which governs the resultant noise at a point in space. Figure 5.31(b) illustrates this process. Thus, if it is desired to relate noise generation to forcing input, and surface vibration measurements are to be used, then $H_A$, which is the frequency response function between a surface movement parameter, say velocity, and the noise level at a point in space, must be known.
Now, estimates of $H_A$ could be obtained by theoretical approaches. Once derived this information could then be used to determine the expected noise response of the structure. However, it must be realized that this would be a non-trivial problem with little guarantee as to the accuracy of the resulting function.

Rather than using analytical methods to derive $H_A$, it would probably be somewhat simpler to measure this function. One could apply surface movement transducers at points of interest and measure the resulting noise using a microphone at points of interest, for a given input force. Of course, the function $H_s$ must be available if the noise generation output characteristics for a given input force are to be determined. The procedure would be:

i) Determine typical values for $H_s$ and $H_A$ from test measurements.

ii) Knowing the frequency spectrum of the impact force, apply $H_s$ to obtain the surface vibration frequency spectrum.

iii) Apply $H_A$ to the surface vibration frequency spectrum obtained in (ii) to get the frequency spectrum of the noise generated at a particular point in space.

Although such a procedure is reasonably straightforward if, for the moment, the difficulty of reliably determining the frequency characteristics of the forcing
function is ignored, it would seem apparent that an even simpler approach is to determine $H_N'$. In this manner the relationship between noise output and forcing function input is obtained directly. Assuming $H_N$ can be reliably measured, then knowledge of the forcing function frequency spectrum allows the noise output spectrum to be easily obtained. This ability would be of great value in "quiet cutter" design since, if for a given tooth spacing the forcing function spectral characteristics can be determined, then the resulting noise generation characteristics can be obtained. The tooth spacing can be varied "on paper" and the resulting noise generation calculated, with this process continuing until an optimum "quiet cutter" is found.

From a practical point of view such a procedure is attractive since it requires only the workpiece and its fixture be available for testing to obtain $H_N'$. The workpiece is almost always available early in the design cycle of a transfer machine, while the fixture, or transfer pallet, can easily be given design and fabrication priority. This means that all necessary testing could take place well before the design for the milling cutters must be finalized.

It is these considerations then, that motivated the following series of measurements.

5.3.3.1 Methodology

For this series of tests the transmission casing was clamped in its transfer pallet as shown in Figure 5.16.
As with the previously described frequency response function measurements, the input points were the 11 points located on the engine mounting face (Points #44 to #54), with force direction as shown in Figure 5.22. The response to the given input force was measured by a free field microphone located 20 mm from the designated response point (Points #32 and #71). The microphone faced the response point and was oriented in such a manner that a normal from the centre of the microphone diaphragm would intersect the workpiece surface at the point of interest and also be, essentially, normal to the workpiece surface at that point.

The noise response was A-weighted by the sound level meter, as this weighting was used on the data from the actual cutting noise test series. The output from the sound level meter was fed directly into channel #2 of the structural analyzer.

The measurements were performed in a standard laboratory type room. The floor was carpeted, the ceiling was acoustical tile, and the walls plaster. Reflections were not expected to degrade the measurements significantly since the measurements are somewhat "self-gating". That is, for the frequency range used in these measurements (0 to 6,400 Hz) the time record is 40 msec. long. Thus any reflected sound, which arrives later than 40 msec. from measurement start, is ignored. This meant that reflections from walls were not a problem. The floor and ceiling were close enough
to have total transit time less than the time record and thus reflections from these objects would be present prior to the end of sample acquisition. However, when it is remembered that the floor is carpeted, the ceiling is made of acoustical tile, and that the microphone/sound level meter combination have reasonably significant directional characteristics, only the lowest frequency components of the reflected noise would be expected to be of any magnitude and these are of little concern in the measurements, particularly considering the fact that A-weighting was in use.

Care was taken to reduce the noise reaching the response microphone from other sources. The analyzer was moved into a separate room, all windows and doors to the test area were closed, tests were performed during quiet background noise periods, and the person using the impact hammer was careful to minimize the reflection effects of his body.

Since rather greater variability was expected in these tests, a total of 10 averages were used to obtain each FRF determined in this test series. This is in contrast to the 5 averages which were considered sufficient when measuring surface vibration.

The quality of each FRF was monitored using the coherence function. Figure 5.32 shows a representative coherence function measurement. Generally the coherence is
acceptable, with the majority of excursions from unity occurring at antiresonances. The relatively poor coherence at the lower frequencies would be expected, since it is difficult to attenuate noise at these frequencies. Thus some energy at these frequencies would reach the microphone from sources other than the transmission case. Also, since the measured sound level is A-weighted, the signal-to-noise ratio for low frequencies would be quite small, again contributing to the relatively poor coherence in this region.

5.3.3.2 Discussion of Results

The results of the noise frequency response function measurements are shown in Figure 5.33 for response at Point #71, and in Figure 5.34 for response at Point #32.

Again, referring to Figure 4.14 it is apparent that the qualitative "strength" of the frequency response measurements correlates well with the actual overall change in noise level as the milling cutter passes over the points of interest on the engine mounting face.

Also, now the range of frequencies of importance tends to agree with the actual cutting values observed in the machining tests described in Chapter IV.

This is encouraging since a test procedure is now available which is relatively simple to perform and which will provide a qualitative indication of overall noise level and typical frequency characteristics of noise generated
under actual machining conditions.

It must be stressed that at this point only qualitative evaluations of the measured frequency response functions are being considered. If, for example, a closer inspection is made of the cutting noise spectra relative to the response functions, certain "deficiencies" become apparent. Figure 5.35 shows the noise frequency response function generated for input at Point #45 and output at Point #32, overlaid with the auto spectrum of the cutting noise produced as the equally spaced (EQ. SP.) mill passed over Point #45 during actual machining tests. This auto-spectrum was obtained using only two averages and thus may have some "random" components present, but will suffice for demonstration purposes.

Note that at those frequencies denoted as "1" through "6" there seems to be a definite relationship between peaks in the noise auto-spectrum and the frequency response function. For these points it would seem that peaks in the FRF result in peaks in the noise auto-spectrum generated during the actual milling sequence. Also, for these frequencies the magnitude of the peaks in the FRF seem to have a direct relationship to the magnitude of the resulting peaks in the auto-spectrum. All this is quite encouraging since it would seem to indicate that FRF data generated using the test method described in this section will provide a direct indication of the noise spectrum.
produced under actual cutting conditions.

Of course, such is not really the case. If those frequencies marked "a" through "h" are considered it becomes apparent that no such direct relationship holds for these spectra. For the case of frequency "a" there exists a large noise response at a point where the FRF is very small. This is again true at "b" and "c". Yet at frequencies "d" through "h" significant peaks in the FRF show no corresponding noise in the milling noise auto-spectrum.

Some of these "anomalous" results might be considered to be a consequence of the small number of averages used to obtain the noise auto-spectrum. Thus, random spectral component variations might be at fault.

In fact, it is unlikely that such errors are responsible to any great extent for the variations observed. The simple answer to these variances is neglect of the frequency content of the forcing function actually causing the noise generation characterized by the auto-spectrum of Figure 5.35. That is, what "comes out" in the form of noise from the transmission case is directly related to "what goes in" in the form of the forcing function's spectral components, if the structural response characteristics of the structure are considered to be invariant.

It must be remembered that the excitation force used to obtain the FRF measurements was a force pulse with typical auto-spectrum as shown in Figure 5.36. It is
obvious that this type of excitation force contains, essentially, all frequencies over the range of interest (0 to 6,000 Hz). The decrease in force level with frequency is not critical, since the nature of the measurement made provides values on a per unit input force basis. Only if the input force level at a particular frequency is low enough to cause signal-to-noise ratio difficulties would this reduction of input level with increasing frequency be of concern. Unacceptable levels of signal-to-noise ratio would be flagged by poor coherence measurements. This "continuous" frequency content is important in obtaining FRF measurements since it is desired that the response function indicate the response of the test structure at all frequencies within the range of interest. Obviously therefore the structure must be excited at all such frequencies during the test.

However, when it is desired to know how the structure will subsequently respond to a particular forcing function the actual frequency content of the forcing function must be utilized to calculate this response.

As an example, consider the simple case illustrated in Figure 5.37. In this instance consider an input (forcing) function which consists of the five harmonically related components shown in Figure 5.37(a). The DC component is not considered since it is of little use in dynamic system analysis. It is assumed that the Dynamic
Compliance FRF of the system has been measured and is as shown in Figure 5.37(b). Notice that this FRF shows the response of the system to all frequencies, within measurement limitations which will be discussed later for the frequency range indicated. Then the actual output of the system, in this case the displacement, is obtained as follows:

Recall from Fourier analysis theory that each frequency component of the forcing function can be represented as,

\[ a_n \cos n\omega t + b_n \sin n\omega t = c_n \cos (n\omega t - \alpha_n) \]  

where,

\[ c_n = \sqrt{a_n^2 + b_n^2} \]  

and,

\[ \alpha_n = \tan^{-1} \left( \frac{b_n}{a_n} \right) \]  

Thus the forcing function can be represented as,

\[ F(t) = \sum_{n=0}^{\infty} c_n e^{j(n\omega t - \alpha_n)} = \sum_{n=0}^{\infty} c_n e^{jn\omega t} \]  

where, \( c_n = c_n e^{(j\alpha_n)} \) is the phasor of the harmonic component.
at the frequency $n\omega$.

Assuming, for illustration purposes, that the system in question has but a single degree-of-freedom, then its frequency response function is,

$$
\frac{X}{F} (j\omega) = \frac{1}{k - \omega^2 m + j\omega c} = \frac{X}{F} e^{-j\phi}
$$

This is the Dynamic Compliance plot shown in Figure 5.37(b).

Note that in a "real-life" situation the Dynamic Compliance function would be far more complex, and would be obtained by measurements on the structure itself.

It is obvious that the equation of motion of the system is now,

$$
m\ddot{x} + C\dot{x} + kx = \sum_{n=0}^{\infty} C_n e^{jn\omega t}
$$

Using the impedance method it can be shown that the response due to a typical component

$$
C_n e^{jn\omega t} = C_n e^{j(n\omega t - \alpha_n)}
$$

of the exciting force $F(t)$ is,

$$
X_n e^{jn\omega t} = X_n e^{j(n\omega t - \alpha_n - \phi_n)}
$$

where,
\[ X_n = C_n \left( \frac{1}{k-n \omega^2 \omega^m + jn\omega c} \right) \]

\[ = \frac{C_n}{\sqrt{k-n \omega^2 \omega^m + jn\omega c}} e^{-j(\alpha_n + \phi_n)} \]

and,

\[ \phi = \tan^{-1}\left( \frac{jn\omega c}{k-n \omega^2 \omega^m} \right) \]

Note that the phasor of the harmonic response is the product of two complex numbers, \( C_n \) and \( 1/(k-n \omega^2 \omega^m + jn\omega c) \). Thus the Fourier spectrum of the system response is the product of the Fourier spectrum of the excitation \( F(t) \) and the system frequency response function (in this instance the Dynamic Compliance). From complex number theory it is known that the magnitude of the product of two complex numbers is the product of the magnitudes of each. Also, the phase angle is the sum of the individual phase angles. Thus at frequency \( n\omega \), the magnitude of the response is \( |C_n| \cdot \left| \frac{1}{k-n \omega^2 \omega^m + jn\omega c} \right| \) which is the product of the frequency spectrum of \( F(t) \) and the magnitude of the system frequency response function. The phase angle, \( -(\alpha_n + \phi_n) \), of the response is the algebraic sum of the phase spectrum of \( F(t) \) and the phase angle of the system frequency response function. The results of this approach are shown in Figure 5.37(c). For the particular application of predicting noise response of a transmission case to the milling cutter input force, the phase angle data
is of no specific use and would normally be neglected.

Note that due to the harmonically discrete nature of the input spectrum, the output spectrum is also harmonically discrete, but the relative magnitude of the components has changed significantly.

Although this example utilized a single degree-of-freedom system for simplicity, the same general approach may be used when working with the more complex systems usually encountered in practice. In these instances the PRF would, as noted earlier, be obtained by measurement. If the frequency components of the forcing function are known, then the response spectrum can be obtained using the straightforward approach outlined here.

Therefore it is expected that the characteristics of the noise generated by the transmission case can be reliably determined if the characteristics of the forcing function can be specified precisely. This is of significance to "quiet cutter" design since such a procedure would give an excellent indication of noise generation for a given insert configuration thus permitting an optimization procedure to be developed. However, the key to such abilities is the recognition of the importance of properly describing the forcing function input to the workpiece.

It was with this objective in mind that the work to be described in Chapter VI was undertaken. This chapter will indicate the significant effect forcing function
characteristics have on system (workpiece) response and will also attempt to identify the difficulties associated with the determination of such characteristics.

5.4 SUMMARY

A modal analysis was performed on the GM 125 Transmission Case for both the freely suspended and "clamped in pallet" configurations. These analyses demonstrated a significant change in structural response resulting from the clamping procedure and therefore indicated that frequency response function measurements should be obtained with the transmission case clamped in its pallet, if such measurements are to be of any use in "quiet cutter" design.

It was demonstrated that although acceleration FRF measurements taken with the case clamped in the pallet could provide a qualitative indication of overall noise level generation during actual machining conditions, the frequency characteristics of the output noise were not well described by such FRF measurements.

Noise FRF measurements were shown to be a simple and practical means of qualitatively determining both relative overall noise level generation during machining and the associated frequency characteristics of this noise.

Finally the importance of being able to describe the characteristics of the forcing function was discussed. Namely, it is possible, when the noise frequency response function measurements described in this chapter are
available, to construct the output noise characteristics for a given workpiece if the input forcing function can be accurately determined. Such an ability would make an optimization scheme for "quiet cutter" design extremely simple and reliable.

Because of this potential for greatly improving "quiet cutter" design, Chapter VI will concentrate on two areas:

i) Demonstrating the significance of forcing function characteristics on workpiece response.

ii) Identifying the difficulties associated with the development of adequate descriptions of the actual cutting force acting on the workpiece during the face-milling process.
Chapter VI

FORCING FUNCTION CHARACTERIZATION

6.1 INTRODUCTION AND OBJECTIVES

The impetus for the work reported in this Chapter comes from the recognition that proper definition of the forcing function produced on the workpiece by a face mill is critical to the success of "quiet cutter" design strategies.

It is desired to demonstrate that the response of the system is greatly influenced not only by the point of application of the force, as has been shown in Chapter V, but that the "shape" of the forcing function is also a significant factor in determining system response.

Many of the results reported in this section will use the work of Wu and associates [7] as the basis of comparison. This approach was taken since this particular "quiet cutter" design criteria has much to recommend it; however, it will be shown that it also has certain failings which, if corrected, could lead to development of a much more powerful design methodology.

