Digital pattern recognition: An investigation of the discriminatory properties exhibited by an information theoretical algorithm when applied to a differential function of pattern.

William Timothy Mackenzie

University of Windsor

Follow this and additional works at: https://scholar.uwindsor.ca/etd

Recommended Citation

This online database contains the full-text of PhD dissertations and Masters’ theses of University of Windsor students from 1954 forward. These documents are made available for personal study and research purposes only, in accordance with the Canadian Copyright Act and the Creative Commons license—CC BY-NC-ND (Attribution, Non-Commercial, No Derivative Works). Under this license, works must always be attributed to the copyright holder (original author), cannot be used for any commercial purposes, and may not be altered. Any other use would require the permission of the copyright holder. Students may inquire about withdrawing their dissertation and/or thesis from this database. For additional inquiries, please contact the repository administrator via email (scholarship@uwindsor.ca) or by telephone at 519-253-3000 ext. 3208.
DIGITAL PATTERN RECOGNITION: AN INVESTIGATION

OF THE DISCRIMINATORY PROPERTIES EXHIBITED

BY AN INFORMATION THEORETICAL ALGORITHM

WHEN APPLIED TO A DIFFERENTIAL

FUNCTION OF PATTERN

BY

WILLIAM TIMOTHY MACKENZIE

A Thesis
Submitted to the Faculty of Graduate Studies Through the
Department of Electrical Engineering (Interdisciplinary
Studies in Communications) in Partial Fulfilment
of the Requirements for the Degree of
Master of Applied Science at the
University of Windsor

Windsor, Ontario

1966
UMI Number: EC52615

INFORMATION TO USERS

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleed-through, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

UMI

UMI Microform EC52615
Copyright 2008 by ProQuest LLC.
All rights reserved. This microform edition is protected against unauthorized copying under Title 17, United States Code.

ProQuest LLC
789 E. Eisenhower Parkway
PO Box 1346
Ann Arbor, MI 48106-1346

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
ABSTRACT

Visual pattern recognition is formalized in terms of an information theoretical algorithm which operates upon a contrast function of pattern data. The contrast function facilitates discrimination ability with a slight decrease in decision time. The information theoretical algorithm computes a measure of the additional information which an unknown pattern provides about each of a set of statistically established pattern classes.

An IBM 1620 computer was used to simulate a classification system based upon these concepts. One hundred per cent of the learned patterns and forty per cent of unlearned random patterns were able to be recognized correctly on the basis of only twelve learned samples of each of thirty-five types of pattern.
ACKNOWLEDGEMENTS

The author is sincerely grateful for the invaluable guidance of Dr. S.N. Kalra, who supervised this research. He wishes also to extend his appreciation to Dr. P.A.V. Thomas and Dr. E. Channer for their direction in computer software techniques, Mr. S. Bayzik who aided in program design and debugging and Mrs. M. Dupuis for her patience and generosity. A special mention is made of Mr. D.C. MacKenzie who assisted in the typing of this manuscript.

This thesis was sponsored by the National Research Council of Canada. The author is indebted to them for their financial support.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>I  INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>A. Related Problems</td>
<td>2</td>
</tr>
<tr>
<td>B. Outline of the Procedure</td>
<td>4</td>
</tr>
<tr>
<td>II  THEORY</td>
<td>8</td>
</tr>
<tr>
<td>A. The Effects of Quantizing Patterns In Terms of their Internal Contrast Structure</td>
<td>8</td>
</tr>
<tr>
<td>1. Contrast Coding</td>
<td>8</td>
</tr>
<tr>
<td>2. The M : N Transformation</td>
<td>13</td>
</tr>
<tr>
<td>B. An Information Theoretical Decision Function</td>
<td>18</td>
</tr>
<tr>
<td>1. Decision Function - Model 1</td>
<td>18</td>
</tr>
<tr>
<td>2. Decision Function - Model 2</td>
<td>20</td>
</tr>
<tr>
<td>III EXPERIMENTAL PROCEDURE</td>
<td>22</td>
</tr>
<tr>
<td>A. Data Acquisition</td>
<td>22</td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
## LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>M Matrix Grouping</td>
<td>15</td>
</tr>
<tr>
<td>II</td>
<td>All Possible M and N Matrices</td>
<td>74</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Signal/Contrast Valued Matrices</td>
<td>11</td>
</tr>
<tr>
<td>2.</td>
<td>Typical Sample Patterns</td>
<td>23</td>
</tr>
<tr>
<td>3.</td>
<td>Recognition Unit Block Diagram</td>
<td>27</td>
</tr>
<tr>
<td>4. - 8.</td>
<td>Identification Trials (mod 1) - (learned)</td>
<td>30 - 34</td>
</tr>
<tr>
<td>9. - 13.</td>
<td>Identification Trials (mod 2) - (learned)</td>
<td>35 - 39</td>
</tr>
<tr>
<td>14. - 27.</td>
<td>Identification Trials (mod 1) - (unlearned)</td>
<td>40 - 53</td>
</tr>
</tbody>
</table>
I INTRODUCTION

Recent advances in the scientific technology of digital computer hardware and the coincident development of computer software have enabled researchers to simulate human recognitory processes which would otherwise most certainly have remained philosophical enigmas. As a result, many new mathematical expressions have evolved concerning visual, aural and other neurological concept formation and detection systems. This paper is principally involved in developing a visual pattern classification algorithm based upon the formalized concepts of information theory and upon the view that pattern content is more fundamentally represented by internal contrasts than by an absolute signal structure.

The design of an automaton capable of classifying random patterns is in answer to a demand for faster and more efficient data processing. Medical pictorial diagnosis, automatic radar pattern detection and remote decisive vehicular control (by environmental pattern analysis) are only some of the eventual goals of such an automaton. Before their realization, however, several outstanding problems are yet to be solved.
A. Related Problems

Of chief concern are coding and identification times and memory capacity requirements. A real time system demands an almost instantaneous machine decision capability in order to cope with sequential events. An off line computer, on the other hand, might require a memory of extraordinarily large capacity to contain the background statistics necessary to produce a correct decision. Fortunately, these requirements are being fulfilled, as was mentioned, by a rapid advancement of computer technology.

Noise is indeed a salient problem. In the sense of a loss of data through quantization, noise is unavoidable; yet, the variability of position of pattern data in the field of view is noise of a type which can almost be completely eliminated by proper normalization techniques.\(^1\) Data which is conjunctive of every pattern in the system vocabulary is noise which may easily be programmed out of the decision process (at the expense of off line computing time). Much background and additive interference can possibly be eliminated by operating on a differential function of the pattern, a principle set forth in this thesis. There is often present, too, completely irrelevant data (random or otherwise) which periodically creates unwanted ambiguities. These may be discontinuities in otherwise continuous pattern structures, smudges, fogginess due to an improper focus, glare arising from intensity saturation and
otherwise undesirable effects. A particular pattern so adversely affected is often inevitably unrecognizable; but, such effects on the overall statistical impression of a large sample of any particular class of pattern is usually negligible.

Random pattern orientation in the field of view presents an interesting challenge to any recognition scheme. This problem is perhaps the one which is most avoided by pattern recognition researchers. An obvious solution would seem to be an inventive transformation which would render the immediately operative data invariant under translation, rotation and magnification. Unfortunately this problem is more complex than it appears and the result is that no such efficient transformation has yet been discovered.

If the recognition system is an adaptive one, the question of degeneration arises. The addition of recognized random inputs to the overall statistics should increase the discriminatory ability of the decision algorithm.

In a statistically based classification system, one must decide when he has acquired enough initial data about the patterns to be discriminated. He is bounded on one hand by the inefficiency of too much data, and on the other by the cost of ambiguous results due to an insufficient statistical sample. Again, in a digital system, a decision must be made as to the coarseness of segmentation of patterns into elements, and the degree of quantization of the signal
energy corresponding to each element. These decisions would not arise in an analogue system; but, the versatility of a digital machine (with the exception of an analogue interface at the input) in both function and memory capacity far outweighs the present speed advantage of analogue recognition systems.

For a more detailed analysis of these and other problems related to pattern recognition research, and how attempts have been made to alleviate them, the reader is invited to refer to a synopsis by Spinard\(^2\) and also to (3).

B. Outline of the Procedure

The classification algorithm under investigation is basically statistical in nature and therefore an amount of prelearning or probabilistic sampling is required. Only after obtaining sufficient background knowledge about a select group of pattern classes can meaningful recognition be accomplished. Hence, there is a need to organize the structural content of patterns into a form most suitable to the techniques employed by the classification algorithm. This involves segmenting the pattern continuum into the minimum number of discrete surface elements which will facilitate an efficient discrimination among the entire set of pattern classes. The number of such elements is critical since there is a proportionate loss of data due to the averaging technique used to measure element signal strength (as in a photo-diode).
Any gradients of light energy over the surface of a single element are therefore extinguished. The resultant average energy of each element is then quantized into one of a predetermined number of signal levels. Thus, patterns are represented by an ordered array of elements and the value of each element belongs to a bounded discrete signal space.

