Single-crossarm stayed column with initial imperfections.

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SINGLE-CROSSARM STAYED COLUMN WITH INITIAL IMPERFECTIONS

by

Kevin Chi-Yuen Wong

A Thesis
submitted to the Faculty of Graduate Studies
through the Department of
Civil Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
The University of Windsor

Windsor, Ontario, Canada

1980
TO MY FAMILY
ABSTRACT

SINGLE-CROSSARM STAYED COLUMN WITH INITIAL IMPERFECTIONS

by

Kevin Chi-Yuen Wong

The elastic buckling strength of a concentrically loaded, pin-ended, slender metal column may be increased many times by reinforcing it with an assemblage of rigidly connected crossarm members and pretensioned stays.

The presence of an initial out-of-straightness has an effect on the strength on the stayed column. This thesis deals with the stability analysis of the imperfect plane single-crossarm stayed column. The effect of pretensioned stays on the buckling strength of the imperfect stayed column is derived. A geometrical study provides the relationship between the applied load, deflections and tension in the stays. The geometrical nonlinear behavior is included in the analysis by using the finite element method. Such an analysis is carried out by a mixed incremental and iterative procedure, is programmed in FORTRAN IV Language and is run on an IBM/360 system.

A plane imperfect stayed column was used in a series of experimental tests. The experimental buckling loads obtained with various initial pretensions were compared with those theoretically predicted. The results of a nonlinear analysis on the imperfect stayed column shows good agreement with the experimental results. These results also indicate that the presence of an initial out-of-straightness significantly reduces the buckling strength and that the type of initial out-of-straightness has a great effect on the mode of deflection and mode of failure.
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English Letters

A_c : cross-sectional area of core
A_ca : cross-sectional area of crossarm
A_s : cross-sectional area of stay
[A] : assembly control matrix
C_l : constant which relates to the optimum pretension and and maximum critical load
E : modulus of elasticity
E_c : modulus of elasticity of core
E_ca : modulus of elasticity of crossarm
E_s : modulus of elasticity of stay
F_i : component force of initial pretension acting on the ends of crossarm
F_r : lateral restraint force
{F} : force matrix of the structure
{F} : force matrix of element in the local coordinate system
G : modulus of rigidity
I : moment of inertia
K : effective length factor
K_c : axial stiffness of core
K_ca : axial stiffness of crossarm
K_s : axial stiffness of stay
[K] : linear elastic stiffness matrix
[K_E] : master elastic stiffness matrix
[K_G] : master geometric stiffness matrix
[K_G^*] : master geometric stiffness matrix for a unit internal force
\([K_{Sec}]\) : secant stiffness matrix

\([K_T]\) : total nonlinear stiffness matrix

\([K_{Tan}]\) : tangent stiffness matrix

\([k]\) : total stiffness matrix for element

\([k_E]\) : elastic stiffness matrix for element in global coordinate system

\([k_e]\) : elastic stiffness matrix for element in local coordinate system

\([k_G]\) : geometric stiffness matrix for element in global coordinate system

\([k_g]\) : geometric stiffness matrix for element in local coordinate system

\(L\) : length of column

\(l\) : length of half column

\(l_{ca}\) : length of crossarm

\(l_s\) : length of stay

\(l_1', m_1\) : direction cosine in x-direction

\(l_2', m_2\) : direction cosine in y-direction

\(P_a\) : applied axial force

\(P_{cr,max}\) : maximum critical load

\(P_E\) or \(P_{Euler}\) : Euler critical load

\(P_f\) : final axial force

\(P_i\) or \(P_I\) : internal axial force

\(P_R\) : unbalanced axial force

\(\{p\}\) : force matrix for element in local coordinate system

\(\{p\}\) : force matrix for element in global coordinate system

\(\{\bar{p}\}\) : total force matrix for the structure in global coordinate system

\(T_i\) : initial stay pretension

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\( T_f \) : final stay tension

\( T_{fl} \) : final stay tension on the left side (convex) of the core

\( T_{fr} \) : final stay tension on the right side (concave) of the core

\( T_{\text{max}} \) : maximum possible pretension

\( T_{\text{min}} \) : minimum effective pretension

\( T_{\text{opt}} \) : optimum pretension

\( T_R \) : residual tension

\([T]\) : transformation matrix

\( U_0 \) : strain energy before any disturbance is applied

\( U_1 \) : strain energy which relates to initial internal stress

\( U_2 \) : strain energy which relates to additional strain

\( U_{\text{max}} \) : maximum displacement which is the convergence criteria for the computer program

\( U_s \) : total strain energy

\( \{U\} \) : displacement matrix for the structural in global coordinate system

\( \{U_T\} \) : total displacement matrix in global coordinate system

\( \{\Delta U\} \) : incremental displacement matrix in global coordinate system

\( u \) : element nodal displacement in x-direction

\( u_s \) : strain energy per unit volume

\( \tilde{u}(x) \) : assumed displacement function in x-direction

\( \{\tilde{u}\} \) : element displacement matrix in local coordinate system

\( V \) : volume

\( v \) : element nodal displacement in y-direction

\( v(x) \) : assumed displacement function in y-direction

\( W_n \) : potential of joint load

\( x \nu \)


Greek Letters

\( \alpha \) : angle between stay and core

\( \delta \) : initial deflection shape of column

\( \Delta_c \) : core shortening due to applied force

\( \Delta_{ca} \) : crossarm shortening due to the decrease in compressive force

\( \Delta_s \) : stay shortening due to the decrease in stay tension

\( \Delta_{sL} \) : stay shortening for the stay on the left (convex) side of the core

\( \Delta_{sr} \) : stay shortening for the stay on the right (concave) side of the core

\( \Delta_m \) : lateral deflection at crossarm level

\( \varepsilon \) : total strain in x-direction

\( \varepsilon_i \) : initial strain in x-direction

\( \varepsilon_f \) or \( \varepsilon_{xx} \) : final strain in x-direction

\( \phi \) : stay diameter

\( \theta \) : angle between stay and crossarm

\( \tilde{\theta} \) : angular displacement

\( \Pi_p \) : total potential energy for element

\( \Pi_p \) : total potential energy for the structure
CHAPTER 1

INTRODUCTION

1.1 General

A stayed column consists of a slender core, rigid crossarms and pretensioned stays. This assemblage can resist the translational and rotational movement at the crossarm level which increases the elastic buckling strength as compared to the Euler load of the simple column. Depending on geometry and the number of crossarms, stayed columns can be classified as single, double and triple-crossarm stayed columns. It is also possible to have an arrangement with more than three crossarms. Generally, three-dimensional space stayed columns are built for practical use. The crossarms are arranged in a cruciform around the core. The two-dimensional stayed columns are used in the laboratory because of the ease with which they can be built and tested. These planar stayed columns must be braced in a plane perpendicular to the one containing the crossarms. Some examples of stayed columns are shown in Figure 1.1.

Before designing a stayed column, it is necessary to have an accurate analysis which predicts the behavior of such a structure. In the following chapters a method has been derived for determining the behavior of these stayed columns.

1.2 Previous Methods of Study

In recent years considerable research has been performed on the analysis of stayed columns. The methods of solution which were generally used are:

(1) The classical method of solving the differential equations.
(2) The stability function method with an eigenvalue approach.

(3) The finite element method with an eigenvalue approach.

All of these methods provide very similar results when determining the maximum critical load for an ideal single-crossarm stayed column. The classical method can be used to determine the critical load of a single-crossarm stayed column, but this method cannot be easily applied to stayed columns with more than one cross-arm. The method using stability functions is very effective, but there are several discrete points at which the functions are discontinuous and this causes some difficulty in the solution process. Since computers are widely and commonly used, the finite element method becomes the most efficient and powerful method. The finite element method will be adopted in this study. The method of solution, however, is different from that used in previous studies.

In most of the previous studies it was assumed that there was only a very small amount of tension left in the stays just prior to buckling. The maximum buckling load was determined without considering the initial pretension. Hafez has derived the linear relationship between the initial pretension and the corresponding buckling load by relating the geometric deformations to the applied axial load. The upper region was bounded by the maximum buckling load while the lower region was bounded by the Euler load. A typical graph illustrating the relationship between initial pretension and critical load is shown in Figure 3.1.

When these theoretical results were compared to the experimental results, there was approximately a 20% difference between the two.

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The difference may be caused by the assumptions or the method of solution. Thus, the original assumptions need to be reviewed and the method of solution may have to be modified.

1.3 The System and Purposes to be Studied

The system which was used in the previous analytical work was an ideal stayed column. The core was considered to be perfectly straight. The bending effect, lateral deformation due to axial load and initial out-of-straightness, were neglected.

In practice, some kind of imperfection such as crookedness or out-of-straightness is likely to be present in any structural member. The presence of initial imperfections has a considerable effect on the buckling strength and the behavior of the stayed column. The stayed column which includes certain initial imperfections is called a real stayed column in this study.

The purposes of this study can be summarized as follows:

(1) To establish an analytical system closely comparable to the real system.

(2) To establish a method of solution which involves using a nonlinear analysis and the finite element method for determining the behavior of such a system.

(3) To generate and verify theoretical results by comparison with experimental results. The behavior of the stayed column includes: (a) the load-deflection relationship; (b) the load-stay tension relationship; (c) the buckling load-initial pretension relationship; (d) the minimum effective, optimum and maximum possible pretensions of the stayed column.
CHAPTER 2

LITERATURE SURVEY

In 1963, Chu and Berge developed a general solution for the elastic critical load of a slender pin-ended column stayed with tension ties arranged in equilateral rosettes around the column and bearing on several intermediate points along the column through hogging frames. The connections between the frames and the column and the connections between the ties and the frames are ideal hinges. There are no initial eccentricities or crookedness in the column. The solution indicated that the maximum critical load would be a four-fold strength increase over the Euler load. At the instance of buckling, if there is more tension remaining in the ties, the critical load will be reduced accordingly. Any increase in the number of symmetrically placed intermediate frames did not affect the strength increase. Experiments were performed and the results showed satisfactory agreement with the predicted results.

To continue the work of Chu and Berge, Mauch and Felton in 1967 developed an analytical foundation for the rational design of these columns. The structural index (i.e., $P/L^2$) has been used, in which $P$ is the compression load and $L$ is the length of the column. This index can be considered as a measure of load intensity. Their analysis indicated that at low values of structural index, columns supported by tension ties offer potential saving of up to 50% of the weight of optimum simple columns.

In 1970, a design-build-test project was performed by the final year Civil Engineering undergraduates at the Royal Military College.
of Canada. The crossarms, instead of being pinned to the column as in Chu and Berges' study, were rigidly welded to the column. The stays were pretensioned so that just prior to buckling their force reduced to zero. Experimental results indicated a buckling load which was a seven-fold increase when compared to the Euler load of a simple unstayed column. No analytical work was performed.

In 1971, Pearson examined the behavior of a pin-ended single-crossarm stayed column with a high slenderness ratio when loaded to its buckling point. In his study the effects on column strength of various stay slopes and pretension forces were examined experimentally only. The results of the tests indicated that buckling strength is directly proportional to these parameters.

In 1972, an experimental study of a pin-ended single-crossarm stayed column was carried out by Clarke. The results of the test pointed out that there was no simple relationship between the buckling load and tie pretension, nor between the buckling load and slopes of ties.

In the same year, another experimental study on the strength of a pin-ended triple-crossarm stayed column was carried out at the Royal Military College by McCaffrey. Test results indicated strength increases ranging from 34 to 45 times that of the Euler critical load of the unstayed column.

In 1975, Smith, McCaffrey and Ellis published a paper, in which they developed an analytical solution to predict the critical load of a pin-ended single-crossarm stayed column, associated with two modes of failure. The differential calculus approach was adopted in the analytical process. The influence of various parameters were also

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In 1976, Temple developed an analytical solution to predict the critical loads and the corresponding buckling shape for single, double and triple-crossarm stayed column by employing the stability function method and the eigenvalue approach.

In the same year, Khosla developed a third analytical solution procedure by using the finite element method for determining the maximum critical load of a pin-ended single-crossarm planar stayed column. This method, based on including higher order terms in the strain-displacement relationship, leads to the consideration of the geometric behavior. This method can be applied to any planar stayed column with the solution being the critical loads and buckling modes. The results were the same as those predicted by Temple. No experimental work was performed for checking the results.

In 1977, Hathout extended the work of Temple and Khosla, and developed a geometric stiffness matrix for a three-dimensional beam-column element. The matrix was developed based on retaining the quadratic terms in the expression for strain energy. Two methods were employed to predict the critical load and the corresponding buckling shape for any type of stayed column. The methods are: (1) The analysis basis on the stability function and the eigenvalue approach, and (2) The geometrically analysis by the finite element and the eigenvalue approach. The effects of different parameters on the behavior of a single-crossarm stayed column were demonstrated. The parameters which were considered in the study were the numbers of crossarm member, end support conditions, sectional properties of the
member, stays size and the length of crossarm. No experimental work was performed.

In the same year, Hafez\textsuperscript{10} derived the complete relationship between the critical load and the initial pretension for an ideal pin-ended single-crossarm stayed column. The minimum effective pretension, the optimum pretension and the maximum possible pretension were determined. A geometric study of the column was used in the analytical process. The maximum critical load was obtained by the same finite element method as used by Hathout. The influence of various parameters on the behavior of the stayed column were demonstrated. The results indicated that the optimum pretension and the maximum critical load would change when the diameter of stays, modulus of elasticity of stays and the crossarm length were varied. A model of a pin-ended single-crossarm planar stayed column was tested. Test results showed satisfactory agreement with the theoretical results when the initial pretension in the stays is small. There was, however, a difference of about 20% between the theoretical and experimental critical loads at larger initial pretensions.
CHAPTER 3

ANALYTICAL STUDY OF STAYED COLUMNS

3.1 General

The stayed column, as stated in Chapter 1, is a slender metal column reinforced with rigid crossarms and pretensioned stays. The critical load of the stayed column will increase many times over the Euler load of the simple column. The amount of the increase is related to parameters such as the initial stay pretension, the size and material properties of members and the geometry of the stayed column.

The stayed column which is perfectly straight is called an ideal stayed column. All the components are symmetrical about both axes. Due to the symmetry, the horizontal component forces on the column from the stays are in equilibrium. There is no lateral or rotational movement of the core because this movement is prevented by the forces in the stays. When the applied axial load is increased, the tension in the stays will decrease uniformly. When all the tension in the stays is reduced to zero, the column is unable to support the applied load and buckling will then occur. The behavior of an ideal pin-ended single-crossarm stayed column was analyzed previously by Hafez10. The method of geometrical analysis of such an ideal stayed column will be mentioned later in this chapter.

In practice, some kind of imperfection such as crookedness or out-of-straightness will be present in the members of the stayed column. The stayed column which has a core with an initial out-of-straightness is called a real stayed column. For a real stayed
column, lateral deflection will occur along the core when it is subjected to any applied axial load. Although the tension in the stays resists the translational and rotational movements at the cross-arm level, it cannot completely resist the movements due to the bending effect. Such a bending effect is caused by the combined actions of the applied load and lateral deflection which is called the P-Δ effect. Due to such effects, the buckling strength of the real stayed column, as compared to an ideal stayed column, will be reduced and the behavior will change if the P-Δ effects are considered. In this section, a geometrical study will be described, including a method of nonlinear analysis. A planar pin-ended single-crossarm stayed column will be used in the following discussion.

The failure load of the stayed column is sometimes referred to as the critical load and sometimes as the buckling load. In this report the critical load is reserved for the load at which neutral equilibrium is possible for an ideal stayed column according to a linear analysis. The buckling load is reserved for the load at which the core undergoes significant lateral displacements when the column is an imperfect one.

3.2 Ideal Stayed Column

3.2.1 Assumptions for an Ideal Stayed Column

When analyzing the behavior of an ideal stayed column, certain assumptions were made. They are:

(1) The single-crossarm stayed column is completely symmetrical and ideally concentrically loaded. (No eccentricity or crookedness).

(2) The connections between the crossarm members and column are perfectly rigid. The connections between the stays and the
column, and between the stays and crossarm members are assumed to be ideal hinge.

(3) The failure load of an ideal stayed column is termed the critical load which is obtained by considering the neutral equilibrium of the system.

(4) The maximum critical load is assumed to be the load obtained by the finite element method and the eigenvalue approach.

(5) The critical load is reached as the tension in the stays goes to zero for the stayed column having initial pretension less than the optimum pretension.

(6) The axial deformation of the crossarm and of the column have been neglected when deriving the maximum critical load, but they are used when determining the tension in the stays.

3.2.2 Basic Definitions of Pretension Applicable to an Ideal Stayed Column

Several terms have to be defined before the analysis is carried out. The definitions are:

(1) Minimum Effective Pretension: It is the minimum initial pretension in the stays which remains effective until the Euler load of the core has been reached. If the initial pretension is equal to or less than the minimum effective pretension, at a certain load less than or equal to the Euler load the stays will become slack and the core will act like a simple column. Hence, the stayed column has the same critical load as the Euler column. There is no practical advantage in using a pretension less than the minimum effective pretension.

(2) Optimum Pretension: It is the initial pretension in the
stays which disappears completely just after the load in the column reaches the maximum critical load. All stays remain effective until the maximum critical load has been reached.

