Shear and bending deformation of sandwich parallelogrammic panels.

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SHEAR AND BENDING DEFORMATION
OF
SANDWICH PARALLELOGRAMMIC PANELS

A Thesis
Submitted to the Faculty of Graduate Studies through the
Department of Civil Engineering in Partial Fulfilment
Of the Requirements for the Degree of
Master of Applied Science at
The University of Windsor

by
Amit K. Guha-Thakurta
B.E., The University of Calcutta, 1961
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1968

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ABSTRACT

Clamped parallelogrammic sandwich panels are analysed by splitting the panel deflection into two parts; one due to shear and the other due to bending. An 'exact' solution to the governing differential equations is found. Results for maximum deflection, centre and edge moments are obtained for various skew angles, aspect ratios, flexural and shear rigidities of panel. The influence of the above independent variables on the behaviour of the panel is investigated. The results from experimental model tests are compared and good agreement is shown for the maximum centre deflections. However, favourable comparison for centre moments is found only for panels with a very rigid core. The validity of the small deflection theory is examined in the light of the theoretical and experimental results.

Tests to rupture revealed sudden reversal of strain around the obtuse corner region after progressive yielding of the edges had taken place.
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CHAPTER I

INTRODUCTION

Sandwich panels are widely used in the construction of aircraft and missiles because of the relatively high strength of the material per unit weight. The core is usually made of a light material, e.g., styrafoam, paper (honeycomb), balsa wood, plastic or cellular assembly of any suitable sheet. The core has to be adequately rigid and well bonded to the skin which is usually metallic. The purpose of the core is to stabilize the face plate in compression and to provide enough strength for the structure to withstand the loading that an aircraft is usually subjected to. Apart from the enhanced structural strength, sandwich plates are advantageous also from the point of view of aerodynamic drag.

Rectangular sandwich plates subjected to uniform transverse load have been analysed by various investigators by different analytical methods, e.g., strain energy method, Principle of Virtual Work and the use of the Calculus of Variations for the derivation of the governing differential Equations and the appropriate boundary conditions. However, no solutions are yet available for parallelogrammic sandwich panels under similar conditions.

The work embodied in this thesis consists of:

i) a theoretical analysis on the bending and shear deformation of clamped parallelogrammic sandwich panels subjected to a uniformly distributed lateral load; and

ii) experimental investigation of the bending behaviour of several clamped parallelogrammic sandwich panels with different core material.
The theoretical analysis is based on small deflection theory of Sandwich Plates. The total deflection of a point on the sandwich plate is divided into two parts i.e. $W_B$, due to bending and $W_S$, due to shear. An "exact solution" for the governing differential equation for the bend deflection $W_B$, is found. The arbitrary constants contained in the deflection function, $W_B$, are solved from the boundary conditions by expanding them into Fourier Sine and Cosine series. Thus the deflection functions $W_B$ and $W_S$ are explicitly known for every point on the skewed clamped sandwich plate and the necessary moment and shear expressions are then obtained from the deflection functions.

The experimental investigation consisted of the loading of sandwich plates of three different core materials e.g. paper, styrafoam and plywood, three different dimensions i.e. 20 in. x 30 in., 16 in. x 14 in., and 15 in. x 12 in. and two different skew angles i.e. 30° and 50° by uniformly distributed load with the four sides clamped. The choice of the three different core materials was made to study the effect of varying core rigidity on the behaviour of the sandwich plates. The softer the core the more pronounced is the effect of transverse shear deformation on the deflection of the sandwich plates. Meta-film strain rosette gauges were installed and moments and stresses were calculated from the strain measurements. Dial indicators were also installed to record the lateral deflection at chosen points on the skewed plates.

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CHAPTER II
REVIEW OF LITERATURE

Reissner, in 1948, carried out a rigorous analysis on the finite deflections of rectangular sandwich plates. He developed the basic differential equations for finite transverse deflections of sandwich panels neglecting the transverse normal stresses in the core and the variation of the face stresses over the thickness of the face layers. He also analyzed the range of linearity of deflections and showed that this range decreases rapidly with the softness of the core relative to the faces.

Taylor, in 1948, developed a set of modified plate equations from the classical theory of plates applicable to the solution of the rectangular sandwich plates. He showed that the total deflection can be divided into two parts: (a) deflection due to bending, and (b) deflection due to transverse shear. His equations show the basic relationship between these two kinds of deflections so that the total deflection could be determined by solving the governing differential equation for deflection due to bending.

Libove and Batdorf, in 1949, developed a general small deflection theory for the elastic behaviour of orthotropic rectangular sandwich plates. This theory is applicable to all types of rectangular sandwich plates and is based on seven physical constants (five stiffnesses and two Poisson ratios) of sandwich plates. These physical constants could be derived analytically or could be determined experimentally. The energy expressions as well as the differential equations for the sandwich plate were developed.

Hoff, in 1950, derived a set of differential equations for the bending and buckling of rectangular sandwich plates. The differential
equations were derived by means of the principle of virtual displacement from the essential parts of the strain energy stored in the sandwich plate. The set of differential equations thus deduced was further reduced to a single sixth order partial differential equation for the solution of a rectangular sandwich plate.

Ikeda, in 1955, developed a theory for bending isotropic flat rectangular sandwich plates taking into account the contributions of the individual stiffnesses of the faces and the core. The solution of the basic differential equations gives the deflection and the stress function, from which the expressions for moments and stresses could be developed.

Thurston, in 1957, applied the Lagrangian multiplier method to Hoff's energy expressions for sandwich plates and derived equations for evaluating the deflections and buckling loads of clamped rectangular sandwich plates.

Cheng, in 1962, developed a system of differential equations for small deflections of a rectangular sandwich plate by means of the variational theorem of complementary energy in conjunction with Lagrangian multipliers. These equations, in turn, yield a governing linear differential equation of sixth order for the deflection $W$.

Eringen, in 1951, carried out a rigorous analysis of the bending and buckling of rectangular sandwich plates having homogeneous core by the use of the theorem of minimum potential. In this theory he considered the three dimensional stress distribution in the core thus taking care of the flattening and the bending rigidity of the core. The bending rigidities of the individual faces were also taken into account.
CHAPTER III

THEORETICAL ANALYSIS

The theoretical analysis consists of the application of the method of partial deflections. A sandwich plate, unlike a homogeneous plate, undergoes considerable deflection due to the transverse shear deformation of the core over and above the deflection due to bending. Thus the total deflection of the plate is divided into two parts, $W_B$, the deflection due to bending, and $W_s$, the deflection due to transverse shear deformation of the core material. The assumptions of this theory are as follows:

1) The facings and the core are assumed to be isotropic materials.
2) The core stiffnesses associated with plane stress components in the plane of the plate are negligibly small.
3) The facings are treated as solid membranes i.e. negligible bending rigidity, of equal thickness.
4) The linear theory of elasticity is assumed to be valid.
The Relationship Between $W_B$ & $W_S$

Fig. A illustrates the forces acting on a rectangular sandwich plate and the various geometrical dimension of a rectangular sandwich plate, e.g. face thickness, $t$, core thickness $c$.

The shears $Q_x$ & $Q_y$ are given in rectangular co-ordinates in terms of the bend deflection, $W_B$, as in the classical theory of plates, as follows:

$$Q_x = -D_o \frac{\partial}{\partial x} \left( \frac{\partial^2 W_B}{\partial x^2} + \frac{\partial^2 W_B}{\partial y^2} \right)$$  \hspace{1cm} (1)

$$Q_y = -D_o \frac{\partial}{\partial y} \left( \frac{\partial^2 W_B}{\partial x^2} + \frac{\partial^2 W_B}{\partial y^2} \right)$$  \hspace{1cm} (2)

where $Q_x$, $Q_y$ & $D_o$ are defined in Appendix I.

The shears $Q_x$ and $Q_y$ can also be expressed in terms of the shearing strains of the core as follows:

$$Q_x = (Gt) \frac{\partial W_s}{\partial x}$$  \hspace{1cm} (3)

$$Q_y = (Gt) \frac{\partial W_s}{\partial y}$$  \hspace{1cm} (4)

where $G$ = modulus of rigidity of the core in shear

$\frac{\partial W_s}{\partial x} =$ shearing strain in the core parallel to $Y$-axis

$\frac{\partial W_s}{\partial y} =$ shearing strain in the core parallel to $X$-axis.

From Eqs. (1), (2), (3) & (4), the shear deflection, $W_S$, can be expressed in terms of the bend deflection, $W_B$, in rectangular co-ordinates as follows:

$$W_S = -\left( \frac{D_o}{G} \right) \left( \frac{\partial^2 W_B}{\partial x^2} + \frac{\partial^2 W_B}{\partial y^2} \right) = -\frac{D_o}{G} (\nabla^2 W_B)$$  \hspace{1cm} (5)

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.
The governing differential equation for \( W_B \) (from small deflection theory of plates) in rectangular co-ordinates is:

\[
\nabla^4 W_B = \frac{q}{D_0} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)
\]

where \( \nabla = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \)

and \( q \) is defined in Appendix I.

The governing differential equations for the sandwich plate, Eqs.(5) \& (6) become when transformed to dimensionless skew coordinates \( \eta \) and \( \xi \):

\[
W_B, \eta \eta \eta \eta + p^4 W_B, \xi \xi \xi \xi + 2(1+2s^2)p^2 W_B, \eta \eta \eta \eta
- 4s(p^3W_B, \xi \xi \eta + pW_B, \xi \xi \eta\eta) = \frac{(bc)^4q}{D_0} \quad \ldots \ldots(7)
\]

\[
W_S = -\frac{D_0}{cG(bc)^c} \left(W_B, \eta \eta - 2s \xi \eta W_B, \xi \xi + p^2 W_B, \xi \xi \xi\right) \quad \ldots \ldots(8)
\]

where \( D_0, c, G, c, s, p, b, q \) are defined in Appendix I.

For an exact solution to Eq.(7), the same method of solution is followed as given in Ref.9. An outline of this method is presented here.

For the complementary function of Eq. 7, i.e., the general solution of

\[
W_B, \eta \eta \eta \eta + p^4 W_B, \xi \xi \xi \xi + 2(1+2s^2)p^2 W_B, \eta \eta \eta \eta
- 4s(p^3W_B, \xi \xi \eta + pW_B, \xi \xi \eta\eta) = 0 \quad \ldots \ldots(7a)
\]
a Fourier series is assumed as follows:

\[ W_b = A_{10} + \sum_{n=1}^{\infty} e^{\Lambda_n} \left( P_n \sin \alpha \eta + Q_n \cos \alpha \eta \right) \]  

(7b)

in which \( n \) is an integer,

\( \alpha n = n \pi \),

\( \Lambda_n = \) characteristic number,

\( A_{10} = \) a constant independent of \( n \).

By substituting Eq. (7b) in Eq. (7a) and equating the coefficients of \( \sin \alpha \eta \) & \( \cos \alpha \eta \) two equations for \( P_n \) & \( Q_n \) are obtained. From these two equations the characteristic double roots are obtained as follows:

\[ \Lambda_n = H_n (\pm i \alpha \pm c) \]

where \( i = \sqrt{-1} \), \( H_n = \alpha \eta / \mu \)

because of the multiplicity of the roots of the characteristic equation, \( e^{\Lambda_n} \) in Eq. (7b) is replaced by a linear combination of \( e^{\Lambda_n} \) and \( e^{\Lambda_n} \).

Thus

\[ C_n e^{\Lambda_n} = C_{\text{sh} \xi \eta \text{sh}} \cos \xi \eta \text{sh} + C_{\text{sh} \xi \eta \text{sh}} \sin \xi \eta \text{sh} \]

\[ + C_{\text{sh} \xi \eta \text{sh}} \sin \xi \eta \text{sh} \cos \xi \eta \text{sh} + C_{\text{sh} \xi \eta \text{sh}} \sin \xi \eta \text{sh} \cos \xi \eta \text{sh} \]

\[ + C_{\text{sh} \xi \eta \text{sh}} \sin \xi \eta \text{sh} \cos \xi \eta \text{sh} \]

in which \( C_\text{sh}, C_{\text{sh} \eta \text{sh}}, \ldots C_{\text{sh} \eta \text{sh}} \) are arbitrary constants and \( \xi \eta, \xi \eta \text{sh}, \xi \eta \text{ch} \) are defined in Appendix I.
The consideration of the inherent polar symmetry of the deflection function requires only even functions to appear in it.

Therefore,
\[ C_{20} = C_{210} = C_{211} = C_{220} = 0 \]  \hspace{1cm} (7a)

Hence,
\[ A_{n} e^{\lambda n \xi} = C_{n} \cos \lambda n \xi \cos \lambda n \eta + C_{n} \sin \lambda n \xi \sin \lambda n \eta \]
\[ + C_{n} \cos \lambda n \xi \sin \lambda n \eta + C_{n} \sin \lambda n \xi \cos \lambda n \eta \]  \hspace{1cm} (7b)

From \( A_{n} \) it can be shown that
\[ P_{n} e^{\lambda n \xi} = -C_{n} \cos \lambda n \xi \cos \lambda n \eta + C_{n} \sin \lambda n \xi \sin \lambda n \eta \]
\[ - C_{n} \cos \lambda n \xi \sin \lambda n \eta + C_{n} \sin \lambda n \xi \cos \lambda n \eta \]  \hspace{1cm} (7c)

For a more general representation of the deflection function \( W_{b} \), a Fourier series in \( \sin \lambda n \xi \) and \( \cos \lambda n \xi \) is also considered; thus the complete form for \( W_{b} \) is:
\[ W_{b} = A_{10} + \sum_{n=1}^{\infty} \left[ B_{n} e^{\lambda n \xi} \left( R_{n} \sin \lambda n \eta + S_{n} \cos \lambda n \eta \right) \right. \]
\[ \left. + \tilde{B}_{n} e^{\lambda n \xi} \left( R_{n} \sin \lambda n \eta + S_{n} \cos \lambda n \eta \right) \right] \]  \hspace{1cm} (7d)

in which \( R_{n}, S_{n}, \tilde{B}_{n} \) are analogous to \( P_{n}, Q_{n}, B_{n} \).

The particular integral of Eq. (7) can be taken as
\[ \left( \frac{1}{2} j \right) \left[ \int_{-\pi}^{\pi} \left( 1 - e^{2 \pi x} + (i - 2 x \pi + \pi^2) \right) e^{i\xi \eta} d\xi \right] \]

Also in order to have the same number of
arbitrary constants in $w_B$ as there are equations, a biharmonic function of the following form:

$$A_2 e^{\eta^2} + A_4 e^{\xi^2} + A_5 e^{(\eta^2 - \xi^2/p^2)}$$

is added to the deflection function, $w_B$, in which $A_2$, $A_4$, $A_5$ are arbitrary constants. Thus an exact solution to Eq. (7), taking into consideration the inherent panel symmetry, can be expressed as:

$$w_B = \frac{k_0}{4} \left[ A_{10} + A_{14} e^{\eta^2} + A_{15} e^{\xi^2} + A_{16} (e^{\eta^2} + e^{\xi^2/p^2}) + 2 \sum_{n=1}^{\infty} \left( C_{mn} \sin \xi n + C_{mn} \sin \eta n \right) \cosh \eta n 
+ (C_{2n} \sin \xi n + C_{2n} \sin \eta n) \sinh \eta n 
+ (C_{3n} \cos \xi n + C_{3n} \cos \eta n) \cosh \xi n 
+ (C_{4n} \sin \xi n + C_{4n} \sin \eta n) \sinh \xi n \right]$$

Where:

$$\xi n = s \lambda_1 \eta + \alpha_1 \xi$$

$$\eta n = s \lambda_2 \eta + \alpha_2 \eta$$

and the terms e.g. $\lambda_1, \lambda_2, \ldots$ etc are defined in Appendix I.

