Fatigue Crack Growth in Steel used in Oil and Gas Pipelines

Jorge Silva
University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation
Silva, Jorge, "Fatigue Crack Growth in Steel used in Oil and Gas Pipelines" (2011). Electronic Theses and Dissertations. 5391.
https://scholar.uwindsor.ca/etd/5391

This online database contains the full-text of PhD dissertations and Masters’ theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000 ext. 3208.
Fatigue Crack Growth in Steel used in Oil and Gas Pipelines

By

JORGE SILVA

A Thesis
Submitted to the Faculty of Graduate Studies
c through
the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for
the Degree of Master of Applied Science at the
University of Windsor

Windsor, Ontario, Canada
2011

© 2011 JORGE SILVA
Fatigue Crack Growth in Steel used in Oil and Gas Pipelines

By

Jorge Silva

APPROVED BY:

Dr. J. Sokolowski, Outside Department Reader
Mechanical, Automotive and Materials Engineering

Dr. S. Cheng, Department Reader
Department of Civil and Environmental Engineering

S. Das, Advisor
Department of Civil and Environmental Engineering

N. Biswas, Chair of Defense
Department of Civil and Environmental Engineering
DECLARATION OF ORIGINALITY

I hereby certify that I am the sole author of this thesis and that no part of this thesis has been published or submitted for publication.

I certify that, to the best of my knowledge, my thesis does not infringe upon anyone’s copyright nor violate any proprietary rights and that any ideas, techniques, quotations, or any other material from the work of other people included in my thesis, published or otherwise, are fully acknowledged in accordance with the standard referencing practices. Furthermore, to the extent that I have included copyrighted material that surpasses the bounds of fair dealing within the meaning of the Canada Copyright Act, I certify that I have obtained a written permission from the copyright owner(s) to include such material(s) in my thesis and have included copies of such copyright clearances to my appendix.

I declare that this is a true copy of my thesis, including any final revisions, as approved by my thesis committee and the Graduate Studies office, and that this thesis has not been submitted for a higher degree to any other University or Institution.
ABSTRACT

Oil and gas pipelines are subject to cyclic loads and can develop fatigue cracks. Particularly, pipes with longitudinal fatigue cracks are of utmost importance since they are associated with bursting. With this issue in mind a method to predict fatigue crack growth was sought out. Hence, the objective of the current study was to develop a model to conservatively estimate fatigue life of a full-scale pipe specimen with a longitudinal fatigue crack. Fracture mechanics concepts in conjunction with a statistical model and both experimental and numerical techniques were used to construct this model. Nine compact tension specimens, one full-scale pipe specimen with longitudinal crack and a finite element model were used in this study. The fatigue life predicted by the model was compared to the fatigue life of a full-scale pipe specimen and reasonable results were found.
DEDICATION

I dedicate this work to my family, friends, Ernesto for his example and Serena.
ACKNOWLEDGEMENTS

I owe many thanks to my supervisor Dr. Das, to whom I am indebted to for his immense help, patience and priceless advice. Thank you very much for your detailed revisions and for treating my problems as if they were yours. I would like to thank the rest of my committee: Dr. Cheng, Dr. Sokolowski for their guidance on my proposal and thesis, and the time spent reading my thesis. Thank you Dr. Biswas for being the chair of my committee.

Also, many thanks to Lucian Pop, Matt St. Louis and Patrick Seguin for all their assistance, and for sharing their knowledge and expertise with me. Thanks to Hanif Ghaednia for his friendship, and genuine help in this work. I owe many thanks to my family for their patience, guidance, support and belief in me. Finally, I would like to say many thanks to Serena Tang for her tireless support and dedication. This work reached its true potential because of you and I’ll never forget it.
# TABLE OF CONTENTS

DECLARATION OF ORIGINALITY .................................................................................. iii
ABSTRACT ................................................................................................................ iv
DEDICATION .............................................................................................................. v
ACKNOWLEDGEMENTS ............................................................................................ vi
LIST OF TABLES .......................................................................................................... x
LIST OF FIGURES ....................................................................................................... xi

## CHAPTER

### I. INTRODUCTION

1.1 General ..................................................................................................................... 1
1.2 Pipes subjected to cyclic loads .................................................................................. 1
1.3 Objectives of the study .............................................................................................. 3
1.4 Methodology ............................................................................................................ 3
1.5 Scope of work ........................................................................................................... 3
1.6 Thesis outline ........................................................................................................... 4

### II. REVIEW OF LITERATURE

2.1 Theoretical Background .......................................................................................... 6
2.1.1 Stress intensity factor ......................................................................................... 7
2.1.2 Effect of stress ratio .......................................................................................... 10
2.1.3 Paris equation ..................................................................................................... 13
2.2 Full-scale pipe tests ................................................................................................ 17
2.3 Methods to predict fatigue life ................................................................................ 18
2.4 Stress intensity factor solutions .............................................................................. 26
2.4.1 General ................................................................................................................. 26
2.4.2 Newman and Raju equations ............................................................................. 26
2.4.3 Stress intensity solutions used in investigations of pipes ................................ 30
2.5 Effect of notch ......................................................................................................... 34
2.6 Semi-elliptical crack shape .................................................................................... 35
2.7 Methods of inspection ........................................................................................... 38
2.8 Summary .................................................................................................................. 43

### III. DESIGN AND METHODOLOGY

3.1 General ....................................................................................................................... 46
3.2 Test specimens .................................................................47
  3.2.1 Selection of specimen parameters ..................................47
  3.2.2 Preparation of the specimen ........................................48
  3.2.3 Selection of boundary conditions ..................................50
  3.2.4 Test variables .........................................................51
  3.2.5 Designation of specimens ..........................................51
  3.2.6 Mechanical properties ...............................................52
3.3 Experimental setup .......................................................54
  3.3.1 Load cell and fatigue testing frame ...............................58
  3.3.2 Linear voltage displacement transducer ..........................58
  3.3.3 Electronic resistance strain gauges ...............................59
  3.3.4 Data acquisition system ............................................60
  3.3.5 Dinolite digital microscope .......................................61
  3.3.6 Extensometer .........................................................62
3.4 Test procedure ..............................................................63
3.5 Summary ............................................................................68

IV. ANALYSIS OF RESULTS
  4.1 General ...........................................................................69
  4.2 CT specimen data ..........................................................69
  4.3 Statistical approach to Paris equation constants .................82
  4.4 Finite element model ....................................................91
  4.5 Stress intensity factor estimation ...................................94
  4.6 Determination of $\beta$-factor ........................................96
  4.7 Development of fatigue life model for pipe .......................99
  4.8 Validation of fatigue model ............................................100
  4.6 Summary ........................................................................107

V. CONCLUSIONS AND RECOMMENDATIONS
  5.1 General ...........................................................................109
  5.2 Conclusions .....................................................................109

APPENDICES
  Additional crack length versus number of cycles curves for CT specimens ......113
  A.1 Crack length versus number of cycles ...............................113
  A.2 CT specimens with longitudinal notch ..............................114
  A.3 CT specimens with circumferential notch ...........................116
Additional crack growth rate versus stress intensity factor range plots for CT specimens

B.1 CT specimens with longitudinal notch .................................................. 119
B.2 CT specimens with circumferential notch ........................................... 121
B.3 Linear region for CT specimens with longitudinal notch ........ 124
B.4 Linear region for CT specimens with circumferential notch ..... 126

Fatigue estimates of CT specimens compared to test data ......................... 129

C.1 Life estimates of CT specimens with longitudinal notch .... 129
C.2 Life estimates of CT specimens with circumferential notch ..... 131

REFERENCES ............................................................................................ 134

VITA AUCTORIS .................................................................................... 140
LIST OF TABLES

Table 2.1 Constants for da/dN vs. ΔK curves for various classes of steel for $R \approx 0$ ........ 14

Table 2.2 Crack size comparison at crack center for three techniques (Satyarnarayan, et al. 2007) ................................................................. 42

Table 3.1: Specimen matrix .............................................................................. 52

Table 3.2: Material Properties........................................................................... 53

Table 4.1 Test matrix for CT specimens ................................................................. 71

Table 4.2: CTC5 data ...................................................................................... 74

Table 4.3: Values of Paris equation constants for CT specimens ......................... 79

Table 4.4: Descriptive statistics of cycles to failure ............................................. 83

Table 4.5: Steps used for determination of C and m ........................................... 87

Table 4.6: Summary of Paris equation constants obtained through statistical model .. 89

Table 4.7: Estimation of full-scale specimen ....................................................... 101
LIST OF FIGURES

Figure 2.1: Crack loading modes ................................................................. 7
Figure 2.2: Polar coordinate ................................................................. 7
Figure 2.3: Small scale yielding (Anderson 1991) ............................................ 10
Figure 2.4: One possible mechanism of fatigue crack growth (Broek 1988) ........ 11
Figure 2.5: Crack growth regions in metal (Fatigue crack growth) ..................... 13
Figure 2.6: Fatigue crack growth of pressure vessel steel (Dowling 1999) .......... 15
Figure 2.7: Effect of R on crack growth (Dowling 1999) ................................. 15
Figure 2.8: Fatigue test setup (Singh, et al. 2008) ........................................ 17
Figure 2.9: Schematic of test set up (Singh et al. 2008) .................................... 18
Figure 2.10: Pipe cross section - notch length (2c) and notch depth (a) (Singh et al. 2008) ................................................................. 18
Figure 2.11: Typical FAD (Cheaitini 2005) .................................................. 19
Figure 2.12: Process of fatigue life prediction (Luo et al. 2004) ................. 21
Figure 2.13: Variation of crack depth with cycles (Singh et al. 2008) ............... 23
Figure 2.14: Variation of crack depth with cycles (Saxena and Chouhan 2009) .... 24
Figure 2.15: Pipe with flaw on outer surface (Yoo and Kotoji 1999) .............. 24
Figure 2.16: Model to evaluate stress intensity factor (Yoo and Kotoji 1999) .... 25
Figure 2.17: Calculated and experimental leak life (Yoo and Kotoji 1999).......... 25
Figure 2.18: Surface crack in a finite plate (Newman and Raju 1981) ............ 27
Figure 2.19: Cracked plate subject to tension or bending loads (Newman and Raju 1981) ................................................................. 28
Figure 2.20: Internal surface crack in a cylinder ................................................................. 29
Figure 2.21: Stress intensity factors for different models (Shen et al. 2006) .................. 31
Figure 2.22: Variation of \( K_{cy} / K_{pl} \) with \( a/c \) ratio for short external surface cracks
   (Gordon 1988) .............................................................................................................. 32
Figure 2.23: Stress concentration factor vs. notch relative radius (Carpinteri and
   Vantadori 2003) .......................................................................................................... 35
Figure 2.24: Beach marks in pipe under bending moment (Satyarnaran et al. 2007)...... 37
Figure 2.25: Photographs of fatigue fracture surface (Luo et al. 2004)....................... 37
Figure 2.26: Dye penetrant technique .............................................................................. 39
Figure 2.27: Crack profile at 0 cycles (Satyarnarayan et al. 2007)................................. 42
Figure 2.28: Crack center growth as measured by three techniques (Satyarnarayan et al.
   2007) ......................................................................................................................... 43
Figure 3.1: Dimensions of CT specimen in mm .............................................................. 48
Figure 3.2: Full-scale specimen set up ............................................................................ 49
Figure 3.3: CT specimen ............................................................................................... 50
Figure 3.4: Tensile stress-strain behaviour of pipe steel .............................................. 53
Figure 3.5: Schematic of full-scale ................................................................................ 54
Figure 3.6: CT specimen set up .................................................................................... 55
Figure 3.7: INSTRON Model 1332 fatigue testing frame ............................................. 56
Figure 3.8: Hydraulic power unit and FlexTest GT Digital Controller ......................... 57
Figure 3.9: CT specimen grips (all dimensions in mm) .................................................. 57
Figure 3.10: Digital microscope in place ....................................................................... 58
Figure 3.11: LVDT inside of pipe .................................................................................. 59
Figure 3.12: Strain gauges near notch................................................................. 60
Figure 3.13: Wheatstone bridge........................................................................ 60
Figure 3.14: Datascan module and computer ...................................................... 61
Figure 3.15: Crack close up aided by digital microscope...................................... 62
Figure 3.16: Notch ............................................................................................... 64
Figure 3.17: Specimens cut near notch.............................................................. 65
Figure 3.18: SEM lab at University of Windsor ................................................... 65
Figure 3.19: SEM photograph of notch and crack.............................................. 66
Figure 3.20: Control computer ........................................................................... 67
Figure 3.21: CT specimen at completion of test.................................................. 68
Figure 4.1: CT specimen at unstable crack growth stage .................................... 71
Figure 4.2: Crack length vs. number of cycles for CTC5..................................... 73
Figure 4.3: Scatter data used for crack growth rate plot for CTC5...................... 74
Figure 4.4: CT specimen ..................................................................................... 76
Figure 4.5: Crack growth rate vs. stress intensity factor range of CTC5............. 77
Figure 4.6: Linear portion of CTC5.................................................................... 78
Figure 4.7: Data for all nine CT specimens ......................................................... 80
Figure 4.8: Fatigue crack growth rate vs. $\Delta K$ (Yoo and Kotoji, 1999).............. 81
Figure 4.9: Regression of crack growth data at cumulative probability of 5% ...... 89
Figure 4.10: Fatigue estimate of CTC1................................................................. 90
Figure 4.11: Finite element model ....................................................................... 91
Figure 4.12: Elements near notch ..................................................................... 92
Figure 4.13: Top collar ...................................................................................... 93
Figure 4.14: Bottom collar boundary condition ...................................................... 93
Figure 4.15: Load-deformation data of test and FE model compared ........................ 93
Figure 4.16: Stresses from finite element model (Broek, 1989) ............................... 97
Figure 4.17: Apparent stress intensity (Broek, 1989) ............................................ 97
Figure 4.18: Extrapolation of K .............................................................................. 98
Figure 4.19: Beta factor ......................................................................................... 98
Figure A.1: Crack length from notch vs. number of cycles for all nine CT specimens . 113
Figure A.2.1: Specimen CTL1 plot of crack length from notch vs. number of cycles... 114
Figure A.2.2: Specimen CTL2 plot of crack length from notch vs. number of cycles... 114
Figure A.2.3: Specimen CTL3 plot of crack length from notch vs. number of cycles... 115
Figure A.2.4: Specimen CTL4 plot of crack length from notch vs. number of cycles... 115
Figure A.3.1: Specimen CTC1 plot of crack length from notch vs. number of cycles... 116
Figure A.3.2: Specimen CTC2 plot of crack length from notch vs. number of cycles... 116
Figure A.3.3: Specimen CTC3 plot of crack length from notch vs. number of cycles... 117
Figure A.3.4: Specimen CTC4 plot of crack length from notch vs. number of cycles... 117
Figure A.3.5: Specimen CTC5 plot of crack length from notch vs. number of cycles... 118
Figure B.1.1: Crack growth rate vs. stress intensity range for specimen CTL1 ........... 119
Figure B.1.2: Crack growth rate vs. stress intensity range for specimen CTL2 ........... 120
Figure B.1.3: Crack growth rate vs. stress intensity range for specimen CTL3 ........... 120
Figure B.1.4: Crack growth rate vs. stress intensity range for specimen CTL4 ........... 121
Figure B.2.1: Crack growth rate vs. stress intensity range for specimen CTC1 ........... 121
Figure B.2.2: Crack growth rate vs. stress intensity range for specimen CTC2 ........... 122
Figure B.2.3: Crack growth rate vs. stress intensity range for specimen CTC3 ........... 122
Figure B.2.4: Crack growth rate vs. stress intensity range for specimen CTC4.............. 123
Figure B.2.5: Crack growth rate vs. stress intensity range for specimen CTC5.............. 123
Figure B.3.1: Crack growth rate vs. stress intensity range (linear region) for CTL1 .... 124
Figure B.3.2: Crack growth rate vs. stress intensity range (linear region) for CTL2 .... 124
Figure B.3.3: Crack growth rate vs. stress intensity range (linear region) for CTL3 .... 125
Figure B.3.4: Crack growth rate vs. stress intensity range (linear region) for CTL4 .... 125
Figure B.4.1: Crack growth rate vs. stress intensity range (linear region) for CTC1 ..... 126
Figure B.4.2: Crack growth rate vs. stress intensity range (linear region) for CTC2 ..... 126
Figure B.4.3: Crack growth rate vs. stress intensity range (linear region) for CTC3 ..... 127
Figure B.4.4: Crack growth rate vs. stress intensity range (linear region) for CTC4 ..... 127
Figure B.4.5: Crack growth rate vs. stress intensity range (linear region) for CTC5 ..... 128
Figure C.1.1: Fatigue estimates compared vs. CTL1 test data ........................................ 129
Figure C.1.2: Fatigue estimates compared vs. CTL2 test data ........................................ 129
Figure C.1.3: Fatigue estimates compared vs. CTL3 test data ........................................ 130
Figure C.1.4: Fatigue estimates compared vs. CTL4 test data ........................................ 130
Figure C.2.1: Fatigue estimates compared vs. CTC1 test data ........................................ 131
Figure C.2.2: Fatigue estimates compared vs. CTC2 test data ........................................ 131
Figure C.2.3: Fatigue estimates compared vs. CTC3 test data ........................................ 132
Figure C.2.4: Fatigue estimates compared vs. CTC4 test data ........................................ 132
Figure C.2.5: Fatigue estimates compared vs. CTC5 test data ........................................ 133
CHAPTER I
INTRODUCTION

1.1 General
Fatigue crack is a serious concern for the oil and gas pipeline industry and hence, there is strong interest in quantifying fatigue damage on pipelines. Because flaws cannot be ruled out and the risk of catastrophic failure such as bursting at a crack location exists, damage tolerance becomes a valuable tool. Damage tolerance has been defined by Broek (1989) as the ability of the structure to sustain damage in the form of cracks without catastrophic consequences, until such time that the damaged component can be repaired. Even though this was written with aircraft requirements in mind the same philosophy can be applied to oil and gas pipelines and other structural components. With the aid of fracture mechanics various investigations were undertaken to estimate fatigue life of steel pipes. However, much of the existing literature involves pipes with circumferential cracks/notches. Thus, the longitudinal crack which is regarded as being more critical has not been studied as thoroughly.

1.2 Pipes subjected to cyclic loads
Fatigue crack growth is defined as the weakening or breakdown of a material subjected to cyclic stresses such as pressure variations which oil and gas pipelines must endure in the field. To simulate cyclic stresses, researchers have tested pipes with four-point bending cyclic load. These studies employed circumferential notch to create stress concentration at predetermined locations. At the notch location, crack growth in the pipe wall thickness direction was monitored by techniques such as alternating current potential drop (ACPD),
conventional ultrasonic, phased array ultrasonic, and beach marking using SEM. Test results of fatigue crack growth were compared to the predictions based on fracture mechanics concepts such as the stress intensity factor and Paris equation. Although handbooks on stress intensity solutions which apply to many configurations exist, the solution for a stress intensity factor for a flaw in a pipe is still a complex problem. Various fatigue crack growth solutions were proposed, however, they were found to be limited to specific cases. Therefore, to use these solutions, important assumptions such as the shape of the flaw were made by researchers.

