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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RÉCUÉ
TRANSITION OF GRAVITY TO SURCHARGED FLOW IN SEWERS

A Dissertation submitted to the Faculty of Graduate Studies through the Department of Civil Engineering in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy at the University of Windsor

by

Mostafa Ahmed Mohamed Hamam
B.Sc. (Honour), M.A.Sc.

Windsor, Ontario
Canada
1982
IN THE NAME OF ALLAH, THE COMPASSIONATE, THE MERCIFUL

Praise is only for Allah, the Lord of the Universe, the All-Compassionate, the All-Merciful, the Master of the Day of Judgement.

Thee alone we worship and to Thee alone we pray for help. Show us the straight way, the way of those whom Thou has blessed; who have not incurred Thy wrath, nor gone astray.

"The Opening Chapter of the Holy Quran"
"To Amal, Marwa and Omar
ABSTRACT

This Thesis deals with transients that occur when gravity flow is suddenly changed to pressure flow by the occurrence of a surge in the form of a travelling hydraulic jump in the line. The pressure head fluctuations associated with this transient have been studied. The transition is complicated by the mixture of air and water in the pipe. The transients in this two-phase air-water flow have been considered in this study. Some of the factors affecting these pressure transients are: pipe size, pipe shape, flow velocity, Froude number, relative depth of flow, alignment of the pipe, pipe material, venting arrangements and boundary conditions such as pumps, interceptors, and drop pipes.

A mathematical model is developed to describe the mechanics of surcharging of a sewer; the model predicts the surge velocity, the surcharge at the surge front relative to sewer crown, the water level in the sump well or manhole, the velocity of the water surface in the sump well, the distance travelled by the surge front and, the transient pressure rise in the air-water two-phase flow.

The mathematical model results are compared with the experimental results from the present study. The mathema-
tical model results are in general agreement with the experimental data but the experimental pressure fluctuations were approximately 50% of the theoretical predictions. The random nature of the instability of the air-water interface and the extent of the blockage contributed to disagreement between the theory and the model prediction.
ACKNOWLEDGEMENTS

All Praise is due to Allah, the Sustainer of the heavens and the earth, Who gave me the strength and the guidance to complete this work.

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CHAPTER I

INTRODUCTION

1.1 Objective

The purpose of this dissertation is to investigate the mechanics of the transition of open-channel to surcharged flow in sewers and to develop a mathematical model to simulate this transition. In order to achieve this objective an experimental study was conducted for pipes with square and circular cross-sections to verify the mathematical model.

1.2 Definition of the Problem

Storm, combined and sanitary sewers are usually designed for free surface or gravity flow; however, under certain conditions, such as during severe floods, the system may become partially or completely surcharged. An examination of the hydraulic characteristics of pipes shows that, near the crown of the pipe, the sewer capacity abruptly decreases. Such a rapid change in the flow and velocity will cause transient pressures in the system. The transient phenomenon is similar to waterhammer in
forcemains but is strongly influenced by the two-phase character of the flow at the transition. Figures 1.1a and 1.1b show that the transition of gravity to surcharged flow in sewers can take place in six stages.

Stage (1): Gravity Flow - Under normal conditions the flow in sewers is a gravity flow but in some cases there may be some air flow in the system. The relative velocity between the water phase and the air phase may produce a wavy surface.

Stage (2): Surge Formation - If, for any reason, a surge is produced in a sewer line with open-channel flow, that surge will propagate with a speed of $V_w$ pushing the air in front of it and producing either a co-current or counter-current stratified air-water flow. The relative velocity between the water phase and the air phase may produce surface waves.

Stage (3): Formation of Interface Instability - When the relative velocity of the air ($|\dot{V}_a| + |V^+_1|$) - for counter-current air-water flow - reaches the interface instability limit at which the waves become unstable, that is the wave height will increase rapidly until it touches the crown of the sewer.

Stage (4): Transition to Surcharged Flow - Interface instability results in a complete blockage of the air flow
and a pressure wave or waterhammer phenomenon occurs in the two-phase mixture which may be classified in one of three regimes: Stratified Flow, Bubble Flow, or Slug Flow.

Stage (5): Release of Trapped Air - When the trapped air in the pipe reaches the upstream end of the pipe, the high pressure inside the air bubble is suddenly reduced to atmospheric pressure, the release of this trapped air may produce a transient pressure in the system.

Stage (6): Surcharged Flow - After the release of all trapped air in the pipe, the pipe becomes full of water, i.e., a short period of time may be required for the decay of transients after which the flow in the pipe will be a surcharged with a quasi-steady hydraulic grade line.

Abrupt changes from gravity to surcharged flow may result from the following:

(a) failure of intermediate or outfall pump;
(b) inadequate pumping capacity in a pumped sewer system;
(c) unsteady inflows to the system, (e.g., flows from forcemains or severe storms);
(d) improper vertical or horizontal alignment;
(e) the presence of drop-pipes;
(f) the presence of interceptors or other control devices;
(g) blockage of the line, e.g., the clogging of an outlet screen, the collapse of a portion of the line; and
(h) presence of inverted siphons.

**Typical Examples of Occurrence:**

The following are some typical examples where the transition of gravity to surcharged flow occurs and may be accompanied by excessive surcharge causing damage to the system, e.g.: basement flooding, popping-off of manhole covers, and bursting or collapsing of a part of the sewer line.

**Example (1):**

Figure 1.2 shows a surge and a possible interface instability which may take place in a pumped sewer system due to pump failure or inadequate pump capacity. In this example the water level in the sump well (S) rises causing a surge to travel upstream in the sewer with a velocity ($V_w$) pushing the air column ahead of it with almost the same velocity ($V_a \approx V_w$), and creating a counter-current air-water flow in the sewer. If the relative velocity of air ($|\dot{V}_a| + |\dot{V}_1|$) - for counter-current air-water flow - exceeds the interface instability velocity, $V_I$ (see Chapter IV for details) the interface waves may increase in height until the sewer is blocked. After
blockage occurs, the trapped air and water will cause pressure waves in the sewer. These pressure transients associated with the transition of gravity to surcharged flow may be very large and hazardous.

Example (2):

Figure 1.3 shows a typical example of transient pressure phenomenon which was documented at the City of Hamilton, Ontario, Canada (5). A large sewer (10 ft. by 10 ft. =3 m by 3 m) of bed slope $S_o = 0.02$ with a stream interceptor weir and an upstream drop-pipe experienced severe pressure shocks during high flows. The pressure rise was sufficient to blow off a welded manhole cover and cause basement flooding. A parallel sewer was built for the house connections in the area to prevent the basement flooding which had occurred several times.

Example (3):

Figure 1.4 shows another case of pressure transients which was observed in the Town of Amherstburg, Ontario, Canada, where the discharge from a pumping station into another sewer caused a surge and resulted in the blowing off of a manhole cover several blocks away.

In addition to the previous examples there are several other cities in Canada that have recorded problems with manhole covers popping off during severe storms (24).
The author believes that many of these occurrences may have been caused by pressure transients associated with the transition of gravity to surcharged flow.

The writer feels that the mathematical model along with the design information based on the experimental study, which is presented in this dissertation should improve the design of sewer systems.

This dissertation deals with transients that occur when gravity flow is suddenly changed to pressure flow by the occurrence of a surge in the line.

1.3 Motivation

The simulation programmes currently available (e.g., 12, 24, 35 and 42) for sewers are adequate for unsteady gravity flow and in some cases (18 and 35) for single phase surcharged flow. The existing simulations assume that there is a smooth transition from gravity to surcharged flow conditions and the air phase is not important. Because of the transition may not be smooth, abrupt changes may result in the system. These changes are expected to be similar to waterhammer in two-phase air-water flow.

To the best of the writer's knowledge and as noted by Yen (43) in March 1980, the available models for analyzing storm sewer systems are either purely open-channel flow or completely pressurized conduit flow
models. No models are currently available that accurately simulate the transition from open-channel to pressurized flow and that was the essential motivation behind this study.

This thesis presents a numerical model for the transition of open-channel to surcharged flow in sewers which can be used to check the design of a sewer system against the pressure transients due to this transition. The thesis also provides design information based on the experimental study for the transition of gravity to surcharged flow in sewers.

1.4 The Approach in General

In this project the transition of open-channel to surcharged flow in sewers is investigated experimentally under different flow conditions for conduits with rectangular and circular cross-sections.

The wave theories were used to study the wave formation for the open-channel flow stage. The incompressible flow continuity and momentum equations were used to predict the surge velocity and pressure rise for the surge formation stage. The wave theory of Helmholtz (17) was used to predict the interface instability velocities. The equations of continuity and momentum for compressible flow in pipes were solved using the
method of characteristics to study the pressure transients. A numerical solution using Runge-Kutta method was used to analyze the surge movement throughout the pipe. The escape of entrained air in the system causes a rapid change in the boundary conditions, e.g., head and velocity which in turn introduce additional transients in the sewer. The transient flow equations for full pipe flow (33) can be used for waterhammer analysis if necessary.

The mathematical model is developed for a single rectangular or circular conduit connected to a sump well. To verify the mathematical model the results are compared with the experimental data of the present study for the transition of open-channel to surcharged flow in sewers.

The experimental investigation was carried out in the hydraulic laboratory of the University of Windsor, Windsor, Ontario, during the period 1977-1981. The IBM 370/3031 computer facilities at the University of Windsor were used in executing all the computations in the mathematical model.
CHAPTER II

LITERATURE REVIEW

2.1 Introduction

The nature of the transition from gravity to surcharged flow is one of the least understood aspects of flow in sewers. To the best of the writer's knowledge, none of the available literature has adequately covered this phenomenon and the design and/or the simulation programmes currently available (12, 24, 35 and 42) do not include the nature of the transition which can be abrupt rather than smooth. A smooth transition from gravity to surcharged flow is the assumption which is commonly used in the available literature.

In this chapter the available literature related to the subject of this study, is presented. The review includes literature on the six stages introduced in Chapter I, of the transition. The review includes literature on:

(a) gravity flow in sewers;
(b) surge formation;
(c) interface instability;
(d) two-phase flow in sewers;
(e) surcharged flow in sewers, and
(f) relevant studies.

2.2 Gravity Flow in Sewers

The gravity flow in sewers may be treated as open-channel flow with a free surface subjected to atmospheric pressure, which may be classified as follows:

(a) steady uniform flow;
(b) steady gradually varied flow;
(c) steady rapidly varied flow, e.g., with a free hydraulic jump;
(d) gradually varied unsteady flow; and
(e) rapidly varied unsteady flow, e.g., with a travelling hydraulic jump.

Many references (e.g., Chow (8) and Streeter and Wylie (32)) have dealt with gravity flow in channels. Many of the available storm sewer design and/or simulation models can handle gravity (open-channel) flow with various degrees of sophistication, e.g., SWMM (35).

It is a common practice to design sewers with a free water surface (open-channel flow), laid to slopes which will ensure adequate self-cleansing velocities. Sometimes a sewer must be constructed as a pressure conduit with the pipe depressed below the hydraulic grade line, but pressure sewers should be avoided wherever possible,
for many reasons, e.g., the difficulty of maintaining scouring velocities over the wide range of discharge encountered.

The hydraulics of sewers is discussed in detail in manuals for Design and Construction of Sanitary and Storm Sewers, e.g., the manual prepared by a joint committee of ASCE and WPCF (1).

Continuity and energy equations are commonly used for flow and energy grade lines calculations in sewers. Friction formulae, such as Kutter and Manning equations, are commonly applicable to sewers of all shapes flowing either full or partly full. Many designers provide a factor of safety in the design of sanitary sewers such that small sewers up to 12 to 15 in. (305 mm to 381 mm) in diameter will not flow more than one-half full at the peak design flow. Larger sanitary sewers may be designed to carry flows from more than half full at the peak design flow, up to about seven-tenths of the diameter for sewers of 30 in. (762 mm) and larger in size. Storm sewers and combined sewers may also be designed to be partly full at the design peak flow, but probably the most common practice is to design these sewers to be full at the design flow.

In 1978, Mussalli (28) investigated the size determination of partly full conduits. He presented design criteria for sizing partly-full conduits to allow for
surging and air bulking. He studied the sealing, or the transition from free-surface to pipe flow, in the horizontal or inclined conduits. This sealing caused structural vibrations because of unsteady plug flow conditions and undesirable variation of the water flow. Sealing is mostly related to supercritical partly full flow which occurs in the conduits of shaft spillways, drop structures, drainage, and blowdown structure. He concludes that sealing of the horizontal or inclined conduits depends on the Froude number of the flow. To maintain partly full flow in the conduit, more space above the water flow area is needed as Froude number increases. Additional space is required for wavy surface flow or for aerated flow. The proper choice of deflector and bend is an important factor (especially with short-tube control) in determining the depth of flow in the conduit. His results also show that with short-tube control, air entrainment into the water flow hastens the transition to pipe control and that ventilating from the upstream of the conduit delays the transition. With weir control, air entrainment dampens the wave action by forming a frothy surface and delays the transition to pipe control. At high air concentrations, the transition to pipe flow is smooth in contrast to the conditions at low air concentrations, where the transition is sudden and violent. Figure 2.1 shows a typical pressure and piezometric head at roof of a tested conduit prior to and at sealing.
2.3 Surge Formation

2.3.1 Surges in Open Channel Flow

Introduction

Bores or sharp-fronted surges are not infrequent occurrences in canals and rivers. Relatively rapid increases in flow through hydraulic turbines can cause sudden increases in the depth of flow, occasionally resulting in a surge or bore in the tailrace. Sudden reduction in flow through a turbine can create a surge or bore that propagates upstream in a headrace. Failure of hydraulic structure can produce catastrophic results in the form of a huge wall of water that rushes downstream. The tidal bore is known to occur in a few estuaries throughout the world because of estuary conditions and high local tides. In any case the question of simulating the propagation of a bore or surge in a channel is of high engineering significance.

Chow (8) gave an equation for the surge velocity of rapidly varied uniformly progressive flow shown in Fig. 2.2, the general form can be written as:

\[
V_w = C \pm V_1
\]

(2.1)

where

\[
C = \left[ \frac{(A_2 \bar{Y}_2 - A_1 \bar{Y}_1)g}{A_1 (1 - A_1/A_2)} \right]^{1/2}
\]

(2.2)
\[ v_w = \text{surge velocity}; \]

\[ C = \text{the velocity of the wave w.r.t. the velocity of the initial flow (celerity)}; \]

\[ \bar{y}_1 = \text{distance between the pressure centre and the water surface at section 1}; \]

\[ \bar{y}_2 = \text{distance between the pressure centre and the water surface at section 2}; \]

\[ A_1 = \text{area of flow at section 1}; \]

\[ A_2 = \text{area of flow at section 2}; \text{ and} \]

\[ V_1 = \text{velocity of flow at section 1 (initial flow)}. \]

Figure 2.3 shows four types of rapidly varied uniformly progressive flows. Equations 2.1 and 2.2 can be used to describe a moving hydraulic jump. In Fig. 2.3, the initial and the final stages of a passing surge are assumed to be uniform. Such surges occur frequently in channels of small slope. In inclined channels, four types of surges can occur as shown in Fig. 2.4.

Positive surges are described by

\[
\sqrt{\frac{V_1 - V_2}{g}} = \frac{\bar{y}_1 + \bar{y}_2}{C(\frac{1}{\bar{y}_1})} \quad (2.3)
\]

where,

\[ y_1 \text{ and } y_2 = \text{flow depths at sections 1 and 2 respectively}; \]
$V_1$ and $V_2$ = flow velocities at sections 1 and 2 respectively; and

$h$ = surge height.

When the height of surge is small compared with the depth of flow ($y_1 = y_2$) Equation 2.3 may be written as

$$V_1 - V_2 = \pm \frac{hg}{C} \quad (2.4)$$

Chow presents several cases of positive surges such as:

1. surge due to sudden stoppage of flow,
2. meeting of two surges, and
3. surge crossing a step.

For negative surges of moderate or small height compared with the depth of flow, the equations derived for a positive surge can be applied to approximate the propagation of the negative surge. Analysis of a negative surge with a relatively large height is also given.

Chow presented a brief study of the following:

1. surges in power canals,
2. surges in navigation canals,
3. surges through channel transitions,
4. surges at channel junctions, and
5. pulsating flow.

2.3.2 Surfges in Closed Conduits With Full Pipe Flow

Streeter and Wylie (33) presented a description of the
waterhammer phenomenon and methods of computations. For example, the method of characteristics is recommended for general use in computer solutions. They discussed the different means of surge and waterhammer control such as using surge tanks on the long pipelines, or supplying a quick-opening bypass valve.

The waterhammer in sewage forcemains due to power failure in the sewage pumping station is discussed by Metcalf and Eddy Inc. (26). The velocity of the pressure wave is given by

\[ a = \frac{1440}{1 + \frac{K_D}{e}} \]  \hspace{1cm} (2.5)

where,

- \( a \) = velocity of pressure wave, m/s,
- \( D_p \) = pipe diameter, m,
- \( e \) = pipe-wall thickness, m,
- \( E \) = modulus of elasticity of pipe material MPa, and
- \( K \) = bulk modulus of water, MPa.

They state that the values of "a" normally lie in the range from 1067 to 1250 m/s, for different types of sewage pipes. Swing check valves with outside lever and weight set to assist closure, are recommended for small and medium size pumps. Spring loaded check valves may also be used. In large pumping stations, check valves should
be positively controlled using cone valves or butterfly valves.

Many of the mathematical studies for the cavitation in pipelines due to waterhammer (3, 10, 11 and 31) have not been verified by experiments.

The studies of Brown (6) are the most interesting, because his theory was verified by prototype experiments. He studied the problem of water column separation at a discrete point. Usual waterhammer theory and experiment disagreed, and consequently small gas pockets were introduced at the grid points of the graphical computational scheme. In this way the overall celerity was decreased, especially at low pressures. By introducing certain assumptions on the total amount of gas in the pockets it was possible to obtain a close agreement between theory and experiment.

In 1971, Kalkwijk and Kranenburg (20) studied the cavitation in horizontal pipelines due to waterhammer. They presented a simplified computational procedure in which the idea of a single long cavity is abandoned. Their computations were in good agreement with their experimental results. Two attempts have been made by them to describe the phenomena of cavitation as a consequence of waterhammer. The first approach, based on the behaviour of nuclei or gas bubbles, failed at the
point where the radii of the bubbles exceed a critical value. At this size the bubbles become unstable and apparently the characteristics become imaginary. From this critical point the equations can no longer describe the process. The advantage of this theoretical approach was that it showed a considerable decrease of the velocity of propagation at low pressures. The second approach, being in fact a crude schematization of the problem, distinguished between regions with and without cavitation. In the first region the celerity was reduced to zero, while in the other section the undisturbed celerity was supposed to hold. Their experiments showed that this method gives a reasonable description of the overall process. Problems arose when it appeared that dispersion of the pressure wave changed the position and the time of occurrence of the cavitation region and the height of the shock. After the extensive cavitation had disappeared the remaining pressure waves were much more pronounced in the computation than they were in the experiment. Figure 2.5 shows the measured and computed pressures at some locations along a 200 m horizontal polyvinyl chloride pipeline (D_p = 81 mm) following the closure of a piston valve at the upstream end of the pipeline.

In 1971, Burton and Nelson (7) studied surges and
air entrainment in pipelines. They reported on experiences of the Bureau of Reclamation in the development and design of water conveyance systems. There were two major closed conduit systems that utilize upstream control have been constructed. These are the Coachella Valley, Distribution System in Southern California and the Canadian River Project in Northwestern Texas. The writers explored the operational problems encountered in these two systems, laboratory and field tests conducted both before and after construction, and the solutions which were adopted for surge and air entrainment problems during design and construction.

They presented a discussion of future systems of the same general type, including methods of analysis for surging and air entrainment, and types of structures which may be utilized.

In 1980, Watt, Hobbs and Boldy (38) studied the hydraulic transients following rapid valve closure at the downstream end of a 15 m pipeline \( (D_p=50 \text{ mm}) \). They compared their experimental results with two methods of digital computer simulation. The first technique is based on the well-known method of characteristics and the solution provides a good correlation with the experimental results even though end conditions placed a lower limit on the size of the time step. The second
method adopts the finite element approach without introducing explicitly prescribed end conditions. The solution using the Galerkin criterion also gives good correlation with the experimental results and has the additional benefit of being stable for time steps exceeding those permissible by the method of characteristics. The comparison of their results from each method is shown alongside the experimental data in Fig. 2.6.

2.3.3 Surges in Closed Conduits With Gravity Flow

Hamam (13) investigated the surge characteristics (velocities and surcharge heads) in storm sewers of a rectangular cross-section starting with gravity flow and assuming a stable air-water interface. He also studied the effect of different ventilation arrangements on the surge characteristics.

In 1981 Hamam and McCorquodale (14) studied experimentally the surge characteristics (velocities and surcharges) in storm sewers of a rectangular cross-section due to a rapid closure of a control gate (or when very little sump storage is provided) for sewers with gravity flow. Figures 2.7 and 2.8 show their experimental results compared with the analytical values for the relative surge velocities \( \frac{V}{V_1} \) and the relative surcharge heads \( \frac{Z}{V_1^2/2g} \), respectively for various slopes. The
surges they studied are representative of the extreme case of a sewer pump failure, when very little sump storage is provided or rapid closure of a control gate. However, the travelling hydraulic jump theory was in good agreement with their experimental observations except when interface instability occurred and/or the high pressures caused leakage at the outfall gate.

In 1981, Meadows and Chestnut (25) used culvert hydraulics to explain how the open-channel flow in one portion of the system can be affected by surcharging in other portion.

2.4 Interface Instability

Introduction

In sewer systems, these may be a two-phase air-water flow as explained in Chapter I. The air-water interface instability is affected mainly by the relative velocity of the two phases, the pipe shape, and the air fraction in the system.

The stability of the interface between gas and liquid in parallel streams originated from the studies of Kelvin (22) and Helmholtz (17), in which the fluid is taken as inviscid and the velocities as constant in each region. Yih (44) solved the stability problem of parallel laminar flow with a free surface.
It is well known (19 and 27) that a high relative velocity at an air-water interface causes waves and instabilities to form. For example, the Helmholtz theory predicts instability between two infinite inviscid fluids at a relative velocity of

\[ V_H = \frac{1}{\sqrt{\frac{\rho_a}{\rho_w}}} C_o \]  

(2.6)

where

- \( \rho_w \) = density of water,
- \( \rho_a \) = density of air, and
- \( C_o \) = celerity of surface waves (e.g., due to gravity).

In 1978, Kometani and Seidel (23) investigated the stability of stratified two-phase flow using the Orr-Sommerfeld equation. The influence of the surface tension was taken into consideration. The mean velocity profile was also considered by approximating it with two quartic polynomials. The principal techniques of numerical analysis they used are the Newton-Raphson method and the Runge-Kutta method. They found the equi-stability curves using a computer. They reported that in solving the Orr-Sommerfeld equation the velocity profile of each stream must be assumed. They found that mathematically, the central issue is the eigenvalue problem.
resulting from the boundary conditions. They compared their results with the results of Kaplan (21), Wazzan (39), and Radbill (29) who obtained the eigenvalues of the Blasius boundary layer.

2.5 Two-Phase Flow in Sewers

Introduction

The air-water two-phase flow regimes expected to occur in sewers, may be one of three flow regimes: (1) bubble; (2) stratified; and (3) slug. Two-phase water-solid flow regime may be found also in sewers but generally will not cause severe complications in the form of pressure fluctuations and instabilities as the two-phase air-water flow regime does.

In 1968, Rhodes and Scott (30) studied the pressure wave phenomena in gas-liquid mixtures, including analytic predictions and experimental data of the wave front velocity or the acoustic speed in the following flow regimes: bubble, slug, stratified, annular, and droplet dispersed. Their analyses and experiments clearly demonstrate the dependence of propagation velocity on flow regime. They demonstrated the transmission characteristics of pressure waves across liquid-gas interfaces by considering the response time of an idealized slug flow mixture to a step change in pressure. Their study indicates the necessity for con-
sidering acoustic pressure propagation.

They used 51 mm (2 in.) stainless steel pipe to investigate the bubble, stratified, and slug flow regimes (these flow regimes are the expected flow regimes in sewers during the transition to full pipe flow). Quartz crystal piezo-electric transducers having frequency response characteristics of 0-200,000 Hz were used to detect the wave signals. The amplitude-time history of a pressure pulse is displayed on an oscilloscope as measured by the transducers. The average void fraction is determined by measuring the hydrostatic pressure difference at the ends of the tested section of the pipe. The pressure pulse was generated by bursting a 0.001 in. thick aluminum diaphragm.

They give the following equation to compute the propagation velocity (speed of sound) in two-phase bubble flow regime:

\[ a_{TP} = \sqrt{a^2 \left[ a^2 + \frac{a(1-a)}{\rho_g} \right]^{-1/2}} a_g \]  

(2.7)

For stratified flow regime the following equation is given:

\[ a_{TP} = a_g \sqrt{1 + \frac{(1-a)\rho_g}{a \rho_f}} \]  

(2.8)
and for slug flow regime the following equation is given:

\[ a_{TP} = \frac{a_g a_L}{\alpha_a a_L + (1 - \alpha_a) a_g} \]  \hspace{1cm} (2.9)

where,

- \( a_L \) = speed of sound in liquid,
- \( a_g \) = speed of sound in gas,
- \( \alpha_a \) = air fraction,
- \( \rho_g \) = gas density, and
- \( \rho_L \) = liquid density

Figure 2.9 shows their experimental wave front propagation velocities for the three flow regimes investigated: bubble, stratified, and slug flow together with the analytical predictions using Equations 2.7, 2.8 and 2.9 respectively.