The boundary conditions for a clamped parallelogrammatic sandwich panel are:

$$w_T = w_B + w_s = 0 \quad @ \; \eta = +1 \quad \ldots \quad (10)$$

$$w_T = w_B + w_s = 0 \quad @ \; \xi = +1 \quad \ldots \quad (11)$$
\[ W_0, \xi = 0 \quad \& \quad \xi = +1 \quad \text{i.e.,} \]
\[ p W_0, \xi - s W_0, \eta = 0 \quad \& \quad \xi = +1 \quad \ldots \quad (12) \]
\[ W_0, \xi = 0 \quad \& \quad \eta = +1, \text{ where } \xi \text{ is the outwardly drawn normal, i.e.,} \]
\[ W_0, \eta = s p W_0, \xi = 0 \quad \& \quad \eta = +1 \quad \ldots \quad (13) \]

The substitution of Eq. (9) in each of the above four boundary conditions and upon expansion in a Fourier Sine and Cosine series and equating the coefficients of the latter to zero (since they are independent) yield 8 equations; hence there would be 12 equations to solve for the 12 constants in Eq. (9).

In the process of satisfying the various boundary conditions, cumbersome and rather lengthy algebra is encountered. This is taken care of by using various symbols e.g., \( \phi_1, \phi_2, \ldots \), etc., and \( R_0, R_1, R_2, \ldots, R_{10}, R_{11}, R_{12}, \ldots \), etc to represent the lengthy algebraic expressions, defined in Appendix I.

The total deflection, \( W_1 \), could be expressed in terms of them by virtue of Eqs. 10, 11, as follows:
\[ W_T = W_B + \frac{W}{b} = W_B + B_1 \frac{W}{b} \eta + B_2 \frac{W}{b} \eta^2 + B_3 \frac{W}{b} \eta^3 \]

where \( B_1, B_2, B_3 \) are defined in Appendix I.

Thus Eq.(10), for example yields:

\[ A_10 + A_{30} + A_{40} \left( \frac{\omega^2}{2} \right) + \left( 1 - 2 \frac{\omega^2}{2} + \frac{\omega^4}{4} \right) / \rho^4 + A_{50} \left( 1 - \frac{\omega^2}{2} / \rho^4 \right) + A_{50} (12 B_2)
\]

\[ + A_{30} (2B_2) + 8B_2 + A_{40} (2B_2) - 4B_3 / \rho^4 + 12B_3 \frac{\omega^2}{2} / \rho^4 - (12B_3 \frac{\omega^2}{2} / \rho^4) A_{50}\]

\[ + \sum_{n=1}^{\infty} \left[ (-1)^{n+1} C_{2n} \sin \xi_{2n} \sin \xi_{2n} + C_{2n} \sin \xi_{2n} \sin \xi_{2n} + (2c H \nu B_2) \xi_{2n} \sin \xi_{2n} \sin \xi_{2n}\right]
\]

\[ + (- \Phi_{25} C_{1n} + \Phi_{25} C_{2n} + R_{32} C_{3n} + R_{32} C_{4n}) \sin \xi_{2n} + \left( \Phi_{25} C_{1n} + \Phi_{25} C_{2n} + R_{32} C_{3n} + R_{32} C_{4n} \right) \cos \xi_{2n}\]

Eqn. (14) can be written as:

\[ f_0(\xi) + \sum_{n=1}^{\infty} \left[ f_{2n}(\xi) + A_{2n} \sin \xi_{2n} + B_{2n} \cos \xi_{2n} \xi_{2n} \right] = 0 \quad (15)\]

where,

\[ f_0(\xi) = A_{10} + A_{30} + A_{40} \left( \frac{\omega^2}{2} \right) + \left( 1 - 2 \frac{\omega^2}{2} + \frac{\omega^4}{4} \right) / \rho^4 + A_{50} \left( 1 - \frac{\omega^2}{2} / \rho^4 \right) + A_{50} (12 B_2)
\]

\[ + 8B_2 + A_{40} (2B_2) - 4B_3 / \rho^4 + 12B_3 \frac{\omega^2}{2} / \rho^4 - (12B_3 \frac{\omega^2}{2} / \rho^4) A_{50}\]

\[ f_{2n}(\xi) = (-1)^{n+1} C_{2n} \sin \xi_{2n} \sin \xi_{2n} + C_{2n} \sin \xi_{2n} \sin \xi_{2n} + (2c H \nu B_2) \xi_{2n} \sin \xi_{2n} \sin \xi_{2n}\]

\[ + (2c H \nu B_2) \xi_{2n} \sin \xi_{2n} \sin \xi_{2n}\]

\[ A_{2n} = - \Phi_{25} C_{1n} + \Phi_{25} C_{2n} + R_{32} C_{3n} + R_{32} C_{4n}\]

\[ B_{2n} = \Phi_{25} C_{1n} + \Phi_{25} C_{2n} + R_{32} C_{3n} + R_{32} C_{4n}\]

Expanding \( f_0(\xi) \) and \( f_{2n}(\xi) \) in a Fourier cosine series,

\[ f_0(\xi) = \sum_{k=1}^{\infty} c_k \cos k \pi \xi\]

\[ f_{2n}(\xi) = \sum_{k=1}^{\infty} b_{2n} \cos k \pi \xi\]
Eqn. (15) then becomes:

\[ b_{00} + \sum_{n=1}^{\infty} b_{0n} \cos(n \pi x) + \sum_{n=1}^{\infty} a_{0n} + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} a_{mn} \cos(m \pi x) + \sum_{n=1}^{\infty} A_{n0} \sin(n \pi y) + \sum_{n=1}^{\infty} B_{n0} \sin(n \pi y) = 0 \]

By changing the "dummy" summation index \( n \) to \( m \) and combining terms, the above Eqn. can be written as:

\[ b_{00} + \sum_{n=1}^{\infty} a_{0n} + \sum_{n=1}^{\infty} (b_{0n} + B_{n0} + \sum_{m=1}^{\infty} A_{mn}) \cos(m \pi x) + \sum_{n=1}^{\infty} A_{n0} \sin(n \pi y) = 0 \]

To satisfy the above equation identically:

\[ b_{00} + \sum_{n=1}^{\infty} a_{0n} = 0 \]
\[ b_{0n} + B_{n0} + \sum_{m=1}^{\infty} A_{mn} = 0 \]
\[ A_{n0} = 0 \]

Writing out these equations in full would give the following:

\[ A_{10} + Y_1 A_{10} + Y_2 A_{10} + Y_3 + Y_4 A_{50} \]
\[ + \sum_{n=1}^{\infty} (R_{35}v C_{5n} + R_{36}v C_{6n} + R_{37}v C_{7n} + R_{38}v C_{8n}) = 0 \]
\[ \sum_{n=1}^{\infty} (C_{5} + R_{28}v C_{27n} + C_{6} + R_{29}v C_{29n} + C_{7} + R_{30}v C_{30n}) = 0 \]
\[ + \sum_{n=1}^{\infty} (A_{40} - 2/p^4 + (128z/p^4)(1-\lambda_{50})^2 + 24 E_{10} - \lambda_{50}^2) \]
\[ + \frac{1}{2} C_{1n} + \frac{1}{2} C_{2n} + C_{3n} + C_{4n} + C_{5n} = 0 \]
\[ - \frac{1}{2} C_{1n} + \frac{1}{2} C_{2n} - C_{3n} + C_{4n} + C_{5n} = 0 \]
Similarly by employing exactly identical expansion procedure with other boundary conditions i.e., boundary conditions (1), (2) & (3) respectively, the following 9 equations are obtained:

\[ A_{10} + Y_5 A_{50} + Y_6 A_{30} + Y_7 A_{40} + Y_8 \sum_{n=1}^{23} (R_{47n} C_{1n} + R_{87n} C_{2n} + R_{247n} C_{3n} + R_{257n} C_{4n} + R_{267n} C_{5n}) = 0 \quad \text{(1)} \]

\[ \phi_{24w} C_{5n} + \phi_{25w} C_{4n} + R_{41w} C_{1n} + R_{42w} C_{2n} + I_{40w} = \frac{1}{2} (2n^2 + 1 + A_{50}) \]

\[ + I_{7w} \sum_{n=1}^{24} (A_{30} - 2 + 1 + B_2 (U + A_{50})) \]

\[ + \sum_{n=1}^{26} (R_{43w} C_{1n} + R_{44w} C_{2n} + R_{45w} C_{3n} + R_{46w} C_{4n}) = 0 \quad \text{(2)} \]

\[ - \phi_{21w} C_{5n} + \phi_{22w} C_{4n} + R_{37w} C_{1n} + R_{40w} C_{2n} = 0 \quad \text{(3)} \]

\[ A_{40} = \left( \frac{2}{\mu^4} \right) A_{50} \quad \text{(4)} \]

\[ R_{7m} C_{5n} + R_{10m} C_{4n} + R_{7m} C_{1n} + R_{10m} C_{2n} = 0 \quad \text{(5)} \]

\[ R_{7m} C_{5n} + R_{10m} C_{4n} + R_{7m} C_{1n} + R_{10m} C_{2n} = 0 \quad \text{(6)} \]

\[ + \sum_{n=1}^{26} (R_{57m} C_{1n} + R_{67m} C_{2n} + R_{77m} C_{3n} + R_{87m} C_{4n}) = 0 \quad \text{(7)} \]

\[ A_{30} = (-2) A_{50} \quad \text{(8)} \]

\[ R_{13m} C_{1n} + R_{14m} C_{2n} + R_{15m} C_{3n} + R_{16m} C_{4n} = 0 \quad \text{(9)} \]

\[ R_{17m} C_{1n} + R_{18m} C_{2n} + R_{19m} C_{3n} + R_{20m} C_{4n} \]

\[ + \sum_{n=1}^{26} (R_{21m} C_{1n} + R_{22m} C_{2n} + R_{23m} C_{3n} + R_{24m} C_{4n}) = 0 \quad \text{(10)} \]

From Eqs (6), (9), (2), (3), (10) it is possible to evaluate \( A_{00}, A_{30}, A_{40}, A_{50} \) in terms of \( C_{1n} \)--- \( C_{4n} \) as follows:

\[ A_{50} = Y_{10} + \sum_{n=1}^{24} (R_{47n} C_{1n} + R_{48n} C_{2n} + R_{47n} C_{3n} + R_{48n} C_{4n} - R_{57n} C_{5n} - R_{36n} C_{6n} - R_{37n} C_{7n} - R_{35n} C_{8n}) / Y_{7} \]

\[ A_{40} = \left( \frac{2}{\mu^4} \right) A_{50} \]

\[ A_{30} = (-2) A_{50} \]
\[ A_{10} = -\sum Y_6 A_{10} + Y_1 A_{40} + Y_2 + Y_5 A_{50} + \sum_{n=1}^{\infty} (R_{41} C_n + R_{42} C_2 + R_{43} C_3 + R_{44} C_4 + R_{45} C_5 + R_{46} C_6 + R_{47} C_7 + R_{48} C_8 + R_{49} C_9 + R_{50} C_{10}) \]

Eqns. (17), (18), (20), (21), (23), (24), (26), (27) could be rewritten by substituting the values of \( A_0 \), \( A_30 \), \( A_{40} \) and \( A_{50} \) which are now in terms of \( C_1 \) ---- \( C_8 \) and eight simultaneous equations comprising \( C_1 \) ---- \( C_8 \) could be formed as follows:

\[ \sum_{n=1}^{\infty} (R_{54} C_n + R_{55} C_2 + R_{56} C_4 + R_{57} C_5 + R_{58} C_6 + R_{59} C_7 + R_{60} C_8 + R_{61} C_9 + R_{62} C_{10}) = R_{53} \]

\[ \sum_{n=1}^{\infty} (R_{63} C_n + R_{64} C_2 + R_{65} C_3 + R_{66} C_4 + R_{67} C_5 + R_{68} C_6 + R_{69} C_7 + R_{70} C_8 + R_{71} C_9 + R_{72} C_{10}) = R_{62} \]

\[ \sum_{n=1}^{\infty} (R_{73} C_n + R_{74} C_2 + R_{75} C_3 + R_{76} C_4 + R_{77} C_5 + R_{78} C_6 + R_{79} C_7 + R_{80} C_8 + R_{81} C_9 + R_{82} C_{10}) = R_{71} \]

\[ \sum_{n=1}^{\infty} (R_{83} C_n + R_{84} C_2 + R_{85} C_3 + R_{86} C_4 + R_{87} C_5 + R_{88} C_6 + R_{89} C_7 + R_{90} C_8 + R_{91} C_9 + R_{92} C_{10}) = R_{80} \]
Equations (28) - (35) can be arranged in a matrix form for the solution of the coefficients $C_n$, ..., $C_n$ as follows:

$$
\begin{bmatrix}
R_{13}S_n & R_{14}S_n & R_{15}S_n & R_{16}S_n & 0 & 0 & 0 & 0 \\
-\phi S_n & \phi S_n & \phi S_n & \phi S_n & 0 & 0 & 0 & 0 \\
R_{54} & R_{55} & R_{56} & R_{57} & R_{58} & R_{57} & R_{56} & R_{55} \\
R_{64} & R_{65} & R_{66} & R_{67} & R_{68} & R_{67} & R_{66} & R_{65} \\
R_{74} & R_{75} & R_{76} & R_{77} & R_{78} & R_{77} & R_{76} & R_{75} \\
R_{84} & R_{85} & R_{86} & R_{87} & R_{88} & R_{87} & R_{86} & R_{85} \\
0 & 0 & 0 & 0 & R_{14}S_n & R_{15}S_n & R_{16}S_n & R_{17}S_n \\
0 & 0 & 0 & 0 & -\phi S_n & \phi S_n & \phi S_n & \phi S_n \\
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
C_8 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
$$

where $S_n = \text{Kronecker delta}$

i.e., $S_n = 0$ for $m \neq n$ and $S_n = 1$ for $m = n$.

Once the eight constants dependent on $n$, $C_n$, ..., $C_n$ are found, the constants $A_{10}$, $A_{20}$, $A_{30}$, $A_{40}$, $A_{50}$ can be easily established from Eqs. (16), (19), (22) and (25). Hence the deflection functions $W_b$ and $W_s$ are explicitly known all over the plate and hence the moments and shears are readily obtained.
Expressions for Moments and Shears

The moments \( M \) and shears at a joint in a sandwich plate, in rectangular co-ordinates \( X \) and \( Y \), are given by:

\[
M_x = -D_e \left( W_{bb,xx} + v W_{bb,yy} \right)
\]
\[
M_y = -D_e \left( W_{bb,yy} + v W_{bb,xx} \right)
\]
\[
M_{xy} = D_e (1-v) W_{bb,xy}
\]
\[
Q_x = -D_e \left( W_{bb,yy} + W_{bb,xxx} \right)
\]
\[
Q_y = -D_e \left( W_{bb,xx} + W_{bb,yyy} \right)
\]

By transforming the rectangular co-ordinates into dimensionless skew co-ordinates \( \eta, \xi \), the expressions for moments and shears become:

\[
M_x = -D_e \left( a_1 W_{bb,\eta\eta} + a_2 W_{bb,\xi\xi} + a_3 W_{bb,\eta\xi} \right) / (bc)^2
\]
\[
M_y = -D_e \left( b_3 W_{bb,\eta\eta} + b_2 W_{bb,\xi\xi} + b_1 W_{bb,\eta\xi} \right) / (bc)^2
\]
\[
M_{xy} = D_e (1-v) c (-\eta W_{bb,\eta\eta} + \xi W_{bb,\xi\xi} ) / (bc)^2
\]
\[
Q_x = -c (d_4 W_{bb,\eta\eta\eta} + d_3 W_{bb,\xi\xi\xi} + d_2 W_{bb,\eta\eta\xi} + d_1 W_{bb,\xi\xi\eta} + d_0 W_{bb,\eta\xi\xi}) D_r / (bc)^3
\]
\[
Q_y = -c (W_{bb,\eta\eta\eta} + c W_{bb,\xi\xi\xi} + e_2 W_{bb,\eta\eta\xi} + e_1 W_{bb,\xi\xi\eta}) D_e / (bc)^3
\]

