The calculation of the loss ratio of pipe-type cable systems using the finite element numerical technique.

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THE CALCULATION OF THE LOSS RATIO
OF PIPE-TYPE CABLE SYSTEMS
USING THE FINITE ELEMENT NUMERICAL
TECHNIQUE

by

Richard John Hohendorf

A Thesis
submitted to the Faculty of Graduate Studies
through the Department of
Electrical Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
The University of Windsor

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ABSTRACT

The ratio of effective A. C. resistance to D. C. resistance, the loss ratio, is calculated for a single isolated conductor and for three conductors in various arrangements in a magnetic steel pipe.

The finite element numerical method is used to solve the dual problem to the solution of Maxwell's field equations. A simple but effective technique is used to specify the boundary condition for the single conductor and an approximate condition, based on skin depth, is used for three conductors in the pipe.

A real and imaginary part functional is presented and shown to be equivalent to the Maxwell field equation, including an eddy current term. The calculations are performed using both a complex arithmetic and a real arithmetic formulation and the program listings appear in the Appendix, including a useful iterative method for solving matrix equations.

Results for the single conductor case agree well with analytic calculations and the results for the three conductor pipe arrangements compare reasonably to measured values.
ACKNOWLEDGEMENTS

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CHAPTER 1

Introduction

1.1 Loss Ratio

One of the parameters of major importance in the selection of the correct cable size for a specific application is the resistance of the cable. The direct current resistance of the cable is easily calculated with a knowledge of only the conductor conductivity and the conductor geometry. However, under alternating current excitation, it has long been known that the resistance of a cable increases to a value greater than the d. c. value. This alternating current resistance has been termed the 'effective resistance' and the ratio of effective resistance to d. c. resistance is known as the 'loss ratio'.

Fundamentally, the increase in resistance is due to an unequal distribution of current over the conductor cross section caused by the magnetic fields of the alternating current. For d. c. excitation, the current is distributed evenly over the conductor cross-section, but under a. c. excitation, the current is concentrated in the outer regions of the conductor and the conductor cross-section is inefficiently used.
For proper cable size selection, a knowledge of the loss ratio is important because, since the d. c. resistance is easily calculated, a knowledge of the loss ratio implies a knowledge of effective, or alternating current, resistance.

Investigations have previously been undertaken to calculate the loss ratio for various cable systems. An analytic solution has been obtained for the single conductor case where axial and rotational symmetry exist. For more than one conductor, the rotational symmetry no longer exists, and an analytic solution ceases to be feasible. Efforts have been made by several investigators to derive semi-empirical expressions to describe the loss ratio for varying conductor sizes and geometries.

In this work, efforts were expended to formulate a numerical solution for the problem of finding the loss ratios of multi-conductor cable systems of varying geometries. For this reason, the electromagnetic equations describing the vector magnetic potential were solved approximately by using the finite element method, and the loss ratio was calculated from the resultant knowledge of vector magnetic potential values.

A specific type of cable system was selected for this work so that the results of the numerical calculations could be compared to measured values which were recently obtained in an industrial laboratory. The cable system in question
is a three-conductor, three phase pipe-type cable. The three cables are compact, segmental, stranded conductor cables loosely enclosed in a quarter inch thick, ten inch diameter magnetic steel pipe. Figure 1 illustrates the compact segmental cables and shows the two most common pipe configurations.

1.2 Scope and Limitations

For this work, it is requisite that the loss ratio be computed for many different cable sizes and configurations within the pipe. In the finite element method, the cable and pipe geometry is modelled by a triangular grid, with large numbers of triangles in regions where field quantities are expected to exhibit a rapid variation with a change in position. Thus, it was essential to find some manner in which the large number of grid points could be easily and quickly specified for each cable configuration of interest.

However, limits exist as to the number of grid points that can be employed for the modelling of the cable system. Each grid point results in an additional equation that must be solved and as the grid points increase in number, the set of elements in the matrix increases accordingly. For matrices of size greater than 100 by 100, solution methods such as Gaussian elimination break down because of round-off errors. For this reason, the cable configurations must be modelled somewhat coarsely. This means that round cable
FIGURE 1 CABLE GEOMETRY

(a) TYPICAL CABLE CROSS SECTION

(b) CONFIGURATIONS
or pipe contours are portrayed by fewer straight line segments than may be desirable. For example octagonal figures are used to represent the round conductors. Also, it is impossible to model the stranding of the conductors without an enormous number of grid points, so the stranded conductor is modelled by a solid conductor. A stranding factor is used to modify the conductor conductivity and thus account for the stranding.

A limit also exists as to the amount of computer time that can reasonably be used for each loss ratio computation and this time constraint results in the exclusion of a number of techniques that might otherwise have been used to specify boundary conditions for the electromagnetic equations.

It is, therefore, the goal of this work to endeavour to achieve the best possible results for loss ratio calculations within the limits which exist for cable system modelling and computer time and storage requirements.
CHAPTER 2
Loss Ratio Theory

2.1 Component Losses

Several investigators have broken down the extra losses which arise under alternating current excitation into constituent parts which are attributed to different phenomena. It was hoped that a simple theory could be found to predict each of the component losses and thus obtain the effective resistance. The theory behind the various components is explained in an American Institute of Electrical Engineers Committee Report (1).

The cable power dissipation is classified into three main types: eddy losses, proximity losses and pipe losses. All three types may occur in each part of the cable. However, the power dissipated in some cable sections is minimal and the most important losses are described below:

The extra losses that occur in a single isolated conductor under a.c. excitation are termed the conductor skin effect losses. These are related to the non-uniform distribution of current which is caused by the electromagnetic fields due to the current in the conductor itself.

If two or more conductors are brought close together, the losses increase to a value greater than those due to
skin effect. This increase is termed the conductor proximity effect and is the result of an intensification of the non-uniformity of the current distribution due to the magnetic fields of the neighbouring conductor.

The non-uniformity of current results in current flow mainly in the outer regions of the conductor, and the area of the conductor is inefficiently used, resulting in higher power dissipation than under d. c. excitation, where the current is evenly distributed. In small conductors, the current distribution is almost uniform, resulting in a small loss ratio. In very large conductors, it is thought that, under a. c. excitation, current flows in quasi-periodically positioned radial bands.

When conductors are placed in a magnetic steel pipe, the electromagnetic field is strengthened and distorted by the influence of the pipe material. This results in a further increase in the conductor losses and is known as the magnetic pipe effect.

These three conductor component losses, skin effect, proximity effect and pipe effect, contribute to the bulk of the extra a. c. losses for the cable system. However, in certain instances, other cable system component parts can contribute significantly. The shield assemblies of the three cables are periodically in contact with each other and form three phase circuits which allow the flow of
current which is induced by the magnetic fields due to the conductor currents. This circulating current flow results in increased power dissipation. An additional loss contribution is termed the shield-assembly proximity effect and occurs when local eddy currents exist in the shield. However, this component is minor in modern cables.

The one remaining significant loss element is that occurring in the pipe itself. In addition to the extra power dissipation caused in other cable parts by the pipe, there exist eddy current losses in the pipe itself which are not negligible. Pipe hysteresis loss may also exist, but this is usually small when compared to eddy loss. The use of non-magnetic pipes would not significantly reduce dissipation in the pipe unless the material had high resistivity to suppress eddy currents, high permeability to reduce the depth of current penetration and a low hysteresis loss characteristic. However, a pipe of such a material would be prohibitively expensive.

2.2 Loss Calculation Methods

Several investigators have attempted to derive expressions for the loss ratio of a magnetic pipe-type cable system using analytical, semi-empirical and empirical means.

Perhaps the earliest investigator was Lord Kelvin, who derived an analytical solution for the problem of alternating current flow in an isolated cylindrical conductor.
The solution is in terms of Bessel functions of the first kind and a similar derivation is presented by Stevenson (2). The skin effect is calculated as a function of the frequency of the current, and the permeability, conductivity, and radius of the conductor.

The proximity effect was investigated in two papers by Arnold (3), (4). In one paper, the proximity effect due to a single phase line and its return is considered and a modified Bessel function solution is presented with the stranding of the cable treated by means of a factor which modifies the resistivity. In the second paper, Arnold's formulae are extended to include multicore cables and to account for effects due to the lead sheath and armouring. The paper handles conductor stranding by considering the cross-conductor conductivity to be about one-half the normal conductivity for concentric stranded conductors and about eighty per cent for compact segmental conductors. The equations in the paper are somewhat complicated and are empirical modifications of solutions developed for simpler geometries.

Arnold's papers did not really refer to cables in steel pipes but research into the resistance of cables in steel pipes was reported by Wiseman (5) and Meyerhoff and Eager (6). Wiseman reported on loss ratio measurements that were carried out on a number of cables, and he used a
modified version of Arnold's equations to account for the magnetic pipe in calculations of the loss ratio. The Meyerhoff and Eager measurements were done on compact segmental cables. Some of the equations presented are again modifications of Arnold's equations, but some new derivations are also performed to calculate the pipe loss. An empirical factor was included in the equations to account for cable segmentation.

In an effort to consolidate the work of the foregoing and other researchers, and to standardize the equations for the loss ratio of segmental conductors in steel pipes, a report was written by the American Institute of Electrical Engineers (1). This report also investigates possible design variations for reducing the cable losses. Another paper by Neher and McGrath (7) investigates all the aspects of cable rating and includes a section on the calculation of A.C. resistance. Stranding factors for various cables are furnished and functions used in determining the losses are presented in tabular and graphical form. The paper by Neher and McGrath is the paper that is most usually quoted in recent literature for the calculation of loss ratio.

Attempts at the numerical solution of the loss ratio problem are not abundant in the literature, but Stoll (8) has used the finite difference method to calculate eddy currents and Silvester (9), (10), (11), (12) has developed
a modal analysis to calculate the a.c. resistance of certain types of conductors. Silvester's papers use an entirely different method than the technique employed in this work. However, the analysis is never applied to pipe-type cables and it is difficult to determine whether or not only the skin effect loss component is treated. Additional study of these papers is probably warranted to see if the application to pipe-type cables is valid.

2.3 Factors Affecting Loss Ratio

In the literature, many factors which could affect the loss ratio are discussed, but various researchers are not always in agreement on the effect or the importance of the factors.

Conductor conductivity and radius, current frequency and cable geometry are factors on which almost everyone agrees. Increased conductivity, radius and frequency results in a higher loss ratio and the loss ratio is usually higher for a cradled configuration than for a triangular configuration in a pipe. Wiseman (5) does report one measurement where the opposite is true.

For other factors, agreement among researchers is not good. This is true for the effect of current and temperature. Some researchers, such as Arnold, state that the only consequences of increased current are those effects caused by the increase in temperature resulting from the
increased loss. Other authors reason that, as the current is increased, more and more of the cross-sectional area is used up, thus decreasing the skin effect. A reduction of the loss ratio with an increase in current at a particular temperature is attributed to the improved current distribution at the higher current level.

With regard to the effect of temperature, it appears that an even greater difference of opinion exists. Wiseman (5) and the AIEE Committee (1) report that the difference between the a.c. and d.c. resistance does not change with temperature. As the heat level increases both the a.c. and d.c. resistance increase by the same amount and the loss ratio decreases. However, recent measurements indicate that temperature plays a much larger role and that heat cycling occurs. When a cable is heated to a high value (about 150°C) and then brought back to room level the loss ratio at room temperature is found to have increased to a value greater than that before heating. This holds true for various starting temperatures and the loss ratio at a given temperature is a function of the maximum conductor heating that occurs after installation. It is thought that the inter-strand contact resistance is permanently altered by the heating to high temperatures. Subsequent temperature cycling to the same high level does not significantly change the loss ratio and the overall change appears independent of the current magnitude at a specific temperature.
The decrease in transverse strand resistance is suspected of increasing the loss ratio by making it easier for the currents to travel in the outer portions of the cable and thus increasing the skin effect. Other researchers have also noticed the relationship between inter-strand resistance and the loss ratio. Arnold (4), who introduced a stranding factor to account for the decrease in resistance in a stranded cable, noted that the stranding factor depends on the surface condition of strands, the lay of strands, core impregnation and the tightness of insulation. Meyerhoff and Eager (6) noted that the stranding factor was influenced by cable treatment and the type of binder used. The AIEE Committee found that such uncontrollable factors as strand oxidation and handling have an effect and that even cables manufactured by the same company exhibited different characteristics. This Committee recommended that the a.c. resistance could be reduced by insulating strands to increase the transverse resistance and they found that coating the strands with varnish did reduce the loss ratio.

Other cable design parameters can also have an effect. Wiseman (5) reported that the loss ratio was less for segmental than for concentric stranded cables. And Meyerhoff and Eager (6) noted that it was dependent on the number of segments that were insulated. They also stated that the construction of the cable shield affects the loss ratio.
with high losses occurring in cables with low resistance shields and suggestions were presented for methods of increasing the shield assembly resistance.

The AIEE Committee examined a large number of possible loss ratio parameters and concluded that the loss ratio was not affected by the phase sequence of the current, by twisting the cables or by placing oil in the pipe (1). They did report that the effective resistance increased with the use of skid wires made of magnetic steel instead of the usual brass or copper, and that, although losses did not change with the use of aluminum or other non-magnetic pipe materials, they did become larger with increasing pipe size for a cradled configuration and decreased with increasing pipe size for a close triangular configuration.

In the present loss ratio analysis, the computer program models some of the parameters, but not all. The temperature is assumed to be constant at room temperature but variations could be implemented by changing conductor conductivities. The current is read into the program and both magnitude and phase can be adjusted to any value. Stranding variations can be implemented by altering the stranding factor. The shield assembly is not modelled but provision is made for the representation of conductor segmentation. Pipe material can be modified by changing the conductivity and permeability characteristics and the pipe can be made any size and thickness. The cables can be placed in any
spatial geometry, either with or without the pipe and the conductors can be any size. Provision is also made for any number of conductors and the fineness of the finite element grid can be adjusted over a fairly wide range.

These features ensure that the programme is sufficiently flexible to be useful in studying an extensive variety of practical cable configurations.
CHAPTER 3
Numerical Methods

3.1 Available Methods

Two numerical methods are widely used for the solution of differential equations and accompanying boundary conditions. The oldest method is the finite difference method where the differential operator itself is approximated. A rectangular grid is drawn over the region of interest and the differential equation is approximated by writing difference equations between neighbouring grid points. A set of equations results which is usually solved by over-relaxation iterative techniques.

The disadvantage of this method is that, for programming ease, the grid must be uniform and rectangular over the entire region of interest. This means that for irregular or curved boundaries, the grid pattern must be very fine and the boundary must be approximated in a step-wise fashion. Advances in the finite difference technique have been developed to allow an irregular grid and to permit special equations to be written at the boundaries, but this results in significant programming difficulties.

Many of the problems inherent in the use of the finite
difference method can be eliminated by the use of a different numerical procedure, the finite element method. Here, a regular grid is not required, but instead, a triangular grid is used with small triangles in regions of high interest and larger triangles elsewhere. With a judicious choice of the number and size of triangles, even complicated boundaries can be faithfully modelled, and this, basically, is the most important reason for the preference of the finite element method over the finite difference method.

3.2 The Finite Element Method

The finite element method has evolved from the work of a large number of people who were active in a variety of fields. Perhaps the most direct development of the method resulted from work on the stress analysis of trusses and beams. When a finite number of connection points did not exist in a structure under analysis, the structure was divided into sections with imaginary connection points and an approximate stress analysis was done using these points. Gradually, it was realized that this technique was related to the Rayleigh-Ritz variational technique and a mathematical basis for the finite element method was derived.

The application to electromagnetic and other fields was first demonstrated in a now famous paper by Zienkiewicz and Cheung in 1965 (13). In this paper, an energy functional was minimized jointly over each subregion in the area of interest
It was shown that the minimization of the functional was equivalent to the solution of Poisson's equation, and the finite element method was thus made available for applications in heat flow, electromagnetic phenomena and other continuous field problems.

The first step in attacking any finite element problem is to divide the domain into triangular sub-regions. The nodal points of the triangles are the variables of the problem and the field in each triangle is expressed in terms of nodal values. Usually, interpolation polynomials of order one or greater are used to characterize the field in the triangles and to ensure that the field is continuous between adjoining triangles. In this work, first order equations are used throughout, but there would be no substantial difficulty in increasing the order.

Consider a typical triangle.

The potential is assumed to be everywhere of the form

\[ \phi = A + Bx + Cy \]
Triangle \( i,j,k \) has nodal potentials \( \Phi_i \), \( \Phi_j \) and \( \Phi_k \) which must be determined to solve the problem. At each of the triangle nodes the potential must satisfy the general form

\[
\Phi_i = A + Bx_i + Cy_i \\
\Phi_j = A + Bx_j + Cy_j \\
\Phi_k = A + Bx_k + Cy_k
\]

The three variables \( A, B \) and \( C \) can be found by solving the three equations with the use of Cramer's rule. In Matrix form:

\[
\begin{bmatrix}
\Phi_i \\
\Phi_j \\
\Phi_k
\end{bmatrix} = \begin{bmatrix}
1 & x_i & y_i \\
1 & x_j & y_j \\
1 & x_k & y_k
\end{bmatrix} \begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
\]

\[
A = \frac{\begin{vmatrix}
\Phi_i & x_i & y_i \\
\Phi_j & x_j & y_j \\
\Phi_k & x_k & y_k
\end{vmatrix}}{\begin{vmatrix}
1 & x_i & y_i \\
1 & x_j & y_j \\
1 & x_k & y_k
\end{vmatrix}}
\]

\[
A = \frac{\Phi_i (x_jy_k - x_ky_j) + \Phi_j (x_ky_i - x_iy_k) + \Phi_k (x_iy_j - x_jy_i)}{x_jy_k - x_ky_j + x_ky_i - x_iy_k + x_iy_j - x_jy_i}
\]
If \[ a_i = x_jy_k - x_ky_j \]
\[ a_j = x_ky_i - x_iy_k \]
\[ a_k = x_iy_j - x_jy_i \]
then
\[
A = \frac{a_i \Phi_i + a_j \Phi_j + a_k \Phi_k}{2\Delta}
\]

Similarly
\[
B = \frac{\Phi_i(y_j - y_k) + \Phi_j(y_k - y_i) + \Phi_k(y_i - y_j)}{2\Delta}
\]
\[
C = \frac{\Phi_i(x_k - x_j) + \Phi_j(x_i - x_k) + \Phi_k(x_j - x_i)}{2\Delta}
\]

where \( \Delta \) = triangle area

By defining
\[ b_i = y_j - y_k, \quad c_i = x_k - x_j, \]
\[ b_j = y_k - y_i, \quad c_j = x_i - x_k, \]
\[ b_k = y_i - y_j, \quad c_k = x_j - x_i, \]
one can write
\[
\Phi(x,y) = \frac{(a_i + b_ix + c_iy) \Phi_i + (a_j + b_jx + c_jx) \Phi_j + (a_k + b_kx + c_ky) \Phi_k}{2\Delta}
\]

Thus, once the nodal potentials \( \Phi_i, \Phi_j \) and \( \Phi_k \) are known, the potential anywhere in the triangle is given in terms of the nodal potentials and the nodal positions.

If shape functions are defined as
\[
N_i = \frac{a_i + b_i x + c_i y}{2\Delta}
\]
then the potential for one element can be written in matrix
form:
\[ \Phi = \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} \Phi_i \\ \Phi_j \\ \Phi_k \end{bmatrix} \]

It should also be noted that, since the potential is defined as a linear function of the nodal values along a triangle edge, the potential along a triangle edge is uniquely specified and continuity between adjoining triangles is assured.

Therefore, the entire finite element problem boils down to selecting the nodal potential values in such a manner that the defining equations and boundary conditions are satisfied. This is done by a transformation into a variational form. Instead of solving the field equation directly, an expression, known as a functional, is sought such that the minimization of this functional is equivalent to solving the field equation. It may not always be easy, or even possible, to find a functional for every field equation, but when it exists, its equivalence to the field equation can be shown and the finite element method can be used.

As an example consider the equation
\[ \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu J \]
The functional whose minimization is equivalent to this equation's solution is

\[ x = \int \left\{ \left| \frac{\partial A}{\partial x} \right|^2 + \left| \frac{\partial A}{\partial y} \right|^2 - 2 \mu JA \right\} \, dx \, dy \]

A full test for equivalence appears in Appendix 1, but a quick check for equivalence is obtained by applying Euler's criteria:

\[ \frac{\partial F}{\partial w} - \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial w_x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial F}{\partial w_y} \right) = 0 \]

where \( w_x = \frac{\partial w}{\partial x} \) and \( w_y = \frac{\partial w}{\partial y} \), and where \( w \) is the field variable and \( F \) is the integrand of the functional.

For the above functional, \( W \equiv A, \) so

\[ \frac{\partial F}{\partial w} = -2 \mu A \quad \frac{\partial F}{\partial w_x} = 2 \frac{\partial A}{\partial x} \quad \frac{\partial F}{\partial w_y} = 2 \frac{\partial A}{\partial y} \]

Therefore Euler's equation gives

\[ -2 \mu J - \frac{\partial}{\partial x} \left( 2 \frac{\partial A}{\partial x} \right) - \frac{\partial}{\partial y} \left( 2 \frac{\partial A}{\partial y} \right) = 0 \quad \text{or} \]

\[ \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu J \quad \text{which is the original field equation.} \]

This, together with the derivation in Appendix 1, shows the equivalence between the solution of the field equation and the minimization of the functional. Thus, to solve the desired equation, it is necessary only to adjust the potential values until the functional is minimized. This is most easily done by differentiating the functional with
respect to each of the nodal values and setting the resulting expressions to zero. This results in a set of \( N \) equations in \( N \) unknowns where \( N \) is the number of nodes. First the expression derived for the potential is placed in the functional and this is then differentiated with respect to nodal values.

\[
x = \int \left\{ \left[ \frac{\partial A}{\partial x} \right]^2 + \left[ \frac{\partial A}{\partial y} \right]^2 \right\} \, dx \, dy - 2 \int \mu J A \, dx \, dy
\]

From the expression for potential,

\[
\frac{\partial A}{\partial x} = \begin{bmatrix} b_i & b_j & b_k \\ \frac{1}{2\Delta} \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix}
\]

\[
\frac{\partial A}{\partial y} = \begin{bmatrix} c_i & c_j & c_k \\ \frac{1}{2\Delta} \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix}
\]

\( \Delta \) is the triangle area

\[
\frac{\partial x}{\partial A_i} = \int \left\{ \left[ b_i \frac{1}{2\Delta^2} \right] + a_i \frac{c_i}{2\Delta^2} \right\} \, dx \, dy - 2 \int \mu J N_i \, dx \, dy = 0
\]

In Appendix 2, it is shown that \( \int N_i \, dx \, dy \) equals \( \frac{\Delta}{3} \).
Therefore, since \( \int dx dy = \text{Area} = \Delta \)

\[
\frac{\partial x}{\partial A_1} = \left[ b_i \frac{b_i}{\Delta} + c_i \frac{c_i}{\Delta} \right] \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} - \frac{J \mu \Delta}{3} = 0
\]

By similarly differentiating with respect to the other nodes, three equations in three unknowns result, which can be put in matrix form

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
A_i \\
A_j \\
A_k
\end{bmatrix}
= \begin{bmatrix}
\frac{\psi J \Delta}{3} \\
+ \frac{\psi J \Delta}{3} \\
+ \frac{\psi J \Delta}{3}
\end{bmatrix}
\]

where \( S_{ij} = \frac{b_i b_j + c_i c_j}{4} \)

After this matrix equation is written for each element in the region, these sub-matrices are used to form the master matrix for the entire problem. Each entry in the sub-matrix is placed into the master matrix in a position given by the overall node numbering of the triangle vertices. For example, a triangle with nodes 7, 21, and 23 will have its value of \( S_{11} \) placed in row 7, column 7 of the master matrix and \( S_{13} \) will be placed in row 7, column 23. Since adjoining triangles share the same nodes, many...
positions in the master matrix will have contributions from more than one triangle. This provides the interaction needed to assure a continuity of solution.

Once the contributions from all the triangles have been added to the master matrix, it is only necessary to impose the existing boundary conditions before the equation is solved and the solution is obtained.

