THEORETICAL AND EXPERIMENTAL STUDY OF THE RADIAL HYDRAULIC JUMP.

ABDELKAWI. KHALIFA

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
https://scholar.uwindsor.ca/etd/4366

This online database contains the full-text of PhD dissertations and Masters' theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000ext. 3208.
The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us a poor photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30. Please read the authorization forms which accompany this thesis.

THIS DISSERTATION HAS BEEN MICROFILMED EXACTLY AS RECEIVED

Ottawa, Canada
K1A 0N4

NL-335 (Rev. 8/80)
THEORETICAL AND EXPERIMENTAL
STUDY OF THE RADIAL
HYDRAULIC JUMP

A Dissertation
Submitted to the Faculty of Graduate Studies through the
Department of Civil Engineering in Partial Fulfilment of
the Requirements for the Degree of
Doctor of Philosophy at the
University of Windsor

by

Abdelkawi Khalifa
B.Sc. (Honour), M.Sc.

Windsor, Ontario
Canada

1980
TO MY PARENTS, AIDA, RANIA AND YESMEEN
ABSTRACT

In this thesis the free radial hydraulic jump phenomenon is investigated experimentally under different flow conditions. The free surface, bed pressure, sequent depth and velocity distribution are measured. The study also includes the role of the entrained air. Statistical analysis is used to analyze the experimental data and regenerate the necessary characteristic design charts for the free radial hydraulic jump.

The macroscopic momentum balance is applied to the expanding section to drive the general equation for the free radial hydraulic jump. The energy equation is used to obtain the jump energy loss.

The same experimental procedure and theory are applied to the submerged radial hydraulic jump to study its characteristics under different submergence and operating conditions.

Since the macroscopic theory does not give the flow pattern within the jump, a mathematical model based on the strip integral method of solving the integral momentum and continuity equations is developed. The mathematical model is developed first for the free rectangular jump and it is extended to simulate the free radial hydraulic jump. The model predicts the water surface profile, hydraulic grade profile, decay of the maximum velocity, variation of the
surface velocity, variation of the energy coefficient $\alpha$, variation of the momentum coefficient $\beta$, sequent depth ratio, jump length and the energy loss.

The mathematical model results are compared with those of the previous researchers experimental data for the rectangular hydraulic jump, and with the experimental data from the present study for the free radial hydraulic jump. The mathematical model results show good agreement with the experimental data.
ACKNOWLEDGEMENTS

I wish to express my deep appreciation and gratitude to my supervisor, Dr. J. A. McCorquodale for his continuous and patient guidance as well as generous aid and constructive criticism throughout the completion of this work.

I am greatly indebted to Dr. J.K. Bewtra and Dr. S. P. Chee for their concern and encouragement.

I also wish to express my thanks to the Central Research staff and the Civil Engineering technicians, Mr. G. Michalczuk and Mr. P. Feimer for their help and co-operation during the experimental investigation.

I also thank the computer staff at the University of Windsor for their assistance and cooperation.

The suggestions of the Comprehensive Examination Committee are sincerely appreciated.

I am grateful to the National Research Council whose financial support made this work possible.

My thanks also go to my wife, Aida, for her assistance, patience, and understanding. I deeply appreciate her sharing.

Last, but not least, I am deeply grateful to my parents for their sacrifices, blessings and moral support.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ABSTRACT</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF PHOTOGRAPHS</td>
<td>xxi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xxii</td>
</tr>
</tbody>
</table>

## I. INTRODUCTION

1.1 Objectives 1

1.2 Definition of the Problem 1

1.3 Motivation 2

1.4 The Approach in General 3

## II. LITERATURE REVIEW

2.1 Introduction 6

2.2 The Rectangular Hydraulic Jump

2.2.1 Forward Flow 12

2.2.2 Backward Flow 13

2.2.3 Submerged Jump 14

2.3 Air Entrainment 17

2.4 Turbulence Characteristics 21

2.5 Radial Hydraulic Jump 22

2.6 Critical Evaluation of the Available Literature 28

## III. THEORETICAL DEVELOPMENTS

3.1 General Comment 30

3.1.1 The Macroscopic Approach 30
### Table of Contents Cont'd

<table>
<thead>
<tr>
<th>3.1.2. The Microscopic Approach</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2 Macroscopic Approach</td>
<td></td>
</tr>
<tr>
<td>3.2.1 Free Radial Hydraulic Jump</td>
<td>31</td>
</tr>
<tr>
<td>3.2.2 Submerged Radial Hydraulic Jump</td>
<td>34</td>
</tr>
<tr>
<td>3.3 The Microscopic Momentum Balance</td>
<td></td>
</tr>
<tr>
<td>3.3.1 The Inner Layer</td>
<td>36</td>
</tr>
<tr>
<td>3.3.2 The Outer Layer</td>
<td>37</td>
</tr>
<tr>
<td>3.3.3 Equations of Motion</td>
<td>37</td>
</tr>
<tr>
<td>3.3.4 Strip-Integral Method</td>
<td>39</td>
</tr>
<tr>
<td>3.3.5 Integral Equations</td>
<td>39</td>
</tr>
<tr>
<td>3.3.6 Shear Functions</td>
<td>40</td>
</tr>
<tr>
<td>3.3.7 Formulation for the Free Rectangular Hydraulic Jump</td>
<td>41</td>
</tr>
<tr>
<td>3.3.7.1 Procedure</td>
<td>41</td>
</tr>
<tr>
<td>3.3.7.2 Integral Continuity Equation (0 to $h$)</td>
<td>41</td>
</tr>
<tr>
<td>3.3.7.3 Integral Momentum Equation (0 to $\delta$)</td>
<td>43</td>
</tr>
<tr>
<td>3.3.7.4 Integral Momentum Equation ($\delta$ to $h$)</td>
<td>45</td>
</tr>
<tr>
<td>3.3.7.5 Integral Momentum Equation ($\delta$ to $y_*$)</td>
<td>47</td>
</tr>
<tr>
<td>3.3.7.6 Initial Conditions</td>
<td>52</td>
</tr>
<tr>
<td>3.3.8 Formulation for the Free Radial Hydraulic Jump</td>
<td>53</td>
</tr>
</tbody>
</table>
Table of Contents Cont'd.

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.8.1</td>
<td>Equation of Motion</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Reynolds Equation (r-component)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Continuity Equation</td>
<td></td>
</tr>
<tr>
<td>3.3.8.2</td>
<td>Procedure</td>
<td>56</td>
</tr>
<tr>
<td>3.3.8.3</td>
<td>Integral Continuity Equation</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>(0 to h)</td>
<td></td>
</tr>
<tr>
<td>3.3.8.4</td>
<td>Integral Momentum Equation</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>(0 to δ)</td>
<td></td>
</tr>
<tr>
<td>3.3.8.5</td>
<td>Integral Momentum Equation</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>(δ to h)</td>
<td></td>
</tr>
<tr>
<td>3.3.8.6</td>
<td>Integral Momentum Equation</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>(δ to y*)</td>
<td></td>
</tr>
<tr>
<td>3.3.8.7</td>
<td>Initial Conditions</td>
<td>64</td>
</tr>
<tr>
<td>3.3.9</td>
<td>The Numerical Solution</td>
<td>65</td>
</tr>
<tr>
<td>IV.</td>
<td>THE EXPERIMENTAL STUDIES</td>
<td>68</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>68</td>
</tr>
<tr>
<td>4.2</td>
<td>The Experimental Apparatus</td>
<td>68</td>
</tr>
<tr>
<td>4.3</td>
<td>Experimental Procedure</td>
<td>70</td>
</tr>
<tr>
<td>4.4</td>
<td>Experimental Ranges</td>
<td>72</td>
</tr>
<tr>
<td>4.5</td>
<td>Experimental Results</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>4.5.1 Free Radial Hydraulic Jump</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>4.5.2 Submerged Radial Hydraulic Jump</td>
<td>73</td>
</tr>
<tr>
<td>V.</td>
<td>ANALYSIS AND DISCUSSION OF THE EXPERIMENTAL DATA</td>
<td>75</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>75</td>
</tr>
<tr>
<td>5.2</td>
<td>The Free Radial Hydraulic Jump</td>
<td>75</td>
</tr>
</tbody>
</table>
Table of Contents Cont'd.

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2.1 Typical Flow Pattern</td>
<td>75</td>
</tr>
<tr>
<td>5.2.2 The Sequent Depth</td>
<td>75</td>
</tr>
<tr>
<td>5.2.3 The Jump Length</td>
<td>76</td>
</tr>
<tr>
<td>5.2.4 The Water Surface and Hydraulic Grade Profiles</td>
<td>77</td>
</tr>
<tr>
<td>5.2.5 The Relative Energy Loss</td>
<td>79</td>
</tr>
<tr>
<td>5.2.6 Entrained Air</td>
<td>79</td>
</tr>
<tr>
<td>5.2.7 The Uplift Force</td>
<td>80</td>
</tr>
<tr>
<td>5.3 The Submerged Radial Hydraulic Jump</td>
<td>81</td>
</tr>
<tr>
<td>5.3.1 A Typical Flow Pattern</td>
<td>81</td>
</tr>
<tr>
<td>5.3.2 The Sequent Depth</td>
<td>82</td>
</tr>
<tr>
<td>5.3.3 The Jump Length</td>
<td>83</td>
</tr>
<tr>
<td>5.3.4 The Water Surface and Hydraulic Grade Profiles</td>
<td>83</td>
</tr>
<tr>
<td>5.3.5 Relative Energy Loss</td>
<td>85</td>
</tr>
<tr>
<td>5.3.6 Entrained Air</td>
<td>86</td>
</tr>
<tr>
<td>5.4 Experimental Errors</td>
<td>86</td>
</tr>
<tr>
<td>VI. EVALUATION OF THE THEORY</td>
<td>89</td>
</tr>
<tr>
<td>6.1 Introduction</td>
<td>89</td>
</tr>
<tr>
<td>6.2 The Macroscopic Approach</td>
<td>89</td>
</tr>
<tr>
<td>6.2.1 Free Radial Hydraulic Jump</td>
<td>89</td>
</tr>
<tr>
<td>6.2.2 Submerged Radial Hydraulic Jump</td>
<td>90</td>
</tr>
<tr>
<td>6.3 The Numerical Model</td>
<td>92</td>
</tr>
<tr>
<td>6.3.1 Boundary Conditions</td>
<td>92</td>
</tr>
<tr>
<td>6.3.2 Initial Conditions</td>
<td>92</td>
</tr>
</tbody>
</table>

Viii
Table of Contents Cont'd

6.3.3 The Input Data 93
6.3.4 Evaluation of the Free Rectangular Hydraulic Jump 94
6.3.5 Verification of the Free Rectangular Hydraulic Jump Model 95
6.3.6 Evaluation of the Free Radial Hydraulic Jump Model 99
6.3.7 Verification of the Free Radial Hydraulic Jump Model 100
6.4 Errors and Limitations in the Mathematical Models 103
   6.4.1 Idealization Limitations 103
   6.4.2 Discretization Errors 103
   6.4.3 Computation Errors 103

VII. DISCUSSION, MODIFICATION AND APPLICATIONS 104

7.1 General 104
7.2 Stability and Sensitivity Analyses of the Numerical Model 104
7.3 Advantages of the Proposed Numerical Model 106
7.4 Applications 107

VIII. CONCLUSIONS AND RECOMMENDATIONS 109

8.1 The Free Rectangular Hydraulic Jump 109
8.2 The Free Radial Hydraulic Jump 110
8.3 Conclusions on the Experimental and Macroscopic Studies 111
8.4 Recommendations 113
Table of Contents Cont'd.

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPENDIX A</td>
<td>Turbulent Shear Model</td>
<td>117</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>Measuring Instruments</td>
<td>121</td>
</tr>
<tr>
<td>APPENDIX C</td>
<td>Figures</td>
<td>123</td>
</tr>
<tr>
<td>APPENDIX D</td>
<td>Photographs</td>
<td>214</td>
</tr>
<tr>
<td>APPENDIX E</td>
<td>Tables</td>
<td>221</td>
</tr>
<tr>
<td>APPENDIX F</td>
<td>Computer Program Flow Chart, Listing and Output</td>
<td>270</td>
</tr>
<tr>
<td>APPENDIX G</td>
<td>References</td>
<td>293</td>
</tr>
<tr>
<td>APPENDIX H</td>
<td>Nomenclature</td>
<td>302</td>
</tr>
<tr>
<td>VITA AUCTORIS</td>
<td></td>
<td>311</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>2.1</td>
<td>Definition Sketch for Submerged Rectangular Hydraulic Jump after Rajaratnam [50]</td>
<td>124</td>
</tr>
<tr>
<td>3.1</td>
<td>Definition Sketch for the Free Radial Hydraulic Jump</td>
<td>125</td>
</tr>
<tr>
<td>3.2</td>
<td>Definition Sketch for the Submerged Radial Hydraulic Jump</td>
<td>126</td>
</tr>
<tr>
<td>3.3</td>
<td>Mean Velocity Distribution</td>
<td>127</td>
</tr>
<tr>
<td>3.4</td>
<td>Definition Diagram for Initial Conditions</td>
<td>128</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental Arrangement</td>
<td>129</td>
</tr>
<tr>
<td>4.2</td>
<td>Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 5.6$ and Radius Ratio, $r_o = 1.5$</td>
<td>130</td>
</tr>
<tr>
<td>4.3</td>
<td>Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 4.4$ and Radius Ratio, $r_o = 1.4$</td>
<td>131</td>
</tr>
<tr>
<td>4.4</td>
<td>Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 6.1$ and Inlet Depth Factor, $\psi = 10.75$</td>
<td>132</td>
</tr>
<tr>
<td>4.5</td>
<td>Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 4.1$ and Inlet Depth Factor, $\psi = 5.3$</td>
<td>133</td>
</tr>
<tr>
<td>5.1</td>
<td>Typical Flow Pattern for the Free Radial Hydraulic Jump Froude Number, $F_1 = 5.6$ and Radius Ratio, $r_o = 1.5$</td>
<td>134</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.2</td>
<td>Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 4.4$ and Radius Ratio, $r_o = 1.4$</td>
<td>135</td>
</tr>
<tr>
<td>5.3</td>
<td>Sequent Depth Ratio as a Function of the Initial Froude Number, $F_1$, and Radius Ratio $r_o$</td>
<td>136</td>
</tr>
<tr>
<td>5.4</td>
<td>The Free Radial Jump Length Ratio, $(r_2 - r_1)/y_2$, as a Function of the Initial Froude Number, $F_1$</td>
<td>137</td>
</tr>
<tr>
<td>5.5</td>
<td>Dimensionless Surface Profiles of the Free Radial Hydraulic Jump</td>
<td>138</td>
</tr>
<tr>
<td>5.6</td>
<td>Dimensionless Hydraulic Grade Profiles at the Bed of the Free Radial Hydraulic Jump</td>
<td>139</td>
</tr>
<tr>
<td>5.7</td>
<td>Comparison Between the Various Proposed Water Surface Profiles</td>
<td>140</td>
</tr>
<tr>
<td>5.8</td>
<td>Relative Energy Loss, $\Delta E/E_1$, as a Function of the Initial Froude Number</td>
<td>141</td>
</tr>
<tr>
<td>5.9</td>
<td>Entrained Air Fraction as a Function of Longitudinal Position and the Initial Froude Number</td>
<td>142</td>
</tr>
<tr>
<td>5.10</td>
<td>Comparison Between the Entrained Air for the Free Radial Hydraulic Jump and the Free Rectangular Hydraulic Jump</td>
<td>143</td>
</tr>
<tr>
<td>5.11</td>
<td>Percent Difference Between Uplift on Hydraulic Grade Profile and Uplift Based on the Average Free Surface</td>
<td>144</td>
</tr>
</tbody>
</table>
List of Figures Cont'd

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.12</td>
<td>Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 6.1$ and Inlet Depth Factor, $\psi = 10.75$</td>
<td>145</td>
</tr>
<tr>
<td>5.13</td>
<td>Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 4.1$ and Inlet Depth Factor, $\psi = 5.3$</td>
<td>146</td>
</tr>
<tr>
<td>5.14</td>
<td>Theoretical and Experimental Inlet Depth Factors, $\psi$, as a Function of Initial Froude Number and Tail Depth Ratio, $\gamma'$</td>
<td>147</td>
</tr>
<tr>
<td>5.15</td>
<td>Theoretical and Experimental Tailwater Depth Ratio as a Function of Initial Froude Number, $F_1$, and Inlet Depth Factor, $\psi$</td>
<td>148</td>
</tr>
<tr>
<td>5.16</td>
<td>The Jump Length Ratio, $(r_2 - r_1)/y_1$, as a Function of the Initial Froude Number, $F_1$, and Inlet Depth Factor, $\psi$</td>
<td>149</td>
</tr>
<tr>
<td>5.17</td>
<td>Experimental Dimensionless Water Surface Profiles for Submerged Radial Hydraulic Jump</td>
<td>150</td>
</tr>
<tr>
<td>5.18</td>
<td>Experimental Dimensionless Hydraulic Grade Line Profiles for Submerged Radial Hydraulic Jump</td>
<td>151</td>
</tr>
<tr>
<td>5.19</td>
<td>Comparison of Theoretical and Experimental Relative Energy Loss, $\Delta E/E$, as a Function of the Initial Froude Number and Inlet Depth Factor, $\psi$</td>
<td>152</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>5.20</td>
<td>The Refraction Error</td>
<td>153</td>
</tr>
<tr>
<td>6.1</td>
<td>Sequent Depth Ratio as a Function of Initial Froude Number, $F_1$, and Radius Ratio, $r_0$.</td>
<td>154</td>
</tr>
<tr>
<td>6.2</td>
<td>The Relative Energy Loss, $\Delta E/E_1$, as a Function of Initial Froude Number, $F_1$, and Radius Ratio, $r_0$.</td>
<td>155</td>
</tr>
<tr>
<td>6.3</td>
<td>Theoretical and Experimental Tailwater Depth Ratio, $y'$, as a Function of Initial Froude Number, $F_1$, and Inlet Depth Factor, $\psi$.</td>
<td>156</td>
</tr>
<tr>
<td>6.4</td>
<td>Theoretical and Experimental Inlet Depth Factors, $\psi$, as a Function of Initial Froude Number, $F_1$, and Tail Depth Ratio, $y'$.</td>
<td>157</td>
</tr>
<tr>
<td>6.5</td>
<td>Theoretical Relative Depth Ratio Versus Observed Relative Depth Ratio for the Submerged Radial Hydraulic Jump</td>
<td>158</td>
</tr>
<tr>
<td>6.6</td>
<td>Comparison of Theoretical and Experimental Relative Energy Loss, $\Delta E/E_1$, as a Function of the Initial Froude Number and Inlet Depth Factor, $\psi$.</td>
<td>159</td>
</tr>
<tr>
<td>6.7</td>
<td>The Dimensionless Water Surface Profiles of the Free Rectangular Hydraulic Jump for Various Initial Froude Number, $F_1$.</td>
<td>160</td>
</tr>
<tr>
<td>6.8</td>
<td>The Dimensionless Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump for Various Initial Froude Number, $F_1$.</td>
<td>161</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>6.9</td>
<td>Theoretical Depth Ratio, $y_2/y_1$, as a Function of Initial Froude Number, $F_1$, for the Rectangular Jump</td>
<td>162</td>
</tr>
<tr>
<td>6.10</td>
<td>The Jump Length Ratio, $L_3/y_2$, as a Function of the Initial Froude Number, $F_1$, for the Free Rectangular Hydraulic Jump</td>
<td>163</td>
</tr>
<tr>
<td>6.11</td>
<td>The Roller Length Ratio, $L_1/y_2$, as a Function of the Initial Froude Number, $F_1$, for the Rectangular Hydraulic Jump</td>
<td>164</td>
</tr>
<tr>
<td>6.12</td>
<td>Typical Theoretical Velocity Distribution along the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$</td>
<td>165</td>
</tr>
<tr>
<td>6.13</td>
<td>The Decay of the Maximum Velocity for the Free Rectangular Hydraulic Jump</td>
<td>166</td>
</tr>
<tr>
<td>6.14</td>
<td>The Variation of the Surface Velocity for the Free Rectangular Hydraulic Jump</td>
<td>167</td>
</tr>
<tr>
<td>6.15</td>
<td>The Growth of the Boundary Layer for the Free Rectangular Hydraulic Jump</td>
<td>168</td>
</tr>
<tr>
<td>6.16</td>
<td>The Variation of $\alpha$ with, $x/y_1$, for the Rectangular Hydraulic Jump</td>
<td>169</td>
</tr>
<tr>
<td>6.17</td>
<td>The Variation of $\beta$, with, $x/y_1$, for the Free Rectangular Hydraulic Jump</td>
<td>170</td>
</tr>
<tr>
<td>6.18</td>
<td>The Relative Energy Profiles of the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers</td>
<td>171</td>
</tr>
</tbody>
</table>
List of Figures Cont'd

6.19  The Relative Energy Loss as a Function of the Initial Froude Number for the Free Rectangular Hydraulic Jump  172

6.20  Comparison of the Water Surface and the Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump  173

6.21  Comparison of the Dimensionless Water Surface Profiles for the Rectangular Hydraulic Jump  174

6.22  Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump  175

6.23  Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump  176

6.24  Comparison of the Jump Length Ratio, $L_J / Y_2$, for the Free Rectangular Hydraulic Jump  177

6.25  Comparison of the Roller Length, $L_C / Y_2$, for the Free Rectangular Hydraulic Jump  178

6.26  The Sequent Depth Ratio, $Y_J / Y_1$, as a Function of the Initial Froude Number, $F_1$, for the Free Rectangular Hydraulic Jump  179

6.27  Comparison of the Decay of the Maximum Velocity for the Free Rectangular Hydraulic Jump  180
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.28</td>
<td>Comparison of the Surface Velocity for the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$</td>
<td>181</td>
</tr>
<tr>
<td>6.29</td>
<td>Comparison of the Boundary Layer Growth for the Free Rectangular Hydraulic Jump</td>
<td>182</td>
</tr>
<tr>
<td>6.30</td>
<td>Comparison of the Relative Energy Profiles of the Free Rectangular Hydraulic Jump</td>
<td>183</td>
</tr>
<tr>
<td>6.31</td>
<td>Comparison of the Velocity Distribution along the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$</td>
<td>184</td>
</tr>
<tr>
<td>6.32</td>
<td>The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number, $F_1 = 4$ and Sequent Radius Ratio, $r_o = 1.2$ and 1.8</td>
<td>185</td>
</tr>
<tr>
<td>6.33</td>
<td>The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number, $F_1 = 6$ and Sequent Radius Ratio, $r_o = 1.2$ and 1.8</td>
<td>186</td>
</tr>
<tr>
<td>6.34</td>
<td>The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number, $F_1 = 8$ and Sequent Radius Ratio 1.2 and 1.8</td>
<td>187</td>
</tr>
</tbody>
</table>
List of Figures Cont'd

6.35  The Theoretical Depth Ratio, $y_2/y_1$, as a Function of the Initial Froude Number, $F_1$ and the Radius Ratio, $r_o$, for the Free Radial Hydraulic Jump

6.36  The Jump Length Ratio, $L_j/y_2$, as a Function of the Initial Froude Number, $F_1$, and Radius Ratio, $r_o$, for the Free Radial Hydraulic Jump

3.37  The Roller Length Ratio, $L_r/y_2$, as a Function of the Initial Froude Number and Radius Ratio, $r_o$, for the Free Radial Hydraulic Jump

3.38  The Theoretical Dimensionless Velocity Distribution along the Centre Line of the Free Radial Hydraulic Jump

3.39  The Decay of the Maximum Velocity for the Free Radial Hydraulic Jump

3.40  The Variation of the Surface Velocity for the Free Radial Hydraulic Jump

3.41  The Growth of the Boundary Layer for the Free Radial Hydraulic Jump

3.42  The Variation of the Energy Coefficient, $\alpha$, for the Free Radial Hydraulic Jump

3.43  The Variation of the Momentum Coefficient, $\beta$, for the Free Radial Hydraulic Jump
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.44</td>
<td>The Relative Energy Profiles of the Free Radial Hydraulic Jump</td>
<td>197</td>
</tr>
<tr>
<td>6.45</td>
<td>The Relative Energy Loss as a Function of the Initial Froude Number and Radius Ratio for the Free Radial Hydraulic Jump</td>
<td>198</td>
</tr>
<tr>
<td>6.46</td>
<td>Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 3.9$ and Radius Ratio, $r_o = 1.5$</td>
<td>199</td>
</tr>
<tr>
<td>6.47</td>
<td>Comparison of the Water Surface and Hydraulic Grade Profile for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 5.6$ and Radius Ratio, $r_o = 1.4$</td>
<td>200</td>
</tr>
<tr>
<td>6.48</td>
<td>Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 7.8$ and Radius Ratio, $r_o = 1.8$</td>
<td>201</td>
</tr>
<tr>
<td>6.49</td>
<td>Comparison of the Sequent Depth Ratio for the Free Radial Hydraulic Jump</td>
<td>202</td>
</tr>
<tr>
<td>6.50</td>
<td>Comparison of the Jump Length Ratio, $(r_2 - r_1) / y_2$, for the Free Radial Hydraulic Jump</td>
<td>203</td>
</tr>
<tr>
<td>6.51</td>
<td>The Relative Jump Length, $L_y / y_1$, as a Function of the Sequent Radius Ratio, $r_o$, and Initial Froude Number, $F_1$</td>
<td>204</td>
</tr>
</tbody>
</table>
List of Figures Cont'd

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.52</td>
<td>Comparison of the Decay of the Maximum Velocity for the Free Hydraulic Jump</td>
<td>205</td>
</tr>
<tr>
<td>6.53</td>
<td>Comparison of the Velocity Distribution for the Free Radial Jump for Initial Froude Number, $F_1 = 4.4$ and Radius Ratio, $r_o = 1.2$</td>
<td>206</td>
</tr>
<tr>
<td>6.54</td>
<td>Comparison of the Velocity Distribution for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 5.6$ and Radius Ratio, $r_o = 1.5$</td>
<td>207</td>
</tr>
<tr>
<td>7.1</td>
<td>The Effect of the Turbulent Shear in the Water Surface Profile</td>
<td>208</td>
</tr>
<tr>
<td>7.2</td>
<td>The Effect of the Turbulent Shear in the Decay of the Maximum Velocity</td>
<td>209</td>
</tr>
<tr>
<td>7.3</td>
<td>The Effect of the Turbulent Shear in the Surface Velocity</td>
<td>210</td>
</tr>
<tr>
<td>7.4</td>
<td>The Effect of Air Entrained in the Water Surface Profile</td>
<td>211</td>
</tr>
<tr>
<td>A.1</td>
<td>The Entrained Air Distribution</td>
<td>212</td>
</tr>
<tr>
<td>A.2</td>
<td>Pressure Distribution</td>
<td>213</td>
</tr>
<tr>
<td>Photograph</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.1</td>
<td>Top view of the Experimental Apparatus</td>
<td>215</td>
</tr>
<tr>
<td>4.2</td>
<td>The Arrangement of the Piezometer Tappings</td>
<td>216</td>
</tr>
<tr>
<td>4.3</td>
<td>The Manometer Board</td>
<td>217</td>
</tr>
<tr>
<td>4.4</td>
<td>Side View of the Model</td>
<td>218</td>
</tr>
<tr>
<td>4.5</td>
<td>Entrance Transition</td>
<td>219</td>
</tr>
<tr>
<td>4.6</td>
<td>Two Channel Electromagnetic Current Meter</td>
<td>220</td>
</tr>
<tr>
<td>4.7</td>
<td>Differential Pressure Transducer and the Strain Indicator</td>
<td>220</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Free Radial Hydraulic Jump Parameters</td>
<td>222</td>
</tr>
<tr>
<td>4.2</td>
<td>Measured Mean Velocity Distribution for the Free Radial Hydraulic Jump</td>
<td>227</td>
</tr>
<tr>
<td>4.3</td>
<td>Measured Mean Velocity Distribution for the Free Radial Hydraulic Jump</td>
<td>228</td>
</tr>
<tr>
<td>4.4</td>
<td>Measured Mean Velocity Distribution for the Free Radial Hydraulic Jump</td>
<td>229</td>
</tr>
<tr>
<td>4.5</td>
<td>Measured Mean Velocity Distribution for the Free Radial Hydraulic Jump</td>
<td>230</td>
</tr>
<tr>
<td>4.6</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.7</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free</td>
<td>231</td>
</tr>
<tr>
<td></td>
<td>Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.9</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.10</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td>Radial Hydraulic Jump</td>
<td></td>
</tr>
</tbody>
</table>
List of Tables Cont'd

4.11 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 233

4.12 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 234

4.13 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 234

4.14 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 235

4.15 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 235

4.16 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 236

4.17 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 236

4.18 Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump 237
List of Tables Cont'd.

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.19</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>237</td>
</tr>
<tr>
<td>4.20</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>238</td>
</tr>
<tr>
<td>4.21</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>238</td>
</tr>
<tr>
<td>4.22</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>239</td>
</tr>
<tr>
<td>4.23</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>239</td>
</tr>
<tr>
<td>4.24</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>240</td>
</tr>
<tr>
<td>4.25</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>240</td>
</tr>
<tr>
<td>4.26</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>241</td>
</tr>
<tr>
<td>4.27</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>241</td>
</tr>
</tbody>
</table>
List of Tables Cont'd

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.28</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>242</td>
</tr>
<tr>
<td>4.29</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>242</td>
</tr>
<tr>
<td>4.30</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump</td>
<td>243</td>
</tr>
<tr>
<td>4.31</td>
<td>Submerged Radial Hydraulic Jump Parameters</td>
<td>244</td>
</tr>
<tr>
<td>4.32</td>
<td>Measured Mean Velocity Distribution for the Submerged Radial Hydraulic Jump</td>
<td>247</td>
</tr>
<tr>
<td>4.33</td>
<td>Measured Mean Velocity Distribution for the Submerged Radial Hydraulic Jump</td>
<td>248</td>
</tr>
<tr>
<td>4.34</td>
<td>Measured Mean Velocity Distribution for the Submerged Radial Hydraulic Jump</td>
<td>249</td>
</tr>
<tr>
<td>4.35</td>
<td>Measured Mean Velocity Distribution for the Submerged Radial Hydraulic Jump</td>
<td>250</td>
</tr>
<tr>
<td>4.36</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>252</td>
</tr>
<tr>
<td>4.37</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>253</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.38</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>254</td>
</tr>
<tr>
<td>4.39</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>255</td>
</tr>
<tr>
<td>4.40</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>256</td>
</tr>
<tr>
<td>4.41</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>257</td>
</tr>
<tr>
<td>4.42</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>258</td>
</tr>
<tr>
<td>4.43</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>259a</td>
</tr>
<tr>
<td>4.44</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>259b</td>
</tr>
<tr>
<td>4.45</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>260</td>
</tr>
<tr>
<td>4.46</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td>261</td>
</tr>
<tr>
<td>Page</td>
<td>Table Description</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>--------------------</td>
<td></td>
</tr>
<tr>
<td>4.47</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.48</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.49</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.50</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.51</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.52</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.53</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
<tr>
<td>4.54</td>
<td>Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump</td>
<td></td>
</tr>
</tbody>
</table>
I. INTRODUCTION

1.1 Objective

The objective of this thesis is to investigate the mechanics of the radial hydraulic jump and to develop a mathematical model to simulate the internal structure of the radial hydraulic jump. In order to achieve this objective an experimental study of the free and submerged radial hydraulic jump was necessary.

1.2 Definition of the Problem

The phenomenon of the hydraulic jump occurs when the flow changes its regime from supercritical to subcritical and in the process passes through abrupt changes in depth accompanied by high turbulence and air entrainment.

The hydraulic jump appears in rectangular basins downstream from overflow structures (spillways) and underflow structures (sluicegates). It may appear in diverging sections, for example, the "St. Anthony Falls" stilling basins and downstream of the critical flow flumes. A practical application of the diverging stilling basin is the W. Darcy McKeaugh Dam and Diversion which is now under construction near Wallaceburg, Ontario. In this study the term "radial hydraulic jump" is used for the jump which occurs in a planwise diverging section.

The hydraulic jump characteristics are affected by the
tailwater depth, i.e., a free jump or a submerged jump can be produced or depending on the operating mode, the jump may be swept out of the stilling basin.

There are many practical applications of the hydraulic jump. It may be used: 1) to dissipate energy in water flowing over dams, weirs and other hydraulic structures and thus prevent scouring downstream from the structures; 2) to recover head or raise the water level on the downstream side of a measuring flume and thus maintain high water level in the channel for irrigation or other water distribution purposes; 3) to increase the weight on an apron and thus reduce the uplift pressure under a masonry structure by raising the water depth on the apron; and 4) to mix chemicals used for water purification and to aerate water for city water supplies.

1.3 Motivation

It has been almost five centuries since the hydraulic jump was described by Leonardo da Vinci [59] and in that period hundreds of papers have been published on this subject. Most of these papers have dealt with the macroscopic descriptions of the jump [4, 5, 6, 7, 9, 10, 13, 16, 19]. This is mainly due to the fact that the existing theories of the fluid turbulence required to describe the hydraulic realities are very complex, necessitating the use of high speed computers. As a consequence, the analytical treatment has resulted in oversimplification of the problem. Several important factors involved such as entrained air, turbulence, and skin friction have been neglected.
There are several reasons for studying the internal flow in the hydraulic jump. Most stilling basins are designed on the basis of small scale model tests. The Froude law is normally used for scaling the data. However, the relative effects of viscosity, entrained air, and entrance boundary layers are significantly different in the model and in the prototype. For example, the size of air bubbles and the detention time for air in the jump do not follow the Froude law.

The writer feels that an adequate mathematical model of the internal flow in the hydraulic jump should improve the scaling up of data from physical models to predict prototype performance.

This thesis presents a numerical model for the free rectangular hydraulic jump and free radial hydraulic jump. The thesis also provides design information based on the experimental study for the free and submerged radial hydraulic jumps.

1.4 The Approach in General

In this project the free radial hydraulic jump phenomenon is investigated experimentally under different flow conditions. The free surface, bed pressure, sequest depth and velocity distribution are measured. The study also includes the role of the entrained air. Statistical analysis is used to analyze the experimental data and regenerate the necessary characteristic design charts for the
free radial hydraulic jump.

The theoretical analysis is based on applying continuity and momentum principles to the expanding section. The energy equation is used to obtain the jump energy loss. The same experimental procedure and theory are applied to the submerged radial hydraulic jump to study its characteristics under different submergence and operating conditions.

Since the macroscopic theory does not give the flow pattern within the jump, a mathematical model based on the strip integral method of solving the momentum and continuity equations is developed. The mathematical model is based on the assumption that the velocity distribution along the jump can be represented by two functions, the 1/7th power law for the inner layer, and the error function for the outer layer.

The momentum equation is integrated for three strips. These equations, and the integral continuity equation, can be solved to predict four unknowns: the surface velocity, $U_s$, the maximum velocity, $U_m$, the boundary layer thickness, $\delta$, and the water depth, $y$, along the jump.

The Prandtl mixing length theory is used to describe the turbulent shear. An expression suggested by Rajaratnam [48] is used for the shear stress at the bed. With the turbulent shear function and the bed shear function, the integral momentum and continuity equations will lead to a set of four first order non-linear differential equations. These
equations are solved by using Runga-Kutta method.

The mathematical model is developed first for the free rectangular jump and then it is extended to simulate the free radial hydraulic jump. To verify the mathematical model the results are compared with the experimental data of previous researchers for the rectangular hydraulic jump, and with the experimental data for the present study for the free radial hydraulic jump.

The experimental investigation was carried out in the hydraulic laboratory of the University of Windsor, Windsor, Ontario, during the period 1977-1979. The IBM 370/3031 computer facilities at the University of Windsor were used in executing all the computations in the mathematical model.
II. LITERATURE REVIEW

2.1 Introduction

The phenomenon of the hydraulic jump has been used since the early days of water resource development. Long before the phenomenon was analyzed mathematically, the hydraulic jump was used as an energy dissipating device on hydraulic structures such as diversion dams and canal regulators.

Although most hydraulic jump stilling basins are rectangular in plan [23, 25, 27 and 77] there are several stilling basins that have diverging side walls. These include stilling basins for sluiceways and critical flow measuring devices, e.g., the Parshall Flume. A few arrangements for stilling basins with diverging side walls have been reported by the U.S. Army corps of Engineers [77,78]. One version of the St. Anthony Falls stilling basin [11], and W. Darcy McKeough Dam and Diversion that is now under construction near Wallaceburg, Ontario [33], incorporate diverging side walls.

In this chapter the available literature related to the subject of this thesis, is presented. The review includes literature on:

a) the rectangular hydraulic jump
b) air entrainment in the hydraulic jump
c) turbulence characteristics in the hydraulic jump
d) the radial hydraulic jump
2.2 The Rectangular Hydraulic Jump

The first study of the hydraulic jump was made by Leonardo da Vinci [60] in 1480. His work, like many of his followers was based on experiments conducted in small horizontal flumes.

A theoretical analysis of the hydraulic jump was first worked out by Belanger [11, 60] in 1828. He applied the one dimensional momentum principle in conjunction with the requirement of continuity. In the absence of the entrained air effect and of bed friction force, this led to the well known equation:

\[
\frac{y_2}{y_1} = \frac{1}{2} \left( -1 + \sqrt{1 + \frac{8F_1^2}{y_1}} \right) \quad \ldots 2.1
\]

in which,

\( y_1 \) = depth upstream of the jump;
\( y_2 \) = depth downstream of the jump;
\( F_1 = \frac{U_1}{\sqrt{gy_1}} \) = the initial Froude number; and
\( U_1 \) = average initial velocity

Since the concept of fluid turbulence and boundary resistance were not understood in Belanger's time, his equation did not incorporate these aspects. Most of the later researchers have observed that his equation yields slightly higher sequent depth values than actually occur.

In 1935, Bakhmeteff and Matzke [6] provided a greater insight into the complexity of the problem. They did a detailed study of the relationship among depth, length and
Froude number, in terms of a one dimensional treatment of the continuity, momentum and energy equations. But they neglected the effect of the friction force on the boundaries. They traced the jump surface profile for Froude numbers 2, 3, 4, 5.5 and 8.6. Moreover, they presented the sequent depth ratio \( y_2/y_1 \), the relative jump length, \( L_3/y_2 \), and the relative energy loss, \( \Delta E/E_1 \), as a function of the initial Froude number \( F_1 \).

In 1963, Govinda Rao and Rajaratnam [18] studied the submerged rectangular hydraulic jump. They obtained a theoretical equation by applying the momentum and continuity equations to the beginning and the end sections of the jump. Their equation reads:

\[
\psi = \left[ \frac{(1 + S)^2}{4} \left( \sqrt{1 + 8F_1^2} - 1 \right)^2 - 2F_1^2 + \frac{4F_1^2}{(1 + S) \left( \sqrt{1 + 8F_1^2} - 1 \right)} \right]^{1/2} 
\]

... 2.2

in which,

\( \psi \) = the ratio between the back-up depth, \( y_3 \), to the supercritical depth, \( y_1 \), Fig.(2.1);

\( S \) = submergence = \( (y_4 - y_2)/y_2 \);

\( y_2 \) = sequent depth for the free jump;

\( y_4 \) = tail water depth of submerged jump; and

\( F_1 \) = supercritical Froude number.

Furthermore, they applied the energy equation to the beginning and the end of the jump to obtain the relative energy loss equation:
\[ E_L = -\frac{1}{2} \left( \sqrt{1 + 8F_1^2} / (1 + S)^2 \right)^2 + \frac{F_1^2}{2} \left[ \frac{4}{(1 + S) \left( \sqrt{1 + 8F_1^2} - 1 \right)} \right]^2 \]

where, 
\[ E_L = \text{energy loss} \]
\[ E_1 = \text{energy at beginning of the jump; and} \]
\[ F_1 \text{ and } S \text{ are defined earlier.} \]

Equations 2.2 and 2.3 have been experimentally verified for different Froude numbers. Rao and Rajaratnam [18] pointed out that the relative energy loss in the submerged jump can be either more or less than the corresponding free jump, depending on the Froude number and the submergence. In addition, they demonstrated experimentally that the high velocities continue along the bed for a considerable distance, thereby potentially causing scour downstream of a stilling basin. Hence they recommended that the submerged jump should not be used for energy dissipation purposes. They concluded that the coincidence between the water surface and the bed pressure profiles for the submerged jump are due to the absence of air. Their study indicated that an intermediate dip in the water surface is distinct at small submergences, and that water surface levels off slowly at high submergences.

In 1965, Rajaratnam [48] studied the free rectangular hydraulic jump as a plane turbulent wall jet under adverse pressure gradient in a confined depth of flow. He omitted
both the entrained air effect and the effect of the backward flow in the surface roller. He reasoned that the momentum and the energy of the backward flow are small compared to the forward flow.

Rajaratnam [48] used a Preston tube to measure the variation of the shear stress at the bed along the jump. Rajaratnam defined the shear stress by the following empirical equation,

\[ \frac{\tau_0}{\rho} = 0.0425 \frac{U_m^2}{\left( \frac{U_m \delta}{\nu} \right)^{2.5}} \]

\[ \tau_0 = \text{shear stress on bed}; \]
\[ \delta = \text{inner layer thickness}; \]
\[ U_m = \text{maximum mean velocity at a vertical}; \]
\[ \nu = \text{kinematic viscosity}; \text{ and} \]
\[ \rho = \text{fluid density}. \]

From Eqn. (2.4), Rajaratnam evaluated the bed friction force

\[ P_f = \int_0^L \tau_0 \, dx \]

He used Eqn. (2.5) to modify the Belanger's equation to account for the effect of the shear force at bed. Thus,

\[ \left( \frac{Y_2}{Y_1} \right)^3 \frac{Y_2}{Y_1} \left[ 1 - \epsilon + 2F_1^2 \right] + 2F_1^2 = 0 \]

in which,

\[ \epsilon = \text{the correction for the bed shear}. \]
Equation (2.6) concurs with the earlier experimental results of Bakhmeteff and Matzka [6], Safranez [63], Bradley and Peterka [9].

Rajaratnam extended the wall jet theory to describe the water surface \((y/y_1)\) and the energy profile \(E/E_1\) as follows:

\[
\left(\frac{y}{y_1}\right)^2 = 1 + 2F_1^2 f' \left(\frac{x}{y_1}\right) - \frac{F_1^2}{4} \left(\sqrt{1 + 8F_1^2} - 1\right) \int_0^\lambda C'_f \, d\lambda \quad \ldots \quad 2.7
\]

and,

\[
\frac{E}{E_1} = \frac{\frac{y}{y_1} + \frac{F_1^2}{2} f'' \left(\frac{x}{y_1}\right) .0632}{1 + F_1^2/2} \quad \ldots \quad 2.8
\]

in which,

\(x\) = distance from the beginning of the jump;
\(y_1\) = supercritical depth;
\(y\) = depth at distance \(x\);
\(f', f''\) = functions of \(F_1\),
\(F_1\) = supercritical Froude number;
\(E\) = energy at distance \(x\);
\(E_1\) = energy at the beginning of the jump;
\(C'_f\) = skin friction coefficient; and
\(\lambda\) = non-dimensional distance = \(x / y_2\).

The results he obtained from Eqn. (2.7) for the water surface were slightly above the observed water surface. This resulting discrepancy may have been caused by omitting the surface roller effect.

It is worthwhile to mention that the data presented by Rajaratnam were for a jump formed near the sluice gate, with a possibly thin boundary layer at the beginning of the jump.
Rajaratnam confirmed experimentally, by colour injection at the bed, that the boundary layer did not separate from the bed during his test.

Rajaratnam [48] concluded that the water depth at the end of the roller \( y_r \) is always less than the sequent depth \( y_2 \), and that the ratio \( y_r / y_2 \) varies linearly with the Froude number, \( F_1 \), in the form,

\[
\frac{y_r}{y_2} = 0.834 + 0.012 F_1 
\]

in which,

\( y_r \) = depth at the end of the roller; and
\( y_2 \) = sequent depth.

In 1965, Rajaratnam [50] studied the submerged jumps as a case of plane turbulent jet under an adverse pressure gradient over which a backward flow had been placed. Based on his experimental results, Rajaratnam presented an analysis of the forward flow in the submerged jump as a plane wall jet. Then the backward flow was evaluated using the result of Henry [21]. Finally, he joined these two terms to predict the characteristics of the submerged jump.