Evaluating the derivatives of \( W_{bb} \) in the above formulas, the resulting expressions for the moments and shears are as follows:

\[
M_x = \left( -\frac{2}{\beta^2} \right) \left( \frac{b c}{2} \right)^2 \left[ \sum_{n=1}^{\infty} C_{\xi,n} \left( (a_1 C_{\eta,n} + a_2 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_4 + 2 d_2) \right) + \sum_{n=1}^{\infty} C_{\eta,n} \left( (a_2 C_{\eta,n} + a_3 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_3 + 2 d_1) \right) \right]
\]

\[
+ \sum_{n=1}^{\infty} C_{\xi,n} \left( (a_1 C_{\eta,n} + a_2 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_4 + 2 d_2) \right)
\]

\[
+ \sum_{n=1}^{\infty} C_{\eta,n} \left( (a_2 C_{\eta,n} + a_3 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_3 + 2 d_1) \right)
\]

\[
+ \sum_{n=1}^{\infty} C_{\xi,n} \left( (a_1 C_{\eta,n} + a_2 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_4 + 2 d_2) \right)
\]

\[
+ \sum_{n=1}^{\infty} C_{\eta,n} \left( (a_2 C_{\eta,n} + a_3 C_{\xi,n}) + \frac{2 \pi^2}{(bc)^2} (d_3 + 2 d_1) \right)
\]

\[
+ \frac{\gamma \left( \beta^2 + \pi^2 \right)}{(bc)^2} \left[ 2 A_{\xi,n} - 4 \left( 1 + i \alpha \gamma \right) + \gamma \left( \beta^2 + \pi^2 \right) \left( A_{\eta,n} + A_{\xi,n} \right) \right]
\]

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\[ M_y = \left( \frac{q}{48} \right) (bc)^2 \sum_{n=1}^{\infty} \left[ C_{n+} \frac{\bar{\xi}_{n} \eta_{n}}{\bar{\xi}_{n}} \left( c_{1} C_{5,1} n + b_{2} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( -a_{1} C_{6,1} - b_{2} C_{3,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( -a_{1} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( -a_{3} C_{3,1} + a_{4} C_{4,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( -a_{4} C_{3,1} - a_{3} C_{4,1} \right) \right] \\
+ \left( (c^2 + \eta^2) \frac{2 2 A_{50}}{3} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) + \varphi^2 \frac{2 2 A_{50} - \varphi^2 + 12 \varphi^4}{\eta^2} (1 - A_{50}) \right] \\
M_{xy} = \left( \frac{q}{48} \right) (bc)^2 \left[ 1 - v \right] \sum_{n=1}^{\infty} \left[ S_{n} \frac{\bar{\xi}_{n} \eta_{n}}{\bar{\xi}_{n}} \left( c_{1} C_{5,1} n + C_{2} C_{7,1} \right) \\
+ \frac{C_{n+} \bar{\xi}_{n} \eta_{n}}{\bar{\xi}_{n}} \left( -C_{6,1} + C_{2} C_{3,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( -C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{4} C_{1,1} + C_{3} C_{2,1} + C_{5} C_{3,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( C_{5} C_{3,1} + C_{4} C_{4,1} \right) \right] \\
- \sin \left[ 2 A_{50} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) \right] \right] \\
Q_{x} = \left( \frac{q}{48} \right) (bc)^2 \sum_{n=1}^{\infty} \left[ C_{n+} \frac{\bar{\xi}_{n} \eta_{n}}{\bar{\xi}_{n}} \left( C_{1} C_{5,1} n + C_{2} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{3,1} + C_{2,1} + C_{4,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( C_{4} C_{3,1} - a_{2} C_{4,1} \right) - 24 \eta \left( 1 + A_{50} \right) \right] \\
+ \left( (c^2 + \eta^2) \frac{2 2 A_{50}}{3} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) + \varphi^2 \frac{2 2 A_{50} - \varphi^2 + 12 \varphi^4}{\eta^2} (1 - A_{50}) \right] \\
Q_{y} = \left( \frac{q}{48} \right) (bc)^2 \sum_{n=1}^{\infty} \left[ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{1} C_{5,1} n + C_{2} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{3,1} - a_{2} C_{4,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( a_{1} C_{3,1} - a_{2} C_{4,1} \right) \right] \\
+ \left( (c^2 + \eta^2) \frac{2 2 A_{50}}{3} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) + \varphi^2 \frac{2 2 A_{50} - \varphi^2 + 12 \varphi^4}{\eta^2} (1 - A_{50}) \right] \right] \\
Q_{z} = \left( \frac{q}{48} \right) (bc)^2 \sum_{n=1}^{\infty} \left[ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{1} C_{5,1} n + C_{2} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{3,1} - a_{2} C_{4,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( a_{1} C_{3,1} - a_{2} C_{4,1} \right) \right] \\
+ \left( (c^2 + \eta^2) \frac{2 2 A_{50}}{3} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) + \varphi^2 \frac{2 2 A_{50} - \varphi^2 + 12 \varphi^4}{\eta^2} (1 - A_{50}) \right] \right] \\
Q_{w} = \left( \frac{q}{48} \right) (bc)^2 \sum_{n=1}^{\infty} \left[ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{1} C_{5,1} n + C_{2} C_{7,1} \right) \\
+ \sin \bar{\xi}_{n} \sin \bar{\eta}_{n} \left( C_{3,1} - a_{2} C_{4,1} \right) + \frac{\bar{\eta}_{n}}{\bar{\xi}_{n}} \left( a_{1} C_{3,1} - a_{2} C_{4,1} \right) \right] \\
+ \left( (c^2 + \eta^2) \frac{2 2 A_{50}}{3} - 4 + 12 \eta^2 \left( 1 + A_{50} \right) + \varphi^2 \frac{2 2 A_{50} - \varphi^2 + 12 \varphi^4}{\eta^2} (1 - A_{50}) \right] \right] \]
It is important from the point of studying the behaviour of sandwich plates to evaluate the centre deflection and the centre moments. These can be readily obtained by substituting $\eta = \zeta = 0$ in the general expressions for deflections and moments.

Thus at the centre of the plate:

\[
W_B = \sum_{n=1}^{\infty} \left( C_{5n} + C_{10n} \right) + A_{10} + 1 + 1/p^4 \left\{ \frac{(bc)^4}{48D_0} \right. \\
W_S = \sum_{n=1}^{\infty} \left[ (2cH_n B_n) C_{7n} + (2cK_n B_n) C_{3n} + B_n (2A_{2n} - 4) \right] + 2A_{40} - 9/p^2 \left\{ \frac{(bc)^4}{48D_0} \right. \\
N_x = (-\frac{9}{48})(bc)^2 \left[ \sum_{n=1}^{\infty} \left\{ (a_1 C_{5n} + a_2 C_{7n} + (a_3 C_{10n} + a_4 C_{2n} + C_{6n} - 25 C_{4n}) \right. \\
+ (c_1 + c_4)^2 (2A_{2n} - 4) + p^2 (2A_{40} - 4/p^2) \right\} \\
N_y = (-\frac{9}{48})(bc)^2 \left[ \sum_{n=1}^{\infty} \left\{ (-a_1 C_{5n} + b_2 C_{7n}) + (a_3 C_{10n} + b_4 C_{2n} + b_5 C_{3n} + b_7 C_{4n}) \right. \\
+ (c_1 + c_4)^2 (2A_{2n} - 4) + p^2 (2A_{40} - 4/p^2) \right\} \\
M_{xy} = \left( \frac{9}{48} \right) (bc)^2 c(1-v) \left[ \sum_{n=1}^{\infty} \left\{ (-a_1 C_{6n} - c_2 C_{8n}) + (a_3 C_{10n} - c_4 C_{2n} + c_6 C_{3n} - 5 C_{4n}) \right. \\
- 5 (2A_{2n} - 4) \right] \\
Q_x = 0 \\
Q_y = 0
\]
(a) **Materials & Apparatus:**

Three different kinds of sandwich plates were tested. These were sandwich plates made up of:-

Aluminium faces and
1) honeycombed paper core
2) styrafoam core
3) plywood core

The sizes and angles of skew of the first two kinds were:

a) 30 in. X 20 in., 30° skew
b) 16 in. X 14 in., 50° skew
c) 15 in. X 12 in., 30° skew.

The size and the angle of skew of the plywood core plate was 16 in. X 14 in. and 50° respectively. Altogether seven plates, three from each of the first two kinds and one from the third kind, were tested. Six strain rosette gauges, three on top and three on the bottom of the plates, were installed on each plate. The thickness of core was 1.0 inch for all plates, the thickness of the faces being 0.025 inch in every case. All rosette gauges were of the 3-gauge 45° rectangular type, having a gauge factor of 2.05 and a resistance of 120 ohms. Terminal strips (type T-50) were used to connect the lead wires to the gauge tabs. The wire leads were at least 12'-0 feet long and were made of No.26 stranded copper wire and with vinyl insulation.

Each sandwich plate was stiffened around the edges by 3 in. X 1 in. plywood stiffeners glued in between the aluminum faces so that the edges
did not get crushed and distorted under clamping conditions. To provide
support for the plates, frames shaped to suit the dimensions and skew
angles of sandwich plates were made from standard steel channels 12 in.
depth and weighing 20.7 lbs./ft. The larger frame, i.e. one for the plates
30 in. X 20 in. @ 30° skew was provided with 1/4 in. thick vertical
stiffeners welded to the web of the channels to ensure against any possible
twisting of the frame. To simulate the built-in edge condition, the
sandwich plate was sandwiched between the flange of the channels and a
1 inch thick cold-rolled steel pressure plate, 3 inches wide. The plate
was cut in such a way that it provided a clamping edge of three inches
while the width of the flange of the channel was also three inches. An
aluminum plate having the same outside dimensions and skew as the model
sandwich plate was put on top of the cold rolled frame and the whole
assembly of the model sandwich plate, pressure plate, aluminum cover plate
were clamped to the channels of the supporting frame by strong U-shaped
steel clamps. The sandwich plate was loaded by air pressure through a hole
drilled in the side of the pressure frame, the air chamber being formed by
the enclosed space between the pressure plate and the bottom of the
aluminum cover plate. This chamber was made airtight by providing gaskets
on top and bottom surface of the pressure plate. Strains were measured by
means of a switch and balancing unit and a digital strain indicator which
permitted the strains to be printed out directly in micro-inch per inch.
Several dial gauges were used to measure the deflections at various points
on the bottom of the model plates.

(b) Procedure & Results:

Each plate was cleaned and the gauge locations on top and bottom
surfaces of the plates were precisely marked. The installation of the gages followed a standard procedure as described below:

i) the spot where the rosette was to be placed was first wiped clean with acetone, sanded with silicon carbide paper.

ii) The gage location was again cleaned with a metal conditioner and then with a neutralizer.

iii) The rosette and the terminal strips were properly lined and Eastman 910 cement was applied to stick the assembly onto the plate.

iv) The gage and the terminal were left to dry for one minute during which time pressure was applied to the gage by means of the thumb.

v) The insulation of both ends of the lead wires was stripped and the bare copper strands twisted. Each lead wire consisted of three copper strands, one strand was soldered to one tab of the terminal strip while the other two were twisted together and soldered to the other tab of the terminal. The tabs of the terminal strips were then connected in turn with the tabs of the gages by means of thin copper jumper wires.

vi) To provide waterproof and mechanical protection, besides the usual highly insulating gage coating, a special transparent glue was applied to cover the entire gage and part of the lead wires.

Gage lead wires from 6 rosette gages were soldered to four 5-channel receptacles especially provided for the switch and balance unit. The unit strains were automatically printed out by a printing unit attached to the strain indicator.

All the gages and dial indicators were "Zeroed in" before loading of each plate and the zero readings were recorded for the strains and deflections as well. For each increment of loading similar readings were
taken and recorded. The average of the loading and unloading readings was computed and the resulting strains tabulated in Table II. Three strain values $e_a$, $e_b$, and $e_c$ (see Fig. 3) were obtained from each rosette gage. From these the values of the principal stresses and principal moments were calculated by the following formulae:

\[
\sigma_{\text{max.}} = \frac{E}{2} \left( \frac{e_a + e_c}{1-\nu} \pm \frac{1}{1+\nu} \sqrt{2(e_a-e_c)^2 + 2(e_b-e_a)^2} \right) \quad (36)
\]
\[
\sigma_{\text{min.}} = \frac{E}{2} \left( \frac{e_a + e_c}{1-\nu} \mp \frac{1}{1+\nu} \sqrt{2(e_a-e_c)^2 + 2(e_b-e_c)^2} \right) \quad (37)
\]
\[
\phi = \frac{1}{2} \tan^{-1} \left( \frac{2(e_a-e_c)}{e_a-e_c} \right) - 45^\circ \quad (38)
\]

where $\phi$ gives the direction in which the maximum stress occurs, this angle being measured counter-clockwise from the X-axis.

The moments are then given by:

\[
M_{\text{max.}} = t(2+\nu) \sigma_{\text{max.}} \quad (39)
\]
\[
M_{\text{min.}} = t(2+\nu) \sigma_{\text{min.}} \quad (40)
\]
TABLE I

Detailed Description of the Experimental Sandwich Panels

\( a = \text{Half of the width of a sandwich panel} \)
\( b = \text{Half of the length of a sandwich panel} \)
\( p = \frac{b}{a}, \text{ Aspect Ratio} \)
\( \Theta = \text{Angle of skew} \)

The thickness of each face, \( t = 0.012'' \) for all panels
The core thickness of sandwich plates, \( c = 1.0'' \) for all panels

<table>
<thead>
<tr>
<th>Panel No.</th>
<th>a (in)</th>
<th>b (in.)</th>
<th>p</th>
<th>\Theta</th>
<th>Core Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
<td>15.0</td>
<td>1.5</td>
<td>30°</td>
<td>Paper (Honeycomb)</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>15.0</td>
<td>1.5</td>
<td>30°</td>
<td>Styrafoam</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
<td>7.5</td>
<td>1.25</td>
<td>30°</td>
<td>Paper (Honeycomb)</td>
</tr>
<tr>
<td>4</td>
<td>6.0</td>
<td>7.5</td>
<td>1.25</td>
<td>30°</td>
<td>Styrafoam</td>
</tr>
<tr>
<td>5</td>
<td>7.0</td>
<td>8.0</td>
<td>1.14</td>
<td>50°</td>
<td>Paper (Honeycomb)</td>
</tr>
<tr>
<td>6</td>
<td>7.0</td>
<td>8.0</td>
<td>1.14</td>
<td>50°</td>
<td>Styrafoam</td>
</tr>
<tr>
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**Average Strains at Different Gage Locations for Panel 1**

All Strains are in Micro-inch per inch

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TABLE II (CONT'D)

Average Strains at Different Gage Locations for Panel 7

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**TABLE III**

**CONVERGENCE OF THEORETICAL SOLUTION FOR CENTRE DEFLECTIONS AND CENTRE MOMENTS**

For all cases $p = 1.0$ $t = .025$in $E = 10^7$ p.s.i. $G = 10^3$ p.s.i. $c = 1.0$ in.  
N = Number of harmonics used in the theoretical solution

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TABLE IV

\[ E = 10^7 \text{ p.s.i., Modulus of Elasticity of the faces} \]
\[ \nu = 0.32, \text{ Poisson's Ratio for the face material} \]
\[ q = \text{Intensity of lateral load, p.s.i.} \]
\[ t = \text{thickness of each individual face, in} \]
\[ \bar{c} = \text{thickness of core, in.} \]

\[ C_s = \frac{E_t (\bar{c} + t)^2}{2 (1-\nu^2)} \]

Core - softness of the sandwich plates

Total Deflection \[ W_T = W_B + W_S = \frac{W_T q}{48} \], in.

\[ M_x \text{ at Centre} = M_x \times \frac{q}{48}, 1\text{ b-in./in.} \]

\[ M_y \text{ at centre} = M_y \times \frac{q}{48}, 1\text{ lb-in/in.} \]

\[ M_{xy} \text{ at Centre} = M_{xy} \times \frac{q}{48}, \text{ lb-in/in} \]

\[ M_{\text{max. at Centre}} = M_{\text{max.}} \times \frac{q}{48}, \text{ lb-in./in.} \]

\[ M_{\text{min. at Centre}} = M_{\text{min.}} \times \frac{q}{48}, \text{ lb-in/in} \]

Max. Edge Moment, \[ (M_{\text{max. edge}}) = M_{\text{edge}} \times \frac{q}{48}, \text{ lb-in./in} \]
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<td>SKEW</td>
<td>CORE SHEAR MODULUS G(psi)</td>
<td>CORE SOFTNESS Cs</td>
<td>DEFLECTION AND MOMENT COEFFICIENTS</td>
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CHAPTER V

COMPARISON AND DISCUSSION OF RESULTS

Analytical results for deflections at the centre, $W_B$ and $W_s$ and the maximum moments at the centre for various harmonics are shown in Table III. The analytical solution of the governing differential equations given in Chapter III involves summation of the elements of the matrix up to $n$ terms, where $n$ is theoretically infinite. In Table III, however, three finite values of $n$ i.e., $n = 8, 10$ and $12$ are tried. Inspection of the figures in this table indicate good convergence for the centre deflections and the maximum moment at the centre. The maximum difference between the results for $n = 10$ and $n = 12$ for deflections do not exceed $1.2$ parts in $100$ parts and $1.8$ parts in $100$ parts for the maximum moment at the centre.