It is important to note that a consistent ordering of these elements be maintained to preserve the very 'essence' of pattern.

To complete the coding process, the above array requires an additional transformation. The reasons for and the effects of this transformation will be discussed in Chapter II. It is sufficient to mention here that a pattern is not merely the communication of a group of parallel signals. Rather, it is meant to convey some concept described by the interrelationship of light energies over the entire pattern field. To realize this idea in terms of the signal valued array, an arbitrary element is first chosen from the array. The value of each element following this one is subtracted in turn from the value of the selected element. The manner in which the remaining elements of the array are selected is also arbitrary. It is important to note, however, that once the starting point and the scheme for exhausting the rest of the elements of the array have been
decided, that scheme and no other must absolutely define the transformation. What results is an ordered sequence of contrast valued digits. This process gives rise to a new bounded and discrete signal space. The elements of that space shall be the basis of frequency distributions characterizing individual pattern classes.

If the pattern were segmented into \( x \) elements, then the corresponding contrast valued sequence would have \( x - 1 \) digits. When accumulating the statistics of a specific class of pattern, each of the \( x - 1 \) digits is treated independently. The number of occurrences (over a wide sample of that class) of each value in the contrast valued signal space is then tabulated first for one of the \( x - 1 \) digits, and then for another, and so on until \( x - 1 \) distributions are formed. Such statistics are accumulated from known patterns and each group of distributions is accordingly labelled.

A very similar process is followed in the recognition of an unknown pattern. After quantizing and transforming the pattern into a contrast value sequence of digits the occurrence of the contrast value in each of the \( x - 1 \) digit positions is compared to the frequency of occurrence of the same contrast value in the corresponding digit position of a particular prelearned pattern class. A measure of this correspondence within the same digit position
is computed, one for each of the \( x - 1 \) digit positions. The measure is derived from Information Theory and will be discussed in detail in Chapter II.

Finally, the \( x - 1 \) measures are summed to give an indication of the total additional information which the unknown sample provides about the particular prelearned pattern class. A weighting function may be included in the above summation. Similar totals are computed for each of the remaining prelearned pattern classes and the unknown sample is identified as the pattern class which yields the highest total measure.

This recognition system is adaptive in the sense that the statistics of an identified pattern may be added to those of the identified class.
II THEORY

A. The Effects of Quantizing Patterns In Terms of their Internal Contrast Structure

The content of an optical pattern is perhaps best described by the variations of light energy over the visual plane. These light energy gradients seem more closely related to the concepts which a pattern strives to communicate. It is logical, therefore, to code patterns in a manner which emphasizes their contrast structure.

1. Contrast Coding

Let a pattern be segmented into an array of a X b discrete elements. In addition, let the signal values which an element may assume be limited to a discrete set, S. The elements of S shall consist of zero (0) and the first q positive real integers.

\[ S: (s_0, s_1, \ldots, s_q) \]

Now generate a set, C, of contrast values from the elements of the set S, according to the law,
The elements of \( \mathbb{C} \), therefore, include zero (0) and the first \( q \) positive and negative real integers.

\[
\mathbb{C} : ( c_{-q}, \ldots, c_0, \ldots, c_q )
\]

Just as a pattern can be represented by a matrix of discrete signal values, it can also be represented by a different matrix of contrast values. If the former matrix is denoted as \( \mathbf{M} \) and the latter is denoted as \( \mathbf{N} \), then the matrix \( \mathbf{N} \) may be generated from the matrix \( \mathbf{M} \) by the following equation,

\[
(n^u, v) = (m^u, v) - (m_i, j)
\]

\[
u, v = 1, 1; \ldots, a, b
\]

\[
i, j = 1, 1; \ldots, a, b
\]

\( n^u, v \) are the elements of the matrix, \( \mathbf{N} \), while \( m^u, v \) and \( m_i, j \) are the elements of the matrix \( \mathbf{M} \). Matrix \( \mathbf{N} \) therefore has \((a \times b)^2 \) discrete contrast valued elements. \((n^u, v)\) represents...
the contrast between the value of the element of row \( u \) and column \( v \) of matrix \( M \), and the value of the element of row \( i \) and column \( j \) of matrix \( M \). Each elemental value, \( (n_{i,j}) \), of matrix \( M \) generates a contrast value between itself and each of the other elements of \( M \) forming the \( i,j \)th row of matrix \( N \). See Figure 1, for a detailed representation of a generalized signal valued matrix and the resultant generated contrast valued matrix.

The following relationships are given for the \( N \) matrix of Figure 1. (See the example in Appendix B)

\[
(n_{u,v}^i,j) = -(n_{u,v}^i,j) \tag{3}
\]

\[
(n_{u,v}^i,j) = (n_{u,v}^{i,j}) = 0 \tag{4}
\]

\( i, j = 1, 1; \ldots \ldots a, b \)

\( u, v = 1, 1; \ldots \ldots a, b \)

and

\[
(n_{u,v}^i,j) = (n_{u,v}^{i,j}) - (n_{u,v}^{(i,j)-1}) \tag{5}
\]

\( i, j = 1, 2; \ldots \ldots (a, b)-1 \)

\( u, v = (i, j)+1; \ldots \ldots a, b \)

(ie. elements enclosed by the diagonal and the first row of \( N \))

Note: \((i, j)-1\) denotes the row preceding row \( i, j \) in matrix \( N \).

\((i, j)+1\) denotes the row following row \( i, j \) in matrix \( N \).

An example of equation (5) may be found in Appendix B.
Figure 1. Signal Valued Matrix

\[
\begin{array}{ccccccc}
m_{1,1} & m_{1,2} & \cdots & m_{1,b} \\
m_{2,1} & & & & \cdots & m_{2,b} \\
& & & & & & M \\
& & & & & & \\
m_{a,1} & m_{a,2} & \cdots & m_{a,b} \\
\end{array}
\]

Contrast Valued Matrix (generated from M)

\[
\begin{array}{ccccccc}
n_{1,1} & n_{1,2} & \cdots & n_{1,b} \\
n_{1,1} & n_{1,2} & \cdots & n_{1,b} & \cdots & n_{1,b} \\
& & & & & & N \\
& & & & & & \\
n_{a,1} & n_{a,2} & \cdots & n_{a,b} \\
& & & & & & \\
& & & & & & \\
& & & & & & \\
n_{a,1} & n_{a,2} & \cdots & n_{a,b} \\
\end{array}
\]
Equation (4) indicates that the diagonal elements of matrix N are always zero and therefore are insignificant in representing the pattern from which the matrix was derived. Equation (3) shows that the elements below the diagonal may be generated by those above the diagonal, and hence only one of these groups of elements is significant in representing the original pattern. From equation (5) it can be shown that the elements above the diagonal (excluding those in the first row of matrix N) may be generated by the aXb -1 elements,

\[ n^1,2; \ldots \ldots \ldots n^a,b \]  

(6)

Hence, the N matrix in Figure 1. can be uniquely specified by the elements, (6).

Suppose that a pattern is coded by the contrast valued sequence of elements, (6). Its N sequence (6) has one less element to consider in the decision process than does the signal valued matrix. An M matrix element could have q + 1 possible signal values; an N sequence digit position may assume any one of 2q + 1 values. That is to say, the 2q + 1 elements of the set, C, represent the contrast valued signal space of an N sequence, while the q + 1 elements of the set, S, represent the signal valued space of an M matrix. This more than doubles the base length of
each frequency distribution characterizing a pattern class. The advantage is that a pattern in contrast digit form has a greater dimensionality \((2q + 1) - (q + 1) = q\) with less time required for classification than in M matrix form. A disadvantage is an increase in the memory capacity required by pattern class frequency distributions. The greater dimensionality is in agreement with the idea that a contrast coded pattern more directly communicates concepts than does a signal coded pattern.

The concept-dimensionality analogy is not to be misinterpreted as a mathematical definition of visual pattern concepts. Nor does it imply that more 'information' can be drawn from a pattern by contrast coding. It is merely a technique which is designed to facilitate pattern discrimination and to meet the more demanding requirements of high speed data processing.