2.2.1 Forward Flow

Rajaratnam [48] found that \( \delta \) and \( U_m \) varied linearly with \( x \) as given by Eqns. (2.10, 2.11).

\[
\frac{\delta}{y_1} = C' + m' \frac{x}{y_1} 
\]

... 2.10
\[
\frac{U_m}{U_1} = C'' - m'' \frac{x}{y_1} \quad \ldots 2.11
\]

in which,
\[
\delta_1 = \text{depth at } U = U_m/2;
\]
\[
U_m = \text{maximum velocity};
\]
\[
U_1 = \text{initial velocity};
\]
\[
C', m' = \text{functions of Froude number}; \text{ and}
\]
\[
C'', m'' = \text{functions of the submergence}.
\]

Rajaratnam used the dimensionless velocity distribution
which is described by Eqns. (2.10 and 2.11) to obtain the
forward continuity, momentum and energy equations as follows:

\[
\frac{Q_f}{Q_1} = .96 \left( C' + m' \frac{x}{y_1} \right) \left( C'' - m'' \frac{x}{y_1} \right) \quad \ldots 2.12
\]

\[
(P + M)_f = .67 \rho U_1^2 y_1 \left( C'' - m'' \frac{x}{y_1} \right)^2 \left( C' + m' \frac{x}{y_1} \right)
+ \gamma \delta_2 \left( \delta_3 + \delta_2/2 \right) \quad \ldots 2.13
\]

\[
E_f = \frac{.495}{2} \rho U_1^3 y_1 \left( C'' - m'' \frac{x}{y_1} \right)^3 \left( C' + m' \frac{x}{y_1} \right) \ldots 2.14
\]

2.2.2 Backward Flow

Rajaratnam [50] used the measurements of the
backward flow velocity of Liu [30] and Henry [21] to plot
dimensionless velocity distributions for the backward flow.
He found a considerable scatter due to the high level of
turbulence in this region. But he fitted this data using
the following simple relation,
\[
\frac{U_s}{U_1} = -0.27 \sin \pi \alpha'
\] ...

in which,

\(U_s\) = surface velocity;

\(\alpha'\) = \(x/L_\alpha\); and

\(L_\alpha\) = roller length

A theory similar to the one used for the forward flow was used to obtain the backward flow, momentum and total energy equation. Rajaratnam found that the momentum and energy for the backward flow were very small, and could be neglected.

2.2.3 **Submerged Jump**

The forward flow and the backward flow were joined together to predict the net forward flow, momentum and the total specific energy for the submerged jump as follows:

\[
q = 0.96q_1 \ (C' + m'x/y_1) \ (C'' - m''x/y_1)
\]

\[
+ F (\alpha') (\psi - 1)
\] ...

\[
P + M = 0.67 \rho U_1^2 y_1 \ (C'' - m''x/y_1)^2 \ (C' + m'x/y_1)
\]

\[
\frac{1}{4} + \frac{1}{2} \gamma (\delta_2 + \delta_3)^2
\] ...

\[
E/y_1 = \frac{0.495}{2} F_1^2 \ (C' - m'x/y_1) \ (C' + m'x/y_1)
\]

\[
+ (\delta_2 + \delta_3)y_1
\] ...

in which,

\(F (\alpha') = -0.184 f(\alpha') \sin \pi \alpha'\);

\(\delta_2\) = depth of the forward flow; and
\( \delta_3 = \) depth of the backward flow

Rajaratnam [50] extended this theory to derive an equation to predict the water surface profiles as;

\[
y/y_1 = g(\alpha') \left( 1 + \xi \right) y_2/y_1 + (\psi - 1) f(\alpha'). ... 2.20
\]

in which,

- \( f(\alpha') \) and \( g(\alpha') \) are empirically determined functions
- \( S = \) submergence = \((y_2 - y_4)/y_2\);
- \( \psi = \) initial depth factor = \( y_3/y_1 \);
- \( y_1 = \) depth of supercritical stream;
- \( y_2 = \) subcritical sequent depth for free jump;
- \( y_3 = \) back up depth, just downstream the gate; and
- \( y_4 = \) tail water depth for submergence jump.

The above theory works reasonably well for the energy equation. However, it predicts lower values for the water surface at \( \alpha' = 0.5 \), i.e., at the middle of the roller. In order to obtain the water surface profile from the above equations, one must have both the tailwater depth, \( y_2 \), and the submergence, \( S \).

In 1968, Rajaratnam [44,49] used the existing data on bed-pressure and water surface profiles to develop a generalized profile for the free rectangular hydraulic jump. He also found that the water surface and the bed pressure profiles could be presented by,

\[
y/y_2 = \bar{A}_1 \left( x/y_2 \right)^2 + \bar{A}_2 \left( x/y_2 \right) ... 2.21
\]

in which,
$A_1$ and $A_2$ are functions of $F_1$;

$x = \text{distance in } x \text{ direction};$ and

$y_2 = \text{sequent depth}.$

In 1972, Resch and Leutheusser [56] demonstrated the importance of the upstream flow characteristics on mean jump flow. Their work showed that the structure of the hydraulic jump with the fully developed upstream flow was quite different from that obtained with nearly potential flow upstream of the jump. The latter condition occurs, in practice, if the jump was formed just downstream of the sluice gate. In this case, the supercritical flow is always made up of two distinct layers, a standard boundary layer, and a potential flow layer above it. They pointed out that the jump in the fully developed upstream flow has a low sequent depth ratio, $y_2/y_1$, and a longer length than the jump with the undeveloped upstream flow. For example, for Froude number, $F_1 = 8$, the jump length ratio, $L_j/y_2 = 16$ for developed upstream flow and for the same Froude number, $L_j/y_2 = 12$ for undeveloped while $L_j/y_2 = 6$ from U.S.B.R [11].

It should be noted that the determination of the jump length tends to be subjective and thus some discrepancy from one study to another should be expected.

In 1975, Narayanan [41] treated the hydraulic jump as a plane turbulent wall jet of an incompressible fluid, of finite width, in an adverse pressure gradient. He presented the velocity distribution by two functions, as follows:
\[
\frac{u}{U_1} = \left[ \frac{y}{\delta} \right]^n \quad \text{For } 0 \leq y \leq \delta \quad \cdots 2.22
\]
\[
u = -U_0 + \frac{U_1 + U_o}{2} \left[ 1 + \cos \pi \frac{y - \delta}{h - \delta} \right] \quad \text{For } \delta < y \leq h \quad \cdots 2.23
\]
in which,

\( U_0 \) = surface velocity;
\( U_1 \) = maximum velocity;
\( n \) = exponent in Blasius equation; and
\( \delta \) = inner layer thickness.

He applied momentum integral techniques to determine the longitudinal variation for (a) the surface profile (b) the maximum velocity, (c) the growth of the boundary and (d) the surface velocity. He also predicted the length of the roller, and the jump length. His results were in general agreement with the available experimental data for \( U_0 \) and \( U_m \) but not \( \delta \).

The theory presented in this thesis provides an improvement to the theory of Narayanan by assuming an error function instead of cosine function for the outer layer velocity distribution. It also accounts for the surface velocity boundary condition and entrained air effect. Further, the turbulent shear is evaluated based on the Prandtl mixing theory, which is calibrated from the experimental results of Rouse [61].

2.3 Air Entrainment

It is well known that air is entrained in the free
hydraulic jump, due to the breaking of large numbers of wave-
lets on the surface and due to the entrainment of air where the
roller meets the supercritical inflow. Air is dispersed to
the lower regions by turbulent mixing and finally the
numerous air bubbles rise to the surface because of the
buoyancy. The mechanism of air entrainment is not yet
completely understood. Reliable field data on the concentra-
tion of entrained air are extremely sparse. However, some
important laboratory experiments were carried out by
Straub and Anderson [73] in 1958, for the air-entrained flow.
They found that the fully developed aerated flow consisted
of two regions. The first, was a lower region in which the
air bubbles were distributed in the water by the turbulent
fluctuations and the second was the upper region in which the
water drops moved in a stream of air. They presented the air
concentration for the lower region as follows:

\[ \overline{C}_x = \overline{C}_1 \left[ \frac{y}{y_T - Y_1} \right]^z \] ... 2.24

in which,

- \( y_T \) = depth of the lower region;
- \( \overline{C}_x \) = air concentration;
- \( \overline{C}_1 \) = concentration at \( y = y_T/2 \);
- \( z = u_b / B \kappa U_* \);
- \( u_b \) = rising air bubbles velocity;
- \( B \) = the ratio of the parameter for bubble transfer
to that for momentum transfer;
- \( \kappa \) = Von Karman's constant; and
\( U_* = \text{shear velocity.} \)

In the upper region, the water drops are distributed according to the Gaussian law, and the concentration is given by the expression,

\[
\frac{1 - \overline{C}_T}{1 - \overline{C}_T} = \frac{2}{h \sqrt{\pi}} \int_{y'}^{\infty} \exp \left[ -(y'/h)^2 \right] \, dy'
\]

in which,

\( \overline{C}_T = \text{concentration at transition depth } y = y_T; \)
\( h = \text{mean height to which water molecules are projected above } y_T; \) and
\( y' = \text{depth above the surface of the lower region.} \)

Based on experimental data of Straub and Anderson [73], Rajaratnam [48] studied the pre-entrained hydraulic jump assuming that:

a) the supercritical stream consists only of the lower stream, which has a depth \( y_T, \) an average air concentration \( \overline{C}_T, \) and moves with a mean velocity \( \overline{u}. \)

b) the air concentration at the end of the jump is zero.

With these assumptions, he applied the momentum equation for the pre-entrained jump to obtain the following:

\[
\phi^3 - [(1 - \overline{C}_T) + 2 (1 - \overline{C}_T) F_{l*}^2 r'^2] \phi + 2n^2 (1 - \overline{C}_T) F_{l*}^2 r'^2 = 0
\]

\[
\phi = \frac{Y_2}{y_T};
\]
\( F_{l*} = \overline{u}/\sqrt{g y_T}; \)
\( \bar{u} = q/\bar{y}; \)
\( \Gamma = \text{experimental coefficient} = 1.12; \)
\( \eta = \bar{y}/[(1 - \bar{C}_T) y_T] = 1.14; \)
\( \bar{y} = \text{mean depth of flow that would exist if all of the} \)
\( \text{entrained air were removed}; \)
\( q = \text{discharge of water}; \) and
\( \bar{C}_T = \text{mean air concentration}. \)

Based on this study, Rajaratnam recommended that the sequent depth should be increased by 10% above the sequent depth obtained from the models so that the pre-entrained jump will not be swept out of the basin.

In 1962, Rajaratnam [46] measured the air concentration using a probe which works on the principle that the presence of air in water changes the electrical resistance between the two probe terminals. He obtained the air concentration along the jump for Froude numbers 2.42 to 8.72. He observed that, at any given section, the concentration decreases towards the bed. Further, he found that the air concentration along the jump increases rapidly toward a maximum value, then decreases rapidly for some distance, and decreases more slowly until it becomes zero at a section approximately 1.6 times the jump length downstream from the jump. Rajaratnam also found that the maximum air concentration, \( C_m \), is related to the Froude number by the equation,

\[ C_m = F_1^{1.35} \]  \( \ldots \) 2.27

where,
\( F_1 = \text{Froude number} \)
\[ C_m = \% \text{ maximum air concentration} \]

In 1974, Resch et al. [55] used a hot film to measure the air concentration in the roller zone of the hydraulic jump. They presented the results as air concentrations to Froude number \( F_1 = 2.8 \) and 6.0. The tests were carried out for the fully developed and the undeveloped supercritical flow. They found that the fully developed supercritical inflow retained its air content longer than the undeveloped flow. Also, they found that the air concentration was maximum at the surface, and at the beginning of the jump, and that it decreased rapidly toward the bed, and the end of the jump.

2.4 **Turbulence Characteristics**

In 1958, Rouse et al. [61] undertook a study of the hydraulic jump problem, combining an analytical approach using the fundamental equations of fluid motion along with experiments on an air-model of the hydraulic jump. Simulating the hydraulic jump, in an air duct shaped according to the experimental profile of the jump, overcame the difficulty of using the hot wire which was unsuitable for use in water. The large number of entrained air bubbles which form fluid discontinuities also make the use of the hot film very difficult. As a result of this model, Rouse et al. [61] were able to determine the turbulence production, the dissipation by viscous action and the connection of the turbulence through the body of the jump, for Froude numbers 2, 4 and 6. The velocity distributions along the jump were also measured for Froude numbers 2, 4 and 6. The energy loss was calculated as
well.

In 1972, Resch and Leutheusser [55, 56, 57] used the hot film anemometer to measure the turbulence characteristic in the hydraulic jump for Froude numbers 2.85 and 6, for the fully developed and the undeveloped supercritical flows. From this study they found that the internal and the external structure of the jump depends on the boundary layer development in the supercritical flow. They pointed out that the turbulence levels in the hydraulic jump are higher in the fully developed flow, than those in the undeveloped flow.

The Leutheusser study indicated that the Reynolds stresses increase as the Reynolds number increases and that the Reynolds stresses decrease as the Froude number increases.

2.5 Radial Hydraulic Jump

Little attention has been given to the analytical aspects of the hydraulic jump in diverging channels, despite the fact that diverging sections have often been used in hydraulic structures [11, 33, 34, 77, 78]. Lack of theoretical development on this type of jump can be attributed to two factors:

1) lack of knowledge of free surface radial flow,

2) the complicated geometry of the diverging sections commonly used in hydraulic structures.

Sloping floors, impact blocks, steps, and other such arrangements in the diverging section made the hydraulic jump too complicated to be solved analytically. It has been common practice in such cases to resort to laboratory experiments.
In the last decade, a few attempts have been made to study free surface radial flow, and the hydraulic jump associated with that flow. These studies are reviewed in this section.

A series of experiments were conducted at Massachusetts Institute of Technology to study the surface profiles, and the hydraulic jump associated with free surface radial flow. These studies were undertaken by Gagnon [17] and Sadler [62]. Gagnon, whose study was primarily about the circular jump, applied the momentum and continuity principles to the entire circular ring containing the jump. He treated the momentum equation as a scalar rather than a vector equation. As a result, he did not introduce the side pressure force in his analysis.

Sadler [62] also studied the hydraulic jump in radial flow. He assumed the length of the jump, \( L_J \), to be constant, \( K_L \), times the height of the jump, i.e. \( L_J = K_L (y_2 - y_1) \). He proposed \( K_L = 4 \). He applied the continuity and the momentum principles in a manner similar to Gagnon. Sadler [62] plotted a series of curves which relate the characteristic of the radial hydraulic jump, for a particular discharge and for different values of bed roughness.

In 1964, Watson [80] studied the radial spread of water in a thin layer, terminated by a circular jump. He analyzed the motion of water by means of the boundary layer theory. He obtained a relationship between the dimensionless
parameters pertinent to the hydraulic jump, the first fluid properties, and the geometry for both laminar and turbulent flows. The dimensionless parameters which Watson proposed are,

$$R_Np = \frac{Q}{va} ; \frac{Q^2}{r_1^2 g y_2^3} ; \frac{r_1 y_2^2 g a^2}{Q^2} ; \frac{r_1}{a} R_Np$$

where,

- $R_Np = \text{jet Reynolds number};$
- $Q = \text{discharge};$
- $r_1 = \text{radius at the toe of the jump};$
- $a = \text{radius of the impinging jet};$
- $g = \text{gravitational accelerations};$ and
- $y_2 = \text{depth at the end of the jump}$

The dimensionless parameter, $Q^2/ r_1^2 g y_2^3$ is proportional to the Froude number at the heel of the jump. The effect of this term was found to be so small that it can be ignored. Watson assumed a jump of zero length, and presented his final results in the form of graphs of $\log [(r_1 y_2^2 g a^2/ Q^2) + (a^2/ 2\pi^2 r_1 y_2)]$ versus $\log [(r_1/a)(R_Np^{-1/3})]$ for both laminar and turbulent flow.

Watson [80] also briefly considered the circular jump from a vertical jet. He postulated that for a large radius, compared to the radius of the impinging jet, the depth of water on the plane is small, and the motion is almost radial, with speed $U_j$ equal to the jet speed. Therefore, the discharge $Q$, can be expressed,

$$Q = 2\pi r y_j U_j = \pi a^2 U_j$$

... 2.28
or,

\[ y = \frac{a^2}{2r} \]  \quad ... 2.29

This indicates that the depth of water flowing radially, on a horizontal plane, is independent of the discharge in a frictionless flow.

Applying momentum and continuity to the entire ring containing the jump, Watson obtained the equation for the circular jump as:

\[ \frac{r_1 y_2^2}{Q^2} \left( 1 - \frac{g y_2 a^4}{2Q^2} \right) = 1 \quad \text{(2.30)} \]

This can be solved for \( y_2 \), once the values of \( Q, r_1 \), and \( a \) are known. In deriving the above equation, the pressure force due to the depth of water at the toe of the jump was neglected. No side pressure force was included.

Watson [80] conducted experiments to verify his theoretical predictions and states, "although a wide scatter, they appear to be consistent with the assumptions of the theory." He conducted experiments on a two foot diameter glass plate, with discharge ranging between 0.00043 and 0.0158 cfs. The experimental conditions were such that radius of the toe of the jump had values between 1.8 in. (25.4 mm) and 7.0 in. (178 mm) and depth \( y_2 \) had values between 0.13 in. (3.3 mm) and 0.65 in. (16.5 mm).

In 1969 Koloseus and Ahmad [28] studied the circular hydraulic jump. They assumed that (a) the liquid was
incompressible, (b) the flow was steady, (c) the frictional shear was negligible, (d) the energy coefficient \( \alpha \) and momentum coefficient \( \beta \) were equal to one, (e) there was no air entrainment within the jump, and (f) the water surface profile of the jump was a straight line. They applied the momentum and continuity equations to infinitesimal elements of the jump, as shown in Fig. (3.1) which led to the equation,

\[
\frac{y_o^3}{2r_o + 1} - \frac{r_o - 1}{2r_o + 1} y_o^2 - \frac{r_o + 6 F_1^2 + 2}{2r_o + 1} y_o + \frac{6 F_1^2}{r_o (2r_o + 1)} = 0
\] ... 2.31

in which,

\[ r_o = \text{radius ratio} = \frac{r_2}{r_1} ; \]
\[ y_o = \text{depth ratio} = \frac{y_2}{y_1} ; \] and
\[ F_1 = \text{Froude number}. \]

They also applied the energy equation to predict the relative energy loss, \( H_L/y_1 \),

\[ H_L/y_1 = \frac{1}{12r_o y_o} \left[ r_o (2r_o + 1) y_o^3 - (r_o^2 + 9r_o - 1) y_o^2 - (r_o^2 - 9r_o - 1) y_o - (r_o + 2) \right] \] ... 2.32

They reported that their experimental results showed trends similar to those of the theoretical equations for the sequent depth and energy. They also found that the circular jump length was less than the corresponding rectangular jump length. However, they found that the relative jump length
\( L_J / y_2 = 3.5 \) to 4.5 depended on Froude number, \( F_1 \) and radius ratio, \( r_0 \).

In 1971, Arbhabhirama and Abella [3] studied the hydraulic jump in a gradually expanding channel. They assumed that the surface profile of the jump was quarter-elliptical with the horizontal major semi-axis equal to the length of the jump \( L_J \), and the vertical minor semi-axis equal to the difference between the two sequent depths \( (y_2 - y_1) \). Their equation of the profile is given by:

\[
\left( \frac{y - y_1}{y_2 - y_1} \right)^2 + \left( \frac{x - L_J}{L_J} \right)^2 = 1.0
\]  

... 2.33

They applied the momentum and continuity equations, in a manner similar to Kholoseus and Ahmad [28] and obtained the following equation for the circular hydraulic jump:

\[
r_o y_o = \frac{1}{2 (\sqrt{1 + 8F_e^2} - 1)}
\]  

... 2.34

in which,

\[
F_e^2 = F_1^2 r_o + C'_p
\]

and,

\[
C'_p = \frac{r_o y_o (r_o - 1)}{(r_o y_o - 1)} \left[ \frac{r_o y_0^2}{3} + 0.118 y_0 + 0.048 \right] + \frac{1}{2}
\]  

... 2.35

Arbhabhirama and Abella carried out experimental tests for the hydraulic jump, within gradually expanding channels with angles of divergence of 5° 27', 7° 13', 9° 3.5', 10° 05', 11° 49' and 13° 04'. They presented their data as a set of figures which relate the characteristic of the hydraulic jump
for particular Froude numbers.

In 1975, Arbhabhirama and Wan[1] studied the characteristics of a circular hydraulic jump created by the radial flow from a circular jet, which spread as a thin layer on a smooth plate. Because the supercritical radial flow was very thin, they considered the boundary layer effects in their study. They assumed that the boundary layer develops radially, from the stagnation point at the center of the jet, and its thickness increased until it reached the surface of the flow. Their study included the laminar and the turbulent boundary layer for supercritical flow. They found that the characteristics of the circular jump were therefore affected by the Reynolds number, the tailwater conditions, and the Froude number.

They applied the momentum and the continuity principles, in a manner similar to Koloseus and Ahmed [28]. They assumed that the jump water surface was linear and they estimated the value of the momentum coefficient, $\beta_1$, based on the velocity distribution of Watson [80] for the supercritical flow. They obtained values for $\beta_1 = 1.6$, for the laminar boundary layer, and $\beta_1 = 1.0$ for the turbulent boundary layer. The momentum equation showed that the tailwater depth was strongly affected by the $\beta_1$ values, i.e., by the laminar boundary layer.

2.6 Critical Evaluation of the Available Literature

An examination of the extensive publications, on the subjects of rectangular and radial hydraulic jumps, reveals that there is no complete numerical model available to
simulate the internal structure of the rectangular or the radial hydraulic jumps.

A considerable amount of the research on the hydraulic jump is limited to the macroscopic approach which deals with the flow outside the jump body. With this approach, the friction force and the air entrainment effect are generally ignored.

The air entrainment effect is necessary to evaluate the pressure force, and the side pressure force in the radial hydraulic jump. In previous studies this effect was ignored. The damping of the turbulent shear due to the air entrainment effect in the jump has not been defined yet.

A mathematical model has not yet been developed to simulate the internal structure of the jump and account for the entrained air effect, the turbulent shear effects, turbulent pressure effects, the bed friction force and arbitrary inflow conditions. It was felt that the above mentioned topics warranted further research with the objective of developing a more complete model for the internal flow in hydraulic jumps. In order to achieve this goal, an extensive experimental programme was carried out on the free radial and submerged radial jumps. The results of this experimental programme were required in order to verify the proposed mathematical models for the radial hydraulic jump.

Rajaratnam's [48] skin shear function as well as the Prandtl mixing length hypothesis were used with the integral momentum and continuity equations to develop the mathematical model.
Other mathematical turbulence models such as $K\ell$ and $K\varepsilon$ and $K-w-g$ models [29] were considered but these need long computer time to simulate the problem. However, in the present study the Prandtl's mixing length hypothesis was sufficient.
III. THEORETICAL DEVELOPMENTS

3.1 General Comment

The developments in this chapter are treated in two parts:

a) The Macroscopic Approach in which the continuity and momentum principles are applied between the inlet section and the end section of the jump to develop general equations for the free radial hydraulic jump and for the submerged radial hydraulic jump.

b) The Microscopic Approach in which the strip integral method is used to integrate the Reynolds equation and the continuity equation for selected strips to describe the internal structure of the jump. This model was developed first for the rectangular jump and then extended to solve the radial hydraulic jump. The rectangular case was treated first because of the extensive experimental data available, particularly on the following topics: turbulence, characteristics, entrained air, bed friction and the water surface profiles, which are needed to verify the model.

3.2 Macroscopic Approach

In deriving the equation of the free and submerged radial hydraulic jump the following assumptions were made:

a) The liquid is incompressible;

b) The flow is steady;

c) The Frictional shear along the floor, over the length of the jump is negligible;
d) The pressure distribution is hydrostatic before and after the jump;

e) The initial kinetic energy correction factor, $\alpha_1$, is unity;

f) The initial momentum coefficient, $\beta_1$, is unity; and

g) The surface profile of the radial jump is a second degree polynomial

3.2.1 Free Radial Hydraulic Jump

The free radial hydraulic jump equation can be obtained by applying the momentum and the continuity equations to the element of the jump shown in Fig. (3.1) considering the $r$-axis as a reference direction:

$$ P_1 + 2P_s \sin \theta/2 - P_2 = \rho Q [U_2\beta_2 - U_1\beta_1] $$ \hspace{1cm} (3.1)

in which,

$$ P_1 = \text{the hydrostatic pressure force at Section 1 in the radial direction} = \gamma \gamma_2 r_1 \sin \theta/2 $$ \hspace{1cm} (3.2)

$$ P_2 = \text{The hydrostatic pressure force at Section 2 in the radial direction} = \gamma \gamma_2 r_2 \sin \theta/2 $$ \hspace{1cm} (3.3)

$$ P_s = \text{The side pressure force} = \int_{r_1}^{r_2} \frac{\gamma \gamma_2}{2} \, dr $$ \hspace{1cm} (3.4)

$$ U_1 = \text{The velocity at Section 1} = \frac{0}{2r_1 \gamma_1 \sin \theta/2} $$ \hspace{1cm} (3.5)
\[ u_2 = \text{The velocity at Section 2} \]

\[ = \frac{Q}{2r_2y_2 \sin \theta/2} \quad \ldots \quad 3.6 \]

The effective surface profile was assumed to be of the form,

\[ Y = AR^2 + BR \quad \ldots \quad 3.7 \]

in which,

\[ Y = \frac{Y - Y_1}{Y_2 - Y_1} ; \quad R = \frac{r - r_1}{r_2 - r_1} \quad \ldots \quad 3.8 \]

A and B are functions of initial Froude number, \( F_1 \) and the entrained air fraction. Substitution of Eqn. (3.7) into Eqn. (3.4) gives,

\[ P_s = \frac{Yr_1 y_1}{2} \left\{ (Y_0 - 1)^2 F' + (Y_0 - 1) G + r_0 - 1 \right\} \]

\[ \ldots \quad 3.9 \]

in which,

\[ r_0 = \frac{r_2}{r_1} ; \quad Y_0 = \frac{Y_2}{Y_1} \]

\[ F' = \frac{1}{(r_0 - 1)^4} \left\{ \frac{r_0^5 - 1}{5} A^2 + \frac{r_0^4 - 1}{4} (r_0 - 1) [2AB \]

\[ - \frac{4A^2}{r_0 - 1} ] + \frac{r_0^3 - 1}{3} [6A^2 + B^2 (r_0^2 - 2r_0 + 1) \]

\[ + 6AB (1 - r_0)] + \frac{r_0^2 - 1}{2} [-4A^2 + 6AB (r_0 - 1) \]

\[ \ldots \quad 3.10 \]
\[ + B^2 \left( -2r_o^2 + 4r_o - 2 \right) + (\dot{r}_o - 1) \left[ A^2 + 2AB \left( 1 - r_o \right) \right] \]
\[ + B^2 \left( r_o^2 - 2r_o + 1 \right) \]
\[ \text{and,} \]
\[ G' = \frac{1}{(r_o - 1)^2} \left\{ \frac{2A}{3} \left( r_o^3 - 1 \right) + \frac{r_o^2 - 1}{2} \left[ -4A + 2B(r_o - 1) \right] \right\} \]
\[ + (r_o - 1) \left[ 2A + 2B \left( 1 - r_o \right) \right] \]

Substitution of Eqns. (3.2), (3.3), (3.5), (3.6) and (3.9) into Eqn. (3.1) leads to:

\[ Y_o^3 \left[ r_o^2 - F'r_o \right] + Y_o^2 \left[ 2r_o F' - Gr_o \right] + Y_o \left[ Gr_o - F'r_o \right] \]
\[ - r_o^2 - 2r_o \beta_1 F_1^2 \left( F_1^2 \right) + 2F_1^2 \beta_2 = 0.9 \]

in which,

\[ F_1 = \frac{U_1}{\sqrt{gY_1}} \]

Equation (3.12) is the general equation of the free radial hydraulic jump; it involves seven variables \( r_o \), \( y_o \), \( F_1 \), \( \beta_1 \), \( A \) and \( B \). The coefficients \( A \) and \( B \) introduce the effect of the entrained air and the effective free surface profile; while \( \beta_1 \) and \( \beta_2 \) are the entrance and exit momentum coefficients respectively.

It is interesting to note that for \( r_o = 1 \), Eqn. (3.12) reduces to,

\[ Y_o^3 - (1 + 2F_1^2) Y_o + 2F_1^2 = 0 \]
which is the equation of the free rectangular jump.

The energy loss in the radial hydraulic jump can be obtained from applying the energy and continuity equations between Sections 1 and 2, Fig. (3.1) as follows:

\[
\Delta E = E_1 - E_2 = \alpha_1 \frac{U_1^2}{2g} + y_1 - \alpha_2 \frac{U_2^2}{2g} - y_2 \quad \cdots \quad 3.14
\]

\[
\frac{\Delta E}{E_1} = 1 - \left[ \frac{\alpha_2 \frac{F_1^2}{2} + 2r_o^2 y_0^3}{r_o^2 y_0^2 \left( \frac{F_1^2}{2} \alpha_1 + 2 \right)} \right] \quad \cdots \quad 3.15
\]

in which,

\( \Delta E \) = the energy head lost in the jump; and

\( E_1 \) = the specific energy at the beginning of the jump

3.2.2 Submerged Radial Hydraulic Jump

The submerged radial hydraulic jump equation is obtained by applying the continuity and the momentum equations to the element in Fig. (3.2) i.e.,

\[
P_1 + 2 P_S \sin \theta/2 - P_2 = \rho \Omega (U_2 - U_1) \quad \cdots \quad 3.16
\]

in which,

\( P_1 \) = the hydrostatic pressure force at Section 1 in the radial direction = \( \gamma y_3^2 r_1 \sin \theta/2 \) \quad \cdots \quad 3.17

\( P_2 \) = the hydrostatic pressure force at Section 2 in the radial direction = \( \gamma y_4^2 r_2 \sin \theta/2 \) \quad \cdots \quad 3.18

\( P_S \) = the side pressure force at Section 2 in the radial direction = \( \frac{Y}{2} B_0 \frac{Y^2}{r_2 - r_1} \) \quad \cdots \quad 3.19
\( B_* \) = correction factor for the side force.
\[
\bar{y} = \text{representative depth} = \sqrt{y_3 y_4} \quad \cdots \quad 3.20
\]
\( U_1 = \text{velocity at Section 1} = \frac{Q}{2 r_1 y_1 \sin \theta / 2} \quad \cdots \quad 3.21\]
\( U_2 = \text{velocity at Section 2} = \frac{Q}{2 r_2 y_2 \sin \theta / 2} \quad \cdots \quad 3.22\]

Substitution of Eqn. (3.17) to (3.22) into Eqn. (3.16) leads to,
\[
\psi = -0.5A_* + \sqrt{.25A_*^2 + r_o y_o^2 + 2F_1^2 \left( \frac{1}{r_o y_o} - 1 \right)} \quad \cdots \quad 3.23
\]
in which,
\[
\psi = \frac{y_3}{y_1} ; \quad y_o = \frac{y_4}{y_1} ; \quad r_o = \frac{r_2}{r_1} ; \quad F_1 = \frac{U_1}{\sqrt{g y_1}} \quad \text{and} \quad A_* = y_o (r_o - 1) B_*
\]

Equation (3.23) is the general dimensionless equation for the submerged radial hydraulic jump, and it includes four variables: inlet depth factor \( \psi \); tailwater depth ratio, \( y_o \); radius ratio, \( r_o \); and initial Froude number, \( F_1 \).

The energy loss in the submerged radial hydraulic jump can be obtained by applying the energy and continuity equations between Sections 1 and 2 shown in Fig. (3.2), this gives,
\[
\frac{\Delta E}{E_1} = 1 - \left[ \frac{0.5F_1^2 + r_o^2 y_o^3}{(r_o y_o)^2 (0.5F_1^2 + \psi)} \right] \quad \cdots \quad 3.24
\]
in which,
\[
\Delta E = \text{energy loss in the jump} \quad \text{and} \quad E_1 = \text{energy at the beginning of the jump}.
\]
3.3 The Microscopic Momentum Balance

In this section the free rectangular hydraulic jump is treated first, then the theory is extended to solve the free radial hydraulic jump.

The following analysis of the hydraulic jump is based on the assumption that the flow region downstream of the potential core (of the incoming jet) can be divided into two distinct layers [40, 41] i.e.

a) the inner layer

and,

b) the outer layer

3.3.1 The Inner Layer

This is the layer which is adjacent to the wall, where the effects of the wall predominate. The flow in the inner layer of a hydraulic jump behaves in a manner similar to the flow in the inner layer of a turbulent wall jet [24, 65]. Therefore the velocity distribution can be approximated by a function such as the 1/7th power law or,

$$ u = U_m \left( \frac{y}{\delta} \right)^{1/7} \quad 0 < y < \delta \quad \ldots \quad 3.25 $$

in which,

$u =$ velocity at $y$;

$U_m =$ maximum velocity;

$\delta =$ distance from the boundary to the point of maximum velocity; and

$y =$ the vertical distance measured from the bed
3.3.2 The Outer Layer

This is the layer away from the wall in which the inertial and gravity forces predominate over the viscous forces, and the effect of the wall on the flow field is negligible. Based on the experimental data of Rajaratnam [46, 48] and Rouse [61], the velocity distribution in this region can be represented by an error function as follows:

\[ u = -U_o + (U_o + U_m) e^{4c \left[ \frac{(y-\delta)}{(h-\delta)} \right]^2} \quad ; \delta < y < h \ldots 3.26 \]

in which,

\[ U_o = \text{the velocity at } y = \infty \]
\[ h = \text{depth of flow; and} \]
\[ c = \text{constant obtained by Rajaratnam [51]} \]
\[ = -0.693 \]

3.3.3 Equations of Motion

The Reynold's equation for the two dimensional flow in the Cartesian Co-ordinate System can be written as [24, 31, 51, 65].

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \left( \frac{\partial u'^2}{\partial x} + \frac{\partial u'v'}{\partial y} \right) \]

in which,

\[ u = \text{the mean time velocity component in the x direction;} \]
\[ v = \text{the mean time velocity component in the y direction;} \]
\[ u' = \text{the fluctuating component of the velocity in the x direction;} \]
\[ v' = \text{the fluctuating components of the velocity in the y direction;} \]
\[ \rho = \text{the mass density} \]

\[ \nu = \text{the kinematic viscosity of the fluid; and} \]

\[ p = \text{hydrostatic pressure} \]

The velocity gradient in the \( y \) direction is generally much greater than the velocity gradient in the \( x \) direction. Therefore \( \partial^2 u / \partial x^2 \) will tend to be much less than \( \partial^2 u / \partial y^2 \) and may be neglected [24, 65]. Similarly, the fluctuating velocity gradient \( \partial u' \partial^2 / \partial x \) will also tend to be small and it is neglected [24, 65]. These assumptions simplify the Reynolds equation to:

\[
\frac{u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\partial u' v'}{\partial y} \quad \cdots \quad 3.28
\]

Combining the second and third terms on the right hand side, the above equation can be rewritten as:

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} - \rho u' v' \right) \quad \cdots \quad 3.29
\]

or,

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \quad \cdots \quad 3.30
\]

in which,

\[ \tau = \text{total shear} = \tau_1 + \tau_t \]

\[ \tau_1 = \text{viscous shear stress} = \mu \frac{\partial u}{\partial y} \quad \text{and} \]

\[ \tau_t = \text{turbulent shear stress} = -\rho u' v' \]

Assuming a hydrostatic pressure distribution at any
vertical section along the jump, then the pressure \( p \) can be replaced by \( \gamma h \). Finally the equation of motion in the \( x \) direction can be rewritten as:

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial h}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y}
\]  

... 3.31

3.3.4 Strip-Integral Method

The strip-integral method is simply one approach to integrating the equations of motion by dividing the velocity distribution into strips and satisfying an integral of the momentum equation for each strip. This method has been used to solve many boundary layer problems [38]. The method used here is a refinement of that proposed by Narayanan [41].

3.3.5 Integral Equations

The method of the integral equations originally was developed as a tool for systematically reducing partial differential equations to ordinary differential equations in order to facilitate the solution [38].

The integral equations can be formed by multiplying the differential equation by \( d\gamma \) and integrating with respect to \( y \). Thus the integral equation of motion and the integral continuity equation, with appropriate limits, can be rewritten respectively as,

\[
\int u \frac{\partial u}{\partial x} \, dy + \int v \frac{\partial u}{\partial y} \, dy = -g \int \frac{\partial h}{\partial x} \, dy + \frac{1}{\rho} \int \frac{\partial \tau}{\partial y} \, dy \quad \ldots \ 3.32
\]

\[
\int \frac{\partial u}{\partial x} \, dy + \int \frac{\partial v}{\partial y} \, dy = 0 \quad \ldots \ 3.33
\]
3.3.6 Shear Functions

In order to complete the analysis of the integral equation, it is necessary to employ auxiliary equations to describe the shear stress in the last term in Eqn. (3.32). In this study two equations were used to obtain the shear stress. An equation for the shear at the bed was suggested by Rajaratnam [48], i.e.,

\[
\frac{\tau_0}{\rho} = \frac{0.0424}{[(U_m \delta)/\nu]^{1.25}} U_m^2 \quad \frac{U}{2} \quad \ldots \quad 3.34
\]

in which,

\( \tau_0 \) = shear stress at the bed;  
\( \rho \) = mass density;  
\( U_m \) = maximum velocity;  
\( \delta \) = inner layer thickness; and  
\( \nu \) = kinematic viscosity.

The Prandtl mixing length theory [12, 31] was used in the outer layer, i.e.,

\[
\frac{\tau_f}{\rho} = -u'v' = \ell^2 \frac{\partial u}{\partial y} \left| \frac{\partial u}{\partial y} \right| \quad \ldots \quad 3.35
\]

in which,

\( \ell \) = mixing length = \( bD_* \);  
\( D_* \) = constant  
\( \frac{\partial u}{\partial y} \) = the velocity gradient.

With these auxiliary Eqns. (3.34) and (3.35) and the velocity distribution from Eqns. (3.25) and (3.26), the partial
integration of the equation of motion can be completed. The velocity distribution can be evaluated at any section along the jump if the integral momentum equation is applied to a number of strips, equal to the number of unknowns minus one, the integral continuity equation is applied for all the domain and a set of initial conditions.

3.3.7 Formulation for the Free Rectangular Hydraulic Jump

3.3.7.1 Procedure

To develop the mathematical model for the free rectangular jump, the continuity equation is integrated from 0 to \( h \), and the momentum equation is integrated for three strips from 0 to \( \delta \), \( \delta \) to \( h \) and \( \delta \) to \( y_* \) (\( y_* \) is the point of maximum shear \( (\partial u^2)/(\partial y^2) = 0 \)) as shown in Fig. (3.3).

Equations (3.34) and (3.35) were used to evaluate the shear stress at the bed and at \( y_* \) respectively. Eqns. (3.25) and (3.26) were used to describe the velocity distribution profile, which is required: 1) to permit the effects of nonuniformity in mass and momentum across the layer to be considered in the governing integral equations and 2) to permit the calculation of the shear stress from Eqns. (3.34) and (3.35).

3.3.7.2 Integral Continuity Equation (0 to \( h \))

Integrating Eqn. (3.33) from 0 to \( h \),

\[
\int_0^h \frac{\partial u}{\partial x} \, dy + \int_0^h \frac{\partial v}{\partial y} \, dy = 0
\]

... 3.36
or,
\[ \int_{0}^{h} \frac{\partial v}{\partial y} \, dy = - \int_{0}^{h} \frac{\partial u}{\partial x} \, dy \] 
\[ \quad \ldots \text{3.37} \]

Integrating the left hand side of the Eqn. (3.37)
\[ v \left[ \int_{0}^{h} \right] = - \int_{0}^{h} \frac{\partial u}{\partial x} \, dy \] 
\[ \quad \ldots \text{3.38} \]

and applying the boundary conditions,
\[ v = 0 \text{ at } y = 0 \]

and,
\[ v = U_s \frac{dh}{dx} \text{ at } y = h \]

Equation (3.38) becomes,
\[ \left[ -U_o + (U_m + U_o) e^{4c} \right] \frac{dh}{dx} = - \int_{0}^{h} \frac{\partial u}{\partial x} \, dy \] 
\[ \quad \ldots \text{3.39} \]

in which, \( U_s \) is the surface velocity, which can be obtained from the outer layer velocity distribution, Eqn. (3.26).