Analytical results for centre deflections, maximum moments at the centre and the maximum edge moments for different skews, aspect ratios, core rigidities and face thicknesses are given in Table IV. Plots of the ratio of shear deflection to bend deflection i.e. $W_s/W_B$ versus skew angles for various aspect ratios, core rigidities and face thicknesses are shown in Figures 6(a) and 6(b). Plots of maximum centre moments
versus skew angles for various aspect ratios, core rigidities and face thicknesses are shown in Figures 7, 8, 9, 10, 11 and 12.

Inspection of Table IV and Fig. 6(a) and 6(b) shows that the effect of the skew is to decrease the centre deflection. Although the panel deflection at the centre decreases with increasing skew, the shear deflection becomes more and more pronounced. Table IV and Figures 7, 8, 9, 10, 11 and 12 also show that the skew produces a decrease in the maximum centre and edge moments. Another effect of skew is to displace the location of the maximum edge moment towards the obtuse corner of the panel as shown in Figures 13, 14, 15 and 16.

From Table IV and Figures 6(a) and 6(b) it can be further seen that the effect of aspect ratio is to decrease the panel deflection at the centre. Like Skew, an increase in aspect ratio produces an increase in the contribution of the shear deflection (see the ascending curves for increasing aspect ratios in Figures 6(a) and 6(b).) The maximum centre and edge moments (see Table IV, Figures 7, 8, 9, 10, 11 and 12) decrease with increase in the aspect ratio. The effect of the aspect ratio is more appreciable than that of skew on the change in the values of the deflections, maximum centre and edge moments.

To investigate the effect of core rigidity on the behaviour of the sandwich panels, the term "core-softness" is introduced here. Let the core-softness be denoted by
Referring to Figures 6(a) and 6(b), it can be seen that the effect of increasing $C_s$ is to increase the panel deflection at the centre, and simultaneously producing rapid increase in the ratio $W_s/W_B$. For example, the magnitude of the shear deflection, $W_s$, increases from approximately 2% of the bend deflection, $W_B$, for a very low $C_s$ value to 122% of $W_B$ for high values of $C_s$. (See the plot for $C_s = 614$, $p = 1.0$, Figure 6(a) and 6(b).) Inspection of Table IV and Figures 7, 8, 9, 10, 11 and 12 reveals also that increase in core-softness value produces an increase in the maximum centre moment. It should be noted, however, that the rate of such an increase is not very appreciable as was found for total centre deflection and the ratio $W_s/W_B$. The variation in the maximum edge moment was rather unexpected. See Table IV. It can be observed that for smaller skews an increase in $C_s$ has the effect of diminishing the maximum edge moment; however, for skews of approximately 45° and larger this effect is reversed. See also Figure 17.

The effect of increasing the face thickness is to decrease the panel deflection but the contribution of the shear deflection, $W_s$, is increased. (Refer to Table IV and Figures 6(a) and 6(b).) It is also seen from Table IV that an increase in the face thickness reduces the bend deflection, $W_B$, but the shear deflection, $W_s$, increases. Furthermore, the centre moment increases with increase in face thickness (see Figures 7,
8, 9, 10, 11 and 12); also the maximum edge moment decreases for skews up to 30° and increases for skews of approximately 45° and over with increase in face thickness. A typical plot, showing this behaviour, of the ratio of maximum edge moment to the maximum centre moment is shown in Figure 17. Reference to Figures 13, 14, 15 and 16 also reveal high concentrated edge moments at the corners of the panels.

There is some interaction between the different independent variables e.g. skew, aspect ratio, core rigidity and face thickness when each of these are allowed to vary. For rigid cores, i.e., cores with low $C_s$ values, the contribution of shear deflection at the centre increases with increasing skew and aspect ratio. The rate of this increase diminishes for higher $C_s$ values, becoming quite rapid with increasing aspect ratio (see Figures 6(a) and 6(b)).

The comparison of experimental and theoretical results for centre deflections of panels 1 to 6 are shown in Fig. 18 whereas the comparison of experimental and theoretical results for centre deflections and centre moment $M_x$ for panel 7 (plywood core) are shown in Figs. 19 and 20. For low intensities of lateral loads Fig. 18 exhibits good agreement between the experimental and theoretical centre deflections, the theoretical results being on the conservative side. The value of the shear rigidity of the styrafoam core was based on the five point loading test (see Ref. 25) and that of the paper core was based on the single block loading test (see Ref. 25). It was found that theoretical centre deflections based on the core shear rigidity obtained from the five point loading test...
(see Ref. 25) were not comparable to the experimental results. This kind of discrepancy may be due to the nature of the paper core material not being a perfectly continuous one. The area of contact between the paper core and the skins was around 5 percent of the skin surface. Furthermore, the bonding between the core and the skin not being uniform may have given rise to some sort of irregularity in the behaviour of the paper core.

The agreement between the theoretical and experimental centre deflections for low intensities of loads is to be expected since with greater intensity of loading the behaviour of the plate is not in keeping with small deflection theory. The correspondence between the experimental and theoretical moments were not good for any of the first six panels. Figs. 19 and 20, however, show very good agreement between the experimental and theoretical values of centre deflection and centre moment $M_x$ for panel 7 with the plywood core. This is, of course, to be expected, since a sandwich panel with a rigid core like plywood would behave more closely to the small deflection theory than a soft core. The comparison of experimental and theoretical values of $M_y$ and $M_{xy}$ for the plywood core, however, was not good. This could have been due to the fact that in the present analysis the core stiffnesses related to plane stress components in the plane of the plate are neglected. This, obviously, does not apply for the plywood core and hence could be a cause of discrepancy.

Certain pertinent remarks could be made at this stage in a general way in connection with the discrepancies found. The deformations of a sandwich panel do not vary linearly with respect to the load as assumed by small deflection theory for any appreciable load. Further, the range
of linearity decreases rapidly with increasing core-softness.

In conclusion it is interesting to mention that the behaviour of all the first six panels was observed to the point where they ruptured. At a certain stage of loading the panels suffered permanent deformations. The strains around the obtuse corner region were suddenly reversed after progressive yielding of the edges had occurred. Fig. 21 shows a photograph of a ruptured panel wherein a sharp fracture could be observed along the edges and extending very closely to the obtuse corner region.
CHAPTER VI

CONCLUSIONS

The following conclusions can be drawn from the study presented herein:

1. Maximum centre moment increases with the core softness, $C_s$.

2. Maximum edge moment decreases with increase in the core softness, $C_s$, and the aspect ratio for low values of skew. Thus the increase in the core softness, $C_s$, has a relieving effect on the maximum edge moment for low values of skew whereas it has an augmenting effect on the maximum centre moment for all values of skew. Furthermore, this moment is located usually around the middle of the longer side and tends to be displaced toward the obtuse corner with increase in skew.

3. Shear deflection becomes predominant with increasing values of $C_s$ and for very soft cores, i.e. cores having $C_s > 292$, shear deflection could be 22% greater than the bend deflection.

4. Skew produces an increase in the contribution of shear deflection.

5. For relatively rigid cores, i.e. cores having low $C_s$ values, the increase in aspect ratio augments the shear deflection but this effect diminishes with increasing values of $C_s$.

6. Maximum centre moment decreases with an increase in the aspect ratio.

7. Significant concentrated moments at the corners were observed for panels with soft cores.

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The comparison of experimental and theoretical results show good correspondence of the centre deflections for small loads. The correspondence of the moments is rather poor except for the very rigid plywood core. This, however, is expected in the sense that with increase in $C_s$, the core softness, the range of linearity of small deflection theory upon which the present theory is based, diminishes very quickly. For a better correspondence of moments and deflections of panels with very soft cores a general theory based on nonlinear flexural behaviour of sandwich panels is desirable.
Fig. 1

Top Surface of Panels

Showing the location of Strain gages (Typical)
FIG. 2

Bottom Surface of Panels
Showing the location of Strain gages (Typical)
All co-ordinates are in dimensionless co-ordinates, $\eta$, $\xi$.

Location for strain gages are the same for top and bottom surfaces of the plate. (Typical)
RATIO OF SHEAR DEFLECTION TO BEND DEFLECTION AT CENTRE, $W_s/W_b$

$C_s = 292.6$

$C_s = 14.6$

$C_s = 1.46$

$\theta = 0.025 \text{ in.}$

$\theta = 15^\circ$ to $60^\circ$

$P = 1.0$
$P = 1.5$
$P = 2.0$

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The diagram illustrates the ratio of shear deflection to bend deflection at the center of a structure, as a function of the angle of skew, θ. The curves are labeled with different values of C_s (6.14, 3.07, 3.07) and p (1.0, 1.5, 2.0). The graph shows how the ratio changes with the angle of skew. The figure is reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
\[ (M_{\text{MAX}})_{\text{CENTRE}} = \left(\frac{M_{\text{MAX}}}{M_{\text{CENTRE}}}\right) \times \frac{q}{48} \]

- \( t = 0.025 \text{ in.} \)
- \( p = 1.0 \)
- \( C_s = 1.46 \)
- \( C_s = 14.6 \)
- \( C_s = 292.6 \)
\[
\left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}}_{\text{CENTRE}} \right) = \left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}}_{\text{CENTRE}} \right) x \frac{q}{48}
\]

\( t = 0.05 \text{ in.} \)

\( p = 1.0 \)

\( C = 3.07 \)
MOMENT COEFFICIENTS FOR MAX. CENTRE MOMENTS, \( \left( \frac{M_{\text{MAX}}}{M_{\text{MAX, CENTRE}}} \right) \)

\[
\left( \frac{M_{\text{MAX}}}{M_{\text{MAX, CENTRE}}} \right) = \left( \frac{M_{\text{MAX}}}{M_{\text{MAX, CENTRE}}} \right) \times \frac{q}{48}
\]

- \( t = 0.025 \text{ in.} \)
- \( p = 1.5 \)
- \( C = 1.46 \)
- \( C = 14.6 \)

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\[ \left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}_{\text{CENTRE}}} \right) = \left( \frac{M}{M_{\text{MAX}}_{\text{CENTRE}}} \right) \times \frac{q}{48} \]

- \( t = 0.025 \text{in.} \)
- \( p = 1.5 \)
- \( C_s = 1.46 \)
- \( C_s = 14.6 \)
- \( C_s = 292.6 \)
\[ \left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}} \right)_\text{CENTRE} = \frac{q}{48} \]

- \( t = 0.05 \text{ in.} \)
- \( p = 1.5 \)
- \( C_s = 3.07 \)
- \( C_s = 30.7 \)
- \( C_s = 614 \)
\[
\left( \frac{M_{\text{max}}}{C_{\text{centre}}} \right) = \left( \frac{M_{\text{max}}}{C_{\text{centre}}} \right) \times \frac{q}{48}
\]

\( t = 0.05 \text{ in.} \)

\( p = 1.5 \)

\( C_s = 3.07 \)

\( C_s = 30.7 \)

\( C_s = 614 \)
\( \left( \frac{M_{\text{MAX}}}{M_{\text{MAX, CENTRE}}} \right) = \left( \frac{M_{\text{MAX}}}{M_{\text{MAX, CENTRE}}} \right) \times \frac{q}{48} \)

- \( t = 0.025 \text{ in.} \)
- \( p = 2.0 \)
- \( \sigma = 1.46 \)

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\[
\left( \frac{M_{\text{MAX}}}{\text{CENTRE}} \right) = \left( \frac{M_{\text{MAX}}}{\text{CENTRE}} \right) \times \frac{q}{48}
\]

\[
t = 0.025 \text{in.}
\]
\[
p = 2.0
\]
\[
C_s = 1.46
\]
\[
C_s = 14.6
\]
\[
C_s = 292.6
\]

Fig. 11

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\[ \left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}_{\text{CENTRE}}} \right) = \left( \frac{M_{\text{MAX}}}{M_{\text{MAX}}_{\text{CENTRE}}} \right) x \left( \frac{q}{48} \right) \]

- \( t = 0.05 \)
- \( p = 2.0 \)
- \( s = 3.07 \)
- \( c = 3.07 \)

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Moment coefficients for max. centre moments: 

\[ \left( \frac{M_{\text{MAX}}}{\text{CENTRE}} \right) = \left( \frac{M_{\text{MAX}}}{\text{CENTRE}} \right) \times \frac{q}{48} \]

- \( t = 0.05 \)
- \( p = 2.0 \)
- \( C_S = 3.07 \)
- \( C_S = 3.07 \)
- \( C_S = 614 \)

Angle of skew \( \theta \)

FIG. 12

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\[(M_{\text{MAX}})_{\text{EDGE}} = -(\overline{M}_{\text{MAX}})_{\text{EDGE}} \times \frac{q}{48}\]

\[p = 1.0\]
\[\theta = 15^\circ\]

LOCATION OF MAX. EDGE MOMENT, \(\eta = -0.2\)

DIMENSIONLESS SKEW COORDINATE, \(\eta\)

**FIG. 13**

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\[(M_{\text{max}})_{\text{edge}} = -(M_{\text{max}})_{\text{edge}} \times \frac{q}{48}\]

\(p = 1.0\)

\(\theta = 30^\circ\)

**DIMENSIONLESS SKEW COORDINATE, \(\eta\)**

**Fig. 14**

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$M_{\text{MAX}} \_{\text{EDGE}} = - (M_{\text{MAX}} \_{\text{EDGE}} \times \frac{a}{48})$

$\rho = 1.0$

$\theta = 60^\circ$

Location of Max. Edge Moment, $\eta = -0.65$

Dimensionless Skew Coordinate, $\eta$

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Figure 17

Angle of skew, \( \theta \)

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FIG. 19

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Experimental results for $(M_x)_C$ at the centre... $X$

**Figure 20**

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APPENDIX I

NOMENCLATURE AND SYMBOLS USED IN THE
THEORETICAL ANALYSIS
$A_{10}, A_{30}, A_{40}, A_{50} = \text{Arbitrary constants}$

$2a = \text{Skew width of the sandwich plate}$

$B_1 = \frac{2 s \gamma D_o}{b^2 c^2 G \bar{c}}$

$B_2 = -\frac{D_o}{b^2 c^2 G \bar{c}}$

$B_3 = -\frac{b^2 D_o}{b^2 c^2 G \bar{c}}$

$2b = \text{Skew length of sandwich plate}$

$C_m, C_{2m}, \ldots = \text{Arbitrary constants}$

$C_3 = \frac{E_f t (\bar{e} + t)^2}{2(1-v^2)G\bar{c}}$, Core-softness

$c = \cos \theta$

$\bar{e} = \text{Thickness of core, in.}$

$\cosh = \text{Hyperbolic Cosine (Cosh)}$

$D_o = \frac{E_f t (\bar{e} + t)^2}{2(1-v^2)}$, bending rigidity of sandwich plate

neglecting individual rigidities of the faces & the core.