3.3 Features of the Finite Element Method

From the simple description of the finite element method described above, sophistications have been applied to make the method applicable to a greater variety of problems.

For greater accuracy, the use of higher order polynomial approximations to the variation within each triangle is recommended. Zienkiewicz (14) gives a good discussion of the derivation, use, and ramifications of higher order finite elements. In order to facilitate the solving of problems with curved boundaries, several persons have developed finite elements with curved sides. Silvester and Rafinejad (15) and Ergatoudis, Iron and Zienkiewicz (16) directly employed elements with curved sides whereas Richards and Wexler (17) espoused a technique where straight-sided elements were used but integration was carried out over the curved boundary.

In addition, several authors have written about the
use of three-dimensional elements. Zienkiewicz, Bahrani and Arlett (18), (19) showed the use of tetrahedral elements as a base for building up larger eight-cornered element blocks and Ziekiewicz and Parekh (20) demonstrated the use of curved three-dimensional elements. The same paper also illustrated a possible treatment of transient problems by using finite elements in space and time. Finite elements of order one greater than the number of spatial dimensions are used and the extra element order is used to time-step the solution.

One other development that results in an even greater range of applications for the finite element technique is the use of an alternate procedure for deriving the finite element equations. This is useful for those instances where a functional cannot be found which is equivalent to the field equation which is to be solved. Several authors (14), (20), (21) have described the process wherein the weighted residual of the field equation is minimized. Usually, element shape functions are used as the weighting functions and a transformation is performed to decrease the order of the resulting system of equations. This technique greatly increases the number of problems that can be tackled. As a result, the finite element method can be used for a large variety of problems if a method for the treatment of boundary conditions can be specified.
Boundary conditions may be incorporated into the problem in either of two ways: an additional term may be added to the functional to satisfy the conditions or the final matrix may be modified after it has been formed.

Dirichlet conditions are usually applied by modifying the master matrix. For a node which is set at some potential value, the row values of the node are set to zero and the diagonal term is set to unity. The fixed potential value is then placed in the corresponding place in the source matrix. For example:

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
\Phi_1 \\
\Phi_2 \\
\Phi_3
\end{bmatrix}
= 
\begin{bmatrix}
J_1 \\
J_2 \\
J_3
\end{bmatrix}
\]

To fix the potential value of the second node to a value of $23$, the matrix becomes

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
0 & 1 & 0 \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
\Phi_1 \\
\Phi_2 \\
\Phi_3
\end{bmatrix}
= 
\begin{bmatrix}
J_1 \\
23 \\
J_3
\end{bmatrix}
\]

However, this leads to an asymmetric matrix. The matrix may be made symmetric again by the following technique.

\[
\begin{bmatrix}
S_{11} & 0 & S_{13} \\
0 & 1 & 0 \\
S_{31} & 0 & S_{33}
\end{bmatrix}
\begin{bmatrix}
\Phi_1 \\
\Phi_2 \\
\Phi_3
\end{bmatrix}
= 
\begin{bmatrix}
J_1 - S_{12} \times 23 \\
J_2 \\
J_2 - 23 \times S_{32}
\end{bmatrix}
\]
If convenient, the row and column may be eliminated and the order of the matrix reduced.

Although this is the usual way of imposing Dirichlet conditions, Hazel and Wexler (22), present a term which may be added to the functional to account for the Dirichlet conditions directly. In the same paper, the authors illustrate how the mixed or Cauchy boundary condition is handled. The condition \( \frac{\partial \phi}{\partial n} + \sigma(s) \phi(s) = h(s) \)

is imposed by including a term of the form

\[
\int [\sigma(s) \phi^2 - 2h(s)\phi] ds \quad \text{in the functional.}
\]

By letting \( \sigma(s) \) be equal to zero, Neumann conditions can be applied. The homogeneous Neumann condition, \( \frac{\partial \phi}{\partial n} = 0 \), is special to the finite element method because it is a natural boundary condition for the method such that no additional term in the functional is needed. In the absence of another applied boundary condition, the homogeneous Neumann condition is in effect. This can be seen by letting \( \sigma(s) \) and \( h(s) \) go to zero in the above equations. The functional term vanishes and the differential equation becomes \( \frac{\partial \phi}{\partial n} \bigg|_s = 0 \), which is the homogeneous Neumann condition.

3.4 Electromagnetic Equations

Before the correct functional can be found and the finite element method applied, the correct electromagnetic
equations which describe the physical problem must be determined. For this work, it was decided to formulate the equations in terms of the vector magnetic potential and the derivation of the equations was found in a paper by Stoll (8).

The basic electromagnetic equations, in the absence of displacement currents are:

\[ \nabla \times \mathbf{H} = \mathbf{J}_{\text{tot}} = \mathbf{J}_s + \mathbf{J}_e \]
\[ \nabla \cdot \mathbf{E} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \cdot \mathbf{J}_e = 0 \]
\[ \mathbf{B} = \mu \mathbf{H} \]
\[ \mathbf{J}_e = \sigma \mathbf{E} \]

Here \( \mathbf{J}_s \) is the source current, \( \mathbf{J}_e \) is the eddy current, and \( \mathbf{E} \) is the partial field related to the eddy current. The vector magnetic potential, \( \mathbf{A} \), can be defined such that

\[ \nabla \times \mathbf{A} = \mathbf{B} \]
\[ \nabla \cdot \mathbf{A} = 0 \]

The first equation is consistent with \( \nabla \cdot \mathbf{B} = 0 \), since the divergence of the curl of any vector field is identically zero. The second equation, together with the equation \( \nabla \cdot \mathbf{J}_e = 0 \), indicates that both the vector magnetic potential and the eddy current must sum to zero over the conductor area. This is seen by integrating the point form...
of the divergence expression over the volume of interest
\[ \int_{\text{vol}} \nabla \cdot \vec{A} \, dv = 0 \]

Applying the divergence theorem to this integral gives
\[ \int_{\partial V} \vec{A} \cdot d\vec{s} = 0 \]

where \( d\vec{s} \) is the area vector of the cable cross-section.
This expression indicates that the average value of vector
magnetic potential must be zero over the cable cross-
section area.

To derive the second order differential equation
consider:
\[ \nabla \times \vec{E} = - \frac{\partial}{\partial t} \vec{B} = - \frac{\partial}{\partial t} \nabla \times \vec{A} \]
\[ \vec{E} = - \frac{\partial}{\partial t} \vec{A} \quad ( + \nabla \phi \quad \text{in general} ) \]

Since
\[ \sigma \vec{E} = J_{E}, \quad \nabla \times \vec{H} = J_{e} + J_{s}, \quad \text{and} \quad \vec{B} = \mu \vec{H} \]
\[ \nabla \times \frac{1}{\mu} \vec{B} = - \sigma \frac{\partial \vec{A}}{\partial t} + J_{s} \]
or
\[ \nabla \times \frac{1}{\mu} \nabla \times \vec{A} = - \sigma \frac{\partial \vec{A}}{\partial t} + J_{s} \]

For a long conductor, it is assumed that there is no
variation of \( A_{z} \) in the z-direction and that the component
of vector magnetic potential in the z-direction is the only
component that exists. Therefore,
\[ \nabla \times \vec{A} = \frac{\partial A_{z}}{\partial y} \hat{a}_{x} - \frac{\partial A_{z}}{\partial x} \hat{a}_{y} \]
Using this result,
\[
\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = -\sigma \frac{\partial \mathbf{A}_z}{\partial t} + \mathbf{J}_s
\]

Therefore,
\[
\frac{\delta}{\delta x} \frac{1}{\mu} \frac{\partial \mathbf{A}_z}{\partial x} + \frac{\delta}{\delta y} \frac{1}{\mu} \frac{\partial \mathbf{A}_z}{\partial y} = -\sigma \frac{\partial \mathbf{A}_z}{\partial t} - \mathbf{J}_s
\]

The equation is now a scalar equation since all the components are in the z-direction.

It remains only to specify boundary conditions before the differential equation can be solved. However, for this problem, there are no real boundary conditions that can be applied. The vector potential cannot be specified to be constant anywhere and it does not go to zero at infinity. The only equation that can really be used to implement a boundary specification of sorts is \( \nabla \cdot \mathbf{A} = 0 \). This can be used as a check. If the boundary conditions are specified in some manner, they can be adjusted until the divergence equation is satisfied. When the iterated boundary conditions allow the divergence equation to be satisfied, then the boundary condition is the correct one.

Other boundary conditions may be able to be specified in the presence of special materials, such as high permea-
bility steels, or other techniques may be applied to simulate the boundary condition with the use of Green's functions or to calculate them by means of iterative techniques (22), (24), (25), (26), (27), (28).

3.5 Matrix Solution Techniques

The application of the finite element method to an electromagnetic field problem results in a large set of simultaneous equations which can be put in matrix form. The solution of the equations requires a major portion of the computer time used in the problem and, as a result, the efficiency of the solution technique is of considerable importance. Both the matrix entries and the solution vector are complex quantities and either complex computer operations must be used or two systems of real equations must be solved. In this work, both of these methods were used, with the complex arithmetic method being used predominantly.

The complex matrix equation in this work was solved by using a complex gaussian elimination subroutine called CSOLVE*. The subroutine uses a maximum pivot strategy and was used at various times in single and double precision form. CSOLVE was used in conjunction with another subroutine, CMPRUV, which was used to improve the values obtained from CSOLVE by adjusting the solution values iteratively. For each matrix row, the largest element was

* See Appendix 4
chosen and the corresponding solution vector entry was set to zero and recalculated by subtracting, from the right hand side, the vector product of the row and the solution vector and by then dividing the result by the previously found largest entry. The procedure was continued until successive solution vector changes were sufficiently small. However, the residual of the solution obtained using CSOLVE alone was invariably good enough so that no iterative adjustment was necessary, and the use of subprogram CMPRUV was dropped from later program runs.

When the real and imaginary part method of solution was used, a completely different concept was used in the solution subprogram.

In this procedure, called CONGRH, a system of equations, $Ax = b$, is solved by an iterative search technique which hunts along conjugate directions and minimizes the distance between the search vector and the solution vector at each step. If round-off errors are insignificant, the search terminates at the solution after $N$ iterations where $N$ is the order of the system.

The program CONGRH is a modification of a program written by Dr. M. Shridhar and the theoretical basis can be found in a paper by F. S. Beckman (29). The advantages of this solution method are that both less computer storage area and less computer time are needed to solve the problem.
since no zero matrix entries are stored. For one problem, a comparison between the use of CONGRH and the use of SIMQ, an IBM library subroutine, showed that CONGRH was ten times faster and required only one-third the core area. The only difficulty with the method is that it does not always converge and it is often difficult to determine why convergence is not achieved for a specific problem. A listing of the program appears in Appendix 4.

3.6 Loss Ratio From Vector Magnetic Potential

The solution of the finite element method matrix system results in a set of vector magnetic potential values for various locations within the cable system. From these potentials, a value for the loss ratio can be obtained by using the finite element equations.

The power per unit length for a conductor of conductivity $\sigma$, is given by

$$\int_{R} \frac{|\bar{J}|^2}{\sigma} \, d[\text{Area}] = \int_{R} \frac{\bar{J}^* \bar{J}}{\sigma} \, d[\text{Area}]$$

where $\bar{J}$ is the total current density.

$$\bar{J} = \bar{J}_s - j \omega \sigma \bar{A} \quad \frac{\partial \bar{A}}{\partial t} = j \omega \bar{A} \text{ for a sinusoidal input}$$

$J_s$ is the source current

$$p_{ac} = \frac{1}{\sigma} \int_{R} (\bar{J}_s - j \omega \sigma \bar{A}) \cdot (\bar{J}_s - j \omega \sigma \bar{A})^* d[\text{Area}]$$

$$= \frac{1}{\sigma} \int_{R} (\bar{J}_s \bar{J}_s^* - j \omega \sigma \bar{A} \bar{J}_s^* + j \omega \sigma \bar{A}^* \bar{J}_s + \sigma \omega^2 \bar{A}^2 \bar{J}_s^* \bar{J}_s) d[\text{Area}]$$
The loss ratio is given by the ratio of the a.c. power to the source current power

\[
\text{Loss Ratio} = \frac{P_{ac}}{\int_{R} \frac{J_s \cdot \bar{J}_s}{s} \, d\text{Area}} = \frac{P_{ac}}{\frac{P_s}{s}}
\]

\[
= 1 + \frac{\sigma \omega^2}{P_s} \int \bar{A} \cdot \bar{A}^* \, d\text{Area} + \frac{j\omega}{P_s} \int \left\{ \bar{A} \cdot \bar{J}_s - \bar{J}_s \cdot \bar{A} \right\} \, d[\text{Area}]
\]

The integrations in the loss ratio calculations are easily performed since the source current density is assumed constant and the vector potential in any triangle is expressed in polynomial form. The integration can be performed for each triangle in turn and the results summed to obtain the overall loss ratio. Since the integrands are polynomial functions, two methods exist for determining the exact result. The integration may be performed analytically, or Gaussian quadrature of a sufficient order to assure exactness may be used. Either method is equally easy, and the analytic method was used in this work.
CHAPTER 4

Single Conductor Results

4.1 Small Radius

The first calculations using the finite element method to compute a.c. resistance were done for a single, solid, small conductor with a radius of one centimeter. This conductor size was chosen so that the finite element solution could be compared with results that were available from analytic and finite difference calculations.

For this conductor size, the eddy current term in the field equation was neglected because a functional was available for this modified field equation in the paper by Zienkiewicz and Cheung (13). Without the eddy current term, \( \sigma w \frac{\delta A}{\delta t} \), the field equation is

\[
\frac{\delta^2 A}{\delta x^2} + \frac{\delta^2 A}{\delta y^2} = -w \overline{J}
\]

The corresponding functional to be minimized is

\[
x = \int \left\{ \left| \frac{\delta A}{\delta x} \right|^2 + \left| \frac{\delta A}{\delta y} \right|^2 - 2w \overline{J} A \right\} \, dx \, dy
\]

The boundary conditions used for the computations were those suggested in the paper by Stoll (8). Because of rotational symmetry in the problem, and because all boundary
points are also symmetrically placed, the potential is the same on all boundary points. Stoll suggests that the boundary potential be fixed at some arbitrary value and that the potential in the conductor be calculated using this boundary condition. The average value of potential is then calculated over the conductor area and this average value is subtracted from each of the computed potential values and the loss ratio is calculated using these adjusted values.

With the average value of potential subtracted out, the average value of potential over the conductor area is zero. This assures that the eddy currents go and return in the same path and that the following equation is satisfied:

\[ \nabla \cdot \mathbf{A} = 0 \]

The results obtained, using this method of specifying the boundary condition, and using the modified equation, compared favourably with analytic and finite difference results.

**TABLE 1**

Comparison Between Loss Ratio Calculation Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Analytic</th>
<th>Finite Difference</th>
<th>Finite Element</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0145</td>
<td>1.0142</td>
<td>1.0146</td>
</tr>
</tbody>
</table>

These results were obtained for an isolated, solid, round conductor with a conductivity of 5.80 \times 10^7 \text{ mhos per meter} and a radius of 1 centimeter. The triangular mesh for
the finite element method consisted of 196 triangles with 97 nodes. This mesh comprised five conducting rings and one insulating ring divided into sixteen segments as shown in Figure 2.

Theoretically, the accuracy of the finite element method can be increased, so that the analytic result is reached, simply by making the triangular mesh sufficiently fine, but this accuracy is not needed. The closeness of the result obtained using the relatively coarse mesh is sufficient to lend confidence to the belief that the finite element method can be used to calculate the loss ratio to a usable accuracy.

4.2 Larger Conductor Radius

After the success of the calculations for the small radius conductor, various parameters were varied to determine their effect on the answer. The conductors are divided into triangles by specifying the number of radial rings and the number of segmental divisions and the grid fineness is adjusted by changing these variables. The rings may be specified to be either conducting or insulating, the position of the rings can be adjusted, and the outer boundary can be placed at any position. Also, the outer conductor radius is adjusted so that the cross-sectional area of the straight-sided conductor model is the same as the corresponding circular conductor. In addition, the cable can be
Figure 2  Single Conductor Element Patterns

Unsegmented

Segmented
specified to be either a solid conductor or a segmental conductor by separating cable quarters by a strip of insulation. Results are presented in Table 2.

From the foregoing data, the effects of the changes in the various parameters can be seen. An increase in radius or conductivity increases the loss ratio. An increase in the number of triangles due to an increase in the number of segments results in increased accuracy. The number of insulating rings does not affect the answer at all, but a change in the position of the conducting rings (location of break-points) does change the answer slightly. Finally, it can be seen that the segmented cable has a lower loss ratio than the solid cable (1.56 compared to 1.59).

When these results were compared to the corresponding analytical results, it was found that results for small conductor cables had good correlation. However, as the conductor radius and conductivity increased, the numerical results deviated more and more from the analytic data.

For the analytic calculation of the loss ratio, formulae and tables are usually presented as a function of a variable \( m_r \), which is defined as

\[
m_r = \frac{r}{\sqrt{2 \pi f \sigma \mu}}
\]

where
- \( r \) - conductor radius
- \( f \) - source current frequency
- \( \mu \) - conductor permeability
- \( \sigma \) - conductor conductivity

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<table>
<thead>
<tr>
<th>Radius</th>
<th># Segments</th>
<th>Rings</th>
<th>Conductivity x10^7 Ω/m</th>
<th>Loss Ratio</th>
<th>Special</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>16</td>
<td>5 / 6</td>
<td>3.54</td>
<td>1.014601</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>8</td>
<td>4 / 7</td>
<td>3.54</td>
<td>1.013538</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>16</td>
<td>5 / 6</td>
<td>3.54</td>
<td>1.01348</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>8</td>
<td>4 / 7</td>
<td>3.54</td>
<td>1.013502</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>8</td>
<td>4 / 5</td>
<td>3.54</td>
<td>1.013502</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>8</td>
<td>4 / 5</td>
<td>3.54</td>
<td>1.013590</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>12</td>
<td>4 / 7</td>
<td>3.54</td>
<td>1.015406</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>12</td>
<td>4 / 5</td>
<td>3.54</td>
<td>1.014285</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>8</td>
<td>4 / 5</td>
<td>5.80</td>
<td>1.03625</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 5</td>
<td>5.80</td>
<td>1.03835</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 5</td>
<td>5.80</td>
<td>1.590352</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 5</td>
<td>5.80</td>
<td>1.622965</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>8 / 8</td>
<td>5.80</td>
<td>1.536107</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 5</td>
<td>3.54</td>
<td>1.232067</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 6</td>
<td>3.54</td>
<td>1.232067</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>12</td>
<td>4 / 7</td>
<td>3.54</td>
<td>1.232067</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>8</td>
<td>4 / 7</td>
<td>5.80</td>
<td>1.55032</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>8</td>
<td>4 / 7</td>
<td>5.80</td>
<td>1.561762</td>
<td></td>
</tr>
<tr>
<td>.791 in</td>
<td>8</td>
<td>4 / 7</td>
<td>5.80</td>
<td>1.55209</td>
<td></td>
</tr>
</tbody>
</table>

Note: The listing N/M under rings means that N of the M rings are conducting.
In an attempt to obtain a clearer view of what was happening as the conductor size increased, it was decided to plot both the analytically and numerically calculated loss ratios as a function of \( mr \), as shown in Figure 3.

**TABLE 3**

Comparison of Numerical and Analytical Loss Ratio Data for Solid Circular Conductors

<table>
<thead>
<tr>
<th>Radius</th>
<th>Conductivity</th>
<th>( mr )</th>
<th>Loss Ratio Range</th>
<th>Analytic L. R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>3.54E7</td>
<td>1.295</td>
<td>1.0135 - 1.0154</td>
<td>1.0145</td>
</tr>
<tr>
<td>.791 in</td>
<td>5.80E7</td>
<td>3.33</td>
<td>1.53 - 1.62</td>
<td>1.43</td>
</tr>
<tr>
<td>1 cm</td>
<td>5.80E7</td>
<td>1.658</td>
<td>1.036 - 1.038</td>
<td>1.038</td>
</tr>
<tr>
<td>.791 in</td>
<td>3.54E7</td>
<td>2.602</td>
<td>1.23</td>
<td>1.20</td>
</tr>
</tbody>
</table>

From the table and the graph, the deviation of numerical loss ratio values from the values obtained analytically is clearly evident as the radius and conductivity increase. Upon evaluation, it was decided that this difference was due to the deletion of the eddy current term from the field equation and the functional, and it was therefore decided to formulate a functional to include the eddy term. The functional chosen was

\[
x = \int \left\{ \left| \frac{\delta \bar{A}}{\delta x} \right|^2 + \left| \frac{\delta \bar{A}}{\delta y} \right|^2 - 2 \mu \frac{\delta \bar{A}}{\delta z} + \sigma \mu \bar{A} \frac{\delta \bar{A}}{\delta t} \right\} \ dx \ dy
\]

For the time harmonic case, the term \( \frac{\delta \bar{A}}{\delta t} \) can be replaced by the expression \( j \omega \bar{A} \), since \( \bar{A} = \bar{A}_0 e^{j \omega t} \) and \( \frac{\delta \bar{A}}{\delta t} = j \omega \bar{A}_0 e^{j \omega t} \).
FIGURE 3: ANALYTICAL AND NUMERICAL LOSS RATIO (NO EDDY TERMS)
Since $e^{j\omega t}$ is common to all terms in the equation, it can be removed and the functional becomes

$$x = \int \left\{ \left| \frac{\partial \overline{A}_o}{\partial x} \right|^2 + \left| \frac{\partial \overline{A}_o}{\partial y} \right|^2 - 2 \mu J \overline{A}_o + j\omega \mu \sigma \overline{A}_o^2 \right\} \, dx \, dy$$

By applying Euler's equation, $\frac{\partial F}{\partial A} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial A_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial A_y}\right) = 0$,

$$\frac{\partial F}{\partial A} = -2\mu J + 2j\omega \mu \sigma \overline{A}, \quad \frac{\partial F}{\partial A_x} = 2 \frac{\partial \overline{A}}{\partial x}, \quad \frac{\partial F}{\partial A_y} = 2 \frac{\partial \overline{A}}{\partial y}.$$

$$\therefore -2\mu J + 2j\omega \mu \sigma \overline{A} - 2 \frac{\partial^2 \overline{A}}{\partial x^2} - 2 \frac{\partial^2 \overline{A}}{\partial y^2} = 0$$

This is the required field equation with eddy terms included.

The chosen functional must be minimized with respect to the nodal potentials and, since this has been done for the other terms already, this must be done only for the eddy term, $j\omega \mu \int \overline{A}_o^2 \, dx \, dy$.

By inserting the finite element expression for $\overline{A}$, this integration is easily evaluated for each triangle. The derivation appears in Appendix 3.

Using the new finite element equations, the loss ratio was again calculated for the large conductor. However, once again the results were not close to the known analytic values. After many possible reasons for the discrepancy were examined and rejected, it was decided that the method of specifying the boundary conditions was no longer valid for the extended functional. This was because, with the
addition of the eddy current term, the vector magnetic potential values became complex numbers instead of real numbers as was the case previously. Therefore, although the potential values on the boundary were still the same everywhere on the boundary, the relative size of the real and imaginary parts of the boundary value could not be arbitrarily fixed. Since there was no way to determine what the complex boundary constant should be, it was decided to try an iterative search technique whose purpose was to alter the boundary constant with the goal of minimizing the average value of potential over the conductor area.

The first technique used was a simple one. The initial boundary value was set at zero, the matrix equation was solved, and the average potential value was determined. This average value was then subtracted from the initial boundary value and the result was used as the new boundary condition. The system was solved again and the process repeated until the average potential value became sufficiently small. This technique is a negative feedback approach and the results obtained were very good. The search pattern, when plotted, was shown to be an inward spiral and convergence was achieved in every case examined. This was the case no matter what initial boundary constant was selected and this can be seen in the plot showing the real part and the imaginary part of the boundary constant plotted in the
complex plane with the number of iterations for a particular conductor size as a parameter. The particular search pattern is not only a function of the parameter \( m_r \). It also depends on the grid pattern used to model the conductor and the position of the boundary.