Substituting for \( u \) from Eqn. (3.25) and (3.26) into Eqn. (3.39) led to:
\[ \left[ -U_o + (U_m + U_o) e^{4c} \right] \frac{dh}{dx} = - \int_{0}^{\delta} \frac{\partial u}{\partial x} U_m \left[ \frac{Y}{\delta} \right]^{1/7} \, dy - \int_{0}^{h} \frac{\partial u}{\partial x} \]
\[ \quad \left. \left[ (-U_o + (U_o + U_m) e^{4c} (Y-\delta)/(h-\delta) ) \right]^{2} \right] \, dy \] 
\[ \quad \ldots \text{3.40} \]

Completing the differentiation of Eqn. (3.40)

with respect to \( x \), led to the first order differential
equation in the following form:

\[ A'_1 \frac{dh}{dx} + B'_1 \frac{d\delta}{dx} + C'_1 \frac{dU_o}{dx} + D'_1 \frac{dU_m}{dx} = 0 \]  \[ \text{... 3.41} \]

in which,

\[ A'_1 = -\frac{8c}{3} e^{4c} U_t S (4c, 5/2) + U_s \]
\[ B'_1 = \frac{8c}{3} e^{4c} U_t S (4c, 5/2) - U_t \cdot (e^{4c} + 1) - \frac{nU_m}{n+1} \]
\[ C'_1 = -h + \delta + e^{4c} (h - \delta) S (4c, 3/2) \]
\[ D'_1 = \frac{\delta}{n+1} + e^{4c} (h - \delta) S (4c, 3/2) \]
\[ U_t = U_o + U_m \]

\[ S(Kc, c') = 1 - \frac{Kc}{c'} + \frac{(Kc)^2}{c'(c'+1)} - \frac{(Kc)^3}{c'(c'+1)(c'+2)} + \ldots \]

3.3.7.3 Integral Momentum Equation for the Strip

(0 to \(\delta\))

Integrating Eqn. (3.31) from 0 to \(\delta\),

\[ \int_0^\delta u \frac{\partial u}{\partial x} \, dy + \int_0^\delta v \frac{\partial v}{\partial y} \, dy = -g \int_0^\delta \frac{\partial h}{\partial x} \, dy + \frac{1}{\rho} \int_0^\delta \frac{\partial \tau}{\partial y} \, dy \]  \[ \text{... 3.42} \]

Integrating the second term on the left hand side of Eqn. (3.42) by parts, gives;

\[ \int_0^\delta v \frac{\partial u}{\partial y} \, dy = uv \int_0^\delta - \int_0^\delta u \frac{\partial v}{\partial y} \, dy \]  \[ \text{... 3.43} \]

Applying the boundary conditions:

\[ u = v = 0 \quad \text{at} \quad y = 0, \quad \text{and} \]
\[ u = U_m, \quad v = V_\delta \quad \text{at} \quad y = \delta \]
gives,
\[ = U_m V_\delta - \int_0^\delta u \, dv \quad \ldots \quad 3.44 \]
Substituting for \( v \) from the continuity Eqn. (3.36) gives,
\[ = -U_m \int_0^\delta \frac{\partial u}{\partial x} \, dy + \int_0^\delta u \frac{\partial u}{\partial x} \, dy \quad \ldots \quad 3.45 \]
Substituting Eqn. (3.45) into Eqn. (3.42) leads to
\[ \int_0^\delta u \frac{\partial u}{\partial x} \, dy - U_m \int_0^\delta \frac{\partial u}{\partial x} \, dy + \int_0^\delta u \frac{\partial u}{\partial x} \, dy = -\varrho \int_0^\delta \frac{\partial h}{\partial x} \, dy \]
\[ + \frac{1}{\rho} \int_0^\delta \frac{\partial \tau}{\partial y} \, dy \quad \ldots \quad 3.46 \]
Substituting for \( u \) from Eqn. (3.25) into Eqn. (3.46), then integrating with respect to \( y \), and applying the boundary conditions,
\[ \tau = 0 \quad \text{at} \quad y = \delta \quad , \quad \text{and} \]
\[ \tau = \tau_0 \quad \text{at} \quad y = 0 \]
with \( \tau_0 \) obtained from Eqn. (3.34), Eqn. (3.46) becomes:
\[ A_2' \frac{dh}{dx} + B_2' \frac{d\delta}{dx} + D_2' \frac{du_m}{dx} + E_2' = 0 \quad \ldots \quad 3.47 \]
in which,
\[ A_2' = g \delta \]
\[ E'_2 = \frac{nU_m^2}{n+1} - \frac{2nU_m^2}{2n+1} \]
\[ D'_2 = \frac{2U_m}{2n+1} - \frac{U_m}{n+1} \]
\[ E'_2 = 0.0424 \left( \frac{U_m}{\nu} \right) \frac{U_m}{2} \]

3.3.7.4 Integral Momentum Equation for the Strip \((\delta - h)\)

Integrating Eqn. (3.32) from \(\delta\) to \(h\):

\[
\int_{\delta}^{h} u \frac{\partial u}{\partial x} dy + \int_{\delta}^{h} v \frac{\partial u}{\partial y} dy = -g \int_{\delta}^{h} \frac{\partial P}{\partial x} dy + \frac{1}{\rho} \int_{\delta}^{h} \frac{\partial \tau}{\partial x} dy
\]

and proceeding in a similar fashion to Section 3.3.7.3 with the boundary conditions,

\[ u = U_s \quad \text{and} \quad v = V_h \quad \text{at} \quad y = h \]
\[ u = U_m \quad \text{and} \quad v = V_\delta \quad \text{at} \quad y = \delta \]

then Eqn. (3.48) becomes,

\[
2 \int_{\delta}^{h} u \frac{\partial u}{\partial x} dy - U_s \int_{\delta}^{h} \frac{\partial u}{\partial x} dy + U_m \int_{\delta}^{h} \frac{\partial u}{\partial y} dy = -g \int_{\delta}^{h} \frac{\partial h}{\partial x} dy + \frac{1}{\rho} \int_{\delta}^{h} \frac{\partial \tau}{\partial x} dy
\]

It is assumed that the bulking effect resulting from the air entrainment will increase the pressure force in
the layer δ to h. Therefore, a correction factor is added to the first term on the right hand side of Eqn. (3.49) as follows:

\[
\text{The increase in the pressure gradient/unit mass} = g \frac{C_0}{2} \left[ \frac{\partial h}{\partial x} (2h - \delta) - \frac{3\delta}{\partial x} h \right] \quad \ldots \quad 3.50
\]

in which \( C_0 \) is average air concentration at a distance \( x \) along the jump. It can be evaluated by using the experimental data of Rajaratnam (46) which can be approximated by:

\[
C'_0 = 0.066 F_1 \frac{X}{L_J} \text{ for } \frac{X}{L_J} < 0.2 \quad \ldots \quad 3.51
\]

\[
C'_0 = 0.0115 F_1 \left( 1 - \frac{X}{1.6L_J} \right) \text{ for } \frac{X}{L_J} > 0.2 \quad \ldots \quad 3.52
\]

Substituting from Eqn. (3.25) and Eqn. (3.26) into Eqn. (3.49) and integrating with respect to \( y \), and applying the boundary conditions,

\[
\tau = 0 \quad \text{at} \quad y = \delta
\]

and,

\[
\tau = 0 \quad \text{at} \quad y = h
\]

Eqn. (3.52) becomes,

\[
A'_3 \frac{dh}{dx} + B'_3 \frac{d\delta}{dx} + C'_3 \frac{du}{dx} + D'_3 \frac{du}{dx} = 0 \quad \ldots \quad 3.53
\]

in which,

\[
A'_3 = g \left( h - \delta \right) + \frac{8}{3} U_s U_t c e^{4c} S \left( 4c, 5/2 \right) + U_s^2
\]

\[
+ \frac{16}{3} U_s U_t c e^{4c} S \left( 4c, 5/2 \right) - \frac{16}{3} U_t^2 e^{8c} S \left( 8c, 5/2 \right)
\]

\[
+ g/2 \left( 2h - \delta \right) C'_0
\]
\[ B_3 = \frac{U_m U_s n}{n + 1} - \frac{U_m 2n}{n + 1} - \frac{8}{3} U_s U_t c e^{4c} S \quad (4c, 5/2) \]

\[ + U_s U_t (e^{4c} - 1) - \frac{16}{3} U_s U_t c e^{4c} S \quad (4c, 5/2) \]

\[ + 2U_s U_t (e^{4c} - 1) + 2 U_t^2 \left( \frac{8c}{3} e^{8c} \right) S \quad (8c, 5/2) \]

\[-.5 (e^{8c} - 1)] - g C_\circ h/2 \]

\[ C_3 = U_s (h - \delta) - U_s e^{4c} (h - \delta) S \quad (4c, 3/2) + 2 U_s (h - \delta) \]

\[-2(U_t + U_s) e^{4c} (h - \delta) S (4c, 3/2) \]

\[ + 2U_t e^{8c} (h - \delta) S (8c, 3/2) \]

\[ D_3 = -\frac{U_s \delta}{n + 1} + \frac{U_m \delta}{n + 1} - U_s e^{4c} (h - \delta) S \quad (4c, 3/2) \]

\[-2U_s e^{4c} (h - \delta) S \quad (4c, 3/2) + 2U_t e^{8c} (h - \delta) S \quad (8c, 3/2) \]

3.3.7.5 Integral Momentum Equation for the Strip

(\(\delta\) to \(y^*_x\))

Integrating Eqn. (3.32) from \(\delta\) to \(y^*_x\)

\[ \int_\delta^{Y^*_x} u \frac{\partial u}{\partial x} dy + \int_\delta^{Y^*_x} v \frac{\partial u}{\partial y} dy = -g \int_\delta^{Y^*_x} \frac{\partial h}{\partial x} dy + \frac{1}{\rho} \int_\delta^{Y^*_x} \frac{\partial \tau}{\partial y} dy \]

\[ \ldots \quad 3.54 \]

and proceeding in a similar fashion as in Section 3.3.7.3 with
the boundary conditions:
\[ u = U_m, \quad v = V_0 \] at \( y = \delta \) and,
\[ u = -U_0 + U_t e^{-0.5}, \quad v = V_{y*} \] at \( y = y_* = \sqrt{1 - 8c} (h - \delta) + \delta \)
yields,
\[
2 \int_0^{Y_*} u \frac{\partial u}{\partial x} \, dy - \left( -U_0 + U_t \cdot e^{\frac{h}{L}} \right) \int_0^{Y_*} \frac{\partial u}{\partial x} \, dy + U_m \int_0^{\delta} \frac{\partial u}{\partial x} \, dy
= -g \int_0^{Y_*} \frac{\partial h}{\partial x} \, dy + \frac{1}{\rho} \int_0^{Y_*} \frac{\partial \tau}{\partial y} \, dy \quad \ldots \quad (3.55)
\]

The increase in the pressure gradient terms due to the entrained air in this layer, as mentioned in the previous integral equation, is an increase in pressure gradient/unit mass
\[
= \frac{\partial^2}{\partial x^2} \left[ \frac{\partial h}{\partial x} (2h - \delta) - \frac{2\delta}{\partial x} h \right] \quad \ldots \quad (3.56)
\]
This term should be added to the first term on the right hand side in Eqn. (3.55) to include the effect of the entrained air on the pressure gradient terms. Consider the remaining term in Eqn. (3.55) which is,
\[
\frac{1}{\rho} \int_0^{Y_*} \frac{\partial \tau}{\partial y} \, dy \quad \ldots \quad (3.57)
\]
In order to evaluate this term the turbulent shear at \( y_* \) from Prandtl mixing length formula can be utilized, i.e.
\[
\frac{\tau_t}{\rho} = \chi^2 \frac{\partial u}{\partial y} \left| \frac{\partial u}{\partial y} \right| \quad \ldots \quad (3.58)
\]
in which \( l = \text{mixing length} = D_\ast (h - \delta) \).

Substituting for \( u \) from Eqn. (3.26) and differentiating the right hand side of Eqn. (3.58) with respect to \( y \) leads to,

\[
\frac{\tau_{\tau a}}{\rho} = \kappa^2 \left[ \frac{h - \delta}{2} \right]^2 \left[ U_t \epsilon \right]^2 4c \left[ \frac{y - \delta}{h - \delta} \right]^2 \frac{8c}{(h - \delta)^2} D_\ast
\]

... 3.59

The shear stress at \( y_\ast \) becomes:

\[
\left( \frac{\tau_{\tau a}}{\rho} \right)_{y_\ast} = -8 c U_t^2 D_\ast / \epsilon
\]

... 3.60

where \( D_\ast = 0.048 \) which is obtained from the experimental data from the air model studied by Rouse et al. (61).

Another correction was applied to Eqn. (3.60) for the effect of entrained air. The entrained air affects the turbulent shear in two opposing ways. It is well known that increasing the stability of the water column, as measured by Richardson number, \( R_i \), reduces the intensity of turbulent mixing (8, 42); on the other hand, air bubbles rising from a high velocity to a low velocity region will increase the vertical momentum transfer.

The vertical turbulent mixing coefficient can be taken as a function of Richardson's number which is defined as,

\[
R_i = g \left[ \frac{\rho}{\rho} \right] \left( \frac{\partial u}{\partial y} \right) \left( \frac{\partial u}{\partial y} \right)
\]

... 3.61
in which,

\[ \rho = \text{density at } y \]

\[ \bar{\rho} = \text{the mean density} \]

The vertical air distribution obtained by Resch et al. [55] and the variation of air concentration along the jump measured by Rajaratnam [46] were used to evaluate the density gradient \((\partial \rho / \partial y)/(\partial y)\) from which Richardson's number can be represented by the following function: (See Appendix A):

\[ R_{iy*} = g \frac{C_o}{-8c} \frac{h (N + 1) N e}{(N+1)^2 (U_o + U_m)} \]

... 3.62

in which,

\[ N = \left( \frac{U_m + U_s}{U_1} \right)^2 \quad \text{for } \frac{x}{L_J} < 0.2 \]

\[ N = \frac{16 u_b}{U_m + U_s} \quad \text{for } \frac{x}{L_J} > 0.2 \]

\[ u_b = \text{The rising air bubble velocity} \]

Odd and Roger [42] found that the damped mixing length \( \ell_p \) could be represented in the form,

\[ \ell_p = l / \sqrt{1 + E \cdot R_{iy*}} \]

... 3.63

in which,

\[ E_* = 160 \]

The rising air bubbles will tend to increase the vertical momentum transfer. This effect will depend on the bubble velocity \( u_b \) as well as the horizontal fluid velocity,
for this study the following empirical relationship was found,

\[ 4.3 \left( \frac{U_m + U_s}{U_b} \right)^2 \leq E_\star \leq 160 \quad \ldots \quad 3.64 \]

From Eqn. (3.60), Eqn. (3.62) and Eqn. (3.63) the turbulent shear at \( y_\star \) can be evaluated as follows:

\[ \left[ \frac{\tau_{to}}{\rho} \right]_{y_\star} = -8 U_t^2 c D/\left( e \sqrt{1 + B_* R_i} \right) \quad \ldots \quad 3.65 \]

Substituting for \( u \) from Eqn. (3.25) and Eqn. (3.26) into Eqn. (3.55) then integrating with respect to \( y \), and applying the boundary conditions,

\[ \tau = 0 \quad \text{at} \quad y = \delta \quad \text{and} \]

\[ \tau = \tau_{y_\star} \quad \text{at} \quad y = Y_\star \]

Eqn. (3.55) becomes,

\[ A'_4 \frac{dh}{dx} + B'_4 \frac{d\delta}{dx} + C'_4 \frac{du_m}{dx} + D'_4 \frac{du_n}{dx} = E'_4 \quad \ldots \quad 3.66 \]

in which,

\[ A'_4 = g \frac{h - \delta}{C_*} - e^{-\frac{h}{C_*}} S \left( -\frac{1}{2}, \frac{5}{2} \right) \left[ -2 U_s U_t + U_s U_n \right] \]

\[ - \frac{2U_t^2}{3eC_*} S \left( -1, \frac{5}{2} \right) + g \frac{C_o}{8} (2h - \delta) \]

\[ B'_4 = U_s Y_\star \frac{U_n}{n+1} - \frac{U_m^2}{n + 1} + U_t Y_\star \left[ e^{-\frac{h}{C_*}} S \left( -\frac{1}{2}, \frac{5}{2} \right) + e^{-\frac{h}{C_*}} - 1 \right] \]

\[ -1 + e^{-\frac{h}{C_*}} + 2 U_s U_t \left[ \frac{e^{-\frac{h}{C_*}}}{3eC_*} S \left( -\frac{1}{2}, \frac{5}{2} \right) + e^{-\frac{h}{C_*}} - 1 \right] \]

\[ -2U_t^2 \frac{S(-1, \frac{5}{2})}{3eC_*} - U_t^2 \left( \frac{1}{e} - 1 \right) - g \frac{C_o}{8} \frac{h}{8} \]
\[ C' = U_{Y*} \frac{(h - \delta)}{C*} - U_{Y*} e^{-\frac{1}{2}} \frac{(h - \delta)}{C*} S \left(-\frac{1}{2}, \frac{3}{2}\right) \]
\[ + \frac{2U_{e*}}{C*} (h - \delta) - 2(U_e + U_t)e^{-\frac{1}{2}} \frac{(h - \delta)}{C*} S \left(-\frac{1}{2}, \frac{3}{2}\right) \]
\[ + \frac{2U_t}{C*} S \left(-1, \frac{3}{2}\right) (h - \delta) \]
\[ D' = \frac{-U_{Y*} \delta}{n + 1} + \frac{U_m \delta}{n + 1} - (?U_e U_{Y*}) e^{-\frac{1}{2}} \frac{(h - \delta)}{C*} S \left(-\frac{1}{2}, \frac{3}{2}\right) \]
\[ + 2U_t \frac{(h - \delta)}{C*} S \left(-1, \frac{3}{2}\right) \]
\[ E'_4 = -\tau_{y*}/\rho \]
\[ C* = \sqrt{-8c} \]

\[ U_{Y*} = -U_e + U_t e^{-\frac{1}{2}} \]

Equations (3.41), (3.47), (3.53) and (3.66) are four non-linear first order differential equations in four unknowns \( h, \delta, U_e, \) and \( U_m \). In order to solve these four equations, four initial conditions are needed.

3.3.7.6 Initial Conditions

It was assumed that the velocity distribution represented by Eqns. (3.25) and (3.26) was only applicable downstream of the potential core of an incoming jet. The potential core is assumed to be extended for a distance \( 4y_1 \) downstream from the beginning of the jump, Section 1 in Fig. (3.4) (48, 51, 61). At Section 1 in Fig. (3.4) the velocity distribution was assumed uniform with velocity, \( U_1 \) and depth, \( y_1 \).
To initialize the solution, the continuity and momentum equations should be satisfied between Sections 1 and 3 in Fig. (3.3) as follows:

\[ U_1 y_1 = \int_0^h u \, dy \bigg|_{x = 4y_1} \quad \ldots \quad 3.67 \]

\[ \frac{\gamma y_1^2}{2} - \frac{\gamma h^2}{2} - C_f \frac{U_1^2}{2} x \rho = -\rho y_1 U_1 \dot{v} + \rho \int_0^h u^2 \, dy \bigg|_{x = 4y_1} \quad \ldots \quad 3.68 \]

in which,

- \( C_f \) = skin friction coefficients
- \( h \) = depth at Section 3 (Fig. 3.4)

Substituting for \( U \) from Eqns. (3.25) and (3.26) into Eqns. (3.67) and (3.68), Eqn. (3.67) and (3.68) can be solved for \( U_0 \) and \( h \), assuming \( \delta = 0.5 y_1 \) (48) and \( U_m = U_1 \) at the end of the potential core (51).

These values for \( U_0 \), \( U_m \), \( \delta \) and \( h \) were used as an initial condition to solve Eqns. (3.41), (3.47), (3.53) and (3.66) numerically using a fifth order Runge-Kutta Merson method. The details of the numerical solution is given after reviewing the theory for the free radial hydraulic jump.

3.3.8 Formulation for the Free Radial Hydraulic Jump

In this section the equations of motion of the free radial hydraulic jump are developed.
3.3.8.1 Equation of Motion

The Reynolds and continuity equations in the cylindrical system \((r, \phi, z)\) for steady axisymmetric flow as given by Schlichting (65) are:

a) Reynolds Equation (r-Component)

\[
\frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_r^2}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} \right] - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} \]

\[
- \left[ \frac{\partial}{\partial r} \left( \frac{v_r}{r} \right) \frac{v_r^2}{r} + \frac{\partial}{\partial z} \left( \frac{v_r}{r} \right) \frac{v_z}{r} \right] + \frac{\nu^2}{r} - \frac{\nu^2}{r^2}. \]

\[... 3.69\]

b) Continuity Equation

\[
\frac{\partial}{\partial r} \left( r v_r \right) + \frac{\partial}{\partial z} \left( r v_z \right) = 0 \]

\[... 3.70\]

in which,

\(v_r\) = the mean velocity component in the \(r\)-direction

\(v_z\) = the mean velocity component in the \(z\)-direction

\(v_\phi\) = the mean velocity component in the \(\phi\)-direction

\(v_r'\) = the velocity fluctuations in the \(r\)-direction

\(v_z'\) = the velocity fluctuations in the \(z\)-direction

\(v_\phi'\) = the velocity fluctuations in the \(\phi\)-direction

The radial hydraulic jump is treated as a two dimensional problem; thus \(v_\phi = 0\), and all the terms containing \(v_\phi\) and its derivatives disappear from Eqns. (3.69, 3.70). Furthermore, the velocity gradients in the radial direction are much smaller than those in the axial direction and the
fluctuating velocity component \( v'_r \) is small from which 
\( (v'_r)^2/r \) and \( (\partial (v'_r)^2)/\partial r \) can be neglected (24, 51). These 
asumptions simplify the Reynolds equation to:

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial r} + v \frac{\partial^2 v_r}{\partial z^2} - \frac{1}{\rho} \left( \nu \frac{\partial v_r}{\partial z} - \rho v'_r v'_z \right) \quad ... 3.71
\]

Combining the second and the third terms of 
the right hand side, the above equation can be rewritten as:

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial v_r}{\partial z} - \rho v'_r v'_z \right) \quad ... 3.72
\]
or,

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad ... 3.73
\]
in which,

\[
\tau = \text{total shear} = \tau_1 + \tau_t
\]

\[
\tau_1 = \text{viscous shear} = \mu \frac{\partial u}{\partial z}
\]

\[
\tau_t = \text{turbulent shear} = -\rho v'_r v'_z
\]

Assuming that the pressure distribution along 
the jump is hydrostatic, then the pressure term \( p \), can be 
replaced by \( \gamma h \) and Eqn. (3.72) can be rewritten as:

\[
v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -g \frac{\partial h}{\partial r} + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad ... 3.74
\]

For convenience, let \( y \) = the axial distance 
and let the velocity components in the axial and radial 
direction be \( v \) and \( u \) respectively. With these substitutions,
the equations of motion become:
\[ u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y} = -g \frac{\partial h}{\partial r} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad \cdots 3.75 \]

and,

\[ \frac{\partial u}{\partial r} + \frac{\partial v}{\partial y} = 0 \quad \cdots 3.76 \]

Finally Eqns. (3.76) and (3.77) can be used to model the free radial hydraulic jump.

3.3.8.2 Procedure

A similar procedure to the one used for the rectangular hydraulic jump is used to develop the mathematical model for the free radial hydraulic jump.

The continuity Eqn. (3.76) is integrated from 0 to \( h \), and the momentum Eqn. (3.75) is integrated for three strips from 0 to \( \delta \), \( \delta \) to \( h \) and \( \delta \) to \( y \), as follows:

3.3.8.3 Integral Continuity Equation (0 - h)

Integrating Eqn. (3.76), one obtains:

\[ \int_0^h \frac{\partial v}{\partial y} \, dy = - \int_0^h \frac{\partial u}{\partial r} \, dy \quad \cdots 3.77 \]

Applying the boundary conditions:

\[ v = 0 \quad \text{at} \quad y = 0 \quad \text{and,} \]

\[ v = U_s \frac{\partial h}{\partial r} \quad \text{at} \quad y = h \]

in which, \( U_s \) is the surface velocity which can be obtained from the outer layer velocity distribution Eqn. (3.26).

Substitution for \( u \) from Eqns. (3.25) and (3.26) into Eqn. (3.77) leads to:
\[
\begin{align*}
&\left[ -U_0 + U_t e^{4c} \right] \frac{dh}{dr} = -\int_\delta^h \frac{\partial}{\partial r} r U_m \left( \frac{y}{\delta} \right)^n dy - \int_\delta^h \frac{\partial}{\partial r} \left[ 4c \left( \frac{y-\delta}{h-\delta} \right) \right]^2 dy \\
& \quad \left[ -U_0 + U_t e^{4c} \right] dy
\end{align*}
\]

Integrating Eqn. (3.78) with respect to \( y \) leads to a first order differential equation in the form:

\[
A_1 \frac{dh}{dr} + B_1 \frac{d\delta}{dr} + C_1 \frac{dU_0}{dr} + D_1 \frac{dU_m}{dr} + E_1 = 0 \quad \ldots \quad 3.79
\]

in which,

\[
\begin{align*}
A_1 &= \frac{-8c}{3} e^{4c} S (4c, 5/2) r U_t + U_s r \\
B_1 &= \frac{-nrU_m}{n+1} + r U_t \left[ \frac{8c}{3} e^{4c} S (4c, 5/2) - e^{4c} + 1 \right] \\
C_1 &= -r (h - \delta) + r e^{4c} (h - \delta) S (4c, 3/2) \\
D_1 &= \frac{r^6}{n+1} + r e^{4c} (h - \delta) S (4c, 3/2) \\
E_1 &= \frac{-U_m \delta}{n+1} + U_0 (h - \delta) U_t e^{4c} (h - \delta) S (4c, 3/2)
\end{align*}
\]

### 3.3.8.4 Integral Momentum Equation for the Strip \((0 \text{ to } \delta)\)

Integrating Eqn. (3.75) from 0 to \( \delta \) gives:

\[
\int_0^\delta u \frac{\partial u}{\partial x} dy + \int_0^\delta v \frac{\partial u}{\partial y} dy = -g \int_0^\delta \frac{\partial h}{\partial r} dy + \frac{1}{\rho} \int_0^\delta \frac{\partial \tau}{\partial y} dy 
\]

\[ \ldots \quad 3.80 \]
Applying the boundary conditions:

\[ u = v = 0 \quad \text{at} \quad y = 0 \quad \text{and} \]
\[ u = U_m, \quad v = V_\delta \quad \text{at} \quad y = \delta \]

and substituting for \( v \) from the continuity Eqn. (3.76) yields,

\[
\int_0^\delta u \frac{\partial u}{\partial r} \, dy - \frac{U_m}{r} \int_0^\delta \frac{\partial ru}{\partial r} \, dy + \frac{1}{r} \int_0^\delta u \frac{\partial ru}{\partial r} \, dy
\]

\[
- \rho g \int_0^\delta \frac{\partial h}{\partial r} \, dy + \frac{1}{\rho} \int_0^\delta \frac{\partial \tau}{\partial y} \, dy \quad \ldots \quad 3.81
\]

Substituting for \( u \) from Eqn. (3.25) into Eqn. (3.81), then integrating with respect to \( y \), and applying the boundary conditions:

\[ \tau = 0 \quad \text{at} \quad y = \delta \quad \text{and} \]
\[ \tau = \tau_0 \quad \text{at} \quad y = 0 \]

with \( \tau_0 \) obtained from Eqn. (3.34), Eqn. (3.81) becomes:

\[ A_2 \frac{dh}{dr} + B_2 \frac{d\delta}{dr} + D_2 \frac{du_m}{dr} + E_2 = 0 \quad \ldots \quad 3.82
\]

in which,

\[ A_2 = g\delta \]
\[ B_2 = \frac{nU_m^2}{n+1} - \frac{2nU_m^2}{2n+1} \]
\[ D_2 = -\frac{U_m\delta}{n+1} + \frac{2U_m\delta}{2n+1} \]
\[ E_2 = -\frac{U_m^2 \delta}{r (n + 1)} + \frac{U_m^2 \delta}{r (2n + 1)} + \left(\frac{U_m \delta}{v}\right)^{2.5} \frac{U_m^2}{2} \]

3.3.8.5 Integral Momentum Equation for the Strip

(δ to h)

Integrating Eqn. (3.75) from δ to h gives:

\[ \int_{\delta}^{h} u \frac{\partial u}{\partial x} dy + \int_{\delta}^{h} v \frac{\partial u}{\partial y} dy = -g \int_{\delta}^{h} \frac{\partial h}{\partial y} dy + \frac{1}{\rho} \int_{\delta}^{h} \frac{\partial \tau}{\partial y} dy \]

Proceeding in a similar fashion as Section 3.3.8.4 with the boundary conditions,

\[ u = U_s, \quad v = V_h \quad \text{at} \quad y = h \]

\[ u = U_m, \quad v = V_\delta \quad \text{at} \quad y = \delta \]

yields,

\[ -\frac{U_s}{r} \int_{\delta}^{h} \frac{\partial u}{\partial x} dy + \frac{U_m}{r} \int_{\delta}^{h} \frac{\partial u}{\partial r} dy + \frac{1}{r} \int_{\delta}^{h} u \frac{\partial u}{\partial r} dy \]

\[ + \int_{\delta}^{h} u \frac{\partial u}{\partial r} dy = -g \int_{\delta}^{h} \frac{\partial h}{\partial y} dy + \frac{1}{\rho} \int_{\delta}^{h} \frac{\partial \tau}{\partial y} dy \]

The increase in the pressure gradient term due to the entrained air in this layer, as mentioned in Section 3.3.7.5, equals the increase in the pressure gradient / unit mass, i.e.,
\[ C_0 = \frac{gC_0}{2} \left[ \frac{\partial h}{\partial x} \left( 2h - \delta \right) - \frac{\partial \delta}{\partial x} h \right] \] ... 3.85

in which, \( C_0 \) is the average air concentration at a distance \( r \) along the jump. It can be evaluated by using the experimental data for the present study which can be approximated by:

\[ C_0 = 0.15 \frac{r - r_1}{L_J} F_1 \quad \text{for} \quad x/L_J < 0.2 \] ... 3.86

\[ C_0 = 0.07 F_1^{1.85} \frac{L_J}{L_J} \left( \frac{r - r_1}{L_J} \right)^2 \quad \text{for} \quad x/L_J \geq 0.2 \] ... 3.87

Equation (3.85) should be added to the pressure gradient term in Eqn. (3.84), to correct the effect of the entrained air.

On substitution for \( u \) from Eqn. (3.25) and Eqn. (3.26) into Eqn. (3.84), then integrating with respect to \( y \), and applying the boundary conditions:

\[ \tau = 0 \quad \text{at} \quad y = \delta \quad \text{and}, \]

\[ \tau = 0 \quad \text{at} \quad y = h \]

Eqn. (3.84) becomes,

\[ A_3 \frac{dh}{dr} + B_3 \frac{d\delta}{dr} + C_3 \frac{dU_s}{dr} + D_3 \frac{dU_t}{dr} + E_3 = 0 \] ... 3.88

in which,

\[ A_3 = g (h - \delta) + U_s^2 + \frac{16}{3} U_s U_t \left( e^{4c} S (4c, 5/2) \right) + \frac{8}{3} U_s U_t c \left( e^{4c} S (4c, 5/2) - \frac{16}{3} c e^{8c} S (8c, 5/2) \right) \]
+ g/2 (2h - δ) \dot{C}_s

B_3 = \frac{U_m U_n}{n + 1} - \frac{U_m^2 n}{n + 1} - \frac{8}{3} U_s U_t C e^{4C} S (4c, 5/2)

+ U_s U_t (e^{4C} - 1) - 2U_0 U_t \left[ \frac{8C}{3} e^{4C} S (4c, 5/2) - e^{4C} - 1 \right] + 2U_t^2 \left[ \frac{8C}{3} e^{8C} S (8c, 5/2) - \frac{1}{2} e^{8C} + \frac{1}{2} \right] - g C_s h/2

C_3 = U_s (h - δ) - U_s e^{4C} (h - δ) S (4c, 3/2)

+ 2U_0 (h - δ) - 2(U_0 + U_t) e^{4C} (h - δ) S (4c, 3/2)

+ 2U_t e^{8C} (h - δ) S (8c, 3/2)

D_3 = \frac{U_s^2 \delta}{n + 1} + \frac{U_m^2 \delta}{n + 1} + U_s e^{4C} (h - δ) S (4c, 3/2)

- 2U_0 e^{4C} (h - δ) S (4c, 3/2) + 2U_t e^{8C} (h - δ)

S (8c, 3/2)

E_3 = \frac{U_m^2 \delta}{x(n + 1)} + \frac{U_0^2}{r} (h - δ) + \frac{2U_0}{r} U_t e^{4C} (h - δ) S (4c, 3/2)

+ \frac{U_t^2}{r} e^{8C} (h - δ) S (8c, 3/2) - \frac{U_s U_m \delta}{(n + 1) x} + \frac{U_s}{r} \frac{U_0}{x} (h - δ)

S (4c, 3/2) - U_s \frac{U_t}{r} e^{4C} (h - δ) S (4c, 3/2)
3.3.8.6 Integral Momentum Equation for the Strip ($\delta$ to $y_*$)

Integrating Eqn. (3.75) from $\delta$ to $y_*$

$$
\int_{\delta}^{y_*} u \frac{\partial u}{\partial r} \, dy + \int_{\delta}^{y_*} v \frac{\partial u}{\partial y} \, dy = -g \int_{\delta}^{y_*} \frac{\partial h}{\partial r} \, dy + \frac{1}{\rho} \int_{\delta}^{y_*} \frac{\partial \gamma}{\partial y} \, dy
$$

... 3.89

Proceeding in a similar fashion to Section 3.3.8.4 and applying the boundary conditions,

$$
u = U_m, \quad v = V_\delta \quad \text{at} \quad y = \delta \quad \text{and},$$

$$u = U_\gamma \quad \text{at} \quad y = y_*$$

$v = v_\gamma \quad \text{at} \quad y = y_* = \sqrt{1/-8c} (h - \delta) + \delta$

Eqn. (3.89) becomes,

$$
\frac{u_\gamma - U_t}{r} e^{0.72c} \int_{\delta}^{y_*} \frac{\partial u}{\partial r} \, dy - \frac{U_m}{r} \int_{\delta}^{y_*} \frac{\partial u}{\partial r} \, dy
$$

$$+ \frac{1}{r} \int_{\delta}^{y_*} u \frac{\partial u}{\partial r} \, dy + \int_{\delta}^{y_*} u \frac{\partial u}{\partial r} \, dy = -g \int_{\delta}^{y_*} \frac{\partial h}{\partial r} \, dy
$$

$$+ \frac{1}{\rho} \int_{\delta}^{y_*} \frac{\partial \gamma}{\partial y} \, dy$$

... 3.90

Due to the increase in the pressure gradient as mentioned in section 3.3.8.5, the following term should be added to the pressure gradient term in Eqn. (3.90):

Increase in the pressure gradient/unit mass

$$
= \frac{g C_p}{8} \left[ \frac{\partial h}{\partial r} (2h - \delta) - \frac{\partial \delta}{\partial r} h \right]
$$

... 3.91
Substituting for \( u \) from Eqn. (3.25) into Eqn. (3.90), integrating with respect to \( y \) and applying the boundary conditions,

\[
\begin{align*}
\tau &= 0 \quad \text{at} \quad y = \tau \\
\tau &= \tau_{y*} \quad \text{at} \quad y = y_*
\end{align*}
\]

with \( \tau_{y*} \) obtained from Eqn. (3.65), Eqn. (3.90) becomes,

\[
A_4 \frac{dh}{dr} + B_4 \frac{dU_y}{dr} + C_4 \frac{dU_x}{dr} + D_4 \frac{dU_m}{dr} + E_4 = 6.0 \quad \ldots \quad 3.92
\]

\[
A_4 = g \frac{h - \delta}{C_*} - \frac{e^{-\frac{1}{2}}}{3C_*} S (-\frac{1}{2}, 5/2) \left[ U_t U_{y*} + 2U_o U_t \right] + 2/3 \frac{U_t^2}{eC_*} S (-1, 5/2) + \frac{9}{8} C_o (2h - \delta)
\]

\[
B_4 = U_{y*} \frac{n + 1}{n + 1} - \frac{U_t^2}{n + 1} + U_{y*} U_t \left[ \frac{e^{-\frac{1}{2}}}{3C_*} S (-\frac{1}{2}, 5/2) - 1 + e^{-\frac{1}{2}} \right] + 2U_o U_t \left[ \frac{e^{-\frac{1}{2}}}{3C_*} S (-\frac{1}{2}, 5/2) + e^{-\frac{1}{2}} - 1 \right] - 2U_t^2 \frac{S(-1, 5/2)}{3 e C_*} - U_t^2 (1/e - 1) - g C_o h/8
\]

\[
C_4 = U_{y*} \frac{h - \delta}{C_*} - \frac{1}{e} U_{y*} e^{-\frac{1}{2}} \frac{h - \delta}{C_*} S (-\frac{1}{2}, 3/2) + \frac{2U_o}{C_*} (h - \delta)
\]

\[
- 2 \left( U_o + U_t \right) e^{-\frac{1}{2}} \frac{h - \delta}{C_*} S (-\frac{1}{2}, 3/2) + \frac{2U_t}{eC_*} S (-1, 3/2)
\]

\[
(h - \delta)
\]

\[
D_4 = \frac{U_{y*}}{n + 1} + \frac{U_m}{n + 1} - (2U_o + U_{y*}) e^{-\frac{1}{2}} \frac{(h-\delta)}{C_*} S (-\frac{1}{2}, 3/2)
\]
\[ E_4 = -\frac{U_t}{r} \left[ \frac{U_m}{n+1} - U_o \frac{(h - \delta)}{C_*} + U_t e^{-\frac{1}{2}} \frac{(h - \delta)}{C_*} S \right. \]
\[ \left. + \frac{U_m^2 \delta}{r(n+1)} + \frac{U_o}{r} \frac{(h - \delta)}{C_*} - \frac{U_o U_t}{r} e^{-\frac{1}{2}} \frac{(h - \delta)}{C_*} S \right) \]
\[ + \frac{U_t^2 (h - \delta)}{r e C_*} S (1, 3/2) - \frac{\tau V_t}{\rho} \]

Equations (3.79), (3.82) (3.88) and (3.92) are four first order non-linear differential equations in four unknowns, \( h, \delta, U_o \) and \( U_m \). In order to solve these four equations one initial condition is required for each dependent variable.

3.3.8.7 Initial Conditions

As mentioned in Section 3.3.7.6, the velocity distribution presented by Eqn. (3.25) and (3.26) is only applicable downstream of the potential core. The potential core is assumed to extend for a distance equal to \( y_1 \) from the beginning of the jump (Section 1, Fig. (3.3)). At Section 1, Fig. (3.3) the velocity distribution was assumed uniform with velocity \( U_1 \) and depth \( y_1 \). To initialize the solution, the continuity and momentum equations should be satisfied between Sections 1 and 3, Fig. (3.3) as follows:

a) Continuity

\[ r_1 U_1 y_1 = r \int_0^h u \, dy \]
b) **Momentum**

\[ \gamma y_1^2 \frac{r_1}{2} \sin \theta /2 + \gamma y_1 \ h \ (r-r_1) \sin \theta /2 - \tau_0 \frac{\theta}{2} (r^2 - r_1^2) \]

\[ - \gamma h^2 r \sin \theta /2 = \rho r \int_0^\delta u^2 dy + \rho r \int_0^h u^2 dy - r_1 U_1^2 y_1 \]

... 3.94

in which,

\[ \tau_0 = \text{the bed shear stress} \]

Substituting for \( u \) from Eqn. (3.25) and Eqn. (3.26) into Eqn. (3.93) and Eqn. (3.94) permits Eqns. (3.93) and (3.94) to be solved for \( u_o \) and \( h \) assuming \( \delta = 0.2y_1 \) and \( U_m = U_1 \) at the end of the potential core.

These values for \( U_o, U_m, \delta \) and \( h \) were used as an initial condition to solve Eqn. (3.79), (3.82), (3.85) and (3.92) numerically, using a fifth order Runge-Kutta Merson method.

3.3.9 **The Numerical Solution**

The set of ordinary differential Eqns. (3.79), (3.82) (3.85) and (3.92) can be expressed in matrix form as:

\[
\begin{bmatrix}
A_1 & B_1 & C_1 & D_1 \\
A_2 & B_2 & C_2 & D_2 \\
A_3 & B_3 & C_3 & D_3 \\
A_4 & B_4 & C_4 & D_4
\end{bmatrix}
\begin{bmatrix}
\frac{dh}{dr} \\
\frac{d\delta}{dr} \\
\frac{dU_o}{dr} \\
\frac{dU_m}{dr}
\end{bmatrix}
= \begin{bmatrix}
E_1 \\
E_2 \\
E_3 \\
E_4
\end{bmatrix}
\]

... 3.95
or,

\[
\begin{bmatrix}
\frac{dh}{dr} \\
\frac{d\delta}{dr} \\
\frac{dU_o}{dr} \\
\frac{dU_m}{dr}
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 & C_1 & D_1 \\
A_2 & B_2 & C_2 & D_2 \\
A_3 & B_3 & C_3 & D_3 \\
A_4 & B_4 & C_4 & D_4
\end{bmatrix}^{-1} \begin{bmatrix}
E_1 \\
E_2 \\
E_3 \\
E_4
\end{bmatrix}
\]

In which the coefficient matrix \([A]\) and the vector \([E]\) are functions of \(h, \delta, U_o,\) and \(U_m\) as given in the previous sections. Therefore the above equations can be rewritten as:

\[
\frac{dh}{dr} = f_1 (h, \delta, U_o, U_m) \quad \ldots \quad 3.97
\]

\[
\frac{d\delta}{dr} = f_2 (h, \delta, U_o, U_m) \quad \ldots \quad 3.98
\]

\[
\frac{dU_o}{dr} = f_3 (h, \delta, U_o, U_m) \quad \ldots \quad 3.99
\]

\[
\frac{dU_m}{dr} = f_4 (h, \delta, U_o, U_m) \quad \ldots \quad 3.100
\]

The above four equations are first order non-linear differential equations. These equations can be solved by the Runge-Kutta method, using initial values for \(h^n, \delta^n, U_o^n,\) and \(U_m^n\) to calculate new values for \(h^{n+1}, \delta^{n+1}, U_o^{n+1},\) and \(U_m^{n+1}\) at step \(r^{n+1} = r^n + \Delta r.\)

The new values can be used as the initial values for the next step. The details of this method are given in (82).
The flow chart and the listing of the computer programs are given in Appendix F.
IV. THE EXPERIMENTAL STUDIES

4.1 Introduction

The principal purpose of the extensive experimental program, conducted for the present study, was first to provide design information for the free radial hydraulic jump and the submerged radial hydraulic jump and second to provide adequate evaluation and verification of a mathematical model for the free radial hydraulic jump.

This chapter contains a description of the laboratory equipment, the experimental procedure, the range of the experimental data and the experimental results.

4.2 The Experimental Apparatus

The tests were carried out in a flume made from plexiglass walls with aluminum and steel supports at 1.82 m. intervals in the longitudinal direction. The flume is 14.0 m long, and 0.46 m. wide. The walls of the flume are 0.91 m. high in the first 1.82 m. from the upstream side, while the rest of the walls are 0.61 m deep. There are two plexiglass walls, 1.3 m. long and 0.61 m. deep, placed in the flume to form the expanding transition with a total angle of 13.5°, as shown in Fig. (4.1) and Photograph 4.1. The flaring walls are located on an elevated plexiglass floor. The elevated floor is 1.9 m. long by 0.46 m. wide with 14 piezometers tapings, 3.2 mm in diameter, along the centerline, that are connected through plastic tubes, 10 mm in diameter, to the manometer board (see
Photograph (4.2) to indicate the bed pressure. The piezometer tappings are distributed, along the centerline so, that two tappings are located upstream the expanding section, and 12 tappings are located in the expanding section as shown in Fig. (4.1) and Photograph (4.2). The space between the first six tappings is 75 mm, and the space between the last eight tappings is 150 mm. The manometer board as shown in Photograph (4.3) contains 14 glass tubes 10 mm in diameter, and it is fixed against the flume walls as shown in Fig. (4.1) and photograph (4.4). A radial gate made of sheet metal 2.5 mm thick, 900 mm radius and 212 mm arch length is located near the beginning of the expansion as shown in Photograph (4.1) to produce a supercritical and radial flow. The entrance transition, upstream of the radial gate, is made from two quarter cylinders of sheet metal, and a plywood ramp with a 5:1 slope as shown in Photograph (4.5). Another plywood ramp with a 5:1 slope is located downstream of the expansion section as a transition between the elevated floor and the flume floor. A tail gate is located at the end of the flume to control the sequent depth.

A centrifugal pump having a rated discharge of 220 L/S and rated head 6.7 m was used to deliver the flow to the flume. Another centrifugal pump having a rated discharge of 15 L/S which delivered the flow to a 15.25 m. constant head taken was used for low flows. The flow rate was measured by an electromagnetic flow meter calibrated to approximately ± 1.25 L/S for high flows. For low flows, a manometer
connected to a venturimeter calibrated to ± 0.125 L/S was used. The flow was regulated by a control valve on the feed pipe.

The depth measurements were made by electric point gauges with a precision of ± 0.25 mm. A 16 mm Bolex movie camera was used to obtain the free surface and simultaneous manometer readings from which the average free surface and the hydraulic grade profiles along the bed were obtained.

A two channel electromagnetic water current meter Photograph (4.6) calibrated to about ± 2% of the full scale deflection was used to measure the low velocity. A 3.2 mm I.D. pitot tube connected to 0.069 N/m² differential pressure transducer which was connected to a strain indicator as shown in Photograph (4.7), was used to determine the velocity profiles in the supercritical flow. The pitot tube was also connected through a T-fitting to a differential manometer, which was used to calibrate the strain indicator, at the beginning of each test. The make and model for all the equipment are given in Appendix B.

A dye injection system was used to trace the points of zero velocity, the end of the roller and to check the boundary layer separation along the bed and the walls.

4.3 Experimental Procedure

A typical test was carried out as follows:

1) The radial gate opening was determined by using machined blocks of different heights.
2) The pump was started, and the flow was delivered to the head tank and the test flume.

3) The desired flow rate was established with the help of the control valve on the feed pipe.

4) The tailgate was adjusted until the free jump started at the desired position in the expanding section, or until the radial gate was properly submerged, in case of submerged hydraulic jump tests.

5) The air bubbles trapped in the piezometers and the pitot tube were removed.

6) The radii to the beginning and the end of the jump were measured, (the end of the jump was taken to be the section at which the flow becomes and continues to be approximately horizontal.)

7) The water surface elevations at the beginning and at the end of the jump were measured using the electric point gauge.

8) In the case of the submerged jump test, the depth downstream the gate was also measured.

9) A 16 mm movie with 5.5 m/s speed was taken for the water surface and the manometer board.

10) The dye was injected to trace the points of zero velocity, the end of the roller, and to check the boundary layer separations.
11) The pitot tube was used to measure the velocity distribution at the beginning of the jump.

12) The electromagnetic water current meter was used to measure the velocity distribution, further downstream. The reasons that the electromagnetic current meter was not used to measure the velocity in supercritical flow zones are:

   a) The metal gate might have affected the magnetic field, and b) the supercritical depth was very small in comparison, to the probe of the electromagnetic current meter.

13) The flow rate chart was read.

14) The test was repeated for different discharges and different gate openings.

15) The movie film was continuously projected frame by frame at approximately 1:1 scale. The water surface and the manometer readings were traced from which the mathematical averages were obtained for the water surface and the hydraulic grade profiles.

4.4 Experimental Ranges

The ranges of the experimental data are as follows:

Flow rate, \( Q = 3.1 \) L/S to \( 30.0 \) L/S

Froude number \( F_1 = 1.5 \) to \( 9.0 \)

Reynolds number, \( R_n = 2.6 \times 10^4 \) to \( 5.4 \times 10^5 \)

Sequent depth ratio, \( \psi = 2 \) to \( 10 \)

Sequent radius ratio, \( r_o = 1.2 \) to \( 1.9 \)

Inlet depth factor, \( \psi = 2 \) to \( 9 \) (in case of submerged jump)
4.5 Experimental Results

4.5.1 Free Radial Hydraulic Jump

The data of the free radial hydraulic jump are documented in Tables (4.1 to 4.30). All length dimensions are given in mm, the flow rate in L/S and the velocity in m/s.

Table (4.1) lists the results of the free radial hydraulic jump tests for the variables: flow rate, Q, initial water depth, \( y_1 \), radius to the beginning of the jump \( r_1 \), radius to the end of the jump \( r_2 \) and the radius to the end of the roller, \( r_3 \).

Tables (4.2) to (4.5) present the velocity distribution at different radii and different initial Froude numbers for the free radial hydraulic jump.

Tables (4.6) to (4.30) give the data obtained from the movie, for average water surface profiles and the hydraulic grade profiles for different initial Froude numbers.

Figures (4.2, 4.3) show a typical flow pattern for the free radial hydraulic jump, water surface profile, hydraulic grade profile and the velocity distribution along the jump. These data are for a Froude number \( F_1 = 6.7 \) and 7.6.

4.5.2 Submerged Radial Hydraulic Jump

The data of the submerged radial hydraulic jump are documented in Tables (4.31) to (4.54). All length dimensions are given in mm, the flow rate in L/S and the velocity in m/s.

Table (4.31) lists the results of the submerged radial hydraulic jump parameters, flow rate, \( Q \), water depth
downstream from the gate, $y_3$, the gate opening, $G_0$, radius 
at the beginning of the gate, $r_1$, radius at the jump end, $r_2$, 
and radius to the end of roller, $r_3$.

Tables (4.32) to (4.35) present the velocity 
distribution along the centerline at different radii, 
different initial Froude numbers, $F_1$, and different inlet 
depth factors, $\psi$.

Tables (4.36) to (4.54) list the water surface 
profiles and the hydraulic grade profiles for different 
initial Froude numbers and inlet depth factor, $\psi$.

Figures (4.4 and 4.5) shows a typical flow pattern for 
the submerged radial hydraulic jump along with a water surface 
profile, hydraulic grade profile, and the velocity distribution 
along the centerline of the jump. The data presented in 
(Fig. 4.4) are for initial Froude number, $F_1 = 5.0$, and inlet 
depth factor, $\psi = 5.2$. The data presented in Fig. (4.5) 
was for initial Froude number, $F_1 = 6$ and inlet depth factor 
$\psi = 6.8$. 
V. ANALYSIS AND DISCUSSION OF THE EXPERIMENTAL DATA

5.1 Introduction

In this chapter the experimental data for the free radial hydraulic jump and the submerged radial hydraulic jump are used to obtain empirical equations for the water surface profiles, the hydraulic grade profiles, the sequent depth ratio and the jump length.