$E_f = \text{Modulus of Elasticity of the faces}$

$H_n = \alpha_n / p$

$G = \text{Modulus of shear rigidity of the core material}$

$K_n = \alpha_n / p$

$M_x, M_y = \text{Bending moments per unit length of sections of the sandwich plate perpendicular to X and Y axes respectively, positive when producing compression on the top face of the plate.}$
\( m_{xy} = \text{Twisting moment per unit length of section of the sandwich plate perpendicular to } X\text{-axis, positive when producing compression at the top face of the plate in direction of line } x = y \)

\( m_{\text{max}}, m_{\text{min}} = \text{Maximum & Minimum moments} \)

\( m = \text{Integer} \)

\( n = \text{Integer} \)

\( \rho = b/a, \text{ the skew aspect ratio} \)

\( q_x, q_y = \text{Shearing forces parallel to } Z\text{-axis per unit length of section of the sandwich plate perpendicular to } X \text{ and } Y \text{ axes respectively.} \)

\( q = \text{Uniform load per unit area} \)

\( s = \sin \theta \)

\( S_h = \text{Hyperbolic Sine} (\sinh) \)

\( t = \text{Thickness of each face plate} \)

\( W_0, W_s = \text{Deflections of the sandwich plate due to bending and transverse shear deformation, respectively.} \)
\[ Y_1 = 1 + 2E_2 \]
\[ Y_2 = \frac{1}{3} + 2E_3 \]
\[ Y_3 = 8 \left( E_2 + \frac{1}{15}p_4 \right) \]
\[ Y_4 = \left\{ 1 - \frac{1}{15}p_4 \right\} + 12E_2 - \frac{4E_3}{p_4} \]
\[ Y_5 = \left\{ \frac{1}{15} - \frac{1}{p_4} + 2E_2 - \left( \frac{12}{p_4} \right)B_3 \right\} \]
\[ Y_6 = Y_3 + 2E_2 \]
\[ Y_7 = 1 + 2B_3 \]
\[ Y_8 = 8 \left( \frac{1}{15} + \frac{B_3}{p_4} \right) \]
\[ Y_9 = 8 \left\{ \frac{-1}{15} - \frac{1}{15}p_4 \right\} + B_2 + \left( \frac{1}{p_4} \right)B_3 \]
\[ Y_{10} = \left( Y_8 - Y_3 \right) / Y_9 \]

X, Y, Z = Rectangular Co-ordinates:
\[ \alpha_{uv} = n \pi \]
\[ \delta_{uv} = Kronecker delta \]
\[ \xi_{uv} = s \eta_{uv} \xi + \alpha_{uv} \eta \]
\[ \eta, \xi = Dimensionless skew co-ordinates \]
\[ N_{1uv} = c K_{uv} \]
\[ N_{2uv} = s K_{uv} \]
\[ \theta = Skew angle \]
\[ \xi_{uv} = s k_{uv} \eta + \alpha_{uv} \xi \]
\[ v = Poisson's Ratio for the material of the face of the sandwich plate \]
\[ \epsilon_{1uv} = C H_{uv} \xi \]
\[ \epsilon_{2uv} = s H_{uv} \xi \]
DEFINITE INTEGRALS

With \( u_1 = sH_n + m\pi \); \( u_2 = sH_n - m\pi \); \( u_3 = cH_n \); \( u_4 = sH_n \)

\[ I_{1mn} = \int_{-1}^{+1} \frac{\cosh n\xi \cos \xi \cos m\pi \xi}{1 + \xi^2} \, d\xi \]
\[ = \left( u_1 \cosh u_3 \sin u_1 + u_3 \cosh u_3 \cos u_1 \right) / \left( u_1^2 + u_1^2 \right) \]
\[ + \left( u_2 \cosh u_3 \sin u_2 + u_3 \cosh u_3 \cos u_2 \right) / \left( u_2^2 + u_2^2 \right) \]

\[ I_{2mn} = \int_{-1}^{+1} \frac{s\sinh n\xi \sin \xi \cos m\pi \xi}{1 + \xi^2} \, d\xi \]
\[ = \left( -u_1 \sinh u_3 \cos u_1 + u_3 \sinh u_3 \sin u_1 \right) / \left( u_1^2 + u_1^2 \right) \]
\[ + \left( -u_2 \sinh u_3 \cos u_2 + u_3 \sinh u_3 \sin u_2 \right) / \left( u_2^2 + u_2^2 \right) \]

\[ I_{3mn} = \int_{-1}^{+1} \frac{\cosh n\xi \sin \xi \cos m\pi \xi}{1 + \xi^2} \, d\xi \]
\[ = \left[ \left( u_1^2 + u_1^2 \right) \left( u_2 \sin u_1 \cosh u_3 + u_2 \cosh u_1 \sin u_3 \right) + \left( u_2^2 + u_2^2 \right) \sin u_1 \cosh u_3 \right] / \left( u_2^2 + u_2^2 \right)^2 \]
\[ + \left[ \left( u_2 + u_2^2 \right) \left( u_3 \sin u_2 \cosh u_3 - u_2 \cosh u_2 \sin u_3 \right) + 2u_2 \cosh u_3 \sin u_2 \cosh u_3 \right] \]
\[ + \left( u_2^2 - u_2^2 \right) \sin u_2 \cosh u_3 \right] / \left( u_2^2 + u_2^2 \right)^2 \]

\[ I_{4mn} = \int_{-1}^{+1} \frac{\sinh n\xi \cos \xi \cos m\pi \xi}{1 + \xi^2} \, d\xi \]
\[ = \left[ \left( u_1^2 + u_1^2 \right) \left( u_1 \sin u_1 \sinh u_3 + u_2 \cosh u_1 \sin u_3 \right) \right] / \left( u_1^2 + u_1^2 \right)^2 \]
\[ + \left[ \left( u_2 + u_2^2 \right) \cos u_1 \sh u_3 - 2u_1 \cos u_1 \sh u_3 \right] / \left( u_2^2 + u_2^2 \right)^2 \]
\[ + \left[ \left( u_2 + u_2^2 \right) \sin u_1 \sh u_3 + u_2 \cosh u_1 \sh u_3 \right] / \left( u_2^2 + u_2^2 \right)^2 \]
\[ + \left( u_2^2 - u_2^2 \right) \cos u_2 \sh u_3 - 2u_2 \cosh u_2 \sh u_3 \right] / \left( u_2^2 + u_2^2 \right)^2 \]
\[
I_{7w} = \int_{-1}^{+1} \frac{\pi \sin \delta \sin \theta \cos \varphi \cos \varphi \cos \varphi}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
= 2 \left[ \left( u_3^2 + u_3^2 \right) (u_3 \sin u_4 \cos u_3 + u_3 \cos u_4 \cos u_3) + 2 u_3 u_4 \cos u_3 \cos u_4 \left[ (u_3^2 - u_3^2) \sin u_4 \cos u_3 \right] \right] \frac{1}{(u_3^2 + u_3^2)^2}
\]

\[
I_{8w} = \int_{-1}^{+1} \frac{\pi \sin \delta \sin \theta \cos \varphi \cos \varphi \cos \varphi}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
= 2 \left[ \left( u_3^2 + u_3^2 \right) (u_3 \sin u_4 \cos u_3 + u_3 \cos u_4 \cos u_3) + 2 u_3 u_4 \sin u_3 \cos u_4 \cos u_3 \right] \frac{1}{(u_3^2 + u_3^2)^2}
\]

\[
I_{9w}(a) = \int_{-1}^{+1} \frac{\pi \cos \theta \cos \varphi \cos \varphi \cos \varphi}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
= (-1)^{t \cdot \pi} \frac{1}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
\]

\[
I_{10w}(w) = \int_{-1}^{+1} \frac{\pi \cos \theta \cos \varphi \cos \varphi \cos \varphi}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
= (-1)^{w \cdot \pi} \frac{1}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
\]

\[
I_{30w}(w) = \int_{-1}^{+1} \frac{\pi \cos \theta \cos \varphi \cos \varphi \cos \varphi}{(\cos \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \, d\tau
= \frac{2}{\pi} (-1)^{w \cdot \pi} \left[ 1 - \frac{6}{(\pi \varphi \cos \varphi \sin \delta \sin \theta) \sin \varphi \cos \varphi} \right]
\]

With \( u_1 = \pm K \), \( u_2 = \pm K \), \( u_3 = \pm K \), \( u_4 = \pm K \), the expressions for the following definite integrals:

\[
I_{12mn}, I_{13mn}, I_{14mn}, I_{15mn}, I_{16mn}, I_{17mn}, I_{18mn}, I_{19mn}
\]

are identical to those for \( I_{10w}, I_{20w}, I_{30w}, I_{40w}, I_{50w}, I_{60w}, I_{70w}, I_{80w} \) respectively.
\[\begin{align*}
I_{12mn} &= \int_{-1}^{1} \cos \eta_{2n} \cos \eta_{2m} \cos \pi \eta \, d\eta \\
I_{13mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta \\
I_{14mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta \\
I_{15mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta \\
I_{20mn} &= \int_{-1}^{1} \sin \eta_{2n} \cos \eta_{2m} \sin \pi \eta \, d\eta \\
I_{21mn} &= \int_{-1}^{1} \cos \eta_{2n} \sin \eta_{2m} \sin \pi \eta \, d\eta \\
I_{22mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \sin \pi \eta \, d\eta \\
I_{23mn} &= \int_{-1}^{1} \sin \eta_{2n} \cos \eta_{2m} \sin \pi \eta \, d\eta \\
I_{24mn} &= \int_{-1}^{1} \cos \eta_{2n} \sin \eta_{2m} \sin \pi \eta \, d\eta \\
I_{16mn} &= \int_{-1}^{1} \sin \eta_{2n} \cos \eta_{2m} \cos \pi \eta \, d\eta \\
I_{17mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta \\
I_{18mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta \\
I_{19mn} &= \int_{-1}^{1} \sin \eta_{2n} \sin \eta_{2m} \cos \pi \eta \, d\eta
\end{align*}\]
\[ I_{24,m} = \int_{-1}^{+1} \sin \xi \cos \xi \sin m \pi \xi d\xi \]
\[ = (-u_1 \cos u_1 \sin u_2 \sin u_3 + u_3 \sin u_1 \cos u_3) / (u_1^2 + u_3^2) \]
\[ + (u_2 \cos u_2 \sin u_3 - u_3 \sin u_2 \cos u_3) / (u_2^2 + u_3^2) \]
\[ I_{25,m} = \int_{-1}^{+1} \sin \xi \sin \xi \sin m \pi \xi d\xi \]
\[ = (u_2 \sin u_2 \cos u_3 + u_3 \cos u_2 \sin u_3) / (u_2^2 + u_3^2) \]
\[ - (u_1 \sin u_1 \cos u_3 + u_3 \cos u_1 \sin u_3) / (u_1^2 + u_3^2) \]
\[ I_{26,m} = \int_{-1}^{+1} \sin \xi \cos \xi \sin m \pi \xi d\xi \]
\[ = \left[ (u_1^2 + u_2^2) (u_1 \sin u_1 \sin u_2 + u_3 \cos u_2 \cos u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ + \left[ (u_2^2 + u_3^2) (u_2 \sin u_2 \cos u_3 - 2 u_2 \sin u_3 \sin u_2 \cos u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ - \left[ (u_1^2 + u_3^2) (u_1 \sin u_1 \sin u_3 + u_3 \cos u_1 \cos u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ I_{27,m} = \int_{-1}^{+1} \cos \xi \sin \xi \cos \xi \sin m \pi \xi d\xi \]
\[ = \left[ (u_1^2 + u_3^2) (u_3 \sin u_1 \sin u_3 - u_3 \cos u_1 \cos u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ + \left[ (u_2^2 + u_3^2) (u_3 \sin u_2 \cos u_3 - 2 u_3 \sin u_2 \cos u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ - \left[ (u_1^2 + u_3^2) (u_3 \sin u_3 \cos u_1 - u_3 \cos u_1 \sin u_3) \right] / (u_1^2 + u_3^2)^2 \]
\[ I_{28,m} = \int_{-1}^{+1} \cos \xi \sin \xi \sin \xi \sin m \pi \xi d\xi \]
\[ = 2 (u_4 \sin u_4 \cos u_3 + u_3 \cos u_4 \sin u_3) / (u_4^2 + u_3^2) \]
\[ I_{29,m} = \int_{-1}^{+1} \sin \xi \sin \xi \sin \xi \sin m \pi \xi d\xi \]
\[ = 2 (u_4 \sin u_4 \cos u_3 + u_3 \cos u_4 \sin u_3) / (u_4^2 + u_3^2) \]

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\[
\begin{align*}
\phi'_{1} &= (-1)^{n} (e^2 \rho) \\
\phi_{2} &= (-1)^{n} (s \alpha \rho) \\
\phi_{3} &= (-1)^{n} (s \alpha \rho) \\
\phi_{4} &= (-1)^{n} \alpha \rho \ e^{2} \\
\phi_{5} &= (-1)^{n} s \alpha \rho \\
\phi_{6} &= (-1)^{n} s \rho \\
\phi_{7} &= (-1)^{n} (2 \alpha \rho) B_{3} \\
\phi_{8} &= (-1)^{n} (2 \alpha \rho) B_{2} \\
\phi_{9} &= C \rho (c H) \sin (s H) \\
\phi_{10} &= S \rho (c H) \cos (s H) \\
\phi_{11} &= S \rho (c H) \sin (s H) \\
\phi_{12} &= C \rho (c H) \cos (s H) \\
\phi_{13} &= C \rho (c K) \sin (s K) \\
\phi_{14} &= S \rho (c K) \cos (s K) \\
\phi_{15} &= S \rho (c K) \sin (s K) \\
\phi_{16} &= C \rho (c K) \cos (s K)
\end{align*}
\]
\[ R'_{\text{a}} \]

\[ R_{1\text{a}} = -c \Phi_{23\text{a}} \alpha_{\text{a}} \]

\[ R_{2\text{a}} = c \Phi_{24\text{a}} \alpha_{\text{a}} \]

\[ R_{3\text{a}} = -p \Phi_{23\text{a}} - c \Phi_{21\text{a}} \alpha_{\text{a}} \]

\[ R_{4\text{a}} = p \Phi_{24\text{a}} + c \Phi_{22\text{a}} \alpha_{\text{a}} \]

\[ R_{9\text{a}} = c \Phi_{22\text{a}} \alpha_{\text{a}} \]

\[ R_{10\text{a}} = c \Phi_{21\text{a}} \alpha_{\text{a}} \]

\[ R_{11\text{a}} = p \Phi_{22\text{a}} + c \Phi_{24\text{a}} \alpha_{\text{a}} \]

\[ R_{12\text{a}} = p \Phi_{21\text{a}} + c \Phi_{23\text{a}} \alpha_{\text{a}} \]

\[ R_{13\text{a}} = (cK_{\text{a}}) \Phi_{20\text{a}} \]

\[ R_{14\text{a}} = (cK_{\text{a}}) \Phi_{25\text{a}} \]

\[ R_{15\text{a}} = \Phi_{20\text{a}} + (cK_{\text{a}}) \Phi_{28\text{a}} \]

\[ R_{16\text{a}} = \Phi_{25\text{a}} + (cK_{\text{a}}) \Phi_{27\text{a}} \]

\[ R_{17\text{a}} = (-cK_{\text{a}}) \Phi_{27\text{a}} \]

\[ R_{18\text{a}} = (cK_{\text{a}}) \Phi_{28\text{a}} \]

\[ R_{19\text{a}} = -\Phi_{27\text{a}} - (cK_{\text{a}}) \Phi_{25\text{a}} \]