From the plot, it can be seen that, usually, about ten iterations were required to achieve completion of the first curl of the spiral and so achieve a bound for the solution value. For a more exact answer, the number of iterations can be increased. However, with this technique it is necessary to solve the matrix of equations for each iterative step and this is very time consuming. This fact means that the number of iterations is limited in practice and prevents this technique from being used for unsymmetric problems where more than one boundary constant must be found.

However, for this symmetric problem results could be determined for various cable sizes within a reasonable amount of time.

The effect of changing the various parameters can be seen from the chart. The segmented cables can be seen to produce lower loss ratios than the solid conductors. However, the validity of the segmental results is questionable because the rotational symmetry that was assumed in the boundary specification technique disappears when the conductor is segmented, and the boundary specification is no longer correct.
### TABLE 4

<table>
<thead>
<tr>
<th>Radius (in.)</th>
<th># of Rings</th>
<th># of Segm.</th>
<th># of Iter.</th>
<th>Conduct. x10^7 N/m</th>
<th>CVA x10^-10 wb/m</th>
<th>Loss Ratio</th>
<th>Special</th>
</tr>
</thead>
<tbody>
<tr>
<td>.791</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>3.54</td>
<td>.0016</td>
<td>1.1905</td>
<td></td>
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<td>.791</td>
<td>4</td>
<td>8</td>
<td>20</td>
<td>5.80</td>
<td>.0001</td>
<td>1.4217</td>
<td></td>
</tr>
<tr>
<td>.842</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>5.80</td>
<td>.0007</td>
<td>1.4938</td>
<td></td>
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<td>4</td>
<td>8</td>
<td>12</td>
<td>5.80</td>
<td>.0002</td>
<td>1.3993</td>
<td>Segmented</td>
</tr>
<tr>
<td>.751</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>5.51</td>
<td>.009</td>
<td>1.3128</td>
<td>Segmented</td>
</tr>
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<td>.791</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>5.51</td>
<td>.0116</td>
<td>1.3735</td>
<td>Segmented</td>
</tr>
<tr>
<td>.751</td>
<td>4</td>
<td>8</td>
<td>8</td>
<td>5.80</td>
<td>.0097</td>
<td>1.3409</td>
<td>Segmented</td>
</tr>
<tr>
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<td>5</td>
<td>8</td>
<td>6</td>
<td>5.80</td>
<td>.06</td>
<td>1.4456</td>
<td>Arbitrary Starting Point</td>
</tr>
<tr>
<td>.842</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>5.80</td>
<td>.038</td>
<td>1.5059</td>
<td></td>
</tr>
<tr>
<td>.991</td>
<td>4</td>
<td>8</td>
<td>20</td>
<td>5.80</td>
<td>.001</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td>.791</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>5.80</td>
<td>.01</td>
<td>1.4319</td>
<td>Under-relaxation</td>
</tr>
<tr>
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<td>8</td>
<td>10</td>
<td>2.90</td>
<td>.0005</td>
<td>1.1048</td>
<td></td>
</tr>
<tr>
<td>.393</td>
<td>4</td>
<td>8</td>
<td>10</td>
<td>3.54</td>
<td>.0000</td>
<td>1.013</td>
<td></td>
</tr>
</tbody>
</table>

Note: CVA is the average potential value over the conductor
However, the unsegmented cable results should be correct, and they can be compared with corresponding analytic results.

**TABLE 5**

Comparison of Numerical and Analytical Loss Ratio

Data for Solid Circular Conductors

<table>
<thead>
<tr>
<th>Radius (in)</th>
<th>Conduct. ( \mu \text{m} )</th>
<th>mr</th>
<th>Num. Loss Ratio</th>
<th>Analytic L. R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>.791</td>
<td>3.54E7</td>
<td>2.60</td>
<td>1.19</td>
<td>1.20</td>
</tr>
<tr>
<td>.791</td>
<td>5.80E7</td>
<td>3.33</td>
<td>1.42</td>
<td>1.43</td>
</tr>
<tr>
<td>.842</td>
<td>5.80E7</td>
<td>3.55</td>
<td>1.50</td>
<td>1.51</td>
</tr>
<tr>
<td>.991</td>
<td>5.80E7</td>
<td>4.17</td>
<td>1.75</td>
<td>1.74</td>
</tr>
<tr>
<td>.791</td>
<td>2.90E7</td>
<td>2.36</td>
<td>1.13</td>
<td>1.14</td>
</tr>
<tr>
<td>.393</td>
<td>3.54E7</td>
<td>1.29</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

From this data and from the corresponding graph, it can be seen that a much better agreement exists between the analytical and numerical results with the eddy current term included.

However, in spite of the success of the calculation technique, as evidenced by these results, it was decided to investigate several possible ways to improve the efficiency of the method.

First, the basic technique was applied, but with the boundary changes either under or over-relaxed. This did not
seem to have any effect on the result. Next, a different method of varying the boundary constant was tried. The program was reorganized so that a Hooke and Jeeves search could be used to vary the boundary constant while minimizing the average potential. Although this search method also converged to the right answer, it was found to be slower than the negative feedback approach.

Other program modifications were also implemented to vary the way in which the eddy current term was handled, since an independent confirmation of the correctness of the chosen functional was not available. As an alternate method, it was decided to treat the eddy current term as part of the source current term. Initially, the problem was solved without an eddy term. Then, using these results, a tentative value for the eddy term was calculated and added to the source current. This process was repeated in conjunction with a Hooke and Jeeves search to determine the boundary constant and convergence was quite rapid to the correct answer. A program listing appears in Appendix 4.

The source current approach was also tried with negative feedback boundary specification and convergence was also good here.

One final eddy term variation was tried in which a simplified method of approximating the eddy term was employed. The potential in a triangle was assumed to be the
average of the three nodal potentials. This method gave fairly accurate results but has no firm theoretical basis.

Overall, the results of the calculations for the loss ratio of a single solid conductor were good and compared very well with the analytic results available.
CHAPTER 5

Three Conductor Results

5.1 Formulation Difficulties

The major goal of this work is the calculation of the loss ratio for three phase cables in a magnetic steel pipe. There are, however, many difficulties encountered in going from the single phase calculations to this new problem.

First, it was necessary to devise a computer program to formulate the element pattern for the extended problem. It was necessary that the three conductors and the pipe be modelled with sufficient flexibility to allow the positions of the conductors to be shifted as required to model various cable configurations. In addition it was necessary to be able to vary the grid fineness without forming greatly elongated triangles. Such a program has been written to automatically generate the finite elements. A listing and several computer-generated plots illustrating its capability appear in Appendices 4 and 6. Basically, the position and size of the conductors are first specified and triangles are formed within the conductor cross-section. Segmentation can be implemented if desired. Then, a rectangular grid, with variable grid size is placed over the region and the positions of grid points on the boundary are modified to

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conform to the circular contour of the pipe. In doing this, the best fit is made between the grid points and the conductor elements in order to obtain good triangles in the area around the conductors. Finally, triangles are formed in one or more circular layers to represent the pipe. Extensive use has proven this program to be adequate for forming the grid for almost any desired cable configuration.

As soon as this pattern generation program was operational, the finite element method was applied to the three phase problem. Initially, a zero potential boundary condition was assumed with no pipe present. The eddy term was not included and the result was quite accurate when applied to small diameter conductors.

However, for larger conductors, the inclusion of the eddy term is necessary. This complicates both the functional and the boundary conditions.

With the addition of the eddy term, the vector magnetic potential becomes complex and the terms in the functional are also complex. The squared magnitude of a complex number equals the number times its conjugate and this results in the addition of new variables to account for the modelling of the conjugate vector magnetic potential. In addition, it has not been possible to find a functional such that the Euler equation is satisfied for both the vector potential and its conjugate, without adding on a
term that is equal to zero. However, it was suggested that this problem might possibly be eliminated if all the equations were expressed in terms of real and imaginary parts and a functional was formulated to satisfy the necessary criteria. For the field equation
\[ \frac{\partial^2 \overline{A}}{\partial x^2} + \frac{\partial^2 \overline{A}}{\partial y^2} = -\mu \overline{J} + j \omega \sigma \mu \overline{A}, \]
the real and imaginary part equations are:
\[ \frac{\partial^2 \overline{A}_R}{\partial x^2} + \frac{\partial^2 \overline{A}_R}{\partial y^2} = -\mu \overline{J}_R - \omega \sigma \mu \overline{A}_I \quad \text{and} \]
\[ \frac{\partial^2 \overline{A}_I}{\partial x^2} + \frac{\partial^2 \overline{A}_I}{\partial y^2} = -\mu \overline{J}_I + \omega \sigma \mu \overline{A}_R, \]
where \( \overline{A}_R \) is the real part of the vector potential and \( \overline{A}_I \) is the imaginary part.

The functional chosen to correspond to these equations is:
\[ x = \int \left\{ \left| \frac{\partial \overline{A}_R}{\partial x} \right|^2 + \left| \frac{\partial \overline{A}_R}{\partial y} \right|^2 - \left| \frac{\partial \overline{A}_I}{\partial x} \right|^2 - \left| \frac{\partial \overline{A}_I}{\partial y} \right|^2 \right\} \ dx \ dy - 2 \sigma \mu \overline{A}_R \overline{A}_I + 2 \mu \overline{J}_I \overline{A}_I - 2 \mu \overline{J}_R \overline{A}_R \]

Applying Euler's equation twice, gives
\[ -2 \mu \sigma \omega \overline{A}_I - 2 \mu \overline{J}_R - 2 \frac{\partial^2 \overline{A}_R}{\partial x^2} - 2 \frac{\partial^2 \overline{A}_R}{\partial y^2} = 0 \quad \text{and} \]
\[ -2 \mu \sigma \omega \overline{A}_R + 2 \mu \overline{J}_I + 2 \frac{\partial^2 \overline{A}_I}{\partial x^2} + 2 \frac{\partial^2 \overline{A}_I}{\partial y^2} = 0. \]
These two equations correspond exactly to the real and imaginary part equations stated previously. A more detailed proof of the equivalence is presented in Appendix 1.

The use of the real and imaginary part functional results in the need for twice as many variables as with the complex formulation and the final solution matrix is four times the size. However, this problem may be reduced by using a solution technique which does not store zero elements. In this work, a conjugate gradient routine was used for this reason. This subprogram, CHGHH, is a search routine that solves a matrix equation, \( A x = B \), by minimizing the quadratic, \( y = \frac{1}{2} x^T A x - B^T x \). At the point of minimization, the gradient of the quadratic is zero:

\[
\nabla y = A x - B = 0 \quad \therefore \quad A x = B
\]

Thus, the values of \( x \) that minimize the quadratic are also those values that satisfy the matrix equation.

It was, however, also desired to continue using the complex formulation and this was done by examining the matrix which resulted from employing the real and imaginary part functional. When this functional was minimized, the matrix equation that resulted had the following form, for each triangular element:
The $S_{ij}$ terms have been previously defined and the $E_{ij}$ terms are the corresponding entries resulting from the minimization of the eddy current term. $A_R$, $A_I$; and $J_R$, and $J_I$ are the real and imaginary parts of the vector potential and source current density respectively.

This matrix can be obtained by manipulating the complex number formulation of the problem. Consider the following matrix equation and then separate it into real and imaginary parts:

$$
\begin{bmatrix}
S - jE & A_R - jA_I \\
-J & J_R - jJ_I
\end{bmatrix} = k
\begin{bmatrix}
J_R - jJ_I
\end{bmatrix}
$$

This matrix has the same form and the same entries as the real and imaginary part matrix. Thus, the correct matrix can be obtained using complex arithmetic if the source
current is conjugated and the eddy current term is subtracted from the original C-matrix to form a complex S-matrix. The resulting answer must also be conjugated to get the correct value of vector potential.

From program test runs, it has been found that both formulations give the same answer. It has also been observed that the matrix equation used for the single conductor calculations is simply the complex conjugate of the modified complex equation described above.

However, from observations of both three phase and single phase calculations, it was noticed that the sign of the right hand side and the sign of the eddy current term do not affect the answer unless Dirichelet boundary conditions are specified. This is logical since the sign of the right hand side only determines the current direction and the eddy current term is in quadrature.

Thus, from these results, it can be concluded that the problem of the determination of the correct functional for this particular application has been resolved through the use of a functional in real and imaginary part form and by the complex formulation derived from this functional.

5.2 Boundary Conditions

The remaining major difficulty with the three phase calculations is the specification of a boundary condition. There is no real condition that can be specified since there
is no equipotential surface and nothing can actually be said about the potential except that the average value over the conductor area should be zero. It is even very difficult to impose the far field boundary condition since the vector magnetic potential does not approach zero at infinity. Therefore some approximations were necessary in order to establish a useable condition.

For the first calculations with the cables in air, a Dirichelet condition was set at a distance far from the cables by treating the conductors as filamentary conductors and summing the contributions by superposition. However, this method is not accurate, especially for large conductors.

It is also not possible to set the boundary condition by using a search technique such as the one used in the single conductor calculations because there is no simple symmetry in the three phase case. This means that a multi-variable search would be necessary with many cost function evaluations and the computer time used would be of the order of hours.

The possibility does exist, however, that a technique based on work already described in the literature may soon be developed to handle the problem. Silvester and Hsieh (25), Hsieh (26), Csendes (28), and McDonald and Wexler (27) have all written papers on the use of an exterior element to specify boundary conditions for unbounded problems.
The methods basically use Green's functions to relate the field in the exterior regions to the potential on the finite element boundary and so derive a relationship between the exterior field energy and the boundary potentials. This is equivalent to specifying a boundary condition. The first three papers seem to be applicable only for electrostatic problems since parts of the derivations are based on assumptions valid only for these problems. However, the work described in the final paper is more general in scope and, in the future, may be able to be applied to this problem.

An initial investigation into these methods has been begun, but the application of this theory to the loss ratio calculations rests with future work, because of the complexity involved in applying exterior element methods to the time harmonic diffusion equation. It was therefore decided to concentrate on solutions for the case of three conductors enclosed in a steel pipe by using an approximate boundary condition. This approximate pipe boundary specification is based on the skin depth of the pipe material. The skin depth is that distance wherein the magnitude of the incident wave is reduced by a factor of \( e^1 \); and the approximate formula for skin depth is:

\[
\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}
\]

The particular pipe modelled had a thickness of 0.25 inches.
and a conductivity of $8.2 \times 10^6$ mhos per meter. The permeability varied throughout the pipe but the average relative permeability was 100. Therefore the skin depth was:

$$\delta = \frac{1}{\sqrt{60 \pi^2 \times 4 \times 0.82 \times 100}} = 0.00227 \text{ meters}$$

$$\delta = 0.0895 \text{ inches}.$$  

The pipe is approximately three times the skin depth. This means that the flux density, $B$, at the outer pipe boundary has been reduced in magnitude by a factor of $e^2$ or about 20. Since the flux density at the outer pipe boundary is only 1/20 of the flux density at the inner pipe wall, the flux density can be taken to be approximately zero at the outer wall of the pipe. Since the flux density is assumed to be zero, the curl of vector magnetic potential is zero at the outer pipe boundary.

$$\nabla \times A = 0$$

Since it is assumed that only a z-component exists for vector potential, the curl expands to: (in cylindrical coordinates)

$$\frac{1}{r} \frac{\delta A}{\delta \phi} \hat{\phi} - \frac{\delta A}{\delta r} \hat{r} = 0$$

To satisfy the equation, both components must be zero. Therefore $\frac{\delta A}{\delta r} = 0$ and $\frac{\delta A}{\delta \phi} = 0$. The second condition indicates that the potential is the same everywhere on the boundary. This is an indication of the amount of approximation that is inherent in accepting that the flux density
is zero when it is not really so. From intuition, it is apparent that the potential should not be the same everywhere on the boundary, especially for the cradled configuration. For this work, this second equation is ignored and only the first one is applied. However, it is reasonable that the use of this chosen condition involves about the same amount of approximation that was implied by the use of the unused equation, and that much of the deviation of answers obtained using this boundary condition, from answers obtained by measurement, may be due to the inexactness of the boundary specification.

The boundary constraint that is used is the homogenous Neumann condition $\frac{\partial A}{\partial r} = 0$. The application of this condition is accomplished without effort in the finite element method since this condition is 'natural' in the method. This means that it is automatically satisfied, in the absence of any other boundary specification, simply by writing the basic element equations.

5.3 Permeability Specification

All that remained, before the in-pipe calculations could be performed, was to determine a way to specify the permeability variation throughout the pipe. Measurements on the variation of permeability with flux density were available for the particular pipe material of interest. To get them into a form useable in a computer program, the
data was curve-fitted to a polynomial expression using a library least-squares curve fitting subroutine. Because of the large range of the data, it was broken into five parts and fitted by five polynomials. In this form, the value of relative permeability can be determined for any value of flux density and the flux density can be calculated from a knowledge of vector potential.

An iterative technique is needed to determine the permeability variation. First, a constant value of permeability is used for all the triangles which make up the pipe. Using this permeability, the matrix equation is written and solved to yield the vector magnetic potential. From the vector potential, the flux density is calculated and a new value of permeability, which is different for each triangle, is determined. The matrix equation is written and solved again with the new permeability variation and the process is repeated until the vector potential is sufficiently similar for successive iterations. In practice, only two iterations were necessary to obtain the correct permeability variation and it was even found that there was no great difference in the vector potential values when constant permeability values, as compared with varying permeability values, were used. The permeability of the pipe triangles did not affect the vector potential in the conductors very much. This can be seen in the program printouts in Appendix 4.
5.4 Results

The following tables list the results obtained for the loss ratio for the three-conductor configuration using the boundary approximations already described.

**TABLE 6**

Loss Ratio in Air Using Filamentary Superposition

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Special Notes</th>
<th>Radius</th>
<th>Loss Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real &amp; Imag.</td>
<td>Used CONGRH</td>
<td>.833</td>
<td>2.02</td>
</tr>
<tr>
<td>Complex</td>
<td>Unconjugated</td>
<td>.833</td>
<td>2.03</td>
</tr>
<tr>
<td>Real &amp; Imag.</td>
<td>Used SIMQ</td>
<td>.833</td>
<td>2.02</td>
</tr>
<tr>
<td>Real &amp; Imag.</td>
<td>Using CONGRH</td>
<td>.393</td>
<td>1.83</td>
</tr>
</tbody>
</table>

The final entry in the table illustrates the change in loss ratio that results with a variation in the conductor radius.

These results are consistent and compare reasonably well with available measured values. In each case, the use of the stranding factor reduces the loss ratio considerably, but the use of segmentation, the thickness of the pipe and a change of conductor radius does not alter the loss ratio as appreciably as might be expected. This may be because the boundary specification has an overpowering effect on the potential distribution.

For each case with similar parameters, the loss ratio for the cradled configuration is consistently higher than that for the close triangular configuration, and this is as
TABLE 7
Pipe Loss Ratios Using Neumann Boundary Conditions

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Special Notes</th>
<th>Radius (inches)</th>
<th>Numerical Loss Ratio</th>
<th>Measured Loss Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centered Triang.</td>
<td>Constant Permeab.</td>
<td>.791</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>Centered Triang.</td>
<td>L. P. R. &amp; Im. Form.</td>
<td>.833</td>
<td>2.02</td>
<td></td>
</tr>
<tr>
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<td>T. P. SF.</td>
<td>.833</td>
<td>1.86</td>
<td></td>
</tr>
<tr>
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<td>T. P.</td>
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<td>2.09</td>
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<td>T. P. SF.</td>
<td>.707</td>
<td>1.86</td>
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<tr>
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<td>.707</td>
<td>1.95</td>
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</tr>
<tr>
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<td>Tn. P.</td>
<td>.745</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td>Cradled</td>
<td>Tn. P. S.F. Segmented.</td>
<td>.745</td>
<td>1.88</td>
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</tr>
<tr>
<td>Close Triangle</td>
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<td>1.90</td>
<td></td>
</tr>
<tr>
<td>Close Triangle</td>
<td>T. P. SF.</td>
<td>.745</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>Close Triangle</td>
<td>Tn. P. SF.</td>
<td>.745</td>
<td>1.88</td>
<td></td>
</tr>
<tr>
<td>Close Triangle</td>
<td>Tn. P. S.F. Segmented.</td>
<td>.745</td>
<td>1.87</td>
<td>1.49-1.70</td>
</tr>
<tr>
<td>Close Triangle</td>
<td>Unconjugated Form.</td>
<td>.745</td>
<td>1.87</td>
<td></td>
</tr>
</tbody>
</table>

Note: L.P.-Large Pipe 15 inches in diameter  
T.P.-Thick 1-inch pipe  
Tn. P.-Thin .25 inch pipe  
S.F.-Stranding Factor .435  
Normal Pipe Diameter 10 inches
was anticipated. However, based on measured values, the difference is not as great as was expected and the loss ratio for the close triangular arrangement is much higher than that obtained by measured values. Once again, the lack of a greater difference between the cradled and triangular loss ratios is probably due to the fact that both have the same specified boundary condition. This condition probably forces the potential values of the two cases to be much more similar than they would be if the correct boundary potential distribution were known. Recent measurements on 2000 Kc mil. segmental cables (corresponding to a .745 inch corrected radius) indicate a range in loss ratio, at room temperature, from 1.78 to 1.95 for the cradled arrangement and from 1.49 to 1.70 for the close triangular configuration.

The range is due to the dependence of loss ratio on current, and heat cycling and corresponds to calculated loss ratio values of 1.88 for cradled and 1.87 for close triangular arrangements obtained in this work.

It should be noted that a computational adjustment was necessary to prevent all answers from being completely unreasonable. As noted previously the loss ratio can be specified as a ratio of the total current to the source current loss:

\[
\text{Loss Ratio} = \frac{\int \left| \mathbf{J} - j\sigma \mathbf{A} \right|^2 \, dxdy}{\int \left| \mathbf{J}_s \right|^2 \, dxdy}
\]
\[
= 1 + \frac{\int \left\{ \omega^2 \sigma^2 \left| A \right|^2 + i \omega \sigma \left[ \overline{A^* J_s} - \overline{A J_s^*} \right] \right\} \, dx \, dy}{\int \left| J_s \right|^2 \, dx \, dy}
\]

It is the cross-term that causes the difficulty. In the single conductor case this term is found to be zero and thus is no problem. However, in the three conductor case, the real parts of the bracketed term cancel, leaving a term which, when included, adds to the remaining terms in such a way that they cancel out and the loss ratio becomes much less than unity. This is clearly an impossibility and for this reason it was necessary to ignore the contribution of this imaginary term.

In the single conductor calculations the imaginary term vanishes because the average value of vector potential is zero over the conductor. In the three conductor case, the average of the vector magnetic potential is also zero over the three conductors but this average is weighted by a different value of current in each conductor, and, although the currents also have a zero average value, the weighting is such that the integral is prevented from being zero.

Although this indicates what actually happens in the performance of the calculations, it does not explain the fundamental theoretical reasons for the difficulty involved in including the cross term. A reference in a paper by Stoll (8) also indicates that this cross term is ignored, but it is not clear why, or if this omission is valid in general or only in specific cases.
6.1 Single Conductor Results

The single conductor loss ratio results compare very favourably to analytical results and can provide very good accuracy if a sufficiently fine grid is used.

From the accuracy and consistency of the results, it can be concluded that a workable functional and an acceptable method of formulating the boundary conditions have been developed. The results also confirm the soundness of the basic finite element procedure and attest to the success of the solution routines.

In terms of specific results, the effect on the loss ratio due to a change in a number of parameters was observed. The finer the triangular grid, the greater the accuracy. However, the accuracy also greatly depends on the judiciousness of the choice of the ring break points which determine where the finest portion of the grid is placed. However, the number of rings outside of the conductor and the position of the outer boundary does not affect the loss ratio. In terms of physical cable parameters, the loss ratio increases with an increase in radius and conductivity and
decreases when the conductor is segmented. As expected, a change in radius results in a larger change in loss ratio than does a corresponding percentage variation in conductivity.

To obtain these results, a variety of techniques were employed. Overall, it appears that the simple negative feedback approach for specifying the boundary is computationally most efficient. The Hooke and Jeeves search also worked, but the added complexity was not necessary. However, the approach using the iterative source current variation was appealing since it gave a physical indication of how the distribution of the source current becomes modified to cause an increase in the loss ratio.