The experimental water surface profiles and the hydraulic grade profiles are used to predict the amount of the entrained air in the radial hydraulic jump.

Finally the sources of the experimental errors are discussed.

5.2 The Free Radial Hydraulic Jump

5.2.1 Typical Flow Pattern

Figures (5.1) and (5.2) show typical flow patterns for the free radial hydraulic jump for different initial Froude numbers, $F_r$, and sequent radius ratios, $r_o$. These figures show that the water surface profile is higher than the hydraulic grade profile. The main reason for this is the entrained air in the jump. The two figures also show that the jet expands gradually at the beginning of the jump and rapidly at the end.

5.2.2 The Sequent Depth

Multiple linear regression analysis for the first data set listed in Table (4.1) yielded an empirical formula
for the sequent depth ratio, \( y_0 \), and sequent radius ratio, \( r_0 \), as a function of the initial Froude number, \( F_1 \). Different combinations of these variables were tried, and the following empirical formula gave the best correlation coefficient, \( R^* \).

\[
\begin{align*}
y_0 &= 0.3 \; r_0 + 0.65 \; F_1 \; (1 + \frac{1}{r_0}) \\
R^* &= 0.98 \quad 1.2 \leq r_0 \leq 1.8 \\
&\quad 1.5 \leq F_1 \leq 9
\end{align*}
\] ... 5.1

Equation (5.1) is presented in Fig. (5.3), for sequent radius ratios, \( r_0 = 1.2, 1.5 \) and \( 1.85 \). Figure (5.3) also shows a comparison between the sequent depth ratio obtained from Eqn. (5.1) for the free radial hydraulic jump and the sequent depth ratio for the free rectangular hydraulic jump. This indicates that the sequent depth ratio for the free rectangular hydraulic jump is higher than that for the free radial hydraulic jump for \( F_1 > 3 \), which agrees with the finding of Koloseus and Ahmed [28]. However, their equation for the sequent depth was based on a linear profile and their sequent depth ratio was approximately 10% higher than those predicted by Eqn. (5.1).

Figure (5.3) also shows that the sequent depth ratio \( y_0 \) decreases as the sequent radius ratio \( r_0 \) increases, for the same Froude number.

5.2.3 The Jump Length

The experimental data for the jump length show a wide scatter. However, a similar statistical analysis of the observed jump length led to the following empirical
formula;
\[ \frac{L_J}{y_2} = 4.7 - 4.2/F_1 \quad 1.2 \leq r_0 \leq 1.85 \quad \ldots \quad 5.2 \]
\[ 1.5 \leq F_1 \leq 9 \]

with correlation coefficient \( R_* = 0.71 \)
in which,
\[ L_J = r_2 - r_1 = \text{jump length} \]
\[ y_2 = \text{the sequent depth; and} \]
\[ F_1 = \text{initial Froude number}. \]

Equation (5.2) is compared in Fig. (5.4) with the U.S. Bureau of Reclamation Curve for the rectangular jump [76, 77, 78]. It was observed that for a given Froude number, \( F_1 \), the radial jump length is approximately 70\% of the corresponding rectangular jump length. It is interesting to note that the volume of the radial jump is almost the same as the rectangular jump for the same initial conditions. Thus, advantages of the radial jump are the lower sequent depth and the wider lateral distribution of the outflow. Koloseus and Ahmed [28] obtained a circular jump length ratio of about 3.5 for \( F_1 \) greater than 12, which indicates a decrease in \( L_J/y_2 \) at higher values of \( F_1 \).

5.2.4 The Water Surface and Hydraulic Grade Profiles

A statistical correlation was made of the measured water surface and hydraulic grade profiles listed in Tables (4.6 to 4.30). It was found that the best equations for the dimensionless water surface profile and hydraulic grade line
are, respectively:

\[ Y = A_{o1} R^2 + B_{o1} R; \quad R_* = 0.92 \quad \ldots \quad 5.3 \]

\[ \frac{P/Y - y_1}{y_2 - y_1} = A_{o2} R^2 + B_{o2} R; \quad R_* = 0.95 \quad \ldots \quad 5.4 \]

in which,

\[ P/Y = \text{The pressure head at the bed} \]

\[ R_* = \text{Correlation Coefficient} \]

\[ A_{o1} = -1.8 + 0.171 F_1 \quad \ldots \quad 5.5 \]

\[ A_{o2} = 0.36 + 0.016 F_1 \quad \ldots \quad 5.6 \]

\[ B_{o1} = 2.8 - 0.171 F_1 \quad \ldots \quad 5.7 \]

\[ B_{o2} = 1.3 - 0.025 F_1 \quad \ldots \quad 5.8 \]

\[ R = (r - r_1)/(r_2 - r_1) \quad \ldots \quad 5.9 \]

\[ Y = (y - y_1)/(y_2 - y_1) \quad \ldots \quad 5.10 \]

Equations (5.3) and (5.4) are presented in Figs. (5.5) and (5.6) respectively. Figures (5.5) and (5.6) indicate that the water surface profiles and the hydraulic grade profiles are functions of the initial Froude number. The dimensionless water surface profiles and the hydraulic grade profiles are high for low initial Froude number and they decrease as the initial Froude number increases. Figures (5.5) and (5.6) also show the difference between the water surface and the hydraulic grade profiles. This difference is mainly due to the entrained air in the jump. Figure (5.7) shows a comparison between the water surface profiles proposed by Koloseus and Ahmed [28], and Arbhabhirama and Abella [3].
Figure (5.7) indicates that the free surface profiles obtained in this study are above the linear profile and slightly below the elliptic profile. The effect of entrained air is to shift the effective profile towards the linear curve.

The constants $A$ and $B$ in Eqn. (3.7) are obtained from weighting the constants $A_01$, $A_02$, $B_01$ and $B_02$ according to the entrained air fraction at the apparent end of the jump as follows:

$$A = A_{01} F_p + A_{02} (1 - F_p) \quad \ldots 5.11$$

$$B = B_{01} F_p + B_{02} (1 - F_p) \quad \ldots 5.12$$

in which,

$$F_p = \text{entrained air fraction} = (y - P/\gamma)/(y_2 - y_1),$$

obtained from Fig. (5.9) at $R = 1.0$ or from $F_p = 0.034 \times F_1$.

5.2.5 The Relative Energy Loss

The energy equation was used to obtain the relative energy loss based on the experimental data. The relative energy loss was plotted in Fig. (5.8) for $\alpha_1 = \alpha_2 = 1.0$ and $r_0 = 1.8$. Figure (5.8) indicates that the relative energy loss in the radial hydraulic jump with $r_0 = 1.8$ is about 15% higher than in the rectangular jump.

5.2.6 Entrained Air

Figure (5.9) shows the average air concentration along the jump for different Froude numbers. Figure (5.9)
indicates that the air concentration increases rapidly at the beginning of the jump and reaches its maximum value at about half the jump length; then it decays slowly until it reaches zero at about 1.6 times the jump length. Figure (5.9) indicates that the air concentration increases as the Froude number increases. Figure (5.10) shows a comparison between the amount of the entrained air for the free rectangular hydraulic jump obtained by Rajaratnam [46] and the amount of entrained air for the free radial hydraulic jump of the present study. It is clear that the maximum air concentration in the rectangular hydraulic jump occurs closer to the beginning of the jump than in the radial jump. The maximum air concentration in the radial jump is greater than in the rectangular jump for the same Froude number. One reason for this is the violent surging at the surface of the radial hydraulic jump. Figure (5.10) also shows that the free radial hydraulic jump entrained the air for a relatively longer distance than the free rectangular hydraulic jump.

5.2.7 The Uplift Force

As shown in Fig. (5.11) the possible uplift obtained from the actual bed pressure profile is 40 to 60% more than that for the uplift based on the free surface profile, for the stilling basin with tailwater drains. Therefore, if the pressure profile is used in the design of the stilling basin floor slab thickness, or an alternative passive constraint system, it would be increased by 40 to 60% over that
indicated by the apparent free surface. This result is based on the model data; it is possible that the prototype jump would entrain relatively more air, and thus the increase in uplift on the slab could even be greater. Other researchers, Kalinski and Robertson [26], Straub and Anderson [72] and Nouma [14] have used physical models to predict air entrainment due to flow on chutes and hydraulic jumps in tunnels.

The U.S. Army Corps of Engineers [76] found that the prototype air demand in tunnels tended to be higher than model predictions; however, the process involved in tunnel flow consists of entrainment due to jump induced turbulence and subsequent transport. It is well known that the air-in-water transport phenomenon cannot be adequately modelled by a Froude law scale model.

5.3 The Submerged Radial Hydraulic Jump

5.3.1 A Typical Flow Pattern

Figures (5.12) and (5.13) show typical flow patterns for the submerged radial hydraulic jump for different initial Froude numbers and inlet depth factors. These figures show that the hydraulic grade profile at the beginning of the jump is higher than the water surface profile due to the centrifugal force exerted by the flow from the radial gate; however, near the end of the jump the hydraulic grade profile coincides with the water surface profile. This result is different from the free radial jump where the hydraulic grade profile at the bed is lower
than the water surface due to the significantly higher amounts of entrained air in the free jump. Figures (5.12) and (5.13) also show that the end of the roller corresponds closely to the end of the expansion, and the jet expands gradually at the beginning and rapidly at the end.

5.3.2 The Sequent Depth

The geometric mean assumption, that was used in Eqn. (3.20) to obtain the representative depth for the estimation of the side force resulted in theoretical solutions which deviated slightly from the observed values. This deviation depended on $F_1$. The experimental data involving $(y_0, \psi, r_0, F_1)$ were used to find an expression for the factor $B_*$, to correct the integrated side force. This analysis led to,

$$B_* = 1.04 - .074 \sqrt{F_1} \quad \ldots \quad 5.13$$

The constants in Eqn. (5.13) gave the minimum deviation between Eqn. (3.23) and the experimental data.

The general equation for the submerged radial hydraulic jump Eqn. (3.23), with the correction factor $B_*$ has been used to produce a set of charts for inlet depth factor, $\psi$, as a function of the initial Froude number, $F_1$, the tailwater depth ratio, $y_0$, and several sequent radius ratios, $r_0$. Figures (5.14) and (5.15) show some of these charts for $r_0 = 1.6$. The experimental data for $r_0$, ranging between 1.5 to 1.7, are also shown in Figs. (5.14) and (5.15).
5.3.3 The Jump Length

Figure (5.16) shows that the length of the submerged radial hydraulic jump is greater than the free radial hydraulic jump, except for low submergence and \( F_1 \geq 4\). The following empirical equation was obtained for the submerged radial jump:

\[
\frac{r_2 - r_1}{y_1} = -0.002 (F_1 + \psi) + 3.75 (F_1 + \psi) - 5.89
\]

\[R_* = 0.91\]

in which,

- \( r_2 \) = radius to the end of the jump;
- \( r_1 \) = radius to the beginning of the jump;
- \( y_1 \) = supercritical water depth;
- \( \psi \) = inlet depth factor = \( y_3/y_1 \); and
- \( R_* \) = correlation coefficient.

5.3.4 The Water Surface and Hydraulic Grade Profiles

A statistical analysis was made of 60 tests for \( 2 < \psi < 10 \) and \( 2.5 < F_1 < 8 \) in order to establish dimensionless equations to describe the water surface profile and the hydraulic grade profile at the bed of the submerged radial hydraulic jump. The following results gave the best fit for the dimensionless water surface profile, \( \psi > 1 \).

\[
Y = A_R Y^4 + B_R Y^3 + C_R Y^2 + D_R Y + E_Y
\]

\[R_* = 0.92\]
in which,

\[ Y = (y - y_1)/(y_2 - y_1) \]  \( \ldots \)  \( 5.16 \)

\[ R = (r - r_1)/(r_2 - r_1) \]  \( \ldots \)  \( 5.17 \)

\[ A_Y = -0.01134 G_*^2 + 0.642 G_* - 3.04 \]  \( \ldots \)  \( 5.18 \)

\[ B_Y = 0.0266 G_*^2 - 1.54 G_* + 7.18 \]  \( \ldots \)  \( 5.19 \)

\[ C_Y = -0.0197 G_*^2 + 1.16 G_* - 5.0 \]  \( \ldots \)  \( 5.20 \)

\[ D_Y = 0.005 G_*^2 - 0.305 G_* + 1.582 \]  \( \ldots \)  \( 5.21 \)

\[ E_Y = 0.048 \psi + 0.385 \]  \( \ldots \)  \( 5.22 \)

\[ G_* = F_1 \psi \]  \( \ldots \)  \( 5.23 \)

A similar fourth degree equation was developed to describe the dimensionless pressure head along the bed, i.e.,

\[ Y_p = A_p R^4 + B_p R^3 + C_p R^2 + D_p R + E_p \]  \( \ldots \)  \( 5.24 \)

\[ \sqrt{R_*} = 0.95 \]

in which,

\[ Y_p = (P/Y - Y_1)/(y_2 - y_1) \]  \( \ldots \)  \( 5.25 \)

\[ A_p = 0.0172 G_*^2 - 1.13 G_* + 24.23 \]  \( \ldots \)  \( 5.26 \)

\[ B_p = -0.0412 G_*^2 - 2.67 G_* - 57.5 \]  \( \ldots \)  \( 5.27 \)

\[ C_p = 0.034 G_*^2 - 2.19 G_* + 47.23 \]  \( \ldots \)  \( 5.28 \)

\[ D_p = -0.0116 G_*^2 + 7.28 G_* + 2.05 \]  \( \ldots \)  \( 5.29 \)
\[ E_p = 0.00116 \ G_\star^2 + 0.0736 \ G_\star + 2.05 \] ... 5.30

Figures (5.17) and (5.18) compare some typical experimental curves for the dimensionless water surface and the dimensionless hydraulic grade profiles, respectively. Figure (5.17) also shows a comparison between the dimensionless water surface profiles for the submerged rectangular hydraulic jump presented by Govinda Rao and Rajaratnam [18] and the typical water surface profiles for the submerged radial hydraulic jump. Both jumps show the same behaviour at the beginning. However, in the central portion of the jump the water surface profile for the rectangular jump is higher than that for the radial submerged hydraulic jump.

5.3.5 Relative Energy Loss

The energy equation has been used to obtain the relative energy loss in the submerged radial hydraulic jump based on the experimental data. In Fig. (5.19) the relative energy loss, \( \Delta E/E_1 \), is plotted as a function of the initial Froude number, \( F_1 \), the inlet depth factor, \( \psi \), and the sequent radius ratio, \( r_o \). The relative energy loss for the free rectangular hydraulic jump given by Bakhmeteff and Matyke [6] and the relative energy loss for the submerged rectangular hydraulic jump by Govinda Rao and Rajaratnam [18] are also presented in Fig. (5.19). Figure (5.19) shows that the relative energy for either the free or the submerged radial hydraulic jump is higher than that for the free or submerged rectangular hydraulic jump for corresponding \( \psi \) and \( F_1 \).
However, at certain large Froude numbers and small submergences the submerged radial jump can have a relative energy loss higher than the free radial hydraulic jump. Rajaratnam [50] obtained the same results for the rectangular jump.

5.3.6 Entrained Air

It was observed that there was almost no air entrainment in the submerged radial hydraulic jump. Even the slightest submergence of the gate eliminated most of the air entrainment associated with the free jump.

5.4 Experimental Errors

The sources of the experimental errors in this study may be summarized as follows:

1) The low discharges were measured by means of a manometer connected to a venturi meter. The calibration error for the venturi meter is estimated at approximately ± 2%. The flowmeter could be read within an accuracy of ± 1.0 USGPM (0.063 L/S).

2) The high discharges were measured by an electromagnetic flowmeter. The combined calibration error for the electromagnetic flowmeter and the chart reading error is approximately ± 0.022 cfs (0.63 L/S).

3) A two-channel electromagnetic water current meter (Cushing meter) was calibrated to about ± 2% of full scale deflection. This instrument was used to measure the low velocities. A pitot tube connected to a differential pressure
transducer was used to measure the high velocities, with an estimated maximum error of about $\pm 0.15$ ft/sec (45 mm/s) for $U > 1$ ft/sec.

4) The depth measurements were made by a point gauge with reading accuracy of $\pm 0.01$ inches (0.25 mm).

5) The error in setting the gate opening was approximately $\pm 0.01$ inches (0.254 mm).

6) The length measurements were made by a scale with a reading accuracy of $\pm 0.1$ inch ($\pm 2.54$ mm).

7) A random error is introduced in the process of averaging the free surface and the hydraulic grade profiles from the movies. This was estimated to be $\pm 4\%$.

8) The error associated with the observed readings of the jump length and roller length, due to the fluctuation of the water surface or flow reversal position is estimated by $\pm 10\%$.

9) In order to keep the errors due to photographic interpretation to a minimum, a meter scale was located in front of the flow section for each photograph. Scale variations from point to point in a photograph were determined by measuring the known regular grid intervals marked on the plexiglass window. A small error could result from the correction for the refraction of the light through the plexiglass viewing window. The refraction error, as illustrated in Fig. (5.20), results in the upper profiles
being overestimated by less than 0.5 mm and lower profiles being underestimated by about 1 mm. This correction was applied to the data.
VI. EVALUATION OF THE THEORY

6.1 Introduction

In this chapter the macroscopic theory for the free radial hydraulic jump and the submerged radial hydraulic jump are presented and compared with the corresponding experimental data of this study. Also the numerical model results for the free rectangular hydraulic jump are presented and compared with the corresponding experimental results from other researchers. The numerical model results for the free radial hydraulic jump are compared with the experimental data of this study.

Finally, the sources of errors and the limitations of the models are discussed.

6.2 The Macroscopic Approach

6.2.1 Free Radial Hydraulic Jump

The general equation for the free radial hydraulic jump, Eqn. 3.12, is used to plot the sequent depth $y_0$, as a function of the initial Froude number, $F_1$, for sequent radius ratios, $r_0 = 1.0, 1.3$ and $1.85$ in Fig. (6.1). Eqn. (3.12) includes the effects of the entrained air and the actual water surface profile, which adds accuracy to the calculated sequent depth ratio, $y_0$. The scatter of the experimental data in Fig. (6.1) can be explained by the experimental error in measuring the tailwater depth which is about $\pm 4\%$. 

89
On comparison of the least squares fitting of the experimental data, and the results of the theoretical equation, Eqn. (3.12) in Fig. (6.1), it is noted that both results show that the sequent depth ratio $y_o$, decreases as the sequent radius ratio, $r_o$, increases. This is due to the increase of difference between the end force and the side pressure force. Figure (6.1) indicates that the theoretical sequent depth ratio, $y_o$, which is obtained from Eqn. (3.12) is slightly higher than the observed sequent depth ratio, $y_o$. This could be due to neglecting the bed friction effect in the theory.

The energy equation along with Eqn. (3.12) is used to plot the relative energy loss, $\Delta E/E_1$, for the free radial hydraulic jump, assuming $a_1 = a_2 = 1.0$, for $r_o = 1.2$ and $1.8$. The experimental data for the relative energy loss are in good agreement with the calculated values as shown in Fig. (6.2). Figure (6.2) shows that the relative energy loss decreases as the sequent radius ratio, $r_o$, decreases. Figure (6.2) also shows that the relative energy loss for the radial hydraulic jump is higher than the relative energy loss for the rectangular hydraulic jump, and depends on the sequent radius ratio, $r_o$. For the same Froude number, $F_1$, the relative energy loss for the free radial hydraulic jump, with $r_o = 1.8$, is about 15% greater than the relative energy loss for the free rectangular hydraulic jump.

6.2.2 **Submerged Radial Hydraulic Jump**

The general equation for the submerged radial
hydraulic jump, Eqn. (3.23), and the experimental data were used to obtain the best value for the correction factor $B_x$. This correction factor, combined with Eqn. (3.23) is used to produce Fig. (6.3) for $y_4/y_1$, the ratio between the tailwater depth and the initial jet depth, as a function of the initial Froude number, $F_1$, and inlet depth factor, $\psi$, for several sequent radius ratios, $r_o$.

Equation (3.23) was used again to produce another chart, Fig. (6.4) for the inlet depth factor, $\psi$, as a function of $y_4/y_1$ and Froude number, $F_1$, for several sequent radius ratios, $r_o$. Using Fig. (6.3) and Fig. (6.4) the inlet depth factor $\psi$ for any particular case can be obtained.

Figure (6.4) shows that at Froude number, $F_1 = 1.0$, the inlet section factor, $\psi$, is equal to the sequent depth ratio $y_i'$. This means that the water surface is flat, i.e., there is no jump. For inlet section factor, $\psi = 1.0$, the sequent depth ratio is equal to the sequent depth ratio for the free radial hydraulic jump. Figure (6.5) shows a very good agreement between the observed $y_i'$ and the predicted $y_i'$ from Eqn. (3.23).

In Fig. (6.6) the energy equation is used to plot the relative energy loss, $\Delta E/E_1$, as a function of the inlet section factor, $\psi$, and Froude number, $F_1$, assuming $\alpha_1 = \alpha_2 = 1.0$ and $r_o = 1.8$. Figure (6.6) compares the relative energy loss for the free and submerged radial hydraulic jumps and the submerged rectangular hydraulic jump as given by Rajaratnam (48). It is clear that the relative energy loss is higher for the submerged radial hydraulic jump than for the free and
submerged rectangular hydraulic jump at the same Froude number. Figure (6.6) also shows that the relative energy loss of the submerged radial hydraulic jump could be more or less than the free radial hydraulic jump, depending on the particular value of the Froude number, $F_1$, and inlet depth factor, $\psi$. Figure (6.6) shows that there is good agreement between the observed and the theoretical relative energy loss, $\Delta E/E_1$, for different inlet depth factors, $\psi$, and sequent radius ratios, $r_s$.

6.3 The Numerical Model

As mentioned in Chapter 3, the numerical model was developed first for the free rectangular jump, and then extended to solve the free radial hydraulic jump.

6.3.1 Boundary Conditions

The numerical model needs one boundary condition at the beginning of the jump, i.e., the velocity distribution. In this study the velocity distribution is assumed uniform at the beginning of the jump for both the free rectangular and the free radial hydraulic jumps, i.e., $U_1 = \text{constant for } 0 \leq y \leq y_1$. The surface and bottom boundary conditions have been introduced in the vertical integrations.

6.3.2 Initial Conditions

The experimental observations indicate that, in the axial direction of the jump, one could divide the jump flow into two distinct regions. In the first region, close to the beginning of the jump is known commonly as the flow development
region. As the turbulence penetrates inward towards the middle of the jet, a wedge is formed known as the potential core, the core of fluid with irrotational flow which is surrounded by a mixing layer on top and a wall shear layer on the bottom [51]. In the second region which is known as the fully developed flow region, the turbulence has penetrated to the jet, and as a result, the potential core has disappeared. For the free rectangular hydraulic jump, the length of the core is about $4y_1$ [48]. For the free radial hydraulic jump the length of the core is approximately $y_1$.

To initialize the solution, the continuity and the momentum equations should be satisfied between the beginning and the end of the core as shown in Chapter 3.

The values for $U_0$, $U_m$, $\delta$ and $h$ at the end of the potential core are used as an initial condition to solve the integral momentum and continuity equations at each space step, $\Delta x$.

6.3.3 The Input Data

The input data required for the numerical model are:

1) $R_1$, the index for the rectangular or radial hydraulic jump. For the rectangular hydraulic jump, $R_1 = 0$. For the radial hydraulic jump, $R_1 = $ the radius at the beginning of the jump.

2) $F_1 = $ initial Froude number

3) $\Delta x = $ the space increment for the Runge-Kutta method.
6.3.4 Evaluation of the Free Rectangular Hydraulic Jump Model

Figures (6.7) to (6.19) show some typical numerical solutions for the free rectangular hydraulic jump. The dimensionless surface profiles, for the initial Froude number, \( F_1 \), 4, 6 and 8 are predicted in Fig. (6.7). The hydraulic grade profiles are plotted for initial Froude numbers, \( F_1 \) = 4, 6 and 8 in Fig. (6.8). The sequent depth ratio, \( y_2/y_1 \), is plotted as a function of the initial Froude number in Fig. (6.9). The predicted jump length, \( L_j/y_1 \), is plotted as a function of the initial Froude number, \( F_1 \), in Fig. (6.10). In Fig. (6.11) the predicted roller length is plotted as a function of the initial Froude number, \( F_1 \). In Fig. (6.12) the dimensionless velocity distribution, along the centerline of the jump is plotted for initial Froude number, \( F_1 \) = 6. The decay of the maximum dimensionless velocity \( U_m/U_1 \) is plotted in Fig. (6.13) for Froude numbers \( F_1 \) = 4 to 10. The dimensionless surface velocity is plotted in Fig. (6.14) for initial Froude number, \( F_1 \) = 4, 6 and 8.

Figure (6.15) shows the growth of the boundary layer for different Froude numbers. The variation of the energy coefficient, \( a \), with \( x/y_1 \) is plotted in Fig. (6.16) for initial Froude numbers, \( F_1 \) = 4, 6 and 8. Fig. (6.17) shows the variation of the momentum coefficient, \( \beta \), with \( x/y_1 \) for initial Froude numbers, \( F_1 \) = 4, 6 and 8.

The relative energy profiles are plotted for Froude numbers, \( F_1 \) = 4, 6 and 8 in Fig. (6.18). The relative
energy loss, $\Delta E/E_1$, as a function of the initial Froude number, $F_1$, is plotted in Fig. (6.19).

Typical input and output of the computer program for the free rectangular hydraulic jump are given in Appendix F.

6.3.5 Verification of the Free Rectangular Hydraulic Jump Model

The verification of the proposed numerical model was achieved by utilizing representative cases from the experimental results of Rouse et al. [61], Rajaratnam [46, 48 and 49] and Bakmetyeff and Matzke [6]. The comparisons between the experimental and the numerical results are shown in Fig. (6.20 to 6.31).

Figure (6.20) shows a very good agreement between the predicted water surface profiles and those of Rouse et al. [61] and Rajaratnam [39] for Froude numbers, $F_1 = 4$ and 6. Figure (6.20) also shows that the water surface profile for Froude number $F_1 = 6$ predicted, by the Narayanan model [40] is higher than the experimental data of Rouse. The theoretical dimensionless water surface profiles for Froude number, $F_1 = 4$, 6 and 8 are plotted in Fig. (6.21) and it was found that these profiles can be collapsed on the general profile, that was obtained by Rajaratnam [44].

Figure (6.22) shows a comparison between the water surface profiles obtained by Bakmetyeff and Matzke [6] for Froude numbers, $F_1 = 5.5$ and 8.6, and the corresponding water surface profiles predicted by the numerical model. The water
surface profile predicted by the model is higher than the water surface profile of Bakhmeffe and Oratzke [6] at the beginning of the jump, while it is lower at the jump end. The water surface predicted by Bakhmeffe and Matzke, at the end of the jump, is equal to the theoretical no friction sequent depth, which does not agree with Rajaratnam [46] or the present study. The water surface profiles for Froude numbers, \( F_1 = 4.4 \) and 9.05 obtained by Rajaratnam are compared with the corresponding water surface profiles obtained by the present model in Fig. (6.23). This figure shows a good agreement for Froude number, \( F_1 = 4.4 \), while for Froude number \( F_1 = 9.05 \), the water surface profile predicted from the model is slightly higher at the beginning of the jump, but near the jump end there is good agreement. As shown in Figs. 6.20, 6.22 and 6.23, the water surface profiles and the hydraulic grade profiles do not coincide due to the entrained air, which agrees with the findings of Ramamoorthy [53], and Rajaratnam [45].

Figure (6.24) shows good agreement between the jump length of the present study and that given by Rajaratnam [46]. However, it shows slightly lower values than the U.S.B.R. [76, 78] for Froude numbers, \( F_1 = 3 \) to 5 and slightly higher values for Froude numbers greater than 5. Figure (6.25) shows that the predicted roller length is slightly greater than the experimental value obtained by Rajaratnam [46], while it is about 30\% higher than the air model data of Rouse. This can be explained by the affect
of the shear on the top surface in the air model.

Figure (6.26) shows that the sequent depth, \( y_s \), for a particular initial Froude number, \( F_1 \), is slightly less than the sequent depth obtained from the Belanger equation (11), but agrees closely with the findings of Rajaratnam [46].

Figure (6.27) shows that the decay of the maximum velocity is not significantly affected by the Froude number, \( F_1 \), i.e., the decay of the maximum velocity for different values of Froude numbers collapses on a single curve. This curve shows a very good agreement with the air model of Rouse et al. [61] for about the first 2/3 of the jump length. However, near the end it is closer to the curve of Rajaratnam [48]. Figure (6.27) also shows that the decay of the maximum velocity predicted by Narayanan [40] model is slightly less than the proposed model. In Fig. (6.28) there is a comparison between the predicted surface velocity and the surface velocity of Rouse [61]. Figure (6.28) shows that the predicted surface velocity is slightly smaller than the one obtained by Rouse [61]. Figure (6.28) also shows that the surface velocity predicted by the Narayanan [40] model agrees very well with Rouse's data.

Figure (6.29) shows a significant deviation from the predicted and the measured boundary layer thicknesses, \( \delta \), as obtained by Rouse et al. [61] and Rajaratnam [48]. However, the values for \( \delta \) predicted by the Narayanan model agree with the present study at the beginning of the jump; however, the Narayanan model shows a decrease in \( \delta \) near the end of the jump,
which should not occur. The reasons for this are not immediately clear. The present study does not show any separation at the bed, which agrees with a statement by Rajaratnam [48], i.e., "At this point it should be noted that in all the data presented in this study the jump was formed near the sluice gate with a possibility of a thin boundary layer at the beginning of the jump. As a result, the boundary layer may not have separated from the bed, and hence, at every point, the boundary shear stress would have contributed to a reduction in the subcritical sequent depth".

Figure (6.30) shows that a rapid drop in the relative energy profile at the beginning of the jump is in good agreement with the air model of Rouse et al. [61], for Froude number $F_1 = 4$ and 6. The relative energy profile by Rajaratnam for Froude number, $F_1 = 9.05$ also is compared with the corresponding relative energy profile predicted by the numerical model of the present study in Fig. (6.30). The predicted value is slightly higher at the beginning of the jump, but it coincides near the end of the jump.

Figure (6.31) shows a good agreement between the theoretical velocity distribution and the measured velocity distribution of Rouse et al. [61] for initial Froude number, $F_1 = 6$, at the beginning of the jump but there is a slight deviation near the end of the jump.

Figure (6.16) shows the variation of the energy coefficient, $\alpha$, along the jump. The values for $\alpha$ increase rapidly at the beginning of the jump reaching a maximum
value at \( x/y_1 = 10 \) and then it decreases gradually to reach approximately 1.05 near the jump end. This agrees with the statement of Rouse et al. [61], "However, although the correction factor compensating for the use of the mean velocity is usually assumed to be unity, its magnitude within the nonuniform zone was found to rise to as high as 4; on the other hand, at the end of transition it was within 5% of unity in every case".

6.3.6 Evaluation of the Free Radial Hydraulic Jump Model

Figures (6.32 to 6.45) show some typical numerical solutions for the free radial hydraulic jump. The surface profiles and the hydraulic grade profiles for the initial Froude numbers, \( F_1 = 4, 6 \) and 8 and sequent radius ratios, \( r_o = 1.2 \) and 1.8 are depicted in Figs. (6.32, 6.33 and 6.34). In Fig. (6.35) the sequent depth, \( y_o \), is plotted as a function of the initial Froude number, \( F_1 \), for sequent radius ratios, \( r_o = 1.2 \) and 1.8.

The predicted jump length, \( L_j/y_2 \), is plotted as a function of the initial Froude number, \( F_1 \), and sequent radius ratio, \( r_o = 1.05, 1.2 \) and 1.8 in Fig. (6.36). In Fig. (6.37) the predicted roller length is plotted as a function of the initial Froude number, \( F_1 \), and sequent radius ratios, \( r_o = 1.05, 1.2 \) and 1.8.

In Fig. (6.38) the dimensionless velocity distribution along the jump is plotted for initial Froude number, \( F_1 = 6 \) and sequent ratio, \( r_o = 1.3 \). The decay of the dimensionless maximum velocity, \( U_m/U_1 \), is plotted in Fig. (6.39) for Froude
numbers, \( F_1 = 4 \) to 10 and sequent radius, \( r_o = 1.2 \) and 1.8.

The dimensionless surface velocity is plotted in Fig. (6.40) for initial Froude numbers \( F_1 = 4, 6 \) and 8 and sequent radius ratios, \( r_o = 1.2 \) and 1.8. Figure (6.41) shows the growth of the boundary layer for different Froude numbers, \( F_1 = 4, 6 \) and 8 and sequent radius ratios, \( r_o = 1.2 \) and 1.8.

The variation of the energy coefficient, \( \alpha \), with \( (r_r_1)/y_1 \) is plotted in Fig. (6.42) for Froude numbers, \( F_1 = 4, 6 \) and 8, and sequent radius ratios, \( r_o = 1.2 \) and 1.8. Figure (6.43) shows the variation of the momentum coefficient, \( \beta \), with \( (r-r_1)/y_1 \) for Froude numbers, \( F_1 = 4, 6 \) and 8 and sequent radius ratios, \( r_o = 1.2 \) and 1.8.

The relative energy profiles are plotted for Froude numbers, \( F_1 = 4, 6 \), and 8 and sequent radius ratios, \( r_o = 1.2 \) and 1.8 in Fig. (6.44). The relative energy loss, \( \Delta E/E_1 \), as a function of Froude number, \( F_1 \), and sequent radius ratio, \( r_o \), is plotted in Fig. (6.45).

Typical input and output data of the computer programs for the free radial hydraulic jump are given in Appendix F.

6.3.7 Verification of the Free Radial Hydraulic Jump Model

The verification of the proposed numerical model was achieved by utilizing representative cases from the experimental results of the present study. The comparisons between the experimental and numerical results are shown in Figs. (6.46 to 6.54).
Figure (6.46) shows a good agreement between the predicted water surface profile and the experimental water surface profile for the initial Froude number, $F_1 = 3.9$ and sequent radius ratio, $r_o = 1.5$. Figure (6.47) shows a close agreement between the predicted water surface profile and the experimental water surface profile for the initial Froude number, $F_1 = 5.6$ and sequent radius ratio, $r_o = 1.4$. Figure (6.48) shows that the predicted water surface profile for Froude number, $F_1 = 7.6$ and sequent radius ratio, $r_o = 1.8$ is slightly higher than the experimental surface profile.

Figure (6.49) shows very good agreement between the predicted sequent depth ratio, $y_o$, and the corresponding experimental data for the sequent radius ratios, $r_o = 1.2$ and 1.8. It also shows that the sequent depth for the radial hydraulic jump is smaller than the sequent depth for the corresponding rectangular jump.

The comparison in Fig. (6.50) shows that most of the experimental data for the free radial jump length fall between the two curves for sequent radius ratios, $r_o = 1.2$ and 1.8. However, the scatter in the experimental data can be explained by the error in measuring the jump length. The measured jump length is subject to a possible error $\pm 10\%$. As mentioned in Chapter 4, Figure (6.50) also shows that the jump length is strongly affected by sequent radius ratio, $r_o$. The jump length ratio, $L_J/y_2$, decreases as $r_o$ decreases until $r_o$, reaches the value of $r_o = 1.1$, then it rapidly approaches the rectangular hydraulic jump curve at $r_o = 1.05$. Figure (6.50)
shows that for a given Froude number $F_1$ and sequent radius ratio $r_o = 1.2$ the free radial hydraulic jump length is approximately 70% of the corresponding rectangular hydraulic jump length. However, for the radius ratio $r_o = 1.8$ the free radial hydraulic jump length is approximately 82% of the corresponding rectangular hydraulic jump length. Figure (6.51) shows that $L_J/y_2$ reaches a maximum value at $r_o = 1.0$, i.e., for the rectangular hydraulic jump, then it decreases as $r_o$ increases reaching a minimum value at a sequent radius ratio, $r_o = 1.1$.

Figure (6.52) shows that the maximum velocity, predicted from the model, agrees with the experimental data at the beginning of the jump but is slightly higher than the experimental results near the jump end. This can partly be explained by the experimental error of measuring the low velocity. Figure (6.40) shows that the surface velocity $U_s$ increases as the Froude number increases. Figure (6.40) also shows that the surface velocity changes its direction faster for small sequent radius ratios.

Figure (6.41) shows that the boundary layer grows faster for small Froude numbers and small sequent radius ratios. Figure (6.41) also shows that the predicted rate of the growth of the boundary layer is higher for the rectangular jump than for the radial jump.

Figures (6.53 and 6.54) show a comparison between the predicted velocity distribution and the experimental velocity distribution for $F_1 = 4.4$, $r_o = 1.2$ and $F_1 = 5.6$, $r_o = 1.5$. 
The agreement is very good in the high velocity regions. The low velocity regions were not accurately measured by the Pitot tube.

6.4 Errors and Limitations in the Mathematical Models

The errors and limitations in the mathematical models may be classified as follows:

6.4.1 Idealization Limitations

It is assumed in the models that the flow at the inlet section is uniform, which is not strictly correct. The turbulence component of the model used in this study was calibrated using Rouse's data which was based on one phase flow. The empirical friction model which is used to evaluate the bed shear is for a turbulent boundary layer only, i.e., the effect of a laminar layer is neglected. The velocity was assumed constant in the potential core; furthermore, the effect of the centrifugal force was neglected.

6.4.2 Discretization Errors

The space increment size, Δx, was examined by assigning different values to Δx. It was found that Δx ≤ 0.5y1 gave similar results.

6.4.3 Computation Errors

Double precision accuracy was used in performing the computation. Single precision resulted in underflow errors.
VII. DISCUSSION, MODIFICATION AND APPLICATIONS

7.1 General

In this chapter, a discussion of the stability analysis of the numerical model is presented. The advantages of the numerical model are discussed. Modifications are proposed to increase the accuracy of the numerical model for high Proude numbers. Practical applications of the numerical model are given.

7.2 Stability and Sensitivity Analyses of the Numerical Model

To study the convergence and stability of the numerical solution, several runs were made using space increments $\Delta x$, of $0.1y$, $0.2y$, $0.3y$, $0.5y$, and $y$. It was found that by using the space increment $\Delta x < 0.5y$, the convergence and stability requirements were satisfied.

A few runs of the mathematical model were made for different values of the turbulent shear. It was found that the solution was very responsive to changes in the turbulent shear, e.g., a 100% increase in the turbulent shear value caused a 50% decrease in the jump length and a 50% decrease in the turbulent shear value caused a 50% increase in the jump length.

The numerical investigation shows that the water surface profile, the decay of the maximum velocity and the surface velocity were also sensitive to the turbulent shear function. Figure (7.1) indicates that the water surface rises rapidly for high turbulent shear values, while it rises gradually for
low turbulent shear values. Figure (7.2) shows that the maximum velocity decayed faster with high turbulent shear than for low turbulent shear. Figure (7.3) indicates that the surface velocity changes its direction in a shorter distance for high turbulent shear, i.e., the length of the roller decreases as the turbulent shear increases. The runs presented in Figs. (7.1), (7.2) and (7.3) are for the rectangular hydraulic jump model with initial Froude number $F_1 = 6$.

The numerical model was run with and without the effect of the entrained air on both the hydrostatic pressure and the turbulent shear. It was observed that eliminating the air from the model would result in an increase in the jump length and a decrease in the water surface profile. However, Fig. (7.4) indicates that the water surface profile, without the entrained air effect, fell between the water surface and the hydraulic grade profiles for those cases which included the effect of the air, for the same initial Froude number $F_1$.

A few runs of the numerical model were made to investigate the effect of the bed shear. It was found that the solution is not strongly affected by changes in the bed shear value. It was observed that an increase in the bed shear by 50\%, resulted in an 8\% decrease in the jump length and a 1\% decrease in the sequent depth. Furthermore, the model predicted a sequent depth equal to the sequent depth obtained by Belanger Eqn. (2.1), if the bed shear was equated to zero.
7.3 Advantages of the Proposed Numerical Model

The present model needs only the boundary conditions at the beginning of the jump, which are the initial Froude number, $F_1$, the initial depth, $y_1$, and the index, $R_1$, to differentiate between the rectangular and the radial hydraulic jump. With these informations the model will predict the following design parameters: water surface profile, bed pressure profile, sequent depth, jump length, the surface velocity and the decay of the maximum velocity. The computation time to simulate one hydraulic jump is 30 seconds with the IBM 370/3031. The model is written in FORTRAN.

The numerical model presented in this thesis, for the free rectangular hydraulic jump, provides an improvement to that of Narayanan by (a) utilizing an error function instead of cosine function for the outer layer velocity distribution, (b) fixing the boundary layer velocity distribution exponent $n$ for the inner layer equal to $1/7$ which eliminates one of the momentum equations, i.e., the present model solves four equations instead of five as in the Narayanan model, which speeds up the numerical solution, (c) accounting for the vertical component of the surface velocity, which Narayanan neglected in his model, (d) evaluating the turbulent shear based on the Prandtl mixing length theory, which is calibrated from the experimental results of Rouse et al [61] and (e) including the entrained air effect on the turbulent shear and on the hydrostatic pressure.
The present model is supported by the good agreement with the experimental data of Rouse [61] and Rajaratnam [46, 48] for the free rectangular hydraulic jump, and with the experimental data of the present study for the free radial hydraulic jump. However, there is a slight deviation from the experimental data for higher Froude numbers, which can be explained by the fact that the turbulent pressure and centrifugal pressure were neglected; these have a more significant effect at high Froude numbers.

7.4 Applications

The proposed numerical model, as presented, is strictly applicable to the free rectangular hydraulic jump and the free radial hydraulic jump.

The proposed numerical model is formulated to predict the internal flow structure of the free rectangular hydraulic jump and the free radial hydraulic jump. It determines the velocity distribution, the energy loss, the water surface profile, the hydraulic grade profile, the roller length, the jump length, the surface velocity, the decay of the maximum velocity, the growth of the boundary layer, the energy coefficients, $\alpha$, and the momentum coefficient, $\beta$.

The applications of the current research as mentioned in Chapter I, are the design of the rectangular stilling basin, and the study of the internal structure of the rectangular hydraulic jump. It can also be applied to the diverging structures such as the "St. Anthony Falls" stilling basins and critical flow flumes. Another application of the
present study was the W. Darcy McKeough Dam and Diversion which is now under construction near Wallaceburg, Ontario.

The numerical model can be used as a sub-program with an external model to study the forces acting on the baffle blocks in a stilling basin. It can also be used to estimate the exit velocity for the stilling basin in order to design the rip rap downstream of the basin.
VIII. CONCLUSIONS AND RECOMMENDATIONS

The conclusions of this thesis are considered in three parts:

1) those related to the numerical model for the free rectangular hydraulic jump;

2) those related to the numerical model for the free radial hydraulic jump; and

3) those related to the experimental and macroscopic studies of the free and submerged radial hydraulic jumps.

8.1 The Free Rectangular Hydraulic Jump

The strip integral method has been developed as a general procedure for integrating the momentum and the continuity equations. It was successfully used in this study to predict the internal flow structure for the free rectangular jump as described by other researchers.

The free rectangular hydraulic jump velocity distribution can be represented by two functions, namely, the $1/7$ power law for the inner layer and an error function for the outer layer.

The numerical model confirms that decay of the maximum velocity is not affected by the initial Froude number. However, the surface velocity is shown to depend on the initial Froude number.

The length of the jump and the roller length are very sensitive to the turbulent shear in the outer layer. The bed shear has only a slight effect on the sequent depth ratio.
and the jump length. The jump length and the sequent depth decrease as the bed shear increases. The length of the jump and the length of the roller are very sensitive to the turbulent shear in the outer layer. The jump length and the roller length increase as the turbulent shear decreases.

The predicted energy coefficient, $\alpha$, and the momentum coefficient, $\beta$, are very high in the non-uniform zone at the start of the jump but they reach values very close to unity near the end of the jump. The maximum values of the energy and momentum coefficients in the nonuniform zone are functions of the initial Froude number. They increase as Froude number increases.

The models predicted that 85% of the relative energy loss occurs in the first half of the jump length. The relative energy loss, $\Delta E/E_1$, increases as the initial Froude number increases.

8.2 The Free Radial Hydraulic Jump

The strip integral method was extended to integrate the momentum and the continuity equations in cylindrical co-ordinates in order to predict the internal flow structure for the free radial hydraulic jump. The experimental portion of this study was used to verify the numerical model.

The free radial hydraulic jump velocity distribution can also be represented by the same two functions as used in the rectangular jump, namely, the $1/7$th power law for the inner layer and the error function for the outer layer.
The decay of the maximum velocity is sensitive to the sequent radius ratio, \( r_0 \), but is not affected significantly by the initial Froude number. The surface velocity is a function of the initial Froude number and the sequent radius ratio.

The length of the free radial hydraulic jump is a function of the sequent radius ratio, \( r_0 \). It reaches a maximum value at \( r_0 = 1 \) and a minimum value at \( r_0 = 1.1 \).