6.2 SYSTEM RESPONSE -- DUHAMEL'S INTEGRAL

Early in this study it was recognized that the force exerted by a milling cutter insert as it engaged, traversed, and then disengaged the workpiece, would be of
finite duration. Although this seems obvious, in fact the "quiet cutter" design approach of Wu assumes force impulses, that is, forces of infinitely small time duration.

Since it was desired to investigate the effect of exciting a structure with force pulses, in contrast to force impulses, and since much of this work would ultimately be performed on an analogue computer, it was considered useful to derive an analytical time domain solution for the excitation of a single degree-of-freedom system by an arbitrary pulse train. Such a tool would then provide a means of verifying the analogue computer set-up results, and also permit an analytical investigation of the importance of insert spacing on system response.

It was decided to work with a single degree-of-freedom system since this was expected to provide reasonable insight into general system response without providing a confusing complexity of information. Also, many "real" systems have single, predominant modes of vibration and the response of this type of system is well represented by a single degree-of-freedom system.

The forcing function was presumed to consist of a "train" of force pulses, each of which may be of different duration. The magnitude of each pulse could be different, however the magnitude of an individual pulse was assumed constant. In other words, the pulse train was assumed to consist of an arbitrary number of rectangular pulses. The
assumption that the force remains constant during the engagement of an insert is not unrealistic for workpieces with widths of cut very much less than the cutter diameter [12]. This is certainly true for the transmission case used in this study and is also true for many production situations.

Since all real systems have inherent damping, the single degree-of-freedom system was assumed to be damped.

Figure 6.1 shows the model of the system and the assumed general forcing function acting on the system.

For such a damped system, assumed to be initially at rest, Duhamel's Integral is [4],

\[ u(t) = \frac{1}{m\omega_d} \int_0^t P(\tau) e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) d\tau \]  \hspace{1cm} 6.1

It is desired to evaluate this expression for the general forcing function so that the response of the system at any point in time may be obtained, based on the system parameters and the actual forcing function acting on the system.

Limiting the range of interest to \( t_0 \leq t \leq t_1 \), that is, the system response anywhere within the time during which the first forcing pulse is acting, gives,

\[ u(t) = \frac{1}{m\omega_d} \int_{t_0}^t P(\tau) e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) d\tau \]  \hspace{1cm} 6.2
\[\begin{align*}
&= \frac{1}{m \omega_d} \int_{t_0}^{t} p_1 e^{-\xi \omega_n (t-\tau)} \sin \omega_d (t-\tau) d\tau \\
&= \frac{1}{m \omega_d} \left[ p_1 \int_{t_0}^{t} e^{-\xi \omega_n (t-\tau)} \sin \omega_d \tau \cos \omega_d \tau d\tau \\
&\quad - p_1 \int_{t_0}^{t} e^{-\xi \omega_n (t-\tau)} \cos \omega_d \tau \sin \omega_d \tau d\tau \right] \\
&= \frac{1}{m \omega_d} \left[ e^{-\xi \omega_n t} (\sin \omega_d t) \int_{t_0}^{t} e^{\xi \omega_n \tau} \cos \omega_d \tau d\tau \\
&\quad - e^{-\xi \omega_n t} (\cos \omega_d t) \int_{t_0}^{t} e^{\xi \omega_n \tau} \sin \omega_d \tau d\tau \right] \\
&= \frac{1}{m \omega_d} \left[ e^{-\xi \omega_n t} (\sin \omega_d t) p_1 \right]
\begin{bmatrix}
\int_{t_0}^{t} e^{\xi \omega_n \tau} (\cos \omega_d \tau + \omega_d \sin \omega_d \tau) d\tau \\
-\int_{t_0}^{t} e^{\xi \omega_n \tau} (\cos \omega_d \tau - \omega_d \cos \omega_d \tau) d\tau \\
\end{bmatrix}
\begin{bmatrix}
t\\
t_0
\end{bmatrix}
\]

But,

\[(\xi \omega_n)^2 + (\omega_d)^2 = \omega_n^2\]

\[u(t) = \frac{1}{m \omega_d} e^{-\xi \omega_n t} \left[ \sin \omega_d t \left( -p_1 \left[ e^{\xi \omega_n t_0} (\zeta \omega_n \cos \omega_d t_0 \\
+ \omega_d \sin \omega_d t_0) - e^{\xi \omega_n (\zeta \omega_n \cos \omega_d t_0 + \omega_d \sin \omega_d t)} \right) \\
- \cos \omega_d t \left( -p_1 \left[ e^{\xi \omega_n t_0} (\zeta \omega_n \sin \omega_d t_0 - \omega_d \cos \omega_d t_0) \\
- \right] \right) \right] \]
Equation 6.8 now provides an expression for the response of the system for times $t_0 < t < t_1$.

To extend the interval of interest to the time during which the second pulse is present, i.e., for $t_1 < t < t_2$ then the integral,

$$u(t) = \frac{1}{m\omega_d} \int_{t_0}^{t} p(\tau) e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) \, d\tau \quad 6.2$$

must again be evaluated.

In this instance it can be broken up into the evaluation of two integrals as,

$$u(t) = \frac{1}{m\omega_d} \left[ \int_{t_0}^{t} p_1 e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) \, d\tau + \int_{t_0}^{t} p_2 e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) \, d\tau \right] \quad 6.9$$

These integrals may now be evaluated as before with the resultant expression,

$$u(t) = \frac{1}{m\omega_d} \frac{1}{\omega_n^2} e^{-\zeta \omega_n t} \left[ \sin \omega_d t \left( -p_2 e^{-\zeta \omega_n t_1} (\zeta \omega_n \cos \omega_d t_1 + \omega_d \sin \omega_d t_1) \right) + \omega_d \sin \omega_d t_1 \right] e^{-\zeta \omega_n t} (\zeta \omega_n \cos \omega_d t + \omega_d \sin \omega_d t)$$
\[ P_1 \left[ e^{\zeta \omega_n t_0} (\zeta \omega_n \cos \omega_d t_0 + \omega_d \sin \omega_d t_0) \right. \\
- e^{\zeta \omega_n t_1} (\zeta \omega_n \cos \omega_d t_1 + \omega_d \sin \omega_d t_1) \left. \right] \]

\[ \cos \omega_d t \left[ -P_2 \left[ e^{\zeta \omega_n t_1} (\zeta \omega_n \sin \omega_d t_1 - \omega_d \cos \omega_d t_1) \right. \\
- e^{\zeta \omega_n t} (\zeta \omega_n \sin \omega_d t - \omega_d \cos \omega_d t) \left. \right] \right] \\
- P_1 \left[ e^{\zeta \omega_n t_0} (\zeta \omega_n \sin \omega_d t_0 - \omega_d \cos \omega_d t_0) \right. \\
- e^{\zeta \omega_n t_1} (\zeta \omega_n \sin \omega_d t_1 - \omega_d \cos \omega_d t_1) \left. \right] \]

6.10

If this process of deriving the expressions governing the response of the system at times within subsequent force pulse durations is continued, it soon becomes apparent that a general expression can be obtained.

For the response time of interest within the force pulse interval "I", where I = 1, 2, 3, ..., q, it can be shown that,

\[ u(t) = \frac{1}{m \omega_d^2} e^{-\zeta \omega_n t} \sin \omega_d t \left[ -P_q \left[ e^{\zeta \omega_n t_{q-1}} (\zeta \omega_n \cos \omega_d t_{q-1} + \omega_d \sin \omega_d t_{q-1}) \right. \\
+ \omega_d \sin \omega_d t_{q-1} - e^{\zeta \omega_n t} (\zeta \omega_n \cos \omega_d t + \omega_d \sin \omega_d t) \left. \right] \\
- \Sigma_{i=1}^{q-1} P_i \left[ e^{\zeta \omega_n t_{i-1}} (\zeta \omega_n \cos \omega_d t_{i-1} + \omega_d \sin \omega_d t_{i-1}) \right. \right] \]
\[ e^{\frac{\omega_n t}{T}} \{(\omega_n \cos \omega_d t + \omega_n \sin \omega_d t)\} \]
\[ \cos \omega_d t \left( -p_q e^{\frac{\omega_n t}{T}(\omega_n \sin \omega_d t - \omega_d \cos \omega_d t)} \right. \]
\[ - \omega_d \cos \omega_d t_{q-1} \left. - e^{\frac{\omega_n t}{T}(\omega_n \sin \omega_d t - \omega_d \cos \omega_d t)} \right) \]
\[ \sum_{i=1}^{q-1} p_i \left[ e^{\frac{\omega_n t}{T-1}(\omega_n \sin \omega_d t_{i-1} - \omega_d \cos \omega_d t_{i-1})} \right. \]
\[ - e^{\frac{\omega_n t}{T-1}(\omega_n \sin \omega_d t_i - \omega_d \cos \omega_d t_i)} \right) \]

where,

the \( \Sigma \) terms are taken as zero for \( q = 1 \)

Equation 6.11 now permits the determination of the response of a single degree-of-freedom system to any arbitrary force pulse train, no matter how complex, as long as it may be characterized as being made up of rectangular pulses. The only requirement is that, for each instant in time for which the response is desired, it must be known which force pulse (i.e., pulse 1, 2, 3 ... \( q \)) is acting on the system. This is not particularly difficult to accomplish.

Equation 6.11 is easy to code for solution on a digital computer, and the simple algebraic nature of the expression results in rapid and accurate determination of system response.
6.2.1 Validation of System Response Solution

Having obtained Equation 6.11 it was desired to confirm the validity of this expression. Conceptually, it would be possible to achieve such verification by using an analog computer solution for comparison purposes. However it was ultimately decided to verify analogue computer solutions using Equation 6.11. It was therefore necessary to develop a check on this expression which did not resort to analogue computer techniques.

To achieve this, it was decided to use an existing simulation package, "System/360 Continuous System Modelling Program". The CSMP package ran on an IBM 3033 mainframe digital computer. It is written primarily in Fortran and requires approximately 100K bytes of core storage, with all calculations performed in single-precision, floating-point arithmetic.

Since CSMP employs numerical integration techniques to solve the differential equations of motion, a choice of integration method must be made from among the eight available in the program. This decision would normally be made on the basis of accuracy required versus allowable run time. Generally, as the more "accurate" methods are chosen, a significant increase in run time will be incurred for a given problem. In the present study, the fourth-order Runge-Kutta method with variable integration interval was used. In this technique Simpson's Rule is used
for error estimation. Although this method is rather "overhead intensive", the expected accuracy is quite high. Since it was desired to verify the accuracy of Equation 6.11, this seemed a reasonable choice, particularly since a large number of simulations would not be required. A detailed description of the capabilities of CSMP is contained in reference [42].

To test the validity of Equation 6.11 a number of simulations were run for damped single degree-of-freedom systems. These systems were excited by various force pulse trains which met the restrictions noted in the development of Equation 6.11. The results obtained from these CSMP runs were in excellent agreement with those obtained from the Duhamel's Integral approach (Equation 6.11).

As an example, a single degree-of-freedom system with \( \omega_n = 100 \text{ Hz}, \zeta = .1, \) and \( m = 1 \text{ kg}, \) was excited by the forcing function shown in Figure 6.2. The results of a comparison of these two solution methodologies is shown in Table 6.1. This table summarizes the system displacement determined using each method and the relative percentage error of the Duhamel Integral solution relative to the CSMP results. The CSMP program is shown in Appendix I. The percentage errors are extremely small. Such results are typical of those obtained from a variety of forcing functions. It was therefore concluded that Equation 6.11 is a reliable and accurate means of determining single
degree-of-freedom system response to a "rectangular" pulse train forcing function.

At this point it may well be asked why it was necessary to derive and utilize a solution methodology based on Duhamel's Integral if tools such as CSMP are already available to perform the desired analyses. Basically, the answer lies in the fact that the CSMP routine is implemented on a mainframe computer with all the implications of cost and long turnaround times associated with such installations. The Duhamel Integral routine can easily be run on existing microcomputers with no significant penalty in accuracy. This results in a combination of minimal cost and excellent turn-around time. In addition, even if, for some reason, a mainframe computer must be used, the Duhamel Integral routine is typically ten times faster than the equivalent CSMP solution, again resulting in economic benefits.

With the Duhamel Integral procedure available, not only can it be used to verify analogue computer solutions, but it can also be used to study the effect of force pulse spacing on system response. Although in this study only a minimal amount of work was done on the latter, due to a considered decision to investigate such phenomena using a hybrid digital-analogue computer system, the results obtained from some preliminary investigations will now be reviewed.
6.2.2 Investigations of the Effect of Force Pulse Spacing on System Response Using the Duhamel Integral Method

One possible area of interest is to investigate the effect of changing pulse spacing while holding pulse width constant. It will be recognized that such a situation is analogous to varying the insert spacing on a face mill, while cutting a part of "simple" geometry. In this instance "simple" means that the part geometry is such that the included angle, \( \psi \), shown in Figure 6.3 remains constant as the mill passes over the workpiece.

Results from such a study, for a fixed pulse width of 0.001 seconds, are shown in Figure 6.4. The response is shown in terms of the dimensionless parameter "R", which is simply the ratio of the system displacement at a given time to that of the static displacement. Such a parameter can be obtained from Equation 6.11 simply by changing the coefficient \( \frac{1}{\mu \omega_d^2 \omega_n^2} \) to \( \frac{1}{\omega_d} \), as long as each pulse is assumed to have the same magnitude. The results shown are for two pulses only, with the first pulse always beginning at time, \( t = 0 \). The "gap" between the pulses is changed by varying the interval "I_2".

The curves of Figure 6.4 obviously indicate that for spacing \( I_2/T_n = 0.40 \), the system response is far less than for, say, spacing \( I_2/T_n = 0.10 \). Of course, making a judgement between \( I_2/T_n = 0.30 \) and \( I_2/T_n = 0.40 \) becomes rather more difficult.
For this reason a program was developed to permit determination of the area under the absolute value of the response curve. This permitted a quantitative ranking of system response to a given pulse train. The listing for such a program is contained in Appendix II. The absolute value was chosen since the overall response of the system is best characterized (at least for the situation where workpiece vibration, and resulting noise generation, are of concern) by the magnitude of excursions in both the positive and negative directions from rest conditions. Calculation of areas provides a convenient means of comparing responses over a given interval of time. The area was calculated using Simpson's One-Third Rule as indicated in the program listing.

Figure 6.5 shows the results of using such an evaluation approach for a system excited by 2 pulses, each of equal magnitude and with constant width of 0.003 seconds. The slightly unusual units for the area (i.e. "seconds") results from the dimensionless nature of the response variable, R. In this instance the area under the absolute value of the response curve was determined from \( t = 0 \) to \( t = 0.040 \) seconds, for each gap width.