Overall, the single conductor analysis results were very successful and this was of some significance since these results provided the basis for the three conductor calculations.

6.2 Three Conductor Results

The results obtained in the three conductor loss ratio calculations are reasonable, but the accuracy is limited by the inexactness of the boundary condition specification.

The finite element pattern generation program appears to be able to satisfactorily model almost any imaginable pipe and conductor arrangement as long as the grid lines are judiciously located. Results also indicate that the use of
the real and imaginary part functional and the derived modified complex formulation has solved the problem of finding the correct functional corresponding to the complete field equation.

With regard to the matrix equation solution routines, the complex gaussian elimination scheme has proven to be very stable and gives good results, even when the matrix order is greater than 150. However, for a high order matrix, a very great amount of core area is needed and much of this is wasted by storing zero entries. For this reason, the use of the conjugate gradient subroutine becomes significant. Large matrices can be solved quickly with comparatively little core storage, since zero entries are not retained. In other programs, similar to this work, matrices of order greater than 2200 have been solved in less than five minutes of CPU time using less than 300 k-bytes of memory. Using an elimination subroutine, more than 20,000 k-bytes of storage would have been required and the round-off errors probably would have rendered the solution useless. Thus, the use of the conjugate gradient solution routine opens up many more areas in which the finite element method can be employed. Much finer modelling, with many more grid points, can be used and, because of the rapid solution time, iterative techniques, such as those used for boundary specification, can be employed where they would
otherwise not be feasible. There only remains to perform some work on the convergence of the method since convergence did not occur in every case tested and a satisfactory explanation of this failure is not yet available.

The results of the iterative permeability specification appear to be consistent with intuitive expectations. The permeability varies from triangle to triangle in the pipe, but is constant throughout any one triangle. It has been found, however, that the specification of permeability has not been as significant as was believed. Results obtained using a reasonably arbitrary constant permeability have differed from results using the variable permeability by an average of less than one per cent. It should be noted that the permeability measurements for the pipe material sample used were those obtained under direct current conditions and that pipe hysteresis was not modelled or treated in the program at all. Nonetheless, this would not account for the lack of dependence of the loss ratio on permeability of the pipe.

However, it may be that the manner in which the boundary is specified accounts for the lack of sensitivity. Normally, it might be expected that the permeability variation in the pipe would exert a considerable influence on the boundary conditions prevailing; but, if the boundary condition is fixed, as in this work, the permeability
variation may not be able to control the potential distribution in the expected manner.

The fixedness of the boundary is also likely to be responsible for the closeness in results for the cradled and the close triangular arrangements. It is reasonable that as the cable configuration changes, the boundary conditions should change, but with the boundary specifications used in this work, this is not possible.

In spite of the boundary approximations used, the results are reasonable and compare favourably with other methods of calculation.

The loss ratio calculations for cables in air gave results that are higher than expected results but this is due to the inexact boundary values that were applied. For the cables in the pipe, the calculated loss ratio for the cradled configuration was close to measured values but the loss ratio for the close triangular arrangement was calculated to be higher than expected. The thickness of the pipe did not significantly affect the results, since the boundary condition was the same in both cases. The use of a stranding factor significantly decreased the loss ratio result but the application of segmentation did not have a great effect.

The effect of a change of a number of variables can be handled by the program. Loss ratio decreases with
increasing conductor radius and conductivity and also changes with the position of the cables or the size of the pipe. No provision has been made for the variation in loss ratio with a change in temperature, but this could be implemented by changing the conductor conductivity. The magnitude and phase of the source current can be altered to any value but provision has not been made to account for the change in conductivity due to the heating caused by larger currents. Also the effect of heat cycling is not considered but could be handled by modifying the stranding factor. Some cable components, such as the shield assembly, are not modelled and the inclusion of these parts in the cable model is one of the things that could be done to improve the program in the future.

6.3 Summary of Results

Good accuracy was obtained for single isolated conductors of all sizes by using the finite element method and including an eddy current term in the functional. A valid boundary condition was specified by iterative techniques which minimized the average vector potential over the conductor cross-section. An alternate formulation procedure illustrated the modification of the source current distribution responsible for the increased losses.

For the three conductor and pipe calculation, an auto-
matic pattern generation program was written which permits the modelling of any size conductor in various arrangements. Grid fineness is easily modified either in the conductors, the pipe or in the air, and the cables may be represented as either solid or segmented conductors.

A real and imaginary part functional was developed and implemented, both in a real and imaginary part and in a modified complex formulation. Preliminary evaluation of an alternate matrix solution routine was undertaken in an effort to reduce both computer storage and computation time. Approximate boundary conditions were applied to the three conductor case and work was initiated on the specification of a more accurate boundary constraint. Pipe permeability was expressed in a polynomial form and several loss calculations were made and compared to measured values.

6.4 Future Work

A number of areas exist in this work which are worthy of consideration for additional study in the future. The most immediate focus of effort should be directed toward the determination of a more accurate boundary condition.

Several avenues exist for the attacking of this problem. With the use of the conjugate gradient routine to decrease the matrix solution time, a modified search routine, similar to that used for the single conductor case, might be feasible. However, the most promising method of defining
a boundary condition appear to be that involving the exterior element concept. The paper by McDonald and Wexler (27) provides a foundation from which it should eventually be possible to obtain a valid boundary condition specification which should improve the results significantly.

Other modifications to improve the computational accuracy include the use of an increased number of grid points to more accurately model the cable geometry. Also, an increase in the order of the defining equations for the electromagnetic vector potential would lead to greater accuracy. The best possible positioning of the triangles of the grid could also improve the results and this might best be accomplished by the use of an interactive graphics terminal. With this type of device, a program to generate the triangular subdivision of the region of interest would not be necessary since the grid would be directly and easily constructed by the user.

In addition to this work undertaken to achieve greater accuracy, a comprehensive study should be initiated in an attempt to more completely resolve the difficulty that arose with respect to the cross-term that had to be omitted in the loss ratio calculations.

If most of this work can be successfully accomplished, it seems certain that a method for confidently evaluating the loss ratio for a pipe-type cable will result.
APPENDIX 1

REAL AND IMAGINARY PART FUNCTIONAL VERIFICATION

The full field equation in terms of vector magnetic potential is given by
\[
\frac{\partial^2 \overline{A}}{\partial x^2} + \frac{\partial^2 \overline{A}}{\partial y^2} = -\mu \overline{J} + j\omega \sigma \mu \overline{A}
\]

Both \(\overline{A}\) and \(\overline{J}\) are complex quantities and the field equation can be separated into real and imaginary parts.
\[
\frac{\partial^2 (\overline{A}_R + j\overline{A}_I)}{\partial x^2} + \frac{\partial^2 (\overline{A}_R + j\overline{A}_I)}{\partial y^2} = -\mu (\overline{J}_R + j\overline{J}_I) + j\omega \sigma \mu (\overline{A}_R + j\overline{A}_I)
\]
\[
\frac{\partial^2 \overline{A}_R}{\partial x^2} + \frac{\partial^2 \overline{A}_R}{\partial y^2} = -\omega \overline{J}_R - \omega \sigma \mu \overline{A}_I \quad \text{(Real)}
\]
\[
\frac{\partial^2 \overline{A}_I}{\partial x^2} + \frac{\partial^2 \overline{A}_I}{\partial y^2} = -\omega \overline{J}_I + \omega \sigma \mu \overline{A}_R \quad \text{(Imaginary)}
\]

For the minimization of a functional to be equivalent to the basic field equation, it must be shown that the functional reduces to the real and imaginary equations written here. Let the proposed functional be:
\[
x = \frac{1}{2} \int \left[ \left( \frac{\partial \overline{A}_R}{\partial x} \right)^2 + \left( \frac{\partial \overline{A}_R}{\partial y} \right)^2 - \left( \frac{\partial \overline{A}_I}{\partial x} \right)^2 - \left( \frac{\partial \overline{A}_I}{\partial y} \right)^2 \right] \, dx \, dy
+ \int \left[ -\omega \sigma \mu \overline{A}_R \overline{A}_I + \mu \overline{J}_I \overline{A}_R - \mu \overline{J}_R \overline{A}_I \right] \, dx \, dy
\]

Now the functional must be minimized. Let the optimal
(minimum) solution be \( A_R^* \) and \( A_I^* \) and let the solution be perturbed a small amount from the optimum:

\[
A_R = A_R^* + \xi \eta_R(x,y) \quad A_I = A_I^* + \xi \eta_I(x,y)
\]

\( \xi \) is a small number and \( \eta(x,y) \) is an arbitrary function.

Substituting these values into the functional gives:

\[
x = \frac{1}{2} \left[ \left( \frac{\delta}{\delta x} (A_R^* + \xi \eta_R(x,y)) \right)^2 + \left( \frac{\delta}{\delta y} (A_R^* + \xi \eta_R(x,y)) \right)^2 \right. \]

\[
- \left( \frac{\delta}{\delta x} (A_I^* + \xi \eta_I(x,y)) \right)^2 - \left( \frac{\delta}{\delta y} (A_I^* + \xi \eta_I(x,y)) \right)^2 \] dx dy

\[
+ \int \left[ -\mu \omega (A_R^* + \xi \eta_R(x,y))(A_I^* + \xi \eta_I(x,y)) + \mu J_R (A_R^* + \xi \eta_R(x,y)) \\
- \mu J_I (A_I^* + \xi \eta_I(x,y)) \right] dx dy
\]

Now take a variation on the functional with respect to and set the functional to zero as \( \xi \) approaches zero

\[
\delta x = \int \left[ \frac{\delta A_R^*}{\delta x} \frac{\delta \eta_R}{\delta x} + \frac{\delta A_R^*}{\delta y} \frac{\delta \eta_R}{\delta y} - \frac{\delta A_I^*}{\delta x} \frac{\delta \eta_I}{\delta x} - \frac{\delta A_I^*}{\delta y} \frac{\delta \eta_I}{\delta y} \\
- \omega \mu \omega_{A_R^*} - \omega \mu \omega_{A_I^*} + \mu J_R \eta_R - \mu J_I \eta_I \right] dx dy = 0
\]

Terms such as \( \int \frac{\delta A_R^*}{\delta x} \frac{\delta \eta_R}{\delta x} dx dy \) may be integrated by parts

\[
\int \frac{\delta A_R^*}{\delta x} \frac{\delta \eta_R}{\delta x} dx dy = \frac{\delta A_R^*}{\delta x} \eta_R \bigg|_{x_1}^{x_2} - \int \frac{\delta^2 A_R^*}{\delta x^2} \eta_R dx dy
\]

Since \( \eta_R(x,y) \) and \( \eta_I(x,y) \) are arbitrary, they can be chosen to be zero at the endpoints. Thus,

\[
\int \frac{\delta A_R^*}{\delta x} \frac{\delta \eta_R}{\delta x} dx dy = - \int \frac{\delta^2 A_R^*}{\delta x^2} \eta_R dx dy
\]

Therefore the functional can be rewritten as:
\[ \delta X = \int \left[ -\frac{\delta^2 A_R}{\delta x^2} \eta_R - \frac{\delta^2 A_R}{\delta y^2} \eta_R + \frac{\delta^2 A_I}{\delta x^2} \eta_I + \frac{\delta^2 A_I}{\delta y^2} \eta_I ight. \\
\left. - \omega \eta_R A_I^* - \omega \eta_I A_R^* + \mu J_R \eta_I - \mu J_I \eta_R \right] \, dx \, dy = 0 \]

Rearranging and collecting like terms gives

\[ \delta X = \int \left[ \eta_R \left( -\frac{\delta^2 A_R}{\delta x^2} - \frac{\delta^2 A_I}{\delta y^2} - \omega \mu A_I^* - \mu J_R \right) \right. \\
\left. + \eta_I \left( \frac{\delta^2 A_I}{\delta x^2} + \frac{\delta^2 A_R}{\delta y^2} - \omega \mu A_R^* + \mu J_I \right) \right] \, dx \, dy = 0 \]

Since \( \eta_R \) and \( \eta_I \) are arbitrary, the two bracketed terms are zero. Thus, if the minimum of the functional is found, then

\[ \frac{\delta^2 A_R}{\delta x^2} + \frac{\delta^2 A_R}{\delta y^2} = -\mu J_R - \omega \mu A_I \]

\[ \frac{\delta^2 A_I}{\delta x^2} + \frac{\delta^2 A_I}{\delta y^2} = -\mu J_I + \omega \mu A_R \]

These equations correspond exactly to the real and imaginary parts of the field equation previously derived and this proves the equivalence between the minimized functional and the field equation.
APPENDIX 2
INTEGRAL EVALUATION

One integral occurs in finite element derivations a sufficient number of times so that its evaluation once and for all is warranted:

\[ I = \int \int N_i \, dx \, dy \]

\[ N_i = \frac{a_i + b_i x + c_i y}{2\Delta} \quad \Delta \text{ is triangle area} \]

It is well known that for a triangle

\[ \int x \, dx \, dy = \bar{x} \Delta \, dx \, dy \quad \text{and} \quad \int y \, dx \, dy = \bar{y} \Delta \, dx \, dy \]

where \( \bar{x} \) and \( \bar{y} \) are the centroid values.

\[ I = \int \int \left\{ \frac{a_i + b_i x + c_i y}{2\Delta} \right\} \, dx \, dy = \frac{a_i + b_i \bar{x} + c_i \bar{y}}{2} \]

Substituting for \( a_i \), \( b_i \) and \( c_i \)

\[ I = \frac{x_i y_k - x_k y_i + (y_j - y_k) \left\{ \frac{x_i + x_j + x_k}{3} \right\} + (x_i - x_j) \left\{ \frac{y_i + y_j + y_k}{3} \right\}}{2} \]

\[ I = \frac{1}{6} \left[ (x_j y_k - x_k y_j) + (y_j x_i - x_j y_i) + (x_k y_i - y_k x_i) \right] \]

\[ \det \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} = 1/3 \quad \text{and} \quad 2 = 1/3 \Delta \]

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APPENDIX 3

EVALUATION OF THE EDDY CURRENT TERM

The contribution of the eddy current term towards the total functional is given by the integral:

\[ x_E = j \omega \mu \int A'^2 \, dx \, dy \]

\[ A = \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} \]

\[ N_i = \frac{a_i + b_i x + c_i y}{2\Delta} \]

To obtain the master matrix entries the functional must be differentiated with respect to each of the nodal potentials.

\[ \frac{dx_E}{\delta A_i} = 2j \omega \mu \int N_i \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} \, dx \, dy \]

This expression can be separated into three terms, each of which will be added to the appropriate place in the master matrix.

\[ B_{ij} = \left[ 2j \omega \mu \int N_i N_j \, dx \, dy \right] A_j \]

The expression in brackets is added to the \( i \)th row and the \( j \)th column of the master matrix after the integration is performed.
\[ E_{ij} = \frac{i2\sigma p}{4\Delta^2} \int (a_i + b_i x + c_i y)(a_j + b_j x + c_j y) \, dx \, dy \]

\[ = \frac{i2\sigma p}{4\Delta^2} \int \left( a_i a_j + a_i b_j x + a_i c_j y + a_j c_i y + a_j c_i y + b_i b_j x^2 \right. \]

\[ + c_i c_j y^2 + b_i c_j xy + c_i b_j xy \left) \, dx \, dy \right. \]

\[ = \frac{i2\sigma p}{2\Delta} \left[ a_i a_j + \overline{x}(a_i b_j + a_j b_i) + \overline{y}(a_i c_j + a_j c_i) \right. \]

\[ + (b_i c_j + c_i b_j) \left( \frac{\overline{x}y + x_1 y_1 + x_2 y_2 + x_3 y_3}{12} \right) \]

\[ + b_i b_j \left( \frac{\overline{x}^2 + x_1^2 + x_2^2 + x_3^2}{12} \right) + c_i c_j \left( \frac{\overline{y}^2 + y_1^2 + y_2^2 + y_3^2}{12} \right) \]

\[ \overline{x} \text{ and } \overline{y} \text{ are triangle centroid co-ordinates} \]

\[ x_1, y_1 \text{ are nodal co-ordinates minus centroid co-ordinates} \]
APPENDIX 4

Computer program listings

The first listing is for a single conductor with a negative feedback boundary specification.
$jom XXXXXXXXY IMPLICIT COMPLEX*16(C), REAL*8(A-B-C-D-H,0-Z)
DIMENSION XO(3),YO(3),R(3)
DIMENSION IFADDX(4),IFADDY(4),SIGMN(56),ISEG(8)
DIMENSION RADUS(16),X(3),Y(3),NODE(56),SIGMM(56),SIGM(56),BI
REAL*8 MU(56),R0
COMMON X,Y,NODE,NSEG,NRAD,SIGMM,BIGJ,MU
DIMENSION PTNk(10)
DATA I FADDX/1.0,-1.0,0,0/ IFADDY/0.0,1.0,0,0/
COMPLEX*16 SIGMN(56),BIGJ1,BIGJ2
REAL*8 PI=3.14159265359D0
C READ SEGMENTATION CONSTANTS, THICKNESS IS IN INCHES, OTHER
C CONSTANTS ARE 4.6 AND 3 FOR SEGMENTATION, ELSE 0
READ(5,83) THIK,NSEG,IFSEG,KSEG
C READ NUMBER OF CABLES, CENTRE OF REGION AND RADIUS OF REGION
READ(5,7C0) NPMAX,XORG,YORG,R0
THIK2=THIK/2.0
THIK=THIK2
C UNSEGMENTED CABLE
C (II) COMPUTE COORDINATES OF EACH NODE, X(N) AND Y(N) IN METERS
C REQUIRED INPUTS: NRAD, NSEG, RADIUS(I)
C RESULTS: X(N), Y(N)
** ANGLE = 2.5D0*3.14159265359D0/NSEG
N=2
DO 6 I=1,NRAD
.THETA=0.00
DO 6 J=1,NSEG
X(N)=RADUS(N)*DCOS(THETA)*1.00-2*2.540100*R0(L)
Y(N)=RADUS(N)*DSIN(THETA)*1.00-2*2.540100*R0(L)
N=N+1
6 THETA=THETA+ANGLE
C SET COORDINATES OF NODE 1
X(1)=0.00
Y(1)=0.00
C (III) DETERMINE NODE NUMBERS OF EACH ELEMENT, NCDE(I,J)
C REQUIRED INPUTS: NRAD, NSEG
C RESULTS: NCDE(I,J) (I=ELEMENT NUMBER; J=L,M,N (1,2,3))
C (I) DETERMINE NODE NUMBERS OF ELEMENTS IN 1ST (INNER) RING
DO 1 J=1,NSEG
SIGMM(J)=SIGMA
NODE(J,1)=1
NODE(J,2)=J+2
1 NODE(J,3)=J+1
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\text{NDP3}(\text{NSEG},2)=2 \\
\text{GO TO 651}

\text{SEGMENTED CABLE- INNER RING}

650 \text{CONTINUE}
\text{DO 111 J}=1,4 \\
\text{X}(\text{NDNO})=\text{XO}(\text{L})+((-1)**((\text{J}+4)/2)\ast \text{THIK2}) \\
\text{Y}(\text{NDNO})=\text{YO}(\text{L})+((-1)**((\text{J}+3)/2)\ast \text{THIK2}) \\
111 \text{NDNO}=N\text{NDNO}+1 \\
\text{DO 72 J}=1,4 \\
\text{ISEG}(2*\text{J}-1)=\text{NDNO}-6+\text{J} \\
\text{IF(}\text{ISEG}(2*\text{J}-1).LT.\text{NDINIT) ISEG}(2*\text{J}-1)=$\text{NDINIT}+3 \\
72 \text{ISEG}(2*\text{J})=\text{NDNO}-5+\text{J} \\
\text{DTHETA}=\pi/\text{NSEG} \ast 2.D0 \\
\text{TA}=0DTHETA \\
\text{NPTS}=\text{NSEG}/4 \\
\text{DO 123 } \text{M}=1,\text{NCCN} \\
\text{ADD}=\text{RO}(\text{L})\ast \text{RADIUS}(\text{M}) \\
\text{DO 124 } \text{J}=1,4 \\
\text{X}(\text{NDNO})=\text{X}(\text{ISEG}(2*\text{J}-1))+$\text{ADD}\ast \text{IFADDX}(\text{J}) \\
\text{Y}(\text{NDNO})=\text{Y}(\text{ISEG}(2*\text{J}-1))+$\text{ADD}\ast \text{IFADDY}(\text{J}) \\
123 \text{NDNO}=N\text{NDNO}+2 \\
\text{NPTS}=\text{NPTS}-1 \\
\text{IF(NPTS.EQ.0) GO TO 123} \\
\text{DO 125 } \text{K}=1,\text{NPTS} \\
\text{X}(\text{NDNO})=\text{RO}(\text{L})\ast \text{DCOS}(\text{THETA})\ast \text{RADIUS}(\text{M})+$\text{X}(\text{ISEG}(2*\text{J})) \\
\text{Y}(\text{NDNO})=\text{RO}(\text{L})\ast \text{DSIN}(\text{THETA})\ast \text{RADIUS}(\text{M})+$\text{Y}(\text{ISEG}(2*\text{J})) \\
\text{THETA}=\text{THETA}+DTHETA \\
125 \text{NDNO}=N\text{NDNO}+1 \\
\text{THETA}=\text{THETA}+DTHETA \\
124 \text{CONTINUE} \\
123 \text{CONTINUE} \\
\text{VIST}=\text{NCCN}+1 \\
\text{IF(VIST.GT.NRAD) GO TO 782} \\
\text{DO 375 } \text{K}=\text{NIST},\text{NRAD} \\
\text{THETA}=0D0 \\
\text{DO 376 } \text{K}=1,\text{NSEG} \\
\text{Y}(\text{NDNO})=\text{YO}(\text{L})$+$\text{RO}(\text{L})\ast \text{RADIUS}(\text{J})\ast \text{DCOS}(\text{THETA}) \\
\text{X}(\text{NDNO})=\text{XO}(\text{L})$+$\text{RO}(\text{L})\ast \text{DSIN}(\text{THETA})\ast \text{RADIUS}(\text{J}) \\
\text{THETA}=\text{THETA}+DTHETA \\
375 \text{NDNO}=N\text{NDNO}+1 \\
782 \text{CONTINUE} \\
\text{NALL}=N\text{NDNO}-1 \\
\text{DO 365 } \text{J}=1,N\text{ALL} \\
\text{X}(\text{J})=\text{X}(\text{J})\ast 2.54010^{-2} \\
365 \text{Y}(\text{J})=\text{Y}(\text{J})\ast 2.54010^{-2} \\
630 \text{FORMAT(F10.5,3(Z12))} \\
\text{C FORM TRAINGLE FOR INNER RADIUS} \\
\text{DO 83 } \text{J}=1,2 \\
\text{NODE(}J,1)=$\text{NDINIT} \\
\text{NODE(}J,2)=\text{J}+\text{NDINIT}+1 \\
\text{NODE(}J,3)=\text{J}+\text{NDINIT} \\
\text{SIGMM(}J,0)=$0.D0 \\
83 \text{J0}=J+1 \\
\text{NDNOW}=\text{NDINIT}+4 \\
\text{DO 41 } \text{J}=1,\text{NSEGC} \\
\text{NODE(J,1)}=\text{NDNOW} \\
\text{NODE(J,2)}=\text{J}+\text{NDINIT}+J-2 \\
\text{NODE(J,3)}=\text{J}+\text{NDINIT}+J-1 \\
\text{SIGMM(J,0)}=$0.D0 \\
\text{IF(NODE(J,2).LT. NDINIT) NODE(J,2)=$NDINIT+3 \\
\text{J0}=J+1 \\
\text{NUM}=\text{NPTS}+1 \\
\text{DO 51 } \text{K}=1,\text{NUM} \\
\text{SIGMM(J0)=$SIGMA} \\
\text{IF(K.EQ.0) SIGMM(J0)=$0.D0} \\
\text{NODE(J0,1)}=\text{NDINIT}+J-1 \\
\text{NODE(J0,2)}=\text{NDNOW} \\
\text{IF((J.EQ.NUM).AND.(J.EQ.NSEG)) NODE(J,?)=$NDINIT+3 \\
\text{NODE(J,3)}=\text{NDNOW} \\
\text{NDNOW}=\text{NDNOW}+1 \\
51 \text{J0}=J+1 \\
41 \text{CONTINUE} \\
651 \text{CONTINUE} \\
\text{C FORM OTHER CONDUCTING RINGS} \\
\text{C} \\
\text{C} \\
\text{C} \\
\text{C}
C 2) DETERMINE NODE NUMBERS FOR RINGS 2, 4, ETC.
   IF(NSEG+NSFG/NSEG
   NTR=NSEG+NSFG
   <1=((NSEG+NSFG)⁺(2*NPRAD-1)+IFSEG)*((NSEG+NSFG+1)+IFSEG
   K2=1+NINIT+KSEG
   DO 2 I=2,NRAD+1
   IF((I.GT.NCOND).AND.(NSEG+NEG.0)) GO TO 1370
   DO 3 J=1,NTR
   NODE(K1,1)=K2
   NODE(K1,2)=K2+NSG+1+NSFG
   NODE(K1,3)=K2+NSG+NSFG
   K1=K1+1
   NODE(K1,1)=K2
   K2=K2+1
   NODE(K1,2)=K2
   NODE(K1,3)=K2+NSG+NSFG
   3 K1=K1+1
   NODE(K1-1.2)=K2-NSEG-NSEG
   NODE(K1-1.3)=K2
   NODE(K1-2.2)=K2
   K1=K1+2*NSG+2*NSFG
   2 <2=K2+NSG+NSG
   C (3) DETERMINE NODE NUMBERS FOR RINGS 3, 5, ETC.
   1370 K1=3*NTR+IFSEG+JSTART+1
   <2=NSEG+1+NINIT+SEG+KSEG
   DO 4 I=3,NRAD+2
   IF((I.GT.NCOND).AND.(NSEG+NEG.0)) GO TO 370
   DO 5 J=1,NTR
   NODE(K1,1)=K2
   K2=K2+1
   NODE(K1,2)=K2
   NODE(K1,3)=K2+NSG+1+NSFG
   K1=K1+1
   NODE(K1,1)=K2
   NODE(K1,2)=K2+NSG+NSFG
   NODE(K1,3)=K2+NSG-1+NSFG
   5 K1=K1+1
   NODE(K1-1,1)=K2-NSEG-SEG
   NODE(K1-1,2)=K2
   NODE(K1-2,2)=K2-NSEG-SEG
   K1=K1+2*SEG+2*SEG
   4 <2=K2+NSFG+SEG
   GO TO 65
   370 K0=K1
   C FORM FIRST INSULATING RING FOR SEGMENTED CABLE
   K1=K2
   KODREV=NCOND/2
   KODREV=NCOND/2*KODREV
   IF(KODREV.EQ.0) K1=K1+NTR
   IF(KODREV.EQ.0) J0=J0-2*NTR
   LST=K1-NTR
   DO 371 K0=1,4
   NODE(J0,1)=LST
   NODE(J0,2)=LST+1
   NODE(J0,3)=K1
   J0=J0+1
   LST=LST+1
   DO 372 J=1,NSFG
   NODE(J0,1)=LST
   NODE(J0,2)=LST+1
   NODE(J0,3)=K1
   J0=J0+1
   NODE(J0,1)=K1
   NODE(J0,2)=LST+1
   NODE(J0,3)=K1+1
   J0=J0+1
   LST=LST+1
   372 K1=K1+1
   C FORM INSULATING LAYERS FOR SEGMENTED CABLE
   DO 373 K=NSTT,NRAD
   NODE(J0,1)=K1
   NODE(J0,2)=K1+NSFG
   NODE(J0,3)=K1+1
   J0=J0+1