The length of the free radial hydraulic jump and the length of the roller are very sensitive to the turbulent shear but the bed shear has a slight effect on the sequent depth and the jump length.

The energy coefficient, \( \alpha \), and the momentum coefficient, \( \beta \), are functions of the initial Froude number, \( F_1 \), and the sequent radius ratio, \( r_0 \). The relative energy loss is a function of the initial Froude number and the sequent radius ratio, \( r_0 \). In the free radial hydraulic jump, 85% of the relative energy loss occurs in the first half of the jump length.

8.3 Conclusions on the Experimental and Macroscopic Studies

From the analysis of the experimental and the macroscopic theory for the free radial hydraulic jump and the submerged radial hydraulic jump, the following conclusions are made:

1) Macroscopic predictive equations were developed to describe the sequent depth, \( Y_0 \), as well as the water surface and the bed pressure profiles for a free radial hydraulic jump and were verified experimentally.
2) A general equation for the depth ratio, $y'$, and inlet depth factor, $\psi$, of the submerged radial hydraulic jump has been developed and verified experimentally.

3) The relative energy loss in the free radial hydraulic jump, $r_o = 1.8$, is approximately 15% greater than in the rectangular hydraulic jump at the same initial Froude numbers.

4) The relative energy loss in the submerged radial jump is a function of the inlet depth factor, $\psi$, and the sequent radius ratio, $r_o$. The relative energy loss increases as the inlet depth factor decreases.

5) The sequent depth ratio, $y_o$, of the free radial hydraulic jump is a function of the initial Froude number, $F_1$, and the radius ratio, $r_o$. For the same $F_1$, $y_o$ is less for the radial jump than for the rectangular jump. In other words, $y_o$ decreases as $r_o$ increases for the same Froude number, $F_1$.

6) The momentum coefficient, $\beta_2$, has only a slight effect on the sequent depth ratio.

7) Based on the model floor pressures, the actual uplift depending on the drainage system on the radial stilling basin slab can be 40% to 60% more than that indicated by the water surface profile for the free jump.

8) The length of the free radial hydraulic jump is approximately 30% less than the corresponding rectangular
jump, but its volume is almost the same.

9) The length of the submerged radial hydraulic jump is less than that of the submerged rectangular jump for the same inlet depth factor and initial Froude number.

10) The water surface and the hydraulic grade line profiles for the free radial hydraulic jump do not coincide due to the large amount (approximately 20%) of entrained air in the jump.

11) The water surface and the hydraulic grade profiles for the submerged radial hydraulic jump are similar except near the beginning of the jump where the centrifugal force exerted by the flow from the gate increases the bed pressure above the hydrostatic value. The free surface and bed pressure along the centerline for the free radial hydraulic jump can be fitted by a second degree equation with coefficients that depend on initial Froude numbers. The free surface and the bed pressure along the centerline for the submerged radial hydraulic jump can be fitted by a fourth degree equation with coefficients that depend on submergence and initial Froude numbers.

8.4 Recommendations

The study has given rise to a number of interesting points that warrant further research. Thus, the following investigations are recommended:

1) More experiments are required to obtain the distribution
of the entrained air in the jump.

2) More experiments are required to study the turbulent shear, and the effect of the entrained air on the turbulent shear.

3) More experiments are required to measure the surface velocity for the rectangular and radial jumps.

4) Experiments for the radial hydraulic jump with different flare angles are required to check the bi-stable jet effect.

5) The proposed numerical model can be extended to apply to the following phenomena:
   a) Submerged rectangular hydraulic jump
   b) Submerged radial hydraulic jump
   c) Hydraulic jump in a sloping channel
   d) Hydraulic jump over rough beds
   e) Jets under a pressure gradient
   f) Flow in Sedimentation Tanks
APPENDIX A
A.1 **Turbulent Shear Model**

A Prandtl mixing length model is used in the outer layer $\delta \leq y \leq h$.

$$\tau_t = -\rho u'u' = \rho \delta^2 \left| \frac{du}{dy} \right| \frac{du}{dy} \quad \ldots \quad \text{A.1}$$

using the velocity distribution Eqn. (3.26) the velocity gradient at $y_*$ is, position of maximum $\tau_t$,

$$\frac{du}{dy} = \frac{8c}{(h - \delta)^2} \ U_t \ (y - \delta) \ e^{\frac{4c}{h - \delta}} \quad \ldots \quad \text{A.2}$$

$$\frac{du}{dy} \bigg|_{y_*} = \frac{8c}{(h - \delta)^2} \ U_t \frac{(h - \delta)}{\sqrt{8c}} \ e^{-\frac{c}{2}} \quad \ldots \quad \text{A.3}$$

Assuming $l = D_*$ $(h - \delta)$

then, Eqn. (A.1) becomes,

$$\left( \frac{\tau_t}{\rho} \right)_{y_*} = k^2 \frac{(h - \delta)^2}{4} \left[ \frac{8c}{(h - \delta)^2} \ U_t \frac{h - \delta}{\sqrt{8c}} \ e^{-\frac{c}{2}} \right] \quad \ldots \quad \text{A.4}$$

$$\left( \frac{\tau_t}{\rho} \right)_{y_*} = -8 \ c \ U_t^2 \ \frac{D_*}{e} \quad \ldots \quad \text{A.5}$$

in which,

$c = -0.695$

$D_* = 0.048$ and was obtained from the experimental results of Rouse et al. [61],

$$y_* = \frac{1}{\sqrt{-8c}} \ (h - \delta) + \delta$$

116
\( \tau_{t_0} \) = the turbulent shear at \( y_* \) for a single phase hydraulic jump.

A.2 The Effect of Air in Turbulence

As mentioned earlier, the density stratification reduces turbulent mixing (8, 42). This effect has been correlated with the Richardson Number [42]. Assuming that most of the entrained air in the jump is in the outer layer (\( \delta \) to \( h \)) as shown in Fig. (A.1) and its distribution can be represented by the following function,

\[
\overline{C}_* = C_a \left( \frac{y - \delta}{h - \delta} \right)^N ; \delta < y < h
\]  ... A.6

in which,

\( C_a \) = the air concentration at the surface

\( \overline{C}_* \) = the air concentration at \( y \)

\[
N = \frac{16u_B}{(U_S + U_m)}
\]

\( u_B \) = air bubble rising velocity

\( U_m \) = maximum velocity; and

\( U_S \) = velocity at the water surface

The average air concentration \( C_o \) can be obtained as follows:

\[
C'_o = \frac{1}{h} \int_{\delta}^{h} \overline{C}_* \, dy \]  ... A.7

\[
C'_o = \frac{1}{h} \int_{\delta}^{h} C_a \left( \frac{y - \delta}{h - \delta} \right)^N \, dy \]  ... A.8
\[ C_a = \frac{C_a (h - \delta)}{h (N + 1)} \] ... A.9

then,

\[ C_a = \frac{C_a (h - \delta)}{(h - \delta)^N} \] ... A.10

in which,

\[ C_a = \text{the depth averaged air fraction} \]

\[ \frac{\partial \rho}{\partial y} = \frac{\partial (C_a - C_a^* \rho)}{\partial y} = \frac{3}{3} \left[ C_a - C_a \left( \frac{y - \delta}{h - \delta} \right)^N \right] \] ... A.11

\[ = 0 - \frac{C_a}{(h - \delta)^N} \left( y - \delta \right)^{N-1} \] ... A.12

\[ \left. \frac{\partial \rho}{\partial y} \right|_{Y^*} = \frac{C_a}{(h - \delta)^N} \frac{N (h - \delta)^{N-1}}{(-8c)^{N/2} (N-1)} = \frac{C_a (h - \delta)^{-1}}{(-8c)^{N/2} (N-1)} \] ... A.13

Substitute for \( C_a \) from Eqn. (A.10)

\[ \left[ \frac{\partial \rho}{\partial y} \right]_{Y^*} = \frac{C_a h (N + 1) N}{(-8c)^{0.5(N-1)} (h - \delta)^2} \] ... A.14

using the Richardson Number in the form,

\[ \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial y} \mid \frac{\partial u}{\partial y} \] ... A.15

Substitution of values from Eqn. (A.3) and Eqn. A.14 into Eqn. (A.15) leads to,
\[ R_{iy} = g \frac{C_{o} h (N + 1) N e}{(-8c)^{5(N + 1)} U_t^2} \] ... A.16

Odd and Rodger [42] found that the damped mixing length \( \lambda_p \) could be represented in the form \( \lambda_p = \lambda / \sqrt{1 + E_* R_i} \) ... A.17

therefore,
\[ \tau_{ty*} = \left[ \frac{\tau_{t_*}}{\sqrt{1 + E_* R_i}} \right] y_* \] ... A.18

where empirically,
\[ 43 \left[ \frac{(U_m + U_s)}{U_b} \right]^2 \leq E_* \leq 160 \] ... A.19

A.3 The Air Entrained Effect

The bulking effect resulting from the air entrainment will increase the pressure force in the upper two layers, \( \delta \) to \( h \) and from \( \delta \) to \( y_* \) Fig. (A.2) as follows:

The pressure force without air for \( \delta \) to \( h \) layer, \( P_e = \frac{Y}{2} (h-\delta)^2 \) ... A.20

The pressure force with air, \( P = \frac{Y}{2} (h-\delta) (y-\delta) \) ... A.21
The increase in the pressure force \( \Delta P = P - P_e \) ... A.22

\[ \Delta P = \frac{Y h}{2} (h-\delta) [C_{o}] \] ... A.23

in which,

\[ C_{o} = \text{the air fraction} = \frac{Y - h}{h} \] ... A.24

Adding this correction to the pressure term in the integral momentum equation from \( \delta \) to \( h \) layer gives,
\[
\frac{1}{\rho} \int_{\delta}^{h} \frac{\partial P}{\partial x} \, dy = \frac{1}{\rho} \int_{\delta}^{h} p \, dy \quad \ldots \quad A.25
\]
\[
= \frac{1}{\rho} \int_{\delta}^{h} \frac{\partial P}{\partial x} \, dy = \frac{1}{\rho} \frac{\partial}{\partial x} h \left( \bar{h} - \delta \right) \frac{C_o}{g} \quad \ldots \quad A.26
\]
\[
= g \frac{C_o}{2} \left[ 2h \frac{\partial}{\partial x} \frac{\partial h}{\partial x} - h \frac{\partial \delta}{\partial x} - \delta \frac{\partial h}{\partial x} \right] \quad \ldots \quad A.27
\]
\[
= \frac{gC_o}{2} \left[ \frac{\partial h}{\partial x} \left( 2h - \delta \right) - h \frac{\partial \delta}{\partial x} \right] \quad \ldots \quad A.28
\]

Therefore, the corrected pressure gradient/unit mass
\[
\frac{1}{\rho} \int_{\delta}^{h} \frac{\partial P}{\partial x} \, dy + \frac{gC_o}{2} \left[ \frac{\partial h}{\partial x} \left( 2h - \delta \right) - h \frac{\partial \delta}{\partial x} \right] \quad \ldots \quad A.29
\]

Similarly, the corrected pressure gradient/unit mass from \( \delta \) to \( y^* \)
\[
\frac{1}{\rho} \int_{\delta}^{y^*} \frac{\partial P}{\partial x} \, dy + \frac{gC_o}{2} \left[ \frac{\partial h}{\partial x} \left( 2h - \delta \right) - h \frac{\partial \delta}{\partial x} \right] \quad \ldots \quad A.30
\]
APPENDIX B

MEASURING INSTRUMENTS
MEASURING INSTRUMENTS

The following is a list of the measuring instruments used in this study:

1. Differential Pressure Transducers:
   Model P2104-0001.
   Specifications are found in literature supplied by the manufacturer; Schaevitz Engineering, U.S. Route 130 & Union Avenue, Pennsauken, New Jersey.

2. Strain Indicator:
   Model P-350A
   Specifications are found in the operation manual supplied by the manufacturer; Vishay Instruments Inc., 63 Lincoln Highway, Malvern, Pa., 19355

3. Two Channel Electromagnetic Current Meter:
   Model 580
   Specifications are found in the operation manual supplied by the manufacturer; Cushing Engineering Chicago.

4. Electromagnetic Flow Meter:
   Model 9650C
   Specifications are found in the operation manual supplied by the manufacturer; Foxboro Co. Ltd., Montreal.
Figure 2.1: Definition Sketch for Submerged Rectangular Hydraulic Jump after Rajaratnam [50]
Figure 3.1: Definition Sketch for the Free Radial Hydraulic Jump
Figure 3.3: Mean Velocity Distribution
Figure 3.4: Definition Diagram for Initial Conditions
Figure 4.2: Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 6.7$ and Radius Ratio, $r_0 = 1.5$
Figure 4.3: Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 7.6$ and Radius Ratio, $r_o = 1.6$. 

Depth in Inches

Distance in Inches

Velocity Dist. Scale

M.S. H.G.

Ψ=Q

Jet Expansion
Figure 4.4: Typical Flow Pattern for the Submerged Radial Hydraulic Jump for Initial Froude Number, $F_I = 5.0$, and Inlet Depth Factor, $\psi = 5.2$
Figure 4.5: Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 6$, and Inlet Depth Factor, $\psi = 6.8$
Figure 5.1: Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_l = 5.6$ and Radius Ratio, $r_o = 1.5$
Figure 5.2: Typical Flow Pattern for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 4.4$ and Radius Ratio, $r_o = 1.4$
Figure 5.3: Sequent Depth Ratio as a Function of the Initial Froude Number, $F_1$ and Radius Ratio, $r_0$. 

Experimental Data:
- $1.20 < r_0 < 1.40$
- $1.40 < r_0 < 1.60$
- $1.60 < r_0 < 1.80$
- $1.80 < r_0 < 1.85$

Rect. Jump Least Sq.
Figure 5.4: The Free Radial Jump Length Ratio, $(r_2 - r_1)/y_2$, as a Function of the Initial Froude Number $F_1$
Figure 5.5: Dimensionless Surface Profiles of the Free Radial Hydraulic Jump
Figure 5.6: Dimensionless Hydraulic Grade Profiles at the Bed of the Free Radial Hydraulic Jump
Figure 5.7: Comparison Between the Various Proposed Water Surface Profiles
Figure 5.8: Relative Energy Loss, \( \frac{\Delta E}{E_1} \), as a Function of the Initial Froude Number.
Figure 5.9: Entrained Air Fraction as a Function of Longitudinal Position and the Initial Froude Number
Figure 5.10: Comparison Between the Entrained Air for the Free Radial Hydraulic Jump and the Free Rectangular Hydraulic Jump
Figure 5.11: Percent Difference Between Uplift on Hydraulic Grade Profile and Uplift Based on the Average Free Surface
Figure 5.12: Typical Flow Pattern for Submerged Radial Hydraulic Jump for Initial Froude Number, $F_1 = 6.1$ and Inlet Depth Factor, $\psi = 10.75$
Figure 5.14: Theoretical and Experimental Inlet Depth Factor, $\psi$, as a Function of Initial Froude Number and Tail Depth Ratio, $y_0'$.
Figure 5.15: Theoretical and Experimental Tailwater Depth Ratio as a Function of Initial Froude Number, $F_r$, and Inlet Depth Factor, $\psi$
Figure 5.16: The Jump Length Ratio, $(r_2 - r_1)/y_1$, as a Function of the Initial Froude Number, $F_1$, and Inlet Depth Factor, $\psi$. 
Figure 5.17: Experimental Dimensionless Water Surface Profiles for Submerged Radial Hydraulic Jump
Figure 5.18: Experimental Dimensionless Hydraulic Grade Line Profiles for Submerged Radial Hydraulic Jump
APPENDIX D

PHOTOGRAPHS
Figure 5.19: Comparison of Theoretical and Experimental Relative Energy Loss, \( \Delta E/E_1 \), as a Function of the Initial Froude Number and Inlet Depth Factor, \( \psi \).
Figure 5.20: The Refraction Error
Figure 6.1: Sequent Depth Ratio as Function of Initial Froude Number, $F_1$ and Radius Ratio, $r_0$. 
Figure 6.2: The Relative Energy Loss, $\Delta E/E_1$, as Function of Initial Froude Number, $F_1$ and Radius Ratio, $r_o$. 
Figure 6.3: Theoretical and Experimental Tailwater Depth Ratio, $y_\theta^*$ as a Function of Initial Froude Number, $F_1$, and Inlet Depth Factor, $\psi$
Figure 6.4: Theoretical and Experimental Inlet Depth Factor, ψ, as a Function of Initial Froude Number, F₁, and Tail Depth Ratio, y₀'.
Figure 6.5: Theoretical Relative Depth Ratio Versus Observed Relative Depth Ratio for the Submerged Radial Hydraulic Jump
Figure 6.6. Comparison of Theoretical and Experimental Relative Energy Loss, $\Delta E/E_1$, as a Function of the Initial Froude Number and Inlet Depth Factor, $\psi$
Figure 6.7: The Dimensionless Water Surface Profiles of the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers, $F_1$
Figure 6.8: The Dimensionless Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers, $F_1$
Figure 6.9: Theoretical Depth Ratio, $\frac{y_2}{y_1}$, as Function of Initial Froude Number, $F_1$, for the Rectangular Jump
Figure 6.10: The Jump Length Ratio, $L_j/y_2$, as a Function of the Initial Froude Number, $F_1$, for the Free Rectangular Hydraulic Jump
Figure 6.11: The Roller Length Ratio, $L_r/y_2$, as a Function of the Initial Froude Number, $F_1$, for the Rectangular Hydraulic Jump
Figure 6.6. Comparison of Theoretical and Experimental Relative Energy Loss, $\frac{\Delta E}{E_1}$, as a Function of the Initial Froude Number $F_0$ and Inlet Depth Factor, $\psi$. 

$\text{r_{th}} = 1.8$ 

Experimental Data 

$17 < F_0 \psi < 19$ 

- $2 < \psi < 4$ 

- $4 < \psi < 6$ 

- $6 < \psi < 8$ 

- F. Radial 

- S. Radial 

- F. Rect. Theory 

- S. Rect. Theory 

Equation 124
Figure 6.7: The Dimensionless Water Surface Profiles of the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers, $F_1$
Figure 6.8: The Dimensionless Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers, F_L.
Figure 6.9: Theoretical Depth Ratio, $\frac{y_2}{y_1}$, as Function of Initial Froude Number, $F_1$, for the Rectangular Jump
Figure 6.10: The Jump Length Ratio, $L_J/y_2$, as a Function of the Initial Froude Number, $F_1$, for the Free Rectangular Hydraulic Jump
Figure 6.11: The Roller Length Ratio $L_r/y_2$, as a Function of the Initial Froude Number $F_1$, for the Rectangular Hydraulic Jump
Figure 6.12: Typical Theoretical Velocity Distribution along the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$
Figure 6.13: The Decay of the Maximum Velocity for the Free Rectangular Hydraulic Jump
Figure 6.14: The Variation of the Surface Velocity for the Free Rectangular Hydraulic Jump
Figure 6.15: The Growth of the Boundary Layer for the Free Rectangular Hydraulic Jump
Figure 6.16: The Variation of $\alpha$, with $x/y_1$, for the Freé Rectangular Hydraulic Jump.
Figure 6.17: The Variation of the $\beta$ with $x/y_1$ for the Free Rectangular Hydraulic Jump
Figure 6.18: The Relative Energy Profiles of the Free Rectangular Hydraulic Jump for Various Initial Froude Numbers
Figure 6.19: The Relative Energy Loss as a Function of the Initial Froude Number for the Free Rectangular Hydraulic Jump
Figure 6.20: Comparison of the Water Surface and the Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump.
Figure 6.21: Comparison of the Dimensionless Water Surface Profiles for the Rectangular Hydraulic Jump
Figure 6.22: Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump
Figure 6.23: Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Rectangular Hydraulic Jump
Figure 6.24: Comparison of the Jump Length Ratio, $L_J/Y_2$, for the Free Rectangular Hydraulic Jump
Figure 6.25: Comparison of the Roller Length, \( L_r/Y_2 \), for the Free Rectangular Hydraulic Jump
Figure 6.26: The Sequent Depth Ratio, \( y_2/y_1 \), as a Function of the Initial Froude Number, \( F_1 \), for the Free Rectangular Hydraulic Jump
Figure 6.27: Comparison of the Decay of the Maximum Velocity for the Free Rectangular Hydraulic Jump
Figure 6.28: Comparison of the Surface Velocity for the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$
Figure 6.29: Comparison of the Boundary Layer Growth for the Free Rectangular Hydraulic Jump
Figure 6.30: Comparison of the Relative Energy Profiles of the Free Rectangular Hydraulic Jump
Figure 6.31: Comparison of the Velocity Distribution along the Free Rectangular Hydraulic Jump for Initial Froude Number, $F_1 = 6$
Figure 6.32: The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number, $F_1 = 4$, and Sequent Radius Ratios, $r_o = 1.2$ and $1.8$
Figure 6.33: The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number $F_1=6$ and Sequent Radius Ratios, $r_o = 1.2$ and 1.8
Figure 6.34: The Theoretical Dimensionless Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Froude Number $F_1 = 8$ and Sequent Radius Ratios, $r_o = 1.2$ and 1.8.
Figure 6.35: The Theoretical Depth Ratio, $y_0/y_1$, as a Function of the Initial Froude Number, $F_1$, and the Radiiš Ratio, $r_0$, for the Free Radial Hydraulic Jump.
Figure 6.36: The Jump Length Ratio, $L_j/y_2$, as a Function of the Initial Froude Number, $F_1$, and Radius Ratio, $r_o$, for the Free Radial Hydraulic Jump.
Figure 3.37: The Roller Length Ratio, $L/L_{r2}$, as a Function of the Initial Froude Number and Radius Ratio, $r_{0}/r_{f0}$, for the Free Radial Hydraulic Jump
Figure 6.38: The Theoretical Dimensionless Velocity Distribution along the Centre Line of the Free Radial Hydraulic Jump
Figure 6.39: The Decay of the Maximum Velocity for the Free Radial Hydraulic Jump
Figure 6.42: The Variation of the Energy Coefficient $\alpha$ for the Free Radial Hydraulic Jump
Figure 6.43: The Variation of the Momentum Coefficient $\beta$ for the Free Radial Hydraulic Jump
Figure 6.44: The Relative Energy Profiles of the Free Radial Hydraulic Jump
Figure 6.45: The Relative Energy Loss as a Function of the Initial Froude Number and Radius Ratio for the Free Radial Hydraulic Jump
Figure 6.46: Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Initial Froude Number, \( F_1 = 3.9 \) and Radius Ratio, \( r_o = 1.5 \)
Figure 6.47: Comparison of the Water Surface and Hydraulic Grade for the Free Radial Hydraulic Jump for Initial Froude Number, $F_1 = 5.6$ and Radius Ratio, $r_o = 1.4$
Figure 6.48: Comparison of the Water Surface and Hydraulic Grade Profiles for the Free Radial Hydraulic Jump for Initial Proude Number, $F_1 = 7.8$ and Radius Ratio, $r_o = 1.8$
Figure 6.49: Comparison of the Sequent Depth Ratio for the Free Radial Hydraulic Jump
Figure 6.50: Comparison of the Jump Length Ratio, \( \frac{L_j}{y_2} \), for the Free Radial Hydraulic Jump
Figure 6.51: The Relative Jump Length, $L_j/y_2$, as a Function of the Sequent Radius Ratio, $r_o$, and Initial Froude Number, $F_1$.
Figure 6.52: Comparison of the Decay of the Maximum Velocity for the Free Radial Hydraulic Jump
\[ F_1 = 4.4 \]
\[ r_0 = 1.2 \]

- Experimental Data
- Theoretical

Figure 6.53: Comparison of the Velocity Distribution for the Free Radial Hydraulic Jump for Initial Froude Number, \( F_1 = 4.4 \) and Radius Ratio, \( r_0 = 1.2 \)
Figure 6.54: Comparison of the Velocity Distribution for the Free Radial Hydraulic Jump for Initial Froude Number, \( F_1 = 5.6 \) and Radius Ratio, \( r_o = 1.5 \)
Figure 7.1: The Effect of the Turbulent Shear in the Water Surface Profile
Figure 7.2: The Effect of the Turbulent Shear in the Decay of the Maximum Velocity
Figure 7.3: The Effect of the Turbulent Shear in the Surface Velocity
Figure 7.4: The Effect of Air Entrained in the Water Surface Profile
Figure A.1: The Entrained Air Distribution
Figure A.2: Pressure Distribution
Photograph 4.1: Top View of the Experimental Apparatus
Photograph 4.2: The Arrangement of the Piezometer Tappings
Photograph 4.3: The Manometer Board
Photograph 4.4: Side View of the Model
Photograph 4.5: Entrance Transition
Photograph 4.6: Two Channel Electromagnetic Current Meter

Photograph 4.7: Differential Pressure Transducer and the Strain Indicator
<table>
<thead>
<tr>
<th>RUN</th>
<th>Q (L/s)</th>
<th>Y1 (MM)</th>
<th>Y2 (MM)</th>
<th>R1 (MM)</th>
<th>R2 (MM)</th>
<th>R3 (MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.8</td>
<td>35.6</td>
<td>158.7</td>
<td>1117.6</td>
<td>1549.4</td>
<td>1447.8</td>
</tr>
<tr>
<td>2</td>
<td>24.3</td>
<td>36.1</td>
<td>177.8</td>
<td>990.6</td>
<td>1752.6</td>
<td>1473.2</td>
</tr>
<tr>
<td>3</td>
<td>15.8</td>
<td>40.6</td>
<td>139.7</td>
<td>990.6</td>
<td>1524.0</td>
<td>1371.6</td>
</tr>
<tr>
<td>4</td>
<td>10.8</td>
<td>16.5</td>
<td>139.7</td>
<td>1041.4</td>
<td>1651.0</td>
<td>1498.6</td>
</tr>
<tr>
<td>5</td>
<td>5.4</td>
<td>17.8</td>
<td>82.5</td>
<td>939.8</td>
<td>1270.0</td>
<td>1168.4</td>
</tr>
<tr>
<td>6</td>
<td>18.0</td>
<td>21.6</td>
<td>139.7</td>
<td>990.6</td>
<td>1625.6</td>
<td>1422.4</td>
</tr>
<tr>
<td>7</td>
<td>11.2</td>
<td>21.6</td>
<td>121.7</td>
<td>990.6</td>
<td>1371.6</td>
<td>1295.4</td>
</tr>
<tr>
<td>8</td>
<td>10.9</td>
<td>29.2</td>
<td>101.6</td>
<td>1066.8</td>
<td>1371.6</td>
<td>1270.0</td>
</tr>
<tr>
<td>9</td>
<td>20.2</td>
<td>28.7</td>
<td>184.1</td>
<td>990.6</td>
<td>1727.2</td>
<td>1447.8</td>
</tr>
<tr>
<td>10</td>
<td>20.9</td>
<td>36.8</td>
<td>177.0</td>
<td>1016.0</td>
<td>1600.2</td>
<td>1473.2</td>
</tr>
<tr>
<td>11</td>
<td>20.9</td>
<td>47.5</td>
<td>158.7</td>
<td>990.6</td>
<td>1524.0</td>
<td>1397.0</td>
</tr>
<tr>
<td>12</td>
<td>29.7</td>
<td>47.0</td>
<td>215.9</td>
<td>990.6</td>
<td>1879.6</td>
<td>1727.2</td>
</tr>
<tr>
<td>13</td>
<td>29.7</td>
<td>58.7</td>
<td>199.4</td>
<td>990.6</td>
<td>1778.0</td>
<td>1651.0</td>
</tr>
<tr>
<td>14</td>
<td>37.8</td>
<td>64.3</td>
<td>238.6</td>
<td>990.6</td>
<td>1854.2</td>
<td>1600.2</td>
</tr>
<tr>
<td>15</td>
<td>29.0</td>
<td>68.1</td>
<td>184.1</td>
<td>990.6</td>
<td>1676.4</td>
<td>1447.8</td>
</tr>
<tr>
<td>16</td>
<td>29.0</td>
<td>61.5</td>
<td>190.5</td>
<td>990.6</td>
<td>1574.8</td>
<td>1473.2</td>
</tr>
<tr>
<td>17</td>
<td>29.0</td>
<td>54.6</td>
<td>209.5</td>
<td>990.6</td>
<td>1676.4</td>
<td>1422.4</td>
</tr>
<tr>
<td>18</td>
<td>17.0</td>
<td>44.4</td>
<td>133.3</td>
<td>990.6</td>
<td>1397.0</td>
<td>1320.8</td>
</tr>
<tr>
<td>19</td>
<td>11.1</td>
<td>15.5</td>
<td>137.2</td>
<td>990.6</td>
<td>1574.8</td>
<td>1447.8</td>
</tr>
<tr>
<td>RUN</td>
<td>D(L/S)</td>
<td>Y1(MM.)</td>
<td>Y2(MM.)</td>
<td>R1(MM.)</td>
<td>R2(MM.)</td>
<td>R3(MM.)</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>20</td>
<td>11.1</td>
<td>18.3</td>
<td>130.8</td>
<td>990.6</td>
<td>1600.2</td>
<td>1447.8</td>
</tr>
<tr>
<td>21</td>
<td>11.1</td>
<td>21.8</td>
<td>121.4</td>
<td>990.6</td>
<td>1473.2</td>
<td>1447.8</td>
</tr>
<tr>
<td>22</td>
<td>11.1</td>
<td>23.9</td>
<td>111.8</td>
<td>1066.8</td>
<td>1473.2</td>
<td>1371.6</td>
</tr>
<tr>
<td>23</td>
<td>16.7</td>
<td>21.8</td>
<td>155.7</td>
<td>1117.6</td>
<td>1778.0</td>
<td>1651.0</td>
</tr>
<tr>
<td>24</td>
<td>16.7</td>
<td>26.2</td>
<td>146.8</td>
<td>1117.6</td>
<td>1701.8</td>
<td>1574.8</td>
</tr>
<tr>
<td>25</td>
<td>21.1</td>
<td>29.2</td>
<td>185.4</td>
<td>990.6</td>
<td>1803.4</td>
<td>1524.0</td>
</tr>
<tr>
<td>26</td>
<td>15.4</td>
<td>29.7</td>
<td>149.9</td>
<td>1016.0</td>
<td>1600.2</td>
<td>1473.2</td>
</tr>
<tr>
<td>27</td>
<td>15.4</td>
<td>35.0</td>
<td>138.2</td>
<td>990.6</td>
<td>1778.0</td>
<td>1701.8</td>
</tr>
<tr>
<td>28</td>
<td>22.4</td>
<td>34.0</td>
<td>179.1</td>
<td>1066.8</td>
<td>1778.0</td>
<td>1574.8</td>
</tr>
<tr>
<td>29</td>
<td>22.4</td>
<td>43.2</td>
<td>180.3</td>
<td>990.6</td>
<td>1676.4</td>
<td>1422.4</td>
</tr>
<tr>
<td>30</td>
<td>23.0</td>
<td>40.3</td>
<td>173.2</td>
<td>990.6</td>
<td>1676.4</td>
<td>1422.4</td>
</tr>
<tr>
<td>31</td>
<td>23.3</td>
<td>51.3</td>
<td>162.6</td>
<td>1168.4</td>
<td>1651.0</td>
<td>1524.0</td>
</tr>
<tr>
<td>32</td>
<td>23.3</td>
<td>32.3</td>
<td>190.5</td>
<td>990.6</td>
<td>1930.4</td>
<td>1600.2</td>
</tr>
<tr>
<td>33</td>
<td>10.5</td>
<td>15.0</td>
<td>132.1</td>
<td>1016.0</td>
<td>1625.6</td>
<td>1473.2</td>
</tr>
<tr>
<td>34</td>
<td>10.5</td>
<td>16.5</td>
<td>116.8</td>
<td>1092.2</td>
<td>1676.4</td>
<td>1524.0</td>
</tr>
<tr>
<td>35</td>
<td>13.9</td>
<td>15.7</td>
<td>146.6</td>
<td>1066.8</td>
<td>1752.6</td>
<td>1625.6</td>
</tr>
<tr>
<td>36</td>
<td>13.9</td>
<td>20.3</td>
<td>142.7</td>
<td>1066.8</td>
<td>1676.4</td>
<td>1524.0</td>
</tr>
<tr>
<td>37</td>
<td>19.2</td>
<td>27.9</td>
<td>181.6</td>
<td>990.6</td>
<td>1828.8</td>
<td>1600.2</td>
</tr>
<tr>
<td>38</td>
<td>19.2</td>
<td>31.2</td>
<td>175.8</td>
<td>990.6</td>
<td>1828.8</td>
<td>1600.2</td>
</tr>
<tr>
<td>RUN</td>
<td>O(L/S)</td>
<td>Y1(MM.)</td>
<td>Y2(MM.)</td>
<td>R1(MM.)</td>
<td>R2(MM.)</td>
<td>R3(MM.)</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>40</td>
<td>19.0</td>
<td>32.5</td>
<td>166.1</td>
<td>1092.2</td>
<td>1770.0</td>
<td>1574.0</td>
</tr>
<tr>
<td>41</td>
<td>19.8</td>
<td>40.3</td>
<td>125.5</td>
<td>1016.0</td>
<td>1524.0</td>
<td>1422.4</td>
</tr>
<tr>
<td>42</td>
<td>22.4</td>
<td>38.6</td>
<td>163.8</td>
<td>1066.8</td>
<td>1676.4</td>
<td>1524.0</td>
</tr>
<tr>
<td>43</td>
<td>22.7</td>
<td>53.3</td>
<td>151.1</td>
<td>1066.8</td>
<td>1600.2</td>
<td>1498.6</td>
</tr>
<tr>
<td>44</td>
<td>14.5</td>
<td>23.1</td>
<td>153.9</td>
<td>1104.9</td>
<td>1625.6</td>
<td>1498.6</td>
</tr>
<tr>
<td>45</td>
<td>13.6</td>
<td>19.3</td>
<td>136.1</td>
<td>1206.5</td>
<td>1727.2</td>
<td>1625.6</td>
</tr>
<tr>
<td>46</td>
<td>13.3</td>
<td>10.0</td>
<td>132.3</td>
<td>1219.2</td>
<td>1701.8</td>
<td>1574.8</td>
</tr>
<tr>
<td>47</td>
<td>12.9</td>
<td>18.0</td>
<td>145.0</td>
<td>1117.6</td>
<td>1549.4</td>
<td>1473.2</td>
</tr>
<tr>
<td>48</td>
<td>10.5</td>
<td>24.9</td>
<td>98.0</td>
<td>1270.0</td>
<td>1473.2</td>
<td>1397.0</td>
</tr>
<tr>
<td>49</td>
<td>10.5</td>
<td>23.4</td>
<td>117.1</td>
<td>965.2</td>
<td>1320.8</td>
<td>1244.6</td>
</tr>
<tr>
<td>50</td>
<td>8.9</td>
<td>15.4</td>
<td>79.0</td>
<td>1117.6</td>
<td>1727.0</td>
<td>1625.0</td>
</tr>
<tr>
<td>51</td>
<td>8.9</td>
<td>18.0</td>
<td>91.7</td>
<td>1016.0</td>
<td>1625.6</td>
<td>1524.0</td>
</tr>
<tr>
<td>52</td>
<td>8.1</td>
<td>15.5</td>
<td>73.7</td>
<td>1066.8</td>
<td>1651.0</td>
<td>1524.0</td>
</tr>
<tr>
<td>53</td>
<td>6.4</td>
<td>15.5</td>
<td>61.2</td>
<td>1320.8</td>
<td>1778.0</td>
<td>1676.4</td>
</tr>
<tr>
<td>54</td>
<td>5.0</td>
<td>14.2</td>
<td>57.4</td>
<td>965.2</td>
<td>1219.2</td>
<td>1143.0</td>
</tr>
<tr>
<td>55</td>
<td>3.2</td>
<td>10.4</td>
<td>24.4</td>
<td>1493.6</td>
<td>1574.8</td>
<td>1498.6</td>
</tr>
<tr>
<td>56</td>
<td>10.2</td>
<td>40.9</td>
<td>82.8</td>
<td>1117.6</td>
<td>1244.6</td>
<td>1168.4</td>
</tr>
<tr>
<td>57</td>
<td>10.2</td>
<td>30.7</td>
<td>75.7</td>
<td>1447.8</td>
<td>1651.0</td>
<td>1549.4</td>
</tr>
<tr>
<td>58</td>
<td>10.2</td>
<td>19.3</td>
<td>96.8</td>
<td>1295.4</td>
<td>1574.0</td>
<td>1498.6</td>
</tr>
<tr>
<td>59</td>
<td>10.2</td>
<td>16.0</td>
<td>114.3</td>
<td>1193.8</td>
<td>1651.0</td>
<td>1371.6</td>
</tr>
<tr>
<td>RUN</td>
<td>Q(L/S)</td>
<td>Y1(MM.)</td>
<td>Y2(MM.)</td>
<td>R1(MM.)</td>
<td>R2(MM.)</td>
<td>R3(MM.)</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>80</td>
<td>13.5</td>
<td>38.9</td>
<td>103.1</td>
<td>1016.0</td>
<td>1244.6</td>
<td>1193.8</td>
</tr>
<tr>
<td>81</td>
<td>13.5</td>
<td>37.8</td>
<td>104.4</td>
<td>1079.5</td>
<td>1371.6</td>
<td>1320.0</td>
</tr>
<tr>
<td>82</td>
<td>15.2</td>
<td>40.9</td>
<td>84.1</td>
<td>1003.3</td>
<td>1397.0</td>
<td>1295.4</td>
</tr>
<tr>
<td>83</td>
<td>15.3</td>
<td>35.0</td>
<td>117.1</td>
<td>1143.0</td>
<td>1524.0</td>
<td>1447.8</td>
</tr>
<tr>
<td>84</td>
<td>15.3</td>
<td>32.0</td>
<td>122.0</td>
<td>1295.4</td>
<td>1651.0</td>
<td>1574.8</td>
</tr>
<tr>
<td>85</td>
<td>15.3</td>
<td>30.7</td>
<td>98.0</td>
<td>1244.6</td>
<td>1600.2</td>
<td>1524.0</td>
</tr>
<tr>
<td>86</td>
<td>15.3</td>
<td>35.0</td>
<td>132.3</td>
<td>1041.4</td>
<td>1447.8</td>
<td>1346.2</td>
</tr>
<tr>
<td>87</td>
<td>15.4</td>
<td>26.2</td>
<td>129.8</td>
<td>1244.6</td>
<td>1651.0</td>
<td>1524.0</td>
</tr>
<tr>
<td>88</td>
<td>15.4</td>
<td>33.3</td>
<td>142.5</td>
<td>965.2</td>
<td>1397.0</td>
<td>1270.0</td>
</tr>
<tr>
<td>89</td>
<td>15.4</td>
<td>26.9</td>
<td>142.5</td>
<td>1092.2</td>
<td>1549.4</td>
<td>1447.8</td>
</tr>
<tr>
<td>90</td>
<td>15.8</td>
<td>32.3</td>
<td>150.1</td>
<td>1054.1</td>
<td>1651.0</td>
<td>1524.0</td>
</tr>
<tr>
<td>91</td>
<td>17.3</td>
<td>27.4</td>
<td>155.2</td>
<td>1244.6</td>
<td>2019.3</td>
<td>1765.3</td>
</tr>
<tr>
<td>92</td>
<td>17.6</td>
<td>31.2</td>
<td>167.6</td>
<td>1016.0</td>
<td>1714.5</td>
<td>1549.4</td>
</tr>
<tr>
<td>93</td>
<td>15.4</td>
<td>28.2</td>
<td>134.9</td>
<td>1225.5</td>
<td>1780.0</td>
<td>1574.6</td>
</tr>
<tr>
<td>94</td>
<td>14.5</td>
<td>31.2</td>
<td>145.0</td>
<td>933.4</td>
<td>1549.4</td>
<td>1320.8</td>
</tr>
<tr>
<td>95</td>
<td>11.0</td>
<td>26.2</td>
<td>105.7</td>
<td>1168.4</td>
<td>1600.2</td>
<td>1478.3</td>
</tr>
<tr>
<td>96</td>
<td>10.7</td>
<td>30.5</td>
<td>106.7</td>
<td>939.8</td>
<td>1428.7</td>
<td>1244.6</td>
</tr>
<tr>
<td>97</td>
<td>10.4</td>
<td>27.7</td>
<td>101.9</td>
<td>1155.7</td>
<td>1498.6</td>
<td>1447.8</td>
</tr>
<tr>
<td>RUN</td>
<td>Q(L/S)</td>
<td>Y1(MM.)</td>
<td>Y2(MM.)</td>
<td>R1(MM.)</td>
<td>R2(MM.)</td>
<td>R3(MM.)</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>60</td>
<td>10.2</td>
<td>15.5</td>
<td>137.2</td>
<td>1219.2</td>
<td>1676.4</td>
<td>1524.0</td>
</tr>
<tr>
<td>61</td>
<td>7.1</td>
<td>15.5</td>
<td>104.4</td>
<td>1100.0</td>
<td>1500.0</td>
<td>1450.0</td>
</tr>
<tr>
<td>62</td>
<td>7.1</td>
<td>18.0</td>
<td>100.2</td>
<td>1100.0</td>
<td>1502.0</td>
<td>1450.0</td>
</tr>
<tr>
<td>63</td>
<td>4.9</td>
<td>16.8</td>
<td>66.3</td>
<td>950.0</td>
<td>1290.0</td>
<td>1250.0</td>
</tr>
<tr>
<td>64</td>
<td>13.5</td>
<td>32.0</td>
<td>70.1</td>
<td>1422.4</td>
<td>1676.4</td>
<td>1625.6</td>
</tr>
<tr>
<td>65</td>
<td>10.0</td>
<td>18.8</td>
<td>101.6</td>
<td>1160.4</td>
<td>1600.2</td>
<td>1447.0</td>
</tr>
<tr>
<td>66</td>
<td>10.0</td>
<td>14.2</td>
<td>111.0</td>
<td>1193.8</td>
<td>1676.4</td>
<td>1549.4</td>
</tr>
<tr>
<td>67</td>
<td>11.7</td>
<td>19.0</td>
<td>105.4</td>
<td>1346.2</td>
<td>1701.8</td>
<td>1625.6</td>
</tr>
<tr>
<td>68</td>
<td>11.7</td>
<td>20.6</td>
<td>118.1</td>
<td>1117.6</td>
<td>1600.2</td>
<td>1473.2</td>
</tr>
<tr>
<td>69</td>
<td>13.7</td>
<td>17.0</td>
<td>139.7</td>
<td>1143.0</td>
<td>1676.4</td>
<td>1600.2</td>
</tr>
<tr>
<td>70</td>
<td>13.7</td>
<td>10.3</td>
<td>129.5</td>
<td>1295.4</td>
<td>1727.2</td>
<td>1651.0</td>
</tr>
<tr>
<td>71</td>
<td>8.9</td>
<td>16.8</td>
<td>71.4</td>
<td>1524.0</td>
<td>1790.7</td>
<td>1676.4</td>
</tr>
<tr>
<td>72</td>
<td>8.9</td>
<td>21.0</td>
<td>91.7</td>
<td>952.5</td>
<td>1295.4</td>
<td>1244.6</td>
</tr>
<tr>
<td>73</td>
<td>15.3</td>
<td>30.7</td>
<td>91.7</td>
<td>1371.6</td>
<td>1752.6</td>
<td>1524.0</td>
</tr>
<tr>
<td>74</td>
<td>14.1</td>
<td>24.4</td>
<td>145.0</td>
<td>1098.5</td>
<td>1574.8</td>
<td>1346.2</td>
</tr>
<tr>
<td>75</td>
<td>14.1</td>
<td>25.7</td>
<td>142.2</td>
<td>1168.4</td>
<td>1778.0</td>
<td>1676.4</td>
</tr>
<tr>
<td>76</td>
<td>14.1</td>
<td>30.7</td>
<td>134.9</td>
<td>1016.0</td>
<td>1765.3</td>
<td>1651.0</td>
</tr>
<tr>
<td>77</td>
<td>14.1</td>
<td>32.0</td>
<td>126.0</td>
<td>1028.7</td>
<td>1524.0</td>
<td>1447.0</td>
</tr>
<tr>
<td>78</td>
<td>0.7</td>
<td>18.0</td>
<td>83.1</td>
<td>965.2</td>
<td>1727.2</td>
<td>1574.8</td>
</tr>
<tr>
<td>79</td>
<td>13.5</td>
<td>35.1</td>
<td>101.1</td>
<td>1193.8</td>
<td>1422.4</td>
<td>1320.0</td>
</tr>
</tbody>
</table>
### Table 4.2

**Measured Mean-Velocity Distribution for the Free Radial Hydraulic Jump**

**Run Number:** 28  
**Q = 4.6 L/sec.**  
**R1 = 991. MM.**  
**R2 = 1397. MM.**  
**F1 = 4.40**

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>991.</th>
<th>1016.</th>
<th>1041.</th>
<th>1067.</th>
<th>1092.</th>
<th>1118.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (MM.)</td>
<td>U(M /S)</td>
<td>U(M /S)</td>
<td>U(M /S)</td>
<td>U(M /S)</td>
<td>U(M /S)</td>
<td>U(M /S)</td>
</tr>
<tr>
<td>1.6</td>
<td>1.60</td>
<td>1.40</td>
<td>1.06</td>
<td>.85</td>
<td>.73</td>
<td>.66</td>
</tr>
<tr>
<td>4.1</td>
<td>1.67</td>
<td>1.40</td>
<td>1.06</td>
<td>.85</td>
<td>.73</td>
<td>.66</td>
</tr>
<tr>
<td>6.7</td>
<td>1.67</td>
<td>1.24</td>
<td>.98</td>
<td>.79</td>
<td>.69</td>
<td>.62</td>
</tr>
<tr>
<td>9.2</td>
<td>1.67</td>
<td>1.15</td>
<td>.95</td>
<td>.72</td>
<td>.66</td>
<td>.59</td>
</tr>
<tr>
<td>11.8</td>
<td>0.0</td>
<td>1.00</td>
<td>.92</td>
<td>.69</td>
<td>.62</td>
<td>.52</td>
</tr>
<tr>
<td>12.7</td>
<td>---</td>
<td>1.02</td>
<td>.885</td>
<td>.66</td>
<td>.62</td>
<td>.52</td>
</tr>
<tr>
<td>15.2</td>
<td>---</td>
<td>.885</td>
<td>.81</td>
<td>.66</td>
<td>.59</td>
<td>.52</td>
</tr>
<tr>
<td>17.8</td>
<td>---</td>
<td>.66</td>
<td>.72</td>
<td>.62</td>
<td>.52</td>
<td>.49</td>
</tr>
<tr>
<td>20.3</td>
<td>---</td>
<td>.49</td>
<td>.59</td>
<td>.52</td>
<td>.49</td>
<td>.46</td>
</tr>
<tr>
<td>22.9</td>
<td>---</td>
<td>.33</td>
<td>.52</td>
<td>.52</td>
<td>.46</td>
<td>.35</td>
</tr>
<tr>
<td>25.9</td>
<td>---</td>
<td>0.0</td>
<td>.426</td>
<td>.46</td>
<td>.426</td>
<td>.33</td>
</tr>
<tr>
<td>30.</td>
<td>---</td>
<td>---</td>
<td>.33</td>
<td>.33</td>
<td>.36</td>
<td>.26</td>
</tr>
<tr>
<td>35.6</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.33</td>
<td>.26</td>
<td>.23</td>
</tr>
<tr>
<td>40.6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.26</td>
<td>.2</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
</tr>
<tr>
<td>48.3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
<tr>
<td>50.8</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>R (mm.)</td>
<td>991</td>
<td>1016</td>
<td>1067</td>
<td>1118</td>
<td>1168</td>
<td>1245</td>
</tr>
<tr>
<td>---------</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>Y (mm.)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
</tr>
<tr>
<td>---------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>1.6</td>
<td>2.1</td>
<td>1.80</td>
<td>1.09</td>
<td>.82</td>
<td>.72</td>
<td>.59</td>
</tr>
<tr>
<td>.41</td>
<td>2.1</td>
<td>1.80</td>
<td>1.05</td>
<td>.85</td>
<td>.72</td>
<td>.59</td>
</tr>
<tr>
<td>.67</td>
<td>2.1</td>
<td>1.11</td>
<td>.82</td>
<td>.69</td>
<td>.62</td>
<td></td>
</tr>
<tr>
<td>9.2</td>
<td>2.1</td>
<td>1.64</td>
<td>1.05</td>
<td>.79</td>
<td>.66</td>
<td>.62</td>
</tr>
<tr>
<td>11.8</td>
<td>0.0</td>
<td>1.37</td>
<td>1.02</td>
<td>.79</td>
<td>.66</td>
<td>.62</td>
</tr>
<tr>
<td>12.7</td>
<td>---</td>
<td>1.34</td>
<td>.98</td>
<td>.82</td>
<td>.72</td>
<td>.62</td>
</tr>
<tr>
<td>15.2</td>
<td>---</td>
<td>.98</td>
<td>.85</td>
<td>.72</td>
<td>.62</td>
<td>.59</td>
</tr>
<tr>
<td>17.0</td>
<td>---</td>
<td>.82</td>
<td>.79</td>
<td>.72</td>
<td>.62</td>
<td>.59</td>
</tr>
<tr>
<td>20.3</td>
<td>---</td>
<td>.69</td>
<td>.72</td>
<td>.69</td>
<td>.62</td>
<td>.52</td>
</tr>
<tr>
<td>22.9</td>
<td>---</td>
<td>.49</td>
<td>.69</td>
<td>.66</td>
<td>.59</td>
<td>.52</td>
</tr>
<tr>
<td>25.9</td>
<td>---</td>
<td>.26</td>
<td>.66</td>
<td>.62</td>
<td>.59</td>
<td>.52</td>
</tr>
<tr>
<td>30.</td>
<td>---</td>
<td>0.0</td>
<td>.52</td>
<td>.52</td>
<td>.52</td>
<td>.52</td>
</tr>
<tr>
<td>35.6</td>
<td>---</td>
<td>---</td>
<td>.35</td>
<td>.49</td>
<td>.52</td>
<td>.46</td>
</tr>
<tr>
<td>40.6</td>
<td>---</td>
<td>---</td>
<td>.26</td>
<td>.426</td>
<td>.46</td>
<td>.35</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>.2</td>
<td>.36</td>
<td>.35</td>
<td>.35</td>
</tr>
<tr>
<td>50.8</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.26</td>
<td>.33</td>
<td>.33</td>
</tr>
<tr>
<td>55.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>2</td>
<td>.26</td>
<td>.26</td>
</tr>
<tr>
<td>61.0</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.23</td>
<td>.26</td>
</tr>
<tr>
<td>65.6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
</tr>
<tr>
<td>71.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
<tr>
<td>76.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
### Table 4.4

**Measured Mean-velocity Distribution**

For the Free Radial Hydraulic Jump.