(3) Maximum Possible Pretension: It is the initial pretension in the stays which gives vertical components at the ends of the column large enough to cause buckling without any additional load. This value does not have any practical importance.

(4) Residual Tension: When the initial pretension in the stays is larger than the optimum pretension, the tension in the stays does not go to zero at the instant of buckling. The tension which remains in the stays is the residual tension.

3.2.3 Geometric Analysis for an Ideal Stayed Column

A two-dimensional single-crossarm stayed column with pinned ends at the supports is analyzed to determine its behavior due to any applied load and initial pretension.

When the initial pretension is less than the minimum effective pretension, the tension in all the stays will decrease at the same rate and will vanish before the buckling occurs. Thus, the failure load will be the Euler load.

When the initial pretension is greater than the minimum effective pretension, the tension in the stays will offer some resistance against the translational and rotational movement at the crossarm level. Thus, the critical load of the stayed column is greater than the failure load of the Euler column. For those stayed columns with an initial pretension greater than the minimum effective pretension, but less than the optimum pretension, the tension in stays
will decrease as the load is increased. When the tension reaches zero, the resistance at the crossarm level is lost. Following that, buckling occurs.

In Figure 3.1, the general relationship between the critical load and the initial pretension of an ideal single-crossarm stayed column is shown. Zone I indicates the region of initial pretension is less than the minimum effective pretension. The critical load of the stayed column is governed by the Euler load. Zone II indicates that the critical load is linearly proportional to the increase of initial pretension in the region between the minimum effective and optimum pretensions. Zone III indicates that the critical load of the stayed column will decrease linearly with an initial pretension greater than the optimum pretension, and zone III terminates with the maximum possible pretension. Under the maximum possible pretension, no additional load is required to cause the stayed column to buckle. Details of the change in tension in stays is derived by a geometric analysis which is discussed as follows with reference to Figure 3.2 and Figure 3.3.

In a pretensioned single-crossarm stayed column, the initial axial force \( P_i \) is caused by the initial axial pretension \( T_i \) in the stays. The relationship between these forces is,

\[
P_i = 2T_i \cos \alpha
\]

where \( \alpha \) = the angle between the stays and the core.

The total final axial force \( P_f \) is defined as the force including the applied force \( P_a \) and the final tension component \( T_f \). Thus, \( P_f \) is given by

\[
P_f = P_i + P_a + T_f
\]
\[ P_f = P_a + 2T_f \cos \alpha \]  \hspace{1cm} \cdots \cdots \hspace{1cm} 3.2

Due to the initial pretension, the core is shortened by an amount \( \Delta L_i \), which is given by

\[ \Delta L_i = \frac{2T_i L \cos \alpha}{A_c E_c} = \frac{2T_i \cos \alpha}{K_c} \]  \hspace{1cm} \cdots \cdots \hspace{1cm} 3.3

where \( K_c = \frac{A_c E_c}{L} \), the axial stiffness of the core;

\( A_c \) is the cross-sectional area of the core; \( E_c \) is the modulus of elasticity of the core; and \( L \) is the length of the core.

There is also a horizontal component due to the initial pretension which causes a compression force \( (F_i) \) on the ends of the crossarm which results in a shortening of \( \Delta l_{ca} \):

\[ F_i = 2T_i \sin \alpha \]  \hspace{1cm} \cdots \cdots \hspace{1cm} 3.4

\[ \Delta l_{ca} = \frac{2T_i l_{ca} \sin \alpha}{A_{ca} E_{ca}} = \frac{2T_i \sin \alpha}{K_{ca}} \]  \hspace{1cm} \cdots \cdots \hspace{1cm} 3.5

where \( K_{ca} = \frac{A_{ca} E_{ca}}{l_{ca}} \), the axial stiffness of the crossarm;

\( A_{ca} \) is the cross-sectional area of the crossarm, \( E_{ca} \) is the modulus of elasticity of the crossarm; and \( l_{ca} \) is the length of the crossarm.

When a certain axial force is applied, the tension in the stays will change to \( T_f \), causing a compression force on the ends of the crossarm equal to \( F_f \):

\[ F_f = 2T_f \sin \alpha \]  \hspace{1cm} \cdots \cdots \hspace{1cm} 3.6

Due to the applied force the column will be shortened by \( \Delta c \), which is given by
By substituting the values of $P_f$ and $P_i$ from Equations 3.1 and 3.2 into Equation 3.7, $\Delta_c$ can be rewritten as

$$\Delta_c = \frac{P - 2(T_i - T_f) \cos^2 \alpha}{K_c}$$

Due to the decrease in the compression force, the crossarm is elongated by an amount equal to $\Delta_{ca}$, given by

$$\Delta_{ca} = \frac{2(F_i - F_f) l_{ca}}{A_{ca} E_{ca}} = \frac{F_i - F_f}{K_{ca}}$$

And $\Delta_{ca}$ can also be written in terms of the tension in stays, which is

$$\Delta_{ca} = \frac{2(T_i - T_f) \sin^2 \alpha}{K_{ca}}$$

The stay is shortened due to the change in the stay tension and the amount of shortening $\Delta_s$ is given as

$$\Delta_s = \frac{(T_i - T_f) l_s}{A_s E_s} = \frac{T_i - T_f}{K_s}$$

where $K_s = \frac{A_s E_s}{l_s}$, the axial stiffness of the stay; $A_s$ = the cross-sectional area of the stay; $E_s$ = the modulus of elasticity of the stay; and $l_s$ = the length of the stay.

Due to the axial deformation of the core and crossarm, $\Delta_s$ can also be written in terms of the deformations,

$$\Delta_s = \frac{1}{2} \Delta_c \cos \alpha - \Delta_{ca} \sin \alpha$$

By substituting the values of $\Delta_c$, $\Delta_{ca}$ and $\Delta_s$ from Equations 3.8,
3.10 and 3.11 into Equation 3.12, the relationship between $T_1$, $T_f$, and $P_a$ is obtained as,

$$\frac{T_i - T_f}{K_s} = \frac{[P_a - 2(T_i - T_f)\cos\alpha]\cos\alpha}{2K_c}$$

$$- \frac{[2(T_i - T_f')\sin\alpha]\sin\alpha}{K_{ca}}$$

...... 3.13

The change in tension $(T_i - T_f)$ can then be expressed in terms of the applied force, which is

$$T_i - T_f = P_a\frac{\cos\alpha}{2K_c\left(\frac{\tan^2\alpha}{K_s} + \frac{2\sin^2\alpha}{K_{ca}} + \frac{2\cos^2\alpha}{K_c}\right)}$$

...... 3.14

or

$$T_i - T_f = P_aC_1$$

...... 3.15

where $C_1 = \frac{1}{2K_c\left(\frac{\tan^2\alpha}{K_s} + \frac{2\sin^2\alpha}{K_{ca}} + \frac{2\cos^2\alpha}{K_c}\right)}$ = a constant.

According to this equation, when the tension in the stays disappears, the applied load is the critical load, which is

$$P_{cr} = \frac{T_i - T_f}{C_1} = \frac{T_i - 0}{C_1} = \frac{T_i}{C_1}$$

...... 3.16

The maximum critical load ($P_{cr,max}$) is calculated by using the finite element method. The maximum critical load occurs only if the stays have an initial pretension equal to the optimum pretension. Thus, the optimum pretension can be predicted by

$$T_{opt} = C_1P_{cr,max} = \text{optimum pretension}$$

...... 3.17
3.3 Real Stayed Column

3.3.1 Assumptions for a Real Stayed Column

The stayed column which contains an initial out-of-straightness is called a real column. Before the analytical work was carried out, several assumptions had to be made. These assumptions are not the same as those for an ideal stayed column. The assumptions are:

(1) Due to the presence of imperfections, the single-crossarm stayed column may not be symmetrical about both axes.

(2) The connections between the crossarm members and the core are perfectly rigid. The connections between the stays and the crossarm members, the stays and the core, are assumed to be ideal hinges.

(3) The deformations of all members are taken into consideration when the buckling load and the tension in the stays is determined by geometric analysis and the finite element method.

(4) The angle (a) between the stay and the core is considered to remain unchanged.

(5) In some cases only two stays are effective when buckling occurs, while in some other cases all stays are effective.

(6) All buckling loads of stayed columns with initial pretension in stays are predicted by the finite element method using a nonlinear analytical approach.

3.3.2 Definitions of Pretension Applicable to a Real Stayed Column

The terms which were defined for the ideal stayed column will also be used for the real stayed column, but some are defined in a slightly different manner. The definitions are:
(1) Minimum Effective Pretension: It is defined the same as for the ideal stayed column. For the real stayed column, the minimum effective pretension is slightly greater than the one for the ideal stayed column. If the initial pretension is less than or equal to the minimum effective pretension, the Euler load is the failure load of the column.

(2) Optimum Pretension: The buckling load of the stayed column will be the maximum if the initial pretension is the optimum pretension in the stays.

(3) Maximum Possible Pretension: It is the initial pretension in the stays which gives a large vertical component at the ends of the column. This component will cause a very large lateral deformation without any additional load. The stayed column cannot resist any applied load when the stays are stressed to the maximum possible pretension.

(4) Residual Tension: When a real stayed column buckles a certain tension will be left in the stays which is called the residual tension. For a real single-crossarm stayed column, which has an initial pretension between the minimum effective and optimum pretension, there are residual tensions left in two stays at buckling. When it has the initial pretension greater than the optimum pretension, all stays will have residual tension at the buckling of the stayed column. When it has the initial pretension less than the minimum effective pretension, there will be no residual tension left in the stay at the buckling load.

(5) Buckling Load: The buckling load is the actual buckling
strength of the real stayed column. With various pretensions and stayed column parameters, the buckling load will vary. In the analytical study the buckling load is obtained according to the following definitions. These definitions are:

(a) When the tension in the stays on the concave side of the column reaches zero, the stiffness of these stays will vanish and the column will then collapse. The applied axial load is considered to be the buckling load. This definition is similar to the one for an ideal stayed column in which the column is considered to have buckled when all the tensions in the stays are zero.

(b) Under a certain axial load the stayed column can also be considered to have failed when the lateral deflection or the rotational deflection is large.

3.3.3 Geometrical Analysis for a Real Stayed Column

A two-dimensional single-crossarm real stayed column with pinned ends is analyzed by determining the relationship between the applied load, stay tension and deflections.

When the real stayed column is subjected to a certain applied axial load, two types of deformation take place. These are the axial deformation and the lateral deformation.

(l) Axial Deformation: The real stayed column behaves in the same manner as an ideal stayed column under a compression axial load. The core will be compressed, the stays will be shortened and the crossarm will be elongated. Due to the axial deformation, as shown in Figure 3.3, the shortening of the stays ($\Delta_s$), can be expressed in terms of the shortening of the column ($\Delta_c$) and the elongation of the
crossarm (Δ_{ca}), which is

\[ \Delta_s = \frac{1}{2} \Delta_{ca} \cos \alpha - \Delta_{ca} \sin \alpha \]  

..... 3.12

(2) Lateral Deformation: Due to the presence of imperfections in the column, the lateral deformation is the most important matter to be considered. The lateral deflection will cause different tension in the stays. Assume that the stayed column is deflecting to the left, as shown in Figure 3.4 and Figure 3.5. The amount of lateral deflection at the crossarm level is \( \Delta_m \). It is assumed that the displacement at both ends of crossarm are the same. Then the total shortening of the stay is obtained by combining the results of axial deformation and lateral deformation together. The total shortening of the stay on the convex side (left) of the column is

\[ \Delta_{sl} = \Delta_s - \Delta_m \sin \alpha \]

\[ = \frac{1}{2} \Delta_{ca} \cos \alpha - \Delta_{ca} \sin \alpha - \Delta_m \sin \alpha \]  

..... 3.18

And the total shortening of the stay on the concave side (right) of the column is

\[ \Delta_{sr} = \Delta_s + \Delta_m \sin \alpha \]

\[ = \frac{1}{2} \Delta_{ca} \cos \alpha - \Delta_{ca} \sin \alpha + \Delta_m \sin \alpha \]  

..... 3.19

The amount of stay shortening can also be expressed by the change in stay tension, which is

\[ \Delta_{sl} = \frac{T_i - T_{fl}}{K_s} \]  

..... 3.20

\[ \Delta_{sr} = \frac{T_i - T_{fr}}{K_s} \]  

..... 3.21
where \( T_{fl} \) = the final stay tension on the left side of the column; and \( T_{fr} \) = the final stay tension on the right side of the column. The force system of the real stayed column is shown in Figure 3.6.

The values of \( \Delta_c \), \( \Delta_{ca} \), \( \Delta_{s1} \), and \( \Delta_{sr} \) from Equations 3.8, 3.10, 3.20, and 3.21 are substituted into Equations 3.18 and 3.19 obtaining,

\[
\frac{T_i - T_{fl}}{K_s} = \frac{1}{2} \left[ \frac{P_a - 2(T_i - \frac{T_{fl} + T_{fr}}{2}) \cos \alpha}{K_c} \right] \cos \alpha
- \frac{[2(T_i - T_{fr}) \sin \alpha] \sin \alpha}{K_{ca}} - \Delta_s \sin \alpha \quad \ldots \quad 3.22
\]

\[
\frac{T_i - T_{fr}}{K_s} = \frac{1}{2} \left[ \frac{P_a - 2(T_i - \frac{T_{fl} + T_{fr}}{2}) \cos \alpha}{K_c} \right] \cos \alpha
- \frac{2(T_i - T_{fr}) \sin \alpha \sin \alpha}{K_{ca}} + \Delta_s \sin \alpha \quad \ldots \quad 3.23
\]

By adding Equations 3.22 and 3.23 together,

\[
\frac{2T_i - (T_{fl} + T_{fr})}{K_s} = \frac{[P_a - 2(T_i - \frac{T_{fl} + T_{fr}}{2}) \cos \alpha]}{K_c} \cos \alpha
- \frac{4(T_i - \frac{T_{fl} + T_{fr}}{2}) \sin^2 \alpha}{K_{ca}} \quad \ldots \quad 3.24
\]

from which

\[
P_a = \frac{K_c}{\cos} \left[ 2T_i - (T_{fl} + T_{fr}) \right] \left( \frac{1}{K_s} + \frac{\cos^2 \alpha}{K_c} + \frac{2 \sin^2 \alpha}{K_{ca}} \right) \quad \ldots \quad 3.25
\]

Equation 3.25 gives the relationship of \( P_a \), \( T_i \), \( T_{fl} \), and \( T_{fr} \) for a single-crossarm stayed column for the first mode of deflection.

By altering the order of terms in Equation 3.24, the final stay
tension can be expressed in terms of $P_a$ and $T_i$, which is

$$T_{fl} + T_{fr} = 2T_i - \frac{P_a}{\frac{K_C}{\cos \alpha} + \frac{1}{K_S} + \frac{\cos^2 \alpha}{K_C} + \frac{2 \sin^2 \alpha}{K_{ca}}}$$

...... 3.26

Subtracting Equation 3.23 from Equation 3.22, the following equation is obtained

$$\Delta_m = \frac{(T_{fl} - T_{fr})}{2 \sin \alpha} \left( \frac{1}{K_S} + \frac{\sin^2 \alpha}{K_{ca}} \right)$$

...... 3.27

where the column mid-height deflection ($\Delta_m$) can be expressed by the final tension as in Equation 3.27.

3.4 Geometrical Nonlinear Analysis

3.4.1 General

Generally, two types of nonlinearity are found in structural problems. These are:

(1) Nonlinearity through material properties, and

(2) Nonlinearity through large deformation and geometrical changes in the structure.

In this study the nonlinearity due to the material properties is neglected. Since initial out-of-straightness is always present in real structural members, large deformation and geometrical changes in the structure is an important matter to be considered for the stability problem.

In the linear theory, the relationship between the applied force $\{P\}$ and the displacement $\{U\}$ is given by

$$\{P\} = [K] \{U\}$$

...... 3.28

where $[K]$ is the linear elastic stiffness matrix for the structure.
In nonlinear theory, where displacements are large, Equation 3.28 is no longer valid since (1) the strain-displacement relations of members are nonlinear, and (2) the equation of joint equilibrium needs to be written in terms of the displaced geometry of the structure.

When considering the strain-displacement relationship higher order terms need to be included. In the analysis of a two-dimensional structure the strain-displacement equation is

\[ \varepsilon_{xx} = \frac{d\bar{u}}{dx} + \frac{1}{2} \left( \frac{d\bar{v}}{dx} \right)^2 - \frac{y^2 \ddot{v}}{2} \]

where \( \varepsilon_{xx} \) = the strain in the \( x \) direction; \( \bar{u} \) = the local displacement in the \( x \) direction; and \( \bar{v} \) = the local displacement in the \( y \) direction.

Since the finite element method is used for solving the stability problem, the structure is divided into a number of elements or substructures. The elements are interconnected at a discrete number of nodal points. The general displacement functions are chosen for the elements. The total strain energy of the structure is derived and the minimization of the strain energy will provide the final relationship between the load \( \{P\} \) and the displacement \( \{U\} \), which is

\[ [K_E + K_G] \{U\} = \{P\} \]

where \( [K_E] \) = the master elastic stiffness matrix; and \( [K_G] \) = the master geometric stiffness matrix. The details of the derivation of Equation 3.30 for a planar structure is shown in Appendix A and References 7 and 9.