\[ R_{20\text{a}} = \Phi_{28\text{a}} + (cK_{\text{a}}) \Phi_{26\text{a}} \]
\[ R_{25m} = 4s \left( \bar{I}_{11m} - I_{30m} \right) \]
\[ R_{26m} = -\frac{(R_{25m})}{p^3} \]
\[ R_{31m} = (2cK_m B_2) \Phi_{28m} + \Phi_{26m} \]
\[ R_{32m} = -(2cK_m B_2) \Phi_{25m} - \Phi_{27m} \]
\[ R_{33m} = (2cK_m B_2) \Phi_{26m} + \Phi_{28m} \]
\[ R_{34m} = (2cK_m B_2) \Phi_{27m} + \Phi_{25m} \]
\[ R_{35m} = (-1)^w I_{5m}/2 \]
\[ R_{36m} = (-1)^w I_{6m}/2 \]
\[ R_{37m} = \Phi_{7m} I_{5m}/2 + (-1)^w I_{8m}/2 \]
\[ R_{38m} = \Phi_{7m} I_{6m}/2 + (-1)^w I_{7m}/2 \]
\[ R_{39m} = -(2cH_m) B_3 \Phi_{21m} - \Phi_{23m} \]
\[ R_{40m} = (2cH_m) B_3 \Phi_{22m} + \Phi_{24m} \]
\[ R_{41m} = (2cH_m) B_3 \Phi_{24m} + \Phi_{22m} \]
\[ R_{42m} = (2cH_m) B_3 \Phi_{23m} + \Phi_{21m} \]
\[ R_{47m} = (-1)^w I_{16m}/2 \]
\[ R_{48m} = (-1)^w I_{17m}/2 \]
\[ R_{47n} = \frac{\phi_{12n}}{\rho_{12}} I_{18n} + (-1)^n I_{17n} \]
\[ R_{50n} = \frac{\phi_{12n}}{\rho_{12}} I_{17n} + (-1)^n I_{18n} \]
\[ R_{51n} = 24 I_{10n} + (12 n - 2)(I_{9n})/\rho^4 \]
\[ R_{52n} = 24 \rho^4 (I_{10n}) + (12 n - 2) I_{7n} \]
\[ R_{53n} = (-R_{51n})(1+Y_{10}) \]
\[ R_{62n} = R_{26n} (1-Y_{10}) \]
\[ R_{71n} = R_{51n} (Y_{10} - 1) \]
\[ R_{80n} = (-R_{52n})(1+Y_{10}) \]
\[ R_{5mn} = (-\phi_{1n} I_{21mn} - \phi_{2n} I_{22mn}) \]
\[ R_{6mn} = (\phi_{3n} I_{22mn} - \phi_{4n} I_{23mn}) \]
\[ R_{7mn} = (-\phi_{3n} I_{22mn} - \phi_{4n} I_{23mn} - \phi_{5n} I_{25mn}) \]
\[ R_{8mn} = (\phi_{4n} I_{23mn} - \phi_{5n} I_{24mn} - \phi_{6n} I_{26mn}) \]
\[ R_{21mn} = -\phi_{4n} I_{25mn} - \phi_{5n} I_{26mn} \]
\[ R_{22mn} = \phi_{4n} I_{24mn} - \phi_{5n} I_{25mn} \]
\[ R_{23mn} = -\phi_{4n} I_{25mn} - \phi_{5n} I_{26mn} - \phi_{6n} I_{27mn} \]
\[ R_{24mn} = \phi_{4n} I_{27mn} - \phi_{5n} I_{28mn} - \phi_{6n} I_{29mn} \]
\[ R_{27mn} = \phi_{1n} I_{1mn} + (-1)^m I_{4mn} \]
\[ R_{28mn} = (-1)^m I_{1mn} \]
\[ R_{29mn} = (-1)^m I_{2mn} \]
\[ R_{30mn} = \phi_{1n} I_{2mn} + (-1)^m I_{3mn} \]
\[ R_{43mn} = (-1)^m I_{12mn} \]
\[ R_{44mn} = (-1)^m I_{13mn} \]
\[ R_{45mn} = \phi_{2n} I_{12mn} + (-1)^m I_{15mn} \]
\[ R_{46mn} = \phi_{2n} I_{13mn} + (-1)^m I_{14mn} \]
\[ R_{54mn} = R_{5mn} + \left( \frac{R_{25m} R_{47n}}{Y_I} \right) \]
\[ R_{55mn} = R_{6mn} + \left( \frac{R_{25m} R_{48n}}{Y_I} \right) \]
\[ R_{56mn} = R_{7mn} + \left( \frac{R_{25m} R_{49n}}{Y_I} \right) \]
\[ R_{57mn} = R_{8mn} + \left( \frac{R_{25m} R_{50n}}{Y_I} \right) \]
\[ R_{58mn} = R_{1m} \bar{S}_{mn} - \left( \frac{R_{25m} R_{55n}}{Y_I} \right) \]
\[ R_{59mn} = R_{2m} \bar{S}_{mn} - \left( \frac{R_{25m} R_{56n}}{Y_I} \right) \]
\[ R_{60mn} = R_{3m} \bar{S}_{mn} - \left( \frac{R_{25m} R_{57n}}{Y_I} \right) \]
\[ R_{61mn} = R_{4m} \bar{S}_{mn} - \left( \frac{R_{25m} R_{58n}}{Y_I} \right) \]
\[ R_{63mn} = R_{11m} \bar{S}_{mn} + \left( \frac{R_{47n} R_{26n}}{Y_I} \right) \]
\[ R_{64\nu} = R_{18\nu} S_{18\nu} + \frac{(R_{48\nu} R_{26\nu})}{Y_9} \]
\[ R_{65\nu} = R_{19\nu} S_{19\nu} + \frac{(R_{49\nu} R_{26\nu})}{Y_9} \]
\[ R_{66\nu} = R_{20\nu} S_{20\nu} + \frac{(R_{50\nu} R_{26\nu})}{Y_9} \]
\[ R_{67\nu} = R_{21\nu} - \frac{(R_{35\nu} R_{26\nu})}{Y_9} \]
\[ R_{68\nu} = R_{22\nu} - \frac{(R_{36\nu} R_{26\nu})}{Y_9} \]
\[ R_{69\nu} = R_{23\nu} - \frac{(R_{37\nu} R_{26\nu})}{Y_9} \]
\[ R_{70\nu} = R_{24\nu} - \frac{(R_{38\nu} R_{26\nu})}{Y_9} \]
\[ R_{72\nu} = \frac{R_{27\nu} S_{27\nu}}{Y_9} - \frac{(R_{51\nu} R_{47\nu})}{Y_9} \]
\[ R_{73\nu} = \frac{R_{28\nu} S_{28\nu}}{Y_9} - \frac{(R_{52\nu} R_{48\nu})}{Y_9} \]
\[ R_{74\nu} = \frac{R_{29\nu} S_{29\nu}}{Y_9} - \frac{(R_{53\nu} R_{49\nu})}{Y_9} \]
\[ R_{75\nu} = \frac{R_{30\nu} S_{30\nu}}{Y_9} - \frac{(R_{54\nu} R_{50\nu})}{Y_9} \]
\[ R_{76\nu} = \frac{R_{31\nu} S_{31\nu}}{Y_9} - \frac{(R_{55\nu} R_{51\nu})}{Y_9} \]
\[ R_{77\nu} = \frac{R_{32\nu} S_{32\nu}}{Y_9} - \frac{(R_{56\nu} R_{52\nu})}{Y_9} \]
\[ R_{78\nu} = \frac{R_{33\nu} S_{33\nu}}{Y_9} - \frac{(R_{57\nu} R_{53\nu})}{Y_9} \]
\[ R_{79\nu} = \frac{R_{34\nu} S_{34\nu}}{Y_9} - \frac{(R_{58\nu} R_{54\nu})}{Y_9} \]
\[ R_{81\nu} = \frac{R_{43\nu} S_{43\nu}}{Y_9} - \frac{(R_{52\nu} R_{47\nu})}{Y_9} \]
\[ R_{82\nu} = \frac{R_{44\nu} S_{44\nu}}{Y_9} - \frac{(R_{52\nu} R_{48\nu})}{Y_9} \]
\[ R_{85,ww} = R_{45,ww} + (R_{52,ww} - R_{44,ww}) / Y_f \]
\[ R_{84,ww} = R_{46,ww} + (R_{52,ww} - R_{56,ww}) / Y_f \]
\[ R_{85,ww} = \Phi_{24,ww} S_n^w - (R_{52,ww} - R_{55,ww}) / Y_f \]
\[ R_{86,ww} = \Phi_{23,ww} S_n^w - (R_{52,ww} - R_{56,ww}) / Y_f \]
\[ R_{87,ww} = R_{41,ww} S_n^w - (R_{52,ww} - R_{57,ww}) / Y_f \]
\[ R_{88,ww} = R_{42,ww} S_n^w - (R_{52,ww} - R_{58,ww}) / Y_f \]

**Symbols for \( M_x \)**

\[ a_1 = p^2 \]
\[ a_2 = -2 \sqrt{p} \]
\[ a_3 = (s^2 + \nu c^2) \]
\[ a_4 = 2 sc^3 K_n^2 (1 - \nu) \]
\[ a_5 = 2 sc^2 K_n (1 - \nu) \]
\[ a_6 = 2 c K_n (s^2 + \nu c^2) \]

**Symbols for \( M_y \)**

\[ b_1 = v p^2 \]
\[ b_2 = -2 \sqrt{v} s p \]
\[ b_3 = s^2 + \nu s^2 \]
\[ b_4 = 2 c K_n (s^2 + \nu s^2) \]
SYMBOLS FOR \( M_n \)

\[
\begin{align*}
C_1 &= -c x_n^2 \\
C_2 &= -p x_n \\
C_3 &= -2 s c^2 K_n^2 \\
C_4 &= c K_n^2 (s^2 - c^2) \\
C_5 &= K_n (s^2 - c^2) \\
C_6 &= -2 s c K_n
\end{align*}
\]

SYMBOLS FOR \( Q_x \)

\[
\begin{align*}
d_1 &= 2 p c^2 x_n^2 \\
d_2 &= -2 c^3 K_n^2 \\
d_3 &= -2 s c^2 K_n^2 \\
d_4 &= -s \\
d_5 &= -3 s p^2 \\
d_6 &= (1 + 2 s^2) p
\end{align*}
\]

SYMBOLS FOR \( G_y \)

\[
\begin{align*}
\hat{e}_1 &= p^2 \\
\hat{e}_2 &= -2 s p
\end{align*}
\]
APPENDIX II

LISTING OF THE FORTRAN PROGRAM
USED FOR THEORETICAL ANALYSIS
FLOW DIAGRAM FOR THE COMPUTER PROGRAM
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<table>
<thead>
<tr>
<th>Formula</th>
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<tbody>
<tr>
<td>( \text{CAK} = C \times \text{AK} )</td>
</tr>
<tr>
<td>( \text{CAH} = C \times \text{AH} )</td>
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<tr>
<td>( \text{TP0} = -\text{TP0} )</td>
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<td>( \text{Y}(3,N) = \text{TP0} \times S \times \text{ALF} )</td>
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<td>( \text{SAK} = S \times \text{AK} )</td>
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<tr>
<td>( \text{SAH} = S \times \text{AH} )</td>
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<tr>
<td>( \text{SH} = \sin(\text{SAH}) )</td>
</tr>
<tr>
<td>( \text{CK} = \cos(\text{SAK}) )</td>
</tr>
<tr>
<td>( \text{CH} = \cos(\text{SAH}) )</td>
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<td>( \text{CAH}2 = 2 \times \text{CAH} )</td>
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</tr>
<tr>
<td>( \text{Y}(18,N) = 2 \times (-\text{SAH} \times \text{Y}(14,N) + \text{CAH} \times \text{Y}(13,N)) / \text{AH2} )</td>
</tr>
<tr>
<td>( \text{Y}(19,N) = 2 \times (\text{AH2} \times (\text{CAH} \times \text{Y}(15,N) - \text{SAH} \times \text{Y}(16,N)) + 2 \times \text{SAH} \times \text{CAH} \times \text{Y}(14,N)) / \text{AH2} )</td>
</tr>
<tr>
<td>( \text{Y}(20,N) = -2 \times \text{TP0} \times (\text{ALF}^2 - 6) / (3 \times \text{AK}^2 \times \text{AK}^2) )</td>
</tr>
<tr>
<td>( \text{Y}(21,N) = \text{TP0} \times 2 \times \text{ALF} )</td>
</tr>
<tr>
<td>( \text{Y}(22,N) = \text{TP0} \times (\text{ALF}^2 - 6) / (3 \times \text{AK}^2 \times \text{AK}^2) )</td>
</tr>
<tr>
<td>( \text{Y}(23,N) = -\text{TP0} \times 2 \times \text{ALF} )</td>
</tr>
<tr>
<td>( \text{Y}(24,N) = -2 \times \text{TP0} \times (\text{ALF}^2 - 6) / (3 \times \text{AK}^2 \times \text{AK}^2) )</td>
</tr>
<tr>
<td>( \text{Y}(25,N) = 2 \times (\text{SAK} \times \text{Y}(9,N) + \text{CAK} \times \text{Y}(10,N)) / \text{AK2} )</td>
</tr>
<tr>
<td>( \text{Y}(26,N) = 2 \times (-\text{SAK} \times \text{Y}(10,N) + \text{CAK} \times \text{Y}(9,N)) / \text{AK2} )</td>
</tr>
<tr>
<td>( \text{Y}(27,N) = 2 \times (\text{AK2} \times (\text{CAK} \times \text{Y}(11,N) - \text{SAK} \times \text{Y}(12,N)) + 2 \times \text{CAK} \times \text{SAK} \times \text{Y}(10,N)) / \text{AK2} )</td>
</tr>
<tr>
<td>( \text{Y}(28,N) = 2 \times \text{AK2} \times (\text{SAK} \times \text{Y}(11,N) + \text{CAK} \times \text{Y}(12,N)) - 2 \times \text{CAK} \times \text{SAK} \times \text{Y}(9,N) )</td>
</tr>
<tr>
<td>( \text{Y}(29,N) = -\text{C} \times \text{ALF} \times \text{Y}(15,N) )</td>
</tr>
<tr>
<td>( \text{Y}(30,N) = \text{C} \times \text{ALF} \times \text{Y}(16,N) )</td>
</tr>
</tbody>
</table>
\[ Y(31,N) = -P*Y(15,N) - C*ALF*Y(13,N) \]
\[ Y(32,N) = P*Y(16,N) + C*ALF*Y(14,N) \]
\[ Y(33,N) = C*ALF*Y(14,N) \]
\[ Y(34,N) = C*ALF*Y(13,N) \]
\[ Y(35,N) = P*Y(14,N) + C*ALF*Y(16,N) \]
\[ Y(36,N) = P*Y(13,N) + C*ALF*Y(15,N) \]
\[ Y(37,N) = CAH*Y(10,N) \]
\[ Y(38,N) = CAH*Y(9,N) \]
\[ Y(39,N) = Y(10,N) + CAH*Y(12,N) \]
\[ Y(40,N) = Y(9,N) + CAH*Y(11,N) \]
\[ Y(41,N) = CAH*Y(11,N) \]
\[ Y(42,N) = CAH*Y(12,N) \]
\[ Y(43,N) = Y(11,N) - CAH*Y(9,N) \]
\[ Y(44,N) = Y(12,N) + CAH*Y(10,N) \]
\[ Y(45,N) = 4.*S*(Y(23,N) - Y(24,N)) \]
\[ Y(46,N) = Y(45,N)/P3 \]
\[ Y(47,N) = CAH2*B2*Y(12,N) + Y(10,N) \]
\[ Y(48,N) = -CAH2*B2*Y(9,N) - Y(11,N) \]
\[ Y(49,N) = CAH2*B2*Y(10,N) + Y(12,N) \]
\[ Y(50,N) = TP0*Y(17,N)/2. \]
\[ Y(51,N) = TP0*Y(16,N)/2. \]
\[ Y(52,N) = (Y(7,N) - Y(17,N) + TP0*Y(20,N))/2. \]
\[ Y(53,N) = (Y(7,N) - Y(18,N) + TP0*Y(19,N))/2. \]
\[ Y(54,N) = -CAH2*B3*Y(13,N) - Y(15,N) \]
\[ Y(55,N) = CAH2*B3*Y(14,N) + Y(16,N) \]
\[ Y(56,N) = CAH2*B3*Y(16,N) + Y(14,N) \]
\[ Y(57,N) = CAH2*B3*Y(15,N) + Y(13,N) \]
\[ Y(58,N) = TP0*Y(25,N)/2. \]
\[ Y(59,N) = TP0*Y(26,N)/2. \]
\[ Y(60,N) = (Y(6,N) - Y(25,N) + TP0*Y(28,N))/2. \]
\[ Y(61,N) = (Y(8,N) - Y(26,N) + TP0*Y(27,N))/2. \]
\[ Y(62,N) = 24.*Y(22,N) + (12.*B3-2.*Y(21,N)/P4 \]
\[ Y(63,N) = 24.*P4*Y(22,N) + (12.*B2-2.*Y(21,N) \]
\[ Y(64,N) = Y(45,N)*Y(14,N) \]
\[ Y(65,N) = Y(46,N)*(1. - Y10) \]
\[ Y(66,N) = Y(62,N)*Y(10-1.) \]
\[ Y(67,N) = Y(63,N)*/1. + Y10) \]
\[ Y(68,N) = CAH2*B2*Y(11,N) + Y(9,N) \]