2. **The M : N Transformation**

The number of elements of set \(S\) is \(q + 1\). Then at most \((q + 1)^a^b\) patterns may be represented in M matrix form. Although the same number of patterns are representable in N sequence form, the \((q + 1)^a^b\) possible sequences that may be generated are not unique. That is, several different patterns in M matrix form are transformed into identical N sequences. In that sense contrast coding appears to be disadvantageous; yet, it will be demonstrated...
to be the contrary.

To consider the similarity between patterns which are transformed into the same sequence, first examine the M matrix form in terms of its quantized signal values. Assume that the order of M matrices has been fixed at (a, b) and that the set S has been determined to contain only the elements,

\[ s_0', \ldots, s_q \]

The following argument shall indicate how the totality of M matrices can be partitioned into groups, and how one can determine from these groups what M matrices are transformed into identical N sequences.

All of the discrete signal values which make up a particular matrix, \( M_{Sp} \), can be represented in a set, \( S_{Sp} \), which is a subset of the set, S. If the largest value, \( s_q' \), of S were missing from the set \( S_{Sp} \), then a unit matrix, \( M_1 \) (of order \( (a, b) \)), could be added to \( M_{Sp} \) to obtain a new matrix, \( M_{Sp+1} \).

\[
M_{Sp} + M_1 = M_{Sp+1}
\]  

(7)

If 'p' denotes the type of matrix that can be added, then equation (7) can be written more specifically as,
\[ M_{s1} + M_1 = M_{s1+1} \] (8)

The set, \( S_{s1+1} \), corresponding to the new matrix, \( M_{s1+1} \), would also be a subset of the set, \( S \). The number of possible matrices (of the totality of \( M \) matrices) whose set \( S_{sp} \) does not contain \( s_q \) is \( q^{ab} \). Similarly there are \( (q - 1)^{ab} \) possible \( M \) matrices whose set, \( S_{sp} \), does not contain \( s_q^* s_{q-1}^* \ldots s_{q-(i-1)}^* \) (\(* \) denotes the logical 'and').

To these matrices the duo matrix, \( M_2 \), could be added to obtain a new matrix, \( M_{s2+2} \), whose set, \( S_{s2+2} \), is a subset of the set, \( S \). In general, if \( S_{sp} \) \( (p = 1, \ldots, q) \) does not contain \( s_q^* s_{q-1}^* \ldots s_{q-(i-1)}^* \), then the addition of \( M_i \) to \( M_{sp} \),

\[ M_{sp} + M_i = M_{sp+i} \] (9)

renders a matrix whose set, \( S_{sp+i} \), is a subset of the set, \( S \).

These arguments are condensed in Table I.

**Table I**  M Matrix Grouping

<table>
<thead>
<tr>
<th>( S_{sp} ) Excludes</th>
<th>Distance</th>
<th>No. of Matrices</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>no element</td>
<td>0</td>
<td>((q + 1)^{ab})</td>
<td>(Q_{q+1})</td>
</tr>
<tr>
<td>( s_q )</td>
<td>1</td>
<td>(q^{ab})</td>
<td>(Q_q)</td>
</tr>
<tr>
<td>( s_q^* s_{q-1}^* )</td>
<td>2</td>
<td>((q - 1)^{ab})</td>
<td>(Q_{q-1})</td>
</tr>
<tr>
<td>\ldots...</td>
<td>\ldots...</td>
<td>\ldots...</td>
<td>\ldots...</td>
</tr>
<tr>
<td>( s_q^* s_{q-1}^* \ldots s_0 )</td>
<td>q</td>
<td>1</td>
<td>(Q_1)</td>
</tr>
</tbody>
</table>
From Table I the following is deduced. There are 
$q + 1$ groupings of $M$ matrices where in general each member
of group $Q_i$ is distant by $(q - (i-1))$ from one member of group
$Q_{q+1}$ and is 'distant' by $(q - (i-1)) - 1$ from one member of group
$Q_q$ and so on. Each member of group $Q_i$ is a member of all
groups of higher order. The significance of this partitioning of
the totality of $M$ matrices into such groups is to show that a mem-
ber of group $Q_i$ and one member of each group 'distant' from
the member of $Q_i$ generate the same $N$ sequence. For example,
The row matrices,

$$(1, 2, 0, 1); (2, 3, 1, 2); \text{and} (4, 5, 3, 4)$$

all generate the $N$ sequence,

$$(-1, 1, 0),$$

when $s_q = 5$ and when the order of $M$ matrices is $(1, 4)$.

Since every member of group $Q_q$ is distant from
$Q_{q+1}$, then the number of unique $N$ sequences which can be gen-
erated by the totality of $M$ matrices is, from Table I,

$$(q + 1)^{ab} - q^{ab}$$
Therefore, the number of redundant N sequences which would be generated by the \((q + 1)^{ab}\) M matrices is,

\[ R = q^{ab} \]  

(10)

M matrices of order (1, 4) with elements under set S: (0, 1, 2) are given in Appendix C in order to exemplify equation (10).

It has been shown, then, that in transforming M matrices to N sequences according to an absolute rule there is no discrimination between patterns which are 'distant' from one another in the sense of Table I. What is lost in the transformation is data of the form represented by matrices \(M_i\), where \(i\) denotes the distance. In the author's opinion there is no difference in the visual pattern content between M matrices which are 'distant' by any amount.

There is actually a gain, therefore, by avoiding an unnecessary ambiguity which 'distant' patterns would create among the prelearned class frequency distributions when derived from M matrices. Furthermore, the representation of patterns by contrast valued N sequences should greatly improve the discriminatory ability of the proposed classification system.
B. An Information Theoretical Decision Function

In the proposed classification system, visual patterns are to be represented by a sequence of contrast valued digits. The sequence is ordered and of fixed length and the digits take on discrete values belonging to the finite set \( C \). During the learning of a particular class of pattern, frequency distributions are accumulated over the set \( C \) for each digit position of the sequence. In identifying a random pattern, therefore, some function is desired of the correlation between the unknown digit values and the corresponding digit value frequencies of the known pattern classes.

Two such functions have been proposed. One treats each digit of the contrast coded sequence with equal weight, while the other stresses values which are more highly correlated with corresponding class digit value frequencies.

1. Decision Function - Model 1

Consider the pattern classes which the system is to recognize as the set of events,

\[
Y = \{ y_i \} \quad (11)
\]

\[ i = 1, \ldots, n \]

Let the elements of set \( C \) form a set of events,
\[ X = (x_j) \]  
\[ j = 1, \ldots, m ; \quad m = 2q + 1 \]  

Denote the number of samples used in the learning of a particular class \( y_i \) as \( N_i \). Let the events \( x_j \) be superscripted as, \( x^k_j \), where \( k \) refers to a particular digit position of the \( N \) sequence. Recall that visual patterns are in the form of contrast valued \( N \) sequences. The range of \( k \) is, 

\[ k = 1, \ldots, ab-1 \]

Considering an individual digit position \( k \), the additional information which the \( k^{th} \) digit of an unknown pattern provides about the \( k^{th} \) digit of class \( y_i \) is given by \((4)\),

\[ I_k(y_i/x_j) = \log \left( \frac{P(y_i|x_j)}{P(y_i)} \right) \]  
\[ (13) \]

But,

\[ I_k(y_i/x_j) = I_k(x^k_j/y_i) \]

and

\[ I_k(x^k_j/y_i) = \log \left( \frac{P(x^k_j/y_i)}{P(x^k_j)} \right) \]  
\[ (14) \]

Therefore

\[ I_k(y_i/x^k_j) = \log \left( \frac{P(x^k_j/y_i)}{P(x^k_j)} \right) \]  
\[ (15) \]

\( P(x^k_j/y_i) \) is the conditional probability that digit position \( k \) of pattern class \( y_i \) has contrast value \( x_j \). \( P(x^k_j) \) is the probability that digit position \( k \) has contrast value \( x_j \) in any class of the set \( Y \). If the frequencies corresponding to \( P(x^k_j/y_i) \) and \( P(x^k_j) \) are \( F(x^k_j/y_i) \) and \( F(x^k_j) \).
respectively, then,

\[ \Pr(x_j^k/y_i) = \left( \frac{\Pr(x_j^k/y_i)}{N_i} \right) \]  

(16)

and

\[ \Pr(x_j^k) = \frac{\Pr(x_j^k)}{\sum_{i=1}^{n} (N_i)} \]  

(17)

Therefore,

\[ I_k(y_i/x_j^k) = \log \left[ \frac{\Pr(x_j^k/y_i)/N_i}{\Pr(x_j^k)/(\sum_{i=1}^{n} N_i)} \right] \]  

(18)

Since \( \Pr(x_j^k/y_i), \Pr(x_j^k) \) and \( N_i (i = 1, \ldots, n) \) are all known quantities, then, \( I_k(y_i/x_j^k) \) can be computed directly.