The geometric stiffness matrix \( [K_G] \) depends not only on the
geometry, but also on the initial internal load \((P_I)\) and hence, is also called the initial stress stiffness matrix. This matrix is linearly proportional to the internal force at the start of the loading step, and is given as

\[
[K_G] = P_I[K^*_G]
\] .... 3.31

where \([K^*_G]\) = the geometrical stiffness matrix for a unit of internal force \((P_I = 1)\). Equation 3.30 can be rewritten in the form

\[
[K_E + P_IK^*_G]{u} = \{P\}
\] .... 3.32

Equation 3.32 will account for the nonlinear behavior due to large deformations of the structure. This equation does not, however, include the influence of the equilibrium of geometry and equilibrium of forces. In order to include such effects in the analytical process, consideration must be given to the method of solving such nonlinear equations. These methods will be discussed in the following sections.

3.4.2 General Techniques for Solving Nonlinear Equations

The solution of the nonlinear problem by the finite element method is usually attempted by three basic techniques. These are:

(1) Iterative procedure.

(2) Incremental procedure.

(3) Mixed procedure.

In this section several techniques will be discussed which are commonly used in solving nonlinear problems. For simplification, the nonlinear Equation 3.32 is put into a simpler form, which is

\[
[K_T]{u} = \{P\}
\] .... 3.33

where \([K_T]\) = a nonlinear stiffness matrix and is a function of \({u}\)

23
and \( \{p\} \).

### 3.4.2.a Iterative Procedure

The iterative procedure is a sequence of repeated calculations in which the structure is loaded for each iteration. Several values are adjusted after each iteration until the result converges to a steady value. Many different iterative approaches have been suggested for analyzing the nonlinear equation. Some of the common procedures are:

1. **Tangent Stiffness Method**\(^{13,14,18,20,22}\) (Newton-Raphson Method)
2. **Secant Stiffness Method**\(^{13,14,20}\)
3. **Modified Tangent Stiffness Method**\(^{13,14,20}\)
4. **Modified Secant Stiffness Method**\(^{13,14,20}\)
5. **Tangent-Secant Stiffness Method**\(^20\), and
6. **Successive Substitution Method**\(^23\).

The methods of tangent stiffness and secant stiffness will be discussed in this section. These methods show the basic ideas of how the iterative procedure works.

(i) **Tangent Stiffness Method:** This method is similar to the Newton-Raphson technique. The final solution is obtained as the sum of the successive estimates that are calculated during each cycle of iteration. The technique is illustrated by a schematic diagram in Figure 3.7. The iterative scheme is

\[
\{P^{i-1}_R\} = [K^{i-1}_\text{Tan}]{\Delta U^i} \quad i = 1,2,3,\ldots \quad 3.34
\]

\[
\{U^i_T\} = \sum_1^i {\Delta U^i} \quad \ldots \quad 3.35
\]

where \( \{P^{i-1}_R\} \) = the remaining unbalanced load vector at the \( i \)th cycle.
of iteration; \( P^0 \) = the applied load; \( U_T \) = the total deformation; and \( \Delta U^i \) = the successive estimate after each iteration; and 
\[ K_{Tan}^{i-1} \] = the tangent stiffness matrix of the structure at the \( i \)th cycle.

After each iteration the portion of the total load that is not balanced is calculated and used in the next step to compute an additional increment of displacement. This is repeated until the equilibrium of the total load is satisfied (or \( P_R^i \) approaches zero). The procedure is then completed. When this method is used it is necessary to compute the tangent stiffness matrix at the end of the previous step. Instead of computing the stiffness matrix for each cycle, a modified iterative technique will be employed by just using the initial stiffness matrix. It requires more cycles, however, to complete the whole procedure.

(ii) Secant Stiffness Method: A schematic representation of this procedure is shown in Figure 3.8. The iterative scheme is

\[
U^i = [K_{sec}^i]^{-1} P \quad i = 1, 2, 3, \ldots 
\]

where \( [K_{sec}^i] \) = the secant stiffness matrix; \( U^i \) = the resulted deformation after \( i \)th iteration; and \( P \) = the applied load.

The stiffness is modified after each iterative step. The stiffness matrix which is calculated from the new geometry of the structure is called the secant stiffness matrix. The procedure is completed when \( U^{i-1} \) is close to \( U^i \). The computed secant modulus for each cycle is treated as if they were an elastic constant.

3.4.2.b Incremental Procedure

The basic of the incremental or stepwise procedure is to subdivide
the load into many small partial loads or increments. Usually
these load increments are of equal magnitude, but in general, they
need not be equal. The load is applied one increment at a time and
during the application of each increment the equations are assumed
to be linear. The value of the stiffness matrix is the same within
an incremental load, but is different for each load. The solution
from each step is an incremental displacement. The total displacement
is the sum of all incremental displacements at any stage of loading.
This technique is illustrated in a schematic diagram in Figure 3.9.

The incremental scheme is

\[
\{\Delta U_i^i\} = [K_T^{i-1}]^{-1}\{P_i\} \quad i = 1,2,3,...
\]

The \([K_T]\) matrix is calculated after each step and is often
referred to as the tangent stiffness matrix.

The total load \(\{P_i\}\) is the sum of all the incremental loads
\((\Delta P_i)\), which is

\[
\{P_T\} = \sum_{i}^{1}\{\Delta P_i\} \quad i = 1,2,3,...
\]

and the total displacement is

\[
\{U_T\} = \sum_{i}^{1}\{\Delta U_i\} \quad i = 1,2,3,...
\]

3.4.2.c **Mixed Procedure**

The disadvantage of using the iterative procedure are: (1) No
information is provided concerning the behavior at the intermediate
loads, and (2) There is no assurance that it will converge to the
exact solution, especially for the highly nonlinearly problems.

The advantages of the incremental procedure is its complete
generality, which is applicable to most structures with nonlinear behavior. The method provides a relatively complete picture of the load-deflection behavior. It is, however, difficult to know in advance what increments of load are necessary to obtain good approximation to the exact solution.

The use of the mixed method in which the iterative process at the end of each increment can be carried out until a desired equilibrium or accuracy is attained combines the advantages of both methods, minimizes the disadvantages and can yield higher accuracy of results. Two schematic representations of mixed procedure are shown in Figures 3.10 and 3.11. These are the mixed incremental tangent stiffness procedure and the mixed incremental secant stiffness procedure.

3.4.3 Solutions for the Nonlinear Behavior of the Real Stayed Column

Some structures, such as a cable truss, guyed tower, stayed column, etc., cannot have their complete behavior described by an equation such as Equation 3.32 or 3.33. Due to the changes in the geometry of the structure, the member forces and the stiffness of the structure may vary. Thus, the system is not in an equilibrium state. To solve such problems, it is necessary to use a different technique, such as the incremental and iterative procedure.

For the stayed column, the nonlinear behavior can be determined by the iterative procedure (secant stiffness method) when the applied load is small compared to the critical load. This method cannot solve the problem with high nonlinearity. Thus, the buckling load cannot be predicted by this method.
By using the incremental procedure, a correction to the stiffness matrix and the axial force in the equilibrium equation is taken into account. This method can provide good results for the nonlinear problem if the incremental load is adequately chosen. It is not possible to determine in advance what an adequate incremental load is. The method of the incremental procedure in solving the nonlinear problem is highly recommended by researchers\textsuperscript{13,14,18,19}.

In order to obtain an accurate analytical result, the mixed procedure may be used in the study of stayed columns. The mixed procedure will eliminate the disadvantages of either method. The mixed procedure used in this study of stayed columns is slightly different to the one discussed previously. This technique is illustrated in a schematic diagram in Figure 3.12, and is discussed in the next paragraph.

When using this procedure, the load is separated into a number of incremental steps. Within each loading step, the incremental deflection is calculated repeatedly in a number of cycles by Equation 3.40 until it converges to a steady value. The values of $K_E$, $K_G$ and $P_1$ are modified after each cycle of iteration.

\[
\{\Delta U_j^i\} = [K_E^i] + P_{ij}^i K_G^i \{\Delta P_{aj}\} \quad \cdots \cdots \text{3.40}
\]

where $i = 1, 2, 3, \ldots$ = number of cycles to converge in each step; $j$ = number of loading steps; $[K_E^i]$ = master elastic stiffness matrix of the structure; $[K_G^i]$ = master geometric stiffness matrix of the structure; $P_{ij}^i$ = actual axial force on the core of the stayed column; $\{\Delta U_j^i\}$ = incremental deflection after $i$\textsuperscript{th} cycle of iteration; and
\[ \{ \Delta P \}_a \] = incremental load.

The total deflection is the sum of the incremental deflections, which is

\[ \{ U_T \} = \sum_{1}^{j} \{ \Delta U_j \} \] \hspace{1cm} \ldots \hspace{1cm} 3.41

The total applied load is

\[ \{ P_a \}_j = \sum_{1}^{j} \{ \Delta P_a \}_j \] \hspace{1cm} \ldots \hspace{1cm} 3.42

The load-deflection relationship can be completely described, and the failure load can also be predicted from these results.
CHAPTER 4

COMPUTER SOLUTION

4.1 General

For the nonlinear analysis of a stayed column, it is necessary to develop a computer program to carry out the mixed incremental iterative procedure. A computer program, based on the finite element method, was written to determine the nonlinear behavior of the two-dimensional real stayed column. This program will take care of the effect of all the different parameters which will be discussed later in Chapter 5.

This program requires a certain minimum input of data and will provide enough information to complete analytical results. The results from the program's output will include the deflection at each nodal point and all the member forces for each loading step when the stayed column is subjected to any initial pretension. The buckling load of the stayed column can be determined from the load-stay tension and the load-deflection relationships. A complete relationship between the initial pretension and the buckling load can then be generated.

4.2 Description of the Computer Program

The computer program was written in FORTRAN IV (or WATFIV) language and was run on the IBM/360 system. This program consists of approximately 550 cards and can be separated into two main parts, which are (1) the main program, and (2) the subroutines. The procedure of applying the mixed incremental iterative technique for determining the nonlinear behavior of the real stayed column is
illustrated by a flow chart shown in Appendix B. The details of the program will be discussed in the following sections and a complete listing of the computer program is given in Appendix C.

4.2.1 Main Program

The main program can be analyzed by dividing it into several steps so that the computation operation can be easily followed. These steps are:

(1) Dimensioning: It is necessary to assign a certain storage area for each array.

(2) Input Data: The data read in are:

(a) number of elements (M),
(b) number of nodes (N),
(c) number of degrees of freedom (NDF),
(d) number of elements with significant axial load (NKG),
(f) dimension of the problem (NNL) - it is two for the planar structure,
(g) number of loading conditions (NLC) - it is set at one for this structure,
(h) number of end nodes (MN),
(i) variable correlation table (IVC),
(j) outer and inner diameters of core and crossarm (D11 and D22),
(k) modulus of elasticity (EM) and modulus of rigidity (GM),
(l) coordinates of nodal point (CN),
(m) the initial force at each nodal point (P), and the incremental force (PA), and
(n) initial tension in stays (TI).

(3) Calculate Member Properties: The member properties calculated are:
   (a) moment of inertia (ZI),
   (b) cross-sectional area (AA),
   (c) length (LGTH), and
   (d) direction cosines (DC).

(4) Write the input data and the calculated member properties.

(5) Calculate the axial force (PI) on the core of the stayed column.

(6) Set up the total master stiffness matrix (TMK) through subroutines (ELAKM), (GEOKM), (ELAK) and (GEOK). From which

\[ K_T = K_E + \begin{bmatrix} P_I \end{bmatrix} K_G \].

(7) Calculate the deformation, \( \Delta U = [K_T]^{-1}[p_A] \), through subroutine (SOLVE).

(8) Based on the new geometry, modify the nodal coordinates,

\( \{U_T\} = \Sigma \{\Delta U_i\} \), through subroutine (COOR).

(9) Calculate the forces on each member, \( \{F_i\} = [K_i][T_i]\{U\} \), through subroutine (STRESS).

(10) Calculate the total actual tension in the stays.

(11) Check how many of the stays are effective. The area of the stays will be neglected when the tension is equal to or less than zero.

(12) Check the iterative convergence criteria.

(13) Write output, which consists of
   (a) incremental deformation and new geometry,
   (b) incremental load and total applied load, and
(c) member forces and convert to tension in stays.

(14) Repeat the incremental procedure from step (3.c) to (14) after the previous iterative step has converged.

(15) Exit when all the required information has been obtained.

4.2.2 Subroutines

(1) ELAMK: Calculates the master elastic stiffness matrix.

(2) GEOKM: Calculates the master geometric stiffness matrix.

(3) TRANS: Transposes a matrix.

(4) TRANF: Calculates the transformation matrix.

(5) ELAK: Calculates the element elastic stiffness matrix.

(6) GEOK: Calculates the element geometric stiffness matrix.

(7) SOLVE: Calculates the displacement for each degree of freedom.

(8) DISPLA: Changes the global displacement to local displacement at each end of the member.

(9) STRESS: Calculates the forces acting on each member.

(10) MULT: Multiplies two matrices.

(11) COOR: Calculates the new coordinate of each node which has changed due to the applied force.

4.3 The Advantages and the Limitation of the Computer Program

The computer program is designed to determine the nonlinear behavior of a real stayed column. This program will include the effects of (1) large deflections, (2) equilibrium of forces and geometry, (3) initial imperfections, (4) initial pretension, and number of effective stays. It will also provide (1) a complete description of the load-deflection relationship, (2) load-stay
tension relationship and (3) the prediction of the buckling load. This program will have a very high accuracy in the prediction of results because the mixed procedure is used.

This computer program can only predict the first buckling load and the corresponding buckled shape, and will require more computation time as compared to the iterative or incremental procedure. This program is designed to determine the behavior of the planar single-crossarm stayed column only. This computer program can be modified, however, to determine the behavior of any type of stayed column.
CHAPTER 5

EFFECT OF STAYED COLUMN PARAMETERS

5.1 General

As stated in Chapter 1, the main objective of this study is to develop the complete relationship between the buckling strength and the initial stay pretension for a real stayed column. Thus, the influence of initial imperfection and initial stay pretension are of great importance in this study. There are some other parameters which also have an effect on the behavior of the stayed column. The parameters of a single-crossarm stayed column are summarized as follows and will be discussed later in this chapter. These parameters are:

(1) The cross-sectional and material properties, as well as the length of the column (E, I, A, G, l).
(2) The end support condition (hinged, fixed or free).
(3) The length of the crossarm (lca).
(4) The modulus of elasticity of stay (Es).
(5) The diameter of the stay (φ).
(6) The initial out-of-straightness (y0).
(7) The initial stay pretension (T), and
(8) The numbers of effective stays.

5.2 The Basic Parameters

The strength of any kind of column is directly related to its cross-sectional and material properties (E, I, l). For any simple column, the strength can be determined by the Euler equation, which is

\[ P_{Euler} = \frac{\pi^2EI}{(KL)^2} \]

[5.1]
where \( P_{\text{Euler}} \) = the Euler critical load; \( E \) = the modulus of elasticity of the column; \( I \) = the moment of inertia of the column; \( L \) = the length of the column; and \( K \) = the effective length factor.

All these parameters are considered to be the basic parameters. In this study these parameters are assumed to be the same for all models. The Euler critical load of the simple column is determined in order to make a comparison with the buckling strength of the stayed column.

5.3 Effect of End Support Condition

For any kind of simple column, the end support condition will influence the strength and mode of failure of the column. The effective length factor \( K \) and the behavior of the column will change as the types of end supports are changed. Different types of end support conditions will also have a great influence on the behavior of the stayed column. In this section a single-crossarm stayed column with various types of end supports and their possible modes of failure are demonstrated.

Generally, there are three common types of end supports. They are: (1) hinges, (2) fixed ends, and (3) free ends. Different combinations of end supports on the stayed column are discussed as follows:

(1) The stayed column with both ends hinged is free to rotate at both ends, and is allowed to rotate and translate at the crossarm level. If the restraint against the translational movement at the crossarm level is relatively small, the column will exhibit a failure shape which is a type of triple curvature, defined as mode I failure.
and shown in Figure 5.1.a. If the restraint against the translational movement is very small, it will have a failure shape of single curvature similar to the failure shape of the simple column, which is also classified as mode I failure. When the restraint against the translational movement is large or the restraint against the rotational movement at the crossarm level is small, the stayed column will have a tendency to fail in a double curvature shape, which is defined as mode II failure and is shown in Figure 5.1.b.

(2) The stayed column with one end fixed and the other end free has a relatively low buckling strength. At the free end it is allowed to rotate and translate, and there are no degrees of freedom at the fixed end. It is recognized that there is only one possible mode of instability. Its failure shape is defined as mode III failure and is shown in Figure 5.1.c.

(3) The stayed column with one end fixed and the other end hinged has two possible modes of instability. The column is allowed to rotate at the hinge and also allowed to translate and rotate at the crossarm level. If the lateral restraint at the crossarm level is relatively small the failure mode will be mode IV, as shown in Figure 5.1.d. When the lateral restraint is large or the rotational restraint is small, the stayed column will fail in the shape of mode V, as shown in Figure 5.1.e.