In 1971, Vermeuler and Ryan (36) studied the slug flow in the air-water system near atmospheric pressure in a one-half inch diameter tube. Measurement of flow characteristics, pressure gradient, pressure fluctuations, slug velocity and frequency were made for horizontal and inclined flow. A semi-empirical theory based on a simple model was developed. Pressure losses calculated from this theory require a prior knowledge of the slugging frequency. A comparison of the prediction of the theory and two published correlations shows
that the proposed theory gives better agreement with the data.

Wallis (37) gave the following equation for the motion of the dynamic waves in incompressible two-component flow in a constant area duct (see Fig. 2.10):

$$ u = V_0 \pm \bar{C} $$  \hspace{1cm} (2.10)

where,

$$ \bar{C} = \pm \sqrt{\frac{-(V_1 - V_2)^2}{y_1/\rho_2 H + (1 - y_1/H)/\rho_1 - f V_0}}^{1/2} x \frac{\rho_1}{(1 - y_1/H) + y_1^{1/2}} \left( 1 - y_1/H \right) \frac{\rho_2 H}{y_1} \right)^{1/2} $$  \hspace{1cm} (2.11)

$$ V_0 = \frac{V_1 \rho_1 / (1 - y_1/H) + V_2 \rho_2 H / y_1}{\rho_1 / (1 - y_1/H) + \rho_2 H / y_1} $$  \hspace{1cm} (2.12)

where,

- $V_1, V_2$ = velocities of components 1 and 2 respectively;
- $\rho_1, \rho_2$ = densities of components 1 and 2 respectively;
- $y_1/H$ = volumetric fraction of component 2;
- $\bar{u}$ = wave velocity; and
- $\bar{C}$ = velocity of dynamic wave relative to weighted average velocity.

$f$ represents the net force on the flow produced by a
concentration of gradient, the subscript \( V_\alpha \) is equal to \( \frac{\partial}{\partial z} \). \( V_o \) is defined as the weighted mean velocity. Figure 2.10 shows schematic representation for the case of a dynamic wave propagation in two incompressible stratified components in a horizontal duct, in this case:

\[
f_{V_\alpha} \frac{\partial}{\partial z} (\rho_2 - \rho_1) g H \tag{2.13}
\]

\( H = \) the height of the duct.

Then equation 2.11 becomes:

\[
\bar{C} = \pm \left[ \frac{(V_1 - V_2)^2}{y_1/\rho_2 + (1 - y_1/H)\rho_1} + (\rho_2 - \rho_1) g H \right]^{1/2} \times \\
\left( \frac{\rho_1}{1 - y_1/H} \right)^{-1/2}
\tag{2.14}
\]

The relative velocity is seen to have a destabilizing effect because it decreases the dynamic wave velocity. For sufficiently high relative velocity, \( \bar{C}^2 \) becomes negative and the flow is unstable. This occurs when

\[
(V_1 - V_2)^2 > (\rho_2 - \rho_1) g H \left( \frac{y_1}{\rho_2 H} + \frac{1 - y_1}{\rho_1} \right) \tag{2.15a}
\]

when compressibility effects are important, as in the case of transient pressures, the propagation of dynamic waves is governed by the gradients of three "concentra-
tions (the densities of the two components and the volumetric concentration of one of them). In this case compressibility waves move with velocities:

$$\bar{u} = V \pm C_{cs}$$  \hspace{1cm} (2.16)

where,

$$C_{cs} = \pm \left( \frac{y_1/\rho_1 H + (1-y_1/H)/\rho_1}{y_1/\rho_2 \bar{c}_2^2 + (1-y_1/H)/\rho_1 \bar{c}_1^2} \right)^{1/2}$$  \hspace{1cm} (2.17)

assuming that there is no relative velocity, where $\bar{c}_1^2$ and $\bar{c}_2^2$ are the velocities of compressibility waves in each component separately. The velocity $C_{cs}$ is the velocity of a compressibility wave in the mixture. For truly homogeneous flow there must be sufficient friction between the components to prevent relative motion. If $\rho_2$ is much less than $\rho_1$ and $\bar{c}_2^2$ is less than $\bar{c}_1^2$ then: the compressibility wave velocity in a strictly homogeneous mixture would be:

$$C_{ch} = \left( \frac{\bar{c}_2^2 \rho_2}{(\rho_1 y_1/H)(1-y_1/H)} \right)^{1/2}$$  \hspace{1cm} (2.18)

$$(C_{ch})_{min} = 2 \bar{c}_2 \left( \frac{\rho_2}{\rho_1} \right)^{1/2}$$  \hspace{1cm} (2.19)

For air and water at atmospheric pressure, for example $\bar{c}_2 \approx 335$ m/s (1100 fps), $\frac{\rho_2}{\rho_1} = 0.0012$, $(C_{ch})_{min}$ is then equal to 22.9 m/s (75 fps).
Streeter and Wylie (33), explained the effects on waterhammer of entrained air in the pipeline. They stated that the existence of gas bubbles can greatly reduce the velocity of the pressure wave in a pipeline. They defined a bulk modulus of elasticity for the mixture (liquid and gas) as:

\[
\bar{K} = \frac{K_L}{1 + \left(\frac{\rho_g}{\rho_l}\right)\left(\frac{K_L}{K_g} - 1\right)}
\]  

(2.20)

where,

\[\nu = \text{total volume of the fluid,}\]

\[\nu_g = \text{volume of the gas,}\]

\[\nu_L = \text{volume of the liquid,}\]

\[K_g = \text{bulk modulus of elasticity for the gas, and}\]

\[K_L = \text{bulk modulus of elasticity for the liquid.}\]

They expressed the mixture density as:

\[
\bar{\rho} = \rho_g \frac{\nu_g}{\nu} + \rho_L \left(\frac{\nu_L}{\nu}\right)
\]

(2.21)

where,

\[\rho_g = \text{density of the gas, and}\]

\[\rho_L = \text{density of the liquid.}\]

They defined also a bulk modulus of elasticity for the mixture when the air phase compresses isothermally as:
\[
\tilde{K} = \frac{K_{L}}{1 + (mRT/P_{ab})[K_{L}/P_{ab} - 1]}
\]

(2.22)

where,

- \( m \) = mass of free air present per cubic unit of volume,
- \( p \) = the absolute pressure,
- \( R \) = the gas constant (air), and
- \( T \) = the absolute temperature.

If the amount of gas in the pipeline is small the wave speed can be obtained from the following equation:

\[
a = \sqrt{\tilde{K}/\tilde{\rho}}
\]

(2.23)

where \( \tilde{K} \) and \( \tilde{\rho} \) can be obtained from Equations 2.20 and 2.21.

If the free air present under high pressure conditions, its volume is greatly reduced and the pipe elasticity becomes important, and the wave speed can be calculated from the following equation:

\[
a = \sqrt{\frac{\tilde{K}/\tilde{\rho}}{1 + (K/E)(D_p/e)}}
\]

(2.24)

where,

- \( E \) = modulus of elasticity for the pipe material,
- \( D_p \) = pipe diameter, and
- \( e \) = pipe wall thickness.
For isothermal compression of the air phase the following equation can be used:

\[
a = \sqrt{\frac{K_L/\rho}{1 + K_L D_p/\tau E + (mRT/P_{ab})[(K_L/P_{ab}) - 1]}}
\]  \hspace{1cm} (2.25)

A good agreement was observed between the theory of Streeter and Wylie (33) and their experimental results as shown in Fig. 2.11.

In 1972, Metcalf and Eddy, Inc. (26) discussed the importance of ventilation and air relief. They mentioned that the ventilation of sewers is important to prevent the accumulation of sewer gases that may be explosive or corrosive and to prevent the creation of pressures above or below atmospheric. Some of the factors causing the movement of air in sewers are explained, theoretically and practically. The best locations for ventilation openings are stated. Different methods for ventilation and air relief are given.

In 1973, Babb, Schneider and Thompson (2) studied the air flow in combined intake and shaft spillway. They described certain hydraulic features of the Castaic Forebay Outlet Works near Los Angeles, California, U.S.A. This structure combines the functions of a gated free-standing tower and a shaft spillway. A hydraulic model
was used to assist the designers in developing the structure and the results of model air-entrainment tests are reported.

Their study shows that shaft spillways (also termed drop inlet or morning-glory) are usually designed for conditions of open-channel flow in a level-conduit downstream from the intake. The conduit portal is normally submerged. This type of structure has operated with a submerged outlet in only a few locations such as that at Owyhee Dam in Idaho (4), where a hydraulic jump has been observed to form in the conduit for most discharges. Here, blow back of air trapped upstream from the jump has created spray that has risen to heights of 15 m to 18 m above the level of the spillway crest. In addition, air forced out of the submerged outlet has caused mist to rise to considerable heights downstream from the dam.

In their model study, a large vent was introduced at the soffit of the conduit just downstream from the vertical bend which affords insurance against blow back of air through the tower. Also, an air trap and slotted diffuser are employed at the downstream end of the conduit to smooth out the evacuation of entrained air.

They concluded that due to the lack of similitude for air entrainment and transport, prototype behaviour cannot be predicted exactly from model observations. In recognition of this, the air diffuser slots may have to be adjusted
in the prototype. Auxiliary model tests were conducted with excessively large quantities of air forced into the system in order to test the capability of the structure to adequately handle larger amounts of air.

In 1977, Hamam (13) analyzed the fluid motion in a sewer duct after the bubble formation (see Fig. 2.12 for notations). He derived the following equation of motion:

\[
\frac{d^2 S_b}{dt^2} + \frac{f}{8h} \left( \frac{dS_b}{dt} \right) \left| \frac{dS_b}{dt} \right| + \frac{Ag}{A_b \ell} S_b - \frac{Pat g}{\gamma \ell'} + \frac{Po g}{\gamma \ell' (1 - \frac{A}{V_o} S_b)} = 0 \quad (2.26)
\]

where,

- \( S_b \) = displacement measured in the flow direction,
- \( A \) = area of flow,
- \( A_b \) = contact area between the bubble and the water,
- \( f \) = friction factor,
- \( Pat \) = atmospheric pressure,
- \( Po \) = initial absolute air pressure when the bubble formed,
- \( V_o \) = volume of trapped air when the bubble formed,
- \( \ell' \) = distance of the bubble from the upstream end of the pipe,
\( R_h \) = hydraulic radius of the water flow, and \\
\( \gamma \) = unit weight of fluid.

He gave also the following equation to compute the variation in piezometric pressure:

\[
Z_b = \frac{P_o}{\gamma (1-S_b A/V_o)} - \frac{P_{at}}{\gamma}
\]  
(2.27)

In 1978, Taitel, Lee, and Dukler (34), studied the Transient Gas-Liquid Flow in Horizontal Pipes "Modeling the Flow Pattern Transitions". They presented a theory to predict flow pattern transition under transient flow conditions, and they compared the theory with experiment. Methods are presented for predicting the flow rates at which flow pattern transitions will take place during flow transients. Figure 2.13 shows a comparison of their experimental and theoretical prediction of flow regimes in horizontal tubes.

Streeter and Wylie (33), gave the relation between pressure change (\( \Delta p \)) and velocity change (\( \Delta V \)) due to a water hammer wave in two-phase two-component solid-liquid mixture. The following equation can be used:

\[
\Delta p = \rho_{sl} \Delta V = \rho_{sl} \sqrt{\frac{K_{sl}}{\rho_{sl}}} \Delta V
\]  
(2.28)
where,
\[ K_{s\ell} = \frac{K_s}{1 + \left(\frac{\psi_s}{\psi_S}\right)\left(\frac{K_{\ell}}{K_S} - 1\right)} \], and
\[ \rho_{s\ell} = \rho_s \frac{\psi_S}{\psi} + \rho_{\ell} \frac{\psi_s}{\psi} \]
in which,
- \( \psi \) = total volume of the fluid,
- \( \psi_s \) = volume of the solid,
- \( \psi_{\ell} \) = volume of the liquid,
- \( K_s \) = bulk modulus of elasticity for the solid,
- \( K_{\ell} \) = bulk modulus of elasticity for the liquid,
- \( K_{s\ell} \) = bulk modulus of elasticity for the mixture,
- \( \rho_s \) = density of the solid,
- \( \rho_{\ell} \) = density of the liquid, and
- \( \rho_{s\ell} \) = density of the mixture.

Equation 2.28 is based on neglecting the relative motion of solid particles to liquid.

2.6 Surcharged Flow in Sewers

Wiggert (40) investigated the submerged tunnel conditions for Wettingen System near Zurich, Switzerland. This case in which a tunnel situated in an open channel system is submerged at all times can be treated as a special type of interior boundary condition. Incorporated into the method of characteristics, the momentum equation
describing flow in the tunnel is solved for discrete time interval, in this manner, a surge wave or bore introduced into the system can be transmitted through the tunnel. An experimental study was presented to verify the numerical model.

In 1977, Colyer (9) presented a study on the effect of surcharging on discharge through a pipe. He gave the following equation for the ratio of discharge under surcharge ($Q_s$) to normal pipe-full discharge ($Q_o$):

$$
\frac{Q_s}{Q_o} = \sqrt{1 + \frac{\Delta y}{\Delta h}}
$$

(2.29)

in which,

\[ \Delta h = \text{the fall in pipe level, and} \]

\[ \Delta y = \text{the incremental depth of surcharging.} \]

Figure 2.14 shows a definition sketch of surcharged pipes. Figures 2.15 and 2.16 show discharge in surcharged pipes for $Q_s/Q_o$ greater than 1.0 and less than 1.0 respectively.

In 1980, Yen (43) presented a comprehensive study on the hydraulics of surcharged flow in sewers, where the flow cross-sectional area is constant and equal to the full pipe area $A_p$. The continuity and momentum equations can be written as:
\[ Q = AV \quad \text{(2.30)} \]

\[ \frac{1}{g} \frac{\partial V}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\partial V^2}{g} \right) + \frac{P_a}{\gamma} = -S_f + \frac{1}{\gamma A} \frac{\partial T''}{\partial x} \quad \text{(2.31)} \]

in which,

- \( Q \) = discharge,
- \( t \) = time,
- \( x \) = distance along the pipe longitudinal direction,
- \( A \) = flow cross-sectional area perpendicular to \( x \),
- \( S_f \) = friction slope,
- \( \beta \) = momentum flux correction factor,
- \( T'' \) = force due to internal stresses acting normally to \( A \),
- \( \gamma \) = specific weight of the liquid,
- \( g \) = gravitational acceleration,
- \( V \) = average velocity over the cross-sectional area \( A \), and
- \( P_a \) = piezometric pressure of the flow.

Assuming incompressible flow for a pipe having constant cross-section and flowing full throughout its length, \( \frac{\partial V}{\partial x} = 0 \). By neglecting the spatial variation of \( \beta \) and \( T'' \), integration of Equation 2.31 over the entire length, \( L \), of the sewer pipe (see Fig. 2.17) yields:
\[ \frac{P_a}{\gamma} \text{ entrance} = H_u - H_{\text{exit}} - K_u \frac{v^2}{2g} \]
\[ = L(S_f + \frac{1}{g} \frac{\partial V}{\partial t}) \]  

(2.32)

in which,

- \( H_u \) = total head at the entrance,
- \( H_{\text{exit}} \) = total head at the exit, and
- \( K_u \) = entrance loss coefficient.

The friction slope, \( S_f \), may be estimated by using Manning's formula, or the Darcy-Weisbach formula.

Advanced techniques for solving storm sewer flow problems usually adopt Equations 2.30 and 2.32 to simulate the surcharged flow.

A simplified flow equations for a surcharged pipe flow (using Kinematic Wave Approximation), can be obtained from Equation 2.32 by neglecting the \( \frac{\partial V}{\partial t} \) term. With the downstream junction water surface, \( H_d \), related to \( H_{\text{exit}} \), and knowing that (see Fig. 2.17):

\[ \frac{P_a}{\gamma} \text{ exit} + \frac{v^2}{2g} - K_d \frac{v^2}{2g} = H_{\text{exit}} - K_d \frac{v^2}{2g} \]  

(2.33)

where, \( V \) is the velocity at the pipe exit, and \( K_d \) is the exit loss coefficient.

Substituting by Equation 2.33 into Equation 2.32, yields:
\[ H_u - K_u \frac{y^2}{2g} - S_f L = H_d + K_d \frac{y^2}{2g} \]  

(2.34)

Combining Equations 2.30 and 2.34 and noting that 
\[ A_p = \frac{\pi D_p^2}{4}, \] 
where \( D_p \) is the pipe diameter, one obtains

\[ K^2 Q^2 = H_u - H_d \]  

(2.35)

in which

\[ K^2 = (K_u + K_d + \frac{5.72 g n^2 L}{4 D_p^{4/3}}) \frac{8}{g^{2/3} D_p} \]  

(2.36)

or

\[ K^2 = (K_u + K_d + \frac{f L}{D_p}) \frac{8}{g^{2/3} D_p} \]  

(2.37)

In general the previous equations are solved numerically.

In 1981, Meadows and Chestnut (25) presented analysis of a surcharging storm sewer system. They reported that the analysis of a surcharged sewer system must integrate the principles of open-channel flow, culvert hydraulics and pressurized conduit flow. The procedures used during a case study are discussed. Manning's equation and the continuity principle were used to identify potentially surcharged manholes, culvert hydraulics was used to explain how surcharging in one portion of the system affected flow in other portions, and a hydraulic network analysis program was used to analyze the system under fully sur-
charged conditions.

2.7 Relevant Studies

To the best of the writer's knowledge and as noted by Yen (43), no models are currently available that accurately simulate the transition from open-channel to pressurized flow through the six stage transition outlined in Chapter I. In 1972, however, Wiggert (40) analyzed the transient behaviour in combined free-surface, pressurized systems by making use of computer techniques. He grouped the flow cycle into four phases (Fig. 2.18):

a) The tunnel was fully ventilated, and small free-surface surges could propagate in the tunnel.

b) As input conditions changed, the water level reached the roof or soffit of the tunnel at the entrance.

c) The tunnel acted as an ordinary surge system.

d) Water levels on one or both sides of the tunnel subsided, and negative surge(s) travelled in the tunnel.

It is noted that the first three phases in his analysis are similar to stages (1), (2) and (6) respectively, of the analysis in this study (see Chapter I). Wiggert presented a system of equations in a generalized form so that their application to a variety of situations are possible.
He used the following differential equations, given by Streeter and Wylie (33), for Free-Surface conditions:

Momentum Equation

\[ gy_x + vv_x + v_t + g(S_f - S_o) = 0 \]  \hspace{1cm} (2.38)

Continuity Equation

\[ \frac{A}{w} v_x + vy + v_t = 0 \]  \hspace{1cm} (2.39)

in which,

- \( y \) = flow depth,
- \( v \) = average velocity at a given section,
- \( A \) = cross-sectional area,
- \( w \) = water surface width,
- \( x \) = distance along channel, and
- \( t \) = time.

One-dimensional flow is assumed, as well as a small channel slope and hydrostatic pressure distribution at any vertical section (Fig. 2.19). The Channel slope is \( S_o \) and \( S_f \) is the friction slope.

To apply the method of characteristics, Eqs. 2.38 and 2.39 are combined with a linear multiplier and the partial differential system is transformed into four total differential equations. For the characteristic direction \( C^+ \)
\[
\frac{g}{c} \frac{dv}{dt} + \frac{dv}{dt} + g(S_f - S_o) = 0 
\]
(2.40)

\[
\frac{dx}{dt} - (V+c) = 0 
\]
(2.41)

and for the characteristic direction \( C^- \)

\[
-\frac{g}{c} \frac{dv}{dt} + \frac{dv}{dt} + g(S_f - S_o) = 0 
\]
(2.42)

\[
\frac{dx}{dt} - (V-c) = 0 
\]
(2.43)

where \( c \) is defined as

\[
c = \sqrt{\frac{gA}{T_w}} 
\]
(2.44)

in which,

\( T_w \) = water surface width, and
\( A \) = wetted cross-sectional area.

The system of equations, Eqs. 2.40-2.43 are valid for the entire region of free surface flow.

For the movement of the surge in the conduit (Surge Wave in Tunnel), he gave the following equations (see Fig. 2.20 for notations):

\[
x_1 \frac{dv}{dt} + (w-V_1)(V_1-V_2) = \frac{g}{A_1} \left( A_1 h_o^\prime - A_2 F_2 \right) + gx_1(S_o - S_f) 
\]
(2.45)
\[ w = \frac{A_1 V_1 - A_2 V_2}{A_1 - A_2} \]  

(2.46)

\[ \frac{dx_1}{dt} = w \]  

(2.47)

The friction term, \( S_f \), for both the free-surface and pressurized flow phases is given by

\[ S_f = n^2 \frac{V^4}{R_h^{4/3}} |V| \]  

(2.48)

in which 

\[ n = \text{Manning number, and} \]

\[ R_h = \text{Hydraulic radius.} \]

At the discontinuity or surge front the following equation is given:

\[ h_1 = \frac{A_2}{A_1} y_2 + \frac{1}{g} \left( (w-V_1)(V_1-V_2) \right) \]  

(2.49)

A laboratory study was initiated by Wiggert to obtain accurate and detailed data, so that the analytical models could be verified. The data used was that of the Limmatwerk Wettingen located near Zurch, Switzerland, in which the power plant tail-race tunnel was designed to perform in conjunction with a surge shaft. Figure 2.21 is a schematic representation of the experimental set-up. Figure 2.22, shows a sample of the experimental results.
for the pressure heads behind the surge front along with the computed values shown as solid circles.

In 1978, Mussalli (28) studied the sealing or the transition from free-surface to pipe flow. He concluded that sealing of the horizontal or inclined conduits depends on the Froude number of the flow. Figure 2.1 shows a typical pressure and piezometric heads at roof of his tested conduit prior to and at sealing. Mussalli (28) did not study the interface instability or pressure transients associated with the transition from free-surface to pipe flow which are essential in such situations.
CHAPTER III

THEORETICAL BACKGROUND

3.1 Introduction

The developments in this chapter are treated in six parts following the six stages of the transition of gravity to surcharged flow that were presented in Chapter I (see Fig. 1.1):

a) Gravity Flow (Stage 1) in which the continuity and momentum equations for gradually varied open-channel flow can be used to describe the flow.

b) Surge Movement (Stage 2) in which the flow is treated as rapidly varied and continuity and momentum principles are applied to develop general equations for the surge characteristics (Velocities and Surcharge Heads) with a stable air-water interface.

c) Interface Instability Formation (Stage 3) in which the wave theories and experimental correlations are used to develop general equations for the critical interface instability velocity ($V_{I}$) at which the transition to full pipe or surcharged flow starts.

d) Transition to Surcharged Flow (Stage 4) in which the continuity and momentum principles for
compressible flow in pipes are used with the method of characteristics to develop the instantaneous surcharge heads and flow velocities in the two-phase flow system during the transition of open-channel to full pipe flow in the conduit.

e) **Trapped Air Release (Stage 5)** in most cases the trapped air in the pipe forms large air bubbles and the system can be treated as a conduit with an air chamber or chambers. The momentum equation can be applied on the water mass inside the pipe just after the formation of the bubble to derive an equation for the fluid motion after the bubble formation, and thus to predict the variation in piezometric pressure as the bubble escapes.

f) **Surcharged Flow (Stage 6)** in which the pipe is flowing full with no trapped air and the continuity and momentum equations for full pipe flow can be used.

### 3.2 Gravity Flow in a Sewer

The open-channel phase of the sewer flow can be represented mathematically by a pair of partial differential equations of hyperbolic type (references 42, 43)

\[
\frac{3A}{3t} + \frac{3Q}{3x} = 0 \quad (3.1)
\]
\[
\frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{1}{gA} \frac{\partial}{\partial x} \left( \frac{\partial A}{A} Q^2 \right) + \cos \theta \frac{\partial}{\partial x} (Kh) \\
+(K_1 - K_2) \bar{h} \cos \theta \frac{1}{A} \frac{\partial A}{\partial x} = S_0 - S_f + \frac{1}{\gamma A} \frac{\partial T}{\partial x}
\]

(3.2)

in which,

- \( Q \) = discharge;
- \( t \) = time;
- \( x \) = distance along the pipe longitudinal direction;
- \( A \) = flow cross sectional area perpendicular to \( x \);
- \( \bar{h} \) = flow depth measured normal to \( x \);
- \( \theta \) = angle between the sewer axis and a horizontal plane;
- \( S_0 \) = \( \sin \theta \) = sewer slope;
- \( S_f \) = friction slope;
- \( \beta \) = momentum flux correction factor;
- \( K_1 \) and \( K_2 \) = correction factors for non-hydrostatic pressure distribution;
- \( T \) = force due to internal stresses acting normally to \( A \);
- \( \gamma \) = specific weight of the liquid (assumed incompressible and homogeneous); and
- \( g \) = gravitational acceleration.