- **Run Number:** 100
- **Q = 0.2 L/sec.**
- **Y1 = 12.8 mm, Y2 = 104.1 mm.**
- **R1 = 99.1 mm, R2 = 1422.4 mm.**
- **F1 = 7.80**

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>99.1</th>
<th>1016</th>
<th>1092</th>
<th>1168</th>
<th>1245</th>
<th>1295</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (mm)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>1.6</td>
<td>2.07</td>
<td>2.46</td>
<td>1.27</td>
<td>.92</td>
<td>.79</td>
<td>.76</td>
</tr>
<tr>
<td>4.1</td>
<td>2.80</td>
<td>2.46</td>
<td>1.27</td>
<td>.95</td>
<td>.82</td>
<td>.79</td>
</tr>
<tr>
<td>6.7</td>
<td>2.89</td>
<td>2.29</td>
<td>1.24</td>
<td>.92</td>
<td>.82</td>
<td>.76</td>
</tr>
<tr>
<td>9.2</td>
<td>2.87</td>
<td>2.1</td>
<td>1.2</td>
<td>.92</td>
<td>.79</td>
<td>.69</td>
</tr>
<tr>
<td>11.8</td>
<td>2.87</td>
<td>2.0</td>
<td>1.18</td>
<td>.885</td>
<td>.79</td>
<td>.69</td>
</tr>
<tr>
<td>12.7</td>
<td>0.0</td>
<td>1.86</td>
<td>1.18</td>
<td>.885</td>
<td>.76</td>
<td>.69</td>
</tr>
<tr>
<td>15.2</td>
<td>---</td>
<td>1.60</td>
<td>1.11</td>
<td>.85</td>
<td>.72</td>
<td>.66</td>
</tr>
<tr>
<td>17.9</td>
<td>---</td>
<td>1.34</td>
<td>.90</td>
<td>.82</td>
<td>.72</td>
<td>.66</td>
</tr>
<tr>
<td>20.3</td>
<td>---</td>
<td>.78</td>
<td>.95</td>
<td>.79</td>
<td>.69</td>
<td>.62</td>
</tr>
<tr>
<td>25.7</td>
<td>---</td>
<td>.43</td>
<td>.92</td>
<td>.79</td>
<td>.69</td>
<td>.62</td>
</tr>
<tr>
<td>30.4</td>
<td>---</td>
<td>0.0</td>
<td>.82</td>
<td>.72</td>
<td>.66</td>
<td>.59</td>
</tr>
<tr>
<td>35.6</td>
<td>---</td>
<td>---</td>
<td>.66</td>
<td>.59</td>
<td>.59</td>
<td>.52</td>
</tr>
<tr>
<td>40.6</td>
<td>---</td>
<td>---</td>
<td>.59</td>
<td>.59</td>
<td>.52</td>
<td>.49</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>.35</td>
<td>.35</td>
<td>.35</td>
<td>.36</td>
</tr>
<tr>
<td>50.8</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.33</td>
<td>.33</td>
<td>.33</td>
</tr>
<tr>
<td>55.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.26</td>
<td>.26</td>
<td>.26</td>
</tr>
<tr>
<td>63.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.2</td>
<td>.23</td>
</tr>
<tr>
<td>68.6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1.6</td>
<td>.23</td>
</tr>
<tr>
<td>76.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.2</td>
</tr>
<tr>
<td>88.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TABLE 4.5

MEASURED MEAN VELOCITY DISTRIBUTION
FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 101
Q = 0.8 L/SEC.     Y1 = 15.4 MM.     Y2 = 107.9 MM.
R1 = 940. MM.      R2 = 1372. MM.    F1 = 6.70

<table>
<thead>
<tr>
<th>R(MM.)</th>
<th>Y(MM.)</th>
<th>U(M/S)</th>
<th>U(M/S)</th>
<th>U(M/S)</th>
<th>U(M/S)</th>
<th>U(M/S)</th>
<th>U(M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>940.</td>
<td>1.6</td>
<td>2.69</td>
<td>2.29</td>
<td>.98</td>
<td>.82</td>
<td>.67</td>
<td></td>
</tr>
<tr>
<td>1016.</td>
<td>4.1</td>
<td>2.79</td>
<td>2.36</td>
<td>1.08</td>
<td>.85</td>
<td>.72</td>
<td></td>
</tr>
<tr>
<td>1105.</td>
<td>6.7</td>
<td>2.79</td>
<td>2.39</td>
<td>1.05</td>
<td>.85</td>
<td>.69</td>
<td></td>
</tr>
<tr>
<td>1192.</td>
<td>9.2</td>
<td>2.79</td>
<td>2.0</td>
<td>1.18</td>
<td>.85</td>
<td>.69</td>
<td></td>
</tr>
<tr>
<td>1283.</td>
<td>11.8</td>
<td>1.38</td>
<td>1.86</td>
<td>.95</td>
<td>.76</td>
<td>.66</td>
<td></td>
</tr>
<tr>
<td>15.2</td>
<td>12.7</td>
<td>0.0</td>
<td>1.08</td>
<td>.79</td>
<td>.69</td>
<td>.66</td>
<td></td>
</tr>
<tr>
<td>17.8</td>
<td>15.2</td>
<td>---</td>
<td>.69</td>
<td>.66</td>
<td>.62</td>
<td>.59</td>
<td></td>
</tr>
<tr>
<td>20.3</td>
<td>17.8</td>
<td>---</td>
<td>.62</td>
<td>.62</td>
<td>.62</td>
<td>.59</td>
<td></td>
</tr>
<tr>
<td>22.9</td>
<td>20.3</td>
<td>---</td>
<td>.49</td>
<td>.46</td>
<td>.426</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>25.9</td>
<td>22.9</td>
<td>---</td>
<td>.49</td>
<td>.46</td>
<td>.426</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>30.</td>
<td>25.9</td>
<td>---</td>
<td>.26</td>
<td>.33</td>
<td>.33</td>
<td>.26</td>
<td></td>
</tr>
<tr>
<td>35.6</td>
<td>30.</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
<td>.2</td>
<td>.26</td>
<td></td>
</tr>
<tr>
<td>40.6</td>
<td>35.6</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>45.7</td>
<td>40.6</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.02</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>50.8</td>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.2</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>55.9</td>
<td>50.8</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.2</td>
<td>.2</td>
<td></td>
</tr>
<tr>
<td>63.</td>
<td>55.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>68.6</td>
<td>63.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>76.</td>
<td>68.6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.13</td>
<td></td>
</tr>
<tr>
<td>88.9</td>
<td>76.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.02</td>
<td></td>
</tr>
</tbody>
</table>
**TABLE 4.6**  
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

RUN NUMBER: 126

\[ Q = 18.0 \text{ L/S} \]
\[ Y_1 = 21.6 \text{ MM.} \quad Y_2 = 165.1 \text{ MM.} \]
\[ R_1 = 990.6 \text{ MM.} \quad R_2 = 1625.3 \text{ MM.} \quad F_1 = 7.7398 \]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P.Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1143.0</td>
<td>78.7</td>
<td>43.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>121.9</td>
<td>66.0</td>
</tr>
<tr>
<td>1397.0</td>
<td>160.0</td>
<td>88.9</td>
</tr>
<tr>
<td>1524.0</td>
<td>177.8</td>
<td>106.7</td>
</tr>
<tr>
<td>1651.0</td>
<td>188.0</td>
<td>121.9</td>
</tr>
<tr>
<td>1779.0</td>
<td>177.8</td>
<td>137.2</td>
</tr>
<tr>
<td>1905.0</td>
<td>177.8</td>
<td>147.3</td>
</tr>
</tbody>
</table>

---

**TABLE 4.7**  
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

RUN NUMBER: 127

\[ Q = 10.5 \text{ L/S} \]
\[ Y_1 = 15.0 \text{ MM.} \quad Y_2 = 132.1 \text{ MM.} \]
\[ R_1 = 1016.0 \text{ MM.} \quad R_2 = 1625.6 \text{ MM.} \quad F_1 = 7.6007 \]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P.Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>15.0</td>
<td>15.2</td>
</tr>
<tr>
<td>1143.0</td>
<td>99.1</td>
<td>20.3</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>38.1</td>
</tr>
<tr>
<td>1397.0</td>
<td>134.6</td>
<td>68.0</td>
</tr>
<tr>
<td>1524.0</td>
<td>134.6</td>
<td>95.5</td>
</tr>
<tr>
<td>1651.0</td>
<td>134.6</td>
<td>104.7</td>
</tr>
<tr>
<td>1779.0</td>
<td>134.6</td>
<td>114.3</td>
</tr>
</tbody>
</table>
### TABLE 4.8

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 104**

\[
Q = 10.8 \text{ L/S} \quad Y_1 = 16.5 \text{ MM.} \quad Y_2 = 165.1 \text{ MM.} \\
R_1 = 1041.4 \text{ MM.} \quad R_2 = 1651.0 \text{ MM.} \quad F_1 = 6.6057
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P/γ (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1143.0</td>
<td>71.1</td>
<td>20.3</td>
</tr>
<tr>
<td>1270.0</td>
<td>106.7</td>
<td>40.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>152.4</td>
<td>109.2</td>
</tr>
<tr>
<td>1651.0</td>
<td>160.0</td>
<td>116.9</td>
</tr>
<tr>
<td>1778.0</td>
<td>162.6</td>
<td>121.9</td>
</tr>
<tr>
<td>1905.0</td>
<td>165.1</td>
<td>124.5</td>
</tr>
<tr>
<td>2032.0</td>
<td>165.1</td>
<td>127.0</td>
</tr>
</tbody>
</table>

### TABLE 4.9

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 105**

\[
Q' = 22.7 \text{ L/S} \quad Y_1 = 53.3 \text{ MM.} \quad Y_2 = 151.1 \text{ MM.} \\
R_1 = 1066.8 \text{ MM.} \quad R_2 = 1600.2 \text{ MM.} \quad F_1 = 2.3378
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P/γ (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>53.3</td>
<td>76.2</td>
</tr>
<tr>
<td>1143.0</td>
<td>71.1</td>
<td>91.4</td>
</tr>
<tr>
<td>1270.0</td>
<td>127.0</td>
<td>121.9</td>
</tr>
<tr>
<td>1524.0</td>
<td>152.4</td>
<td>147.3</td>
</tr>
<tr>
<td>1651.0</td>
<td>162.6</td>
<td>154.9</td>
</tr>
<tr>
<td>1778.0</td>
<td>145.1</td>
<td>157.5</td>
</tr>
<tr>
<td>1905.0</td>
<td>167.6</td>
<td>165.1</td>
</tr>
</tbody>
</table>

232
### TABLE 4.10

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 106**

\[
\begin{align*}
Q &= 23.3 \text{ L/s} & Y_1 &= 51.3 \text{ mm} & Y_2 &= 162.6 \text{ mm} \\
R_1 &= 1168.4 \text{ mm} & R_2 &= 1651.0 \text{ mm} & F_1 &= 2.3254 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1270.0</td>
<td>51.3</td>
<td>116.8</td>
</tr>
<tr>
<td>1397.0</td>
<td>172.7</td>
<td>137.2</td>
</tr>
<tr>
<td>1524.0</td>
<td>177.8</td>
<td>132.4</td>
</tr>
<tr>
<td>1651.0</td>
<td>167.6</td>
<td>157.5</td>
</tr>
<tr>
<td>1778.0</td>
<td>167.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1905.0</td>
<td>167.6</td>
<td>165.1</td>
</tr>
</tbody>
</table>

### TABLE 4.11

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 107**

\[
\begin{align*}
Q &= 29.0 \text{ L/s} & Y_1 &= 67.3 \text{ mm} & Y_2 &= 209.5 \text{ mm} \\
R_1 &= 990.6 \text{ mm} & R_2 &= 1676.4 \text{ mm} & F_1 &= 2.2694 \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>67.3</td>
<td>107.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>127.0</td>
<td>129.5</td>
</tr>
<tr>
<td>1270.0</td>
<td>167.6</td>
<td>147.3</td>
</tr>
<tr>
<td>1397.0</td>
<td>193.0</td>
<td>162.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>195.6</td>
<td>177.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>200.7</td>
<td>177.8</td>
</tr>
<tr>
<td>1778.0</td>
<td>205.7</td>
<td>188.0</td>
</tr>
<tr>
<td>1905.0</td>
<td>208.3</td>
<td>190.5</td>
</tr>
<tr>
<td>2032.0</td>
<td>209.5</td>
<td>193.0</td>
</tr>
</tbody>
</table>
### Table 4.12

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 108**

\[ Q = 19.8 \text{ L/s} \quad Y_1 = 48.3 \text{ mm} \quad Y_2 = 150.9 \text{ mm} \]

\[ R_1 = 1016.0 \text{ mm} \quad R_2 = 1524.0 \text{ mm} \quad F_1 = 2.4957 \]

<table>
<thead>
<tr>
<th>( R (\text{mm}) )</th>
<th>( Y (\text{mm}) )</th>
<th>( F/Y (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>48.3</td>
<td>88.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>101.6</td>
<td>111.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>144.8</td>
<td>142.2</td>
</tr>
<tr>
<td>1397.0</td>
<td>152.4</td>
<td>160.0</td>
</tr>
<tr>
<td>1524.0</td>
<td>162.6</td>
<td>167.6</td>
</tr>
<tr>
<td>1651.0</td>
<td>167.6</td>
<td>167.6</td>
</tr>
</tbody>
</table>

### Table 4.13

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 109**

\[ Q = 16.4 \text{ L/s} \quad Y_1 = 40.6 \text{ mm} \quad Y_2 = 152.4 \text{ mm} \]

\[ R_1 = 990.6 \text{ mm} \quad R_2 = 1524.0 \text{ mm} \quad F_1 = 2.7341 \]

<table>
<thead>
<tr>
<th>( R (\text{mm}) )</th>
<th>( Y (\text{mm}) )</th>
<th>( F/Y (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1143.0</td>
<td>40.6</td>
<td>77.5</td>
</tr>
<tr>
<td>1270.0</td>
<td>96.5</td>
<td>101.8</td>
</tr>
<tr>
<td>1397.0</td>
<td>139.7</td>
<td>119.4</td>
</tr>
<tr>
<td>1524.0</td>
<td>149.9</td>
<td>132.1</td>
</tr>
<tr>
<td>1651.0</td>
<td>152.4</td>
<td>137.2</td>
</tr>
</tbody>
</table>
### TABLE 4.15
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

**RUN NUMBER: 111**

\[
\begin{align*}
Q &= 11.1 \text{ L/s} & Y_1 &= 21.8 \text{ mm} & Y_2 &= 121.4 \text{ mm} \\
R_1 &= 990.6 \text{ mm} & R_2 &= 1473.2 \text{ mm} & F_1 &= 4.6965
\end{align*}
\]

<table>
<thead>
<tr>
<th>R (mm.)</th>
<th>Y (mm.)</th>
<th>P/Y (mm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>21.8</td>
<td>35.6</td>
</tr>
<tr>
<td>1143.0</td>
<td>68.6</td>
<td>55.9</td>
</tr>
<tr>
<td>1270.0</td>
<td>96.5</td>
<td>86.4</td>
</tr>
<tr>
<td>1397.0</td>
<td>114.3</td>
<td>106.7</td>
</tr>
<tr>
<td>1524.0</td>
<td>127.0</td>
<td>119.4</td>
</tr>
<tr>
<td>1651.0</td>
<td>139.7</td>
<td>137.2</td>
</tr>
<tr>
<td>1778.0</td>
<td>139.7</td>
<td>142.2</td>
</tr>
</tbody>
</table>

### TABLE 4.14
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

**RUN NUMBER: 110**

\[
\begin{align*}
Q &= 29.0 \text{ L/s} & Y_1 &= 61.5 \text{ mm} & Y_2 &= 190.5 \text{ mm} \\
R_1 &= 990.6 \text{ mm} & R_2 &= 1574.8 \text{ mm} & F_1 &= 2.6004
\end{align*}
\]

<table>
<thead>
<tr>
<th>R (mm.)</th>
<th>Y (mm.)</th>
<th>P/Y (mm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>990.6</td>
<td>61.5</td>
<td>76.2</td>
</tr>
<tr>
<td>1016.0</td>
<td>91.4</td>
<td>71.1</td>
</tr>
<tr>
<td>1143.0</td>
<td>142.2</td>
<td>76.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>203.2</td>
<td>116.9</td>
</tr>
<tr>
<td>1397.0</td>
<td>215.9</td>
<td>137.2</td>
</tr>
<tr>
<td>1524.0</td>
<td>218.4</td>
<td>162.6</td>
</tr>
<tr>
<td>1651.0</td>
<td>218.4</td>
<td>177.8</td>
</tr>
<tr>
<td>1778.0</td>
<td>218.4</td>
<td>177.8</td>
</tr>
<tr>
<td>2032.0</td>
<td>218.4</td>
<td>182.9</td>
</tr>
</tbody>
</table>
### TABLE 4.16
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 112

\[ Q = 16.7 \text{ L/S} \quad Y_1 = 26.2 \text{ MM.} \quad Y_2 = 146.8 \text{ MM.} \]
\[ R_1 = 1117.6 \text{ MM.} \quad R_2 = 1701.8 \text{ MM.} \quad F_1 = 4.7821 \]

<table>
<thead>
<tr>
<th>( R(\text{MM.}) )</th>
<th>( Y(\text{MM.}) )</th>
<th>( P/Y(\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1143.0</td>
<td>26.2</td>
<td>43.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>91.4</td>
<td>66.0</td>
</tr>
<tr>
<td>1397.0</td>
<td>121.9</td>
<td>88.5</td>
</tr>
<tr>
<td>1524.0</td>
<td>142.2</td>
<td>121.9</td>
</tr>
<tr>
<td>1651.0</td>
<td>152.4</td>
<td>137.2</td>
</tr>
<tr>
<td>1779.0</td>
<td>154.9</td>
<td>147.3</td>
</tr>
<tr>
<td>1905.0</td>
<td>154.9</td>
<td>152.4</td>
</tr>
<tr>
<td>2032.0</td>
<td>157.5</td>
<td>152.4</td>
</tr>
</tbody>
</table>

### TABLE 4.17
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 113

\[ Q = 11.2 \text{ L/S} \quad Y_1 = 21.6 \text{ MM.} \quad Y_2 = 121.9 \text{ MM.} \]
\[ R_1 = 990.6 \text{ MM.} \quad R_2 = 1371.6 \text{ MM.} \quad F_1 = 4.8068 \]

<table>
<thead>
<tr>
<th>( R(\text{MM.}) )</th>
<th>( Y(\text{MM.}) )</th>
<th>( P/Y(\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>21.6</td>
<td>22.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>76.2</td>
<td>35.6</td>
</tr>
<tr>
<td>1270.0</td>
<td>106.7</td>
<td>63.5</td>
</tr>
<tr>
<td>1397.0</td>
<td>127.0</td>
<td>88.9</td>
</tr>
<tr>
<td>1524.0</td>
<td>127.0</td>
<td>101.6</td>
</tr>
<tr>
<td>1651.0</td>
<td>127.0</td>
<td>114.3</td>
</tr>
</tbody>
</table>
### TABLE 4.18

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 114**

\( Q = 20.9 \text{ L/S} \quad Y_1 = 47.5 \text{ MM} \quad Y_2 = 158.7 \text{ MM} \)
\( R_1 = 990.6 \text{ MM} \quad R_2 = 1524.0 \text{ MM} \quad F_1 = 2.7547 \)

<table>
<thead>
<tr>
<th>( \text{R (MM.)} )</th>
<th>( \text{Y (MM.)} )</th>
<th>( F/Y (\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>47.5</td>
<td>43.2</td>
</tr>
<tr>
<td>1066.8</td>
<td>66.0</td>
<td>40.6</td>
</tr>
<tr>
<td>1270.0</td>
<td>116.8</td>
<td>81.3</td>
</tr>
<tr>
<td>1397.0</td>
<td>157.5</td>
<td>101.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>162.6</td>
<td>127.0</td>
</tr>
<tr>
<td>1651.0</td>
<td>167.6</td>
<td>137.2</td>
</tr>
</tbody>
</table>

### TABLE 4.19

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 115**

\( Q = 23.0 \text{ L/S} \quad Y_1 = 48.3 \text{ MM} \quad Y_2 = 173.2 \text{ MM} \)
\( R_1 = 990.6 \text{ MM} \quad R_2 = 1675.4 \text{ MM} \quad F_1 = 2.9660 \)

<table>
<thead>
<tr>
<th>( \text{R (MM.)} )</th>
<th>( \text{Y (MM.)} )</th>
<th>( F/Y (\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>48.3</td>
<td>71.1</td>
</tr>
<tr>
<td>1143.0</td>
<td>106.7</td>
<td>81.3</td>
</tr>
<tr>
<td>1270.0</td>
<td>139.7</td>
<td>111.8</td>
</tr>
<tr>
<td>1397.0</td>
<td>160.0</td>
<td>139.7</td>
</tr>
<tr>
<td>1524.0</td>
<td>177.8</td>
<td>157.5</td>
</tr>
<tr>
<td>1651.0</td>
<td>185.4</td>
<td>172.7</td>
</tr>
<tr>
<td>1778.0</td>
<td>193.0</td>
<td>193.0</td>
</tr>
</tbody>
</table>

237
### TABLE 4.20

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 116**

\[
Q = 37.3 \text{ L/S} \quad y_1 = 64.3 \text{ MM.} \quad y_2 = 228.6 \text{ MM.} \\
R_1 = 990.6 \text{ MM.} \quad R_2 = 1854.2 \text{ MM.} \quad F_1 = 3.1731
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P / Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>64.3</td>
<td>101.6</td>
</tr>
<tr>
<td>1143.0</td>
<td>165.1</td>
<td>116.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>188.0</td>
<td>147.3</td>
</tr>
<tr>
<td>1397.0</td>
<td>208.3</td>
<td>157.5</td>
</tr>
<tr>
<td>1524.0</td>
<td>221.0</td>
<td>165.1</td>
</tr>
<tr>
<td>1651.0</td>
<td>233.7</td>
<td>188.0</td>
</tr>
<tr>
<td>1778.0</td>
<td>238.8</td>
<td>200.7</td>
</tr>
<tr>
<td>1905.0</td>
<td>241.3</td>
<td>213.4</td>
</tr>
<tr>
<td>2032.0</td>
<td>241.3</td>
<td>223.5</td>
</tr>
</tbody>
</table>

### TABLE 4.21

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 117**

\[
Q = 22.4 \text{ L/S} \quad y_1 = 43.2 \text{ MM.} \quad y_2 = 180.3 \text{ MM.} \\
R_1 = 990.6 \text{ MM.} \quad R_2 = 1676.4 \text{ MM.} \quad F_1 = 3.4085
\]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P / Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>43.2</td>
<td>76.2</td>
</tr>
<tr>
<td>1143.0</td>
<td>99.1</td>
<td>81.3</td>
</tr>
<tr>
<td>1397.0</td>
<td>137.2</td>
<td>139.7</td>
</tr>
<tr>
<td>1524.0</td>
<td>157.5</td>
<td>152.4</td>
</tr>
<tr>
<td>1651.0</td>
<td>177.8</td>
<td>165.1</td>
</tr>
<tr>
<td>1778.0</td>
<td>180.3</td>
<td>167.6</td>
</tr>
</tbody>
</table>
### TABLE 4.22
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 118**

\[ \begin{align*}
Q &= 19.8 \text{ L/S} \\
Y_1 &= 35.6 \text{ MM} \\
Y_2 &= 158.7 \text{ MM} \\
R_1 &= 1117.6 \text{ MM} \\
R_2 &= 1549.4 \text{ MM} \\
F_1 &= 3.5871
\end{align*} \]

<table>
<thead>
<tr>
<th>(R(\text{MM.}))</th>
<th>(Y(\text{MM.}))</th>
<th>(F/Y(\text{MM.}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1270.0</td>
<td>127.0</td>
<td>78.7</td>
</tr>
<tr>
<td>1397.0</td>
<td>157.5</td>
<td>101.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>165.1</td>
<td>121.9</td>
</tr>
<tr>
<td>1651.0</td>
<td>165.1</td>
<td>134.6</td>
</tr>
<tr>
<td>1778.0</td>
<td>165.1</td>
<td>147.3</td>
</tr>
<tr>
<td>1905.0</td>
<td>165.1</td>
<td>158.7</td>
</tr>
<tr>
<td>2032.0</td>
<td>165.1</td>
<td>158.7</td>
</tr>
</tbody>
</table>

### TABLE 4.23
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 119**

\[ \begin{align*}
Q &= 11.1 \text{ L/S} \\
Y_1 &= 23.9 \text{ MM} \\
Y_2 &= 111.8 \text{ MM} \\
R_1 &= 1066.8 \text{ MM} \\
R_2 &= 1473.2 \text{ MM} \\
F_1 &= 3.8164
\end{align*} \]

<table>
<thead>
<tr>
<th>(R(\text{MM.}))</th>
<th>(Y(\text{MM.}))</th>
<th>(F/Y(\text{MM.}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1143.0</td>
<td>23.9</td>
<td>50.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>106.7</td>
<td>76.2</td>
</tr>
<tr>
<td>1397.0</td>
<td>121.9</td>
<td>96.5</td>
</tr>
<tr>
<td>1524.0</td>
<td>132.1</td>
<td>111.8</td>
</tr>
<tr>
<td>1651.0</td>
<td>134.6</td>
<td>121.9</td>
</tr>
<tr>
<td>1778.0</td>
<td>134.6</td>
<td>127.0</td>
</tr>
</tbody>
</table>
## TABLE 4.24
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER:** 120

\[ Q = 20.9 \text{ L/s} \quad Y_1 = 37.3 \text{ mm} \quad Y_2 = 177.8 \text{ mm} \]
\[ R_1 = 1016.0 \text{ mm} \quad R_2 = 1600.2 \text{ mm} \quad F_1 = 3.8536 \]

<table>
<thead>
<tr>
<th>( R \text{ (MM.)} )</th>
<th>( Y \text{ (MM.)} )</th>
<th>( P/y \text{ (MM.)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>37.3</td>
<td>55.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>106.7</td>
<td>43.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>147.3</td>
<td>91.4</td>
</tr>
<tr>
<td>1397.0</td>
<td>162.6</td>
<td>111.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>177.8</td>
<td>127.0</td>
</tr>
<tr>
<td>1651.0</td>
<td>177.8</td>
<td>142.2</td>
</tr>
<tr>
<td>1778.0</td>
<td>177.8</td>
<td>152.4</td>
</tr>
<tr>
<td>1905.0</td>
<td>177.8</td>
<td>157.5</td>
</tr>
<tr>
<td>2032.0</td>
<td>177.8</td>
<td>160.0</td>
</tr>
</tbody>
</table>

## TABLE 4.25
**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.**

**RUN NUMBER:** 121

\[ Q = 19.8 \text{ L/s} \quad Y_1 = 32.5 \text{ mm} \quad Y_2 = 166.1 \text{ mm} \]
\[ R_1 = 1092.2 \text{ mm} \quad R_2 = 1778.0 \text{ mm} \quad F_1 = 4.1986 \]

<table>
<thead>
<tr>
<th>( R \text{ (MM.)} )</th>
<th>( Y \text{ (MM.)} )</th>
<th>( P/y \text{ (MM.)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1092.2</td>
<td>32.5</td>
<td>55.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>86.4</td>
<td>58.4</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>89.7</td>
</tr>
<tr>
<td>1397.0</td>
<td>160.0</td>
<td>116.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>172.7</td>
<td>132.1</td>
</tr>
<tr>
<td>1651.0</td>
<td>175.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1778.0</td>
<td>177.8</td>
<td>162.6</td>
</tr>
<tr>
<td>1905.0</td>
<td>177.3</td>
<td>167.6</td>
</tr>
</tbody>
</table>
### TABLE 4.26

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

**RUN NUMBER:** 122

\[ Q = 15.4 \text{ L/S} \quad Y_1 = 29.7 \text{ MM.} \quad Y_2 = 149.9 \text{ MM.} \]
\[ R_1 = 1016.0 \text{ MM.} \quad R_2 = 1600.2 \text{ MM.} \quad F_1 = 4.0170 \]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>29.7</td>
<td>35.6</td>
</tr>
<tr>
<td>1143.0</td>
<td>45.7</td>
<td>50.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>91.4</td>
<td>76.2</td>
</tr>
<tr>
<td>1651.0</td>
<td>160.0</td>
<td>142.2</td>
</tr>
<tr>
<td>1778.0</td>
<td>165.1</td>
<td>144.8</td>
</tr>
<tr>
<td>1905.0</td>
<td>165.1</td>
<td>144.8</td>
</tr>
</tbody>
</table>

### TABLE 4.27

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

**RUN NUMBER:** 123

\[ Q = 29.7 \text{ L/S} \quad Y_1 = 47.0 \text{ MM.} \quad Y_2 = 215.9 \text{ MM.} \]
\[ R_1 = 990.6 \text{ MM.} \quad R_2 = 1879.6 \text{ MM.} \quad F_1 = 3.9836 \]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/Y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>47.0</td>
<td>55.9</td>
</tr>
<tr>
<td>1143.0</td>
<td>127.0</td>
<td>74.2</td>
</tr>
<tr>
<td>1397.0</td>
<td>203.2</td>
<td>114.3</td>
</tr>
<tr>
<td>1524.0</td>
<td>218.4</td>
<td>139.7</td>
</tr>
<tr>
<td>1651.0</td>
<td>218.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1779.0</td>
<td>218.4</td>
<td>175.3</td>
</tr>
<tr>
<td>1905.0</td>
<td>218.4</td>
<td>180.3</td>
</tr>
<tr>
<td>2032.0</td>
<td>218.4</td>
<td>182.9</td>
</tr>
</tbody>
</table>
TABLE 4.28
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 124

\[ Q = 20.2 \text{ L/S} \quad Y_1 = 28.7 \text{ MM.} \quad Y_2 = 134.1 \text{ MM.} \]
\[ R_1 = 990.6 \text{ MM.} \quad R_2 = 1727.2 \text{ MM.} \quad F_1 = 5.3872 \]

<table>
<thead>
<tr>
<th>( R (\text{MM.}) )</th>
<th>( Y (\text{MM.}) )</th>
<th>( P/Y (\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>28.7</td>
<td>33.0</td>
</tr>
<tr>
<td>1143.0</td>
<td>81.3</td>
<td>45.7</td>
</tr>
<tr>
<td>1270.0</td>
<td>127.0</td>
<td>71.1</td>
</tr>
<tr>
<td>1397.0</td>
<td>157.5</td>
<td>88.9</td>
</tr>
<tr>
<td>1524.0</td>
<td>177.8</td>
<td>109.2</td>
</tr>
<tr>
<td>1651.0</td>
<td>177.8</td>
<td>127.0</td>
</tr>
<tr>
<td>1778.0</td>
<td>182.9</td>
<td>144.8</td>
</tr>
<tr>
<td>1905.0</td>
<td>182.9</td>
<td>152.4</td>
</tr>
<tr>
<td>2082.8</td>
<td>182.9</td>
<td>160.0</td>
</tr>
</tbody>
</table>

TABLE 4.29
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 125

\[ Q = 23.3 \text{ L/S} \quad Y_1 = 32.3 \text{ MM.} \quad Y_2 = 190.5 \text{ MM.} \]
\[ R_1 = 990.6 \text{ MM.} \quad R_2 = 1778.0 \text{ MM.} \quad F_1 = 5.5019 \]

<table>
<thead>
<tr>
<th>( R (\text{MM.}) )</th>
<th>( Y (\text{MM.}) )</th>
<th>( P/Y (\text{MM.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>32.3</td>
<td>50.8</td>
</tr>
<tr>
<td>1143.0</td>
<td>101.6</td>
<td>71.1</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>91.4</td>
</tr>
<tr>
<td>1397.0</td>
<td>157.5</td>
<td>114.3</td>
</tr>
<tr>
<td>1524.0</td>
<td>170.2</td>
<td>137.2</td>
</tr>
<tr>
<td>1651.0</td>
<td>185.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1778.0</td>
<td>198.1</td>
<td>172.7</td>
</tr>
<tr>
<td>1905.0</td>
<td>198.1</td>
<td>185.4</td>
</tr>
</tbody>
</table>
TABLE 4.30
MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE FREE RADIAL HYDRAULIC JUMP.

RUN NUMBER: 128
Q = 11.1 L/S Y1 = 15.5 MM. Y2 = 137.2 MM.
R1 = 990.6 MM. R2 = 1574.8 MM. F1 = 7.3620

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1016.0</td>
<td>15.5</td>
<td>30.5</td>
</tr>
<tr>
<td>1143.0</td>
<td>81.3</td>
<td>50.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>111.8</td>
<td>78.2</td>
</tr>
<tr>
<td>1397.0</td>
<td>129.5</td>
<td>101.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>137.2</td>
<td>114.3</td>
</tr>
<tr>
<td>1651.0</td>
<td>142.2</td>
<td>127.0</td>
</tr>
<tr>
<td>1778.0</td>
<td>142.2</td>
<td>137.2</td>
</tr>
<tr>
<td>1905.0</td>
<td>142.2</td>
<td>142.2</td>
</tr>
<tr>
<td>RUN</td>
<td>Q (L/s)</td>
<td>G0 (mm.)</td>
</tr>
<tr>
<td>-----</td>
<td>---------</td>
<td>----------</td>
</tr>
<tr>
<td>126</td>
<td>10.4</td>
<td>25.4</td>
</tr>
<tr>
<td>127</td>
<td>7.2</td>
<td>25.4</td>
</tr>
<tr>
<td>128</td>
<td>17.0</td>
<td>50.0</td>
</tr>
<tr>
<td>129</td>
<td>11.3</td>
<td>38.1</td>
</tr>
<tr>
<td>130</td>
<td>14.2</td>
<td>38.1</td>
</tr>
<tr>
<td>131</td>
<td>14.8</td>
<td>50.0</td>
</tr>
<tr>
<td>132</td>
<td>19.2</td>
<td>63.5</td>
</tr>
<tr>
<td>133</td>
<td>22.4</td>
<td>63.5</td>
</tr>
<tr>
<td>134</td>
<td>22.4</td>
<td>69.8</td>
</tr>
<tr>
<td>135</td>
<td>22.4</td>
<td>76.2</td>
</tr>
<tr>
<td>136</td>
<td>27.4</td>
<td>76.2</td>
</tr>
<tr>
<td>137</td>
<td>12.9</td>
<td>38.1</td>
</tr>
<tr>
<td>138</td>
<td>11.0</td>
<td>27.9</td>
</tr>
<tr>
<td>139</td>
<td>11.0</td>
<td>27.9</td>
</tr>
<tr>
<td>140</td>
<td>11.0</td>
<td>27.9</td>
</tr>
<tr>
<td>141</td>
<td>11.0</td>
<td>31.7</td>
</tr>
<tr>
<td>142</td>
<td>11.0</td>
<td>38.1</td>
</tr>
<tr>
<td>143</td>
<td>15.4</td>
<td>38.1</td>
</tr>
<tr>
<td>144</td>
<td>15.4</td>
<td>44.4</td>
</tr>
<tr>
<td>RUN #</td>
<td>D(L/S)</td>
<td>G0(MM.)</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>145</td>
<td>15.4</td>
<td>44.4</td>
</tr>
<tr>
<td>146</td>
<td>15.4</td>
<td>50.9</td>
</tr>
<tr>
<td>147</td>
<td>19.8</td>
<td>50.8</td>
</tr>
<tr>
<td>148</td>
<td>19.8</td>
<td>57.1</td>
</tr>
<tr>
<td>149</td>
<td>19.8</td>
<td>57.1</td>
</tr>
<tr>
<td>150</td>
<td>19.8</td>
<td>57.1</td>
</tr>
<tr>
<td>151</td>
<td>23.3</td>
<td>63.5</td>
</tr>
<tr>
<td>152</td>
<td>23.3</td>
<td>63.5</td>
</tr>
<tr>
<td>153</td>
<td>23.6</td>
<td>63.5</td>
</tr>
<tr>
<td>154</td>
<td>23.6</td>
<td>69.0</td>
</tr>
<tr>
<td>155</td>
<td>24.3</td>
<td>69.0</td>
</tr>
<tr>
<td>156</td>
<td>23.6</td>
<td>76.2</td>
</tr>
<tr>
<td>157</td>
<td>24.3</td>
<td>76.2</td>
</tr>
<tr>
<td>158</td>
<td>27.7</td>
<td>76.2</td>
</tr>
<tr>
<td>159</td>
<td>19.2</td>
<td>50.8</td>
</tr>
<tr>
<td>160</td>
<td>3.5</td>
<td>12.7</td>
</tr>
<tr>
<td>161</td>
<td>4.7</td>
<td>12.7</td>
</tr>
<tr>
<td>162</td>
<td>6.9</td>
<td>12.7</td>
</tr>
<tr>
<td>163</td>
<td>9.3</td>
<td>25.4</td>
</tr>
<tr>
<td>164</td>
<td>5.5</td>
<td>25.4</td>
</tr>
<tr>
<td>RUN #</td>
<td>Q(L/S)</td>
<td>G0(MM.)</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>165</td>
<td>5.5</td>
<td>25.4</td>
</tr>
<tr>
<td>166</td>
<td>10.0</td>
<td>25.4</td>
</tr>
<tr>
<td>167</td>
<td>10.0</td>
<td>31.7</td>
</tr>
<tr>
<td>168</td>
<td>10.7</td>
<td>31.7</td>
</tr>
<tr>
<td>169</td>
<td>10.7</td>
<td>31.7</td>
</tr>
<tr>
<td>170</td>
<td>11.7</td>
<td>31.7</td>
</tr>
<tr>
<td>171</td>
<td>13.5</td>
<td>31.7</td>
</tr>
<tr>
<td>172</td>
<td>13.3</td>
<td>38.1</td>
</tr>
<tr>
<td>173</td>
<td>13.3</td>
<td>38.1</td>
</tr>
<tr>
<td>174</td>
<td>15.1</td>
<td>38.1</td>
</tr>
<tr>
<td>175</td>
<td>9.5</td>
<td>30.1</td>
</tr>
<tr>
<td>176</td>
<td>9.5</td>
<td>50.8</td>
</tr>
<tr>
<td>177</td>
<td>15.1</td>
<td>50.8</td>
</tr>
<tr>
<td>178</td>
<td>14.5</td>
<td>50.8</td>
</tr>
<tr>
<td>179</td>
<td>19.2</td>
<td>50.8</td>
</tr>
<tr>
<td>180</td>
<td>19.2</td>
<td>50.8</td>
</tr>
<tr>
<td>181</td>
<td>15.1</td>
<td>50.8</td>
</tr>
</tbody>
</table>
TABLE 4.32
MEASURED MEAN-VELOCITY DISTRIBUTION
FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 102
Q = 0.2 L/SEC, BD = 25.0 MM, Y3 = 178.0 MM, Y2 = 193.0 MM,
R1 = 914.0 MM, R2 = 1803.0 MM, F1 = 6.