(4) The stayed column with both ends fixed has the highest buckling strength. It is a rigid support and hence, no movement is allowed at the support. There are two possible modes of failure which are defined as mode VI and mode VII failure, as shown in Figure 5.1.f.
Although different types of end supports will change the behavior of the stayed column, only the stayed column with both ends hinged will be considered in this study, since it is the simplest type of end support and it can provide a general understanding of the behavior of the stayed column.

5.4 Effect of Initial Out-of-Straightness (Imperfection)

In most of the previous studies, the column has been considered to be perfectly straight. In recent years a growing appreciation has developed of the profound influence of the so-called initial imperfection on the behavior of structures under compression loading. In the study of stayed columns, there are discrepancies of 10 to 20% between the experimental and the theoretical results. This lead to a consideration of the presence of initial imperfections and their effect on the strength and modes of buckling of the stayed column. In the following sections, the effect of the initial imperfection on the behavior of the real stayed column is discussed and compared with the behavior of the ideal stayed column.

5.4.1 Types of Initial Imperfection

A real column can have any type of initial imperfection. In order to illustrate the relation between the bifurcation phenomenon and the initially imperfect column, a pin ended column with a slight crookedness is considered.

Any initial imperfect shape can be represented by an infinite series. It can be expressed in the form of

$$\delta = \sum_{m=1}^{\infty} y_m \sin \frac{m \pi x}{L}, \quad m = 1, 2, 3, \ldots$$

\[5.2\]
where $\delta$ = the initial imperfection at any point along the core of the stayed column; $y_0$ = the known constant; $x$ = the coordinate along the core; and $L$ = the length of the column.

Some of the common types of initial imperfections and their corresponding buckling shapes are described as follows:

(1) Type I Initial Imperfection: It is the most common type of initial imperfection and has a single curvature shape. Its shape can be expressed by Equation 5.3. The initially imperfect shape and its possible failure shape are shown in Figure 5.2.a.

$$\delta = y_0 \sin \frac{\pi x}{L} \quad \ldots \quad 5.3$$

(2) Type II Initial Imperfection: This type of initial imperfection consists of a double curvature shape and can be expressed by Equation 5.4.

$$\delta = y_0 \sin \frac{2\pi x}{L} \quad \ldots \quad 5.4$$

The initially imperfect shape and its possible failure shape are illustrated in Figure 5.2.b.

(3) Combination of Type I and Type II Initial Imperfection: Many different combinations of type I and type II initial imperfection shapes will occur. Its shape can be expressed by Equation 5.5.

$$\delta = y_{01} \sin \frac{\pi x}{L} + y_{02} \sin \frac{2\pi x}{L} \quad \ldots \quad 5.5$$

The initial and final possible modes of failure are shown in Figure 5.2.c. This indicates the effect of different values of $y_{01}$ and $y_{02}$.

(4) Irregular Type: Some initial imperfections cannot be expressed by any simple equation, but they can be summarized by the
series equation given by Equation 5.2. A couple of irregular types of initial imperfections are shown in Figure 5.2.d.

Although there are so many types of initial imperfection, only the mode I and mode II initial imperfection will be discussed in detail.

5.5 Effect of Length of Crossarm

The end of each crossarm was rigidly welded to side of the core, and the other end was hinged to the stays. The combined action of stays, crossarms and core will offer certain restraint against the translational and rotational movement at the crossarm level. A different length of crossarm will lead to a different mode of failure and buckling strength. The investigation of the behavior of an ideal stayed column with various crossarm lengths was carried out by Hathout. The ratio of half column length to the crossarm length \((l/1_{ca})\) was used in the analytical process.

When the crossarm length is relatively short, the stayed column is expected to have a failure mode which is a type of triple curvature (mode I). As the crossarm length increases, the stayed column tends to fail in a double curvature mode (mode II). These results can be explained by a geometric analysis. Referring to Figure 5.3 and assuming a certain lateral displacement \((\Delta_m)\) at the crossarm level, the lateral restraint force \((F_r)\) provided by the stays is

\[
F_r = 4\Delta_m \cos^2 \theta A_s E_s / 1_s
\]

where \(\theta\) = the angle between the stay and the crossarm.

If the area and modulus of elasticity of stay are constant, the restraint is given by
Equation 5.7 can be expressed in terms of the length of crossarm \( l_{ca} \) and the half column length \( l \) as

\[
\frac{F_r}{A_m} = \frac{\cos^2 \theta}{l_s} \quad \ldots \quad 5.7
\]

It can be observed from Equation 5.8 that the lateral restraint is smaller when the crossarm is shorter and the lateral deflection at the crossarm level will be larger. This will result in mode I failure. As the crossarm length increases, the lateral restraint will also increase and the deflection at the crossarm level will be very small. Due to the force acting at the ends of the crossarm, the crossarm will be bent. Following these actions, the core will be rotated at mid-height of the core, and will exhibit mode II failure.

If an ideal stayed column is considered, there will be no lateral deflection when the applied load is less than the critical load. When it reaches the critical load, the stiffness of the stayed column will vanish and the stayed column will suddenly bow out into a failure mode of the shape of mode I or mode II. Under this critical load the stayed column is in a neutral equilibrium state, and the restraint force from the stays will not be able to resist any additional load, thus, bifurcation will occur. The strength of the stayed column which is used in this study always refers to the maximum critical load with the optimum pretension in the stays.

If an initial imperfection is present, it will have a certain effect on the deflected shape of the stayed column. The deflected
shape will follow the original imperfect shape and will be magnified as the axial force is applied. This indicates that the initial imperfection will have a great effect controlling the mode of deflection and mode of failure. For any real stayed column, the buckling strength will be lower for the real column than for the ideal column. In this study, the mode I and mode II initial imperfection will be used because they are the most common types and will give a general understanding of their effects. In other words, even if the stayed column is supposed to have a buckling shape of mode I or mode II according to the given information of the ideal case, the actual deflected shape and failure mode will not correspond to that of the ideal column. The stayed column will fail in mode I if the initial imperfection has only a type I shape and will fail in mode II if the initial imperfection has only a type II shape. The buckling strength of the stayed column is a function of the initial imperfection, crossarm length, stay size and modulus of elasticity of stay.

If a stayed column has few buckling modes, it will also have few buckling loads. However, the mode of buckling with the lowest buckling load will control the failure shape.

When the load is applied to a real stayed column with an initial imperfection, the deflected shape will follow the initial shape and continue to magnify it. If the stayed column has an ideal type I, ideal type II or any other type of ideal initial imperfection, the initial deflected shape and the final failure shape will be similar to its original initial imperfect shape. Thus, the buckling strength will only relate to the initial imperfection, regardless to what the cross-
arm length, stay size and modulus of elasticity of stay is.

In practice, the initial imperfection has a great effect on the initial deflected shape, but not on the final failure shape. Since a real stayed column does not have an ideal type of imperfection, the final buckling mode and load will be determined as the first neutral equilibrium is reached. For example, a stayed column is theoretically expected to fail in mode II buckling and it also contains a type I initial imperfection, so the initial deflected shape will be the mode I shape, and the deflected shape will suddenly shift to mode II and fail as the applied load reaches the first buckling load. Thus, only the first buckling load is of most importance in the practical application. In this study, both the effect of type I and type II initial imperfections on the modes of deflection and their corresponding buckling loads will be investigated.

5.6 Effect of Stay Size

The stays of the stayed column are hinged to the ends of the crossarm and core. The size of the stay will also effect its behavior because it is directly related to the lateral restraint at the crossarm level. Referring to Equation 5.6, if the crossarm length and the modulus of elasticity of the stay are constant, the lateral restraint provided by the stay per unit lateral deflection is given by

$$\frac{F_r}{\Delta_m} = \frac{A_s}{h}$$

...... 5.9

As the area of the stay increases, the lateral restraint force also increases. Thus, the core is harder to move at the crossarm level. Hence, it is more likely to fail in mode II buckling and vice versa.
For the ideal stayed column, the size of the stay will not affect the intermediate deflected shape, because no deflection occurs in the intermediate steps. For the real stayed column, the size of the stay will be related to the deflection rate. The larger stay size will have a smaller deflection rate and vice versa. An example will be used to explain this effect later in this chapter.

5.7 Effect of the Modulus of Elasticity of the Stays

With different moduli of elasticity in the stays the stayed column will have different buckling strengths and different modes of failure. Referring to Equation 5.6, if the crossarm length and the stay size are constant, the lateral restraint provided by the stay per unit lateral deflection is given by

\[
\frac{F_r}{\Delta m} \propto E_s
\]  

The effect of various moduli of elasticity of the stays on the stayed column is similar to the effect of various stay sizes. When the modulus of elasticity of stay is larger, it will fail in mode II buckling and vice versa. This parameter has no effect on the intermediate deflection on the ideal stayed column, but it does affect the deflection rate of the real stayed column. An example will be used to explain this effect in the following sections.

5.8 Numerical Example

An example of a two-dimensional ideal pin-ended single-crossarm stayed column will be used to demonstrate the behavior of the stayed column. It consists of a ten foot column length and one foot long crossarms. Both the core and the crossarm are made from a steel tube.
of 1.5 in. outer diameter and 1.0 in. inner diameter. The steel rod of 3/16 in. diameter is used as the stay size. The modulus of elasticity of $29.6 \times 10^3$ ksi is used for the stay and the steel tube. This example includes an ideal stayed column, a real stayed column with type I initial imperfection of $y_o = 0.078$ in., and with type II initial imperfection of $y_o = 0.053$ in. In the analysis of the ideal stayed column, the finite element method with an eigenvalue approach was used. In the analysis of the real stayed column, the finite element method with the mixed iterative-incremental procedure is used. The results of various crossarms, various stay sizes and various moduli of elasticity of stay are listed and discussed as follows.

5.8.1 Buckling Strength versus Various Crossarm Lengths

Figure 5.4 illustrates the plot of the buckling strength versus the ratio of half column length to the crossarm length. Ratios of 10/1 to 1/1 are used in the analysis. The solid lines are the results of an ideal stayed column, which include the buckling strength of mode I and mode II failure shape. It reveals that when the ratio of $l_{ca}$ to $l_{ca}$ is less than 5.7/1, mode II buckling will control. As the angle between the stay and the crossarm becomes larger, the lateral restraint is also larger. Hence, at the instant of buckling the stayed column will be forced to fail in mode II. If the failure mode is controlled by mode II buckling, the buckling strength is not very sensitive to a change of the ratio of $l_{ca}$. This is because there is no deformation at the crossarm level, and the maximum deformation will occur at the quarter points of the core. The lateral restraint forces from the stays do not have any direct effect on the deformation at
the quarter point of the core. If the $l/l_{ca}$ ratio is very small, which implies having a very long crossarm, with such a long crossarm and without any support between the two ends, the crossarm will be easily bent and will cause a larger deflection at the quarter point of the core, so that the buckling strength will be lower. For an $l/l_{ca}$ ratio from 10/1 to 5.7/1, the buckling strength varies from 27 to 28.7 kips, but this strength will never be reached because mode I buckling will govern in this region. For the $l/l_{ca}$ ratio from 5.7/1 to 1/1 the buckling strength varies from 28.7 to 25 kips.

When the ratio of $l/l_{ca}$ is greater than 5.7/1, mode I buckling will control. Due to the smaller angle between the stay and the crossarm and the smaller lateral restraint, the core will deflect easily in the lateral direction. Then, the column will be forced to fail in a triple curvature shape. The buckling strength varies greatly with a change of the $l/l_{ca}$ ratio. The buckling strength is only 16 kips for an $l/l_{ca}$ ratio of 10/1, and the buckling strength becomes 34 kips for the $l/l_{ca}$ ratio of 1/1. However, the maximum buckling strength is only 28.7 kips at $l/l_{ca}$ of 5.7/1, because with the smaller ratio the mode II buckling will govern.

In the same figure, the dotted lines represent the results of the buckling strength of the stayed column with two types of initial imperfection. The buckling strength, which is used in these curves, is the maximum buckling strength with the optimum pretension in stays. The buckling strength is defined as the applied load when the stay tension on the concave side of the core approaches zero. The lateral deflection rate will be much larger for the real stayed column, as
compared to the ideal stayed column without any lateral deflection. Hence, the tension decreasing rate on the concave side of the core is faster than those for the ideal one, so that the buckling strength of the real stayed column can never reach the maximum critical strength for the ideal stayed column. The relationship between the buckling strength and the various $l/l_{ca}$ ratios for the real stayed column have the same general shape for the ideal stayed column, except that there is a significant decrease in strength when the initial imperfection is present. In this example, there is a decrease in buckling strength of 5.0 kips for mode I failure and 4.0 kips for mode II failure. In the region of $l/l_{ca}$ ratio from 10/1 to 5/1 the buckling strength varies from 11.0 to 25.0 kips, and its failure mode is controlled by mode I. For the $l/l_{ca}$ ratio from 5/1 to 1/1 the buckling strength varies from 25.0 to 21.0 kips and the failure mode is controlled by mode II buckling.

5.8.2 Buckling Strength versus Various Stay Sizes

Figure 5.5 illustrates how the various stay sizes affect the buckling strength of an ideal and real stayed column with two types of initial imperfections. Stay sizes of 1/16 to 10/16 in. diameter are used in the analysis. The solid lines are the results of an ideal stayed column, which includes the buckling strength of mode I and mode II failure shape. When the stay size is less than 5/32 in. diameter, mode I buckling will control and the buckling strength will decrease very rapidly as the stay size is decreased. The buckling strength will be the Euler load when the stay size is less than 3/32 in. diameter, and there is no practical reason for using such a
small stay size. When the stay size is larger than 5/32 in. diameter, the failure shape will be controlled by mode II buckling. The buckling strength will not vary too much with the various stay sizes. With the stay sizes from 5/32 to 10/16 in. in diameter, the buckling strength varies from 28.0 to 29.0 kips. The two curves intercept at the stay size of 5/32. At this point, the stayed column will fail in either mode I or mode II buckling.

In the same figure, the dotted curves represent the results of the real stayed column with two types of initial imperfection. When the stay size is small, the lateral restraint is also very small. The core is easily bent in the lateral direction, and will fail in a triple or single curvature shape. If mode I buckling controls, the lateral deflection will occur at the level of the crossarms. The deflection rate is inversely proportional to the stay size, since the larger the stay size, the larger the lateral restraint will be. Thus, the buckling strength increases rapidly with a larger stay size. In this example, the buckling strength is 13.0 kips for the stay size of 2/16 in. diameter, and increases to 24.8 kips for the stay size of 5/32 in. in diameter. When the stay size is larger than 5/32 in. in diameter, mode II buckling will control. The buckling strength is not sensitive to the change of the stay size, and the buckling strength only increases to 26.8 kips for the stay size of 10/16 in. diameter. The relationship between the buckling strength and the various stay sizes for the real stayed column have the same general shape for the ideal stayed column. In this example, there is a decrease in buckling strength of 5.0 kips for mode I failure and 3.6 kips for
mode II failure.

5.8.3 Buckling Strength versus Various Moduli of Elasticity of Stay

Figure 5.6 illustrates the relationship between the buckling strength and the various moduli of elasticity of stay for the ideal and real stayed column. The moduli of elasticity of stay of 9,000 to 30,000 ksi are used in the analysis. The solid lines are the results of the ideal stayed column, which includes the buckling strength of mode I and mode II failure shapes. When the modulus of elasticity of stay is less than 17,000 ksi, mode I buckling will control and the buckling strength will decrease very rapidly with a smaller modulus of elasticity of stay. With a moduli of elasticity from 9,000 to 17,000 ksi, the buckling varies from 16.0 to 28.0 kips. When the modululi of elasticity of stay is larger than 17,000 ksi, mode II buckling will control and the buckling strength is not sensitive to a change of the moduli of elasticity of stay. With the moduli of elasticity from 17,000 to 30,000 ksi, the buckling strength varies from 28.0 to 28.7 kips. Two curves intercept at the modulus of elasticity of stay of 17,000 ksi. At this point, the stayed column will fail in either mode I or mode II buckling shape.

In the same figure the dotted lines represent the results of the real stayed column with two types of initial imperfections. Referring to Equation 5.9, the lateral restraint is proportional to the modulus of elasticity of the stay. When the modulus of elasticity of the stay is small, the lateral restraint is also small. The core is then easily bent in the lateral direction and will have a mode I failure shape. If mode I buckling controls, lateral deflection will occur.
along the core. The lateral deflection rate is inversely proportional to the modulus of elasticity of stay. The larger the value of modulus of elasticity, the larger the lateral restraint will be. Thus, the buckling strength increases rapidly with a larger value of the modulus of elasticity of stay. In this example, the buckling strength is 12.0 kips for the modulus of elasticity of stay of 9,000 ksi and increases to 24.0 kips for the modulus of elasticity of stay of 17,000 ksi. When the modulus of elasticity of stay is larger than 17,000 ksi, mode II buckling will control. The buckling strength is not sensitive to a change of the modulus of elasticity of stay, and the buckling strength only increases to 24.7 kips for the modulus of elasticity of stay of 30,000 ksi. The relationship between the buckling strength and the various moduli of elasticity of stay for the real stayed column have the same general shape as for the ideal stayed column. In this example, there is a decrease in buckling strength of 5.0 kips for the mode I failure and 4.0 kips for mode II failure.