At the time that the current study was undertaken no studies were found where the Paris equation was used to estimate fatigue crack growth in oil and gas pipelines with longitudinal crack. This is despite the longitudinal crack being deemed to be more critical than a circumferential crack since the longitudinal crack is associated with bursting and circumferential crack leads to leaking. Fatigue tests as per ASTM E647 (ASTM 2008) are commonly used to determine material constants C and m, used in Paris equation. It is possible that these constants possess a wide range of values. However, no statistical approach was found to be used to calculate C and m values for metal used in pipelines. Also, no statistical approach was found in the open literature to be used in conjunction with fracture mechanics for determining fatigue life of pipes. This could be an interesting and useful approach since fatigue crack growth and fatigue life are stochastic quantities.
1.3 Objectives of the study
The current study was undertaken to develop a model to conservatively estimate fatigue life of a full-scale pipe specimen that has developed cracks in the longitudinal direction of the pipe. Fatigue life was estimated using fracture mechanics concepts in conjunction with a finite element model using statistics. A full-scale pipe specimen was used to validate the proposed fatigue life model.

1.4 Methodology
This work was completed using both experimental and numerical methods. The following are the activities completed.

- Nine tests on CT specimens
- One full-scale test on 762 mm diameter steel pipe
- Finite element analysis
- A statistical approach for determining Paris equation constants, C and m

1.5 Scope of work
This study included the following activities.

- Detailed literature review
- Nine compact tension tests to determine C and m
- One full-scale test on 762 mm diameter and 1.6 m long pipe with longitudinal notch
- Development of appropriate statistical approach for the estimation of realistic C and m values
• Determination of stress intensity factor using a detailed non-linear finite element model in ABAQUS

• Development of a statistical method to be able to use Paris equation type approach for development of a model that can conservatively determine the fatigue crack growth and fatigue life of oil and gas pipes with longitudinal crack

• Validation of fatigue crack growth model using full-scale test data

1.6 Thesis outline
This thesis consists of five chapters. The second chapter provides a detailed literature review. The second chapter also provides a brief theoretical background to establish the fracture mechanics theory and terminology used in the current study. After the theoretical summary, the methods to estimate fatigue life of full-scale specimens is reviewed. Since determination of the stress intensity factor is a complex issue, a detailed review of the methods and approaches used by other researchers to estimate it is provided. Finally, this chapter provides a short overview of the methods used by various researchers to measure fatigue crack growth during experiments of pipes.

The third chapter describes the experimental procedure followed and the test equipment used. Details are given pertaining to compact tension tests and the full-scale specimen test.

Chapter four provides a detailed account of the results of the current study. The statistical model developed to estimate C and m is presented. This chapter also presents the finite
element model used to determine stress intensity factor. Finally, the chapter discusses the manner in which Paris equation was used in the fatigue life estimate. The result of the model is compared to the full-scale specimen fatigue life.

Chapter five provides the main conclusions of the study and provides recommendations for future research work.
CHAPTER II

REVIEW OF LITERATURE

2.1 Theoretical Background

Fracture mechanics can be defined as the study of solids with cracks (Dowling 1999). In particular linear fracture mechanics assumes a material is linear elastic and isotropic (efunda 2011).

Fatigue crack growth can be defined as the weakening or breakdown of a material subjected to cyclic stresses. These stresses can be due to a variation in loads or temperature (Berkeley 2011).

Fracture mechanics uses three modes to describe the type of loading that a crack can be subjected to (Figure 2.1). Mode I, also known as the opening mode, occurs when a load is applied normal to the crack plane. Mode II is the in-plane shear case. Mode III is out-of-plane shear (Anderson 1991). Only mode I was considered in this study since almost all cracks experience mode I loading and even if other modes were originally present, mode I will almost surely prevail (Broek 1988). A simple example of mode I prevalence would be pulling a piece of paper with an oblique tear. Almost immediately the tear will change direction to be perpendicular (mode I) to the maximum tensile stress. Fatigue crack growth is almost certain to be pure mode I (Dexter, 2004).
A concept known as the stress intensity factor is basic to fracture mechanics and will be discussed next.

2.1.1 Stress intensity factor

Using a polar coordinate axis (Figure 2.2) the stress field in any linear elastic cracked body is defined by Equation 2.1

\[
\sigma_{ij} = \left( \frac{k}{\sqrt{r}} \right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m \sqrt{r}^m g^{(m)}(\theta) \tag{2.1}
\]
In Equation 2.1, $\sigma_{ij}$ is the stress tensor while $k$ is a constant and $f_{ij}$ is a dimensionless function of $\theta$. As $r \to 0$, i.e., $r$ approaches the crack tip, the first term $\left( \frac{k}{\sqrt{r}} f_{ij}(\theta) \right)$ approaches infinity. The rest of the terms go to zero or stay finite. Therefore, the stress near the tip is proportional to $\frac{1}{\sqrt{r}}$ where $r$ is the radial distance of a point measured from the crack tip.

Terms $k$ and $f_{ij}$ (Equation 2.1) depend on the mode and angle $\theta$. The $k$ can be replaced by the stress intensity factor, $K$ in the following manner (Anderson 1991).

$$K = k\sqrt{2\pi}$$

(2.2)

Hence, the following can be said.

$$\lim_{r \to 0} \sigma_{ij} = \frac{k}{\sqrt{2\pi r}} f_{ij}(\theta)$$

(2.3)

If the crack plane (i.e. $\theta = 0$) becomes the focus of interest then a formal definition of the stress intensity factor in a mathematical sense is as follows (Dowling 1999).

$$K = \lim_{r, \theta \to 0} (\sigma_y \sqrt{2\pi r})$$

(2.4)

For the sake of convenience the stress intensity factor is usually expressed as shown in the following equation (Dowling 1999).

$$K = \beta \sigma \sqrt{\pi a}$$

(2.5)

The newly introduced factor $\beta$ is the geometry factor, $\sigma$ is the reference stress and $a$ is the crack length. The factor $\beta$ is used to account for different crack and cracked component
geometry. The stress intensity factor can be understood to be the severity of the stress field near the crack tip (Dowling 1999). Sometimes $K$ is expressed $K_I$ to emphasize that this $K$ value is for mode I.

If certain conditions apply fatigue crack growth can be solely described by the stress intensity factor. This concept is known as similarity. In other words the crack tip is uniquely defined by the stress intensity factor. Assume a crack is growing under constant amplitude cyclic stress. At the crack tip a plastic zone will form as a result of stress concentration. If this plastic zone is entirely within an elastic singularity zone (see Figure 2.3) then similarity applies. The elastic singularity zone can be understood to be a region outside the plastic zone where elastic stress field equations (Equations 2.6-2.8) apply. These equations are the elastic solution for stresses near a crack tip (Paris et al. 1961). This occurs if the plastic zone is small.

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right] + \cdots \quad (2.6)
\]

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}\right] + \cdots \quad (2.7)
\]

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} + \cdots \quad (2.8)
\]
2.1.2 Effect of stress ratio

Crack growth occurs as a result of slip and crack tip blunting. Due to stress concentration at crack tips there will be plastic deformation even at very small loads that will blunt the crack tip. Broek (1988) explained that plastic deformation is slip of atomic planes. Upon unloading the crack tip will again become sharp. Figure 2.4 illustrates a possible mechanism of fatigue crack growth. Broek (1988) suggests that other mechanisms are possible; however they are essentially the same. A larger stress during a cycle will cause more opening and a lower minimum stress will create more sharpening. Hence, a larger maximum stress ($\sigma_{\text{max}}$) and a smaller minimum stress ($\sigma_{\text{min}}$) will extend the crack more. The stress ratio, $R$, is introduced to consider this effect (Broek 1988). Hence, for larger value of $R$, crack grows faster.

$$R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$$ (2.9)
The maximum applied stress in a cycle is called $\sigma_{\text{max}}$ and the minimum applied stress in a cycle is known as $\sigma_{\text{min}}$. Stress ratio can also be defined as follows.

$$R = \frac{K_{\text{min}}}{K_{\text{max}}}$$  \hspace{1cm} (2.10)

Figure 2.4: One possible mechanism of fatigue crack growth (Broek 1988)
\( K_{\text{max}} \) is the maximum stress intensity factor and \( K_{\text{min}} \) is the minimum stress intensity factor in a cycle corresponding to \( \sigma_{\text{min}} \) and \( \sigma_{\text{max}} \), respectively.

The stress intensity factor can also be defined in different ways as follows.

\[
R = \frac{K_{\text{min}}}{K_{\text{max}}} = \frac{\beta \sigma_{\text{min}} \sqrt{\pi a}}{\beta \sigma_{\text{max}} \sqrt{\pi a}} = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = \text{stress ratio}
\]  

(2.11)

Therefore,

\[
R = \frac{K_{\text{min}}}{K_{\text{max}}} = \frac{K_{\text{max}} - \Delta K}{K_{\text{max}}}
\]  

(2.12)

The stress intensity factor range, \( \Delta K \), is defined in the following equation.

\[
\Delta K = K_{\text{max}} - K_{\text{min}}
\]  

(2.13)

Hence, the next equation can be shown to be true.

\[
K_{\text{max}} = \frac{\Delta K}{1-R}
\]  

(2.14)

Since a higher \( K_{\text{max}} \) indicates more crack growth it can be concluded from Equation (2.14) that more crack growth can be achieved for a higher \( \Delta K \) and/or a higher \( R \). Hence, the crack growth rate, \( \frac{da}{dN} \), is a function of the stress intensity factor range (\( \Delta K \)) and stress ratio (\( R \)). The \( N \) is number of fatigue load cycles.

\[
\frac{da}{dN} = f(\Delta K, R)
\]  

(2.15)
2.1.3 Paris equation

A typical plot of the regions of fatigue crack growth is shown in Figure 2.5. It is customary to plot stress intensity factor range ($\Delta K$) versus crack growth rate ($da/dN$) on a log-log plot. Paris and Erdogan (1963) observed that in Region II a linear relationship existed between $\Delta K$ and $da/dN$ on a logarithmic plot.

![Figure 2.5: Crack growth regions in metal (Fatigue crack growth)](image)

A linear equation can be expressed in the slope-intercept form as $y = mx + b$. Here, $y = \log (da/dN)$ and $x = \log (\Delta K)$. Hence, the following equation can be written.

$$\log \left( \frac{da}{dN} \right) = m \log (\Delta K) + \log C \quad (2.16)$$

By applying the anti-log the following relationship is obtained.

$$\frac{da}{dN} = C (\Delta K)^m \quad (2.17)$$
This is known as the Paris equation (See Figure 2.6). Although this equation has no physical basis and it was derived from curve fitting, it is widely used and it has been shown to produce accurate results (Broek 1988). In this equation \( \frac{da}{dn} \) is crack growth rate; \( \Delta K \) is the stress intensity factor range; and C and m are material constants.

Parameter C can be understood to be the y intercept and factor m as the slope of the curve (Figure 2.5). Both parameters are material properties. Parameter m describes how sensitive a material’s growth rate is to the stress applied since \( \Delta K = \beta \sigma \sqrt{\pi a} \). A value of approximately three for m has been reported in carbon steel (Singh, et al. 2008). The constant C is more dependent on the material (Broek 1989). The following table provides conservative values of C and m (Dowling 1999).

<table>
<thead>
<tr>
<th>Class of steel</th>
<th>Constants for</th>
<th>( \frac{da}{dn} = C (\Delta K)^m )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( mm/cycle )</td>
<td>( in/cycle )</td>
</tr>
<tr>
<td>Ferritic-pearlitic</td>
<td>6.89 x 10^{-9}</td>
<td>3.6 x 10^{-10}</td>
</tr>
<tr>
<td>Martensitic</td>
<td>1.36 x 10^{-9}</td>
<td>6.6 x 10^{-10}</td>
</tr>
<tr>
<td>Austenitic</td>
<td>5.61 x 10^{-9}</td>
<td>3.0 x 10^{-10}</td>
</tr>
</tbody>
</table>

A fatigue crack growth equation in the Paris equation form only applies to one stress ratio. Plots of log \( \Delta K \) vs. log(\( da/\dot{N} \)) for different stress ratios are parallel. In other words, they have the same slope (m) value. This is illustrated in Figure 2.7. If possible it is advisable to have data for a wide range of stress ratios.
Noting Equation (2.17) is a differential equation, it is possible to find the number of cycles to reach a certain crack length by integration.
\[ N = \int_{a_0}^{a_f} \frac{da}{f(\Delta K, R)} \tag{2.18} \]

where \(a_f\) is the final crack length and \(a_0\) is the initial crack length. If the Paris equation applies then the following can be written.

\[ N = \frac{1}{c} \int_{a_0}^{a_f} \frac{da}{[\beta \left(\frac{a}{W}\right)\Delta \sigma \sqrt{\pi a}]^m} \tag{2.19} \]

If the crack geometry and loading is simple then Equation (2.19) can easily be integrated, but for most cases \(\beta\) is a complex function of \(a/W\) where \(W\) is the width of cracked body in the direction of the crack growth. Since in most cases the \(\beta\) function is complex, numerical methods are used to solve this equation. It is crucial to note that Equation 2.18 applies to a structure under constant amplitude loading. This is rarely the case, though researchers work with constant amplitude loading because of its simplicity as compared to variable amplitude loading. In the variable amplitude loading scenario the crack growth \(da/dN\) is different from one cycle to the next depending on \(\Delta K\) and \(R\). For the variable amplitude loading case, the researcher must account for load interaction (Anderson 1991).

Although the Paris equation was observed to occur in region II it can be extrapolated to region III because life is very short in this region. The Paris equation can also be used in region I, however, results could be nonconservative (Fuchs and Stephens 1980). Section 2.1 was included with the objective of providing the reader a background of basic fracture mechanics theory since current study took a fracture mechanics approach.
2.2 Full-scale pipe tests

A few researchers and research groups have performed full-scale tests on pipe specimens to study fatigue crack (Singh et al., 2003, 2008; Yoo and Kotoji 1999; Saxena and Chouhan 2009; Luo, Xiong and Huo 2004; Cottam 1973). The majority of these studies employed pipes subjected to four-point constant amplitude fatigue bending load (Figures 2.8 and 2.9) having an initial circumferential notch (see Figure 2.10) (Singh et al. 2003, 2008; Yoo and Kotoji 1999; Saxena and Chouhan 2009). The test set up used by Singh et al. (2007) is shown in these figures. Luo et al. (2004) undertook a study with a longitudinal notch under constant amplitude loading while Dover et al. (1980) conducted an investigation on a tubular welded joint under in-plane bending without a notch subjected to variable amplitude loading.

Figure 2.8: Fatigue test setup (Singh, et al. 2008)
2.3 Methods to predict fatigue life

Various methods are available to estimate fatigue life of pipes. One possible alternative is the failure assessment diagram (FAD) technique (Luo et al. 2004). The FAD based approach used plot of the stress intensity on the ordinate and stress on the abscissa. This is done to account for brittle fracture, fully plastic failure, and a combination of both. It is
a way to account for the fact that stress is restricted by collapse and stress intensity is restricted by the material’s toughness (Broek 1988). On the plot, normalized stress (Lr) and normalized crack tip loading (Kr) appear as shown in Figure 2.11.

The point corresponding to a flaw falls within the FAD envelope then the flaw is thought to be safe. Kr and Lr as expressed as follows.

\[
K_r = \frac{K}{K_{IC}} \tag{2.20}
\]

\[
L_r = \frac{\sigma_{ref}}{\sigma_y} \tag{2.21}
\]

where \(K_{IC}\) is the fracture toughness of the material, \(\sigma_{ref}\) is the reference stress, and \(\sigma_y\) is the yield strength (Hosseini et al. 2010). Factor \(K_{IC}\) is critical stress intensity where the material fails locally as a result of a serious combination of stress and strain. Therefore, fracture toughness is a way to quantify a material’s resistance to an applied load if a flaw exists. \(K_{IC}\) is an alternate way to gauge fracture toughness.

![Figure 2.11: Typical FAD (Cheaitini 2005)](image-url)
Luo et al. (2004) used this approach to estimate the fatigue life of full-scale specimens. A flow chart explaining the procedure is shown in Figure 2.12. Various codes employ the FAD-based method such as API 579 (API 2000) and BS 7910 (BS 7910 2000) to assess crack-like flaws (Z.I. Limited; Hosseini, Plumtree and Kania 2010). For example, both API 579 and BS 7910 provide stress intensity factor solutions for a cylinder under internal pressure having a semi-elliptical surface crack. Therefore, it is possible to use the Paris equation to estimate the time needed to reach a final crack size. This final crack size could be the maximum tolerable size calculated using the fracture assessment procedure (TWI 2011).

Unfortunately, the model proposed by Luo et al. (2004) yielded a large error. This model estimated 3,635 cycles whereas the actual fatigue life was approximately 8,000 cycles. It can be seen that the Paris equation was used to develop a model to estimate fatigue life. Specifically the Paris equation was used to compute the increase in cycle count corresponding to an increment in flaw size. Other researchers have used the Paris equation to estimate the remaining fatigue life of full-scale specimens as well. (Singh, et al. 2003, 2008; Yoo and Kotoji 1999; Saxena and Chouhan 2009; Cottam 1973; Dover and Holbrook 1980).

Singh et al. (2007) used Paris equation to predict fatigue life of carbon steel piping components under four-point bending cycles. Singh et al. (2003, 2007) employed compact tension (CT) tests as in ASTM standard E647 (ASTM 2011) to obtain material constants C and m.
Figure 2.12: Process of fatigue life prediction (Luo et al. 2004)
These material constants were obtained by fitting the test data of CT specimens to the form of the Paris equation. Singh et al. (2003) concluded that standard CT specimens can be used directly to analyze pipe crack growth rate.

Singh et al. (2007) obtained fatigue crack growth curves from plots of crack length (a) versus number of cycles (N). The stress intensity factor was calculated from the stress found by using the following relationship.

\[
\sigma = \frac{M}{z} \tag{2.18}
\]

Equation 2.18 was employed since four-point bending load was applied on the test specimen. The geometry factor \( \beta \), as in Equation 2.5, was obtained from Andersson et al. (1996).

Figure 2.13 shows the level of accuracy that Singh et al. (2008) obtained in predicting fatigue crack growth. This study shows it is possible to obtain good results using Paris equation to predict fatigue crack growth in steel pipes. In Figure 2.13, \( a_i \) is the initial notch depth and a is the actual crack length for a specific load cycle. Notches were used to initiate a crack at a known location.

Saxena et al. (2009) conducted a study on a straight pipe with a constant notch subjected to cyclic bending moment. In this study the number of cycles for each crack length extension was calculated and added up to the point of through-thickness cracking to find the total life (Figure 2.14). Figure 2.14 compares the results of crack growth calculated
with the experimental results. This study claims excellent results were obtained through the use of this model. The study also concluded that semi-elliptical stress intensity factor solutions (obtained from ASM (ASM 1996)) should not be used to assess fatigue life when straight pipe has a constant depth crack profile. In this plot, experimental results are compared with those obtained from finite element analysis stress intensity solutions and with those obtained from ASM Handbook Vol. 19 solutions (ASM 1996).