Equation 3.1 is the continuity equation and Eq. 3.2 is the momentum equation. They are derived from the principle
of conservation of mass and Newton's second law, respectively.

In practice Eqs. 3.1 and 3.2 are simplified by assuming $\beta=1$ (uniform velocity distribution over $A$), hydrostatic pressure distribution ($K_1 = K_2 = 1.0$), and neglecting the last term in Eq. 3.2 containing $T''$. The result is the well known complete dynamic wave or Saint Venant equations, which can be expressed in terms of the average velocity, $V$, over the cross-sectional area $A$, i.e.,

$$\frac{\partial h}{\partial t} + A \frac{\partial V}{\partial x} + V \frac{\partial h}{\partial x} = 0$$  \hspace{1cm} (3.3)

$$\frac{1}{g} \frac{\partial V}{\partial t} + \frac{V}{g} \frac{\partial V}{\partial x} + \cos \theta \frac{\partial h}{\partial x} - (S_o - S_f) = 0$$  \hspace{1cm} (3.4)

in which $T_w$ is the water surface width.

The friction slope, $S_f$, is usually estimated by using Manning's formula

$$S_f = n \frac{V^2}{R_h^{4/3}} = n \frac{Q^2}{A \rho R_h^{4/3}}$$  \hspace{1cm} (3.5)

or the Darcy-Weisbach formula,

$$S_f = \frac{fV^2}{8gR_h} = \frac{f}{8gR_h} \frac{Q^2}{A}$$  \hspace{1cm} (3.6)

in which,

$n = \text{Manning's roughness factor;}

f = \text{Weisbach resistance coefficient, and}

R_h = \text{hydraulic radius of the flow.}$
For an open-channel flow in a circular pipe the geometric parameters of the flow cross section are computed as follows (see Fig. 3.1):

\[ A = \frac{D^2}{8} (\phi - \sin \phi) \quad (3.7) \]

\[ R = \frac{D}{4} (1 - \frac{\sin \phi}{\phi}) \quad (3.8) \]

\[ T_w = \frac{D_p \sin \frac{\phi}{2}}{2} \quad (3.9) \]

\[ \bar{h} = \frac{D_p}{2} (1 - \cos \frac{\phi}{2}) \quad (3.10) \]

in which,

- \( D_p \) = diameter of the pipe; and
- \( \phi \) = central angle in radians described by the water surface having a width \( T_w \).

If the flow is assumed steady and uniform, Eq. 3.4 reduces to \( S_o = S_f \) and \( Q = AV \). Hence, from Eq. 3.5 for steady uniform flow using Manning's formula

\[ Q = \frac{c}{m} S_o^{1/2} \frac{D_p^{8/3} (\phi - \sin \phi)^{5/3}}{\frac{\phi}{2}} \quad (3.11) \]

in which the constant \( c = 0.0737 \) for English units and \( 0.0496 \) for SI units.

Correspondingly, the Darcy-Weisbach formula (Eq. 3.6) yields:
\[ Q = \frac{1}{8} \sqrt{\frac{2g S_0}{f}} \quad D^p \quad \frac{5/2}{1/2} \quad (\phi - \sin \phi)^{3/2} \quad (3.12) \]

Advanced techniques for solving storm sewer flow problems usually adopt Eq. 3.3 and 3.4 to simulate the open-channel flow phase of the sewer flow.

Because of the difficulty in solving the Saint Venant equations for unsteady open-channel flows, a number of approximations have been used for engineering problems (42 and 43). A popular simplification is the kinematic wave approximation which has a simplified momentum equation obtained by dropping all but the last two slope terms in Eq. 3.4, i.e.

\[ S_o = S_f \quad (3.13) \]

The friction slope, \( S_f \), may be approximated by the Darcy-Weisbach formula (Eq. 3.6) or Manning's formula (Eq. 3.5). The kinematic wave approximation involves solving Eq. 3.13 with appropriate initial and boundary conditions, together with a continuity equation (Eq. 3.1) which can be integrated over a reach of the sewer pipe having a length \( \Delta L \) to yield

\[ I_{in} - I_{out} = \frac{dV}{dt} \quad (3.14) \]

in which \( I_{in} \) and \( I_{out} \) are the inflow into and outflow from the reach, respectively; \( V \) is the water storage in the reach,
and $dW/dt$ is the time rate of change of storage.

Only for special simple cases can an analytical solution be obtained for the open-channel flow kinematic wave equations (Eqs. 3.1 and 3.13). In general these equations are solved numerically.

3.3 Surge Movement

In deriving the equation of surge movement in a sewer following rapid submergence of the outfall, e.g., due to pump failure at a sump well or inadequate pumping capacity (see Fig. 3.2), the following assumptions were made:

a) The air-water interface is stable;

b) the flow is incompressible;

c) the pressure distribution is hydrostatic at the open-channel flow section and at the sump location;

d) the open-channel flow ($Q_1$) is steady;

e) the outflow from the sump well ($Q_{out}$) is constant and less than the inflow $Q_1$;

f) The pipe has a constant cross-section area and bed slope; and

g) the flow upstream of the surge is assumed to be uniform flow.

The Equations of Motion for Surge (Stage 2).

Figure 3.2 shows a typical surge formation at a sump or manhole. Applying the continuity and momentum equations
from section 1 to 2 gives:

$$V_w(t) = \frac{A_1 V_1 - A_2 V_2(t)}{A_2 - A_1} = \frac{A_1 V_1 - A_p V_2(t)}{A_p - A_1}$$  \hspace{1cm} (3.15)

and

$$Z(t) = A_1 \bar{y}_1 / A_2 - \bar{y}_2 + \frac{A_1 (V_1 - V_2(t))^2}{g (A_2 - A_1)} + \frac{P}{\gamma}$$  \hspace{1cm} (3.16)

where,

- $V_1$ = velocity of flow at section 1;
- $V_2$ = velocity of flow at section 2;
- $A_1$ = area of flow at section 1;
- $A_2 = A_p$ = area of flow at section 2;
- $V_w$ = surge velocity;
- $\bar{y}_1$ = distance between the centre of pressure and the water surface at section 1;
- $\bar{y}_2$ = distance between the centre of pressure and the sewer crown at section 2;
- $P$ = the air (gauge) pressure at the surge front;
- $\gamma$ = specific weight of the water; and
- $Z$ = pressure rise at the surge front measured above the sewer crown.

The continuity equation for the sump is

$$V_2(t) = \frac{A_s}{A_p} \frac{dS}{dt} + Q_{out}/A_p$$  \hspace{1cm} (3.17)

where,
$A_s$ = surface area of the sump;  
$Q_{out}$ = pumping rate out of the sump well;  
$A_p$ = area of the pipe cross section; and  
$S$ = water level in the sump well.

The momentum equation applied to column $L_s(t)$ (section 2 to section 3) gives:

$$F_3 + \tau_o \rho L_s(t) - \gamma A_p L_s(t) S_o - F_2 = -\frac{\gamma A_p}{g} L_s(t) \frac{dv_2(t)}{dt}$$

(3.18)

in which,

$P'$ = wetted perimeter of the pipe at section 2;  
$S_o$ = bottom slope of the conduit; and  
$F_2$ = hydrostatic pressure on the area $A_p$ at section 2  
$$F_2 = -\gamma \int_{z+D_p}^{(z+D_p)} y dA + \gamma A_p (\bar{y}_2 + z(t))$$

(3.19)

where,

$D_p$ = diameter or depth of conduit; and  
$F_3$ = (hydrostatic pressure on the area $A_p$ at section 3).  
$$F_3 = \int_{S-D_p}^{S} y dA + \gamma A_p (\bar{y}_2 + S(t) - D_p + h_p (1-\bar{K}_3))$$

(3.20)

where,

$\bar{K}_3$ = loss coefficient at section 3;
\[ h_v = \text{velocity head at section 2} = \bar{a} \frac{V_2^2(t)}{2g} \quad (3.21) \]

\[ \bar{a} = \text{kinetic energy correction factor} \]
\[ \tau_o = \text{shear resistance of the pipe walls} \]
\[ = \frac{y_f}{8g} V_2(t) |V_2(t)| \quad (3.22) \]

where,
\[ f = \text{friction factor for the conduit.} \]
\[ L_s(t) = \text{distance travelled by the surge front} = \int_{t_0}^{t} V_w(t) \, dt \quad (3.23) \]

Substituting from Eq. 3.15 into Eq. 3.23 gives:
\[ L_s(t) = \int_{t_0}^{t} (E_1 V_2(t) - E_2) \, dt \quad (3.24) \]

where,
\[ E_1 = \frac{-A_p}{A_p - A_1} = \text{constant, and} \]
\[ E_2 = \frac{-A_1 V_1}{A_p - A_1} = \text{constant.} \]

Substituting from Eq. 3.17 into Eq. 3.24 and integrating gives
\[ L_s(t) = E_3 S(t) + E_4 \cdot t + E_5 \quad (3.25) \]

where,
\[ E_3 = E_1 \frac{A_s}{A_p} = \text{constant}; \]
\[ E_4 = E_1 \left( \frac{Q_{out}}{A_p} \right) - E_2 = \text{constant}; \text{ and} \]
\[ E_5 = L_g(t_o) - E_3 S(t_o) - E_4 t_0 = \text{constant}. \]

Differentiating Eq. 3.17 with respect to time we get

\[ \frac{dv_2(t)}{dt} = \frac{A_s}{A_p} \frac{d^2 s}{dt^2} \tag{3.26} \]

Substituting from Eqs. 3.19, 3.20, 3.21, 3.22, 3.25, 3.26
and A.4 (see Appendix A) and simplifying gives

\[ \frac{v_2^2(t)}{2} \left[ C_1 S(t) + C_2 t + C_3 \right] + \frac{C_4}{C_5} S(t) + C_6 t \]
\[ + C_7 v_2(t) + C_8 \left( \frac{ds}{dt} \right)^2 + C_9 \frac{ds}{dt} = \frac{d^2 s}{dt^2} (C_{10} s(t)) \tag{3.27} \]

where,

\[ C_1 = \frac{-f_p}{8gA_p} E_3 = \text{constant}; \]

\[ C_2 = \frac{-f_p}{8gA_p} E_4 = \text{constant}; \]

\[ C_3 = -\frac{a}{2g(1-K_e)} - \frac{f_p}{8gA_p} E_5 + \frac{A_1}{g(A_p-A_1)} = \text{constant}; \]

\[ C_4 = D_p + S_o E_5 + \frac{A_1}{A_p} - \frac{\gamma a}{2gV_w} (K_e + D) \]
\[ + \frac{A_1 V_1 - Q_{out}}{A_p - A_1} + \frac{A_1 V_1^2}{g(A_p - A_1)} = \text{constant}; \]
\[ C_5 = S_0 E_3 - 1 = \text{constant}; \]
\[ C_6 = S_0 E_4 = \text{constant}; \]
\[ C_7 = -2V_1 \frac{A_1}{g(A_p - A_1)} = \text{constant}; \]
\[ C_8 = \frac{A_2 E_3}{g A_p} = \text{constant}; \]
\[ C_9 = \frac{A_2^2 E_4}{g^2 A_p} = \text{constant}; \]
\[ C_{10} = \frac{A_2 E_5}{g A_p} = \text{constant}; \]
\[ G_2 = \frac{\gamma_a}{2g \gamma_w} \left( K + \frac{f L_A}{D} \right) \left( \frac{-A_s}{A_p - A_1} \right)^2 = \text{constant}; \]
\[ G_3 = \frac{\gamma_a}{2g \gamma_w} \left( K + \frac{f L_A}{D} \right) \left( \frac{-A_s}{A_p - A_1} \right) \left( \frac{A_1 V_1 - Q_{out}}{A_p - A_1} \right) = \text{constant} \]

Substituting from Eq. 3.17 into 3.27 and simplifying one obtains

\[
\left( \frac{dS}{dt} \right)^2 (C_{11} S(t) + C_{12} t + C_{13}) + \left( \frac{dS}{dt} \right) (C_{14} S(t) + C_{15} t + C_{16}) + C_{17} S(t) + C_{18} t + C_{19} =
\]
\[
\frac{d^2 S}{dt^2} (C_9 S(t) + C_9 t + C_{10}) \quad (3.28)
\]

where,

\[ C_{11} = C_1 \left( \frac{A_s}{A_p} \right)^2 = \text{constant}; \]
\[ C_{12} = C_2 \frac{A_S}{A_p}^2 = \text{constant}; \]
\[ C_{13} = C_3 \frac{A_S}{A_p}^2 + G_2 = \text{constant}; \]
\[ C_{14} = \frac{2A_S Q_{\text{out}} C_1}{A_p} = \text{constant}; \]
\[ C_{15} = \frac{2A_S Q_{\text{out}} C_2}{A_p} = \text{constant}; \]
\[ C_{16} = \frac{2A_S Q_{\text{out}} C_3}{A_p} + G_3 + C_7 \frac{A_S}{A_p} = \text{constant}; \]
\[ C_{17} = C_1 \frac{Q_{\text{out}}^2}{A_p} + C_5 = \text{constant}; \]
\[ C_{18} = C_2 \frac{Q_{\text{out}}^2}{A_p} + C_6 = \text{constant}, \text{ and} \]
\[ C_{19} = C_3 \frac{Q_{\text{out}}^2}{A_p} + C_4 + C_7 \frac{Q_{\text{out}}}{A_p} = \text{constant}. \]

By definition

\[ \frac{dS}{dt} = V_s \tag{3.29} \]

Equation 3.28 can be written as follows

\[ \frac{d^2S}{dt^2} = \frac{dS}{dt} \dot{\phi}_1 + \frac{dS}{dt} \dot{\phi}_2 + \dot{\phi}_3 \tag{3.30} \]

in which

\[ \dot{\phi}_1 = \frac{C_{11} S(t) + C_{12} t + C_{13} t}{C_3 S(t) + C_9 t + C_{10}} = f_1(S, t) \tag{3.31w} \]
\[ \phi_2 = \frac{C_{14}S(t) + C_{15}t + C_{16}}{C_{9}S(t) + C_{9}t + C_{10}} = f_2(S,t) \]  
\[ \phi_3 = \frac{C_{17}S(t) + C_{18}t + C_{19}}{C_{9}S(t) + C_{9}t + C_{10}} = f_3(S,t) \]  

Substituting by Eq. 3.29 into Eq. 3.30 gives:

\[ \frac{dv}{dt} = v \frac{2}{s} \phi_1 + v_s \phi_2 + \phi_3 \]  

Equations 3.29 and 3.34 can be solved simultaneously using a numerical solution technique (Runge Kutta Method), the solution gives \( S \) and \( \frac{dS}{dt} \) at time \( t \). Knowing \( S \) and \( \frac{dS}{dt} \) at time \( t \), the quantities, \( V_2, V_w, Z, \) and \( L_s \) can be determined at the same time from Eqs. 3.17, 3.15, 3.16 and 3.25 respectively.

3.4 Interface Instability in sewers

It is well known (Ippen (19) and Milne-Thomson (27)) that a high relative velocity at an air-water interface causes waves and instabilities to form. For example, the Helmholtz theory (17) predicts instability between two infinite inviscid fluids at relative velocity of

\[ V_H = \frac{1}{\sqrt{\frac{\rho_a}{\rho_w}}} C_o \]  

where,

\( \rho_w \) = density of water;
\( \rho_a \) = density of air; and
\( C_o \) = celerity of surface waves (e.g., due to gravity).

In a closed conduit the critical interface instability velocity, \( V_I \), can be expected to be a function of \( V_H, y_1/D_p \) and the shape of the pipe, e.g.

\[
\frac{V_I}{V_H} = \text{fcn} \left( \frac{y_1}{D_p}, \text{pipe shape} \right)
\]  

(3.36)

For example, a simple empirical relationship could be of the form:

\[
V_I = V_H^m [K_a (y_1/D_p)^n + C]
\]

(3.37)

where \( K_a, n, \) and \( C \) are constants related to the pipe shape.

The relative depth of flow \( (y_1/D_p) \) is an important factor because of the reduced pressure resulting from the high local air velocity at the wave crests which in turn is expected to speed up the interface instability, at high relative depth of flow \( (y_1/D_p) \). For lower values of \( (y_1/D_p) \) the reduction in the pressure at wave crest locations will not be significant and the relative air velocity, \( (|\hat{V}_a|+|\hat{V}_1|) \) - for counter-current air-water flow - becomes more important to initiating the interface instability.

For the surge shown in Fig. 3.2 the air velocity is

\[
V_a = V_w \text{ at stage 2}
\]

(3.38)
If \(|\vec{V}_w| + |\vec{V}_l| < V_I\), the flow upstream from the surge front should remain gravity flow; however, if \(|\vec{V}_w| + |\vec{V}_l| > V_I\), interface waves may increase in height until the conduit is blocked. After blockage occurs, the trapped air and water must be treated as a compressible mixture.

3.5 Transition to Surcharged Flow

3.5.1 Introduction

When the transition of open-channel to surcharged flow starts (see Fig. 3.3), the sewer pipe can be divided into two parts \(L_1\) and \(L_2\); where throughout the length \(L_1\) there is a full pipe flow with small air bubbles, while throughout the length \(L_2\), a stratified or a mixed slug-stratified two-phase air-water flow is expected.

The continuity and momentum principles can be used with the method of characteristics approach to develop the instantaneous surcharge head and flow velocity in the lengths \(L_1\) and \(L_2\) during the transition.

3.5.2 The Differential Equations for Transient Flows

3.5.2.1 Momentum Equation

The Momentum Equation for liquid flow with air fraction \(\alpha_a\) can be derived for a cylindrical tube in a similar manner to the equation of motion derived by Wylie and Streeter (33). The derived equation is in terms of the hydraulic grade line pressure \(H(x,t)\) and average velocity \(V(x,t)\)
(see Fig. 3.4).

Considering a fluid element of a constant flow area \( A \) (Fig. 3.4) and length \( \delta x \). The conduit is inclined with the horizontal at angle \( \theta \). The forces on the free body in the \( x \)-direction are the surface contact normal pressures on the transverse faces, and shear and pressure components on the periphery. In addition gravity, the body force, has an \( x \)-component, and the shear force \( \tau_0 \) is considered to act in the \(-x\) direction.

With reference to Fig. 3.4, and assuming that the shear stress \( \tau_0 \) is considered to be the same as if the velocity were steady; the momentum equation for two-phase air-water flow of air fraction \( (\alpha_a) \) may be written as follows:

\[
g(1-\alpha_a) H_x + \frac{dv}{dx} + \frac{dv}{dt} + \frac{fv|v|}{2D} = 0 \quad (3.39)
\]

where \( H \) is in metres of water.

In the last equation \( H \) and \( V \) are the dependent variables, and \( x \) and \( t \) are the independent variables.

3.5.2.2 Continuity Equation

The continuity equation for two-phase air-water flow with air fraction \( (\alpha_a) \) can be derived for a cylindrical tube in the same way that Streeter and Wylie presented (see Reference 33). With reference to Fig. 3.5, con-
sidering a control volume of length $\delta x$ at time $t$.

Assuming that the velocity of the pipe wall $(u)=0$, and the flow has no transverse motion, the continuity equation can be written in terms of the head of water $H$ as follows:

$$VH_x + H_t - V \sin \theta + \frac{\bar{a}^2}{q(1-a)} V_x = 0 \quad (3.40)$$

Equation 3.40 is the continuity equation for two-phase air-water flow of air fraction $a_a$, with $V$ and $H$ as dependent variables, and with $x$ and $t$ as the independent variables. The flow and wall properties are introduced.

3.5.3 **Solution of the Continuity and Momentum Equations Using the Method of Characteristics**

The method of characteristics can be used to solve the continuity and momentum equations utilizing the fact that disturbances are propagated at characteristic velocities that depend on the local flow velocity, $V$, and the local speed of sound, $\bar{a}$. These characteristic velocities can be represented as characteristic directions ($\alpha'$ and $\beta'$) on an $x$-$t$ plot as shown in Fig. 3.6.

Having defined the characteristic directions $\alpha'$ and $\beta'$, it is possible to simplify the solution of the continuity and momentum equations (Eqs. 3.40 and 3.39 respectively) by integration along the $\alpha'$ and $\beta'$ characteristic directions.
The equation for the $\alpha$'-direction is

$$\frac{g(1-a)}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{g(1-a)}{a} V \sin \Theta + \frac{fV|V|}{2D} = 0$$

(3.41)

and for the $\beta$'-direction is

$$-\frac{g(1-a)}{a} \frac{dH}{dt} + \frac{dV}{dt} - \frac{g(1-a)}{a} V \sin \Theta + \frac{fV|V|}{2D} = 0$$

(3.42)

where for the length $L_1$ (see Fig. 3.3) with bubble flow regime $a_1 = a_1'$, $D = D_p$, and $\bar{a} = a_1'$, while for the length $L_2$ with stratified or mixed flow regime $a_2 = a_2'$, $D =$ hydraulic depth of gravity flow, and $\bar{a} = a_2'$.

The characteristic directions may be stated as

$$\alpha' = \frac{dt}{dx} = \frac{1}{V+a} = \frac{1}{\bar{a}}$$

(3.43)

$$\beta' = \frac{dt}{dx} = \frac{1}{V-a} = \frac{1}{\bar{a}}$$

(3.44)

since $|\bar{a}| >> |V|$, therefore, $\Delta t = (\Delta t)_{\alpha'} = (\Delta t)_{\beta'} = \alpha'(\Delta x)_{\alpha'} = \beta'(\Delta x)_{\beta'}$. Referring to Fig. 3.6, Eqs. 3.41-3.44 can be reduced to the following finite difference form (neglecting the terms containing $\sin \Theta$ which account for the gravity effect on density and are often small compared
with the friction term):

\[
\text{[along } \alpha'] \ V(I,J+1) = V(I-1,J) - \frac{g(1-a)}{a} \ [H(I,J+1) - \\
H(I-1,J)] - \frac{f\Delta t}{2D} V(I-1,J) |V(I-1,J)|
\]  
(3.45)

\[
\text{[along } \beta'] \ V(I,J+1) = V(I+1,J) + \frac{g(1-a)}{a} \ [H(I,J+1) - \\
H(I+1,J)] - \frac{f\Delta t}{2D} V(I+1,J) |V(I+1,J)|
\]  
(3.46)

Equations 3.45 and 3.46 can be solved simultaneously to get the new values \(V(I,J+1)\) and \(H(I,J+1)\) after one time step \(\Delta t\).

In the application of Eqs. 3.45 and 3.46 the following considerations should be noted (see Figs. 3.3 and 3.6):

(a) For Length \((L_1)\):

\(N_1 = \text{No. of Nodes;}\)

\(\bar{a} = a_1 = \text{speed of sound in bubble flow;}\)

\(\alpha_a = \alpha_{a_1};\)

\((\Delta x)_1 = \text{to be selected; and}\)

\(\Delta t = \frac{(\Delta x)_1}{a_1}.\)

Initial Conditions

\(V(I,J) = V_2;\)

\(H(I,J) = \text{Level of H.G.L.}\)

\(I=1 \text{ to } (N_1+1)\)
(b) For Length $L_2$:

$N_2 =$ No. of Nodes;

$\bar{a} = a_2 =$ speed of sound in stratified flow;

$\alpha_a = a_2/a_1 = \frac{A_p - A_1}{A_p}$;

$\Delta t = \frac{(\Delta x)_1}{a_1}$ (same as for $L_1$); and

$(\Delta x)_2 = \Delta t \cdot a_2$.

Initial Conditions:

$V(I,J) = V_1$ for $I = N_1 + 2$ to $N_1 + N_2$

$V(I,J) = V_1 \frac{A_1}{A_p}$ for $I = N_1 + N_2 + 1$

$H(I,J) =$ water surface elevation for $I = N_1 + 2$ to $N_1 + N_2$

$H(I,J) = D_p + S_o \cdot L$ for $I = N_1 + N_2 + 1$.

where,

$L =$ total length of the pipe.

(c) Boundary Conditions

Upstream boundary conditions, $(I = N_1 + N_2 + 1)$:

$H_s(J+1) = H_s(J) + (Q_1 - Q'_1) \frac{\Delta t}{A_{ss}}$ \hspace{1cm} (3.47)

$H(N_1 + N_2 + 1, J+1) = H_s(J+1) \pm h_{L_1}$ \hspace{1cm} (3.48)

Substituting from Eq. 3.48 into Eq. 3.45 we get
\[ V(N_1 + N_2 + 1, J+1) = V(N_1 + N_2, J) - \frac{g(1-\alpha)}{a_2} \left[ H(N_1 + N_2 + 1, J+1) - H(N_1 + N_2, J) \right] - \frac{f_2 \Delta t}{2D_h} \frac{V(N_1 + N_2, J)}{V(N_1 + N_2, J)} \]  

(3.49)

where,

- \( H_s \) = water level at the upstream open-channel section (or manhole);
- \( Q_1 = V_1 A_1 \) = flow entering the upstream open-channel section (or manhole);
- \( Q_1' = V(N_1 + N_2 + 1, J) \cdot A_p \) = flow leaving the upstream open-channel section (or manhole) and entering the pipe;
- \( A_{ss} \) = surface area of the upstream open channel section (or manhole);
- \( h_{L1} \) = head loss between the upstream open-channel section and the pipe;

where,

\[ h_{L1} = -0.4 \left[ (V(N_1 + N_2 + 1, J))^2 - V_{ss}^2 \right] / 2g \]  

(3.50)

or,

\[ h_{L1} = +0.2 \left[ (V(N_1 + N_2 + 1, J))^2 - V_{ss}^2 \right] / 2g \]  

(3.51)

in which,

\[ V_{ss} = \frac{[Q_1 - V(N_1 + N_2 + 1, J) \cdot A_p]}{A_{ss}} \]  

(3.52)

Equation 3.50 is used if the flow at the previous time step (J) was entering the pipe; and Eq. 3.51 is used if
the flow at the previous time step (J) was entering the upstream open-channel section (or manhole).