5.9 The Affect of the Initial Imperfection on the Deflection Rate

For any ideal stayed column, there is no lateral deflection along the core when any axial force is applied. When the critical load is reached, however, there will be a tremendous change in the geometry of the structure. For the real stayed column, lateral deflection is obtained whenever any axial load is applied. Since there are so many types of initial imperfection, the deflection rate will vary. In this study, the general behavior of the deflection rate for the real stayed column with type I and type II initial imperfection is illustrated in Figures 5.7.a and 5.7.b. In both figures the horizontal axis
represents the lateral deflection, the vertical axis represents the applied axial load and the dotted line on the top of each figure is the critical load of the ideal stayed column. At this critical load the lateral deflection increases indefinitely.

Figure 5.7.a illustrates in general the lateral deflection rate of the real stayed column with type I initial imperfection. The amount of initial imperfection is taken as \( y_\theta \) at the crossarm level, and the complete shape is \( y_\theta \sin \frac{nm}{L} \). The lateral deflection is at the crossarm level. With a larger initial imperfection, the deflection rate will be larger. There are three solid curves in these figures. The top curve represents the deflection rate of the real stayed column with very small initial imperfection and, hence, the buckling strength is very close to the critical load of the ideal stayed column. The solid curve below the top solid curve is the deflection curve of the real stayed column with a larger initial imperfection. The deflection rate is larger and the buckling strength is less. The solid curve at the bottom is the deflection curve of the real stayed column with a relatively large initial imperfection; hence, the deflection rate is relatively large and the buckling strength is considerably lower.

Figure 5.7.b illustrates in general the lateral deflection rate of the real stayed column with type II initial imperfection. The amount of initial imperfection is taken as \( y_\theta \) at the quarter points of the core and the complete shape is \( y_\theta \sin \frac{2nm}{L} \). There will be no lateral deflection at the crossarm level, and the lateral deflection is at the quarter point of the core. In this figure three curves in solid are used to compare the deflection rate with various
initial imperfections. For this real stayed column, the behavior is similar to that with mode I initial imperfection. With a larger initial imperfection, the deflection rate is larger and the buckling strength is lower.

5.10 Affect of Initial Imperfection on the Change in Stay Tension

Referring to Equation 3.14 or 3.15,

\[ T_f = T_i - P \alpha_1 \]  

it is seen that the decrease in the stay tension is linearly proportional to the increase in the applied load. The final tension in the stays can be calculated using Equation 3.15 and the final tension in all the stays is the same.

If any initial imperfection is present, the change in stay tension is no longer linearly proportional to the applied load, and the final tension in all the stays will not be the same. In order to illustrate such an effect, an example is used. The example is the same as the one used in Section 5.8 with a type I initial imperfection and an initial pretension of 600 lbs. in the stays.

In Figure 5.8, the dotted straight line represents the results of the change in stay tension for the ideal stayed column. These results are calculated using Equation 3.15. The rate of decrease in the tension 25.0 lbs/kip and the stay tension approaches zero at the applied load of 24.0 kips, which is the critical load for the ideal stayed column. In the same figure the two solid curves represent the change in stay tension for the real stayed column. The curve on the left is the stay tension on the concave side of the core and it has a larger rate of decrease in the tension. The curve on the right is the
stay tension on the convex side of the core and it has a smaller rate of decrease in the tension. These results can be obtained by using Equations 3.26 and 3.27, with the known deflection, or by using the finite element method with the mixed procedure to calculate the stay tension. When the applied load is relatively small, the rate of decrease in the tension is linear. In this example, the decrease in the tension is 30.0 lbs/kip for the stay on the concave side and 21.3 lbs/kip for the stay on the convex side. The average rate of decrease in the tension is 25.7 lbs/kip, and is almost the same rate as for the ideal stayed column. When the applied load is relatively large, the stay tension on the concave side decreases faster than the initial rate and the tension will vanish at a load of 18.5 kips, which is the buckling load for this real stayed column. The stay tension on the convex side decreases at a slightly slower rate at a higher load, and when the buckling load is reached the stay tension on the convex side increases very rapidly.

5.11 Affect of the Initial Pretension on Deflection Rate

Varying the magnitude of the initial imperfection in the core will have a great influence on the lateral deflection rate of a real stayed column. This effect is easily demonstrated in an example. The example is the same as the one used in Section 5.8 and only type I initial imperfection is considered. Figure 5.9 illustrates the results of the load-deflection curves for a real stayed column with an initial pretension less than the optimum pretension. Referring to Equation 3.32 and Figure 3.6, the total stiffness $[K_T]$ is given as
\[ [K_T] = [K_E + P_I K_G] \]  \[ \text{.... 5.11} \]

where \( P_I \) = the actual internal axial load on the core and is given by

\[ P_I = P_a + (T_{fl} + T_{fr}) \cos \alpha \]  \[ \text{.... 5.12} \]

When the initial pretension is less than the optimum pretension \( T_{fl} \) and \( T_{fr} \) are relatively small and will not have much effect on the value of \( P_I \) and \( K_T \). Hence, the deflection rates are almost the same for the stayed column with various initial pretension in stays. The buckling strength is reached when the stay tension on the concave side reaches zero. If the initial pretension is larger, a larger axial load is required to cause the stay tension to disappear; hence, the buckling strength will increase.

Figure 5.10 illustrates the load-deflection curves for a real stayed column with an initial pretension larger than the optimum pretension. With the larger initial pretension, \( T_{fl} \) and \( T_{fr} \) are also larger and will significantly affect the values of \( P_I \) and \( K_T \). The total stiffness will decrease with a larger initial pretension and then the deflection rate will be larger. The buckling strength is obtained when the deflection rate is very large, but the stay tension will not necessarily be zero. With a larger initial imperfection the buckling strength will be lower.

5.12 The Buckling Strength versus the Initial Pretension of the Stayed Column

Figure 5.11 illustrates the relationship between the buckling strength and the initial pretension for the ideal and real stayed column. This example is the same as the one used in Section 5.8 in which only the type I initial imperfection is considered. The curve
at the top of the figure is the result of an ideal stayed column, and the other two curves are the results for the real stayed column. One of the real stayed columns has an initial imperfection $y_o$ of 0.063 in., and 0.078 in. for the other one. For the ideal stayed column, the minimum effective pretension is 100.0 lbs., and with the initial pretension less than the minimum effective pretension the buckling strength is the Euler load which is equal to 4.0 kips. With the optimum pretension of 725 lbs. in the stays, the maximum critical load is 28.7 kips, and the maximum possible pretension is 14.5 kips. With an initial pretension of 100. lbs. to 725. lbs., the critical load increases linearly from 4.0 kips to 28.7 kips. Furthermore, with an initial pretension of 725. lbs. to 14.5 kips the critical load decreases linearly from 28.7 kips to zero. The second curve, which is just beneath the top curve, is for the real stayed column with a type I initial imperfection and $y_o$ is equal to 0.063 in.. The minimum effective pretension is 105. lbs. and the maximum possible pretension is 14.4 kips. With the optimum pretension of 1200. lbs., the buckling strength is 26.5 kips. The relationship between the buckling strength and the initial pretension for the real stayed column is no longer linear. For the real stayed column with a slightly larger initial imperfection ($y_o = 0.078$ in.), the value of minimum effective and maximum possible pretension do not vary significantly; but, the optimum pretension will increase to 1250. lbs. and the buckling strength will then change to 26.0 kips. These results are represented by the curve in the lowest part of the figure.
5.13 **Numbers of Effective Stays as Related to the Initial Imperfection**

Any real stayed column will bend into the confined plane as the axial load is applied. If the real stayed column has an initial pretension less than the optimum pretension, the stay tension on the concave side will disappear when the buckling load is reached. This buckling strength is the elastic buckling strength. Actually, if the stayed column is subjected to this load, it will not completely fail. Although some of the stays become slack, the stays which are still effective can resist or delay the onset of failure. If the applied load is slightly larger than the elastic buckling load, the lateral deflection and the rate of increase in stay tension for those stays which are effective becomes very large. For a certain applied load, which is beyond the elastic buckling load, the stayed column will fail completely. This failure load can be defined as a post-buckling load, but will not be discussed in this study. However, the number of effective stays is related to the type of initial imperfection. In a two-dimensional single-crossarm stayed column, there are four possible types of arrangements of effective stays. They are: (1) four stays effective for the ideal case, (2) three stays effective, (3) two stays effective, and (4) only one stay effective. These cases are illustrated in Figure 5.12.
CHAPTER 6

EXPERIMENTAL PROCEDURE

6.1 General

A model of a single-crossarm stayed column was built for an experimental test. A series of tests was carried out on this model, in order to verify the theoretical relationship between the initial pretension and the buckling strength. The initial out-of-straightness of the column was measured before the experimental tests and was then applied in the theoretical analysis. The details of the properties of the tested column, the experimental set-up, and test procedure are described in the following sections.

6.2 The Model of Stayed Column

6.2.1 Type of Model

A two-dimensional single-crossarm stayed column was chosen as the test model, as shown in Figure 6.1. There are several advantages in using a planar, instead of a three-dimensional, stayed column. The advantages are as follows:

(1) The deflected shape will be restricted to only one confined plane. Thus, buckling can be easily established.

(2) The model is easier to build and requires less material.

(3) The theoretical buckling strengths are the same for both the planar and the three-dimensional systems.

(4) The experimental test is easier to perform since fewer readings are required for determining the behavior of the stayed column.

6.2.2 Components of the Stayed Column

The planar pin-ended single-crossarm stayed column is the
simplest type of stayed column. It consists of a slender metal core, crossarms, stays and end supports. The properties of these components are described as follows:

(1) Both the core and crossarm were made from a circular steel tube with an outer diameter of 1.50 in. and an inner diameter of 1.00 in., as shown in Figure 6.2. A sample of a steel tube was subjected to a tension test. From the test results the steel tube exhibited an ultimate stress of 70 ksi, a yield stress of 49 ksi, and a modulus of elasticity of $29.1 \times 10^3$ ksi. The end of the crossarm was machined and firmly welded to the sides of the core at mid-height, as shown in Figure 6.3. Two holes were drilled and threaded at the end of each crossarm for holding the stays in place. Two circular plates with a 3.00 in. diameter and 1.00 in. thickness were welded to each end of the column. A groove of 1.00 in. diameter was bored on the outer face of each plate for holding a steel ball, which was used as a hinge support.

(2) Four stays were used in connecting the ends of the core and the crossarm. Different diameters and different moduli of elasticity of stays will affect the mode of failure and the buckling strength of the stayed column. A steel rod with a 3/16 in. diameter was used in this model. The connection between the stay and crossarm is shown in Figure 6.4. The behavior of the steel rod was determined by a tension test. The steel rod had an ultimate strength of 2250 lbs., a yield strength of 1980 lbs., and a modulus of elasticity of $29.3 \times 10^3$ ksi. In the theoretical analysis, the optimum pretension was determined to be 725 lbs. for an ideal stayed column, and 1250 lbs. for a real
stayed column. When a test is conducted with the optimum pretension, the force in stays will not exceed the yield strength.

(3) The support on top was made of a thick steel plate, which was firmly attached to a girder of a test frame in the laboratory. A groove of 1.00 in. was machined onto the plate for fitting the steel ball as a hinge support. The bottom end of the column was placed on top of a load cell. A 1.00 in. steel ball was placed between the load cell and the end of the column as another hinge support, as illustrated in Figure 6.5.

(4) In order to confine the column to the plane of the crossarm and stays, two sets of lateral supports were used. Each set of these supports consisted of two steel angles, two circular steel bars and two threaded rods. Each circular bar was welded to one exterior face of each angle. These angles were placed on two sides of the core in the direction parallel to the confined buckling plane. These lateral supports were set up at quarter points and were connected to the rigid test frame by threaded rods as shown in Figure 6.6. Rotational movement of the column was prevented by placing a small steel plate at the sides of the end of the crossarm, as shown in Figure 6.7.

6.2.3 The Dimensions of the Stayed Column Model

The length of the column was chosen to be 10.0 ft. The choice of this column length mainly depended on the space available in the laboratory and was close to a practical size. The length of each crossarm was 12.0 in. The maximum buckling load for this stayed column could be applied with a hydraulic jack and measured with a load cell available in the laboratory. The amount of the initial out-of-
straightness was measured. This column contained a type I initial imperfect shape with single curvature along the core. The maximum initial imperfection was 0.078 in. at the mid-height.

6.3 Experimental Equipment and Set-Up

Four screw type load cells, as shown in Figure 6.8, were used to set the initial pretension and to measure the change of tension in the stays. The load cell was joined to the stays by a connection shown in Figure 6.9. This connection consisted of two steel blocks. Each of the steel blocks has dimensions of 1.50 in. x 2.50 in. x 0.75 in.. The central part of this block was machined to make it hollow. Two holes of 1/4 in. and 1/2 in. diameter were drilled through the edges of the block. The stay and the load cell were tightened with nuts. The modulus of elasticity of the steel block and the load cell were the same as the steel rod, so that the modulus of elasticity of the whole member, which included the connection and the stay, was the same throughout. A stay with a screw type load cell and steel blocks were calibrated in order to determine the combined stiffness of the member. Before any experimental test, each of the screw type load cells were calibrated by a tensile test. The amount of tension was measured indirectly with a strain indicator. From the results of the calibration a linear relationship between the applied tension and the corresponding strain was obtained. The maximum allowable load of the screw type load cell is 4500 lbs..

A Universal flat load cell, as shown in Figure 6.10, was used to
measure the applied compressive axial load on the column. The maximum capacity of this load cell is 25 kips. Although the theoretical maximum critical load of an ideal stayed column is 28.7 kips, the actual maximum buckling load of the real stayed column is around 25 kips. Thus, the load cell could achieve this load. This load cell was calibrated by a compression test in a Universal Testing Machine. The load cell was connected to the strain indicator, and the amount of load was given by the corresponding strain reading. A small groove was machined on the top of the load cell to fit the steel ball at the end of the column.

A hydraulic jack was used to apply the compressive axial load to the column. It was centered and aligned with the center of the top plate by using a plumb bob. The base of the hydraulic jack was welded to the steel floor to ensure no movement at the base. It was also mounted by a connection having a hollow tube to fit the cylindrical piston of the jack and a rectangular top to fit the base of the flat load cell, also illustrated in Fig. 6.10.

The strain indicator, which consisted of two separated units, was used to determine the amount of the applied compressive load, the initial pretension and the change in stay tension. This strain indicator has ten channels. Four of them were connected to the screw type load cells and one of them was connected to the flat load cell, as shown in Figure 6.11.

A transit was used to measure the amount of initial out-of-straightness of the stayed column. The transit was located at a distance of 105 in. from the core of the stayed column. It was
aligned with the central vertical axis of the core and was perpendicular to the confined buckling plane. Reference angle readings were obtained by the transit sighting at various points along the core. These angle readings were then converted to the amount of initial out-of-straightness of the column. This set-up is shown in Figure 6.12.

Several steel bars of 5.0 in. x 0.75 in. x 0.125 in. were connected to the side of the core by a U-type joint. These bars were located at certain places along the core and were perpendicular to the direction of deflection. Dial gauges were placed on the bars for measuring the lateral deflection, as shown in Figure 6.13.

6.4 Experimental Procedure

Calibration of load cells and the material properties were determined prior to any experimental test. The column was placed between the top support and the flat load cell. The initial out-of-straightness of the core was determined by using the transit. The stay pretension was introduced by adjusting the nuts at the ends of the stays. The pretension had to be applied gradually, so that the difference in stay tension would not cause any further out-of-straightness of the core. The dial gauges, which were used in measuring the lateral deflection, were positioned on the steel bars. The incremental axial load of 1.00 kip was used initially, and changed to 0.5 kips of load increment as the buckling load was approached. The amount of applied load and the amount of stay tension were recorded by the strain indicator. When the stay tension on the concave side of the column reached zero or the deflection was large, the loading was stopped. The load was then released and the readings on the dial gauges and the
strain indicator were checked to determine whether or not they went back to original readings so as to ensure that the column was not overloaded or yielded. The same tests were carried out again with various initial pretensions. Results of these tests were recorded and will be discussed in Chapter 7.
CHAPTER 7

ANALYSIS OF RESULTS

7.1 General

Several experimental tests were performed, and for each test the stayed column was loaded to its buckling strength. Before any experiment was performed the analysis of the ideal stayed column was carried out, since this analysis is much faster than that for the real stayed column and gives a rough idea as to what initial stay pretension should be used in the experiment. The initial stay pretension chosen for the tests were 200. lbs., and 300. lbs. up to 800. lbs. The initial out-of-straightness was measured with a transit, and the shape of the core of the real stayed column was found to contain a type I initial imperfection, which could be expressed by the equation

\[ \delta = 0.078 \sin \frac{\pi x}{L} \text{ in.} \]

The maximum imperfection occurred at the cross-arm level, and is 0.065% of the column length and 3.9% of the outer diameter of the core. A load-deflection curve and a load-stay tension curve were drawn for each test. The relationship between the buckling load and the various initial pretensions was summarized from all the test results. Later, the theoretical analytical results for the real stayed column were obtained and compared to and verified by the experimental results.