Because each digit value is to be considered of equal importance in the overall decision, then the total additional information which an unknown pattern provides about class \( y_i \), is,

\[ I(y_i/x_j) = \sum_{k=1}^{ab-1} I_k(y_i/x_j^k) \]  

(19)

2. Decision Function - Model 2

A weighting factor, \( \Pr(x_j^k/y_i) \), is included in the decision function represented by equation (19), in order to emphasize the larger values of \( I_k(y_i/x_j^k) \) and to de-emphasize the smaller ones. The decision function for a single digit is therefore,
\[ I^*_k(y_i/x_j^k) = \frac{F_r(x_j^k/y_i)}{\log \left( \frac{F_r(x_j^k/y_i)}{N_i} \right) \sum_{i=1}^{n} \frac{F_r(x_j^k)}{\sum N_i}} \] 

The decision function for the total number of digits in the N sequence is,

\[ I^*(y_i/x_j) = \sum_{k=1}^{ab-1} I^*_k(y_i/x_j^k) \]
III EXPERIMENTAL PROCEDURE

A. Data Acquisition

1. Source of Patterns

Although the proposed recognition system is sufficiently
general in theory to handle any type of visual pattern, it is
limited by the unavailability of accurate normalization techniques.
It would have difficulty at this stage in discriminating between
cyclones and hurricanes, for example. Therefore the twenty-six
capital letters of the alphabet and the first nine cardinal numerals
were used to test the classification system.

The patterns were hand written in order to create a
wide variation in their spatial form. A coarse pen was used
so that a medium proportion of the field of view was covered.
Note in Figure 2. that the field of view is a one inch square
grid of twenty-five segments. Figure 2. gives an example of
the type of each pattern used (excluding '10').

2. Normalizing The Pattern

Each sample hand written character was focussed within
a paper based grid ten units high and eight units wide. The choice
of eighty elements, though arbitrary, was thought to be sufficient
Figure 2. Typical Sample Patterns
for an identification of all thirty-five characters. A rectangular
shape was chosen because a large portion of the sample patterns
tended to be greater in height than in width. The pattern was
positioned so that it was tangent to one pair of opposite grid
borders. Then the pattern was centred in the direction of the
other pair of borders.

The percentage of each element covered by the projected
caracter was taken as the signal value of that element. Eighty
such signal values were computed for each pattern.

3. Signal and Contrast Coding

The eighty pattern signal values were first quantized
into discrete values ranging from zero (0) to twenty (20). The
choice of that number of contrast values was again an arbitrary
one. By subtracting from the value of the first element the
value of each of the following elements, row by row, a contrast
valued sequence of seventy-nine digits was formed. This was
in accordance with equation (2). Therefore, each of the seventy-
nine digits had a discrete contrast value in the range ( -20 to +20 ).

For a sample of a signal and contrast value coded
pattern of a different size and quantization, see Appendix B. The
pattern of Appendix B would be represented by the last eight
digits in the first row of matrix N.
B. Computer Simulated Classification Unit

All of the theoretical techniques derived in Chapter II were implemented in several computer programs. It would be too tedious to include all of the program listings and their flow charts here. The major portion of these were to simulate the learning phase of the recognition system in lump form. The first program was written to read all of the data (consisting of 12 samples of each of 35 characters) from prepunched cards into a core memory. It was to pack the data, code it by character and transfer the condensed block to disk storage. Another program was designed to delimit the data already stored on disk, into fields, by flagging the leftmost digit of each value; this enabled the data to be selected and processed individually. The third program quantized the signal data. The fourth generated contrast values for all of the character samples. The fifth program generated frequency distribution tables from the contrast valued data. See Appendix D for a sample of how the distributions were set up in memory and for a comparison of the contrast valued data structure of the thirty-five characters.

The sixth program summed the thirty-five frequency tables into a single table. In the thirty-five tables were stored the values, \( Fr(x_j^k/y_i) \), while in the sum table the unconditional frequencies, \( Fr(x_j^k) \), were to be found. The purpose of the sum table
was to greatly decrease recognition decision time.

The size of these tables necessitated that they be stored in a secondary memory area and be called into core memory only when required. Thus, much time was consumed in seeking and transferring these tables between primary and secondary storage areas.

The machine used was an IBM 1620\textsubscript{II} computer with 40 K core storage capacity and 1311 disk drive assembly. The programs were written in SPS language and compiled into machine language by a Monitor II System assembler. A basic layout of the recognition unit in terms of the IBM 1620 computer is found in Figure 3.

The final computer program, PCP (Pattern Classification Program - also written in SPS) was designed to handle the data of one pattern at a time and to perform all of the functions of the first four programs. In addition, the decision models were implemented and recognition of an unknown pattern could be attempted by using either decision function 1 (equation (19)) or decision function 2 (equation (21)). It should be noted that the natural logarithm was used in the implementation of both decision functions. PCP was adaptive in the sense that the data of an identified pattern could be added to the system frequency tables and other parameters could be updated to account for the added data. With this facility it could have been determined whether
Figure 3. Recognition Unit Block Diagram (as simulated on the 1620II computer)
or not the classification system were degenerative; however, this was not tried.

Of the total number of sample patterns tested by PCP, 140 learned patterns were tested by function model 1, 105 learned patterns were tested by function model 2 and thirty-five unlearned samples were tested by function model 1. No unlearned samples were tested by function model 2.

A complete listing of the PCP program may be found in Appendix A. The comments adjacent to the listing provide an explanation of the functional flow of the program.

C. **Effective Statistical Zero**

Since the number of samples used to learn the pattern classes was very small and due to the digital nature of operation, many of the frequencies used by the decision algorithms were of value zero \(0\). \(\log(0)\) being indeterminate, an effective zero value had to be established. It was decided arbitrarily that the occurrence of a zero represented the least possible information and, therefore, a 'zero' was chosen to give an information measure which was slightly more negative than the value that could have resulted in the most negative case.
IV RESULTS

Of the prelearned sample patterns, one hundred per cent were correctly identified by the function model 1. Only seventy-two per cent of the prelearned patterns were correctly identified by the function model 2. Forty per cent of the unknown patterns were correctly identified by the function model 1; function model 2 was not applied to the unlearned samples.

The time required for the complete identification of a single pattern sample was approximately three minutes. This time could have been substantially reduced by employing a faster computer.

Figures 4. - 8. represent typical results using function model 1 on learned samples. Figures 9. - 13. represent the the information measures obtained by function model 2 from the learned samples. Figures 14. - 27. are typical results obtained by applying function model 1 to unlearned patterns.
Figure 4. Identification Trial (mod 1) - A (learned)
Figure 5. Identification Trial (mod 1) - B (learned)
Figure 6. Identification Trial (mod l) - C (learned)
Figure 7. Identification Trial (mod 1) - D (learned)
Figure 8. Identification Trial (mod 1) - E (learned)
Figure 9. Identification Trial (mod 2) - A (learned)
Figure 10. Identification Trial (mod 2) - B (learned)
Figure 11. Identification Trial (mod 2) - C (learned)
Figure 12. Identification Trial (mod 2) - D (learned)
Figure 13. Identification Trial (mod 2) - E (learned)
Figure 14. Identification Trial (mod 1) - A (unlearned)
Figure 15. Identification Trial (mod 1) - B (unlearned)

[Graph showing pattern class information measure in natural units]
Figure 16. Identification Trial (mod 1) - C (unlearned)
Figure 17. Identification Trial (mod 1) - E (unlearned)
Figure 18. Identification Trial (mod 1) - F (unlearned)
Figure 19. Identification Trial (mod 1) - G (unlearned)
Figure 20. Identification Trial (mod 1) - H (unlearned)
Figure 21. Identification Trial (mod 1) - I (unlearned)
Figure 22. Identification Trial (mod 1) - J (unlearned)
Figure 23. Identification Trial (mod 1) - K (unlearned)
Figure 24. Identification Trial (mod 1) - L (unlearned)
Figure 25. Identification Trial (mod 1) - M (unlearned)
Figure 27. Identification Trial (mod 1) - O (unlearned)
V CONCLUSIONS AND RECOMMENDATIONS

Function model 1 is definitely a successful algorithm. Function model 2, however, is inadequate in its ability to weight the component measures of function model 1. A proper weighting function has yet to be formulated.

The advantages of the contrast transformation are not fully utilized in the simulation program (PCP). The decision time is shorter than it would be for signal coded patterns; but, the small number of samples as well as the method of data collection limit the opportunity for the contrast transformation to be fully effective. It would be interesting to compare contrast coding and signal coding based upon a larger sample of patterns.

The problem of an effective statistical zero has by no means been solved. Further study is required to find out its effect upon a digital pattern recognition system.