10 \( Y(68,N) = CAH2*B2*Y(11,N) + Y(9,N) \)

DO 11 M=1,12

BM=P
BM1=BM*PI
TP0=1.0
DO 11 N=1,12
BAA=N
TP0=-TP0
AH=BAA*PI/P
AK=BAA*PI*P
U1=S*AH+BM1
U2=S*AH-BM1
U3=C*AH
SU1=SIN(U1)
SU2=SIN(U2)
CU1=COS(U1)
CU2=COS(U2)
UI3=U1+U3+U3
U13N=U1-U3+U3

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\[
\begin{align*}
U_{23} &= U_2^* U_2 + U_3^* U_3 \\
U_{23N} &= U_2^* U_2 - U_3^* U_3 \\
U_{13S} &= U_{13}^* U_{13} \\
U_{23S} &= U_{23}^* U_{23} \\
S_03 &= \exp(-U_{3T}) \\
S_{NH} &= \frac{1. - S_03}{2} \\
C_{NH} &= \frac{1. + S_03}{2} \\
PT_1 &= \left( U_{13}^* C_{NH} U_{SU1} + U_{3*SNH} U_{CU1} \right) / U_{13} \\
PT_2 &= \left( U_{23}^* C_{NH} U_{SU2} + U_{3*SNH} U_{CU2} \right) / U_{23} \\
PT_3 &= \left( -U_{1*SNH} U_{CU1} + U_{3*CNH} U_{SU1} \right) / U_{13} \\
PT_4 &= \left( -U_{2*SNH} U_{CU2} + U_{3*CNH} U_{SU2} \right) / U_{23} \\
PT_5 &= \left( U_{13}^* \left( U_{3*SU1*SNH} - U_{1*CU1*CNH} \right) + 2. U_{1*U_{3}*CU1*SNH} \right) / U_{13} \\
PT_6 &= \left( U_{23}^* \left( U_{3*SU2*SNH} - U_{2*CU2*CNH} \right) + 2. U_{2*U_{3}*CU2*SNH} \right) / U_{23} \\
PT_7 &= \left( U_{13}^* \left( U_{1*SU1*SNH} + U_{3*CU1*CNH} \right) + U_{13N} U_{CU1*SNH} \right) / U_{13} \\
PT_8 &= \left( U_{23}^* \left( U_{1*SU2*SNH} + U_{3*CU2*CNH} \right) + U_{23N} U_{CU2*SNH} \right) / U_{23} \\
U_{1(M,N)} &= PT_1 + PT_2 \\
U_{2(M,N)} &= PT_3 + PT_4 \\
U_{3(M,N)} &= PT_5 + PT_6 \\
U_{4(M,N)} &= PT_7 + PT_8 \\
U_{13(M,N)} &= PT_5 - PT_4 \\
U_{14(M,N)} &= PT_2 - PT_1 \\
U_{15(M,N)} &= PT_8 - PT_7 \\
U_{16(M,N)} &= PT_6 - PT_5 \\
U_1 &= S * A_K + B_{M1} \\
U_2 &= S * A_K - B_{M1} \\
U_3 &= C * A_K \\
S_{U1} &= \sin(U_1) \\
S_{U2} &= \sin(U_2) \\
C_{U1} &= \cos(U_1) \\
C_{U2} &= \cos(U_2) \\
U_{13} &= U_{1*U_1*U_3} U_{U_3} \\
U_{13N} &= U_{1*U_1*U_3} U_{U_3} \\
U_{23} &= U_{2*U_2*U_3} U_{U_3} \\
U_{23N} &= U_{2*U_2*U_3} U_{U_3} \\
U_{13S} &= U_{13}^* U_{U_1} \\
U_{23S} &= U_{23}^* U_{U_2} \\
U_{3T} &= 2. U_3 \\
S_03 &= \exp(-U_{3T}) \\
S_{NH} &= \frac{1. - S_03}{2} \\
C_{NH} &= \frac{1. + S_03}{2} \\
PT_1 &= \left( U_{13}^* C_{NH} U_{SU1} + U_{3*SNH} U_{CU1} \right) / U_{13} \\
PT_2 &= \left( U_{23}^* C_{NH} U_{SU2} + U_{3*SNH} U_{CU2} \right) / U_{23} \\
PT_3 &= \left( -U_{1*SNH} U_{CU1} + U_{3*CNH} U_{SU1} \right) / U_{13} \\
PT_4 &= \left( -U_{2*SNH} U_{CU2} + U_{3*CNH} U_{SU2} \right) / U_{23} \\
PT_5 &= \left( U_{13}^* \left( U_{3*SU1*SNH} - U_{1*CU1*CNH} \right) + 2. U_{1*U_{3}*CU1*SNH} \right) / U_{13} \\
PT_6 &= \left( U_{23}^* \left( U_{3*SU2*SNH} - U_{2*CU2*CNH} \right) + 2. U_{2*U_{3}*CU2*SNH} \right) / U_{23} \\
PT_7 &= \left( U_{13}^* \left( U_{1*SU1*SNH} + U_{3*CU1*CNH} \right) + U_{13N} U_{CU1*SNH} \right) / U_{13} \\
PT_8 &= \left( U_{23}^* \left( U_{1*SU2*SNH} + U_{3*CU2*CNH} \right) + U_{23N} U_{CU2*SNH} \right) / U_{23}
\end{align*}
\]
\[ \begin{align*} 
1-2, \ast U2 \ast U3 \ast U2 \ast CN1 \ast U23 S \\
U(5, M, N) = PT1 + PT2 \\
U(6, M, N) = PT3 + PT4 \\
U(7, M, N) = PT5 + PT6 \\
U(8, M, N) = PT7 + PT8 \\
U(9, M, N) = PT3 - PT4 \\
U(10, M, N) = PT2 - PT1 \\
U(11, M, N) = PT8 - PT7 \\
U(12, Y, N) = PT5 - PT6 \\
ALF = 8AA \ast P1 \\
U(17, M, N) = ALF \ast (-Y(1, N) \ast U(10, M, N) - Y(2, N) \ast U(9, M, N)) \\
U(18, M, N) = ALF \ast (Y(1, N) \ast U(9, M, N) - Y(2, N) \ast U(10, M, N)) \\
U(19, M, N) = ALF \ast (-Y(1, N) \ast U(11, M, N) - Y(3, N) \ast U(9, M, N)) \\
U(20, Y, N) = U(12, M, N) \\
U(21, M, N) = -Y(4, N) \ast U(14, M, N) - Y(5, N) \ast U(13, M, N) \\
U(22, M, N) = Y(4, N) \ast U(13, M, N) - Y(5, N) \ast U(14, M, N) \\
U(23, M, N) = -Y(4, N) \ast U(15, M, N) - Y(6, N) \ast U(13, M, N) - Y(5, N) \ast U(16, M, N) \\
U(24, M, N) = Y(4, N) \ast U(16, M, N) - Y(6, N) \ast U(13, M, N) - Y(5, N) \ast U(15, M, N) \\
U(25, M, N) = Y(7, N) \ast U(11, M, N) + TPO \ast U(4, M, N) \\
U(26, M, N) = TPO \ast U(1, M, N) \\
U(27, M, N) = TPO \ast U(2, M, N) \\
U(28, M, N) = Y(7, N) \ast U(2, M, N) + TPO \ast U(3, M, N) \\
U(29, M, N) = TPO \ast U(5, M, N) + U(4, M, N) \\
U(30, M, N) = TPO \ast U(6, M, N) \\
U(31, M, N) = Y(8, N) \ast U(5, M, N) + TPO \ast U(8, M, N) \\
U(32, M, N) = Y(8, N) \ast U(6, M, N) + TPO \ast U(7, M, N) \\
DO 12 M = 1, 96 \\
V(M) = 0.0 \\
DO 12 N = 1, 96 \\
12 BIGN(M, N) = 0.0 \\
DO 13 M = 1, 12 \\
V(M+24) = Y(64, M) \\
V(M+36) = Y(65, M) \\
V(M+48) = Y(66, M) \\
V(M+60) = Y(67, M) \\
BIGN(M, M) = Y(37, M) \\
BIGN(M, M+12) = Y(38, M) \\
BIGN(M, M+24) = Y(39, M) \\
BIGN(M, M+36) = Y(40, M) \\
BIGN(M, M+12) = Y(41, M) \\
BIGN(M, M+12, M+12) = Y(42, M) \\
BIGN(M, M+12, M+24) = Y(43, M) \\
BIGN(M, M+12, M+36) = Y(44, M) \\
BIGN(M, M+72, M+60) = Y(45, M) \\
BIGN(M, M+72, M+48) = Y(46, M) \\
BIGN(M, M+72, M+36) = Y(47, M) \\
BIGN(M, M+84, M+60) = Y(48, M) \\
BIGN(M, M+84, M+48) = Y(49, M) \\
BIGN(M, M+84, M+36) = Y(50, M) \\
BIGN(M, M+84, M+24) = Y(51, M) \\
BIGN(M, M+84, M+12) = Y(52, M) \\
BIGN(M, M+24, M+12) = Y(53, M) \\
BIGN(M, M+24, M+36) = Y(54, M) \\
BIGN(M, M+24, M+48) = Y(55, M) \\
DO 13 N = 1, 12 \\
BIGN(M, M+24, N) = U(17, M, N) + Y(45, N) \ast Y(58, N) / Y9 \\
BIGN(M, M+24, N+12) = U(18, M, N) + Y(45, N) \ast Y(59, N) / Y9 \\
BIGN(M, M+24, N+24) = U(19, M, N) + Y(45, N) \ast Y(60, N) / Y9 
\end{align*} \]
\( \text{BIGM}(M+24, N+36) = U(2C, M, N) + Y(45, M) \times Y(61, N) / Y9 \)

\( \text{BIGM}(M+36, N+48) = U(21, M, N) - Y(50, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+36, N+60) = U(22, M, N) - Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+36, N+72) = U(23, M, N) - Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+68, N+48) = U(26, M, N) + Y(50, N) \times Y(62, M) / Y9 \)

\( \text{BIGM}(M+68, N+60) = U(27, M, N) + Y(51, N) \times Y(62, M) / Y9 \)

\( \text{BIGM}(M+68, N+72) = U(25, M, N) + Y(52, N) \times Y(62, M) / Y9 \)

\( \text{BIGM}(M+68, N+84) = U(24, M, N) + Y(50, N) \times Y(62, M) / Y9 \)

\( \text{BIGM}(M+48, N+48) = U(29, M, N) + Y(50, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+48, N+60) = U(27, M, N) + Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+48, N+72) = U(25, M, N) + Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+48, N+84) = U(23, M, N) + Y(50, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+48) = U(30, M, N) + Y(51, N) \times Y(45, M) / Y9 \)

\( \text{BIGM}(M+60, N+60) = U(29, M, N) + Y(50, N) \times Y(45, M) / Y9 \)

\( \text{BIGM}(M+60, N+72) = U(27, M, N) + Y(51, N) \times Y(45, M) / Y9 \)

\( \text{BIGM}(M+60, N+84) = U(25, M, N) + Y(52, N) \times Y(45, M) / Y9 \)

\( \text{BIGM}(M+36, N+48) = U(32, M, N) + Y(50, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+36, N+60) = U(31, M, N) + Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+36, N+72) = U(29, M, N) + Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+36, N+84) = U(27, M, N) + Y(53, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+48) = U(33, M, N) + Y(50, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+60) = U(32, M, N) + Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+72) = U(30, M, N) + Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+84) = U(29, M, N) + Y(53, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+60) = U(30, M, N) + Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+72) = U(29, M, N) + Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+84) = U(27, M, N) + Y(53, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+60) = U(30, M, N) + Y(51, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+72) = U(29, M, N) + Y(52, N) \times Y(46, M) / Y9 \)

\( \text{BIGM}(M+60, N+84) = U(27, M, N) + Y(53, N) \times Y(46, M) / Y9 \)

\( \text{GO TO 13} \)

\( \text{CALL LEQU(0, BIGM, 96, 96, 7, 1, LUCK)} \)

\( \text{WRITE (6, 14) LUCK} \)

\( \text{FORMAT (I5)} \)

\( A10 = 0.0 \)

\( WB = 0.0 \)

\( WS = 0.0 \)

\( A50 = 0.0 \)

\( DO 21 N = 1, 12 \)
\[ \begin{align*}
\text{BAB} &= \text{N} \\
\text{ALF} &= \text{BAB} \cdot \text{PI} \\
\text{S00} &= \text{A50} + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+12) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+24) \\
1+\text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+36) &= \text{Y}(\text{N}+48) \cdot \text{V}(\text{N}+48) - \text{Y}(\text{N}+60) \cdot \text{V}(\text{N}+60) - \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+72) \\
2-\text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+84) &= \text{CAK} = \text{C} \cdot \text{ALF} \cdot \text{PI} \\
\text{S02} &= \text{EXP}(\text{-CAK}) \\
\text{CAH} &= \text{C} \cdot \text{ALF} / \text{P} \\
\text{S03} &= \text{EXP}(\text{-CAH}) \\
\text{CAK} &= \text{V}(\text{N}) \cdot \text{S} \cdot \text{2} \\
\text{CAH} &= \text{V}(\text{N}+48) \cdot \text{S} \cdot \text{03} \\
\text{WB} &= \text{WB} + \text{CAK} + \text{CAH} \\
\text{A}10 &= \text{A}10 + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+12) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+24) \\
1+\text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+36) &= \text{CAK} = \text{C} \cdot \text{ALF} \cdot \text{PI} \\
\text{S02} &= \text{EXP}(\text{-CAK}) \\
\text{CAH} &= \text{C} \cdot \text{ALF} / \text{P} \\
\text{S03} &= \text{EXP}(\text{-CAH}) \\
\text{CAK} &= \text{V}(\text{N}) \cdot \text{S} \cdot \text{2} \\
\text{CAH} &= \text{V}(\text{N}+48) \cdot \text{S} \cdot \text{03} \\
\text{WB} &= \text{WB} + \text{CAK} + \text{CAH} \\
\text{A}10 &= \text{A}10 + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+12) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+24) \\
1+\text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+36) &= \text{CAK} = \text{C} \cdot \text{ALF} \cdot \text{PI} \\
\text{S02} &= \text{EXP}(\text{-CAK}) \\
\text{CAH} &= \text{C} \cdot \text{ALF} / \text{P} \\
\text{S03} &= \text{EXP}(\text{-CAH}) \\
\text{AMX} &= \text{AMX} + \text{S} \cdot \text{P} \cdot \text{V}(\text{N}+48) \cdot \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+72) \\
1+\text{S} \cdot \text{P} \cdot \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+48) &= \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+48) - \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+60) - \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+72) \\
\text{AMY} &= \text{AMY} + \text{S} \cdot \text{P} \cdot \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+48) + \text{Y}(\text{S} \cdot \text{N}) \cdot \text{V}(\text{N}+72) \\
\end{align*} \]
$1 + S02 \cdot (-Y(3, N) \cdot V(N) + Y(4, N) \cdot V(N+12) + Y(8, N) \cdot V(N+24) + Y(5, N) \cdot V(N+36))$

$22 \quad AMXY = AMXY + S03 \cdot (-Y(9, N) \cdot V(N+60) - Y(10, N) \cdot V(N+84)) \cdot S02 \cdot (Y(11, N) \cdot V(N) - Y(12, N) \cdot V(N+12) + Y(14, N) \cdot V(N+24) - Y(13, N) \cdot V(N+36))$

$AMX = AMX - (S2 + ANU \cdot C2) \cdot (2 \cdot A30 - 4) - 2 \cdot P2 - A40 + P2$

$AMY = AMY - (C2 + ANU \cdot S2) \cdot (2 \cdot A30 - 4) - ANU \cdot 2 \cdot (A40 \cdot P2 - 2) / P2$