7.2 Comparison of the Observed Stayed Tension and the Predicted Results

As mentioned in Section 5.10, when the compressive axial force is applied to the ideal stayed column, the stay tension in all the stays will decrease at the same rate. For the real stayed column, if the initial imperfection is present, the core will tend to bend in the
confined plane when it is subjected to a compressive load. Then, one side of the column becomes a concave shape and the other side becomes a convex shape. The stay tension on the concave side will decrease faster than those on the convex side. When the applied force is relatively small, the bending effect is not very great. Thus, the rate of tension change is almost linear for this model. When the applied load approaches the buckling load, the bending effect will cause a large deflection, which will dominate the change in stay tension. The stay tension on the convex side will decrease much slower than the stay tension on the concave side and eventually will increase when the tension on the other side vanishes or the deflection rate becomes very large. The results which were obtained from the experimental tests were similar to what has been described above. The experimental load-stay tension curves for initial pretensions from 200 lbs. to 800 lbs. are shown in Figures 7.1 to 7.7. In each of these figures there are four dotted curves which represent the tensions in each stay of the stayed column. The stay tension on the concave and on the convex side has two slightly different values, which represent the tension of the stay on the top and bottom on each side. This difference may be caused by experimental error or the stayed column not having an ideal type I initial imperfection. In the same figures, the theoretical results are also illustrated, which are represented by the solid curves. In these figures it must be observed that the relationship between the stay tension and applied axial load is in good agreement. The initial rate of change in stay tension for the experimental and theoretical results are tabulated in Table 7.1.
When the stayed column buckles, there will be a certain tension remaining in the stays, which are still effective. This tension is the residual tension. The experimental and theoretical residual tensions for different tests are listed in Table 7.2.

7.3 **Comparison of the Observed and the Predicted Deflection Shape of the Real Stayed Column**

The load-deflection curves for all tests are also shown in Figures 7.1 to 7.7. The deflection, which is plotted in these figures, is the lateral deflection at the crossarm level. The results showed satisfactory agreement between the experimental and theoretical values. All the experimental load-deflection curves for the various initial pretensions are summarized and plotted in Figure 7.8. It illustrates that the deflection rates are almost the same for all initial pretensions up to an applied axial load of about 80% of the buckling load. These can be compared to the theoretical results in Figure 5.9.

The complete experimental deflected shape along the core of the stayed column for initial pretensions of 600, 700, and 800 lbs. are shown in Figures 7.9 to 7.11. The theoretical results are also shown in these figures. There is good agreement between the theoretical and experimental results.

7.4 **Comparison of the Observed and Predicted Buckling Loads as the Initial Pretension is Varied**

The relationship between the buckling load and various initial pretensions are shown in Figure 7.12. This figure includes the experimental results for the real stayed column and the theoretical results of a real and an ideal stayed column. These results cover
the range of buckling loads of stayed columns with an initial pretension between the minimum effective and optimum pretensions. The experimental results of the real stayed column are in better agreement with the theoretical results of the real stayed column, than with the theoretical results of the ideal stayed column. Comparable values are listed in Table 7.3. In this comparison, the difference between the experimental and the theoretical buckling loads for the real stayed column varies from 4.7% to 8.6%. The difference between the experimental and the theoretical results for the ideal stayed column varies from 11.3% to 20.6%. The maximum difference in buckling loads of 20.6% occurs at the optimum pretension for the ideal stayed column. This indicates that the presence of an initial imperfection has a very significant effect on the buckling strength and mode of deflection of the stayed column. In any analytical or design problem involving stayed columns, this factor cannot be neglected.

7.5 Comparison of the Experimental and the Theoretical Change in Stay Tension

As mentioned in Chapter 3, from the geometric analysis, the relationship between \( P_a, T_i, T_{fl}, T_{fr} \) and \( \Delta_m \) are given in Equations 3.26 and 3.27

\[
T_{fl} - T_{fr} = \frac{2\Delta \sin \alpha}{m} \left( \frac{1}{K_s} + \frac{\sin^2 \alpha}{K_{ca}} \right) 
\]

\[
T_{fl} + T_{fr} = 2T_i - \frac{P_a}{\frac{K_c}{\cos K_s} + \frac{\cos^2 \alpha}{K_c} + \frac{2\sin^2 \alpha}{K_{ca}}} 
\]
In each experimental test, the stayed column was loaded with an axial force and had a certain initial pretension in the stays. The deflection at mid-height was measured. These results are substituted into Equations 3.26 and 3.27 and the calculated stay tension, based on experimental results, can be determined. For the stayed column with an initial pretension in the stays of 800 lbs., the calculated stay tension is shown in Figure 7.13. In this figure, the experimental stay tensions are also shown for comparison. There is a very close agreement between these results. It indicates that if the deflection rate is known, the behavior of the stayed column can easily be determined.
CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 General

This thesis was concerned with the study of the behavior of a single-crossarm stayed column with an initial out-of-straightness. A nonlinear analytical method was employed to predict the deflection shape, buckling load and change in stay tension for stayed columns with various initial pretensions. The influence of the stayed column parameters, such as the crossarm length, stay size, modulus of elasticity of stay, and the type of initial imperfection, on the behavior of the stayed column were investigated. The nonlinear analytical procedure was carried out using a computer program. The theoretical results were verified by the experimental program.

8.2 Conclusion

The conclusions of this study are summarized as follows:

(1) The stayed column will have a buckling load several times higher than the Euler load of a corresponding simple column.

(2) The real stayed column and the ideal stayed column behave differently.

(3) The relation between the initial pretension and the corresponding buckling load is related to three values. These are the minimum effective, optimum and the maximum possible pretensions. When the initial pretension is less than the minimum effective pretension, the buckling strength will be governed by the Euler load, for both the ideal and the real stayed column. The optimum pretension is associated with the maximum buckling strength of the column, and the
maximum possible pretension will cause buckling without the addition of an external applied load.

(4) The relationship between the initial pretension and the corresponding buckling strength will be linear for an initial pretension between the minimum effective and optimum pretension, and between the optimum and the maximum possible pretension for the ideal stayed column. This relationship between the initial pretension and the corresponding buckling load will not be linear for a real stayed column.

(5) When the initial pretension is between the minimum effective and the optimum pretensions, the buckling strength will increase as the initial pretension is increased.

(6) When the initial pretension is larger than the optimum pretension, the buckling strength will decrease as the initial pretension is increased.

(7) The mode of failure for the ideal stayed column is governed by the stayed column parameters, such as crossarm length, stay size, and modulus of elasticity of the stay.

(8) The deflection shape for a real stayed column will be of the same shape as the initial imperfection and will be magnified when axial load is applied. However, the mode of failure will also be related to the crossarm length, stay size and modulus of elasticity of the stay. So, it is possible that the final failure shape will not be similar to the deflection shape.

(9) The deflection rate of the real stayed column is related to the amount of initial imperfection, initial pretension, stay size and modulus of elasticity of stay. If the amount of the initial
imperfection in the core, or the initial pretension in the stay is increased, the lateral and the rotational deflection rate will also increase. If the stay size or the modulus of elasticity of stay is increased, the lateral and the rotational deflection rate will be smaller.

(10) The buckling load of the real stayed column is always less than the critical load of the ideal stayed column. As the amount of the initial imperfection increases, the buckling load is decreased.

(11) The difference between the buckling strength of the ideal and real stayed column is a maximum when the initial pretension is the optimum pretension for an ideal stayed column.

(12) When the crossarm length is relatively short, mode I buckling will govern. The buckling strength will increase rapidly as the crossarm length increases. When the crossarm length is relatively long, mode II buckling will control. The buckling strength will decrease if a longer crossarm length is used.

(13) When the stay size is relatively small, mode I buckling will control. The buckling strength will increase rapidly and linearly as the stay size is increased. When the stay size is relatively large, mode II buckling will control. The buckling strength will increase very slowly as stay size is increased.

(14) When the modulus of elasticity of the stay is relatively small, mode I buckling will control. The strength will increase rapidly as the modulus of elasticity of the stay is increased. When the modulus of elasticity of the stay is relatively large, mode II buckling will control and the increase in buckling strength is very
slow as the modulus of elasticity of the stay is increased.

(15) The minimum effective pretension for the real stayed column is slightly larger than that for the corresponding ideal stayed column. The maximum possible pretension for the real stayed column is slightly less than that for the ideal stayed column. The optimum pretension is quite large for the real stayed column, as compared to that for the ideal stayed column.

(16) There is approximately a 20% difference between the theoretical results of the ideal stayed column and the experimental results. This difference is reduced to less than 10% when the theoretical results for the real stayed column are compared to the experimental results.

(17) The deflection rate, change in stay tension and the buckling strength from the nonlinear analytical method are verified by the experiment program and satisfactory agreement was achieved.

8.3 Future Research

In this study, a nonlinear analysis, using the finite element method with the incremental and iterative procedure, was successfully used to determine the behavior of a two-dimensional single-crossarm stayed column. This approach can be modified to analyze the more complicated stayed columns with double or triple-crossarms, or three-dimensional stayed columns.

The single-crossarm stayed column being considered in this study is loaded axially only. Other types of loads, such as the lateral loads should be studied.

Work can be performed on the study of the inelastic behavior, and the postbuckling behavior of the stayed column.
Fig. 1.1 Examples of Stayed Columns

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Fig. 3.1 General Relationship between Critical Load and Initial Pretension of An Ideal Single-Crossarm Stayed Column
Fig. 3.2 Equilibrium Force System of An Ideal Stayed Column
Fig. 3.3 Change in Length of Stays Due To Axial Deformation

Fig. 3.4 Change in Length of Stays Due To Lateral Deformation
Fig. 3.5 Total Deformation of A Real Stayed Column
(a) Initial condition at end of column

\[ P_i = 2T_i \cos \alpha \]

(b) Final condition at end of column

\[ P_f = P_q + (T_{fl} + T_{fr}) \cos \alpha \]

(c) Initial condition at end of crossarm

\[ F_i = 2T_i \sin \alpha \]

(d) Final condition at end of crossarm

\[ F_{fr} = 2T_{fr} \sin \alpha \]

Fig. 3.6 Equilibrium Force System of A Real Stayed Column
Fig. 3.7 Schematic Diagram of Tangent Stiffness Method

Fig. 3.8 Schematic Diagram of Secant Stiffness Method
Fig. 3.9  Schematic Diagram of Incremental Method
Fig. 3.10 Schematic Diagram of Mixed Incremental Tangent Stiffness Procedure

Fig. 3.11 Schematic Diagram of Mixed Incremental Secant Stiffness Procedure
Fig. 3.12 Schematic Diagram of The Mixed Incremental & Iterative Procedure for The Analysis of The Real Stayed Column
Fig. 5.1 Modes of Buckling Influenced by End Support Conditions
Fig. 5.1

(d) MODE IV

(e) MODE V

(f) MODE VI

(g) MODE VII

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(1) Type I

\[ \delta = y_0 \sin \frac{\pi x}{L} \]

Initial Shape

Deflected Shape

(2) Type II

\[ \delta = y_0 \sin \frac{2\pi x}{L} \]

Initial Shape

Deflected Shape

(3) Type I + Type II

\[ \delta = y_{oi} \sin \frac{\pi x}{L} + y_{ou} \sin \frac{2\pi x}{L} \]

(a) \( y_{oi} \) is small

Initial Shape

Deflected Shape

(b) \( y_{ou} \) is large

Initial Shape

Deflected Shape

(4) Irregular Type

Initial Shape

Deflected Shape

Fig. 5.2 Types of Initial Imperfection & Deflected Shapes
\[ F_r = 2(T_{fl} - T_{fr}) \cos \theta \]
\[ = 2(T_{fl} - T_i) + 2(T_i - T_{fr}) \cos \theta \]
\[ = 4 \Delta_s K_s \cos \theta \]
\[ = 4 \Delta_m \cos^2 \theta A_s E_s / l_s \]

Fig. 5.3 The Lateral Restraint Force
Fig. 5.4  Theoretical Results of the Buckling Load Versus The Half Column Length to Crossarm Length Ratio of The Ideal & Real Single-Crossarm Stayed Column with \( \phi = 3/16" \) & \( E_s = 29600 \) Ksi

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Fig. 5.5 Theoretical Results of The Buckling Load Versus The Stay Diameter of The Ideal & Real Single-Crossarm Stayed Column with $\frac{I}{I_{ca}} = 5$ & $E_s = 29600$ Ksi
Fig. 5.6 Theoretical Results of the Buckling Load Versus the Modulus of Elasticity of the Stay of the Ideal & Real Single-Crossarm Stayed Column with $\phi = 3/16''$ & $1/l_{ca} = 5$
Deflection $u$

(a) Stayed Column with Type I Initial Imperfection

Deflection $u$

(b) Stayed Column with Type II Initial Imperfection

Fig. 5.7 The Deflection Rate Influenced by Initial Imperfection
Fig. 5.8 Change in Stay Tension Versus Applied Load of An Ideal & A Real Stayed Column with $\phi = 3/16\,\text{in}$, $l/l_{ca} = 5$ & $E_s = 29600 \, \text{Ksi}$
Load (Kips)

Stayed column with type I initial imperfection & \( T_i < T_{opt} \)

\[ y_0 = 0.078'' \]

\[ T_j = 1000\text{lbs} \]
\[ T_j = 800\text{lbs} \]
\[ T_j = 600\text{lbs} \]
\[ T_j = 400\text{lbs} \]
\[ T_j = 200\text{lbs} \]

Fig. 5.9 The Deflection Rate Influenced by The Initial Pretension of A Real Single-Crossarm Stayed Column with \( \phi = 3/16'' \), \( l/l_{cd} = 5 \) & \( E_s = 29600 \text{ Ksi} \)

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Fig. 5.10 The Deflection Rate Influenced by The Initial Pretension of A Real Single-Crossarm Stayed Column with $\phi = 3/16''$, $1/l_{csd} = 5$ & $E_s = 29600$Ksi
Fig. 5.11 Theoretical Results of Buckling Load Versus Various Initial Pretension of An Ideal & Two Real Stayed Columns with \( \phi = 3/16'' \), \( l/l_{ca} = 5 \) & \( E_s = 29600 \text{Ksi} \)
Fig. 5.12 Numbers of Effective Stays Due To Various Types of Initial Imperfection
Fig. 6.1 Model of Stayed Column
Fig. 6.2 Circular Steel Tube of Crossarm & Core

$D_o = 1.5''$

$D_{in} = 1.0''$
Fig. 6.3 Connection between Crossarm & Core

Fig. 6.4 Connection between Crossarm & Stay
Fig. 6.5 Bottom Support

Fig. 6.6 Lateral Support
Fig. 6.7 Device for Preventing Rotational Movement

Fig. 6.8 Screw Type Load Cell
Fig. 6.9 Connection between Load Cell & Stay
Fig. 6.10 Hydraulic Jack & Universal Flat Load Cell
Fig. 6.11 Strain Indicator
Fig. 6.12 (a) Transit
Fig. 6.12 (b) Set-Up of Transit

\[ D = 105 \text{ in.} \]
Fig. 6.13 Device for Measuring Lateral Deformation
Fig. 7.1 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for A Real Stayed Column with 200 lbs Initial Pretension

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Experimental results

Theoretical results

Convex Side

Concave Side

Fig. 7.2 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for A Real Stayed Column with 300 lbs Initial Pretension
Fig. 7.3 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for A Real Stayed Column with 400 lbs Initial Pretension
Experimental results

Theoretical results

Fig. 7.4 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for A Real Stayed Column with 500 lbs Initial Pretension

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Fig. 7.5 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for a Real Stayed Column with 600. lbs Initial Pretension
Experimental results
Theoretical results

Fig. 7.6 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stays for A Real Stayed Column with 700 lbs Initial Pretension.
Fig. 7.7 Experimental & Theoretical Results of Load Versus Deflection & Tension in Stay for A Real Stayed Column with 800 lbs Initial Pretension
Fig. 7.8 Experimental Results of Load Versus Deflection for A Real Stayed Column with Different Initial Pretension
Experimental results

Theoretical results

Fig. 7.9 The Deflection Shape of A Real Stayed Column with 600 lbs Initial Pretension
Fig. 7.10 The Deflection Shape of A Real Stayed Column with 700 lbs Initial Pretension
Deflection (in.)