The choice of grid size and the degree of signal quantization are variables that should be investigated further.
BIBLIOGRAPHY


APPENDIX A

PCP

PATTERN CLASSIFICATION PROGRAM

An SPS computer program originally written for an IBM 1620 MK II machine with,

MONITOR II RESIDENT SYSTEM
INDEX REGISTERS
40K CORE MEMORY
1311 DISK STORAGE DRIVES
1622 CARD READ PUNCH

The program requires 25,630 core positions and is operated in conjunction with 36 data tables stored on disk pack memory (satellite - drive #1)
* Declarative Statements

Output  Das  26
        Dac  1,0
A1    Das  80
        Dac  1,0
A2    Das  80
        Dac  1,0
A3    Das  80
        Dac  1,0
A4    Das  80
        Dac  1,0
A5    Das  80
        Dac  1,0
A6    Das  80,0
        Dac  1,0,0
A7    Das  80,0
        Dac  1,0
A8    Das  80
        Dac  1,0
A9    Das  80
        Dac  1,0
A10   Das  80
        Dac  1,0
A11   Das  80
        Dac  1,0
A12   Das  80
        Dac  1,0
Blank Dac  50,
        Dsc  50,000000000000000
        Dsc  11,000000000000000
Note  Das  80,0
        Dac  1,0
None   Das  80,0
        Dac  1,0
        Dac  1,0
Mod    Dac  38,Switch 1 - on for Mod1, off for Mod2
       Note  Dac  44,Switch 3 - on to adjust the System Tables
       Calmes Dac  36,Switch 4 - on for Further Samples
       Badnum Dac  15, Zero Numerator
       Badden Dac  17, Zero Denomenator
       Inf    Dac  1,1( ) = @
       Two    Dac  3, @
       Dec    Dac  2, @
       Exp    Dac  2,E@
       Plus   Dac  2,+@
       Empty  Dac  27,
       Item   Dac  24,System Tables Adjusted

*
<table>
<thead>
<tr>
<th>Stop</th>
<th>Dac</th>
<th>25, Identification Complete. @</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cardin</td>
<td>Dss</td>
<td>80,</td>
</tr>
<tr>
<td>Shrink</td>
<td>Dss</td>
<td>240,</td>
</tr>
<tr>
<td></td>
<td>Area</td>
<td>Dss 160,</td>
</tr>
<tr>
<td></td>
<td>Dummy</td>
<td>Dda 0,0,0,0</td>
</tr>
<tr>
<td>Tabar</td>
<td>Dss</td>
<td>10000,</td>
</tr>
<tr>
<td></td>
<td>Flarea</td>
<td>Dss 790,</td>
</tr>
<tr>
<td></td>
<td>Flarl</td>
<td>Dss 790,</td>
</tr>
<tr>
<td></td>
<td>Sort</td>
<td>Dss 525,</td>
</tr>
<tr>
<td></td>
<td>Out</td>
<td>Dss 350,</td>
</tr>
<tr>
<td></td>
<td>Mul</td>
<td>Ds 5,</td>
</tr>
<tr>
<td></td>
<td>Result</td>
<td>Ds 5</td>
</tr>
<tr>
<td></td>
<td>Prodct</td>
<td>Ds 5</td>
</tr>
<tr>
<td></td>
<td>Quot</td>
<td>Ds 2</td>
</tr>
<tr>
<td></td>
<td>Rem</td>
<td>Ds 2,</td>
</tr>
<tr>
<td></td>
<td>Sub</td>
<td>Ds 2,</td>
</tr>
<tr>
<td></td>
<td>Zero</td>
<td>Dc 8,0,0</td>
</tr>
<tr>
<td></td>
<td>Zr</td>
<td>Dc 7,0</td>
</tr>
<tr>
<td></td>
<td>Fltzer</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Cntrst</td>
<td>Ds 2,</td>
</tr>
<tr>
<td></td>
<td>Smtbcn</td>
<td>Ds 2,</td>
</tr>
<tr>
<td></td>
<td>Idl</td>
<td>Ds 1,</td>
</tr>
<tr>
<td></td>
<td>Id2</td>
<td>Ds 1</td>
</tr>
<tr>
<td></td>
<td>Setind</td>
<td>Ds 1</td>
</tr>
<tr>
<td></td>
<td>Compar</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Sum</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Smallf</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Denu</td>
<td>Ds 2,</td>
</tr>
<tr>
<td></td>
<td>Denomu</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Denl</td>
<td>Ds 3,</td>
</tr>
<tr>
<td></td>
<td>Denoml</td>
<td>Ds 10,</td>
</tr>
<tr>
<td></td>
<td>Patcls</td>
<td>Ds 5,</td>
</tr>
<tr>
<td></td>
<td>Hold</td>
<td>Ds 15,</td>
</tr>
<tr>
<td></td>
<td>Flot</td>
<td>Ds 11,</td>
</tr>
<tr>
<td></td>
<td>Bang</td>
<td>Ds 10</td>
</tr>
<tr>
<td></td>
<td>Bangf</td>
<td>Ds 10</td>
</tr>
<tr>
<td></td>
<td>Tempf</td>
<td>Ds 10</td>
</tr>
<tr>
<td></td>
<td>Wait</td>
<td>Ds 10</td>
</tr>
<tr>
<td></td>
<td>Save</td>
<td>Ds 10</td>
</tr>
</tbody>
</table>

* Card Input Area
* Reduced Input Data
* Quantized Data Area
* Frequency Table Core Area
* Conditional Freq Area
* Unconditional Freq Area
* Sort Area
* Information Area
* Product Areas
* Remainder
* Difference Area
* Fixed Point Zeros
* Floating Pt Zero
* Contrast Data Areas
* Indicators
* Compare Area
* Accumulator
* Effective Statistical Zero
* Number of Samples / Class
* Total Number of Samples
* Identified Class Coded Area
* Temporary Locations

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Sap Ds 10
Can Ds 5
Still Ds 5
Tmp Ds 3
Sav Ds 2
Tempi Ds 2
Holdl Ds 2
Table Dsa Tabar

* Disk Control Flds

Tab Dda 3, 0, 65, Tabar
Dc 1,@
Sumtab Dda 3, 09600, 98, Tabar
Dc 1,@
A Dc 5, 02456,
Three Dc 2, 35
Seven Dc 2, 79

* Constants

Input Unknown Data

Start H
Cf Setind,,
Tdm Setind, 0
Rcty Waty Mod
Rcty H
Bnc1 Noflag
Sf Setind,,

* Reset Mod-2 Indr

Noflag B
Bsba *=12
Tf Denu, Zr-5,,
Tf Denl, Zr-4
Tfm Tab+5, 06000

* Zero Sample Totls

Count Sk Tab
Rdn Tab
Cdn Tab
A Denl, Tabar+3,,
Am Tab+5,100,,
Cm Tab+5,09500,,
Bnz Count
Blxm *=12, -240(Al)
Racd Cardin,,
Racd Cardin
Racd Cardin

* Tally T otl Samples
From Disk-Store d
Freq Tables

Rut ine Td Shrink+240(Al), Cardin+028,,
Td Shrink+241(Al), Cardin+032

Read Data Cards

Compact Input Dat
Td Shrink+242(Al), Cardin+034
Td Shrink+243(Al), Cardin+040
Td Shrink+244(Al), Cardin+044
Td Shrink+245(Al), Cardin+046
Td Shrink+246(Al), Cardin+052
Td Shrink+247(Al), Cardin+056
Td Shrink+248(Al), Cardin+058
Td Shrink+249(Al), Cardin+064
Td Shrink+250(Al), Cardin+068
Td Shrink+251(Al), Cardin+070
Td Shrink+252(Al), Cardin+076
Td Shrink+253(Al), Cardin+080
Td Shrink+254(Al), Cardin+082
Td Shrink+255(Al), Cardin+088
Td Shrink+256(Al), Cardin+092
Td Shrink+257(Al), Cardin+094
Td Shrink+258(Al), Cardin+100
Td Shrink+259(Al), Cardin+104
Td Shrink+260(Al), Cardin+106
Td Shrink+261(Al), Cardin+112
Td Shrink+262(Al), Cardin+116
Td Shrink+263(Al), Cardin+118
Bcxm Rutine-12, 24(Al)
Racdc Cardin
Racdc Cardin
Blxm **12, -240(Al)
Blxm **12, -160(A2)