A decrease in the "response area parameter" (RAP) is taken to indicate a decrease in system response. Figure 6.5 indicates the oscillatory nature of this response area parameter, with a global maximum and minimum, and also
additional local maxima and minima. It is interesting to note that the RAP curve is a damped sinusoid with damped natural frequency equal to that of the system (i.e. $\omega_d \approx \omega_n = 100$ Hz). The "optimum" gap width is obviously 0.002 seconds and the "worst" is 0.007 seconds. If for some reason the optimum gap width could not be used, then the next best choice would be a somewhat smaller or larger gap. If for practical design considerations the gap must be greater than say, 0.006 seconds, then the best gap choice would be .012 seconds (which is the "optimum" gap spacing of .002 seconds, plus the system period of .010 seconds). This ability to choose from a "selection" of gap spacings has practical benefits since, in many cases, physical limitations often prevent specific gap spacings from being employed.

As a point of interest, Figures 6.6 and 6.7 show the actual system response for the "optimum" and "worst" gap spacings, respectively. The significant difference in system response is readily apparent.

Figure 6.8 shows the RAP as a function of gap width for a variety of pulse widths. Again, the damped sinusoidal nature of the curves is quite evident. In addition, this figure shows the marked dependence of "optimum" gap width on pulse width. Recognizing that pulse width will, for the case of a milling cutter, be a function of the width of cut associated with a given workpiece, then
the importance of workpiece geometry in determining optimum insert spacing is readily apparent. This important point will again be illustrated during subsequent analogue computer simulations.

Although only relatively simple applications of the Duhamel Integral Method to the study of insert spacing optimization have been illustrated, the potential for more involved analyses exists. Having developed the basic mechanism for such studies, it was decided to forego further work at this time in favour of simulations on the analogue computer.

However, before reviewing the analog computer results, one more illustration of the use of the Duhamel Integral Method is in order.

6.2.3 Ranking of Milling Cutter Performance

In this instance it was decided to try and rank the expected performance of three of the milling cutters employed in the cutting tests reported in Chapter IV. The cutters of interest were; "EQ. SP:n", "1° STAG" and "1/2° STAG" (as designated in Chapter IV). The response area parameter was to be used as the criterion for ranking. The smaller the RAP value the better the cutter would be expected to perform. Once this ranking had been obtained, it could be compared with the results of the noise tests already performed.

For the simulation, the workpiece had to be
characterized by a damped single degree-of-freedom system. For this purpose the natural frequency was taken as 320 Hz with $\zeta = 0.05$. These values were based on available structural response data and the frequency content of the noise generated during the cutting tests.

The difficulties associated with the "complex" nature of the engine mounting face geometry were avoided by assuming an "equivalent" face of "simple" geometry with the effective width of cut constant at .64 cm.

Having set the effective width of cut, it was a simple matter to calculate the force pulse widths and the gaps between pulses.

For an effective workpiece width, $\Delta$, the length of time a particular insert is in contact with the workpiece (the pulse width) is,

$$W = \frac{\Delta}{\Omega r} \text{ (seconds)}$$  \hspace{1cm} 6.12

assuming the cutter radius is large in comparison with $\Delta$.

Both $\Delta$ and $r$ in consistent units, and $\Omega$ in units of radians per second.

The time from the start of one pulse to the start of the next is,

$$L = \frac{\phi}{\Omega} \text{ (seconds)}$$  \hspace{1cm} 6.13
where $\phi$ is the angle between the adjacent inserts (in radians) producing the pulses. Thus the gap width between any two adjacent pulses is,

$$
G = \left( \frac{\phi}{\pi} - \frac{A_t}{\pi} \right)
$$

$$
= \frac{1}{\pi} \left( \phi - \frac{A_t}{\pi} \right) \text{ (seconds)}
$$

With equation 6.14 available, it was then possible to determine the time limits of the force pulses associated with each cutter configuration, for the cutter rotational speed of 967 rpm.

Each insert on the cutter was assumed to engage the workpiece only once. That is, only one rotation of the cutter was considered. The response area parameter was determined for the time interval from zero (start of cut) to 0.1 seconds. A listing of the program used, with its output data, is contained in Appendix III.

The program generated the following RAP values:

- EQ.SP. = 0.20 (seconds)
- $1^\circ$ STAG = 0.28 (seconds)
- $1/2^\circ$ STAG = 0.31 (seconds)

Remembering that the lower the RAP value, the lower the system response is judged to be, then it is apparent that this analysis indicates that the cutter with equally spaced inserts should be the "quietest" of the three. This
is precisely the result obtained during the actual field tests (see Table 4.2). The other two cutters are shown to provide approximately the same level of excitation to the system and thus would be expected to produce similar noise levels. Again this hypothesis is confirmed by reference to Table 4.2. Although the 1° STAG cutter has a RAP value less than that of the 1/2° STAG cutter, this difference is certainly not large enough to justify any conclusions concerning expected relative performance, particularly when the very approximate nature of the analysis method is considered.

This example, although employing a simple analysis approach, does show the potential value of utilizing the Duhamel Integral Method in the design of "quiet cutters". It will be left to future researchers to incorporate the many enhancements necessary to produce a practical and reliable design tool.

6.3 **HYBRID DIGITAL-ANALOGUE COMPUTING SYSTEM**

Since one of the principle concerns of this Chapter is to demonstrate the significant effect variations in forcing function have on the vibration (and hence noise) response of a given system, it was necessary to determine how such variations could best be demonstrated.

It was decided that the combined use of an analogue computer and a digital computer, in complimentary roles, would provide the desired balance between solution
accuracy and ease of implementation. The analog computer would be used to quickly and reasonably accurately solve the system models' differential equations of motion, while the digital computer would be used to generate the many complex forcing functions necessary in the simulations. Thus each computer is assigned a task for which it is particularly well suited.

Of course, the use of "hybrid" computing systems is not new, with systems having been developed for use in various fields of study as a result of some, or all, of the following motivations [2]:

i) The desire to combine the speed of an analog computer with the accuracy of a digital computer.

ii) The need to incorporate actual system hardware into a digital simulation.

iii) The increased flexibility which can be obtained when using digital memory and control in an analog simulation.

iv) The need to increase the speed of digital computation by utilizing analog subroutines.

v) The ability to process incoming data which is partially discrete and partially continuous.

Naturally, with numerous different motivations for employing hybrid computers a broad range of systems have developed as shown in Figure 6.9.
The term "true hybrid" is usually applied to those combined computing systems containing analog as well as digital hardware in appreciable amounts. The ways in which analog and digital computers can be used in conjunction may be classified into two broad categories:

i) Unilateral operation in which information flows across the interface between the analog and the digital sections in only one direction.

ii) Bilateral operation in which information flows across the digital-analog interface in both directions.

Obviously both methods require conversion equipment at the interface; however, there are interesting differences in how such interfaces must be structured.

Figure 6.10 shows examples of hybrid systems. Figure 6.10(a) and (b) illustrate two types of unilateral systems. Note that only a single type of converter, either analog to digital or digital to analog, is required at each interface. In such systems the digital or analog computer can be regarded as playing the part of a complex and elegant input or output device.

An example of a bilateral hybrid system is shown in Figure 6.10(c). Such systems are characterized by a closed loop formed by the digital computer, the digital-analog conversion devices, the analog computer, and the analog-digital converters. In addition to the major
units illustrated in Figure 6.10(c), bilateral systems also include a number of other important devices. These would include multiplexers and demultiplexers, conversion hold devices, buffers, and finally, timing and control circuitry. Each of these units and sub-units manifests input-output relationships which deviate from the ideal or specified behaviour. Obviously when employing such units it is important to be aware of the effect such non-ideal behaviour has on the overall system dynamics. However, a general examination of such errors is beyond the scope of this discussion, particularly since the hybrid system employed in this study is "unilateral" in concept and does not contain many of the devices listed. Consequently, sources of error and expected magnitudes will be identified and discussed at appropriate points in the following review.

6.3.1 System Development

A schematic of the hybrid computing system developed for this study is shown in Figure 6.11. Note that it is essentially a "true hybrid" system (employing appreciable amounts of digital and analog hardware, albeit on a small scale) in a "unilateral" configuration. In this case the digital computer is used as an input device to the analog computer.

The basic concept behind the system design was to use the digital computer (Apple II+), in conjunction with the D/A converter, to generate the many complex forcing
functions needed for the system simulations. The ease of programming, together with the ability to store created waveforms, makes the digital computer the ideal tool for such tasks. The forcing functions so generated could then be fed into the analog computer (EAI TR20) where the system response could be obtained in real time, or in "scaled" time, if more convenient. This response, in the form of a calibrated voltage, could be observed directly on an oscilloscope or taped for later analysis. The oscilloscope used in the set-up was a dual trace, digital storage scope thus permitting storage and, if desired, plotting of both forcing function and system response. System parameters could be easily varied by simply adjusting precision potentiometers on the analog computer.

6.3.1. Digital Computer

Since the task of the digital computer was to generate the various forcing functions required for the simulations, it was necessary to develop a programme which would drive the D/A converter in the desired manner. To understand the approach taken in developing such a programme, it is first necessary to review certain operational characteristics of the D/A converter.

The D/A converter utilized was an 8-bit device which converted values from 0 to 255 into analog voltages between -5 volts and +5 volts. The number "0" produced a nominal value of -5.00 volts while "255" produced 5.00
volts. A one digit change in the number being converted changed the output voltage by 39 mV. The number being converted had to be put into a particular location in the computer's memory space (to be called, "port", subsequently). Once the loading of the number was complete, the D/A converter would convert the value to an analog output voltage within 16 microseconds. The voltage would be maintained until a new number was placed into the D/A converter "port". Following the necessary 16 microsecond scan time, the new analog voltage would be output.

Thus the desired rectangular pulse trains can be generated by simply putting the correct number (the pulse's magnitude) into the D/A port at the correct time (the pulse's spacing). Although this sounds reasonably simple, it does, in fact, present some problems. In particular, being sure that the number to be converted is placed into the D/A port quickly enough to prevent significant distortion of the pulse widths (time intervals) presents some difficulties. If a high level programming language, such as BASIC, is used to transfer the number to be converted into the D/A port, it is found that such transfers are extremely slow and result in unacceptable errors in the pulse train timing.

To avoid such problems, a machine language programme was developed which transferred the necessary number to the D/A port at precisely, within very close
tolerances, the right time. The program is shown in Appendix IV.

Generally, the programme assumes that somewhere in the computer's memory there is a "forcing function table" which contains the necessary information to construct a forcing function "fragment". This fragment represents the basic period of the forcing function. See Figure 6.12. The forcing function table consists of three "blocks" of data as shown in Figure 6.13. Although the blocks are shown as being contiguous in this figure, they do not, in fact, have to be stored in such a manner in memory. However, each block must start on a "page boundary" [43]. One block contains the numbers to be loaded into the D/A port thus determining the pulse amplitudes. The other two blocks contain the timing information necessary to determine exactly when a value in the D/A port should be changed, that is, they determine the pulse widths.

As indicated in Figure 6.13 each block of data may be made up of "n" pages of data (where n is an integer, obviously restricted to a value which does not exceed available memory space). Since each page of data contains 256 values, then a total of 256 unique rectangular pulses could be generated from a full page of data. For the present studies this was more than sufficient (in fact, only a small fraction of this number was used). Therefore, the program was always run with a page size of 1, with only part
of this space used, as determined by the parameter "PULSE" in line 23 of the programme listing (see Appendix IV).

The multiple page capability will be useful in future studies where the forcing pulse train produced by the milling cutter is not restricted to be periodic (equal to the period of the milling cutter rotation). In these instances the force pulse train would vary during the complete cutting cycle of the mill. For instance, where an 18-insert milling cutter has a process cutting time of 9.4 seconds and a spindle speed of 960 rpm, a total of 2,727 unique force pulses would be generated. A total of 11 pages would then be required for each of the three data blocks in the "forcing function table".

The advantage of such a system lies in the fact that the necessary "forcing function table" can be constructed and saved in memory using a BASIC programme prior to actual forcing function generation. The actual output from the D/A converter is then controlled by the machine language program which is very fast, and results in the generation of a very accurately timed forcing function. Tests using the "Frowler" digital oscilloscope showed the timing interval errors to be on the order of 0.07%.

An example of a simple BASIC programme used to construct a "forcing function table", together with the resulting forcing function, is shown in Figure 6.14. Lines 10 to 120, inclusive, construct the "table" while lines 150
to 205 pass necessary parameters to the machine language programme. Note that such BASIC programmes could be made as complex as desired (perhaps doing all necessary calculations to construct the "table" and generate the necessary parameters, with the user required only to input basic data such as number of inserts on the mill, width of cut, etc.) without degrading the performance of the machine language programme used to generate the forcing function.

6.3.1.2 Analog Computer

The analog computer was to be used to simulate the response of a damped, single degree-of-freedom mechanical system to various forcing function inputs.

The general equation of motion for such system is,

\[ \ddot{x} = \frac{-2p_s}{s} \dot{x} - \frac{p_s^2}{s^2} x + \frac{F(t)}{ms^2} \quad 6.15 \]

where "S" represents the analog computer's time scaling factor. For \( S > 1 \) the computer (or "machine") time is slow by the factor \( 1/S \), while for \( S < 1 \) the machine time is fast by the factor \( 1/S \). Thus, if \( S = 5 \) the machine time will be 1/5 that of real time, whereas for \( S = .5 \) the machine time will be 2 times that of real time.

The analog computer diagram for the solution of Equation 6.15 is shown in Figure 6.15. Note that no "amplitude scaling" is indicated in this figure or in Equation 6.15, although amplitude scaling was, in fact,
employed during the actual simulation runs. However, since the actual values of the amplitude scaling changed with the various forcing functions in use, no particular value is indicated.

For the simulations run in this study the value used for S was always 100, that is, the solutions were run at one hundredth of real time. For purposes of discussion, scaled time values are always referred to as "machine" time. In those instances where no particular time designation is used, it will be understood that "real" time is being considered.

6.3.2 Verification of Hybrid System

Having completed the set-up of the digital and analog elements of the hybrid system, it remained to demonstrate that such a system could produce results of acceptable accuracy. To verify the accuracy of the system it was used to generate the response of particular damped, single degree-of-freedom systems to various force pulse trains. These results were then compared with the "exact" solution obtained by using the Duhamel Integral Method described earlier. The agreement was found to be quite good, except for low response values where relative errors were found to be significant. This is not unexpected for an analogue computer solution as will be discussed later.