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NODE(J0,1)=K1-NSEG
NODE(J0+1,2)=K1+1-NSEG
NODE(J0+1,3)=K1+1
K1=K1+1
374 J0=J0+2
NODE(J0-1,2)=K1-2-NSEG
NODE(J0,1)=K1-NSEG
NODE(J0-1,3)=K1-NSEG
373 CONTINUE
65 NTR2=NTR*2
C SPECIFY ELEMENT CONDUCTIVITIES
JNOW=JSTART+1+IFSEG+NTR
ICOUN=ICOUN
SIG=SIGMA
DO 63 JJ=2,NPAD
IF(((JJ-1) .EQ. NCCN) .AND. (NSEG .NE. 0)) NTR2=NTR2-4
IF(((JJ-2) .EQ. NCCN) .AND. (NSEG .NE. 0)) NTR2=NTR2-4
IF(JJ.GT.NCON) SIG=0.0
DO 64 KK=1,NTR2
JARG=JNOW+(JJ-2)*NTR2+KK-1
IF(JJ.GT.NCON) JARG=JARG+NSEG+NCCN-NSEG
IF(JJ.GT.NCON+1) JARG=JARG+NSEG+NCCN+NSEG-NSEG
64 SIGMM(JARG)=SIG
IF((NSEG .EQ. 0).OR.(JJ .GT. NCCN)) GC TO 63
DO 62 LL=1,NSEG
SIGMM( ICOUN )=0.0
SIGMM( ICOUN+1 )=0.0
62 ICOUN=ICOUN+NSEG*2+2
63 CONTINUE
C ASSIGN CURRENTS AND PERMEABILITIES
CALL MUANDJ(MU,BIGJ,NRAD,NSEG,NCCN,MU1,BIGJ1,BIGJ2,SIGMM,IFSEG,
*SEG,NTR)
NDINIT=(NTR+NRAD+1+KSEG)*NPHASE+1
JSTART=JSTART+(NCON*2-1)*NTR+(NRAD-NCON)*NSEG*2+IFSEG
IF(NRAD.GT.NCON) JSTART=JSTART+NSEG
113 CONTINUE
102 FORMAT(16F5.3)
101 FORMAT(212)
703 FORMAT(3(F7.4))
700 FORMAT(12,3(F10.5))
C INITIALIZE VARIABLES AND CALL SEARCH ROUTINE
VAL=(0.0,0.0,0.0)
AREAL=0.0
AIMAG=0.0
CALL RJHSER(AREAL,AIMAG,VAL)
STOP
END
SUBROUTINE MUANDJ(MU, BIGJ, NRAD, NSEG, NCON, MUL, BIGJ1, BIGJ2, SIGMM, IFSEG, KSEG, NTP)
IMPLICIT COMPLEX*16(C), REAL*8(A-B,C-H,D-Z)
ASSIGNS VALUES OF CURRENT DENSITY (BIGJ) AND PERMEABILITY (MU) TO EACH ELEMENT.
NCON=NUMBER OF RINGS IN CONDUCTOR
BIGJ1=CURRENT DENSITY IN CONDUCTOR
BIGJ2=CURRENT DENSITY IN INSULATION
MUL=MAGNETIC PERMEABILITY
RFAL*8 MU(56), MUL, SIGMM(56)
Ccomplex*16 BIGJ(56), BIGJ1, BIGJ2
NLC0NS=NCON*2*NTR-NTR+IFSEG
NLMNTS=NLC0NS+2*(NRAD-NCON)*NSEG
DO 1 N=1,NLC0NS
MU(N)=MUL
BIGJ(N)=BIGJ1
IF(SIGMM(N).EQ.C.DO) BIGJ(N)=BIGJ2
1 CONTINUE
IF(NLCONS.EQ.NLMNTS) RETURN
NLCONS=NLCONS+1
DO 2 N=NLCONS,NLMNTS
MU(N)=MUL
BIGJ(N)=BIGJ2
2 RETURN
END

SUBROUTINE R JHSER(AREAL, AIMAG, VAL)
COMPLEX*16 VAL
COMPLEX*16 DCONJG, DCMPLX
REAL*8 AREAL, AIMAG
THIS IS A SIMPLE NEGATIVE FEEDBACK SEARCH ROUTINE
VAL IS THE AVERAGE POTENTIAL
AREAL IS THE REAL BOUNDARY VALUE
AIMAG IS THE IMAGINARY BOUNDARY VALUE
OLD BOUNDARY MINUS AVERAGE POTENTIAL EQUALS NEW BOUNDARY
DO 1 J=1,20
AREAL=AREAL-(DCONJG(VAL)+VAL)/2.D0
AIMAG=AIMAG+(DCMPLX(1.D0,1.D0)**((VAL-DCONJG(VAL))/2.D0))
WRITE(6,3) J
3 FORMAT(* ,**** ITERATION NUMBER',2X,12,'*****)
CALL CVI(AREAL, AIMAG, VAL)
1 CONTINUE
RETURN
END

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SUBROUTINE CVI (Areal, aimag, val)
IMPLICIT REAL*8 (A-B, D-H), *I-Z, COMPLEX*16 (C)
COMMON X (33), Y (33), NDF (56, 3), NSEG, NFRD, SIGMA3, DlGJ (56), MU (56)
COMMON/X, XP (50)
DIMENSION YP (8), SP (8, 8), Tmp (8, 3), El (3, 8), VMUL (3), IECT (8), IEC2 (9)
DIMENSION CS (65), S3 (3), D (3), T (3), FAD (10)
COMPLEX*16 EZ, G/J (3, 3), RHS, CV (3), VAL, DCMLX, ALPHA
REAL*8 MU, COADS
COMMON X (33), Y (33), NODF (33), NSEG, SIGMA (33), aIGD (56), MU (36)
COMMON/A 3/ XP (50)
DIMENSION CS (65), S3 (3), D (3), T (3)
COMPLEX*16 EZ, G/J (3, 3), RHS, CV (3), VAL, DCMLX, ALPHA
REAL*8 MU, COADS
COMMON X (33), Y (33), NODF (33), NSEG, SIGMA (33), aIGD (56), MU (36)
COMMON/A 3/ XP (50)
DIMENSION CS (65), S3 (3), D (3), T (3)

THIS SUBROUTINE COMPUTES VECTOR POTENTIAL, CALCULATES LOSS RATIO AND FINDS NEW AVERAGE VALUE

THIS TECHNIQUE IS NOT VALID FOR SEGMENTED CABLES DUE TO A LACK OF ROTATIONAL SYMMETRY
SIGMA = 5.267
NPOT = 1 + (NFRD - 1) * NSEG
NTRA = NSEG
NPOT = NPOT + 8
NLMNT5 = NFRD * 2 * NSEG
NPOT = NPOT + 1
NPOTQ = NPOT * NPOT
NSPG = NPOT * (NPOT - 1)
BETA = 100.
NM = NSEG + 1
NMT = NPOT
C INITIALIZE MASTER MATRIX
DO 7 I = 1, NPOT
DO 7 J = 1, NPOT
7 CSM( I + NPOT * (J - 1)) = (0.D0, 0.D0)
C FORM MATRIX FOR EACH ELEMENT
DO 106 N = 1, NLMNTS
XCENT = (X(NODE(N, 1)) + X(NODE(N, 2)) + X(NODE(N, 3))) / 3.D0
YCENT = (Y(NODE(N, 1)) + Y(NODE(N, 2)) + Y(NODE(N, 3))) / 3.D0
XI = X(NODE(N, 1)) - XCENT
X2 = X(NODE(N, 2)) - XCENT
X3 = X(NODE(N, 3)) - XCENT
Y1 = Y(NODE(N, 1)) - YCENT
Y2 = Y(NODE(N, 2)) - YCENT
Y3 = Y(NODE(N, 3)) - YCENT
J = 2
K = 3
C CALCULATE FINITE ELEMENT CONSTANTS
DO 8 I = 1, 3
R(I) = (X(NODE(N, I)) - X(NODE(N, K)))
Q(I) = (Y(NODE(N, I)) - Y(NODE(N, K)))
T(I) = X(NODE(N, K)) * Y(NODE(N, I)) - X(NODE(N, I)) * Y(NODE(N, K))
I = I + 1
8 K = K + 1
K = 3
ARE = (X(NODE(N, 1)) * Y(NODE(N, 3)) - Y(NODE(N, 2)))
* X(NODE(N, 2)) - (Y(NODE(N, 1)) - Y(NODE(N, 3)))
* X(NODE(N, 3)) * Y(NODE(N, 2)) - Y(NODE(N, 1))
J = 3
C FORM MATRIX
DO 9 I = 1, 3
DO 99 J = 1, 3
SREAL = (Q(I) * Q(J) + R(I) * R(J)) / (ARE * 4.D0)
S(I, J) = DCMLX (SREAL, 0.D0)
IF (SIGMA3(N, I) * E0 * 0.D0) GO TO 98
C CALCULATE EDG TERM
EDCUR = (50.0D0, 1415926530.D0) * XCENT * T(I) * T(J) * XCENT * (X(I) * T(J) + YCENT * (Y(I) * T(J) + R(J)))
* (YCENT * XCENT + (X1 * Y1 + X2 * Y2 + X3 * Y3) / 12.0D0) + (Q(I) * Q(J) * ARE) * (XCENT ** 2 + X1 ** 2 + X2 ** 2 + X3 ** 2) / 12.0D0)
) / (ARE * 2.D0)
S(I, J) = S(I, J) + DCMLX (0.D0, EDCUR)
98 CONTINUE
99 CONTINUE
C CALCULATE SOURCE TERM
RHS = BIGJ (N) * AREA * ML (N) / 3.D0
C PLAC ELEMENT MATRIX INTO MASTER MATRIX
DO 11 I = 1, 3
IF (NODE(N, I) .GT. NPOT) GO TO 10
11 I = I + 1
IF (NODE(N, I) .GT. NPOT) GO TO 10
10 J = J + 1

\[
CSM(NODE(N,J)-1) * NPOT = CSM(NODE(N,I)) + (NODE(N,J)-1) * NPOT
\]

11 CONTINUE

10 CONTINUE

100 CONTINUE

SOLVE MASTER MATRIX

CALL CSOLVE(CSM,CV,NPOT,CDEF)

CALCULATE AVERAGE POTENTIAL

VAL = 0.0

DO 11 J = 1, NPOT

AREA = (X(NODE(N,J)) - X(NODE(N,J-1)))

11 CONTINUE

CONTINUE

118 CONTINUE

CALCULATE LOSS RATIO

\[
DP = (Q, DC, 0.0)
\]

\[
DR = (C, DO, 0.0)
\]

\[
DS = (O, DD, 0.0)
\]

\[
RAD(1) = 2.0
\]

NLCONS = NSEG * (2 * NRAD - 1)

DO 40 N = 1, NLCONS

IF (SIGMM(N) EQ 0.0) GO TO 40

XCENT = X(NODE(N,1)) + X(NODE(N,2)) + X(NODE(N,3))/3.0

YCENT = Y(NODE(N,1)) + Y(NODE(N,2)) + Y(NODE(N,3))/3.0

X1 = X(NODE(N,1)) - XCENT

X2 = X(NODE(N,2)) - XCENT

X3 = X(NODE(N,3)) - XCENT

Y1 = Y(NODE(N,1)) - YCENT

Y2 = Y(NODE(N,2)) - YCENT

Y3 = Y(NODE(N,3)) - YCENT

CW(1) = CV(NODE(N,1))

CW(2) = CV(NODE(N,2))

CW(3) = CV(NODE(N,3))

CTRAS = (CW(1) - CV(NODE(N,1)))

\[
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\]
CTPB = (C (W (1) - C VA) * (Y (NODE (N, 1)) - Y (NODE (N, 1))) + (C (W (2) - C VA) * (Y (NODE (N, 1)) - Y (NODE (N, 1)))) + (C (W (3) - C VA) * (Y (NODE (N, 3)) - Y (NODE (N, 3))))

CTRC = (C (W (1) - C VA) * (X (NODE (N, 1)) - X (NODE (N, 1))) + (C (W (2) - C VA) * (X (NODE (N, 2)) - X (NODE (N, 2)))) + (C (W (3) - C VA) * (X (NODE (N, 3)) - X (NODE (N, 3))))

AREA = (X (NODE (N, 1)) * (Y (NODE (N, 1)) - Y (NODE (N, 1)))) + (X (NODE (N, 2)) * (Y (NODE (N, 2)) - Y (NODE (N, 2)))) + (X (NODE (N, 3)) * (Y (NODE (N, 3)) - Y (NODE (N, 3))))

Vd = ((COABS (CTP3)) ** 2 * (X1**2 + X2**2 + X3**2) / 12) + (X CENT**2)

Do = THE SOURCE CONTRIBUTION

Dr and 05 are CROSS TERMS

DP = THE EDDY CONTRIBUTION

DO IS THE SOURCE CONTRIBUTION

DR AND 05 ARE CROSS TERMS

DP = DP + (6 C.D3*6 C.DO*SIGMA*VP*4.03*PI*PI)

DO = DO + AREA/SIGMA*BIGJ(N)*DCONJG(BIGJ(N))

DR = DR + 4f.Dr*PI*AREA*(C W (1)+C W (2)+C W (3)-3*DO*C VA)*DCONJG(BIGJ(N))

DS = DS + 4f.DO*PI*AREA*BIGJ(N)*DCONJG(C W (1)+C W (2)+C W (3)-3*DO*C VA)

IF (SIGMA(N) < 0.03) GO TO 40C

40C CONTINUE

CPTOT = DP + DO + DCPLX(0, D0, 1, D0) * (DS - DR)

PDC = D0

RADC = (CPTOT + DCONJG(CPTCT)) / (2*D0*D FC)

WRITE(6, 500) RADC

WRITE(6, 501)


WRITE(6, 503) DO, DP, DS, CPTOT

503 FORMAT(9.15X, 'R A T I O O F A C T O D C R E S I S T A N C E I S = ', G14.7)

WRITE(6, 802)

802 FORMAT(9.15X, 'E D D Y L O S S', 20X, 'D C')

WRITE(6, 803) DP, PDC

WRITE(6, 803)

803 FORMAT(9.15X, 'A R E A')

WRITE(6, 803) ATOT

500 FORMAT(9.15X, 'R A T I O O F A C T O D C R E S I S T A N C E I S = ', G14.7)

15 CONTINUE

9999 CONTINUE

RETURN

END
### Iteration Number 3

<table>
<thead>
<tr>
<th>RAW POTENTIAL</th>
<th>RESIDUAL</th>
<th>ADJUSTED POTENTIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVa=(-0.14812490-10, 0.55229310-11)</td>
<td>(-0.00000000)</td>
<td>(-0.00000000)</td>
</tr>
</tbody>
</table>

NEW BOUNDARY

| CVa=(-0.11081660-17, 0.52551440-12) | (-0.00000000) | (-0.00000000) |

NEW BOUNDARY

<table>
<thead>
<tr>
<th>AVERAGE POTENTIAL</th>
<th>RESIDUAL</th>
<th>ADJUSTED POTENTIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVa=(0.45044790-10)</td>
<td>(-0.00000000)</td>
<td>(-0.00000000)</td>
</tr>
</tbody>
</table>
Single conductor with source current variation and a Hooke and Jeeves search.

Mainline is the same as that of the previous program except:

(1) Variable 'VAL' is omitted from statement 9

(2) Statements 241, 242 and 243 are omitted

(3) Statement 244 becomes 'CALL HKNJV(VAL, PTNW, 2)'
SUBROUTINE HKNVJ(VAL,PTNW,N)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION CORD(10),CTEST(10),PTNW(10)
DIMN/N/10 IFWRIT
DELTA=1.*10**-10 I=1
READ(5,27)(CORD(J),J=1,N)
27 FORMAT(1CEF,4)
DELTA=DELTA
IFWRIT=1 CALL FUNCT(CORD,N,VAL)
IFWRIT=0
C TRY FOR A BETTER POINT BY TESTING ALL DIRECTIONS
VALOLD=VAL
5 CALL NEWDIR(N,DELTA,CORD,PTNW,VAL)
IFWRIT=1 CALL FUNCT(PTNW,N,VAL)
IFWRIT=0 WRITE(6,18) VAL,(PTNW(L),L=1,N)
C TEST FOR THE NUMBER OF SUCCESSFUL CO-ORDINATES
20 ITEST=0 DO 2 J=1,N IF(CORD(J) .NE. PTNW(J)) ITEST=TEST+1
2 CONTINUE
C IF NO SUCCESSFUL SEARCHES DECREASE DELTA
25 IF(ITEST .NE. 0) GO TO 4 DELTA=DELTA/2
IF(DABS(DELTA/Delta) .LT. 1.E-3) GO TO 7 GO TO 5
C IF LESS THAN TWO NEW CO-ORDINATES DO A LOCAL SEARCH
4 IF(ITEST .NE. 1) GO TO 3 DO 21 J=1,N
21 CORD(J)=PTNW(J) GO TO 5
C IF MORE THAN ONE NEW POINT ACCELERATE
3 DO 16 J=1,N
16 CTEST(J)=2.*PTNW(J)-CORD(J)
CALL FUNCT(CTEST,N,VNEW)
IF(VNEW .GE. VAL) GO TO 17 DO 19 J=1,N
19 CORD(J)=PTNW(J)
VAL=VNEW CALL NEWDIR(N,DELTA,CTEST,PTNW,VAL)
IFWRIT=1 CALL FUNCT(PTNW,N,VAL)
IFWRIT=0 GO TO 20
7 WRITE(6,18) VAL,(PTNW(L),L=1,N)
18 FORMAT(1CEF,4)
RETURN
END
SUBROUTINE NEWDF(N, DELTA, CC, PTNW, VAL)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION CC(10), PTNW(10)
REAL*8 MAX
DO 2 L = 1, N
2 PTNW(L) = CC(L)
MAX = VAL
DO 1 J = 1, N
1 TNPT = PTNW(J)
IF (VAL .GE. MAX) GO TO 3
MAX = VAL
TNPT = PTNW(J)
3 PTNW(J) = TNPT - DELTA
CALL FUNCT(PTNW, N, VAL)
IF (VAL .GE. MAX) GO TO 1
MAX = VAL
TNPT = PTNW(J)
4 PTNW(J) = TNPT + DELTA
CALL FUNCT(PTNW, N, VAL)
IF (VAL .GE. MAX) GO TO 1
MAX = VAL
TNPT = PTNW(J)
5 RETURN
END

SUBROUTINE FUNCT(Z, NORD, VAB)
IMPLICIT REAL*8(A-B, D-H, O-Z), COMPLEX*16(C)
COMMON X(33), Y(33), NODE(56, 3), NSEG, NRAD, SIGM(55), BIGJ(56), MU(56)
DIMENSION CSM(650), R(3), Q(3), T(3), FADI(1)
COMPLEX*16 BIGJ, S(3,3), RHS, CV(33), VAL, DCMPLX, ALPHA
COMPLEX*16 SJ(3), CK(100)
DIMENSION Z(10)
REAL*8 MU, CDABS
REAL*8 EDCUR(3,3)
COMPLEX*16 D0, DR, DS, CW(3), DCONJ, CRES(33)
COMMON/IO*/ IF WRT
DATA CV/33*(0.00, 0.00)/
DATA VAL/I0.DO, CD.DO)/
AREAL = Z(1)
AIMAG = Z(2)
SIGMA = 5.007
NPO = 1 +(NORD - 1)*NSEG
NPO8 = NPO + 8
NLMNTS = NRAD*2*NSEG-NSEG
NPO1 = NPO + 1
NPO2 = NPO*NPO
ALPHA = DCMPLX(AREAL, AIMAG)
ATOT = 0.0
DO 7 I = 1, NPO
7 CK(J) = CV(J) - VAL
DO 422 J = NPO, 1
422 CK(J) = ALPHA
DO 100 N = 1, NLMNTS
XCENT = (X(NODE(N, 1)) + X(NODE(N, 2)) + X(NODE(N, 3)))/3.0
YCENT = (Y(NODE(N, 1)) + Y(NODE(N, 2)) + Y(NODE(N, 3)))/3.0
X1 = X(NODE(N, 1)) - XCENT
X2 = X(NODE(N, 2)) - XCENT
X3 = X(NODE(N, 3)) - XCENT
Y1 = Y(NODE(N, 1)) - YCENT
Y2 = Y(NODE(N, 2)) - YCENT
Y3 = Y(NODE(N, 3)) - YCENT