<table>
<thead>
<tr>
<th>R (MM)</th>
<th>914</th>
<th>1092</th>
<th>1270</th>
<th>1448</th>
<th>1626</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (MM)</td>
<td>U (M/S)</td>
<td>U (M/S)</td>
<td>U (M/S)</td>
<td>U (M/S)</td>
<td>U (M/S)</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>1.6</td>
<td>2.55</td>
<td>2.55</td>
<td>1.9</td>
<td>1.57</td>
<td>.95</td>
</tr>
<tr>
<td>4.1</td>
<td>2.65</td>
<td>2.59</td>
<td>1.96</td>
<td>1.47</td>
<td>.95</td>
</tr>
<tr>
<td>6.7</td>
<td>2.65</td>
<td>2.3</td>
<td>1.9</td>
<td>1.47</td>
<td>.98</td>
</tr>
<tr>
<td>7.6</td>
<td>2.65</td>
<td>2.1</td>
<td>1.8</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td>11.2</td>
<td>2.65</td>
<td>1.96</td>
<td>1.77</td>
<td>1.28</td>
<td>.98</td>
</tr>
<tr>
<td>12.7</td>
<td>2.65</td>
<td>1.77</td>
<td>1.57</td>
<td>1.17</td>
<td>.91</td>
</tr>
<tr>
<td>15.2</td>
<td>1.2</td>
<td>1.1</td>
<td>1.57</td>
<td>1.0</td>
<td>.85</td>
</tr>
<tr>
<td>20.3</td>
<td>.66</td>
<td>.49</td>
<td>1.17</td>
<td>.98</td>
<td>.81</td>
</tr>
<tr>
<td>25.5</td>
<td>0.0</td>
<td>.39</td>
<td>.98</td>
<td>.71</td>
<td>.78</td>
</tr>
<tr>
<td>30.</td>
<td>---</td>
<td>.26</td>
<td>.78</td>
<td>.59</td>
<td>.72</td>
</tr>
<tr>
<td>37.1</td>
<td>---</td>
<td>0.0</td>
<td>.53</td>
<td>.49</td>
<td>.66</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>.26</td>
<td>.39</td>
<td>.62</td>
</tr>
<tr>
<td>50.0</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.33</td>
<td>.59</td>
</tr>
<tr>
<td>58.1</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
<td>.53</td>
</tr>
<tr>
<td>63.</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.1</td>
<td>.46</td>
</tr>
<tr>
<td>71.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.065</td>
<td>.33</td>
</tr>
<tr>
<td>78.2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.26</td>
</tr>
<tr>
<td>81.3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.2</td>
</tr>
<tr>
<td>80.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
</tr>
<tr>
<td>96.5</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.065</td>
</tr>
<tr>
<td>101.6</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>---</td>
</tr>
</tbody>
</table>
TABLE 4.33
MEASURED MEAN-VELOCITY DISTRIBUTION
FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 103
Q = 7.2 L/sec, d0 = 25.0 mm, y3 = 178.0 mm, y2 = 107.0 mm.
R₁ = 914.0 mm, R₂ = 1473.0 mm, F₁ = 5.

<table>
<thead>
<tr>
<th>R (mm.)</th>
<th>914.0</th>
<th>1003.0</th>
<th>1092.0</th>
<th>1181.0</th>
<th>1270.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (mm.)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
<td>U (m/s)</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>1.6</td>
<td>2.0</td>
<td>1.9</td>
<td>1.9</td>
<td>1.2</td>
<td>0.81</td>
</tr>
<tr>
<td>4.1</td>
<td>2.1</td>
<td>1.81</td>
<td>1.96</td>
<td>1.38</td>
<td>0.81</td>
</tr>
<tr>
<td>6.7</td>
<td>2.1</td>
<td>1.81</td>
<td>1.96</td>
<td>1.57</td>
<td>0.89</td>
</tr>
<tr>
<td>7.6</td>
<td>1.81</td>
<td>1.96</td>
<td>1.83</td>
<td>1.38</td>
<td>0.72</td>
</tr>
<tr>
<td>11.2</td>
<td>1.81</td>
<td>1.71</td>
<td>1.64</td>
<td>0.84</td>
<td>0.60</td>
</tr>
<tr>
<td>12.7</td>
<td>1.81</td>
<td>1.2</td>
<td>1.2</td>
<td>1.0</td>
<td>0.60</td>
</tr>
<tr>
<td>15.2</td>
<td>1.64</td>
<td>0.85</td>
<td>0.85</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>20.3</td>
<td>0.98</td>
<td>0.26</td>
<td>0.59</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>25.9</td>
<td>0.0</td>
<td>0.0</td>
<td>0.26</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>30.0</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>0.26</td>
<td>0.39</td>
</tr>
<tr>
<td>38.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>0.26</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TABLE 4.34
MEASURED MEAN-VELOCITY DISTRIBUTION
FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 184
Q = 10.4 L/SEC.  B0 = .49 MM.  Y3 = 168 MM.  Y2 = 190 MM.
R1 = 914 MM.  R2 = 1651 MM.  F1 = 4.

<table>
<thead>
<tr>
<th>R (MM)</th>
<th>914</th>
<th>1003</th>
<th>1092</th>
<th>1181</th>
<th>1270</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (MM)</td>
<td>U(M/S)</td>
<td>U(M/S)</td>
<td>U(M/S)</td>
<td>U(M/S)</td>
<td>U(M/S)</td>
</tr>
<tr>
<td>1.5</td>
<td>2.08</td>
<td>2.0</td>
<td>1.9</td>
<td>1.57</td>
<td>.91</td>
</tr>
<tr>
<td>4.1</td>
<td>2.1</td>
<td>1.81</td>
<td>1.96</td>
<td>1.64</td>
<td>.95</td>
</tr>
<tr>
<td>6.7</td>
<td>2.1</td>
<td>1.81</td>
<td>1.96</td>
<td>1.64</td>
<td>.98</td>
</tr>
<tr>
<td>7.6</td>
<td>2.1</td>
<td>1.81</td>
<td>1.9</td>
<td>1.5</td>
<td>.95</td>
</tr>
<tr>
<td>10.5</td>
<td>2.1</td>
<td>2.0</td>
<td>1.7</td>
<td>1.2</td>
<td>.81</td>
</tr>
<tr>
<td>12.7</td>
<td>2.1</td>
<td>2.0</td>
<td>1.74</td>
<td>1.17</td>
<td>.78</td>
</tr>
<tr>
<td>15.2</td>
<td>2.1</td>
<td>1.96</td>
<td>1.2</td>
<td>1.17</td>
<td>.75</td>
</tr>
<tr>
<td>20.3</td>
<td>.66</td>
<td>.98</td>
<td>1.17</td>
<td>1.0</td>
<td>.72</td>
</tr>
<tr>
<td>25.9</td>
<td>.39</td>
<td>.39</td>
<td>.72</td>
<td>.72</td>
<td>.72</td>
</tr>
<tr>
<td>30.9</td>
<td>0.0</td>
<td>0.0</td>
<td>.26</td>
<td>.72</td>
<td>.72</td>
</tr>
<tr>
<td>33.1</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.39</td>
<td>.59</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.26</td>
<td>.39</td>
</tr>
<tr>
<td>50.0</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.26</td>
</tr>
<tr>
<td>55.9</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
</tr>
<tr>
<td>63.1</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
</tbody>
</table>
**TABLE 4.35**

**MEASURED MEAN-VELOCITY DISTRIBUTION**

**FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.**

RUN NUMBER: 195  
Q = 0.8 L/SEC.  G0 = 25, MM.  Y3 = 152, MM.  Y2 = 2.45 MM.  
R1 = 914, MM.  R2 = 1828, MM.  F1 = 6.

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>914</th>
<th>1003</th>
<th>1092</th>
<th>111.0</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Y (MM.)</th>
<th>U (M/S)</th>
<th>U (M/S)</th>
<th>U (M/S)</th>
<th>U (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>2.60</td>
<td>2.65</td>
<td>2.36</td>
<td>2.0</td>
</tr>
<tr>
<td>4.1</td>
<td>2.79</td>
<td>2.68</td>
<td>2.45</td>
<td>2.1</td>
</tr>
<tr>
<td>6.7</td>
<td>2.79</td>
<td>2.68</td>
<td>2.45</td>
<td>2.1</td>
</tr>
<tr>
<td>7.6</td>
<td>2.79</td>
<td>2.68</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td>11.0</td>
<td>2.60</td>
<td>2.5</td>
<td>1.83</td>
<td>1.7</td>
</tr>
<tr>
<td>12.7</td>
<td>2.60</td>
<td>2.36</td>
<td>1.57</td>
<td>1.46</td>
</tr>
<tr>
<td>15.2</td>
<td>2.45</td>
<td>2.3</td>
<td>1.17</td>
<td>1.17</td>
</tr>
<tr>
<td>20.3</td>
<td>0.66</td>
<td>0.66</td>
<td>0.85</td>
<td>0.81</td>
</tr>
<tr>
<td>25.9</td>
<td>0.0</td>
<td>0.26</td>
<td>0.33</td>
<td>0.54</td>
</tr>
<tr>
<td>30.0</td>
<td>---</td>
<td>0.0</td>
<td>0.26</td>
<td>0.33</td>
</tr>
<tr>
<td>38.1</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>0.26</td>
</tr>
<tr>
<td>45.7</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TABLE: 435 (CONTINUED)

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>1270</th>
<th>1422</th>
<th>1499</th>
<th>1575</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y (MM.)</td>
<td>U (M /S)</td>
<td>U (M /S)</td>
<td>U (M /S)</td>
<td>U (M /S)</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>1.6</td>
<td>1.74</td>
<td>1.1</td>
<td>.85</td>
<td>.59</td>
</tr>
<tr>
<td>4.1</td>
<td>1.77</td>
<td>1.2</td>
<td>.91</td>
<td>.66</td>
</tr>
<tr>
<td>6.7</td>
<td>1.8</td>
<td>1.3</td>
<td>.91</td>
<td>.66</td>
</tr>
<tr>
<td>7.6</td>
<td>1.64</td>
<td>1.1</td>
<td>.81</td>
<td>.62</td>
</tr>
<tr>
<td>10.2</td>
<td>1.34</td>
<td>1.0</td>
<td>.66</td>
<td>.53</td>
</tr>
<tr>
<td>1.7</td>
<td>1.28</td>
<td>.98</td>
<td>.59</td>
<td>.46</td>
</tr>
<tr>
<td>15.2</td>
<td>1.0</td>
<td>.95</td>
<td>.53</td>
<td>.39</td>
</tr>
<tr>
<td>20.3</td>
<td>.72</td>
<td>.60</td>
<td>.39</td>
<td>.33</td>
</tr>
<tr>
<td>25.9</td>
<td>.54</td>
<td>.49</td>
<td>.33</td>
<td>.26</td>
</tr>
<tr>
<td>30.5</td>
<td>.48</td>
<td>.33</td>
<td>.26</td>
<td>.2</td>
</tr>
<tr>
<td>38.1</td>
<td>.33</td>
<td>.26</td>
<td>.26</td>
<td>.2</td>
</tr>
<tr>
<td>49.6</td>
<td>.26</td>
<td>.26</td>
<td>.26</td>
<td>.2</td>
</tr>
<tr>
<td>50.8</td>
<td>0.0</td>
<td>.13</td>
<td>.13</td>
<td>.13</td>
</tr>
<tr>
<td>55.9</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
<td>.13</td>
</tr>
<tr>
<td>63.0</td>
<td>---</td>
<td>---</td>
<td>.13</td>
<td>.13</td>
</tr>
<tr>
<td>71.1</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
<td>.13</td>
</tr>
<tr>
<td>76.2</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>.13</td>
</tr>
<tr>
<td>81.3</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>0.0</td>
</tr>
</tbody>
</table>
TABLE 4.36

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 186

\( Q = 15.4 \text{ L/s} \quad \theta_0 = 50.0 \text{ mm} \quad Y_3 = 127.0 \text{ mm} \)
\( Y_2 = 182.9 \text{ mm} \quad R_1 = 990.6 \text{ mm} \quad R_2 = 1701.0 \text{ mm} \quad F_1 = 3.9 \)

<table>
<thead>
<tr>
<th>( R (\text{mm}) )</th>
<th>( Y (\text{mm}) )</th>
<th>( F/y (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>127.0</td>
<td>177.8</td>
</tr>
<tr>
<td>965.2</td>
<td>114.3</td>
<td>134.6</td>
</tr>
<tr>
<td>1016.0</td>
<td>101.6</td>
<td>111.8</td>
</tr>
<tr>
<td>1066.8</td>
<td>101.6</td>
<td>106.7</td>
</tr>
<tr>
<td>1117.6</td>
<td>106.7</td>
<td>111.8</td>
</tr>
<tr>
<td>1168.4</td>
<td>111.8</td>
<td>119.4</td>
</tr>
<tr>
<td>1219.2</td>
<td>119.4</td>
<td>124.5</td>
</tr>
<tr>
<td>1270.0</td>
<td>127.0</td>
<td>129.5</td>
</tr>
<tr>
<td>1320.8</td>
<td>134.6</td>
<td>134.6</td>
</tr>
<tr>
<td>1371.6</td>
<td>142.2</td>
<td>142.2</td>
</tr>
<tr>
<td>1422.4</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1473.2</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1524.0</td>
<td>157.5</td>
<td>157.5</td>
</tr>
<tr>
<td>1524.0</td>
<td>157.5</td>
<td>157.5</td>
</tr>
<tr>
<td>1574.8</td>
<td>162.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1625.6</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1676.4</td>
<td>170.2</td>
<td>170.2</td>
</tr>
<tr>
<td>1701.0</td>
<td>182.9</td>
<td>182.9</td>
</tr>
</tbody>
</table>
TABLE 4.37

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 107
Q = 15.4 L/s  G0 = 50.8 MM  Y3 = 114.3 MM
Y2 = 172.7 MM  R1 = 990.6 MM  R2 = 1701.8 MM  F1 = 3.9 \(^{\circ}\)

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/y (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>114.3</td>
<td>190.5</td>
</tr>
<tr>
<td>965.2</td>
<td>111.6</td>
<td>157.5</td>
</tr>
<tr>
<td>1016.0</td>
<td>101.6</td>
<td>132.1</td>
</tr>
<tr>
<td>1066.0</td>
<td>104.6</td>
<td>127.0</td>
</tr>
<tr>
<td>1117.6</td>
<td>114.3</td>
<td>127.0</td>
</tr>
<tr>
<td>1168.4</td>
<td>121.9</td>
<td>132.1</td>
</tr>
<tr>
<td>1219.2</td>
<td>132.1</td>
<td>137.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>137.2</td>
</tr>
<tr>
<td>1371.6</td>
<td>144.0</td>
<td>147.3</td>
</tr>
<tr>
<td>1422.4</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1473.2</td>
<td>157.5</td>
<td>157.5</td>
</tr>
<tr>
<td>1524.0</td>
<td>162.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1574.8</td>
<td>167.6</td>
<td>167.6</td>
</tr>
<tr>
<td>1625.6</td>
<td>170.2</td>
<td>170.2</td>
</tr>
<tr>
<td>1676.4</td>
<td>171.4</td>
<td>171.4</td>
</tr>
<tr>
<td>1727.2</td>
<td>172.7</td>
<td>172.7</td>
</tr>
</tbody>
</table>
TABLE 4.38

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 188

Q = 19.8 L/S  G0 = 57.1 MM,  Y3 = 165.1 MM,
Y2 = 215.9 MM,  R1 = 1000.1 MM,  R2 = 1828.0 MM,  F1 = 4.1

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/A (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>165.1</td>
<td>330.2</td>
</tr>
<tr>
<td>1016.0</td>
<td>162.6</td>
<td>220.6</td>
</tr>
<tr>
<td>1066.0</td>
<td>157.5</td>
<td>170.2</td>
</tr>
<tr>
<td>1117.6</td>
<td>160.0</td>
<td>172.7</td>
</tr>
<tr>
<td>1168.4</td>
<td>165.1</td>
<td>175.3</td>
</tr>
<tr>
<td>1219.2</td>
<td>170.2</td>
<td>177.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>175.3</td>
<td>180.3</td>
</tr>
<tr>
<td>1320.8</td>
<td>180.3</td>
<td>185.4</td>
</tr>
<tr>
<td>1371.6</td>
<td>185.4</td>
<td>185.4</td>
</tr>
<tr>
<td>1422.4</td>
<td>190.5</td>
<td>190.5</td>
</tr>
<tr>
<td>1473.2</td>
<td>193.0</td>
<td>193.0</td>
</tr>
<tr>
<td>1524.0</td>
<td>198.1</td>
<td>198.1</td>
</tr>
<tr>
<td>1625.6</td>
<td>203.2</td>
<td>203.2</td>
</tr>
<tr>
<td>1676.4</td>
<td>205.7</td>
<td>205.7</td>
</tr>
<tr>
<td>1727.2</td>
<td>208.3</td>
<td>208.3</td>
</tr>
<tr>
<td>1778.0</td>
<td>210.0</td>
<td>210.0</td>
</tr>
<tr>
<td>1828.8</td>
<td>215.9</td>
<td>215.9</td>
</tr>
</tbody>
</table>
TABLE 4.39
MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBLERGED RADIAL HYDRAULIC JUMP

RUN NUMBER: 189

Q = 19.8 L/S  g0 = 57.1 MM.  y3 = 127.0 MM.
y2 = 203.2 MM.  r1 = .1000.1 MM.  r2 = 1779.0 MM.  f1 = 4.1

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>P/s (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>127.0</td>
<td>233.7</td>
</tr>
<tr>
<td>965.2</td>
<td>111.8</td>
<td>152.4</td>
</tr>
<tr>
<td>1016.0</td>
<td>101.6</td>
<td>127.0</td>
</tr>
<tr>
<td>1066.8</td>
<td>104.1</td>
<td>124.5</td>
</tr>
<tr>
<td>1117.6</td>
<td>111.8</td>
<td>127.0</td>
</tr>
<tr>
<td>1117.6</td>
<td>111.8</td>
<td>127.0</td>
</tr>
<tr>
<td>1160.4</td>
<td>121.9</td>
<td>132.1</td>
</tr>
<tr>
<td>1219.2</td>
<td>127.0</td>
<td>132.1</td>
</tr>
<tr>
<td>1220.0</td>
<td>137.2</td>
<td>137.2</td>
</tr>
<tr>
<td>1320.8</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1371.6</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1422.4</td>
<td>160.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1473.2</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1524.0</td>
<td>172.7</td>
<td>172.7</td>
</tr>
<tr>
<td>1574.8</td>
<td>177.8</td>
<td>177.8</td>
</tr>
<tr>
<td>1625.6</td>
<td>182.9</td>
<td>182.9</td>
</tr>
<tr>
<td>1676.4</td>
<td>188.0</td>
<td>188.0</td>
</tr>
<tr>
<td>1727.2</td>
<td>195.6</td>
<td>195.6</td>
</tr>
<tr>
<td>1727.2</td>
<td>203.2</td>
<td>203.2</td>
</tr>
</tbody>
</table>
TABLE 4.40

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 190

\[ \begin{align*}
Q &= 10.4 \text{ L/s} \quad G_0 = 25.4 \text{ mm} \quad Y_0 = 101.6 \text{ mm} \\
Y_2 &= 177.8 \text{ mm} \quad R_1 = 952.5 \text{ mm} \quad R_2 = 1727.2 \text{ mm} \quad F_1 = 7.6
\end{align*} \]

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>Y (mm)</th>
<th>P (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>101.6</td>
<td>203.2</td>
</tr>
<tr>
<td>965.2</td>
<td>91.4</td>
<td>111.8</td>
</tr>
<tr>
<td>1016.0</td>
<td>81.3</td>
<td>104.1</td>
</tr>
<tr>
<td>1066.8</td>
<td>86.4</td>
<td>101.6</td>
</tr>
<tr>
<td>1117.6</td>
<td>96.5</td>
<td>101.6</td>
</tr>
<tr>
<td>1168.4</td>
<td>101.6</td>
<td>106.7</td>
</tr>
<tr>
<td>1219.2</td>
<td>104.1</td>
<td>109.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>111.0</td>
<td>116.8</td>
</tr>
<tr>
<td>1320.0</td>
<td>121.9</td>
<td>121.9</td>
</tr>
<tr>
<td>1371.6</td>
<td>127.0</td>
<td>127.0</td>
</tr>
</tbody>
</table>
TABLE 4.41

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 192

Q = 7.2 L/S  G0 = 25.4 MM,  Y3 = 106.7 MM,
Y2 = 142.2 MM,  R1 = 952.5 MM,  R2 = 1727.2 MM,  F1 = 5.3

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/R (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>257.0</td>
<td>106.7</td>
<td>127.0</td>
</tr>
<tr>
<td>264.8</td>
<td>101.3</td>
<td>110.6</td>
</tr>
<tr>
<td>269.0</td>
<td>78.7</td>
<td>96.5</td>
</tr>
<tr>
<td>276.0</td>
<td>78.7</td>
<td>91.4</td>
</tr>
<tr>
<td>289.6</td>
<td>81.3</td>
<td>94.0</td>
</tr>
<tr>
<td>291.4</td>
<td>84.4</td>
<td>99.1</td>
</tr>
<tr>
<td>291.2</td>
<td>94.0</td>
<td>104.1</td>
</tr>
<tr>
<td>290.0</td>
<td>97.1</td>
<td>104.1</td>
</tr>
<tr>
<td>289.2</td>
<td>101.6</td>
<td>109.2</td>
</tr>
<tr>
<td>289.0</td>
<td>109.2</td>
<td>111.0</td>
</tr>
<tr>
<td>287.8</td>
<td>114.3</td>
<td>116.8</td>
</tr>
<tr>
<td>288.4</td>
<td>121.9</td>
<td>121.9</td>
</tr>
<tr>
<td>288.2</td>
<td>127.0</td>
<td>127.0</td>
</tr>
<tr>
<td>288.0</td>
<td>129.5</td>
<td>129.5</td>
</tr>
<tr>
<td>287.8</td>
<td>134.6</td>
<td>134.6</td>
</tr>
<tr>
<td>287.6</td>
<td>137.2</td>
<td>137.2</td>
</tr>
<tr>
<td>287.4</td>
<td>139.7</td>
<td>139.7</td>
</tr>
<tr>
<td>287.2</td>
<td>142.2</td>
<td>142.2</td>
</tr>
</tbody>
</table>
TABLE 4.42

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 193

\( Q = 17.0 \text{ L/s} \)
\( D_0 = 50.0 \text{ MM} \)
\( y_3 = 81.3 \text{ MM} \)
\( y_2 = 177.8 \text{ MM} \)
\( r_1 = 990.6 \text{ MM} \)
\( r_2 = 1651.0 \text{ MM} \)
\( f_1 = 4.2 \)

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>y (MM.)</th>
<th>F/\theta (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>81.3</td>
<td>177.8</td>
</tr>
<tr>
<td>965.2</td>
<td>101.6</td>
<td>142.2</td>
</tr>
<tr>
<td>1016.0</td>
<td>99.1</td>
<td>111.8</td>
</tr>
<tr>
<td>1066.8</td>
<td>94.0</td>
<td>104.1</td>
</tr>
<tr>
<td>1117.6</td>
<td>91.4</td>
<td>109.2</td>
</tr>
<tr>
<td>1168.4</td>
<td>101.6</td>
<td>114.3</td>
</tr>
<tr>
<td>1219.2</td>
<td>111.0</td>
<td>121.9</td>
</tr>
<tr>
<td>1270.0</td>
<td>119.4</td>
<td>129.5</td>
</tr>
<tr>
<td>1320.8</td>
<td>129.5</td>
<td>134.6</td>
</tr>
<tr>
<td>1371.6</td>
<td>139.7</td>
<td>139.7</td>
</tr>
<tr>
<td>1422.4</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1473.2</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1524.0</td>
<td>157.5</td>
<td>157.5</td>
</tr>
<tr>
<td>1574.8</td>
<td>162.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1625.6</td>
<td>170.2</td>
<td>170.2</td>
</tr>
<tr>
<td>1675.0</td>
<td>175.3</td>
<td>175.3</td>
</tr>
</tbody>
</table>
## Table 4.43

Measured water-surface profiles and hydraulic grade profiles for the submerged radial hydraulic jump.

**Run Number:** 194

$Q = 11.3 \text{ L/s}$, $v_0 = 30.1 \text{ mm}$, $y_3 = 101.6 \text{ mm}$, $y_2 = 147.3 \text{ mm}$, $r_1 = 971.5 \text{ mm}$, $r_2 = 1447.8 \text{ mm}$, $F_1 = 4.4$

<table>
<thead>
<tr>
<th>$r$ (mm)</th>
<th>$y$ (mm)</th>
<th>$f$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>101.6</td>
<td>152.4</td>
</tr>
<tr>
<td>965.2</td>
<td>83.0</td>
<td>127.0</td>
</tr>
<tr>
<td>1016.0</td>
<td>76.2</td>
<td>109.2</td>
</tr>
<tr>
<td>1066.8</td>
<td>91.4</td>
<td>106.7</td>
</tr>
<tr>
<td>1117.6</td>
<td>101.6</td>
<td>109.2</td>
</tr>
<tr>
<td>1168.4</td>
<td>114.3</td>
<td>114.3</td>
</tr>
<tr>
<td>1219.2</td>
<td>119.4</td>
<td>119.4</td>
</tr>
<tr>
<td>1270.0</td>
<td>124.5</td>
<td>124.5</td>
</tr>
<tr>
<td>1320.8</td>
<td>127.0</td>
<td>127.0</td>
</tr>
<tr>
<td>1371.6</td>
<td>129.5</td>
<td>129.5</td>
</tr>
<tr>
<td>1422.4</td>
<td>132.1</td>
<td>132.1</td>
</tr>
<tr>
<td>1447.8</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>$R$ (mm.)</td>
<td>$Y$ (mm.)</td>
<td>$P/Y$ (mm.)</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td>914.4</td>
<td>142.2</td>
<td>254.0</td>
</tr>
<tr>
<td>965.2</td>
<td>127.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1016.0</td>
<td>119.4</td>
<td>132.1</td>
</tr>
<tr>
<td>1066.8</td>
<td>114.3</td>
<td>127.0</td>
</tr>
<tr>
<td>1168.4</td>
<td>116.8</td>
<td>132.1</td>
</tr>
<tr>
<td>1219.2</td>
<td>121.9</td>
<td>137.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>139.7</td>
</tr>
<tr>
<td>1320.8</td>
<td>137.2</td>
<td>142.2</td>
</tr>
<tr>
<td>1371.6</td>
<td>149.9</td>
<td>149.9</td>
</tr>
<tr>
<td>1422.4</td>
<td>154.9</td>
<td>154.9</td>
</tr>
<tr>
<td>1473.2</td>
<td>160.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1524.0</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1574.0</td>
<td>172.7</td>
<td>172.7</td>
</tr>
<tr>
<td>1625.6</td>
<td>175.3</td>
<td>175.3</td>
</tr>
<tr>
<td>1676.4</td>
<td>177.8</td>
<td>177.8</td>
</tr>
<tr>
<td>1727.2</td>
<td>182.9</td>
<td>182.9</td>
</tr>
<tr>
<td>1778.0</td>
<td>190.5</td>
<td>190.5</td>
</tr>
</tbody>
</table>
TABLE 4.45

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 196

\(Q = 14.2 \text{ L/s}\) \(G_0 = 38.1 \text{ mm}\), \(Y_3 = 177.8 \text{ mm}\),
\(Y_2 = 215.9 \text{ mm}\), \(R_1 = 971.5 \text{ mm}\), \(R_2 = 1020.8 \text{ mm}\), \(F_1 = 5.6\)

<table>
<thead>
<tr>
<th>(R (\text{mm}))</th>
<th>(Y (\text{mm}))</th>
<th>(F/R (\text{mm}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>177.8</td>
<td>254.0</td>
</tr>
<tr>
<td>965.2</td>
<td>165.1</td>
<td>208.3</td>
</tr>
<tr>
<td>1016.0</td>
<td>144.8</td>
<td>157.5</td>
</tr>
<tr>
<td>1066.8</td>
<td>137.2</td>
<td>149.9</td>
</tr>
<tr>
<td>1117.6</td>
<td>137.2</td>
<td>152.4</td>
</tr>
<tr>
<td>1168.4</td>
<td>139.7</td>
<td>152.4</td>
</tr>
<tr>
<td>1219.2</td>
<td>147.3</td>
<td>154.9</td>
</tr>
<tr>
<td>1270.0</td>
<td>152.4</td>
<td>157.5</td>
</tr>
<tr>
<td>1270.0</td>
<td>152.4</td>
<td>157.5</td>
</tr>
<tr>
<td>1330.8</td>
<td>160.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1371.6</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1422.4</td>
<td>172.7</td>
<td>172.7</td>
</tr>
<tr>
<td>1473.2</td>
<td>177.8</td>
<td>177.8</td>
</tr>
<tr>
<td>1473.2</td>
<td>177.8</td>
<td>177.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>185.4</td>
<td>185.4</td>
</tr>
<tr>
<td>1574.8</td>
<td>190.5</td>
<td>190.5</td>
</tr>
<tr>
<td>1625.6</td>
<td>195.6</td>
<td>195.6</td>
</tr>
<tr>
<td>1676.4</td>
<td>203.2</td>
<td>203.2</td>
</tr>
<tr>
<td>1727.2</td>
<td>208.3</td>
<td>208.3</td>
</tr>
<tr>
<td>1778.0</td>
<td>213.4</td>
<td>213.4</td>
</tr>
<tr>
<td>1828.8</td>
<td>215.9</td>
<td>215.9</td>
</tr>
</tbody>
</table>
### TABLE 4.46

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 197**

\[ Q = 22.4 \text{ L/s}, \quad \theta = 69.6 \text{ MM}, \quad Y_3 = 165.1 \text{ MM}; \]

\[ Y_2 = 215.9 \text{ MM}, \quad R_1 = 1019.2 \text{ MM}, \quad R_2 = 1854.2 \text{ MM}, \quad F_1 = 3.4 \]

<table>
<thead>
<tr>
<th>( R \text{ (MM.)} )</th>
<th>( Y \text{ (MM.)} )</th>
<th>( F/\gamma \text{ (MM.)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>165.1</td>
<td>279.4</td>
</tr>
<tr>
<td>965.2</td>
<td>142.2</td>
<td>208.3</td>
</tr>
<tr>
<td>1016.0</td>
<td>137.2</td>
<td>172.7</td>
</tr>
<tr>
<td>1066.8</td>
<td>137.2</td>
<td>152.4</td>
</tr>
<tr>
<td>1117.6</td>
<td>137.2</td>
<td>154.9</td>
</tr>
<tr>
<td>1168.4</td>
<td>137.2</td>
<td>157.5</td>
</tr>
<tr>
<td>1219.2</td>
<td>139.7</td>
<td>160.0</td>
</tr>
<tr>
<td>1270.0</td>
<td>144.8</td>
<td>162.6</td>
</tr>
<tr>
<td>1320.0</td>
<td>147.3</td>
<td>165.1</td>
</tr>
<tr>
<td>1371.6</td>
<td>152.4</td>
<td>167.6</td>
</tr>
<tr>
<td>1422.4</td>
<td>157.5</td>
<td>172.7</td>
</tr>
<tr>
<td>1473.2</td>
<td>162.6</td>
<td>177.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>167.6</td>
<td>182.9</td>
</tr>
<tr>
<td>1574.8</td>
<td>177.8</td>
<td>188.0</td>
</tr>
<tr>
<td>1625.6</td>
<td>182.9</td>
<td>193.0</td>
</tr>
<tr>
<td>1727.2</td>
<td>190.1</td>
<td>198.1</td>
</tr>
<tr>
<td>1778.0</td>
<td>200.3</td>
<td>208.3</td>
</tr>
<tr>
<td>1828.8</td>
<td>210.8</td>
<td>210.8</td>
</tr>
<tr>
<td>1879.6</td>
<td>215.9</td>
<td>215.9</td>
</tr>
</tbody>
</table>
TABLE 4.47

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 198

$Q = 22.4$ L/s  $G_0 = 76.2$ mm.  $Y_3 = 127.0$ mm.
$Y_2 = 190.5$ mm.  $R_1 = 1020.7$ mm.  $R_2 = 1727.2$ mm.  $F_1 = 2.9$

<table>
<thead>
<tr>
<th>$R$ (mm.)</th>
<th>$Y$ (mm.)</th>
<th>$P$ (mm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>127.0</td>
<td>315.0</td>
</tr>
<tr>
<td>965.2</td>
<td>119.4</td>
<td>132.1</td>
</tr>
<tr>
<td>1016.0</td>
<td>114.3</td>
<td>149.9</td>
</tr>
<tr>
<td>1066.0</td>
<td>114.3</td>
<td>137.2</td>
</tr>
<tr>
<td>1117.6</td>
<td>127.0</td>
<td>137.2</td>
</tr>
<tr>
<td>1168.4</td>
<td>139.7</td>
<td>137.7</td>
</tr>
<tr>
<td>1219.2</td>
<td>144.8</td>
<td>144.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>152.4</td>
<td>154.9</td>
</tr>
<tr>
<td>1320.8</td>
<td>154.9</td>
<td>154.9</td>
</tr>
<tr>
<td>1371.6</td>
<td>162.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1422.4</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1473.2</td>
<td>170.2</td>
<td>170.2</td>
</tr>
<tr>
<td>1524.0</td>
<td>180.3</td>
<td>180.3</td>
</tr>
<tr>
<td>1575.6</td>
<td>185.4</td>
<td>185.4</td>
</tr>
<tr>
<td>1627.4</td>
<td>188.0</td>
<td>188.0</td>
</tr>
<tr>
<td>1727.2</td>
<td>190.5</td>
<td>190.5</td>
</tr>
</tbody>
</table>
TABLE 4.48

MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 199

Q = 27.4 L/s  G0 = 76.2 mm, Y3 = 177.8 mm,
Y2 = 234.9 mm, R1 = 1026.7 mm, R2 = 1701.8 mm, F1 = 3.6

<table>
<thead>
<tr>
<th>R (mm.)</th>
<th>Y (mm.)</th>
<th>F/s (mm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>177.8</td>
<td>355.6</td>
</tr>
<tr>
<td>965.2</td>
<td>170.2</td>
<td>177.8</td>
</tr>
<tr>
<td>1016.0</td>
<td>160.0</td>
<td>182.9</td>
</tr>
<tr>
<td>1066.8</td>
<td>154.9</td>
<td>177.8</td>
</tr>
<tr>
<td>1117.6</td>
<td>152.4</td>
<td>175.3</td>
</tr>
<tr>
<td>1168.4</td>
<td>160.0</td>
<td>177.8</td>
</tr>
<tr>
<td>1219.2</td>
<td>165.1</td>
<td>177.8</td>
</tr>
<tr>
<td>1270.0</td>
<td>172.7</td>
<td>180.3</td>
</tr>
<tr>
<td>1330.8</td>
<td>177.8</td>
<td>182.9</td>
</tr>
<tr>
<td>1371.6</td>
<td>185.4</td>
<td>185.4</td>
</tr>
<tr>
<td>1422.4</td>
<td>190.5</td>
<td>190.5</td>
</tr>
<tr>
<td>1473.2</td>
<td>195.6</td>
<td>195.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>198.1</td>
<td>198.1</td>
</tr>
<tr>
<td>1574.8</td>
<td>203.2</td>
<td>203.2</td>
</tr>
<tr>
<td>1625.6</td>
<td>208.3</td>
<td>208.3</td>
</tr>
<tr>
<td>1676.4</td>
<td>210.8</td>
<td>210.8</td>
</tr>
<tr>
<td>1701.8</td>
<td>234.9</td>
<td>234.9</td>
</tr>
</tbody>
</table>
### Table 4.49

**Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump**

**Run Number:** 200

\[Q = 12.9 \text{ L/s}, \quad G_0 = 38.1 \text{ mm}, \quad Y_3 = 81.3 \text{ mm}, \quad Y_2 = 165.1 \text{ mm}, \quad R_1 = 971.5 \text{ mm}, \quad R_2 = 1778.0 \text{ mm}, \quad F_1 = 5.1\]

<table>
<thead>
<tr>
<th>R (mm.)</th>
<th>Y (mm.)</th>
<th>P/κ (mm.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>91.4</td>
<td>152.4</td>
</tr>
<tr>
<td>965.2</td>
<td>63.5</td>
<td>91.4</td>
</tr>
<tr>
<td>1016.0</td>
<td>55.9</td>
<td>76.2</td>
</tr>
<tr>
<td>1068.8</td>
<td>55.9</td>
<td>81.3</td>
</tr>
<tr>
<td>1117.6</td>
<td>60.6</td>
<td>94.0</td>
</tr>
<tr>
<td>1168.4</td>
<td>81.3</td>
<td>101.6</td>
</tr>
<tr>
<td>1219.2</td>
<td>91.4</td>
<td>109.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>101.6</td>
<td>116.8</td>
</tr>
<tr>
<td>1320.8</td>
<td>111.8</td>
<td>121.9</td>
</tr>
<tr>
<td>1371.6</td>
<td>121.9</td>
<td>127.0</td>
</tr>
<tr>
<td>1422.4</td>
<td>134.6</td>
<td>134.6</td>
</tr>
<tr>
<td>1473.2</td>
<td>139.7</td>
<td>139.7</td>
</tr>
<tr>
<td>1524.0</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1574.8</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1625.6</td>
<td>157.5</td>
<td>157.5</td>
</tr>
<tr>
<td>1676.4</td>
<td>160.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1727.2</td>
<td>162.6</td>
<td>162.6</td>
</tr>
<tr>
<td>1778.0</td>
<td>165.1</td>
<td>165.1</td>
</tr>
</tbody>
</table>
### Table 4.50

**Measured Water Surface Profiles and Hydraulic Grade Profiles for the Submerged Radial Hydraulic Jump.**

**Run Number:** 201

\[ Q = 15.4 \text{ L/s} \quad g_0 = 44.4 \text{ mm} \quad y_3 = 139.7 \text{ mm} \]

\[ y_2 = 190.5 \text{ mm} \quad r_1 = 981.1 \text{ mm} \quad r_2 = 1054.2 \text{ mm} \quad f_1 = 4.8 \]

<table>
<thead>
<tr>
<th>( R (\text{mm}) )</th>
<th>( Y (\text{mm}) )</th>
<th>( F / g (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>139.7</td>
<td>228.6</td>
</tr>
<tr>
<td>965.2</td>
<td>121.9</td>
<td>157.5</td>
</tr>
<tr>
<td>1016.0</td>
<td>101.6</td>
<td>124.5</td>
</tr>
<tr>
<td>1066.0</td>
<td>106.7</td>
<td>121.9</td>
</tr>
<tr>
<td>1117.6</td>
<td>114.3</td>
<td>124.5</td>
</tr>
<tr>
<td>1168.4</td>
<td>121.9</td>
<td>129.5</td>
</tr>
<tr>
<td>1219.2</td>
<td>127.0</td>
<td>134.6</td>
</tr>
<tr>
<td>1270.0</td>
<td>132.1</td>
<td>139.7</td>
</tr>
<tr>
<td>1320.0</td>
<td>137.2</td>
<td>147.3</td>
</tr>
<tr>
<td>1371.6</td>
<td>142.2</td>
<td>157.5</td>
</tr>
<tr>
<td>1422.4</td>
<td>152.4</td>
<td>162.6</td>
</tr>
<tr>
<td>1473.0</td>
<td>161.1</td>
<td>172.7</td>
</tr>
<tr>
<td>1524.0</td>
<td>175.3</td>
<td>175.3</td>
</tr>
<tr>
<td>1575.6</td>
<td>177.8</td>
<td>177.8</td>
</tr>
<tr>
<td>1627.2</td>
<td>182.9</td>
<td>182.9</td>
</tr>
<tr>
<td>1678.0</td>
<td>185.4</td>
<td>185.4</td>
</tr>
<tr>
<td>1729.2</td>
<td>190.0</td>
<td>190.0</td>
</tr>
<tr>
<td>1780.0</td>
<td>190.5</td>
<td>190.5</td>
</tr>
<tr>
<td>1831.2</td>
<td>190.5</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4.51

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 202

\( Q = 15.4 \text{ L/s} \quad g_0 = 44.4 \text{ mm} \quad y_3 = 88.9 \text{ mm} \)
\( y_2 = 177.8 \text{ mm} \quad r_1 = 981.1 \text{ mm} \quad r_2 = 1651.0 \text{ mm} \quad f_1 = 4.8 \)

<table>
<thead>
<tr>
<th>R (mm)</th>
<th>Y (mm)</th>
<th>F/g (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>88.2</td>
<td>127.0</td>
</tr>
<tr>
<td>965.2</td>
<td>81.3</td>
<td>109.2</td>
</tr>
<tr>
<td>1016.0</td>
<td>74.2</td>
<td>104.1</td>
</tr>
<tr>
<td>1066.0</td>
<td>76.2</td>
<td>101.6</td>
</tr>
<tr>
<td>1117.6</td>
<td>81.3</td>
<td>101.6</td>
</tr>
<tr>
<td>1168.4</td>
<td>101.6</td>
<td>106.7</td>
</tr>
<tr>
<td>1219.2</td>
<td>109.2</td>
<td>109.2</td>
</tr>
<tr>
<td>1270.0</td>
<td>121.9</td>
<td>121.9</td>
</tr>
<tr>
<td>1320.0</td>
<td>129.5</td>
<td>129.5</td>
</tr>
<tr>
<td>1371.6</td>
<td>137.2</td>
<td>137.2</td>
</tr>
<tr>
<td>1422.4</td>
<td>144.8</td>
<td>144.8</td>
</tr>
<tr>
<td>1473.2</td>
<td>152.4</td>
<td>152.4</td>
</tr>
<tr>
<td>1524.0</td>
<td>160.0</td>
<td>160.0</td>
</tr>
<tr>
<td>1574.8</td>
<td>165.1</td>
<td>165.1</td>
</tr>
<tr>
<td>1625.6</td>
<td>172.7</td>
<td>172.7</td>
</tr>
<tr>
<td>1651.0</td>
<td>177.8</td>
<td>177.8</td>
</tr>
</tbody>
</table>
**TABLE 4.52**

**MEASURED WATER SURFACE PROFILES AND HYDRAULIC GRADE PROFILES FOR THE SUBMERGED RADIAL HYDRAULIC JUMP.**

**RUN NUMBER: 203**

\[ Q = 11.0 \text{ L/s} \quad 60 = 27.9 \text{ MM} \quad Y_3 = 91.4 \text{ MM}, \]
\[ Y_2 = 152.4 \text{ MM} \quad R_1 = 956.3 \text{ MM} \quad R_2 = 1524.0 \text{ MM} \quad F_1 = 7.0 \]

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F/6 (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>80.9</td>
<td>127.0</td>
</tr>
<tr>
<td>965.2</td>
<td>66.0</td>
<td>76.2</td>
</tr>
<tr>
<td>1016.0</td>
<td>50.8</td>
<td>71.1</td>
</tr>
<tr>
<td>1066.8</td>
<td>55.9</td>
<td>76.2</td>
</tr>
<tr>
<td>1117.6</td>
<td>66.0</td>
<td>83.8</td>
</tr>
<tr>
<td>1168.4</td>
<td>83.8</td>
<td>91.4</td>
</tr>
<tr>
<td>1219.2</td>
<td>101.6</td>
<td>101.6</td>
</tr>
<tr>
<td>1270.0</td>
<td>109.2</td>
<td>109.2</td>
</tr>
<tr>
<td>1320.8</td>
<td>127.0</td>
<td>127.0</td>
</tr>
<tr>
<td>1371.6</td>
<td>134.6</td>
<td>134.6</td>
</tr>
<tr>
<td>1422.4</td>
<td>139.7</td>
<td>139.7</td>
</tr>
<tr>
<td>1473.2</td>
<td>144.8</td>
<td>144.8</td>
</tr>
<tr>
<td>1524.0</td>
<td>152.4</td>
<td>152.4</td>
</tr>
</tbody>
</table>
TABLE 4.53

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 204.

\( Q = 11.0 \) L/S \( G_0 = 31.7 \) MM, \( Y_3 = 88.9 \) MM,
\( Y_2 = 152.4 \) MM, \( R_1 = 962.0 \) MM, \( R_2 = 1651.0 \) MM, \( F_1 = 5.7 \)

<table>
<thead>
<tr>
<th>( R(\text{MM}) )</th>
<th>( Y(\text{MM}) )</th>
<th>( F/\delta(\text{MM}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>88.9</td>
<td>127.0</td>
</tr>
<tr>
<td>965.2</td>
<td>71.1</td>
<td>81.3</td>
</tr>
<tr>
<td>1016.0</td>
<td>50.8</td>
<td>76.2</td>
</tr>
<tr>
<td>1066.8</td>
<td>61.0</td>
<td>81.3</td>
</tr>
<tr>
<td>1117.6</td>
<td>71.1</td>
<td>88.9</td>
</tr>
<tr>
<td>1168.4</td>
<td>83.0</td>
<td>96.5</td>
</tr>
<tr>
<td>1219.2</td>
<td>94.0</td>
<td>106.7</td>
</tr>
<tr>
<td>1270.0</td>
<td>101.6</td>
<td>116.0</td>
</tr>
<tr>
<td>1320.8</td>
<td>111.8</td>
<td>124.5</td>
</tr>
<tr>
<td>1371.6</td>
<td>121.9</td>
<td>129.5</td>
</tr>
<tr>
<td>1422.4</td>
<td>127.0</td>
<td>137.2</td>
</tr>
<tr>
<td>1473.2</td>
<td>134.6</td>
<td>142.2</td>
</tr>
<tr>
<td>1524.0</td>
<td>137.2</td>
<td>147.3</td>
</tr>
<tr>
<td>1574.8</td>
<td>142.2</td>
<td>147.3</td>
</tr>
<tr>
<td>1625.6</td>
<td>144.0</td>
<td>149.9</td>
</tr>
<tr>
<td>1651.0</td>
<td>152.4</td>
<td>152.4</td>
</tr>
</tbody>
</table>
TABLE 4.54

MEASURED WATER SURFACE PROFILES AND
HYDRAULIC GRADE PROFILES FOR THE
SUBMERGED RADIAL HYDRAULIC JUMP.