The results from such a verification test are shown in Figure 6.16. Note that the analog computer
results generally provide a good indication of the shape of the response curve. Also, at high response values the relative errors for the analog data are quite acceptable (around 10% or less). At low response values, say for a time of 0.016 seconds, the relative error becomes quite large. Basically this would be expected due to non-linearities in the analog computer amplifiers, the very low signal-to-noise ratio at these instances, the discrete sampling of the oscilloscope used to store the waveform, and errors in setting the time and amplitude scales on the x-y plotter used to provide a hard copy of the response function. All of these factors become particularly important at points of low system response (low output voltages from the analog computer). In addition, it was estimated that the value of the system response could only be read with a certainty of 1/5 of a division from the graphical copy. For low output levels, say for an R value of about 0.1, this translates into a possible error in reading the graphical value of about ±10%. This factor is thus a major contribution to errors found in the low response values of the system as determined from the hybrid system.

In spite of these relatively large errors at low response levels the hybrid system was still considered to be acceptable for the purposes of this study, since the focus of interest would be in comparing the shapes of the response.
curves and determining the highest response levels for a given type of forcing function. Information of this type is reliably provided by the hybrid system.

6.4 DERIVATION OF FORCING FUNCTION MODEL

Before the hybrid system could be used to demonstrate the relative importance of various parameters (particularly forcing function shape definition) on system response, it was necessary to derive a forcing function model which would permit generation of realistic milling cutter force characteristics.

6.4.1 Variable Level Cutting Force Model

To begin, it was assumed that, at the instant of engagement with the workpiece, the force produced by an individual milling cutter insert would rise instantly to a given force level, remain at that level during the interval of engagement, and then drop immediately back to zero at the moment of disengagement. If at any time more than one insert was engaged in the workpiece, then the resultant force would simply be taken as the sum of the individual forces produced by each insert.

The force produced by an individual insert, during workpiece engagement, was assumed to have the following form [17],

\[ P_i (t) = K f_i^a N^b d^c \]  

6.16
where, $P_i(t)$ is the force produced by the $i$th tooth

$K$ is a constant of proportionality

$f_i$ is the feed for the $i$th tooth (mm)

$N$ is the cutter speed (rpm)

$d$ is the depth of cut (mm)

$a, b, c$ are experimentally determined exponents.

Generally, the exponent "c" is close to 1, i.e., the cutting force is essentially directly proportional to the depth of cut. For the present model development the depth of cut parameter will be lumped into the constant of proportionality since a constant depth of cut will be assumed in all subsequent examples.

The exponent "b" is usually quite small and thus the influence of cutter speed on cutting force is minimal. Again, whatever the residual effect cutter speed may have on cutting force will be lumped into the constant of proportionality. Since only those situations in which cutter speed remains constant will be considered, this is not an unreasonable approach.

At this point, then, the cutting force model takes on the form,

$$P_i(t) = K' f_i^a$$

6.17

where, $K'$ is a constant of proportionality.

The "a" exponent has generally been found to have
values of between 0.6 and 0.8, depending on factors such as workpiece material, tool geometry, etc. For the purposes of this work a value of 0.7 was assumed.

For a milling cutter with equal tooth spacing the feed per tooth is,

$$f_0 = \frac{f_m}{N \times B} \quad 6.18$$

where $f_0$ is the feed per tooth (mm)

$f_m$ is the machine feed rate (mm/min)

$N$ is the milling cutter speed (rpm)

$B$ is the number of teeth

For a cutter with unequal insert spacing the feed for the $i$th tooth is,

$$f_i = \left[ \frac{\theta_i - \theta_{i-1}}{360/B} \right] f_0 \quad 6.19$$

where, $\theta_i$ is the angular location of tooth $i$ on the cutter (degrees)

$\theta_{i-1}$ is the angular location of tooth $i-1$ on the cutter (degrees)

The angular location of each blade on the cutter would be obtained as shown in Figure 6.17. The tooth arbitrarily designated $\theta_0$ would be given the location value 0 degrees.

If Equations 6.17 and 6.19 are combined, the
cutting force acting on the workpiece due to the ith tooth may be written as,

\[
P_i(t) = \begin{cases} 
K' \left( \frac{\theta_i - \theta_{i-1}}{360/B} \right)^{7/2} f_0, & \text{for } (2\pi m + \theta_i) A \leq t < (2\pi m + \theta_i + \psi) A \\
0, & \text{for all other time} \quad i = 0,1,2,\ldots,B-1, m = 0,1,2,\ldots,B-1
\end{cases}
\]

where,

\[
A = \frac{60}{2\pi N}
\]

\[
\psi = \text{workpiece subtended angle (degrees)}
\]

The total force acting on the workpiece at any time is then given by,

\[
P(t) = \sum_{i=0}^{B-1} P_i(t)
\]

The workpiece subtended angle, \(\psi\), noted in Equation 6.20 depends on the cutter diameter (effective cutting diameter) and the workpiece geometry.

Consider the situation shown in Figure 6.18. In this instance the workpiece is parallel to, and centred on, the milling cutter path. The workpiece width is also constant. This would be considered to constitute milling of a workpiece with "simple" geometry since \(\psi\) obviously remains constant. For this case it can easily be determined that,

\[
\sin \left( \frac{\psi}{2} \right) = \frac{W}{d}
\]

or,

\[
\psi = 2 \sin^{-1} \left( \frac{W}{d} \right)
\]
For the instance where the workpiece of constant width is offset from, yet still parallel to, the milling cutter path (see Figure 6.19) it can be shown that,

$$\psi = \sin^{-1}\left[\frac{2M+W}{d}\right] - \sin^{-1}\left[\frac{2M-W}{d}\right]$$  \hspace{1cm} 6.24

Note that Equations 6.23 and 6.24 show that $\psi$ may be easily obtained from workpiece and milling cutter geometry. In both cases illustrated the values of $\psi$ is constant.

In the simulations that follow the milling cutter - workpiece geometry was assumed to be as shown in Figure 6.18. This was done as a result of the relative simplicity of Equation 6.23. However, as shall be seen, limiting simulations to such geometry still allows a significant insight to be gained into the effect of more complex geometries on forcing function shape definition.

Before describing the procedure employed in obtaining the simulation results, a rearrangement of Equation 6.20 will be performed for the sake of convenience. Recalling Equation 6.20,

$$P_i(t) = k'\left(\frac{1 - \Phi - 0.1}{360/B}\right)^0.7$$  \hspace{1cm} 6.20

It is understood that the time dependence is as shown in the original expression and will not, for simplicity of
expression, be carried through this development. Thus,

\[ P_i(t) = \left[ \frac{\theta_i - \theta_{i-1}}{360/B} \right] \times K' f_0 \cdot 7 \]  

6.25

Note that the term \( K' f_0 \cdot 7 \) is, for this cutting force model, the force that would be generated by a given insert on a milling cutter with evenly spaced teeth. Therefore letting,

\[ F = K' f_0 \cdot 7 \]  

6.26

and substituting Equation 6.26 into 6.25 gives,

\[ P_i(t) = \left[ \frac{\theta_i - \theta_{i-1}}{360/B} \right] \cdot F \]  

6.27

which can be rewritten,

\[ \frac{P_i(t)}{F} = \left[ \frac{\theta_i - \theta_{i-1}}{360/B} \right] \cdot 7 \]  

6.28

Denoting \( \frac{P_i(t)}{F} \) as \( f_i(t) \) gives,

\[ f_i(t) = \begin{cases} 
\left[ \frac{\theta_i - \theta_{i-1}}{360/B} \right] \cdot 7, & \text{for } (2\pi m + \theta_i) \leq t < (2\pi m + \theta_i + \psi) \Lambda \\
0, & \text{for all other time}
\end{cases} \]  

6.29

Thus an expression for the "relative cutting force" for a given insert has been derived. This is a dimensionless
value and it indicates the ratio of the actual insert cutting force to the cutting force that would be generated by an insert on an equally spaced milling cutter containing the same number of inserts.

Equation 6.29 is particularly useful for the purposes of simulation since it removes the necessity of explicitly evaluating $K'$. This equation simply defines the actual cutting force acting on the system scaled by a constant factor (for a given cutter). This scaling does not in any way limit the model's ability to make valid comparisons of system response to various parameter changes (such as natural frequency, damping, width of cut, etc.).

The total "relative cutting force" would be given by,

$$ f(t) = \sum_{i=0}^{B-1} f_i(t) $$

6.5 SIMULATION PROCEDURE

To perform a particular simulation run, two major tasks first had to be accomplished:

i) set the required system parameters on the analog computer and,

ii) load the digital computer with the required "forcing function table" to produce the necessary forcing function.

The first task required knowledge of the desired
system parameters (natural frequency and damping ratio) together with the time scaling (always $S = 100$) and an initial estimate of the required amplitude scaling. With this information defined it was then a simple matter of adjusting various potentiometers to the required values.

Before parameters were set on the analog computer, a warm-up period of approximately fifteen minutes was observed. Once the potentiometers had been set, they were checked at regular intervals throughout the test sequences to ensure no drift in values.

The second task was accomplished by applying the force model to the assumed part geometry and insert spacing to obtain the relative cutting force as a function of time. This information could then be entered into the "forcing function table" of the digital computer.

With the hybrid system now ready (and all peripheral equipment such as oscilloscope, plotter and tape recorder calibrated and operating) the analog computer was set to its "operate" condition. However, since no forcing function was yet being generated by the digital computer, no system response was obtained. At this point the tape recorder was started, if it was necessary to store the system response, and all equipment given a final check for proper scale settings, etc. The digital computer was then commanded to begin generation of the forcing function. Both the forcing function input and the system output could be
observed on the digital oscilloscope while at the same time these signals were, if desired, tape recorded. During each simulation run the output from all active amplifiers on the analog computer were checked for overloads or underloads. If either occurred, the amplitude scaling was adjusted and the simulation was re-run.

Generally each simulation was run for approximately two minutes after steady-state response was achieved. This length of time was determined by sample requirements of the FFT analyzer used to obtain frequency spectrum characteristics of the input and output signals. With a frequency bandwidth of zero to 3.125 Hz, a total of fifteen averages could be used in generating the displayed spectrum. This averaging greatly reduced any random noise components. Although a greater number of averages could be obtained simply by increasing the length of the time sample, the desire to minimize any thermal drift effects in the analog computer and the minimal improvement in spectral data which would be obtained, dictated against a longer time sample.

6.6 DISCUSSION OF SIMULATION RESULTS

This section will review the results obtained from the various simulations performed. For the sake of clarity, each major area of interest will be discussed separately.

6.6.1 Comparison of Forcing Functions

It is very instructive if, at this point, the
forcing function definition obtained from the model by Doolan et al. [7], is compared with that obtained from the force pulse model, for a given cutter-workpiece geometry.

Consider a milling cutter of 10.2 cm diameter, rotating at 540 rpm, and containing 6 blades. The model proposed in [7] assumes force impulses of constant magnitude, unaffected by blade spacing. The magnitude being that obtained from an equally spaced insert configuration. The impulsive force occurs at the point of engagement of each milling cutter insert with the workpiece. The relative cutter-workpiece geometry is as shown in Figure 6.18. For these conditions Doolan determined an "optimum" insert spacing as shown in Figure 6.20.

The forcing function can now be defined using the force pulse model, for the insert spacing shown in Figure 6.20 if a workpiece width of, say, 5.1 cm is assumed. Note that the workpiece width is not a consideration in the impulsive force model of [7].

The resulting forcing function definition for each of these models is now compared in Figure 6.21. Although the impulsive forces are shown with finite duration for clarity on the diagram, they are, in fact, of infinitely small duration.

It is obvious from Figure 6.21 that there is a significant difference in the forcing function shape definition between these two models. What would, perhaps,
be an "optimum" insert spacing according to the force impulse model, would very likely not be an "optimum" when considering the forcing function predicted by the force pulse model. Since it is felt that the force pulse model is the more realistic description of cutting force, therefore methods using an impulse force model would be expected to produce poor predictions of actual system response. The field test results reported in Chapter IV tend to support such a conclusion.

If it is desired to see the effect of these two very different forcing function shape definitions on system response, the system characteristics must first be defined. For purposes of illustration, the system was assumed to have a natural frequency, \( p \), of 56.5 Hz and a damping ratio, \( \zeta \), of 0.2. Also, the required impulses were simulated by very short duration pulses. In this case, the impulse was modelled using a pulse of duration 0.00031 seconds real time. This was considered to be a reasonable approximation of an impulse, particularly considering the system's natural frequency.

Figure 6.22 shows the resultant system response for the two different force excitations. The difference in response is quite apparent, with the relative magnitudes of the responses being very significant. Note that, for convenience, the displacement is expressed in arbitrary units since the main interest is in comparing responses.
Results obtained using other system characteristics show the same significant difference in response for the two forcing models, with the response of the system to the impulse force model being typically an order of magnitude lower than for the force pulse model. Also the "shape" of the displacement responses are significantly different, thus indicating a corresponding difference in the spectral content of the response functions.

As noted earlier, considering that the force pulse model is felt to be the more reasonable approximation to the actual cutting force, then it can be appreciated that significant errors in predicting actual system response would be expected when the forcing function is assumed to be impulsive.

For the remainder of this chapter only the force pulse model will be considered when illustrating the importance of various parameters on system response.

6.6.2 Effect of Natural Frequency and Damping on System Response

The effect of changes in system damping, for a given forcing function, are shown in Figure 6.23. It is apparent that increasing damping reduces the system's peak response levels. In addition, the shape of the response curve changes significantly with changes in damping. This obviously is reflected by changes in the spectral content of the system response as shown in Figure 6.24. The results
shown in this figure were obtained using the FFT analyzer on
the output signal from the analog computer.

Figure 6.25 shows the effect of changes to the
natural frequency on system response. Again the forcing
function is the same in each instance. The significant
change in system response is quite evident. The large rise
in system response as the system natural frequency
approaches the cutter rotational speed (540/60 = 9 Hz) is
quite apparent.

Of course, the results noted in this section are
not new, being covered in depth in most fundamental
vibration texts. However, it was desired to highlight the
importance of properly evaluating a system's natural
frequency and damping (or, in the case of a multi
degree-of-freedom system, its natural frequencies and
associated damping). Significant errors can be present in
the experimental determination of these values, particularly
in the values of damping, if care is not taken to ensure a
reasonable level of accuracy. Any errors in establishing
these system parameters are then reflected in errors in
predicting system response.

6.6.3 Effect of Width of Cut on Forcing Function Value

Generally "quiet cutter" design strategies have
dealt with workpieces of "simple" geometry. That is, the
workpiece-milling cutter geometry is assumed to be like that
shown in Figure 6.18 or Figure 6.19. Obviously, in these
instances the width of cut is constant. Of course, in the actual industrial application of "quiet cutter" design, the workpiece is not usually of "simple" geometry and the width of cut is not constant. As an example, refer to Figure 4.7. It is apparent that, as the milling cutter advances through its feed stroke, the width of cut is varying from point to point.

At this point it is desired to demonstrate the importance of the variation in width of cut in determining the effective forcing function acting on the workpiece.