**Downstream Boundary Conditions (I-1)**

\[ S(J+1) = S(J) + \left( Q_{in} - Q_{out} \right) \frac{At}{A_s} \]  

(3.53)

in which,

- \( S \) = water level at the sump well;
- \( Q_{in} \) = flow from the pipe to the sump well;
- \( Q_{out} \) = pumping rate from the sump well, and
- \( A_s \) = surface area of the sump well.

\[ H(1,J+1) = S(J+1) + h_{L2} \]  

(3.54)

where,

\[ h_{L2} = \text{head loss between the pipe and sump well;} \]

\[ h_{L2} = + k_L [V(1,J) - (V(1,J) \cdot A_p)/A_s - Q_{out}/A_s]^2/2g \]  

(3.55)

or,

\[ h_{L2} = -0.40[V(1,J)]^2/2g \]  

(3.56)

where,

\[ k_L = 1.50 \left[ 1.0 - \frac{A_p}{A_s} + \frac{Q_{out} \cdot A_p}{A_1 \cdot V_1 \cdot A_s} \right]^2 \]  

(3.57)

\( k_L \) is the local loss coefficient.

Equation 3.55 is used if the flow at the previous time step (J) was entering the sump well, and Eq. 3.56 is used if the flow at the previous time step (J) was
entering the pipe from the sump well.

(d) At Surge-Front \( I = N_1 + 1 \)

At the surge front or discontinuity, Eqs. 3.45 and 3.46 can be solved simultaneously for the same time step \( \Delta t \) at both sides of the surge front, where Eq. 3.45 is used with the considerations mentioned before for length \( L_1 \) and Eq. 3.46 is used with the previous conditions for the length \( L_2 \). Solving Eqs. 3.45 and 3.46 simultaneously yields,

\[
H(N_1 + 1, J + 1) = \left(1.0 / \left[g\left(\frac{a_1}{a_1} + \frac{a_1}{a_2}\right)\right]\right) \cdot \frac{1-a_1}{a_1} \cdot \frac{1-a_2}{a_2} \cdot \left[\frac{a_1}{a_1} \cdot H(N_1, J) + \frac{a_2}{a_2} \cdot H(N_1 + 2, J) + \right.
\]

\[
\left(\frac{f_1 \Delta t}{2D_p} \cdot V(N_1, J) \right) \cdot \frac{1}{|V(N_1, J)|} + \left(\frac{f_2 \Delta t}{2D_h} \cdot V(N_1 + 2, J) \right) \cdot \frac{1}{|V(N_1 + 2, J)|} \right]
\]

\[
V(N_1 + 1, J) = \left(g(1-a_1) \right) \cdot \frac{1-a_1}{a_1} \cdot (H(N_1 + 1, J + 1) - \left(\frac{f_1 \Delta t}{2D_p} \cdot V(N_1, J) \right) \cdot \frac{1}{|V(N_1, J)|} \right)
\]

\[
(3.58)
\]

\[
H(N_1 + 1) = \left(\frac{f_1 \Delta t}{2D_p} \cdot V(N_1, J) \right) \cdot \frac{1}{|V(N_1, J)|} \right)
\]

\[
(3.59)
\]
3.5.4 Approximate Solution of Transients in Sewers

An approximate equation for the surcharge head following rapid pressurization of the pipeline is

$$\Delta H = \frac{\tilde{a}(V_1 - V_2)}{q} (1 - \alpha)$$  \hspace{1cm} (3.60)

where $V_1$ and $V_2$ are shown in Figs. 1.1 and 3.2.

In a two-phase system, the propagation speed of the pressure waves or speed of sound, $\tilde{a}$, depends on the air fraction in the pipe, $\alpha$, the flow regime, and the elastic characteristics of the pipe and backfill. The following equation for the speed of sound, $\tilde{a}$, is obtained from Wylie and Streeter (33):

$$\tilde{a} = \frac{\sqrt{K'/\rho'}}{\sqrt{1 + (\frac{E}{e})(\frac{K'}{E})}}$$  \hspace{1cm} (3.61)

where,

$$K' = (\alpha)^2 \rho'$$  \hspace{1cm} (3.62)

in which,

- $K'$ = effective bulk modulus of air-water mixture;
- $E$ = modulus of elasticity of the pipe;
- $a$ = effective speed of sound in two-phase (air-water) flow in a rigid pipe;
- $\rho'$ = density of air-water mixture; and
- $e$ = pipe wall thickness.
Equations 2.15, 2.16, and 2.17 can be used to calculate the speed of sound in the pipe $\bar{a}$ for bubble, stratified, and slug flow regimes respectively.

3.6 **Trapped Air Release**

The following section is dealing only with the treatment of trapped air in the pipe in the form of one bubble; however when the air is suddenly released at the upstream end of the pipe it will initiate another pressure wave in the pipe and the transients can be analyzed in a similar manner as in section 3.5.

Applying Newton's second law of motion on the water mass inside the pipe (see Fig. 2.12 for notations) just after the formation of the bubble, yields

$$F = ma \hspace{1cm} (3.63)$$

$$-A[(P_i - P_{at}) + \gamma h_b] - h_f \gamma A = A \frac{\bar{\ell}^2}{g} \gamma \frac{d^2 s_p}{dt^2} \hspace{1cm} (3.64)$$

where,

$A =$ area of flow;

$P_{at} =$ atmospheric pressure;

$P_i =$ pressure of air inside the bubble during the compression of the bubble;

$\bar{\ell} =$ distance of the bubble from the upstream end of the pipe;
\[ h_b = \text{height of the bubble}; \]
\[ h_f = \text{pressure drop due to the friction, and} \]
\[ S_b = \text{distance measured along the bed direction}. \]

Substituting with,
\[ h_f = \frac{f}{S g h} \left( \frac{P}{P_0} \right) \left( \frac{\partial P}{\partial t} \right) \left| \frac{\partial P}{\partial t} \right|; \]
\[ h_b = S_b \left( \frac{A}{A_b} \right), \text{ and} \]
\[ P_i = P_0 \left( \frac{\psi_0}{\psi} \right) = \frac{P_0}{(1 - S_b \frac{A}{A_b})^{\frac{1}{c}}}; \]

in Eq. 3.64 and simplifying
\[ \frac{d^2 S_b}{dt^2} + \frac{f}{S g h} \left( \frac{dS_b}{dt} \right) \left| \frac{dS_b}{dt} \right| + \frac{Ag}{A_b} s_b - \frac{P_{at g}}{\gamma \ell} + \frac{P_0 g}{\gamma \ell} \]
\[ = 0 \quad (3.65) \]

where,
\[ \psi_0 = \text{volume of trapped air when the bubble formed}; \]
\[ P_0 = \text{initial absolute air pressure when the bubble formed}; \]
\[ \psi = \text{volume of bubble during the compression}; \]
\[ A_b = \text{contact area between the bubble and the water, and} \]
\[ \frac{1}{c} = \text{constant between 1 and 1.4 for isothermal and adiabatic conditions, respectively}. \]
Neglecting the friction, Eq. 3.65 can be written as:

\[
\frac{d^2 S_b}{dt^2} + B_1 S_b + B_2 + \frac{B_3}{(1 - B_4 S_b)} = 0 \quad (3.66)
\]

where,

\[
B_1 = \frac{gA}{A_b \ell} ;
\]

\[
B_2 = \frac{p - g}{\gamma \ell} ;
\]

\[
B_3 = \frac{p - g}{\gamma \ell}, \quad \text{and}
\]

\[
B_4 = \frac{A}{\gamma \ell}.
\]

Equation 3.66 can be approximated to

\[
\frac{d^2 S_b}{dt^2} + B_1 S_b + B_2 + B_3 (1 + c B_4 S_b) = 0 \quad (3.67)
\]

or

\[
\frac{d^2 S_b}{dt^2} + B_5 S_b + B_6 = 0 \quad (3.68)
\]

where,

\[
B_5 = B_1 + c B_3 B_4, \quad \text{and}
\]

\[
B_6 = B_2 + B_3.
\]

Assume \(u = B_5 S_b + B_6\) then Eq. 3.68 becomes

\[
\frac{d^2 u}{dt^2} + B_5 u = 0 \quad (3.69)
\]
The solution of Eq. 3.69 is

\[ u = D_1 \cos \sqrt{B_5} t + D_2 \sin \sqrt{B_5} t \]  \hspace{1cm} (3.70)

at \( t = 0 \)

\[ \frac{ds_b}{dt} = V_0 \quad \text{and} \quad s_b = s_{b_0} \]

\[ D_1 = B_5 s_{b_0} + B_6 \quad \text{and} \]

\[ D_2 = \sqrt{B_5} V_0 \]

substituting the values of \( u \), \( D_1 \) and \( D_2 \) in Eq. 3.70

\[ s_b = (s_{b_0} + \frac{B_6}{B_5}) \cos \sqrt{B_5} t + \frac{V_0}{\sqrt{B_5}} \sin \sqrt{B_5} t - \frac{B_6}{B_5} \]  \hspace{1cm} (3.71)

Equation 3.71 represents the general solution for the flow movement after bubble formations. Variation in piezometric pressure with the time is given by

\[ z_b = \frac{P_i - P_{at}}{\gamma} = \frac{P_0}{\gamma(1 - S_b \frac{A}{A_0})} - \frac{P_{at}}{\gamma} \]  \hspace{1cm} (3.72)

where \( s_b \) is calculated at time \( t \) using Eq. 3.71.

3.7 Surcharged Flow in Sewers

In the case of surcharged flow in sewers, the flow cross-sectional area is constant and equal to the full pipe
area $A_p$.

The following equation presented by Colyer (9) (see article 2.6 for details) can be used to analyze the surcharged flow in sewers:

$$\frac{Q_s}{Q_0} = \sqrt{1 + \frac{\Delta V}{\Delta h}} \quad (3.73)$$

Yeš (43) gave the following equation (see article 2.6 for details) which can be used also for surcharged flow in sewers:

$$K^2 Q^2 = H_u - H_d \quad (3.74)$$

where $K^2$ can be estimated using Eq. 2.46 or Eq. 2.47.

In general the previous equations are solved numerically. In this study a simplified model is formulated to simulate approximately the surcharge flows in sewers.

Iterative Hardy-Cross or pipe network type of solutions have also been used in the single phase full pipe flow case. Details are given by Imam (18) and Wood (41).
CHAPTER IV

EXPERIMENTAL STUDIES

4.1 Introduction

The main purpose of the experimental program of this study, was to provide design information on sewer systems subjected to pressure transients. Of special interest was the instability of the air-water interface and the maximum possible surcharge heads due to transients. Another purpose was to provide data for the evaluation and verification of a mathematical model for the transition of gravity to surcharged flow in sewers.

This chapter contains a description of the laboratory equipment, the experimental procedure, the experimental results, and the sources of the experimental errors.

4.2 The Experimental Apparatus

The experimental study of the formation of surges in sewers and the associated transient (waterhammer) pressures was carried out using the apparatus shown in Fig. 4.1 and photographs 4.1, 4.2, and 4.3. It consisted of an interchangeable 12.03 m plexiglass conduit (rectangular, 140 mm by 140 mm or circular with 152 mm diameter).
The discharge was supplied by a centrifugal pump through a head tank. The conduit represented a sewer discharging to a sump which had a gate controlled outlet and could be used to simulate an in-line pumping station. The cross-sectional area of the sump could be varied. A rapid closing gate (shown in Fig. 4.1 and in details in Fig. 4.2) could be used to completely block the outflow from the sewer to simulate a sudden closure of a control valve.

A removable air tight cover was provided with an opening connected to flexible air hose, 12.7 cm in diameter (see photograph 4.4), which was connected to a blower with a circular-disc valve (see photograph 4.5) to control the air flow to the gravity sewer model. A venturi with a pitot-static tube, shown in photograph 4.5, was connected to the suction pipe of the blower for the air flow measurements.

A centrifugal pump having a rated discharge of 220 L/s and rated head of 6.7 m was used to deliver the flow to the overhead tank by means of 10.2 cm diameter pipeline. The flow rate was measured by an electromagnetic flow meter calibrated to approximately ±1.25 L/s for high flows. The flow was regulated by a control valve on the feed pipe. Nine piezometers (see Fig. 4.3) were located along the pipe length to trace the hydraulic grade line in the pipe. Two pressure transducers,
connected to points 1 and 5 for the circular conduit (see Fig. 4.3), or points 1 and 2 for the rectangular conduit (see Fig. 4.1), along with 2 strain indicators (A) and (B) were used for the measurement of pressure transients which were recorded by using a two channel strip chart recorder (see Fig. 4.3 and photograph 4.1).

The conduit was connected at the upstream end to an open-channel section (see Figs. 4.1 and 4.4) with a transition section to provide smooth entrance conditions for the flow. For the rectangular conduit a small steel tank representing a manhole (see photograph 4.1) was also used to replace the open-channel section.

A Sony video recording system was used to record the surge movement in the conduit. A 16 mm Bolex movie camera was also used for the same purpose.

More details are given in Appendix (B) about the measuring instruments.

4.3 Experimental Procedures

4.3.1 Introduction

The experimental study includes 3 types of tests:

a) tests to simulate a pump failure at the sump well;

b) tests to simulate a sudden closure of a control valve at the downstream end of the conduit;

c) tests to simulate a two-phase (air-water) counter-current flow.
A description of the following is also given:

a) setting and measuring the bed slope of the pipe;
b) calibration of the measurement equipment; and
c) checking and measuring leakage at the outflow of the sump well.

4.3.2 Experimental Simulation of a Pump Failure at the Sump Well:

Following the "pump failure" at the sump the water level (S) rises in the sump well producing a surge which travels with a velocity \( V_w \) from the sump location to the upstream end of the conduit. These tests were carried out to study the surge velocities (\( V_w \)) and surcharge heads (\( Z \)), as well as the water level (S) at the sump well with the time (t) for different Sump/pipe area ratios (SPR).

A typical test was carried out as follows:

1) A movable vertical partition was fixed inside the sump well, as shown in Fig. 4.1, for the desired Sump/pipe area ratio.

2) The pump was started, and the flow was delivered to the head tank and the test conduit.

3) The desired flow rate was established with the help of the control valve on the feed pipe.

4) the initial water surface elevation in the pipe
was traced using the nine piezometers located along the pipe length (see Fig. 4.3).

5) The controlled outflow, shown in Fig. 4.1, was closed to simulate a pump failure at the sump well.

6) The water level at the sump (S) was recorded with time (t) using stop watches and a piezometric tube placed on the side of the sump well.

7) The surcharge head (Z) at points 1 and 5 for the circular conduit, shown in Fig. 4.3, or at points 1 and 2 for the rectangular conduit, shown in Fig. 4.1, was recorded on two channel strip chart recorder using two pressure transducers (see photograph 4.6).

8) The circular pipe was divided into 7 reaches between points 2 and 9 (see Fig. 4.3), and the rectangular pipe was divided into 7 equal reaches of 1.83 m each, both pipes were equipped by piezometric tubes fixed on vertical scales at the ends of these reaches. The surge velocity, \( V_w \), was obtained by using a stopwatch to measure the transit time of the surges, over the seven reaches of the pipe.

9) At high relative depths of flow, the air being exhausted in front of the travelling surge caused blockages in the pipe and the subsequent transient pressures for these cases were recorded using the recording system shown in Fig. 4.3 and photograph 4.6.
10) The test was repeated for different discharges, different sump sizes, different bed slopes and for both rectangular and circular pipes.

4.3.3 Experimental Simulation of a Sudden Closure of a Control Valve

After an abrupt closure of the gate located at the downstream end of the pipe (shown in Fig. 4.1) a surge is produced which travels upstream in the pipe with either a stable air-water interface or with unstable air-water interface which may lead to severe pressure transients in the system.

A typical test was carried out as follows:

1) The pump was started, the desired flow rate was established, and the initial water surface elevation in the pipe was traced, as explained previously (Article 4.3.2 - Step No. 2, 3, and 4 respectively).

2) The rapid closing gate (shown in Fig. 4.1) was abruptly closed using the manual closure mechanism shown in Fig. 4.2, to simulate a sudden closure of a control valve at the downstream end of the pipe.

3) The surcharge head \( Z \) and the surge velocity, \( V_w \), were measured as explained before (Article 4.3.2 - Step No. 7 and 8 respectively).

4) In case of air-water interface instability,
the resulting pressure transients in the pipe were recorded using the recording system shown in Fig. 4.3 and photograph 4.6.

5) The test was repeated for different discharges, different bed slopes, and for both rectangular and circular pipes.

4.3.4 Experimental Simulation of Two-Phase Air-Water Flow in Sewers

These tests were conducted to study the instability of the air-water interface for open-channel flow in sewers.

A typical test was carried out as follows:

1) The pump was started, the desired flow rate was established, and the initial water surface elevation in the pipe was traced as explained previously (Article 4.3.2 - Steps No. 2, 3, and 4 respectively). The average water flow area, \( A_1 \), was computed.

2) The controlled outflow at the sump, shown in Fig. 4.5, was submerged to prevent the air from escaping through it during the test.

3) The air flow was passed through the sump (with the sealed cover in place) into the downstream end of the conduit to simulate a counter-current air-water flow case (see Fig. 4.5).

4) The air flow, \( Q_a \), was fixed using the valve on
the delivery pipe of the blower, shown in Fig. 4.5, and the average air velocity, \( V_a \), was computed as \( \frac{Q_a}{A_p - A_l} \).

5) The air-water interface was noted and wave dimensions were recorded.

6) The air flow was adjusted to produce an interface instability at which the wave crests would likely touch the crown of the pipe blocking the air flow and producing a surge generally moving upstream in the pipe.

7) The relative velocity of the air \( (|V_a|^2 + |V_l|^2) \) for this counter-current air-water flow at which the interface instability occurred was recorded.

8) The test was repeated for different discharges, different bed slopes, and for both rectangular and circular pipes.

4.3.5 Slope Adjustment of the Pipe

The following steps were followed to adjust the pipe slope:

1) The pipe was placed first on the flange of a continuous steel beam (W150X22) which was nearly horizontal.

2) The pipe was divided into seven reaches, shown in details in Fig. 4.3, and at the ends of each reach, piezometer tubes were fixed at the side of the pipe.

3) The level of the downstream tank (sump well)
was adjusted by means of 4 screws at the corners supported on 4 steel plates, as shown in Fig. 4.6, resting on the wooden platform designed to carry the tank.

4) The rapid closing gate shown in Fig. 4.1 was used to close the downstream end of the pipe, and the pipe was partially filled with water to produce an accurate horizontal reference plane for the slope adjustment.

5) The slope of the pipe was adjusted by adjusting the slope of the steel beam supporting the pipe, with the help of adjustable steel posts placed at about every 3 m under the steel supporting beam (see photograph 4.7). The posts were fixed to the laboratory ground and were connected to the supporting beam with 25 mm screw for slope adjustment.

**EXAMPLE:** For a certain slope, the difference between the bottom levels at the ends of the first reach was calculated, and the upstream end of this reach was elevated (using the screws of the steel posts) to have the exact slope by subtracting the depth of water at the upstream end from that at the downstream end, and comparing that difference with that desired.

6) This procedure was repeated with the rest of the reaches, and the difference between the bottom levels of the pipe at the downstream end tank and the end of each reach was checked, to obtain the most accurate measurement.
of the slope for the whole pipe.

7) The same procedures were repeated to change the slope for each series of tests.

4.3.6 **Calibration of the Measurement Equipment**

**Flowmeter**

The pump was started for a few minutes to stabilize the flow meter, and then the flow was stopped to determine the zero reading, which may be different from that reading before starting the pump. The zero reading was subtracted from the subsequent flow meter readings to get the proper discharges.

The magnetic flow meter used in the experimental work was calibrated for two slopes. Red dye was injected in the upstream portion of the pipe and the dye velocity was measured for different depths and flow meter readings. The discharge was computed from the following equation:

\[ Q = AV_d \tag{4.1} \]

where,

- \( A \) = average measured flow area along the downstream three reaches of the pipe (see Fig. 4.3), the depths were recorded at points 2, 3, 4 and 5, and the average area \( A \) was calculated based on the average depth at these points;
- \( V_d \) = measured dye velocity, as the average of three
measurements, within the downstream last three reaches of the pipe.

Figure 4.7 shows the calculated discharges using Eq. 4.1, against the flow meter readings for pipe slope, $S_o = 0.0018$ used in the experimental study.

**Pressure Transducers and Strain Indicators**

Two strain indicators, denoted (A) and (B), each connected with a pressure transducer, were used to measure the pressure head ($H$), (see Fig. 4.3). The strain indicators (A) and (B) each connected to a pressure transducer placed at points 1 and 5 respectively, were adjusted to have zero readings when there was no flow in the pipe, i.e., the strain indicators were measuring the atmospheric pressure. The two strain indicators were connected to 2 channel strip recorder as shown in Fig. 4.3. The suitable ranges for the pressure head recording were chosen at about 2 m static water head at points 1 and 5 ($= 200$ mm higher than the maximum possible pressure head in the model).

The readings from the two strain Indicators (A) and (B) shown in Fig. 4.3 (in volts) were plotted for various static water heads in Fig. 4.8. This represents the calibration curves of the readings of the two strain indicators used to record the pressure head ($H$).
4.3.7 Measurement of Leakage at the Outflow of the Sump Well

During the pump failure simulation tests there was a leakage at the outflow of the sump well (see Fig. 4.1) which was measured as follows:

1) The outflow was closed (by the outflow steel gate).

2) The sump was filled by water through the model conduit up to a level of 90 cm from the bottom of the sump which was horizontal.

3) The rapid closing gate was closed.

4) A stopwatch was used to record the time for the water level to drop from level 80 cm above the bottom of the sump well to the 50 cm level (which was the range used in these tests).

5) The rate of leakage was calculated as the volume of water between the two previous levels divided by the time for the water to fall to the lower level.

6) The previous steps were repeated ten times, and the rate of leakage was considered as the average of these ten readings.

4.4 Ranges of Experimental Measurements

The ranges of the experimental data are as follows:

Relative Depth of Water Flow, \( y_r = y_1 / D_0 \) = 0.25 to 1.0.
Air Flow Rate, $Q_a = 0$ to $100 \text{ L/s}$

Sump/Pipe Area Ratio, $\text{SPR} = 0$ to $29.8$

Bed Slope, $S_o = 0.00109$ to $0.0066$

Froude Number, $F_r = 0.5$ to $1.8$

Reynold's Number, $R_n = 8.5 \times 10^4$ to $2.35 \times 10^5$

Relative Elevation of Water at Sump Well,

$S/D_p = 0$ to $5.4$

Relative Length of Pipe, $L/D_p = 80$

Pipe Shapes: Rectangular (140 mm by 140 mm)

Circular ($D_p = 152$ mm)

4.5 Experimental Results

4.5.1 Pump Failure at the Sump Well

Tables 4.1 and 4.2 show the surge velocities along the seven reaches of the circular conduit following the pump failure at the sump well for bed slope, $S_o = 0.00109$, for different relative depths of flow, $y_r = y_1/D_p$, and for Sump to Pipe area ratios (SPR) of 4.46 and 29.8 respectively. Tables 4.3 and 4.4 show these results for bed slope, $S_o = 0.0018$. Tables 4.5 and 4.6 are for bed slope, $S_o = 0.005$.

Figures 4.9 to 4.12 show the change in the sump water level with time - STA.(1) - and pressure transients recorded at STA.(5) for the circular conduit and for bed slope, $S_o = 0.00109$, Sump/ Pipe area, SPR = 4.46 and relative depths of flow, $y_r = y_1/D_p$ of 0.69, 0.76, 0.80
and 0.87 respectively.

Figures 4.13 to 4.15 show the changes in the sump water level with time at STA.(1) and pressure transients recorded at STA.(5) for the circular conduit and for bed slope, $S_0 = 0.0018$, Sump/Pipe area, SPR = 4.46 and relative depths of flow, $y_r = y_1/D_p$ of 0.67, 0.79 and 0.85 respectively.

Figures 4.16 to 4.19 show the change in the sump water level with time at STA.(1) and pressure transients recorded at STA.(5) for the circular conduit and for bed slope, $S_0 = 0.005$, Sump/Pipe area, SPR = 4.46 and relative depths of flow, $y_r = y_1/D_p$ of 0.58, 0.67, 0.71 and 0.75 respectively.

Figures 4.20 and 4.21 show the change in the sump water level with time at STA.(1) and pressure transients recorded at STA.(5) for the circular conduit and for bed slope, $S_0 = 0.00109$, Sump/Pipe area, SPR = 29.8 and relative depths of flow, $y_r = y_1/D_p$ of 0.74 and 0.88 respectively.