Figure 2.13: Variation of crack depth with cycles (Singh et al. 2008)
Yoo and Kotoji (1999) also used Paris equation to estimate fatigue life of steel pipe. However, curvature of the pipe was eliminated by treating the pipe into a plate like specimen. Essentially a pipe (Figure 2.15) was “flattened” into a plate as shown in Figure 2.16. The model assumed that the plate width (2W) was equal to the outer pipe perimeter (2πR) and plate thickness was equal to the pipe wall thickness (t).

![Figure 2.15: Pipe with flaw on outer surface (Yoo and Kotoji 1999)](image-url)
Calculation for load cycles until leak occurred was conducted using Paris equation, Newman-Raju’s formula (Newman and Raju 1981) and the plate model mentioned above. Newman and Raju’s formula is an equation for the stress intensity factor of a semi-elliptical flaw. This equation is briefly discussed in Section 2.4.2. Experimental and calculated values are shown in Figure 2.17. This study claimed that an excellent agreement between calculated and experimental results was obtained.
2.4 Stress intensity factor solutions

2.4.1 General
Stress intensity factor (K) needs to be determined if Paris equation is used for determining the fatigue life. However, determination of K is a complex issue. Since three-dimensional closed form stress intensity factor solutions are not available for pipeline flaws, many determined these from finite element analysis. Others used a closed-form solution for a flawed flat plate and then incorporated a correction factor to account for the pipe curvature effect. It is common to use a flat plate and use a simple correction factor to take account for different geometry (Gordon 1988). Since the stress intensity factor equations proposed by Newman and Raju are mentioned frequently in the literature, a brief summary of the equations mentioned in this chapter is provided next.

2.4.2 Newman and Raju equations
Newman and Raju (1981) developed an empirical stress intensity factor equation for a surface crack with semi-elliptical shape (Figure 2.18). The equation is a function of parametric angle, crack depth, crack length, plate thickness, and plate width for a finite plate subjected to tension or bending loads. This empirical equation for the stress intensity factor for a surface crack in a finite plate subjected to tension and bending loads was fitted to finite elements results from Newman and Raju (1977, 1979) for a/c values from 0.2 to 1.0. To include a wider range of crack geometry (as a/c approaches zero) the results of Gross and Srawley (1965) for a single-edge crack were also used. In Figure 2.18 a is the crack depth and 2c is the crack length. Before this equation was developed,
stress intensity factors were obtained from three-dimensional, finite element analysis, but it was presented in the form of curves or table. Since an equation was deemed to be preferable, the equation of finite plate with a semi-elliptical surface crack was developed. The equation covers a wide range since it applies to any parametric angle (ϕ), ratios of crack depth to crack length \( \left( \frac{a}{c} \right) \) from 0 to 1.0, ratios of crack depth to plate thickness \( \left( \frac{a}{t} \right) \) ranging from 0 to 1.0, and ratios of crack length to plate width \( \left( \frac{c}{b} \right) \) less than 0.5. The equation for stress intensity factor developed by Newman and Raju (1981) is as follows.

\[
K_I = (S_t + HS_b) \sqrt{\pi \frac{a}{q} F \left( \frac{a}{c}, \frac{a}{t}, \frac{c}{b}, \phi \right)} 
\]  

(2.19)

where \( a \) is the depth of surface crack, \( b \) is the half-width of cracked plate, \( c \) is the half-length of surface crack, \( F \) is the stress-intensity boundary-correction factor, \( Q \) is the shape factor for elliptical crack, and \( \phi \) is the parametric angle of the ellipse. Figure 2.19 shows that \( S_t \) is the remote uniform-tension stress and \( S_b \) is the remote bending stress on outer fiber. Finally, the authors claimed stress intensity factor equations presented should be useful for correlating fatigue crack growth rates as well as in computing fracture toughness of surface cracked plates.

Figure 2.18: Surface crack in a finite plate (Newman and Raju 1981)
Newman and Raju (1983) also developed stress-intensity factor influence coefficients ($G_j$) of semi-elliptical surface cracks on the inside or outside of a cylinder. However, the equations are not given for such a wide range of crack geometry as Newman and Raju (1981). The ratio of crack depth to crack length ($\frac{a}{t}$) ranged from 0.2 to 1; the ratio of crack depth to wall thickness ranged from 0.2 to 0.8 ($\frac{a}{t}$); and the ratio of wall thickness to vessel radius was 0.1 or 0.25 ($\frac{t}{R}$). A three dimensional finite element method was used. Crack surfaces were subjected to four stress distributions: uniform, linear, quadratic, and cubic. These four solutions can be superimposed to obtain stress intensity factor solutions for other stress distributions such as that caused by internal pressure. Results given by Newman and Raju (1983) are those for internal pressure. The cylinder used in this study and the surface crack is shown in Figure 2.20. The stress intensity
factor for an internal surface crack in an internally pressurized cylinder is shown in Equation 2.20. The stress intensity factor for an external surface crack in an internally pressurized cylinder is shown in Equation 2.21.

\[ K_I = \frac{pR}{t} \sqrt{\pi \frac{a}{Q} F_i \left( \frac{a}{c'}, \frac{a}{t}, \frac{t}{R}, \phi \right)} \]  

\[ K_I = \frac{pR}{t} \sqrt{\pi \frac{a}{Q} F_e \left( \frac{a}{c'}, \frac{a}{t}, \frac{t}{R}, \phi \right)} \]  

where \( \frac{pR}{t} \) is the "average" hoop stress and \( F_i \) and \( F_e \) are the boundary-correction factor for a surface crack on the inside of an internally pressurized cylinder and the boundary correction factor for a surface crack located on the outside of an internally pressurized cylinder, respectively. All other factors in Equation are the same as defined by Newman and Raju (1981). The expressions for \( F_i \) and \( F_e \) are in terms of the influence factor \( G_j \). Influence coefficients \( (G_j) \) are given in table form as a function of \( \frac{a}{c'}, \frac{a}{t}, \frac{t}{R} \) and \( \phi \). The authors claimed that stress intensity factors presented should be useful in correlating and predicting fatigue crack growth rates and in calculating fracture strength of surface cracked cylinders under various loading conditions.

Figure 2.20: Internal surface crack in a cylinder
2.4.3 Stress intensity solutions used in investigations of pipes

As described in Section 2.3, Yoo and Kotoji (1999) used a model where the pipe was treated as a plate having a surface flaw of depth $a$ and inner surface crack length $2C$ (Figure 2.16). The stress intensity factor before crack penetration was determined using Newman-Raju’s equation (Newman and Raju 1981).

Numerous studies on pipes or cylinders (Dover and Holbrook 1980; Yoo and Kotoji 1999; Singh, et al. 2003, 2007; Shen, et al. 2006; Saxena and Chouhan 2009; Luo et al. 2004; Iranpour and Taheri 2007; Hosseini et al. 2010) used a semi-elliptical shape flaw in their analysis. This is an important assumption which will be discussed in the following paragraphs.

Shen et al. (2006) used the stress intensity factor solution for a plate with a semi-elliptical crack (based on the work of Newman and Raju (1983) where a finite plate was considered) multiplied by a curvature factor. The curvature factor was taken from BS 7910 (British guide to assess cracks in metallic structures). The problem with this approach is this curvature factor was originally developed to predict ductile failure instead of being an elastic multiplier.

In this work, a pipe with a rectangular crack with filleted corners was simulated. It was found that stress intensity factors for a cracked pipe are greatly overestimated by using the curvature factor for deep cracks (where $\frac{a}{t} \geq 0.5$). The $a$ is the crack depth and $t$ is the pipe thickness. Wall thickness of the specimen was 4.8 mm. However, for shallow cracks
\( \left( \frac{a}{c} \leq 0.2 \right) \) the pipe curvature effect is negligible on the stress intensity factor for a rectangular crack with an aspect ratio of \( \frac{a}{c} \leq 0.1 \). The \( c \) is the half crack length. From Figure 2.21 one can see that a plate solution combined with the pipe bulging factor gives reasonable values for \( K \) when \( \frac{a}{c} \leq 0.5 \). The limitation of the model developed by Newman and Raju (1983) is that results are only given for \( \frac{a}{c} \geq 0.2 \). However pipeline flaws often have very small aspect ratios (for example \( \frac{a}{c} = 0.01 \)).

![Figure 2.21: Stress intensity factors for different models (Shen et al. 2006)](image)

It should be noted that the stress intensity factor in the study by Shen et al. (2006) was for rectangular cracks and work by Newman and Raju (1983) was for semi-elliptical cracks.
Rectangular cracks produce stress intensity factor values higher than semi-elliptical (Shen et al. 2006).

Gordon et al. (1989) undertook a study to compare the stress intensity factor solutions for cracks in pipes and thin walled cylinders with the solutions of flat plates. Since proposed finite element solutions only apply to particular crack sizes, aspect ratios, and wall thickness-to-radius ratios, many used flat plate solutions (Gordon 1988). Gordon (1988) used stress intensity factor solutions ($K_{cyl}$) for cylinders (Raju and Newman 1982) as the benchmark for short, external surface cracks in the axial direction. The study compared it to a flat plate ($K_{pl}$) solution (Newman and Raju 1983). Figure 2.22 demonstrates that for any crack depth the error increases as the aspect ratio ($a/c$) decreases. Ratio $a/c$ decreases as the crack becomes finer.

![Variation of $K_{cyl}/K_{pl}$](image)

Figure 2.22: Variation of $K_{cyl}/K_{pl}$ with $a/c$ ratio for short external surface cracks

(Gordon 1988)
For a/c of 0.2 in Figure 2.22 it can be seen that at a/t of 0.2 the flat plate solution estimates K which is approximately 1.05 times larger than the K value obtained from pipe solution. For a/t of 0.8 the flat plate solution is about 0.85 of the pipe solution. Therefore, there is a 5% and 17% error for a/t of 0.2 and a/t of 0.8, respectively. Recalling the Paris equation, it is known that the value of m for metals is approximately three. This would mean that a 5% error in K value turns into a 16% overestimation ($1.05^3 = 1.16$) in the stress intensity factor. An error of 17% error turns into a 39% underestimation ($0.85^3 = 0.61$) in the K value. It is easy to see how significant error in the value of K can be.

Sometimes in fatigue life it is hard to predict what shape the crack will take. According to Broek (1989) flaw assumptions lead to large error. For example, to be conservative some researchers assume a circular flaw instead of an elliptical flaw. This could cause a stress intensity factor value to be two or three times larger than it should be. In fact Saxena and Chouhan (2009) stated the use of a semi-elliptical solution will produce non-conservative results. This study claims that semi-elliptical stress intensity solutions should not be used to estimate the fatigue life of a straight pipe with a constant depth crack profile. It was reported that semi-elliptical crack profiles using ASM results in an over-prediction of fatigue life (ASM 1996).

The reason elliptical cracks are assumed in practical analysis is that geometry factors are not available for irregular cracks. If multiple crack initiation points exist then when they
join a non-elliptical shape will exist. If the flaw indeed has an irregular front, this assumption may cause considerable error in the analysis (Broek 1989).

2.5 Effect of notch

A finite element analysis based study investigated the influence of a circular-arc circumferential notch in a pipe (Satyarnarayan, et al. 2007). An elliptical external surface crack was assumed to exist at the notch. According to Carpinteri et al. (2004) the stress field in a notched specimen is quite different compared to an unnotched specimen with the same surface flaw. The effect of the notch on the stress intensity factor distribution becomes more pronounced for shallower notches. Carpinteri et al. (2004) claimed that a thin walled pipe having a shallow notch subjected to tension can be compared to a plate under tension having an edge notch. For example, Figure 2.23 is a plot comparing the stress concentration factor obtained from the finite element solution for the pipe to the finite element solution obtained for a finite plate with an edge notch under tension as proposed by Cole and Brown (1958). Stress concentration factor is defined as the ratio of the local stress to the nominal stress. The relative notch radius is defined as $\rho^* = \frac{\rho}{t}$ where $\rho$ is the notch radius and $t$ is the pipe wall thickness. Factor $R^*$ is the ratio of the radius to the wall thickness ($R/t$). Since the ratio is ten it can be considered a thin pipe. The factor $\delta$ is the relative notch depth. Hence, in this graph the notch depth penetrates one tenth of the pipe wall. The stress concentration factor was compared because it significantly affects the value of stress intensity factor.
The study found that a good agreement between numerical and approximate values can be achieved especially in the case of a thin-walled pipe for which the stress distribution on the pipe cross-section is more similar to that of a uniformly tensioned plate. The study also concluded that the stress intensity factor is significantly affected by the notch for any crack size and shape.

2.6 Semi-elliptical crack shape

As stated earlier crack shape assumptions can introduce considerable error. Having said that it is important to note that evidence of a semi-elliptical crack shape does in fact exist. Iranpour and Taheri (2007) completed a study to validate the use of a flat plate subjected
to tension instead of a pipe under bending moment. This method reduces cost of monitoring instruments. Iranpour and Taheri (2007) claimed surface cracks with an elliptical shape were found to be a common crack type in pipes subjected to bending moments. This study also determined how the shape of the crack front progressed. This study found that as long as the material microstructure is homogenous and the stress distribution is uniform the crack shape can be assumed to have a semi-circular form. This study claimed that the crack front eventually takes a circular shape as the crack grows even if it started as an elliptical one. The smaller the initial surface flaw, the sooner the crack front changes its shape to a semi-circular one and follows the same trend towards the end of the fatigue life of the structural components. This research found that once a surface flaw reaches a semi-circular shape then a pipe under bending moment can be replaced by a plate under tension.

Several researchers (Yoo and Kotoji 1999; Iranpour and Taheri 2007; Satyarnarayan, et al. 2007; and Wittenberghe, et al. 2011) used beach marks (See Figure 2.24 and Figure 2.25) to verify crack depth values. Semi-elliptical shapes are clearly observed. Beach marks correspond to different periods of crack growth. Researchers introduce beach marks by changing the load amplitude after a predetermined number of load cycles.

A study by Raghanva, et al. (2009) was dedicated to understanding the use of beach marks to determine fatigue crack growth in steel plates. The procedure involves application of active blocks and passive blocks. In the active blocks crack growth is expected to occur while in passive blocks crack growth is negligible. Passive blocks
create beach marks. At regular intervals the stress range was significantly reduced. This is achieved by increasing the minimum load level and leaving the maximum load level unchanged in the passive blocks. The reduced stress range in the passive block is applied for a few thousand cycles. Upon completion of passive block the test returns to the previous stress range of active block. The reduced stress range and number of cycles are not meant to contribute to crack growth but rather to create beach marks.

Figure 2.24: Beach marks in pipe under bending moment (Satyarnaran et al. 2007)

Figure 2.25: Photographs of fatigue fracture surface (Luo et al. 2004)
2.7 Methods of inspection

Researchers used various destructive and nondestructive methods to measure or detect crack growth on material surface or in the direction of depth. Some of these methods include: dye penetrant (Seetharaman, et al. 2000), radiography (Sanyazi and Lobley 2009), conventional ultrasonic (Satyarnarayan et al. 2007), phased array ultrasonic (Satyarnarayan, et al. 2007), eddy current (Sanyazi and Lobley 2009), equipment running on the principle of alternating current potential drop (Singh, et al. 2008; Singh, et al. 2003; Satyarnarayan, et al. 2007; Seetharaman, et al. 2000; and Kiefner and Maxey 2000), equipment running on the principle of direct current potential drop (Kiefner and Maxey 2000), equipment running on the principle of alternating current field measurement (Seetharaman, et al. 2000), stereomicroscope (Yoo and Kotoji 1999), galvanostat crack tester (Luo, Xiong and Huo 2004), beach marking (Yoo and Kotoji 1999; Dover and Holbrook 1980; and Iranpour and Taheri 2007), optical 3D displacement analysis technique (Wittenberghe, et al. 2011) and magnetic particle testing (Hosseini et al. 2010; and Sanyazi and Lobley 2009).

The dye penetrant method is an effective visual technique to find surface cracks and to determine crack length (Black, DeGarmo and Kohser 2007). After the surface is cleaned a penetrant is sprayed, dipped or brushed on the subject. This penetrant is a liquid with very small particles capable of penetrating fine flaws. Next the penetrant is given time for capillary action to pull the penetrant into any flaw. Finally a developer is used. This liquid draws the penetrant back to the surface leaving a mark showing where the flaw exists (Figure 2.26). Hence, this method works well when crack grows on the surface.
However, this method is not suitable when crack grows internally and through the wall thickness of a pipe.

Magnetic particle inspection is based on the fact that if a ferromagnetic material is magnetized and a flaw exists then the magnetic field will be distorted near the flaw. After the material has been exposed to a magnetic field, magnetic particles are spread on the specimen. The particles are drawn to where the lines of magnetic flux break the surface, revealing anomalies that can then be interpreted (Black et al. 2007). Like penetrant method technique, this technique is useful when used on surface flaws.

One of the most popular methods for inspection of cracks in pipes is ultrasonic inspection. High-frequency sound waves are sent through the material and the returned waves are collected and interpreted. These returned sound waves are affected by flaws in the specimen. First, a pulse oscillator creates a burst of alternating voltage. Next, the alternating voltage is applied to a sending transducer. A transducer converts electrical
energy to mechanical vibrations. An acoustic coupling medium is used between the specimen and the transducer because air is a poor transmitter of ultrasonic waves. The next step requires a receiving transducer (can also be the same sending transducer) converting these reflected vibrations into electrical signals. These signals indicate the size of the flaw (Satyarnarayan, et al. 2007). A receiving unit is used to amplify, filter, and process the returned waves.

Alternating current potential drop (ACPD) is another technique. This approach involves applying a varying amplitude current on the specimen while the AC voltage is measured. Basically the specimen’s resistance is measured. Defects change the measured potential difference (Satyarnarayan, et al. 2007). ACPD is different from DCPD (DCPD relies on direct current) because AC currents are mostly restricted to a layer on top of the specimen. This is known as the ‘skin effect’. The advantages of ACDP over DCPD are that a linear relationship between flaw depths exists and a higher sensitivity exists.

Phased array ultrasonic, alternating current potential drop, and conventional ultrasonic technique were compared by Satyarnarayan et al. (2007) with respect to their ability to detect fatigue crack growth in pipes. In this study the results of each method were compared to beach marks in a pipe wall. Beach marks were used as the reference (i.e. as a benchmark). The same test set up was used as the study done by Singh et al. (2008). A 169 mm diameter pipe was subjected to constant amplitude, four-point bending loading cycles. At every 2 mm crack growth in the depth direction, loads were varied (passive blocks) to produce beach marks.
For the phased array technique the crack depth at the surface notch location was measured at seven points along the length of the notch (Figure 2.27). For the ACPD method five points were used. Points were joined to obtain a crack profile. The study reported lack of accessibility and the large curvature of the pipe as being a source of error. Error was calculated as the difference between a depth measured by one of the techniques with respect to mentioned to the depth measured from a beach mark. For the phased array technique flaw size was determined by time of flight technique. Conventional UT stands for conventional ultrasonic. Phased array differs from conventional UT in that phased array technology has the ability to steer, focus and scan beams. For example, the ability to test welds with various angles improves the probability to detect defects. It also simplifies the inspection of components with complex geometry (Olympus 2011).