To begin, consider the cutter shown in Figure 6.20. Any cutter configuration would suffice for illustration purposes, however, the configuration of Figure 6.20 was already available for use, and was thus the most expedient to employ. The cutter is assumed to be running at a speed of 540 rpm. Now, for this fixed cutter geometry and speed, the forcing function acting on the workpiece is determined for various workpiece widths. The results of this exercise are shown in Figure 6.26.

Notice the very significant difference in the effective forcing function as the width of cut is varied. Thus for a workpiece which presents varying widths of cut to the mill, the result can be a significant variation in the excitation force with time, even though the cutter configuration is, obviously, unchanged.

This phenomena is again dramatically illustrated
in the frequency domain, as shown in Figure 6.27. Here, the first fifteen frequency components (determined as illustrated in Appendix V) of the forcing functions are compared. The significant differences in the excitation frequencies are readily apparent.

The forcing functions shown in Figure 6.26 were applied to a single-degree-of-freedom system ($p = 56.5$ Hz; $\zeta = 0.2$) using the hybrid computer system. The system responses are shown in Figure 6.28. The significant variation in system response is quite evident.

It is instructive to now consider the same milling cutter but with equal insert spacing. Figure 6.29 shows the forcing function acting on the workpiece for various workpiece widths. Figure 6.30 presents the first fifteen harmonics of the forcing functions. It is again apparent that the width of cut has a significant effect on the forcing function "seen" by the workpiece. Thus, even for a cutter with equal insert spacing, force modulation occurs as a result of the changes in width of cut. Figure 6.31 shows the various system responses obtained on the hybrid computer. The variation in system response is obvious.

Thus it seems logical to assume that such variations would manifest themselves during the face-milling of "complex" workpieces.

Relating these observations to the noise generation of a workpiece, significant variation in both
noise level and also frequency content would be expected as the cutter encountered varying widths of cut.

Referring to Figure 4.16 exactly these results are observed for the noise radiation from the GM 125 transmission case. It should be noted, however, that some of this variation in workpiece response is also due to the variation in structural response characteristics at the various points of force application. Thus, both changes in structural response due to changes in point of force application, and the changing nature of the exciting force itself, due to width of cut variations, must be considered in obtaining workpiece response at a given instant. At the present time, neither of these factors is realistically represented in "quiet cutter" design strategies. It is suggested that these failings are major contributors to the lack of success generally encountered when applying presently available "quiet cutter" design procedures to "complex" workpieces.

6.7 IMPROVEMENT OF "QUIET CUTTER" DESIGN STRATEGIES

At this point the question of how the results and insights gained in this and previous chapters might be used to improve "quiet cutter" design strategies will be addressed.

To begin, the force pulse model described in Section 6.4 permits a more realistic estimation of the forcing function acting on the workpiece, than does the
force impulse model.

Recognizing the significant effect width of cut has on forcing function value and thus workpiece response, it is obvious that this parameter should be included in any "quiet cutter" design strategy.

The width of cut variation could be accounted for by defining various "sectors" of the cut during which the width of cut is essentially constant. Each of these sectors could then be represented by an "effective" width of cut and the appropriate forcing function could then be generated for each sector using the force pulse model. Figure 6.32 shows how the engine mounting face of the GM 125 transmission case might be sectored.

For each sector of constant "effective" width of cut, a representative noise frequency response function could be obtained using the techniques discussed in Section 5.3.3.

Now for each sector both a representation of the forcing function acting on that sector and its representative noise frequency response function are available. Thus, using the technique described in Section 5.3.3.2, the expected noise generation (in terms of a normalized frequency spectrum) as the cutter passes over each of designated sectors can be determined.

Each of these noise spectrums could then be weighted to reflect the length of time the cutter is within
the bounds of each sector. The total "energy" (the sum of
the square of each spectral component) associated with each
weighted spectrum could then be determined. The sum of the
"energy" values for all the sectors would then be used as
the criterion for ranking different cutter configurations.
The lower this total energy value, the better the cutter
would be expected to perform, that is, the "quieter" the
cutter.

The critical part of the procedure described above
is the determination of the expected noise generation, using
the assumed forcing function and the representative noise
F.R.F. for a given sector. It would be instructive at this
point to try and determine the kind of error which would be
expected with such a procedure. Such a determination can be
made since the actual noise levels generated during the
milling of the engine mounting face are available, as
detailed in Chapter IV.

For illustration purposes consider Sector 6 shown
in Figure 6.32. The noise FRF was taken as that obtained
between an input force applied at Point A and the noise
output obtained at Point 32. (This was a point on the "bell
housing" section of the transmission case. It was chosen
since it provided the greater response of the two output
points for which data was available. See Figures 5.33 and
5.34). This noise FRF was obtained as described in Section
5.3.3.1, and is shown in Figure 6.33. Once the frequency
content of the forcing function is known, Figure 6.33 may be used to determine the expected frequency content of the noise generated during milling of Sector 6.

The force pulse model was used to determine the forcing function value. The value of \( \psi \) for this sector was taken as 3\( ^\circ \), based on the cutter-workpiece geometry. The cutter was assumed to be the "EQ. SP." configuration of Chapter IV. This forcing function was then transformed into a frequency domain representation using the technique described in Appendix V. The result is shown in Figure 6.34.

The frequency axis is numbered in terms of the "insert engagement frequency" (cutter rotational speed \( \times \) the number of cutter inserts) and its harmonics. The number "1" represents the "insert engagement frequency", the number "2" the first harmonic of the "insert engagement frequency", etc. The frequency components representing the cutter rotational frequency and its harmonics are at least two orders of magnitude smaller than the insert engagement frequencies and hence have been excluded from consideration.

The magnitude of Figure 6.34 has been normalized to make the magnitude of the 2nd harmonic equal to 1.

At this stage, the FRF function value was obtained from Figure 6.33 for each of the sixteen frequencies shown in Figure 6.34. The product of the magnitude of each of these FRF values and the corresponding forcing function value (Figure 6.34) results in an estimate of the noise
spectrum generated during milling of Sector 6. The results of this calculation are shown in Figure 6.35, again with the spectrum normalized to give a 2nd harmonic magnitude of 1.

The actual noise spectrum obtained as the cutter passed over Point A is shown in Figure 6.36. If this spectrum is normalized to present the 2nd harmonic of the "insert engagement frequency" with magnitude 1, and then compared to the "expected" spectrum, the result is as shown in Figure 6.37.

Unfortunately, there are some rather significant errors in level estimation at particular frequencies, however the general "shape" of the spectrum is not too badly distorted.

Considering that the "actual" spectrum was obtained using only two averages (and thus may well contain some random variations in level) and that the actual noise measurement was made some distance from the workpiece (and thus the "effective" noise FRF includes a "spatial average" over the workpiece surface, and also the effects of objects in, or near, the sound transmission path) the agreement between "predicted" and "actual" noise generation is quite reasonable.

It is, therefore, felt that the method described does hold promise as a means of improving the prediction of milling cutter noise generation during the machining of workpieces of complex geometry.
It is expected that improvements could be made to the prediction accuracy by employing noise frequency response functions obtained using narrower bandwidths, thus improving frequency resolution and also making the response (noise) measurement farther from the workpiece during laboratory determination of the noise FRF. The latter will provide a "spatial average" of the noise FRF, as larger areas of the workpiece will contribute to the resultant noise level at the microphone. This is more in keeping with the actual machining situation where the noise measured is the "sum" of the noise generated by the whole workpiece.

It should perhaps be noted at this point, that a purely theoretical approach to determining the sound pressure generated by the workpiece (a transmission case) was initially considered. Although such an approach has been shown to provide reasonable results for radiators with relatively simple geometry and straightforward boundary conditions [1] [14], it was judged that the complex geometry and the difficult to quantify boundary conditions associated with a transmission case clamped in a machining fixture made a purely analytical approach impractical.

Subsequently, a technique based on that described by Sas and Snoeys [28] was considered. They showed that it was possible to predict sound generation from a structure using a combination of experimental and analytical techniques. Basically, modal data obtained from the
structure was used to determine the motion of a finite number of points on the surface of the structure. These points were then treated as point sources and the total sound pressure obtained by solving the wave equation with appropriate boundary conditions. Such a technique has the advantage of obtaining the surface motion parameters for the radiator with its actual boundary conditions, no matter how complex. However, only a finite number (the number of measuring points) of sources are accounted for, and they must therefore each represent a radiating area. Also, reflections from the structure itself are difficult to account for in this method. The authors suggest addition of appropriate supplementary sources. For radiators of complex geometry this would be a formidable task, fraught with error. For these reasons, it was felt that such an approach would not provide the desired reliability and simplicity.

Thus it was concluded that the approach described in this section, based on the acquisition of noise FRF, provided a proper balance between simplicity of implementation and reliability in predicting noise emission characteristics during milling operations.

6.8 SUMMARY

Several important developments were detailed in this chapter.

To begin, a general expression for the response of a damped, single degree-of-freedom system to an arbitrary
series of rectangular pulses was developed using the Duhamel's Integral technique. This expression was not only used to verify subsequent analogue computer solutions, but also formed the basis of a simple procedure to rank the expected noise generation from specific milling cutter configurations. The results of this ranking procedure closely agreed with actual noise measurements.

Next, the development of a hybrid digital-analogue computing system was described. This system was used to demonstrate the importance of various parameters (forcing function definition, natural frequency, damping, and width of cut) on system response.

A more realistic forcing function definition, than the often used force impulse model, was derived for a milling cutter. Significant differences in system response were demonstrated for excitation based on the force pulse model versus the force impulse model.

The importance of properly evaluating system (workpiece) parameters, such as natural frequency and damping was also demonstrated. In addition, it was shown that variation in width of cut has a significant influence on forcing function definition.

Several suggestions were made for improving "quiet cutter" design strategy. In particular, a technique of "sectoring" the workface into sections with constant "effective" width of cut was described. This approach
permits the very important width of cut variation to be accounted for in a relatively simple manner. Also, it was demonstrated that noise frequency response functions, when used in conjunction with the force pulse model for predicting insert engagement forces, allowed reasonably accurate prediction of noise emission characteristics as a function of cutter location. The ability to make such predictions is a major enhancement of "quiet cutter" design strategies.
Chapter VII

CONCLUSIONS.

On the basis of the results presented in this study, the following conclusions have been reached.

1) The non-uniform insert pitch configurations which were used in this study did not result in a noise reduction relative to a standard face milling cutter with equally spaced inserts. Thus it is apparent that the use of non-uniform insert pitch does not guarantee a reduction in cutting vibration and noise.

2) Spectral analysis indicated that the non-uniform insert pitch did in fact redistribute the sound energy over a wider bandwidth compared to the evenly spaced cutter. Although this rearrangement of energy was not sufficient to provide a reduction in overall noise level, it did provide a significant reduction in the "noisiness" of the cut. This was the result of a significant decrease in the magnitude of discrete, harmonically related components.

3) The complex nature of the workpiece geometry produced, during each cut, several well defined...
noise emission regimes within each of which the overall level and noise spectrum were unique.

4) It was evident that the most predominant source of noise during the cutting process was the vibration of the workpiece under the influence of the forces exerted by insert engagements. Its complexity and dependence upon numerous variables makes modifications to the excitation source (the milling cutter) the only practical method of reduction.

5) Modal analysis of the transmission case showed significant differences in structural response parameters for the "freely suspended" versus the "clamped in fixture" test conditions. Thus response data should be obtained under conditions which simulate, as closely as possible, the actual machining situation.

6) The significant modal density of the transmission case indicates that attempting to reduce noise generation from the workpiece by increasing the number of fixture clamps or by adding additional external damping pads would be futile. Such a strategy can only be really successful if it can be guaranteed that the modes are shifted away from frequencies associated with the forcing function. The significant modal density, coupled with the harmonically rich forcing function
associated with face milling, guarantees that numerous modes will still be excited with consequent significant noise generation.

7) Although acceleration FRF data can provide a qualitative indication of the relative magnitude of the overall noise levels likely to be produced during the milling process, this type of data does not provide a reasonable indication of the frequency content of the noise generated. Both mobility and dynamic compliance measurements suffer the same failing.

8) Noise FRF measurements provide a qualitative indication of both the relative magnitude of the overall noise level, and also the frequency characteristics of the noise generated under actual machining conditions.

9) Noise FRF measurements clearly illustrated the dependence of workpiece response on the point of application of the forcing function.

10) The Duhamel's Integral technique was used to develop a general expression for the response of a damped single degree-of-freedom system to an arbitrary series of rectangular pulses. This expression was used as the basis of a simple procedure to rank the expected noise generation from specific milling cutter configurations. The
results of this ranking procedure agreed closely with actual noise measurements.

11) A "Force Pulse Model" was developed to describe the variation of cutting force on the workpiece due to milling cutter insert engagements. This model is seen as a significant improvement over the often-used "Force Impulse Model".

12) A hybrid digital-analog computing system was developed to allow simulation of cutter-workpiece interactions. It was demonstrated that proper evaluation of workpiece natural frequency and damping parameters is necessary for reliable response predictions. Also, this computing system provided the means of demonstrating the significant effect that the width of cut parameter has on cutting force and therefore the resulting workpiece response.

13) It was shown that, for a given "sector" on the workface, the measured noise frequency response function can be combined with the forcing function data generated using the analytical force pulse model to provide reasonably accurate predictions of noise emission characteristics. By obtaining such information "sector-by-sector" for a given workface, it is possible to obtain the noise emission characteristics as a function of milling
cutter location during the cutting cycle. The ability to make such predictions is a major advancement in "quiet cutter" design.
Chapter VII

RECOMMENDATIONS

The following recommendations represent both evolutionary enhancements of the present work and also suggest new areas for future study, all of which are expected to greatly enhance "quiet cutter" design strategies.

1) The "Force Pulse Model" and the "Noise Frequency Response Function" technique should be incorporated into a computer based routine to permit automatic optimization (minimum noise generation over a cutting cycle period) of face milling cutter design through use of non-uniform insert pitch.

2) Since it is recognized that the cutting force exerted on a workpiece by insert engagement is only properly represented by a rectangular pulse for the instance of a symmetrical, or nearly symmetrical, cut of narrow width, then it would be appropriate to study the effect of cutting forces of other shapes (sinusoidal pulses, ramp-type pulses, etc.) on workpiece response. It is recommended that such a simulation be undertaken
using the hybrid computing system developed for the present study. To properly effect such a study, a preliminary review has shown that the present 8-bit D/A converter should be replaced by a 12-bit version. Such a modification would be necessary to provide the requisite fidelity in pulse shape generation.

3) In conjunction with point (2) above, the desirability and feasibility of modelling multi-degree-of-freedom systems using the hybrid computing package should be investigated. In particular, it would be desirable to determine if the added complexity of such a set-up is justified by a substantially greater insight into system response characteristics than can be obtained from the simple single degree-of-freedom model.