Figures 4.22 and 4.23 show the change in the sump water level with time at STA.(1) and pressure transient recorded at STA.(5) for the circular conduit and for bed slope, $S_0 = 0.0018$, Sump/Pipe area, SPR = 29.8 and relative depths of flow, $y_r = y_1/D_p$ of 0.69 and 0.89 respectively.
Figures 4.24 and 4.25 show the pressure transients recorded at point 2 (shown in Fig. 4.1) for the rectangular conduit and for bed slope, $S_o = 0.00109$, Sump/pipe area, $SPR = 4.13$ and relative depths of flow, $y_r = y_1/D_p$ of 0.74 and 0.84 respectively.

Photograph 4.8 shows a typical surge travelling in the rectangular conduit following the pump failure at the sump well for relative depth of flow, $y_r = y_1/D_p = 0.54$, bed slope, $S_o = 0.0033$, and $SPR = 4.13$. Photograph 4.9 shows a typical surge travelling in the circular conduit following the pump failure at the sump well for relative depth of flow, $y_r = y_1/D_p = 0.75$, bed slope, $S_o = 0.005$, and $SPR = 29.8$.

4.5.2 Sudden Closure of a Control Valve at the Downstream End of the Pipe

Tables 4.7, 4.8, and 4.9 show the surge velocity, $V_w$, along the circular conduit following the sudden closure of the downstream end control gate for different relative depths of flow, and for bed slope, $S_o$ of 0.00109, 0.0018, and 0.005 respectively.

Figures 4.26 to 4.29 show the pressure recording at points 1 and 5, shown in Fig. 4.3 following the sudden closure of the downstream end of the circular conduit of bed slope, $S_o = 0.00109$, and for relative depths of flow,
\[ y_r = y_1/D_p \] of 0.64, 0.76, 0.83 and 0.87 respectively.

Figures 4.30 to 4.32 show the pressure recording at points 1 and 5 following the sudden closure of the downstream end of the circular conduit of bed slope, \( S_o = 0.0018 \), and for relative depths of flow, \( y_r = y_1/D_p \) of 0.67, 0.79, and 0.86 respectively.

Figures 4.33 to 4.35 show the pressure recording at points 1 and 5 following the sudden closure of the downstream end of the circular conduit of bed slope, \( S_o = 0.005 \), and for relative depths of flow, \( y_r = y_1/D_p \) of 0.72, 0.75, and 0.79 respectively.

Figures 4.36 and 4.37 show the pressure recording at points 1 and 2, shown in Fig. 4.1, following the sudden closure of the gate at the downstream end of the rectangular conduit of bed slope, \( S_o = 0.00109 \) and for relative depths of flow, \( y_r = y_1/D_p \) of 0.66 and 0.77 respectively.

The relative surge velocities and relative surcharge heads resulting from a sudden closure of the downstream end of a pipe of rectangular cross section are presented in Fig. 2.7 and 2.8.

Photograph 4.10 shows a typical surge travelling in the circular conduit following the sudden closure of the downstream end control gate, at relative depth of flow, \( y_r = y_1/D_p = 0.60 \).
4.5.3 **Two-Phase Air-Water Flow Tests**

As the relative velocity of the air at the air-water interface increases, the interface becomes unstable with a wavy surface. The instability becomes fully developed when a wave crest touches the crown of the pipe.

Tables 4.10, 4.11, and 4.12 show the critical interface instability velocities \( \dot{V}_I = |\dot{V}_a| + |\dot{V}_l| \) in the circular conduit for different relative depths of flow, \( y_r = y_1/D_p \), and for bed slopes, \( S_0 = 0.00109 \), \( S_0 = 0.0018 \), and \( S_0 = 0.005 \) respectively.

Tables 4.13, 4.14, and 4.15 show the critical interface instability velocities \( \dot{V}_I = |\dot{V}_a| + |\dot{V}_l| \) in the rectangular conduit for different relative depths of flow \( y_r = y_1/D_p \), and for bed slopes, \( S_0 = 0.00109 \), \( S_0 = 0.0033 \), \( S_0 = 0.0066 \) respectively.

Photograph 4.11 shows an interface instability at early stage in a circular pipe of bed slope of \( S_0 = 0.00109 \), relative water depth \( y_r = y_1/D_p = 0.45 \) and relative air velocity \( (|\dot{V}_a| + |\dot{V}_l|) = 5.88 \text{ m/s} \); photograph 4.12 shows the developed interface instability for the same conditions.

Photographs 4.13 and 4.14 show the interface instability at early stage and fully developed stage respectively, for the circular pipe of bed slope, \( S_0 = 0.005 \), relative depth of water \( y_r = y_1/D_p = 0.35 \), and relative air velocity \( (|\dot{V}_a| + |\dot{V}_l|) = 8.3 \text{ m/s} \).
Photographs 4.15 and 4.16 show early stage and fully developed stage interface instability respectively for the circular pipe of bed slope, \( S_o = 0.005 \), relative depth of water \( y_r = y_1/D_p = 0.43 \), and relative air velocity \( |\vec{v}_a| + |\vec{v}_1| \) = 9.2 m/s.

Photograph 4.17 shows interface instability in rectangular pipe of bed slope, \( S_o = 0.0033 \), relative depth of water, \( y_r = y_1/D_p = 0.26 \) and relative air velocity \( |\vec{v}_a| + |\vec{v}_1| \) = 7.45 m/s.

Photographs 4.18, 4.19 and 4.20 show the effect of increasing the relative air velocity \( |\vec{v}_a| + |\vec{v}_1| \), on the interface instability in the rectangular conduit of bed slope, \( S_o = 0.0066 \) and relative depth of water \( y_r = y_1/D_p = 0.40 \).

The movies and the video recordings were used to study the formation of interface instability and the bubble formation which frequently occurred in very short time. The slow motion replay was necessary to know how the interface instability changes with time and to study the nature of bubble formation and its contraction and expansion in the conduit.

4.5.4 Leakage Rate at the Outflow of the Sump Well

The average leakage rate at the outflow of the sump well measured, as explained in Article 4.3.8, was equal to 1.5 L/s.
4.6 Experimental Errors

The sources of experimental errors in this study may be summarized as follows:

1. The errors that might occur in the measurement of the inflow discharge into the model conduit are estimated to be:

   (a) A calibration error of the electromagnetic flowmeter is ±15 U.S.G.P.M. (±0.00945 m³/s).

   (b) The error of evaluation of flowmeter reading is estimated to be 1 mm on the flowmeter scale, which represents ±12.5 U.S.G.P.M. (±0.000788 m³/s).

2. The error that might occur in each measurement of the surge transient time is estimated to be ±0.2 sec. For surge travel distance of 1.85 m with an average velocity of 2 m/s, the standard error in each measurement of \( V_w \) will be about 22%. The standard error (standard deviation) in the mean of three readings would be about 12.7%. For a surge travel distance of 12 m (the conduit length) with an average velocity of 2 m/s, the standard error in each measurement of \( V_w \) will be about 3%. The standard error in the mean of three readings would be about 2%.

3. The error that might occur in the measurement of initial flow depth \( (y_1) \) is estimated to be 2% of an average depth of flow of 76 mm.

4. The errors that might occur in the measurement of
the pressure head (H) for pressure transients or the sump water level (S) are estimated to be:

(a) A possible chart reading error of \( \pm 3\% \) of an average pressure head of 1.0 m, was assumed for the pressure transducers used to measure the pressure head (H) or the sump water level (S).

(b) A calibration error of \( \pm 1\% \) (of an average pressure head of 1.0 m) for estimating the pressure head (H) or the sump water level (S) from the pressure transducer recordings.

(c) A possible error of \( -2\% \) of a maximum pressure head of 2.0 m (one full span) for pressure transients of maximum frequency of 10 Hz due to the frequency response time of the strain indicators (A) or (B) used to measure the pressure transients.

(d) A possible error of \( -10\% \) of a maximum pressure head of 2.0 m (one full span) for pressure transients of maximum frequency of 10 Hz due to the frequency response time of the two channel chart recorder used to record the pressure transients.

5. The errors that might occur in the measurement of the air velocity in the pipe for the two-phase air-water
flow tests, are estimated to be:

(a) A possible error of -10% was assumed for the leakage of air from the flexible hose and at the sump well.

(b) The error that might occur in the measurement of air velocity by the venturimeter is estimated to be ±6%. This error is due to a possible error of 1 mm (one division) of the scale connected to the pitot-tube used to measure the air flow through the venturimeter connected to the blower (see Fig. 4.5).

6. A possible error in setting the bed slope ($S_o$) of the pipe is estimated to be ±0.5 mm per every reach of the pipe, which represents a slope error of ±0.00025.
CHAPTER V

DESCRIPTION OF COMPUTER PROGRAM
AND DOCUMENTATION

5.1 General

The model presented in this research consists of a computer program with 4 subroutines written in the FORTRAN language. The program requires approximately 200K bytes of storage. The average execution time for the program to perform its role (for one bed slope, one depth, and one sump size) is approximately 8 seconds (CPU), class B, fast core. The program was run on an IBM 3031 computer at the University of Windsor Computer Centre. A listing of the program with definition of terms in the program and subroutines is given in Appendix (F).

The model analyzes a simple case of one single conduit of rectangular or circular cross-section connected at the upstream end with a manhole, or a reservoir, and at the downstream end with a sump well with pumping facilities, as well as a control gate. The model simulates the surge movement in the conduit and the surcharge head at the surge front following the pump failure at the sump well or the sudden closure of the control gate at the downstream end.
of the pipe. The model also predicts the sump well elevation, interface instability limit and the associated pressure transients, and the pressure fluctuations resulting from the bubble formation in the pipe.

5.2 Main Program and Subroutines

Main Program

Figure 5.1 shows a flow chart for the main program. The variables that appear in the flow chart are defined in the subroutines. The subroutines that are called by the main program will be discussed in the subsequent sections.

Subroutine - PUMPFS

This subroutine is used to compute and print out the results of the pump failure simulation with a stable air-water interface. Flag statements are used to stop the computing process at the interface instability or when the surge front reaches the upstream end of the pipe (which represents a manhole or a reservoir). Figure 5.2 shows a flow chart of the subroutine. A listing of the subroutine is given in Appendix (F).

Subroutine - RK5ES

This subroutine computes the sump well water level \( S \) and the rate of rise of the sump water level at the sump well by solving Eqs. 3.29 and 3.34 simultaneously using a numerical solution technique (Runge-Kutta Method). A listing of the subroutine is given in Appendix (F).
Subroutine - Bubble

This subroutine computes and prints out the expansion and contraction of the trapped air (bubble), $S_B$, the resulting surcharge, $z_B$, in the pipe versus the time (Eqs. 3.71 and 3.72). A listing of the subroutine is given in Appendix (F).

Subroutine - TRANS

This subroutine is used to compute and print out the pressure transient results ($T, V(I,J)$ and $H(I,J)$) along the pipe. Figure 5.3 shows a flow chart of the subroutine. A listing of the subroutine is given in Appendix (F).

5.3 Input Data

The following data should be given:

(i) **Data for Sewer:** the shape of the sewer (rectangular or circular), the dimensions of the sewer, the roughness coefficient (Manning's $n$), and the bed slope.

(ii) **Data for Fluids:** densities of the flowing fluids in the sewer (water and air) at the prevailing conditions of temperature and atmospheric pressure.

(iii) **Data for Flow Rates:** inflow at the upstream manhole or reservoir. Inflow, outflow, and pumping rate at the sump well.

(iv) **General Data:** surface area of manhole or reservoir at the upstream end of the pipe, surface area of sump well, street or overflow levels at the boundaries, and estimate of air fraction in the sewer.
5.4 Output

The output of the program is a printout of the input data and one or more of the following:

(a) **Pump Failure Simulation Results**

Following the pump failure at the sump well at the downstream end of the pipe the water level rises at the sump well and a surge starts to travel upstream the pipe. If the air-water interface was stable, or, until the interface instability starts, the printout contains the following terms versus the time (t):

- The relative water level at the sump well,
  
  \[ RS(T) = \frac{S}{D_p} \]

- The relative speed of the water surface at the sump well,
  
  \[ VSR = \frac{dS}{dt} / V_1 = \frac{V_s}{V_1} \]

- The dimensionless time,
  
  \[ PIL = \frac{t V_1}{D_p} \]

- The speed of the surge relative to the water
  
  \[ VWR = (|V_w^+| + |V_1^+|) \]

- The dimensionless travelled distance by the surge front,
  
  \[ RLAR(T) = \frac{L_s}{D_p} \]

- The dimensionless speed of the surge relative to the water
  
  \[ VWR/V_1 = (|V_w^+| + |V_1^+|) / V_1 \]

- The dimensionless velocity of flow behind the surge front,
  
  \[ V2R = V_2 / V_1 \]

Figure 5.4 shows a typical printout for Pump Failure Simulation for the model of bed slope, \( S_0 = 0.00109 \), Froude
number, $F_r = 0.62$, initial relative depth of flow in the pipe $y_r = y_1/D_p = 0.69$, and for Sump to Pipe area ratio $(SPR) = 4.46$.

(b). Expansion and Contraction of Trapped Air (bubble)

Following the air-water interface instability the air is trapped in the form of one bubble or more, the printout contains the expansion and contraction of the bubble, $S_b$, computed by Eq. 3.71, and the resulting surcharge head, $Z_b$, versus the time $(t)$ along the pipe.

Figure 5.5 shows a typical printout for $t$, $S_b$, and $Z_b$ for the model of bed slope, $S_o = 0.0018$, Froude number, $F_r = 0.65$, initial relative depth of flow in the pipe, $y_r = y_1/D_p = 0.86$, and for Sump to Pipe area ratio $(SPR) = 4.46$.

(c) Pressure Transient Results

The moving surge in the pipe which may result from the pump failure at the sump well or the sudden closure of the downstream control gate, may produce interface instability which may in turn produce pressure wave (Pressure Transient) in the system. The printout is the solution of Eqs. 3.45 and 3.46 which was explained in detail in the previous chapter (Chapter IV). The values of $T$, $V(I,J)$, and $H(I,J)$ are given along the pipe length at distances computed from the initial conditions.
Figure 5.6 shows a typical printout for Transient Pressure following interface instability resulting from pump failure at the sump well for the model of pipe bed slope, $S_o = 0.005$, Froude number, $F_r = 1.03$, initial relative depth of flow in the pipe, $y_r = y_1/D_p = 0.75$, and for Sump to Pipe Area Ratio (SPR) = 4.46.

Figure 5.7 shows a typical printout for Transient Pressure following interface instability resulting from the sudden closure of the downstream end control gate of the model, for pipe bed slope, $S_o = 0.00109$, Froude number, $F_r = 0.58$, and for initial relative depth of flow in the pipe, $y_r = y_1/D_p = 0.83$. 
CHAPTER VI

ANALYSIS AND DISCUSSION OF THE EXPERIMENTAL DATA

6.1 Introduction

The experimental data for the surges and pressure transients which may result from a restriction of the downstream flow, possibly due to a pump failure at an in-line pumping station, or from the sudden closure of the downstream control gate, are used to verify the solution explained in Chapter III.

The experimental data for the air-water interface instability study are used to obtain empirical equations for the critical interface instability velocity for rectangular and circular conduits.

A discussion of the convergence and stability of the travelling hydraulic jump numerical model is presented.

6.2 Pump Failure at the Sump Well

The change in the following variables due to the restriction of outflow (e.g., due to pump failure) at the sump well will be discussed:

(a) The speed of the surge front, \( V_w \) along the pipe:
(b) the surcharge head at the surge front, $Z$;
(c) the sump/pipe area ratio (SPR); and
(d) the water level at the sump well.

The complete simulation with the relevant variables following the pump failure is given in Article 5.4 with a typical printout for the example of a pump failure simulation for the model shown in Fig. 5.4.

The discussion in this section will be related to the travelling hydraulic jumps (or surges) with stable air-water interfaces resulting from the pump failure at the sump well. The interface instability and the pressure transients which may result from the pump failure will be discussed in a subsequent section.

6.2.1 Surge Velocity, $V_w$ along the Pipe

The relative surge velocity $|\vec{V}_w - \vec{V}_1|$ is an essential parameter in determining the air-water interface instability in sewers.

Figures 6.1 and 6.2 show typical dimensionless relative surge velocities $(|\vec{V}_w| + |\vec{V}_1|)/|\vec{V}_1|$ along the circular conduit for sump/pipe area ratio (SPR) of 4.46 and 29.8 respectively for counter-current air-water flow. These figures compare some of the experimental data with the corresponding theoretical values predicted by the travelling hydraulic jump model explained in Chapter III, Article 3.3.
Table 6.1 shows the agreement between the experimental dimensionless relative surge velocities \( (\tilde{V}_w + |\tilde{V}_1|)/|\tilde{V}_1| \) and the theory (explained in Chapter III) along the circular conduit for the tests shown in Figs. 6.1 and 6.2. The degree of agreement is based on the following assumptions:

For \( (\tilde{F}) = 0.00 \): 0.30 Excellent Agreement

For \( (\tilde{F}) = 0.30 \): 0.60 Very Good Agreement

For \( (\tilde{F}) = 0.60 \): 1.00 Good Agreement

For \( (\tilde{F}) = 1.00 \): 2.00 Fair Agreement

For \( (\tilde{F}) > 2.00 \): Poor Agreement

Table 6.1 gives an average \( (\tilde{F}) \) of +3.90 along the downstream portion of the conduit \( (L_r = L_s/D_p = 0.0 \) to 40.0) which means there is a poor agreement between the theory and the experiment for the prediction of the dimensionless relative surge velocities \( (|\tilde{V}_w| + |\tilde{V}_1|)/|\tilde{V}_1| \). Table 6.1 gives an average \( (\tilde{F}) \) of -0.023 along the upstream portion of the conduit \( (L_r = L_s/D_p = 40.0 \) to 80.0) which means an excellent agreement between the theory and the experiment. A possible reason for the poor agreement along the downstream portion of the conduit may be because of the effect of gradually varied flow which was noticed.
Table 6.1

The Agreement Between the Experimental Dimensionless Relative Surge Velocities \( \left( \frac{|V_w| + |V_1|}{|V_1|} \right) \) and the Theory Along the Circular Conduit

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Bed Slope ((S_0))</th>
<th>Relative Depth ((y_r))</th>
<th>Sump/Pipe Area Ratio (\text{SPR})</th>
<th>(\frac{\varepsilon^*}{\sigma_n^{**}})</th>
<th>Degree of Agreement</th>
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<td></td>
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<td></td>
<td></td>
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<td>0.64</td>
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<td>+2.65</td>
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</tr>
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<td>+7.00</td>
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</tr>
<tr>
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<td>+5.63</td>
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</tr>
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<td>+1.40</td>
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<td>29.80</td>
<td>-5.420</td>
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</tbody>
</table>

\(\varepsilon^*\) = average difference between the experimental mean curve and the Theory
\(\sigma_n^{**}\) = deviation of experimental results from the experimental mean curve
during the experiment.

Table 6.2 shows the comparison between the experimental average dimensionless relative surge velocities \( \frac{|\ddot{v}_w| + |\ddot{v}_1|}{|\ddot{v}_1|} \) and the theory (explained in Chapter III) in the circular conduit. Table 6.2 gives an average error \( \frac{\ddot{v}_{r, \text{th}} - \ddot{v}_{r, \text{exp}}}{\ddot{v}_{r, \text{th}}} \times 100 \) of about +3.1% which is within the standard error of the experimental results. The agreement between the experimental and the theoretical dimensionless relative surge velocities \( \frac{|\ddot{v}_w| + |\ddot{v}_1|}{|\ddot{v}_1|} \) along the conduit, was based on the assumption of having uniform flow throughout the conduit, which was not the case in all tests.

Figure 6.3 compares the experimental dimensionless surge velocities \( |\ddot{v}_w| + |\ddot{v}_1|/|\ddot{v}_1| \) along the circular conduit for a bed slope \( S_o = 0.0011 \), with the theoretical model prediction (assuming uniform flow) and the theoretical model prediction (considering the gradually varied low profile recorded during the experiment). This figure shows a
Table 6.2
Comparison Between the Experimental Average Dimensionless Relative Surge
\[
\left( \frac{|\vec{V}_w| + |\vec{V}_1|}{|\vec{V}_1|} \right)_{av}
\]
and the Theory in the Circular Conduit

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Bed Slope ((S_0))</th>
<th>Relative Depth ((\gamma_R))</th>
<th>Sump/ Pipe Area Ratio ((SPR))</th>
<th>(\vec{V}_{th}^*)</th>
<th>(\vec{V}_{exp}^{**})</th>
<th>Error ***</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0011</td>
<td>0.64</td>
<td>4.46</td>
<td>2.64</td>
<td>2.49</td>
<td>+5.7</td>
</tr>
<tr>
<td>2</td>
<td>0.0018</td>
<td>0.67</td>
<td>4.46</td>
<td>2.97</td>
<td>2.80</td>
<td>+5.7</td>
</tr>
<tr>
<td>3</td>
<td>0.0050</td>
<td>0.67</td>
<td>4.46</td>
<td>2.83</td>
<td>2.64</td>
<td>+6.7</td>
</tr>
<tr>
<td>4</td>
<td>0.0011</td>
<td>0.66</td>
<td>29.80</td>
<td>2.54</td>
<td>2.40</td>
<td>+5.5</td>
</tr>
<tr>
<td>5</td>
<td>0.0018</td>
<td>0.69</td>
<td>29.80</td>
<td>2.43</td>
<td>2.53</td>
<td>-4.1</td>
</tr>
<tr>
<td>6</td>
<td>0.0050</td>
<td>0.64</td>
<td>29.80</td>
<td>1.93</td>
<td>1.95</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

\[\vec{V}_{th}^* = \left( \frac{|\vec{V}_w| + |\vec{V}_1|}{|\vec{V}_1|} \right)_{av} \] computed from theory

\[\vec{V}_{exp}^{**} = \left( \frac{|\vec{V}_w| + |\vec{V}_1|}{|\vec{V}_1|} \right)_{av} \] computed from experimental data

\[\% Error = \left( \frac{\vec{V}_{th} - \vec{V}_{exp}}{\vec{V}_{th}} \right) \times 100\]
significant improvement in the model prediction when the gradually varied flow profile was considered in the case of the circular conduit of bed slope, \( S_0 = 0.0011 \).

The corrected model (assuming gradually varied flow), shown in Fig. 6.3 resulted in an average \( \frac{c}{\sigma_n} \) of 1.90 which means a fair agreement between the experiment and the corrected model; compared with an average \( \frac{c}{\sigma_n} \) of 2.81 which means a poor agreement between the experiment and the model (assuming uniform flow throughout the conduit).

Figure 6.4 shows the travelling time for the surge between the downstream and the upstream ends of the circular conduit for different relative depths of flow, bed slopes, and sump sizes. Figure 6.4 gives \( \frac{c}{\sigma_n} = -0.72 \), which means good agreement between the experimental travelling time of the surge along the whole conduit and the theory (explained in Chapter III). \( c \) represents the average difference between the experimental mean curve and the theory, and \( \sigma_n \) represents the deviation of experimental results from the experimental mean curve.

6.2.2 Surcharge Head at the Surge Front \( \{Z\} \)

Figure 6.5 shows typical model prediction (explained in Chapter III) for the relative surcharge head at the surge front \( \{Z/D_p\} \) along the circular conduit, and the experimental corresponding \( \{Z/D_p\} \) at point 5 (shown in Fig. 4.3). Figure 6.5 shows that the relative surcharge head at the
surge front \((Z/D_p)\) increases rapidly for all slopes and up to \(L_r=40D_p\) and then remains more or less constant or decreases slightly. Figure 6.5 shows also that \((Z/D_p)\) increases with increasing bed slope \((S_o)\) of the conduit. This is mainly due to the increase of the initial momentum of the flow – and the increasing initial flow which results in a more rapid increase of the water level at the sump well \((S)\) and the relative surcharge head \((Z/D_p)\) at the front of the travelling surge. The experimental data are in good agreement with those predicted by the model (explained in Chapter III).

6.2.3 The Sump/Pipe Area Ratio (SPR)

Figures 6.6 and 6.7 show the values of surge velocities \(V_w\) and surcharge head \(Z\) at the surge front at the first peak, for different Sump/Pipe Area Ratio (SPR), after a pump failure induced surge. The measured experimental values of \(V_w\) and \(Z\) were recorded before the occurrence of the air-water interface instability. Figures 6.6 and 6.7 indicate the attenuating effect of the sump to pipe area ratio, \(SPR=A_s/A_p\) on \(V_w\) and \(Z\). Figures 6.6 and 6.7 show that the travelling hydraulic jump model gives surge velocities \((V_w)\) and surcharge heads \((Z)\) within the standard error of the experimental results.

A comparison of the experimental data and the theory (explained in Chapter III), shows that the relative surge
velocities \( |\vec{v}_w| + |\vec{v}_1| \) increase as the sump size decreases for rectangular or circular conduits of any bed slope, \( S_o \). If the resulting relative surge velocity \( (|\vec{v}_w| + |\vec{v}_1|) \) exceeds the critical interface instability velocity, \( V_i \), the air-water interface becomes unstable. Also, the surcharge head \( (z) \) at the surge front increases with decreasing sump size as shown in Fig. 6.7.