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$$\text{AREA} = (X(NODE(N,1)))(Y(NODE(N,3)) - Y(NODE(N,2)))$$

$$+ X(NODE(N,2))(Y(NODE(N,1)) - Y(NODE(N,3)))$$

$$+ X(NODE(N,3))(Y(NODE(N,2)) - Y(NODE(N,1))) \times 3 \times 1 - 1$$
IF($\text{SIGMM}(N) = 0.0000$) GO TO 400
$\text{XCENT} = (X(NODE(1)) + X(NODE(2)) + X(NODE(3))) / 3.0$
$\text{YCENT} = (Y(NODE(1)) + Y(NODE(2)) + Y(NODE(3))) / 3.0$
$X1 = X(NODE(1)) - \text{XCENT}$
$X2 = X(NODE(2)) - \text{XCENT}$
$X3 = X(NODE(3)) - \text{XCENT}$
$Y1 = Y(NODE(1)) - \text{YCENT}$
$Y2 = Y(NODE(2)) - \text{YCENT}$
$Y3 = Y(NODE(3)) - \text{YCENT}$
$\text{CW}(1) = \text{CVW}(NODE(1))$
$\text{CW}(2) = \text{CVW}(NODE(2))$
$\text{CW}(3) = \text{CVW}(NODE(3))$
$\text{CTRA} = (\text{CW}(1) - \text{CVA}) * (X(NODE(3)) - Y(NODE(2))) + (\text{CW}(2) - \text{CVA}) * (Y(NODE(3)) - X(NODE(2))) - \text{XCENT} + \text{X(NODE(2))}$
$\text{CTRB} = (\text{CW}(1) - \text{CVA}) * (Y(NODE(3)) - X(NODE(2))) + (\text{CW}(2) - \text{CVA}) * (X(NODE(3)) - Y(NODE(2))) - \text{YCENT} + \text{Y(NODE(2))}$
$\text{CTRC} = (\text{CW}(1) - \text{CVA}) * (X(NODE(1)) - Y(NODE(2))) + (\text{CW}(2) - \text{CVA}) * (Y(NODE(1)) - X(NODE(2))) - \text{XCENT} + \text{X(NODE(2))}$
$\text{AREA} = (X(NODE(1)) + X(NODE(3)) - 2.0) / 2.0$
$\text{C} = (\text{X(NODE(1))} - \text{XCENT}) + (\text{Y(NODE(1))} - \text{YCENT})$
$\text{VP} = ((\text{CDABS}(\text{CTRA}))**2 / ((X1)**2 + X2**2 + X3**2) / 12.0) + XCENT**2 + (\text{CDABS}(\text{CTRA}))**2 / ((Y1)**2 + Y2**2 + Y3**2) / 12.0) + YCENT**2$
$\text{DP} = \text{DP} + 60.00 + \text{SIGMA} / \text{SIGMA}$
$\text{DQ} = \text{DQ} + \text{AREA} / \text{SIGMA}$
$\text{DR} = \text{DR} + \text{AREA} * (\text{CW}(1) + \text{CW}(2) + \text{CW}(3)) - 3.0$}
$\text{PDC} = (\text{PI} * \text{RADI(1)} * \text{RADI(1)} * (2.5401 - 2**2)) / \text{SIGMA}$
$\text{WRITE}(6, 500)$
$\text{WRITE}(6, 503)$
$\text{WRITE}(6, 501)$
$\text{WRITE}(6, 500)$
$\text{WRITE}(6, 503)$
$\text{WRITE}(6, 501)$
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$\text{WRITE}(6, 501)$
$\text{WRITE}(6, 500)$
$\text{WRITE}(6, 503)$
$\text{WRITE}(6, 501)$
$\text{WRITE}(6, 500)$
Three conductors in a steel pipe using homogeneous Neumann boundary condition with a complex formulation
FINITE ELEMENT PATTERN GENERATOR FOR A CABLE CONFIGURATION

*     ***
THIS PROGRAM GENERATES THE FINITE ELEMENT PATTERN FOR A MULTI-PHASE
ENCASED CABLE CONFIGURATION, SEVERAL CURRENT-CARRYING CONDUCTORS CAN
BE LOCATED ANYWHERE WITHIN AN OUTER CASING. A STANDARD FINITE
ELEMENT PATTERN IS GENERATED WITHIN EACH CONDUCTOR, AND THE USER CAN SPECIFY
THE NUMBER AND ARRANGEMENT OF ADDITIONAL ELEMENTS FOR COMPLETING THE
PATTERN WITHIN THE REMAINDER OF THE REGION.

 IMPLICIT REAL*8(A-H,O-Z)
 REAL XT(6),YT(6),A
 DIMENSION IFADDX(4), IFADDY(4),SIGMM(40),ISFG(8),XR(3),NSPG(9)
 DATA IFADDX/1.,0.,-1.,0./,IFADDY/0.,1.,0.,-1./
 DIMENSION XT(13),YT(13),RO(3),RADI(3),JEND(3),RADIUS(10),X(200),
 Y(200),NODE(300),3
 DIMENSION DIVH(15),NC(3),DIVV(15),NBF(3),XB(3),YB(3),AROUND(3,25),
 NNN(5),LL(5),NT(5),NSF(3),NGS(3),NST(15),LROW(25),ANG(25)

 CONDUCTOR CONDUCTIVITY
 SIGMA=5.8797
 STRANDING FACTOR
 SIGMA1=SIGMA/4.35
 PIPE CONDUCTIVITY
 SIGMA1=8.19D6

 READ SEGMENTATION THICKNESS IN INCHES, AND 3 CONSTANTS WHICH ARE
 4,6, AND 3 FOR SEGMENTED CABLES AND 0,0,0 FOR UNSEGMENTED CABLES
 READ(5,8)30) THK,NSF,KSEG
 THK2=THK/2.DC

 READ NUMBER OF CONDUCTORS AND CENTRE AND INSIDE RADIUS OF PIPE
 READ(5,700) NOHM AX,XORG,YORG,RÜ

 READ OUTSIDE PIPE RADIUS AND NUMBER OF PIPE SEGMENTS
 READ(5,902) ROUT,NOUT
 WRITE (6,43) NOUT, RO ,  POUT

 PI =3.14159265359D0

 JSTART=0
 J0=1
 NJ0=1
 ND0=NDN=NDNO
 NPHASE=0
 DD 1 L=1,NPHMAX
 NPHASE=NPHASE+1

 READ NUMBER OF RADIAL DIVISIONS, SEGMENTS, AND CONDUCTING
 RADIAL DIVISIONS FOR EACH CONDUCTOR
 READ(5,701) NSEG,NTR,NSG,L,NCON
 NCML=NCN-1
 NSEG=NSG+1
 NTR=NSEG-1
 JMNX=NSG

 DTHETA=2.57/NSEG

 READ RADIAL BREAK-POINTS AS PERCENTAGE OF CONDUCTOR RADIUS
 READ(5,702) K=1,NRAD

 READ POSITION AND RADIUS OF CONDUCTOR
 READ(5,703) XO(L),YO(L),RO(L)

 RAD(L)=RADIUS(NCON)*RO(L)
 X0=X0(L)
 Y0=Y0(L)
 RO=RO(L)
 IF(NSG*N=0.0) GC TO 65C

 CALCULATE NODES FOR UNSEGMENTED CONDUCTOR

 JMNX2=J0+NSG-1
 X(NMOD)=X0
 Y(NMOD)=Y0
 NMOD=MOD+1
 DO 25 1=1,NRAD
 THETA=0
 DO 25 J=1,NSEG
 X(NMOD)=RO*RADIUS(I)*DCOS(THETA)+X0(L)
 Y(NMOD)=RO*RADIUS(I)*DSIN(THETA)+Y0(L)
 NMOD=MOD+1
 THETA=THETA+DTHETA
 NMODS(L)=NMOD

 CALCULATE NODE ORDERING FOR EACH ELEMENT FOR THIS CONDUCTOR

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C INNERNOST CORE OF ELEMENTS

DO 30 J=J0,JMAX
SIGMA(J)=SIGMA
NODE(J,1)=NDINIT
NODE(J,2)=J-J0+1+NDINIT+1
IF(J.EQ.JMAX) NODE(J,2)=NDINIT+1
30 NODE(J,3)=J-J0+1+NDINIT
GO TO 91

650 CONTINUE

C CALCULATE NODES FOR SEGMENTED CONDUCTOR
C INNER RING
DO 111 J=1,4
X(NDN0)=X0(L)+((-1)**((J+4)/2))*THIK
Y(NDN0)=Y0(L)+((-1)**((J+3)/2))*THIK
111 NDN0=NDN0+1

DO 107 J=1,4
ISEG(2*J-1)=NDNO-6+J
IF((ISEG(2*J-1).LT.NSFG) .AND. ISEG(2*J-1).LT.NSFG+3) ISEG(2*J-1)=NDINIT+3
72 ISEG(2*J)=NDNO-5+J
DTHETA=PI/NSEG*2.D0
THETA=DTHETA
NPTS=NSEG/4

C CALCULATE NODE POSITIONS FOR CONDUCTING RINGS
DO 123 M=1,NCM1
ADD=R0(L)*RADIUS(M)
DO 124 J=1,4
X(NDN0)=X0(L)+(ADD)*IFADDX(J)
Y(NDN0)=Y0(L)+(ADD)*IFADDY(J)
124 X(NDN0+1)=X0(L)+(ADD)*IFADDX(J)
Y(NDN0+1)=Y0(L)+(ADD)*IFADDY(J)
NDNC=NDNC+2
NDN0=NDN0+1
NDNO=NDN0+1

125 CONTINUE
THETA=THETA+DTHETA
123 CONTINUE
THETA=0.D0

C CALCULATE NODE POSITIONS FOR OUTER RING
DO 375 K=1,NSEG
X(NDN0)=X0(L)+R0(L)*RADIUS(J)*DCOS(THETA)+RADIUS(J)*DSIN(THETA)
Y(NDN0)=Y0(L)+R0(L)*RADIUS(J)*DCOS(THETA)

375 NDNC=NDNC+1
782 CONTINUE

C FORM TRIANGLES FOR INNER RADIUS
DO 83 J=1,2
NODE(JC,1)=NDINIT
NODE(JC,2)=J+NDINIT+1
NODE(JC,3)=J-1+NDINIT
SIGMA(JC)=C.D0
83 J0=J0+1
NDNOW=NDINIT+4
DO 41 J=1,NSEG
NODE(JC,1)=NDNOW
NODE(JC,2)=NDINIT+J-2
NODE(JC,3)=NDINIT+J-1
SIGMA(JC)=C.D0
IF((NODE(JC,2).EQ.NSEG) .AND. (JFQ.EQ.NSEG)) NODE(JC,2)=NDINIT+4
JO=JO+1
NUM=NUM+1
41 CONTINUE

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EVEN - NUMBERED RINGS (2, 4, 6, ...)

\[ I = (N, E, G, C, N, E, C) \]

\[ N = E, G, C, N, E, C \]

\[ K_1 = N T R + 2 \times N C N - 1 + I F S E G - N S E G \times (N P H A S E - 1) + N T R + 1 + I F S E G \]

\[ K_2 = N D I N I T + N S E G \]

\[ D O 40 I = 2 \times N R A D + 2 \]

IF \((I > N C N) \) \(A N D (N S E G \neq 0)\) GOTO 1370

\[ D O 35 J = 1 \times N T R \]

\[ N O D E (K_1, 1) = K_2 \]

\[ N O D E (K_1, 2) = K_2 + N S E G + 1 + N S E G \]

\[ N O D E (K_1, 3) = K_2 + N S E G + N S E G \]

\[ K_1 = K_1 + 1 \]

\[ N O D E (K_1, 1) = K_2 \]

\[ K_2 = K_2 + 1 \]

\[ N O D E (K_1, 2) = K_2 \]

\[ N O D E (K_1, 3) = K_2 + N S E G + N S E G \]

\[ K_2 = K_2 + N S E G + N S E G \]

ODD - NUMBERED RINGS (3, 5, 7, ...)

\[ K_1 = 3 \times N T R + I F S E G + J S T A R T + 1 \]

\[ K_2 = N S E G + 1 + N D I N I T + N S E G + K S E G \]

\[ D O 60 I = 3 \times N R A D + 2 \]

IF \((I > N C N) \) \(A N D (N S E G \neq 0)\) GOTO 370

\[ D O 55 J = 1 \times N T R \]

\[ N O D E (K_1, 1) = K_2 \]

\[ K_2 = K_2 + 1 \]

\[ N O D E (K_1, 2) = K_2 \]

\[ N O D E (K_1, 3) = K_2 + N S E G - 1 + N S E G \]

\[ K_1 = K_1 + 1 \]

\[ N O D E (K_1, 1) = K_2 \]

\[ N O D E (K_1, 2) = K_2 \]

\[ N O D E (K_1, 3) = K_2 + N S E G - 1 + N S E G \]

\[ K_2 = K_2 + N S E G + N S E G \]

IF \((N S E G \neq 0)\) GOTO 370

GOTO 65

FORM TRIANGLES FOR OUTER RING

\[ J_0 = K_1 \]

\[ K_1 = K_2 \]

\[ K O D R E V = N C N / 2 \]

\[ K O D R E V = N C N / 2 \times K O D R E V \]

IF \((K O D R E V \neq 0)\) \(K_1 = K_1 + N T R \)

IF \((K O D R E V = 0)\) \(J_0 = J_0 + 2 \times N T R \)

\[ L S T = K_1 - N T R \]

\[ D O 371 K = 1, 4 \]

\[ N O D E (J_0, 1) = L S T \]

\[ N O D E (J_0, 2) = L S T + 1 \]

\[ N O D E (J_0, 3) = K_1 \]

\[ J_0 = J_0 + 1 \]

\[ L S T = L S T + 1 \]

\[ D O 372 J = 1 \times N S E 4 \]

\[ N O D E (J_0, 1) = L S T \]

\[ N O D E (J_0, 2) = L S T + 1 \]

\[ N O D E (J_0, 3) = K_1 \]

\[ J_0 = J_0 + 1 \]

\[ N O D E (J_0, 1) = K_1 \]

\[ N O D E (J_0, 2) = L S T + 1 \]

\[ N O D E (J_0, 3) = K_1 + 1 \]

\[ J_0 = J_0 + 1 \]

\[ L S T = L S T + 1 \]

\[ 372 K_1 = K_1 + 1 \]

\[ 371 C O N T I N U E \]

\[ N O D E (J_0, 2, 2) = K_1 - N S E G - N S E G - N S E G \]
C

D

E

F

G

H

I

J

K

L

M

N

O

P

Q

R

S

T

U

V

W

X

Y

Z

103

05/03/73

NODF(JC-1,2)=K1-NSEG-NSSEG=NSGC=NSMG
NODF(JC-1,2)=K1-NSEG

KSTT=NCON+2

65 NTR2=NTR2*

JNCW=NCTA+1+IFSEG+NTR
ICOUN=JNCW
SIG=SIGMA

DO 62 JJ=2,NRAD
IF((JJ-1) .EQ. NCM) .AND. (NSEGC.NE.0)) NTR2=NTR2+4
IF((JJ-2) .EQ. NCM) .AND. (NSEGC.NE.0)) NTR2=NTR2+4
IF(JJ.GT.NCCN) SIG=0.0
DO 64 KK=1,NTR2

C COMPUTE CONDUCTIVITY
JARG=JNCW+(JJ-2)*NTR2+KK-1
IF(JJ.GT.NCM1+1) JARG=JARG+NSEGC*NCM1-NSSEG
IF(JJ.GT.NCM1+1) JARG=JARG+NSEGC*NCM1+NSEGC-NSSEG

64 SIGMN(JARG)=SIG
IF((NSEGC.FLT.0).OR.(JJ.GT.NCON)) GO TO 63
DO 62 ML=1,NSEG
ICOUN=ICOUN+1-66
GO TO 62

66 SIGMN(ICOUN+1)=0.0
62 ICOUN=ICOUN+NSEG*2+2
63 CONTINUE

JSTART=JSTART+NTR*(NCCN-1)-NSEGC+IFSEG
JEND(J)=JSTART

C DRAW ALL ELEMENTS
J0=JSTART+1
NDINIT=1+NTR=NSEG-NSEG)*NPHASE+1
1 CONTINUE

700 FORMAT(12,3(F5.3))
701 FORMAT(3(12))
702 FORMAT(3(F7.4))

C SELECT GRID SIZE
C READ NUMBER OF HORIZ. AND VERT. GRID LINES
READ(5,705)(DIVV(J),J=1,NVERT)
READ(5,705)(DIVH(J),J=1,NHORZ)

C ***NOTE*** A HORIZONTAL GRID LINE SHOULD NOT PASS EXACTLY THROUGH
C A NODE. GRID LINES SHOULD BE PLACED SO THAT AT LEAST ONE LINE
C SEPARATES CONDUCTORS

WRITE(6,800)

600 FORMAT(12,'VERTICAL AND HORIZONTAL GRID POSITIONS')
WRITE(6,46)(DIVV(J),J=1,NVERT)
WRITE(6,46)(DIVH(J),J=1,NHORZ)

46 FORMAT(12,F10.5)

C INITIALIZE BOUNDARY COUNTERS AND FLAGS
DO 207 J=1,NPHASE
NC(J)=1

207 NAF(J)=1

NTH=1
C NUMBER THE NODES
DO 202 J=1,NHORZ
NTV=1
IFLAG=0
DO 203 K=1,NVERT
X0NDO=XORG+PO*DIVV(K)
Y0NDO=YORG+PO*DIVH(J)
C CHECK IF NODE IS IN SHEATH CIRCLE
IF(DSORT((X0NDO-XORG)**2+(Y0NDO-YORG)**2).LE.PO) GO TO 999
C CHECK IF NODE POSITION SHOULD BE MODIFIED
IF((X0NDO).EQ.0.0) KK=K+NTV
JH=J+NTH
XS=XORG+PO*DIVV(KK)
YS=YORG+PO*DIVH(J)
C DO NOT ASSIGN A NUMBER TO THIS NODE
GO TO 203

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MODIFY VALUE OF X FOR THIS NODE

IF(DIVV(K,L,T,N) != X(NDC))
GO TO 203

C MODIFY VALUE OF Y FOR NODE

Y(NDC) = SORT(P**2 - X(NDC)**2)

C TEST IF NODE FALLS WITHIN CONDUCTOR RADIUS

IF(DIVH(J,L,T,N) != Y(NDC))
GO TO 203

C STORE FIRST NODE OF EACH GRID ROW

IF(FLAG.EQ.0) NST(J) = NDNO
FLAG = 1

C TEST IF NODE IS A CONDUCTOR BOUNDARY NODE

JN = J-1
KM = K-1
KP = K+1

IF((JN.EQ.1) .AND. (KM.EQ.0)) .OR. ((JN.EQ.3) .AND. (KM.EQ.2)) .OR. 
((KP.EQ.1) .AND. (KM.EQ.0)) .OR. ((KP.EQ.3) .AND. (KM.EQ.2))
GO TO 103

IF(DSORT((X(KP)-XORG)**2+(Y(KP)-YORG)**2) .LT. RO(L)*9999)
GO TO 209

C ORDER THE BOUNDARY NODES

DO 117 L = 1, NPHASE
DO 108 JJ = 1, 3
DO 108 KK = 1, 3
IF((JM.EQ.1) .AND. (JM.EQ.0)) .OR. ((JN.EQ.3) .AND. (KM.EQ.0))
GO TO 209

IF((JN.EQ.3) .AND. (KM.EQ.0)) .AND. (DSORT((X(KM)-XORG)**2+(Y(KM)-YORG)**2) .LT. RO(L)*9999)
GO TO 209

CONTINUE
GO TO 117

C ORDER THE BOUNDARY NODES

117 CONTINUE

C ORDER THE BOUNDARY NODES

DO 210 L = 1, NPHASE
NC(L) = NC(L)+1

210 CONTINUE

C ORDER THE BOUNDARY NODES

DO 220 L = 1, NPHASE
NC(L) = NC(L)-1

220 CONTINUE

C ORDER THE BOUNDARY NODES

DO 230 L = 1, NPHASE
NC(L) = NC(L)-1

230 CONTINUE

C ORDER THE BOUNDARY NODES

DO 240 L = 1, NPHASE
NC(L) = NC(L)-1

240 CONTINUE
MAIN

DATE = 7/5/304  05/43/38

NAC=1
NR=NODES(L)-NSEG(L)+1
NODES(L)=NR
J1=J0
DO 70 J=1,NTRIA
NP1=NP+1
NPC1=NBC+1
IF(NP1.GT.NODES(L))NP1=NODES(L)
IF(NBCP1.GT.NC(L))NBCP1=1

CALCULATE TEST DISTANCES

D1=DSQRT((X(NR)-X(NBOUN(L,NPC1)))**2+(Y(NR)-Y(NBOUN(L,NPC1)))**2)
D2=DSQRT((X(NRP1)-X(NBOUN(L,NBC)))**2+(Y(NRP1)-Y(NBOUN(L,NBC)))**2)
D3=DSQRT((X(NR)-X(NBOUN(L,NBC)))**2+(Y(NR)-Y(NBOUN(L,NBC)))**2)
D4=DSQRT((X(NRP1)-X(NBOUN(L,NPC1)))**2+(Y(NRP1)-Y(NBOUN(L,NPC1)))**2)
D5=DSQRT((X(NR)-X(NRP1))**2+(Y(NR)-Y(NRP1))**2)

TEST FOR SHORTEST DISTANCES

IF(D1.LE.D2)GO TO 305
IF((D2/D3).GE.((C3+PO(L)*DTHETA*.3)/D3))GO TO 305
IF(D2/(D3+D5).GE.99)GO TO 306
XTEST=X(NRP1)-2500*(X(NRP1)-X(NBOUN(L,NBC)))
YTEST=Y(NRP1)-2500*(Y(NRP1)-Y(NBOUN(L,NBC)))
RTTEST=DSQRT(XTEST-XO(L))**2+(YTEST-YO(L))**2)

CHECK FOR IMPROPER PLACEMENT

IF(RTEST.LE.R0(L))GO TO 306
NODE(J1,1)=NR
NODE(J1,3)=NBOUN(L,NBC)
NR=NR+1
IF(NR.GT.NODES(L))NR=NODES(L)
NODE(J1,2)=NP
IF((J.EQ.NTRIA).AND.(NBC.NE.1))GO TO 710
GO TO 306
NODE(J1,2)=NBOUN(L,NBC)
GO TO 306

DRAW TRIANGLES

JFIN=J1+NTRIA-1
DO 301 J=1,JFIN
SIGMM(J)=0.000
301 CONTINUE
70S FORMAT(15(F4.2))
902 FORMAT(F5.2,13)

FORM TRIANGLES INSIDE STEEL PIPE

J01=J0
NHMI=NHMI+1
DO 400 J=1,NHMI

INITIALIZE FLAGS AND COUNTERS

TEST=(-1)**J
K=J
NCOWN=0
NCOUN=0
F1=1
F2=1
F3=1
IF((Y(NST(J))+GE.*YORG).AND.(Y(NST(J+1))+LT.*YORG))NSTAR=J+1
IF(J.EQ.1)GO TO 410
IF(J.EQ.NHMI)GO TO 411
401 IF((NST(J+1)+NCOUN+1).GE.NFIN)GO TO 400
F1=1
IF(X(NST(J)+NCOWN+1).LE.X(NST(J)+NCOWN))F2=0
IF(X(NST(J+1)+NCOUN+1).LE.X(NST(J)+NCOWN))GO TO 412

CHECK IF TRIANGLES ALREADY FORMED IN THIS REGION

DO 402 L=1,NPHAS

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IF((Y(NST(J+1)) .LE. (Y(L)+PO(L))) .AND. ((X(NST(J+1)+NCOWN) .LE. XO(L) .AND. (X(NST(J+1)+NCOWN+1) .GE. XO(L)) .AND. (Y(NST(J)) .LE. Y(L)+PO(L)) .AND. (X(NST(J)) .LE. X(L)+PO(L)) .AND. (X(NST(J)+NCOWN) .LE. XO(L)) .AND. (X(NST(J)+NCOWN+1) .GE. XO(L)) .AND. (Y(NST(J)) .LE. Y(L)+PO(L)))) GO TO 406