Fig. 7.11 The Deflection Shape of A Real Stayed Column with 800 lbs Initial Pretension

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Theoretical results for an ideal stayed column
Experimental results for a real stayed column
Theoretical results for a real stayed column with type I Initial Imperfection

Fig. 7.12 Experimental & Theoretical Results of Load Versus Initial Pretension for A Real Single-Crossarm Stayed Column
Fig. 7.13 Experimental Measured & Theoretical Calculated Stay Tension of A Real Stayed Column with 600 lbs Initial Pretension
### TABLE 7.1

**RESULTS OF INITIAL RATE OF CHANGING STAY TENSION**

\[
\Delta T/\Delta p \text{ (lbs./kips)}
\]

<table>
<thead>
<tr>
<th>Initial Pretension ( T_i ) lbs.</th>
<th>Stay on Concave Side</th>
<th>Stay on Convex Side</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental Results</td>
<td>Theoretical Results</td>
</tr>
<tr>
<td>200</td>
<td>29.00</td>
<td>30.00</td>
</tr>
<tr>
<td>300</td>
<td>30.83</td>
<td>30.00</td>
</tr>
<tr>
<td>400</td>
<td>29.50</td>
<td>30.50</td>
</tr>
<tr>
<td>500</td>
<td>30.00</td>
<td>30.00</td>
</tr>
<tr>
<td>600</td>
<td>29.16</td>
<td>30.00</td>
</tr>
<tr>
<td>700</td>
<td>29.16</td>
<td>30.00</td>
</tr>
<tr>
<td>800</td>
<td>32.50</td>
<td>30.83</td>
</tr>
</tbody>
</table>

*(These results were determined as the applied load was 6 kips for all cases, except the results of 200 lbs. initial pretension was determined with the applied load of 4 kips)*

### TABLE 7.2

**RESULTS OF RESIDUAL STAY TENSION**

\[
T_R \text{ (lbs.)}
\]

<table>
<thead>
<tr>
<th>Initial Pretension ( T_i ) (lbs.)</th>
<th>Experimental Results</th>
<th>Theoretical Results</th>
<th>Amount Difference (lbs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stay at Top ( T_{st} )</td>
<td>Stay at Bottom ( T_{sb} )</td>
<td>Ave. ( T_{ave} )</td>
</tr>
<tr>
<td>200</td>
<td>60</td>
<td>74</td>
<td>67</td>
</tr>
<tr>
<td>300</td>
<td>90</td>
<td>105</td>
<td>98</td>
</tr>
<tr>
<td>400</td>
<td>155</td>
<td>180</td>
<td>168</td>
</tr>
<tr>
<td>500</td>
<td>130</td>
<td>175</td>
<td>153</td>
</tr>
<tr>
<td>600</td>
<td>225</td>
<td>245</td>
<td>235</td>
</tr>
<tr>
<td>700</td>
<td>270</td>
<td>300</td>
<td>285</td>
</tr>
<tr>
<td>800</td>
<td>415</td>
<td>450</td>
<td>433</td>
</tr>
</tbody>
</table>
### TABLE 7.3

RESULTS OF BUCKLING LOAD, MINIMUM EFFECTIVE, OPTIMUM AND MAXIMUM PRETENSION

<table>
<thead>
<tr>
<th>Initial Pretension</th>
<th>Experimental Results</th>
<th>Theoretical Results of Ideal Stayed Column</th>
<th>Diff.% Ex.-Theo.</th>
<th>Theoretical Results of Real Stayed Column</th>
<th>Diff.% Ex.-Theo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_i$ (lbs.)</td>
<td>(a) Buckling Load (kips)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>7.5</td>
<td>8.0</td>
<td>11.3%</td>
<td>7.0</td>
<td>7.1%</td>
</tr>
<tr>
<td>300</td>
<td>10.6</td>
<td>12.0</td>
<td>11.7%</td>
<td>10.0</td>
<td>6.0%</td>
</tr>
<tr>
<td>400</td>
<td>13.8</td>
<td>16.0</td>
<td>13.8%</td>
<td>12.8</td>
<td>4.7%</td>
</tr>
<tr>
<td>500</td>
<td>17.0</td>
<td>20.0</td>
<td>15.0%</td>
<td>15.8</td>
<td>7.6%</td>
</tr>
<tr>
<td>600</td>
<td>20.0</td>
<td>24.0</td>
<td>16.7%</td>
<td>18.5</td>
<td>8.1%</td>
</tr>
<tr>
<td>700</td>
<td>22.0</td>
<td>28.7</td>
<td>20.6%</td>
<td>21.0</td>
<td>8.6%</td>
</tr>
<tr>
<td>800</td>
<td>24.0</td>
<td>28.7</td>
<td>12.9%</td>
<td>22.6</td>
<td>6.2%</td>
</tr>
</tbody>
</table>

(b) Minimum Effective, Optimum and Maximum Possible Pretension

<table>
<thead>
<tr>
<th>Minimum Effective Pretension (lbs.)</th>
<th>105</th>
<th>100</th>
<th>+ 5%</th>
<th>115</th>
<th>- 8.7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum Pretension (lbs.)</td>
<td>N.I.*</td>
<td>725</td>
<td></td>
<td>1250</td>
<td></td>
</tr>
<tr>
<td>Maximum Possible Pretension (lbs.)</td>
<td>N.I.*</td>
<td>14500</td>
<td></td>
<td>14400</td>
<td></td>
</tr>
</tbody>
</table>

* (N.I. = No information)
APPENDIX A

GENERATION OF THE NONLINEAR LOAD-DISPLACEMENT RELATIONSHIP OF A TWO-DIMENSIONAL STRUCTURE
The displacement matrix for each element is

\[ \{u\}^T = \{u_p, v_p, \theta_p, u_q, v_q, \theta_q\} \quad \ldots \quad 1 \]

The assumed displacement functions along the element are

\[
\begin{align*}
\tilde{u}(x) &= \tilde{u}_p + (\tilde{u}_q - \tilde{u}_p) \frac{x}{L} \\
\tilde{v}(x) &= [1 - 3(\frac{x}{L})^2 + 2(\frac{x}{L})^3] \tilde{v}_p + [\frac{x}{L} - 2(\frac{x}{L})^2 + (\frac{x}{L})^3] \tilde{\theta}_p \\
&\quad + [3(\frac{x}{L})^2 - 2(\frac{x}{L})^3] \tilde{v}_q + [- (\frac{x}{L})^2 + (\frac{x}{L})^3] \tilde{\theta}_q 
\end{align*}
\]  

\ldots \quad 2

The total strain (\(\varepsilon\)) is

\[ \varepsilon = \varepsilon_i + \varepsilon_f \quad \ldots \quad 4 \]

where \(\varepsilon_i = \) initial strain and \(\varepsilon_f = \) final strain

The total strain energy \(U_s\) is

\[ U_s = \int_{V_s} u_s \, dv \quad \ldots \quad 5 \]
where \( u^* \) = strain energy per unit volume; and \( V \) = volume

so that

\[
U_s = \frac{1}{2} \int \varepsilon^2 \, dV
\]

\[
= \frac{1}{2} AEL(e_1)^2 + E \varepsilon_i \int \varepsilon_f \, dV + \frac{1}{2} E \int (\varepsilon_f^*)^2 \, dV
\]

\[
= U_0 + U_1 + U_2
\]

\[
U_0 = \frac{1}{2} AEL(e_1)^2
\]

\[
U_1 = E \varepsilon_i \int \varepsilon_f \, dV
\]

\[
U_2 = \frac{1}{2} E \int (\varepsilon_f^*)^2 \, dV
\]

where \( U_1 \) = the strain energy which relates to the initial stress and \([k_1]\); \( U_2 \) = the strain energy which relates to the additional strain and \([k_E]\); and, \( U_0 \) = the strain energy before any disturbance is applied. \( U_0 \) does not contribute to \([k]\) because this term will vanish when the minimum potential energy theorem is applied. Then,

Equation 7 becomes

\[
U_s = U_1 + U_2
\]

The strain-displacement equation is represented by

\[
\varepsilon_f = \frac{du^*}{dx} + \frac{1}{2} \left( \frac{d^*-1}{dx} \right)^2 - y \frac{d^2v}{dx^2}
\]

Substituting Equation 12 into Equation 11, obtains

\[
U_s = \{E \varepsilon_i A \int_0^L \left( \frac{du^*}{dx} - y \frac{d^2v}{dx^2} \right) \, dx + \frac{1}{2} E \varepsilon_i A \int_0^L \left( \frac{d^2v}{dx^2} \right)^2 \, dx \}
\]
Substituting Equations 2 and 3 and \( c_i = P_i/A_iE \) into Equation 13, the total strain energy can be expressed in the following equation

\[
U_s = \frac{1}{2} \{u^i\}^T [k^E_i + k^G_i] \{u^i\}
\]

where \( \{u^i\} \) = the local displacement matrix for the element; \( \{k^E_i\} \) = the elastic stiffness matrix for the element; and \( \{k^G_i\} \) = the elastic stiffness matrix for the element in the local dimension system, and

\[
[k^E_i] = \begin{bmatrix}
\frac{EA}{L} & -12\frac{EI}{L^2} & 0 & 0 & 0 & 0 \\
-12\frac{EI}{L^2} & 6\frac{EI}{L} & 4\frac{EI}{L} & 0 & 0 & 0 \\
0 & 4\frac{EI}{L} & 4\frac{EI}{L} & 0 & 0 & 0 \\
0 & 0 & 0 & 6\frac{EI}{L^2} & 2\frac{EI}{L} & 0 \\
0 & 0 & 0 & 0 & -6\frac{EI}{L^2} & 4\frac{EI}{L}
\end{bmatrix}
\]

and

\[
[k^G_i] = P_i \begin{bmatrix}
0 & -\frac{6}{5L} & 0 & 1/10 & 2L/15 & 0 & \frac{6}{5L} & 0 & -L/30 & \frac{1}{10} & 2L/15 & 0 & -1/10 & 2L/15
\end{bmatrix}
\]

where \( P_i \) = initial internal axial force.

The total potential energy for the element is given by
\[ \tau_p^i = \frac{1}{2} \{ u_i \}^T [k_i^E + k_i^G] \{ u_i \} - \{ u_i \}^T \{ p_i \} \] \quad \ldots \quad 17

The displacement may be expressed in terms of global coordinate system as

\[ \{ u_i \} = [T^i] \{ u_i \} \] \quad \ldots \quad 18

where \( \{ u_i \} \) = the displacement matrix in the global coordinate system; and \([T^i]\) = the transformation matrix which can be expressed in the form

\[
[T_i] = \begin{bmatrix}
\lambda_1 & m_1 & 0 & 0 & 0 & 0 \\
\lambda_2 & m_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_1 & m_1 & 0 \\
0 & 0 & 0 & \lambda_2 & m_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\] \quad \ldots \quad 19

where \( \lambda_1 \) and \( m_1 \) = the direction cosine in the x-direction; and \( \lambda_2 \) and \( m_2 \) = the direction cosine in the y-direction.

Substituting Equations 18 and 20 into 17, gives

\[ \{ \pi^i \} = \frac{1}{2} \{ u_i \}^T [k_i^E + k_i^G] - \{ u_i \}^T \{ p_i \} \] \quad \ldots \quad 21

where

\[ [k_i^E + k_i^G] = [T^i]^T [k_i^E + k_i^G] [T^i] \] \quad \ldots \quad 22

and

\[ \{ p_i \} = [T^i] \{ \pi^i \} \] \quad \ldots \quad 23
The total potential energy for the whole structure with \( m \) elements can be expressed as

\[
\Pi_p = \text{Strain Energy of Members} - \text{Potential of Loads at Joint} - \text{Potential of Loads of Members}
\]

\[
= \sum_{i=1}^{m} \left[ \frac{1}{2}(u_i)^T \left[ k_E^i + k_G^i \right] (u_i) - (u_i)^T p_i \right] - W_n \quad \ldots \quad 24
\]

\[
= \frac{1}{2}(u)^T \left[ k_E + k_G \right] (u) - (u)^T p - (U)^T F \quad \ldots \quad 25
\]

\[
= \frac{1}{2}(u)^T \left[ k_E + k_G \right] (u) - (U)^T p \quad \ldots \quad 26
\]

where \((U)^T = \{u_1, u_2, \ldots, u_i\}\) \ldots 27

\((p)^T = \{p_1, p_2, \ldots, p_i\}\) \ldots 28

\([k_E + k_G] = \text{Diagonal} [k_{E_1} + k_{G_1}, k_{E_2} + k_{G_2}, \ldots, k_{E_i} + k_{G_i}]\)

\(W_n = \text{potential of loads of member}\)

\([F] = \text{force at each degree of freedom}\)

\((u) = [A](U)\) \ldots 30

\([K_E + K_G] = [A]^T [k_E + k_G] [A]\) \ldots 31

\((p) = [A]^T (p) + (F)\) \ldots 32

\([A] = \text{assembly control matrix}\)

Applying the principle of minimum potential energy obtains

\[
\delta \Pi_p = 0 \quad \ldots \quad 33
\]

\[
(p) = [K_E + K_G](U) \quad \ldots \quad 34
\]
APPENDIX B

A FLOW CHART FOR THE SOLUTION OF THE
NONLINEAR BEHAVIOR OF THE STAYED COLUMN

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START

Input Data:
Geometry properties;
material properties;
load conditions and
initial pretension.

Calculate stiffness matrices and internal axial force.

Calculate incremental deformation and member force in each iterative cycle.

Check if the deformation fits the convergence criteria for the iterative procedure.

Modify geometry properties; stay tension and internal axial force.

Check if the deformation fits the convergence criteria for the incremental procedure.

Calculate the total deformation and total applied load.

Check if the deformation fits the convergence criteria for the incremental procedure.

Print Output

END
APPENDIX C

LISTING OF THE COMPUTER PROGRAM
C* TO CALCULATE THE DEFLECTION RATE OF A TWO-DIMENSIONAL STAYED COLUMN
C* BY FINITE ELEMENT METHOD INCLUDING THE EFFECT OF IMPERFECTION

C*************************************************************
  ** NOTATION **
  ** AA = CROSS SECTION AREA **
  ** ALPHA = ANGLE BETWEEN STAY AND CORE **
  ** CN = COORDINATES OF NODES **
  ** DII = INNER DIAMETER **
  ** D22 = OUTER DIAMETER **
  ** DC = DIRECTION COSINES **
  ** E = MODULUS OF ELASTICITY **
  ** F = MEMBER FORCE **
  ** G = MODULUS OF RIGIDITY **
  ** GLK1 = MASTER ELASTIC STIFFNESS MATRIX **
  ** GLK2 = MASTER GEOMETRIC STIFFNESS MATRIX **
  ** IVC = VARIABLE CORELATION TABLE **
  ** LGTH = LENGTH OF ELEMENT **
  ** M = NUMBER OF ELEMENTS **
  ** MK = MASTER STIFFNESS MATRIX **
  ** MB = NUMBER OF NODES **
  ** NBS = NUMBER OF ELEMENTS WITH BENDING STIFFNESS **
  ** NDF = NUMBER OF DEGREES OF FREEDOM **
  ** NEL = NUMBER OF ELEMENTS WITH SIGNIFICANT AXIAL LOAD **
  ** NLC = NUMBER OF LOADING CONDITION **
  ** MN = DIRECTION OF MEMBER **
  ** P = FORCES ACTING AT EACH DEGREE OF FREEDOM **
  ** PA = INCREMENTAL LOAD **
  ** PA = COLUMN AXIAL LOAD **
  ** TI = INITIAL TENSION **
  ** T2 = TENSION IN THE STAY AT THE TOP AND LEFT SIDE **
  ** T3 = TENSION IN THE STAY AT THE TOP AND RIGHT SIDE **
  ** T4 = TENSION IN THE STAY AT THE BOTTOM AND RIGHT SIDE **
  ** U = DISPLACEMENTS AT EACH DEGREE OF FREEDOM **
  ** UM = DISPLACEMENT OF MEMBER **
  ** ZI = MOMENT OF INERTIA **

C*************************************************************
  ** DIMENSION CARDS AFFECTED BY THE NUMBER OF MEMBERS **
  ** DIMENSION MN(30,2),IVC(30,6),DC(30,2) **
  ** DIMENSION DII(30),D22(30),EI(30),G(30),LGTH(30),ZI(30),AA(30) **
  ** DIMENSION F(30,6,10),AA(30) **

C*************************************************************
  ** DIMENSION CARDS AFFECTED BY THE NUMBER OF NODES **
  ** DIMENSION CN(30),DII(30),E(30),G(30),LGTH(30) **

C*************************************************************
  ** DIMENSION CARDS AFFECTED BY THE NUMBER OF DEGREES OF FREEDOM **
  ** DIMENSION GLK1(50,50),GLK2(50,50),TMX(50,50) **

C*************************************************************
  ** THE FOLLOWING CARDS MUST BE CHANGED WHEN THE NUMBER OF MEMBERS, **
  ** NODES, OR DEGREES OF FREEDOM ARE CHANGED **
  ** REAL MA,LGTH **
  ** REAL NDF=50 **
  ** ME=30 **
  ** NFL=NPDF+10 **
  ** NPDF=0 **
  ** NFL=NPDF+10 **
  ** NPDF=0 **
  ** ALPHA=ATAN(0,1,0) **

C*************************************************************
  ** DIMENSION CARDS AFFECTED BY THE NUMBER OF ELEMENTS **
  ** READ(5,10) M+N+NPDF,NKG,NBS,NN,NLC **
  ** READ(5,15) (MN(1,J),J=1,2),(MN(I,M),I=1,M) **
  ** READ(5,20) (IVC(I,J),J=1,6),(IVC(I,M),I=1,M) **
  ** READ(5,25) ((DG1(I,J),J=1,6),(DG1(I,M),I=1,M)) **
  ** READ(5,30) (CN(1,J),J=1,6),(CN(1,N),J=1,N)) **
  ** READ OUT AND INNER DIAMETERS ( IN NUMERICAL ORDER BY NUMBER) **
  ** READ(5,50) NUM+NS+MF,DO **
  ** READ(5,55) (F(NU,M),M=1,6) **
  ** READ(5,60) (J(NU),N=1,6) **
  ** READ(5,65) TP1,TP2 **
  ** READ(5,70) (I5(NU),N=1,6) **
  ** READ(5,75) (O051 I=NS+MF **