6.2.4 The Water Level at the Sump Well \((S)\)

Figures 4.9, 4.13, and 4.16 compare the experimental sump water level \((S)\) with the theoretical model prediction for a sump to pipe area ratio \((SPR)\) of 4.46 and for conduit bed slopes \((S_o)\) of 0.0011, 0.0018 and 0.0050 respectively. Figures 4.20 and 4.22 compare the experimental sump water level \((S)\) with the theoretical model prediction for sump to pipe area ratio \((SPR)\) of 29.8 and for conduit bed slopes \((S_o)\) of 0.0011 and 0.0018 respectively. These figures show that the theory gives sump water levels \((S)\) within the standard error of the experimental result for the circular model conduit, for different bed slopes, \( S_o \), and for two sump sizes.

6.3 Sudden Closure of the Downstream Control Gate

The discussion in this section will be related to the surges with stable air-water interface resulting from the sudden closure of the downstream control gate. The inter-
face instability and the pressure transients which may result from this sudden closure will be discussed later.

6.3.1 The Surge Velocity \(v_w\)

Tables 6.3 and 6.4 show that the theory (Eq. 3.15) gives surge velocities \(v_w\) within the standard error of the experimental results.

In Tables 6.3 and 6.4, \(\varepsilon\) represents the average difference between the experimental mean curve and the theoretical curve based on Eq. 3.15; \(q_m\) represents the deviation of the mean of the experimental results.

Figure 6.8 compares the experimental data and the theoretical dimensionless relative surge velocities \(\left(\frac{|\bar{V}_w| + |\bar{V}_1|}{|\bar{V}_1|}\right)\), based on Eq. 3.15, throughout the circular conduit for different bed slopes. This figure shows that \(v_w\) increases as the bed slope increases and as the relative depth of flow increases.

6.3.2 Surcharge Head \(z\) at the Surge Front

Tables 6.5 and 6.6 show that the theory (Eq. 3.16) gives surcharge heads \(z\) at the surge front within the standard error of the experimental results providing there is no interface instability.

The Theory (Eq. 3.16) and experiments (Fig. 6.9) show that the surcharge heads \(z\) at the surge front increase when the bed slope, \(S_0\), increases, which is
Table 6.3

The Agreement Between the Experimental Surge Velocities ($V_W$) and the Theory (Eq. 3.15) in the Rectangular Conduit for Different Slopes

<table>
<thead>
<tr>
<th>Bed Slope ($S_o$)</th>
<th>$\left(\frac{e}{\sigma_n}\right)$</th>
<th>Degree of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>+0.93</td>
<td>Good</td>
</tr>
<tr>
<td>0.0033</td>
<td>-0.42</td>
<td>Very Good</td>
</tr>
<tr>
<td>0.0061</td>
<td>+0.18</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

Table 6.4

The Agreement Between the Experimental Surge Velocities ($V_W$) and the Theory (Eq. 3.15) in the Circular Conduit for Different Slopes

<table>
<thead>
<tr>
<th>Bed Slope ($S_o$)</th>
<th>$\left(\frac{e}{\sigma_n}\right)$</th>
<th>Degree of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>-0.12</td>
<td>Excellent</td>
</tr>
<tr>
<td>0.0018</td>
<td>-0.55</td>
<td>Very Good</td>
</tr>
<tr>
<td>0.0050</td>
<td>-0.90</td>
<td>Good</td>
</tr>
</tbody>
</table>
Table 6.5
The Agreement Between the Experimental Surcharge Heads (Z) at the Surge Front and the Theory (Eq. 3.16) in the Rectangular Conduit for Different Slopes

<table>
<thead>
<tr>
<th>Bed Slope ( (S_o) )</th>
<th>( \frac{\epsilon -}{\sigma_n} )</th>
<th>Degree of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>+0.92</td>
<td>Good</td>
</tr>
<tr>
<td>0.0033</td>
<td>+0.88</td>
<td>Good</td>
</tr>
<tr>
<td>0.0061</td>
<td>+0.56</td>
<td>Very Good</td>
</tr>
</tbody>
</table>

Table 6.6
The Agreement Between the Experimental Surcharge Heads (Z) at the Surge Front and the Theory (Eq. 3.16) in the Circular Conduit for Different Slopes

<table>
<thead>
<tr>
<th>Bed Slope ( (S_o) )</th>
<th>( \frac{\epsilon -}{\sigma_n} )</th>
<th>Degree of Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>+0.51</td>
<td>Very Good</td>
</tr>
<tr>
<td>0.0018</td>
<td>+0.35</td>
<td>Very Good</td>
</tr>
<tr>
<td>0.0050</td>
<td>+0.85</td>
<td>Good</td>
</tr>
</tbody>
</table>
expected because of the accompanying increase in the initial flow velocity $V_1$.

The experimental results in all cases are consistently higher than the theory (Eq. 3.16). A possible reason for this difference is a higher than the assumed atmospheric pressure in front of the surge front.

Hamam (13) reported that the blockage of the air vents (at the upstream end of a rectangular conduit) can cause an increase of about 25% in the value of $Z$ as calculated by Eq. 3.16.

6.4 Pressure Transients

The surge travelling along the conduit will push a column of air in front of it and may produce an air-water interface instability as explained in Chapter I. These surges may result from a downstream flow restriction, a pump failure at the sump well, a sudden increase of inflow at the downstream end, or from a sudden closure of a downstream control gate. If a complete blockage results from this air-water interface, the trapped air in the system will be compressed by the moving surge and a pressure wave will be initiated at the surge front end at the blockage.

The suggested model (explained in Chapter III) to calculate these pressure transients along the conduit is based on the following assumptions:
(a) The upstream end of the conduit is completely blocked by the air-water interface instability.

(b) The complete blockage at the upstream end of the conduit will develop when the relative velocity of the travelling surge \(|\vec{V}_w| + |\vec{V}_1|\) reaches the critical interface instability velocity, \(V_I\), calculated by Eq. 3.37.

(c) Based on the experimental observations a minimum travelled distance by the surge of 20 \(D_p\) is required to develop the blockage of the upstream end of the conduit.

(d) The flow regime throughout the length \(L_1\) (see Fig. 3.3) is assumed to be bubble flow with air fraction, \(a_a = 0.15(A_p - A_1)\), and the flow regime throughout the length, \(L_2\), is assumed to be bubble flow with air fraction, \(a_a = (A_p - A_1)/A_p\).

(e) The speeds of sound \(\bar{a}\) and throughout the lengths \(L_1\) and \(L_2\) (shown in Fig. 3.3) are calculated from Eq. 2.7.

The comparison between the experimental transients and the theoretically predicted (see Chapter III) in circular conduit resulting from pump failure at the sump well is shown in Figs. 4.10 to 4.23.

Figures 4.27 to 4.35 show a comparison between the experimental and theoretical pressure transients in the
circular conduit resulting from the air-water interface instability produced by the sudden closure of the downstream end control gate, for different relative depths of flow ($y_r$), and different bed slopes ($S_o$).

Table 6.7 compares the maximum theoretical and experimental amplitudes (see Figs. 4.10 to 4.23) of the pressure transients in the circular conduit resulting from the pump failure at the sump well.

Table 6.8 shows a comparison of the maximum theoretical and experimental amplitudes (shown in Figs. 4.27 to 4.35) of the pressure transients in the circular conduit resulting from the sudden closure of the downstream control gate.

Table 6.7 shows that the ratio $\frac{\text{Max. Amplitude (EXP)}}{\text{Max. Amplitude (Theory)}}$ is ranging between 12% and 52% with an average ratio of 32% for all tests of the pressure transients in the circular conduit resulting from the pump failure at the sump well.

Table 6.8 shows that the ratio $\frac{\text{Max. Amplitude (EXP)}}{\text{Max. Amplitude (Theory)}}$ is ranging between 8% and 38% with an average ratio of 21% for all tests of the pressure transients in the circular conduit resulting from the sudden closure of the downstream end gate.

Tables 6.7 and 6.8 indicate the following:

(a) The agreement between the theory and experiment is poor. The theory gives a maximum amplitude
Table 6.7

Comparison Between the Maximum Theoretical and Experimental Amplitudes of the Pressure Transients in the Circular Conduit Resulting from the Pump Failure at the Sump Well

<table>
<thead>
<tr>
<th>Bed Slope ($S_0$)</th>
<th>Relative Depth ($y_r$)</th>
<th>Sump/Pipe Area Ratio (SPR)</th>
<th>Max. Amplitude in m</th>
<th>Max. Amplitude (EXP)</th>
<th>Max. Amplitude (Theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>0.76</td>
<td>4.46</td>
<td>2.67</td>
<td>0.30</td>
<td>0.12</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.80</td>
<td>4.46</td>
<td>1.41</td>
<td>0.55</td>
<td>0.39</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.87</td>
<td>4.46</td>
<td>1.33</td>
<td>0.55</td>
<td>0.41</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.79</td>
<td>4.46</td>
<td>1.46</td>
<td>0.43</td>
<td>0.29</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.86</td>
<td>4.46</td>
<td>1.40</td>
<td>0.55</td>
<td>0.39</td>
</tr>
<tr>
<td>0.0050</td>
<td>0.75</td>
<td>4.46</td>
<td>1.30</td>
<td>0.40</td>
<td>0.31</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.88</td>
<td>29.80</td>
<td>0.73</td>
<td>0.38</td>
<td>0.52</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.89</td>
<td>29.80</td>
<td>0.77</td>
<td>0.10</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Table 6.8

Comparison Between the Maximum Theoretical and Experimental Amplitudes of the Pressure Transients in the Circular Conduit Resulting from the Sudden Closure of the Downstream Control Gate

<table>
<thead>
<tr>
<th>Bed Slope ($S_o$)</th>
<th>Relative Depth ($Y_r$)</th>
<th>Max. Amplitude in m</th>
<th>Max. Amplitude (EXP)</th>
<th>Max. Amplitude (Theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0011</td>
<td>0.76</td>
<td>2.15</td>
<td>0.33</td>
<td>0.15</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.83</td>
<td>4.41</td>
<td>0.42</td>
<td>0.10</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.87</td>
<td>4.78</td>
<td>1.80</td>
<td>0.38</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.79</td>
<td>2.88</td>
<td>0.33</td>
<td>0.11</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.86</td>
<td>5.09</td>
<td>1.80</td>
<td>0.35</td>
</tr>
<tr>
<td>0.0050</td>
<td>0.72</td>
<td>4.78</td>
<td>0.40</td>
<td>0.08</td>
</tr>
<tr>
<td>0.0050</td>
<td>0.76</td>
<td>5.19</td>
<td>1.38</td>
<td>0.27</td>
</tr>
</tbody>
</table>
that exceeds the standard error of the experimental results.

(b) The experiment gives a better agreement for the pressure transients resulting from the pump failure at the sump well, as compared to the pressure transients resulting from the sudden closure of the downstream end control gate.

(c) The agreement between the experiment and the theory improves as the relative depth of flow, \( y_r = \frac{y_1}{D_p} \), increases, for the same bed slope.

(d) Sudden closure of the downstream end control gate creates the most severe pressure fluctuations.

The pressure fluctuations (transients) are affected to a great extent by the flow regime, in the conduit, following the full blockage caused by the air-water interface instability. The flow regimes in sewers may be: slug, or stratified or bubble or mixed regime.

The theory explained in this study gives the maximum possible pressure fluctuations for any of these flow regimes, which may exist in sewers under certain conditions.

Table 6.9 shows typical example calculations of the maximum theoretical amplitudes of the pressure transients for slug, stratified, and bubble flow regimes, compared with the maximum experimental amplitude for the circular
model conduit of bed slope, \( S_o = 0.0018 \), relative depth of flow, \( y_r = y_1 / D_p = 0.79 \) and sump/pipe area ratio, \( SPR = 4.46 \).

Table 6.9 shows that the most severe pressure fluctuations is for slug flow regime which gives 5.5 times the pressure fluctuations in bubble flow regime for the same conditions. In case of stratified flow regime the pressure fluctuations were 5.1 times the pressure fluctuations for bubble flow regime.

It was difficult to simulate slug flow regime during the experimental study because the conduit was not long enough. The poor agreement between the maximum experimental amplitude and the maximum amplitude predicted by the model may be explained by the following:

(a) There is a possibility of air leakage at the upstream end of the conduit, that is the assumption of the full blockage at the upstream end of the conduit in the model may not be true. This would significantly reduce the pressure in the conduit. The improvement in the agreement at higher relative depth of flow, \( y_r = y_1 / D_p \) for the same bed slope, \( S_o \) supports this explanation.

(b) The pressure transients are affected greatly by the speed of sound in the conduit, which depends on the flow regime as discussed before, and as shown in Fig. 2.9. The speed of sound in the conduit is based on Eqs. 2.7, 2.8 and 2.9 for
Table 6.9

Comparison Between the Maximum Theoretical and Experimental Amplitudes of the Pressure Transients in the Circular Conduit for Different Flow Regimes in the Model and for $S_o=0.0018$, $y_r=0.79$, and SPR=4.46

<table>
<thead>
<tr>
<th>Maximum Experimental Amplitude in m</th>
<th>Maximum Theoretical Amplitude in m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slug Flow</td>
</tr>
<tr>
<td>0.43 ($=2.8D_p$)</td>
<td>8.06</td>
</tr>
<tr>
<td></td>
<td>($=53.0D_p$)</td>
</tr>
</tbody>
</table>
bubble, stratified, and slug flow regimes respectively. The speed of sound is also a function of the air fraction \( \alpha_a \) which was estimated from experimental observations and was very approximate.

(c) The assumption of having homogenous bubble flow regimes throughout the lengths \( L_1 \) and \( L_2 \) (shown in Fig. 3.3) is not true and the air concentration may vary significantly along the conduit.

(d) The speed of sound given by Eqs. 2.7, 2.8, and 2.9 by Rhodes and Scott are based on small amplitude pressure fluctuations, however the pressure fluctuations in this study were considerably larger.

6.5 Air-Water Interface Instability

A statistical analysis was made of 27 tests to determine the constants of Eq. 3.37 for the rectangular conduit (see Fig. 6.10). The best fit for the experimental data shown in Fig. 6.10, gave the following equation for the critical interface instability velocity, \( V_I \) (at which a full blockage of the pipe occurs) in the rectangular conduit:

\[
V_I = V_H (-1.18 (\frac{y_1}{D_p})^{2.545} + 1.18) \quad (6.1)
\]
A similar statistical analysis was made of 19 tests to determine the constants of Eq. 3.37 for the circular conduit for the experimental data shown in Fig. 6.11. The following equation was obtained for the critical interface instability velocity, $V_I$, in the circular conduit:

$$V_I = V_H (-1.66(y_I/D_p)^{3.268} + 1.13) \quad (6.2)$$

Figure 6.12 shows the dimensionless interface instability velocity ($V_I/V_H$) for different relative depths of flow for both rectangular and circular conduits.

The comparison of the critical interface instability velocities ($V_I$) for rectangular and circular conduits (shown in Fig. 6.12) shows that gravity flow in circular conduits is slightly more unstable than that in rectangular conduits, i.e., at the same relative depth of flow ($y_r$) the critical interface instability velocity, $V_I$, for circular conduits is less than that for rectangular conduits.

To have the same rate of flow $Q_I$ in both circular and rectangular conduits of the same cross-sectional area; the relative depth of flow $y_r$ should be, for example, 0.80 and 0.88 respectively. For this case there is no difference in the dimensionless interface instability velocities ($V_I/V_H$) which will be of the same value ($0.36 \equiv 2.34$ m/s) in both circular and rectangular conduits.
6.6 Convergence Stability and Analysis of the Travelling Hydraulic Jump Numerical Model

To study the convergence and stability of the numerical solution, several runs were made for the analysis of the travelling hydraulic jump with stable air-water interface for time increments, $\Delta t$, of 0.05s, 0.10s, 0.20s and 0.40s. It was found that by using the time increment, $\Delta t < 0.20s$, the convergence and stability requirements were satisfied.

For the analysis of pressure transients in the conduit (following the air-water interface instability), several runs were made using space increments, $(\Delta x)_1$, of 1.25 $D_p$, 2.5$D_p$, 5.0$D_p$, 10.0$D_p$, 20.0$D_p$ and 40.0$D_p$, and time increment $(\Delta t)_1 = (\Delta x)_1 / \bar{a}_1$, for the length $L_1$, shown in Figure 3.3. For length $L_2$ (see Figure 3.3) the space increments were adjusted to be $(\Delta x)_2 = (\Delta t)_1 \bar{a}_2$, and the time increment $(\Delta t)_2$ was kept the same as for the length $L_1$ (i.e., $(\Delta t)_1 = (\Delta t)_2$). It was found that by using the space increment $\Delta x < 20.0D_p$, and $\Delta t = \Delta x / \bar{a}$, the convergence and stability requirements were satisfied.
CHAPTER VII

APPLICATIONS

7.1 General

In this chapter, the applicability of the proposed travelling hydraulic jump numerical model is discussed. Typical practical applications of the numerical model are also given.

7.2 Applicability of the Proposed Travelling Hydraulic Jump Numerical Model

The proposed numerical model, as presented, is applicable to a single prismatic pipe of rectangular or circular cross-section (prismatic pipes), connected at each end to a manhole, or sump well, or reservoir, with or without pumping facilities at either end. The initial gravity flow in the conduit should be steady and uniform flow. Other restrictions are presented in Chapter V.

The required input data are explained in Chapter V - Article 5.3. The model can predict the occurrence of air-water interface instabilities and the maximum possible pressure transients during surcharging in a sewer.
7.3 Typical Practical Applications

**Example 1: Surcharging of a Sewer due to Excessive Pumped Inflow at a Manhole**

A typical surcharging problem was observed in the Town of Amherstburg, Ontario, in the sewer system shown schematically, in Fig. 1.4. During severe storms, the discharge, $Q_I$, from a pumping station at the sump well to manhole (2), caused a rapid increase in the water level at manhole (2), producing a surge in sewer (2). The moving surge resulted in the blowing off of a manhole cover several blocks away (at manhole (1)).

The travelling hydraulic jump numerical model was applied to sewer (2), in the system shown in Fig. 1.4. The following data were used for this sewer:

**Data for Sewer (2):**
- Total length, $L=300$ m (between manhole 1 and manhole 2);
- Pipe diameter, $D_p=1.2$ m;
- Bed slope, $S_o=0.0015$; and
- Pipe material is reinforced concrete with Manning's $n=0.0135$.

**Data for Manhole (1) and Manhole (2):**
- Both are of circular cross-section with the same diameter, $D_p=1.2$ m.
Assumptions

The following are the assumptions made for the analysis of this sewer system:

- The initial relative depth of flow, \( y_r = y_1/D_p \) = 0.75 in sewer (2); and

- The discharge \( Q_o \) in sewer (3) going to Detroit River is reduced by 25\% (=0.39 m\(^3\)/s) due to surcharging and entrapped air.

Table 7.1 shows the maximum and the minimum possible pressure transients predicted by the travelling hydraulic jump model for 25\% (=0.39 m\(^3\)/s) reduction in \( Q_o \). The travelled distance by the surge front at interface instability is \( L_1 = 22.5 \text{ m} \).

Table 7.2 shows the maximum and the minimum possible pressure transients predicted for a 50\% (=0.78 m\(^3\)/s) reduction in \( Q_o \). The travelled distance by the surge front at interface instability is \( L_1 = 21.9 \text{ m} \).

Example 1 shows that a prototype sewer of 1.2 m diameter flowing 75\% full, and bed slope, \( S_o = 0.015 \), may experience transient pressures up to 45 m of water and as low as the vapor pressure.
Table 7.1

Maximum and Minimum Possible Pressure Transients Throughout the Amherstburg Sewer System Predicted by the Travelling Hydraulic Jump Model for $Q_o = 0.75 Q_i$.

<table>
<thead>
<tr>
<th>$H_{\text{max}}$ or $H_{\text{min}}$ in m of water</th>
<th>Using Eqs. 3.45 and 3.46 (Pressure Transient Analysis)</th>
<th>Using Eq. 3.60 (Approximate Solution)</th>
<th>Using Eq. 3.71 (Trapped Air Solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Length $L_1 = 22.5$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{\text{max}}$</td>
<td>+19.08</td>
<td>+13.06</td>
<td>+5.74</td>
</tr>
<tr>
<td>$H_{\text{min}}$</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-2.64</td>
</tr>
<tr>
<td>For Length $L_2 = 277.5$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{\text{max}}$</td>
<td>+33.23</td>
<td>+39.90</td>
<td>+5.74</td>
</tr>
<tr>
<td>$H_{\text{min}}$</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-2.64</td>
</tr>
</tbody>
</table>
Table 7.2

Maximum and Minimum Possible Pressure Transients Throughout the Amherstburg Sewer System Predicted by the Travelling Hydraulic Jump Model for $Q_o = 0.50 \, Q_I$

<table>
<thead>
<tr>
<th>$H_{\text{max}}$ or $H_{\text{min}}$ in m of water</th>
<th>Using Eqs. 3.45 and 3.46 (Pressure Transient Analysis)</th>
<th>Using Eq. 3.60 (Approximate Solution)</th>
<th>Using Eq. 3.71 (Trapped Air Solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>For Length $L_1 = 21.9$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{\text{max}}$</td>
<td>+21.53</td>
<td>+14.87</td>
<td>+7.39</td>
</tr>
<tr>
<td>$H_{\text{min}}$</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-2.94</td>
</tr>
<tr>
<td>For Length $L_2 = 278.1$ m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{\text{max}}$</td>
<td>+36.87</td>
<td>+45.44</td>
<td>+7.39</td>
</tr>
<tr>
<td>$H_{\text{min}}$</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-2.94</td>
</tr>
</tbody>
</table>
Example 2: Pressure Transients in a Large Sewer With a Drop-Pipe and Interceptor Weir

Another example of a transient pressure phenomenon was documented in the City of Hamilton, Ontario (5). A large sewer (10 ft. by 10 ft.) with a downstream interceptor weir and an upstream drop-pipe, shown schematically in Fig. 1.3, experienced severe pressure shocks during high flows.

The travelling hydraulic jump model presented in this study was used to predict the maximum and the minimum possible pressure transients in this system.

The following are the data for this system:

Data for the Sewer:
- Total length, L=426 m (between the upstream drop-pipe and the downstream interceptor weir);
- Conduit Cross-section dimensions are 10 ft. by 10 ft.;
- Bed slope, $S_o=0.02$; and
- conduit material is reinforced concrete with
  - Mannings $n=0.0135$.

Data for Upstream Drop-Pipe:
- Cross-section dimensions are 10 ft. by 10 ft.; and
- street level is 13.41 m higher than the bottom level of the sewer at the drop-pipe.

Data for Downstream Interceptor:
- Cross-section dimensions are 10 ft. by 10 ft.; and
- street level is the same as of the upstream drop-pipe.
Assumptions:

The following assumptions were made for the analysis of this sewer system:

- The initial relative depth of flow, $y_r = y_1 / D = 0.75$;
- The discharge, $Q_1 = V_1 A_1$, in the sewer is reduced 25% (=16.48 m$^3$/s) at the location of the downstream interceptor weir due to surcharging and entrained air;
- The air flow was blocked at the upstream drop-pipe after the surcharging of the downstream end.

Table 7.3 shows the maximum and the minimum possible pressure transients predicted by the travelling hydraulic jump model for the Hamilton storm sewer system for initial relative depth of flow, $y_r = 0.75$.

Table 7.3 shows that a large sewer (10 ft. by 10 ft.) of bed slope, $S_o = 0.02$, may experience transient pressure up to 140 m of water with a possibility of minimum pressure of vapor pressure.

These pressures, shown in Examples (1) and (2), will be transmitted along the sewer line and may be superimposed on existing surcharge pressures. Thus, sufficient pressure could be developed to cause basement flooding and lifting of manhole covers as was documented for these two cases.

It should be noted that the predicted pressures are upper and lower limits in an actual sewer the pressures would probably be much less because of incomplete blockages and escape of air when instabilities form.
Table 7.3

Maximum and Minimum Possible Pressure Transients Throughout the Hamilton Storm Sewer System Predicted by the Travelling Hydraulic Jump Model for \( Q_0 = 0.75 \ Q_1 \).

<table>
<thead>
<tr>
<th></th>
<th>Using Eqs. 3.45 and 3.46 (Pressure Transient Analysis)</th>
<th>Using Eq. 3.60 (Approximate Solution)</th>
<th>Using Eq. 3.71 (Trapped Air Solution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{\text{max}} )</td>
<td>+83.54</td>
<td>+41.38</td>
<td>+60.10</td>
</tr>
<tr>
<td>( H_{\text{min}} )</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-4.51</td>
</tr>
<tr>
<td>For Length ( L_1 = 26.2 ) m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( H_{\text{max}} )</td>
<td>+132.75</td>
<td>+140.09</td>
<td>+60.10</td>
</tr>
<tr>
<td>( H_{\text{min}} )</td>
<td>-10.00</td>
<td>-10.00</td>
<td>-4.51</td>
</tr>
<tr>
<td>For Length ( L_2 = 399.8 ) cm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

From the experimental and theoretical analysis presented, the following conclusions are made:

1. Although for practical purposes most flows in sewers can adequately be treated by single phase methods, the transition from gravity to pressure or surcharged flow has been shown to be a two-phase phenomenon.