Crack growth measurement results for beach marks, phased array technique, conventional ultrasonic, and ACPD technique were compared. The comparison is shown in Table 1.1 and Figure 2.28. Hence, this study found ACPD as the best alternative to beach mark method.

Crack growth in the circumferential direction also took place, but not until the crack had penetrated the pipe wall thickness. Table 1.1 seems to indicate that reasonable results are possible to obtain through these three techniques. ACPD yielded the smallest percentage error when compared to the beach marks.
Figure 2.27: Crack profile at 0 cycles (Satyarnarayan et al. 2007)

Table 2.2 Crack size comparison at crack center for three techniques (Satyarnarayan, et al. 2007)

<table>
<thead>
<tr>
<th>No. of cycles</th>
<th>Beach Mark (mm)</th>
<th>Phased Array (mm)</th>
<th>% error</th>
<th>ACPD (mm)</th>
<th>% error</th>
<th>Conventional Ultrasonic (mm)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.4</td>
<td>3.6</td>
<td>5.88</td>
<td>3.4</td>
<td>0</td>
<td>3.5</td>
<td>2.94</td>
</tr>
<tr>
<td>5,000</td>
<td>5.2</td>
<td>6.1</td>
<td>17.31</td>
<td>5.8</td>
<td>7.69</td>
<td>3.8</td>
<td>26.92</td>
</tr>
<tr>
<td>10,000</td>
<td>7.3</td>
<td>7.7</td>
<td>5.48</td>
<td>7.2</td>
<td>1.37</td>
<td>6.7</td>
<td>8.22</td>
</tr>
<tr>
<td>23,000</td>
<td>9.8</td>
<td>11.1</td>
<td>13.26</td>
<td>10.5</td>
<td>7.14</td>
<td>11.6</td>
<td>18.37</td>
</tr>
<tr>
<td>26,000</td>
<td>12.7</td>
<td>13.2</td>
<td>3.93</td>
<td>12.4</td>
<td>2.36</td>
<td>13.5</td>
<td>6.31</td>
</tr>
</tbody>
</table>
Figure 2.28: Crack center growth as measured by three techniques (Satyarnarayan et al. 2007)

It must be stated that accurate results are not always obtained through these monitoring instruments (Raghava, et al. 2009). In the literature it has been reported that crack depth values obtained using the ACPD technique were not reliable. Interestingly some of the same researchers who used ACPD for pipes (Singh, et al. 2008) were also the same ones who found it unreliable at times (Raghava et al. 2009). Although it must be stated that Raghava et al. (2008) performed the study on notched plate specimens and not pipes.

2.8 Summary
This chapter provided a literature review on pipes subjected to fatigue. The chapter was done to present a short theoretical background of fracture mechanics and to present a
review of past work concerning fatigue estimates on steel pipes. The following observations were made.

1. Few full-scale studies concerning pipes subject to fatigue load have been performed. Most of these tests consisted of pipes under cyclic four-point bending load having a longitudinal notch. Crack depths were measured by different techniques such as Phased Array, ACPD and beach marking.

2. Paris equation was found to provide acceptable fatigue estimates in the wall thickness direction.

3. Determination of $K$ is a complex issue. Since three-dimensional closed form stress intensity factor solutions are not available for pipeline flaws, many determined these from finite element analysis. Other researchers used the stress intensity factor solution for a plate and then incorporated a correction factor to account for the pipe curvature effect. A semi-elliptical shape has sometimes been assumed to exist. Solutions by Newman and Raju (1981, 1983) were commonly employed.

4. Error in assumed stress intensity factor solution introduced large error into Paris equation especially at smaller aspect ratios ($a/c$). A semi-elliptical shape was often assumed exist but it is hard to predict what shape the crack will take. According to Broek (1989) flaw assumptions are responsible for large error. One study (Saxena and Chouhan 2009) stated the use of a semi-elliptical solution will produce non-conservative results.
and it claimed that semi-elliptical stress intensity solutions should not be used to estimate the fatigue life of a straight pipe with a constant depth crack profile. A semi-elliptical shape has been assumed to exist.

5. Compact tension specimens as per ASTM E647 have been used to estimate Paris equation constant C and m.
CHAPTER III
DESIGN AND METHODOLOGY

3.1 General

This chapter describes the test specimens, the test setup, and the test procedure used to study fatigue crack growth of steel used in oil and gas pipelines. These tests took place in the Structural Engineering laboratory at the University of Windsor.

Two different types of tests were completed. One involved compact tension (CT) specimens in accordance with ASTM E647 and the other type involved a full-scale pipe specimen. Since it was desired to estimate the number of cycles to reach a certain crack depth in the wall of a full-scale pipe specimen, a number of CT tests were completed to assist in the estimation. One full-scale pipe specimen was used to validate the fatigue crack growth model developed using data from CT specimens and a finite element model. A total of nine CT specimens were used in the study to find Paris equation constants, C and m. This set of specimens was further subdivided into two test series. One test batch consisted of CT specimens with notch orientation in the longitudinal direction of the pipe. The other batch was composed of CT specimens with notch oriented in the circumferential direction. All CT specimens were obtained from the same pipe as the full-scale specimen. All CT specimens were subjected to cyclic loads to study their fatigue behaviour. The test data from CT specimens tests obtained were crack length and number of cycles. Load ranged from 5 kN to 10 kN. Data obtained from these CT tests were used to predict fatigue crack growth of the full-scale specimen.
One full-scale specimen was also used in this study. This specimen was cut from a 762 mm outer diameter pipe. An EDM longitudinal notch was cut into the specimen to create a defect which introduced stress concentration. The objective was to produce a fatigue crack at a predetermined location. Cyclic loads were applied through a fatigue actuator on the top surface of the pipe. Load value in fatigue load cycle ranged from 50 kN to 100 kN.

3.2 Test specimens
This section describes specimen parameters, materials, dimensions, and specimen preparation.

3.2.1 Selection of specimen parameters
The purpose of full-scale testing was to study experimentally the fatigue crack growth behaviour of similar field pipes subjected to cyclic load. Hence, the size and material properties of the pipe specimen were chosen to represent the properties of typical pipes used in the oil and gas pipeline industry.

Compact tension (CT) specimens were used to determine the constants in Paris equation, to obtain fatigue crack growth rate curves and to study the effect of notch orientation of CT specimens on fatigue crack growth. CT specimens were cut from the same pipe as the full-scale specimens. Hence they were made of the same material.
Full-scale specimens were made of pipes with an outer diameter of 762 mm, an 8.5 mm wall thickness and material grade of API 5L X65 (API 2008). The majority of pipelines used in the field have a diameter to thickness (D/t) ratio ranging from 20 to 90. The full-scale specimen had a D/t ratio of 90. Length of full-scale specimen was 1600 mm. The width of CT specimens was 35 mm and the thickness was 8.5 mm which is the pipe wall thickness. Figure 3.1 illustrates a typical CT specimen as per ASTM E647 (2008). The CT specimens were made using a wire EDM machine.

![Figure 3.1: Dimensions of CT specimen (in mm)](image)

3.2.2 Preparation of the specimen

A total of nine CT specimens and one full-scale specimen was prepared. A photograph of the pipe specimen in place is shown in Figure 3.2. Pipes were left with open ends during
testing. An electrical discharge machining (EDM) notch was made on the outer surface of each full-scale specimen. The notch was oriented in the longitudinal direction. Notch depth was approximately 3.8 mm throughout its length which was 100 mm. The purpose was to create a stress concentration to achieve a fatigue crack in that location. Notches were made away from welds to avoid any effect of residual stresses in the vicinity of welds.

ASTM E647 (ASTM 2008) specifies a standard method for conducting fatigue tests. Two different specimens are described in ASTM E647. They are the middle specimen and the compact tension (CT) specimen. Only CT specimens were used in this investigation since all tests were purely loaded in tension. CT specimens also allow for small specimens to be employed. Middle specimens are used if compression-tension loading is required.

Figure 3.2: Full-scale specimen set up
CT tests were completed since the stress intensity factor solution for CT specimens is known and provided in the manual for ASTM E647 (ASTM 2008) and hence, fatigue crack growth curves can be obtained. CT specimens were cut from the same pipes as the full-scale specimen. The width of CT specimens was 35 mm and the thickness was 8.5 mm. Dimension proportions of CT specimens were chosen as per guidelines of ASTM E647 (ASTM 2008). Figure 3.3 shows a photograph of a CT specimen.

![Figure 3.3: CT specimen](image)

### 3.2.3 Selection of boundary conditions

Since this study was carried out in order to investigate the fatigue behaviour of field pipes under cyclic load, the chosen boundary conditions were chosen to best simulate field conditions of a linepipe. As mentioned in the literature review, previous studies have conducted fatigue tests with a notch in the circumferential direction under four-point cyclic bending loading. Since a longitudinal notch is associated with bursting it was
deemed more critical. Hence, a longitudinal notch was used to produce longitudinal cracks. Four-point bending loading was not used because bending moment was not desired. The setup tries to resemble a pipe experiencing increases and decreases in internal pressure. Cyclic loads were applied on the outer surface of the pipe.

3.2.4 Test variables
One of the objectives of the CT tests was to study the effect of the notch orientation on crack growth rate. Therefore notches in CT specimens were created in the longitudinal and circumferential directions. However it was stated that notch orientation with respect to extrusion axis has no significant effect on fatigue crack growth rate (Singh et al. 2008). This study intended to validate this statement. Hence, a number of CT specimens were cut from the 762 mm outer diameter pipe with notches in both the longitudinal and circumferential directions. The stress ratio for CT specimens and the full-scale specimen were identical and the value was 0.5.

3.2.5 Designation of specimens
Each CT specimen was given a unique name as shown in Table 3.1. Names were chosen to recognize major parameters of the experiments. For example, for specimen CTL1 the first two characters denote a CT specimen. The full-scale specimen was designated by the letters FS. The last letter indicates the direction of the notch was in the longitudinal and the number indicated the sequence it was tested. A notch in the circumferential direction was denoted by the letter C. Table 3.1 provides the number of cycles required to fracture. It should be noted that there was a significant variability in these numbers. These
specimens were used to develop a statistical method to find Paris equation constants $C$ and $m$. A statistical method was developed because a high variability of values was shown to exist under the same conditions.

Table 3.1: Specimen matrix

<table>
<thead>
<tr>
<th>Specimen name</th>
<th>Minimum load (kN)</th>
<th>Maximum load(kN)</th>
<th>Notch orientation</th>
<th>Cycles to fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC1</td>
<td>5</td>
<td>10</td>
<td>Circumferential</td>
<td>413507</td>
</tr>
<tr>
<td>CTC2</td>
<td>5</td>
<td>10</td>
<td>Circumferential</td>
<td>413758</td>
</tr>
<tr>
<td>CTC3</td>
<td>5</td>
<td>10</td>
<td>Circumferential</td>
<td>588405</td>
</tr>
<tr>
<td>CTC4</td>
<td>5</td>
<td>10</td>
<td>Circumferential</td>
<td>498985</td>
</tr>
<tr>
<td>CTC5</td>
<td>5</td>
<td>10</td>
<td>Circumferential</td>
<td>546577</td>
</tr>
<tr>
<td>CTL1</td>
<td>5</td>
<td>10</td>
<td>Longitudinal</td>
<td>520223</td>
</tr>
<tr>
<td>CTL2</td>
<td>5</td>
<td>10</td>
<td>Longitudinal</td>
<td>556056</td>
</tr>
<tr>
<td>CTL3</td>
<td>5</td>
<td>10</td>
<td>Longitudinal</td>
<td>421417</td>
</tr>
<tr>
<td>CTL4</td>
<td>5</td>
<td>10</td>
<td>Longitudinal</td>
<td>407666</td>
</tr>
<tr>
<td>FS</td>
<td>50</td>
<td>100</td>
<td>NA</td>
<td></td>
</tr>
</tbody>
</table>

A larger number of CT specimens were tested than are shown here. Some of these specimens were not included since they were used to discover the optimum loads, the best way to measure etc.

3.2.6 Mechanical properties

All specimens were cut from the same material. The mechanical properties of CT specimens and full-scale specimens were identical. Tensile coupon specimens were cut from the pipe specimen and these coupon specimens were oriented in the longitudinal direction of the pipe and far away from any welds to avoid effect of residual stresses. Tensile tests were prepared and performed as per ASTM E 8/E 8M-08 specification. (ASTM 2008) Three tensile coupon specimens were obtained. To measure longitudinal
strain on the reduced area of the tensile specimens an extensometer was placed on the coupon. Load and displacement over 2 inch (50.8 mm) gauge length was recorded. The data were then used in the finite element model. Typical engineering stress-strain behaviour of pipe steel is shown in Figure 3.4. Table 3.3 lists the mechanical properties of the pipe material used.

![Tensile stress-strain behaviour of pipe steel](image)

**Figure 3.4: Tensile stress-strain behaviour of pipe steel**

<table>
<thead>
<tr>
<th>Modulus of Elasticity (MPa)</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>568</td>
<td>650</td>
</tr>
</tbody>
</table>

**Table 3.2: Material Properties**

53
3.3 Experimental setup

This section describes the equipment and the test setup used in this study.

The experimental program was performed in the Structural Engineering Laboratory of the University of Windsor at room temperature. Figure 3.5 shows a schematic of the full-scale test setup. Full-scale specimens rested on three supports. The end supports consisted of a pin and a roller. The middle support was a raised thick steel platform. Cyclic loads were applied on the outer surface of the pipe using a fatigue loading actuator with a capacity of ± 500 kN on the top surface of the pipe. Ends of the full-scale specimen were open.

Figure 3.5: Schematic of full-scale

Figure 3.6 shows the setup of CT specimen testing. A ± 100 kN INSTRON Model 1332 fatigue testing frame and machine were used to carry out these tests (Figure 3.7). MTS
Flextest GT Digital Controller is shown in Figure 3.8. The testing frame machine was operated through an MTS Model 505.60 hydraulic power unit (Figure 3.8). CT specimens were held in place by steel grips shown in Figure 3.6. Dimensions of grips are shown in Figure 3.9. Crack growth was measured using the digital microscope camera shown in Figure 3.10. Both the CT specimens and grips were machined at the University of Windsor according to ASTM E647 specifications.

Figure 3.6: CT specimen set up
Figure 3.7: INSTRON Model 1332 fatigue testing frame
Figure 3.8: Hydraulic power unit and FlexTest GT Digital Controller

Figure 3.9: CT specimen grips (all dimensions in mm)
3.3.1 Load cell and fatigue testing frame
Cyclic loads were applied to the full-scale pipe specimens using a 500 kN compression-tension actuator with an internal load cell. However, cyclic loads were applied on the CT specimens using the ± 100 kN INSTRON Model 1332 fatigue testing frame machine.

3.3.2 Linear voltage displacement transducer
A linear voltage displacement transducer (LVDT) was used to measure pipe deformation as the cyclic loads through the actuator were applied. A LVDT is an instrument used to measure displacement. The LVDT was installed inside the pipe directly underneath the actuator. This LVDT is shown in Figure 3.11.
3.3.3 Electronic resistance strain gauges

Electrical resistance strain gauges were used to measure local strain on the outer surface of the pipe near the notch (Figure 3.12). A strain gauge is attached to an object by an adhesive such as Cyanoacrylate cement. When the strain gauge deforms along with the object the electrical resistance of the strain gauge also changes. Electrical resistance change can be measured using a Wheatstone bridge. Electrical resistance change is related to strain through the gauge factor (Efunda 2011).
A Wheatstone bridge is an electrical circuit used to measure electrical resistance. Strain gauges used in the study were of the quarter bridge type. Figure 3.13 illustrates a Wheatstone bridge. The gauge factor of the strain gauges used was 2.1 and the resistance was 120 Ohms.

3.3.4 Data acquisition system

Strain gauge and LVDT data was recorded in a computer file through Dalite software. A Datascan 7321 module was connected to a computer to collect real time data. Essentially this module acted as voltmeter which was interpreted by the software. Four strain gauges
at were used to measure strain. A total of five channels were used. One channel was reserved for the LVDT. A photograph of the module and computer is shown in Figure 3.14.

Figure 3.14: Datascan module and computer

3.3.5 Dinolite digital microscope
A Dinolite digital microscope model AM 413T was used to monitor crack length of CT specimens. A photograph taken with this microscope is shown in Figure 3.15. Maximum magnification allowed is 200X. However due to the grips blocking contact between the microscope and specimen a magnification of approximately 40X-45X was used. A higher magnification produced blurry images. During the first stages of these tests, CT specimens were removed from the grips so the microscope could be directly placed on it. A magnification of 200X could be used that way, However because the crack was very
fine it was difficult to see the crack. Therefore, readings were taken with the specimen in place since cyclic loads opened up the crack enough to be visible at a magnification of approximately 40X-45X. Testing was not stopped during measurements. However, the load cycle frequency was decreased from 50 Hz to 1 Hz to allow the crack to be visible. The full-scale specimen load cycle frequency was 2 Hz.

![Crack close up aided by digital microscope](image)

**Figure 3.15: Crack close up aided by digital microscope**

3.3.6 Extensometer

A 50.8 mm (2 inch) gauge length axial extensometer was used to record displacement feedback during the three tensile tests following ASTM E 8/E 8M-08 guidelines (ASTM
The extensometer was an MTS Model 634.25F-24. The displacement data was then divided by 50.8 mm to determine the strain.

3.4 Test procedure

Full-scale tests were run using the setup shown in Figure 3.5. First the actuator was lowered to achieve the minimum desired load (50 kN) using manual control. Next the actuator was lowered to reach the maximum desired load (100 kN) using manual control and at a very slow loading rate. After obtaining the corresponding displacements the average displacement was calculated and this was input into the control computer as the “target setpoint”. All full-scale tests were performed in displacement control as it was deemed to be the safest. The displacement ranged from approximately 13 mm to 27 mm.

Next these loads were applied at a frequency of 2.00 Hz as this was the maximum frequency that provided the maximum acceptable error between command (demanded) and output displacement. The error was approximately ± 1 mm but at times it was as much as ± 4 mm.

As mentioned earlier in Section 2.3(Figure 2.19) the presence of beach marks indicate different periods of crack growth. It was attempted to introduce beach marks to relate the number of cycles to crack depth using scanning microscope (SEM). After a number of cycles in active block the minimum load of 50 kN was increased to 80 kN while the maximum load of 100 kN was kept constant in passive block. This was done to create a change in fatigue crack growth without growing the crack any farther. The objective was to create beach marks so that crack growth could be measured with a SEM. Cyclic loads
were applied until a through wall crack developed. This fine crack was found with the aid of liquid penetrant inspection. Once a line indicating the existence of a through wall crack was observed the test was discontinued.

Next, five specimens were cut from the area near the notch. The notch is shown in Figure 3.16. These specimens were taken to the Scanning Electron Microscope Laboratory in the University of Windsor for inspection and determination of beach marks. Figure 3.17 shows where these specimens were cut from. The Scanning Electron Microscope lab is shown in Figure 3.18.