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FORTRAN IV G LEVEL 21

0025   51 D22(I)=00
0026   52 GO TO 40
0027   55 READ(5,50) NUM,MS,MF,I
0029   57 NU5=NU5
0030   57 DI1(I)=1
0031 GO TO 55
0032
C READ MODUL OF ELASTICITY (IN NUMERICAL ORDER BY MEMBER)
0033   50 READ(5,50) NUM,MS,MF,EN
0034 IF (NUM.EQ.0) GO TO 75
0035 DO 62 I=NS,MF
0036 62 E(I)=EN
0037 GO TO 60
0038 IF (NUM.EQ.0) GO TO 56
0039 DO 60 I=NS,MF
0040 60 I(I)=EN
0041 GO TO 75

0042 C READ COORDINATES (IN NUMERICAL ORDER BY NODE, A THEN Y)
0043   87 FORMAT(2F10.4)
0045 CN2=CN2(2,1)
0046 CN3=CN3(3,1)
0047 CN4=CN4(4,1)
0048 READ(5,89) (P(I,J),J=1,11),I=I,N)
0049   89 FORMAT(F12.4)
0050 READ(5,89) T
0051   86 FORMAT(F10.A)
0052 C+++++++++++CALCULATE MEMBER PROPERTIES+++++++++++++++++++++++++++++++++++++++
0053 DO 90 I=1,NBS
0054 90 Z(I)=I(022(I)**4-011(I)**4)/64.
0055 NBS1=NBS+1
0056 DO 92 I=NBS1,N
0057 92 Z(I)=0.
0058 C AREA, LENGTH, AND DIRECTION COSINES
0059   95 A(I)=PI*1(022(I)**2-011(I)**2)/4.
0060 WRITE(6,95) NMPR3
0061   96 FORMAT(11.,5X,1APROBLEM NUMBER,1A7)
0062   97 FORMAT(6,95)
0063 WRITE(6,99) NMPR3
0064   98 FORMAT(10X,6H55CM. OF SX,SmHO. OF SY,55CM. OF SZ,55DEGREES OF X,55DEGREES OF Y,55DEGREES OF Z)
0065 WRITE(6,99)
0066   99 FORMAT(10X,7HMEMBERS,4A,6HNOODES,7X,7HRECDCH,7X,10HCF) PAGE41
0067 WRITE(6,100) M,N,NOF,NN
0068   100 FORMAT(10X,14.,8X,13.,13.,13.,13.,13.,13.)
0070 WRITE(6,101)
0071   101 FORMAT(10X,6HMEMBERS,23X,8MYC TABLE,/) 
0072 WRITE(6,102) (1.,(IVC(I,J),J=1,6),I=1,W)
0074 TRA=TI
0075 TLB=TI
0077 TLA=TI
0078 DO 103 J=1,N
0079 103 CONTINUE
0080 PA=2.
0082 TPA=2.
0084 CONTINUE
0085 US=0.
0086 UG=0.
0088 UY=0.
0088 UZ=0.
0090 T1=IVC(I,J)
0098   105 FORMAT(110HREAD M,N,NOF,NN)
0099 WRITE(6,100)
0100   107 FORMAT(10X,6HINITIAL TENSION,10X,4H4KIPS,/) 
0102 TP=TPA+(TRA+TLA+UST+UG)*COS(ALPHA)
0103 DO 109 I=1,N

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IV G LEVEL 21

107 DO 115 I=1,N
109 SUM=MN(1,1)
110 DO 11 J=1,2
111 SUM=SUM+MN(I,J)*MN(I,J)
112 KK=KK+1
113 WRITE(6,120) KK
114 WRITE(6,122)
115 WRITE(6,120) KK
116 WRITE(6,122)
117 WRITE(6,120) KK
118 WRITE(6,122)
119 WRITE(6,120) KK
120 WRITE(6,122)
121 WRITE(6,120) KK
122 WRITE(6,122)
123 WRITE(6,120) KK
124 WRITE(6,122)
125 WRITE(6,120) KK
126 WRITE(6,122)
127 WRITE(6,120) KK
128 WRITE(6,122)
129 WRITE(6,120) KK
130 WRITE(6,122)
131 WRITE(6,120) KK
132 WRITE(6,122)
133 WRITE(6,120) KK
134 WRITE(6,122)
135 WRITE(6,120) KK
136 WRITE(6,122)
137 WRITE(6,120) KK
138 WRITE(6,122)
139 WRITE(6,120) KK
140 WRITE(6,122)
141 WRITE(6,120) KK
142 WRITE(6,122)
143 WRITE(6,120) KK
144 WRITE(6,122)
145 WRITE(6,120) KK
146 WRITE(6,122)
147 WRITE(6,120) KK
148 WRITE(6,122)
149 WRITE(6,120) KK
150 WRITE(6,122)
151 WRITE(6,120) KK
152 WRITE(6,122)
153 WRITE(6,120) KK
154 WRITE(6,122)
155 WRITE(6,120) KK
156 WRITE(6,122)
157 WRITE(6,120) KK
158 WRITE(6,122)
**FORTRAN IV LEVEL 2**

```
0159  WLTENKG=4
0160  WARBNKGS=5
0161  WLB=NGK=6
0162  TRT=TRT3-FL(T,1,NLC)
0163  TLTL0=T(LT,1,NLC)
0164  TRB=TRB3-FL(WRB,1,NLC)
0165  TLNL0=FL(WLBN,1,NLC)

C***CHECK IF THE STAY IS STILL EFFECTIVE******************************
0166  IF (TRT.GT.0.) GO TO 500
0167  TRT=0.
0168  TRT0=0.
0169  AA(J)=0.
0170  S00 IF (TRSL.GT.0.) GO TO 501
0171  TRS=0.
0172  TRS0=0.
0173  S01 CONTINUE
0174  IF (TLT.GT.0.) GO TO 502
0175  TLT=0.
0176  AA(9)=0.
0177  S02 CONTINUE
0178  IF (TLM.GT.0.) GO TO 503
0179  TLM=0.
0180  AA(10)=0.
0181  S03 CONTINUE
0182  S10 FORMAT(/,10X,4HTLT=,FI0.4,4HKIPS,10X,4HTRT=,FI0.4,4HKIPS)
0183  WRITE(6,510) TLT,TRT
0184  S10 FORMAT(/,10X,4HTLT=,FI0.4,4HKIPS,10X,4HTRT=,FI0.4,4HKIPS)
0185  WRITE(6,520) TLNL0
0186  S20 FORMAT(/,10X,4HTLT=,FI0.4,4HKIPS,10X,4HTRT=,FI0.4,4HKIPS)
0187  WRITE(6,520) TLNL0

C***CONVERGENCE CRITICA FOR THE ITERATIVE PROCEDURE*****************************
0190  ICONT=0
0180  CI=U(J,1)-UJ
0181  C2=U(6,1)-U6
0182  C3=U(9,1)-U9
0183  UJ=U(J,1)
0184  U6=U(6,1)
0185  U9=U(9,1)
0186  IF (ABS(C1).LT.0.0005) GO TO 104
0187  IF (ABS(C2).LT.0.0005) GO TO 104
0188  IF (ABS(C3).LT.0.0005) GO TO 104
0189  IF (((TRT.LE.0.).OR.(TRT.LE.0.).OR.(TLT.LE.0.).OR.(TLM.LE.0.).OR.(TLNL0.LE.0.))
1 IF (ICONT.GE.1) GO TO 1009

C***CONVERGENCE CRITICA FOR THE INCREMENTAL PROCEDURE*****************************
0200  IF (ICONT.GE.1) GO TO 1009
0201  A1=ABS(C1)-CN1
0202  A2=ABS(C2)-CN2
0203  A3=ABS(C3)-CN3
0204  IF (ABS(A1).GT.0.30) GO TO 1009
0205  IF (ABS(A2).GT.0.30) GO TO 1009
0206  IF (ABS(A3).GT.0.30) GO TO 1009
0207  GO TO 111
0208  1000 READ(E,1001) NA
0209  1001 FORMAT(6)
0210  IF (NA.NE.0) GO TO 64
0211  STOP
0212  END

```

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FORTRAN IV G LEVEL

SUBROUTINE ELARM (M, I, A, A, 5, G, EL, OC, NOF, IV C, MK, MEMB, NOF)

THIS SUBROUTINE WILL CALCULATE THE MASTER ELASTIC STIFFNESS MATRIX.

DIMENSION ZI(MEMB, AA, MEMB), EL(MEMB), G(MEMB)

DIMENSION CC(MEMB, 2), IV C(MEMB, 6), MK(NDFD, NOFD), EK(G, 6)

DIMENSION (6, 6), V(6, 6), EKL(6, 6), T(6, 6)

REAL MK

DO 10 J = 1, NOF
    MK(I, J) = 0.0
10

PI = 4. * ATAN(1.0)

DO 100 K = 1, M
    CALL TRANF(K, OC, T, MEMB)
    CALL EL(AK, EL, OC, G, EL, OC, NOF, IV C, MK, MEMB)
    CALL TRANS(T, U, 6, 6)
    CALL MULT(T, EKL, V, 6, 6)
    CALL MULT(V, T, EKL, 6, 6)
    DO 50 J = 1, 6
        IN = IV C(K, J)
        IF (IN.EQ.0) GO TO 50
        IL1 = ABS(I)
        JJ = IL1/I
        KK = IN/IN
        MK(I, J) = MK(I, J) + JJ * KK * EKL(J, J)
50

CONTINUE

DO 100 J = 1, 6
    IN = IV C(I, J)
    IF (IN.EQ.0) GO TO 50
    IL1 = ABS(I)
    JJ = IL1/I
    KK = IN/IN
    MK(I, J) = MK(I, J) + JJ * KK * EKL(I, J)
100

CONTINUE

RETURN
END

SUBROUTINE GEOKM(NKG, EL, OC, NOF, IV C, MK, MEMB, NOF)

THIS SUBROUTINE WILL CALCULATE THE MASTER GEOMETRIC STIFFNESS MATRIX.

DIMENSION EL(MEMB), OC(MEMB, 2), IV C(MEMB, 6), MK(NDFD, NOFD)

DIMENSION GKG(6, 6), GKLI(6, 6), T(6, 6), U(6, 6), V(6, 6)

REAL MK

DO 10 J = 1, NOF
    MK(I, J) = 0.0
10

PI = 4. * ATAN(1.0)

DO 30 K = 1, NKG
    CALL TRANF(K, OC, T, MEMB)
    CALL GECK(K, EL, GKL, MEMB)
    CALL TRANS(T, U, 6, 6)
    CALL MULT(T, GKL, V, 6, 6)
    CALL MULT(V, T, GKL, 6, 6)
    DO 60 J = 1, 6
        IN = IV C(K, J)
        IF (IN.EQ.0) GO TO 60
        IL1 = ABS(I)
        JJ = IL1/I
        KK = IN/IN
        MK(I, J) = MK(I, J) + JJ * KK * GKG(I, J)
60

CONTINUE

DO 30 J = 1, 6
    IN = IV C(I, J)
    IF (IN.EQ.0) GO TO 30
    IL1 = ABS(I)
    JJ = IL1/I
    KK = IN/IN
    MK(I, J) = MK(I, J) + JJ * KK * GKG(I, J)
30

CONTINUE

RETURN
END
SUBROUTINE TRANS(A,V,X,L)

* THIS SUBROUTINE WILL TRANSPOSE A MATRIX *

DIMENSION A(L,K),V(K,L)

DO 10 I=1,K
10 V(I,J)=A(J,I)
RETURN
END

SUBROUTINE TRANSF(K,OC,T,MEM3)

* THIS SUBROUTINE WILL CALCULATE THE TRANSFORMATION MATRICES *

DIMENSION OC(MEM3,2),T(6,6)

DO 20 I=1,6
20 T(I,J)=0.0
CX=OC(K,1)
CY=OC(K,2)
T(1,1)=CX
T(1,2)=CY
T(2,1)=-CY
T(2,2)=CX
T(3,3)=1.0
T(3,4)=CX
T(3,5)=CY
T(4,4)=CX
T(4,5)=-CY
T(5,4)=CX
T(5,5)=CY
T(6,6)=1.0
RETURN
END
SUBROUTINE GEOK(X,EL,KL,MEB)

* THIS SUBROUTINE WILL CALCULATE THE GEOMETRIC STIFFNESS MATRICES *

DIMENSION E(MEB),G(MEB),AA(MEB),EL(MEB),ZI(MEB),SKL(M)

DO 30 J=1,6
30 GKL(1,J)=0.0
GKL(2,2)=6./(5.*EL(K))
GKL(2,3)=0.1
GKL(2,5)=-GKL(2,2)
GKL(2,6)=0.1
GKL(3,3)=2.*EL(K)/15.
GKL(3,5)=0.1
GKL(3,6)=-EL(K)/15.
GKL(5,5)=GKL(2,2)
GKL(5,6)=0.1
GKL(6,6)=2.*EL(K)/15.
DO 40 I=2,6
40 GKL(I,J)=GKL(J,I)
RETURN
END
SUBROUTINE SOLVE(U,TMK,P,NOF,NDFO,NLC,NNL,NL)

* THIS SUBROUTINE WILL CALCULATE THE DISPLACEMENT AT EACH DEGREE OF FREEDOM

DIMENSION U(NDFO,10),P(NDFO,10),TMK(NDFO),NOF,NDFO,NLC

CONTINUE

DO 60 I=1,NDF

DO 50 J=1,NDF

CONTINUE

RETURN

END

SUBROUTINE STRESS(U,PA,DC,NKG,F,E,G,EL,AA,ZZ,MEMB,NDFO,NDFD,NLC)

DIMENSION U(NDFD,10),EKL(6,6),GKL(6,6),TKL(6,6)

DIMENSION UM(NDF,10),EKLI6,6),GKL(6,6),TKL(6,6)

DIMENSION DC(MEMB,6),E(6),G(6),EKL(6,6),F2(6,6)

DIMENSION DC(MEMB,6),E(6),G(6),EKL(6,6),F2(6,6)

CALL DISPLA(U,DC,NDFO,NLC,UM)

CONTINUE

DO 10 I=1,6

CONTINUE

RETURN

END

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SUBROUTINE DISPLAY(U, VIC, MEMB, NOFO, NLC, UM)

* THIS SUBROUTINE WILL CHANGE THE GLOBAL DISPLACEMENT TO LOCAL
* DISPLACEMENT ON EACH MEMBERS' ENDS

DIMENSION U(1,NOFO,10), VIC(MEMB,6), UM(MEMB,6,10)

DO 80 J=1,NLC
  IF(K1) 30,20,10
  10 UM(1,1,J)=U(K1,J)
  20 UM(1,1,J)=0.0
  30 K1=ABS(K1)
  40 UM(1,1,J)=-U(K1,J)
  50 UM(1,2,J)=U(K2,J)
  60 UM(1,2,J)=0.0
  70 K2=ABS(K2)
  80 IF(K2) 70,60,50
  90 UM(1,3,J)=U(K3,J)
  100 UM(1,3,J)=0.0
  110 K3=ABS(K3)
  120 IF(K3) 120,110,100
  130 UM(1,4,J)=U(K4,J)
  140 UM(1,4,J)=0.0
  150 K4=ABS(K4)
  160 IF(K4) 160,150,140
  170 UM(1,5,J)=U(K5,J)
  180 UM(1,5,J)=0.0
  190 K5=ABS(K5)
  200 IF(K5) 200,190,180
  210 UM(1,6,J)=U(K6,J)
  220 UM(1,6,J)=0.0
  230 K6=ABS(K6)
  240 CONTINUE
  250 CONTINUE
  260 CONTINUE
  270 CONTINUE
  280 CONTINUE
  290 CONTINUE
  300 RETURN
END
**FORTRAN IV G LEVEL 21**

**MUL**

**DATE = 080204 19/07/43**

0001 SUBROUTINE MUL(A,B,C,M,K,N)

0002 ***********

0003 DIMENSION A(M,A),B(K,N),C(M,N)

0004 DO 10 I=1,M

0005 C(I,J)=0.

0006 DO 10 J=1,N

0007 C(I,J)=A(I,L)*B(L,J)

0008 10 C(I,J)=C(I,J)+C1

0009 RETURN

0010 END

---

**FORTRAN IV G LEVEL 21**

**COOR**

**DATE = 080204 19/07/43**

0001 SUBROUTINE COOR(CN,U,N,NKG,NOFD,NLC,KK,CN1)

0002 ***********

0003 DIMENSION CN(25,2),U(1,NOFD,10)

0004 DIMENSION CN1(25,2)

0005 I1=3

0006 I2=4

0007 NC=1+NKG

0008 DO 200 I=2,N

0009 IF (I.NE.NC) GO TO 20

0010 I1=I+1

0011 I2=I2+1

0012 GO TO 200

0013 CONTINUE

0014 CN(I,1)=CN1(I,1)+U(I1,NLC)

0015 CN(I,2)=CN1(I,2)+U(I2,NLC)

0016 I1=I+3

0017 I2=I2+3

0018 200 CONTINUE

0019 RETURN

0020 END
BIBLIOGRAPHY


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The author was born on July 20, 1954 in Macau. In 1966, he completed his primary school education at Pui-Cheng Middle School, Macau, and in 1972, he completed his secondary school education at Lingnau Middle School, Hong Kong. Afterwards, he finished one year of study in the field of general technology at St. Clair College, Windsor, Ontario, Canada. Then, he joined the Faculty of Engineering, University of Windsor, Windsor, Ontario, Canada. In May 1977, he received the B.A.Sc. degree with Honours in the Civil Engineering Department from that University.

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