Setflg Sf Shrink+240(Al), , ,
Quant Mm Shrink+242(Al), 2, 10,
Tf Mul, 00099,,
Sf Mul-2
Tf Quot, Mul-1
Td Rem, Mul
Tdm Rem-1, 0, 11
Cm Rem, 5
Bl Noadd
Am Quot, 1, 10
Noadd Tf Area+16l(A2), Quot
Bxm **12, 2(A2)
Bcxm Setflg, 3(A1)
Blxm **12, -158(Al)
Tf Holdl, Area+1
Subtr Tf Sub, Holdl,,
S Sub, Area+16l(A1),
Tf Area+16l(A1), Sub
Bcxm Subtr, 2(Al)

* Flg Compacted Data
* Quantize Data
* Into 21 Levels
* Generate Contrast Value Sequence

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Calculate Mutual Informations

Tf  Smallf-2, Zero,
Tfm Smallf, 2, 10,
Tdm Smallf-9, 1, 11
Sf  Smallf
Tf  Flitzer-2, Zero,
Tfm Flitzer, 99, 10,
Sf  Flitzer
Blxm *+12, -790(A1)

Back
Tfl Flarea+799(A1), Flitzer,

Bcxm Back, 10(A1)
Blxm *+12, -790(A1)

Bac
Tfl Flarl+799(A1), Flitzer
Bcxm Bac, 10(A1)
Blxm *+12, -350(A5)
Blxm *+12, -35(A2)

Nxtcls
Tf  Sav, Three,
Mf  00313, 00310,
A  Sav, 00314,
Mf  00310, 00313
Mm  Sav, 100, 9
Bxm *+12, 1(A2)
Tf  Prodct, 00099
Am  Prodct, 06000
Tf  Tab+5, Prodct,
Sk  Tab,,
Rdn  Tab,,
Cdn  Tab
Tf  Denu, Tabar+3
Blxm *+12, -158(A4)
Blxm *+12, -790(A1)
Blxm *+12, -79(A3)

Bbls 79
Tf  Sav, Seven,
Mf  00318, 00315,
A  Sav, 00319
Mf  00315, 00318
Mm  Sav, 82, 9
Bxm *+12, 1(A3)
Tf  Result, 00099,
Tf  Cntrst, Area+161(A4),
Am  Cntrst, 21,10
Mm  Cntrst, 2, 9
Tf  Prodct, 00099

Set Up
Statistical Zero

Set Up Floating
Pt Zero

Zero Cond and
Uncond Freq Areas

Find Total Number
of Samples for
Class Under Test

Set Up Sctr Addr
Bring in Cond
Freq Table

Set Up Cond Freq
Table Addr

Add Digit Posn
To Addr
A Result, Prodct, , * Add Cntrst Value
A Result, Table, , Posn to Addr
Am Result, 3
Tf Tempi, Result, 11,
Tfl Tempf, Fltzer, , * Extract Freq
Cm Tempi, 9, , From Table
Bh Alter
Td Tempf-9, Tempi
Sf Tempf-9
Tfm Tempf, 1, 10
C Tempf-8, Fltzer-8
Bnz Ll
Tfl Tempf, Fltzer
B7 Ll
Alter Tf Tempf-8, Tempi
Tfm Tempf, 2, 10
Ll Tf Flarea+799(A1), Tempf, , * Flt Pt Value to
* Cond Freq Area
Bxm *|l2, 10(A1)
Bcxm Bbbs79, 2(A4)
Sk Sumtab, ,
Rdn Sumtab, , Cond Freq Table
Cdn Sumtab
Blxm *|l2, -790(A1)
Blxm *|l2, -79(A3)
Blxm *|l2, -158(A4)
Bls79 Tf Sav, Seven, , Set Up Uncond
Mf 00318, 00315, , Freq Addr
A Sav, 00319
Mf 00315, 00318
Mm Sav, 123, 9
Bxm *|l2, l(A3)
Tf Result, 00099, , * Add Digit Posn
to Addr
Tf Smtbcn, Area+16l(A4), ,
Am Smtbcn, 21, 10
Mm Smtbcn, 3, 9
Tf Prodt, 00099
A Result, Prodct, , * Add Cntrst Value
A Result, Table, , Posn to Addr
Am Result, 3
Tf Tempi, Result, 11,
Tfl Tempf, Fltzer, , * Extract Freq
Cm Tempi, 99, , from Table
Bnh Twoo
Tf Tempf-7, Tempi
Tfm Tempf, 3, 10
* Freq to Flt Pt
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Two</td>
<td>Cm</td>
<td>Ml</td>
</tr>
<tr>
<td></td>
<td>Bmh</td>
<td>One</td>
</tr>
<tr>
<td></td>
<td>Mf</td>
<td>Tmpi-1, Tmpi-2</td>
</tr>
<tr>
<td></td>
<td>Tf</td>
<td>Tmpf-8, Tmpi</td>
</tr>
<tr>
<td></td>
<td>Tfm</td>
<td>Tmpf, 2, 10</td>
</tr>
<tr>
<td></td>
<td>B7</td>
<td>Ml</td>
</tr>
<tr>
<td>One</td>
<td>Td</td>
<td>Tmpf-9, Tmpi</td>
</tr>
<tr>
<td></td>
<td>Sf</td>
<td>Tmpf-9</td>
</tr>
<tr>
<td></td>
<td>Tfm</td>
<td>Tmpf-1, 10</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Tmpf-8, Fltzer-8</td>
</tr>
<tr>
<td></td>
<td>Bnz</td>
<td>Ml</td>
</tr>
<tr>
<td></td>
<td>Tfl</td>
<td>Tmpf, Fltzer</td>
</tr>
<tr>
<td>Ml</td>
<td>Tfl</td>
<td>Flarl+799(A1), Tmpf,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>* Flt Pt Value to Uncond Freq Area</td>
</tr>
<tr>
<td></td>
<td>Bxm</td>
<td>*+12, 10(A1)</td>
</tr>
<tr>
<td></td>
<td>Bcxm</td>
<td>B1s79, 2(A4)</td>
</tr>
<tr>
<td></td>
<td>Tf</td>
<td>Tmpf, Fltzer</td>
</tr>
<tr>
<td></td>
<td>Blxm</td>
<td>*+12, -790(A1)</td>
</tr>
<tr>
<td>Comput</td>
<td>Tfl</td>
<td>Sum, Fltzer,</td>
</tr>
<tr>
<td></td>
<td>Tfl</td>
<td>Tmpf, Fltzer</td>
</tr>
<tr>
<td></td>
<td>Tfm</td>
<td>Denomu, 2, 10,</td>
</tr>
<tr>
<td></td>
<td>Tf</td>
<td>Denomu-2, Zero,</td>
</tr>
<tr>
<td></td>
<td>Tfm</td>
<td>Denoml, 3, 10</td>
</tr>
<tr>
<td></td>
<td>Tf</td>
<td>Denoml-2, Zero</td>
</tr>
<tr>
<td></td>
<td>Tf</td>
<td>Denoml-7, Denl</td>
</tr>
<tr>
<td>Cycle</td>
<td>Bnf</td>
<td>Mod2, Setind,</td>
</tr>
<tr>
<td>Mod</td>
<td>Tfl</td>
<td>Compar, Flarea+799(A1)</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Compar-8, Fltzer-8,</td>
</tr>
<tr>
<td></td>
<td>Bnz</td>
<td>Nosame</td>
</tr>
<tr>
<td></td>
<td>Tfl</td>
<td>Compar, Flarl+799(A1)</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Compar-8, Fltzer-8,</td>
</tr>
<tr>
<td></td>
<td>Bnz</td>
<td>Fad</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Mak,</td>
</tr>
<tr>
<td>Fad</td>
<td>Tfl</td>
<td>Bangf, Smallf,</td>
</tr>
<tr>
<td></td>
<td>Fdiv</td>
<td>Bangf, Denomu,</td>
</tr>
<tr>
<td></td>
<td>Fdiv</td>
<td>Bangf, Flarl+799(A1),</td>
</tr>
<tr>
<td></td>
<td>Fmul</td>
<td>Bangf, Denoml</td>
</tr>
<tr>
<td></td>
<td>Fln</td>
<td>Bang, Bangf</td>
</tr>
<tr>
<td></td>
<td>Fadd</td>
<td>Sum, Bang,</td>
</tr>
<tr>
<td></td>
<td>Bnc2</td>
<td>Away</td>
</tr>
<tr>
<td>Rcty</td>
<td>Waty</td>
<td>Badnum</td>
</tr>
<tr>
<td>Rcty</td>
<td>B</td>
<td>Away</td>
</tr>
</tbody>
</table>
Mak  Bnc1  Away
Rcty  Waty  Badden
Rcty  B  Away
Nosame  Fmul  Flarea+799(Al), Denoml,
         Fmul  Flarl+799(Al), Denomu
         Fdiv  Flarea+799(Al), Flarl+799(Al)
         Tfl   Tempf, Flarea+799(Al)
         Fln   Bang, Tempf
         Fadd  Sum, Bang,
         B  Away
         *  Compute Measure
         *  Add Information
Mod2  Tfl   Compar, Flarea+799(Al)
       C    Compar-8, Flitzer-8,
       Bnz  Nosam
       Tfl   Compar, Flarl+799(Al)
       C    Compar-8, Flitzer-8,
       Bnz  Fadd
       B  Kam,
       *  Test Cond Freq
       *  Test Uncond Freq
       *  No Information
       *  Substitute Stat
       Zero and
       Compute Measure
       *  Add Effective Inf
Fadd  Tfl   Tempf, Smallf,
       Fdiv  Tempf, Denomu,
       Fdiv  Tempf, Flarl+799(Al),
       Fmul  Tempf, Denoml
       Fln   Flarl+799(Al), Tempf
       Tfl   Tempf, Smallf
       Fdiv  Tempf, Denomu
       Fmul  Flarl+799(Al), Tempf
       Fadd  Sum, Flarl+799(Al),
       Bnc2  Away
       Rcty  Waty  Badnum
       Rcty  B  Away
Kam  Bnc1  Away
Rcty  Waty  Badden
Rcty  B  Away
Nosam  Tfl   Sap, Flarea+799(Al),
        Fdiv  Sap, Denomu
        Fmul  Flarea+799(Al), Denoml
        Fmul  Flarl+799(Al), Denomu
        Fdiv  Flarea+799(Al), Flarl+799(Al)
        Tfl   Tempf, Flarea+799(Al)
        Fln   Bang, Tempf
        Fmul  Bang, Sap
        *  Compute Measure