Figure 3.16: Notch
Figure 3.17: specimens cut near notch

Figure 3.18: SEM laboratory at University of Windsor
Unfortunately no beach marks could be found in these specimens. Figure 3.19 shows the through wall crack as seen through a scanning electron microscope. This figure shows no presence of beach marks, but the fatigue crack is clearly visible. It is not known if beach marks can be introduced into this type of steel.

Strain gauges were placed perpendicular to the notch in an attempt to find the strain hysteresis as crack grew. These strain gauges were used to calibrate the finite element model. Static strain when no load was applied was recorded as well as strain at the minimum load of 50 kN and maximum load of 100 kN were obtained. Strain data was also recorded once cycling load began. Test data were acquired at a speed of one reading per second. LVDT data was recorded as well in a computer file during static and cyclic loads.

Figure 3.19: SEM photograph of notch and crack
CT specimens tests were conducted using the guidelines of ASTM-E647. Once a specimen was held in place by the modular hydraulic grips, fatigue testing commenced. Care was taken to align each specimen vertical. Since the full-scale specimen was subjected to a 0.5 stress ratio the CT specimens were also subjected to this stress ratio. A minimum load of 5 kN and a maximum load of 10 kN were required and used in all CT specimens. These loads were chosen since they were found to provide the most practical experiments. That is, loads were large enough to finish one test per day and small enough to prevent a quick fracture. Figure 3.20 is a screen shot of the control computer. After a number of cycles (at a frequency of 50 Hz) the digital microscope mentioned before was used to measure the crack length (at a frequency of 1 Hz). The number of cycles associated with this crack length was monitored through the control computer. Measurements were taken a number of times for each specimen. A test for each CT specimen would continue until fracture occurred (Figure 3.21).

Figure 3.20: Control computer
3.5 Summary

This chapter provided the details of the experimental component of this study. All experiments were conducted in the Structural Engineering laboratory at the University of Windsor at room temperature.

Two different types of tests were conducted. One involved tests in accordance to ASTM E647 (ASTM 2008) using compact tension (CT) specimens. The objective was to estimate Paris equation constants $C$ and $m$. The other type of test involved applying cyclic loads through a fatigue actuator on a full-scale pipe specimen having a longitudinal notch until a through-wall crack was developed. A detailed description of the test equipment used and procedure of all CT tests and full-scale specimen test was documented.
CHAPTER IV
ANALYSIS OF RESULTS

4.1 General

This chapter discusses the experimental data and methods of analysis. The objective of this study is to develop a mathematical model that can estimate the number of cycles for a crack to grow a certain depth in a steel pipe. To achieve this goal, crack growth curves and a statistical model for the estimation of representative C and m values (constants in Paris equation) were determined. These were then used with the finite element model to predict the number of cycles when a through-wall crack develops. Finally, the model was validated with the actual test data obtained from a full-scale pipe specimen.

4.2 CT specimen data

The test data of compact tension (CT) specimens were used to find realistic and statistically meaningful values of C and m of Paris equation. The following steps were used to find these values.

1. Nine fatigue tests on CT specimens were completed and fatigue crack lengths at various load cycles were measured. The fatigue life was also determined. Five CT specimens had notch oriented in circumferential direction and remaining four CT specimens had notch in the longitudinal direction (Table 4.1).

2. The stress intensity factor range ($\Delta K$) was determined using Equation 4.1 for all CT specimens.
3. The fatigue crack growth rate (da/dN) was then calculated as detailed in Table 4.2.

4. Plots of $\Delta K$ vs. da/dN were made and it was found that due to data scatter use of raw test data leads to controversial results (Figure 4.3). Hence, best fit data (exponential data) of crack length versus load cycles was used (Figure 4.2) to determine $\Delta K$ and da/dN as shown in Figure 4.5. This plot (Figure 4.5) was then used to determine C and m, the constants of Paris equation.

5. The logarithm of the crack growth rate ($\log(\text{da/dN})$) calculated using best fit data was found to be normally distributed if the number of cycles (N) is normally distributed Equation (4.5b). The number of cycles at fracture was determined to be normally distributed.

6. Since N was normally distributed, it allowed for the use of normal distribution for determining values of C and m.

7. Linear regression used on the Paris equation expressed in linear form (Equation 4.6) since the logarithm of the crack growth rate was assumed to be normally distributed. This was done to calculate Paris equation constants C and m.
Table 4.1 Test matrix for CT specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Notch orientation</th>
<th>Specimen name</th>
<th>Cycles to unstable crack growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circumferential</td>
<td>CTC1</td>
<td>413,507</td>
</tr>
<tr>
<td>2</td>
<td>Circumferential</td>
<td>CTC2</td>
<td>413,758</td>
</tr>
<tr>
<td>3</td>
<td>Circumferential</td>
<td>CTC3</td>
<td>588,405</td>
</tr>
<tr>
<td>4</td>
<td>Circumferential</td>
<td>CTC4</td>
<td>498,985</td>
</tr>
<tr>
<td>5</td>
<td>Circumferential</td>
<td>CTC5</td>
<td>546,577</td>
</tr>
<tr>
<td>6</td>
<td>Longitudinal</td>
<td>CTL1</td>
<td>520,223</td>
</tr>
<tr>
<td>7</td>
<td>Longitudinal</td>
<td>CTL2</td>
<td>556,056</td>
</tr>
<tr>
<td>8</td>
<td>Longitudinal</td>
<td>CTL3</td>
<td>421,417</td>
</tr>
<tr>
<td>9</td>
<td>Longitudinal</td>
<td>CTL4</td>
<td>407,666</td>
</tr>
</tbody>
</table>

Average: 485,177

The explanations of these seven steps are as follows. Number of fatigue cycles to unstable crack growth refers to the number of load cycles when a specimen could not sustain any load imposed on it. At this stage it is possible to see with the naked eye that the specimen quickly tears apart (Figure 4.1).

Figure 4.1: CT specimen at unstable crack growth stage
Figure 4.2 shows a plot of crack length measured from the notch tip of a CT specimen vs. number of cycles for specimen CTC5. Similar plots of the other specimen are provided in Appendix A.

This figure shows the crack lengths measured at various cycle counts from the test. The first reading for this specimen was collected at 399,430 cycles. No crack length measurements could be acquired prior to that because the crack was not yet visible under the digital microscope until about 400,000 cycles. The raw test data was not directly used in the fatigue analysis. This is because the test data were found to be scattered and hence, an exponential trendline was used. The objective of the trendline of Figure 4.2 is explained next.

Figure 4.3 shows the relationship between stress intensity factor range (ΔK) and crack growth rate (da/dN). The procedure of how Figure 4.3 was derived is described in page 76. As can be observed from Figure 4.3, use of raw test data produces unrealistic results. This is because Figure 4.3 states that a higher crack growth rate can apparently be obtained for a smaller stress intensity range (see Point 1 in Figure 4.3 and compare with Point 2) which is opposite to what is believed in fracture mechanics. This translates to the contradictory belief that a smaller crack length leads to a faster crack growth rate. This controversy occurs because of the scatter in the test data of crack lengths measured at various cycle counts (Figure 4.2). Hence, test raw data was not directly used in developing the statistical method of Paris constants C and m. Instead, an exponential trendline (best-fit curve) generated by Excel was used (Figure 4.2).
Figure 4.2: Crack length vs. number of cycles for CTC5

Figure 4.2 shows that an exponential relationship of the trendline for CTC5 closely resembles the actual test data and at the same time, it follows the convention of fracture mechanics: smaller the stress intensity factor range ($\Delta K$), lower the crack growth rate ($da/dN$) is. Hence, this exponential function was used in the development of the statistical method for Paris constants $C$ and $m$. Mohanty et al. (2010) claimed that increase in crack length when plotted vs. number of load cycles is exponential in nature. In fact, the study conducted by Obrtlik and Polak (2000), experimental data were fitted with an exponential function as well. The following paragraphs describe how the crack growth rate ($da/dN$) vs. stress intensity factor range ($\Delta K$) curve for each specimen was plotted. Table 4.2 is used to illustrate the procedure.
Figure 4.3: Scatter data used for crack growth rate plot for CTC5

Table 4.2: CTC5 data

<table>
<thead>
<tr>
<th>Distance from notch (mm)</th>
<th>a (mm)</th>
<th>Cycles, N</th>
<th>a_{avg} (mm)</th>
<th>da/dN (mm/cycle)</th>
<th>∆K (MPa m^{0.5})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>Idealized</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.3</td>
<td>NA</td>
<td>293,073</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>1.7</td>
<td>1.6</td>
<td>376,741</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2.7</td>
<td>2.1</td>
<td>399,430</td>
<td>8.8</td>
<td>0.25</td>
<td>2.25\times10^{-3}</td>
</tr>
<tr>
<td>3.3</td>
<td>3.1</td>
<td>430,470</td>
<td>9.6</td>
<td>0.27</td>
<td>3.15\times10^{-3}</td>
</tr>
<tr>
<td>4.3</td>
<td>4.4</td>
<td>460,214</td>
<td>10.7</td>
<td>0.31</td>
<td>4.58\times10^{-3}</td>
</tr>
<tr>
<td>5.9</td>
<td>5.8</td>
<td>482,104</td>
<td>12.1</td>
<td>0.35</td>
<td>6.29\times10^{-3}</td>
</tr>
<tr>
<td>6.7</td>
<td>7.0</td>
<td>496,856</td>
<td>13.4</td>
<td>0.38</td>
<td>7.87\times10^{-3}</td>
</tr>
<tr>
<td>7.2</td>
<td>8.4</td>
<td>512,148</td>
<td>14.7</td>
<td>0.42</td>
<td>9.48\times10^{-3}</td>
</tr>
<tr>
<td>10.1</td>
<td>10.7</td>
<td>531,600</td>
<td>16.6</td>
<td>0.47</td>
<td>1.18\times10^{-3}</td>
</tr>
<tr>
<td>12.1</td>
<td>12.2</td>
<td>541,886</td>
<td>18.4</td>
<td>0.53</td>
<td>1.41\times10^{-3}</td>
</tr>
</tbody>
</table>

The first column “Distance from notch” lists the crack length measured from the notch tip (That is (a-a_n) in Figure 4.4). The second column “Idealized” refers to the values of crack length based on the exponential line of best fit (trendline) shown in Figure 4.2. at a
specific cycle count. The third column is the crack length \((a)\) which is used in the calculation of the stress intensity factor range \((\Delta K)\). The value of \(a\) in this column is the idealized (second column) value plus seven millimetres \((a_n = 7 \text{ mm in Figure 4.4})\). The reason seven millimetres was added is because the stress intensity factor range solution provided in ASTM E647 (ASTM 2008) measures crack length ‘a’ from the centre of the holes in the CT specimen (Figure 4.4). In this study, the value of \(a_n\) in CT specimens was 7 mm. The solution of \(\Delta K\) for CT specimens provided by ASTM E647 (ASTM E647) is as follows and it was used in this study to calculate \(\Delta K\) for CT specimens. The test material is assumed to be linear elastic, isotropic, and homogeneous.

\[
\Delta K = \frac{\Delta P}{B\sqrt{W}} \left(\frac{2+a}{2(1-a)^2}\right) (0.886 + 4.64a - 13.32a^2 + 14.7a^3 - 5.6a^4)
\] (4.1)

where \(a = a/W\) (see 6\(^{th}\) column in Table 4.2), \(\Delta P\) is the load range applied to CT specimen, \(a\) is crack length, \(B\) is wall thickness, and \(W\) is as defined in Figure 4.4.

Column entitled “Cycles” (4\(^{th}\) column in Table 4.2) lists the number of cycles as counted from the test. The next column “\(a_{avg}\)” is the average crack length during a cycle interval. For example, the value of average crack length between cycle counts of 399,430 and 430,470 was 9.6 mm which was obtained by adding 9.1 mm to 10.1 mm and then dividing the total value by two. The values of “\(a_{avg}\)” were later used to calculate the values in the column entitled \(\alpha\).
The column entitled “α” is the ratio of $a$ to $W$ (as defined earlier). This is calculated as the $a_{avg}$ divided by $W$ which is 35 mm. The crack growth rate column ($da/dN$) is calculated by dividing the change in crack length by the difference in number of cycles. Here $a_{avg}$ was not used. The following is an example on how this is calculated.

$$\frac{da}{dN} = \frac{10.1mm - 9.1mm}{430,470 - 399,430} \approx 3.15 \times 10^{-5} \text{ mm/cycle}$$

The crack growth rate ($da/dN$) versus stress intensity factor range ($\Delta K$) plot of CTC5 is shown in Figure 4.5. Similar plots for other specimens are shown in Appendix B. It should be noted that Figures 4.3 and 4.5 are similar plots. The difference between these two plots is that in Figure 4.5 the $\Delta K$ was calculated considering the trendline.

Figure 4.4: CT specimen
(exponential best-fit) of test data as shown in Figure 4.2 However, in Figure 4.3, the $\Delta K$ and $da/dN$ values were calculated using raw test data. The difference is that Figure 4.3 produced unrealistic results due to scatter while Figure 4.5 provided results in line with what is believed in fracture mechanics. Figure 4.5 was deemed to be more reliable for further analysis of fatigue crack growth data.

![Figure 4.5: Crack growth rate vs. stress intensity factor range of CTC5](image)

Since Paris equation applies to the linear region (Region II) of crack growth curve (Section 2.1.3), parameters C and m are applicable to only the linear region (i.e. curved regions were ignored as shown by the large oval in Figure 4.5). Experimentally, it is very difficult to begin a constant amplitude fatigue crack growth test corresponding to region I crack growth rates. To generate region I one must systematically shed the applied load as the crack. This procedure is known an $\Delta K$-decreasing (Stephens and Fochs 2001). This region was not captured in this study. Figure 4.6 shows the linear region for which factors
C and m were calculated. A power law trendline as shown in Figure 4.6 was used to find these parameters. Hence, the value of C for this specimen was found to be 2.44×10^{-10} and the value of m was found to be 4.14. Similar plots for other CT specimens are shown in Appendix B. Table 4.3 lists the values of C and m that were calculated for all CT specimens. Unfortunately, values of C and m obtained using this method from various CT specimens exhibited a wide scatter (see Table 4.3). Appropriate choice of values of C and m is important fatigue life depends on these values if Paris equation is used to determine fatigue life.

Figure 4.6: Linear portion of CTC5
Table 4.3: Values of Paris equation constants for CT specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Notch orientation</th>
<th>C</th>
<th>m</th>
<th>Specimen name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Circumferential</td>
<td>4.01×10^{-8}</td>
<td>2.56</td>
<td>CTC1</td>
</tr>
<tr>
<td>2</td>
<td>Circumferential</td>
<td>4.82×10^{-10}</td>
<td>4.00</td>
<td>CTC2</td>
</tr>
<tr>
<td>3</td>
<td>Circumferential</td>
<td>4.87×10^{-10}</td>
<td>3.86</td>
<td>CTC3</td>
</tr>
<tr>
<td>4</td>
<td>Circumferential</td>
<td>8.69×10^{-10}</td>
<td>3.65</td>
<td>CTC4</td>
</tr>
<tr>
<td>5</td>
<td>Circumferential</td>
<td>2.44×10^{-10}</td>
<td>4.14</td>
<td>CTC5</td>
</tr>
<tr>
<td>6</td>
<td>Longitudinal</td>
<td>8.78×10^{-10}</td>
<td>3.63</td>
<td>CTL1</td>
</tr>
<tr>
<td>7</td>
<td>Longitudinal</td>
<td>4.18×10^{-9}</td>
<td>3.17</td>
<td>CTL2</td>
</tr>
<tr>
<td>8</td>
<td>Longitudinal</td>
<td>1.09×10^{-8}</td>
<td>2.76</td>
<td>CTL3</td>
</tr>
<tr>
<td>9</td>
<td>Longitudinal</td>
<td>6.76×10^{-10}</td>
<td>3.9</td>
<td>CTL4</td>
</tr>
</tbody>
</table>

Paris equation is as follows.

\[
\frac{da}{dN} = C(\Delta K)^m
\]  

(2.17)

Equation 2.17 indicates that the crack growth rate (da/dN) is directly proportional to parameter C. Table 4.3 shows that a wide range of values of parameter C were obtained from CT specimens. These values ranged from 2.44×10^{-10} to 4.01×10^{-8}. The latter number is approximately 164 times larger than the former. Hence, if the two estimates were prepared with the same m value (same m value would be used for comparison purposes) one equation would indicate a crack growth rate 164 times larger than the other. Values of parameter m also varied and the range was from 2.56 to 4.14. Recall that parameter m describes how sensitive a material’s growth rate is to the stress applied (Section 2.1.4). It can therefore be concluded that the variations in values of C and m have significant effects on the fatigue life estimation. This is evident from Figure 4.7 which shows plots for crack growth rate (da/dN) versus stress intensity range (ΔK) for all CT specimens.
In Figure 4.7, da/dN vs. ΔK for CT specimens with notch oriented in the circumferential and longitudinal directions are plotted. It is interesting to note that no obvious differences were observed between the two groups. Hence, it is concluded that notch direction has no effect on fatigue life for CT specimen used in this study.

Since CT tests were undertaken to obtain C and m values and these values exhibited a large scatter, right choice of C and m values to estimate crack growth in the full-scale pipe specimen is very important. Carl (2007) obtained Paris law constants (C and m) from upper-bound curve for ferritic-pearlitic steels as provided in Appendix F of API 579 (API 2000). Luo et al. (2004) obtained crack growth equation parameters (C and m) from CT tests, but no specific details on the procedure of calculating these values were provided. Singh et al. (2003) evaluated these material constants (C and m) by fitting CT
specimen test data points conforming to the form of Paris equation. However, no details were provided by them either. Yoo and Kotoji (1999) plotted crack growth rate \( (da/dN) \) vs. stress intensity range \( (\Delta K) \) for carbon steel pipe using the formula suggested by Newman-Raju (1981) (see Sections 2.3 and 2.4.2) as shown in Figure 4.8. The entire region of this data shows less scatter when compared to the obtained in the current study. Hence, it seems wise to simply fit the test data to a power function as was adopted by Yoo and Kotoji (1999) and shown by Equation 4.2. Hence, \( C \) was found to be \( 3.2 \times 10^{-10} \) and \( m \) was found to be 3.72. Figure 4.8 lists \( da/dN, dC_d/dN, dC_p/dN \) in the vertical axis. These represent crack growth in different directions. They are in the depth direction, to outer surface after penetration, and to inner surface after crack penetration, respectively.

\[
\frac{da}{dN} = 3.20 \times 10^{-10} (\Delta K)^{3.72} \tag{4.2}
\]

![Figure 4.8: Fatigue crack growth rate vs. \( \Delta K \) (Yoo and Kotoji, 1999)](image)

The exact same procedure as that used by Yoo and Kotoji (1999) was not used in the current study because test data in the current study were considerably scattered. Rather,
in the current study, a statistical procedure as discussed in the next section (4.3) was used to determine appropriate values of C and m for all specimens. The objective was to calculate Paris equation constants (C and m) which statistically represent scattered test data to aid in the estimation of fatigue crack growth and fatigue life of the full-scale pipe specimen.