4) The fact that the insert (and consequently its support structure) is in contact with the workpiece during the actual cutting process, means that both the effective stiffness and damping of the workpiece are, to some degree, modified. Such changes affect the resulting noise generation. It is therefore recommended that a study be undertaken to quantify the effect that insert engagement has on the response characteristics of a workpiece. If necessary, means of incorporating
such effects into the "quiet cutter" design procedure (such as modifications to measured noise frequency response functions, or the manner in which they are obtained) should be developed.

5) A study should be undertaken to determine the optimum manner in which to obtain the noise frequency response functions required for noise emission prediction. In particular, it would be desirable to answer the following questions:

i) How should the microphone position be chosen and should more than one position be used?

ii) If a workpiece's noise response is to be properly defined, how many forcing function input locations are necessary for a given milled surface, and how should they be chosen?

iii) Just how critical is the experimental environment in which the noise FRF are obtained? Is a "quieted" room (versus the far more expensive semi-anechoic room) an acceptable measuring environment?

iv) Is the prediction method seriously compromised by the presence of the milling cutter itself and other machine elements (all of which change the "effective" noise transmission path) which are not included in
the experimental noise FRF measurements?

6) The ability to correctly predict noise emission from a workpiece is dependent upon the ability to accurately predict actual cutting forces. Therefore work should be initiated with the goal of providing a cutting force model with greatly enhanced abilities to correctly predict the actual force exerted by insert engagement in workpieces of complex geometry. The model should be able to predict forces for any reasonable combination of cutting conditions (cutter rotational speed, feedrate and depth of cut), cutter geometry (cutter diameter, number of teeth, tooth spacing, axial rake, radial rake and lead angle), workpiece geometry (particularly in interrupted cuts the possibility of variation in entry-exit angle for each insert), relative position of the cutter and the workpiece, and finally, "run-out" characteristics (due to insert size variations, insert setting, etc.). The model should be coded for execution on a computer to minimize user complexity and to allow easy integration into a "quiet cutter" design optimization scheme.
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Table 4.1: Summary of Test Designations.
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Table 4.2: Summary of the $L_{eq}$ Values Obtained During the Test Sequences.
### Table 5.1: Summary of the Different Forms of Transfer Function for Mechanical Systems.

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**Note:** This table summarizes the different forms of transfer functions used in mechanical systems, highlighting the relationships between force, displacement, velocity, and acceleration.
## Measurement Point Coordinates

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**Table 5.2:** Excerpt from the Measurement Point Coordinates Table.
COMPONENT NO. 1

COORD. SYS. TYPE: RECTANGULAR

ORIGIN OF LOCAL COORD. SYSTEM

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<th>Z</th>
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<td>0.000</td>
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ORIENTATION

ROTATE ABOUT THE X AXIS: 0.000 DEG
ROTATE ABOUT THE Y AXIS: 0.000 DEG
ROTATE ABOUT THE Z AXIS: 0.000 DEG

DIRECTION COSINE MATRIX

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<th>Z'</th>
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Table 5.3: Table Specifying the Location of a Local Coordinate System.
## MODAL RESIDUES

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</table>

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**Table 5.4:** Excerpt from a Typical Residue Table.
| MODE NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| FREQUENCY | 243.750 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| DAMP (%) | 284.315 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M. MASS | 652.944 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M. DAMP | 5.686 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M. STIF | 1.532 K | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M. U | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | Y | -20.8603 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | 2 | Y | -8.1410 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | 3 | Y | -2.9715 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | 4 | Y | -10.4932 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | 5 | Y | 27.3011 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | 6 | Y | 44.5530 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 7 | 7 | Y | 45.0186 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 8 | 8 | Y | -6.9196 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 9 | 9 | Y | -16.8539 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 10 | 10 | Y | -2.7336 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 11 | 11 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 12 | 11 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 13 | 11 | Z | 96.1921 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 14 | 12 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 15 | 12 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 16 | 12 | Z | 9.4455 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 17 | 13 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 18 | 13 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 19 | 13 | Z | -23.6908 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 20 | 14 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 21 | 14 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 22 | 14 | Z | 62.7907 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 23 | 15 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 24 | 15 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 25 | 15 | Z | 189.916 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 26 | 16 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 27 | 16 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 28 | 16 | Z | 288.143 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 29 | 17 | X | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 30 | 17 | Y | 0.0000 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Table 5.5: Excerpt from a Typical Mode Shape Table.
<table>
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<th>MODE</th>
<th>FREQUENCY (Hz)</th>
<th>DAMPING (%)</th>
</tr>
</thead>
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<td>244</td>
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<tr>
<td>2</td>
<td>306</td>
<td>0.132</td>
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<tr>
<td>3</td>
<td>569</td>
<td>0.055</td>
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<tr>
<td>4</td>
<td>688</td>
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<tr>
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Table 5.6: Summary of Modes Identified for the Freely Suspended Transmission Case - 0 to 3200 Hz.
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<th>DAMPING (%)</th>
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<td>20</td>
<td>2,000</td>
<td>1,572</td>
</tr>
<tr>
<td>21</td>
<td>2,053</td>
<td>1,678</td>
</tr>
<tr>
<td>22</td>
<td>2,181</td>
<td>1,722</td>
</tr>
<tr>
<td>23</td>
<td>2,203</td>
<td>1,853</td>
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Table 5.7: Comparison of the Modal Parameters Associated with the Freely Suspended and Clamped Transmission Case.
<table>
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<tr>
<th>TIME (seconds)</th>
<th>R DUHAMEL INTEGRAL</th>
<th>CSMP</th>
<th>RELATIVE % ERROR</th>
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<tr>
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<td>.047934</td>
<td>.047936</td>
<td>(.DUHAMEL-CSMP) / CSMP x 100</td>
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<tr>
<td>.0005</td>
<td>.18328</td>
<td>.18328</td>
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<td>.45479</td>
<td>.45481</td>
<td>-.004</td>
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<td>.0020</td>
<td>.71428</td>
<td>.71431</td>
<td>-.004</td>
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<td>.86244</td>
<td>-.005</td>
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<td>.68352</td>
<td>-.006</td>
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<td>.0050</td>
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<td>.28038</td>
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<td>.0060</td>
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<td>.0080</td>
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<td>.0140</td>
<td>-.27408</td>
<td>-.27407</td>
<td>+.004</td>
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Figure 4.16 Cont'd: Comparison of the Spectral Content of Seven Arbitrarily Defined Regimes.
MODES OF VIBRATION

MATHEMATICAL: Parameters of a linear dynamic model.

PHYSICAL: Structure's predominant motion is a well-defined waveform.

EACH MODE DEFINED BY

<table>
<thead>
<tr>
<th>MODE 1</th>
<th>MODE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.69 Hz</td>
<td>45.25 Hz</td>
</tr>
<tr>
<td>1.25%</td>
<td>2.17%</td>
</tr>
<tr>
<td>1st Bending</td>
<td>2nd Bending</td>
</tr>
<tr>
<td>RESONANT FREQUENCY</td>
<td>DAMPING FACTOR</td>
</tr>
<tr>
<td>MODE SHAPE</td>
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5 AVERAGES

REAL (LEFT SCALE)

IMAGINARY (RIGHT SCALE)

FREQUENCY - Hz

MODE 1

ACCELERANCE - m/Ns²
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APPENDIX I

EXAMPLE CSMP PROGRAMME.
The following programme was run on an IBM 360 digital computer.

***CONTINUOUS SYSTEM MODELING PROGRAM***

*** VERSION 1.3 ***

INITIAL

CONSTANT FNAT=100., ZETA=.1, HASS=1.
INCON YO=0., YDOT=0.
WN=6.283185*FNAT
KM=WN**2
CM=2.*ZETA*WN

DYNAMIC

NOSORT
IF (TIME.GT.0.003) GO TO 1
PLS=WN**2*PULSE(.001,1)*PULS(0...002))
GO TO 2
1 PLS=0.
2 CONTINUE

SORT
YDDOT=-CM*YDOT-KM*Y+PLS
YDOT=INTGRL(YDOT,YDDOT)
Y=INTGRL(YO,YDOT)

TIMER DELT=.00001, OUTDEL=.0001, FINTIM=.02
LABEL PULSE RESPONSE OF A FIRST ORDER SYSTEM
PRTPLOT Y

END
STOP
APPENDIX II

RESPONSE OF A SINGLE DEGREE-OF-FREEDOM SYSTEM USING THE DUHAMEL'S INTEGRAL TECHNIQUE
The following programme was coded in WATFIV and run on an IBM 3631 digital computer.

5JOB WATFIV Xxxxxxxxxx

'TCW6VIB-WK' !

THIS PROGRAM DETERMINES THE AREA UNDER THE OUTPUT RESPONSE CURVE OF A SINGLE DEGREE OF FREEDOM SYSTEM EXCITED BY A FINITE PULSE TRAIN OF ARBITRARY SPACING. THE PROGRAM ALSO PROVIDES THE VALUE OF THE RESPONSE CURVE AT INTERVALS OF 'TLM/VIN' SECONDS.

THE AREA IS DETERMINED ON THE BASIS OF THE ABSOLUTE VALUE OF THE OUTPUT RESPONSE. THE OUTPUT VALUE IS LABELED 'AREA'.

THE FORCE PULSES ARE ALL OF THE SAME MAGNITUDE.

THE PULSE WIDTH AND SPACING CAN BE SET AS DESIRED BY PROVIDING 'PULSE TIME LIMITS' TO BE READ BY THE PROGRAM.

THE DATA REQUIRED IS,

- \( \text{LOOPS} \) = THE NUMBER OF DATA SETS BEING RUN.
- \( \text{N} \) = NUMBER OF 'PULSE TIME LIMITS'.
- \( \text{TP}(N) \) = THE VALUES OF THE 'PULSE TIME LIMITS'.
- \( \text{FNAT} \) = THE UNDAMPED NATURAL FREQUENCY OF THE SYSTEM IN HERTZ.
- \( \text{ZETA} \) = THE DAMPING RATIO OF THE SYSTEM.
- \( \text{TLIM} \) = THE LAST TIME FOR WHICH AN OUTPUT VALUE IS DESIRED.
- \( \text{INT} \) = THE NUMBER OF STRIPS (OR 'INTERVALS') WHICH ARE TO BE USED IN DETERMINING THE AREA UNDER THE RESPONSE CURVE.

- THIS VALUE MUST BE AN EVEN NUMBER.

- THIS DATA IS SPECIFIED IN FREE FORMAT WITH 'LOOPS' ON A SEPARATE DATA CARD (THE FIRST) AND THE REMAINING DATA SETS ON SEPARATE CARDS IN THE ORDER LISTED ABOVE.

- THE MAXIMUM NUMBER OF PULSES IS 6.

DIMENSION TP(11),TP0(11)
COMMON ZETA, WN, WD, TP, IN
READ, LOOPS
DO 52 JL=1, LOOPS
READ 9, TP(JL), TP0(JL), WN(2), ZETA, TLIM, INT
IF(N .eq. 3) THEN
   GO TO 60
ELSE WHERE THERE WOULD BE THREE 'PULSE TIME LIMITS' SPECIFIED.
   GO TO 5
END WHERE THERE WOULD BE THREE 'PULSE TIME LIMITS' SPECIFIED.
FNAT = THE UNDAMPED NATURAL FREQUENCY OF THE SYSTEM IN HERTZ.
ZETA = THE DAMPING RATIO OF THE SYSTEM.
TLIM = THE LAST TIME FOR WHICH AN OUTPUT VALUE IS DESIRED.
INT = THE NUMBER OF STRIPS (OR 'INTERVALS') WHICH ARE TO BE USED IN DETERMINING THE AREA UNDER THE RESPONSE CURVE.

THIS VALUE MUST BE AN EVEN NUMBER.

1 print 102
2 print 103
52 CONTINUE
11 print 107
12 print 108
13 print 101
14 sum=0.
15 delta=tlim/int
16 t=delta
17 do 51 l=2, int
The interval in which 'T' is located is determined and appropriate output is generated.

10 IF(T+GT+TLIM)GO TO 52
19 IF(T+GT+TP(N))GO TO 200
30 IF(T+LE+TP(1))GO TO 25
40 IF(T+GT+TP(1)+AND+T+LT+TP(2))GO TO 5
50 IF(T+GE+TP(2)+AND+T+LT+TP(3))GO TO 6
60 IF(T+GT+TP(3)+AND+T+LT+TP(4))GO TO 7
70 IF(T+GE+TP(4)+AND+T+LT+TP(5))GO TO 8
80 IF(T+GT+TP(5)+AND+T+LT+TP(6))GO TO 9
90 IF(T+GT+TP(6)+AND+T+LT+TP(7))GO TO 10
100 IF(T+GT+TP(7)+AND+T+LT+TP(8))GO TO 11
110 IF(T+GE+TP(8)+AND+T+LT+TP(9))GO TO 12
120 IF(T+GT+TP(9)+AND+T+LT+TP(10))GO TO 13
130 IF(T+GE+TP(10)+AND+T+LT+TP(11))GO TO 14
5
IN=2
32 GO TO 20
33 IN=3
34 GO TO 30
35 IN=4
36 GO TO 20
37 IN=5
38 GO TO 30
39 IN=6
40 GO TO 20
41 IN=7
42 GO TO 30
43 IN=8
44 GO TO 20
45 IN=9
46 GO TO 30
47 IN=10
48 GO TO 20
49 IN=11
50 GO TO 30
51 IN=12
52 GO TO 20
53 R=(1./WD*EXP(ZETA+WN*T))*SEC(WD*T)*(-ZETA+WN+EXP(ZETA+WN*T))
   *(ZETA+WN+CSS(WD*T)+WD*EXP(ZETA+WN*T))
   *(ZETA+WN+CSS(WD*T)-WD*CSS(WD*T)))
54 IN=1
55 PRINT 100,R,T,IN
56 GO TO 50
57 CALL SIGMA(Q3,04)
58 R=(1./WD*EXP(ZETA+WN*T))*SEC(WD*T)*(-Q3-ZETA+WN)-CSS(WD*T)
59 PRINT 100,R,T,IN
60 GO TO 50
61 CALL SIGMA(Q3,04)
62 R=(1./WD*EXP(ZETA+WN*T))*SEC(WD*T)*EXP(ZETA+WN*T)*ZETA+WN
   *CSS(WD*T)+WD*EXP(ZETA+WN*T)-Q3-ZETA+WN
   *CSS(WD*T)+WD*CSS(WD*T)+Q3)
   *(ZETA+WN+CSS(WD*T)-WD*CSS(WD*T))+Q4)
63 PRINT 100,R,T,IN
64 50 CONTINUE
65 IF(MODEL+21)2,2,3
66 2 SUM=SUM+4.4*ABS(R)
67 GU TO 51
68 3 SUM=SUM+2.4*ABS(R)
69 51 T+DELTA
70 AREA=DELTA/3.*SUM+ABS(R))
71 PRINT 100,R,T,IN
72 PRINT 106,AREA
73 IF(JL+EQ+1)GO TO 55
74 IF(AREA+LT+AROPT)GU TO 55
75 GO TO 52
76 55 AROPT=AREA
77 DO 57 I=1,IN
78 TPD(I)=TP(I)
79 57 CONTINUE
80 52 CONTINUE
DIMENSION TP(11)
COMMON ZETA, WN, WD, TP, IN
INH=IN-1
Q03=0.
Q04=0.
DO 1 JP=1, INH
Q01=(-1.)**(JP-1) * EXP(ZETA*WN*TP(JP)) * (ZETA*WN*COS(WD*TP(JP))) +
CWD*SIN(WD*TP(JP)))
Q02=(-1.)**(JP-1) * EXP(ZETA*WN*TP(JP)) * (ZETA*WN*SIN(WD*TP(JP))) -
CWD*COS(WD*TP(JP)))
Q03=Q03+Q01
Q04=Q04+Q02
1 CONTINUE
105 RETURN
ENTRY