2. A typical transition process is shown in Fig. 1.1 and was confirmed by physical model studies. For relative air-water velocity, \( |\vec{V}_w - \vec{V}_1| < V_T \), Eqs. 3.29 and 3.34 are suggested to compute the water level \( S \) and the rate of rise of the water surface \( \frac{ds}{dt} \) at the sump well, at time \( t \), using the numerical solution technique given by Runge-Kutta. The flow velocity, \( V_2 \), at section 2 (see Fig. 3.2), the surge velocity, \( V_w \), the surcharge head at the surge front, \( Z \), and the travelled distance by the surge, \( L_s \), can be determined at the same time from Eqs. 3.17, 3.15, 3.16, and 3.25 respectively. If the relative air-water velocity, \( |\vec{V}_w - \vec{V}_1| > V_T \), Eqs. 3.45 and 3.46 should be solved using the method of characteristics to estimate the maximum possible transient
pressure rise in the sewer pipe.

3. Equation 3.37 is suggested to estimate the critical interface instability velocity, $V_I$.

4. It is found that interface instabilities which normally precede the transition to pressure flow, occur at a lower relative depth ($y_r = y_1 / D_p$) in a circular pipe than in rectangular pipe.

5. The present study shows that the instantaneous pressure head associated with the transition of open-channel to surcharged flow in sewers, can exceed the hydraulic grade line for full pipe flow by several times the pipe diameter. In a prototype sewer, pressures of the order of 40 m are possible. Sub atmospheric pressures are also possible.

6. The sudden transition of gravity to surcharged flow creates the most severe pressure fluctuations in the sewers.

7. The random nature of the instability of the air-water interface and the extent of the blockage contributed to disagreement between the theory and the model prediction.

8. Other factors affecting the pressure transients are: pipe size, pipe shape, flow velocity, Froude number, relative depth, alignment of the pipe, pipe material, venting arrangements and boundary conditions such as pumps, interceptors and drop-pipes.
8.2 **Recommendations**

Based on the present study the following practical ways are recommended to decrease the risk of pressure transients in sewers:

(a) over-sizing the sewer pipes;

(b) providing adequate venting of the line;

(c) provision of adequate standby power for pumped sewer systems;

(d) improved design of interceptors;

(e) improved design of drop pipes;

(f) use of rectangular conduits for large sewers with high velocities;

(g) regular maintenance of the system to avoid collection of debris that could clog outfall screens;

(h) careful analysis and design of pumped inflows to the system;

(i) good grade control and backfilling of the sewer; and

(j) installation of air-jumpers at inverted siphons.

The study gives rise to some potential problems for further research. The following modifications in the sewer model are suggested:

(a) Longer conduits are needed to study the pressure transients in a long reach of stratified air-water flow, and for possible simulation of a slug flow regime;
(b) conduits of steep slopes are needed to study the transition of open-channel to surcharged flow at high Froude numbers;
(c) long vertical pipes should be added to study pressure transients in sewers with long drop-pipes;
(d) a sewer system with more than one pipe, and with intermediate manholes should be used to study the effect of the transition from open-channel to surcharged flow in one pipe on the other pipes of the system as well as on the intermediate manholes;
(e) a special study is needed to study the effect of improper vertical or horizontal alignment on the transition from gravity to surcharged flow in sewers; and
(f) a special study is needed to study the transition of gravity to surcharged flow for unsteady, nonuniform flow conditions.
APPENDIX A

DERIVATION OF THE EQUATION OF AIR (GAUGE) PRESSURE AT THE SURGE FRONT
DERIVATION OF THE EQUATION OF AIR (GAUGE) PRESSURE AT THE SURGE FRONT

The propagating surge of Fig. 3.2, with a speed of \( V_w(t) \) will push the air in front of it with a speed of \( = V_w(t) \), i.e., \( V_a = V_w(t) \), therefore we can write the equation of air (gauge) pressure at the surge front as follows:

\[
P_a = \frac{\gamma_a}{\gamma_w} \left[ K_e \frac{V_w^2(t)}{2g} + \frac{f L_a}{D_h} \frac{V_w^2(t)}{2g} \right] = G_1 V_w^2(t) \tag{A.1}
\]

where,

\( \gamma_a = \) unit weight of air;
\( \gamma_w = \) unit weight of water;
\( K_e = \) exit coefficient for the air flow;

in which

\[
K_e = \frac{1.0}{0.6} \left( \frac{A_a}{A_v} \right)^2
\tag{A.2}
\]

where,

\( A_a = \) area of air flow \( = A_p - A_l \); and
\( A_v = \) area of vents at the upstream end of the sewer pipe.

\( L_a = \) Length of air path \( (\text{Length of pipe} - L_s) \);
\( D_h = 4R_h \) = hydraulic depth;
\( R_h = \) hydraulic radius for air flow;

\( f L_a / D_h \frac{V_w^2(t)}{2g} = \) friction coefficient;

\( R_h / v = \) Reynolds number for air flow;
\( v = \text{kinematic viscosity of air}; \) and

\[
G_1 = \frac{\gamma_a}{2g
\gamma_w} [K_e + \frac{fL_a}{D}] \text{ is constant.}
\]

Equation 3.15 can be written as follows:

\[
V_w(t) = E_1 V_2(t) - E_2 \quad \text{(A.3)}
\]

where,

\[
E_1 = \frac{-A_p}{A_p - A_1} \text{ is constant, and}
\]

\[
E_2 = \frac{A_1 V_1}{A_p - A_1} \text{ is constant.}
\]

Substituting from Eqs. A.1 and 3.17 into Equation A.3 we get

\[
\frac{P}{\gamma} = G_1 V_w^2(t) = G_1 [E_1 V_2(t) - E_2]^2
\]

\[
= G_1 [E_1 \left( A_s \frac{dS}{dt} + \frac{Q_{out}}{A_p} \right) - E_2]^2
\]

\[
\therefore \frac{P}{\gamma} = G_2 \left( \frac{dS}{dt} \right)^2 + G_3 \left( \frac{dS}{dt} \right) + G_4 \quad \text{(A.4)}
\]

where,

\[
G_2 = G_1 E_3^2 = \text{constant;}
\]

\[
E_3 = E_1 A_s = \text{constant;}
\]

\[
G_3 = 2G_1 E_3 E_4 = \text{constant;}
\]
\[ E_4 = E_1 \frac{Q_{\text{out}}}{A_p} - E_2 = \text{constant}; \text{ and} \]

\[ G_4 = G_1 E_4^2 = \text{constant}. \]
APPENDIX B

MEASURING INSTRUMENTS
MEASURING INSTRUMENTS

The following is a list of the measuring instruments used in this study:

1. Pressure Transducers: Model PB 531A. Specifications are found in literature supplied by the manufacturer, Sensor, Inc., 13112 Grenshaw Boulevard, Gardena, California, U.S.A.

2. Strain Indicators: Model P-350A. Specifications are found in the operation manual supplied by the manufacturer, Vishay Instruments Inc., 63 Lincoln Highway, Malvern, Pa., 19355, U.S.A.

3. Honeywell, Two Pen Bench Mount Recorder: Model Electronik 196. Specifications are found in the operation manual supplied by the manufacturer, Honeywell Inc., 1100 Virginia Drive, Fort Washington Pa., 19034, U.S.A.

4. Electromagnetic Flow Meter: Model 9650C. Specifications are found in the operation manual supplied by the manufacturer, Foxboro Co. Ltd., Montreal, Quebec, Canada.
APPENDIX C

FIGURES
**Figure 1.1(a) Transition of Gravity to Surcharged Flow in Sewers.**
Figure 1.1(b) Transition of Gravity to Surcharged Flow in Sewers.
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Figure 2.19 One-Dimensional Flow in Channel of Small Slope ($S_o$) After Wiggert (40).

Figure 2.20 Movement of Surge Wave in Tunnel After Wiggert (40).
Figure 2.21 Schematic of Experimental Set-Up (Dimensions are in Decimeters) After Wiggert (40).

Figure 2.22 Experimental and Computed Pressure Heads Behind the Surge Front (the Computed Values are Shown by Solid Circles) After Wiggert (40).
Cross Section A-A

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Figure 3.3 Initial Conditions at the Beginning of Transition.
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Figure 4.10 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_0=0.00109$, $F_\tau=0.50$, $y_\tau=0.76$, and SPR=4.46.
Figure 4.11 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_0=0.00109$, $F_r=0.59$, $y_r=0.80$, and $SPR=4.46$. 

- Computed Upper Limit
- Computed Lower Limit
- Computed Sump Level
Figure 4.12 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.00109$, $F_r=0.56$, $y_r=0.87$, and SPR=4.46.
Figure 4.13 Change in the Sump Water Level with Time (ST. (l)) and the Pressure Change at ST. (5) for the Circular Conduit and for $S_o = 0.0018$, $F_c = 0.72$, $y_e = 0.67$, and $SPR = 4.46$. 
Figure 4.14 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.0018$, $P_r=0.68$, $Y_r=0.79$, and SPR=4.46.
Figure 4.15 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.0018$, $F_r=0.65$, $Y_r=0.86$, and SPR=4.46.
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Figure 4.17 Change in the Sump Water Level with Time (ST.1) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.005$, $F_r=1.07$, $y_r=0.67$, and SPR=4.46.
Figure 4.18 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.005$, $F_r=1.05$, $y_r=0.71$, and SPR=4.46.
Figure 4.19(a) Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_0 = 0.005$, $F_r = 1.03$, $r_c = 0.75$, and SPR=4.46.
Figure 4.19(b) Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_0=0.005$, $F=1.03$, $\gamma_r=0.75$, and SPR=4.46.
Figure 4.20  Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o=0.00109$, $F_r=0.61$, $y_r=0.74$, and SPR=29.8.
Figure 4.21 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $S_o = 0.00109$, $F_r = 0.56$, $Y_r = 0.88$, and SPR = 29.8.
Figure 4.22 Change in the Sump Water Level with Time (ST.(1)) and the Pressure Change at ST.(5) for the Circular Conduit and for $s_0 = 0.0018$, $f_r = 0.71$, $\gamma_r = 0.69$, and SPR = 29.8.
Figure 4.23 Change in the Sump Water Level with Time (ST. 1) and the Pressure Change at ST. (S) for the Circular Conduit and for $S_n = 0.0018$, $F_r = 0.64$, $y_r = 0.89$, and SPR = 29.8.
Figure 4.24 Change in the Pressure at Point 2 for the Rectangular Conduit and for $S_0 = 0.00109$, $F_r = 0.63$, $y_r = 0.74$, and SPR = 4.13.
Figure 4.25  Change in the Pressure at Point 2 for the Rectangular Conduit and for $S_0=0.00109$, $F_r=0.57$, $y_r=0.84$, and SPR=4.13.
Figure 4.26 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_o=0.00109$, $F_r=0.63$, and $y_L=0.64$. 
Figure 4.27 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_0 = 0.00109$, $F_r = 0.60$, and $y_r = 0.76$. 

\[\text{Computed Upper Limit} \quad \text{Computed Lower Limit}\]
Figure 4.28 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for \( S_0 = 0.00109 \), \( F_r = 0.58 \), and \( y_r = 0.83 \).
Figure 4.29 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_0=0.00109$, $F_r=0.56$, and $y_r=0.87$. 
Figure 4.30 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_o=0.0018$, $P_r=0.72$, and $y_r=0.67$. 
Figure 4.31 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_o=0.001$, $F_r=0.68$, and $Y_r=0.79$. 
Figure 4.32 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_0 = 0.0018$, $F_r = 0.66$, and $y_r = 0.86$. 

- ▼ Computed Upper Limit
- ▲ Computed Lower Limit
- ▲ Transients
Fig. 4.33 Pressure Recording at Points 1 and 5. Following the Sudden Closure of the Downstream End of the Circular Conduit and for \( S_0 = 0.005 \), \( F_r = 1.05 \), and \( y_r = 0.72 \).
Figure 4.34  Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_o=0.005$, $F_r=1.03$, and $y_r=0.75$. 

\[ \text{Computed Upper Limit} \]
\[ \text{Computed Lower Limit} \]

Transients
Figure 4.35 Pressure Recording at Points 1 and 5 Following the Sudden Closure of the Downstream End of the Circular Conduit and for $S_0=0.005$, $F_r=1.01$, and $y_r=0.79$. 
Figure 4.36 Pressure Recording at Point 1 Following the Sudden Closure of the Downstream End of the Rectangular Conduit and for $S_o=0.00109$, $F_r=0.65$, and $y_r=0.66$. 
Figure 4.37 Pressure Recording at Point 2 Following the Sudden Closure of the Downstream End of the Rectangular Conduit and for $S_o = 0.00109$, $P_r = 0.63$, and $y_r = 0.77$. 
Figure 5.1  Flow Chart for the Computer Program.
Figure 5.2. Flow Chart for Subroutine PUMPFS.
Figure 5.3 Flow Chart for Subroutine - TRANS.
Figure 5.3 (continued)

\[
\text{COMPUTE /} \\
\text{(INTERNAL POINTS FOR LENGTH X1,2)} \\
VWH(1,2), H(1,2), i = M + 2, N1 + N2
\]

\[
\text{COMPUTE /} \\
\text{(UPSTREAM BOUNDARY CONDITIONS)} \\
VWH(NWH - 1,2), H(NWH - 1,2)
\]

\[
\text{PRINT /} \\
T, VWH(i,2), H(i,2), \\
i = 1, NWH + 1
\]

\[
\text{IF} \ \frac{T}{T_{\text{MAX}}} > 1.0 \ \text{THEN} \ \text{NO} \\
VWH(i,1) = VWH(i,2) \\
H(i,1) = H(i,2) \\
i = 1, NWH + 1
\]

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Figure 5.5 A Typical Printout (t, S, and Z) for the Expansion and Contraction of Bubble in the Model.
INITIAL CONDITIONS

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Figure 5.6 A Typical Printout for Transient Pressure Following the Pump Failure at the Sump Well.
Figure 5.6 (continued)

RESULTS

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Figure 5.6 (continued)
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### Figure 5.6 (continued)

#### RESULTS

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This table contains results for various elements, including V & E, I, S, E, Fe, Mn, Si, P, As, Sb, Sn, Pb, and Bi. The values are given in a format that suggests a tabular representation of experimental or calculated data.
### RESULTS

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Figure 5.6 (continued)
Figure 5.6 (continued)

## RESULTS

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## Figure 5.7 (continued)

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Legend:
- x1: Variability
- x2: Mean Temperature
- x3: Minimum Temperature
- x4: Maximum Temperature
- x5: Humidity
- x6: Rainfall
- x7: Wind Speed
- x8: Air Pressure
- x9: Sunlight
- x10: Water Level
## RESULTS

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### RESULTS

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</table>

Figure 5.7 (continued)
Figure 6.1  A Typical Dimensionless Relative Surge Velocities
\((|V_w| + |V_1|)/|V_1|\) along the Circular Conduit for
Sump/Pipe Area Ratio (SPR) of 4.46.
Figure 6.2 A Typical Dimensionless Relative Surge Velocities $(\frac{V_s}{V_1} + \frac{\Delta V}{V_1})$ along the Circular Conduit for Sump/Pipe Area Ratio (SPR) of 29.8.
Figure 6.3 Comparison of the Theoretical Dimensionless Relative Surge Velocities ($|v_w| + |v_1|/|v_1|$) along the Circular Conduit Assuming either Uniform Flow, or Gradually Varied Flow.
Figure 6.4 Travelling Time for the Surge Between the Downstream and the Upstream Ends of the Circular Conduit for Different Relative Depths of Flow ($y_r$), and Bed Slopes ($S_o$).
Figure 6.5 Typical Model Prediction for the Relative Surge Head at the Surge Front ($\zeta_p^*$) along the Circular Conduit for Different Relative Depths of Flow ($y_f^*$) and Bed Slopes ($S_b^*$).
Figure 6.6 Dimensionless Relative Surge Velocities \(\frac{|\vec{v}_w| + |\vec{v}_1|}{|\vec{v}_1|}\) along the Rectangular Conduit at the First Peak for Different Sump/Pipe Area Ratios (SPR) After a Pump Failure Induced Surge.
Figure 6.7  Dimensionless Surcharge Heads ($Z/(V_1^2/2g)$) at the Surge Front at the First Peak for the Rectangular Conduit and for Different Sump/Pipe Area Ratios (SPR) after a Pump Failure Induced Surge.
Figure 6.8 Theoretical and Experimental Dimensionless Relative Surge Velocities $(|\vec{V}_w| + |\vec{V}_1|)/|\vec{V}_1|$ Throughout the Circular Conduit Following the Sudden Closure of the Downstream End Control Gate.
Figure 6.9 Theoretical and Experimental Dimensionless Surcharge Heads ($Z/D_p$) at the Surge Front for the Circular Conduit Following the Sudden Closure of the Downstream End Control Gate.
Figure 6.10 Dimensionless Critical Interface Instability Velocities \( \frac{V_I}{V_H} \) in the Rectangular Conduit for Different Relative Depths of Flow \( \frac{y_1}{D_p} \).
Figure 6.11 Dimensionless Critical Interface Instability Velocities ($V_I/V_H$) in the Circular Conduit for Different Relative Depths of Flow ($y_1/D_p$).
Figure 6.12 Comparison of the Dimensionless Critical Interface Instability Velocities ($V_I/V_H$) in Rectangular and Circular Conduits.
APPENDIX D

PHOTOGRAPHS
Photograph 4.1 Upstream End of the Rectangular Conduit and the Overhead Tank.
Photograph 4.2 View of the Rectangular Conduit Looking Downstream.
Photograph 4.3 Downstream End of the Rectangular Conduit and the Sump Well.
Photograph 4.4  Downstream End of the Circular Conduit and the Air Flow Flexible Hose Connected to Tight Cover of Sump Well.
Photograph 4.5 The Air Blower with a Control Valve and a Venturi with a Pitot-Static Tube.
Photograph 4.6  Pressure Transients Recording System (Two Channel Strip Chart Recorder Connected to Two Strain Indicators which are Connected to Two Pressure Transducers).

Photograph 4.7  A Fully Developed Interface Instability in the Circular Conduit for Relative Depth of Water, \( y_r = 0.55 \).
Photograph 4.8  Typical Surge Travelling Upstream the Rectangular Conduit (Left to Right) Following Pump Failure at the Sump Well.

Photograph 4.9  Typical Surge Travelling Upstream the Circular Conduit (Left to Right) Following Pump Failure at the Sump Well.
Photography 4.10  Typical Surge Travelling Upstream the Circular Conduit (Left to Right) Following the Sudden Closure of the Downstream End Control Gate.

Photograph 4.11  A Typical Interface Instability in the Circular Conduit at Early Stage.
Photograph 4.12 A Typical Developed Interface Instability in the Circular Conduit.

Photograph 4.13 An Interface Instability at Early Stage in the Circular Conduit for Relative Depth of Water, $y_r=0.35$. 
Photography 4.14  A Fully Developed Interface Instability in the Circular Conduit for Relative Depth of Water, $y_r = 0.35$.

Photograph 4.15  An Interface Instability at Early Stage in the Circular Conduit for Relative Depth of Water, $y_r = 0.43$. 
Photograph 4.16  A Fully Developed Interface Instability in the Circular Conduit for Relative Depth of Water, $y_r = 0.43$.

Photograph 4.17  An Interface Instability in Rectangular Conduit.
Photograph 4.18 An Interface Instability in Rectangular Conduit at Relative Air Velocity of 6.33 m/s (20.77 fps).

Photograph 4.19 An Interface Instability in Rectangular Conduit at Relative Air Velocity of 6.60 m/s (21.65 fps).
Photograph 4.20 An Interface Instability in Rectangular Conduit at Relative Air Velocity of 7.02 m/s (23.02 fps).
Table 4.1 Surge Velocity, $V_w$, Along the Circular Conduit Following the Pump Failure at the Sump Well for Bed Slope, $S_0=0.00109$, SPR=4.46, and for Different Relative Depths of Flow $Y_r=Y_1/D_P$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $Y_r=Y_1/D_P$</th>
<th>SURGE VELOCITY, $V_w$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
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<td>0.64</td>
<td>0.68</td>
<td>0.90</td>
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<tr>
<td></td>
<td>ST. A.W. Int.*</td>
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<tr>
<td>0.69</td>
<td>0.79</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>ST. A.W. Int.*</td>
<td></td>
</tr>
<tr>
<td>0.76</td>
<td>1.11</td>
<td>1.48</td>
</tr>
<tr>
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<td>ST. A.W. Int.*</td>
<td>Int. Ins.**</td>
</tr>
<tr>
<td>0.80</td>
<td>1.13</td>
<td>1.78</td>
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<tr>
<td></td>
<td>ST. A.W. Int.*</td>
<td>Int. Ins.**</td>
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<tr>
<td>0.87</td>
<td>1.06</td>
<td>2.70</td>
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</table>

*Stable Air-Water Interface
**Interface Instability
***Bubble Formation
Table 4.2 Surge Velocity, $V_w$, Along the Circular Conduit Following the Pump Failure at the Sump Well for Bed Slope $S_0=0.00105$, SPR=29.8, and for Different Relative Depths of Flow, $y_r=y_t/D_p$:

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $y_r = y_t/D_p$</th>
<th>SURGE VELOCITY $V_w$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0.66</td>
<td>0.30</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>ST. A.W. Int.*</td>
<td></td>
</tr>
<tr>
<td>0.71</td>
<td>0.28</td>
<td>0.76</td>
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<tr>
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<td>ST. A.W. Int.*</td>
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<tr>
<td>0.77</td>
<td>0.33</td>
<td>0.95</td>
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<tr>
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<td>ST. A.W. Int.*</td>
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<td>0.80</td>
<td>0.35</td>
<td>1.05</td>
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<td>Int. Ins.**</td>
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<tr>
<td>0.92</td>
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<td>0.78</td>
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*Stable Air-Water Interface,
**Interface Instability
***Bubble Formation
Table 4.3  Surge Velocity, $V_W$, Along the Circular Conduit Following the Pump Failure at the Sump Well for Bed Slope, $S_0=0.0018$, SPR=4.46, and for Different Relative Depths of flow, $y_r=y_1/D_p$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $y_r=y_1/D_p$</th>
<th>SURGE VELOCITY $V_W$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
<td>0.75 1.24 1.11 1.16 1.52 1.61 1.61</td>
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</tr>
<tr>
<td>0.79</td>
<td>1.11 2.06 1.88 1.94 1.61 1.37 1.37</td>
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<td>0.86</td>
<td>1.16 2.59 1.41 1.16 1.09 1.09 1.09</td>
<td>Int. Ins.** and B.F.***</td>
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</table>

* Stable Air-Water Interface
** Interface Instability
*** Bubble Formation
Table 4.4  Surge Velocity, $V_w$, Along the Circular Conduit
Following the Pump Failure at the Sump Well for
Bed Slope, $S_o = 0.0018$, SPR = $29.8$, and for Different
Relative Depths of Flow, $y_r = y_1 / D_p$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $y_r = y_1 / D_p$</th>
<th>SURGE VELOCITY, $V_w$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
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<tbody>
<tr>
<td></td>
<td>1</td>
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<tr>
<td>0.69</td>
<td>0.30</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>ST. A.W. Int.*</td>
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<tr>
<td>0.82</td>
<td>0.34</td>
<td>1.06</td>
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<tr>
<td></td>
<td>ST. A.W. Int.*</td>
<td>Int. Inst.** and B.F.***</td>
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<tr>
<td>0.89</td>
<td>0.34</td>
<td>0.84</td>
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<tr>
<td></td>
<td>Int. Inst.** and B.F.***</td>
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* Stable Air-Water Interface
** Interface Instability
*** Bubble Formation
Table 4.5  Surge Velocity, $V_w$, Along the Circular Conduit Following the Pump Failure at the Sump Well for Bed Slope, $S_o=0.005$, SPR=4.46, and for Different Relative Depths of Flow, $y_r=y_1/D_p$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH</th>
<th>SURGE VELOCITY, $V_w$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
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</thead>
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<tr>
<td>$y_r=y_1/D_p$</td>
<td>1</td>
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<tr>
<td>0.58</td>
<td>0.88</td>
<td>0.95</td>
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<tr>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.67</td>
<td>0.97</td>
<td>1.37</td>
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<tr>
<td>0.75</td>
<td>1.31</td>
<td>1.91</td>
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* Stable Air-Water Interface
** Interface Instability
*** Bubble Formation
- Random Movement
Table 4.6  Surge Velocity, \( V_W \), Along the Circular Conduit
Following the Pump Failure at the Sump Well for
Bed Slope, \( S_0 = 0.005 \), \( SPR = 29.8 \), and for Different
Relative Depths of Flow, \( y_r = y_1/D_p \).