4.3 Statistical approach to Paris equation constants

As mentioned earlier, the crack length versus number of cycles plot was found to be an exponential relationship (Figure 4.2). Hence, crack length and crack growth rate (da/dN) were assumed to take the following forms (Equations 4.3 and 4.4).

\[ a = de^{bN} \]  
\[ \frac{da}{dN} = dbe^{bN} \]

where d and b are constants. For example, for CTC5 specimen (Figure 4.2) constant d is 0.015 and b is 0.000012.

Next, the number of cycles at fracture was examined. Each CT specimen was assumed to fracture at the same fracture toughness. Toughness is described at the highest stress intensity that can be supported by a cracked component made of that material (Broek 1989). Since CT specimens were made of the same material all specimen were assumed to possess the same fracture toughness. Important statistical data for the number of cycles to fracture (see Table 4.1 for specimens and cycles to fracture) for the CT specimens are
shown in Table 4.4. In this table mean is the average number of cycles to failure (see Column 4 of Table 4.1) of all nine CT specimens.

<table>
<thead>
<tr>
<th>Mean (µ)</th>
<th>Skewness</th>
<th>Standard deviation (SD)</th>
<th>µ + SD</th>
<th>µ - SD</th>
<th>µ + 2×SD</th>
<th>µ - 2×SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>485177</td>
<td>0.12</td>
<td>71745</td>
<td>556923</td>
<td>413432</td>
<td>628668</td>
<td>341686</td>
</tr>
</tbody>
</table>

The number of CT specimens within ± one standard deviation from the mean is six out of a total of nine specimens. All specimens were within ± 2 standard deviations from the mean. Therefore, approximately 67% of specimens were within one standard deviation and 100% of specimens were within two standard deviations of the mean. It should also be noted that skewness (indicative of symmetry) was calculated at 0.12, which was considered to be close to zero. Hence, for the reasons mentioned above the distribution of “cycles to fracture (N)” was assumed to be normally distributed.

Equation 4.4 can be written in the following manner.

\[ \ln \left( \frac{da}{dN} \right) = \ln(db e^{bN}) \]  \hspace{1cm} (4.5a)

\[ \ln \left( \frac{da}{dN} \right) = \ln(db) + bN \]  \hspace{1cm} (4.5b)

Referring to the right hand side of Equation 4.5b it can be seen that \( \ln(db) \), and \( b \) are all constants. Hence, the only variable on the right hand side of Equation 4.5b is the number of load cycles \( N \). Therefore, if \( N \) is assumed to be normally distributed then \( \ln(da/dN) \) is also normally distributed. The Paris equation can be expressed as in the form of Equation
2.16. It can also be written in the following form if the base of the logarithm is changed to the natural logarithm as shown in Equation 4.6.

\[
\log \left( \frac{\Delta a}{\Delta N} \right) = m \log(\Delta K) + \log C
\]  

(2.16)

\[
\ln \left( \frac{\Delta a}{\Delta N} \right) = m \ln(\Delta K) + \ln C
\]  

(4.6)

From the same reasons as discussed above it can also be concluded that Equation 2.16 is normally distributed where m and C are constants. Linear regression analysis was then applied to Equation 4.6 to calculate Paris equation constants C and m. The procedure is explained in the following paragraphs.

Table 4.5 illustrates the steps used in this procedure. The first group of rows lists the crack growth rates of all CT specimens corresponding to a particular stress intensity factor range. For example, the crack growth rate (da/dN) of specimen CTC1 at \(\Delta K\) of 16 is \(4.85 \times 10^{-5}\) mm/cycle. Crack growth rate was not calculated using \(a_{avg}\) (Section 4.2). The next group of cell names ‘Sorted’ shows a descending list of crack growth rates for a particular \(\Delta K\). Following this group is the natural logarithm of each crack growth rate, \(\ln(da/dN)\). It should be noted that \(\Delta K\) was calculated using Equation 4.1 and \(da/dN\) was calculated as explained in page 76.

Next, the natural logarithm of crack growth rate for various \(\Delta K\) values is calculated. Three different cumulative probabilities (that is, \(G = 5\%, 50\%, \text{ and } 95\%)\) were considered to obtain a wide range of fatigue life. In other words, these values were chosen to use different degrees of conservatism. This will be explained shortly. In the
third group from the bottom of Table 4.5 is the value of ln(da/dN) (i.e. H) for which a cumulative probability of G is obtained for a certain value of ΔK. Mathematically, this can be expressed in the following way.

\[ P \left( \ln \left( \frac{da}{dN} \right) \leq H \right) = G \]  

(4.7)

Table 4.5 shows values of \( \ln \left( \frac{da}{dN} \right) \) that correspond to cumulative probabilities (G) of 5%, 50%, and 95%. In the case of cumulative probability of 5% and \( \Delta K \) of 15 (first column), there is a 5% probability that ln(da/dN) is equal to or less than -11.24 (that is, \( H = -11.24 \)). This procedure was repeated for values of \( \Delta K \) ranging from 15 to 19. These \( \Delta K \) values were chosen because the CT specimens had a linear region, Region II, (Section 2.1.3) in this range from 15 to 19.

The reason normal distribution was previously checked for is that it was used in Equation 4.7 for calculating H. It should be noted that H is the maximum natural logarithm of crack growth for a certain \( \Delta K \) which corresponds to cumulative probability G (Equation 4.7). For each value of \( \Delta K \), a value of H was found which corresponds to the specified cumulative probability (G). This H was found using the mean and standard deviation of the ln(da/dN) values of the nine specimens for a certain \( \Delta K \). For example, a value of ln(da/dN) of -11.24 has a cumulative probability of 5% assuming normal distribution at \( \Delta K \) of 15.

Before moving forward the physical meaning of the 5% cumulative probability is explained. There is a 5% cumulative probability that the natural logarithm of crack
growth (i.e. \(\ln(da/dN)\)), is equal to or less than -11.24 when \(\Delta K\) has a value of 15 (first column of Table 4.5). In other words, 5% of specimens have a crack growth of \(e^{-11.24}\) equal to 1.314\(\times10^{-5}\) mm/cycle or less if \(\Delta K\) is equal to 15. Referring to Equation 4.7, \(H\) is equal to -11.24 and \(G\) is equal to 5% if \(\Delta K\) is equal to 15.

Rows labeled as X and Y in Table 4.5 are values of \(\ln(\Delta K)\) and \(\ln(da/dn)\), respectively. Linear regression analysis was performed (since Equation 4.6 is linear) for rows labeled X and Y when cumulative probability is 5% and plotted as shown in Figure 4.9. Slope of the line was found to be 4.05 and hence, \(m\) was found to be 4.05. The Y intercept was found to be -22.174 which is \(\ln C\) as in Equation 4.6. Hence, the exponential function was used since it is the inverse function of the natural logarithm. Therefore, \(C\) was calculated as 2.32\(\times10^{-10}\) (that is, \(e^{-22.174}\)). Hence, the values of \(C\) and \(m\) were found to be 2.32\(\times10^{-10}\) and 4.05, respectively if cumulative probability (\(G\)) is chosen as 5% (Table 4.6).

Linear analysis was repeated for cumulative probabilities (\(G\)) of 50% and 95%. Table 4.6 lists the values of \(C\) and \(m\) for all three cumulative probabilities (5%, 50%, and 95%). Similarly to 5% cumulative probability, the physical meaning of 50% is that there is a 50% probability that the natural logarithm of crack growth rate is equal to or less than -10.75 when \(\Delta K\) has a value of 15. That is, 50% of specimens have a crack growth of \(e^{-10.75}\) equals to 2.14\(\times10^{-5}\) mm/cycle or less if \(\Delta K\) equals to 15. Therefore, the higher the cumulative probability the more conservative the estimate is. For example, the \(C\) and \(m\) values for 50% cumulative probability will generate a more conservative fatigue life estimate than if 5% cumulative probability was used to calculate \(C\) and \(m\) values.
Table 4.5: Steps used for determination of C and m

<table>
<thead>
<tr>
<th>Name</th>
<th>da/dN for $\Delta K = 15$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 15.5$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 16$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 16.5$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 17$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 17.5$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 18$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 18.5$ (mm/cycle)</th>
<th>da/dN for $\Delta K = 19$ (mm/cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTC1</td>
<td>4.11E-05</td>
<td>4.47E-05</td>
<td>4.85E-05</td>
<td>5.25E-05</td>
<td>5.66E-05</td>
<td>6.10E-05</td>
<td>6.55E-05</td>
<td>7.03E-05</td>
<td>7.53E-05</td>
</tr>
<tr>
<td>CTC2</td>
<td>2.44E-05</td>
<td>2.78E-05</td>
<td>3.16E-05</td>
<td>3.58E-05</td>
<td>4.03E-05</td>
<td>4.52E-05</td>
<td>5.06E-05</td>
<td>5.65E-05</td>
<td>6.29E-05</td>
</tr>
<tr>
<td>CTC3</td>
<td>1.70E-05</td>
<td>1.92E-05</td>
<td>2.18E-05</td>
<td>2.45E-05</td>
<td>2.75E-05</td>
<td>3.08E-05</td>
<td>3.43E-05</td>
<td>3.81E-05</td>
<td>4.23E-05</td>
</tr>
<tr>
<td>CTC4</td>
<td>1.71E-05</td>
<td>1.92E-05</td>
<td>2.16E-05</td>
<td>2.42E-05</td>
<td>2.69E-05</td>
<td>2.99E-05</td>
<td>3.32E-05</td>
<td>3.67E-05</td>
<td>4.04E-05</td>
</tr>
<tr>
<td>CTC5</td>
<td>1.81E-05</td>
<td>2.07E-05</td>
<td>2.36E-05</td>
<td>2.68E-05</td>
<td>3.03E-05</td>
<td>3.42E-05</td>
<td>3.84E-05</td>
<td>4.31E-05</td>
<td>4.81E-05</td>
</tr>
<tr>
<td>CTL1</td>
<td>1.63E-05</td>
<td>1.84E-05</td>
<td>2.06E-05</td>
<td>2.31E-05</td>
<td>2.57E-05</td>
<td>2.86E-05</td>
<td>3.16E-05</td>
<td>3.49E-05</td>
<td>3.85E-05</td>
</tr>
<tr>
<td>CTL2</td>
<td>2.22E-05</td>
<td>2.46E-05</td>
<td>2.72E-05</td>
<td>3.00E-05</td>
<td>3.30E-05</td>
<td>3.61E-05</td>
<td>3.95E-05</td>
<td>4.31E-05</td>
<td>4.69E-05</td>
</tr>
<tr>
<td>CTL3</td>
<td>1.91E-05</td>
<td>2.09E-05</td>
<td>2.28E-05</td>
<td>2.49E-05</td>
<td>2.70E-05</td>
<td>2.93E-05</td>
<td>3.16E-05</td>
<td>3.41E-05</td>
<td>3.67E-05</td>
</tr>
<tr>
<td>CTL4</td>
<td>2.61E-05</td>
<td>2.97E-05</td>
<td>3.36E-05</td>
<td>3.78E-05</td>
<td>4.25E-05</td>
<td>4.76E-05</td>
<td>5.31E-05</td>
<td>5.91E-05</td>
<td>6.56E-05</td>
</tr>
<tr>
<td>Sorted</td>
<td>4.11E-05</td>
<td>4.47E-05</td>
<td>4.85E-05</td>
<td>5.25E-05</td>
<td>5.66E-05</td>
<td>6.10E-05</td>
<td>6.55E-05</td>
<td>7.03E-05</td>
<td>7.53E-05</td>
</tr>
<tr>
<td></td>
<td>2.61E-05</td>
<td>2.97E-05</td>
<td>3.36E-05</td>
<td>3.78E-05</td>
<td>4.25E-05</td>
<td>4.76E-05</td>
<td>5.31E-05</td>
<td>5.91E-05</td>
<td>6.56E-05</td>
</tr>
<tr>
<td></td>
<td>2.44E-05</td>
<td>2.78E-05</td>
<td>3.16E-05</td>
<td>3.58E-05</td>
<td>4.03E-05</td>
<td>4.52E-05</td>
<td>5.06E-05</td>
<td>5.65E-05</td>
<td>6.29E-05</td>
</tr>
<tr>
<td></td>
<td>2.22E-05</td>
<td>2.46E-05</td>
<td>2.72E-05</td>
<td>3.00E-05</td>
<td>3.30E-05</td>
<td>3.61E-05</td>
<td>3.95E-05</td>
<td>4.31E-05</td>
<td>4.81E-05</td>
</tr>
<tr>
<td></td>
<td>1.91E-05</td>
<td>2.09E-05</td>
<td>2.36E-05</td>
<td>2.68E-05</td>
<td>3.03E-05</td>
<td>3.42E-05</td>
<td>3.84E-05</td>
<td>4.31E-05</td>
<td>4.69E-05</td>
</tr>
<tr>
<td></td>
<td>1.81E-05</td>
<td>2.07E-05</td>
<td>2.28E-05</td>
<td>2.49E-05</td>
<td>2.75E-05</td>
<td>3.08E-05</td>
<td>3.43E-05</td>
<td>3.81E-05</td>
<td>4.23E-05</td>
</tr>
<tr>
<td></td>
<td>1.71E-05</td>
<td>1.92E-05</td>
<td>2.18E-05</td>
<td>2.45E-05</td>
<td>2.70E-05</td>
<td>2.99E-05</td>
<td>3.32E-05</td>
<td>3.67E-05</td>
<td>4.04E-05</td>
</tr>
<tr>
<td></td>
<td>1.70E-05</td>
<td>1.92E-05</td>
<td>2.16E-05</td>
<td>2.42E-05</td>
<td>2.69E-05</td>
<td>2.93E-05</td>
<td>3.16E-05</td>
<td>3.49E-05</td>
<td>3.85E-05</td>
</tr>
<tr>
<td></td>
<td>1.63E-05</td>
<td>1.84E-05</td>
<td>2.06E-05</td>
<td>2.31E-05</td>
<td>2.57E-05</td>
<td>2.86E-05</td>
<td>3.16E-05</td>
<td>3.41E-05</td>
<td>3.67E-05</td>
</tr>
<tr>
<td>Probability</td>
<td>Y = ln(da/dN)</td>
<td>X = ln(ΔK)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>--------------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>-11.24</td>
<td>2.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-11.04</td>
<td>2.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.92</td>
<td>2.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.80</td>
<td>2.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.69</td>
<td>2.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.57</td>
<td>2.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.45</td>
<td>2.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.34</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.24</td>
<td>2.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>-10.75</td>
<td>2.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.57</td>
<td>2.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.46</td>
<td>2.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.35</td>
<td>2.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.24</td>
<td>2.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.14</td>
<td>2.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.04</td>
<td>2.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.94</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.84</td>
<td>2.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95%</td>
<td>-10.26</td>
<td>2.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-10.09</td>
<td>2.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.99</td>
<td>2.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.90</td>
<td>2.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.80</td>
<td>2.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.71</td>
<td>2.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.62</td>
<td>2.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.53</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.45</td>
<td>2.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table shows the da/dN values for different ΔK values and corresponding ln(da/dN) values for 5%, 50%, and 95% probability levels.
Figure 4.9: Regression of crack growth data at cumulative probability of 5%

![Regression of crack growth data](image)

Table 4.6: Summary of Paris equation constants obtained through statistical model

<table>
<thead>
<tr>
<th>Cumulative probability (G)</th>
<th>C</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>2.32×10^{-10}</td>
<td>4.05</td>
</tr>
<tr>
<td>50%</td>
<td>1.04×10^{-9}</td>
<td>3.68</td>
</tr>
<tr>
<td>95%</td>
<td>4.65×10^{-9}</td>
<td>3.31</td>
</tr>
</tbody>
</table>

Figure 4.10 shows estimate of fatigue life for CTC1 for 5%, 50%, and 95% cumulative probability. It should be noted that the line corresponding to 95% cumulative probability was above all other lines. Therefore 95% cumulative probability provided the most conservative fatigue life estimate. The plot also shows the experimental data and it can be found that all test data lies within the range estimated by the method proposed by this study. Fatigue estimates were completed for all specimens and they are shown in Appendix C.
Test data of specimen CTC1 was between 5% and 50%. This particular result was expected the fatigue estimate was closer to 50%. Hence, the fatigue estimate of specimen CTC1 is closer to the “average”. However, the test data for most CT specimens fell closer to the 5% cumulative probability line (Appendix C). In other words, most of the specimens had a higher crack growth rate that is, most specimens were closer to the “weaker” end. This trend could be due to the small number of CT specimens used to develop the statistical method. Perhaps if more CT specimens were tested more samples would lie closer to the 50% line. This needs to be investigated in future research.

Since reasonably good results were obtained for estimating fatigue life for CT specimens, the Paris equation constants calculated through the statistical method were finally chosen and used to determine the crack growth and fatigue life the full-scale pipe specimen.

![CTC1 estimates](image_url)

**Figure 4.10: Fatigue estimate of CTC1**
4.4 Finite element model

A finite-element (FE) model (Figure 4.11) was created to determine the stress intensity factor for the full-scale pipe specimen. The objective was to use the stress intensity factor (K) with Paris equation to estimate fatigue life of the pipe specimen. This section describes the FE model used in this study. The model was developed by Hossein Ghaednia, a Ph. D. student working in the same research group under the supervision of Dr. S. Das.

The primary objective of having a reliable FE model is to determine burst strength of same pipe with dent-crack defect. This model was also used to determine stress intensity factor (K) for the current project. Only half of the pipe was modeled since the pipe was symmetric about its mid span. The model was created to closely reflect the experimental setup (Figure 3.2). On top of the pipe is the actuator and the top collar.

![Figure 4.11: Finite element model](image)
First order solid elements were used to simulate pipe wall. Six elements were used through the wall thickness. First order elements were employed to minimize solution time. Each element was 8 mm x 8 mm x 1.42 mm. The element has eight nodes and uses linear formulation (C3D8R). Elements surrounding the notch (Figure 4.12) were four-node linear tetrahedron solid elements (C3D4). These elements were smallest in size close to the notch to simulate proper stress concentration at the notch tip.

![Figure 4.12: Elements near notch](image)

The actuator and the support were modeled as discrete rigid because they were assumed not to deform. Both the top and bottom collars used the same material as the pipe (Figure 4.13). Collars had a fillet to prevent a stress concentration from occurring. Interaction algorithm was chosen as surface-to-surface contact between all parts (i.e. actuator, bottom collar, top collar, support, and pipe). Figure 4.14 illustrates the boundary condition at the bottom collar. At the middle of the bottom of the collar a width 100 mm (two 50 mm rectangles) was fixed to the bottom of the support.
The FE model was validated with load-deformation data obtained from the test and a good agreement was obtained (Figure 4.15).