--- 335 ---
APPENDIX III

PROGRAMME TO RANK THE PERFORMANCE
OF THREE MILLING CUTTER CONFIGURATIONS
The following programme was coded in WATFIV and run on an IBM 7030 digital computer.

```
JOB WATFIV XXXXXXXX TCM MOORE

THIS PROGRAMME DETERMINES THE AREA UNDER THE OUTPUT RESPONSE CURVE OF A SINGLE DEGREE OF FREEDOM SYSTEM EXCITED BY A FORCE PULSE TRAIN OF ARBITRARY SPACING.

THE AREA IS DETERMINED ON THE BASIS OF THE ABSOLUTE VALUE OF THE OUTPUT RESPONSE. THE OUTPUT VALUE IS LABELED 'AREA'.

THE FORCE PULSES ARE ALL OF THE SAME MAGNITUDE.

THE PULSE WIDTH AND SPACING ARE DETERMINED BY THE PROGRAMME BASED ON CUTTER AND PART PARAMETERS SUPPLIED AS DATA.

THIS PROGRAMME ASSUMES THAT EACH INSERT ON THE CUTTER ENGAGES THE PART ONLY ONCE. THAT IS, ONLY ONE ROTATION OF THE CUTTER IS CONSIDERED.

******************************************************************************* DATA REQUIRED *******************************************************************************

LOOPS = THE NUMBER OF CUTTER CONFIGURATIONS BEING CONSIDERED.
EPFW = EFFECTIVE WIDTH OF PART (UNITS CONSISTANT WITH THOSE OF THE RADIUS).
RADIUS = EFFECTIVE RADIUS OF CUTTER.
OMEGA = ROTATIONAL SPEED OF CUTTER (RPM).
N = NUMBER OF INSERTS IN THE CUTTER.
FRAT = UNDAMPED NATURAL FREQUENCY OF THE SYSTEM (HZ).
ZETA = DAMPING RATIO OF THE SYSTEM.
TLIN = LAST TIME FOR WHICH AN OUTPUT VALUE IS DESIRED.
INT = NUMBER OF STRIPS TO BE USED IN DETERMINING THE AREA UNDER THE RESPONSE CURVE***MUST BE AN EVEN NUMBER***
TANG = ANGLE BETWEEN INSERTS (DEGREES).

*******************************************************************************

THE DATA IS SPECIFIED IN FREE FORMAT IN THE ORDER USED ABOVE. THE VALUES OF TANG FOR EACH CUTTER ARE LISTED IN SUCCESSION.

THE MAXIMUM NUMBER OF INSERTS ON THE CUTTER IS UNLIMITED ALTHOUGH THE 'DIMENSION' STATEMENT MUST BE MODIFIED IF THE NUMBER OF INSERTS EXCEEDS 40.
```
DIMENSION TP(40), TG(40), TANG(40), TANGR(40)
COMMON ZETA, WN, WD, TP, IN
READ, LOOPS, EPFW, RADIUS, WMEGA, N, FNAT, ZETA, TLIM, INT

READ VALUES OF THE INSERT ANGLES FOR EACH CUTTER CONFIGURATION

DO 52 JL=1, LOOPS
   READ, (TANG(I), I=1, N)

GENERATE SYSTEM PARAMETERS

WN=6.283185*FNAT
WD=WN*SORT(1.-ZETA**2)
ALPHA=ARCSIN(ZETA)

DETERMINE PULSE TIME LIMITS BASED ON CUTTER PARAMETERS

CWE=-104720*OMEGA
DO 69 IU=1, N
   TANG(IU)=TANG(IU)/57.295780
   CONTINUE
   N70=2*N-1
   TP(1)=TW
   DO 70 IT=2, N70
      TF(MOD(IT, 2))=80, 80, 81
      TGI(IT/2)=TANGR(IT/2)-EPFW/RADIUS/OMEGA
      TP(1)=TP(1)+TGI(IT/2)
      GO TO 70
   70 CONTINUE
   CONTINUE

PRINT INPUT DATA FOR VERIFICATION PURPOSES

PRINT 102
PRINT 103, LOOPS, N
PRINT 104, EPFW, WMEGA, RADIUS, FNAT, ZETA, TLIM, INT
PRINT 105, (JN, TANG(JM), JM=1, N)

PRINT VALUES OF 'PULSE TIME LIMITS'

PRINT 106
PRINT 107, (JN, TP(JN), JN=1, N70)

DETERMINE THE TIME INCREMENT WHICH WILL BE USED IN CALCULATING SYSTEM KFSCNSE

SUM=0
DELT=TLIM/INT
T=DELT
**DETERMINE THE INTERVAL IN WHICH 'T' IS LOCATED.**

**DO S1 I=2,INT**

**IF(T-CT-0) GO TO S2**

**IF(T=TP(2+N-1) GO TO 200**

**IF(T=LE.*TP(1)) GO TO 25**

**ND=2*N-2**

**DO 58 I=1,ND**

**IF(T=TP(I)*AND.T=TP(I+1)) GO* TO 40**

**58 CONTINUE**

**DO 59 I=2,ND**

**IF(T=GE.*TP(I)*AND.T=LE.*TP(I+1)) GO TO 41**

**59 CONTINUE**

**IN=I+1**

**GO TO 20**

**48 GO TO 20**

**DETERMINE THE SYSTEM RESPONSE AT TIME 'T' BASED ON THE INTERVAL IN WHICH 'T' IS LOCATED.**

**49 K=1./((W0*EXP(ZETA*WN*T)>(*SIN(W0*T)*(-ZETA*WN*EXP(ZETA*WN*T)*SIN(W0*T)*COS(WO*T)+COS(W0*T)*W0*EXP(ZETA*WN*T)))*(SIN(W0*T)*COS(W0*T)+COS(W0*T)*SIN(W0*T))))**

**50 IN=1**

**GO TO 50**

**51 CALL SIGMA(Q3,Q4)**

**52 W=I/(W0*EXP(ZETA*WN*T))**

**53 CALL SIGMA(Q3,Q4)**

**54 GO TO 50**

**55 CALL SIGMA(Q3,Q4)**

**56 CONTINUE**

**DETERMINE AREA UNDER THE RESPONSE CURVE**

**IF(W0*(I-2),2,3**

**SUM=SUM+ABS(R)**

**GO TO 51**

**SUM=SUM+ABS(R)**

**T=T+DELT**

**AREA=DELT*3.*(SUM+ABS(R))**

**PRINT 108**

**PRINT 109,AREA**

**DETERMINE THE OPTIMUM (MINIMUM) AREA GENERATED FROM ALL CUTTER CONFIGURATIONS INVESTIGATED IN THIS RUN (NUMBER OF CONFIGURATIONS=LOOP)**

**IF(JL.EQ.1) GO TO 55**

**IF(AREA.LE.AROPT) GO TO 55**

**GO TO 52**

**55 AROPT=AREA**

**SAVE THE OPTIMUM INSERT ANGLES**
DO 57 I=1,N
TANGO(I)=TANG(I)
CONTINUE
PRINT 110
C PRINT 111,AROPT
PRINT 112,(J,TANGO(J),J=1,N)
C
102 FORMAT(1X, 30, '**** DATA VALUES *******')
103 FORMAT(1X, 'N=12')
104 FORMAT(1X, 'E=10.2', 'R=1.0')
105 FORMAT(1X, 'O=10.2', 'T=10.2', 'C=10.2')
106 FORMAT(1X, 'P=10.2')
107 FORMAT(1X, 'F=10.2')
108 FORMAT(1X, 'D=10.2')
109 FORMAT(1X, 'O=10.2', 'T=10.2', 'C=10.2')
110 FORMAT(1X, 'E=10.2')
111 FORMAT(1X, 'O=10.2', 'T=10.2', 'C=10.2')
112 FORMAT(1X, 'E=10.2')
STOP
END

SUBROUTINE SIGMA

COMMON ZETA, WN, WD, TP, IN
INN=IN-1
Q03=0.
Q04=0.
DO 1 JP=1,INN
Q01=(-1.)**(JP-1)*EXP(ZETA*WN*TP(JP))*ZETA*WN*CCS(WD*TP(JP))
Q02=(-1.)**(JP-1)*EXP(ZETA*WN*TP(JP))*ZETA*WN*SIN(WD*TP(JP))
Q03=Q03+Q01
Q04=Q04+Q02
1 CONTINUE
RETURN
END
***** DATA VALUES *****

LOOPS = 3
N = 18

BPW = 0.250
CWSGA = 967.0
RADIUS = 7.970

ENAT = 320.00
ZETA = 0.005000
TLIN = 0.100000
INT = 500

TANG( 1) = 20.0
TANG( 2) = 20.0
TANG( 3) = 20.0
TANG( 4) = 20.0
TANG( 5) = 20.0
TANG( 6) = 20.0
TANG( 7) = 20.0
TANG( 8) = 20.0
TANG( 9) = 20.0
TANG(10) = 20.0
TANG(11) = 20.0
TANG(12) = 20.0
TANG(13) = 20.0
TANG(14) = 20.0
TANG(15) = 20.0
TANG(16) = 20.0
TANG(17) = 20.0
TANG(18) = 20.0

***** PULSE TIME LIMITS *****

TP( 1) = 0.000310
TP( 2) = 0.000347
TP( 3) = 0.003757
TP( 4) = 0.006894
TP( 5) = 0.007204
TP( 6) = 0.010341
TP( 7) = 0.010651
TP( 8) = 0.013789
TP( 9) = 0.014098
TP(10) = 0.017235
TP(11) = 0.017545
TP(12) = 0.020682
TP(13) = 0.020992
TP(14) = 0.024130
TP(15) = 0.024439
TP(16) = 0.027577
TP(17) = 0.027886
TP(18) = 0.031024
TP(19) = 0.031333
TP(20) = 0.034471
TP(21) = 0.034781
TP(22) = 0.037918
TP(23) = 0.038228
TP(24) = 0.041365
TP(25) = 0.041675
TP(26) = 0.044812
TP(27) = 0.046122
TP(28) = 0.048259
**RESULTS**

\[
\text{AREA}=0.128896
\]

**DATA VALUES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCCPS</td>
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</tr>
<tr>
<td>N</td>
<td>18</td>
</tr>
<tr>
<td>EFFW</td>
<td>0.250</td>
</tr>
<tr>
<td>GMDGA</td>
<td>967.0</td>
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<tr>
<td>RADIUS</td>
<td>7.970</td>
</tr>
<tr>
<td>FNAT</td>
<td>320.00</td>
</tr>
<tr>
<td>ZETA</td>
<td>0.05000</td>
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<tr>
<td>TLIM</td>
<td>0.10000</td>
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<tr>
<td>INT</td>
<td>500</td>
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<tr>
<td>TANG(1)</td>
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<tr>
<td>TANG(2)</td>
<td>21.0</td>
</tr>
<tr>
<td>TANG(3)</td>
<td>21.5</td>
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<tr>
<td>TANG(4)</td>
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</tr>
<tr>
<td>TANG(5)</td>
<td>21.5</td>
</tr>
<tr>
<td>TANG(6)</td>
<td>21.5</td>
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<tr>
<td>TANG(7)</td>
<td>21.0</td>
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<tr>
<td>TANG(8)</td>
<td>20.5</td>
</tr>
<tr>
<td>TANG(9)</td>
<td>20.0</td>
</tr>
<tr>
<td>TANG(10)</td>
<td>19.5</td>
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<tr>
<td>TANG(11)</td>
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<tr>
<td>TANG(12)</td>
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<td>TANG(13)</td>
<td>18.5</td>
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<tr>
<td>TANG(14)</td>
<td>18.5</td>
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<tr>
<td>TANG(15)</td>
<td>18.5</td>
</tr>
<tr>
<td>TANG(16)</td>
<td>19.0</td>
</tr>
<tr>
<td>TANG(17)</td>
<td>19.5</td>
</tr>
<tr>
<td>TANG(18)</td>
<td>20.0</td>
</tr>
</tbody>
</table>
**PULSE TIME LIMITS**

<table>
<thead>
<tr>
<th>TPI</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000310</td>
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<tr>
<td>2</td>
<td>0.003533</td>
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<tr>
<td>3</td>
<td>0.003843</td>
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<tr>
<td>4</td>
<td>0.007153</td>
</tr>
<tr>
<td>5</td>
<td>0.007462</td>
</tr>
<tr>
<td>6</td>
<td>0.010858</td>
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<tr>
<td>7</td>
<td>0.011168</td>
</tr>
<tr>
<td>8</td>
<td>0.014564</td>
</tr>
<tr>
<td>9</td>
<td>0.014874</td>
</tr>
<tr>
<td>10</td>
<td>0.018270</td>
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<tr>
<td>11</td>
<td>0.019579</td>
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<tr>
<td>12</td>
<td>0.021975</td>
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<td>13</td>
<td>0.022285</td>
</tr>
<tr>
<td>14</td>
<td>0.025595</td>
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<tr>
<td>15</td>
<td>0.025804</td>
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<tr>
<td>16</td>
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<tr>
<td>17</td>
<td>0.029439</td>
</tr>
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<td>18</td>
<td>0.032575</td>
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<td>19</td>
<td>0.032885</td>
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<td>0.035836</td>
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<td>0.036246</td>
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<td>22</td>
<td>0.039210</td>
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<tr>
<td>23</td>
<td>0.039520</td>
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<td>0.042309</td>
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<td>0.042709</td>
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<td>27</td>
<td>0.049397</td>
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<td>0.049776</td>
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<td>0.049086</td>
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<td>0.051865</td>
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<td>31</td>
<td>0.052274</td>
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<td>32</td>
<td>0.055239</td>
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<td>33</td>
<td>0.055549</td>
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<td>34</td>
<td>0.055630</td>
</tr>
<tr>
<td>35</td>
<td>0.058910</td>
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</tbody>
</table>