402 CONTINUE

C CHECK GRID DIRECTION
IF(ITEST.EQ.-1) GO TO 404
GO TO 412
C FORM FIRST TRIANGLE OF FIRST ROW
410 NODE(J0,1)=NST(J)
NODE(J0,2)=NST(J+1)+1
NODE(J0,3)=NST(J+1)
J0=J0+1
NCOWN=1
GO TO 401
C FORM FIRST TRIANGLE OF LAST ROW
411 NODE(J0,1)=NST(J)
NODE(J0,2)=NST(J+1)+1
NODE(J0,3)=NST(J+1)
NCOWN=1
J0=J0+1
GO TO 401
C ADJUST COUNTERS FOR GRID NEAR CONDUCTORS
403 NCOWN=NCOWN+1
407 NCOWN=NCOWN+1
IF((X(NST(J)+NCOWN) .LE. X(L)+PO(L))) GO TO 401
IF((X(NST(J+1)+NCOWN) .LT. (XO(L)+PO(L)))) GO TO 403
GO TO 407
406 NCOWN=NCOWN+1
408 NCOWN=NCOWN+1
IF((X(NST(J+1)+NCOWN) .LE. (XO(L)+PO(L)))GO TO 401
IF((X(NST(J)+NCOWN) .LT. (XO(L)+PO(L))) GO TO 406
GO TO 408
C FORM REGULAR TRIANGULAR GRID
404 NODE(J0,1)=NST(J)+NCOWN
NODE(J0,2)=NST(J+1)+NCOWN+1
NODE(J0,3)=NST(J+1)+NCOWN
IF(F3.EQ.1) LROW(J)=JO
F3=0
JO=JO+1
405 IF(F2.EQ.0) GO TO 400
NODE(J0,1)=NST(J)+NCOWN
NODE(J0,2)=NST(J)+NCOWN+1
NODE(J0,3)=NST(J+1)+NCOWN+1
IF(F3.EQ.1) LROW(J)=JO
F3=0
JO=JO+1
NCOWN=NCOWN+1
NCOWN=NCOWN+1
GO TO 401
C FORM REGULAR TRIANGULAR GRID
412 IF(F2.EQ.0) GO TO 400
NODE(J0,1)=NST(J)+NCOWN
NODE(J0,2)=NST(J)+NCOWN+1
NODE(J0,3)=NST(J+1)+NCOWN
JO=JO+1
IF(F3.EQ.1) LROW(J)=JO
F3=0
IF(F1.EQ.1) GO TO 400
NODE(J0,1)=NST(J)+NCOWN+1
NODE(J0,2)=NST(J+1)+NCOWN+1
NODE(J0,3)=NST(J+1)+NCOWN
JO=JO+1
NCOWN=NCOWN+1
NCOWN=NCOWN+1
GO TO 401
C CONTINUE
D0 747 J=J0+1,J0
747 SIGMV(J)=0.000
C SPECIFY CONDUCTIVITY
J'=J'+1
J=J+1
NBEG=NBEG
THETA=THETA
DTHETA=2.00*DPI/NRANG
DO 903 I=1,NOUT
X(NDNO)=ROUT*DOSIN(THETA)
Y(NDNO)=ROUT*DOSIN(THETA)
NDNO=NDNO+1
903 THETA=THETA+DTHETA
KM=1
NN=2
I=1
J=NSTAR
C ORDER NODE NUMBERS OF INNER PIPE BOUNDARY
900 NT(I)=NST(J)-1
I=I+1
J=J+1
IF(J,NE.1) GO TO 900
901 NT(I)=NST(2)-NN
NN=NN+1
I=I+1
J=J+1
IF(NT(I-1),NE,NST(1)) GO TO 901
904 NT(I)=NST(J+1)
I=I+1
J=J+1
IF(J,NE,NHORZ) GO TO 904
905 NT(I)=NST(NHORZ)+NN
I=I+1
NN=NN+1
IF(NT(I-1),NE,NFIN-1) GO TO 905
906 NT(I)=NST(J-1)
I=I+1
J=J-1
IF(J,NE,NSTAR) GO TO 906
NPTS=I-1
NTP1=NPTS+NSOUT
NBE=1
NRE=1
C FORM PIPE TRIANGLES
DO 910 J=1,NTRIA
NRPl=NR+1
NBBPl=NBB+1
IF(NRPl,GT,NBEG+NSOUT-1) NRPl=NBEG
IF(NBPl,GT,NPTS) NBBPl=1
D1=DSQRT((X(NR)-X(NT(NBBPl))),**2+(Y(NR)-Y(NT(NBB))),**2)
D2=DSQRT((X(NRPl)-X(NT(NBB))),**2+(Y(NRPl)-Y(NT(NBB))),**2)
IF(D1.LT.D2) GO TO 922
NODE(JO,1)=NR
NODE(JO,2)=NT(NBB)
NR=NR+1
IF(NR,GT,NBEG+NSOUT-1) NR=NBEG
NODE(JP,3)=NT(NBB)
GO TO 910
922 NODE(JO,1)=NR
NODE(JO,2)=NT(NBB)
NBB=1
IF(NBB,GT,NBEG+NSOUT-1) NBB=NBEG
NODE(JP,3)=NT(NBB)
910 JO=JO+1
C READ NUMBER OF EXTRA RINGS, NUMBER OF RING SEGMENTS, AND RING
C RADII IN INCHES
READ(3,883)NXRNG,(NSRG(J),J=1,9),(RXR(J),J=1,NXRNG)
WRITE(6,801)
801 FORMAT('EXTRA PING RADIUS')
WRITE(6,444)(RXR(J),J=1,NXRNG)
444 FORMAT(' ',8G14.7)
883 FORMAT(10I2,5F10.5)
C FORM EXTRA RINGS
DO 881 J=1,NXRNG
NRRG=NRRG(J)
THETA=THETA
DTHETA=2.00*DPI/NRANG
NXRNG=NXRNG
DO 880 K=1,NNRG
X(NDNO)=RXR(J,K)*DOSIN(THETA)
Y(NDNO)=RXR(J,K)*DOSIN(THETA)
880 CONTINUE
881 CONTINUE
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C CONVERT INCHES TO METERS
DO 277 I=1,NDTOT
X(I)=X(I)*2.5401E-2
Y(I)=Y(I)*2.5401E-2
C WRITE DATA ON TAPE
REWIND
WRITE(2)NDET,JFIN,NCUT,NPHMAX,NTRIA,
*(NODES(L),L=1,3),(NSEGM(L),L=1,3),
*(JEND(L),L=1,3),
*(RADI(I),L=1,NPHMAX),
*(XO(L),L=1,3),
*(YO(L),L=1,3)
WRITE(2)((NODE(J,L),L=1,3),J=1,JFIN),
*(SIGMM(L),L=1,JFIN)
WRITE(2)(X(I),I=1,NDTOT)
WRITE(2)(Y(I),I=1,NDTOT)
WRITE(2)(NXRNG,NRSRG(J),J=1,NXRNG)
ENDFILE2
STOP
END

* ID,EBCDIC,SOURCE,LIST,NODECK,LOAD,NOMAP
* NAME = MAIN + LINECNT = 80
URCE STATEMENTS = 546, PROGRAM SIZE = 29342
AGNOSTICS GENERATED

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SUBROUTINE ORDER (NCCL, ANG, NT)
IMPLICIT REAL*6 (A-H, O-Z)
DIMENSION ANG(25), NT(50)

C C THIS SUBROUTINE ORDERS GRIDNODES WHICH BOUND CONDUCTORS

C
DO 2 K = 1, NCCL
  AMAX = -1.
  DO 1 J = 1, NCCL
    IF (ANG(J) .LT. AMAX) GO TO 1
    AMAX = ANG(J)
  NT(K) = J
  1 CONTINUE
  ANG(NT(K)) = -2.
2 CONTINUE
RETURN
END

16 INSIDE RAD. 5.120000  COUTSIDE RAD. 5.300000
VERTICAL AND HORIZONTAL GRID POSITIONS
  -1.00000 -0.50000 -0.30000  0.0  0.30000  0.55000  1.00000
  1.00000  0.50000  0.35000 -0.30000 -0.60000 -0.75000 -1.00000
EXTRA RING RADII
  5.350000  5.400000
C  SPECIFY BOUNDARY RADIUS
C  READ DATA FROM TAPES
C  INITIALIZE PIPE MATERIALITY
C  CALCULATE APPROXIMATE BOUNDARY POTENTIAL FOR USE IF DESIRED
C  READ NUMBER OF COND. RINGS, PERMEABILITY, AND COND. AND INSULATION
C  CURRENT
C  READ (5,22) NCCN,MU1,BIGJ1,BIGJ2
BIGJ1=Bigj1=0.0, (PI* (RAD(1)+0.254011)**2)
NCCD(L)=NCCN
22 FORMAT(1X,2I5,8E10.5)
WRITE (6,33) NCCN,MU1,BIGJ1,BIGJ2
33 FORMAT(1X,*NUMBER OF RINGS WITHIN THE CONDUCTOR*:*,IS/5X,1*CURRENT DENSITY*:*,IS/5X,1*WITHIN THE CONDUCTOR*:*,IS/5X,1*WITHIN T
HE CONDUCTOR*:*,2(G12.5)/*5X,*WITHIN THE INSULATION*:*,2(G12.5)*/X")
WRITE (6,34) NLCN(L)=NLCN(L)+JEND(L)-1
34 FORMAT(1X,*X AT CENTRE IS*:*,IS/5X,*Y AT CENTRE IS*:*,IS/5X)
IF(L,NE.1) NLCN(L)=NLCN(L)+JEND(L)-1
IF(L,E.1) NSEG=JEND(L)+1
NF2=JEND(L)
C  ASSIGN PERMEABILITIES
C (IV) GET VALUES OF PERMEABILITY AND CURRENT DENSITY FOR EACH ELEMENT
CALL MUAND(JM1,BIGJ,NSEG,NF12,SM1,BIGJ1,BIGJ2)
777 CONTINUE
CVA(4)=0.000000
ATCT(4)=C0.00
BIGJ2=0.000000
SM1=0.000000
777 CONTINUE
C  ASSIGN PIPE PERMEABILITY
CALL MUAND(JM1,BIGJ,NSEG,NF12,SM1,BIGJ1,BIGJ2)
C  IFPIPE IS ZERO THEN PIPE IS NOT PRESENT
C
C  MAIN DATE = 763 1 6  C6/32/19

C  NOTST IS THE NUMBER OF UNFIXED NODES
NOTST =
IF(IEPIPE.NE.4) NOTST=IEPIPE/IEPIPE
NPCT=NODT+N5OUT *(NOTST-1)
NPCT1=NPCT+1
NPOTSO=NOTST*NPCT
WRITE (6,800)
800 FORMAT(' , ', 'APPROXIMATE BOUNDARY POTENTIALS' )
DO 750 J=1,N5OUT
750 WRITE(6,551) COUN(J)
501 FORMAT(' , 2(G14.7))
DO 15 JJ=1,2
DO 441 L=1,4
C  ATCT(L) = C . D 0
DO 7 I=1,NPOT
DO 7 J=1,NPOTL
7 CSM(I+MPCT*(J-1))=(O. D 0)
C (VI) MAIN LOOP-CALCULATE MATRIX ENTRIES FOR EACH ELEMENT
C AND PLUS THEM INTO MATRIX EQUATION (SM(A)=(RM)
C REQUIRE inputs: NAD,NSFG,NODE,X,Y,MU,BIGJ
C RESULTS:CSM
NLNMTS=JFIN
DO 100 N=1,NLMNTS
C (1) CALCULATE Q(I) AND P(I)
XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.00
YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.00
X1=X(NODE(N,1))-XCENT
X2=X(NODE(N,2))-XCENT
X3=X(NODE(N,3))-XCENT
Y1=Y(NODE(N,1))-YCENT
Y2=Y(NODE(N,2))-YCENT
Y3=Y(NODE(N,3))-YCENT
J=2
K=3
DO 8 I=1,3
Q(I)=(Y(NODE(N,K))-Y(NODE(N,J))
R(I)=(X(NODE(N,J))-X(NODE(N,K))
T(I)=X(NODE(N,J))*Y(NODE(N,K))-X(NODE(N,K))*Y(NODE(N,J))
J=J+1-J/3*K
8 K=K+1-K/3*K
C (2) CALCULATE ELEMENT MATRIX ENTRIES
AREA=(X(NODE(N,1))*Y(NODE(N,3))-Y(NODE(N,2)))+X(NODE(N,2))*Y(NODE(N,1))-Y(NODE(N,3))
C=+X(NODE(N,2))*Y(NODE(N,1))-Y(NODE(N,3))
C=(X(NODE(N,3))*Y(NODE(N,2))-Y(NODE(N,1)))*5.0-1
DO 9 I=1,3
DO 99 J=1,3
SREAL=(Q(I)*Q(J)+R(I)*R(J))/AREA*4.00
S(I,J)=DCMPLX(SREAL,0.00)
IF(SIGMM(N).LT.0.00) GO TO 93
EDCUR=(60.00*P*MU(N)**SIGMM(N)/(T(I)*T(J)+XCENT**2+Q(I)**2)+(Q(I)**2)+(R(I)**2)+(Q(I)*R(I))**2+(Q(I)*XCENT**2+Y1**2+Y2**2+Y3**2)
*12.00*12.00)+S(I,J)*(T(I)**2+Q(I)**2+R(I)**2+T(J)**2)*12.00
S(I,J)=S(I,J)-DCMPLX(0.00,EDCUR)
58 CONTINUE
S(I,J)=S(I,J)/MU(N)
C CONTINUE
C (3) CALCULATE RIGHT HAND SIDE OF ELEMENT MATRIX
P=SIGMM(RIG(J))*(AREA)*3.00
C (4) ADD ENTRIES INTO BIG MATRIX EQUATION: (SM(A)=(RM)
DO 10 I=1,3
IF(NODE(N,I)).GT.NPOT GC TO 10
DO 11 J=1,3
CSM(NODE(N,I)+NODS(N,J)-1)*NPOT=CSM(NODE(N,I)+NODS(N,J)-1)*NPOT
11 CONTINUE
C (5) WRITE VALUES
WRITE(6,920)
920 FORMAT(' , 'VECTOR POTENTIAL VALUES' )
C CORRECTION OF FUNCTIONAL
CALL CSFLEX(CSM,CV,NPOT,CYST)
WRITE(6,864)
864 FORMAT( •  ','V E CTOR POTENTIAL VALUES')
C  COFRECTICN OF UNCTIC
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DO 920 J = 1, NPOT
   CV(J) = CONJG(CV(J))
   NZHU = NCAM + NREG + 1
   NPOT1 = NPOT + 1
   NPOTS = NPOT + 1
   THIS SECTION RE-EVALUATES SM THEN EVALUATES THE RESIDUAL
   IF DESIRED CMPRUV IS USED HERE
   DO 200 N = 1, NUMNTS
      AREA = (X(NODE(N, 1)) - X(NODE(N, 2)))
      + (X(NODE(N, 2)) - X(NODE(N, 3)))
      + (X(NODE(N, 3)) - X(NODE(N, 1)))
      IF(SIGMM(N) .LE. 0.0) GO TO 29
      C
      C CALCULATE AVERAGE POTENTIAL
      C
      L = (N-1)/JEND(L)+1
      IF(L .GT. 4) L = 4
      ATOT(L) = ATOT(L) + AREA
      DO 247 I = 1, 3
         CVA(L) = CVA(L) + CV(NODE(N, I)) * AREA / 3.
         GO TO 29
      247 CONTINUE
      950 CONTINUE
      29 CONTINUE
      IF(N .NE. JFIN) GO TO 200
      C
      C THIS SECTION IS USED TO RECONSTRUCT THE MASTER MATRIX IF
      C CMPRUV IS USED
      DO 18 I = 1, 3
         IF(NODE(N, I) .GT. NPOT) CV(NODE(N, I)) = CV(NODE(N, I) + NPOT)
         DO 21 J = 1, 3
            SM(NODE(N, I) + (NODE(N, J) - 1) * NPOT) = SM(NODE(N, I) + (NODE(N, J) - 1) * NPOT)
         21 CONTINUE
      18 CONTINUE
      RHS = CONJG(BIGJ(N)) * AREA * MU(N) / 28
      DO 230 I = 1, 3
         IF(NODE(N, I) .GT. NPCT) GO TO 20
         DO 220 J = 1, 3
            CSM(NODE(N, I) + (NODE(N, J) - 1) * NPOT) = CSM(NODE(N, I) + (NODE(N, J) - 1) * NPOT)
         220 CONTINUE
      230 CONTINUE
      C
      C PUT CALL TO SUBROUTINE CMPRUV HERE IF DESIRED
      DO 510 L = 1, 4
      510 WRITE(*, IP5) CVA(L)
      1050 FORMAT(10I4) CVA = (0, G14.7, 0, G14.7, 0)
      11 CONTINUE
      12X, 'FAN POTENTIAL', 26X, 'RESIDUAL', 22X, 'ADJUSTED POTENTIAL
      CVA(1) = CVA(1) * ATOT(1) + CVA(2) * ATOT(2) + CVA(3) * ATOT(3) + CVA(4) * ATOT(4)
      CVA(1) = CVA(1) / (ATOT(1) + ATOT(2) + ATOT(3) + ATOT(4))
      NEQ = NEQ + MAX + 1
main

calculate loss ratio

do 411 l=1,nfnd

if (ipipe.eq.0.0) and (l.eq.nend) go to 421
if (l.ne.1) nbg = jend(l-1) + !
if (l.eq.nend) nbg = jend(l) + !
if (l.eq.nend) nfi = jend(l)
if (l.ne.nend) write(5,504) nend, ifpipe, jend(l), nlcon(l), nfin

504 format(' ',1,12,12,14,14,14)
do 400 n=nfg,nfin

ip(sign(n)) .eq.0.0 go to *33
xcfnt=x(node(n,1))+x(node(n,2))+x(node(n,3))/3.0
ycfnt=(y(node(n,1))+y(node(n,2))+y(node(n,3))/3.0

xi=x(node(n,1))-xcfnt
x2=x(node(n,2))-xcfnt
x3=x(node(n,3))-xcfnt
y1=y(node(n,1))-ycfnt
y2=y(node(n,2))-ycfnt
y3=y(node(n,3))-ycfnt

cw(1)=c(node(n,1))-cva(1)
cw(2)=c(node(n,2))-cva(1)
cw(3)=c(node(n,3))-cva(1)
cwa=(cw(1))*(x(node(n,3))*y(node(n,2))-x(node(n,2))*y(node(n,3)))*s*1
vp=(|cdabs(cwa)|**2*(x1**2+x2**2+x3**2)/12.0+ycfnt**2)+(cdabs(cwa))**2*(1.0+y(node(n,1))**2+x1**2+x2**2+x3**2)/12.0+ycfnt**2)
+
(cตรา*conjug(c埽)+c埽*conjug(c埽)+c埽*conjug(c埽)+c埽*conjug(c埽)+c埽*conjug(c埽))**2*((x1*y1+x2*y2+x3*y3)**2/12.0+ycfnt**2)
+x1*ycfnt*y1+x2*ycfnt*y2+x3*ycfnt*y3)

505 format(' ',1,14,2,14,7)
dq is the d.c. loss

dq=dq+area/sigm(n)*sigj(n)*conjug(bigj(n))

dq=s*a*i*1*area*signj(bigj(n))*conjug(cwa(cwa+1)+cwa(2)+cwa(3))

dr=dr+s*a*i*1*area*(cw(1)+cw(2)+cw(3))

401 continue

dp is the eddy loss

dp=dp+360.0*dc*sign(n)**1*v**4.0**d*p*1*1

400 continue

write(3,803)

803 format(' ',5x,'d.c. loss',*25x,'i4imaginary cross terms',*30x,total loss)

write(6,533) do,dr,ds,cftct

p2c=c

dt=dt+do

d=dp*dt+dp

if (l.ne.nend) racdc=(cpto+dc*conjug(cpto))/2.0*dpdc

if (l.ne.nend) racdc=1.0+dp/dq

503 format(' ',a(14,7))

if (l.ne.nend) write(5,51) racdc

write(3,844)

844 format(' ',*1,1*3,eddy loss1)

write(6,551) dp,fp,dc,dot(l)

401 continue

racdc=1.0+dp/dt
dt=dt+do

write(6,551) racdc

end
DO 16 J=1, NFI1
DO 17 K=1, 3
XT(K)=X(NODE(J, K))
YT(K)=Y(NODE(J, K))
17 CT(K)=CV(NODE(J, K))-CVA(A)
CALL MUADJS(XT, YT, CT, VAL, J)
16 MU(J)=.1257E-5*VAL
15 CONTINUE
500 FORMAT( 'RATIO OF AC TO DC RESISTANCE IS = ', G14.7)
9999 CONTINUE
STOP
END
SUBROUTINE CSOLVE (CA, CX, N, CDET)
IMPLICIT COMPLEX(C)
REAL CABS
REAL*8 CABS
DIMENSION IPIV(2*N), JPIV(N*C), CA(1), CX(1)
LOGICAL F1, F2

THIS SUBROUTINE SOLVES A SYSTEM OF COMPLEX SIMULTANEOUS EQUATIONS BY GAUSSIAN ELIMINATION

WRITTEN BY R.H. ALEXANDER

USES MAXIMUM PIVOT STRATEGY

F1 = .FALSE.,
F2 = .FALSE.,
NM1 = N-1
NP1 = N+1
DO 5 I = 1, NP1
JPIV(I) = I
CDET = (1.0D0, 0.0D0)
DO 100 I = 1, N
IP1 = I+1
CELMAX = CA((JPIV(I)-1)*N+IPIV(I))
DO 25 I = 1, N
DO 20 J = I, N
IF (CABS(CA((JPIV(J)-1)*N+IPIV(I))) .LE. CABS(CELMAX)) GO TO 20
ISAVE = JPIV(I)
JPIV(I) = JPIV(J)
JPIV(J) = ISAVE
ISAVE = JPIV(I)
IPIV(I) = ISAVE
CELMAX = CA((JPIV(I)-1)*N+IPIV(I))
20 CONTINUE
25 CONTINUE
IF (F1 .OR. F2) GO TO 26
CDET = CDET * CELMAX
IF (CABS(CDET) .LE. 1.0D-77) F1 = .TRUE.,
IF (CABS(CDET) .GT. 1.0D77) F2 = .TRUE.,
IF (F1 .OR. F2) K = I
26 CONTINUE
DO 30 J = IP1, NP1
CA((JPIV(J)-1)*N+IPIV(I)) = CA((JPIV(J)-1)*N+IPIV(I))/CELMAX
CA((JPIV(I)-1)*N+IPIV(I)) = (1.0D0, 0.0D0)
DO 35 I = 1, N
DO 30 J = I, NP1
CA((JPIV(J)-1)*N+IPIV(I)) = CA((JPIV(J)-1)*N+IPIV(I)) - CA((JPIV(J)-1)*N+IPIV(I))
CA((JPIV(I)-1)*N+IPIV(I)) = (1.0D0, 0.0D0)
30 CONTINUE
35 CONTINUE
100 CONTINUE
DO 105 J = 1, N
105 CX(J) = CA(N*N+J)
IF (F1) GO TO 110
IF (F2) GO TO 109
RETURN
109 WRITE(6,108) K, K, N, N, CDET
RETURN
110 WRITE(6,111) K, K, N, N, CDET
111 FORMAT(13, 13, 13, 'ELEMENT OF THIS ' , 13, 'X', 13, ' SYSTEM = (' , 2G12.5, 13, 2G12.5, *))
RETURN
END
THIS SUBROUTINE MAY BE USED TO ITERATIVELY ADJUST THE VALUES OBTAINED BY CSOLVE.

WRITTEN BY P.M. ALEXANDER

FOR EACH ROW, SOLUTION ENTRY CORRESPONDING TO LARGEST ROW ELEMENT IS ADJUSTED BY SUBTRACTING RESIDUAL OF OTHER ROW ELEMENTS FROM THE RIGHT HAND SIDE.