Figure 4.13: Top collar

Figure 4.14: Bottom collar boundary condition

Figure 4.15: Load-deformation data of test and FE model compared
4.5 Stress intensity factor estimation

In previous studies researchers have assumed a semi-elliptical shape to exist (Section 2.3 and 2.4). For example Yoo and Kotoji (1999) treated the pipe specimen as a flat plate with a semi-elliptical surface flaw. Newman-Raju’s formula (Newman and Raju 1981) was used to calculate the stress intensity factor. The current study did not use this approach because it is hard to predict what shape the crack will take and flaw assumptions can lead to large error (Broek 1989). In addition, Saxena and Chouhan (2009) found that using a semi-elliptical solution produces non-conservative results. Saxena and Chouhan also advised against using semi-elliptical stress intensity solutions for estimating fatigue life of a straight pipe with a constant depth crack profile. Hence, it was concluded that stress intensity factor should be calculated with the aid of a finite element model.

Using data provided by the finite element model the stress intensity factor was calculated. The following steps were used to calculate K.

1. Using stress values provided by finite element model ($\sigma_{FEM}$) the stress intensity factor was found by extrapolating Equation 4.9 to the crack tip.

2. To obtain $\beta$ (Equation 2.5) the stress intensity factor was calculated at different crack lengths. Next $\beta$ was calculated as a function of $a/W$ using Equation 4.10 and the stress intensity factor (Equation 2.5) was calculated.

A stress intensity factor solution was obtained by making use of the stress values near the crack tip as provided by the finite element model. The finite element model did not
directly provide the stress intensity factor (K) and hence stress values near the crack tip were used instead to determine the K value. The universal crack tip stress field solution provides the crack tip stress perpendicular to the crack plane (σ) as follows (Broek, 1989).

\[ \sigma = \frac{K}{\sqrt{2\pi x}} \]  (4.8)

where, K is the stress intensity factor and x is the distance of location where stress (σ) is being calculated. Distance of location x is measured from the crack tip.

Hence, the stress intensity factor can be calculated with Equation 4.9.

\[ K = \sigma_{FEM} \sqrt{\frac{2\pi x_{FEM}}{}} \]  (4.9)

However, Equation 4.8 is only valid for a very small value of x which is the distance measured from the notch (crack) tip (Figure 4.16). Unfortunately, since high stress gradient is present at the crack tip, considerable error generates in the calculated value of σ of the FE model. This error arises primarily due to refinement of elements at the crack tip. Broek (1989) therefore recommended solving Equation 4.8 using the calculated stresses (σ₁, σ₂, σ₃,…) at arbitrarily chosen locations (x = x₁, x₂, x₃,…) from the notch tip (Figure 4.16). However, none of these values is the correct one and hence, none of these values of can be directly used for the calculation of K. Hence, an indirect approach as discussed next was used.
Equation 4.9 is solved at these locations for K (Figure 4.17) and plotted. A straight line is drawn through the points (1,2,3,4) and extrapolated at the crack tip or point 5 (at x=0). This is considered as an appropriate value of K. Figure 4.18 shows the apparent stress intensity factor for actual crack geometry. The plot was obtained by plotting the apparent stress intensity factor (Equation 4.9) and extrapolating data as discussed above.

4.6 Determination of β-factor

The last step is to find the β factor with Equation 4.10. This is Equation 2.5 rearranged. The geometry factor is calculated to find the equation of the stress intensity factor.

\[
\beta = \frac{K}{\sigma_{ref} \sqrt{\pi a}}
\]  

(4.10)

The factor \(\sigma_{ref}\) is the reference stress. It is not important which reference stress is chosen as long as the same reference stress is used in analysis. For convenience the reference stress was chosen to be the applied actuator load divided by the surface area of the actuator. Hence, the value of reference stress was found to be 1.43 MPa when 50 kN is applied. Since the maximum load applied was 100 kN, then \(\Delta\sigma\) was also 1.43 MPa.

For crack length (a) of 3.75 mm the stress intensity factor (K) was calculated to be 9.77 MPa \(\sqrt{m}\) (Figure 4.18). This procedure of finding the apparent stress intensity factor (K) and extrapolating to the crack tip (at x = 0) was repeated along the pipe wall thickness (Figure 4.19) for crack lengths (a) of 4.25 mm and 4.5 mm. Pipe wall thickness (W) was unchanged and it was 8.5 mm. Hence, the values of a/W obtained were Figure 4.19 shows the plot of the beta factor (β) versus the ratio of crack length to pipe wall thickness (a/W).
However, value of crack length (a) was changed to develop the plot. A trendline was used to solve a function for factor $\beta$.

Figure 4.16: Stresses from finite element model (Broek, 1989)

Figure 4.17: Apparent stress intensity (Broek, 1989)
Figure 4.18: Extrapolation of $K$

\[ y = -2662.3x + 9.7702 \]
\[ R^2 = 0.9609 \]

Crack length = 3.75 mm

Figure 4.19: Beta factor

\[ y = 1300.8x^2 - 1162.8x + 322.92 \]
\[ R^2 = 1 \]
Hence, the following equation was used to find the value of $\beta$ in this study.

$$\beta = 1300.8 \left( \frac{a}{W} \right)^2 - 1162.8 \left( \frac{a}{W} \right) + 322.92 \quad (4.11)$$

### 4.7 Development of fatigue life model for pipe

Next the final form of the Paris equation is developed. Paris equation is the following.

$$\frac{da}{dN} = C(\Delta K)^m \quad (2.17)$$

The values of $C$ and $m$ values for 5% cumulative are used in Equation 4.12. This is because the test data of most CT specimens was closest to the fatigue estimate corresponding to the values of $C$ and $m$ obtained from 5% cumulative probability. See Appendix C for the fatigue estimates of all CT specimens of the current study. Therefore, $C$ has value of $2.35 \times 10^{-10}$ and $m$ has value of 4.05. The reference stress was calculated as 1.43 MPa. To complete the Paris equation, the stress intensity factor is required which is expressed in Equation 4.12. Equation 4.11 is used to solve for $\beta$.

$$\Delta K = \beta \Delta \sigma \sqrt{\pi a} \quad (4.12a)$$

$$\Delta K = 1.43 \left[ 1.43 \left( 1300.8 \left( \frac{a}{W} \right)^2 - 1162.8 \left( \frac{a}{W} \right) + 322.92 \right) \right] \sqrt{\pi a} \quad (4.12b)$$

Therefore, the Paris equation was obtained by combining Equations 4.11 and 4.12 with $C$ and $m$ values obtained from the statistical method described in Section 4.3. Hence, the following form of the Paris equation was developed and used to estimate fatigue life of the full-scale pipe specimen in the current study. Equation 4.13 is valid for 5% cumulative probability. Ratio $a/W$ can also be represented by $\alpha$ in this equation.
\[
\frac{da}{dN} = 2.35 \times 10^{-10} \left\{ 1.43 \left( \left( \frac{a}{W} \right)^2 - 1162.8 \left( \frac{a}{W} \right) + 322.92 \right) \right\}^{\frac{1}{3.05}} \quad (4.13)
\]

Table 4.7 is a spreadsheet that explains how the number of cycles to reach a certain depth were calculated for the full-scale pipe specimen using this approach. An explanation this table follows in the next paragraph.

### 4.8 Validation of fatigue model

First, the original crack length (a) was chosen. In this case the notch depth was taken to be the original notch length (3.75 mm) or first crack. Since integration was being performed in this table, small crack length increments (\(\Delta a\)) were used (one percent of current crack length). This small increments were listed under the column titled \(\Delta a\) (mm).

Column \(\alpha\) refers to crack length (a) divided by pipe wall thickness (W=8.5 mm). As mentioned earlier, a reference stress of 1.43 MPa was used (see Equation 4.13). The \(\beta\) function (Equation 4.11) obtained from the finite element model was input under the column called \(\beta\). The stress intensity factor (K) was calculated using Equation 4.12. The values in the column titled “da/dN” were calculated using Equation 4.13. Finally, the increment of cycles (\(\Delta N\)) corresponding to this increase in length (\(\Delta a\)) was achieved by assuming the following equation.

\[
\frac{\Delta a}{\Delta N} \approx \frac{da}{dN} \quad (4.14)
\]

\[
\Delta N \approx \frac{\Delta a}{da} \quad (4.15)
\]
Table 4.7: Estimation of full-scale specimen

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>Δa (mm)</th>
<th>α</th>
<th>Δσ</th>
<th>β</th>
<th>ΔK (MPa m₀.₅)</th>
<th>da/dN (mm/cycle)</th>
<th>ΔN</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.75</td>
<td>0.04</td>
<td>0.44</td>
<td>1.43</td>
<td>63.10</td>
<td>9.77</td>
<td>0.0000024</td>
<td>15655.78</td>
<td></td>
</tr>
<tr>
<td>3.79</td>
<td>0.04</td>
<td>0.45</td>
<td>1.43</td>
<td>63.06</td>
<td>9.81</td>
<td>0.0000024</td>
<td>15537.77</td>
<td></td>
</tr>
<tr>
<td>3.83</td>
<td>0.04</td>
<td>0.45</td>
<td>1.43</td>
<td>63.07</td>
<td>9.86</td>
<td>0.0000025</td>
<td>15370.26</td>
<td></td>
</tr>
<tr>
<td>3.86</td>
<td>0.04</td>
<td>0.45</td>
<td>1.43</td>
<td>63.13</td>
<td>9.92</td>
<td>0.0000025</td>
<td>15153.38</td>
<td></td>
</tr>
<tr>
<td>3.90</td>
<td>0.04</td>
<td>0.46</td>
<td>1.43</td>
<td>63.25</td>
<td>9.99</td>
<td>0.0000026</td>
<td>14887.93</td>
<td></td>
</tr>
<tr>
<td>3.94</td>
<td>0.04</td>
<td>0.46</td>
<td>1.43</td>
<td>63.42</td>
<td>10.07</td>
<td>0.0000027</td>
<td>14575.34</td>
<td></td>
</tr>
<tr>
<td>3.98</td>
<td>0.04</td>
<td>0.47</td>
<td>1.43</td>
<td>63.65</td>
<td>10.16</td>
<td>0.0000028</td>
<td>14217.72</td>
<td></td>
</tr>
<tr>
<td>4.02</td>
<td>0.04</td>
<td>0.47</td>
<td>1.43</td>
<td>63.94</td>
<td>10.25</td>
<td>0.0000029</td>
<td>13817.79</td>
<td></td>
</tr>
<tr>
<td>4.06</td>
<td>0.04</td>
<td>0.48</td>
<td>1.43</td>
<td>64.29</td>
<td>10.36</td>
<td>0.0000030</td>
<td>13378.84</td>
<td></td>
</tr>
<tr>
<td>4.10</td>
<td>0.04</td>
<td>0.48</td>
<td>1.43</td>
<td>64.70</td>
<td>10.48</td>
<td>0.0000032</td>
<td>12904.73</td>
<td></td>
</tr>
<tr>
<td>4.14</td>
<td>0.04</td>
<td>0.49</td>
<td>1.43</td>
<td>65.18</td>
<td>10.61</td>
<td>0.0000033</td>
<td>12399.72</td>
<td></td>
</tr>
<tr>
<td>4.18</td>
<td>0.04</td>
<td>0.49</td>
<td>1.43</td>
<td>65.72</td>
<td>10.75</td>
<td>0.0000035</td>
<td>11868.48</td>
<td></td>
</tr>
<tr>
<td>4.23</td>
<td>0.04</td>
<td>0.50</td>
<td>1.43</td>
<td>66.33</td>
<td>10.90</td>
<td>0.0000037</td>
<td>11315.94</td>
<td></td>
</tr>
<tr>
<td>a (mm)</td>
<td>Δa (mm)</td>
<td>α</td>
<td>Δσ</td>
<td>β</td>
<td>ΔK (MPa m$^{0.5}$)</td>
<td>da/dN (mm/cycle)</td>
<td>ΔN</td>
<td>N</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>---</td>
<td>-----</td>
<td>------</td>
<td>------------------</td>
<td>------------------</td>
<td>-----------------</td>
<td>------</td>
</tr>
<tr>
<td>4.27</td>
<td>0.04</td>
<td>0.50</td>
<td>1.43</td>
<td>67.02</td>
<td>11.07</td>
<td>0.0000040</td>
<td>181084</td>
<td></td>
</tr>
<tr>
<td>4.31</td>
<td>0.04</td>
<td>0.51</td>
<td>1.43</td>
<td>67.77</td>
<td>11.25</td>
<td>0.0000042</td>
<td>191831</td>
<td></td>
</tr>
<tr>
<td>4.35</td>
<td>0.04</td>
<td>0.51</td>
<td>1.43</td>
<td>68.60</td>
<td>11.45</td>
<td>0.0000045</td>
<td>201998</td>
<td></td>
</tr>
<tr>
<td>4.40</td>
<td>0.04</td>
<td>0.52</td>
<td>1.43</td>
<td>69.50</td>
<td>11.65</td>
<td>0.0000049</td>
<td>211580</td>
<td></td>
</tr>
<tr>
<td>4.44</td>
<td>0.04</td>
<td>0.52</td>
<td>1.43</td>
<td>70.48</td>
<td>11.88</td>
<td>0.0000053</td>
<td>220575</td>
<td></td>
</tr>
<tr>
<td>4.49</td>
<td>0.04</td>
<td>0.53</td>
<td>1.43</td>
<td>71.54</td>
<td>12.12</td>
<td>0.0000057</td>
<td>228987</td>
<td></td>
</tr>
<tr>
<td>4.53</td>
<td>0.05</td>
<td>0.53</td>
<td>1.43</td>
<td>72.69</td>
<td>12.37</td>
<td>0.0000062</td>
<td>236824</td>
<td></td>
</tr>
<tr>
<td>4.58</td>
<td>0.05</td>
<td>0.54</td>
<td>1.43</td>
<td>73.92</td>
<td>12.64</td>
<td>0.0000068</td>
<td>244098</td>
<td></td>
</tr>
<tr>
<td>4.62</td>
<td>0.05</td>
<td>0.54</td>
<td>1.43</td>
<td>75.24</td>
<td>12.93</td>
<td>0.0000075</td>
<td>250826</td>
<td></td>
</tr>
<tr>
<td>4.67</td>
<td>0.05</td>
<td>0.55</td>
<td>1.43</td>
<td>76.64</td>
<td>13.24</td>
<td>0.0000082</td>
<td>257025</td>
<td></td>
</tr>
<tr>
<td>4.71</td>
<td>0.05</td>
<td>0.55</td>
<td>1.43</td>
<td>78.14</td>
<td>13.57</td>
<td>0.0000090</td>
<td>262719</td>
<td></td>
</tr>
<tr>
<td>4.76</td>
<td>0.05</td>
<td>0.56</td>
<td>1.43</td>
<td>79.73</td>
<td>13.91</td>
<td>0.0000100</td>
<td>267929</td>
<td></td>
</tr>
<tr>
<td>4.81</td>
<td>0.05</td>
<td>0.57</td>
<td>1.43</td>
<td>81.43</td>
<td>14.28</td>
<td>0.0000111</td>
<td>272681</td>
<td></td>
</tr>
<tr>
<td>4.86</td>
<td>0.05</td>
<td>0.57</td>
<td>1.43</td>
<td>83.22</td>
<td>14.67</td>
<td>0.0000124</td>
<td>277002</td>
<td></td>
</tr>
<tr>
<td>a (mm)</td>
<td>Δa (mm)</td>
<td>α</td>
<td>Δσ</td>
<td>β</td>
<td>ΔK (MPa m^{0.5})</td>
<td>da/dN (mm/cycle)</td>
<td>ΔN</td>
<td>N</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>4.91</td>
<td>0.05</td>
<td>0.58</td>
<td>1.43</td>
<td>85.11</td>
<td>15.07</td>
<td>0.0000139</td>
<td>3539.011</td>
<td>280918</td>
</tr>
<tr>
<td>4.95</td>
<td>0.05</td>
<td>0.58</td>
<td>1.43</td>
<td>87.11</td>
<td>15.51</td>
<td>0.0000155</td>
<td>3188.848</td>
<td>284457</td>
</tr>
<tr>
<td>5.00</td>
<td>0.05</td>
<td>0.59</td>
<td>1.43</td>
<td>89.21</td>
<td>15.96</td>
<td>0.0000175</td>
<td>2865.38</td>
<td>287646</td>
</tr>
<tr>
<td>5.05</td>
<td>0.05</td>
<td>0.59</td>
<td>1.43</td>
<td>91.43</td>
<td>16.44</td>
<td>0.0000197</td>
<td>2567.902</td>
<td>290511</td>
</tr>
<tr>
<td>5.10</td>
<td>0.05</td>
<td>0.60</td>
<td>1.43</td>
<td>93.76</td>
<td>16.94</td>
<td>0.0000222</td>
<td>2295.487</td>
<td>293079</td>
</tr>
<tr>
<td>5.16</td>
<td>0.05</td>
<td>0.61</td>
<td>1.43</td>
<td>96.21</td>
<td>17.47</td>
<td>0.0000252</td>
<td>2047.028</td>
<td>295375</td>
</tr>
<tr>
<td>5.21</td>
<td>0.05</td>
<td>0.61</td>
<td>1.43</td>
<td>98.78</td>
<td>18.03</td>
<td>0.0000286</td>
<td>1821.286</td>
<td>297422</td>
</tr>
<tr>
<td>5.26</td>
<td>0.05</td>
<td>0.62</td>
<td>1.43</td>
<td>101.47</td>
<td>18.61</td>
<td>0.0000325</td>
<td>1616.926</td>
<td>299243</td>
</tr>
<tr>
<td>5.31</td>
<td>0.05</td>
<td>0.62</td>
<td>1.43</td>
<td>104.28</td>
<td>19.22</td>
<td>0.0000371</td>
<td>1432.557</td>
<td>300860</td>
</tr>
<tr>
<td>5.37</td>
<td>0.05</td>
<td>0.63</td>
<td>1.43</td>
<td>107.23</td>
<td>19.86</td>
<td>0.0000424</td>
<td>1266.762</td>
<td>302292</td>
</tr>
<tr>
<td>5.42</td>
<td>0.05</td>
<td>0.64</td>
<td>1.43</td>
<td>110.30</td>
<td>20.53</td>
<td>0.0000485</td>
<td>1118.123</td>
<td>303559</td>
</tr>
<tr>
<td>5.47</td>
<td>0.05</td>
<td>0.64</td>
<td>1.43</td>
<td>113.52</td>
<td>21.24</td>
<td>0.0000556</td>
<td>985.2493</td>
<td>304677</td>
</tr>
<tr>
<td>5.53</td>
<td>0.05</td>
<td>0.65</td>
<td>1.43</td>
<td>116.87</td>
<td>21.97</td>
<td>0.0000638</td>
<td>866.7874</td>
<td>305663</td>
</tr>
<tr>
<td>5.58</td>
<td>0.06</td>
<td>0.65</td>
<td>1.43</td>
<td>120.37</td>
<td>22.74</td>
<td>0.0000733</td>
<td>761.4413</td>
<td>306529</td>
</tr>
<tr>
<td>a (mm)</td>
<td>Δa (mm)</td>
<td>α</td>
<td>Δσ</td>
<td>β</td>
<td>ΔK (MPa m$^{0.5}$)</td>
<td>da/dN (mm/cycle)</td>
<td>∆N</td>
<td>N</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---</td>
<td>----</td>
<td>---</td>
<td>-------------------</td>
<td>------------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>5.64</td>
<td></td>
<td>0.06</td>
<td>0.66</td>
<td>1.43</td>
<td>124.01</td>
<td>23.55</td>
<td>0.0000844</td>
<td>667.9802</td>
</tr>
<tr>
<td>5.70</td>
<td></td>
<td>0.06</td>
<td>0.67</td>
<td>1.43</td>
<td>127.80</td>
<td>24.39</td>
<td>0.0000973</td>
<td>585.2463</td>
</tr>
<tr>
<td>5.75</td>
<td></td>
<td>0.06</td>
<td>0.68</td>
<td>1.43</td>
<td>131.75</td>
<td>25.27</td>
<td>0.0001123</td>
<td>512.1588</td>
</tr>
<tr>
<td>5.81</td>
<td></td>
<td>0.06</td>
<td>0.68</td>
<td>1.43</td>
<td>135.86</td>
<td>26.19</td>
<td>0.0001298</td>
<td>447.7164</td>
</tr>
<tr>
<td>5.87</td>
<td></td>
<td>0.06</td>
<td>0.69</td>
<td>1.43</td>
<td>140.13</td>
<td>27.14</td>
<td>0.0001501</td>
<td>390.9973</td>
</tr>
<tr>
<td>5.93</td>
<td></td>
<td>0.06</td>
<td>0.70</td>
<td>1.43</td>
<td>144.56</td>
<td>28.14</td>
<td>0.0001737</td>
<td>341.1576</td>
</tr>
<tr>
<td>5.99</td>
<td></td>
<td>0.06</td>
<td>0.70</td>
<td>1.43</td>
<td>149.16</td>
<td>29.18</td>
<td>0.0002013</td>
<td>297.4292</td>
</tr>
<tr>
<td>6.05</td>
<td></td>
<td>0.06</td>
<td>0.71</td>
<td>1.43</td>
<td>153.94</td>
<td>30.27</td>
<td>0.0002333</td>
<td>259.116</td>
</tr>
<tr>
<td>6.11</td>
<td></td>
<td>0.06</td>
<td>0.72</td>
<td>1.43</td>
<td>158.90</td>
<td>31.40</td>
<td>0.0002707</td>
<td>225.5904</td>
</tr>
<tr>
<td>6.17</td>
<td></td>
<td>0.06</td>
<td>0.73</td>
<td>1.43</td>
<td>164.04</td>
<td>32.58</td>
<td>0.0003142</td>
<td>196.2884</td>
</tr>
<tr>
<td>6.23</td>
<td></td>
<td>0.06</td>
<td>0.73</td>
<td>1.43</td>
<td>169.37</td>
<td>33.80</td>
<td>0.0003649</td>
<td>170.7049</td>
</tr>
<tr>
<td>6.29</td>
<td></td>
<td>0.06</td>
<td>0.74</td>
<td>1.43</td>
<td>174.89</td>
<td>35.08</td>
<td>0.0004240</td>
<td>148.3897</td>
</tr>
<tr>
<td>6.35</td>
<td></td>
<td>0.06</td>
<td>0.75</td>
<td>1.43</td>
<td>180.60</td>
<td>36.41</td>
<td>0.0004928</td>
<td>128.9422</td>
</tr>
<tr>
<td>6.42</td>
<td></td>
<td>0.06</td>
<td>0.76</td>
<td>1.43</td>
<td>186.52</td>
<td>37.79</td>
<td>0.0005730</td>
<td>112.007</td>
</tr>
<tr>
<td>a (mm)</td>
<td>Δa (mm)</td>
<td>α</td>
<td>Δσ</td>
<td>β</td>
<td>ΔK (MPa m(^{0.5}))</td>
<td>da/dN (mm/cycle)</td>
<td>ΔN</td>
<td>N</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>----------------------</td>
<td>------------------</td>
<td>-----</td>
<td>------</td>
</tr>
<tr>
<td>6.48</td>
<td>0.06</td>
<td>0.76</td>
<td>1.43</td>
<td>192.65</td>
<td>39.22</td>
<td>0.0006664</td>
<td>97.26995</td>
<td>311775</td>
</tr>
<tr>
<td>6.55</td>
<td>0.07</td>
<td>0.77</td>
<td>1.43</td>
<td>198.99</td>
<td>40.71</td>
<td>0.0007752</td>
<td>84.45364</td>
<td>311872</td>
</tr>
<tr>
<td>6.61</td>
<td>0.07</td>
<td>0.78</td>
<td>1.43</td>
<td>205.54</td>
<td>42.26</td>
<td>0.0009019</td>
<td>73.31378</td>
<td>311956</td>
</tr>
<tr>
<td>6.68</td>
<td>0.07</td>
<td>0.79</td>
<td>1.43</td>
<td>212.32</td>
<td>43.88</td>
<td>0.0010495</td>
<td>63.63569</td>
<td>312030</td>
</tr>
<tr>
<td>6.75</td>
<td>0.07</td>
<td>0.79</td>
<td>1.43</td>
<td>219.32</td>
<td>45.55</td>
<td>0.0012213</td>
<td>55.23096</td>
<td>312093</td>
</tr>
<tr>
<td>6.81</td>
<td>0.07</td>
<td>0.80</td>
<td>1.43</td>
<td>226.56</td>
<td>47.29</td>
<td>0.0014212</td>
<td>47.93459</td>
<td>312148</td>
</tr>
<tr>
<td>6.88</td>
<td>0.07</td>
<td>0.81</td>
<td>1.43</td>
<td>234.03</td>
<td>49.09</td>
<td>0.0016539</td>
<td>41.60222</td>
<td>312196</td>
</tr>
<tr>
<td>6.95</td>
<td>0.07</td>
<td>0.82</td>
<td>1.43</td>
<td>241.75</td>
<td>50.96</td>
<td>0.0019247</td>
<td>36.10778</td>
<td>312238</td>
</tr>
<tr>
<td>7.02</td>
<td>0.07</td>
<td>0.83</td>
<td>1.43</td>
<td>225.63</td>
<td>47.80</td>
<td>0.0014849</td>
<td>47.26926</td>
<td>312274</td>
</tr>
</tbody>
</table>
The cracked region near the notch in the full-scale pipe test was inspected using SEM and it was found that the fatigue crack penetrated approximately 7.0 mm before fracture occurred. That was the reason why the estimate of final crack length in Table 4.6 is 7.02 mm.