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Away

Tnnsfr

Gobak

Remain

Fadd Sum, Bang,,
Bcxm Cycle, 10(A1)
Tfl Out+359(A5), Sum
Bcxm Nxtcls, 10(A5)
Rcty
Waty Stop
Rcty

* Add Information

* Output Mutual Informations

* Zero Column Indrs

* Set Up Alpha Flds

Index Cls Measurs

* Sort Measures

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
| Bcm  | Gobak, -1(B1)                      | * Reset Indicators |
| Bsb  | *+12                               |                   |
| Blxm | *+12, -525(A1)                     |                   |
| Blxm | *+12, 13(A5)                       |                   |
| Blxm | *+12, 13(A6)                       |                   |
| Blxm | *+12, 12(A7)                       |                   |
| Tf   | Still, A                           |                   |
| Cf   | Id1,,                              |                   |
| Cf   | Id2                                |                   |
| **Return** | Tfl Flot, Sort+539(A1),,       | * Sorted Measures |
|       | Sm Flot, 1,10,                      | to Data Out Area  |
|       | Mf Flot-10, Flot-9                 |                   |
| Tnf  | Output+22, Flot-9,                | * Data Left One Dig |
| Tdm  | Output+20, 0                       |                   |
| Tdm  | Output+19, 0                       |                   |
| Sf   | Flot-8                             |                   |
| C    | Flot-2, Zr,,                      | * Test Mantisa Sign |
| Bnl  | Go                                 |                   |
| Cf   | Flot-2                             |                   |
| Tdm  | Output+19, 2,                      | * Mantisa Negative |
| Tdm  | Output+20, 0                       |                   |
| B7   | Go+24                              |                   |
| **Go** | Tdm Output+19, 1,,                 | * Mantisa Positive |
|       | Tdm Output+20, 0                   |                   |
| Tnf  | Output+38, Flot-2,                | * Mant to Out Area |
| Cm   | Flot, 0,10                        | * Test Expon Sign  |
| Bnl  | Notlow                             |                   |
| Cf   | Flot                               |                   |
| Tdm  | Output+41, 2,                      | * Expon Negative   |
| Tdm  | Output+42, 0                       |                   |
| B7   | Notlow+24                          |                   |
| **Notlow** | Tdm Output+41, 1,,               | * Expon Positive   |
|       | Tdm Output+42, 0                   |                   |
| Tnf  | Output+46, Flot,                  | * Expon to Out Area |
| Mf   | Sort+528(A1), Sort+525(A1)        |                   |
| Tnf  | Output+6, Sort+529(A1),          | * Index to Out Area |
|      | Bnf *+24, Id1,,                   | * Column 1 Check   |
|      | B Yes2-36                          |                   |
| Bcm  | Noi, -1(A5)                        | * Column 1 Complete |
|      |                                 |                   |
| **Yes1** | Sf Id1,,                         |                   |
|      | Tf Still, A                        |                   |
|      | Am Still, 56                       |                   |
|      | Bnf *+24, Id2,,                    | * Column 2 Check   |
|      | B Yes3-12                          |                   |
| Bcm  | No2, -1(A6)                        |                   |
|      |                                 |                   |
| **Yes2** | Sf Id2,,                         | * Column 2 Complete |
|      |                                 |                   |

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Tf  Still, A
Am  Still, 108
Bcxm No3, -(A7)

Yes 3
Trnm Still, Empty-1, 6,
Wacd A1,,
Wacd A2,,
Wacd A3,,
Wacd A4,,
Wacd A5
Wacd A6
Wacd A7
Wacd A8
Wacd A9
Wacd A10
Wacd A11
Wacd A12
Wacd Cardin,,,
Wacd Cardin,,,
B7  Art

No3
Trnm Still, Output-1, 6,
Am  Still, 162
B7  Gas

No2
Trnm Still, Output-1, 6
Am  Still, 162
B7  Gas

Nol
Trnm Still, Two-1, 6
Am  Still, 4
Trnm Still, Output-1, 6
Am  Still, 158

Gas
Bxm Return, 15(A1)

*  Column 3 Complete
*  Punch Information
  Measures In
  Descending Order
  of Magnitude

*  Punch Sample
  Coded Cards

*  Columnnator Routine

*  Frequency Table Adjustment Option

*  Get Class Code
  of Max Measure

*  Set Up Cond Freq
  Table Addr
  Table to Core

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
Rdn Tab
Cdn Tab
Am Tabar+3, 1, 10
Blxm *12, -158(A1)
Blxm *12, -79(A3)

Again

Tf Sav, Seven
Mf 00318, 00315
A Sav, 00319
Mf 00315, 00318
Mm Sav, 82, 9
Bxm *12, 1(A3)
Tf Result, 00099,
Tf Cntrst, Area+16l(A1),
Am Cntrst, 21, 10
Mm Cntrst, 2, 9
Tf Prodct, 00099
A Result, Prodct,
A Result, Table,
Am Result, 3
Am Result, 1, 610,
Bcxm Again, 2(A1)
Sk Tab,,
Wdn Tab,,
Cdn Tab
Sk Sumtab,,
Rdn Sumtab,,
Cdn Sumtab
Blxm *12, -158(A1)
Blxm *12, -79(A3)

Contin

Tf Sav, Seven,
Mf 00318, 00315
A Sav, 00319
Mf 00315, 00318
Mm Sav, 123, 9
Bxm *12, 1(A3)
Tf Result, 00099,
Tf Smtbcn, Area+16l(A1),
Am Smtbcn, 21, 10
Mm Smtbcn, 3, 9
Tf Prodct, 00099
A Result, Prodct,
A Result, Table,
Am Result, 3
Am Result, 1, 69,
Bcxm Contin, 2(A1)
Sk Sumtab,,

* Update Sample Qty

* Add Digit Posn to Addr

* Add Cntrst Value Posn to Addr

* Update Cond Freq

* Rewrite Table Onto Disk

* Uncond Freq Table to Core

* Set Up Uncond Freq Addr

* Add Digit Posn to Addr

* Add Cntrst Value Posn to Addr

* Update Uncond Freq

* Rewrite Table
APPENDIX B

EXAMPLE OF EQUATION (5)

Consider a pattern which is segmented into nine discrete elements. Let the set S of signal values be,

\[ S: \{0, 1, 2, 3\} \]

Assume the following M matrix,

\[
\begin{pmatrix}
3 & 1 & 2 \\
2 & 2 & 0 \\
2 & 0 & 1 \\
\end{pmatrix}
\]

Using equation (2), the following N matrix is generated,

\[
\begin{pmatrix}
0 & 2^* & 1 & 1 & 1 & 3 & 1 & 3 & 2^* \\
-2 & 0 & -1 & -1 & -1 & 1 & -1 & 1 & 0' \\
-1 & 1 & 0 & 0 & 0 & 2 & 0 & 2 & 1 \\
-1 & 1 & 0 & 0 & 0 & 2 & 0 & 2 & 1 \\
-1 & 1 & 0 & 0 & 0 & 2 & 0 & 2 & 1 \\
-3 & -1 & -2 & -2 & -2 & 0 & -2 & 0 & -1 \\
-1 & 1 & 0 & 0 & 0 & 2 & 0 & 2 & 1 \\
-3 & -1 & -2 & -2 & -2 & 0 & -2 & 0 & -1 \\
-2 & 0 & -1 & -1 & -1 & 1 & -1 & 1 & 0 \\
\end{pmatrix}
\]