**RESULTS**

AREA = 0.312451

**DATA VALUES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOOPS</td>
<td>3</td>
</tr>
<tr>
<td>N</td>
<td>18</td>
</tr>
<tr>
<td>EFFW</td>
<td>0.250</td>
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<tr>
<td>CMAGA</td>
<td>967.0</td>
</tr>
<tr>
<td>RADIUS</td>
<td>7.970</td>
</tr>
<tr>
<td>FNAT</td>
<td>320.00</td>
</tr>
<tr>
<td>ZETA</td>
<td>0.050000</td>
</tr>
<tr>
<td>TLW</td>
<td>0.100000</td>
</tr>
<tr>
<td>INT</td>
<td>0.500</td>
</tr>
</tbody>
</table>

| TANG(1) | 21.0 |
| TANG(2) | 16.0 |
| TANG(3) | 23.0 |
| TANG(4) | 19.0 |
| TANG(5) | 15.0 |
| TANG(6) | 25.0 |
| TANG(7) | 27.0 |
| TANG(8) | 17.0 |
TANG( 9) = 21.0
TANG(10) = 20.0
TANG(11) = 24.0
TANG(12) = 19.0
TANG(13) = 16.0
TANG(14) = 22.0
TANG(15) = 20.0
TANG(16) = 15.0
TANG(17) = 23.0
TANG(18) = 17.0

****** PULSE TIME LIMITS ******
TP( 1) = 0.000310
TP( 2) = 0.003619
TP( 3) = 0.009329
TP( 4) = 0.006377
TP( 5) = 0.006687
TP( 6) = 0.010341
TP( 7) = 0.010651
TP( 8) = 0.013616
TP( 9) = 0.013926
TP(10) = 0.016201
TP(11) = 0.016511
TP(12) = 0.020510
TP(13) = 0.020820
TP(14) = 0.025164
TP(15) = 0.025473
TP(16) = 0.028094
TP(17) = 0.029402
TP(18) = 0.031718
TP(19) = 0.032020
TP(20) = 0.035460
TP(21) = 0.035260
TP(22) = 0.039297
TP(23) = 0.039906
TP(24) = 0.042571
TP(25) = 0.042981
TP(26) = 0.045329
TP(27) = 0.045630
TP(28) = 0.049121
TP(29) = 0.049431
TP(30) = 0.052568
TP(31) = 0.052878
TP(32) = 0.055153
TP(33) = 0.055463
TP(34) = 0.059117
TP(35) = 0.059427

****** RESULTS ******
AREA = 0.276494

****** OPTIMUM VALUES ******
AREA = 0.198896
TANG( 1) = 20.0
TANG( 2) = 20.0
TANG( 3) = 20.0
TANG( 4) = 20.0
TANG( 5) = 20.0
TANG( 6) = 20.0
TANG( 7) = 20.0
TANG( 8) = 20.0
TANG( 9) = 20.0
TANG(10) = 20.0
TANG(11) = 20.0
TANG(12) = 20.0
TANG(13) = 20.0
TANG(14) = 20.0
TANG(15) = 20.0
TANG(16) = 20.0
TANG(17) = 20.0
TANG(18) = 20.0
APPENDIX IV

ASSEMBLY LANGUAGE PROGRAMME TO OUTPUT

A FORCING FUNCTION TO THE ANALOG COMPUTER
The following assembly language program was run on an APPLE II+ microcomputer equipped with a FOUNTAIN COMPUTER D/A converter card. The program was assembled using the APPLE COMPUTER TOOL KIT.

```
SOURCE FILE: WAVER
----- NEXT OBJECT FILE NAME IS WAVER.OBJ

0000: 1  ORG $0000 ; PROGRAM ORIGIN
0001: 2  TABLE EQU $1B
0002: 3  TBLLOC EQU $1D
0003: 4  TBLSIZ EQU $1E ; DEFINE THE
0004: 5  PAGCNT EQU $1F ; ... ZERO PAGE
0005: 6  TABLI EQU $EB ; ... LOCATIONS USED
0006: 7  TABLR EQU $ED
0007: 8  TABLCI EQU $FA
0008: 9  TABLCR EQU $FB
0009: 10 COUNTR EQU $FC
000A: 11 COUNTI EQU $FD
000B: 12 DELY EQU $FE
000C: 13 PULS EQU $FF
000D: 14
000E: 4C 0A 50 15 JMP START
0010: 16
0011: 17 CHNL DS 1 ; CHAN. # PARAMETER
0012: 18 TPAGE DS 1 ; TABLE PAGE ADDR PAR.
0013: 19 TSIZE DS 1 ; TABLE SIZE IN PAGES
0014: 20 TPAGEI DS 1 ; TABLE(INT) ADDR PAR
0015: 21 TPAR DS 1 ; TABLE(REM) ADDR PAR
0016: 22 DELAY DS 1 ; DELAY IN WAIT S/R
0017: 23 PULSE DS 1
0018: 24
0019: 25
001A: A9 00 26 START LDA #00 ; INITIALIZE THE
001B: 85 1B 27 STA TABLE ; ... TABLE POINTERS,
001C: 85 EB 23 STA TABLI ; ... SET LOW
001D: 85 ED 29 STA TABLR ; ... BYTE TO ZERO
001E: AD 04 30
001F: 85 1C 33 STA TABLE+1 ; ... TO PAGE #
0020: 85 1D 34 STA TBLLOC ; STORE TABLE START
0021: 35
0022: 36
0023: AD 08 37 LDA DELAY ; MOVE DELAY PARAMETER
0024: 85 FE 38 STA DELY ; ... TO ZERO PAGE
0025: 39
0026: 36
0027: AD 08 37 LDA TPAGEI ; SET HIGH BYTE
0028: 85 FE 38 STA TABLI+1 ; ... TO PAGE #/(INT)
0029: 85 FA 43 STA TABLCI ; STORE TABLEI START
002A: 44
002B: 45
```
LDA TPAGER ;SET HIGH BYTE
STA TABLR+1 ;TO PAGE#(R)
STA TABLCR ;STORE TABLER START
LDA TSIZE ;INIT PAGE CNT IN
STA TBLSZ ;ZERO PAGE LOC
LDX $ANL ;PUT CHANNEL # INTO X
LDA TBLSZ ;GET # OF PAGES
STA PAGCNT ;AND STORE INTO COUNT(ZERO PAGE)
LDA TBLLOC ;GET TABLE ADDRESS
STA TABLE+1 ;AND STORE INTO POINTER (HIGH BY
LDA TABLCI ;GET TABLEI ADDRESS
STA TABLI+1 ;AND STORE INTO POINTER
LDA TABLCR ;GET TABLER ADDRESS
STA TABLR+1 ;AND STORE INTO POINTER
LDA PULSE ;TRANSFER PULSE TO ZERO PAGE
STA PULS
LDY $00 ;SET INDEX TO ZERO
LDA (TABLI),Y ;GET TABLEI VALUE (HIGH BYTE)
STA COUNTI ;STORE AS COUNT,HIGH
LDA (TABLR),Y ;GET TABLER VALUE (LOW BYTE)
STA COUNTR ;STORE AS COUNT,LOW
LDA (TABLE),Y ;GET A TABLE VALUE
STA $C080,X ;AND OUTPUT TO D/A,
LDA DELY ;SET DELAY
LDA $98 ;BACK TO WORK!
$06C:A5$ $06E:D0$ $070:C6$ $072:C6$ $074:$ $074:A5$ $076:05$ $078:F0$ $07A:4C$ $07D:C4$ $07F:F0$ $081:$ $081:$ $082:$ $082:4C$ $083$ CHNL FD COUNTR FC COUNTR 8003 DELAY FE DELY 8072 LBL 8048 LOOP 8049 LOOPA 8059 NEXT 1F PAGCNT 8009 PULSE FF PULS 8084 START FA TABLCI FB TABLCT 1B TABLE EB TABLI ED TABLR 1D TBLOCC 1E TBLS12 8006 TPAGE 8006 TPAGEI 8007 TPAGER 8005 TSIZE 807D ZERO 1B TABLE 1D TBLOCC 1E TBLS12 1F PAGCNT EB TABLI ED TABLR FA TABLCI 8003 CHNL 8004 TPAGE 8005 TSIZE 8006 TPAGEI 8049 LOOPA 8048 LOOP 8059 NEXT 8072 LBL 8070 FD DEC COUNTI ...BIT COUNTER 8072 LBL DEC COUNTR 8074: 102 BNE LBL ...THE SIXTEEN 104 LBL DEC COUNTR 8074: 106 LDA COUNTR ;CHECK IF COUNTER 107 ORA COUNTI ;IS ZERO 108 BEQ ZERO ;YES, JUMP OUT! 109 JMP NEXT ;OTHERWISE, BACK AGAIN 110 ZERO CPY PULS- ;ALL PULSES OUTPUT? 111 BEQ LOOPA ;YES, START AGAIN 112 ;NO, CONTINUE TO NEXT PULSE 113 ;INCR. INDEX TO GET NEXT TABLE VALUE 114 ;INCR. INDEX TO GET NEXT TABLE VALUE 115 INY. 116 ;BACK FOR NEXT TABLE VALUE
APPENDIX V

FOURIER SERIES COEFFICIENTS
It is well known that the general expression for the Fourier Series of a periodic waveform, as shown in Figure V.1, is given by,

\[ P(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\Omega t) + \sum_{n=1}^{\infty} b_n \sin(n\Omega t) \]

where,

\[ \Omega = \frac{2\pi}{T} \text{ (rad/sec)} \]

\[ a_0 = \frac{1}{T} \int_{\tau}^{\tau+T} P(t) dt \]

\[ a_n = \frac{2}{T} \int_{\tau}^{\tau+T} P(t) \cos(n\Omega t) dt \quad \text{for } n \neq 0 \]

\[ b_n = \frac{2}{T} \int_{\tau}^{\tau+T} P(t) \sin(n\Omega t) dt \]

(\(\tau\) is any arbitrary time)

Now, for the case of an arbitrary, periodic forcing function made up of a series of rectangular pulses, as shown in Figure V.2, the Fourier coefficients can be rewritten as,
\[ a_y = \frac{1}{T} \left( \int_{t_0}^{t_1} P(t) \, dt + \int_{t_1}^{t_2} P(t) \, dt + \ldots + \int_{t_{I-1}}^{t_I} P(t) \, dt \right) \]

\[ a_n = \frac{2}{T} \left( \int_{t_0}^{t_1} P(t) \cos (n\Omega t) \, dt + \int_{t_1}^{t_2} P(t) \cos (n\Omega t) \, dt \right. \]

\[ \left. + \ldots + \int_{t_{I-1}}^{t_I} P(t) \cos (n\Omega t) \, dt \right) \]

\[ b_n = \frac{2}{T} \left( \int_{t_0}^{t_1} P(t) \sin (n\Omega t) \, dt + \int_{t_1}^{t_2} P(t) \sin (n\Omega t) \, dt \right. \]

\[ \left. + \ldots + \int_{t_{I-1}}^{t_I} P(t) \sin (n\Omega t) \, dt \right) \]

Since the forcing function is constant within each interval the Fourier coefficients become:

\[ a_0 = \frac{1}{T} \sum_{i=0}^{I} (t_{i+1} - t_i) P_i \]

\[ a_n = \frac{2}{Tn\Omega} \sum_{i=0}^{I} P_i \left[ \sin (n\Omega t_{i+1}) - \sin (n\Omega t_i) \right] \]

\[ b_n = -\frac{2}{Tn\Omega} \sum_{i=0}^{I} P_i \left[ \cos (n\Omega t_{i+1}) - \cos (n\Omega t_i) \right] \]
Recalling that \( \Omega = \frac{2\pi}{T} \), the Fourier coefficients may finally be written as,

\[
a_0 = \frac{\Omega}{2\pi} \sum_{i=0}^{I} (t_{i+1} - t_i) P_i
\]

\[
a_n = \frac{1}{n\pi} \sum_{i=0}^{I} P_i \left[ \sin (n\Omega t_{i+1}) - \sin (n\Omega t_i) \right]
\]

\[
b_n = -\frac{1}{n\pi} \sum_{i=0}^{I} P_i \left[ \cos (n\Omega t_{i+1}) - \cos (n\Omega t_i) \right]
\]

The amplitude of a particular spectral component is then obtained from

\[
A_n = \sqrt{a_n^2 + b_n^2}
\]

It is a simple matter to code the above expressions for solution on a digital computer.
Figure V.1: General Periodic Function.

Figure V.2: Example of Typical Periodic Forcing Function.
APPENDIX VI

WORKPIECE AND TOOLING MATERIAL SPECIFICATIONS
WORKPIECE
Supplier: General Motors corporation
Part Number: 3637019
Material: Cast Aluminum (EMS-2E)

MILLING CUTTERS
Supplier: Valenite Metals
Part Number: 107L-D26725
Inserts: SEC 631-J (Tungsten Carbide)
Wedges: HFMW-60
Wedge Screws: LS-30

All inserts were brand-new at the start of each cutting sequence.
REFERENCES


VITA AUCTORIS

1949 - Born in Windsor, Ontario, Canada on January 20th.


1972 - Received the Degree of Bachelor of Applied Science in Mechanical Engineering from the University of Windsor, Windsor, Ontario, Canada in May.

1972 - Employed as a Manufacturing Engineer by the Ford Motor Company of Canada, Windsor, Ontario, Canada.

1974

1974 - Returned to the University of Windsor for Advanced Degree Studies in September.

1975 - Received the Degree of Master of Applied Science in Mechanical Engineering from the University of Windsor, Windsor, Ontario, Canada in October.

- Began Doctoral Studies at the University of Windsor.

1977 - Appointed to the Faculty of Engineering, Department of Mechanical Engineering of the University of Windsor as a Sessional Lecturer in June.

1981 - Employed as a Senior Research Engineer at the F. Jos Lamb Co. Ltd., Windsor, Ontario, Canada in August.

- Part-time Sessional Lecturer at the University of Windsor.

1984 - Appointed to the Faculty of Applied Science, Department of Mechanical Engineering, Queen's University at Kingston, Ontario, Canada in August.

Recent Publications


1984 - "Modal and Signature Analysis Methods Applied to the Cutting Noise and Vibration Problems of High Volume Transfer Machines", Moore, T.N. and Reif, Z.F., 8th Machinery Dynamics Seminar (NRC).


1980 - "Noise Exposure of Truck Drivers", Reif, Z.F., Moore, T.N. and Steevensz, A.E., Transactions of the Society of Automotive Engineers.