RUN NUMBER: 205

Q = 11.0 L/S  G0 = 38.1 MM.  Y3 = 101.6 MM,
Y2 = 152.4 MM.  R1 = 971.5 MM.  R2 = 1727.2 MM.  F1 = 4.3

<table>
<thead>
<tr>
<th>R (MM.)</th>
<th>Y (MM.)</th>
<th>F (MM.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>914.4</td>
<td>101.6</td>
<td>142.2</td>
</tr>
<tr>
<td>965.2</td>
<td>91.4</td>
<td>116.8</td>
</tr>
<tr>
<td>1016.0</td>
<td>70.7</td>
<td>104.1</td>
</tr>
<tr>
<td>1066.0</td>
<td>70.7</td>
<td>101.6</td>
</tr>
<tr>
<td>1117.6</td>
<td>83.0</td>
<td>101.6</td>
</tr>
<tr>
<td>1168.4</td>
<td>96.5</td>
<td>144.3</td>
</tr>
<tr>
<td>1219.2</td>
<td>96.5</td>
<td>144.3</td>
</tr>
<tr>
<td>1270.0</td>
<td>101.6</td>
<td>112.4</td>
</tr>
<tr>
<td>1320.6</td>
<td>111.8</td>
<td>124.5</td>
</tr>
<tr>
<td>1371.6</td>
<td>116.8</td>
<td>127.0</td>
</tr>
<tr>
<td>1422.4</td>
<td>124.5</td>
<td>132.1</td>
</tr>
<tr>
<td>1473.2</td>
<td>134.6</td>
<td>134.6</td>
</tr>
<tr>
<td>1524.0</td>
<td>139.7</td>
<td>139.7</td>
</tr>
<tr>
<td>1574.0</td>
<td>144.8</td>
<td>144.8</td>
</tr>
<tr>
<td>1625.6</td>
<td>147.3</td>
<td>147.3</td>
</tr>
<tr>
<td>1676.4</td>
<td>149.9</td>
<td>149.9</td>
</tr>
<tr>
<td>1727.2</td>
<td>152.4</td>
<td>152.4</td>
</tr>
</tbody>
</table>
APPENDIX F

COMPUTER PROGRAM FLOW CHART, LISTING AND OUTPUT
COMPUTER PROGRAM. FLOW CHART

START

Read N
Number of Simulations

L = 1, N

Read:
- Radius to the beginning of the jump, RI
- Initial depth, D_I
- Exponent for the inner layer velocity, XN
- Air Rising Velocity, U_0
- Gravitational acceleration, G
- Space increment, DRI
- Coefficient of upper layer velocity distribution, C

RI > 0

Rectangular Jump

Establish Conditions at the end of the potential core.

Yes Radial Jump

Set D_n = 0.2 D_I
U_m = U_I
R_n = R_I + D_I

Calculate the sum of the series
[Call subroutine series]
\[ H_0 = D + x \]

Calculate \( U_{re} \) using the continuity equation

Momentum Equation Satisfied

No

Yes

Normalize \( H_0, D, U_{re}, U_{re}^* \) using \( D \) and \( U_0 \) as characteristic length and velocity

Calculate approximately jump length, \( XL \), using Macroscopic momentum balance.
Simulation length = 1.5 \( XL \)
Solve the Governing Equations (4 Nonlinear differential equations) using Runge Kutta method.

[Call Subroutine RKSES]

Calculate the coefficients of the 4 nonlinear differential equations

[Call Subroutine VECTOR]

Calculate the inverse of the matrix coefficients of the 4 nonlinear differential equations

[Call Subroutine MINVRS]

\[ R = R_0 + DRI \]
Calculate \( \alpha, \beta \) and \( E/E_1 \)
check the momentum and continuity
[Call Subroutine ENERGY]

PRINT \( R, H_y, H, \delta, U_m, U_s, U_s' \), \( \alpha, B \) and \( E/E_1 \)

Has the Simulated Length Been Exceeded

Yes
PLOT/
\( H_y, H, \delta, U_m, U_s, U_s' \), \( \alpha, B \) and \( E/E_1 \)
Against \( R \)

No

\( L = N \)

Yes
STOP
Set $D_0 = 0.5 D_I$

$U_m = U_I$

$R_0 = R_I + 4D_I$

Calculate the sum of the series
[Call subroutine series]

$H_0 = D_I$

$H_0 = D_I - XX$

Calculate $U_{oo}$ using the continuity equation

Momentum Equation Satisfied

Yes

No
Normalize $H_0$, $D_0$, $U_{00}$, $U_{R0}$, using $D_1$ and $U_1$ as characteristic length and velocity.

Calculate approximately jump length, $XL$, using Macroscopic momentum
Simulation length = 1.5 $XL$

$I = 1, 6$

$R = R_0 + DRI$

Solve the Governing Equations (4 Nonlinear differential equations) using Runge Kutta method.
[Call Subroutine RK5ER]
Calculate the coefficients of the 4 nonlinear differential equations
[Call Subroutine VECTOR]

Calculate the inverse of the matrix coefficients of the 4 nonlinear differential equations
[Call Subroutine MINVRS]

Calculate $a$, $b$ and $E/E_1$
check the momentum and continuity
[Call Subroutine ENERGY]

PRINT $r$, $H_0$, $H$, $L$, $U_m'$, $U_a$
$U_s$, $a$, $B$ and $E/E_1$
Has the Simulated Length Been Exceeded

PLOT:

H, H', 8, \( U_m \), \( U_s \), \( U_g \), a, B and \( E/E_1 \) Against R

No

L = N

Yes

STOP

278
COMPUTER PROGRAM LISTING

// RANIA JOB (R12300G2C2/5,5), 'KHALIFA', CLASS=F
// EXEC FORTGLG, REGION=300K
// FORT.SYSIN DD *
C
******************************************************************************************
C * THIS PROGRAM SIMULATES BOTH THE FREE RECTANGULAR AND RADIAL*
C * HYDRAULIC JUMPS
C * THE PROGRAM SOLVES THE MOMENTUM AND CONTINUITY EQUATIONS
C * BY USING RUNGE-KUTTER METHOD.
C * XN= THE EXP. FOR INNER LAYER VELOCITY
C * D1,DO= INNER LAYER THICKNESS IN FEET.
C * UM, UMO= MAX VELOCITY IN FEET/SEC.
C * U0, U00= MEAN SURFACE VELOCITY IN FEET/SEC.
C * MEAN VELOCITY U.S., THE JUMP IN FEET/SEC.
C * RI= RADIUS AT DI IN FEET
C * RI=0.0 TO SIMULATE FREE RECTANGULAR JUMP
C * RI=RO= TO SIMULATE FREE RADIAL JUMP
C * THE UM AT RO IN FEET/SEC.
C * DI= INITIAL DEPTH
C * G = GRAVITY ACC.
C * F1= FROUDE NUMBER
C * C = COEFF. OF THE UPPER LAYER VELOCITY DISTRIBUTION
C * CO= AIR CONCENTRATION.
C * S1-S10= SERIES
C * CF1,CF2=BED FRICTION
C * UB=AIR RISING VELOCITY (FT/SEC)
C * DRI = SPACE INCREMENT

******************************************************************************************
DIMENSION ANH(4), AN(4), ANUO(4), ANUM(4), REL(900), U(300)
DIMENSION ALFA(300), BET(300), HG(300), UUS(300)
DIMENSION HH(310), DDI(300), UUO(300), UUM(300), RR(300)
REAL*8 Y(25), W(25), IY(25)
EXTERNAL VECTOR
COMMON/AREA1/F1,UI,RI,DI,CO,R
COMMON/AREA2/ALPHA,BETA,EE1,EE2
COMMON/AREA3/TS,US,CF1
COMMON/AREA4/E1,E2,E3,E4,E31,E41
COMMON/AREA5/XL
COMMON/AREA6/S1,S2,S3,S4,S5,S9,S10,SS1
COMMON/AREA7/C,CC
COMMON/AREA8/G,XN,UT,UB
COMMON/AREA9/U,D0,H0,UMO,U00,REL1
READ(5,100) N
CALL PLOTID('KHALIFA','12300G2C7')
CALL XLIMIT(160.)
100 FORMAT(I1)
DO 50 LL=1,N
**BOUNDARY CONDITION**

```
READ(5,300)RI,DI,F1,XN,UB,G,DRI,C
300 FORMAT(BF10.3)
UI=F1*SQR(G*DI)
PRINT 2000
PRINT 2200*RI,DI,F1,UI,XN,DRI

**INITIAL CONDITION**

CC=-C
DO = .2*DI
IF(RI.EQ.0.0) DO = .5*DI
R0 = RI+DI
IF(RI.EQ.0.0) R0=RI+.5*DI
CALL CSERIS
UMO=UI
XK = 0.0
30 CONTINUE
XK = XK +.0001
H0 = DI + XK
CF2=.003
E1=1./EXP(4.*CC)
E2=1./EXP(8.*CC)
E3=1./EXP(CC)
E4=1./EXP(2.*CC)
E31=1./EXP(.723*CC)
E41=1./EXP(.723*.2*CC)
UOO=(UI*DI-UMO*D0/(XN+1.)) -UMO*E1*(H0-D0)*S3)/(D0-H0)+E1
*(H0-D0)*S3)
UT=UOO+UMO
FRF=1.-CF2*F1*F1/DI*R0-H0*H0/DI/DI
FMO=2.*G/DI/DI*(UMO*UMO*D0/(2.*XN+1.)+UOO*UOO*(H0-D0))
F2=2.*UOO*UT*E1*(H0-D0)*S3+UOO*UT*E2*(H0-D0)*S4)-2.*F1*F1
IF(RI.EQ.0.0) GO TO 40
UOO = (RI*UI*DI-R0*UMO*D0/(XN+1.)) -UMO*R0*E1*(H0-D0)*S3)
$/(R0*(H0-D0)+E1*R0*(H0-D0)*S3)
FRF = 1.+H0/DI*(R0/R0-RI-1.)-(H0-H0)*R0/DI/DI/R0
F2= CF2*F1*F1/2./2./DI/R0*(R0*R0-R0*RI)
FMO = 2.*R0/G/R0/DI/DI*(UMO*UMO*D0/(2.*XN+1.)+UOO*UOO*(H0-D0))
F2=2.*UOO*UT*E1*(H0-D0)*S3+UOO*UT*E2*(H0-D0)*S4)-2.*F1*F1
40 CONTINUE
FX=FRF-FMO
IF(ABS(FX).GT.1) GO TO 30
PRINT 2240
PRINT 2300,R0,H0,DO,UMO,UMO,DR
```

**U0,UM,RO,HO ARE DIMENSIONLESS W.R.S. TO UI AND DI.**

DR=DRI/DI
H0=HO/DI
DO=DO/DI
UOO=U00/UI
UMO=UMO/UI
RO=RO/DI
R=RO
CALL ENERGY
PRINT 2400
**PRINT** 2300,R0,H0,D0,U00,UM0,DR

C

************ Printing Formats ************

2000 FORMAT(14X,'The Boundary Condition',//)
2200 FORMAT(10X,'RI=' ,'F6.3,5X','DI=' ,'F6.3,5X','F1=' ,'F6.3,5X',
   'UI=' ,'F6.3,5X','XN=' ,'F6.3,5X','DRI=' ,'F6.3,//)
2400 FORMAT(14X,'The Initial Condition',//)

C

2300 FORMAT(10X,'R0=' ,'F6.3,5X','H0=' ,'F6.3,5X','DO=' ,'F6.3,5X',
   'U00=' ,')6X','UM=' ,')6X',' UM=' ,')6X',' UO=' ,')6X',
   'DI=' ,')6X',' D1=' ,')6X',' DR=' ,')6X',
   'XN=' ,')6X',' DN=' ,')6X',' DN=' ,')6X',
   'F1=' ,')6X',' F2=' ,')6X',' CF=' ,')6X',
   'Y='+')6X',' C1=' ,')6X',' C2=' ,')6X',
   'TS=' ,')6X',' VS=' ,')6X',

C

2400 FORMAT(14X,'Dimensionless Initial Condition',//)

Y2 = -5*DI+5*DI*SQRT(1.08.*F1*F1)
XL = 5.*Y2
RA = 1.5

IF(RI.NE.0.0) Y2 = DI*(3*RA+0.3*F1*(1+1./RA))
IF(RI.NE.0.0) XL = Y2*(4.7-4.2/F1)

M = XL/DR*1.5

C

**** CHOOSE THE DO LOOP LENGTH.

PRINT 2500

2500 FORMAT(14X,'5X','R0','8X','H0','6X','DO','6X','U00','6X','UM','6X','YY',
   '6X','US','6X','ALPHA','6X','BETA','6X','EE2','6X','CF1','6X','TS')

L = 4
X = R0
Y(1) = H0
Y(2) = DO
Y(3) = U00
Y(4) = UM0
D0 = 1 I = 1,M

CALL RKSES (X,Y,DY,DR,L,VECTOR)
HO = Y(1)
DO = Y(2)
U00 = Y(3)
UM0 = Y(4)
RR(I) = RI+X
HH(I) = Y(1)*(1+C0)
ID(I) = Y(2)
U00(I) = Y(3)
HG(I) = Y(1)
UUM(I) = Y(4)
UUS(I) = US
WRITE (6,2)X,Y(J),J=1,4),HH(I),US,ALPHA,BETA,EE2,CF1,TS

CALL ENERGY
ALFA(I) = ALPHA
BET(A) = BETA
Q = UM0*D0/(XN+1.)-U00*(HO-DO)+(U00+UM0)*E1*(HO-DO)*S3
REL(I) = REL1

2 FORMAT(12F9.3)
1 CONTINUE

C

----- THESE INFORMATIONS ARE NEEDED FOR PLOTTER CA/C03 -----
FIRSTY = 0.0
DELTAY = 5.
FIRSTX = RI/DI
DELTAY = 2.
LINTYP = 1
INTEQ = 3
NDX = 1
NDY = 1
MOVE = -1
CALL CALCO3(RR, HH, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
CALL CALCO3(RR, HG, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
DELTAY = .2
CALL CALCO3(RR, REL, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
DELTAY = 1.5
FIRSTY = 0.0
CALL CALCO3(RR, ALPA, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
CALL CALCO3(RR, BET, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
DELTAY = .2
FIRSTY = .0
CALL CALCO3(RR, DD, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
DELTAY = .25
CALL CALCO3(RR, UUM, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
FIRSTY = -.5
DELTAY = .25
CALL CALCO3(RR, UUS, I, MOVE, XS, YS, FIRSTX, DELTAY, FIRSTY
  +, DELTAY, LINTYP, INTEQ, NDX, NDY)
MOVE = 10.
CONTINUE
200 CONTINUE
50 CONTINUE
CALL PLTEND(10.)
STOP
END

**************************************************************************
** THIS SUBROUTINE CALCULATES THE COEFFICIENT OF THE 4 TH **
** ORDINARY DIFFERENTIAL EQUATIONS **
**************************************************************************
SUBROUTINE VECTOR(X, Y, W, N)
REAL*8 X, Y(25), W(25)
DIMENSION A(4, 4), IR(4), IC(4), B(4)
COMMON/AREA1/F1, UI, RI, DI, CO, R

282
COMMON/AREA2/ALPHA,BETA,EE1,EE2
COMMON/AREA3/TS,US,CF1
COMMON/AREA4/E1,E2,E3,E4,E31,E41
COMMON/AREA5/XL
COMMON/AREA6/S1,S2,S3,S4,S8,S9,S10,SS1
COMMON/AREA7/C,CC
COMMON/AREA8/G,XN,UT,UB
R = X
H = Y(1)
D = Y(2)
U0 = Y(3)
UM = Y(4)
CC = -C
UT = U0+UM
CALL CSEIRIS
X1 = B.*C/3.
X2 = H-D
X3 = C/3.*E3*S8
US = U0+UT*E1
AA = .51
BR = .425
CY = .85

C --------- THE AIR CONCENTRATION FRACTION ---------
XX = (D*DI-R1) / XL
IF(XX.LE.2) CO = (6.6*F1**1.35*XX)/100.
IF(XX.GT.0.2) CO = 1.15*F1**1.35*(1.-XX/1.6)/100.
IF(XX.LE.6. AND.RI.NE.0.0) CO = .015*XXX**1.85
IF(XX.GT.6. AND.RI.NE.0.0) CO = F1**1.85* .01/EXP(.5*XXX**2)**.7
IF(CO.LT.0.0) CO = 0.0
CO0 = CO*1.0

C ------- THE BED FRICTION -------
CF1 = .003/(UM*UI*DI**2)**.250

C ------- THE TURBULENT SHEAR -------
TS = .036*CC*UT**UT
IF(XX.LE.2) AK = (UM+US)**2
IF(XX.GT.2) AK = 16.*UB/UI/UT
RNI = CO*6*H*DI*(AK+1.)*AK*EXP(1.)/((-9.*C)**(5.*(AK+1.)))/UT/UT
TS = TS/SORT(1.+4.3*UI**2*(UM+US)**2/RNI/UB**2)

240 CONTINUE
C*******************************
C**** THE MOMENTUM EQU. O--D
C*******************************
C*******************************
C**** A(1,1) --------- THE COEFF. OF DH/DR.
C**** A(1,2) --------- THE COEFF. OF DU/DR.
C**** A(1,3) --------- THE COEFF. OF DUO/DR.
C**** A(1,4) --------- THE COEFF. OF DUM/DR.
C*******************************
C**** B(1) --------- THE CONSTANT.
C*******************************
C*******************************
C**** A(1,1) = G*DI/UI/UT
A(1,2) = XN*UM*UM/(XN+1.)-2.*XN*UM*UM/(2.*XN+1.)
A(1,3) = .00
A(1,4) = 2.*UM*O/(2.*XN+1.)-UM*O/(XN+1.)
B(1) = CF1*UM*UM/2.
C*******************************
C*******************************
C*******************************
C********************
C*****A(2,1)---- THF COFF. OF DH/DR.
C*****A(2,2)---- THE COFF. OF DD/DR.
C*****A(2,3)---- THE COFF. OF DUO/DR.
C*****A(2,4)---- THE COFF. OF DUM/DR.
C*****B(2) ---- THE CONSTANT.
C****** THE EFFECT OF ENTRAINED AIR IN THE HYDRASTATIC PRESSURE--
AD=G*K*(2.*H*D)*C00*DI/UI/UI/2.
BD=G*K*H*C00*DI/UI/UI/2.
AD1=G*K*C00*1.25*(2.*H*D)*DI/UI/UI
BD1=G*K*C00*1.25*H*DI/UI/UI
A(2,1)=G*X2*DI/UI/UI*US*US+2.*UT*UT*X1*E1*S2-2.*UT*UT*X1*E1*S2+AD
A(2,2)=-UM*UM*XN/(XN+1.)*-2.*UT*UT*(X1*E1*S2-2+E1.+1.)
++2.*UT*UT*(X1*E2*S1-5*E2-5)*BD
A(2,3)=2.*U0*X2-2.*U0*UT*X1*X2*S3-2.*UT*U2*X2*S4
A(2,4)=UM*D/(XN+1.)*-2.*U0*E1*X2*S3+2.*UT*E2*X2*S4
B(2)=0.0
C**************************
C*** THE MOMENTUM EQU. Y*
C**********************************************************
C*****A(3,1)---- THF COFF. OF DH/DR.
C*****A(3,2)---- THE COFF. OF DD/DR.
C***** A(3,3)---- THE COFF. OF DUO/DR.
C*****A(3,4)---- THE COFF. OF DUM/DR.
C*****B(3) ---- THE CONSTANT.
C**********************************************************
UN=-U0+UT*E31
A(3,1)=G*DI*X2*BB/UI/UI+X3*BB*(UT*UN+2.*U0*UT)
--2.*UT*UT*E41/3.*C*SS1*AA+AD1
A(3,2) =UM*(XN*UM/(XN+1.)*UT*(-X3*BB+E31-1.))*-UM*UM*
*XN/(XN+1.)*+2.*U0*UT*-(X3*BB+E31-1.)*
--2.*UT*UT*(-E41/3.*C*SS1*AA+5*E41-5)*BD1
A(3,3)=UN*BB*(X2-E31*X2*S3)+U0*X2*CY-E31*X2*S9*(U0+UT)*CY
++UT*E41*X2*S10*CY
A(3,4)=UN*(U0/((XN+1.)*E31*X2*BB*SP)+UM*D/(XN+1.)*
--U0*E31*X2*S9*CY+UT*E41*X2*S10*CY
B(3)=-TS
C*** CONTINUITY EQU. . O._H.
C**********************************************************
C*****A(4,1)---- THE COFF. OF DH/DR.
C*****A(4,2)---- THE COFF. OF DD/DR.
C*****A(4,3)---- THE COFF. OF DUO/DR.
C*****A(4,4)---- THE COFF. OF DUM/DR.
C*****B(4) ---- THE CONSTANT.
C**********************************************************
A(4,1)=X1*E1*S2*UT+US
A(4,2)=XN*UM/(XN+1.)*+UT*(X1*E1*S2-2+E1.+1.)
A(4,3)=X2+E1*X2*S3
A(4,4)=D/(XN+1.)*+E1*X2*S3
B(4)=0.0
B(2)=0.0
IF(R1,NE.0.0) GO TO 40
A(2,1) = A(2,1) -US*A(4,1)
A(2,2) = A(2,2) -US*A(4,2)
A(2,3) = A(2,3) - US*A(4,3)
A(2,4) = A(2,4) - US*A(4,4)
IF (RI,ER,0.0) GO TO 50
B(2) = UM*UM*D/R/(XN+1.) - U0*U0/R*X2+2.*U0*UT/R*E1*X2*S3
-UT*UT/R*E2*X2*S4
B(1) = B(1) + UM*UM*D/R/(XN+1.) - UM*UM*D/R/(2.*XN+1.)
A(4,1) = -X1*E1*S2*R*UT+US*R
A(4,2) = -XN*R*UM/(XN+1.) + R*UT*(X1*E1*S2-E1+1.)
A(4,3) = R*X2+R*E1*X2*S3
A(4,4) = R*ID/(XN+1.) + R*E1*X2*S3
B(4) = -UM*D/(XN+1.) + U0*X2-UT*E1*X2*S3
A(2,1) = A(2,1) - US/R*A(4,1)
A(2,2) = A(2,2) - US/R*A(4,2)
A(2,3) = A(2,3) - US/R*A(4,3)
A(2,4) = A(2,4) - US/R*A(4,4)
B(2) = B(2) - US/R*B(4)
B(3) = UN/R*(UM*I/D/(XN+1.) - U0*X2/2.*CY+UT*E31*X2/2.*CY*S9)
--UM*UM*D/R/(XN+1.) - U0*U0/R*X2/2.*CY+U0*UT/R*E31*X2*CY*S9
--UT*UT*E41/R*X2/2.*CY*S10-TS
50 CONTINUE
CALL MINVRS(A,4,4,DET,IER,IR,IC)
DHR = B(1)*A(1,1) + B(2)*A(1,2) + B(3)*A(1,3) + B(4)*A(1,4)
DDR = B(1)*A(2,1) + B(2)*A(2,2) + B(3)*A(2,3) + B(4)*A(2,4)
DOR = B(1)*A(3,1) + B(2)*A(3,2) + B(3)*A(3,3) + B(4)*A(3,4)
DMR = B(1)*A(4,1) + B(2)*A(4,2) + B(3)*A(4,3) + B(4)*A(4,4)
W(1) = DHR
W(2) = DDR
W(3) = DOR
W(4) = DMR
RETURN
END

SUBROUTINE CSERIES
C******** THIS CALCULATES THE SUM OF THE SERIES INCLUDE, CC
C********************************************************************************************
DIMENSION S1(20),S2(20),S3(20),S4(20),S8(20),S9(20),
& S10(20),SS11(20)
COMMON/AREA6/S1,S2,S3,S4,S8,S9,S10,SS1
COMMON/AREA7/C,CC
CC1=.723*CC
S1=1.0
S2=1.0
S3=1.0
S4=1.0
S8=1.0
S9=1.0
S10=1.0
SS1=1.
S11(1)=1.
S22(1)=1.
S33(1)=1.
S44(1)=1.
S88(1)=1.
S99(1)=1.
S1010(1)=1.
SS11(1)=1.
DO 100 I=2,20
S11(I) = S11(I-1) + 8.0C/(FLOAT(I)*0.5)
S22(I) = S22(I-1) + 4.0C/(FLOAT(I)*0.5)
S33(I) = S33(I-1) + 4.0C/(FLOAT(I)*0.5)
S44(I) = S44(I-1) + 8.0C/(FLOAT(I)*0.5)
S88(I) = S88(I-1) + 2.0C/(FLOAT(I)*0.5)
S99(I) = S99(I-1) + 2.0C/(FLOAT(I)*0.5)
S1010(I) = S1010(I-1) + 2.0C/(FLOAT(I)*0.5)
S111(I) = S111(I-1) + 2.0C/(FLOAT(I)*0.5)
S1 = S1 + S11(I)
S2 = S2 + S22(I)
S3 = S3 + S33(I)
S4 = S4 + S44(I)
S8 = S8 + S88(I)
S9 = S9 + S99(I)
S10 = S10 + S1010(I)
S11 = S11 + S111(I)
100 CONTINUE
RETURN
END

*****************************************************************************
* THIS SUBROUTINE CALCULATES THE FOLLOWING:                           *
* 1. THE RELATIVE ENERGY                                              *
* 2. THE ENERGY COEFFICIENT ALPHA                                     *
* 3. THE MOMENTUM COEFFICIENT BETA                                    *
*****************************************************************************
SUBROUTINE ENERGY
COMMON/AREA1/F1, UI, RI, DI, CO, R
COMMON/AREA2/C, CC
COMMON/AREA3/ALPHA, BETA, EE1, EE2
COMMON/AREA4/G, XN, UT, UR
COMMON/AREA5/U, DO, H0, UMO, U00, REL1
DIMENSION U (300)
U(1)=0.0
YY = 0.0
TUB = 0.0
TUBM=0.0
DY = .05
TUDQ=0.0
N=H0/DY+1.
DO 100 I = 2, N
DY = H0/(N-1)
YY = YY + DY
UT = U00+UMO
IF(YY.LT.DO) U(I)=UMO*(YY/DO)**.142
IF(YY.GT.DO) U(I) = -U00+UT/EXP(4.0C*((YY-DO)/(H0-DO))**2)
UDD = (U(I-1)+U(I))/2.
UD = UDD*UDD*DY
TUD = TUD + UD
UM = UDD*UDD*DY
TUBM = TUBM + UMM
UDD= UDD*DY
TUDQ =TUDQ+UDD
100 CONTINUE
ALPHA=TUD*HO*HO
BETA = TUDM*HO

286
VA = UI/HO
IF (RI.NE.0) VA = VA*RI/DI/R
HV = ALPHA*VA*VA/2/G
EE1 = UI*UI/2/G+DI
EE2 = HV+HO*DI
DE = EE1-EE2
REL = EE2/EE1
XM = 1.94*UI*UI/HO*DI*BETA + 31.2*(HO*DI)**2
RETURN
END

C
C ******************************************************************************
C * THIS SUBROUTINE CALCULATES THE MATRIX INVERSE  * 
C ******************************************************************************
SUBROUTINEMINVRSA, IA, MA, DETA, IER, IR, IC
DIMENSION IA, IA, IR(MA), IC(MA)
IER = 0
DO11 I = 1, MA
IR(I) = 0
DO8 IC(I) = 0
DO4 DETA = 1.0
DO123 J, K, L = 1, MA
DO8 I = 1, MA
IF (K.EQ.I) GOTO10
P0V1 = A(K, J)
DO9 K = 1, MA
IF (L.EQ.I) GOTO11
A(K, L) = A(K, L) - PIV1*A(I, L)
A(K, J) = PIV1
CONTINUE
P0V1 = A(I, J)
DO11 K = 1, MA
A(K, J) = -PIV*A(K, J)
A(I, J) = PIV1
CONTINUE
DO16 I = 1, MA
K = IC(I)
M = IR(I)
IF (K.EQ.I) GOTO16
DETA = -DETA
DO14 L = 1, MA
TEMP = A(K, L)
A(K, L) = A(I, L)
A(I, L) = TEMP
DO15 L = 1, MA
TEMP=A(L,M)
A(L,M)=A(L,I)
15 A(L,I)=TEMP
IC(M)=K
IR(K)=M
16 CONTINUE
RETURN
17 IER=1
RETURN
END
SUBROUTINESUBMXS(A,IA,JA,MA,NA,IR,IC,I,J)
DIMENSIONA(IA,JA),IR(NA),IC(NA)
I=0
J=0
TEST=0.0
DO5K=1,MA
IF(IR(K).NE.0)GOTO5
DO4L=1,NA
IF(IC(L).NE.0)GOTO4
X=ABS(A(K,L))
IF(X.LT.TEST)GOTO4
I=K
J=L
TEST=X
5 CONTINUE
CONTINUE
RETURN
END

********************************************************************
* SUBROUTINE RUNGE-KUTTA METHOD
********************************************************************
EXTERNALVECTOR
REAL*DY(Y(25),Y(25),W(25),G(6,25),Z(25),T,T1,H,BI,A(6,5)/.D0,.5D0,1.875D0,2*D0,.1428571428571429*2*D0,.625D0,0.D0,-1.875D0,.571
*42857142857143*0.D0,.5D0,.375D0,.8571428571428571*4*0.D0,.5625D0,
*-1.7142857142857143*0.D0+1.142857142857143/B(6)/.7777777777777777
*8B-1*0.D0,.3555555555555555,.333333333333333,.3555555555555555555
*7777777777778D-1/C(6)/.D0,.5D0,.25D0,.5D0,.75D0,1.D0/
T1=T0
H=H0
T=T1
CALLVECTOR(T,Y,W,N)
BI=B(1)
D02I=1,N
G(I,T)=H*W(I)
2 DY(I)=G(I,T)*BI
D03I=2,6
IM=I-1
T=T1+C(I)*H
D04K=1,N
Z(K)=Y(K)
D04J=1,IM
4 Z(K)=Z(K)+A(I,J)*G(J,K)
CALLVECTOR(T,Z,W,N)
BI=B(I)
DO3J=1,N
G(I,J)=H*W(J)
3
DY(J)=DY(J)+G(I,J)*BI
DO5I=1,N
5
Y(I)=Y(I)+DY(I)
T0=T1+H
RETURN
END

/*
//GO.PLOTTER DD UNIT=PLOTTER
//GO.SYSIN DD *
1
3.25  .042   6.
26  S.W. OF HYDRAULIC JUMP
20   X/DI
20   Y/DI
26  H.G. OF HYDRAULIC JUMP
20   X/DI
20   H/DI
20  R.E.LOSS
20   X/DI
20   D/E1
20  ALPHA
20   X/DI
20  ALPHA
20  BETA
20   X/DI
20  BETA
25GROWTH OF INNER LAYER
20   X/DI
20 DD/DI
29DECAY OF MAXIMUM VELOCITY
20   X/DI
20 UM/UI
20SURFACE VELOCITY
20   X/DI
20 US/UI
*/

C
<table>
<thead>
<tr>
<th>K</th>
<th>R1</th>
<th>H1</th>
<th>R2</th>
<th>H2</th>
<th>R3</th>
<th>H3</th>
<th>Y1</th>
<th>Y2</th>
<th>US</th>
<th>ALFA</th>
<th>BETA</th>
<th>LE2</th>
<th>CF1</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.500</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>3.500</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>5.000</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>7.500</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>9.000</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>10.500</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>12.000</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>13.500</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
<tr>
<td>15.000</td>
<td>2.430</td>
<td>0.596</td>
<td>0.402</td>
<td>0.259</td>
<td>2.425</td>
<td>-0.442</td>
<td>3.651</td>
<td>2.661</td>
<td>0.645</td>
<td>0.002</td>
<td>0.048</td>
<td>0.293</td>
<td>0.062</td>
<td>0.034</td>
</tr>
</tbody>
</table>

**COMPUTER PROGRAM OUTPUT**

**THE BOUNDARY CONDITION**

\[ K_1 = 0.000 \quad R_1 = 0.042 \quad F_1 = 6.000 \quad H_1 = 6.978 \quad H_2 = 0.142 \quad H_3 = 0.021 \]

**THE INITIAL CONDITION**

\[ K_0 = 0.160 \quad R_0 = 0.077 \quad H_0 = 0.021 \quad U_0 = 3.549 \quad U_0 = 6.978 \quad H_0 = 0.021 \]

**DIMENSIONLESS INITIAL CONDITION**

\[ K_0 = 4.000 \quad R_0 = 1.471 \quad H_0 = 0.500 \quad U_0 = 0.597 \quad U_0 = 1.000 \quad H_0 = 0.500 \]
APPENDIX G

REFERENCES
REFERENCES


47. Rajaratnam, N., The Forced Hydraulic Jump, Water Power 16,
1964, 14-19 and 61-65.


67. Scott-Moncrieff, A., Behaviour of Water Jet in a Diverging
    Shallow Open Channel, Fifth Australian Conference on
    Hydraulics and Fluid Mechanics, Christ Church, New Zealand,

68. Sigalla, A., Measurements of Skin Friction in a Plane
    Turbulent Wall Jet, Journal of the Royal Aeronautical

69. Sharma, Hari, R., Air Entrainment in High Head Gated
    Conduits, ASCE Journal, Hydraulic Division, V. 102 N 11,

70. Shames, I., Mechanics of Fluids, McGraw-Hill Book Company,

71. Silvester, R., Hydraulic Jump in all Shapes of Horizontal
    90, HY 1, 1964, pp. 23-55.

72. Straub, L.G., and Anderson, A.G., Experiments of Self-

73. Straub, L.G. and Anderson, A.G., Self-Aerated Flow in
    Open Channels, Transactions, ASCE, Vol. 125, Part I,

74. Squire, H.B., Radial Jets, in: H. Goerter and W. Tollmien
    (Editors), 50 Jahre Grenzschichtforschung Vieweg,
    Graunschweig, 1955, pp. 47-54.

75. U.S. Army Corps of Engineers, Engineering and Design,
    Structural Design of Spillways and Outlet Works, Engineering

76. U.S. Army Corps of Engineers, Hydraulic Design Criteria
Hydraulic Design Chart 050-1, 1964.

77. U.S. Bureau of Reclamation, Research Studies on Stilling
Basins, Energy Dissipators and Associated Appurtenances,
Hydraulic Laboratory Report No. Hyd-399, June 1, 1955.

78. U.S. Department of Interior Bureau of Reclamation,

79. Vennard, K. John and Street, Robert, Elementary Fluid

pp. 481-499.

81. Wilson, E., Turner, A., Boundary Layer Effects on Hydraulic

82. Wylie, C., Differential Equations, McGraw-Hill Book Co.,
APPENDIX H

NOMENCLATURE
\[ \text{A} = \text{Coefficient function of the initial Froude number and entrained air fraction} \]

\[ A_\ast = Y_1 (r_0 - 1)B_\ast \]

\[ A_1, A_2, A_3, A_4 = \text{Integral equations coefficients for the free radial hydraulic jump} \]

\[ A_1', A_2', A_3', A_4' = \text{Integral equations coefficients for the free rectangular hydraulic jump} \]

\[ \overline{A}_1 = \text{Coefficient function of the initial Froude number} \]

\[ \overline{A}_2 = \text{Coefficient function of the initial Froude number} \]

\[ A_{01}, A_{02} = \text{Coefficient function of the initial Froude number} \]

\[ [A] = \text{Coefficient matrix} \]

\[ \text{A}_p, \text{A}_y = \text{Coefficient function of the initial Froude number and the inlet depth factor} \]

\[ a = \text{Radius of the impinging jet} \]

\[ B = \text{Coefficient function of the initial Froude number and entrained air fraction} \]

\[ B = \text{The ratio of the parameter for bubble transfer to that for momentum transfer.} \]

\[ (\text{Chapter II}) \]

\[ B\ast = \text{Correction factor for the side pressure force} \]

\[ B_{01}, B_{02} = \text{Coefficient function of the initial Froude number} \]

\[ B_1, B_2, B_3, B_4 = \text{Integral equations coefficients for the free radial hydraulic jump} \]
\( B_1', B_2', B_3', B_4' \) = Integral equations coefficients for the free rectangular hydraulic jump

\( B_Y', B_p \) = Coefficient function of the initial Froude number and the inlet depth factor

\( C_* \) = \( \sqrt{-8c} \)

\( \bar{C}_* \) = Air concentration at \( y \)

\( C_o \) = Average air concentration \( = (h'-h)/h' \)

\( C_1', C_2', C_3', C_4' \) = Integral equations coefficients for the free radial hydraulic jump

\( C'_o \) = Air concentration at \( (h'-h)/h \)

\( C'_1', C'_2', C'_3', C'_4' \) = Integral equations coefficients for the free rectangular Hydraulic jump

\( C' \) = Coefficient

\( C'' \) = Coefficient

\( \bar{C}_1 \) = Air concentration at \( y = \frac{y}{T}/2 \)

\( C_a \) = Air concentration at the water surface

\( C_f' \) = Skin friction coefficient

\( C_m \) = % maximum air concentration

\( C_p \) = Coefficient function of the initial Froude number and the inlet depth factor

\( C_T \) = Air concentration at the transition depth \( y = \frac{y}{T} \)

\( \bar{C}_T \) = Mean air concentration (Chapter II)

\( C_Y' \) = Coefficient function of the initial Froude number and the inlet depth factor

\( c \) = Constant \( = -0.693 \)

\( D_* \) = Constant \( = 0.048 \)
$D_1, D_2, D_3, D_4$ = Integral equations coefficients for the free radial hydraulic jump

$D_1', D_2', D_3', D_4'$ = Integral equations coefficients for the free rectangular hydraulic jump

$D_p', D_y$ = Coefficients functions of the initial Froude number and inlet depth factor

$E$ = Specific energy at distance $x$

e = Base of natural logarithms

$E_*$ = Empirical coefficient

$E_1$ = The specific energy at the beginning of the jump

$E_2$ = The specific energy at the end of the jump

$E_1', E_2', E_3', E_4'$ = Integral equation coefficient for the free radial hydraulic jump

$E_1', E_2', E_3', E_4'$ = Integral equations coefficient for the free rectangular hydraulic jump

$E_f$ = Total energy of the forward flow

$E_L$ = Energy Loss

$E_p', E_y$ = Coefficients function of the initial Froude number and inlet depth factor

$[E]$ = Coefficient Vector

$F(a')$ = Function

$F'$ = Spacing parameter

$F_1$ = Supercritical Froude number

$F_{11}$ = Entrained air supercritical Froude number

$F_P$ = Entrained air fraction
\( f'(\cdot), f''(\cdot) \) = Function

\( G' \) = Spacing parameter

\( G_x \) = Coefficient function of the initial Froude number and the inlet depth factor

\( G_o \) = Gate opening

\( g \) = Acceleration due to gravity

\( g(\alpha') \) = Function

\( H_L \) = Head loss

\( h \) = Effective water depth at distance \( x \)

\( h(x) \) = Water depth at distance \( x \)

\( \lambda \) = Mixing length

\( \lambda_b \) = Damped mixing length

\( L_J \) = Jump length

\( L_r \) = Roller length

\( M \) = Momentum rate

\( m' \) = Coefficient

\( m'' \) = Coefficient

\( N \) = Coefficient, function of \( U_b, U_m, U_s \) and \( U_l \)

\( n \) = Exponent in Blasius's equation

\( P \) = Total pressure force

\( p \) = Hydrostatic pressure

\( P_1 \) = Pressure force at the beginning of the jump

\( P_2 \) = Pressure force at the end of the jump

\( P_f \) = Integrated bed shear

\( P_s \) = Side pressure force

\( Q \) = Total discharge
\( q \) = Discharge/unit length
\( q_i \) = Initial discharge/unit length
\( q_f \) = Forward discharge/unit length
\( R \) = \((r - r_i)/(r_2 - r_1)\)
\( R_i \) = Richardson number
\( R_N \) = Reynolds number
\( R_{NP} \) = Jet Reynolds number
\( R_* \) = Correlation coefficient
\( r \) = Radial distance from jump centre
\( r_o \) = Radius ratio, \( r_2/r_1 \)
\( r_i \) = Radius to beginning of the jump
\( r_2 \) = Radius to the end of the jump
\( S \) = Submergence factor = \((y_4 - y_2)/y_2\)
\( S(KC, C') \) = Series
\( TEL \) = Total energy line
\( U_J \) = Jet velocity
\( U_o \) = Velocity at \( y = \infty \)
\( U_1 \) = Average velocity at the beginning of the jump
\( U_2 \) = Average velocity at the end of the jump
\( U_m \) = Maximum velocity
\( U_s \) = Surface velocity
\( u \) = The main time velocity component in the x-direction
\( u' \) = Turbulent fluctuation in the x-direction
\( \overline{u'v'} \) = Turbulent shear stress
\( \overline{u} \) = Velocity used in aerated flow
$u_b$ = The rising air bubble velocity

$U_t$ = $U_o + U_m$

$U_{\gamma*}$ = Velocity at depth $y = y_*$

$u_*$ = Shear velocity

$v$ = The mean time velocity component in the $y$-direction

$v'$ = Turbulent fluctuation in the $y$-direction

$v_r$ = The mean time velocity component in the $r$-direction

$v'_r$ = The velocity fluctuations in the $r$-direction

$v_z$ = The mean time velocity component in the $z$-direction

$v'_z$ = The velocity fluctuations in the $z$-direction

$v_\phi$ = The mean time velocity component in the $\phi$-direction

$v'_\phi$ = The velocity fluctuations in the $\phi$-direction

$V_h$ = The mean time velocity in the $y$-direction at $y = h$

$V_\delta$ = The mean time velocity component in the $y$-direction at $y = \delta$

$V_{\gamma*}$ = The mean time velocity in the $y$-direction at $y = y_*$

$x$ = Distance from the beginning of the jump

$y$ = $(y - y_1)/(y_2 - y_1)$

$y$ = Water depth at distance $x$

$y_*$ = $\sqrt{1/8c} (h - \delta) + \delta$
\( y' \) = Depth above the surface of the lower region

\( \bar{y} \) = Representative depth, \( \sqrt{y_3 \cdot y_4} \)

\( \bar{y}_1 \) = Mean depth of flow

\( y_o \) = Sequent depth ratio for free jump = \( y_2/y_1 \)

\( y'_o \) = Sequent depth ratio for submerged jump = \( y'_4/y_1 \)

\( y_1 \) = Supercritical water depth

\( y_2 \) = Water depth at the end of the jump (free jump)

\( y_3 \) = Backed up depth of the submerged jump

\( y_4 \) = Tail water depth of the submerged jump

\( y_p \) = Dimensionless pressure head

\( y_r \) = Water depth at the end of the roller

\( y_T \) = Depth of lower region

\( z \) = \( U_b/\beta^* \cdot k_U* \)

\( a \) = Energy coefficient

\( a' \) = Parameter; angle with the horizontal = \( x/L_r \)

\( a_1 \) = Energy coefficient at the beginning of the jump

\( a_2 \) = Energy coefficient at the end of the jump

\( \Gamma \) = Experimental coefficient

\( \beta \) = Momentum coefficient

\( \beta^* \) = The ratio of parameter for bubble transfer to that for the momentum transfer

\( \beta_1 \) = Momentum coefficient at the beginning of the jump
β₂ = Momentum coefficient at the end of the jump
γ = Specific weight of fluid
ΔE = Energy loss
ΔP = Increase in the pressure force
δ = Boundary layer thickness (inner layer thickness)
δ₁ = Length scale for wall jet
δ₂ = Depth of forward flow
δ₃ = Depth of backward flow
ε = Correction factor for the bed shear
η = Spacing parameter
θ = The total angle of divergence
κ = Von Karman's constant
λ = Non-dimensional distance = x/y₂
μ = Coefficient of dynamic viscosity
ν = Coefficient of kinematic viscosity
ρ = Mean density of fluid
τ = Total shear stress
τ₀ = Shear stress on bed
τ₁ = Viscous shear stress
τ₂ = Turbulent shear stress
τ₃ = Turbulent shear stress for single phase flow
τₜ₀ = Shear stress at depth y = yₜ
φ = y₂/yₜ
ψ = Initial depth factor = y₃/y₁
VITA AUCTORIS

1946    Born on October 4th in Nekla Island, Egypt
1964    Matriculated from Elnokrashe High School
        Cairo, Egypt
1969    Graduated with a Bachelor of Science in Civil
        Engineering, Ain Shams University, Cairo, Egypt
1969    Appointed as an instructor of Civil Engineering
        at Ain Shams University, Cairo, Egypt
1973    Obtained a Master of Science in Civil Engineering,
        Ain Shams University, Cairo, Egypt
1976    Obtained a Master of Science in Petroleum Engineering,
        University of Pittsburgh, Pittsburgh, Pennsylvania
1976    Registered as a Ph.D. candidate in Civil
        Engineering at the University of Windsor,
        Windsor, Ontario