<table>
<thead>
<tr>
<th>RELATIVE DEPTH ( y_r = y_1/D_p )</th>
<th>SURGE VELOCITY ( V_W ) m/s ALONG THE PIPE REACHES</th>
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<tbody>
<tr>
<td>0.51 m</td>
<td>0.27 0.45 0.59 0.64 0.64 0.66 0.66</td>
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<tr>
<td>Free Surge</td>
<td>Full Surge and ST. A.W. Int.*</td>
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<tr>
<td>0.64</td>
<td>0.40 0.70 0.90 1.27 1.19 1.32 1.32</td>
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<tr>
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</tr>
<tr>
<td>0.69</td>
<td>0.39 0.70 0.82 1.09 1.12 1.00 1.00</td>
</tr>
<tr>
<td>ST. A.W. Int.*</td>
<td>Int. Ins.** and B.F.***</td>
</tr>
<tr>
<td>0.71</td>
<td>0.38 0.65 0.82 1.13 1.16 0.93 0.93</td>
</tr>
<tr>
<td>ST. A.W. Int.*</td>
<td>Int. Ins.** and B.F.***</td>
</tr>
<tr>
<td>0.74</td>
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</tbody>
</table>

* Stable Air-Water Interface
** Interface Instability
*** Bubble Formation
- Random Movement
Table 4.7  Surge Velocity, $V_w$, Along the Circular Conduit
Following the Sudden Closure of the Downstream
End Control Gate for Bed Slope, $S_o=0.00109$, and
for Different Relative Depths of Flow, $y_r=y_1/D_p$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $y_r = y_1/D_p$</th>
<th>SURGE VELOCITY, $V_w$ (m/s)</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.61</td>
<td>1.06 1.18 1.13 1.11 1.46</td>
<td>1.54 1.54</td>
</tr>
<tr>
<td>0.66</td>
<td>1.50 1.95 1.76 2.00 1.85</td>
<td>1.09 2.00</td>
</tr>
<tr>
<td>0.73</td>
<td>2.13 3.02 3.13 3.10 1.03</td>
<td>1.39 2.00</td>
</tr>
<tr>
<td>0.81</td>
<td>3.40 4.53 4.70</td>
<td></td>
</tr>
</tbody>
</table>

*Stable Air-Water Interface
**Interface Instability
***Bubble Formation

ST. A.W. Int. = Stable Air-Water Interface
ST. A.W. Int. = Stable Air-Water Interface
Int. Ins. and B.P. = Interface Instability and Bubble Formation
Table 4.8  Surge Velocity, \( V_w \), Along the Circular Conduit Following Sudden Closure of the Downstream End Control Gate for Bed Slope, \( S_o=0.0018 \), and for Different Relative Depths of Flow, \( y_r=y_1/D_p \).

<table>
<thead>
<tr>
<th>RELATIVE DEPTH ( y_r=y_1/D_p )</th>
<th>SURGE VELOCITY, ( V_w ) m/s</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0.63</td>
<td>1.06</td>
<td>1.65</td>
</tr>
<tr>
<td></td>
<td>*<em>ST. A.W. Int.</em></td>
<td></td>
</tr>
<tr>
<td>0.68</td>
<td>1.31</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>*<em>ST. A.W. Int.</em></td>
<td></td>
</tr>
<tr>
<td>0.74</td>
<td>2.13</td>
<td>2.59</td>
</tr>
<tr>
<td></td>
<td>*<em>ST. A.W. Int.</em></td>
<td><strong>Int. Ins.</strong></td>
</tr>
<tr>
<td>0.78</td>
<td>2.43</td>
<td>3.62</td>
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<tr>
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<td>*<em>ST. A.W. Int.</em></td>
<td><strong>Int. Ins.</strong></td>
</tr>
<tr>
<td>0.85</td>
<td>4.25</td>
<td>4.53</td>
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<tr>
<td></td>
<td>*<em>ST. A.W. Int.</em></td>
<td><strong>Int. Ins.</strong></td>
</tr>
</tbody>
</table>

*Stable Air-Water Interface,
**Interface Instability
***Bubble Formation
Table 4.9 Surge Velocity, $V_w$, Along the Circular Conduit Following the Sudden Closure of the Downstream End Control Gate for Bed Slope, $S_o=0.005$, and for Different Relative Depths of Flow, $y_r = y_1/D_p$.

<table>
<thead>
<tr>
<th>RELATIVE DEPTH $y_r = y_1/D_p$</th>
<th>SURGE VELOCITY $V_w$ m/s</th>
<th>ALONG THE PIPE REACHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.47</td>
<td>0.76 0.63 0.70 0.72 0.79 0.76 0.76</td>
<td>ST. A.W. Int.*</td>
</tr>
<tr>
<td>0.56</td>
<td>1.34 1.19 1.13 1.34 1.39 1.50 1.50</td>
<td>ST. A.W. Int.*</td>
</tr>
<tr>
<td>0.60</td>
<td>1.65 1.38 1.33 1.82 1.75 2.03 2.03</td>
<td>ST. A.W. Int.*</td>
</tr>
<tr>
<td>0.67</td>
<td>2.36 2.51 2.07 2.45 2.28 2.08 2.08</td>
<td>ST. A.W. Int.* Int. Ins** and B.F.***</td>
</tr>
<tr>
<td>0.71</td>
<td>3.40 3.62 3.76 - - -</td>
<td>S.T. A.W. Int.* Int. Ins** and B.F.***</td>
</tr>
</tbody>
</table>

*Stable Air-Water Interface
**Interface Instability
***Bubble Formation
- Random Movement
Table 4.10  Critical Interface Instability Velocity, 
$V_I = V_a + V_l$ m/s in Circular Conduit for
Different Relative Depths of Flow $y_r$, and
for Bed Slope, $S_o = 0.00109$.

<table>
<thead>
<tr>
<th>$y_r = y_1/D_p$</th>
<th>$V_I = V_a + V_l$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>6.78</td>
</tr>
<tr>
<td>0.37</td>
<td>6.20</td>
</tr>
<tr>
<td>0.52</td>
<td>4.99</td>
</tr>
<tr>
<td>0.59</td>
<td>3.89</td>
</tr>
<tr>
<td>0.87</td>
<td>0.64</td>
</tr>
</tbody>
</table>
Table 4.11 · Critical Interface Instability Velocity, $V_I = V_a + V_l$ m/s in Circular Conduit for Different Relative Depths of Flow, $Y_r$, and for Bed Slope, $S_o = 0.0018$.

<table>
<thead>
<tr>
<th>$Y_r = Y_l/D_p$</th>
<th>$V_I = V_a + V_l$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>6.64</td>
</tr>
<tr>
<td>0.44</td>
<td>6.27</td>
</tr>
<tr>
<td>0.45</td>
<td>5.98</td>
</tr>
<tr>
<td>0.48</td>
<td>5.83</td>
</tr>
<tr>
<td>0.61</td>
<td>3.16</td>
</tr>
<tr>
<td>0.63</td>
<td>5.50</td>
</tr>
<tr>
<td>0.84</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Table 4.12  Critical Interface Instability Velocity, $V_\text{I} = V_a + V_\text{l}$ m/s in Circular Conduit for Different Relative Depths of Flow, $Y_r$, and for Bed Slope, $S_0 = 0.005$.

<table>
<thead>
<tr>
<th>$Y_r = Y_1/D$</th>
<th>$V_\text{I} = V_a + V_\text{l}$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>7.62</td>
</tr>
<tr>
<td>0.43</td>
<td>7.42</td>
</tr>
<tr>
<td>0.50</td>
<td>7.52</td>
</tr>
<tr>
<td>0.58</td>
<td>7.27</td>
</tr>
<tr>
<td>0.67</td>
<td>4.61</td>
</tr>
<tr>
<td>0.73</td>
<td>4.50</td>
</tr>
<tr>
<td>0.87</td>
<td>1.09</td>
</tr>
</tbody>
</table>
Table 4.13  Critical Interface Instability Velocity, $V_I = V_a + V_l$ m/s in Rectangular Conduit for Different Relative Depths of Flow, $Y_r$, and for Bed Slope, $S_o = 0.0011$.

<table>
<thead>
<tr>
<th>$Y_r = Y_1/D_p$</th>
<th>$V_I = V_a + V_l$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>6.44</td>
</tr>
<tr>
<td>0.36</td>
<td>7.28</td>
</tr>
<tr>
<td>0.44</td>
<td>6.44</td>
</tr>
<tr>
<td>0.48</td>
<td>6.57</td>
</tr>
<tr>
<td>0.61</td>
<td>4.42</td>
</tr>
<tr>
<td>0.67</td>
<td>5.59</td>
</tr>
<tr>
<td>0.73</td>
<td>4.55</td>
</tr>
<tr>
<td>0.81</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Table 4.14 Critical Interface Instability Velocity, $V_I = V_a + V_1$ m/s in Rectangular Conduit for Different Relative Depths of Flow, $Y_r$, and for Bed Slope, $S_0 = 0.0033$.

<table>
<thead>
<tr>
<th>$Y_r = y_1/D_p$</th>
<th>$V_I = V_a + V_1$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>6.89</td>
</tr>
<tr>
<td>0.36</td>
<td>6.57</td>
</tr>
<tr>
<td>0.43</td>
<td>6.24</td>
</tr>
<tr>
<td>0.50</td>
<td>6.34</td>
</tr>
<tr>
<td>0.59</td>
<td>5.85</td>
</tr>
<tr>
<td>0.64</td>
<td>5.66</td>
</tr>
<tr>
<td>0.69</td>
<td>5.20</td>
</tr>
<tr>
<td>0.77</td>
<td>3.77</td>
</tr>
<tr>
<td>0.82</td>
<td>0.91</td>
</tr>
</tbody>
</table>
Table 4.15  Critical Interface Instability Velocity, \( V_I = V_a + V_l \) m/s in Rectangular Conduit for Different Relative Depths of Flow, \( y_r \), and for Bed Slope, \( S_o = 0.0066 \).

<table>
<thead>
<tr>
<th>( y_r = y_l / D_p )</th>
<th>( V_I = V_a + V_l ) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29</td>
<td>7.80</td>
</tr>
<tr>
<td>0.39</td>
<td>8.06</td>
</tr>
<tr>
<td>0.44</td>
<td>6.96</td>
</tr>
<tr>
<td>0.49</td>
<td>4.36</td>
</tr>
<tr>
<td>0.57</td>
<td>3.84</td>
</tr>
<tr>
<td>0.66</td>
<td>3.64</td>
</tr>
<tr>
<td>0.71</td>
<td>5.53</td>
</tr>
<tr>
<td>0.76</td>
<td>3.90</td>
</tr>
<tr>
<td>0.81</td>
<td>5.59</td>
</tr>
<tr>
<td>0.91</td>
<td>1.30</td>
</tr>
</tbody>
</table>
APPENDIX F

LISTING OF COMPUTER PROGRAM AND SUBROUTINES
CONTINUE

DATA FOR RECTANGULAR CROSS SECTION PIPE

DP = 0.14
B = 0.14
AP = 0.9
A2 = 0.9
A3 = 0.9
B3 = 0.9
A4 = 0.9

V1 = (1.0/IN) * (HR = (2.0/3.0)) * (50 = 0.5)

V = V1 / DP
Y2 = Y1 / 0.2
Y2 = Y2 / 0.2

IF (REC EQ 1.0) GO TO 13

CONTINUE

DATA FOR CIRCULAR CROSS SECTION PIPE

DP = 0.152
RP = DP / 2.0
A2 = RP + RP
AP = 0.2
A3 = 0.2
A4 = 0.2
B3 = 0.2

THETAR = 2.0 * (ARCSIN (RP - Y1) / RP)
AL1 = (1.0 / 2.0) * THETAR - SIN (THETAR) * DP

THETAR = (RP + Y1) / 0.2

V1 = (1.0 / IN) * (HR = (2.0/3.0)) * (50 = 0.5)

SD = 4.0 * RP * (SIN (0.5 * THETAR)) / (3.0 * (THETAR - SIN (THETAR)))

EC = SD - RP - Y1

V = V1 * (RP + Y1) / 10.0 * ((2.0 + THETAR) - SIN (2.0 * THETAR))

GCA = (V / 2) * (EC / 2)

EC2 = EC / 2

IF (REC EQ 1.0) GO TO 13

CONTINUE

CALCULATIONS

DDM = 0.013
DI = 0.0
DOUT = 0.0
AL1 = (AP - A1) / A1

IF (DPM = DP) GO TO 12

CLERT = ((A2 + Y2 - A2) / (A1 + (1.0 - A1/A2)))

V = CLERT * V1

CHECK OF SWEP HYDRAULIC JUMP

IF (V > VLT - 0.2) GO TO 10

IF (V < VLT + 0.2) GO TO 10

IF (REC EQ 1.0) DNEQ = 1.0

IF (REC EQ 1.0) DNEQ = 1.0

IF (REC EQ 1.0) DNEQ = 1.0

GO 17 JUMP + 450 + Z

END

SW = 1/1000.0

XLD = 1.0
IZS=OP
XYZ=3
YZ2=9/3.0
V=CLR=SHIFT
V2=1.0/AP+VW=(A1-AP)+(V1+A1)
VNR2=(AP/A5)-QOUT/AS
IF (VNR.GT.0.0) GO TO 19
17 CONTINUE
18 CONTINUE
V2N=1.0/AP+VW=(A1-AP)+(V1+A1)
VNR2=(AP/AS)-QOUT/AS
X=V=(V+V1)/(V-V2)/V2=(V1*V2)/(V1-V2)*=V1*V2=V
19 CONTINUE
XLT=1=QG
XLT1=1=QG/OP
R2=2/OP
VVR=V+V1
VNR=V/2
V=V/2
R=V/2
S=V/2
R=V/2
S=V/2
WRITE (0.23)
20 FORMATT(0.X,10HROUDE NUMBER =.10X,THRESHOLD NUMBER = .10X,
C7H9V = 1/3)
WRITE (0.24)
21 FORMATT(0.X,6.61X,10.4,15X,F5.2)
WRITE (0.25)
22 FORMATT(8.X,4.3HINITIAL DEPTH .3X,4HACTIVE SEC.6X,5HRES(T),7X,3HVS.
C0X,3HPR1,9X,3HVR/7X,3HVAR(T),4X,11HMLT 3RCHG,3X,4HVAR1,7X,3HVR/4X,6X,3HMLT
WRITE (0.25)
23 FORMATT(0.X,7)
WRITE (0.27)
24 FORMATT(0.27)
25 FORMATT(0.27)
26 CALL NURM3
27 CALL BUBBLE
112 CONTINUE
STOP
END
SUBROUTINE SUBBLE

COMMON/ARRAY1,XL2,TX,PAT,POV,VO
IF (X1EQC0.0) CO=1.0
IF (AD1EQC0.0) CQ=1.0
G1=0.0
WRITE (6,3)
FORMAT(1X,100)
WRITE (6,3)
FORMAT(1X,100)
DO 2 I=1,100
 SI=I
 TE=(BI-1.0)/20.0
 VOLUM=XL2*I*(AP=A1)
 AB=XL2*I*(A)=0.1
 BN=(C#AP)/(AB*X1)
 GN=-(AT-1)/(Q=WXL)
 SN=1+BN(G1)/(Q=WXL)
 S4=AP/VCLA
 S5=BI=OP=3=E
 S6=S5=3
 S8=SGD=(36/65)*(2*(S5)*T5)=(VG/SQRT(65))*(SQR(S5)*T5)
 C31=(SQRT(85)+T5)-66/66
 2=PO/11.0-(36*(AP=A1))/VCLA)-S5/2
 WRITE(6,4) TE,SB,TS
 CONTINUE
1 CONTINUE
RETURN
END
H(=2))=HUS$H(0)=AE1/(G(1,0)=AL1))=(VUS=VOS)FRI=AE1/
C(G(1,0)=AL1))=(VUS=ABS(VUS)=VOS=ABS(VOS))

**THE POINT AT THE SURGE FRONT AT (=N+1)**

H(N+1,2)=H(V(1,0)/G(1,0)=AL1)/AE1+(1,0-AL2)/AE1=1/(G(1,0)=AL1
C1)/AE1=H(N+1,1)+(G(1,0)=AL2)/AE1=H(N+1,1)=H(V(N,N+1)=V(N,N+1))
C1)=CAVH(N,N+1)=ABS(V(N,N+1))=CAVH(N,N+1)=ABS(V(N,N+1))
C1)=CAVH(N,N+1)=ABS(V(N,N+1))

**THE POINTS IN FRONT OF THE SURGE FRONT (=N+2 TO V(N,N+1)**

**DO** 65 (=N+2,NM)

**VUS**=V(N,N+1)

**HUS**=H(N+1,1)

**VH(N,N+1)**=(VUS=VOS)=1/(G(1,0-AL2)/AE2)+(HUS=HOS)=FRI=VUS

**CAVH(N,N+1)=ABS(V(N,N+1))**

**H(N+2)=$$=H(V(1,0)/G(1,0)=AL2)/AE2=H(V(N,N+1)=A

**V(N,N+1)=(VUS=VOS)=ABS(V(N,N+1))

**DO** 66 (=N+2,NM)

**CONTINUE**

**VUS**=V(N,N+1)

**HUS**=H(N+1,1)

**VH(N,N+1)**=(VUS=VOS)=1/(G(1,0-AL2)/AE2)+(HUS=HOS)=FRI=VUS

**CAVH(N,N+1)=ABS(V(N,N+1))**

**H(N+2)=$$=H(V(1,0)/G(1,0)=AL2)/AE2=H(V(N,N+1)=A

**V(N,N+1)=(VUS=VOS)=ABS(V(N,N+1))

**DO** 67 (=NM,NM)

**WRITE** (4,100) T

**WRITE** (4,101) T

**WRITE** (4,102) T

**WRITE** (4,103) T

**WRITE** (4,104) T

**CONTINUE**

**HOSS(1)=HOS5(2)

**HUS(1)=HUS5(2)

**IF** (V(N,N+1)=V(N,N+1)) GC TC 63

**RETURN**

**END**
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APPENDIX H

NOMENCLATURE
A  Area of flow
A_1  Area of flow at section 1
A_2  Area of flow at section 2
A_a  Area of air flow
A_b  Contact area between the bubble and the water
A_p  Full conduit area
A_s  Surface area of the sump well
A_{ss}  Surface area of the upstream open-channel section (or manhole)
A_v  Area of vents at the upstream end of the sewer pipe
a  Effective speed of sound in two-phase (air-water) flow in a rigid pipe
\bar{a}  Effective speed of sound in two-phase (air-water) flow in an elastic pipe
a_g  Speed of sound in gas
a_l  Speed of sound in liquid
a_{TP}  Speed of sound in two-phase flow
B_1 \text{ to } B_6  \text{ Constants}
C  Velocity of wave with respect to velocity of initial flow (celerity)
\bar{C}  Velocity of dynamic wave relative to weighted average velocity
\bar{C}_1  Velocity of compressibility waves in component 1
\bar{C}_2  Velocity of compressibility waves in component 2
C_1 \text{ to } C_{19}  \text{ Constants}
C_{ch}  Velocity of compressibility wave in strictly homogeneous mixture
$C_{cs}$  Velocity of compressibility wave in mixture

$C_0$  Celerity of surface waves

$c$  Constant between 1.0 and 1.4 for isothermal and adiabatic conditions respectively

$D_h$  Hydraulic depth of gravity flow

$D_P$  Diameter of conduit

$E$  Modulus of elasticity of pipe material

$E_1$ to $E_5$  Constants

e  Pipe wall thickness

$F$  Hydrostatic pressure on area $A$

$F_1$  Hydrostatic pressure on area $A_1$

$F_2$  Hydrostatic pressure on area $A_2$

$F_3$  Hydrostatic pressure on area $A_3$ or $A_P$

$F_r$  Froude number

$f$  Friction factor for the conduit (=Weisbach resistance coefficient)

$G_1$ to $G_4$  Constants

g  Acceleration due to gravity

$H$  Pressure head

$H$  Height of duct

$H_{exit}$  Total head at the exit

$H_s$  Water level at the upstream open-channel section (or manhole)

$H_{u}$  Total head at the entrance

$h$  Surge height

$h$  Flow depth measured normal to $x$

$h_b$  Height of the bubble
$h_f$ Pressure drop due to friction

$h_{L1}$ Head loss between the upstream open-channel section and the pipe

$h_{L2}$ Head loss between the pipe and the sump well

$h_v$ Velocity head at section 2

$I_{in}$ Inflow into channel reach

$I_{out}$ Outflow from channel reach

$K$ Bulk modulus of water

$\hat{K}$ Bulk modulus of air-water mixture

$\hat{K}_L$ Loss coefficient

$K_1, K_2$ Correction factors for non-hydrostatic pressure distribution

$K_d$ Exit loss coefficient for water flow

$K_e$ Exit loss coefficient for air flow

$K_g$ Bulk modulus of elasticity for gas

$K_L$ Bulk modulus of elasticity for liquid

$K_s$ Bulk modulus of elasticity for solid

$K_{sl}$ Bulk modulus of elasticity for solid-liquid mixture

$K_u$ Entrance loss coefficient

$L$ Length of pipe

$L_a$ Length of air path

$L_s$ Distance travelled by the surge front

$l$ Distance of the bubble from the upstream end of the pipe

$m$ Mass of free air present per cubic unit of volume

$N$ Number of nodes

$n$ Manning's roughness coefficient
P  Air pressure at the surge front
P'  Wetted perimeter of the pipe
P_a  Piezometric pressure of the flow
P_ab  Absolute pressure
P_at  Atmospheric pressure
P_i  Pressure of air inside the bubble during the compression of the bubble
P_o  Initial absolute pressure when the bubble formed
Q  Discharge
Q_1  Discharge at section 1 (gravity flow discharge)
Q_1'  Discharge leaving the upstream open-channel section (or manhole) and entering the pipe
Q_2  Discharge at section 2
Q_a  Air flow
Q_in  Discharge entering the sump well
Q_out  Discharge leaving the sump well
Q_o  Normal pipe full discharge
Q_s  Discharge under surcharge
R  Gas constant
R_h  Hydraulic radius of the flow
R_n  Reynold's number
RLAR  = L_s/D_p = dimensionless travelled distance by the surge front
RS  = S/D_p = relative water level at the sump well
S  Water level in the sump well
S_b  Displacement measured in the flow direction (=bed direction)
$S_f$  Friction slope

$S_o$  Bottom slope of the conduit

$SPR = \frac{A_s}{A_p}$  ratio of the surface area of sump well to the pipe area

$\overline{T}$  Absolute temperature

$T''$  Force due to internal stresses acting normally to A

$T_w$  Water surface width

$t$  time

$u$  The velocity of the pipe wall

$\overline{u}$  Wave velocity

$V$  Average velocity of flow

$V_1$  Velocity of flow at section 1

$V_2$  Velocity of flow at section 2

$V_a$  Velocity of air phase in the pipe

$V_d$  Measured dye velocity

$V_H$  Interface instability velocity predicted by Helmholtz Theory ($\approx 6.5$ m/s)

$V_I$  Critical interface instability velocity in the pipe

$V_o$  Weighted mean velocity

$V_w$  Surge velocity

$VSR = \frac{V_s}{V_1}$  relative speed of the water surface at the sump well

$VWR = |\vec{V}_w| + |\vec{V}_1|$  speed of surge relative to water (for counter-current air-water flow)

$\psi$  Total volume of fluid

$\psi_g$  Volume of gas
\( \psi \) Volume of liquid
\( \psi_s \) Volume of solid
\( \psi_0 \) Volume of trapped air when the bubble formed
\( x \) Distance along channel (or pipe) longitudinal direction
\( y \) Depth of flow
\( y_1 \) Distance between the pressure center and the water surface at section 1
\( y_2 \) Distance between the pressure center and the water surface at section 2
\( y_1 \) Depth of flow at section 1
\( y_2 \) Depth of flow at section 2
\( y_r = \frac{y_1}{D_p} \) Relative depth of open-channel flow
\( z \) Pressure rise at the surge front measured above the sewer crown
\( z_b \) Variation in piezometric pressure due to the expansion and contraction of the bubble
\( \alpha \) Kinetic energy correction factor
\( \alpha' \) Characteristic direction
\( \alpha_a \) Air fraction (void fraction)
\( \beta \) Momentum flux correction factor
\( \beta' \) Characteristic direction
\( \gamma \) Unit weight of fluid
\( \gamma_a \) Unit weight of air
\( \gamma_w \) Unit weight of water
\( \Delta h \) The fall in pipe level
\( \Delta t \) Time interval
\( \Delta x \)   
Space interval  

\( \Delta y \)   
Depth of surcharging  

\( \Delta z \)   
Variation in piezometric pressure  

\( \theta \)   
Angle between the sewer axis and a horizontal plane  

\( \nu \)   
Kinematic viscosity  

\( \pi \)   
= \( \frac{22}{7} \) = constant  

\( \rho \)   
Density of air-water mixture  

\( \rho_1 \)   
Density of component 1  

\( \rho_2 \)   
Density of component 2  

\( \rho_a \)   
Density of air  

\( \rho_g \)   
Density of gas  

\( \rho_l \)   
Density of liquid  

\( \rho_{SL} \)   
Density of solid-liquid mixture  

\( \rho_w \)   
Density of water  

\( \tau_0 \)   
Shear resistance of pipe walls  

\( \phi \)   
Central angle of water surface
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1949 Born on the 17th of March in Alexandria, Egypt.

1967 Matriculated from Ramleh High School, Alexandria, Egypt.

1972 Graduated with a Bachelor of Science (Honour) in Civil Engineering, Alexandria University, Alexandria, Egypt.

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