(\( ^* - n^{1,2}_{1,1} \); \( ^* - n^{3,3}_{1,1} \); \( ^* - n^{3,3}_{1,2} \) )
According to equation (5),

\[(n^3, 3) = (n^3, 3) - (n^1, 2) \cdot (l, 2) - l \cdot (1, 2) \cdot l - 1\]

\[= (n^3, 3) - (n^1, 2)\]

\[= 2 - 2 = 0 = (n^3, 3)_1, 2\]
APPENDIX C

EXAMPLE OF EQUATION (10)

Equation (10) gives the number of redundant N sequences which would be generated by the transformation in Chapter II. Consider M matrices of order (1, 4),

\[ a = 1 \]
\[ b = 4 \]

Define the set S,

\[ S: (0, 1, 2) \]
\[ s_q = 2 \]

Let the order of the N sequence generation be row by row starting with \( m_{1,1} \). Then the total number of possible M matrices is,

\[ (q + 1)^{ab} = 3^4 = 81 \]
The number of redundant matrices according to equation (10) is,

\[ R = q^{ab} = 2^4 = 16 \]

The number of unique N sequences is,

\[ (q + 1)^{ab} - q^{ab} = 81 - 16 = 65 \]

From Table II on the following page, the number of unique N sequences (those without an *) is seen to be 65, in agreement with the general formula,

\[ U = (q+1)^{ab} - q^{ab} \quad (22) \]

Therefore the number of redundant matrices is,

\[ R = 81 - 65 = 16, \]

the number predicted by equation (10).
### Table II  All Possible M and N Matrices

<table>
<thead>
<tr>
<th>M</th>
<th>N</th>
<th>M</th>
<th>N</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>000</td>
<td>*0001</td>
<td>00-1</td>
<td>0002</td>
<td>00-2</td>
</tr>
<tr>
<td>1000</td>
<td>111</td>
<td>*1001</td>
<td>110</td>
<td>1002</td>
<td>11-1</td>
</tr>
<tr>
<td>2000</td>
<td>222</td>
<td>2001</td>
<td>221</td>
<td>2002</td>
<td>220</td>
</tr>
<tr>
<td>0100</td>
<td>-100</td>
<td>*0101</td>
<td>-10-1</td>
<td>0102</td>
<td>-10-2</td>
</tr>
<tr>
<td>1100</td>
<td>011</td>
<td>*1101</td>
<td>010</td>
<td>1102</td>
<td>01-1</td>
</tr>
<tr>
<td>2100</td>
<td>122</td>
<td>2101</td>
<td>121</td>
<td>2102</td>
<td>120</td>
</tr>
<tr>
<td>0200</td>
<td>-200</td>
<td>0201</td>
<td>-20-1</td>
<td>0202</td>
<td>-20-2</td>
</tr>
<tr>
<td>1200</td>
<td>-111</td>
<td>1201</td>
<td>-110</td>
<td>1202</td>
<td>-11-1</td>
</tr>
<tr>
<td>2200</td>
<td>022</td>
<td>2201</td>
<td>021</td>
<td>2202</td>
<td>020</td>
</tr>
<tr>
<td>0010</td>
<td>0-10</td>
<td>*0011</td>
<td>0-1-1</td>
<td>0012</td>
<td>0-1-2</td>
</tr>
<tr>
<td>1010</td>
<td>101</td>
<td>*1011</td>
<td>100</td>
<td>1012</td>
<td>10-1</td>
</tr>
<tr>
<td>2010</td>
<td>212</td>
<td>2011</td>
<td>211</td>
<td>2012</td>
<td>210</td>
</tr>
<tr>
<td>0110</td>
<td>-1-10</td>
<td>*0111</td>
<td>-1-1-1</td>
<td>0112</td>
<td>-1-1-2</td>
</tr>
<tr>
<td>1110</td>
<td>001</td>
<td>*1111</td>
<td>000</td>
<td>1112</td>
<td>00-1</td>
</tr>
<tr>
<td>2110</td>
<td>112</td>
<td>*2111</td>
<td>111</td>
<td>2112</td>
<td>110</td>
</tr>
<tr>
<td>0210</td>
<td>-2-10</td>
<td>0211</td>
<td>-2-1-1</td>
<td>0212</td>
<td>-2-1-2</td>
</tr>
<tr>
<td>1210</td>
<td>-101</td>
<td>*1211</td>
<td>-100</td>
<td>1212</td>
<td>-10-1</td>
</tr>
<tr>
<td>2210</td>
<td>012</td>
<td>*2211</td>
<td>011</td>
<td>2212</td>
<td>010</td>
</tr>
<tr>
<td>0020</td>
<td>0-20</td>
<td>0021</td>
<td>0-2-1</td>
<td>0022</td>
<td>0-2-2</td>
</tr>
<tr>
<td>1020</td>
<td>1-11</td>
<td>1021</td>
<td>1-10</td>
<td>1022</td>
<td>1-1-1</td>
</tr>
<tr>
<td>2020</td>
<td>202</td>
<td>2021</td>
<td>201</td>
<td>2022</td>
<td>200</td>
</tr>
<tr>
<td>0120</td>
<td>-1-20</td>
<td>0121</td>
<td>-1-2-1</td>
<td>0122</td>
<td>-1-2-2</td>
</tr>
<tr>
<td>1120</td>
<td>0-11</td>
<td>*1121</td>
<td>0-10</td>
<td>1122</td>
<td>0-1-1</td>
</tr>
<tr>
<td>2120</td>
<td>102</td>
<td>*2121</td>
<td>101</td>
<td>2122</td>
<td>100</td>
</tr>
<tr>
<td>0220</td>
<td>-2-20</td>
<td>0221</td>
<td>-2-2-1</td>
<td>0222</td>
<td>-2-2-2</td>
</tr>
<tr>
<td>1220</td>
<td>-1-11</td>
<td>*1221</td>
<td>-1-10</td>
<td>1222</td>
<td>-1-1-1</td>
</tr>
<tr>
<td>2220</td>
<td>002</td>
<td>*2221</td>
<td>001</td>
<td>*2222</td>
<td>000</td>
</tr>
</tbody>
</table>
APPENDIX D

PATTERN CLASS FREQUENCY DISTRIBUTION

The curves on the following pages represent the statistics gathered from twelve samples of each of thirty-five pattern classes. Each column corresponds to a particular digit position. The frequencies of occurrence of forty-one contrast values (ranging from -20 to +20) are represented in each curve. Of the seventy-nine digit positions which define an N-sequence, fifty-two are shown.
<table>
<thead>
<tr>
<th>CONTRAST VALUE</th>
<th>CONTRAST VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>-20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>+20</td>
<td>+20</td>
</tr>
</tbody>
</table>

**DIGIT POSITION**

<table>
<thead>
<tr>
<th>PATTERN CLASS</th>
<th>FREQUENCY (12 units maximum / class)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
PATTERN CLASS
FREQUENCY (12 units maximum / class)
<table>
<thead>
<tr>
<th>Digit Position</th>
<th>19</th>
<th>20</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Contrast Value

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>PATTERN CLASS</th>
<th>FREQUENCY (12 units maximum / class)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CONTRAST VALUE</th>
<th>CONTRAST VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>+20</td>
</tr>
<tr>
<td>-20</td>
<td>+20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>Digit Position</th>
<th>25</th>
<th>26</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Pattern Class Frequency (12 units maximum / class)**

**Contrast Value**

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
<table>
<thead>
<tr>
<th>PATTERN CLASS</th>
<th>FREQUENCY (12 units maximum / class)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |

<table>
<thead>
<tr>
<th>CONTRAST VALUE</th>
<th>CONTRAST VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>-20</td>
</tr>
</tbody>
</table>
VITA AUCTORIS

1943 Born on February 25, in Belleville, Ontario.

1956 Graduated from Central Public School, Windsor, Ontario.

1961 Completed Senior Matriculation at Hon. W.C. Kennedy Collegiate Institute, Windsor, Ontario.

1965 Graduated from the University of Windsor, Windsor, Ontario; B.A.Sc. in Engineering Science.

1966 Candidate for the degree of M.A.Sc. in Electrical Engineering (Interdisciplinary Studies in Communications) at the University of Windsor, Windsor, Ontario.