NSO=N*N
XNORM=0.D0
RNORM=0.D0
DO 2 I =1,N
CRES(I)=CA(NSQ+I)
DO 1 J=1,N
1 CRES(I)=CRES(I)-CA((J-1)*N+I)*CX(J)
XNORM=XNORM+(CAES(CX(I)))**2
2 RNORM=RNORM+(CAES(CRES(I)))**2
RNORM=DSORT(RNORM)
XNORM=DSORT(XNORM)
IF(RNORM/XNORM .LE. CRIT ) GO TO 100
ITER=1
3 XNORM=0.D0
DO 5 I =1,N
CXOLD(I)=CX(I)
CMA=0.D0
DO 9 J=1,N
IF(CDABS(CA(N+(J-1)*N)) .LT. CMAX) GO TO 9
CMAX=CDABS(CA(I+(J-1)*N))
JMAX=J
9 CONTINUE
CX(JMAX)=CA(NSQ+I)
DO 4 J=1,N
2 CX(JMAX)=CX(JMAX)-CA((J-1)*N)*CX(J)
XNORM=XNORM+(CAES(CX(J)))**2
4 CONTINUE
DO 5 I =1,N
IF(CDABS((CX(I)-CXOLD(I))/XNORM) .GT. CRIT) GO TO 7
6 CONTINUE
GO TO 10
7 IF(ITER .GT. ITMAX) GO TO 230
8 CXOLD(I)=CX(I)
ITER=ITER+1
GO TO 3
10 RNORM=0.D0
DO 12 I =1,N
CRES(I)=CA(NSQ+I)
11 CRES(I)=CRES(I)-CA((J-1)*N)*CX(J)
12 RNORM=RNORM+(CAES(CRES(I)))**2
RNORM=DSORT(RNORM)
WRITE(6,13) XNORM, RNORM, ITER, CRIT
13 FORMAT(5X,'CONVERGENCE WITH XNORM=',G12.5,' AND RNORM=',G12.5, ' AF
ITER ' ,ITMAX,' ITERATIONS'/10X,'ERROR CRITERION=',G14.7)
RETURN
160 WRITE(6,101) XNORM, RNORM
101 FORMAT(5X,'NO ITERATIONS NECESSARY. XNORM=',G12.5, ' ; RNORM=',G12.5, ' ')
RETURN
260 WRITE(6,201) CRIT, ITMAX, XNORM, RNORM
201 FORMAT(5X,'CONVERGENCE NOT ATTAINED TO WITHIN ',G12.5,' BETWEEN
SUCCESSIVE ITERATIONS FOR ALL SOLUTION ELEMENTS WITHIN ',ITMAX,' ITERA
TIONS'/10X,'XNORM=',G12.5, '; RNORM=',G12.5, ' ')
RETURN
END
SUBROUTINE MUADJ(MU, RIGJ, NBEG, NFI, SIGMM, NUI, RIGJ1, RIGJ2)
C ASSIGNS VALUES OF CURRENT DENSITY (RIGJ) AND PERMEABILITY (MU)
C TO EACH ELEMENT.
C NCON=NUMBER OF RINGS IN CONDUCTOR
C RIGJ1=CURRENT DENSITY IN CONDUCTOR
C RIGJ2=CURRENT DENSITY IN INSULATION
C MU=MAGNETIC PERMEABILITY
C IMPLICIT COMPLEX*16(C), REAL*3(A-D, H-O, 7)
C REAL MU(A-D), MU1
C REAL*3 SIGMM(4**0)
COMPLEX RIGJ(A-D), RIGJ1, RIGJ2
DO 1 N = NBEG, NFI
MU(N) = MU1
RIGJ(N) = RIGJ1
IF (SIGMM(N) .EQ. 0.) RIGJ(N) = RIGJ2
1 CONTINUE
3 RETURN
END

SUBROUTINE MUADJS(XT, YT, CT, VAL, L)
DIMENSION XT(2), YT(3), B(3), C(3)
COMPLEX CT(3), CONJG, GRADX, GRADY

C THIS SUBROUTINE CALCULATES FLUX DENSITY IN EACH TRIANGLE AND
C COMPUTES CORRESPONDING ELEMENT PERMEABILITY

J=2
K=3
DO 18 I = 1, 3
B(I) = YT(K) - YT(J)
C(I) = XT(J) - XT(K)
J = J + 1
K = K + 1
18 K = K - 1
AREA = XT(1) * B(1) + XT(2) * B(2) + XT(3) * B(3)
GRADX = (0.0, 0.0)
GRADY = (0.0, 0.0)
DO 2 J = 1, 3
GRADX = GRADX + B(J) * (CT(J)) / AREA
2 GRADY = GRADY + C(J) * (CT(J)) / AREA
TEMP = GRADX * CONJG(GRADX) + GRADY * CONJG(GRADY)
C MAGNITUDE OF FLUX DENSITY
ST = SQRT(TEMP)
S = ST * 1.EA
C VAL IS RELATIVE PERMEABILITY
IF (S .LT. 45.60) VAL = 60.
IF (S .LT. 15.79) VAL = 64.7661 + 0.779276 * 5 - 4.935729 * 5**2
WRITE(*, 3) S, VAL, L

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1. Number of rings within the conductor: 3

2. Magnetic permeability: $\mu = 735\times 10^{-15}$

3. Current density: within the conductor: $D = 0.711\times 10^{-6}$

   within the insulation: $0.0$

X at centre is 0.0, Y at centre is -1.820

- Number of rings within the conductor: 3
- Magnetic permeability: $\mu = 705\times 10^{-15}$
- Current density: within the conductor: $D = 0.711\times 10^{-6}$
- within the insulation: $0.0$

X at centre is -1.800, Y at centre is -2.900

- Number of rings within the conductor: 3
- Magnetic permeability: $\mu = 735\times 10^{-15}$
- Current density: within the conductor: $D = 0.711\times 10^{-6}$
- within the insulation: $0.0$

X at centre is 1.800, Y at centre is -2.900

- Number of rings within the conductor: 3
- Magnetic permeability: $\mu = 705\times 10^{-15}$
- Current density: within the conductor: $D = 0.711\times 10^{-6}$
- within the insulation: $0.0$

Approximate boundary potentials:

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Approximate boundary potentials

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Vector potential values:

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Determinant calculated to the (11, 11) element of this 193x198 system.
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**Imaginary Cross Terms**

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**ATIO of AC to DC Resistance**

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**ILL-CONDITIONED DETERMINANT CALCULATED TO THE (11,11) ELEMENT OF THIS 194X193 SYSTEM**

**VECTOR POTENTIAL VALUES**

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2. 0.24630651F-04 - 0.7445522F-04
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6. 0.24630651F-04 - 0.7445522F-04
7. 0.24630651F-04 - 0.7445522F-04
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**Total Loss**

<table>
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<th>Item</th>
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<tbody>
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Three conductors in air using a computed boundary condition and a real and imaginary formulation with a conjugate gradient solution routine.
INTEGER NDF2(13),NSFGM(3)
DIMENSION CVR(3),CVI(3),CV(360),CVI(175),CFP(360)
DIMENSION A(720)
DIMENSION RADIUS(14),SCUF(3),SC(3),NSDFG(3),NSFGM(3)
* INTG, IEND=V1 (v),E7D=V1 (v),IEND= (3),NLCDG (7),NCOD (3)
REAL (X(22),Y(22),FPI(3),J,364,J,364),XO(2),Y(3)
REAL MUI3(35),MUI,ATDF(4),MUD
COMPLEX MUIJ(35),MIG1,MIG2,CSINC,SAZ,PA,PHR,ACSN(13),CTE,
COMPLEX CTRA,CTR(3),C72,CONJG(3),CPDN(13),CTE
DIMENSION XT(3),YT(3)

C (I) READ IN SPECIFICATIONS OF NODE PATTERN:
REIND2
READ (2) NDTRT,JFIN,NSOUT,NPHMAX,NTRIA, (NODES(L),L=1,3), (NSFGM(L),
L=1,3), (JEND(L),L=1,7), (XOIL(L),L=1,7), (YI(L),L=1,3)
* READ (2) ((NODE(J,L),L=1,3),J=1,JFIN), (SICMM(J),J=1,JFIN)
READ (2) (X(I),I=1,NDTCT)
READ (2) (Y(I),I=1,NDTCT)
READ (2) (XRI,NSNode,J=1,JFIND)
WRITE (5,444) NDTRT,JFIN,NSOUT,NPHMAX,NTRIA, (NODES(L),L=1,NPHMAX),
* (NSFGM(L),L=1,NPHMAX), (JEND(L),L=1,3), (RADIL(L),L=1,3)
444 FORMAT(13,2(E13,2))
NLVNT=(JFIN+1)*3
NSEG1=
NUMOUT=NDTCT-NSOUT
PI3=3.14159265358979
MUG=0.00000012570
DTHF=TA=2.DC*PI/NSOUT
H=TA=2.DC
C COMPUTE APPROXIMATE BOUNDARY VALUES
DO 777 L=1,NPHMAX
ATOT(L)=0,
C READ NUMBER COND. RINGS, PcRMLA J IL ITY, AND COND. AND INSULATOR
READ (5,22) NCCN, VUL, PI1G11, PI1G12
NCCN=1
22 FORMAT(12,FI1,6.5)
WRITE (6,33) NCCN, VUI, PI1G11, PI1G12
33 FORMAT(15X,'NUMBER OF RINGS WITHIN THE CONDUCTOR: ',15/5X,
1'MAGNETIC PERMEABILITY: ',15G12.5/5X,'CURRENT DENSITY: ',15G12.5/5X)
NLCN(L)=2*NCCN(L)-1=NSFGM(L)
IF (L/=1) NLCN(L)=NLCN(L)+JEND(L-1)
IF (L/=1) NREG=EDIL(L-1)+1
NL1=NLCN(L)
NL2=JEND(L)
C (IV) GET VALUES OF PERMEABILITY AND CURRENT DENSITY FOR EACH ELEMENT
CALL MUA NDF (MUI,BIGJ, NREG, NL1,NL2,MUI1,BIGJ1,BIGJ2)
110 CONTINUE
777 CONTINUE
C SPECIFY PIPE CURRENT AND PERMEABILITY
BIGJ=0.02,0.02
BIGJ1=0.02,0.02
N3EG=EPN(NPHMAX)+1
NL1=JFIN-NTRIA
NL2=JFIN
CALL MUA NGD(MUI,BIGJ,NREG,NL1,NL2,MUI1,BIGJ1,BIGJ2)
C (V) SET ON AND RMS TO ZERO. PM(1)=CSM(1+NPOTSO).
C IF PIPE IS 0 FOR NE PIPE. ELSE NEW ZERO INTEGER
READ (5,355) IFPIE
IF (IFPIE/=0) IFPIE
NFI1=NFI1+1
NITST=1
IF (IFPIE/=0) IFPIE
NPOT=NPOT+1
NP=IFPIE
**INITIALIZE LOCATION, VALUE AND SOLUTION ARRAYS**

DO 1,J=1,NPCT
L2(J) =
IF(J.GT.3)GOTO1201
C4(J)=0.
1201 CONTINUE

**FORMULATE FINITE ELEMENT MATRIX ENTRIES**

C DO 100 N=1,NLMNTS
XCENT=((X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
YCENT=((Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
X1=X(NODE(N,1))-XCENT
X2=X(NODE(N,2))-XCENT
X3=X(NODE(N,3))-XCENT.
Y1=Y(NODE(N,1))-YCENT
Y2=Y(NODE(N,2))-YCENT
Y3=Y(NODE(N,3))-YCENT.

C (1) CALCULATE Q(I) AND F(I)
AREA=((X(NODE(N,1))*Y(NODE(N,3))-Y(NODE(N,2)))+
(X(NODE(N,2))*Y(NODE(N,1))-Y(NODE(N,3)))+
(X(NODE(N,3))*Y(NODE(N,2))-Y(NODE(N,1))))*6.D-1
J=2
K=3
DO 8 I=1,3
Q(I)=(Y(NODE(N,K))-Y(NODE(N,J))
F(I)=(X(NODE(N,J))-X(NODE(N,K))
T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
J=J+1
K=K+1
8 K=K+1*K/3*3

C (2) CALCULATE ELEMENT MATRIX ENTRIES
C COMPUTE CONDUCTOR AREAS
A(N)=AREA.
IF(N.GT.LGEN(NPH.MAX))GOTO20
D3500L=1.3
IF(N.GT.NLCON(L,A)AND.N.GT.JEND(L))GOTO20
ATOT(L)=ATOT(L)+AREA
GOTO20.

550 CONTINUE

C (3) CALCULATE RIGHT HAND SIDE OF ELEMENT MATRIX
RHS=SIGJ(N)*AREA/3.D0*MU(N)
RHS=SIGJ(N)*MU(N)
RHS=SIGJ(N)*MU(N).

C (4) ADD ENTRIES INTO BIG MATRIX EQUATION: (SM)(A)=(FM)
IF(SIGMM(N).LE.0.0)GOTO1987
NSNOW=NSFA.
1857 CONTINUE

C CHECK FOR UNSTABLE ENTRIES
C COMPUTE LOCATION WHERE NON-ZERO VALUE OCCURS
L=NODE(N,J)-NDEF*DOU(J)
IF(L.EQ.0)JEND(1PH.MAX))NSFIN=NSFA-4
357 NSFIN=NSFA.
IF (LOG (NN) .EQ. LOCTMF) GO TO 851
850 CONTINUE
IF (SIGMM (N) .EQ. 0) GO TO 11
VAL (K) = VAL (K) + S (I, J)
VAL (K+1) = VAL (K+1) - S (I, J)
IF (NSWV .EQ. NSFAR) GO TO 854
NSFAR = NSFAR + 2
GO TO 11
C
ADD NEW ENTRIES TO MASTER MATRIX -CONDUCTOR
852 LOC (NSFAR) = LOCTMF
VAL (NSFAR) = S (I, J)
NSFAR = NSFAR + 1
LOC (NSFAR) = LOCTMF + NPSG + NPOT
VAL (NSFAR + 1) = EDCUR (I, J)
NSFAR = NSFAR + 1
LOC (NSFAR + 1) = LOCINSFAR
VAL (NSFAR + 2) = S (I, J)
NSFAR = NSFAR + 2
GO TO 11
C
ADD TO EXISTING MATRIX ENTRIES
851 VAL (K) = VAL (K) + S (I, J)
VAL (K+1) = VAL (K+1) - S (I, J)
IF (SIGMM (N) .EQ. 0) GO TO 11
VAL (K+2) = VAL (K+2) - EDCUR (I, J)
VAL (K+3) = VAL (K+3) - EDCUR (I, J)
GOTO 11
C
ADD NEW ENTRIES TO MASTER MATRIX -INSULATOR
854 LOC (NSFAR) = LOCTMF
VAL (NSFAR) = S (I, J)
LOC (NSFAR) = LOCINSFAR + NPSQ + NPOT
VAL (NSFAR + 2) = S (I, J)
NSFAR = NSFAR + 2
GO TO 11
C
INSERT BOUNDARY CONDITIONS INTO MATRIX
211 CTFM = CCON (NODE (N, J) - NUMCUT)
CBR (NODE (N, J) + NPOT) = CB (NODE (N, J) + NPOT) + S (I, J) * AMG (CTEMP)
11 CONTINUE
C
INSERT SOURCE ENTRIES
CB (NODE (N, J)) = CB (NODE (N, J)) + RHS
CB (NODE (N, J) + NPOT) = CB (NODE (N, J) + NPOT) - AMG (RHS)
10 CONTINUE
IF (NSWV .EQ. NSFAR) NSFAR = NSFAR + 4
C
END
NSFAR = NSFAR - 1
WRITE (6, 50) (ATGL (L) + 1, 2)
END
C
FORMAT (1x, "THE NUMBER OF NON-ZERO ENTRIES IS", I6)
129

CALL CONJUGATE GRADIENT SOLUTION ROUTINE
CALL CUGRHFLV, LCC, NGE, CAP, CVR, CHK, NPO2, TOL, IER
IF (IER .EQ. 0) GOTO 222
WRITE (6,222)
222 CONTINUE
1. CHECK MATRIX GSM FOR ERROR 1
333 CONTINUE
L=1
NGE=1
CALCULATE AVERAGE POTENTIAL VALUES
DO 246 LL=1,3
LLL
CVAP(LL)=0.
CVAI(LL)=0.
IF (LL .NE. 1) NVEG=JEND(LL-1)+1
NFIP=NLCCF(LL)
DO246=N=NGE,NFIN
AREA=A(N)
DO 242 I=1,3
IF (NODE(N,I).LE.NPCT) GO TO 645
CVR(NODE(N,I))=REAL(CBOUN(NODE(N,I)-NUMIN))
CVR(NODE(N,I)+NPCT)=AIMAG(CBOUN(NODE(N,I)-NUMAT))
645 CONTINUE
CVAP(L)=CVAP(L)+CVR(NODE(N,I))*AREA/3.
CVAI(L)=CVAI(L)+CVR(NODE(N,I)+NPCT)*AREA/3.
242 CONTINUE
CVAP(L)=CVAP(L)/ATOT(L)
CVAI(L)=CVAI(L)/ATOT(L)
WRITE (6,247) L,CVAP(L),CVAI(L)
246 CONTINUE
CALCULATE AVERAGE VALUE FOR PIPE
ATOT(4)=0.
CVAP(4)=0.
CVAI(4)=0.
IF (IPRIDE .EQ. 0) GO TO 443
DO 441 N=NP1,1,NFIN
ATOT(4)=ATOT(4)+A(N)
DO 442 K=1,3
CVAP(4)=CVAP(4)+A(K)*CVR(NODE(N,K))
442 CONTINUE
CVAP(4)=CVAP(4)/ATOT(4)
CVAI(4)=CVAI(4)/ATOT(4)
443 CONTINUE
WRITE(6,247) L, CVAP(4), CVAI(4)
WRITE(6,250)
890 FORMAT(' ', 'VECTCP POTENTIAL VALUES')
DO 212 I=1,NPOT
CVI(I)=CVR(I)+NPOT
212 WRITE(6,247) I, CVI(I), CVR(I+NPOT)
L=1
247 CONTINUE
COMPUTE OVERALL AVERAGE POTENTIAL
CVAP(1)=CVAP(1)*ATOT(1)+CVAP(2)*ATOT(2)+CVAP(3)*ATOT(3)+CVAP(4)*ATOT(4)
CVAP(1)=CVAP(1)/ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
CVAI(1)=CVAI(1)/ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
CVAI(1)=CVAI(1)/ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
CALCULATE LESS FATIO
DO 301 J=1,NEND
DO =0.01,1.01
DO =0.01,1.01
DO =0.01,1.01
301 IF ((J .EQ. JEND) .AND. (J .EQ. JEND)) GOTO 304
IF (J .EQ. JEND) NREG=JEND(J-1)+1
IF (J .EQ. JEND) NREG=JEND(J-1)+1
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IF ( J, NE = NODE(N) ) NODE = NODE(N) + 1
DO 20 J = 1, NODE(N)
END

10 A = 0

20 WRITE (6, 501) A
END

81 FORMAT (1, 'CALCULATE NEW PIPE PERMEABILITY')

DO 16 J = NODE(1), NODE(2)
16 WRITE (6, 500) (XT(K), YT(K), CT(K))
END

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SUBROUTINE CNGPH(A,L,NNZRO,X,J,N,UP3,1ER)

C C INITIALIZE RESIDUAL RC=E-AX
R(J)=B(J)
P(J)=0.D0
DR=H(J)*R(J)+DR
Y(J)=J+0
T(J)=J+0
C C INITIALIZE SOLUTION VECTOR
1 X(J)=0.D0
DO 12 JC=1,N
12 DO 15 J=1,NNZRO
KP=(L(J)-1)/N+1
KC=L(J)-(KR-1)*N
C C INITIALIZE DIRECTION PO=A'RC
P(KC)=P(KC)+A(J)*P(KR)
C C CALCULATE TO=A'ROT(KC)=T(KR)+A(J)*P(KC)
IF(KR.EQ.KO)GO TO 15
15 CONTINUE

RSQ=0.D0
DO 16 J=1,N
16 RSQ=RSQ+P(J)*R(J)
RTET=1.D0/RSQ
DO 17 J=1,N
17 XLSQ(J)=X(J)/RSQ
DO 10 JJ=1,N
DO4 J=1,NNZRO
KP=(L(J)-1)/N+1
KC=L(J)-(KR-1)*N
C C CALCULATE V=A'P
Y(KR)=Y(KR)+A(J)*P(KC)
IF(KR.EQ.KO)GO TO 4
Y(KO)=Y(KO)+A(J)*P(KC)
4 CONTINUE
SUM=0.D0
LAMBDCA=0.D0
GAMMA=0.D0
C C LAMBDCA=(AP)**2
C C GAMMA=(A'R)**2
C C LAMBDA=(AP)**2
DO 6 J=1,N
SUM=SUM+P(J)*Y(J)
GAMMA=GAMMA+T(J)*T(J)
6 LAMBDCA=LAMBDCA+Y(J)*Y(J)
C C ALPHA=(A'R)**2/((AP)**2)
ALPHA=GAMMA/LAMBDCA
DO 8 J=1,N
T(J)=J+0
X(J)=X(J)+ALPHA*F(J)
C C NEW RESIDUAL P=R-ALPHA*A'AP
8 R(J)=R(J)-ALPHA*Y(J)
DO 7 J=1,N
KR=(L(J)-1)/N+1
KC=L(J)-(KR-1)*N
C C CALCULATE T=A'R
T(KC)=T(KC)+A(J)*R(KP)
IF(KP.EQ.KC)GO TO 7
T(KP)=T(KP)+A(J)*R(KP)
7 CONTINUE
RSQ=0.D0
SUM1=0.D0
C C CALCULATE SUM1=(A'E)**2
DO 14 J=1,N
SUM1=SUM1+P(J)*R(J)
14 SUM1=SUM1+T(J)*T(J)
CTOT=1.D0/RTET
NEITA=SUM1/GAMMA
C C P=E'+HTAP NEW DIRECTION
DO 19 J=1,N

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CNGRH

132

XLSQ(J) = XLSQ(J) + X(J) / EPS
Y(J) = P(J)

P(J) = T(J) + BETA * P(J)
SUM1 = SUM1 / N
WRITE(6,13) SUM1, SUM

13 FORMAT(' ', 10X, G15.8, 10X, G15.8)
IF(SUM1.LE.EPS) GOTO 11
10 CONTINUE
C RE=INITIALIZE
D03 J=1, NNZPD
KP = (L(J)-1)/N+1
KC = L(J) - (KR-1) * N

C Y=AX
Y(KP) = Y(KP) + A(J) * X(KC)
IF(KP.EQ.KC) GOTO 3
Y(KC) = Y(KC) + A(J) * X(KP)
3 CONTINUE
D05 J=1, N
C RESIDUAL P=B-AX
R(J) = B(J) - Y(J)
XLSQ(J) = XLSQ(J) / STCT
WRITE(6,13) XLSQ(J), X(J)
T(J) = 0.0
P(J) = 0.0
5 Y(J) = 0.0
12 CONTINUE
IF(SUM1.GT.EPS) IER=1
D0 18 J=1, N
18 X(J) = XLSQ(J)
11 CONTINUE
RETURN
END

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APPENDIX 5
FLUX DENSITY FROM VECTOR POTENTIAL

Once the nodal potentials have been determined by solving the matrix equation, the flux density is easily found. The potential in each triangle is given by

\[ \overline{A}(x,y) = \left( \frac{a_i + b_i x + c_i y}{2\Delta} \right) A_i + \left( \frac{a_j + b_j x + c_j y}{2\Delta} \right) A_j + \left( \frac{a_k + b_k x + c_k y}{2\Delta} \right) A_k \]

\[ B = \nabla \times A = \frac{\partial A_i}{\partial y} \hat{a}_x - \frac{\partial A_i}{\partial x} \hat{a}_y \]

\[ |\overline{B}| = \frac{1}{2\Delta} \sqrt{\left( b_i A_i + b_j A_j + b_k A_k \right)^2 + \left( c_i A_i + c_j A_j + c_k A_k \right)^2} \]

where the \( b_i \)'s and \( c_i \)'s have been previously defined.
APPENDIX 6

Typical three cable finite element patterns.
CLOSE TRIANGULAR CONFIGURATION

THICK PIPE
Cradled Configuration in a Thick Pipe
THIN PIPE - CRADLED
SEGMENTED CONDUCTORS
BIBLIOGRAPHY


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