$AMXY = (AMXY - S \cdot (2 \cdot A30 - 4)) \cdot C \cdot (1 - ANU)$

$PP1 = ((A - AMY) / 2) \cdot S2$

$PP2 = AMXY \cdot S2$

$AMNT = \sqrt{PP1 + PP2}$

$AMMAX = (AMX + AMY) / 2 + AMNT$

$AMMIN = (AMX + AMY) / 2 - AMNT$

$PIPE = 2 \cdot AMXY / (AMX - AMY)$

$\alpha = 28.64789 \times \tan(PIPE)$

$\omega = \omega \times AF1$

$\omega = \omega \times AF2$

$AMXX = AMX \times AF1$

$AMMY = AMY \times AF2$

$AMMXX = AMMXX \times AF2$

$AMMIN = AMMIN \times AF2$

$WRITE(6, 151) P, THETA, THICK, EMOPGe, CBAR, BE, ANU, DG, \omega, WS, AMX, AMY, AMXY, AMMXX, AMMIN, ALPHAX$

$151 \quad FORMAT(6E16.8/5E16.8/6E16.8/)

$READ(5, 201) (TEAT(1), I=1, 21)$

$201 \quad FORMAT(21F3.1 )$

$163 \quad DO 101 L=1, 21$

$\eta = TEAT(L)$

$AMX = 0.0$

$AMY = 0.0$

$AMXY = 0.0$

$QX = 0.0$

$QY = 0.0$

$DO 93 N=1, 12$

$BAB = N$

$ALF = BAB * P$

$CAK = C * ALF / P$

$CAH = C * ALF / P$

$CAK1 = ALF * (C + S * AXI / P)$

$CAK2 = ALF * (AXI + S * ETA + P)$

$SU1 = \sin(CAK1)$

$CU1 = \cos(CAK1)$

$SU2 = \sin(CAK2)$

$CU2 = \cos(CAK2)$

$UI = CAH * (1 - AXI)$

$U2 = CAH * (1 + AXI)$

$CH = \exp(-U1)$

$SH = \exp(-U2)$

$CNH = (CH + SH) / 2.$

$SNH = (CH - SH) / 2.$

$UI = CAH * (1 - ETA)$

$U2 = CAH * (1 - ETA)$

$CH = \exp(-U2)$

$SH = \exp(-U2)$

$CNH = (CH + SH) / 2.$

$SNH = (CH - SH) / 2.$
AMX = AMX + CU1 * CNH * Y(1,N) * V(N+48) + Y(2,N) * V(N+84) + AXI * CU1 * SNH * V(N+72) + SU1 * CNH *
3 - Y(5,N) * V(N+36) + SU2 * SNK * Y(4,N) * V(N) + Y(3,N) * V(N+12) + Y(5,N) *
4 * V(N+24) + Y(6,N) * V(N+36) + ETA * CU2 * SNK * Y(3,N) * V(N+24) - Y(4,N) *
5 * V(N+36) + ETA * SU2 * CNK * Y(4,N) * V(N+24) + Y(3,N) * V(N+36)
AMY = AMY + CU1 * CNH * Y(1,N) * V(N+48) + Y(7,N) * V(N+72) + SU1 * SNH *
1 - Y(1,N) * V(N+60) + Y(7,N) * V(N+84) - AXI * Y(1,N) * (CU1 * SNH * V(N+72) +
2 + SU1 * CNH * V(N+84) + CU2 * CNK * (-Y(3,N) * V(N) + Y(4,N) * V(N+12) + Y(8,N) *
3 * V(N+24) + Y(5,N) * V(N+36) + SU2 * SNK * (-Y(4,N) * V(N) - Y(3,N) * V(N+12)
4 - Y(5,N) * V(N+24) + Y(8,N) * V(N+36) + ETA * CU2 * SNK * (-Y(3,N) * V(N+24)
5 + Y(4,N) * V(N+36) - ETA * SU2 * CNK * (Y(4,N) * V(N+24) + Y(3,N) * V(N+36)
AMXY = AMXY + SU1 * SNH * Y(9,N) * V(N+48) + Y(10,N) * V(N+72) - CU1 * CNH *
1 * Y(9,N) * V(N+60) + Y(10,N) * V(N+84) + AXI * Y(9,N) * (-CU1 * SNH * V(N+84) +
2 + SU1 * CNH * V(N+72) + CU2 * CNK * (Y(11,N) * V(N) - Y(12,N) * V(N+12) + Y(14,N) *
3 * V(N+24) - Y(13,N) * V(N+36) + SU2 * SNK * Y(12,N) * V(N) + Y(11,N) * V(N+12)
4 + Y(13,N) * V(N+24) + Y(14,N) * V(N+36) + ETA * CU2 * SNK * Y(11,N) * V(N+24)
5 - Y(12,N) * V(N+36) + ETA * SU2 * CNK * Y(12,N) * V(N+24) + Y(11,N) * V(N+36)
QX = QX + Y(15,N) * (CU1 * SNH * V(N+72) + SU1 * CNH * V(N+84)) + SU2 * CNK * Y(16,N) *
1 + V(N+24) + Y(17,N) * V(N+36) + CU2 * SNK * Y(17,N) * V(N+24) - Y(16,N) * V(N+36)
93 - QY = QY + Y(15,N) * (-SU1 * CNH * V(N+72) + CU1 * SNH * V(N+84)) + SU2 * CNK *
1 * (Y(17,N) * V(N+24) - Y(16,N) * V(N+36)) - CU2 * SNK * Y(16,N) * V(N+24)
2 + Y(17,N) * V(N+36))
AMX = AMX - (S2 + ANU * C2) * (2. * A30 -4. + 12. * ETA * ETA * (1. + A50))
1 - ANU * (2. * A40 * P2 - 4. - 12. * AXI * AXI * (1. - A50)) / P2
QN = QX + S + QY * C
PP1 = ((AMX - AMY) / 2.) ** 2
PP2 = AMXY * 2
AMNT = SQRT(PP1 + PP2)
AMMAX = (AMX + AMY) / 2. + AMNT
AMMIN = (AMX - AMY) / 2 - AMNT
PIPE = 2. * AMXY / (AMX - AMY)
ALPHA = 28.64789 * ATAN(PIPE)
AMX = AMX * AF2
AMY = AMY * AF2
AMXY = AMXY * AF2
AMMAX = AMMAX * AF2
AMMIN = AMMIN * AF2
QX = QX * AF3
QY = QY * AF3
QN = QN * AF3
101 WRITE(6,154) AMX, AMY, AMXY, AMMAX, AMMIN, ALPHA, QX, QY, CN, AXI, ETA
154 FORMAT(6E16.8/5E16.8,/) 
DO 3 L=1,21
ETA = 1.0
AXI = ETA (L)
AMX = 0.0
AMY = 0.0
AMXY = 0.0
QX = 0.0
QY = 0.0

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DO 2  N = 1, 12
B A = N
A L F = B A *( P )
C A H = C *( A L F / P )
C A K 1 = A L F *( E T A + S *( A X I / P )
C A K 2 = A L F *( A X I + S * E T A / P )
S U 1 = S I N ( C A K 1 )
C U 1 = C O S ( C A K 1 )
S U 2 = S I N ( C A K 2 )
C U 2 = C O S ( C A K 2 )
U 1 = C A H *( 1. - A X I )
U 2 = C A H *( 1. + A X I )
C H = E X P (- U 1)
S H = E X P (- U 2)
C N H = ( C H + S H ) / 2.
S N H = ( C H - S H ) / 2.
U 1 = C A K *( 1. - E T A )
U 2 = C A K *( 1. + E T A )
C H = E X P (- U 1)
S H = E X P (- U 2)
C N K = ( C H + S H ) / 2.
S N K = ( C H - S H ) / 2.
A M X = A M X * C U 1 * C N H *( Y ( 1, N ) * V ( N + 4 8 ) + Y ( 2, N ) * V ( N + 7 2 ) ) + S U 1 * S N H *
1 *( Y ( 1, N ) * V ( N + 4 0 ) + Y ( 2, N ) * V ( N + 4 4 ) ) + A X I *( C U 1 * S N H * V ( N + 7 2 ) + S U 1 * C N H *
2 *( V ( N + 8 0 ) ) + Y ( 1, N ) + C U 2 * C N K *( Y ( 3, N ) * V ( N ) - Y ( 4, N ) * V ( N + 1 2 ) + Y ( 6, N ) * V ( N + 2 4 )
3 *( Y ( 5, N ) * V ( N + 3 6 ) ) + S U 2 * S N K *( Y ( 4, N ) * V ( N + 2 4 ) + Y ( 3, N ) * V ( N + 1 2 ) + Y ( 5, N ) *
A M Y = A M Y * C U 1 * C N H *( - Y ( 1, N ) * V ( N + 4 8 ) + Y ( 7, N ) * V ( N + 7 2 ) ) + S U 1 * S N H *
1 *( - Y ( 1, N ) * V ( N + 4 0 ) + Y ( 7, N ) * V ( N + 8 4 ) ) + A X I *( Y ( 1, N ) * ( C U 1 * S N H * V ( N + 7 2 )
2 * S U 1 * C N H * V ( N + 8 4 ) ) + C U 2 * C N K *( - Y ( 3, N ) * V ( N ) + Y ( 4, N ) * V ( N + 1 2 ) + Y ( 6, N ) *
4 *( Y ( 5, N ) * V ( N + 2 4 ) + Y ( 8, N ) * V ( N + 3 6 ) ) + E T A * C U 2 * S N K *( Y ( 1, N ) * V ( N + 4 8 )
5 *( Y ( 4, N ) * V ( N + 3 6 ) ) - E T A * S U 2 * C N K *( Y ( 1, N ) * V ( N + 4 8 ) + Y ( 8, N ) * V ( N + 3 6 ) )
A M X Y = A M X Y * S U 1 * S N H *( Y ( 9, N ) * V ( N + 4 8 ) + Y ( 1 0, N ) * V ( N + 7 2 ) ) - C U 1 * C N H *
1 *( Y ( 9, N ) * V ( N + 4 0 ) + Y ( 1 0, N ) * V ( N + 8 4 ) ) + A X I *( Y ( 9, N ) * ( - C U 1 * S N H * V ( N + 8 4 )
2 * S U 1 * C N H * V ( N + 8 4 ) ) + C L 2 * C N K *( Y ( 1 1, N ) * V ( N ) - Y ( 1 2, N ) * V ( N + 1 2 ) + Y ( 1 4, N ) *
3 *( V ( N + 2 4 ) - Y ( 1 3, N ) * V ( N + 3 6 ) ) + S U 2 * S N K *( Y ( 1 2, N ) * V ( N ) + Y ( 1 1, N ) * V ( N + 1 2 )
4 *( Y ( 1 3, N ) * V ( N + 2 4 ) + Y ( 1 4, N ) * V ( N + 3 6 ) ) + E T A * C U 2 * S N K *( Y ( 1 1, N ) * V ( N + 4 8 )
5 *( Y ( 1 2, N ) * V ( N + 3 6 ) ) + E T A * S U 2 * C N K *( Y ( 1 2, N ) * V ( N + 4 8 ) + Y ( 1 1, N ) * V ( N + 3 6 ) )
Q X = Q X + Y ( 1 5, N ) * ( C U 1 * S N H * V ( N + 7 2 ) + S U 1 * C N H * V ( N + 8 4 ) ) + S U 2 * C N K *( Y ( 1 6, N ) *
1 *( V ( N + 2 4 ) + Y ( 1 7, N ) * V ( N + 3 6 ) ) + C U 2 * S N K *( Y ( 1 7, N ) * V ( N ) - Y ( 1 6, N ) * V ( N + 3 6 )
2 *( Q Y = Q Y + Y ( 1 5, N ) * ( - S U 1 * C N H * V ( N + 7 2 ) + C U 1 * S N H * V ( N + 8 4 ) ) + S U 2 * C N K *
3 *( Y ( 1 7, N ) * V ( N + 2 4 ) - Y ( 1 6, N ) * V ( N + 3 6 ) ) - C U 2 * S N K *( Y ( 1 6, N ) * V ( N + 2 4 )
4 *( Y ( 1 7, N ) * V ( N + 3 6 ) ) + S U 1 * S N K *( Y ( 1 2, N ) * V ( N + 2 4 ) + Y ( 1 1, N ) * V ( N + 3 6 ) )
Q X = Q X + Y ( 1 5, N ) * ( - S U 1 * C N H * V ( N + 7 2 ) + C U 1 * S N H * V ( N + 8 4 ) ) + S U 2 * C N K *
5 *( Y ( 1 7, N ) * V ( N + 2 4 ) - Y ( 1 6, N ) * V ( N + 3 6 ) ) - C U 2 * S N K *( Y ( 1 6, N ) * V ( N + 2 4 )
2 *( Q X = Q X + Y ( 1 5, N ) * ( - S U 1 * C N H * V ( N + 7 2 ) + C U 1 * S N H * V ( N + 8 4 ) ) + S U 2 * C N K *
3 *( Y ( 1 7, N ) * V ( N + 2 4 ) - Y ( 1 6, N ) * V ( N + 3 6 ) ) - C U 2 * S N K *( Y ( 1 6, N ) * V ( N + 2 4 )
4 *( Y ( 1 7, N ) * V ( N + 3 6 ) ) + S U 1 * S N K *( Y ( 1 2, N ) * V ( N + 2 4 ) + Y ( 1 1, N ) * V ( N + 3 6 ) )
1-2. * A 4 0 * P 2 *( 4. - 1 2. * A X I * A X I *( 1. - A 5 0 ) ) / P 2
1-2. * A 4 0 * P 2 *( 4. - 1 2. * A X I * A X I *( 1. - A 5 0 ) ) / P 2
Q Y = Q Y + Y ( 1 5, N ) * ( - A N U * S * A 3 0 - 4. + 1 2. * E T A * E T A *( 1. + A 5 0 ) ) * C *
Q N = Q X * S * Q Y * C
P P 1 = ( ( A M X - A M Y ) / 2. ) ** 2
P P 2 = A M X Y ** 2
AMNT = SQRT(P1 + P2)
AMMAX = (AMX + AMY) / 2 + AMNT
AMMIN = (AMX + AMY) / 2 - AMNT
PIPE = 2 * AMXY / (AMX - AMY)
ALPHA = 28.64789 * ATAN(PIPE)
AMX = AMX * AF2
AMY = AMY * AF2
AMXY = AMXY * AF2
AMMAX = AMMAX * AF2
AMMIN = AMMIN * AF2
QX = QX * AF3
QY = QY * AF3
QN = QN * AF3
3 WRITE (6,154) AMX, AMY, AMXY, AMMAX, AMMIN, ALPHA, QX, QY, QN, AXI, ETA
GO TO 373
END
THE FIRST LOCATION NOT USED BY THIS PROGRAM IS 46027.
REFERENCES

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Deflections of a simply supported rectangular sandwich plate subjected to transverse loads, NACA TN 2581, 1951.
VITA AUCTORIS

1937  Amit Kumar Guha-Thakurta was born in Calcutta, India, November 2, 1937.

1947  In January, 1947, he entered South Suburban (Branch) School, Calcutta, India, where he completed his primary and secondary education.

1954  In March, 1954 he passed the School Final Examination, Board of Secondary Education, West Bengal, India.

1956  In March, 1956 he passed the Intermediate Science Examination, University of Calcutta.

1961  In May, 1961, he graduated with a Bachelor in Engineering in Civil Engineering, University of Calcutta, from Bengal Engineering College, Shibpore.

1962  In August, he was employed by the Department of Irrigation and Waterways, Government of West Bengal, India, as an Assistant Engineer and worked there for four months.

1965  In the middle of December, 1961, he joined the Pask Corporation, and worked as an Engineer trainee for six months on structural design of Industrial buildings.

1965  In September, he joined the Steel Design Cell of Heavy Engineering Corporation Ltd., Ranchi, India.

1965  He resigned from Heavy Engineering Corporation, in December, 1964 and came to Windsor, in January 1965

Continued
to accept an offer of employment in the Engineering office of Canadian Bridge Co., Windsor, Canada.

In August 1966 he resigned from Canadian Bridge Co., and enrolled at the University of Windsor for the degree of Master of Applied Science in Civil Engineering.