Unfortunately, since no beach marks could be found under the SEM an assumption was made as to when the fatigue crack initiated (Region I). Since the notch was 3.75 mm deep and pipe wall thickness was 8.5 mm, which means $a/t$ is high ($\frac{a}{t} = \frac{3.75}{8.5} \approx 0.44$) a high stress concentration was expected to occur. Hence, the fatigue crack was assumed to immediately begin or soon after fatigue load cycles began on the pipe specimen. Thus, the assumption is made that crack initiation (Region I) was very small. The full-scale specimen was subjected to an effective total load cycle count of 370,000. (i.e. only active cycles were counted as discussed in Section 2.6). It should be noted that 370,000 cycles was the number of cycles when the through wall crack was first observed. It is not known when exactly the crack penetrated the entire pipe wall although through-wall crack was not found at 350,000 cycles. Hence, the number of cycles required for the crack to grow through the entire wall thickness in the pipe specimen was between 350,000 cycles and 370,000 cycles.

The proposed model of Equation 4.9 estimated the fatigue life at 312,000 cycles using $C$ and $m$ values corresponding to 5% cumulative probability. Estimated fatigue life using $C$ and $m$ values corresponding to 50% cumulative probability and 95% cumulative probability were 174,000 cycles and 97,000, cycles respectively. Hence, the model
predicted a conservative life. The error in life prediction is between 11% and 15% for the best estimate (at 5% cumulative probability) of fatigue life.

It should be noted that if the fatigue life model was developed with 50% or 95% cumulative probability then the model would have been even more conservative (174,000 and 97,000 cycles).

4.6 Summary
This chapter discussed the results of fatigue tests on API 5L X65 steel pipes that were completed with the objective to predict fatigue crack growth in a full-scale specimen with the aid of a finite element model. The following provides a summary of this chapter.

1. Notch orientation of CT specimens had no effect on crack growth rate and K-value.

2. Paris equation constants, C and m, obtained from CT tests exhibited a wide scatter. Hence, it was not obvious which values of C and m were to be used to estimate fatigue crack growth rate of the full-scale specimen. Hence, a statistical method was developed and used to find statistically ‘representive’ values of C and m.

3. A finite element model was developed to simulate a full-scale test. The purpose was to calculate a stress intensity factor function. Then the function along with Paris equation and the constants (C and m) obtained were used to develop a mathematical model for
predicting fatigue crack growth in the full-scale pipe specimen. The fatigue crack growth rate model developed is shown in Equation 4.13.

4. The number of cycles for the full-scale specimen to develop a through-wall crack was between 350,000 and 370,000 cycles. The number estimated (using 5% cumulative probability C and m values) by the model was 312,000 cycles. The numbers of cycles estimated corresponding to 50% and 95% cumulative probability were 174,000 and 97,000, respectively and hence, yielded even more conservative predictions for fatigue life. Hence, it was found that the fatigue crack growth model is conservative even though the model uses 5% cumulative probability.

5. However, the model for crack growth prediction needs to be validated with other full-scale pipe tests data before it is accepted for use in the field.
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

5.1 General
The current study was conducted to investigate fatigue crack growth in steel used in oil and gas pipelines. Specifically, the main objective was to develop a model that is able to estimate fatigue life of a full-scale pipe specimen. In doing so, a fracture mechanics approach was adopted. This chapter concludes the main findings and provides recommendations for future research.

5.2 Conclusions
Based on the work completed the following conclusions were made. These findings apply only to the specimens used in the current study.

1. A statistics based method was developed to estimate Paris equation constants, C and m, after performing fatigue tests as per ASTM E647 (ASTM 2008) for steel used in oil and gas pipelines. The method was developed because after conducting a number of fatigue tests on compact tension specimens it was found that the values of C and m exhibit a large scatter. Hence, a statistical method was developed to estimate “representative” values of C and m which were then used to develop a model for estimating fatigue crack growth rate (da/dN) and fatigue life of the full-scale pipe specimen.
2. Notch orientation of CT specimens with respect to pipe axis showed no effect on crack growth rate.

3. A fatigue crack growth model (formula) in the form of Paris equation was developed with the aid of the statistical model and a finite element model. The finite element model was used to calculate the stress intensity factor (K) of the full-scale pipe specimen. Therefore, the statistical method provided the values of C and m and the finite element model provided the stress intensity factor (K). The fatigue crack growth model developed in this study is as follows.

$$\frac{da}{dN} = 2.35 \times 10^{-10} \left\{ 1.43 \left( 1300.8 \left( \frac{a}{W} \right)^2 - 1162.8 \left( \frac{a}{W} \right) + 322.92 \right) \right\} \sqrt{\pi a}^{4.05}$$  \hspace{1cm} (4.13)

4. The number of cycles for the full-scale test specimen to develop a through-wall crack was found to be between 350,000 and 370,000 cycles. The number of cycles estimated (using C and m values corresponding to 5% cumulative probability) by the model (Equation 4.13) was 312,000. Hence, the fatigue crack growth model yielded an error between 11% and 15% for its best estimate (5% cumulative probability).

5.2 Recommendations

The following recommendations are made regarding future research works.

1. More tests using compact tension tests are recommended to improve the statistical method. Crack length readings can be taken at a predetermined number of cycles with the
aid of the present log-normal model. The objective is to measure cracks at longer crack
intervals to minimize measurement errors.

2. More compact tension specimen tests are recommended at different stress ratios to
obtain a wider range in the test data pool.

3. The finite element model could possibly be improved by using singularity elements to
capture more accurately the crack tip singularity. Alternatively, very fine elements at and
near the crack tip can be implemented which however, can increase computer processing
time exponentially.

4. Due to the small size of the compact tension specimens and the fine size of cracks, it is
recommended to use a digital microscope that can focus at a higher magnification.

5. It is recommended to conduct the load shedding procedure (described in ASTM E647)
to generate region I fatigue crack growth data. The objective is to estimate threshold
value, $\Delta K_{th}$, for which very slow crack growth occurs. This could aid in determining what
actuator loads to apply to initiate crack growth for notches of various depths.

6. A larger number of full-scale pipe tests are recommended as well as more fatigue crack
measurements in each specimen before a through-wall crack develops. Measurements
could be done with the aid of the phased array ultrasonic method and/or benchmarking if
possible. The objective is to obtain more data to validate or improve the fatigue crack growth model of the current study.

7. To assist in fatigue life estimates it is recommended to find the fracture toughness of the material.

8. It is recommended to develop a stress intensity factor solution for a wider range of cracks in pipes. Clearly this is a complex task, but it is worthwhile to improve current approaches.

9. Finally, the model proposed in this thesis should be compared to procedures of API 579 and BS 7910 for assessing crack flaws.
APPENDICES

APPENDIX A

Additional crack length versus number of cycles curves for CT specimens

This appendix provides plots of crack length from notch tip vs. number of cycles of compact (CT) tension tests.

A.1.1 Crack length versus number of cycles

![Graph of Crack length vs. Number of cycles for CT specimens]

Figure A.1.1: Crack length from notch vs. number of cycles for all nine CT specimens
A.2 CT specimens with longitudinal notch

Figure A.2.1: Specimen CTL1 plot of crack length from notch vs. number of cycles

\[ y = 0.09e^{0.0000096N} \]

\[ R^2 = 0.962 \]

Figure A.2.2: Specimen CTL2 plot of crack length from notch vs. number of cycles

\[ y = 0.019000e^{0.000012N} \]

\[ R^2 = 0.963 \]
Figure A.2.3: Specimen CTL3 plot of crack length from notch vs. number of cycles

\[ y = 0.200 e^{0.000091N} \]
\[ R^2 = 0.961 \]

Figure A.2.4: Specimen CTL4 plot of crack length from notch vs. number of cycles

\[ y = 0.010000 e^{0.000017N} \]
\[ R^2 = 0.941 \]
A.3 CT specimens with circumferential notch

Figure A.3.1: Specimen CTC1 plot of crack length from notch vs. number of cycles

\[ y = 0.006e^{0.000019N} \]
\[ R^2 = 0.921 \]

Figure A.3.2: Specimen CTC2 plot of crack length from notch vs. number of cycles

\[ y = 0.020000e^{0.000015N} \]
\[ R^2 = 0.927 \]
Figure A.3.3: Specimen CTC3 plot of crack length from notch vs. number of cycles

Figure A.3.4: Specimen CTC4 plot of crack length from notch vs. number of cycles
Figure A.3.5: Specimen CTC5 plot of crack length from notch vs. number of cycles

\[
y = 0.015e^{0.00012N} \\
R^2 = 0.947
\]
APPENDIX B

Additional crack growth rate versus stress intensity factor range plots for CT specimens

This appendix shows the crack growth rate \((\frac{da}{dN})\) vs stress intensity factor range \((\Delta K)\) for specimens CTL1, CTL2, CTL3, CTL4, CTC1, CTC2, CTC3, CTC4, and CTC5.

B.1 CT specimens with longitudinal notch

![Graph showing crack growth rate vs. stress intensity range for specimen CTL1](image)

Figure B.1.1: Crack growth rate vs. stress intensity range for specimen CTL1
Figure B.1.2: Crack growth rate vs. stress intensity range for specimen CTL2

Figure B.1.3: Crack growth rate vs. stress intensity range for specimen CTL3
Figure B.1.4: Crack growth rate vs. stress intensity range for specimen CTL4

Figure B.2.1: Crack growth rate vs. stress intensity range for specimen CTC1

B.2 CT specimens with circumferential notch
Figure B.2.2: Crack growth rate vs. stress intensity range for specimen CTC2

Figure B.2.3: Crack growth rate vs. stress intensity range for specimen CTC3
Figure B.2.4: Crack growth rate vs. stress intensity range for specimen CTC4

Figure B.2.5: Crack growth rate vs. stress intensity range for specimen CTC5
B.3 Linear region for CT specimens with longitudinal notch

Figure B.3.1: Crack growth rate vs. stress intensity range (linear region) for CTL1

\[ y = 0.000000000878x^{3.63} \]
\[ R^2 = 0.987 \]

Figure B.3.2: Crack growth rate vs. stress intensity range (linear region) for CTL2

\[ y = 0.000000004182x^{3.17} \]
\[ R^2 = 0.989 \]
Figure B.3.3: Crack growth rate vs. stress intensity range (linear region) for CTL3

Figure B.3.4: Crack growth rate vs. stress intensity range (linear region) for CTL4
B.4 Linear region for CT specimens with circumferential notch

Figure B.4.1: Crack growth rate vs. stress intensity range (linear region) for CTC1

Figure B.4.2: Crack growth rate vs. stress intensity range (linear region) for CTC2
Figure B.4.3: Crack growth rate vs. stress intensity range (linear region) for CTC3

Figure B.4.4: Crack growth rate vs. stress intensity range (linear region) for CTC4

\[ y = 0.000000000487x^{3.86} \quad R^2 = 0.988 \]

\[ y = 0.00000000087x^{3.65} \quad R^2 = 0.975 \]
Figure B.4.5: Crack growth rate vs. stress intensity range (linear region) for CTC5

$y = 0.00000000024x^{4.14}$

$R^2 = 0.983$
APPENDIX C

Fatigue estimates of CT specimens compared to test data

C.1 Life estimates of CT specimens with longitudinal notch

Figure C.1.1: Fatigue estimates compared vs. CTL1 test data

Figure C.1.2: Fatigue estimates compared vs. CTL2 test data
Figure C.1.3: Fatigue estimates compared vs. CTL3 test data

Figure C.1.4: Fatigue estimates compared vs. CTL4 test data
C.2 Life estimates of CT specimens with circumferential notch

Figure C.2.1: Fatigue estimates compared vs. CTC1 test data

Figure C.2.2: Fatigue estimates compared vs. CTC2 test data
Figure C.2.3: Fatigue estimates compared vs. CTC3 test data

Figure C.2.4: Fatigue estimates compared vs. CTC4 test data
Figure C.2.5: Fatigue estimates compared vs. CTC5 test data
REFERENCES


Efunda 2011., “Linear elastic fracture mechanics”,
http://www.efunda.com/formulae/solid_mechanics/fracture_mechanics/fm_lefm.cfm,
viewed in May 2011.

Fatigue crack growth.


Gross, Bernard and John E. Srawley, Stress-intensity factors for single-edge notch specimens in bending or combined bending and tension by boundary collocation of a stress function. NASA TN D-2603 (1965)


Newman, J.C., and Raju, I.S., 1979, “Analyses of surface cracks in finite plates under tension or bending loads” NASA TP-1578


Olympus, 2011, “Knowledge-What are the advantages?”


Z.I. Limited, 2011

http://www.zentech.co.uk/zencrack_example_fad.htm, viewed on May 17, 2011
VITA AUCTORIS

Jorge Silva was born in Guatemala City, Guatemala. After obtaining his degree of Bachelor of Applied Science from the University of Windsor in 2009 he obtained a degree of Master of Applied Science also from the University of Windsor.